Augmented inertial navigation using cold atom sensing

Alexander M Phillips[a], Michael J Wright[a], Marton Kiss-Toth[b], Iain Read[b], Isabelle Riou[b], Stephen Maddox[b], Simon Maskell[a], and Jason F Ralph[a]

[a]Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool, L69 3GJ, UK
[b]Teledyne e2v, Chelmsford, CM1 2QU, UK.

ABSTRACT

Inertial sensing based on cold atom technologies has been proposed as a possible answer to the limited accuracy of current inertial navigation systems. Cold atom technologies offer measurements of inertial quantities that have unprecedented precision and accuracy. However, sensor accuracy is only one of the factors that limit the performance of purely inertial navigation systems. This paper reviews the possible benefits that cold atom quantum sensing may offer in navigation, and discusses a specific example where cold atom gravity gradiometers can be used to augment a standard inertial navigation system through gravitational map-matching.

Keywords: Cold atoms, inertial navigation, quantum sensing, gravity map matching, gravity gradiometer

1. INTRODUCTION

Quantum sensing using cold atoms has the potential to dramatically improve the accuracy and precision of inertial measurements.[1,2] In recent years, there have been significant advances in cold atom technology, with matter wave interferometers being used to measure inertial quantities – acceleration[3–8] and angle rates[9,10,12] – which could be used in an inertial navigation system (INS).[13] The hope is that sensitive cold atom sensors can provide ultra-precise measurements and lead to significant improvements in the accuracy of navigational data generated by conventional, classical inertial navigation systems. Such a goal does place significant demands on the cold atom sensors, however.[14] Even with perfect sensors, the navigation solution of a purely inertial navigation system will always drift away from the true values, due to the inherent instability of the problem. Better inertial sensors can help reduce the rate at which the navigation solution drifts, but they will never remove this problem entirely.

An alternative approach to direct replacement of classical inertial sensors is to use cold atom sensors to measure properties associated with environmental parameters, such as the local gravity[15,16] or gravity gradients.[17–19] In this mode, cold atom systems could be used to measure the properties of the local gravity fields and use this information to determine the position of the vehicle relative to some reference database.[20–23] Position fixing is familiar from other methods currently used to augment classical INSs, such as satellite navigation systems (Global Navigation Satellite System, GNSS: GPS, GLONASS, BeiDou, Galileo) or terrain referenced navigation (TRN) used by military systems.[24,25] Whilst extracting the gravity signal from other confounding factors when a vehicle is moving is potentially difficult, this approach does offer some benefits in terms of its robustness in situations where the other external information is not available; either due to deliberate jamming or interference with reference signals (GNSS systems are very low power and relatively easy to jam) or due to a lack of suitable terrain features (very flat terrain or over/across bodies of water).

This paper discusses the different options for using cold atom sensors to augment conventional classical navigation systems, in their current form and possibilities for the longer term. It begins by providing an overview of the issues associated with inertial drift and the different forms of position fixing that have been used or are being considered for use to augment inertial navigation systems. The paper then goes on to consider the properties of a specific augmentation method; the use of gravity gradiometers for map-matching, where simulations show that gravity gradiometers can provide a benefit over pure inertial navigation systems if the gravity reference database has sufficient resolution.

Further author information: Send correspondence to J. F. Ralph, E-mail: jfralph@liverpool.ac.uk

2. DEAD RECKONING AND INERTIAL DRIFT

Standard inertial navigation systems are a form of dead reckoning[13] – that is, the measurement of time derivatives of position, which are integrated to provide a change in position. An INS contains three accelerometers and three gyroscopes, which measure accelerations and angle rates. These are then integrated to provide estimates for velocity, position and vehicle attitude. An INS has many practical benefits. It provides high frequency estimates for the motional states of a vehicle, often operating at frequencies of several hundred Hz, which can be used as part of the vehicle control system. Most modern INSs are strapdown systems. They are fixed to the vehicle body and do not require a stabilised platform on which the sensors are mounted. Angle rates measured by three-axes gyroscopes are integrated to provide attitude information, which is used to resolve the measurements of the accelerometers into a navigation frame (that is, a reference frame that is either the local North-East-Down axes or a global Earth-centred frame). Once the accelerometer measurements (which are strictly measurements of specific force rather than acceleration[13]) have been resolved into the navigation frame, they can be integrated to provide velocity and position. Each of the integrated quantities requires an initial value to be defined because the integration only provides the change in the integrated quantity relative to the initial value (effectively, this is a 'constant of integration', familiar from standard indefinite integrals). The integration of angle rates to provide attitude requires that the initial orientation of the vehicle is known. The integration of the specific force requires the initial position and the initial velocity, but it also requires an estimate of the local gravity, since the specific force includes the effect of gravity. In addition, it will also contain the effect of the Coriolis force due to the rotation of the Earth; this second term is small relative to gravity but must be considered when dealing with accurate navigation systems. Both of these effects must be removed before integrating to find the velocity and position estimates.

