# Dynamic Characteristics Analysis of a Coupled Multi-crack Rotor System

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**Abstract.** This paper establishes a coupled model for multi-crack rotor using Timoshenko beam element with six degrees of freedom, and derives the stiffness matrix in the equations of motion accounting for the coupling between multiple cracks (the interaction between cracks). Then the effects of crack orientation angles (the relative angle between cracks, γ) on dynamic characteristics of the coupled multi-crack rotor near 1/3 and 1/2 subcritical speeds are analysed. The coupling between cracks induces more complex nonlinear dynamic characteristics such as large magnitudes of the super-harmonic components, which can be used as the indicators of early crack and for multi-crack identification. This work has a promotive significance for the application of the model-based method in the field of multi-crack detection of actual rotors.

**Keywords:** coupled multi-crack rotor; super-harmonic; orbit; crack orientation angle; early crack; multi-crack detection.

## 1. Introduction

Cracks are one of the most critical and fundamental damage in rotors, which may lead to a sudden and catastrophic failure of equipment. So it is of great significance to detect cracks in a timely manner and accurately, especially in the early stage of crack propagation in order to maintain healthy and stable operations of rotors and avoid failure of rotating machines. At present, the crack detection methods can be categorised as the model-based, the signal-based [1, 2], and the artificial intelligence-based methods [3]. The model-based method can directly extract features of cracks from the vibration responses based on the finite element(FE) model of a cracked rotor and identify detailed crack parameters, even study the relationship between crack parameters and vibration characteristics, so it is widely used in online monitoring of structural health and fault diagnosis of rotors [4].

In the model-based method, the effects of crack are usually regarded as a stiffness variation, additional equivalent loads [5], and massless rotational springs [6] in the FE model. Stiffness variation, reflecting the gradual crack breathing behaviour, is sensitive to the change of crack parameters. Chandra and Sekhar [7] considered the crack effect as additional flexibility in FE model and detected crack, misalignment and rub-impact faults in a rotor based on the startup vibration signals. Darpe et al. [8, 9] analysed the coupled bending, longitudinal and torsional vibration response features of a cracked rotor and studied the lateral interaction effect of two cracks. AL-Shudeifat et al. [10, 11] presented two new breathing functions in formulating the stiffness matrix and detected early cracks by the large vibration amplitudes at the backward whirl. Gayen et al. [12] studied the effects of crack location, orientation, size, and power-law gradient index on natural frequencies and critical speeds for a multi-crack shaft. The slope discontinuity was utilized to identify multiple cracks in a shaft by Singh and Tiwari [13, 14]. Lu et al. [15, 16] utilized the discontinuities in the characteristic proper orthogonal mode and the local shape distortions in the super-harmonic characteristic deflection shapes to localize multiple cracks in rotors. Lu and Chu [17] determined the crack location and depth in a shaft through the acoustic emission signals and the optimal solution of the difference between simulation and experimental vibration data.

The above-mentioned researches make full use of the advantages of the model-based method in crack detection of rotors, but almost all lack detailed description of the coupling between multiple cracks in six degrees of freedom, restricting the application of the model-based method for multi-crack rotors. However, the coupling between cracks always occurs in multi-crack rotors especially under the coupled bending and torsional vibrations, leading to more complex nonlinear vibration responses, which contain important fault features for accurate multi-crack identification. Therefore, a coupled multi-crack rotor model considering the coupling effects between cracks is proposed based on fracture mechanics in this paper. The remainder are constructed as follows. In Section 2, the stiffness matrix in the equations of motion is derived to account for the coupling between cracks in six degrees of freedom for a coupled multi-crack rotor. In Section 3, the system dynamics characteristics with respect to different crack orientation angles are analysed. Finally, the conclusions are summarized in Section 4.

## 2. Stiffness matrix of a coupled multi-crack rotor

The rotor system is composed of two discs, two ball bearings, and a shaft including two transverse breathing cracks with a relative angle *γ*, as shown in figure 1(a). The physical parameters of the rotor system are given in table 1.

The cracked shaft is discretized into 60 Timoshenko beam elements with length of *l* and radius of *R*, each element contains two nodes having six degrees of freedom each. The two cracks are assumed to have a certain distance to prevent their stress fields from interfering with each other [9]. ,  are the forces applied at the two crack cross-sections, shown in figure 1(a).

