Simplified Models and Beyond: Matching Models of New Physics up to One-loop Level

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ABSTRACT

In this thesis, we study models of new physics generically by extending the Standard Model (SM) particle content with an arbitrary number of heavy fermions and bosons. Without additional constraints, such an extension does not describe a renormalisable quantum field theory and cannot be used to derive predictions for physical processes. We use the constraints from the Slavnov-Taylor Identities (STIs) that are derived from the vanishing Becchi-Rouet-Stora-Tyutin (BRST) transformation of suitable vertex functions. Within this setup, we calculate one-loop matching corrections for the flavour changing neutral current transitions with the emission of virtual vector bosons. We present explicit results for the Wilson coefficients of the weak effective Lagrangian for leptonic, semileptonic and radiative meson decays in a generic renormalisable model. We explicitly show that our results are finite and independent of the gauge-fixing parameters, and provide `Mathematica` code that implements our results in an easily usable form. The results of our calculations are published in Ref. [1].

Our generic one-loop results are not limited to flavour observables and they are applicable to other new physics interactions as well, such as dark matter (DM) interactions with the SM particles that are one-loop induced. We study these interactions where new vector-like fermions and complex scalars mediate DM interactions with the SM sector at one-loop level. For the sake of simplicity, we consider the phenomenology of tree-level DM interaction in the last part of the thesis. We study the direct-detection rate for axial-vectorial DM scattering off nuclei in an SU(2)×U(1) invariant effective theory and compare it with the LHC reach. Current constraints from direct detection experiments are already bounding the mediator mass to be in the TeV range for WIMP-like scenarios. This motivates a consistent and systematic exploration of the parameter space to map out possible regions where the rates could be suppressed. We do indeed find such regions and proceed to construct consistent UV models that generate the relevant effective theory. We then discuss the corresponding constraints from both collider and direct-detection experiments on the same parameter space. We find a benchmark scenario, where even for future XENONnT experiment, LHC constraints will have a greater sensitivity to the mediator mass Ref. [2].

Contents

1 INTRODUCTION

A modern view of elementary particles and interactions between them is encapsulated in the Standard Model (SM) of particle physics. The particle content of the SM consists of *fermions* that are quantas of matter fields, and *gauge bosons* that mediate the fundamental forces. In addition there is a single fundamental scalar, *Higgs boson* [3]. It is an experimental evidence that the fermions exist in three families or generations [4].

It has been well established that the fundamental interactions can be formulated as gauge theories. The idea originates in studies of the gravitational and electromagnetic interactions as gauge theories which are based on the Lorentz group $\mathrm{SO}(3,1)$ and the compact 'internal' phase group $\mathrm{U}(1)$, respectively. Later, gauge theories for the strong and weak nuclear interactions have been constructed successfully [5, 6]. In the gauge theory formulation, interactions occur via the exchange of integer-spin particles, gauge bosons. The gauge symmetry of the standard model is $\mathrm{SU}(3)_{\mathrm{color}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$, where $\mathrm{SU}(3)_{\mathrm{color}}$ is the symmetry group of the strong interaction. The $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$ defines the symmetry group of the unified electroweak interaction which is expected to be spontaneously broken to give a physically consistent description of the weak and electromagnetic forces.

Although, the standard model provides a successful description of most data in particle physics experiments, there are hints of its incompleteness coming from astrophysics and deviations from its predictions in flavour observables. These and other facts motivate to extend the SM with new particles and to construct models of new physics (NP) Beyond the SM (BSM). In general, shortcomings of the SM can be viewed as theoretical, experimental. One theoretical shortcoming of the SM is the so called flavour puzzle. The SM only describes the masses and mixing of the observed quarks and leptons in terms of fundamental parameters. Models of NP can address this flavour puzzle by aiming to predict the associated parameters in terms of fewer more fundamental ones. Also, assignments of quantum numbers (*e.g.*, hypercharge) to particles can not be explained within the SM, even though we know that they have to fulfil the anomaly cancellation condition. The list can be extended by questions regarding the gauge structure of the SM, namely, the hierarchy problem (why $m_W \ll M_{\mathrm{Planck}}$?) and unification of the gauge couplings constants [7, 8]. The experimental limitations of the SM can be linked to the fact that the SM predictions in flavour observables depart from the corresponding experimental results. The recent experimental results on lepton flavour non-universality in rare B-meson decays [9] and on the anomalous magnetic moment of the muon [10] have reaffirmed and strengthened the pre-existing tensions with the corresponding SM predictions. Furthermore, there are observational reasons for NP which come from astrophysical and cosmological studies. In the standard model, there is no suitable candidate for Dark Matter (DM) whose existence is strongly confirmed by measurements at the level of clusters of galaxies [11]. In addition, the Standard Model is incapable to explain the nature of dark energy and matter-antimatter asymmetry in the Universe [12, 13].

The theoretical approaches to study new physics can be categorised into three groups:

- The effective field theories (EFT), for example, the standard model effective field theory (SMEFT), where effects of NP are included in higher-dimensional operators that are constructed by the SM fields. Even though EFT method allows to study the NP effects in a model independent way, a number of interaction operators to be considered can be significantly large. For instance, following the Ref. [14], the SMEFT Lagrangian at dimension-6 is
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i=1}^{2499} \frac{C_i}{\Lambda^2} \mathcal{O}_i,$$
where Λ is a dimensionful scale and C_i are the dimensionless Wilson coefficients. The dimension-6 operators $\mathcal{O}_i$ in the Warsaw basis can be found in [15].

- The explicit models such as supersymmetric extension of the SM or a composite Higgs model. These ultraviolet-complete models have been proposed as possible solutions of the hierarchy problem. Usually, explicit models contain a large amount of parameters.

- Simplified models which cover a large set of model space, and are widely used for new physics searches at colliders. They are specifically designed to involve only a few new parameters directly related to collider physics observables: particle masses (and their decay widths), production cross-sections, and branching fractions [16]. However, they are often neither renormalisable nor unitary.

In this work, we propose an alternative approach to study NP which can be viewed as renormalisable generalisation of simplified models. Particularly, we consider a generic extension of the SM particle content into an arbitrary number of vectors, scalars, and fermions whose masses are above the electroweak scale. In order to implement constraints on such generic extensions, the perturbative unitarity property of the scattering amplitudes is utilised. Specifically, the required cancellation of unbounded high-energy growth of amplitudes leads to certain relations among the coupling constants that are common to all models. With the help of these relations, one can understand and perform the renormalisation of the observables in a general way [17–19]. These relations can be derived by means of Slavnov-Taylor identities [20], that are a generalised version of the Ward identities for the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [21–23].

As a practical implementation of these relations, we perform an one-loop matching calculation for generic models of new physics which are proposed as possible explanations of flavour anomalies such as anomalies observed in $B-$meson decays. In the SM, this process is loop-induced; hence, it is reasonable to expect that physics beyond the standard model to also contribute from the one-loop level if present. The SM contribution to B-meson decays is well described by the weak effective Lagrangian [24]. The same is true for many of the SM extensions if they involve particles with masses above the electroweak scale. However, matching onto the effective theory is tedious and generally has to be repeated for every new model. The tediousness is exacerbated if one wants to, additionally, check that the result is gauge-independent and that all UV divergences properly cancel. In order to facilitate these processes, we do the matching procedure generically, where the coupling constant sum rules are used for the renormalisation of the one-loop diagrams, as well as to achieve gauge invariant results. In the result, the Wilson coefficients depend on a minimal set of physical parameters and are guaranteed to be finite and gauge independent. Moreover, we provide easy-to-use code to obtain the Wilson coefficients in general perturbatively unitary models, available at

`https://wellput.github.io`.

The code can be integrated with other automated tools, which are designed for the evaluation of different experimental observables in flavour physics, such as `flavio` [25].

In fact, the derived one-loop results are not limited to flavour observables, and they are applicable to other NP models as well, such as DM models where DM interactions with the SM particles are one-loop induced. Furthermore, for the sake of simplicity, we do a phenomenological analysis for DM models where DM couples with the SM particles at tree level. As a DM candidate, we consider Weakly Interacting Massive Particles (WIMPs) that are theoretically attractive candidates for a particle explanation of the DM content of the Universe. The interactions of WIMPs with light SM particles can be tested experimentally via their elastic scattering off nuclei in ground-based direct detection experiments, in specific signatures at the Large Hadron Collider (LHC) involving visible SM objects and missing energy [26], or by observing their annihilation products with astrophysical experiments.

Up to date the leading liquid xenon based direct detection experiments LUX [27] and XENON1T [28] have not detected DM, thereby severely constraining the interaction rates between WIMPs and the SM. This strongly constrains the possible parameter space of WIMP quark interactions – and with it also the production mechanisms in collider experiments – and requires a suppression mechanism for the direct detection rates. In this work, we focus on axial vector DM interactions, where the direct detection rate is naturally suppressed [29].

To study the impact of SM gauge invariance we use the effective theory above the electroweak scale from Ref. [30] that couples SU(2)×U(1) invariant SM fields with axial-vector DM currents. Such a theory naturally arises in models where the DM candidate is a Majorana fermion and the coupling of the dark and visible sectors is mediated by a tree-level neutral vector boson exchange, typically referred to as Z' and often related to an additional U(1) gauge symmetry. In such models, the vectorial couplings between the DM candidate and the Z' vanish because Majorana fermions are self-conjugate under charge conjugation while the vector current is odd. In addition, the extra U(1) gauge symmetry in these models results in constraints from anomaly cancellation, similar to those studied in Ref. [31] for Dirac dark matter. Experimentally, neutral gauge bosons that interact with the SM quark current can be searched for at the LHC, for instance via dijet final states [32]. Null results from these searches generally require mediator masses to be at the TeV scale. The large separation between the Z' mass and the momentum transfer in direct detection experiments justifies an effective field theory description of the interaction between the DM and the baryons and mesons. Matching the Z' exchange model to our effective field theory, we find that a significant suppression for the DM direct detection rate is only possible for very specific combinations of the effective theory parameters.

