Distributionally Robust Energy Consumption Scheduling of HVAC Considering the Uncertainty of Outdoor Temperature and Human Activities

Yingjie Wang, Qi Zeng, Zhongbei Tian, Yuefang Du, Pingliang Zeng, *Senior Member, IEEE* and Lin Jiang, *Member, IEEE*

***Abstract*—Achieving a low or zero carbon target is to reduce the energy demand and improve the energy efficiency of electricity consumers. One of the main electricity consumers in power systems is heating, ventilation, and air conditioning systems (HVACs), which cost around 30% of the total usage in commercial buildings. This paper investigates the scheduling problem of HVAC energy consumption taking into account two uncertainties: the outdoor temperature and human activities. The distributionally robust optimisation approach (DROA) is extended to deal with these two uncertainties which are modelled by the proposed disjoint layered ambiguity sets according to the historical data. Based on the proposed DROA method, the distributionally robust chance constraints (DRCCs) will be formulated as a nonlinear optimisation problem, converted into a linear optimisation problem using duality theorem and solved using SeDuMi solver. The simulation results are used to compare with the existing methods, which shows that the proposed DROA can decrease 2.81% and 0.14% of the electricity cost in comparison with the traditional RO method and the DROA based on a nest layered ambiguity set, respectively. Also, the proposed DROA decreases the number and maximum of violations from the comfort level of users. The multi-zone HVAC system model is used in the case study to verify the proposed DROA with the disjoint ambiguity set. The consecutive simulation results illustrate that the proposed DROA approach can provide a stable performance in a three-day scheduling period.**

***Index Terms*—Distributionally robust optimisation, energy consumption scheduling, HVAC, demand response.**

NOMENCLATURE

$\theta_t^{\mathbf{ref}}$	The indoor temperature at time t.
$T_{t-1}^{\mathbf{out}}$	The actual outdoor temperature at time slot t-1.
μ_t	The forecast outdoor temperature at time slot t.
ϕ_t	The forecast of the number of people indoors.
N_t	The number of people indoors.
q^{H}	A stable heat source.
η_1	The heat transfer coefficient of the human body.
p_t^{ij}	The probability of $\omega_t \in B_t^{ij}$.
B_t^{ij}	The two-dimensional interval that encompasses the outdoor temperature and the number of humans indoors.
B_t	The maximum interval.
$\mathbb{P}_t^{ij}$	The probability distributions of T_t^{out} and N_t.
$\mathcal{P}_t^0(B_t^{ij})$	The set of all the probability distributions included in B_t.
$\mathcal{P}_t^1$	The proposed disjoint layered ambiguity set that is built using the probabilities of the disjoint uncertainty intervals.
m	The number of the outdoor temperature intervals in the ambiguity set.
θ_t	The indoor temperature at time t.
$\theta^{\mathbf{max}}$	The upper bound of the comfort indoor temperature zone.
$\theta^{\mathbf{min}}$	The lower bound of the comfort indoor temperature zone.
T	The scheduling horizon.
Δt	The time period.
q_{t-1}	The energy consumption of HVAC at time period $t-1$.
$q_t^{\mathbf{ref}}$	The reference power of HVAC.
$\mathrm{q}^{\mathbf{max}}$	The maximum energy consumption of HVAC.
C, R, η	The coefficients of the HVAC system.
$\mathrm{e_t}$	The electricity price at time period t.
ϵ	The violation probability of HVAC power consumption.
β, $\boldsymbol{y}$, λ_i	Auxiliary variables.
l_t^i, u_t^i	The lower and upper bounds for 'T_t^{out}'.
k_t^j, h_t^j	The lower and upper bounds for 'N_t'.
β, $\boldsymbol{y}$	Auxiliary variables.
λ_{ij}, $\boldsymbol{y}$	Dual variables.

Y. Wang, Q. Zeng, Z. Tian and L. Jiang are with the Department of Electrical Engineering and Electronics, The University of Liverpool, Liverpool L69 3GJ, U.K. (Corresponding author: Zhongbei Tian, Zhongbei.Tian@liverpool.ac.uk).

Y. Du is with the School of Mechanical and Electrical Engineer- ing, University of Electronic Science and Technology of China, Chengdu, 611731, China. (e-mail: yuefangdu@hotmail.com).

P. Zeng is with the School of Automation, Hangzhou Dianzi University, Hangzhou, 310018, China (e-mail: plzeng@hotmail.com).

I. Introduction

Nowadays, energy has already become an essential part of our daily lives. It has been predicted that energy consumption across the world will rise by approximately 35% in the next decade due to increasing consumer demands [1]. The increasing energy demand is often accompanied by energy conservation concerns and it has become a challenge to zero carbon emissions target [2], [3]. According to [4]–[6], commercial sectors is considered that around 80% of people's time is spent indoors and HVAC systems occupy more than 20% of the total energy consumption in the USA. In the Southeastern cities of China during summer time, there are 30% to 40% of

total load demand consumed by the residential air conditioning load [7]. For achieving the zero carbon target, it is necessary to reduce energy demand and to improve energy management in commercial, industrial and residential buildings by scheduling the energyst Energy Consumption Scheduling of HVAC based on consumption of HVAC intelligently [8].

The building energy use is influenced by two factors: building system and occupant behaviour. Researchers have obtained numerous achievements in the area of building system, such as [1], which proposed the building energy consumption optimisation based on the factors of the physical design, e.g., building features and the effect of outdoor temperature instead of the interaction effect from buildings to occupants. On the other hand, there is only tiny minority of the researchers studying occupant behaviour [9], [10]. [11] provides the American Society of Heating, Refrigeration, and Air-Conditioning Engineers (ASHRAE) Standard 90.1, which only concentrates on building systems such as HVAC, building envelopes system and lighting system, and it does not focus on the occupant behaviour. In [9], the author point out that the Net Zero Energy (NZE) buildings can not achieve the objective because the occupants are not willing to follow the original architectural concept of the NZE buildings. In building performance simulation, some representative profiles of the deterministic occupant status are utilised, which leads to incorrect simulation results of the total energy use in buildings [12]. A fixed operation schedule according to some customer surveys or certain standards such as ASHRAE is utilised by some conventional building systems [13] and results in energy waste and the uncomfortableness of occupants [14].

Therefore, the detecting and modelling of occupant behaviours are necessary for the energy consumption of buildings by scheduling the energy consumption of HVAC [15]. The accurate estimation of occupant behaviours ensures the consistency between the simulation results of the user's energy consumption and the user's actual energy consumption [16]. Also, the accurate estimation of occupant behaviours can result in significant savings in the HVAC energy consumption schedule [1]. Besides, the information collection of occupant behaviours can improve building performance and services to all occupants [12]. In order to address the inaccurate estimation of occupant behaviours and low energy efficiency of the HVAC system, this paper investigates the scheduling problem of HVAC energy consumption taking into account two uncertainties: the outdoor temperature and human activities (occupant behaviour). According to existing research, the human comfort level is highly affected by the error of the outdoor temperature forecast [5] which is one of the main concerns in the HVAC energy consumption scheduling [17].

To tackle the uncertainty of the forecast error to the scheduling, many papers focus on the optimisation-based method [18] - [19]. For example, a neural network (NN) based approach and the stochastic optimisation approach (SOA) to calculate the consumed HVAC energy are proposed in [18], [20], respectively. However, these methods require numerous data of the historical temperature or a specific probability distribution of the outdoor temperature, which are not practical in real implementation. Furthermore, the decision is vulnerable to risk if the consumed energy is scheduled based on the temperature with a pre-defined distribution. On the other hand, a robust optimisation approach (ROA) is proposed in [19] to deal with the forecast error by considering the worst situation in the scheduling. ROA does not require the exact probability distribution of the temperature, however, this method is too conservative to calculate the solution of the scheduling because the ROA sacrifice its performance for enhancing the robustness to the maximum forecast error.

Different from SOA and ROA, the distributionally robust optimisation approach (DROA) utilises historical data to build an ambiguity set that presents the probability information of the outdoor temperature such that the uncertain variables do not need to be assumed while reducing the conservativeness by incorporating observed probabilistic information [21]. Existing research [5] about the DROA was proposed based on a nest layered ambiguity set which divides the probability distribution into multiple subintervals. This method is proved to be able to deal with the uncertainty of the outdoor temperature. However, the nest layered ambiguity set is interlaced together and the repeated used subintervals assume that the distribution of the uncertain temperature is symmetrical [22].