The errors in the estimates of position, velocity and attitude are all coupled, and all accumulate over time in an unaided INS. The initial errors in position, velocity and attitude are not corrected by the integrated measurements since the integrals only provide changes in the states rather than direct measurements of the quantities. This is true even if the sensors are perfect, with no measurement noise. The fact that the attitude estimates are used to resolve the specific forces, means that any initial attitude error or subsequent gyroscope measurement errors will inevitably corrupt the estimates of the velocity and position. The initial velocity error will not be corrected without a direct measurement of velocity, and will continue to be added to the position after each time step. Additional acceleration errors due to inaccurate attitude and force resolution, accelerometer measurement noise, and gravity compensation will cause the velocity errors to increase and have a knock on effect on the position estimates. In addition, conventional inertial sensors will also have other sources of errors: sensor bias, sensor bias drift, non-orthogonality due to sensor mis-alignment, sensor scaling variations, acceleration dependent errors due to structural flexure in the sensor housing, errors due to temperature variations, and numerical integration errors. Providing a better sensor to remove some of these effects is worthwhile if it can reduce the rate at which errors accumulate, but it will not solve the underlying problem, which comes from the instability of the estimation process that relies on the integration of derivatives. Cold atom sensors will not resolve all of these issues.

Currently, the best way to improve the performance of a purely inertial navigation solution is through an accurate calibration of the errors in the sensors.[26,27] Many of the errors in conventional inertial sensors are either static (non-orthogonality and scaling errors) or vary slowly with time (sensor bias, which often has a static component and a component that drifts slowly around this fixed value). High-performance navigation systems will often be subject to multi-position tests to allow these constant errors to be calibrated and then corrected for in software.[26,27] Lower quality sensors are less likely to be calibrated in this way because it is time-consuming and expensive.

The current generation of quantum cold atoms sensors is able to provide ultra-precise measurements of physical quantities: measurements can be good enough to test fundamental physics.[28,29] As such, it is reasonable to ask whether cold atom sensors can be used to improve the performance of conventional INSs via improved calibration either of the sensor errors or the environmental parameters;[30] which could be off-line (as with conventional multi-position tests) or online, in parallel with a conventional INS. Offline calibration using very high performance conventional sensors is already very good and it requires that the calibration and the calibrated sensors are subjected to known input stimuli; including being oriented, reoriented and rotated about well-defined axes in the test equipment. The current generation of cold atom sensors are relatively large and heavy. They are unlikely to perform well in size, weight and power (SWaP) when compared to conventional INSs. In addition, the sensors tend to have limited duty cycles, limited operating frequencies and limited dynamic ranges, meaning that they are not ideal for offline calibration and may only be suitable for use in online calibration in fairly restricted applications where the platform motion is relatively benign and size, weight and power are not limiting factors. Of course, the first generation of cold atom sensors are scientific instruments and not generally engineered as practical, ruggedised devices for deployment in the field (although great efforts are currently being made in this direction, see section 4). Following generations of quantum sensors are likely to be smaller, compact and provide more practical alternatives to conventional inertial sensors.[12] With future developments in mind, it is worth considering how cold atom sensors can be integrated into an operational INS.[14] However, for the sake of brevity, this paper concentrates on a more practical application that could make use of current technologies: using cold atom sensors to provide position fixes.

3. POSITION FIXING AND INERTIAL AUGMENTATION

The alternative to navigation via inertial dead reckoning is to provide a series of position fixes which allows a platform to verify its location at instants in time. The advantage of directly measuring position is that it is not susceptible to inertial drift due to the accumulation of (uncorrected) errors. The negative aspects of position fixing are that it requires an external database against which the position can be measured. Traditionally, this would have been some form of map, but the standard today is a satellite based navigation system, such as GPS, BeiDou or Galileo. Navigation satellites broadcast radio-frequency signals that provide satellite position and timing information, from which a GNSS receiver can calculate its own position – the signals and the satellites together providing the reference against which the location is determined. An unaided INS requires no such external reference. It is autonomous. An unaided INS also provides high frequency navigation information, which is useful for vehicle control systems as well as navigation. A position fixing method based on an external reference could provide high frequency updates of position, but ultimately this is not always beneficial because a finite resolution database or slowly changing reference signals will generate measurements that will be highly correlated. This means that higher frequency updates will not necessarily provide better navigation data.