The structure of the thesis is as following: the first two chapters are dedicated to the background theory. Particularly, in Sec. 2, we briefly overview some topics of the quantum field theory which are relevant to our further discussions, and represent the form of the generic Lagrangian in Sec. 2.6 along with sum rules in Sec. 2.6.1. Then, in Sec. 3 we discuss the effective theory approach that is relevant to the weak decays of hadrons. In addition, in this chapter we review some mathematical tools used in the evaluation of one-loop amplitudes. The practical implementation of the sum rule relations in the calculation and renormalisation of transition amplitudes will be extensively discussed in Sec. 4. Also, in this chapter we present applications of our one-loop matching results to the beyond the standard model phenomenology, Sec. 4.4. Sec. 5 is about dark matter phenomenology, where we emphasise the complementarity between dark matter direct detection and collider searches, Sec. 5.3.3. The summary and conclusions of the thesis are presented in Sec. 6. In addition, the work contains appendices. Namely, in App. A the list of effective operators for the photon penguin is given. Furthermore, explicit forms of one-loop functions are provided in App. B. Finally, App. C contains some details of dark matter phenomenology.

2 THEORETICAL FOUNDATION

Quantum Field Theory (QFT) is the mathematical and conceptual framework for contemporary elementary particle physics and it a synthesis of classical field theory, special relativity and quantum mechanics. The key point of QFT is that the fields are fundamental and particles are derived concepts, emerging only after performing a proper quantisation.
There are persuasive arguments to support this idea: Firstly, at a distance shorter than the Compton wavelength, $\lambda = \hbar/(mc)$, a notion of single point-like particle breaks down, *i.e.*, it is highly probable that a particle localised at this distance is surrounded by a swarm of particle-antiparticle pairs. Hence, a relativistic form of the one-particle Schrödinger equation can not be used safely at short distances. Secondly, indistinguishability of particles from the same species. For example, an electron produced in colliders on Earth is the same as the electron captured from the cosmic rays, they are indistinguishable. Thus, swapping two particles of the same species leaves the state completely unchanged — apart from a possible minus sign. This minus sign determines the statistics of the particle, Bose statistics (no minus sign) for integer spin particles, and Fermi statistics (with minus sign) for half-integer spin particles. In quantum mechanics, these statistics are implemented by hand. On the other hand, in QFT, this relationship between spin and statistics is a consequence of the framework [33].

In this chapter, we discuss some concepts of the QFT starting from the construction of a simple gauge theory, Sec. 2.1. Further, we discuss quantisation of classical field theories in Sec. 2.2, particularly path-integral quantisation will be in focus, Sec. 2.2.1. The gauge theories necessitate the introduction of gauge-fixing terms in the effective Lagrangian which spoil the gauge symmetry. A new symmetry of the theory can be defined that is known as Becchi-Rouet-Stora-Tyutin symmetry, and field transformations under this symmetry are discussed in Sec. 2.3. The generalised Ward identities of this symmetry are given in Sec. 2.3.1. Further, we will talk about the gauge invariant way to generate masses for fermions and gauge bosons in Sec. 2.4 along with generalised Ward identities for broken theories, Sec. 2.4.1. The chapter is concluded by presenting the explicit form of the generically extended the standard model Lagrangian, Sec. 2.6, and discussing its renormalisability which can be achieved with the help of sum rules, Sec. 2.6.1. A simple application of these sum rules is demonstrated in Sec. 2.6.2.

2.1 Gauge principle

The fundamental interactions among elementary particles are well described by gauge theories where local/gauge symmetries are used to generate dynamics. The prototype gauge theory is quantum electrodynamics (QED) which describes an electron interacting with light. QED is constructed by requiring that the Dirac's free electron theory to be invariant under the Abelian U(1) gauge symmetry.

The Lagrangian for a free electron field $\psi(x)$ is given as

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x), \tag{2.1}$$

and it is invariant under the phase change which corresponds to a global U(1) symmetry

$$\psi(x) \to \psi'(x) = e^{-i\alpha}\psi(x), \tag{2.2}$$
$$\bar{\psi}(x) \to \bar{\psi}'(x) = \bar{\psi}(x)e^{i\alpha}, \tag{2.3}$$

where α is space-time independent. However, the invariance of the Lagrangian is spoilt if α is replaced by $\alpha(x)$, because the derivative term will have a non-trivial transformation,

$$\begin{aligned} \bar{\psi}(x)\partial_\mu\psi(x) \to \bar{\psi}'(x)\partial_\mu\psi'(x) &= \bar{\psi}(x)e^{i\alpha}\partial_\mu\left(e^{-i\alpha}\psi(x)\right) \\ &= \bar{\psi}(x)\partial_\mu\psi(x) - i\bar{\psi}(x)\left(\partial_\mu\alpha(x)\right)\psi(x). \end{aligned} \tag{2.4}$$

It is the second term that ruins the invariance. To restore the symmetry of the Lagrangian the partial derivative ∂_μ is replaced by a gauge covariant derivative D_μ, which transforms as

$$D_\mu\psi(x) \to [D_\mu\psi(x)]' = e^{-i\alpha}D_\mu\psi(x). \tag{2.5}$$

Thereby the term $\bar{\psi}(x)D_\mu\psi(x)$ is gauge invariant, particularly the action of the covariant derivative on the field leaves the transformation of the field unaltered. The covariant derivative is defined as

$$D_\mu\psi(x) = \left(\partial_\mu + ieA_\mu\right)\psi(x), \tag{2.6}$$

where e is a free parameter which will be associated with the electric charge. The gauge field A_μ has a transformation property

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x). \tag{2.7}$$

In order to make the gauge field a proper dynamical variable, the Lagrangian should be supplemented with terms containing derivatives of A_μ and the straightforward gauge-invariant choice is

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{2.8}$$

where $F_{\mu\nu}$ is gauge invariant by itself and has the following form

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x). \tag{2.9}$$

Therefore the total Lagrangian for QED can be written as

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x). \tag{2.10}$$

Some facts about this Lagrangian can be highlighted: *i)* It does not contain a mass term for the photon, because $A_\mu A^\mu$ is gauge non-invariant. *ii)* In QED the electron-photon interaction is encoded in the covariant derivative $D_\mu \psi$ which can be built from the transformation property of the electron field. Generally speaking, the coupling of the photon to any matter field is determined by its transformation property under the symmetry group. This is known as universality. *iii)* There is no self-interaction of the gauge field, which means that the photon does not carry $\mathrm{U}(1)$ charge and without a matter field, the theory is a free field theory [34].

In an analogous way to QED, the gauge theories for the weak and the strong interactions can be formulated. The local symmetries for constructing weak and strong gauge theories are non-Abelian $\mathrm{SU}(2)_{\text{isospin}}$ and $\mathrm{SU}(3)_{\text{colour}}$ symmetries, respectively. Unlike QED, in these theories self-interactions of the gauge fields are present.

2.2 Field quantisation

The quantisation of a classical field can be done in alternative ways. The first one is *canonical quantisation* which is defined in the Hamiltonian formalism. In the canonical quantisation the field $\phi(\vec{x})$ and its momentum conjugate $\pi(\vec{x})$ are operator valued functions of space obeying the equal time commutation relations,

$$\begin{aligned} &[\phi(\vec{x}), \phi(\vec{y})]_{\mp} = [\pi(\vec{x}), \pi(\vec{y})]_{\mp} = 0, \\ &[\phi(\vec{x}), \pi(\vec{y})]_{\mp} = i\delta^{(3)}(\vec{x} - \vec{y}), \end{aligned} \tag{2.11}$$

where minus and plus signs for the commuting and anti-commuting fields, respectively. Although the mathematical formulation of this approach is more intuitive, the symmetries of the underlying theory can not be seen explicitly in the Hamiltonian of a system. Moreover, the quantisation of the gauge fields is not straightforward in this formalism, because the canonically conjugate variable to the gauge field A_μ,

$$\Pi_\mu \equiv \frac{\partial \mathcal{L}}{\partial \dot{A}^\mu} = -F_{0\mu}, \tag{2.12}$$

has vanishing zeroth component, i.e. $\Pi_0 = 0$. Consequently, the propagator of the gauge fields cannot be defined appropriately. The problem can be solved by adding a gauge fixing term to the Lagrangian and following the carefully defined procedure. Nevertheless, the Feynman rules for calculating scattering amplitudes that involve gauge bosons are derived more easily in the path-integral formulation. The path-integral quantisation is based on the *Feynman's path-integral* formalism [35], which is a preferred method for relativistic field theories, particularly the gauge theories. Since the fundamental quantity of the path integral quantisation is the Lagrangian, rather than the Hamiltonian, it explicitly illustrates all symmetries of a theory. Moreover, the analogy between quantum field theory and statistical mechanics is well revealed in this method. Despite these virtues, the path-integral formalism cannot be accepted as a complete description of quantum field theories. There are some shortcomings, namely Heisenberg operators and state vector space cannot be discussed directly. Also, in this framework the proof of 'unitarity' of the scattering matrix ($S-$matrix) is not obvious because the customary physical in-state space is so restrictive that it is not an invariant subspace of the $S-$matrix [36].

