Maximising with-profit pensions without guarantees

M. Carmen Boado-Penas* Julia Eisenberg† Paul Krühner ‡
University of Liverpool TU Wien WU Vienna

Abstract

Pension providers are currently running into trouble mainly due to the ultra-low interest rates and the guarantees associated to some pension benefits. In this paper, we aim to reduce the pension volatility and provide adequate pension levels – with no guarantees – through a new pension design. Under this design, the individual's premium is split into an individual and a collective account, both invested in funds. When the return from the individual fund exceeds a predefined corridor, a certain number of units is transferred to or from the collective account. In this way, the volatility of the individual fund is smoothed. By controlling the corridor width, we maximise the total accumulated capital at retirement.

Key words: Pensions, Collective mechanism, Optimisation, Redistribution index, Volatility smoothing.

2010 Mathematical Subject Classification: Primary 93E20
Secondary 91B30, 91B08, 90B50

JEL Classification: C61, G22, G52, J26

1 Introduction

Pensions are in constant flux as insurers need to reinvent their products in an environment with continuous increases in longevity and ultra-low interest rates. At the same time employees desire security in retirement in the sense that they could get the retirement income they expect due to their past and current contributions into a pension scheme.

With-profits contracts (or participating policies in the US) were historically a significant part of the UK life insurance product palette. With-profits contract generally consists of a benefit if the individual dies within the term (term insurance) and a lump sum if the policyholder survives within the term (pure endowment). This allows the policyholder to build up funds for a specific purpose such as an income in retirement. The important

* ✉ carmenbo@liverpool.ac.uk
† jeisenbe@fam.tuwien.ac.at
‡ peisenbe@wu.ac.at

feature in this type of contracts is that additional periodic return can be given to the policyholder. In order to remove the short-term volatility of policyholder's payout value different smoothing mechanisms are applied in practice, see Goecke [7] and Guillén et al. [8]. A with-profits investment can either be conventional or unitised with the latter buying units in the with-profits fund. In the past, with-profits contracts often contained guarantees, like for example minimum guaranteed return, which allowed just for low-risk investments resulting in a lower expected value of the final accumulated amount.[1]

In order to meet consumer's needs in terms of stability after retirement, the dynamic hybrid life insurance offers guarantees achieved by a periodical rebalancing process between three funds (the policy reserves, a guarantee fund and and equity fund), Bohnert et al. [3]. However, the investments are still made on low and average-risk products. In the case of Constant Proportion Portfolio Insurance (CPPI) strategies, the investor chooses a multiplier and a floor below which he does not want the value of the portfolio to fall. The aim of the CPPI design is to keep the exposure to equities as a constant multiple of the floor. In a bull market the CPPI strategies perform better than buy-and-hold strategies. However, when the markets are falling in price CPPIs are not good strategies, doing it also relatively poor in a flat market, see Perold and William [11]. Currently, under the ultra-low interest rate economic environment, which significantly reduces the long-term benefits, the insurers try to avoid guarantees associated to pension products. Over the past few decades, in occupational pensions, traditional defined benefit (DB) plans are gradually losing their dominance and there has been a shift towards defined contribution (DC) pensions, where the investment risk is completely shifted from the insurer to the clients. Under a DC scheme the level of the pension is uncertain and, in general, without higher contribution rates will not produce decent benefits.

For workplace and private pension plans, collective defined contribution (CDC) schemes offer a middle ground between DB and DC plans. Under CDC, contributions are pooled and managed on a collective basis, and members own a proportional share of the aggregated collective investment rather than individuals share of the underlying assets as for the case of individual DC.[2] The plan also has a target pension amount — rather than a contractual guarantee-based on a long term and mixed risk investment plan. The way CDC adjusts the level of current and prospective pensions mean that there is an element of cushioning (smoothing) of volatility and much better long-term protection as the risk is shared by the members. This is because investment risk is adjusted over time and longevity risk is pooled across the membership. However, CDC entails some significant challenges particularly regarding the communication of the benefit calculation to members[3], complex governance decision for trustees and high running costs that are likely to

[1] For an overview of new life insurance products, see Gatzert and Schmeiser [6].

[2] Collective pension schemes are also the dominant form of saving for retirement in countries such as the Netherlands and Norway – countries recognised as having among the best pension systems in the world according the Melbourne Mercer Global Pension Index (2019). See Bovenberg [4], Hoevenaars and Ponds [9], Ponds and Van Riel [12] and Binsbergen et al. [1], amongst others. In the UK, the Pension Scheme Act (2015) sets up a new legislate framework for private pensions encouraging shared risk pension schemes and collective benefits.

[3] Members have no control over the attribution of losses and surpluses.

make it suitable only for larger schemes.

There is a clear need of security in retirement, i.e. satisfactory and stable pension benefits but at the same time the pension providers do not want to offer guarantees under the current low-interest rate environment. In this paper, the traditional guarantees are replaced with low volatility so that a more return-oriented approach is followed and, therefore, higher expected returns are obtained for the policyholder.
With the aim of reducing the pension volatility and providing satisfactory pension levels, in this paper, we analyse a new pension design (first described in Boado et al. [2]) in the accumulation phase where the individual's premium is split into two accounts: individual account and collective account. Similar to unitised with-profit products, the premia in both accounts are invested in funds (the same or different ones). Depending on the performance of the individual fund, some units are transferred from or to the collective fund. In this way, the collective account acts as a smoothing mechanism for individual accounts. At retirement age, the individual receives a lifelong pension (or a lump sum payment) linked to her individual account and a portion of the collective account according to a so-called redistribution index, that is, a weight that identifies the part of the collective account belonging to a client according to her premium payment evolution. This paper builds upon Boado-Penas et al. [2] and addresses the mathematical aspects to determine the optimal corridor for the exchange of units between the individual and collective accounts so that the final accumulated capital at retirement age is maximised. In January 2020, the aforementioned pension design has been reportedly adapted by Die Mobiliar, a Swiss insurance company, see [10]. Given its very recent market authorisation no studies have been conducted to determine the optimal corridor to maximise the first pension amount or the total accumulated capital at retirement age. In the present paper, we aim to address the latter problem.

Following this introduction, the next section of the paper describes the proposed pension model in the accumulation phase. Section 3 describes the mathematical model of the product and defines the target functional to maximise. Depending on the chosen help/gain-sharing procedures the optimal strategy for individual accounts has a different structure. First, we analyse a relatively simple case when the collective fund can never be empty, which corresponds to the full guarantee case. Second, we assume that it is not possible to get any help from the collective account if the total number of help units required by individual accounts exceeds the number of units in the collective account. Section 4 concludes and makes suggestions for further research.

2 The Model

This section presents the mathematical formulation to determine the optimal corridor for the exchange of units between the individual and the collective account so that the total saved amount for the individual at retirement age is maximised. As optimisation criteria, we discuss the reasonability and mathematically feasibility of the optimal mean and optimal mean-variance.

For simplicity, we assume that the individual and collective accounts hold shares from the same fund whose price per share H_t is modelled by a Geometric Brownian motion, i.e.

$$H_t = e^{x+\mu t+\sigma W_t}, \quad t \geq 0$$

with W being a standard Brownian motion and e^x the initial value of the fund, and the return of the fund is expressed as

$$\rho_t := \frac{H_t}{H_{t-1}} - 1 \ .$$

We denote by k and $-k$ the *corridor boundaries* of the individual fund, and $k \in [0,1]$. The returns exceeding the upper corridor boundary are partially distributed from the individual account to the collective account while the losses (negative returns falling out the lower corridor boundary) are partially compensated from the collective account.

