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A relaxed binary quadratic function negative-determination lemma and its application to neutral systems with interval time-varying delays and nonlinear disturbances

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ABSTRACT
This paper considers the stability problem of neutral systems with interval time-varying delays and nonlinear disturbances. Firstly, an augmented vector containing two double integral terms is introduced into the Lyapunov-Krasovskii functional (LKF). In this case, a binary quadratic function with discrete and neutral delay arises in the time derivative. To gain the negativity condition of such function, by taking full advantage of the idea of partial differential of the binary quadratic function and Taylor's formula, a relaxed binary quadratic function negative-determination lemma with two adjustable parameters is proposed, which contains the existing lemmas as its special cases and shows the great potential of reducing conservatism for the case where the tangent slope at the endpoint is far from zero. Then, based on the improved lemma, more relaxing stability criteria have been obtained via an augmented LKF. Finally, two classic numerical examples are given to attest the effectiveness and strengths of the obtained stability criteria.

KEYWORDS
Stability, neutral systems, generalized reciprocally convex combination lemma, a relaxed binary quadratic function negative-determination lemma

1. Introduction

As the widespread phenomenon in practical systems, the time-delay, which inevitably appears in neural networks, robotic systems, automatic control, power systems and so on (Lee & Bhattacharya , 2015; Sakthivel et al. , 2015; Shangguan et al. , 2021; Yao et al. , 2015), has become a significant consideration due to its potentially harmful impact on stability of system. Therefore, the issue that stability analysis of the systems with time-delays has increasingly aroused much concern over the past decades (Lee & Park , 2018; Seuret & Gouaisbaut , 2015; Zhang et al. , 2018b).

Particularly, in some practical control systems, time-delay is not only contained in the state, but also in its state derivative. These kinds of systems are generally referred to as neutral systems with time-delay (Park & Won , 2000), which can describe the

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dynamics of things more accurately. What needs to be pointed out is that the stability problems become more complicated due to the existence of the delayed state derivative. As a result of their successful applications in all sorts of fields such as models of current and voltage fluctuations in lossless transmission lines, automatic control, population dynamics and ecological models (Brayton , 1966; Lu & Ge , 2004; Niculescu , 2001), the interest has gone up rapidly in the stability of neutral systems with time-delay.

For the purpose of gaining the stability conditions of neutral systems, until now, the Lyapunov-Krasovskii functional (LKF) method is the most classic way together with the linear matrix inequality (LMI) technique (Kim , 2011; Lien , 2001). Initially, for the case where the discrete and neutral delay are constant, some effective methods have been extended from traditional time-delay system to neutral system with constant time-delay to study the stability, such as model transformation (Fridman , 2001; Ivanescu et al. , 2003), the Newton-Leibniz formula along with Park's inequality (Li & Liu , 2009), the free-weighting-matrix approach (He et al. , 2004; Qian et al. , 2010) and so on. Furthermore, several delay-partitioning ideas (Du et al. , 2009; Han , 2009; Lakshmanan et al. , 2013) are also proposed to construct LKF so as to reduce conservatism. Note that the Du et al. (2009); Han (2009); He et al. (2004); Ivanescu et al. (2003); Lakshmanan et al. (2013); Li & Liu (2009); Qian et al. (2010) only discuss the stability of neutral systems with constant time-delays. It is universally acknowledged that the time-delay is generally a differentiable function in the practical systems, called time-varying delay. Moreover, because of model inaccuracy, noise and environmental changes, nonlinear disturbances for neutral systems are inevitable in practical dynamic systems (Krishnasamy & Balasubramaniam , 2015; Yang et al. , 2007). Therefore, the research mentioned above is far from enough. It has an important significance to study the stability of neutral systems with time-varying delays and nonlinear disturbances, which has focused on considerable attention.

In recent years, plentiful stability results on the neutral systems with time-varying delays and nonlinear disturbances have been reported. In Balasubramaniam et al. (2012); Wang et al. (2014), by separating the delay interval into several subintervals unevenly and selecting different weight matrices for each subinterval, a delay-partitioning LKF is constructed to ensure a larger stability region. In Ren et al. (2016), a second-order reciprocally convex inequality is first introduced to estimate double integral terms. Furthermore, ground on the interconnected information between discrete delay and neutral delay, the triple integral terms (Wang et al. , 2017) and the quadruple integral term (Zhang et al. , 2018a) were also introduced into the LKF, which contribute to further increase the stability regions. It is not difficult to notice that in Balasubramaniam et al. (2012); Wang et al. (2017); Zhang et al. (2018a), the lower bound of discrete delay and neutral delay is limited to zero, which may lead to relatively conservative stability conditions.

As mentioned in Chen & Zhao (2015); Kwon et al. (2014), in dynamic systems, like networked control systems, time-varying delay may be a variation range whose lower bound is not limited to zero, which is known as interval time-varying delay. Most recently, a great attention has been devoted to the stability of the neutral systems with interval time-varying delays and nonlinear disturbances (Chen et al. , 2020; Cheng et al. , 2013; Lakshmanan et al. , 2011; Liu , 2016; Mohajerpoor et al. , 2017; Ramakrishnan & Ray , 2011; Yu & Lien , 2008; Zhang & Yu , 2012). By introducing the central point of interval time-varying delay and dividing delay interval into two equal length subintervals, Cheng et al. (2013) proposed a piecewise delay method to obtain some stability conditions. In Liu (2016), the delay interval is segmented into two subintervals with an unequal length, and the robust stability conditions of the uncertain neutral system with interval time-varying delays were acquired. However, it is obvious that the computational complexity increase as the number of segments increases.

In order to take system information and time-varying delay information into account enough, and reduce computational complexity, some augmented LKFs have been brought up successively, which have flexibility and extensibility. In Lakshmanan et al. (2011); Mohajerpoor et al. (2017), an augmented LKF containing triple integral terms was proposed, combined Wirtinger-based single and double integral inequalities with reciprocally convex approach, new stability criteria in terms of LMIs were presented. Further, by taking full account of the state time-varying delay, neutral time-varying delay and the relationship between upper and lower bounds, a novel augmented LKF was raised in Chen et al. (2020). The integral terms were dealt with by using the integral intervals decomposition method and some integral inequalities (Chen et al. , 2017), which give rise to the advent of two nonlinear time-varying delay square terms in its time derivative. When this occurs, the determination of negative-definite conditions of such binary quadratic function is a crucial step to obtain stability criteria expressed as LMIs.

For the sake of obtaining strict LMI-based conditions, a few jobs on the negative-determination of the binary quadratic functions have so far been achieved. In Chen et al. (2021), a new nonlinear optimization technology was adopted to linearize the nonlinear time-varying delay square terms when its quadratic coefficients are negative. In Liu et al. (2021), by taking full advantage of the property of partial differential, the convexity/concavity and the tangents' slope characteristic, a binary quadratic function negative-determination lemma is developed, which can be put into use to the case where the plus-minus sign of the quadratic coefficients are misty. Notice that the lower bound of time-varying delay variables are fixed at zero in the negative-determination lemmas of Chen et al. (2021) and Liu et al. (2021), which makes it infeasible for the case of interval time-varying delay. For this purpose, a new quadratic inequality technology (Chen et al. , 2020) was presented to handle the nonlinear interval time-varying delay square terms. However, the methods mentioned above only consider the slope characteristic of tangent lines at the endpoints of time-varying delay variables, which make it conservative in a large part when the slope of the endpoint tangent line is far from zero.

As a matter of fact, up till now, numerous studies have been done on the negative-determination of the quadratic function with a time-varying delay. The quadratic function is the result of introducing double integral term into augmented vector. In Kim (2016); Long et al. (2020a, 2021), some simple quadratic function negative-determination lemmas were developed by using the convex/concave property. Subsequently, Zeng et al. (2020) proposed a hierarchical negative-determination lemma by considering the idea of delay-partitioning, in which each subinterval of a curve is constrained by two tangent lines. Further, in order to reduce conservatism, a parameter-adjustable-based negative-determination lemma was put forward in Zhang et al. (2020). Besides, an improved negative-definiteness determination method (Long et al. , 2020b) is brought up by using Taylor's formula and the interval-decomposition technique. Obviously, in the methods (Long et al. , 2020b; Zeng et al. , 2020; Zhang et al. , 2020), not only the slope characteristic of the endpoint tangent lines are considered, but also even more. It can be seen that, due to the introduction of some adjustable parameters, the methods (Long et al. , 2020b; Zeng et al. , 2020; Zhang et al. , 2020) work well for the case where the tangent slope at the endpoints is much greater than zero. With the development of various quadratic function negative-determination lemmas, the research on the stability of time-varying delay systems has made significant breakthroughs. These ideas offer the motivation that how to improve the negative-determination method of the binary quadratic function.

To conclude, from the above discussions, it is shown that the current methods (Chen et al. , 2021, 2020; Liu et al. , 2021) are still conservative in different degrees and may be improved, so that a further study of the negative-determination method of binary quadratic function is extremely essential. In addition, the current results mostly employed the free-weighting-matrix approach, Wirtinger-based integral inequalities and reciprocally convex approach to bound the LKF's derivative, which are relatively conservative to a certain degree. This drives the current study.

Inspired by the above-mentioned ideas, this paper focuses on improving the negative-determination method of binary quadratic functions that appears in the time derivative. In order to reduce conservatism, a relaxed binary quadratic function negative-determination lemma with two adjustable parameters is first presented by giving enough thought to the partial differential property of the binary quadratic function and Taylor's formula, which shows its advantages when the tangent slope at the endpoint is much bigger than zero. Moreover, by taking more augmented terms into consideration, which involve two double integral terms of discrete delay and neutral delay state vectors, a novel augmented LKF is constructed. Then, by employing the presented negative-determination lemma, single/multiple integral inequalities and generalized reciprocally convex combination lemma, two stability criteria with strict LMIs are derived. Finally, two representative examples verified the availability of the proposed methods and demonstrated their superiority compared with previous results.

Notations: Throughout this paper, $\mathcal{R}^n$ shows the n-dimensional Euclidean space; the identity (zero) matrix is represented by I (0); the symbol $*$ stands for the symmetric block in a matrix; $Y > 0$ refers to a positive-definite matrix; $\mathrm{Sym}\{Y\} = Y + Y^T$; $\mathrm{col}\{y_1, y_2, \ldots, y_n\} = [y_1^T, y_2^T, \ldots, y_n^T]^T$; and the $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix.