2.2.1 Path-integral formalism

In the Feynman path-integral formalism the transition amplitude is defined as the sum (a functional integral) over all possible paths between initial and final states, weighted by the exponential of i times the action for the particular path [37]. Here we will focus on the part of the $S-$matrix element that involves only gauge fields,

$$\langle \text{out}|\text{in}\rangle \sim \int \mathcal{D}[\mathbf{A}] e^{iS[\mathbf{A}]}, \tag{2.13}$$

where $\mathbf{A} = (A^a_\mu)$. The measure $\mathcal{D}[\mathbf{A}] = \prod_{x,\mu,a} dA^a_\mu(x)$ involves at each space-time point a product over all group and vector components of the field $A^a_\mu(x)$ and the action functional $S[\mathbf{A}] = \int d^4x\mathcal{L}$. The Lagrangian for non-Abelian gauge theories with a symmetry group G of dimension dim$G$ (=number of group parameters) is

$$\mathcal{L}_{\text{gauge}}(x) = -\frac{1}{4}F^a_{\mu\nu}(x)F^{a,\mu\nu}(x), \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu, \tag{2.14}$$

where f^{abc} are the group's structure constants and g denotes a coupling constant, the group indices are $a, b, c \in \{1, \ldots, \text{dim}G\}$. The $\mathcal{L}_{\text{gauge}}(x)$ can be decomposed into two parts, $\mathcal{L}_{\text{gauge}} = \mathcal{L}_0 + \mathcal{L}_1$, where $\mathcal{L}_0$ consists of quadratic terms in A^a_μ and defines the propagator function. $\mathcal{L}_1$ includes trilinear and quartic self-interactions of gauge fields.

Using the partial integration, the $\mathcal{L}_0$ part of the integral in Eq. (2.13) could be given as

$$\int \mathcal{D}[\mathbf{A}] e^{i\int d^4x\left[\frac{1}{2}A^a_\mu(g^{\mu\nu}\Box - \partial^\mu\partial^\nu)A^a_\nu\right]}, \tag{2.15}$$

where the operator $K^{\mu\nu}(x) = (g^{\mu\nu}\Box - \partial^\mu\partial^\nu)$ is not invertible, because it has an eigenvector with the eigenvalue which is equal to zero. Therefore the required condition for the existence of the Gaussian integral, $(A_\mu K^{\mu\nu}A_\nu) > 0$, is not satisfied and the result is singular.

The cause of the issue is that the above integration is taken over a continuous infinity of physically equivalent field configurations. The problem can be solved by isolating the part of the functional integral, which counts each physical configuration only once and it can be accomplished following the procedure proposed by L.D. Faddeev and V.N. Popov [38]. Namely, some local functions of the gauge fields, $\mathbf{C}\{\mathbf{A}; x\} = (C^a\{\mathbf{A}; x\})$, should be introduced with the following constraint

$$\mathbf{C}\{\mathbf{A}; x\} = \mathbf{c}(x) \tag{2.16}$$

as a *gauge-fixing condition.* Here $\mathbf{c}(x) = (c^a(x))$ are arbitrary functions of x. For example, $C^a\{\mathbf{A}; x\} = \partial^\mu A^a_\mu$, $c^a(x) = 0$ corresponds to the *Lorentz gauge.* With this gauge fixing condition the path integral must be performed on the restricted paths which satisfy (2.16). This constraint may be explicitly taken into account by using the functional $\Delta[\mathbf{A}]$,

$$\Delta[\mathbf{A}] \equiv \int \left[\mathcal{D}g\delta\left(\mathbf{C}\{\mathbf{A}_g;x\} - \mathbf{c}(x)\right)\right]^{-1}, \tag{2.17}$$

where $\mathcal{D}g$ is an invariant measure of a group element g of the gauge symmetry. $\mathbf{A}_g$ is a transformed version of $\mathbf{A}$ under the group element g. $\Delta[\mathbf{A}]$ is invariant under the transformation g, $\Delta[\mathbf{A}] = \Delta[\mathbf{A}_g]$. Furthermore, Eq. (2.17) as an identity is inserted in (2.13),

$$\int \mathcal{D}g \int \mathcal{D}[\mathbf{A}]\Delta[\mathbf{A}]\delta\left(\mathbf{C}\{\mathbf{A}_g;x\} - \mathbf{c}(x)\right)e^{iS[\mathbf{A}]}. \tag{2.18}$$

Since the integrand in (2.18) does not depend on g and the integral over $\int \mathcal{D}g$ can be factored out which is an infinite constant. By this factorisation one obtains a well-defined transition amplitude. Using the gauge-fixing condition enforces the measure of the path integral to be deformed resulting in an appearance of $\Delta[\mathbf{A}]$.

In order to derive Feynman rules, the $\Delta[\mathbf{A}]$ in (2.18) should be exponentiated and the Lagrangian should be redefined. Considering the gauge field transformation $\mathbf{A}_g \sim \mathbf{A} + \theta^a\delta^a\mathbf{A}$ and assuming that $\mathcal{D}g \simeq \mathcal{D}\theta$ for $g \simeq 1$, Eq. (2.17) with the condition $\mathbf{C}\{\mathbf{A};x\} = \mathbf{c}(x)$ can be rewritten as

$$\Delta[\mathbf{A}] = \left[\int \mathcal{D}\theta\delta\left(\delta^a\left(C^b\{\mathbf{A};x\}\right)\theta^a - c^b(x)\right)\right]^{-1} = \det\left(\delta^a\left(C^b\{\mathbf{A};x\}\right)\right). \tag{2.19}$$

Therefore, $\Delta[\mathbf{A}]$ in Eq. (2.18) is replaced by the functional determinant,

$$\det\left(\delta^a\left(C^b\{\mathbf{A};x\}\right)\right) = \int \mathcal{D}[\mathbf{u}]\mathcal{D}[\bar{\mathbf{u}}]e^{-i\int d^4x\,\bar{u}^a(x)\delta^a(C^b\{\mathbf{A};x\})u^b(x)}. \tag{2.20}$$

where auxiliary fields $\mathbf{u} = (u^a(x))$ and $\bar{\mathbf{u}} = (\bar{u}^a(x))$ and they are known as *Fadeev-Popov (FP) ghost* fields. They transform as scalars under the Lorentz group and belong to the adjoint representation of the gauge group. These fields are anti-commuting (which is necessary to obtain $\det$ instead of $1/\det$) and they have the property of Grassmann numbers.

The delta function $\delta\left(\mathbf{C}\{\mathbf{A};x\} - \mathbf{c}(x)\right)$ in Eq. (2.18) also can be exponentiated if one averages it over the parameter $\mathbf{c}$ with weight $\exp\left[-i\left(c^a\right)^2/2\xi\right]$,

$$\int \mathcal{D}[\mathbf{c}]\delta\big(C^a\{\mathbf{A};x\} - c^a(x)\big)e^{-i\int d^4x (c^a)^2/2\xi} = \int \mathcal{D}Be^{i\int d^4x\left[B^aC^a\{\mathbf{A};x\} + \frac{\xi}{2}B^aB_a\right]}, \tag{2.21}$$

where a new scalar field B is introduced to linearise the gauge term, and ξ is a gauge-fixing parameter [39]. Finally, the transition amplitude can be written as

$$\langle \text{out}|\text{in}\rangle \sim \int \mathcal{D}g \int \mathcal{D}[\mathbf{A}]\mathcal{D}[\bar{\mathbf{u}}]\mathcal{D}[\mathbf{u}]\mathcal{D}[\mathbf{B}]e^{i\int d^4x\mathcal{L}_{\text{eff}}}, \tag{2.22}$$

where the effective Lagrangian $\mathcal{L}_{\text{eff}}$ for a quantised gauge theory reads

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{ghost}}, \tag{2.23}$$

where fermion, gauge and gauge-fixing parts are given as

$$\mathcal{L}_{\text{fermion}} = \bar{\psi}_i(x)\left(\not{D}_{ij} - m_i\delta_{ij}\right)\psi_j(x), \tag{2.24}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^a_{\mu\nu}(x)F^{a,\mu\nu}(x), \tag{2.25}$$

$$\mathcal{L}_{\text{fix}} = B^aC^a\{\mathbf{A};x\} + \frac{\xi}{2}\left(B^a\right)^2, \tag{2.26}$$

$$\mathcal{L}_{\text{ghost}} = -\bar{u}^a(x)\delta^a\left(C^b\{\mathbf{A};x\}\right)u^b(x). \tag{2.27}$$

In the covariant global symmetric gauge,

$$C^a\{\mathbf{A};x\} = \partial^\mu A^a_\mu \qquad \text{and} \qquad \delta^a\left(C^b\{\mathbf{A};x\}\right) = \partial^\mu D^{ab}_\mu, \tag{2.28}$$

where the covariant derivative is $D^{ab}_\mu = \delta^{ab}\partial_\mu + gf^{abc}A^c_\mu$. Doing the integration by parts, the Lagrangian for the ghost fields is changed as $\mathcal{L}_{\text{ghost}} = \left(\partial^\mu\bar{u}^a(x)\right)D^{ab}_\mu u^b(x)$, and this Lagrangian is local as long as $C^a\{\mathbf{A};x\}$ is local in the fields.

The path-integral framework is appropriate for perturbative calculations. The Feynman rules can be obtained directly from the effective Lagrangian (2.23). The non-physical parts of the Lagrangian, $\mathcal{L}_{\text{fix}}$ and $\mathcal{L}_{\text{ghost}}$, cancel out in the final results for physical observables such as $S-$matrix elements.

