Technical note on the determination of strength reduction factors using elastic analysis

Mike Bather

University of Liverpool

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Introduction

In structural engineering literature, the correlation between, knot ratios and MoR, has been interpreted intuitively as a causal relationship, with knots defined as '*strength reducing defects*' (McKenzie and Zhang, 2007, p. 7) and the key influence of their presence being ''*their effective reduction of the modulus of the section*' (Ozelton and Baird, 1976, p. 29).

The assumption of causal correlation between knots and MoR has led to some visual grading codes to treat knots sizes as being directly related to MoR. For instance, the American code of practice ASTM D245 treats face knots as voids and bases adjusted strength reduction factors directly on reductions of elastic section moduli due to the presence of knots/voids (ASTM, 2019). Strength ratios associated with knots in bending members have been derived as the ratio of moment-carrying capacity of a member with cross section reduced by its largest knot to the moment-carrying capacity of the member free of knots. This gives the anticipated reduction in bending strength due to the knot. For simplicity, D245 treats all knots on the wide face as being either knots along the edge of the piece (edge knots) or knots along the centreline of the piece ('centerline knots').

This approach is at least partially adopted by structural engineers assessing in situ timber (Yeomans, 2019) and can be compounded by further misunderstandings of the methodology of visual grading and strength classification (Yeomans, 2003).

The purpose of this technical note is to document the rationale behind a brief investigation into the idea of knots acting as voids within timber joists and to report how effective this approach is at estimating the bending strength of timber joists.

How the knots are modelled as voids

In brief, the notion of knots acting as voids is investigated by calculating strength reduction factors. This analysis is based on an approximate model of rectangular timber joists containing voids in place of knots. A spreadsheet was developed using Excel which attempts to model the complexity of the wide range of knot sizes, orientations, types and locations in rectangular timber joists orientated with the wide face vertical and subjected to normal vertical loading. The stages of the analysis are given in outline:

1. Notionally split each joist into four equal quadrants (top left, top right, bottom left and bottom right)
2. Calculate the elastic section modulus for the whole section (z_{full}), and two left quadrants combined and the two right quadrants combined
3. Ignore knots in the two top quadrants (as all knots in the upper half of the joist are assumed to act in compression with no deduction in cross sectional area)
4. Model the remaining knots as rectangular vertical or horizontal voids within each lower quadrant in turn (see Figure 1)
5. Calculate the position of the new neutral axis of the reduced cross section for the left and then the right hand sides of the joist in turn
6. Calculate the reduced second moment of area (I) of each side of the joist

7. Calculate the reduced section modulus of the joist (z_{red}) of each side of the joist
8. Calculate the bending strength capacity of the reduced section (SR) as a percentage or decimal by combining the results from the left and right sides of the joist and comparing with z_{full}

The equations used to calculate the elastic section modulus and elastic bending stresses within rectangular beams are given below (for a beam of vertical height h and of horizontal width b in vertical bending and subject to a vertical bending moment M):

$$Second\ moment\ of\ area\ (I) = \frac{width\ (b) \times height\ (h)^3}{12}$$

$$I = \frac{b \times h^3}{12} \tag{1}$$

$$Bending\ stress\ (\sigma)\ at\ any\ point\ in\ the\ height\ of\ a\ joist = \frac{bending\ moment\ (M) \times y}{second\ moment\ of\ intertia\ (I)}$$

$$\sigma = \frac{My}{I} \tag{2}$$

$$y = distance\ from\ the\ neutral\ axis\ to\ the\ point\ of\ interest$$

At the neutral axis, $y = 0$ and so bending stresses here are also zero. For a symmetrical section such as a rectangular timber beam, considering the maximum stresses at the outermost fibres of the beam (in either tension at the bottom or compression at the top):

$$y = \frac{h}{2}$$

So, to calculate the maximum stresses in the top and bottom of a beam, y can be substituted into the equation for bending stress and making use of the concept of an elastic section modulus:

$$Elastic\ section\ modulus\ (z) = \frac{I}{y} = \frac{b \times h^3}{12} \times \frac{1}{y} = \frac{b \times h^3}{12} \times \frac{2}{h}$$

$$z = \frac{b \times h^2}{6} \tag{3}$$

The original equation for bending stress can be re-written in a slightly simpler format, often used by structural engineers:

$$Bending\ stress\ (\sigma) = \frac{bending\ moment\ (M)}{elastic\ section\ modulus\ (z)}$$

$$\sigma = \frac{M}{z} = \frac{M \times 6}{b \times h^2}$$

The strength reduction ratio (SR) is the ratio of the reduced bending strength to the original bending strength, expressed as a percentage.