In this paper, the DROA based on the disjoint layered ambiguity set is proposed. Furthermore, the ambiguity set is extended into two dimensions: outdoor temperature and human activity. The proposed DROA divides the distribution into several independent disjoint subintervals and the probability information with respect to each subinterval has a low correlation to others. This advantage further reduce the conservativeness of the optimisation of the HVAC energy consumption scheduling. The nonlinear optimisation problem is expressed as a nonlinear problem with DRCCs and is reformulated into a linear problem via duality theorem and solved by the SeDuMi solver provided by the Yalmip toolbox. In comparison, the DROA based on a nest layered ambiguity set [5] and the ROA are also implemented, whose electricity cost, and the number and maximum of violations from a user's thermal comfort zone are used to compare with the proposed approach. In addition, the multi-zone HVAC system model and consecutive simulation are also implemented and considered in the case studies to verify the effectiveness and the advantage of the proposed approach in complex models and consecutive scenarios.

The main contributions of this paper are shown below:

- The HVAC energy consumption scheduling problem and the corresponding solution methodology have been extended and consider the aforementioned two uncertainties.
- The proposed DROA is developed to deal with these two uncertainties, which are modelled by the proposed disjoint layered ambiguity sets. The historical data of outdoor temperature and the number of humans in a conference room are utilised in this proposed method.
- The HVAC energy consumption scheduling problem is formulated and solved as a linear programming problem after reformulating DRCCs.

The remaining part of the paper is organized as follows. The HVAC energy consumption scheduling is formulated as

an optimisation problem with DRCC considering two uncertainties is introduced in Section II and Section III presents the solution methodology to this nonlinear optimisation problem. The simulation results are given in Section IV to verify the effectiveness of the proposed DROA under two uncertainties and the conclusions are given in V.

II. Problem Formulation considering double uncertainties

In this section, the HVAC energy consumption scheduling considering two uncertainties, i.e., forecast errors of the temperature and human activities is formulated as a nonlinear optimisation problem. Then, the proposed disjoint layered ambiguity sets are built based on the uncertainties. Finally, the HVAC's energy consumption optimisation model is demonstrated.

A. HVAC system model considering temperature and human activity

1) HVAC system model: The HVAC system model is given by the followed equations which has been illustrated in existing paper [5], [22]:

$$\theta_t^{\text{ref}} = \theta_{t-1} - \frac{\Delta t}{C \cdot R} \cdot (\theta_{t-1} - T_{t-1}^{\text{out}} + \eta \cdot R \cdot q_{t-1}) \tag{1}$$

where Δt is the time period; θ_t^{ref}, T_{t-1}^{out}, q_{t-1} are the indoor temperature at time t, the actual outdoor temperature and the power consumption of HVAC, respectively. The $\mathrm{C(kWh/^{\circ}F)}$, $\mathrm{R(^{\circ}F/kW)}$, and η are thermal capacity, thermal resistance and coefficient of the functioning of HVAC, respectively. HVAC system model with the constant values of $\mathrm{C(kWh/^{\circ}F)}$, η, $\mathrm{R(^{\circ}F/kW)}$ cannot represent the real thermal dynamics. However, the time varying parameters of HVAC systems is complex for some control and optimisation applications. This simplification is acceptable for the proposed methodology, and used by the recent research of the HVAC optimisation problem [3], [5], [23], [24].

2) Human activity model: The human is a heat source and the effect of indoor human activity needs to be included in the HVAC's energy scheduling [25]. In this paper, the human activity (ΔT_a) is defined as the indoor temperature variation caused by humans, which is shown as follows:

$$\Delta T_a = \eta_1 \cdot R \cdot N_t \cdot q^H \tag{2}$$

where η_1 is the heat transfer coefficient of the human body, q^H is the heat generated by one human body (100 watt/hour), N_t is the number of humans.

3) HVAC considering human activity: Integrate the above two models together, a completed indoor temperature model that combines the weather forecast error, the HVAC system and human activities is built as follows:

$$\theta_t^{\text{ref}} = \theta_{t-1} - \frac{\Delta t}{C \cdot R} \cdot (\theta_{t-1} - T_{t-1}^{\text{out}} + \eta \cdot R \cdot q_{t-1} + \eta_1 \cdot R \cdot N_{t-1} \cdot q^H) \tag{3}$$

The influence of the outdoor temperature forecast error and the difference of the number of people indoors can be eliminated by DROA due to the fact that the energy consumption of HVAC is constantly adjusted according to the forecast error of the outdoor temperature and the number of people indoors.

$$q_t = q_t^{\text{ref}} + \frac{1}{\eta \cdot R} \cdot (T_t^{\text{out}} - \mu_t) - \frac{\eta_1}{\eta} \cdot q^H \cdot (N_t - \phi_t) \tag{4a}$$

$$\mathbb{P}_t\{q_t \geq 0\} \geq 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{4b}$$

$$\mathbb{P}_t\{q_t \leq q^{\max}\} \geq 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{4c}$$

where $\mathrm{q_t^{ref}}$ and $\mathrm{q^{max}}$ are the reference power and the maximum power consumption of HVAC, respectively. μ_t and ϕ_t are expressed as the forecast of the outdoor temperature and the number of humans, respectively.

In conventional approaches, the constraint of $0 \leq q_t \leq q_t^{\max}$ was used for HVAC power consumption scheduling [17]. However, [5] points out that $0 \leq q_t \leq q_t^{\max}$ is too conservative to schedule the HVAC's power consumption. This results in higher electricity cost for users. To reduce the conservativeness, distributionally robust chance constraints (DRCCs) is used in this paper following the research in [5] and [21]. The DRCCs approach introduces two extra constraints (4b) and (4c) into the model. These two constraints make the power consumption q_t possible to excess its limit with the possibility of ε, which reduces the conservativeness but also makes the original problem nonlinear. Section III will discuss the transformation of this nonlinear problem into the linear.

B. Modified optimisation model of HVAC's energy consumption scheduling

The objective of this modified HVAC energy consumption scheduling optimisation model considering double uncertainties is to minimise the electricity cost and satisfy customer's indoor temperature thermal comfort zone by scheduling the power consumption of HVAC q_t^{ref}. The parameters of the model, i.e. the electricity price and the consumer's preset comfort indoor temperature zone, is provided by [5], [22]. The HVAC's energy consumption optimisation model considering the uncertainties of outdoor temperature and human activities is formulated with the proposed disjoint layered ambiguity set as follows:

$$\min_{q_t^{\text{ref}}} \mathbb{E}\{\sum_{t=1}^{T}(e_t \cdot q_t \cdot \Delta t)\} \tag{5a}$$

$$\text{s.t.} \quad \theta_t^{\text{ref}} = \theta_{t-1} - \frac{\Delta t}{C \cdot R} \cdot (\theta_{t-1} - T_{t-1}^{\text{out}} + \eta \cdot R \cdot q_{t-1} + \eta_1 \cdot R \cdot N_{t-1} \cdot q^H) \tag{5b}$$

$$q_t = q_t^{\text{ref}} + \frac{1}{\eta \cdot R} \cdot (T_t^{\text{out}} - \mu_t) - \frac{\eta_1}{\eta} \cdot q^H \cdot (N_t - \phi_t) \tag{5c}$$

$$\theta^{\min} \leq \theta_t^{ref} \leq \theta^{\max} \tag{5d}$$

$$\mathbb{P}_t\{q_t \geq 0\} \geq 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{5e}$$

$$\mathbb{P}_t\{q_t \leq q^{\max}\} \geq 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{5f}$$

where $\mathrm{e_t}$ is denoted as the electricity price at time period t. The comfortable indoor temperature zone boundaries are limited to $\theta^{\max}$ and $\theta^{\min}$ respectively. The scheduling horizon is expressed as T.

C. Disjoint layered ambiguity set

The distributionally robust optimisation approach (DROA) is a data-driven method and it does not rely on a specific probability distribution, in contrast, it can obtain probability information obtained from historical data. By introducing the soft margin of the energy limit and incorporating observed probabilistic information, the DROA can reduce the conservativeness significantly [21]. In this subsection, the historical data of the outdoor temperature and the number of humans is used to generate a disjoint layered ambiguity set, which contains the probability information of these two variables.