2.3 BRST symmetry

The introduction of the gauge-fixing term into the theory destroys its manifest gauge invariance. However, a symmetry of the effective Lagrangian without affecting gauge invariance of the physical quantities can be redefined by an extension of the gauge transformation to the ghost fields, which is known as *Becchi-Rouet-Stora-Tyutin* (BRST) symmetry [21–23].

The BRST transformation for gauge bosons and fermions with transformation parameter $\delta\theta^a(x) = g\delta\lambda u^a(x)$ is a gauge transformation and it is given by

$$\begin{aligned}
\delta_{\text{BRST}} A_\mu^a(x) &= \delta\lambda D_\mu^{ab} u^b(x) = \delta\lambda \left(\partial_\mu \delta^{ab} - g f^{abc} A_\mu^c(x)\right) u^b(x) \equiv \delta\lambda s A_\mu^a(x), \\
\delta_{\text{BRST}} \psi_i(x) &= \delta\lambda u^a(x) ig \left(T^a \psi(x)\right)_i \equiv \delta\lambda s \psi_i(x), \\
\delta_{\text{BRST}} \bar{\psi}_i(x) &= -\delta\lambda u^a(x) ig \left(\bar{\psi}(x) T^a\right)_i \equiv \delta\lambda s \bar{\psi}_i(x),
\end{aligned} \tag{2.29}$$

where $\delta\lambda$ is an infinitesimal, Grassmann-valued constant that anticommutes with the ghost fields $u^a(x)$ and $\bar{u}^a(x)$. The *BRST operator* s is defined as the left derivative with respect to $\delta\lambda$ of the BRST transformed fields. The above transformations leave the fermionic and gauge parts of the effective Lagrangian invariant, *i.e.*,

$$\delta_{\text{BRST}} \mathcal{L}_{\text{gauge}} = \delta_{\text{BRST}} \mathcal{L}_{\text{fermion}} = 0. \tag{2.30}$$

The BRST transformation of the ghost fields should be selected in the way that it must satisfy

$$\delta_{\text{BRST}} \left(\mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{ghost}}\right) = 0, \tag{2.31}$$

and the following choices for ghost and the scalar B field are consistent with this requirement,

$$\begin{aligned}
\delta_{\text{BRST}} u^a(x) &= -\delta\lambda \frac{1}{2} g f^{abc} u^b(x) u^c(x) \equiv \delta\lambda s u^a(x), \\
\delta_{\text{BRST}} \bar{u}^a(x) &= \delta\lambda B^a \equiv \delta\lambda s \bar{u}^a(x), \\
\delta_{\text{BRST}} B^a &= 0 \equiv \delta\lambda s B^a(x).
\end{aligned} \tag{2.32}$$

The proof of the statement in Eq. (2.31) is more involved, since transformation behaviours of gauge-fixing and ghost terms are complicated, this is well-explained in Ref. [37].

In fact, BRST symmetry is a global symmetry and according to *Noether's theorem* [40], the invariance of the effective Lagrangian (2.23) under the BRST transformations implies the existence of a conserved current J_μ^{BRST}. Thus, the charge that generates the BRST transformation is

$$Q_{\text{BRST}} = \int d^3x J_0^{\text{BRST}}(x). \tag{2.33}$$

An important property of the BRST charge Q_{BRST} is the *nilpotency*,

$$Q^2_{\text{BRST}} = 0. \tag{2.34}$$

The consequence of this property is reflected in the proof of $\delta_{\text{BRST}}\mathcal{L}_{\text{eff}} = 0$. The *physical state* $|\text{phys}\rangle$ should be defined such that it must satisfy the following condition

$$Q_{\text{BRST}}|\psi\rangle_{\text{phys}} = 0. \tag{2.35}$$

With the help of the nilpotency of Q_{BRST} it is possible to show that the space spanned by $|\text{phys}\rangle$ has the positive semidefinite norm [36].

The physical state condition in Eq. (2.35) ensures the unitarity of the physical $S-$ matrix, the $S-$ matrix restricted on the physical space.

Actually, in the physical space the ghost fields u^a, $\bar{u}^a$ and the auxiliary field B^a (the scalar component of A^a_μ) appear only together with longitudinal component, A^a_L, of A^a_μ and this combination has zero norm. Thus u^a, $\bar{u}^a$, B^a and A^a_L effectively decouple from the physical states. The mechanism in which four unphysical components of fields may appear in the physical space only as a combination of zero-norm states is called the quartet mechanism [39].

2.3.1 Slavnov-Taylor identities

In the path-integral formulation of the gauge theories the invariance of the local action and the integration measure under field transformations allows to derive a set of useful identities. These identities can be written in either functional form or as relations between Green's functions. For non-Abelian gauge theories, the BRST invariance of the theory generates a generalised version of the Ward identites [41] known as *Slavnov-Taylor identities* (STIs) [42, 43].

These identities are crucial for the proof of renormalisability of the gauge theories, as well as to show the unitarity and gauge-independence of the scattering matrix.

The generating functional of Green's functions including the source terms for different fields is

$$Z\{\boldsymbol{J}, \boldsymbol{J}_u, \boldsymbol{J}_{\bar{u}}, \boldsymbol{J}_\psi, \boldsymbol{J}_{\bar{\psi}}, \boldsymbol{K}, \boldsymbol{K}_u, \boldsymbol{K}_\psi, \boldsymbol{K}_{\bar{\psi}}\} = \int \mathcal{D}[\boldsymbol{A}]\mathcal{D}[\boldsymbol{u}]\mathcal{D}[\bar{\boldsymbol{u}}]\mathcal{D}[\boldsymbol{\psi}]\mathcal{D}[\bar{\boldsymbol{\psi}}]e^{i\int d^4x\mathcal{L}_{\text{tot}}}, \tag{2.36}$$

where the BRST invariant total Lagrangian is

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{eff}} + J^{a,\mu} A_\mu^a + J_u^a u^a - \bar{u}^a J_{\bar{u}}^a + J_\psi^i \psi_i - \bar{\psi}_i J_{\bar{\psi}}^i \\ + K^{a,\mu} s A_\mu^a + K_u^a s u^a + K_\psi^i s\psi_i + (s\bar{\psi}_i) K_{\bar{\psi}}^i. \tag{2.37}$$

The sources $J_{\bar{u}}^a$, J_u^a, $J_{\bar{\psi}}^i$, J_ψ^i and K^a are Grassmann-valued variables. The fact that $\mathcal{L}_{\text{tot}}$ should have a vanishing ghost number assigns a ghost number -1 for K^a, K_ψ^i and $K_{\bar{\psi}}^i$ and -2 for K_u^a.

The BRST invariance of the total Lagrangian and the integration measure with the nilpotency of BRST operator, *i.e.*, $s^2 A_\mu^a = s^2\psi = s^2\bar{\psi} = s^2 u^a = s^2 C^a = 0$ and $s^3 \bar{u}^a = 0$ gives the following relation

$$0 = \int \mathcal{D}[\boldsymbol{A}]\mathcal{D}[\boldsymbol{u}]\mathcal{D}[\bar{\boldsymbol{u}}]\mathcal{D}[\boldsymbol{\psi}]\mathcal{D}[\bar{\boldsymbol{\psi}}] e^{i\int d^4x \mathcal{L}_{\text{tot}}} \\ \times \int d^4x \left[J^{a,\mu} s A_\mu^a - J_u^a s u^a - (s\bar{u}^a) J_{\bar{u}}^a - J_\psi^i s\psi_i - (s\bar{\psi}_i) J_{\bar{\psi}}^i \right]. \tag{2.38}$$

By writing the fields in terms of derivatives with respect to the corresponding sources, *Slavnov-Taylor identities for the generating functional* T can be obtained,

$$0 = \int d^4x \left[J_\mu^a \frac{\delta}{i\delta K_\mu^a} - J_u^a \frac{\delta}{i\delta K_u^a} + \frac{1}{\xi} C^a \left\{ \frac{\delta}{i\delta \boldsymbol{J} x}; x \right\} J_{\bar{u}}^a - J_\psi^i \frac{\delta}{i\delta K_\psi^i} \right. \\ \left. - \frac{\delta}{i\delta K_{\bar{\psi}}^i} J_{\bar{\psi}}^i \right] T\{\boldsymbol{J}, \boldsymbol{J}_u, \boldsymbol{J}_{\bar{u}}, \boldsymbol{J}_\psi, \boldsymbol{J}_{\bar{\psi}}, \boldsymbol{K}, \boldsymbol{K}_u, \boldsymbol{K}_\psi, \boldsymbol{K}_{\bar{\psi}}\}, \tag{2.39}$$

where, $T\{\boldsymbol{J}\} = Z\{\boldsymbol{J}\}/Z\{0\}$ and for connected diagrams, this expression is modified by using $T_c = \log T$.

In order to obtain STIs for Green's functions, one can differentiate (2.39) with respect to the external sources and after that equate the sources to zero. As a result the following relation is obtained

$$\frac{\delta_{\text{BRST}}}{\delta\lambda} \left\langle T \prod_l \Psi_{I_l} \right\rangle = s \left\langle T \prod_l \Psi_{I_l} \right\rangle = 0, \tag{2.40}$$

where T denotes the time-ordered product of fields and Ψ_{I_l} is a general notation for all kinds of fields, {gauge, ghost, fermion,...}, the index I comprises all indices of the actual fields including the space-time ones.