2. Problem formulation and preliminary

Consider the following neutral systems with nonlinear disturbances:

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-\tau(t)) = Ax(t) + Bx(t-h(t)) + g_1(x(t),t) \\ \qquad\qquad\qquad\qquad\quad + g_2(x(t-h(t)),t) + g_3(\dot{x}(t-\tau(t)),t) \\ x(t) = \phi(t), \dot{x}(t) = \varphi(t), \qquad \forall t \in [-\max\{h_2, \tau_2\}, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector; $A, B, C \in \mathcal{R}^{n\times n}$ are known constant matrices with appropriate dimensions; $\phi(t)$ and $\varphi(t)$ express the initial conditions; $h(t)$ and $\tau(t)$ represent the discrete and neutral delays in the state, respectively. And they satisfy the following conditions

$$\begin{aligned} &0 \le h_1 \le h(t) \le h_2, \qquad |\dot{h}(t)| \le \mu_1. \\ &0 \le \tau_1 \le \tau(t) \le \tau_2, \qquad |\dot{\tau}(t)| \le \mu_2. \end{aligned} \tag{2}$$

where h_i, τ_i and $\mu_i, i = 1, 2$ are constants. Let $h_{21} = h_2 - h_1$ and $\tau_{21} = \tau_2 - \tau_1$. The functions $g_i(\cdot, t) \in \mathcal{R}^n$, $i = 1, 2, 3$, are unknown nonlinear disturbances satisfying $g_i(0, t) = 0$ that are continuous in t and locally Lipschitz in their first argument, and are assumed to satisfy the following conditions for any given scalars $\gamma_i \geq 0$, $i = 1, 2, 3$,

$$
\begin{aligned}
&g_1^T(x(t),t)g_1(x(t),t) \leq \gamma_1^2 x^T(t)x(t),\\
&g_2^T(x(t-h(t)),t)g_2(x(t-h(t)),t) \leq \gamma_2^2 x^T(t-h(t))x(t-h(t)),\\
&g_3^T(\dot{x}(t-\tau(t)),t)g_3(\dot{x}(t-\tau(t)),t) \leq \gamma_3^2 \dot{x}^T(t-\tau(t))\dot{x}(t-\tau(t)).
\end{aligned} \tag{3}
$$

The following lemmas are employed to estimate the quadratic integral terms in the derivative of LKF, as shown below respectively.

Lemma 2.1. *(Chen et al. , 2017) Let x be a differentiable signal in $[\alpha, \beta] \to \mathcal{R}^n$, for a symmetric matrix $R \in \mathcal{R}^{n\times n}$, and any matrices F_{11}, F_{12}, $F_{13} \in \mathcal{R}^{4n\times n}$, then the following inequalities hold:*

$$
-\int_\alpha^\beta x^T(s)Rx(s)ds \leq \varpi^T(\alpha,\beta)\Upsilon_1\varpi(\alpha,\beta) \tag{4}
$$

$$
-\int_\alpha^\beta \dot{x}^T(s)R\dot{x}(s)ds \leq \varpi^T(\alpha,\beta)\Upsilon_2\varpi(\alpha,\beta) \tag{5}
$$

$$
-\int_\alpha^\beta \int_\theta^\beta \dot{x}^T(s)R\dot{x}(s)dsd\theta \leq \varpi^T(\alpha,\beta)\Upsilon_3\varpi(\alpha,\beta) \tag{6}
$$

where

$$
\begin{aligned}
\Upsilon_1 =& -(\beta-\alpha)(\Theta_{11}^T R\Theta_{11} + 3\Theta_{12}^T R\Theta_{12} + 5\Theta_{13}^T R\Theta_{13})\\
\Upsilon_2 =& (\beta-\alpha)\epsilon_1^T(F_{11}R^{-1}F_{11}^T + \frac{1}{3}F_{12}R^{-1}F_{12}^T + \frac{1}{5}F_{13}R^{-1}F_{13}^T)\epsilon_1\\
&+ \mathrm{Sym}\left\{\epsilon_1^T F_{11}\Theta_{21} + \epsilon_1^T F_{12}\Theta_{22} + \epsilon_1^T F_{13}\Theta_{23}\right\}\\
\Upsilon_3 =& -2\Theta_{31}^T R\Theta_{31} - 16\Theta_{32}^T R\Theta_{32} - 54\Theta_{33}^T R\Theta_{33}\\
\varpi(\alpha,\beta) =& \Bigg[x^T(\beta) \quad x^T(\alpha) \quad \frac{1}{\beta-\alpha}\int_\alpha^\beta x^T(s)ds \quad \frac{1}{(\beta-\alpha)^2}\int_\alpha^\beta\int_\theta^\beta x^T(s)dsd\theta\\
&\quad \frac{1}{(\beta-\alpha)^3}\int_\alpha^\beta\int_\theta^\beta\int_u^\beta x^T(s)dsdud\theta\Bigg]^T\\
&\Theta_{11} = \lambda_3, \quad \Theta_{12} = \lambda_3 - 2\lambda_4, \quad \Theta_{13} = \lambda_3 - 6\lambda_4 + 12\lambda_5,\\
&\Theta_{21} = \lambda_1 - \lambda_2, \quad \Theta_{22} = \lambda_1 + \lambda_2 - 2\lambda_3, \quad \Theta_{23} = \lambda_1 - \lambda_2 + 6\lambda_3 - 12\lambda_4,\\
&\Theta_{31} = \lambda_1 - \lambda_3, \quad \Theta_{32} = \frac{1}{2}\lambda_1 + \lambda_3 - 3\lambda_4, \quad \Theta_{33} = \frac{1}{3}\lambda_1 - \lambda_3 + 8\lambda_4 - 20\lambda_5,\\
&\epsilon_1 = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4]^T, \quad \lambda_j = [0_{n\times(j-1)n} \quad I_n \quad 0_{n\times(5-j)n}]^T, \quad j = 1, 2, \cdots, 5.
\end{aligned}
$$

Lemma 2.2. *(Park et al. , 2015) Let x be a continuously differentiable signal in $[\alpha, \beta] \to \mathcal{R}^n$, for a symmetric matrix $R \in \mathcal{R}^{n\times n} > 0$, the following inequality holds:*

$$
\int_\alpha^\beta \dot{x}^T(s)R\dot{x}(s)ds \geq \frac{1}{\beta-\alpha}\widetilde{\chi}_1^T R\widetilde{\chi}_1 + \frac{3}{\beta-\alpha}\widetilde{\chi}_2^T R\widetilde{\chi}_2 + \frac{5}{\beta-\alpha}\widetilde{\chi}_3^T R\widetilde{\chi}_3 \tag{7}
$$

where

$$
\widetilde{\chi}_1 = x(\beta) - x(\alpha), \qquad \widetilde{\chi}_2 = x(\beta) + x(\alpha) - \frac{2}{\beta-\alpha}\int_\alpha^\beta x(s)ds
$$

$$\widetilde{\chi}_3 = x(\beta)-x(\alpha)+\frac{6}{\beta-\alpha}\int_\alpha^\beta x(s)ds - \frac{12}{(\beta-\alpha)^2}\int_\alpha^\beta\int_\theta^\beta x(s)dsd\theta$$

Lemma 2.3. *(Seuret et al. , 2018) For a real scalar $\alpha \in (0,1)$, the symmetric matrix $R \in \mathcal{R}^{n\times n} > 0$ and any matrices with suitable dimensions S_1, S_2 and E, the following inequality holds:*

$$E^T\begin{bmatrix} \frac{1}{\alpha}R & 0 \\ 0 & \frac{1}{1-\alpha}R \end{bmatrix}E \geq E^T\begin{bmatrix} (2-\alpha)R & 0 \\ 0 & (1+\alpha)R \end{bmatrix}E + \text{Sym}\left\{E^T\begin{bmatrix} (1-\alpha)S_1^T \\ \alpha S_2^T \end{bmatrix}\right\} \\ -\alpha S_1 R^{-1}S_1^T - (1-\alpha)S_2R^{-1}S_2^T \tag{8}$$

The following Lemma 2.4 is an improved result to ensure the negativity of binary quadratic functions, which is indispensable to obtain next main results.

Lemma 2.4. *Consider a binary quadratic function $f(h(t),\tau(t)) = a_6h^2(t) + a_5h(t) + a_4\tau^2(t) + a_3\tau(t) + a_2h(t)\tau(t) + a_1$, where $a_i \in \mathcal{R}$, $i = 1, 2, \cdots, 6$, $h(t) \in [h_1, h_2]$ and $\tau(t) \in [\tau_1, \tau_2]$. For $a_6a_4 \geq 0$, $f(h(t), \tau(t)) < 0$ holds for $\forall h(t) \in [h_1, h_2]$ and $\forall\tau(t) \in [\tau_1, \tau_2]$ if the following conditions hold for any given α, β within $[0, 1]$:*

$$\begin{array}{ll} C_{1a}: f(h_1,\tau_1) < 0 & C_{1b}: f(h_1,\tau_2) < 0 \\ C_{1c}: f(h_2,\tau_1) < 0 & C_{1d}: f(h_2,\tau_2) < 0 \end{array} \tag{9}$$

$$\begin{array}{l} C_{2a}: -\alpha^2(h_2-h_1)^2a_6 - \beta^2(\tau_2-\tau_1)^2a_4 + f(h_1,\tau_1) < 0 \\ C_{2b}: -\alpha^2(h_2-h_1)^2a_6 - (1-\beta)^2(\tau_2-\tau_1)^2a_4 + f(h_1,\tau_2) < 0 \\ C_{2c}: -(1-\alpha)^2(h_2-h_1)^2a_6 - \beta^2(\tau_2-\tau_1)^2a_4 + f(h_2,\tau_1) < 0 \\ C_{2d}: -(1-\alpha)^2(h_2-h_1)^2a_6 - (1-\beta)^2(\tau_2-\tau_1)^2a_4 + f(h_2,\tau_2) < 0 \end{array} \tag{10}$$

Proof. Firstly, when $a_6 > 0$, $a_4 \geq 0$, $f(h(t),\tau(t))$ is a convex function. Consequently, on the basis of convex polyhedron method, if conditions C_{1a}, C_{1b}, C_{1c} and C_{1d} hold, then $f(h(t),\tau(t)) < 0$ for $\forall h(t) \in [h_1, h_2]$, $\forall\tau(t) \in [\tau_1, \tau_2]$ is guaranteed.