$$(SR) = \frac{strength\ capacity\ of\ section\ reduced\ by\ knots}{strength\ capacity\ of\ whole\ section} \times 100 \qquad (4)$$

The bending strength capacity of a section is directly related to the maximum bending stresses that develop within the section in bending. The ratio between the reduced elastic section modulus and the full elastic section modulus are exactly the same as the above strength ratio.

$$SR = \frac{z_{red}}{z_{full}} \times 100 \qquad (5)$$

From these equations, it is seen that the bending stress distribution within a rectangular beam is linear, varying from maximum compression stresses at the top (under normal vertical bending) and maximum tension stresses at the bottom. Bending stresses close to the neutral axis (i.e. the centre line of the beam) are small, reducing to zero at the position of the neutral axis. This approach therefore provides a nuanced method of accounting for knots differently according to their location in the joist.

Several Excel spreadsheets were written to carry out the necessary calculations for the strength ratios (SRs). A surprising degree of complexity is required to create the spreadsheets due to the many ways in which knots affect the cross section of a joist in relation to the knots' surface dimensions.

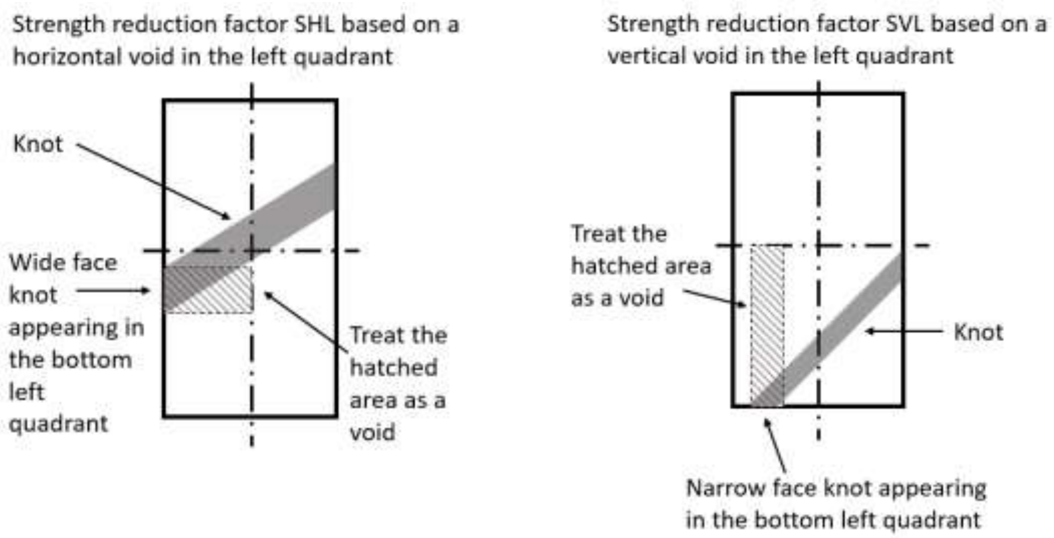

Figure 1. Modelling of knots as vertical or horizontal voids in the bottom left quadrant

The surface dimensions of each of the several different knot types defined by the visual grading code INSTA142 (Dansk Standard, 2009) were found to relate differently to the cross sectional areas of knots within joists, on occasion leading to similarly sized knots in similar positions having quite different strength reduction factors. For instance, referring to Figure 2, compare the inner arris knot with the outer arris knot at the bottom left hand corner of the joist. The variety of different knot conditions accounted for in the spreadsheets includes narrow horizontal edge knots, wide vertical face knots, inner arris knots and outer arris knots, wide face through knots and full width narrow face knots. Additionally, the presence or absence of the pith complicates matters. In short, strength reductions are based on the left and right elastic section moduli calculated separately for knots present in each of the two lower quadrants of the rectangular cross section through the joist. The two strength reductions are combined to provide an estimate for the joist as a whole. Knots (and voids) within a quadrant are modelled as either purely horizontal or vertical, and based on these notional voids, strength reduction factors are calculated for the horizontal case (SHL) and the vertical case (SVL), see Figure 1.