A generic form of the Slavnov-Taylor identities in Eq. (2.40) can be applied for the case when there are one anti-ghost field and an arbitrary number of other physical and on-shell fields. Using the BRST transformation behaviour of the anti-ghost field (2.32) and considering the connection between states and asymptotic fields, the following identity can be derived

$$0 = \left\langle T C^a \prod_l \Psi_{I_l}^{\text{as,phys}} \right\rangle , \tag{2.41}$$

where the gauge-fixing function $C^a = C^a\{\mathbf{A}; x\}$. If there are additional gauge-fixing terms, with the help of the equation of motion for the anti-ghost fields,

$$i \left\langle T \frac{\delta}{\delta \bar{u}^a(x)} X \right\rangle = - \left\langle T \left(s C^a(x) \right) X \right\rangle ,$$

where X stands for any product of fields, the above identity can be generalised as

$$\left\langle T \frac{1}{\xi_a} C^a(x) \left(\prod_m C^{a_m}(x_m) \right) \prod_l \Psi_{I_l}^{\text{as,phys}} \right\rangle_{\text{c}} = 0. \tag{2.42}$$

Here, the subindex c means the connected Green's functions. There are other identities which are extracted from the BRST invariance of the effective action, which are called *Lee identities* [37]. These identities are used to derive physical fields and their properties in the quantised gauge theories, in analogy to the classical case where physical fields and their properties are extracted from the Lagrangian.

2.4 Spontaneous symmetry breaking

It is the experimental fact that the mediators of the weak interactions are massive particles. However, the underlying theory of this interaction, the gauge theory of the weak interaction, does not allow to add explicit mass terms to the Lagrangian, since it destroys gauge invariance of the theory. Also fermion masses can not be added directly to the effective Lagrangian, for example in the electroweak theory the mass term for fermions $\bar{\psi}_L \psi_R$ is a doublet representation of the SU(2) symmetry group and it is gauge non-invariant.

Currently, the only way to introduce masses of fields without violating gauge invariance is via the *Higgs mechanism* which is based on the *spontaneously symmetry breaking* (SSB) [44, 45]. The idea behind the SSB originates from the elementary particle physics where Nambu and Jona-Lasinio first introduced the concept in connection with the chiral symmetry [46].

First, consider a Lagrangian that is invariant under a continuous symmetry group G. According to Noether theorem, there exists a conserved charge, Q, that generates this symmetry and commutes with the Hamiltonian, H. In the case of quantum mechanics, there is one unique ground state $|0\rangle$, a state with lowest energy, which is a simultaneous eigenstate of H and Q. On the other hand, in quantum field theory, it is possible to have a set of degenerate ground states which transform into one another under the group G. Although all these ground states are equivalent, each of them selects a direction in representation space. By choosing one of them as the ground state of the theory, one breaks the symmetry G spontaneously. Therefore SSB is characteristic for theories involving infinite degrees of freedom, such as quantum field theory [39].

A classic example of the spontaneously breakdown of a symmetry is the *linear σ model* described by the Lagrangian

$$\mathcal{L}_\sigma = \frac{1}{2} \left(\partial_\mu \phi_i\right) \left(\partial^\mu \phi_i\right) - V\left(\boldsymbol{\phi}\right), \qquad V\left(\boldsymbol{\phi}\right) = \frac{\lambda}{4} \left(\phi_i \phi_i - \frac{\mu^2}{\lambda}\right)^2, \tag{2.43}$$

where $\boldsymbol{\phi} = (\phi_i)$ denotes real scalar fields which form $N-$component real vectors and self-interactions of these fields are given by the potential $V\left(\boldsymbol{\phi}\right)$. The Lagrangian is symmetric under the global symmetry group $G = \mathrm{SO}(N)$, the rotation group in N dimensions. The potential $V\left(\boldsymbol{\phi}\right)$ has a non-zero local minimum,

$$\phi_{i0}\phi_{i0} = v_i v_i = \frac{\mu^2}{\lambda} = v^2 > 0. \tag{2.44}$$

This expression defines only the norm of the ground state that is v and its direction is arbitrary which is typically chosen to be pointing to Nth direction,

$$\boldsymbol{\phi}_0 = \boldsymbol{v} = (0, ..., 0, v) \,. \tag{2.45}$$

Now, the ground state is invariant under the orthogonal transformations of the first $N-1$ coordinates, *i.e.*, under the subgroup $H = \mathrm{SO}(N-1)$. The dimensions of the original group G and its subgroup H are dim$G = N(N-1)/2$ and dim$H = (N-1)(N-2)/2$, correspondingly. In accordance to the *Goldstone theorem* [46] the difference of dim$G$-dim$H = N-1$ defines the number of massless Goldstone bosons. Furthermore, the field vector with small excitations from the ground state can be written as $\boldsymbol{\phi} = (\boldsymbol{\pi}, \sigma + v)$ where $\boldsymbol{\pi} = (\pi_1, ..., \pi_{N-1})$ are Goldstone fields and σ is a massive field. With this field vector, the Lagrangian is

$$\mathcal{L}_\sigma = \frac{1}{2}\left(\partial_\mu \sigma\right)\left(\partial^\mu \sigma\right) - \frac{1}{2}\left(2\mu^2\right)\sigma^2 + \frac{1}{2}\left(\partial_\mu \boldsymbol{\pi}\right)\left(\partial^\mu \boldsymbol{\pi}\right) + \mathcal{L}_{\text{int}}\left(\sigma, \boldsymbol{\pi}\right), \tag{2.46}$$

where $\mathcal{L}_{\text{int}}\left(\sigma, \boldsymbol{\pi}\right)$ is the interaction Lagrangian of the σ and $\boldsymbol{\pi}$ fields, and the mass of σ is $\sqrt{2}\mu$.

When $N = 4$, the group SO(4) is locally isomorphic to the group $\mathrm{SU}(2) \times \mathrm{SU}(2)$ which characterises the chiral symmetry. In the SSB of the chiral symmetry the three Goldstone bosons are associated with $\pi^0, \pi^\pm$ pions and the massive field with the σ meson [37].

In similar manner the spontaneous breakdown of local symmetries can be formulated, which generates masses for gauge bosons. For instance, one can consider the gauged version of the Lagrangian in Eq. (2.43) with added gauge dynamics. The gauge fields minimally couple to the scalar fields, *i.e.*, $\partial_\mu \rightarrow D_\mu = \partial_\mu - igA^a_\mu T^a$,

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \frac{1}{2}\left(D_\mu \phi_i\right)\left(D^\mu \phi_i\right) - V\left(\boldsymbol{\phi}\right), \tag{2.47}$$

where $F^a_{\mu\nu}$ is the gauge field-strength tensor and the T^a are the generators of G. The form of the potential $V\left(\boldsymbol{\phi}\right)$ will not change. The gauge fields transform according to the adjoint representation of the gauge group G and the scalar multiplet $\boldsymbol{\phi}$ transforms according to a real representation with generators T^a.

After the potential gets non-zero vacuum expectation value $\boldsymbol{v}$, the mass terms for the gauge bosons are derived from the gauge invariant kinetic term of the scalar fields,

$$\frac{1}{2}\left(D_\mu v_i\right)\left(D^\mu v_i\right) = \frac{1}{2}g^2\left(i\hat{T}^{\hat{a}}_{ij}v_j\right)\left(i\hat{T}^{\hat{b}}_{ik}v_k\right)A^{\hat{a}}_\mu A^{\hat{b},\mu} = \frac{1}{2}M^2_{\hat{a}\hat{b}}A^{\hat{a}}_\mu A^{\hat{b},\mu}, \tag{2.48}$$

where $\hat{T}^{\hat{a}}$, $\hat{a} = \mathrm{dim}H + 1, \ldots, \mathrm{dim}G$ are generators of the coset space G/H.

In general, the Lagrangian of the spontaneously broken gauge theories contains the interactions of gauge bosons with unphysical Goldstone fields. This mixing can be eliminated by properly selecting the gauge-fixing term. For example, in a generalised renormalisable gauge (R_ξ-gauge) [47, 48] the gauge-fixing Lagrangian has the following form

$$\mathcal{L}_{\text{fix}} = -\sum_v (2\xi_v)^{-1} F_{\bar{v}} F_v \,, \qquad F_v = \partial_\mu V_v^\mu - \sigma_v \xi_v M_v \phi_v. \tag{2.49}$$

Here V_v denotes a gauge field which has a mass M_v and its associated would-be-Goldstone boson is ϕ_v. A coefficient σ_v is equal to ± 1 for real fields and $\pm i$ for complex fields and the notation ξ_v is for the gauge-fixing parameter. The index v simply indicates gauge bosons of a theory, for example in the electroweak theory v can be charged or neutral weak bosons, or a photon.

The value of the gauge-fixing parameter ξ in Eq. (2.49) is defined in the range from 0 to ∞: *i)* $\xi \to \infty$ corresponds to the Landau gauge in QED; *ii)* $\xi = 1$ is the 't Hooft-Feynman gauge where the gauge boson propagator is proportional to $g_{\mu\nu}$ and the unphysical scalar boson propagator has a pole at $k^2 = M_v^2$. The advantage of the 't Hooft-Feynman gauge is the low degree of divergence of individual Feynman diagrams and the simple form of the vector boson propagators. The slight disadvantage is the presence of unphysical scalars and ghosts which increase the number of diagrams that must be considered; *iii)* the unitary gauge or U-gauge is obtained in the limit $\xi \to 0$. In this gauge the unphysical scalar bosons do not exist.