Let denote by $V_t^j = \eta_t^j H_t$ the value of the j-th individual account at time t where η_t^j is the number of shares that belongs to the j-th individual at time t.

The value of of the collective account at time t is given by $C_t = \theta_t H_t$ where θ_t denotes the number of shares at time t in the collective account. Note that the individual 1-step return from time $t-1$ to t, without readjustments of the number of shares in the time interval $[t-1, t]$, is given by

$$\frac{\eta_{t-1}^j H_t}{\eta_{t-1}^j H_{t-1}} - 1 = \rho_t,$$

i.e. the individuals have the same return as the fund as long as their number of shares is not changed.
The mathematical formulation of the with-profit procedure is as follows for some $1 \leq a \leq b$:

- If $\rho_t > k$, then we say that the fund over-performed and a fraction $\frac{1}{b}$ $(b \geq 1)$ of the surplus is transferred from the individual to the collective account

  $$\frac{1}{b}\Big(H_t - H_{t-1}(1+k)\Big)\eta_{t-1} = \frac{1}{b}V_{t-1}\Big(\frac{H_t}{H_{t-1}} - 1 - k\Big) = \frac{1}{b}V_{t-1}\big(\rho_t - k\big),$$

  i.e. one transfers $\frac{1}{b}\Big(1 - \frac{H_{t-1}}{H_t}(1+k)\Big)\eta_{t-1}$ units of the fund into the collective account.

- If $\rho_t < -k$, then we say that the fund under-performed and in this case the individual account creates a claim for the collective account and gets

  $$\frac{1}{a}\Big(H_{t-1}(1-k) - H_t\Big)\eta_{t-1} = \frac{1}{a}V_{t-1}\Big(1 - k - \frac{H_t}{H_{t-1}}\Big) = \frac{1}{a}V_{t-1}\big(-k - \rho_t\big)\ ,$$

  i.e $\frac{1}{a}\Big(\frac{H_{t-1}}{H_t}(1-k) - 1\Big)\eta_{t-1}$ units will be transferred into the individual account, where $a \in [1, b]$.

Hence, the over- and under-performances will be less severe as one gets compensated for under-performance and shares the gains in case of an over-performance. From an individual point it is desirable that $b > a$, i.e. relatively less shares are transferred from the individual account in case of over-performance than shares are transferred into the individual account in case of under-performance. It is clear that, compared to the case with no exchange mechanism between individual and collective account, the realised volatility will be reduced. Note that the value $k = 0$ means that the exchange mechanism is used at each time step with non-zero return, i.e. growth and shrinking of the individual account is linearly dampened by the amounts $\frac{1}{b}$, $\frac{1}{a}$ respectively. The other extreme value $k = 1$ means that the fund has to grow/shrink by more than 100% per time step for the exchange mechanism to be triggered. Since shrinking by more than 100% is not possible (100% shrinking is already ruin) this means that $k = 1$ has only an effect on very extreme growth — which we believe is neglectable for practical use. Theoretically, one could also allow $k \in (1, \infty)$, where $k = \infty$ would simply turn off the exchange mechanism.

The exchange mechanism described above faces a problem if the collective account is plundered by individual accounts too often. It would mean that (some) individual accounts are not profitable and need a continuous support. For this reason, we restrict the choice of a barrier to those $k \in [0, 1]$ where the collective account does not loose money in expectation. This leads to the following profitability condition:

Profitability condition: The set of admissible $k \in [0, 1]$ is given by those k fulfilling

$$\mathbb{E}\Big[\frac{1}{a}\Big(1 - k - \frac{H_t}{H_{t-1}}\Big)^+ - \frac{1}{b}\Big(\frac{H_t}{H_{t-1}} - 1 - k\Big)^+\Big] \leq 0 \,. \tag{1}$$

This can be easily reformulated as:

$$\frac{1}{a}\mathbb{E}\Big[\Big(1 - k - \frac{H_t}{H_{t-1}}\Big)^+\Big] \leq \frac{1}{b}\mathbb{E}\Big[\Big(\frac{H_t}{H_{t-1}} - 1 - k\Big)^+\Big].$$

The idea of profitability is not new. For instance, in ruin theory, the net profit condition, that states that the expected total loss should be strictly smaller than the expected earnings from premia payments, is required.[4]

At the end of the accumulation phase, at the retirement point, the total saved amount will consist of two parts: the total saved amount from the individual account and a part from the collective account.

3 Maximisation of the Total Saved Capital

From our model setup it is clear that the total accumulated capital will depend on the transactions between the individual and the collective accounts as explained in the previous section. Therefore, the way the transactions are performed will impact the

[4]See Dickson [5].

optimisation procedures. In this section we consider three scenarios.

First, we assume that the collective account always has enough units to cover all claims from the individual account. This means, in particular, that the insurance product we consider offers a guarantee in the sense that if the real collective account becomes empty or does not have a sufficient number of units to cover the drawdowns of individual accounts – a third party (the state or the employer) pays the deficit by investing into the collective account.

In the second scenario, in case of an insufficient number of units in the collective account to cover the individual claims, no units at all are transferred from the collective fund.

Third, we use the so-called redistribution index J, to be explained in Section 3.3 in order to specify the number of units that can be transferred to an individual in case of a deficit in the collective account.

3.1 The collective fund is never empty

In this section, we seek to maximise the expected total saved amount at the retirement point[5], given that the collective account is never empty. It means that the insurance company promises to provide a sufficient coverage for individual accounts if needed. This case can be seen as a pension product with guarantees. The product described in the previous sections provides some stability and better returns than the traditional products with guarantees.

Following the Markovian structure of the fund, the expected saved capital in a single period can be optimised independently from the past.

In order to find the optimal choice of the corridor boundary k_t for the period $[t-1,t]$, we will make use of a recursive backward search, starting our considerations in the period $[T-1,T]$.

The wealth in the individual account of a policyholder V, in the case we refer to a representative individual, consists of the part $\gamma \in [0,1]$ of the premia π_{ind} paid into the individual accounts (the proportion $1-\gamma$ of the premia is paid into the collective account, the investment returns, and the amount transferred to or from the collective account.

$$V_t = \gamma\pi_{\text{ind}} + \eta_{t-1}H_t - \mathbb{1}_{[\rho_t > k_t]}\frac{1}{b}V_{t-1}\Big(\rho_t - k_t\Big) + \mathbb{1}_{[\rho_t < -k_t]}\frac{1}{a}V_{t-1}\Big(-k_t - \rho_t\Big) \,. \tag{2}$$

The wealth of the collective fund is described by its investment returns, the part $1-\gamma$ of all the premia π_{all}, and the gains or losses from the transactions with all the individual accounts

$$C_t = (1-\gamma)\pi_{\text{all}} + \theta_{t-1}H_t + \sum_j \left\{ \mathbb{1}_{[\rho_t > k_t^j]}\frac{1}{b}V_{t-1}^j\Big(\rho_t - k_t^j\Big) + \mathbb{1}_{[\rho_t < -k_t^j]}\frac{1}{a}V_{t-1}^j\Big(k_t^j + \rho_t\Big) \right\},$$

[5] The monetary amounts (the premia paid) are transformed into fund shares almost immediately after the payment. Therefore, the money being invested into the fund is automatically discounted – not by an interest rate, but by the value of the fund serving as a benchmark. It means, the present value of the premia is represented by the value of the fund.

