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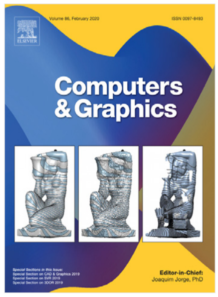

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SHREC'21: Quantifying shape complexity

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ABSTRACT

This paper presents the results of SHREC'21 track: Quantifying Shape Complexity. Our goal is to investigate how good the submitted shape complexity measures are (*i.e.* with respect to ground truth) and investigate the relationships between these complexity measures (*i.e.* with respect to correlations). The dataset consists of three collections: 1800 perturbed cube and sphere models classified into 4 categories, 50 shapes inspired from the fields of architecture and design classified into 2 categories, and the data from the Princeton Segmentation Benchmark, which consists of 19 natural object categories. We evaluate the performances of the methods by computing Kendall rank correlation coefficients both between the orders produced by each complexity measure and the ground truth and between the pair of orders produced by each pair of complexity measures. Our work, being a quantitative and reproducible analysis with justified ground truths, presents an improved means and methodology for the evaluation of shape complexity.

1. Introduction

Shape complexity is studied across several fields such as psychology [1], design [2, 3], computer vision [4]. In the context of 3D shapes, it has the potential to be useful in shape retrieval [5, 6], measuring neurological development and disorders [7, 8], in determining the processes and costs involved for manufacturing products [2, 9], etc. Early work on shape complexity appears in the literature of experimental psychology as well as in literature related to design and aesthetics. The classical aesthetic notions of "unity" and "variety" [10], or comparably, "order" and "complexity" [11] are directly connected to the complexity of spatial objects. One of the first measures of complexity for polygonal shapes can be found in [11]. Attneave [1] conducted human experiments to seek correlations of shape complexity with scale, curvedness, symmetry and number of turns. On the basis of the variety in the responses from human subjects, Attneave states that shape complexity is ill-

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[1] Track organizers

defined. With the premise of circles being the simplest shapes, a natural candidate for the quantification of shape complexity is P^2/A. In several works ([1, 3]) it is used as a measure of the complexity along with other indicators. In most other works [4, 12, 13, 14, 15, 16], tools from information theory, on top of various geometric features are used to quantify complexity. Work that relates complexity to algorithmic information theory and is applied to objects of art and design can also be found in Stiny and Gips [17]. Rossignac [18] provides a classification of shape complexity that focuses on measuring different aspects of computer representations for 3D shapes. The variety of approaches taken in the quantification of shape complexity further supports the claim that complexity can obtain a variety of meanings based on the approach that one chooses to take in a particular research area and for the particular task at hand.

There is a lack of benchmark datasets for shape complexity. Even the methodologies in the literature need improvements. For example, in many cases just visual results are reported without quantitative analysis [19, 13]. The methods are neither compared to other methods nor evaluated in terms of statistical consistency. In this track paper, we aim to account for and investigate different aspects of complexity that can help other researchers to develop and test their methods. In particular, we investigate how good the submitted shape complexity measures are (*i.e.* with respect to ground truth) and investigate the relationships between these complexity measures (*i.e.* with respect to pairwise correlations). Due to the ill-defined nature of complexity, a linear order may not make sense. Hence, we propose to explore complexity using multiple tasks and multiple shape collections.

The first collection is composed of subgroups obtained by introducing additive or subtractive noise to two basic shapes: sphere and cube. The purpose is to investigate the relation of complexity to noise level. The second collection is composed of artificial 3D shapes constructed by transforming and combining multiple elements, and evaluated by experts to provide ground truth. The purpose is to investigate the complexity methods in relation to perceptual categories. The final collection is an already existing 3D shape dataset which was originally developed as a segmentation benchmark. We repurpose this data and use the segmentation ground truth as a means to investigate 3D shape complexity via a proxy (secondary) task. The main contributions of this work are as follows:

- Generation of two novel shape collections with associated ground truth, and repurposing of a previous segmentation benchmark for assessing complexity measures.
- Systematic evaluation of the performance of a selection of both 2D and 3D classical and recent shape complexity measures.
- Assessment of similarities and differences between different measures by using pairwise correlations and clustering based on their performance with respect to multiple ground truths.

Note that due to the ill-defined nature of shape complexity our dataset contains three shape collections with different characteristics. Each collection in the dataset contains a different type of 3D object with ground truth defined and obtained in a different manner.

The paper is organized as follows: In Section 2 the dataset is introduced. In Section 3 the ground truths and the evaluation strategy are explained. In Section 4 the short descriptions of the participating methods are included. In Section 5 the the evaluation results for each collection in the dataset are presented. Finally, Section 6 is Discussion and Section 7 is Conclusion.

2. Dataset

The used dataset consists of three collections each aiming to account for a different aspect of shape complexity. The first two collections are created synthetically, and the third is an existing collection consisting of natural shapes. The ground truth for the first collection is based on the parameters used in creating the collection. For the second collection, the ground truth is provided by two design experts on the final design object. The purpose of the third collection is to investigate how estimated complexity is related to the number of parts perceived by humans, which we hypothesise is related to shape complexity.

2.1. Collection 1 – Perturbed basic shapes

In this collection we aim to explore the correlation between shape complexity and magnitude of perturbations of a cube and a sphere.

A cube of side length 199 voxels and a sphere of radius 100 voxels are stochastically perturbed additively and subtractively, separately. This forms four families (additively perturbed cubes, subtractively perturbed cubes, and so on). The algorithm used in perturbing a shape introduces a perturbation at a random location on the shape's boundary in each application. The algorithm has two parameters: *i)* width (w) determining the area of effect of the perturbations and *ii)* number of times of application (c) determining how many times a local perturbation is introduced. Both parameters are set to three different values, $w \in \{3, 4, 5\}$ and $c \in \{25, 50, 75\}$. This results in a group of nine shapes. A sample group for an additively perturbed cube is displayed in Fig. 1. Fifty such groups form a family.

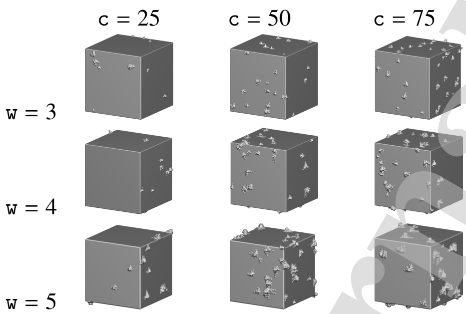

Fig. 1. A sample group of additively perturbed cubes.

The shapes in this collection were distributed to the participants as both volumetric data and triangular meshes.

2.2. Collection 2 – Parametric shape families

The second collection is made up of two distinct families of shapes and each family contains twenty five shapes. The shapes in this collection are inspired from 3D models and designs that are commonly found in the fields of architecture and urban design. The shapes were created with two primary objectives in mind. The first objective was to have shapes that vary parametrically in terms of a few spatial features. The set of spatial features is different for each family (see below), but in both families, these features guide the generation of the shapes in a systematic way via algorithms. The second objective was to have shapes that on the one hand are spatially "rich", in the sense that they can be deployed in a variety of realistic design scenarios and problems, and on the other hand, are abstract enough to not suggest fixed typological interpretations or the shapes of everyday objects. While such shapes make the task of measuring complexity significantly more challenging, they present an opportunity for a broader exploration of what constitutes complexity of spatial objects.