The choice of the gauge does not affect an expression for physical quantities such as $S-$matrix elements since they are independent on the gauge-fixing parameter. The gauge invariance of the $S-$matrix elements is one of the consequences of the following two fundamental relations: *i)* the $S-$matrix element is gauge-independent, namely $\frac{\partial S}{\partial \xi} \equiv 0$; *ii)* the propagator of a gauge boson $D_{\mu\nu}$ and the propagator of its associated would-be-Goldstone boson D satisfies the identity $\frac{\partial}{\partial\xi} D_{\mu\nu}(k) \equiv -\frac{k_\mu k_\nu}{M_v^2} \frac{\partial}{\partial\xi} D(k)$ [49].

2.4.1 STIs for spontaneously broken gauge theories

For an arbitrary fundamental spontaneously broken gauge theory the Slavnov-Taylor identities discussed in Sec 2.3.1 can be used in two distinct ways. The first one is the *Goldstone-boson equivalence theorem*, which relates the amplitude of the gauge bosons to the amplitude of their associated would-be-Goldstone bosons in the processes involving high energetic, longitudinal vector bosons [50–52]. The second application of the STIs is that they allow to derive definite *sum rules*, *i.e.*, relations between coupling constants of physical fields [20].

Furthermore, let us consider the Slavnov-Taylor identity encoded in Eq. (2.41) with the gauge-fixing operator given in Eq. (2.49). To be more precise, V_v^μ can be chosen to be the mass eigenstate of the charged electroweak gauge bosons, $W^\pm$. In momentum space the Eq. (2.41) is

$$0 = \left\langle T\left(k^\mu W_\mu^\mp(k) \pm \xi_W M_W \phi_\mp(k)\right) \prod_l \Psi_{I_l}^{\text{as,phys}}(k_l) \right\rangle, \tag{2.50}$$

where k and k_l denote incoming momenta. According to the Lehmann-Symanzik-Zimmermann reduction formula the $S-$matrix element is obtained from the truncated Green's functions [53]. Therefore one has to truncate the external fields, put them on-shell, and restrict to physical fields. Although the truncation of the physical legs can be done straightforwardly, the truncation of the gauge-fixing leg is more involved, because the gauge boson is contracted with a derivative. A complete form of the propagator for W and ϕ bosons is

$$G_{(\mu\nu)}^\mp = \begin{pmatrix} G_{\mu\nu}^{W^\mp W^\pm} & G_\mu^{W^\mp \phi^\pm} \\ G_\nu^{\phi^\mp W^\pm} & G^{\phi^\mp \phi^\pm} \end{pmatrix} = \begin{pmatrix} g_{\mu\nu}^T G_T^{WW} + g_{\mu\nu}^L G_L^{WW} & \pm k_\mu G_L^{W\phi} \\ \pm k_\nu G_L^{W\phi} & G^{\phi\phi} \end{pmatrix}, \tag{2.51}$$

where the projectors on transverse and longitudinal parts are

$$g_{\mu\nu}^T = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right), \qquad g_{\mu\nu}^L = \frac{k_\mu k_\nu}{k^2}. \tag{2.52}$$

Writing the propagators of the gauge-fixing legs explicitly, Eq. (2.50) can be rewritten as

$$\begin{aligned} 0 &= \begin{pmatrix} k^\mu \\ \pm\xi_W M_W \end{pmatrix}^{\boldsymbol{T}} G_{(\mu\nu)}^\mp \begin{pmatrix} \left\langle T\underline{W}^{\pm,\nu} \prod_l \Psi_{I_l}^{\text{as,phys}} \right\rangle \\ \left\langle T\underline{\phi}^\pm \prod_l \Psi_{I_l}^{\text{as,phys}} \right\rangle \end{pmatrix} \\ &= \begin{pmatrix} k^\nu \left(G_L^{WW} + \xi_W M_W G_L^{W\phi}\right) \\ \pm\left(k^2 G_L^{W\phi} + \xi_W M_W G^{\phi\phi}\right) \end{pmatrix}^{\boldsymbol{T}} \begin{pmatrix} \left\langle T\underline{W}_\nu^\pm \prod_l \Psi_{I_l}^{\text{as,phys}} \right\rangle \\ \left\langle T\underline{\phi}^\pm \prod_l \Psi_{I_l}^{\text{as,phys}} \right\rangle \end{pmatrix}, \end{aligned} \tag{2.53}$$

where truncated fields are denoted by underlines and this expression yields

$$k^\nu \left\langle T\underline{W}_\nu^\pm \prod_l \Psi_{I_l}^{\text{as,phys}} \right\rangle = \pm M_W A^\pm(k^2) \left\langle T\underline{\phi}^\pm \prod_l \Psi_{I_l}^{\text{as,phys}} \right\rangle. \tag{2.54}$$

The proportionality factor $A^\pm(k^2)$ is defined as

$$A^\pm(k^2) = -\frac{k^2 G_L^{W\phi} + \xi_W M_W G^{\phi\phi}}{M_W\left(G_L^{WW} + \xi_W M_W G_L^{W\phi}\right)}. \tag{2.55}$$

Taking into account the relation between the propagator matrix, $G^{\mp}_{\mu\nu}$, and the matrix of the corresponding vertex functions, $\Gamma^{\mp}_{\mu\nu}$ [1],

$$G^{\mp}_{(\mu\lambda)}\Gamma^{\pm,(\lambda\nu)} = i\begin{pmatrix}\delta^{\nu}_{\mu} & 0\\ 0 & 1\end{pmatrix}, \tag{2.56}$$

it is possible to write a simpler form for the factor $A^{\pm}(k^2)$, *i.e.*,

$$A^{\pm}(k^2) = \frac{k^2/\xi_W + \Gamma^{WW}_L}{M_W\left(M_W + \Gamma^{W\phi}_L\right)} = \frac{\tilde{\Gamma}^{WW}_L}{M_W\tilde{\Gamma}^{W\phi}_L}. \tag{2.57}$$

It is obvious that the factor $A^{\pm}$ is gauge-dependent. At leading order it equals 1 and one-loop corrections are given by self-energies,

$$A^{\pm}(k^2) = 1 - \frac{\Sigma^{WW}_L(k^2)}{M_W^2} - \frac{\Sigma^{W\phi}(k^2)}{M_W}, \tag{2.58}$$

where the self energies Σ^{WW}_L and $\Sigma^{W\phi}$ are defined by splitting off the tree-level contributions, $\tilde{\Gamma}^{WW}_L = M_W^2 - \Sigma^{WW}_L$ and $\tilde{\Gamma}^{W\phi} = M_W^2 + \Sigma^{W\phi}$.

In the case of the Z boson the similar result can be obtained with carefully treating the mixing of the Z with the photon [37].

[1] A Legendre transformation of connected Green's function $T_c\{J\}$, *i.e.*, for a field $a(x) = \frac{\delta T_c\{J\}}{i\delta J(x)}$ the generating functional of vertex functions is $i\Gamma\{a\} = -i\int d^4x J(x)a(x) + T_c\{J\}$.

2.5 Anomalies

A *quantum anomaly* is a phenomenon where a symmetry of the classical theory is destroyed or modified by radiative corrections. In the case of global symmetries, the corresponding anomalies may have physical meaning. For example, an anomaly associated with axial or chiral currents of global symmetries determine the matrix element for the decay of the neutral π meson: $\pi^0 \to \gamma\gamma$. On the other hand anomalies associated with local gauge symmetries violate the validity of the classical Ward identities which are important to prove renormalisability and unitarity of a theory. Thus, in the presence of anomalies, unitarity and gauge invariance of the $S-$matrix are not preserved. This means that anomalous theories are not physically allowable.

In this section, we focus on the *chiral* or *Adler, Bell and Jackiw* (ABJ) [54, 55] anomaly which states that the axial charge of a massless fermion is not conserved in the presence of parallel electric and magnetic fields. As a starting point, let us consider massless QED which has both vector and axial symmetry. The infinitesimal action of the vector $\psi \to e^{i\epsilon}\psi$ and the axial $\psi \to e^{i\epsilon\gamma_5}\psi$ rotations are

$$\delta\psi = i\epsilon\psi, \qquad \delta\bar{\psi} = -i\epsilon\bar{\psi}, \tag{2.59}$$

$$\delta\psi = i\epsilon\gamma_5\psi, \qquad \delta\bar{\psi} = i\epsilon\bar{\psi}\gamma_5 \tag{2.60}$$

with corresponding currents

$$j^\mu = \bar{\psi}\gamma^\mu\psi, \tag{2.61}$$

$$j_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi, \tag{2.62}$$

respectively. In the classical theory, according to Noether's theorem both currents are conserved, *i.e.,* $\partial_\mu j^\mu = 0$ and $\partial_\mu j_5^\mu = 0$. However it is well-known that in the quantum theory $\partial_\mu j_5^\mu \neq 0$, instead

$$\partial_\mu j_5^\mu = \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}, \tag{2.63}$$

where $F_{\mu\nu}$ is the electromagnetic field strength. This is the expression for the chiral anomaly and its derivation can be shown by either the *perturbative* method or the *Fujikawa* method [56]. According to the Fujikawa approach, in the derivation of the Ward identity it is not enough for the action to be invariant under a symmetry: the path integral measure must also be invariant. Namely, for fermions the measure $\int \mathcal{D}\psi\mathcal{D}\bar{\psi}$ picks up a Jacobian factor when variables are changed, *i.e.,* $\psi \to \psi'$ and $\bar{\psi} \to \bar{\psi}'$. The Jacobian of the vector transformation, Eq. (2.59), vanishes at the leading order in ϵ, because of the sign difference in the transformation behaviour of ψ and $\bar{\psi}$ fields. On the other hand, the Jacobian of the axial transformation, Eq. (2.60), gives rise to the anomaly.