The proposed DROA based on a disjoint layered ambiguity set considers the forecast error of the outdoor temperature and the forecast error of the number of people in a conference room. The proposed ambiguity set is established as follows,

$$\mathcal{P}_t^1 = \left\{ \mathbb{P}_t \in \mathcal{P}_t^0(\mathcal{B}_t) \left| \begin{array}{l} \mathbb{E}_{\mathbb{P}_t}\{\boldsymbol{\omega}_t\} = \boldsymbol{\rho}_t \\ \boldsymbol{\omega}_t = (T_t^{\text{out}}, N_t)^{\text{T}}, \boldsymbol{\rho}_t = (\mu_t, \phi_t)^{\text{T}} \\ \mathbb{P}_t\{\boldsymbol{\omega}_t \in \mathcal{B}_t^{ij}\} = p_t^{ij}, i = 1, \cdots, m, \\ j = 1, \cdots, n \\ \mathcal{B}_t = \{\boldsymbol{\omega}_t \in \mathcal{R}^2 | \\ \qquad T_t^{\text{out(min)}} \le T_t^{\text{out}} \le T_t^{\text{out(max)}} \\ \qquad N_t^{\text{min}} \le N_t \le N_t^{\text{max}}\} \\ \mathcal{B}_t^{ij} = \{\boldsymbol{\omega}_t \in \mathcal{R}^2 | \\ \qquad l_t^i \le T_t^{\text{out}} \le u_t^i \\ \qquad k_t^j \le N_t \le h_t^j\} \\ \forall i, j \le m, \mathcal{B}_t^{ij} \subseteq \mathcal{B}_t, P_t = 1 \\ \sum_{i=1}^{m}\sum_{j=1}^{n} \mathcal{B}_t^{ij} = \mathcal{B}_t, \sum_{i=1}^{m}\sum_{j=1}^{n} p_t^{ij} = P_t \end{array} \right. \right\} \tag{6}$$

where p_t^{ij} denotes the probability of $\omega_t \in \mathcal{B}_t^{ij}$. The number of outdoor temperature intervals and the number of human activities intervals in the ambiguity set are expressed as m and n respectively. $\mathcal{B}_t^{ij}$ is the two-dimensional interval which includes the outdoor temperature and the number of humans in the conference room. The probability distributions of T_t^{out} and N_t are represented by $\mathbb{P}_t^{ij}$. $\mathcal{B}_t$ is the maximum interval. The set of all the probability distributions included in $\mathcal{B}_t$ is expressed by $\mathcal{P}_t^0(\mathcal{B}_t^{ij})$. $\mathcal{P}_t^1$ is the proposed disjoint layered ambiguity set that is built using the probabilities of the disjoint outdoor temperature subintervals and the number of humans in the conference room. For $\mathcal{P}_t^1$, the lower and upper bounds for 'T_t^{out}' are shown below:

$$\begin{array}{l} l_t^i = T_t^{\text{out(min)}} + (i-1) \cdot \frac{T_t^{\text{out(max)}} - T_t^{\text{out(min)}}}{m} \\ u_t^i = T_t^{\text{out(min)}} + i \cdot \frac{T_t^{\text{out(max)}} - T_t^{\text{out(min)}}}{m}, \quad i = 1, 2, \cdots, m. \end{array} \tag{7}$$

For $\mathcal{P}_t^1$, the lower and upper bounds for 'N_t' are shown below:

$$\begin{array}{l} k_t^j = N_t^{min} + (j-1) \cdot \frac{N_t^{max} - N_t^{min}}{m} \\ h_t^j = N_t^{min} + j \cdot \frac{N_t^{max} - N_t^{min}}{m}, \quad j = 1, 2, \cdots, n. \end{array} \tag{8}$$

Fig. 1 and Fig. 2 depict the example of a disjoint layered ambiguity set with $m = n = 3$ for $\mathcal{P}_t^1$ and a nest layered ambiguity set with $m = n = 5$ for $\mathcal{P}_t^1$ respectively.

Unlike [22], the human activity model has been built first according to the number of people in a conference room, and the modified model of the HVAC system is established with the consideration of this human activity model. Based on historical data of the forecast of human activities and outdoor temperature, the proposed disjoint layered ambiguity set is built.

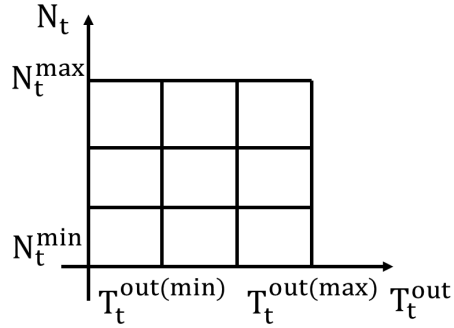

Fig. 1. Disjoint layered ambiguity set

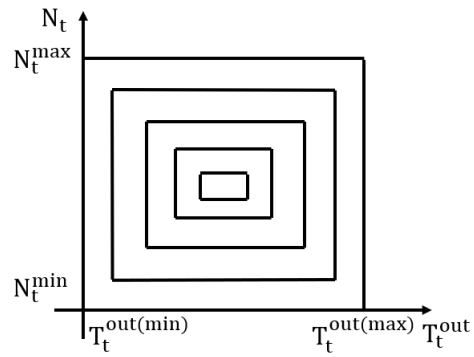

Fig. 2. Nest layered ambiguity set

III. Reformulation of DRCCs

As mentioned, to reduce the conservativeness of the optimisation problem, the DRCCs are introduced and makes the HVAC energy consumption scheduling problem a nonlinear problem. In this section, a reformulation via Theorem 1 is utilised to convert the nonlinear optimisation problem into a linear problem, i.e., constraints (5e) and (5f) are converted to (13a) - (13i) such that the HVAC scheduling problem can be solved by a linear solver.

Equations (5e) and (5f) are transformed uniformly into

$$\mathbb{P}_t\{\boldsymbol{a}_t \cdot \boldsymbol{\omega_t} \le c_t\} \ge 1 - \varepsilon, \ \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{9}$$

where $\boldsymbol{a}_t$ and c_t are calculated in Table I for (4b) and (4c). The values of $\boldsymbol{a}_t$ and c_t are different from [22] because the equation for real-time power consumption of HVAC q_t is modified in this paper based on the uncertainties of outdoor temperature and number of humans in a conference room. It has been proven that [26]

$$\mathbb{P}\text{-CVaR}_\varepsilon(L(\boldsymbol{\xi})) = \inf_{\beta \in \mathcal{R}} \left\{\beta + \tfrac{1}{\varepsilon}\mathbb{E}_{\mathbb{P}}\{(L(\boldsymbol{\xi}) - \beta)^+\}\right\} \tag{10}$$

CVaR has an appealing feature, which is the convex and conservative approximation of the DRCCs [27]. It has been proven that [26]

$$\begin{aligned}&\mathbb{P}_t \text{ - CVaR}_\varepsilon(\boldsymbol{a_t}\cdot\boldsymbol{\omega_t}-c_t)\le 0\\ &\Rightarrow \mathbb{P}_t\{\boldsymbol{a_t}\cdot\boldsymbol{\omega_t}\le c_t\}\ge 1-\varepsilon\end{aligned} \tag{11}$$

On the basis of (10), equation (9) can be met if:

$$\mathbb{P}_t\text{-CVaR}_\varepsilon(\boldsymbol{a_t}\cdot\boldsymbol{\omega_t}-c_t)\le 0,\ \forall \mathbb{P}_t\in\mathcal{P}_t^1 \tag{12}$$

where $\boldsymbol{a_t}$ and $\boldsymbol{\omega_t}$ are row and column vectors with two elements respectively. The equivalent linear constraint reformulation is admitted by equation (12) due to the convexity of CVaR [27].

TABLE I
EXPRESSIONS OF $\boldsymbol{a}_t$ AND c_t

s.t.	$\boldsymbol{a}_t$	c_t
(5e)	$[-\frac{1}{\eta\cdot R}\quad \frac{\eta_1}{\eta}\cdot q^H]$	$-q_t^{\text{ref}}+\frac{1}{\eta\cdot R}\cdot\mu_t-\frac{\eta_1}{\eta}\cdot q^H\cdot\phi_t$
(5f)	$[\frac{1}{\eta\cdot R}\quad -\frac{\eta_1}{\eta}\cdot q^H]$	$q_t^{\text{ref}}-q^{\max}-\frac{1}{\eta\cdot R}\cdot\mu_t+\frac{\eta_1}{\eta}\cdot q^H\cdot\phi_t$