The shapes in both families are generated with the built-in scripting language of the Rhinoceros 3D software package (Robert McNeel & Associates, USA). All shapes are represented as watertight triangle meshes and were distributed to the workshop's participants in this format.

In the first family, the shapes are generated by stacking cuboids. The main spatial features that control the generation are the number of cuboids and the length of the side faces of each cuboid. An additional rotational factor is used for eleven out of twenty five shapes in the family. In more detail, for each shape in the family the opposite faces of each cuboid are equal and all cuboids have constant height (heights are adjusted automatically based on the number of cuboids, for a fixed total height). The sizes of the cuboids are controlled by varying the length of the side faces according to the function of a predefined curve (either a sine or a Bezier curve). For the shapes that are controlled by a rotational factor, variation is also achieved by varying the angle of rotation of the cuboids around a central vertical axis. The resulting shapes can be understood as design objects ranging anywhere between pedestals and columns, to high-rise buildings and towers.

In the second family, the shapes are generated by aggregating three, four or five cuboids within a predefined rectangular area in the plane. All the resulting shapes form connected configurations (that is to say, there are no gaps between neighboring

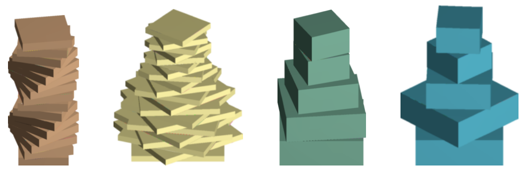

Fig. 2. Sample shapes from the first family of Collection 2.

cuboids) and the cuboids are merged in a single solid. The main spatial features that control the generation are the number of cuboids, the locations of the cuboids within the rectangular area and their individual heights. The lengths of the side faces of all cuboids are equal. Variation in the way cuboids are aggregated in the plane is achieved mainly by varying the locations and the heights of the cuboids. This is done by randomly sampling values from the allowable ranges specified for the locations and the heights. The resulting shapes can be understood as designs ranging anywhere between furniture and stairs, to volumes of buildings that form city blocks.

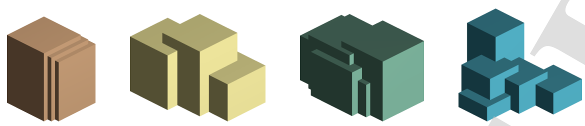

Fig. 3. Sample shapes from the second family of Collection 2.

2.3. Collection 3 – Manually segmented shapes

Collection 3 is the dataset for Princeton mesh segmentation benchmark [20]. We use this set with the primary objective of exploring how shape complexity measures correlate with the uniformity of the number of segments of the segmentations of the shape. The benchmark consists of 380 shapes across 19 categories and their human-generated segmentations. As opposed to the synthetic shapes in the first two collections, the shapes in the benchmark are natural. As such, they have a particular semantic content, which may affect the perception of complexity. The availability of manual segmentations for this collection makes it an ideal candidate to be used in exploring complexity by using segmentation as a proxy task.

The shapes in this collection were distributed to the participants as triangular meshes.

2.4. 2D Collections

Most of the shape quantifying methods in the literature work exclusively in 2D. To include such methods into this study we have created the 2D analogues of the shapes. We create twelve 2D silhouettes of each shape in the above collections from the views determined by the azimuthal angles ($\{0°, 30°, 60°, \ldots 330°\}$) and the elevation angle ($30°$). The resulting silhouettes of a shape are similar in size, thus, the collections do not pose a challenge in terms of scale-invariance. The contributing 2D methods report the averaged score over the twelve silhouettes as the measure of complexity for the corresponding 3D shape.

The families consisting of subtractively perturbed spheres and cubes are excluded because the resulting silhouettes highly depend on whether the perturbations appear on the 2D boundary of a given view or not, rather than the controlling parameters.

3. Ground truths and evaluation

3.1. Collection 1

For the first collection, the two parameters `w` and `c` used in creating the shapes constitute the ground truth. We expect the complexity scores to increase as either of the parameters increase. The performance of the methods are measured in a controlled experiment manner: we keep one of the parameters fixed and let the other vary. The performance of a method is then measured by averaging the Kendall rank correlation coefficient over the groups. This results in six measures of performance (one for each value of the parameters) for a family.

3.2. Collection 2

In Collection 2 the ground truth is provided manually. Two experts on the topics of Computer-Aided Design and 3D Shape Modeling determined the ground truth complexities of the shapes in the two families. The shapes in each family were presented to an expert in a random presentation order.

The evaluation of the experts was based on a qualitative comparison of the shapes in each collection that aimed to determine how simple or difficult it would be to model or execute a shape in three-dimensional space in a finite number of steps.

The shapes in a family were divided into groups of shapes of comparable executional difficulty. Shapes in different groups were considered incommensurate from this standpoint.

For the first family, the evaluation produced the following five groups in which shapes are listed in increasing order of complexity:

Group 1: (16, 14, 12, 17, 18, 19, 20)
Group 2: (15, 13, 24, 22)
Group 3: (23, 21, 25)
Group 4: (11, 10, 7, 1, 2)
Group 5: (9, 8, 4, 5, 6, 3)

For the second family, the evaluation produced the following six groups in which shapes are considered to be of equal complexity:

Group 1: (18, 24, 17, 20)
Group 2: (25, 21, 22, 19)
Group 3: (5, 3, 2)
Group 4: (23, 16, 15)
Group 5: (6, 4, 14, 12, 13)
Group 6: (7, 9, 10, 1, 11, 8)

As the shapes from different groups are incommensurate, we provide ground truths only for the shapes in the same group. While many qualitative notions of complexity could equally apply to shapes of the kind we use in this collection, we consider the aforementioned notion of executional complexity one of the best determinants of complexity for 3D models representing design objects (i.e. objects that can be used for design purposes). It is also an approach to the characterization of complexity that has not been investigated in the literature.

Since we have a total order on the groups of the first family, we measure the performance of the methods using the Kendall rank correlation coefficient between the complexity order indicated by the assigned complexity scores and the ground truth. For the second collection, we measure the uniformity of in-group scores. The scores are first normalized to the range [0, 1]. The pairwise absolute differences of the normalized scores are summed to yield the performance measure of a group. Note that the lower score indicates a better performance, in contrast to the rest of the performance measures.

3.3. Collection 3

The shapes in the third collection are segmented by both humans and computer algorithms in [20]. We consider the data collected from humans to be an indicator of a shape's complexity. The fact that these human annotations differ is consistent with the ill-posed nature of specifying both segmentation and complexity. For each shape, there are 11 human-generated segmentations and 7.90 segments, on average. We use two ground truths: one is the order induced on the shapes by the mean number of segmentations (μ) and the other is the order acquired by the standard deviation (σ) of the number of segments. For each ground truth, we calculate *i)* Kendall rank correlation coefficient over all the shapes in the collection which we refer to as $\tau_{\mu_{\text{all}}}$ and $\tau_{\sigma_{\text{all}}}$ in Table 4 *ii)* the averaged coefficients $\frac{1}{N}\sum_i \tau_i$ where τ_i is the correlation coefficient for the ith category, referred to as $\tau_{\mu_{\text{cat}}}$ and $\tau_{\sigma_{\text{cat}}}$.