It can be noticed that the right-hand side of (2.63) is itself a total derivative, $\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} = 4\partial_\mu\left(\epsilon^{\mu\nu\rho\sigma}A_\nu\partial_\rho A_\sigma\right)$. Using this expression the result in Eq. (2.63) can be generalised straightforwardly for non-Abelian gauge theories with massless chiral fermions,

$$\partial_\mu j_5^{\mu\, a} = \frac{g^2}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\big[T^a\big\{T^b, T^c\big\}\big]\big(\partial_\mu A_\nu^b\big)\big(\partial_\rho A_\sigma^c\big), \tag{2.64}$$

where g is a gauge coupling constant, T^a-generators of the gauge group and the trace is taken over group indices. In general, a chiral or Weyl fermion in a representation $\boldsymbol{R}$ contributes a term to the anomaly proportional to the totally symmetric group factor

$$d^{abc}(\boldsymbol{R}) = \mathrm{Tr}\big[T^a\big\{T^b, T^c\big\}\big]. \tag{2.65}$$

Furthermore, left and right-handed fermions contribute to the anomaly with opposite signs. The requirement for anomaly cancellation is then

$$d^{abc}(\boldsymbol{R}) = 0. \tag{2.66}$$

For example, in Sec. 5, we propose a dark matter model where we extend the SM gauge group by an additional $\mathrm{U}(1)'$ symmetry whose generator is denoted by Y'. For this case, the anomaly cancellation conditions read

$$\begin{aligned}
\mathrm{SU}(3)_c^2 \times \mathrm{U}(1)' &\implies \mathrm{Tr}\big[\{\lambda^a, \lambda^b\}Y'\big] = 0\\
\mathrm{SU}(2)_W^2 \times \mathrm{U}(1)' &\implies \mathrm{Tr}\big[\{\tau^a, \tau^b\}Y'\big] = 0\\
\mathrm{U}(1)_Y^2 \times \mathrm{U}(1)' &\implies \mathrm{Tr}\big[Y^2 Y'\big] = 0\\
\mathrm{U}(1)_Y \times \mathrm{U}(1)'^2 &\implies \mathrm{Tr}\big[Y Y'^2\big] = 0\\
\mathrm{U}(1)'^3 &\implies \mathrm{Tr}\big[Y'^3\big] = 0\\
\text{Gauge-gravity} &\implies \mathrm{Tr}\big[Y\big] = \big[Y'\big] = 0.
\end{aligned} \tag{2.67}$$

Here, Y is the usual hypercharge and generators of $\mathrm{SU}(3)_c$ and $\mathrm{SU}(2)_W$ groups are denoted by λ^a and τ^a, respectively. In principle, there are two more conditions, $\mathrm{SU}(3)_c \times \mathrm{U}(1)'^2$ and $\mathrm{SU}(2)_W \times \mathrm{U}(1)'^2$, which are automatically satisfied due to the traceless generators of $\mathrm{SU}(3)_c$ and $\mathrm{SU}(2)_W$ groups.

2.6 A generic extension of the standard model

In this section following Ref. [20] we discuss about a generic extension of the Standard Model in an arbitrary number of heavy fermions (ψ), physical scalars (h), and vector bosons (V_μ) whose masses are equal to or above the electroweak scale.

A part of the interaction Lagrangian comprising only heavy fields is

$$\begin{aligned}\mathcal{L}_{\text{int}} =& \sum_{f_1 f_2 s_1 \sigma} y^{\sigma}_{s_1 \bar{f}_1 f_2} h_{s_1} \bar{\psi}_{f_1} P_\sigma \psi_{f_2} + \sum_{f_1 f_2 v_1 \sigma} g^{\sigma}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{\psi}_{f_1} \gamma^\mu P_\sigma \psi_{f_2} \\ &+ \tfrac{i}{6} \sum_{v_1 v_2 v_3} g_{v_1 v_2 v_3} \Big(V_{v_1,\mu} V_{v_2,\nu}\, \partial^{[\mu} V^{\nu]}_{v_3} + V_{v_3,\mu} V_{v_1,\nu}\, \partial^{[\mu} V^{\nu]}_{v_2} + V_{v_2,\mu} V_{v_3,\nu}\, \partial^{[\mu} V^{\nu]}_{v_1} \Big) \\ &+ \tfrac{1}{2} \sum_{v_1 v_2 s_1} g_{v_1 v_2 s_1}\, V_{v_1,\mu} V^{\mu}_{v_2} h_{s_1} - \tfrac{i}{2} \sum_{v_1 s_1 s_2} g_{v_1 s_1 s_2}\, V^{\mu}_{v_1} \Big(h_{s_1}\, \partial_\mu h_{s_2} - \big(\partial_\mu h_{s_1}\big)\, h_{s_2} \Big) \\ &+ \tfrac{1}{6} \sum_{s_1 s_2 s_3} g_{s_1 s_2 s_3}\, h_{s_1}\, h_{s_2}\, h_{s_3} + \tfrac{1}{24} \sum_{s_1 s_2 s_3 s_4} g_{s_1 s_2 s_3 s_4}\, h_{s_1}\, h_{s_2}\, h_{s_3}\, h_{s_4}\, .\end{aligned} \tag{2.68}$$

Here the coefficient $\sigma \in \{L, R\}$ denotes the two chiralities of fermions, with chirality projection operators $P_{L/R} = (1 \mp \gamma_5)/2$ and the indices f_i, s_i, and v_i denote the different physical fermion, scalar, and vector fields, respectively, and run over all particles in a given multiplet of the gauge group $\text{U}(1)_{\text{EM}} \times \text{SU}(3)_{\text{color}}$. Spinor indices are suppressed in the notation. Note, the three and four point interactions of scalars are not included in Ref. [20]. The trilinear couplings of scalars are relevant in the calculations of scalar mediated transition amplitudes at one-loop level.

The non-interacting part of the Lagrangian is given by the standard kinetic terms, an R_ξ gauge fixing term expressed in Eq. (2.49) for each massive vector, and a 't Hooft-Feynman gauge-fixing term for the photon field. Particularly, the QED interaction follows from the kinetic terms [57]:

$$\mathcal{L}_{\text{kin}} \supset \bar{f} i \not{D}_\mu f - \tfrac{1}{2} \left| D_\mu v_\nu - D_\nu v_\mu \right|^2 - \tfrac{1}{4} \left| F_{\mu\nu} + ieQ_v \left(\bar{v}_\mu v_\nu - v_\mu \bar{v}_\nu \right) \right|^2 + (D_\mu h_s)^\dagger (D^\mu h_s)\, , \tag{2.69}$$

where the covariant derivatives that act on a field f of charge Q_f is

$$D_\mu f = \left(\partial_\mu - ieQ_f A_\mu \right) f. \tag{2.70}$$

With this choice the trilinear interactions with the photon field can be defined as

$$g^{\sigma}_{\gamma \bar{f} f} = eQ_f, \qquad g_{v \bar{v} \gamma} = eQ_v \qquad \text{and} \qquad g_{\gamma s \bar{s}} = eQ_s, \tag{2.71}$$

where Q_v and Q_s denote the charge quantum numbers of the vector $V_{v,\mu}$ and the scalar h_s, respectively, and the bar denotes the coupling with a charge conjugated fields.

The massive vectors V_μ in Eq. (2.68) are assumed to be gauge bosons of an arbitrary fundamental gauge symmetry and their masses are generated via the spontaneous symmetry breaking. All coupling constants in the interaction Lagrangian are only the model-dependent ones [2]; couplings of unphysical fields such as the would-be Goldstone bosons associated with the spontaneous symmetry breaking can be redefined in terms of other physical couplings with the usage of the Slavnov-Taylor identities which will be discussed in the next section.

Owing to the $\mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{SU}(3)_{\mathrm{color}}$ gauge invariance non-vanishing couplings can be given only for index combinations which allow the fields to form an uncharged singlet. For example, a non-vanishing coefficient $y^{\sigma,abc}_{s_1 \bar{f}_1 f_2}$ implies the charge relation $Q_{s_1} + Q_{f_2} = Q_{f_1}$, and

$$y^{\sigma,dbc}_{s_1 \bar{f}_1 f_2} T^e_{s_1,da} + y^{\sigma,abd}_{s_1 \bar{f}_1 f_2} T^e_{f_2,dc} = T^e_{f_1,bd} y^{\sigma,adc}_{s_1 \bar{f}_1 f_2}. \tag{2.72}$$

In addition, the hermiticity of the Lagrangian imposes restrictions on the couplings. For example, one can express the couplings of negatively charged scalars and gauge bosons to fermions through the couplings of the corresponding positively charged particles, particularly

$$\begin{aligned} & y^{\sigma}_{s_1 \bar{f}_2 f_1} = \left(y^{\bar{\sigma}}_{\bar{s}_1 \bar{f}_1 f_2}\right)^*, \quad g^{\sigma}_{v_1 \bar{f}_2 f_1} = \left(g^{\sigma}_{\bar{v}_1 \bar{f}_1 f_2}\right)^*, \\ & g_{v_1 v_2 s_1} = \left(g_{\bar{v}_1 \bar{v}_2 \bar{s}_1}\right)^*, \quad g_{v_1 s_1 s_2} = -\left(g_{\bar{v}_1 \bar{s}_2 \bar{s}_2}\right)^*, \quad g_{v_1 v_2 v_3} = -\left(g_{\bar{v}_1 \bar{v}_2 \bar{v}_3}\right)^*, \end{aligned} \tag{2.73}$$

Without additional constraints, the Lagrangian of Eq. (2.68) does not describe a renormalisable quantum field theory and cannot be used to derive predictions for physical processes that are finite and gauge independent. The necessary constraints arise from using the Slavnov Taylor Identities (STIs) derived in Ref. [20]. It is observed that these STIs are sufficient to constrain the relevant couplings for $\Delta F = 1$ flavour changing transitions that are generated at one-loop order. In addition, the STIs determine the unphysical Goldstone couplings in terms of the physical couplings. For instance, the Feynman rule of the photon interactions can be read of from the generic Lagrangian by replacing appropriate scalar fields s by ϕ and noting that the STIs derived in Ref. [20] imply $g_{v\bar{v}\gamma} = g_{\gamma\phi\bar{\phi}}$. This allows us to express all contributions of Goldstone bosons in terms of physical couplings.