Theorem 1: For the purpose of constructing the proposed disjoint layered ambiguity set $\mathcal{P}_t^1$, the equation (12) can be satisfied if there exist $\lambda_{ij}\in\mathcal{R}$, i=1, $\cdots$, m, j=1, $\cdots$, n, $\boldsymbol{y}\in\mathcal{R}^2$, $\beta\in\mathcal{R}$, such that

$$\beta+\frac{1}{\varepsilon}\cdot(\boldsymbol{\rho}_t^{\mathrm{T}}\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\cdot P_t^{ij})\le 0 \tag{13a}$$

$$[l_t^i\quad k_t^j]\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\ge 0 \tag{13b}$$

$$[l_t^i\quad h_t^j]\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\ge 0 \tag{13c}$$

$$[u_t^i\quad k_t^j]\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\ge 0 \tag{13d}$$

$$[u_t^i\quad h_t^j]\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\ge 0 \tag{13e}$$

$$[l_t^i\quad k_t^j]\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}-([l_t^i\quad k_t^j]\boldsymbol{a}_t-c_t-\beta)\ge 0 \tag{13f}$$

$$[l_t^i\quad h_t^j]\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}-([l_t^i\quad h_t^j]\boldsymbol{a}_t-c_t-\beta)\ge 0 \tag{13g}$$

$$[u_t^i\quad k_t^j]\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}-([u_t^i\quad k_t^j]\boldsymbol{a}_t-c_t-\beta)\ge 0 \tag{13h}$$

$$[u_t^i\quad h_t^j]\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}-([u_t^i\quad h_t^j]\boldsymbol{a}_t-c_t-\beta)\ge 0 \tag{13i}$$

Proof: To start with, equation (12) is equivalent to:

$$\begin{aligned}&\sup_{\mathbb{P}_t\in\mathcal{P}_t^1}\inf_{\beta\in\mathcal{R}}\left\{\beta+\frac{1}{\varepsilon}\mathbb{E}_{\mathbb{P}_t}\{(a_t\cdot\omega_t-c_t-\beta)^+\}\right\}\\ &=\inf_{\beta\in\mathcal{R}}\left\{\beta+\frac{1}{\varepsilon}\sup_{\mathbb{P}_t\in\mathcal{P}_t^1}\mathbb{E}_{\mathbb{P}_t}\{(a_t\cdot\omega_t-c_t-\beta)^+\}\right\}\ \le 0\end{aligned} \tag{14}$$

The exchange of inf and sup is supported by a stochastic saddle point theorem [28]. Equation (14) can be reformulated if the following part as the worst-case expectation is reconsidered:

$$\sup_{\mathbb{P}_t\in\mathcal{P}_t^1}\ \mathbb{E}_{\mathbb{P}_t}\{(a_t\cdot\omega_t-c_t-\beta)^+\} \tag{15}$$

The proposed disjoint ambiguity set can be built based on several possible distributions due to the unknown probability distribution of the number of people indoors and outdoor temperature. Equation (15) can be formed as an infinite dimensional linear optimisation problem, which is expressed as:

$$\min\int_{\mathcal{B}_t}-(a_t\cdot\omega_t-c_t-\beta)^+\mathbb{P}_t(d\omega_t) \tag{16a}$$

$$\text{s.t.}\int_{\mathcal{B}_t}\omega_t\mathbb{P}_t(d\omega_t)-\rho_t=0 \tag{16b}$$

$$\int_{\mathcal{B}_t}I_{\mathcal{B}_t^{ij}}\mathbb{P}_t(d\omega_t)-p_t^{ij}=0 \tag{16c}$$

Constraint (16) can be rewritten using duality theorem, which introduces dual variables λ_{ij} and $\boldsymbol{y}$.

$$\begin{aligned}\min\int_{\mathcal{B}_t}&-(a_t\cdot\omega_t-c_t-\beta)^+\mathbb{P}_t(d\omega_t)+\boldsymbol{y}\cdot(\int_{\mathcal{B}_t}\omega_t\mathbb{P}_t(d\omega_t)\\ &-\rho_t)+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\cdot(\int_{\mathcal{B}_t}I_{\mathcal{B}_t^{ij}}\mathbb{P}_t(d\omega_t)-p_t^{ij})\end{aligned} \tag{17}$$

which equals to

$$\begin{aligned}\min -y\cdot\rho_t-\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}P_t^{ij}+\int_{\mathcal{B}_t}[\boldsymbol{y}\cdot\omega_t+\sum_{i=1}^{m}\sum_{j=1}^{n}\\ \lambda_{ij}\cdot I_{Bt}^{ij}-(a_t\cdot\omega_t-c_t-\beta)^+]\mathbb{P}_t(d\omega_t)\end{aligned} \tag{18}$$

Notice that $I_{\mathcal{B}_t^{ij}}=0$ if $\omega_t\notin\mathcal{B}_t^{ij}$. Then the constraint (16) becomes:

$$\inf_{\lambda_{ij},\boldsymbol{y}}\rho_t\cdot\boldsymbol{y}+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\cdot p_t^{ij} \tag{19a}$$

$$\text{s.t. }\lambda_{ij}\in\mathcal{R},\ \boldsymbol{y}\in\mathcal{R}^2,\ i=1,\cdots,m,\ j=1,\cdots,n \tag{19b}$$

$$\begin{aligned}&\inf_{\omega_t\in\mathcal{B}_t}\left\{\boldsymbol{y}\cdot\omega_t+\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\cdot I_{\mathcal{B}_t^{ij}}-(a_t\cdot\omega_t-c_t-\beta)^+\right\}\\ &\ge 0.\end{aligned} \tag{19c}$$

The derived constraint (19) is a finite two-dimensional HVAC energy consumption scheduling optimisation problem and $\mathcal{B}_t$ is divided to $m\times n$ mutually disjoint sets $\mathcal{R}_t^{ij}=\mathcal{B}_t^{ij}$, $i=1,\cdots,m$, $j=1,\cdots,n$, (19c) becomes the following:

$$\begin{aligned}&\inf_{\omega_t\in R_t^{ij}}\left\{\boldsymbol{y}\cdot\omega_t+\textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}-(a_t\cdot\omega_t-c_t-\beta)^+\right\}\\ &\ge 0,\qquad\forall i=1,\cdots,m,\ \forall j=1,\cdots,n\end{aligned} \tag{20}$$

Furthermore, (20) converts to

$$\begin{aligned}&\inf_{\omega_t\in\mathcal{B}_t^{ij}}\left\{\boldsymbol{y}\cdot\omega_t+\textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}-(a_t\cdot\omega_t-c_t-\beta)^+\right\}\\ &\ge 0,\qquad\forall i=1,\cdots,m,\ \forall j=1,\cdots,n\end{aligned} \tag{21}$$

The equation (21) can be met if and only if $\forall i=1,\cdots,m$, $\forall j=1,\cdots,n$, the linear reformulation of equation (12) has been proved and the HVAC optimisation model can be solved by linear programming. ■

IV. Simulation results

A. Case study formulation

The HVAC energy consumption has been scheduled for 12 hours ahead with the time slot T given as 24. The scheduling cycle is between 9 am and 9 pm, and samples have been taken every half an hour (the total time slots are 24) [20]. The parameters of the HVAC energy consumption scheduling system is shown in Table II. The forecast of outdoor temperature is based on the historical temperature data taken from Austin on the 6th of August 2013 between 9 am to 9 pm [29].

Required first for the proposed method is the historical data of outdoor temperature to encode probabilistic information. To best simulate the historical data of outdoor temperature and drive the ambiguity set $\mathcal{P}_t^1$, 10000 samples have been generated from the forecast of outdoor temperatures, following the normal distribution with the standard deviation given by 2.5 [19].

TABLE II
Parameters of HVAC Energy Consumption Scheduling [5]

Parameter	Value	Units
C	0.33	kWh/°F
R	13.5	°F/kW
η	2.2	
q^{max}	3.5	kW
Δt	30	minutes
m	15	
n	15	
θ^{min}	60	°F
θ^{max}	70	°F

Required second for the proposed method is historical data of the number of humans (NH) in the conference room with different time slots. This is shown below in the Table III and is derived from the data given by the University of Liverpool [30]. To accurately simulate the historical data of the number of humans and drive the ambiguity set $\mathcal{P}_t^1$, 10000 samples have been generated from the forecast of number of humans, following normal distribution with the standard deviation given by 1. The scheduling of the HVAC reference power consumption (q_t^{ref}) considers the forecast error of the outdoor temperature and the number of humans in a conference room. The real-time HVAC power consumption can be modified according to both of these uncertainties and q_t^{ref} to maintain the indoor temperature within the human comfort zone.

The optimisation problem: the proposed DROA considering the uncertainty of outdoor temperature and the human activity in a conference room is implemented using MATLAB with YALMIP toolbox [31] and solved by the SeDuMi [32]. The model of the HVAC system is implemented together with the DRCC as the constraints of the optimisation problem. The optimisation solves the reference value of the consumed energy qt. The simulation environment is MATLAB. The model of the HVAC system is implemented as a discrete recursive model using the solved q_t as input.

B. Performance comparison of the three methods

The proposed DROA based on a disjoint layered ambiguity set is in comparison with two different methods to solve the HVAC energy consumption scheduling problem considering two uncertainties, which are weather forecast error and human activities. The latter is the fluctuating number of humans within a conference room given in different time slots. For practicality, the three methods are named M1, M2 and M3.