4. Methods

We present the examined methods in this section. A total of 19 methods are presented in 6 groups:

1. A multi-scale measure of complexity for arbitrary dimensional discrete shapes [16] by M. F. Arslan, § 4.1,
2. Alpha-shape complexity [21] by J. Gardiner and C. Brassey, § 4.2
3. Discrepancy [15] by A. Genctav, § 4.3,
4. PARCELLIN distance [14] by M. Genctav, § 4.4,
5. 2D multi-view based shape convexity measures C_1, C_2, [22], [23] by P. L. Rosin § 4.5
6. 2D multi-view based shape complexity measures [24], [4], C_{CRE} ([25]), [26] and C_σ, [27], [28], [29], C_{PC} ([19]) from the literature, § 4.6.

4.1. *A Multi-scale Measure of Complexity for Arbitrary Dimensional Discrete Shapes (M.F. Arslan)*

Assuming the space (of any dimensions) in which the shape S is embedded has uniform grid, we solve the following partial differential equation (PDE) inside S

$$\left(\Delta_\infty - \frac{1}{\rho^2}\right) f_S = -1 \text{ subject to } f_S\big|_{\partial S} = 0 \qquad (1)$$

where Δ_∞ is the Laplace operator in L^∞. The term $\Delta_\infty f$ is the minimizer of $\int |\nabla f|^p$ as $p \to \infty$. The parameter ρ is chosen as the maximum of the L^∞ distance transform of S (the field is referred to as t from now on). This choice ensures the robustness of solutions under changes in scale.

We construct f_S using the iterative scheme given in [16]. However, as the 3D shapes in the collections contain high number of voxels, instead of applying the convergence conditions used there, we start with a guided initial assumption and solve for a fixed number of steps. For the shapes in Collection 1, we solve for 200 steps, whereas for the shapes in Collection 2 and 3 (sampled to fit into a rectangle of total volume 300×300×300 voxels) we solve for 600 steps. The guided initial assumption is the analytical solution of (1) for an axis-aligned origin-centered rectangle whose value at the point $(x, y) \in S$ is given as:

$$f_S^{(0)}(x,y) = \rho^2 - \rho^2 \frac{e}{e^2+1} \times \left(\exp\left\{\frac{\max\{|x|,|y|\}}{\rho}\right\} + \exp\left\{\frac{-\max\{|x|,|y|\}}{\rho}\right\}\right).$$

In vague terms, f_S can be regarded as a well-behaving distance transform. The discrepancy between f_S and t is due to the smoothed propagation of the level sets of f_S in comparison to those of t. We use the entropy of the values of $\hat{f}_S$ (f_S normalized to $[0, 1]$) collected from a level set $t = t_0$ to measure the discrepancy at the scale t_0. We construct a pseudo-probability distribution acquired from $\hat{f}_S\big|_{t=t_0}$ by partitioning it into 1024 bins and normalizing it to have a total sum of 1. The entropy of this distribution gives the complexity of the shape at the scale $t = t_0$.

The submitted scores are the summation of the complexities at scales $t \leq 0.1$.

4.2. Alpha-shape Complexity (J. Gardiner and C. Brassey)

Model pre-processing. All data collections were pre-processed to produce the prerequisite 3D point clouds for subsequent alpha-shape complexity analysis. Each model's original volume and a reference length (to be used as a metric of the model's scale) were also calculated during pre-processing. Models in Collection 1 were stored in a 3D voxel format, and the points clouds were produced by taking the row, column and depth of each voxel's location as the x, y and z coordinates respectively and randomly down-sampling to 100,000 points. Previous analyses [30, 21, 31] have found 100,000 points to be a good compromise between retaining sufficient detail of the original model and minimising calculation times. The volume of each model in Collection 1 was calculated as the sum of the number of voxels in the original model. The models in Collections 2 and 3 were stored as watertight surface meshes. Point clouds were produced by generating a spatially random distribution of 100,000 points inside each mesh (Fig. 4). The volume of each model in Collections 2 and 3 was calculated as the volume of the original watertight mesh.

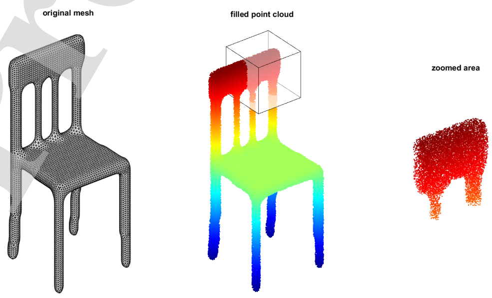

Fig. 4. Example generation of a point cloud from an original watertight mesh by filling the internal volume of the mesh with a random distribution of 100,000 points.

Across all models, a reference length was calculated to be used for model scaling within the complexity algorithm. Reference length was calculated as the mean of the distance from 10,000 random points (10% of the point cloud) to their nearest 100 neighbors.

Alpha-shape complexity algorithm. Once model point clouds, and their associated volumes and reference lengths, had been calculated, the alpha-shape complexity algorithm (originally developed for analysing biological datasets lacking homologous landmarks [30, 21, 31]) was run. Alpha-shapes [32] are a suite of shapes fitted to underlying point clouds, with the 'tightness' of their fit being determined by the value of the radius α.

For large values of α, the fit is coarse and tends to a convex hull as α approaches infinity (Fig. 5). For smaller values of α the fit conforms tightly to the underlying 'shape' of the object, until the single alpha shape fit breaks down and begins to form multiple separate objects as α approaches the smallest distance between any two points within the point cloud, where no fit will be achieved.

To calculate the shape complexity of each model, ten separate alpha-shapes were fitted to each point cloud across a range of α values (Fig. 5), from highly refined (corresponding to fine scale complexity) to very coarse (corresponding to gross scale complexity). To account for differences in the absolute size of models, the α used for each model was scaled by the reference length such that

$$\alpha_m = k \times l_{\text{ref}}$$

where α_m is the model-specific alpha radius, k the refinement coefficient and l_{ref} the point cloud reference length calculated in pre-processing. For the ten alpha-shape fits calculated, the same ten values of k (equally spaced on a logarithmic scale) were used, ensuring fits are equivalent across each model despite differences in absolute scale. Alpha-shapes were fitted using the "alphavol" function of Jonas Lundgren (www.mathworks.co.uk/matlabcentral/fileexchange/28851-alpha-shapes).

Following shape fitting, 'volume ratios' were calculated for each of the fits as the ratio of alpha-shape volume to the original model's volume (calculated in pre-processing). Relatively larger volume ratios therefore correspond to greater complexity in the model at any given scale. To further boil down the results of the alpha-shape analysis, the ten volume ratios produced for each model were subject to principal component analysis (PCA) as a dimension reduction technique. PCAs were run for each collection separately and the first two principal component scores were taken as the complexity metrics for each model. All data pre-processing and analysis were performed in Matlab R2020b (Mathworks Inc., Natick, USA).