 (1)

 (2)

And the forces  could be denoted by :

 (3)

where  is called the coupling matrix between  and :

 

(a) (b)

**Figure 1.** Model of coupled multi-crack rotor system, (a) System details, (b) Cross section of crack.

**Table 1.** Physical parameters of rotor system.

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
| Shaft diameter, *D* | 0.01 m | Polar moment of inertia of disc 1 | 5.76×10−4 kg·m2 |
| Density of shaft (40Cr) | 7.87×103 kg·m-3 | Diametrical moment of inertia of disc 1 | 3.18×10−4 kg·m2 |
| Shaft length | 0.6 m | Polar moment of inertia of disc 2 | 5.84×10−4 kg·m2 |
| Young’s modulus, *E* | 2.11×1011 N·m-2 | Diametrical moment of inertia of disc 2 | 3.23×10−4 kg·m2 |
| Shear modulus, *G* | 8.26×1010 N·m-2 | Disc eccentricity | 2.0×10-5 m |
| Poisson’s ratio, *v* | 0.277 | Rayleigh damping coefficient, *a* | 0.684 |
| Gravitational acceleration | 9.8 m·s-2 | Rayleigh damping coefficient, *b* | 2.80×10-5 |
| Mass of disc 1 | 0.759 kg | First critical speed, *Ω*0 | 3229 rpm |
| Mass of disc 2 | 0.770 kg |  |  |

 (4)

Considering the coupling between cracks, the additional displacement vector  due to crack-i at crack-ii is written as:

 (5)

where  is the additional displacement vector at crack-i:

 (6)

So the total displacement vector at crack-ii is written as:

 (7)

where,  is the displacement vector of the uncracked element and  is the additional displacement vector at crack-ii, expressed as follows:

 (8)

 (9)

Based on Castigliano’s theorem and fracture mechanics concepts, and the additional flexibility coefficient  due to crack-i at crack-ii is derived as:

 (10)

where  is the additional displacement due to crack-i at crack-ii in the *i*-th direction,  is the force at the crack-ii cross-section along the *j*-th coordinate; .

From equation (5),  satisfies

 (11)

Here,  are the additional displacements at crack-i, the detailed expressions can be obtained from [8].

It can be obtained from equation (3)

 (12)

Substituting equation (12) into the additional displacements  at crack-i,  can be rewritten as:

 (13)

 (14)

 (15)

 (16)

 (17)

 (18)

where  and ,  is the shaft cross-sectional shape coefficient,  is the Poisson ratio,  is the torsional angle of a crack relative to the shaft axis,  is the area of crack region,  is the area of crack region at crack-i.  is the crack depth at any distance  from the centre along the crack edge and  is the total height of the strip with width , as figure 1(b). Here , , and , ,  are given as [8]:

 (19)

 (20)

 (21)

 (22)

The flexibility values  could be obtained from equation (10), then the flexibility matrix  is denoted:

 (23)

The total flexibility matrix of crack-ii element  is calculated as:

 (24)

where the flexibility matrix of the uncracked element  and the additional flexibility matrix  at crack-ii can be found in detail from [8] based on strain energy release rate theory [18].

Thus, the stiffness matrix of the crack-ii element can be described by :

 (25)

Here, the transformation matrix  is given by [8]:

 (26)

The above derivation process gives a detailed description of the coupling of multiple cracks in six degrees of freedom. Equations (24)-(25) show that the coupling between cracks increases the system flexibility and alters the system time-varying stiffness, further resulting in the changes of eigenmode and eigenfrequency of rotor systems. These changes in modal parameters can be used for crack detection and parameters identification, especially for the early cracks due to the high sensitivity for modal parameters to the small structure changes.

## 3. Dynamic characteristics analysis

The simulated vibrations of a coupled multi-crack rotor with different crack orientation angles are carried out. The vertical and horizontal responses, and the orbits at the 21st node (at the disc 1) near the rotating speed of 1/3*Ω*0 and 1/2*Ω*0 [19-21] are analysed compared with the results of an uncoupled multi-crack rotor [15, 16, 22]. The two transverse breathing cracks (crack-i and crack-ii) are located at the 26th and 36th elements with the same depth of 0.2*D*. In the following text, the coupled multi-crack rotor is referred to as the coupled rotor, and the uncoupled multi-crack rotor is abbreviated as the uncoupled rotor.

The vibration responses and the harmonic components at the 21st node for the coupled and uncoupled rotors are investigated when the crack orientation angle *γ* = 0, *π*/2, *π* rad and the unbalance orientation angle *β* = 0, as figures 3-5. It can be found that almost all displacements for the coupled rotor are larger than those of the uncoupled rotor in time domain responses.