TABLE III
The number of humans in the conference room with different time slots [30]

Time	9:00am-9:30am	9:30am-10:00am	10:00am-10:30am
NH	57	66	75
Time	10:30am-11:00am	11:00am-11:30am	11:30am-12:00pm
NH	69	78	91
Time	12:00pm-12:30pm	12:30pm-13:00pm	13:00pm-13:30pm
NH	101	126	142
Time	13:30pm-14:00pm	14:00pm-14:30pm	14:30pm-15:00pm
NH	143	127	95
Time	15:00pm-15:30pm	15:30pm-16:00pm	16:00pm-16:30pm
NH	86	88	105
Time	16:30pm-17:00pm	17:00pm-17:30pm	17:30pm-18:00pm
NH	104	137	150
Time	18:00pm-18:30pm	18:30pm-19:00pm	19:00pm-19:30pm
NH	153	129	101
Time	19:30pm-20:00pm	20:00pm-20:30pm	20:30pm-21:00pm
NH	99	84	66

- M1: The traditional ROA method consider the worst condition and it does not allow any violation of the indoor temperature, as shown by the followed robust constraints.

$$q_t \geq 0, \forall \omega_t \in \mathcal{B}_t^{ij} \tag{22a}$$
$$q_t \leq q^{\max}, \forall \omega_t \in \mathcal{B}_t^{ij}. \tag{22b}$$

- M2: The DROA approach with a nest layered ambiguity set considers probabilistic information of two uncertainties, which is proposed in [5].
- M3: The proposed DROA approach based on a disjoint layered ambiguity set.

An example will be given to clearly illustrate the calculation of the ambiguity set. For simplification, the ambiguity set for the first time slot (t=1) is calculated. An assumption has been made as only 10 sets of historical data for t=1 and 3 layers for each dimension have been utilised (In fact, 10000 sets of historical data will be utilised in the following case studies). The outdoor temperature is T= [98.04, 100.11, 96.38, 98.43, 95.09, 94.32, 102.85, 98.39, 95.16, 101.37], the human number N_t is [57, 66, 75, 69, 78, 91, 101, 126, 142, 143]. Firstly, we will divide the data into 3 groups according to equations (7) and (8). Then the outdoor temperature becomes T= [[102.85, 101.37], [100.11, 98.43, 98.39, 98.04], [96.38, 95.16, 95.09, 94.32]] and the human number become N_t = [[143, 142, 126], [101, 91], [78, 69, 75, 66, 57]]. Secondly, the probability of the T and N_t belonging to each group are [0.2, 0.4, 0.4] and [0.3, 0.2, 0.5], respectively. Finally, the probability information in the ambiguity set is shown below:

$$\begin{bmatrix} 0.06 & 0.04 & 0.1 \\ 0.12 & 0.08 & 0.2 \\ 0.2 & 0.08 & 0.2 \end{bmatrix} \tag{23}$$

The meaning of p_{ij} is the probability of a situation whose outdoor temperature lies on the ith group and human activity lies on the jth group.

M1 and M2 are in comparison with M3 considering the aforementioned two uncertainties. Fig. 3 illustrate the power consumption of HVAC and indoor temperature for method M3. For the purpose of keeping the indoor temperature at a user's thermal comfort level according to Fig. 3, HVAC schedules its power consumption at a high level with low electricity prices from 1 to 5 time slots, and the indoor temperature starts to decline from 1 to 5 time slots as well. As the electricity price is high between 5 and 16 time slots, the power consumption of HVAC remains low to reduce the electricity costs of the user. From time slot 11 to time slot 24, the indoor temperature remains the same at $70°\mathrm{F}$ because the users do not need to spend more money to decrease indoor temperature. In order to obtain the lowest electricity cost, HVAC will remain lowest power level for satisfying the upper limit of the indoor temperature and the optimal solution is to keep the indoor temperature at $70°\mathrm{F}$.

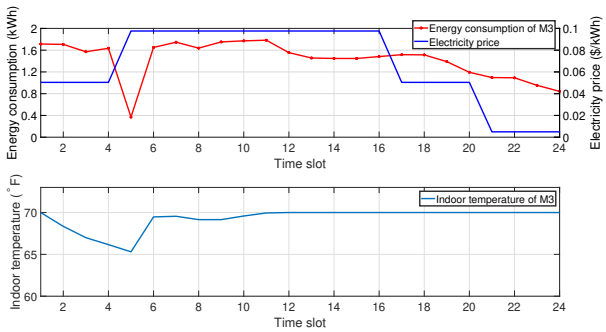

Fig. 3. Power consumption and indoor temperature for M3

Fig. 4 demonstrates the changes of outdoor temperature and the number of humans in a conference room (human activities). In the upper figure, the forecast temperature is obtained from historical temperature data. A sample of the outdoor temperature (test temperature) is obtained according to the mean value, which is the normal distribution of the forecast outdoor temperature and the standard deviation of 2.5.

In the lower figure, the forecast NH is expressed as the forecast of number of humans in a conference room, which is obtained from historical data. Similarly, a sample of the number of humans (test NH) is obtained based on the mean value, which is the normal distribution of the number of humans indoors and the standard deviation of 1. These two test samples, the forecast temperature, and the forecast of number of humans (forecast NH) in a conference room are utilised in the power consumption schedules of HVAC.

Fig. 5 and Fig. 6 show real-time power consumption and accumulated power consumption for M1 - M3 against the time slot. In Fig. 5 and Fig. 6, M1 has the greatest consumption of power when compared to the other two methods because the highest power consumption of M1 reaches at 2.4 kWh at time slots 2 and 3 in Fig. 5 and the accumulated power consumption of M1 is higher than the other two methods at almost every time slot in Fig. 6. Both of the power consumption of M2 and M3 fluctuates from time slot 1 to 24 but the accumulated power consumption of M3 is lower in comparison to M2. The accumulated power consumption for M2 is 34.32 kWh and the proposed M3 is 33.92 kWh at time slot 24 in Fig. 6.

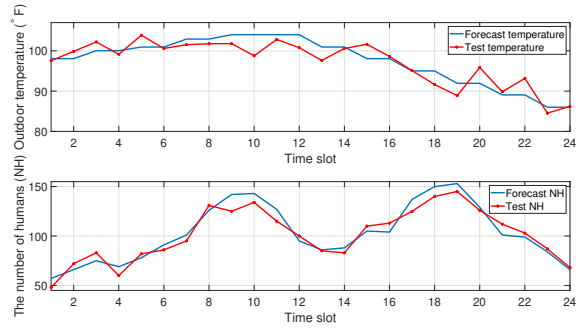

Fig. 4. Outdoor temperature and the number of humans

Fig. 7 demonstrates the accumulated electricity cost for M1 - M3. It is apparent from the figure that the accumulated electricity cost for M1 is higher than M2 and M3 at almost every time slot and M1 pays the highest electricity cost in a scheduling cycle. In comparison to M2, the accumulated electricity cost for the proposed M3 is lower at the majority of the time slots.

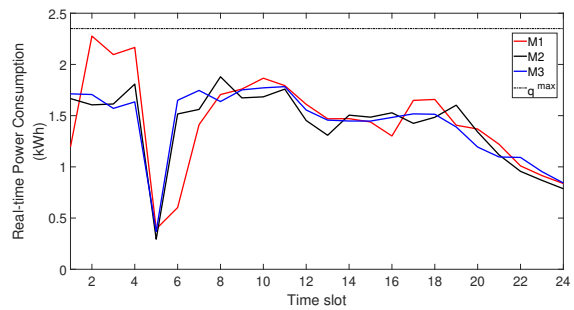

Fig. 5. The real-time power consumption for M1, M2, and M3

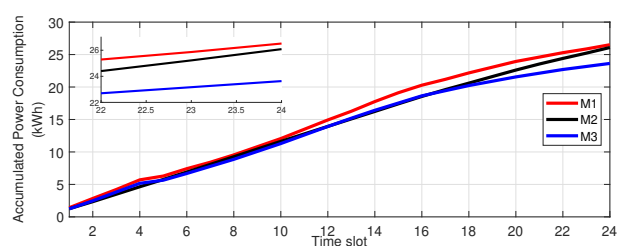

Fig. 6. Accumulated power consumption for M1, M2, and M3

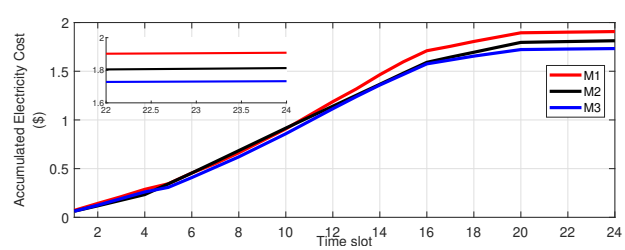

Fig. 7. Accumulated electricity cost for M1, M2, and M3

Fig. 8 depicts the comparison between reference and real-time power consumption for M1 - M3 against the time slot. The real-time power consumption for each time slot does change according to real-time outdoor temperature and the number of humans in a conference room. According to the

equation (4a), the real-time power consumption of HVAC for M1 - M3 can be calculated. As can be seen from this figure, the real-time power consumption is limited in the range of 0 to $q^{\max}$ (which is shown in dash black line) and it is different to reference power consumption q^{ref} under three methods.