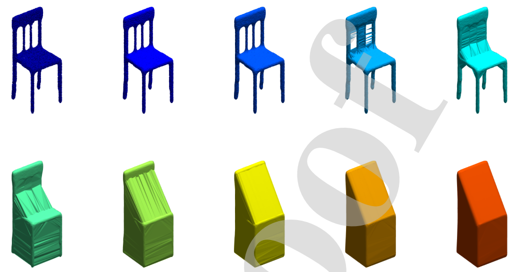

Fig. 5. Example chair model illustrating the ten alpha-shape fits used in the complexity analysis, ranging from highly refined (top left) to very coarse (bottom right). Particular underlying details of the model's point cloud are resolved at the different scales of fit, from only gross shape at the coarsest fits to details of the chair's legs and back at finer scales.

4.3. Discrepancy (A. Genctav)

Discrepancy [15] is a field $D : \Omega \rightarrow R$ defined over shape domain Ω. At each shape point p, it measures local deviation from a reference disk shape. Radius of the reference disk is defined using a global shape property A that is radius of maximally inscribed circle of Ω. The deviation is measured indirectly using an auxiliary field, which is selected as solution of the following screened Poisson equation:

$$\left(\Delta - \frac{1}{A^2}\right) v = 0 \text{ subject to } v\Big|_{\partial\Omega} = 1 \qquad (2)$$

The auxiliary field is obtained numerically for Ω and analytically for the reference disk. For each shape point p, discrepancy $D(p)$ is computed as the difference between value of the auxiliary field at p and value of the auxiliary field at the corresponding point of p in the reference disk. For each shape point p, the corresponding point in the reference disk is specified using its minimal distance to the shape boundary $\partial\Omega$. Due to uniform inhomogeneous boundary condition in (2), the auxiliary field is circularly symmetric for the reference disk, so it takes the same value at each point with the same minimal distance to the boundary. For more information including the implementation details, the reader is referred to [15].

Discrepancy is uniformly zero for a perfect disk and, hence, the entropy is 0. As shape deviated from a disk, discrepancy takes its highest positive values on central regions and lowest negative values on appendages, protrusions and boundary detail, and the entropy increases.

In this work, we used discrepancy entropy for measuring shape complexity. As discrepancy is presented in [15] for 2D shapes, our method works on the 2D views of 3D shapes.

4.4. PARCELLIN Distance (M. Genctav)

In this method, following the idea presented in [14], (3) is solved simultaneously with different source functions, which are designed for exploration of the shape volume, subject to homogeneous Dirichlet boundary conditions.

$$(\Delta - \alpha)\,\Phi_i = -f_i \tag{3}$$

where Δ denotes the Laplace operator, and α is a small damping parameter introduced for numerical conditioning and $i = 1, 2, \ldots, n$.

By design, each source function represents an initial hypothesis for a decomposition of the shape volume into central and outer regions which correspond to positive and negative sets in the steady state distribution, respectively. Specifically, the source function selected for the ith solution is

$$f(x)_i = \mathrm{sign}\,(d(x) - i \times s)$$

where $s = 1/n$ is the step size and $d(x)$ is the normalized signed distance between the location x and the boundary point nearest to x. The normalization is performed by dividing the raw distances to their maximum value.

Once a set of $n = 70$ equations are solved, the shape information contained in the solutions Φ_i are aggregated by assigning each shape location the number of solutions in which the location falls into the outer region, i.e. attains a negative value.

Finally, to obtain a measure of complexity, the entropy is computed on the 70 bin histogram of function values near shape boundary.

4.5. 2D Multi-view Based Shape Convexity (P.L. Rosin)

Several methods for measuring convexity were tested as it is hypothesised that a convexity measure can act as a shape complexity measure. It is likely that oscillations and irregularities in a shape's boundary which lead to scores indicating lower convexity will also indicate high complexity.

The method [22] measured convexity by applying a polygonal convexification process which applies a *flip* operation that reflects a polygon's concavities about their corresponding edges (termed *lids*) in the convex hull. The process is guaranteed to converge to a convex polygon in a finite number of flips. To ensure repeatability for similar shapes, the order of flipping is standardised. At each iteration the maximum deviation between each pocket and its lid is determined, and the pocket with the largest deviation is selected for flipping. Convexity is measured as the ratio of the areas of the original and convexified polygon. An alternative version is also considered, in which the pocket is flipped and also has the order of its vertices reversed (a *flipturn*). To improve computational efficiency and also reduce sensitivity to digitisation effects, polygonal approximation is first applied to the shape boundaries [33] using a small error tolerance (0.5).

The convexity measure [23] of shape S is given by

$$C(S) = \min_{\theta \in [0, 2\pi]} \frac{\mathcal{P}_2(\mathcal{R}(S,\theta))}{\mathcal{P}_1(S,\theta)}$$

where $\mathcal{P}_1(S,\theta)$ denotes the l_1 perimeter of S after rotation by angle θ, and $\mathcal{P}_2(\mathcal{R}(S,\theta))$ is the l_2 (Euclidean) perimeter of the minimum area bounding rectangle of S. Polygonal approximation is first applied to the shape boundaries, using an error tolerance of 2.

The two standard convexity measures in the literature are included: If we denote the convex hull of polygon S by $\mathbf{CH}(S)$ then the measures are defined as

$$C_1(S) = \frac{\mathbf{area}(S)}{\mathbf{area}(\mathbf{CH}(S))}$$

and

$$C_2 = \frac{\mathcal{P}_2(\mathbf{CH}(S))}{\mathcal{P}_2(S)}.$$

4.6. 2D Multi-view Based Shape Complexity

The method [24] attempts to capture global and local aspects of a shape in order to measure its complexity. It uses a linear combination of three quantitative terms: the number of notches (non-convex vertices) normalized to be in the range $[0, 1]$, and two terms similar to C_1 and C_2.

The method [4] uses the entropy of boundary turning angles (i.e. the subtended angle at each point); this is used as a discrete alternative to curvature. We use Sturges' rule to select the bin size for estimating the probability distribution. An alternative version was tested, where cumulative residual entropy [25] was used instead of Shannon entropy, and is denoted as C_{CRE}.

Another approach that considers curvature is [26], who use the sum of absolute Gaussian curvature to calculate the complexity of curved surface shapes. We apply a version to two-dimensional shapes, and also calculate the standard deviation of signed curvature as another measure of complexity, denoted as C_σ. In order to make these measures scale invariant, the shapes are first scaled to a fixed area (100,000) and uniformly sampled along the interpolated boundary at a fixed resolution (single unit steps).

The method [27] measures k-regularity (i.e. wiggliness or fractal dimension) of curves based on a ratio between lines lengths (distances between points on the curve) at different scales. The local value at point p_i of S is given as

$$r_{s,k}(S)(i) = \frac{\|p_{i+ks} - p_i\|}{\sum_{j=1}^{k} \|p_{i+js} - p_{i+(j-1)s}\|}$$

and the k-regularity of the shape is the mean value of $r_{s,k}(S)(i)$ over S. Our experiments used the values $s = 2$ and $k = 3$.