The benefits of a high frequency inertial sensor and a lower frequency position fixing system are clear, providing the benefits of both approaches at the expense of a slightly more complicated system. Such combinations are common, particularly combinations of INS and GNSS. An INS that is augmented by GNSS provides a stable, high frequency navigation solution which is based on a global reference signal. The problem with such systems is that GNSS is a very low power signal, and – in the standard commercial configuration – is vulnerable to jamming (by the transmission of noise in the relevant radio waveband) and spoofing (deliberate manipulation of the GNSS signals to modify the location information that it provides).[31] Military systems have additional methods to reduce the risk associated with jamming and spoofing, including additional GNSS frequencies, but they are not completely insensitive to malicious interference and most commercial systems are not as robust to these effects. As a result, other position fixing methods using alternative references have been developed. Some military aircraft and weapon systems measure terrain features and correlate these terrain features against digital terrain elevation data (DTED) to determine location.[25] An advantage of this approach is that large regions of terrain are relatively constant and databases of their features are straightforward to construct and to maintain. The disadvantage of using terrain features is that there are large regions of the Earth that do not have suitable features to correlate against: e.g. flat ground or open water. For military systems, an additional disadvantage is that the terrain measurements require an active sensor, which can expose the platform to detection by potential adversaries.

In addition to terrain features, other reference/correlation systems have been developed that can use visual ground features and a camera system (operating either in the visible or infrared wavebands – image based navigation), terrestrial radio signals (either navigation specific broadcast radio (e.g. LORAN) or serendipitous 'signals of opportunity' from other broadcast radio signals), astronomical features ('star tracking'), or a combination of several sources. All techniques relying on an external reference have physical limitations, e.g. the lack of suitable features in particular areas or in certain environmental conditions (e.g. weather). By combining several reference systems with complimentary capabilities these issues can be mitigated, at the expense of making the navigation system more complex and potentially require the platform to be equipped with a number of additional sensors. In addition, there are often significant costs associated with the initial construction of the reference database and with updating and maintaining a database.

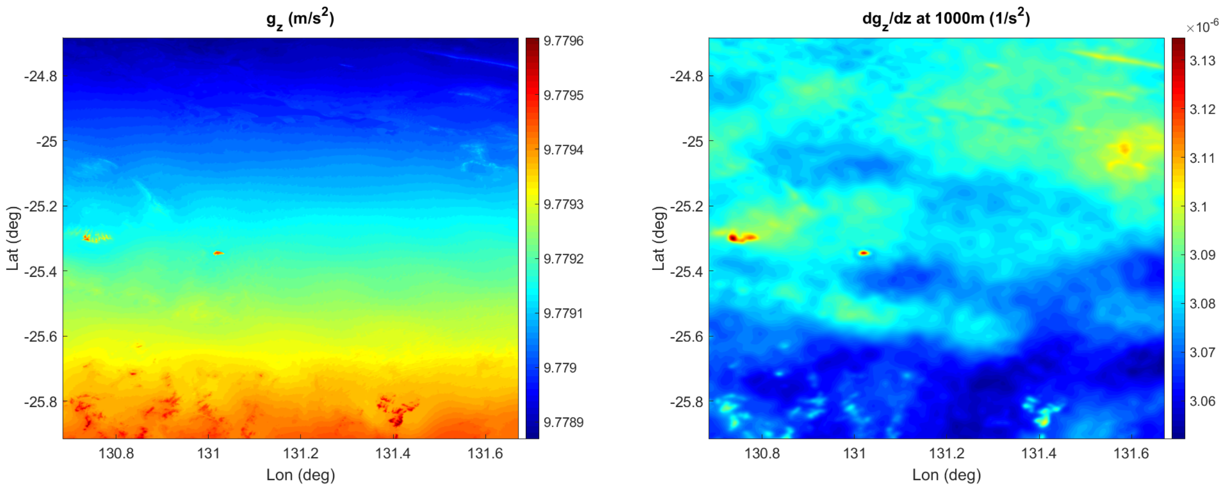
Combining complementary sensors/references is beneficial, but there are still situations where none of the methods mentioned are suitable. In maritime applications outside the littoral environment, there are no land or ground features to use. In situations where GNSS and other radio navigation signals are not available and weather prohibits star tracking, another solution would be required. This is true for underwater vehicles in particular, where conventional position fixing methods are problematic. For these applications, cold atom sensors could have a role. The ability to use an atom interferometer to measure the local gravity provides a signal that can be used to correlate with a gravity database and to fix the position of the vehicle in a similar manner to the way that terrain features are used.[20–23] The benefits of using gravity are that global (or near global) databases of local gravitational variations are available in the open domain,[32] and that the features that are contained in the database are persistent. As with terrain features, gravity is difficult to manipulate or alter to mask key features.