Note that we first use θ_{t-1}, i.e. the number of shares in the collective account from the time $t-1$. Only after the unit exchanges and premium payments, the number of shares will be adjusted to θ_t.

For an individual, it is desirable to maximise the wealth of their individual account at the retirement point, which would imply a higher value of the initial pension. Clearly, high expectation on the value of the saved wealth with a strong negative skewness is not desirable and it is much more preferable to have low down-side risk. While the "goodness" of the wealth at retirement can be measured in various ways (e.g. mean-variance optimisation, mean-semivariance optimisation or utility maximisation) we offer an analysis based on mean-"realised volatility" optimisation later on. This is related to mean-variance optimisation but not the same[6]. We return to the more simple mean maximisation which leads to following optimisation criterion:

$$A(k_1, \ldots, k_T) := \mathbb{E}\big[V_T\big] \tag{3}$$

which is to be maximised over all possible choices of corridor boundaries $k_1, \ldots, k_T$ at every point of time for every individual[7] where the corridor boundary k_t is decided at time $t-1$, i.e. $\mathcal{F}_{t-1}$-measurable.

Due to the simplifying assumption that the collective account does not ruin, the choices for the corridor boundaries are not influenced by the choices for other individuals. In order to show this we define the function

$$\begin{aligned}\Psi_1(k) :&= \mathbb{E}\Big[\rho_T - \frac{1}{b}(\rho_T - k)^+ + \frac{1}{a}(-\rho_T - k)^+\Big] \\ &= \mathbb{E}\Big[\frac{V_T - V_{T-1} - \gamma\pi_{\text{ind}}}{V_{T-1}}\Big]\end{aligned} \tag{4}$$

for $k \in [0,1]$. Note that the distribution of the return ρ_t does not depend on the time-point t due to homogeneity assumptions of the underlying fund. Consequently, the time-point T in the definition of Ψ_1 can be replaced by any time-point $t = 1, \ldots, T$ while Ψ_1 stays the same. We make an observation regarding the maximum of Ψ_1 first.

Lemma 3.1
Define

$$\Xi(k) := \mathbb{E}\Big[\rho_t + \frac{1}{a}\Big(-\rho_t - k\Big)^+ - \frac{1}{b}\Big(\rho_t - k\Big)^+\Big], \tag{5}$$

then the maximum of Ξ is attained either at $k = 1$ or at the minimal k allowed by the profitability condition. If the profitability condition is not assumed, then the maximum is attained in either 0 *or* 1.

[6] Note that realised volatility can be calculated from a single path whereas variance estimation requires several scenarios to be meaningful.

[7] The optimal corridor boundaries will be chosen by the insurance company for the individuals and not by the individuals themselves.

Proof: Let f denote the density of the random variable $\frac{H_T}{H_{T-1}}$, which is given by

$$f(y) = \frac{1}{\sqrt{2\pi} y\sigma} e^{-\frac{(\ln(y)-\mu)^2}{2\sigma^2}} . \tag{6}$$

because H is a geometric Brownian motion. The derivatives of Ξ are given by

$$\begin{aligned}\Xi'(k) &= -\frac{1}{a}\int_0^{1-k} f(y)\,\mathrm{d}y + \frac{1}{b}\int_{1+k}^{\infty} f(y)\,\mathrm{d}y,\\ \Xi''(k) &= \frac{1}{a} f(1-k) - \frac{1}{b} f(1+k) .\end{aligned}$$

therefore

$$\begin{aligned}\Xi'(1) &= \frac{1}{b}\int_2^{\infty} f(y)\,\mathrm{d}y > 0,\\ \Xi''(0) &= \Big(\frac{1}{a} - \frac{1}{b}\Big) f(1) \geq 0,\\ \Xi''(1) &= -\frac{1}{b} f(2) < 0,\end{aligned}$$

where the second inequality holds because $1 \leq a \leq b$. Continuity of Ξ'' yields that the set $\{k \in [0,1] : \Xi''(k) = 0\}$ is non-empty and closed. For $k^0 \in [0,1]$ with $\Xi''(k^0) = 0$ we find that $\frac{1}{a} f(1-k^0) = \frac{1}{b} f(1+k^0)$ and, hence, we have

$$\Xi'''(k^0) = \frac{f(1-k^0)}{a\sigma^2(1-k^0)(1+k^0)}\Big[2\sigma^2 - 2\mu + (1+k^0)\ln(1-k^0) + (1-k^0)\ln(1+k^0)\Big].$$

The expression in quadratic brackets $B(k) := [2\sigma^2 - 2\mu + (1+k)\ln(1-k) + (1-k)\ln(1+k)]$ above is strictly decreasing in k, converging to $-\infty$ if k approaches 1, and has at most one zero point. We show that there is $k^0 \in [0,1]$ such that Ξ'' is positive on $[0,k^0]$ and negative on $[k^0,1]$.

Case 1: B has no zero in $[0,1]$. Then $B(k) < 0$ for any $k \in [0,1]$ and Ξ'' is strictly decreasing near any of its zeros and, hence, has exactly one zero $k^0 \in [0,1]$. Consequently, Ξ'' is positive on $[0,k^0]$ and negative on $[k^0,1]$.

Case 2: B has a unique zero $\bar{k} \in [0,1]$. For any zero k^1 of Ξ'' with $k^1 < \bar{k}$ we find that Ξ'' is strictly increasing near k^1 and, hence, $k^1 = 0$. Since $Xi''(0) \geq 0$ and $\Xi''(1) < 0$ there must be a zero k^0 of Ξ'' in $[\bar{k},1]$ and 0 is the only possible zero strictly before $\bar{k}$. If $k^0 = \bar{k}$ is the only such zero, then Ξ'' is of the desired shape. Hence, we may assume that $k^0 > \bar{k}$. Then we have $\Xi'''(k^0) < 0$ and, hence Ξ'' is strictly decreasing near k^0 which implies that there is at most one such zero and Ξ'' must be strictly positive on $(\bar{k},k^0]$ and, hence, has the desired shape.

Thus, we have that Ξ'' is positive on $[0,k^0]$ and negative on $[k^0,1]$ for some $k^0 \in [0,1]$. Consequently, Ξ' attains its maximum in k^0, Ξ' is increasing on $[0,k^0]$ and decreasing on $[k^0,1]$. Since $\Xi'(1) > 0$ we find that Ξ' has at most one zero $k^1 \in [0,k^0)$. Consequently, either Ξ is decreasing on $[0,k^1]$ and increasing on $[k^1,1]$ or Ξ is increasing on $[0,1]$. The claim follows. □

Proposition 3.2
Let $k^* \in [k_{\min}, 1]$ be the unique maximum of the function Ψ_1, where $k_{\min}$ is the smallest value for k allowed by the profitability condition (1).[8]
Then the optimal choice for the corridor boundary is given by k^*, which is time- and contract-independent in the sense that k^* will be optimal at any t and for any contract. Moreover, it holds either $k^* = k_{\min}$ or $k^* = 1$.