The strength reduction factors SHL and SVL are combined according to the configuration of the knot. SHL alone is used for: (i) inner arris knots, (ii) full width narrow face knots and (iii) wide face through knots. SVL alone is used for: narrow horizontal edge knots (with no knot presence in the wide vertical face of the same quadrant and with no inner arris knot in the adjacent quadrant). The worst case of SVL or SHL is adopted for: (i) outer arris knots (i.e. pith present) and (ii) knots present in both the wide face and the narrow face of the same quadrant. Where there is an inner arris knot present in the adjacent quadrant and there is not a full width narrow face knot, then the narrow face knot is ignored. This is best understood by reference to Figure 2.

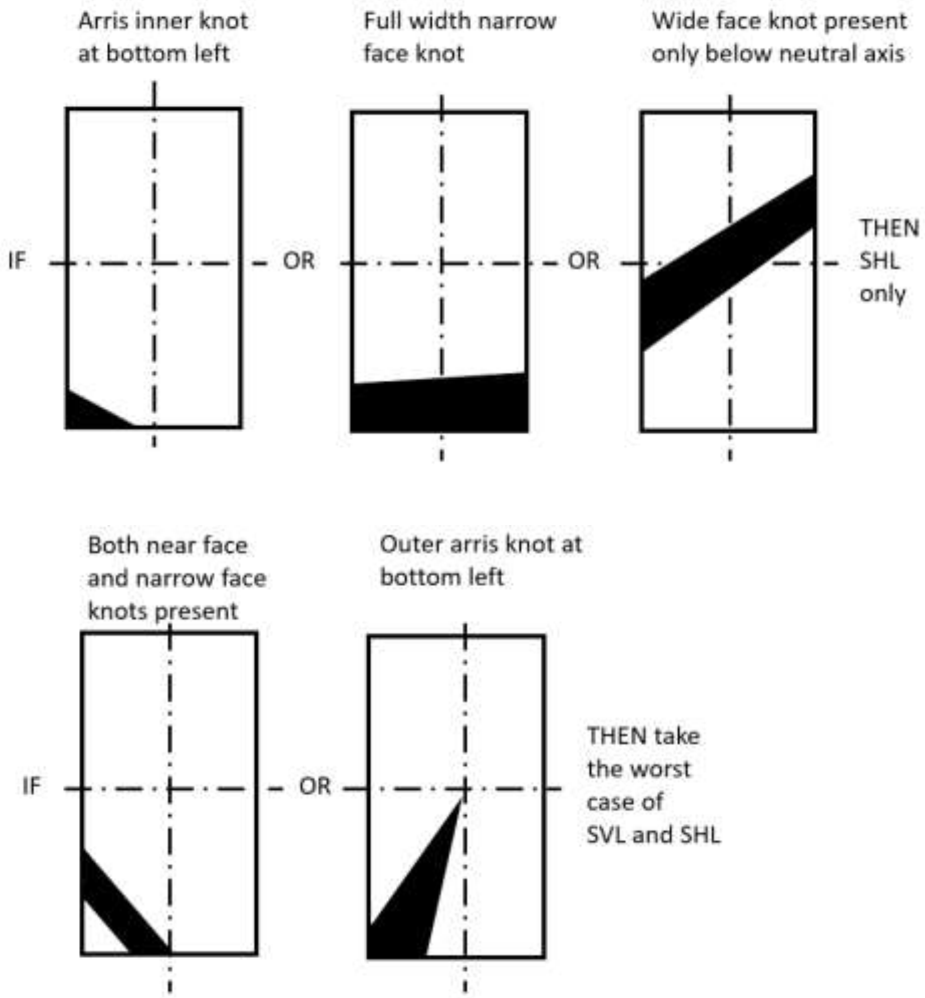

Figure 2. Illustration of just some of the rules for combining SVL and SHL strength reduction factors

While this approach is an improvement on ASTM D245, it is still clear to see that it is an approximate approach that could be refined further to increase its theoretical accuracy. However, even in its approximate form, this work is sufficient to determine if the assumption that knots do not transmit tension and so act as voids is reasonable. The results of the analysis presented here can be applied to data sets of timber joists that have had their knots measured and have been testing to destruction in a laboratory. This comparative work is reported on in the thesis “The assessment of the mechanical and physical properties of in situ timber” by Mike Bather (2021).