Secondly, in the case of $a_6 \leq 0$, $a_4 < 0$, based on the partial differential property, $h(t)$ is considered as the function variable firstly, and $\tau(t)$ is perceived as the constant. For $a_6 \leq 0$, $f(h(t),\tau(t))$ is concave. Then, it is expanded to its Taylor's series at $h(t) = h_0$ as

$$\begin{aligned} & f(h(t),\tau(t)) \\ &= \frac{f(h_0,\tau(t))}{0!} + \frac{f'(h_0,\tau(t))}{1!}(h(t)-h_0) + \frac{f''(h_0,\tau(t))}{2!}(h(t)-h_0)^2 \\ &\doteq (2a_6h_0 + a_5 + a_2\tau(t))h(t) - a_6(h_0)^2 + a_4\tau^2(t) + a_3\tau(t) + a_1 \\ &\quad + a_6(h(t)-h_0)^2, \end{aligned} \tag{11}$$

where $h_0 = (1-\alpha)h_1 + \alpha h_2$, $\alpha \in [0,1]$. By relaxing the square term in equality (11), $a_6(h(t)-h_0)^2$, the following holds:

$$\begin{aligned} f(h(t),\tau(t)) &\leq (2a_6h_0 + a_5 + a_2\tau(t))h(t) - a_6(h_0)^2 + a_4\tau^2(t) + a_3\tau(t) + a_1 \\ &:= g_1(h(t),\tau(t)), \end{aligned} \tag{12}$$

Since $g_1(h(t),\tau(t))$ is a first-degree polynomial function. Thereupon, $f(h(t),\tau(t)) < 0$,

$\forall h(t) \in [h_1, h_2]$, holds if

$$g_1(h_1, \tau(t)) = -\alpha^2(h_2 - h_1)^2 a_6 + f(h_1, \tau(t)) < 0, \tag{13}$$
$$g_1(h_2, \tau(t)) = -(1-\alpha)^2(h_2 - h_1)^2 a_6 + f(h_2, \tau(t)) < 0. \tag{14}$$

Later on, on top of that the $\tau(t)$ is taken as a variable of the functions $g_1(h_1, \tau(t))$ and $g_1(h_2, \tau(t))$. Similarly, they are expanded to their Taylor's series at $\tau(t) = \tau_0$, respectively, where $\tau_0 = (1-\beta)\tau_1 + \beta\tau_2$, $\beta \in [0, 1]$. Then, under $a_4 < 0$, inequality (13) leads to conditions C_{2a} and C_{2b}, and inequality (14) leads to conditions C_{2c} and C_{2d}. Thus, if conditions C_{2a}, C_{2b}, C_{2c} and C_{2d} hold, then $f(h(t), \tau(t)) < 0$ is guaranteed.

Finally, for $a_6 a_4 \geq 0$, if the plus-minus sign of a_6 and a_4 are unknown, then the above-mentioned both cases should be combined. Therefore, $f(h(t), \tau(t)) < 0$ is guaranteed, under $a_6 a_4 \geq 0$, if conditions (9) and (10) hold. The proof is completed. $\square$

Remark 1. If only conditions C_{2a}, C_{2b}, C_{2c} and C_{2d} are introduced, and set $\alpha = 1$, $\beta = 1$, then Lemma 2.4 reduces to Lemma 3 of Chen et al. (2020). Besides, if $h_1 = \tau_1 = 0$ is added, Lemma 2.4 is equivalent to Lemma 3 of Chen et al. (2021). Moreover, by taking $h_1 = \tau_1 = 0$, $\alpha = 1$, $\beta = 1$ or $\alpha = 0$, $\beta = 0$, then Lemma 2.4 becomes Lemma 4 of Liu et al. (2021). Thus, the Lemmas of Chen et al. (2021, 2020); Liu et al. (2021) can be viewed as special cases of Lemma 2.4. Obviously, the methods mentioned above only consider the slope characteristic of tangent lines at the endpoints, which make it conservative when the slope of the endpoint tangent line is far from zero. In Lemma 2.4, the introduction of two adjustable parameters α and β can provide more freedom to linearize $f(h(t), \tau(t))$ and make the reduction of conservatism promising.

Remark 2. On the strength of the idea of partial differential and Taylor's formula, two adjustable parameters α and β are introduced to decrease the conservatism. The proof of the lemma 2.4 shows that these terms, $-\alpha^2(h_2-h_1)^2 a_6$, $-(1-\alpha)^2(h_2-h_1)^2 a_6$, $-\beta^2(\tau_2 - \tau_1)^2 a_4$ and $-(1-\beta)^2(\tau_2 - \tau_1)^2 a_4$, are non-negative in the case of $a_6 \leq 0$, $a_4 < 0$, and that they are additionally added to conditions C_{2a}, C_{2b}, C_{2c} and C_{2d}. The aim was to give more freedom and weaken the influence of the non-negative terms in order to reduce the conservatism. With α and β changing within $[0, 1]$, the sizes of the non-negative terms vary. Therefore, choosing suitable values of α and β within $[0, 1]$ can further reduce the conservatism.

3. Main results

For the sake of simplicity, several related notations are denoted:

$$h_{1t} = h(t) - h_1, \quad h_{2t} = h_2 - h(t), \quad \tau_{1t} = \tau(t) - \tau_1, \quad \tau_{2t} = \tau_2 - \tau(t),$$
$$u(a, b, t) = \int_{t-a}^{t-b} \frac{x(s)}{a-b} ds, \quad v(a, b, t) = \int_{t-a}^{t-b} \int_{\theta}^{t-b} \frac{x(s)}{(a-b)^2} ds d\theta,$$
$$w(a, b, t) = \int_{t-a}^{t-b} \int_{\theta}^{t-b} \int_{u}^{t-b} \frac{x(s)}{(a-b)^3} ds du d\theta,$$
$$\bar{\chi}_0(a, b, t) = \mathrm{col}\big\{u(a, b, t),\ v(a, b, t)\big\}, \quad \chi(a, b, t) = \mathrm{col}\big\{\bar{\chi}_0(a, b, t),\ w(a, b, t)\big\},$$

$$
\begin{aligned}
&\xi(t) = \Big[\xi_1(t),\ \xi_2(t),\ \xi_3(t),\ \xi_4(t),\ \xi_5(t)\Big]^T,\\
&\xi_1(t) = \mathrm{col}\big\{x(t),\ x(t-h_1),\ x(t-h(t)),\ x(t-h_2),\ x(t-\tau_1),\ x(t-\tau(t)),\ x(t-\tau_2)\big\},\\
&\xi_2(t) = \mathrm{col}\big\{\dot{x}(t-\tau_1),\ \dot{x}(t-\tau(t)),\ \dot{x}(t-\tau_2),\ \dot{x}(t-h_1)\big\},\\
&\xi_3(t) = \mathrm{col}\big\{\chi(h_1,0,t),\ \chi(h(t),h_1,t),\ \chi(h_2,h(t),t),\ \chi(\tau_1,0,t),\ \chi(\tau(t),\tau_1,t),\ \chi(\tau_2,\tau(t),t)\big\},\\
&\xi_4(t) = \mathrm{col}\big\{\bar{\chi}_0(\tau(t),0,t),\ h_2^2 v(h_2,0,t),\ \tau_2^2 v(\tau_2,0,t)\big\},\\
&\xi_5(t) = \mathrm{col}\big\{g_1(x(t),t),\ g_2(x(t-h(t)),t),\ g_3(\dot{x}(t-\tau(t)),t),\ \dot{x}(t-h_2),\ \dot{x}(t-h(t))\big\},\\
&e_i = [0_{n\times(i-1)n}\ \ I_n\ \ 0_{n\times(38-i)n}]^T,\ i = 1,2,\cdots,38.
\end{aligned}
\tag{15}
$$

Base on Lemma 2.4, the following main result of the stability of neutral system (1) with nonlinear disturbances is stated.

Theorem 3.1. *For given scalars $0 \le h_1 < h_2$, $0 \le \tau_1 < \tau_2$, μ_1, μ_2, γ_i, $(i = 1, 2, 3)$, and α, $\beta \in [0,1]$, under the nonlinear disturbances fulfilling conditions (3), the neutral system (1) with any discrete and neutral delays satisfying (2) is asymptotically stable if there exist positive definite matrices $P_0 \in \mathcal{R}^{12n\times 12n}$, $P_i \in \mathcal{R}^{2n\times 2n}$, $(i = 1,\ldots, 4)$, $Q_i \in \mathcal{R}^{2n\times 2n}$, $(i = 1,\ldots, 6)$, $R_1 \in \mathcal{R}^{2n\times 2n}$, $R_i \in \mathcal{R}^{n\times n}$, $(i = 2,\ldots, 6)$, $Z_i \in \mathcal{R}^{n\times n}$, $(i = 1,\ldots, 4)$, any matrices $S_i \in \mathcal{R}^{38n\times 3n}$, $(i = 1,\ldots, 4)$, $F_{ij} \in \mathcal{R}^{4n\times n}$, $(i = 1, 2, 3,\ j = 1, 2, 3)$, $L_1, L_2 \in \mathcal{R}^{6n\times n}$, and real scalars $\rho_i \ge 0$, $(i = 1, 2, 3)$, such that the following LMIs are satisfied:*

$$
\begin{aligned}
&\begin{bmatrix} \Xi(h_1,\tau_1) & \Gamma_{11} \\ * & \Delta_1 \end{bmatrix} < 0, \qquad \begin{bmatrix} \Xi(h_1,\tau_2) & \Gamma_{12} \\ * & \Delta_1 \end{bmatrix} < 0,\\
&\begin{bmatrix} \Xi(h_2,\tau_1) & \Gamma_{13} \\ * & \Delta_1 \end{bmatrix} < 0, \qquad \begin{bmatrix} \Xi(h_2,\tau_2) & \Gamma_{14} \\ * & \Delta_1 \end{bmatrix} < 0,
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&\begin{bmatrix} \Xi(h_1,\tau_1) - \alpha^2 h_{21}^2 \Psi_h - \beta^2 \tau_{21}^2 \Psi_\tau & \Gamma_{21} \\ * & \Delta_2 \end{bmatrix} < 0,\\
&\begin{bmatrix} \Xi(h_1,\tau_2) - \alpha^2 h_{21}^2 \Psi_h - (1-\beta)^2 \tau_{21}^2 \Psi_\tau & \Gamma_{22} \\ * & \Delta_2 \end{bmatrix} < 0,\\
&\begin{bmatrix} \Xi(h_2,\tau_1) - (1-\alpha)^2 h_{21}^2 \Psi_h - \beta^2 \tau_{21}^2 \Psi_\tau & \Gamma_{23} \\ * & \Delta_2 \end{bmatrix} < 0,\\
&\begin{bmatrix} \Xi(h_2,\tau_2) - (1-\alpha)^2 h_{21}^2 \Psi_h - (1-\beta)^2 \tau_{21}^2 \Psi_\tau & \Gamma_{24} \\ * & \Delta_2 \end{bmatrix} < 0.
\end{aligned}
\tag{17}
$$