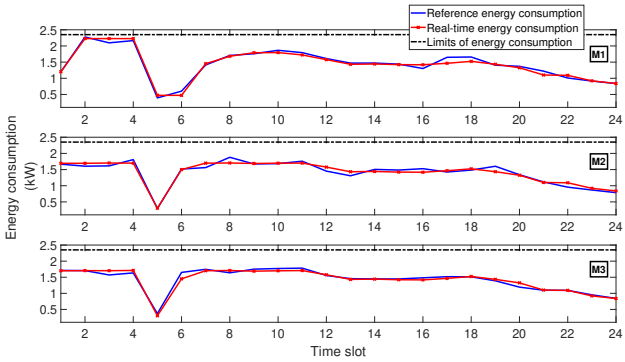

Fig. 8. The comparison between reference and real-time power consumption

Fig. 9 shows the comparison between reference and actual indoor temperatures for M1 - M3. The methods provide a reference temperature that equates exactly to the actual indoor temperature and all stay in the comfort temperature zone outlined in the figure. At the scheduling stage, reference indoor temperature can be calculated based on the historical data and reference power consumption q^{ref} of HVAC. At the verification stage, weather forecast error can be adjusted and eliminated based on the equation (4a) and real-time power consumption of HVAC q^{real} is calculated. Therefore, the actual indoor temperature returns to the reference indoor temperature.

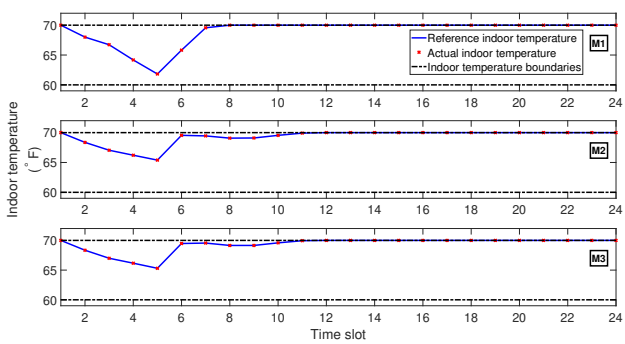

Fig. 9. Reference and actual indoor temperatures

The electricity cost of users, the number and the maximum of violations from the thermal comfort level of users (Num_VioT and Max_VT (°F)), and the total cost reduction (CR) are given in Table IV, which illustrates the performances of three methods in a scheduling cycle. As shown in the table, M3 has better performance where the electricity cost are reduced by 2.81% and 0.14% compared with M1 and M2, respectively. Furthermore, M3 have less violations compared to M2.

C. Sensitivity analysis of M3

It was decided to test the proposed algorithm's performance considering the uncertainties of outdoor temperature and human activities under different parameters. The following case studies have been carried out: performances of the proposed method M3 when violation probability of HVAC power consumption (ϵ), the number of layers in the ambiguity set (m) and human comfort indoor temperature zone are different. During the scheduling cycle between 9 am and 9 pm on the 6th of August 2013, the electricity cost, the number and the maximum of violations from the thermal comfort level of users (Num_VT and Max_VT), the maximum of violations of real-time power consumption at the lower boundary (M_VEL and M_VEH), and the total time used for programming, are shown within Tables V, VI, and VII respectively.

TABLE IV
PERFORMANCES OF THE THREE METHODS IN A SCHEDULING CYCLE

Method	Cost ($)	Num_VioT	Max_VT (°F)	CR (%)
M1	1.2174	0	0	0
M2	1.1849	18	1.1882	2.67
M3	1.1832	11	0.4856	2.81

Performance of the proposed method M3 when ϵ is different: Table V outlines the performances of M3 when ϵ is different, in which it is tested with both 10,000 samples of the number of humans in a conference room, and the outdoor temperature. The initial conditions are the human comfort indoor temperature zone within [60°F, 70°F], ϵ = 0.005, and m = n = 15. Table V illustrates that when ϵ is increasing from 0.005 to 0.085, the number of violations from the thermal comfort level of users is greatly increased while the cost of electricity reduces. The reason for this is that the probabilities of DRCCs of the power consumption constraints (4b) and (4c) should be bigger than 1- ϵ, which means more violations from the boundaries are allowed when ϵ is ascending.

TABLE V
PERFORMANCE OF THE PROPOSED METHOD BASED ON DIFFERENT ϵ

ϵ	Cost($)	Num_VT	Max_VT(°F)	M_VEL	T(s)
0.005	1.1832	11	0.4856	0.0782	3.6287
0.025	1.1789	90	0.5413	0.1051	3.4257
0.045	1.1772	148	0.5794	0.1659	3.3526
0.065	1.1769	297	0.6079	0.2146	3.2571
0.085	1.1765	319	0.7641	0.2321	3.1664

Performance of the proposed method M3 when human comfort indoor temperature zones are changed: Table VI outlines the performances of M3 when human comfort indoor temperature zones are changed, in which it is tested with both 10,000 samples of the number of humans in a conference room and the outdoor temperature. The initial conditions are as previously defined. Table VI illustrates that, as the human comfort indoor temperature boundary is decreased, electricity costs start to increase. The human comfort indoor temperature zones are one of the constraints in power consumption scheduling of HVAC. Also, the boundaries of this constraint become more compact. That leads to a slight increase in the electricity cost as the optimisation result in order to satisfy all the constraints.

Performance of the proposed method M3 when m is different: Table VII outlines the performances of M3 when m is different, in which it is tested with both 10,000 samples of the number of humans in a conference room, and the outdoor temperature. The initial conditions are as previously defined.

TABLE VI
PERFORMANCE OF THE PROPOSED METHOD BASED ON DIFFERENT HUMAN COMFORT INDOOR TEMPERATURE ZONES (TZS)

TZ(°F)	Cost($)	Num_VT	Max_VT(°F)	M_VEL	T(s)
[60-70]	1.1832	11	0.4856	0.0782	3.6287
[62-70]	1.1835	9	0.3951	0.0764	3.6522
[64-70]	1.1838	8	0.2133	0.0734	3.1525
[66-70]	1.1840	6	0.1087	0.0677	3.2108
[68-70]	1.1843	3	0.0746	0.0658	3.8337

Table VII illustrates that the indoor temperature resides largely within the comfortable zone outlined. When more probabilistic information on the number of people indoors and outdoor temperature is considered (m and n are increasing) the computation time to solve this schedule problem is increasing while the user's electricity cost is reducing.

TABLE VII
PERFORMANCE OF THE PROPOSED METHOD BASED ON DIFFERENT M

m	Cost($)	Num_VT	Max_VT(°F)	M_VEL	T(s)
15	1.1832	11	0.4856	0.0782	3.6287
35	1.1830	16	0.5547	0.0837	3.8854
55	1.1829	9	0.6369	0.0868	4.3267
75	1.1829	23	0.2887	0.0904	4.6834
95	1.1826	18	0.1458	0.0951	4.9689

D. Performances of the three methods considering multi-zone commercial building HVAC system

AHU, VAV boxes and chiller are included in the HVAC systems' typical configuration of a multi-zone commercial building. N zones' commercial HVAC systems are considered here with i = 1, ..., n (i expresses each temperature zone), a VAV box is associated with each temperature zone and its temperature evolution is described by the following simple model [33],

$$C^i \frac{dT^i(t)}{dt} = \frac{T_o(t) - T^i(t)}{R^i} + q^i(t) + q_x^i(t) \tag{24}$$

$$q^i(t) = -q_c^i(t) + q_h^i(t) \tag{25}$$

$$q_c^i(t) = c_a \dot{m}^i(t) \left(T^i(t) - T_c(t)\right) \tag{26}$$

$$q_h^i(t) = c_a \dot{m}^i(t) \left(T_s^i(t) - T_c(t)\right) \tag{27}$$

$$\dot{m} = \sum_{i=1}^{n} \dot{m}^i \tag{28}$$

where $T^i(t)$ is zone temperature, R^i and C^i are thermal resistance and thermal capacitance at different temperature zones respectively; $T_o(t)$ is the outdoor temperature; $q_c^i(t)$ is the cooling power produced by the cooling coil and $q_h^i(t)$ is the reheating power provided by the VAV box; $q_x^i(t)$ is the external disturbance from occupancy; the specific heat of air is denoted as c_a and the discharge air temperature is expressed as $T_c(t)$; $\dot{m}^i(t)$) is the supply airflow rate. [33] describes these parameters in detail. Note that the consumed power q^i in the paper refers to the energy for cooling/heating the room instead of the power for the fan. Therefore, the energy consumed for each zone hold:

$$q^{\max} \geq \sum_{i=1}^{n} q^i \tag{29}$$

The parameters of the scenario are given by Table VIII. Three zones are considered in the multi-zone HVAC system and the results of the DROA optimisation and simulation are given by the following figures.