Proof: We have for A defined in (3)

$$A(k_1, \ldots, k_T) = \mathbb{E}[V(T)] = \sum_{t=1}^{T} \mathbb{E}[\Delta V_t]$$

where $\Delta X_t := X_t - X_{t-1}$ for any process X and $t \geq 1$. We will see that there is a choice of corridor boundaries which maximises each summand and, hence, maximises the sum. Rearranging the terms, we get

$$V_t - V_{t-1} = \Delta V_t = \gamma\pi_{\text{ind}} + V_{t-1}\frac{V_t - V_{t-1} - \gamma\pi_{\text{ind}}}{V_{t-1}}$$

which yields by the tower property and the definition of Ψ_1 in (4):

$$\begin{aligned}\mathbb{E}[\Delta V_t] &= \gamma\pi_{\text{ind}} + \mathbb{E}\left[V_{t-1}\mathbb{E}\left[\frac{V_t - V_{t-1} - \gamma\pi_{\text{ind}}}{V_{t-1}}\Big|\mathcal{F}_{t-1}\right]\right] \\ &= \gamma\pi_{\text{ind}} + \mathbb{E}\left[V_{t-1}\Psi_1(k_t)\right]\end{aligned}$$

for any $t = 1, \ldots, T$. The later equality holds true due to the iid property of the returns. Since V_{t-1} is positive, we find that the maximiser k^0 of Ψ_1 is the optimal choice for k_t when maximising $\mathbb{E}[\Delta V_t]$. Thus, we have

$$\sup_{k_1,\ldots,k_T} \mathbb{E}[\Delta V_t] = \gamma\pi_{\text{ind}} + \Psi_1(k^0) \sup_{k_1,\ldots,k_{t-1}} \mathbb{E}\left[V_{t-1}\right].$$

Consequently, we find that $k_t = k^0$ is the optimal choice. Lemma 3.1 yields that $k^0 \in \{0, 1\}$. $\square$

As we have seen in the proof above, by looking at the terms depending on k it is sufficient to maximise

$$M_1(k) := \mathbb{E}\Big[\frac{1}{a}\Big(1 - k - \frac{H_T}{H_{T-1}}\Big)^+ - \frac{1}{b}\Big(\frac{H_T}{H_{T-1}} - 1 - k\Big)^+\Big]. \tag{7}$$

As our target is to smooth the evolution of individual portfolios we need some kind of penalty for high volatility in order to obtain a maximum. Such a penalty function can be the expected realised volatility of the fund. Here, we use the relative realised variance which means we optimise

$$\mathfrak{A}(k_1, \ldots, k_T) := \mathbb{E}[V_T] - \alpha\mathbb{E}\left[\sum_{t=1}^{T} \frac{1}{V_{t-1}}(V_t - V_{t-1} - \gamma\pi_{\text{ind}})^2\right]$$

[8] Note that with $k = 1$ the profitability condition is always met and, hence, $k_{\min} \leq 1$.

for some risk-aversion $\alpha > 0$ where we optimise the corridor boundaries $k_1, \dots, k_T$ at every point of time. For α the insurance company may choose the desired proportion between the mean and the realised volatility.[9]

We observe the following identity

$$\frac{V_t - V_{t-1} - \gamma\pi_{\text{ind}}}{V_{t-1}} = \rho_t + \frac{1}{a}\Big(- \rho_t - k\Big)^+ - \frac{1}{b}\Big(\rho_t - k\Big)^+,$$

and define equivalently to Ψ_1 given in (4):

$$\Psi_2(k) := \mathbb{E}\Big[\Big(\frac{V_t - V_{t-1} - \gamma\pi_{\text{ind}}}{V_{t-1}}\Big)^2\Big],$$

for any $k \in [0,1]$ which allow to simplify the optimisation problem.

Lemma 3.3
It holds

$$\mathfrak{A}(k_1, \dots, k_T) = \sum_{t=1}^{T} \mathbb{E}\Big[V_{t-1}\big(\Psi_1(k_t) - \alpha\Psi_2(k_t)\big)\Big]$$

for any choice of corridor boundaries for the times $1, ...T$*:* $k_1, \dots, k_T$*. In particular, it is optimal to choose* k_1 *such that the expression*

$$\Psi_1(k_1) - \alpha\Psi_2(k_1)$$

is maximised and to let then $k_1 = ... = k_T$ *for all individuals.*

Proof: We have

$$\begin{aligned}
\mathfrak{A}(k_1, \dots, k_T) &= \mathbb{E}\Big[\sum_{t=1}^{T} \Delta V_t\Big] - \alpha\mathbb{E}\left[\sum_{t=1}^{T} \frac{1}{V_{t-1}}(V_t - V_{t-1} - \gamma\pi_{\text{ind}})^2\right] \\
&= \sum_{t=1}^{T} \mathbb{E}\left[V_{t-1}\left\{\mathbb{E}\left[\frac{V_t - V_{t-1} - \gamma\pi_{\text{ind}}}{V_{t-1}}\middle|\mathcal{F}_{t-1}\right]\right.\right. \\
&\qquad \left.\left. - \alpha V_{t-1}\mathbb{E}\left[\left(\frac{V_t - V_{t-1} - \gamma\pi_{\text{ind}}}{V_{t-1}}\right)^2\middle|\mathcal{F}_{t-1}\right]\right\} + \gamma\pi_{\text{ind}}\right] \\
&= \sum_{t=1}^{T} \Big(\mathbb{E}\big[V_{t-1}\big(\Psi_1(k_t) - \alpha\Psi_2(k_t)\big)\big] + \gamma\pi_{\text{ind}}\Big).
\end{aligned}$$

This shows that an optimal choice for k_T is a maximum of the function $\Psi_1 - \alpha\Psi_2$. A simple induction shows that $k_1 = \dots = k_T$ with the above choice of k_1 is optimal. $\square$

[9]We would like to note that the realised volatility is easily observed from the data while other risk measures such as the variance can only be obtained if something like "independent copies of the same experiment" are at hand. However, real data usually provides only a single path.

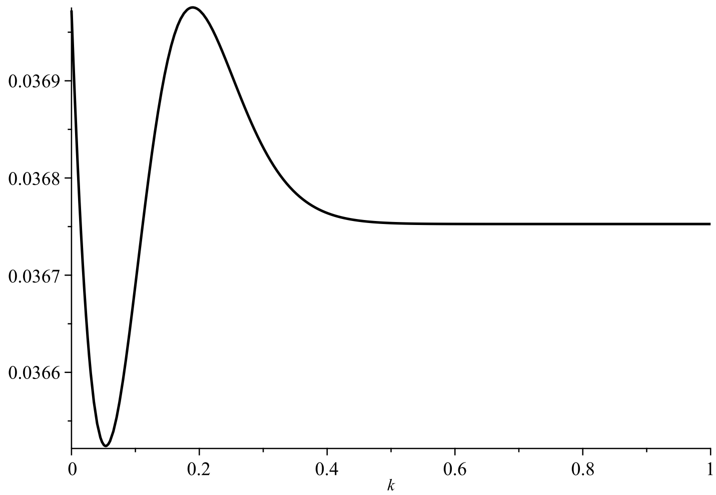


Figure 1: The function $M_2(k)$ for $\sigma = 0.092367$, $\mu = 0.06$, $\alpha = 2$, $a = 2$ and $b = 4$ yielding $M_2(0) = M_2(0.1897)$. The profitability condition (1) is fulfilled on $[0, 1]$.