where

$$
\begin{aligned}
&\Xi(h(t),\tau(t)) = \sum_{i=1}^{6} \Xi_i,\\
&\Xi_1 = \mathrm{Sym}\left\{\Pi_1^T P_0 \Pi_2\right\},\\
&\Xi_2 = \mu_1\left(\Pi_{3a}^T P_1 \Pi_{3a} - \Pi_{3a}^T P_2 \Pi_{3a}\right) + \mu_2\left(\Pi_{3c}^T P_3 \Pi_{3c} - \Pi_{3c}^T P_4 \Pi_{3c}\right)\\
&\qquad + \mathrm{Sym}\left\{h(t)\Pi_{3a}^T P_1 \Pi_{3b} + h_{2t}\Pi_{3a}^T P_2 \Pi_{3b} + \tau(t)\Pi_{3c}^T P_3 \Pi_{3d} + \tau_{2t}\Pi_{3c}^T P_4 \Pi_{3d}\right\},\\
&\Xi_3 = \Pi_3^T(Q_1 + Q_3 + Q_4 + Q_6)\Pi_3 + \Pi_{3e}^T(Q_2 - Q_1)\Pi_{3e} - \Pi_{3f}^T Q_2 \Pi_{3f}\\
&\qquad - (1-\mu_1)\Pi_{3g}^T Q_3 \Pi_{3g} + \Pi_4^T(Q_5 - Q_4)\Pi_4 - \Pi_5^T Q_5 \Pi_5 - (1-\mu_2)\Pi_6^T Q_6 \Pi_6,
\end{aligned}
$$

$$
\begin{aligned}
\Xi_4 &= h_1^2\Pi_3^T R_1\Pi_3 + h_{21}^2 e_1^T R_2 e_1 + e_s^T(h_{21}^2 R_3 + \tau_1^2 R_4 + \tau_{21}^2 R_5 + \tau(t)R_6 + \frac{h_1^2}{2}Z_1 + \frac{\tau_1^2}{2}Z_3)e_s \\
&\quad + \frac{h_{21}^2}{2}e_{11}^T Z_2 e_{11} + \frac{\tau_{21}^2}{2}e_8^T Z_4 e_8 - h_{21}h_{1t}(H_{21}^T R_2 H_{21} + 3H_{22}^T R_2 H_{22} + 5H_{23}^T R_2 H_{23}) \\
&\quad - h_{21}h_{2t}(H_{31}^T R_2 H_{31} + 3H_{32}^T R_2 H_{32} + 5H_{33}^T R_2 H_{33}) \\
&\quad - (H_{11}^T R_1 H_{11} + 3H_{12}^T R_1 H_{12} + 5H_{13}^T R_1 H_{13}) - (H_{41}^T R_4 H_{41} + 3H_{42}^T R_4 H_{42} + 5H_{43}^T R_4 H_{43}) \\
&\quad - 2\Pi_{21}^T Z_1\Pi_{21} - 16\Pi_{22}^T Z_1\Pi_{22} - 54\Pi_{23}^T Z_1\Pi_{23} - 2\Pi_{31}^T Z_3\Pi_{31} - 16\Pi_{32}^T Z_3\Pi_{32} - 54\Pi_{33}^T Z_3\Pi_{33} \\
&\quad - 2\Pi_{41}^T Z_2\Pi_{41} - 16\Pi_{42}^T Z_2\Pi_{42} - 54\Pi_{43}^T Z_2\Pi_{43} - 2\Pi_{51}^T Z_2\Pi_{51} - 16\Pi_{52}^T Z_2\Pi_{52} - 54\Pi_{53}^T Z_2\Pi_{53} \\
&\quad - 2\Pi_{61}^T Z_4\Pi_{61} - 16\Pi_{62}^T Z_4\Pi_{62} - 54\Pi_{63}^T Z_4\Pi_{63} - 2\Pi_{71}^T Z_4\Pi_{71} - 16\Pi_{72}^T Z_4\Pi_{72} - 54\Pi_{73}^T Z_4\Pi_{73} \\
&\quad + (1-\mu_2)\mathrm{Sym}\left\{\Pi_{11}^T F_{11}H_{51} + \Pi_{11}^T F_{12}H_{52} + \Pi_{11}^T F_{13}H_{53}\right\} \\
&\quad + h_{2t}\mathrm{Sym}\left\{\Pi_{12}^T F_{21}H_{61} + \Pi_{12}^T F_{22}H_{62} + \Pi_{12}^T F_{23}H_{63}\right\} \\
&\quad + \tau_{2t}\mathrm{Sym}\left\{\Pi_{13}^T F_{31}H_{71} + \Pi_{13}^T F_{32}H_{72} + \Pi_{13}^T F_{33}H_{73}\right\} + \Omega, \\
\Xi_5 &= \mathrm{Sym}\left\{N_1L_1\left(h_1^2 e_{13} + h_{1t}^2 e_{16} + h_{2t}^2 e_{19} + h_{2t}h_{1t}e_{15} + h_{21}h_1 e_{12} - e_{32}\right)\right\} \\
&\quad + \mathrm{Sym}\left\{N_2L_2\left(\tau_1^2 e_{22} + \tau_{1t}^2 e_{25} + \tau_{2t}^2 e_{28} + \tau_{2t}\tau_{1t}e_{24} + \tau_{21}\tau_1 e_{21} - e_{33}\right)\right\}, \\
\Xi_6 &= \rho_1\left(\gamma_1^2 e_1^T e_1 - e_{34}^T e_{34}\right) + \rho_2\left(\gamma_2^2 e_3^T e_3 - e_{35}^T e_{35}\right) + \rho_3\left(\gamma_3^2 e_9^T e_9 - e_{36}^T e_{36}\right), \\
\Psi_h &= \mathrm{Sym}\left\{\Upsilon_{h_a}^T P_0\Upsilon_{h_b} + N_1L_1(e_{16} + e_{19} - e_{15})\right\}, \\
\Psi_\tau &= \mathrm{Sym}\left\{\Upsilon_{\tau_a}^T P_0\Upsilon_{\tau_b} + N_2L_2(e_{25} + e_{28} - e_{24})\right\}, \\
\Psi_{h_0} &= -\Pi_{12}^T(F_{21}Z_2^{-1}F_{21}^T + \frac{1}{3}F_{22}Z_2^{-1}F_{22}^T + \frac{1}{5}F_{23}Z_2^{-1}F_{23}^T)\Pi_{12}, \\
\Psi_{\tau_0} &= -\Pi_{13}^T(F_{31}Z_4^{-1}F_{31}^T + \frac{1}{3}F_{32}Z_4^{-1}F_{32}^T + \frac{1}{5}F_{33}Z_4^{-1}F_{33}^T)\Pi_{13}, \\
\Upsilon_{h_a} &= \mathrm{col}\left\{\underbrace{0,\ldots,0}_{10},\quad e_{18} - e_{15},\quad 0\right\}, \qquad \Upsilon_{h_b} = \mathrm{col}\left\{\underbrace{0,\ldots,0}_{7},\quad e_{15} - e_{18},\quad \underbrace{0,\ldots,0}_{4}\right\}, \\
\Upsilon_{\tau_a} &= \mathrm{col}\left\{\underbrace{0,\ldots,0}_{11},\quad e_{27} - e_{24}\right\}, \qquad \Upsilon_{\tau_b} = \mathrm{col}\left\{\underbrace{0,\ldots,0}_{9},\quad e_{24} - e_{27},\quad 0,\quad 0\right\}, \\
\Omega &= -\begin{bmatrix}\Pi_7\\ \Pi_8\end{bmatrix}^T \begin{bmatrix}\left(2-\frac{h_{1t}}{h_{21}}\right)\tilde{R}_3 & 0\\ 0 & \left(1+\frac{h_{1t}}{h_{21}}\right)\tilde{R}_3\end{bmatrix}\begin{bmatrix}\Pi_7\\ \Pi_8\end{bmatrix} - \mathrm{Sym}\left\{\begin{bmatrix}\Pi_7\\ \Pi_8\end{bmatrix}^T\begin{bmatrix}\frac{h_{2t}}{h_{21}}S_1^T\\ \frac{h_{1t}}{h_{21}}S_2^T\end{bmatrix}\right\} \\
&\quad - \begin{bmatrix}\Pi_9\\ \Pi_{10}\end{bmatrix}^T \begin{bmatrix}\left(2-\frac{\tau_{1t}}{\tau_{21}}\right)\tilde{R}_5 & 0\\ 0 & \left(1+\frac{\tau_{1t}}{\tau_{21}}\right)\tilde{R}_5\end{bmatrix}\begin{bmatrix}\Pi_9\\ \Pi_{10}\end{bmatrix} - \mathrm{Sym}\left\{\begin{bmatrix}\Pi_9\\ \Pi_{10}\end{bmatrix}^T\begin{bmatrix}\frac{\tau_{2t}}{\tau_{21}}S_3^T\\ \frac{\tau_{1t}}{\tau_{21}}S_4^T\end{bmatrix}\right\}, \\
N_1 &= \mathrm{col}\left\{e_{13},\ e_{16},\ e_{19},\ e_{18},\ e_{15},\ e_{32}\right\}, \qquad N_2 = \mathrm{col}\left\{e_{22},\ e_{25},\ e_{28},\ e_{27},\ e_{24},\ e_{33}\right\}, \\
\Gamma_{11} &= [S_2,\ S_4,\ \Gamma_a], \quad \Gamma_{12} = [S_2,\ S_3,\ \Gamma_b], \quad \Gamma_{13} = [S_1,\ S_4,\ \Gamma_a], \quad \Gamma_{14} = [S_1,\ S_3,\ \Gamma_b], \\
\Gamma_{21} &= [\Gamma_{11},\ \Gamma_{c1},\ \Gamma_{c3}],\ \Gamma_{22} = [\Gamma_{12},\ \Gamma_{c1},\ \Gamma_{c4}],\ \Gamma_{23} = [\Gamma_{13},\ \Gamma_{c2},\ \Gamma_{c3}],\ \Gamma_{24} = [\Gamma_{14},\ \Gamma_{c2},\ \Gamma_{c4}], \\
\Gamma_a &= \left[\varepsilon_1\Pi_{11}^T F_{11}, \varepsilon_1\Pi_{11}^T F_{12}, \varepsilon_1\Pi_{11}^T F_{13}\right], \qquad \Gamma_b = \left[\varepsilon_2\Pi_{11}^T F_{11}, \varepsilon_2\Pi_{11}^T F_{12}, \varepsilon_2\Pi_{11}^T F_{13}\right], \\
\Gamma_{c1} &= \left[\alpha h_{21}\Pi_{12}^T F_{21}, \alpha h_{21}\Pi_{12}^T F_{22}, \alpha h_{21}\Pi_{12}^T F_{23}\right],\ \Gamma_{c3} = \left[\beta\tau_{21}\Pi_{13}^T F_{31}, \beta\tau_{21}\Pi_{13}^T F_{32}, \beta\tau_{21}\Pi_{13}^T F_{33}\right], \\
\Gamma_{c2} &= \left[(1-\alpha)h_{21}\Pi_{12}^T F_{21}, (1-\alpha)h_{21}\Pi_{12}^T F_{22}, (1-\alpha)h_{21}\Pi_{12}^T F_{23}\right], \\
\Gamma_{c4} &= \left[(1-\beta)\tau_{21}\Pi_{13}^T F_{31}, (1-\beta)\tau_{21}\Pi_{13}^T F_{32}, (1-\beta)\tau_{21}\Pi_{13}^T F_{33}\right],
\end{aligned}
$$