The method [28] computes the fractal dimension of a curve by estimating a shape's perimeter using a series of ruler lengths. We use the hybrid (Clark) method which is a combination of two other methods, the fast and exact algorithms. Fractal dimension is then estimated as the slope of the regression line computed for log versus log plots of ruler length versus perimeter.

Another fractal approach (the averaged mass dimension method) is given by [29] who use a version of box counting but replace the box with a circular neighborhood. To obtain a more robust line fit, we find the line with least median absolute error rather than least mean squared error.

We have implemented a method, denoted as C_{PC}, that is a simplified version of [19]. Their insight was that simple shapes lead to similar views whereas complex ones result in dissimilar views. In our version this is measured by performing a pairwise comparison of the boundaries of all the views for a given model, and returning the mean score across all comparisons. Arkin *et al.*'s [34] method for comparing polygons is used since it is invariant under translation, rotation, and scaling.

5. Results

Since ground truths provide only the order information, we are interested in the order relations rather than linear relationship between actual values that could be measured by Pearson correlation coefficient or any other parametric relation. Even for pairwise comparison of measures in Section 6, order correlation seems as a more meaningful measure rather than some preassumed parametric relation which may or may not exist. Hence, we use only Kendall rank correlation as a robust rank correlation measure. We report Kendall rank correlation coefficients between the participating methods and the ground truths in Tables 1-4. In the tables, we mark the scores of the best performing methods with red, the second best performers with green, and the third best performers with blue.

5.1. Collection 1

The Kendall rank correlation coefficient (τ) for the additively perturbed cubes and spheres are given in Table 1. For the cubes [16], and for the spheres [15] induce the correct order on all considerations. In both cases, [14] and [21]-1 follow very closely. Some of the methods ([4], [27], C_{CRE}, [23], [22]-1, [22]-2 and C_2) achieve strong correlations when the parameter c is varied, yet the correlations weaken when the parameter w is varied. This suggests that it is easier to account for the number of perturbations than it is for the magnitude of perturbations. Likewise, performances of some of the methods are sensitive to the base shape. Notable are [15], [28], [29] and C_1.

Comparing 3D methods with 2D ones, we see that 3D methods [16], [14] and [21]-1 consistently score highly whereas the best performing 2D methods performs only partially well. For example, [15] only achieves high scores for the sphere-related tasks and [24], [4], C_{CRE}, C_{PC}, and so on, score high only when the width parameter w is kept fixed.

Four submissions have been run on the subtractively perturbed cubes and spheres. The performances of these are re-

Table 1. The averaged Kendall τ for the additively perturbed cubes (the first value) and spheres (the second value).

Method	w = 3	w = 4	w = 5	c = 25	c = 50	c = 75
[16]	1.00 / 1.00	1.00 / 1.00	1.00 / 0.99	1.00 / 1.00	1.00 / 1.00	1.00 / 1.00
[14]	0.87 / 0.93	0.95 / 0.97	0.93 / 1.00	0.89 / 0.93	0.92 / 1.00	0.96 / 1.00
[21]-1	0.99 / 0.97	0.99 / 1.00	0.99 / 1.00	1.00 / 0.99	1.00 / 1.00	1.00 / 1.00
[21]-2	0.17 / 0.57	0.24 / 0.80	0.52 / 0.88	-0.09 / 0.35	0.03 / 0.69	0.39 / 0.69
[15]	0.25 / 1.00	0.31 / 1.00	0.68 / 1.00	0.13 / 1.00	0.39 / 1.00	0.52 / 1.00
[29]	0.67 / 0.91	0.84 / 0.99	0.80 / 0.97	0.84 / 0.68	0.95 / 0.71	0.93 / 0.68
[28]	0.45 / 0.97	0.65 / 0.99	-0.11 / 0.96	0.44 / 0.83	0.43 / 0.89	0.04 / 0.84
[27]	-0.97 /-1.00	-0.96 /-1.00	-0.97 /-1.00	-0.29 /-0.08	-0.29 / 0.05	-0.24 / 0.28
C_{CRE}	0.97 / 1.00	0.95 / 1.00	0.96 / 1.00	0.33 / 0.12	0.47 /-0.07	0.47 /-0.35
[4]	0.93 / 1.00	0.96 / 1.00	0.96 / 1.00	0.20 / 0.28	0.39 / 0.17	0.32 /-0.21
[26]	0.79 / 0.87	0.87 / 0.93	0.87 / 0.93	0.83 / 0.83	0.88 / 0.83	0.88 / 0.79
C_σ	0.51 / 0.72	0.60 / 0.80	0.64 / 0.87	0.68 / 0.60	0.68 / 0.63	0.69 / 0.64
C_1	-0.73 /-0.97	-0.83 /-0.97	-0.76 /-0.95	-0.77 /-0.89	-0.81 /-0.95	-0.93 /-0.96
C_{PC}	0.93 / 0.96	0.91 / 0.93	0.88 / 0.97	0.61 / 0.53	0.71 / 0.51	0.64 / 0.36
[24]	0.93 / 1.00	0.95 / 0.99	0.91 / 0.99	0.73 / 0.69	0.83 / 0.77	0.79 / 0.64
[23]	-1.00 /-0.99	-0.96 /-0.99	-0.91 /-0.96	-0.68 /-0.59	-0.77 /-0.68	-0.76 /-0.61
C_2	-0.95 /-0.99	-0.96 /-0.99	-0.93 /-0.99	-0.67 /-0.53	-0.80 /-0.63	-0.77 /-0.51
[22]-1	-0.96 /-0.99	-0.93 /-0.99	-0.92 /-0.99	-0.59 /-0.68	-0.76 /-0.76	-0.76 /-0.64
[22]-2	-0.96 /-0.99	-0.95 /-0.99	-0.92 /-0.99	-0.65 /-0.68	-0.76 /-0.76	-0.75 /-0.63
MA	0.79 / 0.94	0.83 / 0.96	0.82 / 0.97	0.60 / 0.65	0.68 / 0.69	0.68 / 0.68

ported in Table 2. For the cubes, [14] ranks the first in all measurements, and for the spheres there is no clear winner.

In the last rows of Tables 1 & 2 we provide the mean of the absolute scores, denoted as *MA*. The mean absolute scores show that the most challenging case is the cubes with c = 25 for both the additive and subtractive cases. We also note that for the additive perturbations it is significantly harder for the considered methods to correlate with the ground truth when the parameter *w* is varied.

Table 2. The averaged Kendall τ for the subtractively perturbed cubes (the first value) and spheres (the second value).