4. GRAVITY GRADIOMETRY AND COLD ATOM SENSORS

Classical gravity sensors have been used in demonstrations of map-matching for navigation,[22,23] but cold atom sensors offer significant benefits in terms of sensitivity and the removal of sensor biases. The construction of cold atom gravity sensors is a very new technology and moving from the laboratory to use in the field is not a simple process. To this end, Teledyne e2v, working with the University of Birmingham,[33] have developed a gravity gradient sensor for a number of different markets including civil engineering, defence and space and for a range of applications covering geophysical surveying, fundamental metrology and navigation. Teledyne e2v's approach is to focus on differential measurement schemes which allow common mode vibration and motion to be rejected. The system relies on atom interferometry whereby two clouds of 87Rb atoms separated by a baseline of about one metre are laser-cooled at a temperature of a few micro-Kelvin and then dropped. The matter-waves associated with these cold atoms are split, reflected and recombined using a set of three Raman pulses which form spatially separated Mach-Zehnder interferometers. The phase-shift at the output of each interferometer is proportional to the acceleration of the free-falling atoms with respect to the laser reference frame. In this configuration, many noise sources, such as platform vibration, are common to both interferometers and can be cancelled to achieve a much higher signal-to-noise ratio. The resultant sensor measurement is one component of the gravitational gradient tensor, $\frac{dg_z}{dz}$ (the 'zz' component), the vertical derivative of the vertical component of the gravitational acceleration.

Significant challenges still remain in preparing cold atom interferometry devices for most real-world navigation environments. Moving vehicles are generally subjected to vibration, rotation and accelerations which can all cause loss of signal, signal biases or noise. However, studies have shown that cold atom devices can be adapted to these noisy environments using compensation or correlation methods.[33] Cold atom gravimeters (or accelerometers) are extremely sensitive to inertial effects. The measurement is generally taken using a reference mirror and the vibrations experienced by the mirror are seen on the atomic measurement. In this case, the sensor will not work at all without a compensation or a correlation method. However, as gravity gradiometers use a differential measurement between two atomic clouds, this allows noise that is common between the two atomic clouds to be rejected leading to lower sensitivity to vibration. Compensation is still needed for high acceleration environments to avoid contrast loss but the compensation requirements can be relaxed when compared to a single interferometer acting as a gravimeter. Another consideration is that the inclusion of compensation mechanisms is often at the cost of the compactness. In addition to the environmental challenges faced by cold atom interferometers, there are also some limitations linked to the sensor characteristics and to the way the data is collected. In order to use a cold atom sensor, a measurement sequence needs to be used to load, prepare, interrogate and detect the atoms. By their very nature cold atom sensors have dead time between measurements and limited repetition rates. There is also clear trade-off between the repetition rate of the instrument and the sensitivity which needs to be optimised. Additionally, there are also limits on the duration of the interferometer cycle due to the rotations and vibrations of the platform encountered during the measurement.

In order to understand the application of cold atom gravity gradiometers in real world navigation use cases, Teledyne e2v has developed a mathematical simulation toolkit, called GRAVITAS, which has been used in the work presented here. This toolkit allows a cold atom gravity gradiometer to be configured and to pass through a given gravitational field, taking into account the effects of noise in the environment and in the instrument and providing a representative raw data output. This enables rapid iterations through sensor design as well as modelling behaviour of a given instrument in different environments. The GRAVITAS toolkit provides the opportunity to adjust the key instrument design parameters such as the interrogation time, the number of atoms taking part in the measurement, the temperature of the atoms, the gradiometer baseline and the diffraction order. As inputs, the model takes the true gravity field for both sensor heads in the time domain (the signal to be measured) and the full dynamic tensor in the time domain (3 accelerations, 3 rotations) in order to model the inertial effects on the system. These inputs are generated from gravity models and a defined trajectory through the gravity field. The GRAVITAS model then calculates the effect of gravity and inertial effects using a physical model based on atom interferometer transfer functions and outputs the gravity gradient measurement in the time domain. This output is then passed to the map-matching algorithm. The toolkit is built in Python and follows a modular object-oriented design, with modules representing physical or abstract gradiometer components, abstract data types and physical noise sources. The modular nature of the architecture allows the model to be easily extended to other type of sensors such as horizontal accelerometers and gyroscopes, and the ability to refine the model by adding other noise sources such as variable magnetic fields and instrument noise.



(a) (b)

Figure 1. Section of gravity map, near Uluru (Australia), showing: (a) gravity (z) values and (b) gravity (zz) gradients calculated from the SRTM2gravity database[32] at an altitude of 1000m.