$$
\begin{aligned}
&\Delta_1 = diag\{-\tilde{R}_3, -\tilde{R}_5, -R_6, -3R_6, -5R_6\},\\
&\Delta_2 = diag\{-\tilde{R}_3, -\tilde{R}_5, -R_6, -3R_6, -5R_6, -Z_2, -3Z_2, -5Z_2, -Z_4, -3Z_4, -5Z_4\},\\
&\tilde{R}_3 = diag\{R_3,\ 3R_3,\ 5R_3\}, \qquad \tilde{R}_5 = diag\{R_5,\ 3R_5,\ 5R_5\},\\
&e_s = Ae_1 + Be_3 + Ce_9 + e_{34} + e_{35} + e_{36}, \quad \varepsilon_1 = \sqrt{(1-\mu_2)\tau_1}, \quad \varepsilon_2 = \sqrt{(1-\mu_2)\tau_2},\\
&\Pi_1 = \mathrm{col}\left\{e_s,\ e_{11},\ e_{37},\ e_8,\ e_{10},\ e_s - (1-\mu_2)e_9,\ e_1 - e_2,\ e_2 - e_4,\ e_1 - e_5,\ e_5 - e_7,\ E_{h_1},\ E_{\tau_1}\right\},\\
&\Pi_2 = \mathrm{col}\left\{e_1,\ e_2,\ e_4,\ e_5,\ e_7,\ e_1 - e_6,\ h_1e_{12},\ E_{h_2},\ \tau_1e_{21},\ E_{\tau_2},\ e_{32},\ e_{33}\right\},\\
&E_{h_1} = h_2e_1 - h_1e_{12} - h_{1t}e_{15} - h_{2t}e_{18}, \qquad E_{h_2} = h_{1t}e_{15} + h_{2t}e_{18},\\
&E_{\tau_1} = \tau_2e_1 - \tau_1e_{21} - \tau_{1t}e_{24} - \tau_{2t}e_{27}, \qquad E_{\tau_2} = \tau_{1t}e_{24} + \tau_{2t}e_{27},\\
&\Pi_3 = \mathrm{col}\left\{e_1,\ e_s\right\}, \qquad \Pi_7 = \mathrm{col}\left\{e_2 - e_3, e_2 + e_3 - 2e_{15}, e_2 - e_3 + 6e_{15} - 12e_{16}\right\},\\
&\Pi_4 = \mathrm{col}\left\{e_5,\ e_8\right\}, \qquad \Pi_8 = \mathrm{col}\left\{e_3 - e_4, e_3 + e_4 - 2e_{18}, e_3 - e_4 + 6e_{18} - 12e_{19}\right\},\\
&\Pi_5 = \mathrm{col}\left\{e_7,\ e_{10}\right\}, \qquad \Pi_9 = \mathrm{col}\left\{e_5 - e_6, e_5 + e_6 - 2e_{24}, e_5 - e_6 + 6e_{24} - 12e_{25}\right\},\\
&\Pi_6 = \mathrm{col}\left\{e_6,\ e_9\right\}, \qquad \Pi_{10} = \mathrm{col}\left\{e_6 - e_7, e_6 + e_7 - 2e_{27}, e_6 - e_7 + 6e_{27} - 12e_{28}\right\},\\
&\Pi_{11} = \mathrm{col}\left\{e_1,\ e_6,\ e_{30},\ e_{31}\right\}, \quad \Pi_{12} = \mathrm{col}\left\{e_2,\ e_3,\ e_{15},\ e_{16}\right\}, \quad \Pi_{13} = \mathrm{col}\left\{e_5,\ e_6,\ e_{24},\ e_{25}\right\},\\
&\Pi_{3a} = \mathrm{col}\left\{e_1,\ e_3\right\}, \qquad \Pi_{3b} = \mathrm{col}\left\{e_s,\ (1-\mu_1)e_{38}\right\}, \qquad \Pi_{3c} = \mathrm{col}\left\{e_1,\ e_6\right\},\\
&\Pi_{3d} = \mathrm{col}\left\{e_s,\ (1-\mu_2)e_9\right\}, \qquad \Pi_{3e} = \mathrm{col}\left\{e_2,\ e_{11}\right\}, \qquad \Pi_{3f} = \mathrm{col}\left\{e_4,\ e_{37}\right\},\\
&\Pi_{3g} = \mathrm{col}\left\{e_3,\ e_{38}\right\}, \quad H_{13} = \mathrm{col}\left\{h_1(e_{12} - 6e_{13} + 12e_{14}),\ e_1 - e_2 + 6e_{12} - 12e_{13}\right\},\\
&H_{11} = \mathrm{col}\left\{h_1e_{12},\ e_1 - e_2\right\}, \qquad H_{12} = \mathrm{col}\left\{h_1(e_{12} - 2e_{13}),\ e_1 + e_2 - 2e_{12}\right\},\\
&H_{21} = e_{15}, \quad H_{22} = e_{15} - 2e_{16}, \quad H_{23} = e_{15} - 6e_{16} + 12e_{17},\\
&H_{31} = e_{18}, \quad H_{32} = e_{18} - 2e_{19}, \quad H_{33} = e_{18} - 6e_{19} + 12e_{20},\\
&H_{41} = e_1 - e_5, \quad H_{42} = e_1 + e_5 - 2e_{21}, \quad H_{43} = e_1 - e_5 + 6e_{21} - 12e_{22},\\
&H_{51} = e_1 - e_6, \quad H_{52} = e_1 + e_6 - 2e_{30}, \quad H_{53} = e_1 - e_6 + 6e_{30} - 12e_{31},\\
&H_{61} = e_2 - e_3, \quad H_{62} = e_2 + e_3 - 2e_{15}, \quad H_{63} = e_2 - e_3 + 6e_{15} - 12e_{16},\\
&H_{71} = e_5 - e_6, \quad H_{72} = e_5 + e_6 - 2e_{24}, \quad H_{73} = e_5 - e_6 + 6e_{24} - 12e_{25},\\
&\Pi_{21} = e_1 - e_{12}, \quad \Pi_{22} = \frac{1}{2}e_1 + e_{12} - 3e_{13}, \quad \Pi_{23} = \frac{1}{3}e_1 - e_{12} + 8e_{13} - 20e_{14},\\
&\Pi_{31} = e_1 - e_{21}, \quad \Pi_{32} = \frac{1}{2}e_1 + e_{21} - 3e_{22}, \quad \Pi_{33} = \frac{1}{3}e_1 - e_{21} + 8e_{22} - 20e_{23},\\
&\Pi_{41} = e_2 - e_{15}, \quad \Pi_{42} = \frac{1}{2}e_2 + e_{15} - 3e_{16}, \quad \Pi_{43} = \frac{1}{3}e_2 - e_{15} + 8e_{16} - 20e_{17},\\
&\Pi_{51} = e_3 - e_{18}, \quad \Pi_{52} = \frac{1}{2}e_3 + e_{18} - 3e_{19}, \quad \Pi_{53} = \frac{1}{3}e_3 - e_{18} + 8e_{19} - 20e_{20},\\
&\Pi_{61} = e_5 - e_{24}, \quad \Pi_{62} = \frac{1}{2}e_5 + e_{24} - 3e_{25}, \quad \Pi_{63} = \frac{1}{3}e_5 - e_{24} + 8e_{25} - 20e_{26},\\
&\Pi_{71} = e_6 - e_{27}, \quad \Pi_{72} = \frac{1}{2}e_6 + e_{27} - 3e_{28}, \quad \Pi_{73} = \frac{1}{3}e_6 - e_{27} + 8e_{28} - 20e_{29}.
\end{aligned} \tag{18}
$$

Proof. Consider the following LKF:

$$
V(x_t) = \sum_{i=1}^{5} V_i(x_t) \tag{19}
$$

where

$$
\begin{aligned}
V_1(x_t) &= \chi_0^T P_0 \chi_0(t) \\
V_2(x_t) &= h(t)\chi_1^T(t)P_1\chi_1(t) + h_{2t}\chi_1^T(t)P_2\chi_1(t) \\
&\quad +\tau(t)\chi_2^T(t)P_3\chi_2(t) + \tau_{2t}\chi_2^T(t)P_4\chi_2(t) \\
V_3(x_t) &= \int_{t-h_1}^{t} \eta^T(s)Q_1\eta(s)ds + \int_{t-h_2}^{t-h_1} \eta^T(s)Q_2\eta(s)ds + \int_{t-h(t)}^{t} \eta^T(s)Q_3\eta(s)ds \\
&\quad + \int_{t-\tau_1}^{t} \eta^T(s)Q_4\eta(s)ds + \int_{t-\tau_2}^{t-\tau_1} \eta^T(s)Q_5\eta(s)ds + \int_{t-\tau(t)}^{t} \eta^T(s)Q_6\eta(s)ds \\
V_4(x_t) &= h_1\int_{-h_1}^{0}\int_{t+\theta}^{t} \eta^T(s)R_1\eta(s)dsd\theta + h_{21}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} x^T(s)R_2x(s)dsd\theta \\
&\quad +h_{21}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s)R_3\dot{x}(s)dsd\theta + \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_4\dot{x}(s)dsd\theta \\
&\quad +\tau_{21}\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{x}^T(s)R_5\dot{x}(s)dsd\theta + \int_{-\tau(t)}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_6\dot{x}(s)dsd\theta \\
V_5(x_t) &= \int_{t-h_1}^{t}\int_{\theta}^{t}\int_{u}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsdud\theta + \int_{t-h_2}^{t-h_1}\int_{\theta}^{t-h_1}\int_{u}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s)dsdud\theta \\
&\quad + \int_{t-\tau_1}^{t}\int_{\theta}^{t}\int_{u}^{t} \dot{x}^T(s)Z_3\dot{x}(s)dsdud\theta + \int_{t-\tau_2}^{t-\tau_1}\int_{\theta}^{t-\tau_1}\int_{u}^{t-\tau_1} \dot{x}^T(s)Z_4\dot{x}(s)dsdud\theta
\end{aligned}
$$

with

$$
\begin{aligned}
\chi_0(t) = \mathrm{col}\Bigg\{ & x(t), x(t-h_1), x(t-h_2), x(t-\tau_1), x(t-\tau_2), \int_{t-\tau(t)}^{t} \dot{x}(s)ds, \int_{t-h_1}^{t} x(s)ds, \\
& \int_{t-h_2}^{t-h_1} x(s)ds, \int_{t-\tau_1}^{t} x(s)ds, \int_{t-\tau_2}^{t-\tau_1} x(s)ds, \int_{t-h_2}^{t}\int_{\theta}^{t} x(s)dsd\theta, \ \int_{t-\tau_2}^{t}\int_{\theta}^{t} x(s)dsd\theta \Bigg\},
\end{aligned}
$$

$$
\chi_1(t) = \mathrm{col}\left\{x(t),\ x(t-h(t))\right\}, \quad \chi_2(t) = \mathrm{col}\left\{x(t),\ x(t-\tau(t))\right\}, \quad \eta(t) = \mathrm{col}\left\{x(t),\ \dot{x}(t)\right\},
$$