Method	w = 3	w = 4	w = 5	c = 25	c = 50	c = 75
[16]	1.00 / 0.99	1.00 / 0.99	1.00 / 1.00	0.67 / 0.92	0.89 / 1.00	0.91 / 0.97
[14]	1.00 / 0.89	1.00 / 0.88	1.00 / 0.93	0.97 / 0.97	0.97 / 1.00	0.99 / 1.00
[21]-1	0.68 / 0.81	0.81 / 0.95	1.00 / 1.00	0.77 / 0.83	0.91 / 1.00	0.97 / 1.00
[21]-2	0.21 / 0.48	0.48 / 0.83	0.87 / 0.96	0.37 / 0.61	0.69 / 0.80	0.81 / 0.95
MA	0.72 / 0.79	0.82 / 0.91	0.97 / 0.97	0.70 / 0.83	0.87 / 0.95	0.92 / 0.98

5.2. *Collection 2*

For the first family of Collection 2, the top five methods are C_{CRE}, C_{PC}, [27], [4] and C_2 according to the summed scores given in Table 3. Note that all of these are 2D methods, three of which are convexity measures. The best performing 3D method is [14] and places the sixth. [26] and [21]-2 perform the poorest on this family both having almost no correlations with the ground truth considering all of the groups.

For each group except the fifth, there is at least one method that completely agrees (or disagrees) with the ground truth. It seems that none of the considered methods is able to capture the notion of complexity that induces the order given by the ground truth for Group 5. The *MA* scores are in alignment with this, indicating that Group 5 is the most challenging group. We also note that the highest *MA* is attained by Group 3 that consists only of three elements which is the minimum number required to attain a non-trivial Kendall rank correlation coefficient (τ).

Strangely, some of the methods ([28], [29], [15] and [21]-2) have both strongly positive and strongly negative correlations with the ground truth.

Table 3. The Kendall τ for the first family (the first value) and the non-uniformity measurements for the second family (the second value) of Collection 2.

Method	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Sum
[16]	0.33 / 2.91	0.67 / 2.08	0.00 / 0.16	1.00 / 0.66	0.47 / 2.53	– / 2.75	2.47 / 11.09
[14]	0.81 / 1.82	1.00 / 1.31	0.00 / 0.02	0.80 / 0.28	0.33 / 2.78	– / 0.52	2.94 / 6.73
[21]-1	-0.52 / 0.88	-0.33 / 1.65	-1.00 / 0.41	0.80 / 0.06	0.60 / 0.75	– / 1.05	-0.46 / 4.79
[21]-2	0.62 / 0.87	0.33 / 1.83	-1.00 / 0.53	-0.60 / 0.09	0.73 / 0.49	– / 1.46	0.09 / 5.27
[15]	-0.81 / 0.86	0.33 / 1.60	1.00 / 0.38	-0.20 / 0.63	-0.60 / 0.67	– / 3.33	-0.28 / 7.47
[29]	-0.81 / 1.37	0.67 / 1.91	1.00 / 0.31	0.40 / 0.35	0.47 / 1.00	– / 1.62	1.72 / 6.56
[28]	-0.43 / 1.57	-0.67 / 0.71	1.00 / 1.93	0.00 / 0.08	0.33 / 0.12	– / 0.24	0.24 / 4.65
[27]	-1.00 / 1.75	-1.00 / 2.15	-1.00 / 1.08	-0.40 / 0.34	-0.47 / 1.43	– / 5.55	-3.87 / 12.31
C_{CRE}	0.90 / 1.72	1.00 / 2.10	1.00 / 0.99	0.60 / 0.26	0.47 / 0.86	– / 4.28	3.97 / 10.21
[4]	0.81 / 2.01	0.67 / 2.98	1.00 / 1.76	0.80 / 1.63	0.47 / 0.80	– / 6.04	3.74 / 15.23
[26]	-0.71 / 1.06	-0.33 / 0.84	0.33 / 0.24	0.40 / 0.21	0.33 / 0.80	– / 1.33	0.02 / 4.47
C_σ	-0.81 / 1.44	0.00 / 1.29	0.33 / 0.17	0.40 / 0.17	0.33 / 0.61	– / 1.19	0.26 / 4.87
C_1	0.05 / 1.31	-0.33 / 1.88	-0.33 / 0.10	-0.80 / 0.09	-0.73 / 0.45	– / 0.59	-2.15 / 4.41
C_{PC}	0.62 / 1.19	0.67 / 1.49	1.00 / 0.20	1.00 / 0.14	0.60 / 1.13	– / 0.62	3.89 / 4.78
[24]	0.43 / 1.17	0.33 / 1.66	1.00 / 0.16	0.60 / 0.10	0.47 / 0.33	– / 0.92	2.83 / 4.33
[23]	-0.52 / 0.98	-0.33 / 1.37	-1.00 / 0.10	-0.60 / 0.25	-0.47 / 0.75	– / 1.42	-2.92 / 4.88
C_2	-0.43 / 1.28	-0.33 / 1.41	-1.00 / 0.19	-0.80 / 0.18	-0.47 / 0.29	– / 1.02	-3.03 / 4.37
[22]-1	0.14 / 0.93	-0.33 / 1.59	-1.00 / 0.11	-0.80 / 0.20	-0.47 / 0.43	– / 1.00	-2.46 / 4.27
[22]-2	-0.05 / 1.12	-0.33 / 1.85	-1.00 / 0.13	-0.80 / 0.16	-0.60 / 0.43	– / 0.82	-2.78 / 4.51
MA	0.57 / 1.38	0.51 / 1.67	0.79 / 0.47	0.62 / 0.31	0.49 / 0.88	– / 1.88	2.11 / 6.59

For the second family of Collection 2, we start by remarking that the reported scores indicate better performances when they are close to 0, in contrast with the other reported scores. Similar to the case in the first family, 2D methods take the lead (listed from best to worst: [22]-1, [24], C_2, C_1, [26]), with the best performing 3D method ([21]-1) placing the 8th. The worst performing method is [4]. This is interesting because it is also the third best performing method in the first family. In a similar manner, we note that there is no overlap between the top five performers of the two families except for C_2.

The top three performers for both families are 2D methods based on the summed scores. The highest scoring 3D method for the first family is [14] and [21]-1 for the second family.

5.3. *Collection 3*

The Kendall rank correlation coefficients computed for Collection 3 are reported in Table 4. The best performers are [21]-2, [26], [4] and [4] for $\tau_{\mu_{\text{cat}}}$, $\tau_{\mu_{\text{all}}}$, $\tau_{\sigma_{\text{cat}}}$ and $\tau_{\sigma_{\text{all}}}$, respectively.

All of the methods, except [21]-1, perform better when the correlations are computed over the whole collection, regardless of the ground truth.

For the tasks of this collection we observe that [21]-2 outperforms [21]-1. This is interesting because [21]-1 is a better performer for the majority of tasks involving the other two collections. Since these two are the first two principal components of the method of [21], this suggests that the segmentation accounts for an aspect of shape complexity different than those of the other collections.

We note that all of the top three performers are 2D methods, except for $\tau_{\mu_{cat}}$.

Table 4. Kendall τ when the ground truth is the mean and the standard deviation of the number of segments of the human segmentations.