5. GRAVITY MAP-MATCHING

The gravitational structure of the Earth at a large scale is complex. It is determined by the local topography and by the density of the materials near the Earth's surface (including bodies of water). Although the problem is complex, it is also well studied. Standard global gravity databases exist and are freely available. For example, the EM2008 gravity model[34] is a global gravity database which is based on measurements taken from a combination of terrestrial, satellite and airborne surveys. The resolution of this database is 1 nautical mile (one minute of arc at the Earth's surface). This model defines the current standard Earth geoid for variations in local mean sea level relative to the standard Earth ellipsoid (WGS84). More detailed databases do exist but these are all defined relative to this global standard. In particular, the recent development of the SRTM2gravity model[32] has provided a gravity model that has a minimum resolution of 90 metres. It covers nearly all of the land masses, but not sea areas. This model provides estimated small scale, local corrections to EGM2008. The corrections are not from measured data – measured data at such small scales would be prohibitive to collect for a global database – the values are calculated from the local topography and average rock density values, using a method

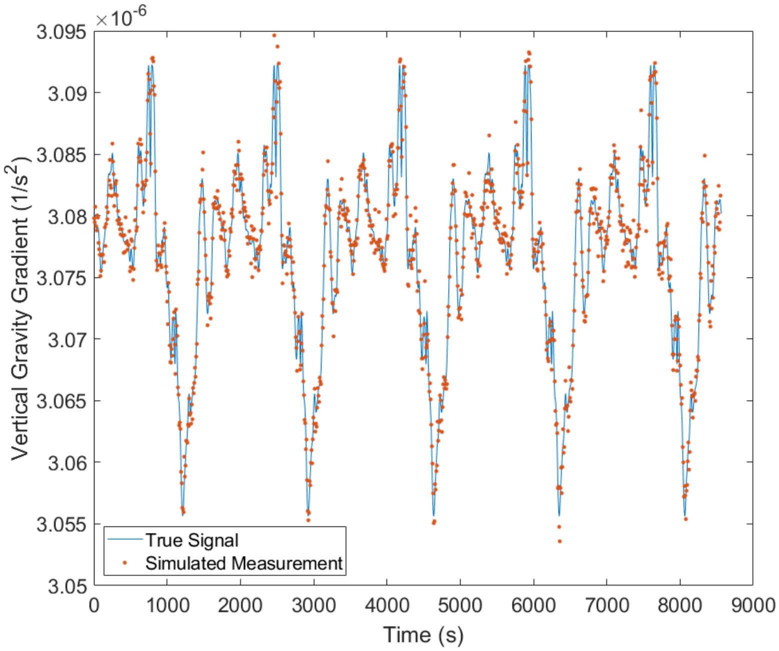


Figure 2. Example of simulated gravity gradient signal (red dots) and 'true' gravity gradient (blue solid line) for a circular trajectory around Uluru (Australia) calculated from the SRTM2gravity database[32] at an altitude of 1000m at 250 knots.

called 'forward modelling'.[32] As such, the actual values of the local gravity provided by this model are likely to be fairly accurate on average but there will be some variations from the true gravity values in practice.

Given a suitable gravity database, it is possible to do gravity map-matching in two ways, either matching the actual values of the gravity at each location or matching the gravity gradients at each location. Intuitively, it may seem that matching the gravity values is the easier option. However, most correlation based map-matching methods tend to use gradient or feature based measurements to correlate against. The reason for this is that many sensor measurements contain a bias signal, and correlation based metrics tend to be sensitive to biases signals. In addition, for the specific case considered here, a cold atom gravity gradiometer can be less sensitive to mechanical vibrations than a cold atom gravity sensor (see section 4), and the availability of the high-resolution (forward modelled) SRTM2gravity database means that using the gradient information should provide features that are robust even in locations where the absolute value of the gradient contains bias errors.

The navigation system model used for this work is based on the standard INS equations given in reference,[13] with a Kalman state estimation method,[35] which provides state estimates for position, velocity and attitude as well as estimates for the expected errors, which allows the inertial position to be fused with the position fix provided by gravity map-matching. The INS model is combined with a kinematic vehicle model – which can be adapted to represent the main motional characteristics of aircraft, maritime or land vehicles, including vibrational characteristics. The aim is to provide a representative environment and realistic dynamical states, within which the performance of the INS and the gravity gradiometer can be assessed. The INS typically operates at around 400Hz and it can be configured to represent a range of INS performance characteristics: tactical grade, aviation grade and maritime grade.[13] The gravity gradiometer model used here is part of Teledyne e2v's GRAVITAS toolkit. It provides simulated gravity measurements at around 1Hz and around 20 of these measurements are used to estimate the vertical (zz) gravity gradient, dg_z/dz, which can be done using separate batches or a sliding window using an ellipse fitting method.[36] The underlying gravity database used to generate the simulated gradient measurements is the SRTM2gravity model,[32] and the reference database is generated from the same gravity model, but with the ability to vary the resolution of the data to investigate the minimum resolution required to achieve a reliable position fix. Other effects can also be added to this – such as small signal bias