From the preceding lemma we know that the optimal choice of corridor boundary k is the maximiser of the following (time-independent) functional:

$$\begin{aligned} M_2(k) &:= \Psi_1(k) - \alpha\Psi_2(k) \\ &= \Bigg\{\mathbb{E}\Big[\rho_T + \frac{1}{a}\Big(-\rho_T - k\Big)^+ - \frac{1}{b}\Big(\rho_T - k\Big)^+\Big] \\ &\quad - \alpha\mathbb{E}\Big[\Big(\rho_T + \frac{1}{a}\Big(-\rho_T - k\Big)^+ - \frac{1}{b}\Big(\rho_T - k\Big)^+\Big)^2\Big]\Bigg\}, \end{aligned}$$

Remark 3.4
The function Ψ_2 is strictly increasing. Lemma 3.1 shows that Ψ_1 is first decreasing until reaching its minimum and increasing thereafter. Consequently, $\Psi_1 - \alpha\Psi_2$ is first decreasing and may start to increase at a later time but this cannot be before the minimum of Ψ_1. Thus, the maximum of $\Psi_1 - \alpha\Psi_2$ as a function on $[0, 1]$ is either attained in 0 or after the minimum of Ψ_1.
It might happen, confer Figure 1, that the maximum of $\Psi_1 - \alpha\Psi_2$ cannot be uniquely defined, i.e. the set $\operatorname{argmax}\{\Psi_1 - \alpha\Psi_2\}$ contains at least two elements, say $k_1 < k_2$. Recall that maximising the value of an individual account corresponds to the maximisation of the function M_1 defined in (7) and leads to a bang bang strategy. Since, the individual accounts yield the main basis for the calculation of the initial pension, we choose k_1 if M_1 is decreasing and k_2 if M_1 is increasing in order to optimise the value of individual accounts.

Example 3.5
Let us assume the following parameters: $a = 2$ and $b = 4$. In Figures 2, 3, 4 we use the following parameter sets:

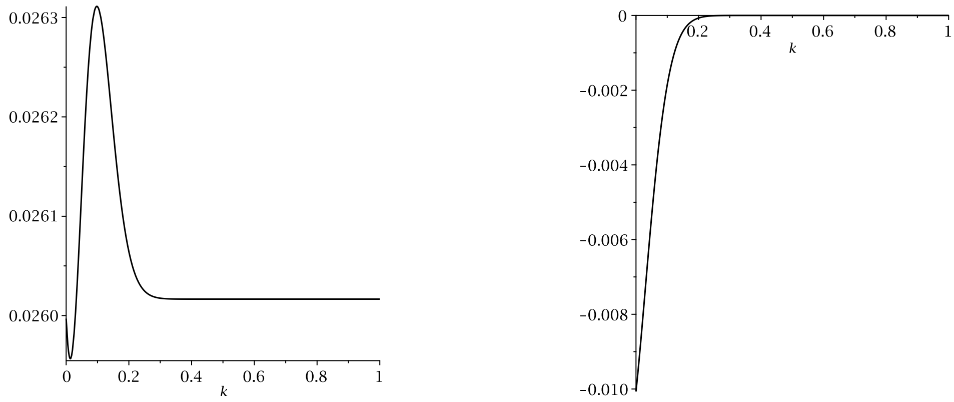


Figure 2: $M_2(k)$ (left picture) and profitability condition (right picture) for $\mu = 0.045$, $\sigma = 0.06$ and $\alpha = 3.5$.

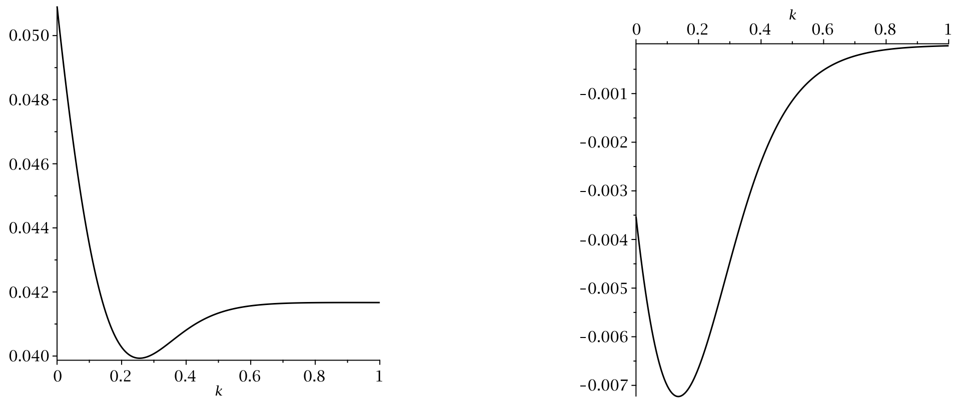


Figure 3: $M_2(k)$ (left picture) and profitability condition (right picture) for $\mu = 0.045$, $\sigma = 0.2$ and $\alpha = 0.5$.

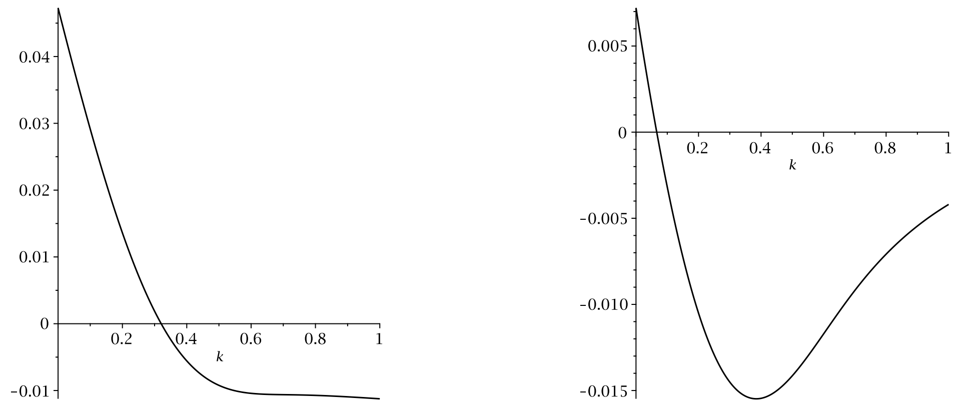


Figure 4: $M_2(k)$ (left picture) and profitability condition (right picture) for $\mu = 0.01$, $\sigma = 0.4$ and $\alpha = 0.5$.

- *Figure 2:* $\mu = 0.045$, $\sigma = 0.06$ *and* $\alpha = 3.5$.
The maximum of $M_2(k)$ *is attained at* $k = 0.09785$. *The profitability condition* (1) *is fulfilled for all* $k \in [0, 1]$.
The existence of a unique maximum is due to the fact that the volatility is quite low compared to the return. But on the other hand, the penalising factor α *increases the risk awareness.*

- *Figure 3:* $\mu = 0.045$, $\sigma = 0.2$ *and* $\alpha = 0.5$.
For this set of parameters, the function $M_2(k)$ *attains its maximum at* $k = 0$. *The volatility of* 20% *is too high for the given return. It is optimal to get help from the collective account continuously (and to share the gains continuously), i.e.* $k = 0$, *as the evolution of the fund seem to be not very optimistic.*
The right picture in Figure 3 represents the profitability condition and indicates that all $k \in [0, 1]$ *are admissible.*

- *Figure 4:* $\mu = 0.01$, $\sigma = 0.4$ *and* $\alpha = 0.5$.
In this third case, the volatility even increases to 40%. *The maximum of* M_2 *is again attained at* $k = 0$. *However, the profitability condition allows just* $k \in [0.0664, 1]$ *as we can see in the right picture. Therefore, one would take* $k = 6.64\%$ *as the optimal barrier parameter.*

■

3.2 No help if the collective account does not have sufficient number of units

In this section, we assume that in the case that the collective account does not have enough units in order to cover all claims at a particular point in time, no single claim will be paid, i.e. the exchange mechanism to help the individual accounts is void for that time point. The evolution of the wealth for an individual j is given by

$$\begin{aligned} V_t^j &= \gamma\pi_{\text{ind}} + \eta_{t-1}H_t - \frac{1}{b}V_{t-1}^j\Big(\rho_t - k^j\Big)^+ \\ &\quad + \frac{1}{a}V_{t-1}^j\Big(-\rho_t - k^j\Big)^+ \mathbb{1}_{\left[\theta_{t-1}(1+\rho_t)\geq \frac{1}{a}\sum_{i=1}^{n}\eta_{t-1}^i\left(-k^i-\rho_t\right)^+\right]}, \end{aligned}$$

where n is the total number of the contracts in the insurance pool, $k^1, ..., k^n$ and $\eta_{t-1}^1, ..., \eta_{t-1}^n$ are the corridor boundaries and the number of shares in the individual accounts at that time respectively.