Taking the derivative of $V_1(x_t)$, $V_2(x_t)$ and $V_3(x_t)$, we respectively acquire

$$
\dot{V}_1(x_t) = 2\chi_0^T(t)P_0\dot{\chi}_0(t) = \xi^T(t)\Xi_1\xi(t) \tag{20}
$$

$$
\begin{aligned}
\dot{V}_2(x_t) &= \dot{h}(t)\chi_1^T(t)P_1\chi_1(t) + 2h(t)\chi_1^T(t)P_1\dot{\chi}_1(t) - \dot{h}(t)\chi_1^T(t)P_2\chi_1(t) \\
&\quad +2h_{2t}\chi_1^T(t)P_2\dot{\chi}_1(t) + \dot{\tau}(t)\chi_2^T(t)P_3\chi_2(t) + 2\tau(t)\chi_2^T(t)P_3\dot{\chi}_2(t) \\
&\quad -\dot{\tau}(t)\chi_2^T(t)P_4\chi_2(t) + 2\tau_{2t}\chi_2^T(t)P_4\dot{\chi}_2(t) \\
&\le \xi^T(t)\Xi_2\xi(t)
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\dot{V}_3(x_t) &\le \xi^T(t)\big[\Pi_3^T(Q_1+Q_3+Q_4+Q_6)\Pi_3 + \Pi_{3e}^T(Q_2-Q_1)\Pi_{3e} - \Pi_{3f}^TQ_2\Pi_{3f} \\
&\quad -(1-\mu_1)\Pi_{3g}^TQ_3\Pi_{3g} + \Pi_4^T(Q_5-Q_4)\Pi_4 - \Pi_5^TQ_5\Pi_5 - (1-\mu_2)\Pi_6^TQ_6\Pi_6\big]\xi(t) \\
&= \xi^T(t)\Xi_3\xi(t)
\end{aligned} \tag{22}
$$

Then, the derivative of $V_4(x_t)$ and $V_5(x_t)$ can be respectively acquired as

$$\begin{aligned}\dot{V}_4(x_t) &= \xi^T(t)[h_1^2\Pi_3^T R_1\Pi_3+h_{21}^2 e_1^T R_2 e_1+e_s^T(h_{21}^2 R_3+\tau_1^2 R_4+\tau_{21}^2 R_5+\tau(t)R_6)e_s]\xi(t)\\ &\quad +\theta_1+\theta_2+\theta_3 \qquad (23)\\ \dot{V}_5(x_t) &= \xi^T(t)[e_s^T(\frac{h_1^2}{2}Z_1+\frac{\tau_1^2}{2}Z_3)e_s+\frac{h_{21}^2}{2}e_{11}^T Z_2 e_{11}+\frac{\tau_{21}^2}{2}e_8^T Z_4 e_8]\xi(t)+\delta_1+\delta_2 \qquad (24)\end{aligned}$$

where

$$\begin{aligned}\theta_1 &= -h_1\int_{t-h_1}^{t}\eta^T(s)R_1\eta(s)ds - h_{21}\int_{t-h_2}^{t-h_1}x^T(s)R_2x(s)ds - \tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R_4\dot{x}(s)ds,\\ \theta_2 &= -h_{21}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_3\dot{x}(s)ds - \tau_{21}\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_5\dot{x}(s)ds,\\ \theta_3 &= -(1-\dot{\tau}(t))\int_{t-\tau(t)}^{t}\dot{x}^T(s)R_6\dot{x}(s)ds,\\ \delta_1 &= -\int_{t-h_1}^{t}\int_{\theta}^{t}\dot{x}^T(s)Z_1\dot{x}(s)dsd\theta - \int_{t-h(t)}^{t-h_1}\int_{\theta}^{t-h_1}\dot{x}^T(s)Z_2\dot{x}(s)dsd\theta\\ &\quad -\int_{t-h_2}^{t-h(t)}\int_{\theta}^{t-h(t)}\dot{x}^T(s)Z_2\dot{x}(s)dsd\theta - \int_{t-\tau_1}^{t}\int_{\theta}^{t}\dot{x}^T(s)Z_3\dot{x}(s)dsd\theta\\ &\quad -\int_{t-\tau(t)}^{t-\tau_1}\int_{\theta}^{t-\tau_1}\dot{x}^T(s)Z_4\dot{x}(s)dsd\theta - \int_{t-\tau_2}^{t-\tau(t)}\int_{\theta}^{t-\tau(t)}\dot{x}^T(s)Z_4\dot{x}(s)dsd\theta,\\ \delta_2 &= -h_{2t}\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)Z_2\dot{x}(s)ds - \tau_{2t}\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^T(s)Z_4\dot{x}(s)ds.\end{aligned}$$

By using inequalitiy (4) of Lemma 2.1, the estimation of θ_1 can be derived as follows

$$\begin{aligned}\theta_1 \le \xi^T(t)\big[&-(H_{11}^T R_1 H_{11}+3H_{12}^T R_1 H_{12}+5H_{13}^T R_1 H_{13})\\ &-h_{21}h_{1t}(H_{21}^T R_2 H_{21}+3H_{22}^T R_2 H_{22}+5H_{23}^T R_2 H_{23})\\ &-h_{21}h_{2t}(H_{31}^T R_2 H_{31}+3H_{32}^T R_2 H_{32}+5H_{33}^T R_2 H_{33})\\ &-(H_{41}^T R_4 H_{41}+3H_{42}^T R_4 H_{42}+5H_{43}^T R_4 H_{43})\big]\xi(t) \qquad (25)\end{aligned}$$

Based on Lemma 2.2 and Lemma 2.3, θ_2 is computed as follows

$$\begin{aligned}\theta_2 &\le -\xi^T(t)\Big[\frac{h_{21}}{h_{1t}}\Pi_7^T\tilde{R}_3\Pi_7+\frac{h_{21}}{h_{2t}}\Pi_8^T\tilde{R}_3\Pi_8+\frac{\tau_{21}}{\tau_{1t}}\Pi_9^T\tilde{R}_5\Pi_9+\frac{\tau_{21}}{\tau_{2t}}\Pi_{10}^T\tilde{R}_5\Pi_{10}\Big]\xi(t)\\ &\le \xi^T(t)(\Omega+\Phi)\xi(t) \qquad (26)\end{aligned}$$

where Ω is given in (18) and

$$\Phi = \frac{h_{1t}}{h_{21}}S_1\tilde{R}_3^{-1}S_1^T + \frac{h_{2t}}{h_{21}}S_2\tilde{R}_3^{-1}S_2^T + \frac{\tau_{1t}}{\tau_{21}}S_3\tilde{R}_5^{-1}S_3^T + \frac{\tau_{2t}}{\tau_{21}}S_4\tilde{R}_5^{-1}S_4^T$$

By using inequalities (6) and (5) of Lemma 2.1, the estimation of δ_1, θ_3 and δ_2 can be respectively got as follows

$$\begin{aligned}\delta_1 \leq \xi^T(t)\big[&-2\Pi_{21}^T Z_1\Pi_{21}-16\Pi_{22}^T Z_1\Pi_{22}-54\Pi_{23}^T Z_1\Pi_{23}-2\Pi_{31}^T Z_3\Pi_{31}-16\Pi_{32}^T Z_3\Pi_{32}\\ &-54\Pi_{33}^T Z_3\Pi_{33}-2\Pi_{41}^T Z_2\Pi_{41}-16\Pi_{42}^T Z_2\Pi_{42}-54\Pi_{43}^T Z_2\Pi_{43}\\ &-2\Pi_{51}^T Z_2\Pi_{51}-16\Pi_{52}^T Z_2\Pi_{52}-54\Pi_{53}^T Z_2\Pi_{53}-2\Pi_{61}^T Z_4\Pi_{61}-16\Pi_{62}^T Z_4\Pi_{62}\\ &-54\Pi_{63}^T Z_4\Pi_{63}-2\Pi_{71}^T Z_4\Pi_{71}-16\Pi_{72}^T Z_4\Pi_{72}-54\Pi_{73}^T Z_4\Pi_{73}\big]\xi(t) \qquad (27)\end{aligned}$$

$$\begin{aligned}\theta_3+\delta_2 \leq \xi^T(t)\big[&(1-\mu_2)\tau(t)\Pi_{11}^T(F_{11}R_6^{-1}F_{11}^T+\frac{1}{3}F_{12}R_6^{-1}F_{12}^T+\frac{1}{5}F_{13}R_6^{-1}F_{13}^T)\Pi_{11}\\ &+(1-\mu_2)\mathrm{Sym}\{\Pi_{11}^T F_{11}H_{51}+\Pi_{11}^T F_{12}H_{52}+\Pi_{11}^T F_{13}H_{53}\}\\ &+h_{2t}h_{1t}\Pi_{12}^T(F_{21}Z_2^{-1}F_{21}^T+\frac{1}{3}F_{22}Z_2^{-1}F_{22}^T+\frac{1}{5}F_{23}Z_2^{-1}F_{23}^T)\Pi_{12}\\ &+h_{2t}\mathrm{Sym}\{\Pi_{12}^T F_{21}H_{61}+\Pi_{12}^T F_{22}H_{62}+\Pi_{12}^T F_{23}H_{63}\}\\ &+\tau_{2t}\tau_{1t}\Pi_{13}^T(F_{31}Z_4^{-1}F_{31}^T+\frac{1}{3}F_{32}Z_4^{-1}F_{32}^T+\frac{1}{5}F_{33}Z_4^{-1}F_{33}^T)\Pi_{13}\\ &+\tau_{2t}\mathrm{Sym}\{\Pi_{13}^T F_{31}H_{71}+\Pi_{13}^T F_{32}H_{72}+\Pi_{13}^T F_{33}H_{73}\}\,\big]\xi(t) \qquad (28)\end{aligned}$$