Visualisation of the effect of knots (acting as voids)

It should be remembered that knots close to the centre line of a joist have a smaller effect on its strength than knots close to the lower edge. Table 1 and the graph shown in Figure 3 show how strength reduction varies according to knot size and position. The blank cells in the table represent combinations of knot size and position that are physically not possible to exist.

Table 1. Strength reduction factors for wide face through knots varying in vertical position within joists

Knot size as %	Knot size (mm)	Strength reduction factors for 50mm thick x 100mm high timber joist								
		Knot centre line position - distance below centre line of timber joist (mm)								
		0	5	10	15	20	25	30	35	40
5	5	100	99	98	97	95	94	91	89	86
10	10	99.9	98	96	94	91	87	82	78	72
15	15	99.7	97	94	90	85	79	73	66	58
20	20	99.2	96	92	86	79	72	63	53	43
25	25	98.4	94	89	81	73	63	52	40	
30	30	97.3	92	85	76	66	54	41	26	
35	35	95.7	89	81	70	57	44	28		
40	40	93.6	86	76	63	49	33	15		
45	45	90.9	82	70	55	39	21			
50	50	87.5	77	63	47	28	8			

Both the graph (see Figure 3) and the table show the accelerating reduction in the bending strength of a joist as the knot size increases and its distance from the centre of the joist increases. The strength reduction is greatest where both of these factors combine.