TABLE VIII
THE PARAMETERS OF THE SCENARIO

Parameters	C (kWh/°F)	R (°F/kW)	η
Zone 1	0.33	13.5	2.2
Zone 2	0.35	13.2	2.3
Zone 3	0.35	13.2	2.3

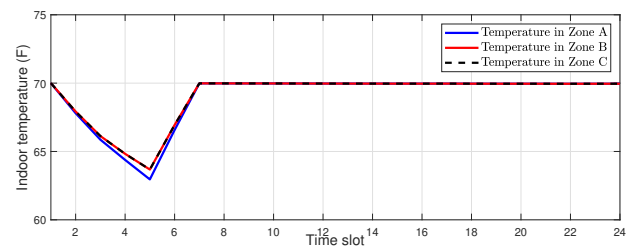

Fig. 10. The indoor temperature of each zone when the M3 is used

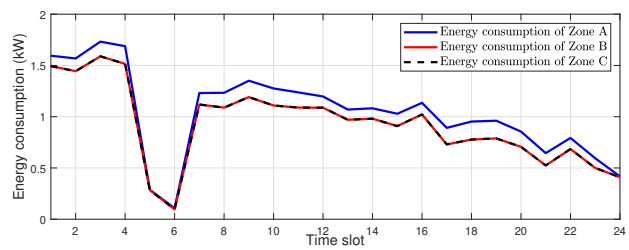

Fig. 11. Actual value of power consumption for each zone when the M3 is used

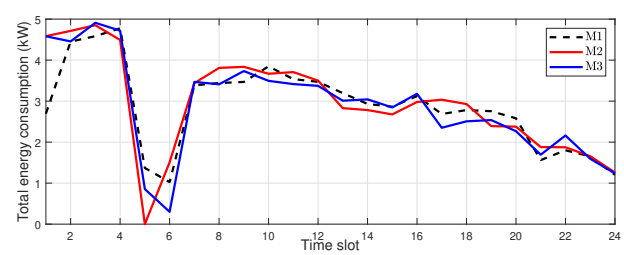

Fig. 12. Total power consumption in comparison of M1, M2 and M3

Fig.10 and Fig.11 show the indoor temperature and the actual consumed power for each zone when the M3 is used, respectively. Fig.12 and Fig.13 show the advantage of the proposed DROA based on disjoint layered ambiguity set by comparing the total power consumption and the accumulative electricity price of three methods. As shown in the Fig.13, the accumulative electricity price for M3 is about $2.38, however, those of the M1 and M2 are about $2.44. This data shows that the proposed method can reduce the electricity cost by 2.5%. This case study also shows that the proposed method is able to deal with the uncertainty in a multi-zone scenario.

E. Performances of the M3 in consecutive cycles

The proposed DROA's performance is derived from consecutive cycles between 9 am on August the 6th to 9 pm

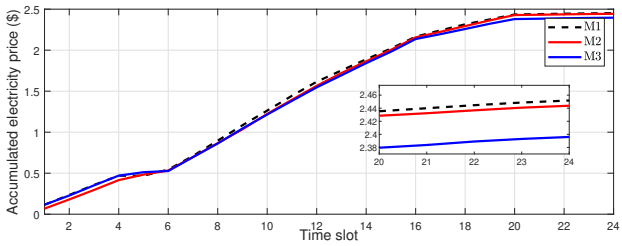

Fig. 13. Accumulative electricity price of three methods

on August the 9th 2013. There are six consecutive cycles in total and in each scheduling cycle. The scheduling period is 12 hours ($\Delta t = 0.5h$, T = 24). The initial conditions are the human comfort indoor temperature zone within [60°F, 70°F], ϵ = 0.005, and m = n = 15. The starting indoor temperature is 70 °F, and the end of day indoor temperature is taken as the starting temperature for the next day.

Fig. 14 illustrates the power consumption of HVAC and indoor temperature for method M3 in continuous cycles. For the purpose of keeping the indoor temperature at a user's thermal comfort level, HVAC starts to schedule its power consumption at relatively low prices from 1 to 4 time slots based on Fig. 14, and therefore the indoor temperature starts to decline from 1 to 5 time slots. As the electricity price is high between 5 and 16 time slots, the power consumption of HVAC remains low to reduce the electricity costs of the users.

The similar pattern has been followed in continuous schedule cycles. There are six drops in the indoor temperature. The first drop is different from the other five drops because of the difference of the electricity price. HVAC continues to schedule its power consumption at the lowest price, it leads to two significant drops in indoor temperature to minimise the electricity cost for users.

Fig. 15 depicts that the real-time power consumption for each time slot does change according to real-time outdoor temperature and the number of humans in a conference room in continuous cycles. Fig. 16 illustrates the comparison between reference and actual indoor temperatures. Method M3 provides a reference temperature that equates exactly to the actual indoor temperature and stays within the comfortable temperature zone outlined in the figure.

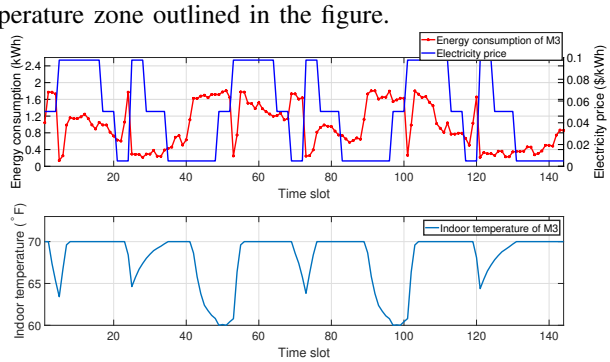

Fig. 14. Power consumption and indoor temperature for M3 in continuous cycles

Table IX outlines the three methods' performances in continuous cycles, in which it is tested with both 10,000 samples of the the number of humans in a conference room, and

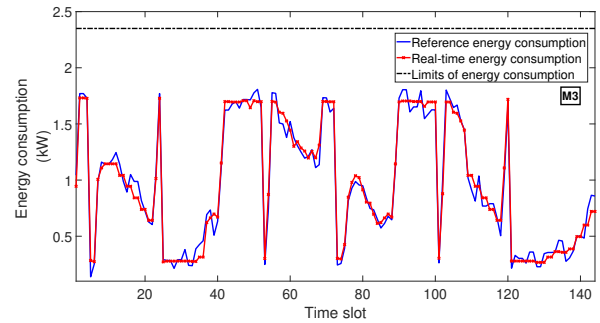

Fig. 15. The adjusted power consumption for M3 in continuous cycles

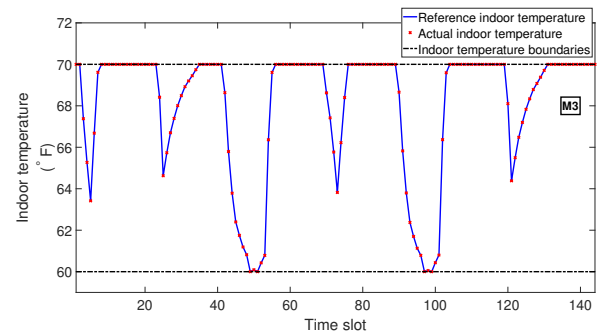

Fig. 16. Reference and actual indoor temperatures for M3 in continuous cycles

outdoor temperature. The initial conditions are the human comfort indoor temperature zone within [60°F, 70°F], ϵ = 0.005, and m = n = 15. Table IX illustrates that the proposed method M3 pays less electricity cost compared with M2 in continuous cycles. Also, Num_VioT, Max_VT, and M_VEL for M3 are smaller as well compared with M2. Although the total number of violations from human comfortable indoor temperature for M1 is zero, the highest electricity cost is paid by RO in continuous cycles.