Thus, combining Eqs. (23) and (24) and inequalities (25)-(28), we have

$$\dot{V}_4(x_t)+\dot{V}_5(x_t) \leq \xi^T(t)\left(\Xi_4+\Theta\right)\xi(t) \qquad (29)$$

where

$$\begin{aligned}\Theta = \Phi &+ (1-\mu_2)\tau(t)\Pi_{11}^T(F_{11}R_6^{-1}F_{11}^T+\frac{1}{3}F_{12}R_6^{-1}F_{12}^T+\frac{1}{5}F_{13}R_6^{-1}F_{13}^T)\Pi_{11}\\ &+ h_{2t}h_{1t}\Pi_{12}^T(F_{21}Z_2^{-1}F_{21}^T+\frac{1}{3}F_{22}Z_2^{-1}F_{22}^T+\frac{1}{5}F_{23}Z_2^{-1}F_{23}^T)\Pi_{12}\\ &+ \tau_{2t}\tau_{1t}\Pi_{13}^T(F_{31}Z_4^{-1}F_{31}^T+\frac{1}{3}F_{32}Z_4^{-1}F_{32}^T+\frac{1}{5}F_{33}Z_4^{-1}F_{33}^T)\Pi_{13}\end{aligned}$$

For any matrices L_1 and L_2, the following hold

$$0 = 2\xi^T(t)N_1L_1\left[h_1^2e_{13}+h_{1t}^2e_{16}+h_{2t}^2e_{19}+h_{2t}h_{1t}e_{15}+h_{21}h_1e_{12}-e_{32}\right]\xi(t) \qquad (30)$$

$$0 = 2\xi^T(t)N_2L_2\left[\tau_1^2e_{22}+\tau_{1t}^2e_{25}+\tau_{2t}^2e_{28}+\tau_{2t}\tau_{1t}e_{24}+\tau_{21}\tau_1e_{21}-e_{33}\right]\xi(t) \qquad (31)$$

Besides, from Eq. (3), for any given scalars $\rho_i \geq 0$, $i=1,2,3$, we can obtain

$$\begin{aligned}&\rho_1\left[\gamma_1^2x^T(t)x(t)-g_1^T(x(t),t)g_1(x(t),t)\right] \geq 0,\\ &\rho_2\left[\gamma_2^2x^T(t-h(t))x(t-h(t))-g_2^T(x(t-h(t)),t)g_2(x(t-h(t)),t)\right] \geq 0,\\ &\rho_3\left[\gamma_3^2\dot{x}^T(t-\tau(t))\dot{x}(t-\tau(t))-g_3^T(\dot{x}(t-\tau(t)),t)g_3(\dot{x}(t-\tau(t)),t)\right] \geq 0. \qquad (32)\end{aligned}$$

So, combining formulas (20)-(22) and (29)-(32), $\dot{V}(x_t)$ can be bounded as

$$\dot{V}(x_t) \leq \xi^T(t)\bar{\Xi}\xi(t) \qquad (33)$$

where $\bar{\Xi}=\Xi+\Theta$, with Ξ and Θ are shown in (18) and (29), respectively. Notice that some $h^2(t)$-dependent and $\tau^2(t)$-dependent terms are involved in $\bar{\Xi}$. Hence, $\dot{V}(x_t)$ can be expressed as $f(h(t),\tau(t))$ in Lemma 2.4, with

$$f(h(t),\tau(t)) := \xi^T(t)\bar{\Xi}\xi(t) \tag{34}$$
$$a_6 := \xi^T(t)(\Psi_h + \Psi_{h_0})\xi(t) \tag{35}$$
$$a_4 := \xi^T(t)(\Psi_\tau + \Psi_{\tau_0})\xi(t) \tag{36}$$

where Ψ_h, Ψ_τ, Ψ_{h_0} and Ψ_{τ_0} are defined in (18), and the signs for a_6 and a_4 are the same, but unknown. Therefore, by Lemma 2.4, $\dot{V}(x_t) < 0$ can be guaranteed if the following conditions (37) and (38) hold

$$\begin{aligned}
&\Xi(h_1,\tau_1)+\Theta(h_1,\tau_1) < 0,\\
&\Xi(h_1,\tau_2)+\Theta(h_1,\tau_2) < 0,\\
&\Xi(h_2,\tau_1)+\Theta(h_2,\tau_1) < 0,\\
&\Xi(h_2,\tau_2)+\Theta(h_2,\tau_2) < 0.
\end{aligned} \tag{37}$$

and

$$\begin{aligned}
&\Xi(h_1,\tau_1)-\alpha^2h_{21}^2\Psi_h-\beta^2\tau_{21}^2\Psi_\tau+\Theta(h_1,\tau_1)-\alpha^2h_{21}^2\Psi_{h_0}-\beta^2\tau_{21}^2\Psi_{\tau_0} < 0,\\
&\Xi(h_1,\tau_2)-\alpha^2h_{21}^2\Psi_h-(1-\beta)^2\tau_{21}^2\Psi_\tau+\Theta(h_1,\tau_2)-\alpha^2h_{21}^2\Psi_{h_0}-(1-\beta)^2\tau_{21}^2\Psi_{\tau_0} < 0,\\
&\Xi(h_2,\tau_1)-(1-\alpha)^2h_{21}^2\Psi_h-\beta^2\tau_{21}^2\Psi_\tau+\Theta(h_2,\tau_1)-(1-\alpha)^2h_{21}^2\Psi_{h_0}-\beta^2\tau_{21}^2\Psi_{\tau_0} < 0,\\
&\Xi(h_2,\tau_2)-(1-\alpha)^2h_{21}^2\Psi_h-(1-\beta)^2\tau_{21}^2\Psi_\tau+\Theta(h_2,\tau_2)\\
&\qquad\qquad\qquad -(1-\alpha)^2h_{21}^2\Psi_{h_0}-(1-\beta)^2\tau_{21}^2\Psi_{\tau_0} < 0.
\end{aligned} \tag{38}$$

It is evident that there are some nonlinear terms Θ, Ψ_{h_0} and Ψ_{τ_0} in the above conditions (37) and (38). To tackle this, on the basis of the Schur complement, the equivalent conditions for inequalities (37) and (38) can be obtained as LMIs (16) and (17). As a consequence, LMIs (16) and (17) ensure the asymptotical stability of the neutral system (1) subjects to (2) and (3). This completes the proof. □

Remark 3. A fresh augmented LKF, $V(x_t)$, is developed in Theorem 3.1, which not only introduces four delay-product-type terms in $V_2(x_t)$, but also involves some augmented vectors. Particularly, the augmented vector $\chi_0(t)$ considering the two double integral terms $\int_{t-h_2}^{t}\int_{\theta}^{t} x(s)dsd\theta$ and $\int_{t-\tau_2}^{t}\int_{\theta}^{t} x(s)dsd\theta$ as augmented variables is introduced in LKF (19), which are not involved in the LKF of Chen et al. (2020). The addition of those terms provides more information of the state-related vectors with discrete and neutral delay, and makes the proposed LKF be more comprehensive, which will be conducive to reduce the conservatism.

Remark 4. The two vector zero-valued equalities (30) and (31) containing discrete and neutral delay, $h(t)$ and $\tau(t)$, which give enough thought to the relationship between the system state vectors, are builded respectively. For example, for discrete delay $h(t)$, the relationship between vectors $\frac{1}{h_1^2}\int_{t-h_1}^{t}\int_{\theta}^{t}x(s)dsd\theta$, $\frac{1}{h_{1t}^2}\int_{t-h(t)}^{t-h_1}\int_{\theta}^{t-h_1}x(s)dsd\theta$, $\frac{1}{h_{2t}^2}\int_{t-h_2}^{t-h(t)}\int_{\theta}^{t-h(t)}x(s)dsd\theta$, $\frac{1}{h_{1t}}\int_{t-h(t)}^{t-h_1}x(s)ds$, $\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds$ and $\int_{t-h_2}^{t}\int_{\theta}^{t}x(s)dsd\theta$ is taken into consideration. Since adding the equalities (30) and (31) into the bound of LKF derivative, the expression of relations between state vectors can be strengthened and the information of cross terms can be increased, it has the ability to acquire stability criteria with lower conservatism.

Remark 5. What is noteworthy is that due to the introduction of the two double integral augmented terms and two vector zero-valued equalities (30) and (31), and the usage of the inequality (5) to estimate Z_2-dependent and Z_4-dependent terms, the derived stability condition (33) contains two delay square terms, $(\Psi_h + \Psi_{h_0})h^2(t)$ and $(\Psi_\tau + \Psi_{\tau_0})\tau^2(t)$. In Theorem 3.1, the two nonlinear time-varying delay terms are directly treated by Lemma 2.4. As a consequence, Theorem 3.1 with two adjustable parameters α and β is obtained. For proper preset α and β, the conservatism of Theorem 3.1 is reduced.

For the case where the discrete delay derivative μ_1 is unknown, if let $P_1 = P_2 = Q_3 = 0$ in the Theorem 3.1, then the following Corollary 3.2 is directly acquired.

Corollary 3.2. *For given scalars $0 \le h_1 < h_2$, $0 \le \tau_1 < \tau_2$, μ_2, γ_i, $(i = 1, 2, 3)$, and α, $\beta \in [0, 1]$, under the nonlinear disturbances fulfilling conditions (3), the neutral system (1) with any discrete and neutral delays satisfying (2) is asymptotically stable if there exist positive definite matrices $P_0 \in \mathcal{R}^{12n\times 12n}$, P_3, $P_4 \in \mathcal{R}^{2n\times 2n}$, $Q_i \in \mathcal{R}^{2n\times 2n}$, $(i = 1, 2, 4, 5, 6)$, $R_1 \in \mathcal{R}^{2n\times 2n}$, $R_i \in \mathcal{R}^{n\times n}$, $(i = 2,\ldots, 6)$, $Z_i \in \mathcal{R}^{n\times n}$, $(i = 1,\ldots, 4)$, any matrices $S_i \in \mathcal{R}^{37n\times 3n}$, $(i = 1,\ldots, 4)$, $F_{ij} \in \mathcal{R}^{4n\times n}$, $(i = 1, 2, 3,\ j = 1, 2, 3)$, $L_1, L_2 \in \mathcal{R}^{6n\times n}$, and real scalars $\rho_i \ge 0$, $(i = 1, 2, 3)$, such that the following LMIs are satisfied:*

$$\begin{aligned}
&\begin{bmatrix} \bar{\Xi}(h_1,\tau_1) & \Gamma_{11} \\ * & \Delta_1 \end{bmatrix} < 0, \qquad \begin{bmatrix} \bar{\Xi}(h_1,\tau_2) & \Gamma_{12} \\ * & \Delta_1 \end{bmatrix} < 0, \\
&\begin{bmatrix} \bar{\Xi}(h_2,\tau_1) & \Gamma_{13} \\ * & \Delta_1 \end{bmatrix} < 0, \qquad \begin{bmatrix} \bar{\Xi}(h_2,\tau_2) & \Gamma_{14} \\ * & \Delta_1 \end{bmatrix} < 0,
\end{aligned} \tag{39}$$