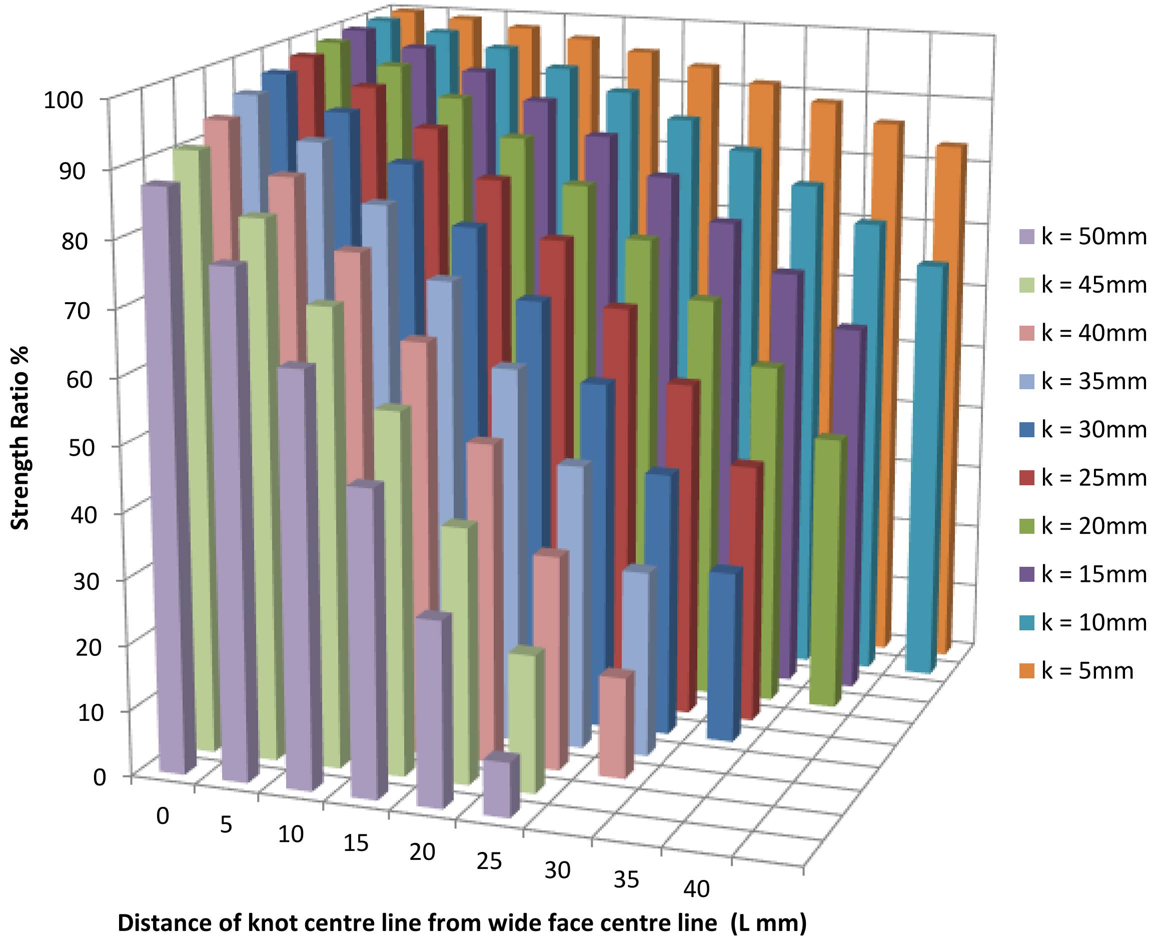

Figure 3. Strength ratios (as percentages) affected by position of knot below centre line of 50mm x 100mm joist (L mm) and size of knot (k mm)

Incidentally, based on horizontal through knots of K diameter and with bottom edge located distance h2 from the bottom edge of the joist, then the reduction in the elastic modulus of the section (SR) is governed by a single mathematical equation.

SR := 600*(1/12*(1-h2-K)^3+(1-h2-K)*(1/2+1/2*h2+1/2*K-((1-h2-K)*(1/2+1/2*h2+1/2*K)+1/2*h2^2)/(1-K))^2+1/12*h2^3+h2*(((1-h2-K)*(1/2+1/2*h2+1/2*K)+1/2*h2^2)/(1-K)-1/2*h2)^2)*(1-K)/((1-h2-K)*(1/2+1/2*h2+1/2*K)+1/2*h2^2)

This equation is unattractive to structural engineers using hand calculations but relatively straightforward for use in Excel. The equation gives a 3 dimensional surface of results for SR as it varies with K and h2, refer to Figure 4.

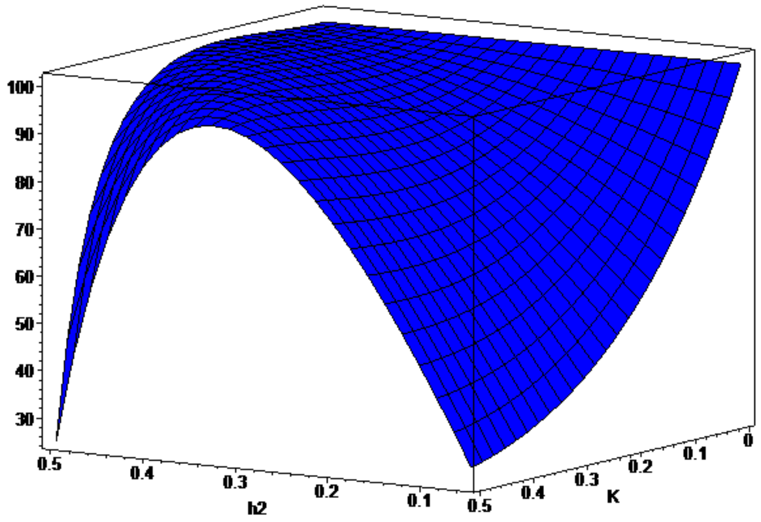

Figure 4. Chart of strength reduction factors for wide face through knots varying in vertical position within joists

Conclusions

This technical note explains the approaches taken in creating a model for the effect of knots on the bending strength of timber joists.

The derivation of formulae and spreadsheets to allow the effects of knots acting as voids within timber joists allows the effect to be investigated. This is done so and reported on in the thesis “The assessment of the mechanical and physical properties of in situ timber” by Mike Bather (2021).

The complexity of the calculations and the interaction of both knot position and knot size do not accord well with the simplified approach taken by the US visual grading code ASTM D245.

References

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