Method	$\tau_{\mu_{\text{cat}}}$	$\tau_{\mu_{\text{all}}}$	$\tau_{\sigma_{\text{cat}}}$	$\tau_{\sigma_{\text{all}}}$
[16]	0.148	0.346	0.072	0.234
[14]	0.041	0.354	0.018	0.203
[21]-1	0.110	0.105	0.055	-0.006
[21]-2	0.151	0.417	0.065	0.262
[15]	-0.022	0.251	0.013	0.138
[29]	0.061	0.401	0.027	0.202
[28]	0.140	0.375	0.087	0.167
[27]	-0.082	-0.458	-0.089	-0.282
C_{CRE}	0.110	0.585	0.083	0.331
[4]	0.132	0.600	0.117	0.350
[26]	0.131	0.671	0.066	0.283
C_{σ}	0.075	0.540	0.077	0.244
C_1	-0.037	-0.255	-0.037	-0.078
C_{PC}	0.099	0.464	0.078	0.217
[24]	0.041	0.326	0.029	0.123
[23]	-0.105	-0.486	-0.056	-0.219
C_2	-0.112	-0.501	-0.084	-0.227
[22]-1	-0.069	-0.372	-0.085	-0.168
[22]-2	-0.079	-0.395	-0.090	-0.188
MA	0.092	0.416	0.065	0.206

6. Discussion

Despite the lack of full shape information, 2D methods are observed to perform unexpectedly well when compared with 3D methods, especially for Collection 3. However, it should be noted that for the results presented in this paper, the participating 3D methods also do not make use of the full shape information because some of them ([14, 21]) down-sample the shapes in Collection 1 and they all need to voxelize the shapes in Collection 2 and 3. Also, [16] solves for only a limited number of steps for all of the collections due to the size and the number of the shapes and relatively costly computation.

One of the interesting observations is that [4], [27] and [26] perform poorly under the changes of the parameter w despite their high scores under the changes of the parameter c. This might be explained by the fact that changing the width parameter w has a greater impact on the local changes in curvature than the parameter c and that these methods are inherently curvature-dependent. Similar results for the methods that measure convexity can be explained in a similar manner since convexity can be related to curvature for the examples in our datasets. Note also that this observation highlights the non-triviality of the noisy collections.

Assessing the results for Collection 1 suggests the use of different methods for different use cases. For example, [16], [21]-1 can be used in applications involving additive perturbations and [15] can be used in applications involving noisy spheres. Provided one has information about the type of the noise present in their use cases, one can settle for [4], [27], C_2, [22], or [23]. For overall robustness [14] can be preferred. The results for Collection 2 suggest that classical measures supported by psychology experiments are still better alternatives for quantifying perceptual complexity as judged from the final product of the design process (*i.e.* ignoring the generation level complexity). For Collection 3, we observe that the performances of the methods improve significantly when the entire collection is considered. In this sense, we can say that the task of correlating complexity with the segmentation is harder when the shapes are from the same category.

In Fig. 6, two 2D embeddings of the evaluated complexity measures using Stochastic Neighborhood Embedding (t-SNE) [35] are depicted. For each measure a high dimensional feature vector is formed using the Kendall rank correlation coefficients reported in Tables 1-4. For the plot on the left, 17-dimensional feature vectors (whose components are the twelve τ scores from Table 1 and five τ scores from Table 3) are used. For the plot on

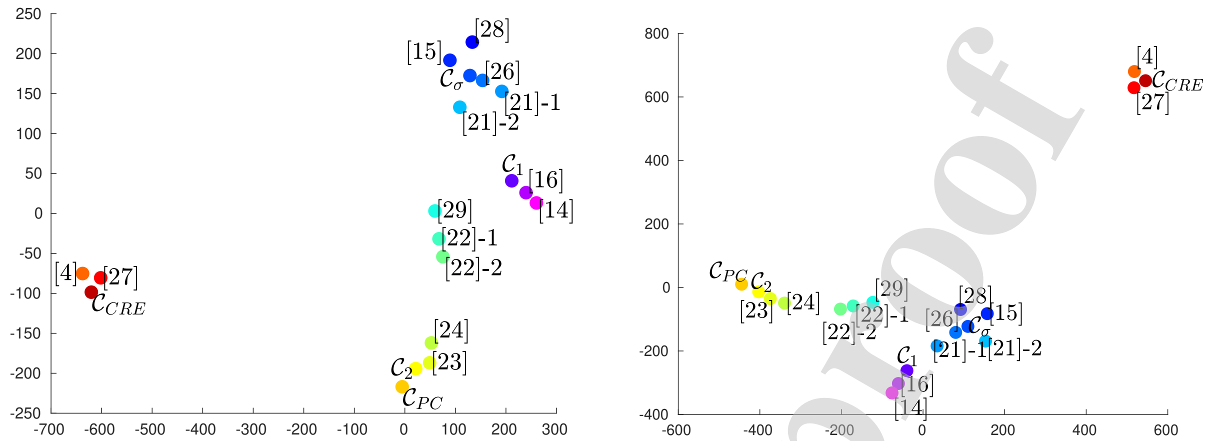

Fig. 6. Clustering of the methods in τ-based feature space: 2D embedding applied to 17 (left) and 21 (right) τ-values

the right 21-dimensional feature vectors are used by augmenting the 17-dimensional vectors with four additional τ scores from Table 4. We considered the scores from Collection 3 as optional because we feel that the nature of this collection is different from those of the first two collections. Note that we negate the τ scores of the methods, [27], C_1, [23], C_2, [22]-1 and [22]-2 as they serve as measures of simplicity rather than complexity. In both plots the perplexity parameter is set to 2. Nevertheless, we have observed that doubling or even quadrupling the perplexity parameter does not make a significant qualitative change except that the spread gets larger. Notice that [4], [27] and [26] form a distinct cluster. Another interesting observation from Fig. 6 is that the two methods [15] and [16], both employing real valued fields computed using a common Partial Differential Equation, are not close in the τ-based feature space. This is because these methods use different metrics. The choice of the metric makes [16] an ideal method for noisy cubes whereas the other is better suited for noisy spheres.

In addition to correlations between the ground truths and the order induced by the measures, we believe that the correlations among the orders induced by the measures convey insight into the ill-defined concept of shape complexity. Hence, we report in Fig. 7 Kendall rank correlation coefficients (τ) for each pair of methods over the dataset. Specifically, for Collection 1 we compute the mean of τ_i ($i \in \{1, 2, \ldots, 50\}$) for the groups of each family, for Collection 2, we compute τ over the families (*i.e.* disregarding the groups), and for Collection 3 we compute both the mean of τ_{cat} over the categories and τ_{all} over the whole collection. Here also we negate the scores of the methods, [27], C_1, [23], C_2, [22]-1 and [22]-2. The results show that the methods correlate the most to each other over the additively perturbed spheres. This could be explained by noting that the different approaches of the methods towards complexity, such as uniformity of curvature, convexity, or the agreement of the shape with the underlying grid, more or less agree for this family. Similar clusters to the ones seen to emerge in Fig. 6 can be identified, such as [4], [27] and C_{CRE} or the cluster consisting of convexity measures. Yet, for example, the second family of Collection 2 provides a means of distinguishing [4] from C_{CRE} and [27]. The same family also allows us to observe the differences between the behaviors of the 3D methods. Similarly, the results acquired for Collection 3 by comparing shapes from the same categories show that [23] and C_2 are more close to each other than they are to [22]-1 and [22]-2, and vice versa. Together, these provide support for our claim that the three collections account for different aspects of shape complexity.