$$\begin{aligned}
&\begin{bmatrix} \bar{\Xi}(h_1,\tau_1) - \alpha^2 h_{21}^2\Psi_h - \beta^2\tau_{21}^2\Psi_\tau & \Gamma_{21} \\ * & \Delta_2 \end{bmatrix} < 0, \\
&\begin{bmatrix} \bar{\Xi}(h_1,\tau_2) - \alpha^2 h_{21}^2\Psi_h - (1-\beta)^2\tau_{21}^2\Psi_\tau & \Gamma_{22} \\ * & \Delta_2 \end{bmatrix} < 0, \\
&\begin{bmatrix} \bar{\Xi}(h_2,\tau_1) - (1-\alpha)^2 h_{21}^2\Psi_h - \beta^2\tau_{21}^2\Psi_\tau & \Gamma_{23} \\ * & \Delta_2 \end{bmatrix} < 0, \\
&\begin{bmatrix} \bar{\Xi}(h_2,\tau_2) - (1-\alpha)^2 h_{21}^2\Psi_h - (1-\beta)^2\tau_{21}^2\Psi_\tau & \Gamma_{24} \\ * & \Delta_2 \end{bmatrix} < 0.
\end{aligned} \tag{40}$$

where

$$\begin{aligned}
&\bar{\Xi}(h(t),\tau(t)) = \Xi_1 + \bar{\Xi}_2 + \bar{\Xi}_3 + \Xi_4 + \Xi_5 + \Xi_6, \\
&\bar{\Xi}_2 = \mu_2\left(\Pi_{3c}^T P_3\Pi_{3c} - \Pi_{3c}^T P_4\Pi_{3c}\right) + \mathrm{Sym}\left\{\tau(t)\Pi_{3c}^T P_3\Pi_{3d} + \tau_{2t}\Pi_{3c}^T P_4\Pi_{3d}\right\}, \\
&\bar{\Xi}_3 = \Pi_3^T(Q_1 + Q_4 + Q_6)\Pi_3 + \Pi_{3e}^T(Q_2 - Q_1)\Pi_{3e} - \Pi_{3f}^T Q_2\Pi_{3f} + \Pi_4^T(Q_5 - Q_4)\Pi_4 \\
&\qquad - \Pi_5^T Q_5\Pi_5 - (1-\mu_2)\Pi_6^T Q_6\Pi_6, \\
&\bar{\xi}(t) = \Big[\xi_1(t),\ \xi_2(t),\ \xi_3(t),\ \xi_4(t),\ \bar{\xi}_5(t)\Big]^T, \\
&\bar{\xi}_5(t) = \mathrm{col}\big\{g_1(x(t),t),\ g_2(x(t-h(t)),t),\ g_3(\dot{x}(t-\tau(t)),t),\ \dot{x}(t-h_2)\big\}, \\
&\bar{e}_i = [0_{n\times(i-1)n}\ I_n\ 0_{n\times(37-i)n}]^T,\ i = 1, 2, \cdots, 37.
\end{aligned}$$

In addition, other notations are given in Theorem 3.1.

Table 1. The MAUBs of h_2 for various h_1 and γ_3. ($\gamma_1 = 0$)

h_1	Methods	$\gamma_1 = 0$	
		$\gamma_3 = 0$	$\gamma_3 = 0.1$
0.1	Chen et al. (2020)	1.3207	0.9822
	Theorem 3.1	1.6241 ($\alpha = 0.11$, $\beta = 0.58$)	1.1697 ($\alpha = 0.13$, $\beta = 0.60$)
0.5	Chen et al. (2020)	1.3763	1.0312
	Theorem 3.1	1.6692 ($\alpha = 0.40$, $\beta = 0.53$)	1.2208 ($\alpha = 0.41$, $\beta = 0.53$)

Table 2. The MAUBs of h_2 for various h_1 and γ_3. ($\gamma_1 = 0.1$)

h_1	Methods	$\gamma_1 = 0.1$	
		$\gamma_3 = 0$	$\gamma_3 = 0.1$
0.1	Chen et al. (2020)	1.1591	0.8472
	Theorem 3.1	1.3463 ($\alpha = 0.12$, $\beta = 0.57$)	0.9885 ($\alpha = 0.15$, $\beta = 0.63$)
0.5	Chen et al. (2020)	1.2084	0.8941
	Theorem 3.1	1.3882 ($\alpha = 0.46$, $\beta = 0.54$)	1.0390 ($\alpha = 0.15$, $\beta = 0.59$)

4. Numerical examples

Two numerical examples are provided to evaluate the performance of the presented Lemma 2.4 and stability criteria.

Example 4.1. Consider the neutral system (1) with nonlinear disturbances having the following constant matrices:

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

Suppose that $\mu_1 = \mu_2 = 0.5$, $\tau_1 = 0$, $\tau_2 = h_2$ and $\gamma_2 = 0.1$. By taking different values as $h_1 = \{0.1,\ 0.5\}$, $\gamma_1 = \{0,\ 0.1\}$ and $\gamma_3 = \{0,\ 0.1\}$, the maximum allowable upper bounds (MAUBs) h_2 for the discrete delay $h(t)$ are obtained by LMIs (16) and (17) in Theorem 3.1, which are made a comparison with the numerical results presented in existing literature (Chen et al. , 2020). For various values of γ_1, the MAUBs h_2 are described in Tables 1 and 2, repectively. (The values of α and β are gained by incrementing with a step size of 0.01 from 0 to 1 and choosing the best one that minimizes the conservatism of Theorem 3.1.)

Tables 1 and 2 clearly show that the numerical results obtained in Theorem 3.1 are superior to that of Chen et al. (2020). Moreover, it can be noted that the MAUBs h_2 is downed for larger values of γ_1 and γ_3.

Example 4.2. Consider the neutral system (1) with nonlinear disturbances with the parameters below:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

Suppose that $\gamma_1 = 0.1$, $\gamma_2 = \gamma_3 = 0.05$, $\tau_1 = 0$, $\tau_2 = h_2$ and $\mu_1 =$ unknown. Given $\mu_2 = \{0,\ 0.6\}$ and $h_1 = \{0,\ 0.5,\ 1\}$, Table 3 reveals the comparison of the MAUBs h_2

Table 3. The MAUBs of h_2 for various μ_2 and h_1.

μ_2	Methods	h_1		
		0	0.5	1
0	Yu & Lien (2008)	1.8842	2.3032	2.8032
	Ramakrishnan & Ray (2011)	3.7024	4.1064	4.6064
	Lakshmanan et al. (2011)	4.4239	4.7392	5.0992
	Cheng et al. (2013)	5.0237	5.2164	5.9862
	Mohajerpoor et al. (2017)	6.6601	-	7.0600
	Chen et al. (2020)	-	7.3501	7.8499
	Corollary 3.2	9.9205 ($\alpha = 0.55$, $\beta = 0.53$)	10.3543 ($\alpha = 0.53$, $\beta = 0.50$)	10.8541 ($\alpha = 0.54$, $\beta = 0.52$)
0.6	Yu & Lien (2008)	1.6904	2.1094	2.6094
	Ramakrishnan & Ray (2011)	3.3150	3.7189	4.2189
	Lakshmanan et al. (2011)	3.9563	-	4.6391
	Cheng et al. (2013)	4.6235	-	5.0052
	Mohajerpoor et al. (2017)	6.4020	-	6.5010
	Chen et al. (2020)	-	6.5839	7.0839
	Corollary 3.2	8.7436 ($\alpha = 0.52$, $\beta = 0.67$)	9.1701 ($\alpha = 0.54$, $\beta = 0.46$)	9.6673 ($\alpha = 0.54$, $\beta = 0.50$)

Table 4. The MAUBs of h_2 for various μ_2 and h_1.

μ_2	Methods	h_1		
		1	0.6	0.1
0.2	Chen et al. (2020)	8.0702	7.6701	7.1736
	Corollary 3.2	10.7244 ($\alpha = 0.56$, $\beta = 0.55$)	10.3415 ($\alpha = 0.56$, $\beta = 0.51$)	9.8793 ($\alpha = 0.55$, $\beta = 0.43$)
0.5	Chen et al. (2020)	7.5186	7.1186	6.6713
	Corollary 3.2	10.1257 ($\alpha = 0.56$, $\beta = 0.48$)	9.7460 ($\alpha = 0.55$, $\beta = 0.32$)	9.2755 ($\alpha = 0.54$, $\beta = 0.29$)

for the discrete delay $h(t)$ obtained by LMIs (39) and (40) in Corollary 3.2 for various discrete delay lower bounds h_1 and neutral delay derivatives μ_2 over the existing ones (Chen et al. , 2020; Cheng et al. , 2013; Lakshmanan et al. , 2011; Mohajerpoor et al. , 2017; Ramakrishnan & Ray , 2011; Yu & Lien , 2008), where " $-$ " denotes that the MAUBs for corresponding cases are not provided.

Suppose that $\gamma_1 = 0.1$, $\gamma_2 = \gamma_3 = 0.05$, $\tau_1 = 0.5$, $\tau_2 = 1$ and $\mu_1 =$ unknown. For $\mu_2 = \{0.2,\ 0.5\}$ and $h_1 = \{1,\ 0.6,\ 0.1\}$, Table 4 reports the MAUBs h_2 for the discrete delay $h(t)$ calculated by Corollary 3.2 for different values of μ_2 and h_1, and compares them against the results presented in existing literature (Chen et al. , 2020). Based on the comparison of the MAUBs h_2, it is noted that Corollary 3.2 provides less conservative results than those of Chen et al. (2020).

As can be seen from Tables 3 and 4, it is clear that the MAUB h_2 is dropped by increasing neutral delay derivatives μ_2. However, by increasing the lower bound h_1 of discrete delay, the MAUB h_2 is improved. And the results calculated from Corollary 3.2 are notably much larger than that of Chen et al. (2020); Cheng et al. (2013); Lakshmanan et al. (2011); Mohajerpoor et al. (2017); Ramakrishnan & Ray (2011); Yu & Lien (2008).

It is clear from Tables 1-4 that the experimental results derived from Theorem 3.1 and Corollary 3.2 can generate the larger MAUBs h_2 than before. Therefore, the stability criteria obtained by applying the LKF (19) and Lemma 2.4 have greater superiority in realizing less conservative stability regions.

5. Conclusion

A new augmented LKF in which two double integral terms of discrete delay and neutral delay state vectors are considered as augmented terms has been firstly built. Then, the single/multiple integral inequalities, together with generalized reciprocally convex combination lemma, have been employed to estimate the time derivative. To linearize the $h^2(t)$-dependent and $\tau^2(t)$-dependent terms appearing in the time derivative, based on Taylor's formula, a relaxed negative-determination lemma has been raised, which introduces two adjustable parameters. Two delay-dependent stability criteria have been put forward by using the proposed method. Lastly, the advantages of the derived stability conditions have been testified by two typical examples. It is worth noting that the proposed negative-determination lemma is still conservative, and future work on avoiding this defect is meaningful.

Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

Disclosure statement

No potential conflict of interest was reported by the authors.

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