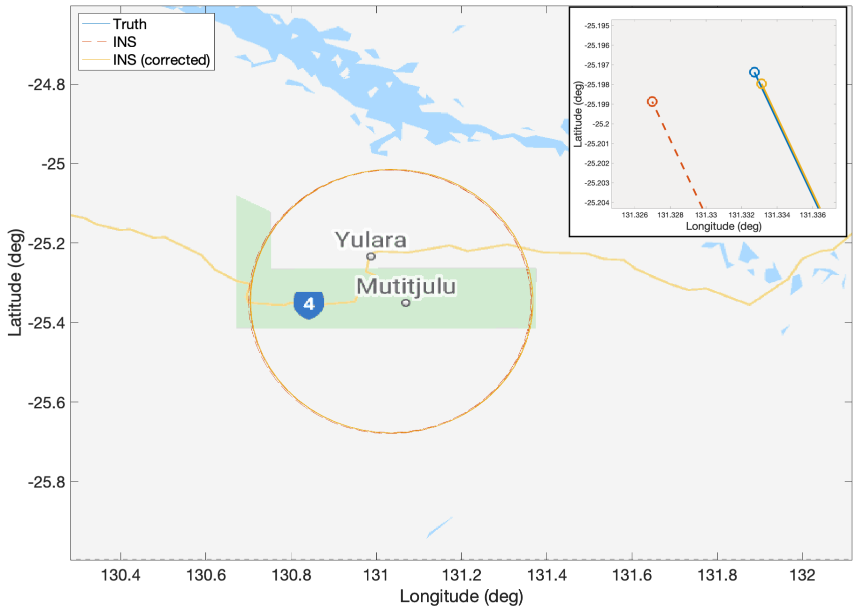


Figure 3. Example of a simulated circular trajectory around Uluru (Australia) with inset figure showing the effect of the inertial draft for an aircraft at an altitude of 1000m at 250 knots (blue = truth, red dash = INS only, yellow = INS corrected with gravity map matching), using the performance characteristics for an aircraft grade INS[13]

values, localised clutter, or reference misalignment errors – but these are not considered here. In each case, the gravity gradients are calculated from the gravity models using a finite integration method and the SRTM2gravity corrections to the EGM2008 gravity anomalies.[37,38]

The sensor model produces simulated gravity gradient measurements at regular intervals along the vehicle's trajectory. Once a sufficient number of gravity gradient measurements have been obtained (around 20 points are sufficient in most cases) gravitational map-matching is performed by calculating the normalised cross-correlation of the measured gradient signal against the gravity gradient map. Specifically, the measurements are compared with the gradient map at the positions from the INS solution which correspond to the time points at which the gradient measurements were obtained. A hill-climbing search is performed to determine the shift in position that maximises the correlation of the measured signal with the gradient map, and a bivariate Gaussian distribution is fitted to the correlation surface to characterise the uncertainty of this estimate, which is used with the expected state covariance from the INS model to fuse the two solutions using a Kalman update (although more sophisticated fusion methods, such as particle filters have also been explored). Map-matching corrections which would contradict the INS – that is, where the position of maximum correlation is far from the estimated INS position (Mahalanobis distance greater than 3) – or where the maximum correlation is less than some threshold (95%), are taken to be false matches and discarded.

6. EXAMPLE RESULTS

As an example, simulated results are shown for an aircraft flying multiple circular loops at 1000m altitude around Uluru in Australia at a constant speed of 250 knots (see Figure 3). The trajectory is benign in that it does not contain any abrupt manoeuvres that might affect the gravity gradiometer performance. The INS parameters are set to be consistent with a conventional aviation grade INS,[13] which drifts in the horizontal plane by about one nautical mile per hour of flight. The area has been selected because of the nature of the terrain – the ground in this area is very flat – and the altitude is at the normal limit of terrain referenced navigation systems, which

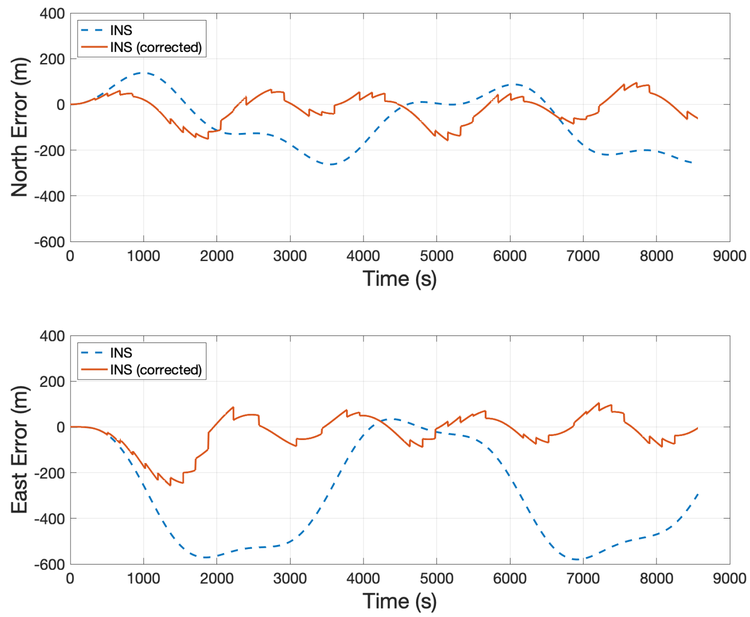


Figure 4. An example of INS errors (North-East) for a simulated circular trajectory around Uluru (Australia) for an aircraft at an altitude of 1000m at 250 knots, uncorrected (blue dashed line) and corrected (red solid line) using gravity map-matching