Note that here we index the corridor boundaries by individuals rather by time but new thresholds can be chosen dynamically at discrete time points. The individual account under consideration is indexed by j. On the left hand side of the indicator we find $1/a$ times the value of the collective account before any units are transferred and on the right hand-side the total volume of all individual losses exceeding the individual thresholds. If the left hand side in the indicator is not bigger, then there is insufficient wealth to

cover for the corresponding part of the individual excess losses. In that case, no one gets any help at all to prevent the collective account to become negative.
Our target is to optimise the expected return minus the relative realised quadratic variation for each individual, i.e. for each individual we aim at optimising

$$\mathbb{E}[V_t^j] - \alpha \mathbb{E}\left[\sum_{t=1}^{T} \frac{1}{V_{t-1}^j}(V_t^j - V_{t-1}^j - \gamma\pi_{\mathrm{ind}})^2\right].$$

The problem here is the cross-dependence among all individuals. One possibility could be that all individuals use the same barrier, chosen by the insurance company. We make a precise error analysis in the sense that we single out how much an individual can improve and show that this depends only on the fraction of its wealth compared to the collective wealth, which in a large community should be rather small.
We try to find a common choice of barriers such that no individual has an improved target value if all barriers are increased or decreased a bit.
Our main result of this section, Theorem 3.7 below, states that it is optimal to choose the same barrier for all individuals and dynamically increase the barriers if the amount in the collective account is relatively low compared to the total wealth of all individual accounts. We would like to stress that we use the same risk aversion level α for all individuals. This fact leads to the same corridor boundary for all individuals, i.e. all individuals are treated the same. We believe that it is impossible to have a fair risk-sharing agreement respecting different individual risk aversion levels.
Following the same arguments as in the previous sections we can see that it is optimal to optimise at each time step separately, i.e. at time t for each individual j we need to optimise

$$\mathbb{E}[U_t^j - \alpha(U_t^j)^2]$$

where

$$U_t^j = \rho_t - \frac{1}{b}(\rho_t - k^j)^+ + \frac{1}{a}\left(-k^j - \rho_t\right)^+ \mathbb{1}_{\left[\theta_{t-1}(1+\rho_t) > \frac{1}{a}\sum_{i=1}^{n} \eta_{t-1}^i \left(-k^i - \rho_t\right)^+\right]}$$

and $k^1, \dots, k^n$ are to be chosen $\mathcal{F}_{t-1}$-measurable in $[k_{\min}, 1]$ where $k_{\min}$ is the minimal allowed value by the profitability condition. Our target criterion implies that an optimal choice of corridor boundary k^j is either the minimal allowed value $k_{\min}$, the maximal possible value 1 or it satisfies

$$\partial_{k_j} \mathbb{E}[U_t^j - \alpha(U_t^j)^2] = 0.$$

Remark 3.6
If a (possibly non-optimal) choice of thresholds has been made, then the indicator, as a function of ρ_t is decreasing and, hence, there is some constant $z^*(k^1, \dots, k^n)$ such that

$$\mathbb{1}_{\left[\theta_{t-1}(1+\rho_t) \geq \frac{1}{a}\sum_{i=1}^{n} \eta_{t-1}^i \left(-k^i - \rho_t\right)^+\right]} = \mathbb{1}_{[\rho_t \geq z^*(k^1,\dots,k^n)]}.$$

Also we have

$$z^*(k^1,\dots,k^n) = -\frac{a\theta_{t-1} + \sum_{i\in I}\eta^i_{t-1}k^i}{a\theta_{t-1} + \sum_{i\in I}\eta^i_{t-1}} \in [-1,0]$$

$$I := \left\{ j = 1,\dots,n : a\theta_{t-1}(1-k^j) - \sum_{i=1}^n \eta^j_{t-1}(k^j - k^i)^+ \geq 0 \right\}$$

Note that z^* is Lipschitz-continuous and its absolutely continuous derivative is given by

$$\partial_{k^j} z^*(k^1,\dots,k^n) = \frac{-\eta^j_{t-1}\mathbb{1}_{\{j\in I\}}}{a\theta_{t-1} + \sum_{i\in I}\eta^i_{t-1}}.$$

In the particular case that all k^i are equal we find $I = \{1,\dots,n\}$ and, hence, the following simplifications

$$z^*(k^1,\dots,k^1) = -\frac{a\theta_{t-1} + \sum_{i=1}^n \eta^i_{t-1}k^1}{a\theta_{t-1} + \sum_{i=1}^n \eta^i_{t-1}},$$

$$\partial_{k^j} z^*(k^1,\dots,k^1) = \frac{-\eta^j_{t-1}}{a\theta_{t-1} + \sum_{i=1}^n \eta^i_{t-1}}.$$

This reveals that if the same barrier k is chosen for all individuals and if the j-th individual has negligible amount compared to the total amount of all other individuals plus the collective amount, then the ∂_{k^j}-derivative is negligible as well.

We can now formulate the main result of this section. Basically, we try to optimise the choice of k under the constraint that all k^j have to be equal. This does not allow to optimise for every individual but we quantify that each individual cannot improve by much if every individual has a small wealth in the scheme compared to the total wealth of the scheme.

Theorem 3.7
Define $z(k) := -\frac{a\theta_{t-1} + k\sum_{i=1}^n \eta^i_{t-1}}{a\theta_{t-1} + \sum_{i=1}^n \eta^i_{t-1}}$ for $k \in [0,1]$ and

$$N(c,k) := \mathbb{E}[h(c,k) - \alpha h(c,k)]$$

where $h(c,k) := \rho_t - \frac{1}{b}(\rho_t - k)^+ + \frac{1}{a}(-k-\rho_t)^+\mathbb{1}_{\{\rho_t > c\}}$ for $k \in [0,1]$ and $c \in [-1,0]$. For a given value of $c \in [-1,0]$ we denote the maximiser of $N(c,\dots)$ by $k(c)$.
Assume that there is $\bar{k} \in [0,1]$ such that $\bar{k} = k(z(\bar{k}))$.
Then choosing the barrier k, for each individual at time $t-1$ is near to the optimal in the sense that changing the barrier k^j for the j-th individual does not improve its performance by more than

$$\|\rho_t\|_\infty \frac{(1/a + \alpha)\eta^j_{t-1}}{a\theta_{t-1} + \sum_{i=1}^n \eta^i_{t-1}}$$

where $\|\rho_t\|_\infty$ denotes the maximum of the continuous density of ρ_t.