The correlations for Collection 3 are generally lower than those for Collections 1 and 2. This could be a consequence of either the data being more challenging, or else that the proxy task does not map strongly to complexity. This needs further study, and can be explored in future work.

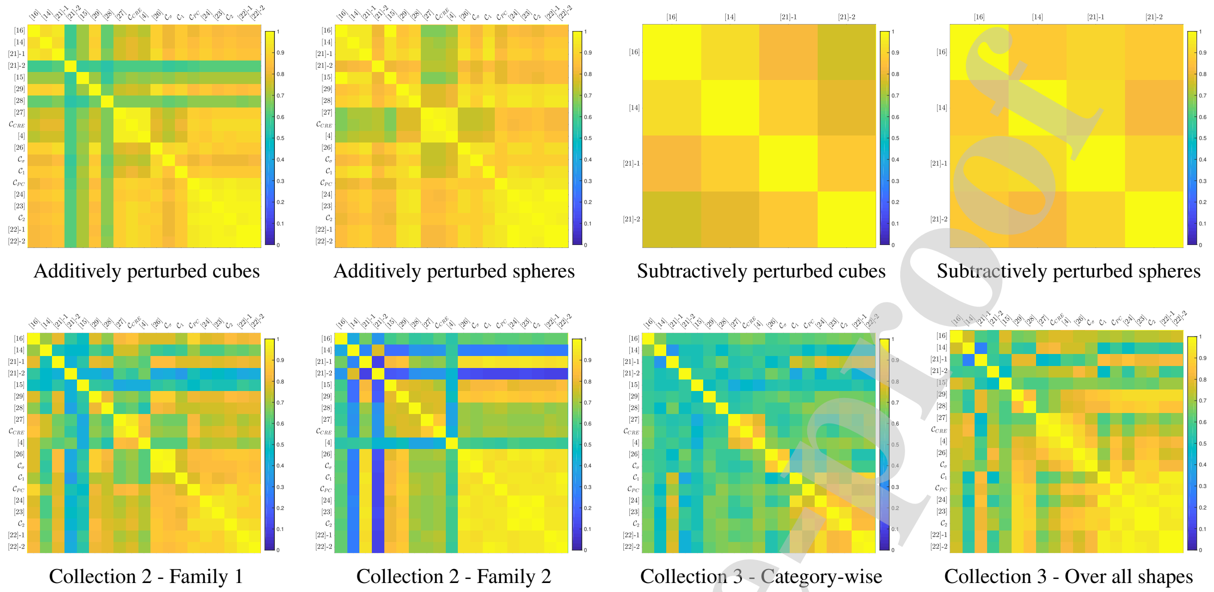

Fig. 7. Correlations between pairs of methods

7. Conclusion

We have introduced a novel 3D dataset to evaluate shape complexity measures. Using this dataset we not only evaluated the methods with respect to ground truth but also with respect to each other under a rich variety of ordering tasks in order to see how they are related in the context of shape complexity. To evaluate methods with respect to each other, we clustered measures in the *tau*-based feature space, and displayed pairwise rank correlations between orders induced by all pair of methods.

We conclude the paper by noting that the evaluation methodology of the paper is a significant improvement on the current literature in the sense that the reported scores are quantitative with justified ground truths and the analysis is reproducible. Since the research in 3D shape complexity is still in its infancy, we believe that this work will encourage further explorations of the field.

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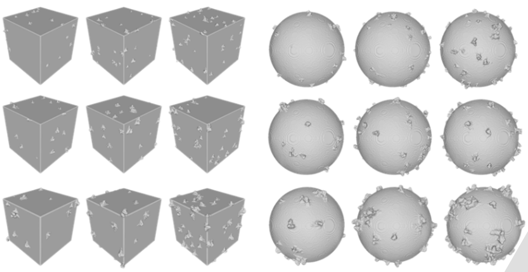
Research Highlights

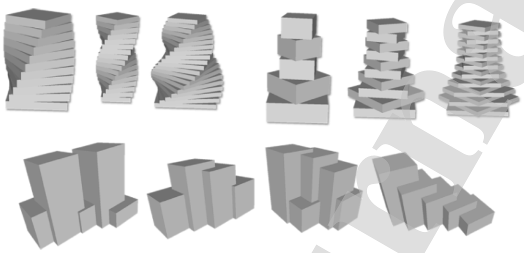
- Two novel datasets with associated ground truth are introduced.
- Segmentation is used as a proxy task for measuring complexity.
- The performance of 2D and 3D complexity measures are systematically compared and evaluated.

Data Contribution

Collection 1

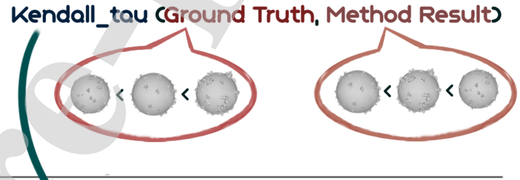


Collection 2



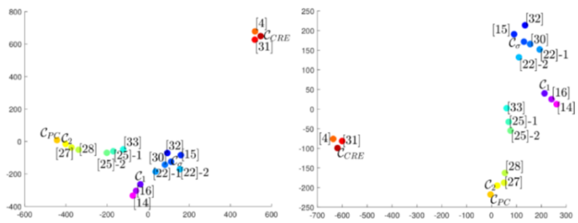
Methodology

Analysis with Ground Truth



Method	w = 3	w = 4	w = 5	c = 25	c = 50	c = 75
[16]	1.00 / 1.00	1.00 / 1.00	1.00 / 0.99	1.00 / 1.00	1.00 / 1.00	1.00 / 1.00
[14]	0.87 / 0.93	0.95 / 0.97	0.93 / 1.00	0.89 / 0.93	0.92 / 1.00	0.96 / 1.00
[22]-1	0.99 / 0.97	0.99 / 1.00	0.99 / 1.00	1.00 / 0.99	1.00 / 1.00	1.00 / 1.00
[22]-2	0.17 / 0.57	0.24 / 0.80	0.52 / 0.88	-0.09 / 0.35	0.03 / 0.69	0.39 / 0.69
[15]	0.25 / 1.00	0.31 / 1.00	0.68 / 1.00	0.13 / 1.00	0.39 / 1.00	0.52 / 1.00
[33]	0.67 / 0.91	0.84 / 0.99	0.80 / 0.97	0.84 / 0.68	0.95 / 0.71	0.93 / 0.68
[32]	0.45 / 0.97	0.65 / 0.99	-0.11 / 0.96	0.44 / 0.83	0.43 / 0.89	0.04 / 0.84
[31]	-0.97 / -1.00	-0.96 / -1.00	-0.97 / -1.00	-0.29 / -0.08	-0.29 / 0.05	-0.24 / 0.28

Cross-method Analysis



Title page for SHREC'21: Quantifying Shape Complexity

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The authors declare no conflicts