 

(a) (b)

 

(c) (d)

coupled rotor uncoupled rotor

**Figure 2.** Multi-crack rotor responses with *γ* = 0, (a)Vertical response at 1/3*Ω*0, (b) Vertical response at 1/2*Ω*0,

(c) Horizontal response at 1/3*Ω*0, (d) Horizontal response at 1/2*Ω*0.

Figure 2 shows the vibration responses of the coupled and uncoupled rotors, and the 2X and 3X amplitudes(where X represents the rotating frequency of rotor) of the coupled rotor are nearly twice as those of the uncoupled rotor when *γ* = 0, the 1X amplitudes of the former are also larger than those of the latter.

 

(a) (b)

coupled rotor uncoupled rotor

**Figure 3.** Multi-crack rotor vertical responses with *γ* = *π*/2 rad, (a)1/3*Ω*0, (b)1/2*Ω*0.

As *γ* is *π*/2 rad, it can be seen from figure 3 that there is almost no 2X components for the coupled rotor, and the 3X component of the coupled rotor is nearly three times larger than that of the uncoupled rotor.

 

(a) (b)

coupled rotor uncoupled rotor

**Figure 4.** Multi-crack rotor vertical responses with *γ* = *π* rad, (a)1/3*Ω*0, (b)1/2*Ω*0.

From figure 4, the 1X of the coupled rotor are smaller than those of the uncoupled rotor, and there are only 1X and 2X components for the coupled rotor in the frequency spectra near 1/3*Ω*0 and 1/2*Ω*0.

Figure 5 indicates that the crack orientation angle has a great influence on the system harmonic components, especially for the coupled rotor. When *γ* changes from 0 to *π* rad, both the 1X and 3X components for the coupled and uncoupled rotors decrease gradually, the 2X components decrease first and then increase for these two rotors. This is because the two cracks change from simultaneous opening to partial opening, and then to one opening and another closing as the crack orientation angle increases from 0 to *π* rad.

 

(a) (b)

coupled rotor uncoupled rotor

**Figure 5.** Variation of harmonic components with crack orientation angle, (a)1/3*Ω*0, (b)1/2*Ω*0.

The shape characteristics of orbit, such as the concave and inner loop, can be used as important information for crack diagnosis of rotors. Figure 6 represents the orbits with different crack orientation angles when rotating speeds are near 1/3*Ω*0 and 1/2*Ω*0. Most of the orbits for the coupled rotor is little larger than the uncoupled rotor due to the increased system flexibility by the multi-crack coupling. The orbits with *γ* = 0 for the coupled rotor are well consistent with the results of a cracked rotor with a crack depth of 1.6mm in [23].

The orbit shapes under 1/2*Ω*0 for the coupled rotor are in accordance with the results of a Jeffcott rotor with two cracks in [9]. From figure 6 that the orbit shapes for the coupled rotor are generally more complex, which is resulted from the coupling of the two cracks. It can be also concluded from above time and frequency domain responses that the crack orientation angle has a great influence on the dynamic characteristics of a multi-crack rotor, especially on the super-harmonic components and the orbits. The large magnitudes of the super-harmonic components induced by the coupling of cracks can be used as the indicators of early crack and for multi-crack identification.

(I) 1/3*Ω*0

  

(a) (b) (c)

(II) 1/2*Ω*0

  

(d) (e) (f)

coupled rotor uncoupled rotor

**Figure 6.** Orbits with different crack orientation angles, (I)1/3*Ω*0: (a) *γ* = 0, (b) *γ* = *π*/2 rad, (c) *γ* = *π* rad; (II)1/2*Ω*0: (d) *γ* = 0, (e) *γ* = *π*/2 rad, (f) *γ* = *π* rad.

## 4. Conclusions

In the paper, a coupled multi-crack rotor model considering the coupling between cracks is established and the effects of crack orientation angles on dynamic characteristics of the coupled two-crack rotor are analysed.

(1) The coupling between cracks in six degrees of freedom changes the system time-varying stiffness, resulting in more complicated nonlinear dynamic characteristics of the multi-crack rotor system.

(2) Most of the vibration responses and orbits for the coupled rotor are larger than those of the uncoupled rotor because of the coupling between cracks, and the amplitude of the super-harmonic components for the coupled rotor are nearly twice as those of the uncoupled rotor in most cases, which can be used for the crack diagnosis of rotors, especially for the early crack detection.

This work is of great significance to the dynamic characteristics research and the multiple cracks detection for the practical multi-crack rotor system, which can promote the application of model-based method in the field of rotor fault diagnosis. The focus of future work is the early detection and quantitative identification for multiple cracks in rotors based on above theoretical research.

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