Proof: We choose the barriers $k^i = \bar{k}$ for any other individual $i \neq j$. For the j-th individual we are supposed to maximise the function

$$\mathbb{E}[U_t^j - \alpha(U_t^j)^2]$$

over the possible values of $k^j \in [0,1]$ and its maximiser is denoted by $\bar{k}^j$. We simply write $z^*(k^j)$ when we mean $z^*(k^1,\dots,k^n)$ as a function of k^j and the other $k^i = \bar{k}$. Define $c := z(\bar{k}) = z^*(\bar{k})$. $\bar{k}$ is the maximiser of the function $N(c,\cdot)$ and

$$\begin{aligned}&|N(c,k^j) - \mathbb{E}[U_t^j - \alpha(U_t^j)^2]|\\ &\qquad \le \mathbb{E}\left[\frac{1}{a}\mathbb{1}_{\{\rho_t<-k^j,\rho_t\in[c,z^*]\}} + \alpha(|U_j^t| + |N(c,k^j)|)\frac{1}{a}\mathbb{1}_{\{\rho_t<-k^j,\rho_t\in[c,z^*(\bar{k}^j)]\}}\right].\end{aligned}$$

Since U, N are bounded by 1 on $\{\rho_t < 0\}$ we find

$$|N(c,k^j) - \mathbb{E}[U_t^j - \alpha(U_t^j)^2]| \le (1/a+\alpha)P(\rho_t \in [c, z^*(\bar{k}^j)]).$$

Remark 3.6 yields $|c - z^*(\bar{k}^j)| \le |\bar{k}^j - \bar{k}|\frac{\eta_{t-1}^j}{a\theta_{t-1}+\sum_{i=1}^n \eta_{t-1}^i} \le \frac{\eta_{t-1}^j}{a\theta_{t-1}+\sum_{i=1}^n \eta_{t-1}^i}$ and the result follows. □

The theorem suggests a simple algorithm to find a nearly optimal choice, namely to choose a sequence $\bar{k}_n$ and c_n recursively via $\bar{k}_0 = 1$, $c_0 = 0$ (or any other starting values) and define recursively

$$\begin{aligned}c_{n+1} &:= z(\bar{k}_n),\\ \bar{k}_{n+1} &:= k(c_{n+1})\end{aligned}$$

for any $n \in \mathbb{N}$. The value c_n is the threshold where it is expected that such a downfall of the underlying fund makes it impossible to cover all the losses from the individual accounts and $\bar{k}_n$ the barrier chosen for all the individuals.[10]

3.3 Using a redistribution index if the collective account does not have sufficient number of units

Another possibility to handle the situation of insufficient number of units in the collective account is to use the so-called redistribution index.

At the retirement point T, every new pensioner gets not only the capital from their individual account but also a proportion from the collective account. This proportion is determined via the redistribution index which we denote by J_{T-1}^j for the j-th contract. The time $T-1$ indicates that J_{T-1}^j is determined at time $T-1$ to avoid arbitrage.

[10] Also, using the mean field game theory, where each individual has contributed an actually negligible amount compared to the whole collective, optimisation of the barrier in this particular case is roughly the same as ignoring the possibility of the collective fund to become empty. There, due to the fact that the value functions for all individuals are equal, the optimality can only be attained by choosing the same barrier.

The index is updated during the accumulation phase when premia payments are made and is constant otherwise, which means that it is discrete in nature and develops over time, i.e. at any time t one can determine J_t^j.

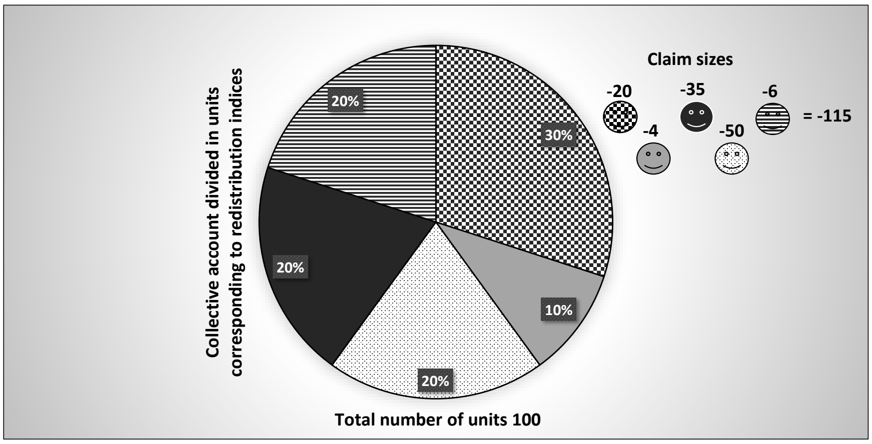
The redistribution index can be also used in order to determine the number of units to be transferred into an individual account in case of under-performance if the collective account does not have enough units to cover all occurred claims. This is critical as there is a lack of analysis in both academic research and regulation with respect to the strategy the insurance company should adopt in this case. We intend to fill in this gap through some possible scenarios in this section.
If the collective account does not have enough units, the policies with a deficit can, for instance, claim a number of units corresponding to their redistribution index. It means the individual account has the following value

$$\begin{aligned}V_t &= \gamma\pi_{\text{ind}} + \eta_{t-1}H_t - \frac{1}{b}V_{t-1}\Big(\frac{H_t}{H_{t-1}} - 1 - k\Big)^+ \\ &+ \frac{1}{a}V_{t-1}\Big(1 - k - \frac{H_t}{H_{t-1}}\Big)^+ \mathbb{1}_{\left[a\theta_{t-1}\frac{H_t}{H_{t-1}} > \sum\limits_{i=1}^{n}\eta_{t-1}^i\left(1-k^i-\frac{H_t}{H_{t-1}}\right)^+\right]} \\ &+ \min\left\{J_{t-1}\theta_{t-1}H_t, \frac{1}{a}V_{t-1}\Big(1 - k - \frac{H_t}{H_{t-1}}\Big)^+\right\} \\ &\quad \times \mathbb{1}_{\left[a\theta_{t-1}\frac{H_t}{H_{t-1}} \leq \sum\limits_{i=1}^{n}\eta_{t-1}^i\left(1-k^i-\frac{H_t}{H_{t-1}}\right)^+\right]},\end{aligned}$$

where $k^j = k$. In Section 3.2, we proved that the optimal corridor corridor boundary for the return, k, is the same for all contracts in the pool of contributors. It means, if the fund go down all individual accounts will produce claims simultaneously. However, the claim sizes depend on the number of shares in the individual accounts and differ from contract to contract. Therefore, some contracts might produce claims smaller than the number of units in the collective account corresponding to their redistribution index and vice versa. If the regulation requirements allow to entirely empty the collective account, the following recursive procedure can be applied, see also Figure 5:

- Settle all individual claims that are below their redistribution part.
  In Figure 5 the claims amounting to 4, 6 and 20 have the redistribution indices 0.1 (yielding 10 shares), 0.2 (yielding 20 shares) and 0.3 (yielding 30 shares) respectively. It means these claims can be settled immediately.

- Adjust the redistribution indices of the remaining claims to the new number of claims and settle those that are now below their redistribution part.
  In Figure 5, after settling "small" claims in the first step, the collective account has $100 - 4 - 6 - 20 = 70$ shares on its disposal. The redistribution indices of the remaining two claims, amounting to 35 and 50 shares, equaled to 0.2 in the first step and should be adjusted due to the new claim number of 2. Therefore, the new redistribution indices are given by 0.5 yielding 35 shares. Thus, one claim can be completely covered.