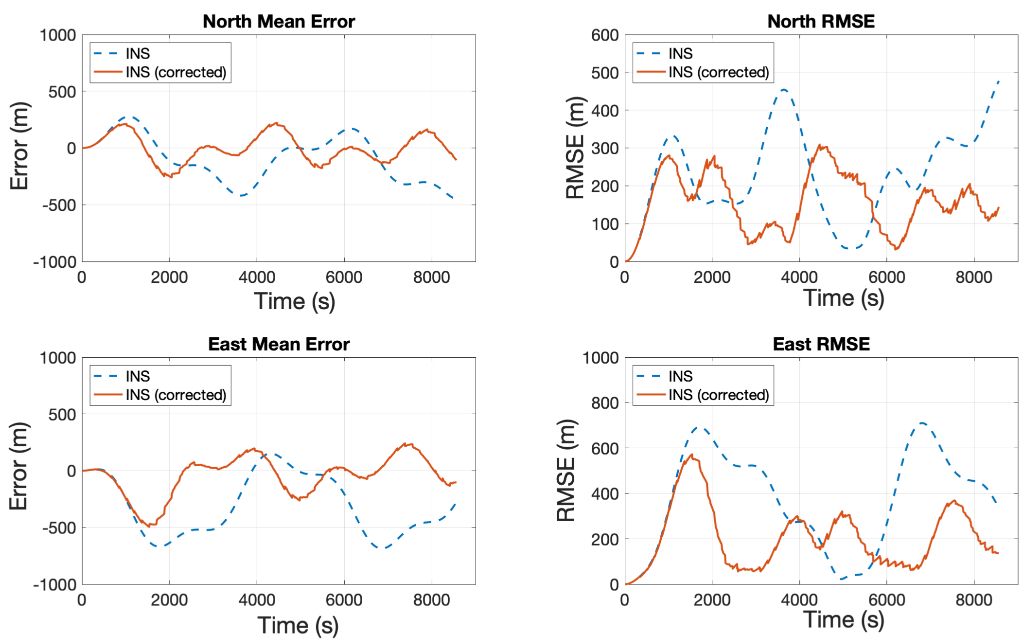


Figure 5. Average INS errors (North-East) for ten simulated circular trajectories around Uluru (Australia) for an aircraft at an altitude of 1000m at 250 knots, uncorrected (blue dashed line) and corrected (red solid line) using gravity map-matching

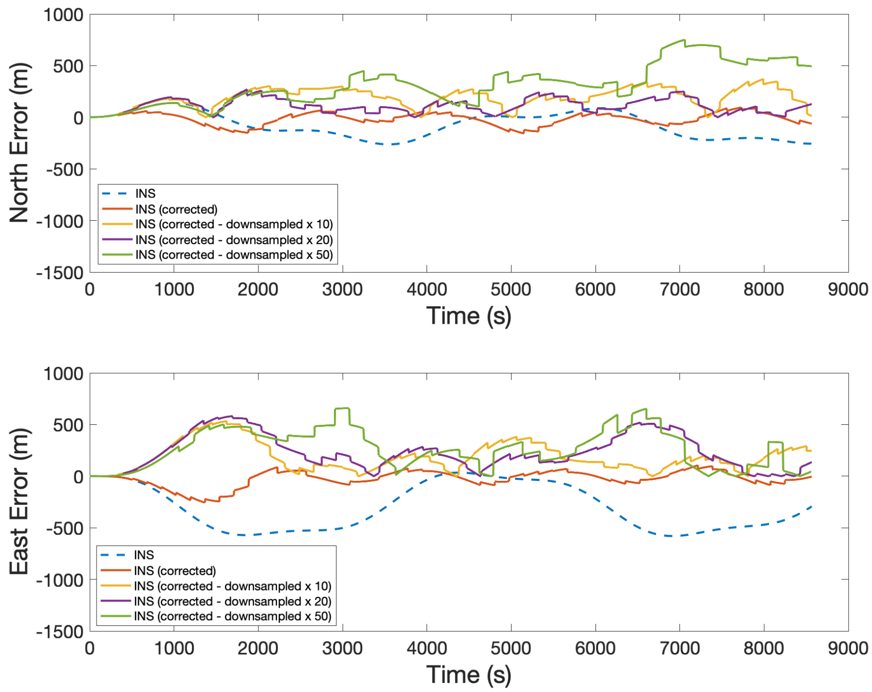


Figure 6. An example of INS errors (North-East) for a simulated circular trajectory around Uluru (Australia) for an aircraft at an altitude of 1000m at 250 knots, showing the effect of databases of different resolution: uncorrected (blue dashed line), corrected using 90m resolution database (red solid line), corrected using x10 database (yellow solid line), corrected using x20 database (purple solid line), corrected using x50 database (green solid line)