TABLE IX
THE THREE METHODS' PERFORMANCES IN CONTINUOUS CYCLES

	Cost ($)	Num_VT	Max_VT (°F)	M_VEL	T (s)
M1	3.4724	0	0	0	2.3195
M2	3.4483	15	1.0014	0.1446	2.3664
M3	3.4465	8	0.4929	0.1331	2.3181

F. *Effects of human activity*

In this subsection, the effectiveness of the proposed method considering human activities as the fluctuation of the number of humans in a conference room will be demonstrated. The comparison will be made between the proposed method and the traditional method which considers human activities as the constant indoor temperature variations caused by human.

As can be seen from Figure 17, there are three dimensions in this figure: Ratio_Schedule, Ratio_Real Human and Perc_NotVioHum. Ratio_Schedule is referred to the applied constant human activity in a scheduling period in the traditional method [5], [22]. If the Ratio_Schedule is equal to 0.5, the constant human activity in the traditional method has reduced to its half value. Perc_NotVioHum refers to the percentage of test samples of indoor temperature which are

within human comfortable temperature zones [60°F, 70°F]. Ratio_Real Human denotes the number of humans in a conference room from 1 (the original number of humans indoors) to 0 (there is no one indoors) in the simulation stage. Perc_NotVioHum is calculated by firstly adopting the reference power consumption using the traditional method with the Ratio_Schedule, and secondly calculating Perc_NotVioHum in the simulation using this reference power consumption and the Ratio_Real Human.

As shown in Figure 17, when Ratio_Real Human = 1, 'Perc_NotVioHum' is rising with the Ratio_Schedule increasing from 0 to 1. It means that if we did not consider the human activity at HVAC scheduling, the indoor temperature will not be within the comfortable zone (at about 20% probability). Even if when Ratio_Schedule = 0.5, the Perc_NotVioHum is still about 0.6. It means that the traditional method to deal with human activity cannot provide a satisfactory performance unless the applied constant human activity in a scheduling period is exactly equal to the real human activity.

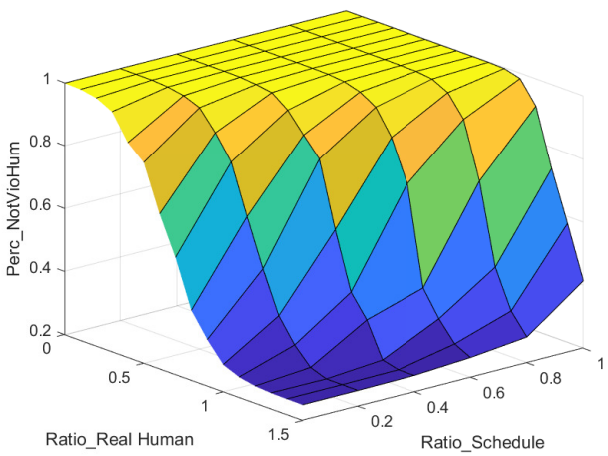

Fig. 17. The traditional method considering the effect of human activity

Figure 18 illustrates the proposed method with the consideration of human activities as the fluctuation of the number of humans in a conference room. In contrast to the traditional method, 'Perc_NotVioHum' for the proposed method is closed to 1 as the ratio increases from 0 to 1.

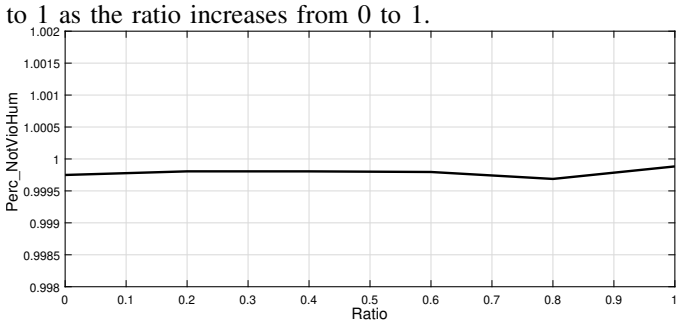

Fig. 18. The proposed method considering the effect of human activity

V. Conclusion

This paper proposes a DROA based on disjoint layered ambiguity set to optimise the energy consumption in the HVAC energy scheduling problem. Furthermore, two uncertainties are considered in the problem: the weather predicted error and human activities, which are two of the main factors that influence the indoor temperature, and cause a violation of human optimum comfort temperature level. The HVAC energy scheduling problem considering two uncertainties is formulated as an optimisation problem with nonlinear DRCCs. Then, these two nonlinear constraints are transformed into linear inequalities via duality theorem.

The simulation results considering the multi-zone HVAC system model and the consecutive time period demonstrate that the proposed DROA approach can provide better performance compared to existing research. Numerical results illustrate that the proposed DROA can decrease 2.81% and 0.14% of the electricity cost, and the number and maximum of violations are smaller compared with the two aforementioned methods. The comparison simulation demonstrates that the traditional method considers the impact of human activity as fixed indoor temperature variations caused by human activities, which leads to inaccurate optimisation results. However, the proposed DROA constructing the ambiguity set using the historical data of the number of people indoors can accurately consider the impact of human activity on the HVAC energy consumption scheduling and provide better performance on avoiding violation of human optimum comfort temperature level.

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Yingjie Wang received the Ph.D. degree in Electrical Engineering and Electronics from the University of Liverpool, Liverpool, UK, in 2021.

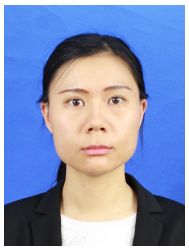
He is now a Research Associate in the Smart Grid and Renewable Energy Group in the Department of Electrical Engineering and Electronics at the University of Liverpool. His research interests include energy consumption scheduling in HVAC and smart grid.

Qi Zeng received the B.Eng in South China University of Technology and received the M.Sc in Electrical Engineering and Electronics from the University of Liverpool. He is currently pursuing for the Ph.D. degree in Electrical Engineering and Electronics from the University of Liverpool, UK.

His research is focused the dynamic state estimation, stability, control and the optimisation of the power system.

Zhongbei Tian received the B.Eng in Huazhong University of Science and Technology, Wuhan, China, in 2013. He received the B.Eng. and PhD degree in Electrical and Electronic Engineering from the University of Birmingham, Birmingham, U.K., in 2013 and 2017.

He is currently a Lecturer in Electrical Energy Systems at the University of Liverpool. His research interests include railway traction power system modelling and analysis, energy-efficient train control, energy system optimisation, and sustainable transport energy systems integration and management.



Yuefang Du received the B.Eng degree from the University of Electronic Science and Technology of China, Chengdu, China, in 2013, and the Ph.D. degree from the University of Liverpool, U.K., in 2017, both in electrical engineering.

She worked as a Postdoctoral Research Fellow with the Nanyang Technological University, Singapore, in 2018. Currently, she is a lecturer with the School of Mechanical and Electrical Engineering, University of Electronic Science and Technology, China. Her research interests include demand response in smart grid, optimal power system scheduling, and renewable energy.

Pingliang Zeng (senior member, IEEE) is a professor at Hangzhou Dianzi University, Hangzhou, China, and a national special expert. He is a Chartered Engineer (CEng), Fellow of the IET, and a member of IEC TC122 and TC8. He obtained his bachelor and PhD degrees, both in electrical engineering, respectively from Huazhong University of Science and Technology, China, in 1984, and Strathclyde University, UK, in 1990. Upon attaining doctorate degree, he joined National Grid Company (NGC), UK, where he held a number of important positions including system operator incentives and strategy manager, and network design manager. In 2012, he joined China Electric Power Research Institute as the chief expert in Power System Analysis and Planning, where he led a research team undertaking a number of key projects including National High Technology Development ("863" Plan), SGCC special projects, and key Sino-UK joint research projects funded by National Natural Science Foundation of China on integration of EV and energy storage in power systems. He joined Hangzhou Dianzi University in August 2017. He has published one book and about 100 academic papers. His current research interests include power system planning under uncertainty, system operations, renewable energy

Lin Jiang (Member, IEEE) received the B.S. and M.S. degrees from the Huazhong University of Science and Technology (HUST), Hubei, China, in 1992 and 1996, and the Ph.D. degree from the University of Liverpool, Liverpool, U.K., in 2001, all in electrical engineering.

He worked as a Postdoctoral Research Assistant with the University of Liverpool from 2001 to 2003, and Postdoctoral Research Associate with the Department of Automatic Control and Systems Engineering, University of Sheffield, from 2003 to 2005. He was a Senior Lecturer with the University of Glamorgam from 2005 to 2007 and moved to the University of Liverpool in 2007. Currently, he is a Reader with the University of Liverpool. His current research interests include control and analysis of power system, smart grid, and renewable energy.