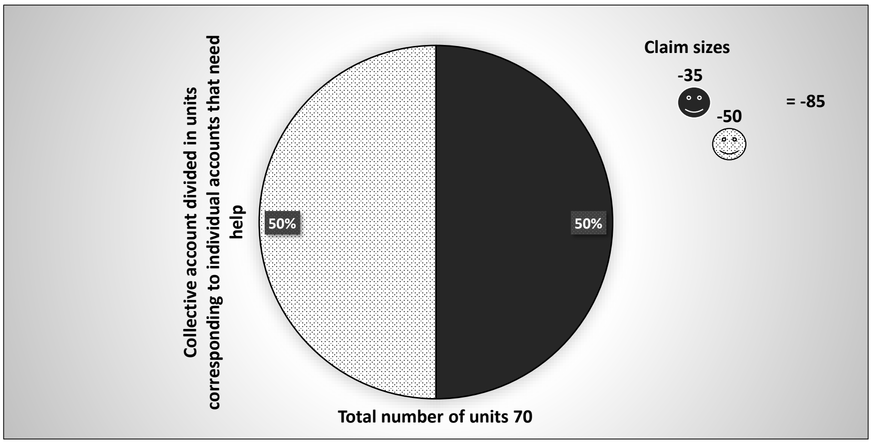


Figure 5: Redistribution of the shares from the collective account by a recursive procedure.

- Proceed until all remaining claims exceed their redistribution part and eventually settle them.
  In our example, the collective account has now 35 shares. The new redistribution index of the claim amounting to 50 is now 1. This contract gets just 35 shares from the collective account which is now empty.

The above procedure serves just as an example and targets to showcase a possibility to handle the individual claims. Therefore, the presented numbers cannot be considered as realistic quantities. Also, it should be noted that redistributing all shares from the collective account between individual accounts might leave the next retiring cohort with small amounts of capital resulting from the collective account compared to the amount of premia they paid in if the size of the collective is not large enough. This would clearly violate the concept of fairness and require intergenerational smoothing mechanisms.
On the one hand, the procedure of getting help from the collective account is a question

of product design but on the other hand, it should also be in line with the regulations in place to achieve/provide intergenerational fairness and fulfil sustainability requirements. The above described recursion could also be applied on the returns of the collective account so that the main capital remains untouched. However, this procedure will contradict the primary mission of the collective account – to serve as a backup for the individual accounts.

Concerning the mathematical implementation of the scenarios described above, the method is similar to the one described in Section 3.2. Neither the value function nor the optimal strategy can be calculated explicitly.

4 Conclusions

The life insurance companies are continuously creating new products to cope up not only with longevity but also with the period of protracted low interest rates. It is well-known that low interest rates affect investment opportunities and, in particular, have a significant adverse effect on insurers whose liabilities includes some benefit promises such as guarantees.

With the aim of offering an adequate level of benefits to the policyholders and at the same time preserving the long-term solvency of the plan, this paper analyses a new pension design applied to the accumulation phase from a mathematical point of view. Under the proposed design, we seek to maximise the accumulated capital at retirement by investing the premia into two funds: an individual and a collective. The collective fund acts as a buffer where some units are transferred to (from) the individual account when the performance of the individual fund is below (above) a particular barrier.

We prove that, in the case of symmetric corridor boundaries for the corridor $[-k, k]$ and if the collective account never ruins, the optimal k for the exchange of units between the individual and the collective account is given either by the lowest barrier allowed by the profitability condition or by 1. If the barriers are asymmetric, we might have cases where no satisfactory results are obtained because the profitability condition is not fulfilled.

In order to incorporate the possible risk into the target functional to maximise, we include a penalty function given by the expected realised volatility of the fund. We also show that in some occasions the maximum might not be unique. However, as the individual account is the main basis to calculate the initial pension, we choose the barrier that optimises the expected value of the individual account for the policyholder.

Due to the lack of analysis in both academia research and practice, this paper also analyses the possibility of not having enough units to be transferred to the individual account. The first solution analysed in this paper is the non-transfer of any units if the number of units is not enough to cover all claims. Due to the cross dependence among all individuals we make a precise error analysis where the same barrier is used by every policyholder. The same barrier is a suboptimal strategy but the explicit solution of an optimal k would require lot of computation and the improvement for the policyholder would be negligible, i.e. the maximised amount at retirement would barely increase.

Secondly, we describe a redistribution index that could cover some of the deficit of the individual claims and could be applied through a recursive procedure.
This paper presents an innovative and attractive way to smooth the volatility of the fund in the accumulation phase. Hence, the proposed product design should be beneficial to both the life insurers – as there are no benefit promises – and policyholders – as the amount of accumulated capital is more secure than in the case of risky investments and much higher than in the case of non-risky investments.
Finally, based on the model presented, at least three important directions for future research can be identified. First, another challenge in the accumulation phase is the maximisation of the retirement capital through an optimal splitting strategy of the premia into the two funds, i.e. individual and collective. Another avenue for future research would be to explore the redistribution index so that it ensures the intergenerational fairness among the members' plan. Third, it would be interesting to set up bounds for the pension amount during the retirement phase so that the retirees have a stable benefit level over time.

Acknowledgements

The research of Julia Eisenberg was funded by the Austrian Science Fund (FWF), Project number V 603-N35.

All authors would like to thank the unknown referees for their careful reading of our manuscript and their insightful comments and suggestions.

References

[1] J.H. Van Binsbergen, D. Broeders, M. De Jong, and R.S.J. Koijen. Collective pension schemes and individual choice. *Journal of Pension Economics & Finance*, 13(2):210–225, 2014.

[2] M.C. Boado-Penas, J. Eisenberg, A. Helmert, and P. Krühner. A new approach for satisfactory pensions with no guarantees. *European Actuarial Journal*, 10:3–21, 2020. DOI: 10.1007/s13385-019-00220-2.

[3] A. Bohnert, P. Born, and N. Gatzert. Dynamic hybrid products in life insurance: Assessing the policyholders' viewpoint. *Insurance: Mathematics and Economics*, 59(1):87–99, 2014.

[4] A.L. Bovenberg. Dutch stand-alone collective pension schemes: The best of both worlds? In Z. Bodie, L.B. Siegel, and R.N. Sullivan, editors, *The Future of Life-Cycle Saving and Investing: The Retirement Phase*, pages 69–76. The CFA Institute Research Foundation, 2009.

[5] D.C.M. Dickson. *Insurance: risk and ruin.* Cambridge University Press, New York, 2005.

[6] N. Gatzert and H. Schmeiser. New Life Insurance Financial Products. In Dionne Georges, editor, *Handbook of Insurance, second edition*, pages 1061–1095. Springer, New York, second edition edition, 2013.

[7] O. Goecke. Pension saving schemes with return smoothing mechanism. *Insurance: Mathematics and Economics*, 53(3):678–689, 2013.

[8] M. Guillén, P.L. Jørgensen, and J.P. Nielsen. Return smoothing mechanisms in life and pension insurance: Path-dependent contingent claims. *Insurance: Mathematics and Economics*, 38(2):229–252, 2006.

[9] R.P.M.M. Hoevenaars and E.H.M. Ponds. Valuation of intergenerational transfers in funded collective pension schemes. *Insurance: Mathematics and Economics*, 42:578–593, 2008.

[10] Die Mobiliar. Vorausschauende Sparversicherung, January 2020.

[11] André Perold and William F. Sharpe. Dynamic strategies for asset allocation. *Financial Analysts Journal*, 44(1):16–27, 1988.

[12] E.H.M. Ponds and B. Van Riel. Sharing risk: the Netherlands' new approach to pensions. *Journal of Pension Economics & Finance*, 8(1):91–105, 2009.