are normally reliant on radar altimeters (Radalt), which typically have a maximum height above terrain of a few thousand feet.

For any pure INS, the vertical accuracy of the navigation solution is the most sensitive to inertial drift due to errors in gravitational compensation (see section 2), which may mask the effects of horizontal inertial drift. To reduce the effect of the vertical drift, simulated barometric altimeter (Baro Alt) measurements are used to correct the vertical channel. In addition, the horizontal errors also contain the effect of Schuler oscillations, which have a 84 minute period (approximately) and arise from coupling between the curvature of the Earth and the inertial axes of the INS.[13] The larger oscillations shown in Figure 4 are due to Schuler oscillations, nearly two Schuler periods are shown over the 2.5 hour period of the simulated data, and the background drift over time is the underlying inertial drift of the INS. The errors for the INS corrected by the gravity gradient map matching are also shown - the jagged 'steps' showing the points at which a position correct is applied. The error for the corrected/augmented INS is closely confined to within about ± 200 metres of the correct location (Figure 5), which is related to the accuracy/resolution of the SRTM2gravity database.[32] Figure 6 shows the effect of reducing the resolution of the database. The results shown in Figure 4 are shown together with results generated from alternative databases constructed by downsampling the gravity gradient data to reduce the effective resolution of the reference. The original results used a database calculated from the 90m resolution SRTM2gravity database of gravitational disturbances to the EGM2008. As the resolution of the database is reduced, the accuracy of the position fixes deteriorates, as expected. What is noticeable, however, is that the reduction in resolution also creates more false matches with the database. Examining the original results, each of the 'steps' in the error curve (corresponding to a position update) tend to reduce the position error, whereas reducing the resolution by a factor of 10 starts to produce noticeable steps that increase the position error, and these steps are more noticeable as the resolution is reduced further. These effects are caused by false matches between the signal (which still retains high resolution measurements) and the reference database. This mis-match of scales can cause the matching algorithm to find an apparent maximum correlation that is far from the true location – causing the estimated position to deteriorate rather than improve. A reference database that has the right level of detail and resolution for the sensor and speed profile of the platform is therefore critical to the reliability of the method.

7. DISCUSSION AND CONCLUSIONS

The use of inertial sensors for navigation is well established but prone to difficulties with inertial drift. The position drift seen in inertial navigation systems is an inherent property of dead reckoning. Integration of measurements of time differentials of position causes errors to accumulate over time. With no absolute position reference, there is no mechanism to correct this accumulation. Because of small errors in other factors within the integration processing (gravity and Coriolis compensation, numerical artefacts) even a perfect sensor will lead to position drift. Therefore, there is potential for cold atom sensors to reduce but not to remove inertial drift. Cold atom sensors can provide ultra-precise measurements for acceleration and rotation, and could – in principle — approach the level predicted for a perfect sensor. However, the current generation of inertial sensors are large when compared to the state-of-the-art conventional inertial sensors, and their all round performance is somewhat limited by their dynamic range and measurement frequency. The underpinning technologies needed to manipulate and measure cold atoms have many applications and are developing very rapidly. It is likely that future inertial cold atom sensors will be able to compete or surpass the accuracy of current commercial inertial sensors. When cold atom sensors can do this, they will be viable technologies for improved inertial navigation systems – reducing, but not altogether removing, the effect of position drift. Until such sensors are available, cold atom sensors could be used to calibrate conventional sensors to remove some of the larger sources of error (such as sensor bias and bias drift).

Dead reckoning navigation has many attractive qualities, but it performs best when augmented with direct measurements of position with respect to some external reference or database. As an alternative to cold atom sensors as pure inertial sensors, they can also be used in combination with a map to measure position using a series of gravity or gravity gradient measurements. In the example presented, a physics-based model for a cold atom gravity gradiometer has been used to demonstrate the utility of gravitational gradient map-matching in augmenting the performance of a model inertial navigation system. Even for a region where the ground is relatively flat, the gravity signal is still within the range of sensitivity of a cold atom gravity gradiometer. The resultant signal is sufficient to provide a good correlation against a gravity map and to provide a position fix that is accurate to the resolution of the underlying gravity database.

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