[2]The couplings in (2.68) are defined such that the Feynman rules are given, after multiplication by a factor of i, in terms of the usual Lorentz structures in the conventions of `FeynArts` [58].

2.6.1 Sum rules

In order to derive sum rule relations between coupling constants we use the Slavnov-Taylor identities for the truncated Green's functions with one or more gauge-fixing functions [20]. When more than one gauge-fixing term is considered the truncation of the gauge-fixing legs will not cause difficulties, because the $A(k^2)$ factor is equal to one at tree level as it is discussed in Sec. 2.4.1.

Futhermore, we can use the STIs derived in Ref [20] for one, two and three gauge-fixing operators, separately. This allows to express all possible three point coupling constants involving would-be-Goldstones bosons ϕ_i in terms of couplings between physical fields,

$$\begin{aligned} g_{v_1\phi_2\phi_3} &= \sigma_{v_2}\sigma_{v_3}\frac{M_{v_2}^2+M_{v_3}^2-M_{v_1}^2}{2M_{v_2}M_{v_3}}g_{v_1v_2v_3}, & g_{\phi_1\phi_2s_1} &= -\sigma_{v_1}\sigma_{v_2}\frac{M_{s_1}^2}{2M_{v_1}M_{v_2}}g_{v_1v_2s_1}, \\ g_{v_1v_2\phi_3} &= -i\sigma_{v_3}\frac{M_{v_1}^2-M_{v_2}^2}{M_{v_3}}g_{v_1v_2v_3}, & g_{\phi_1s_1s_2} &= i\sigma_{v_1}\frac{M_{s_1}^2-M_{s_2}^2}{M_{v_1}}g_{v_1s_1s_2}, \\ g_{v_1\phi_2s_1} &= -i\sigma_{v_2}\frac{1}{2M_{v_2}}g_{v_1v_2s_1}, & g_{\phi_1\phi_2\phi_3} &= 0, \end{aligned} \tag{2.74}$$

$$y^{\sigma}_{\phi_1\bar{f}_1f_2} = -i\sigma_{v_1}\frac{1}{M_{v_1}}(m_{f_1}g^{\sigma}_{v_1\bar{f}_1f_2} - g^{\bar{\sigma}}_{v_1\bar{f}_1f_2}m_{f_2}),$$

where m_{f_i}, M_{v_i} and M_{s_i} denote masses of fermions, vectors and scalars, respectively. The coefficients σ_v can have the values $\pm i$ for complex fields and ± 1 for real fields.

Next we consider the STIs for four-point vertices which has two different consequences. Firstly, if the diagram contains a four-point coupling, the resulting relation allows to write this coupling in terms of three-point couplings. In this manner, all four-point couplings with at least one Goldstone or vector boson can be derived. For example, a four-point vertex of vectors can be defined as a product of trilinear vector boson couplings,

$$g_{v_1v_2v_3v_4} = \sum_{v_5}\left(g_{v_1v_4v_5}g_{v_2v_3\bar{v}_5} + g_{v_1v_3v_5}g_{v_2v_4\bar{v}_5}\right), \tag{2.75}$$

or a vertex of two vectors and two scalars can be written as

$$g_{v_1v_2s_3s_4} = \sum_{v_5} g_{v_1v_5s_4}g_{v_2\bar{v}_5s_3}\frac{1}{4M_{v_5}^2} - \sum_{s_5} g_{v_1s_3s_5}g_{v_2s_4\bar{s}_5} + \text{symm}\left(v_1, v_2\right). \tag{2.76}$$

The second consequence of the STIs for four-point vertices is that if the diagram does not contain a four-point coupling, the STIs provide additional relations among three-point couplings. For example, the Lie-algebra structure of the vector and fermion couplings is represented by the following two sum rules

$$\sum_{v_5}(g_{v_1v_2v_5}g_{v_3v_4\bar{v}_5} + g_{v_2v_3v_5}g_{v_1v_4\bar{v}_5} + g_{v_3v_1v_5}g_{v_2v_4\bar{v}_5}) = 0, \tag{2.77}$$

$$\sum_{v_3} g^{\sigma}_{v_3\bar{f}_1f_2}g_{v_1v_2\bar{v}_3} = \sum_{f_3}(g^{\sigma}_{v_1\bar{f}_1f_3}g^{\sigma}_{v_2\bar{f}_3f_2} - g^{\sigma}_{v_2\bar{f}_1f_3}g^{\sigma}_{v_1\bar{f}_3f_2}), \tag{2.78}$$

where the first equation reflects the *Jacobi identity*. To show this, let us consider generators of a compact Lie group G which satisfy

$$\left[T^a, T^b\right] = if^{abc}T^c,$$

where $a, b, c = 1, \ldots,$dim$G$ and f^{abc} are the fully antisymmetric structure constants. The factor of i is taken to ensure that the generators are Hermitian: $(T^a)^\dagger = T^a$. Above commutation relation with the identity

$$\left[T^a, \left[T^b, T^c\right]\right] + \left[T^b, \left[T^c, T^a\right]\right] + \left[T^c, \left[T^a, T^b\right]\right] = 0$$

implies that the structure constants obey [53]

$$f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0. \tag{2.79}$$

Eq. (2.78) relates the structure constants of fermion and vector representations. Moreover, this relation implies the unitarity of the quark mixing matrix for an universal and diagonal gauge boson couplings to fermions.

There are two more sum rules which provide non-trivial constraints,

$$\begin{aligned}\sum_{s_1} g_{v_1v_2\bar{s}_1}y^{\sigma}_{s_1\bar{f}_1f_2} = &\sum_{v_3}\frac{M^2_{v_1}-M^2_{v_2}}{M^2_{v_3}}g_{v_1v_2\bar{v}_3}\left(m_{f_1}g^{\sigma}_{v_3\bar{f}_1f_2} - g^{\bar{\sigma}}_{v_3\bar{f}_1f_2}m_{f_2}\right)\\ &+\sum_{f_3}\Bigg(-m_{f_1}\left(g^{\sigma}_{v_2\bar{f}_1f_3}g^{\sigma}_{v_1\bar{f}_3f_2} + g^{\sigma}_{v_1\bar{f}_1f_3}g^{\sigma}_{v_2\bar{f}_3f_2}\right)\\ &-m_{f_2}\left(g^{\bar{\sigma}}_{v_2\bar{f}_1f_3}g^{\bar{\sigma}}_{v_1\bar{f}_3f_2} + g^{\bar{\sigma}}_{v_1\bar{f}_1f_3}g^{\bar{\sigma}}_{v_2\bar{f}_3f_2}\right)\\ &+2m_{f_3}\left(g^{\bar{\sigma}}_{v_2\bar{f}_1f_3}g^{\sigma}_{v_1\bar{f}_3f_2} + g^{\bar{\sigma}}_{v_1\bar{f}_1f_3}g^{\sigma}_{v_2\bar{f}_3f_2}\right)\Bigg),\end{aligned} \tag{2.80}$$

$$\sum_{s_1} g_{v_1 s_2 \bar{s}_1} y^{\sigma}_{s_1 \bar{f}_1 f_2} = -\sum_{v_3} \frac{1}{2M^2_{v_3}} g_{v_1 \bar{v}_3 s_2} \Big(m_{f_1} g^{\sigma}_{v_3 \bar{f}_1 f_2} - g^{\bar{\sigma}}_{v_3 \bar{f}_1 f_2} m_{f_2} \Big) + \sum_{f_3} \Big(g^{\bar{\sigma}}_{v_1 \bar{f}_1 f_3} y^{\sigma}_{s_2 \bar{f}_3 f_2} - y^{\sigma}_{s_2 \bar{f}_1 f_3} g^{\sigma}_{v_1 \bar{f}_3 f_2} \Big). \tag{2.81}$$

The full set of sum rules can be found in [20].

2.6.2 The sum rules for the electroweak theory

The standard model electroweak theory is a non-Abelian gauge field theory based on the local $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry [59–61]. The independent parameters of the theory are: g and g' that are $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge coupling constants, correspondingly, μ^2 and λ that are parameters in the Higgs potential and the fermion-scalar Yukawa coupling constant, y_i. Another important parameter is the vacuum expectation value of the Higgs field, v which can be expressed in terms of other free parameters. To describe the physical world the gauge symmetry $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ should be broken spontaneously to $\mathrm{U}(1)_{\mathrm{EM}}$ and this breaking generates masses for gauge bosons and fermions. Consequently, the above mentioned independent parameters can be translated into different sets of physical parameters, one choice is $\{e, M_Z, M_W, m_{f_i}, M_h\}$ where e is an elementary charge and $M_{Z/W}$ are masses of weak force mediators, m_{f_i} is a fermion mass and M_h is the mass of the Higgs scalar.

In this section we hypothesise that we know nothing about the design of the electroweak theory, except its particle content which may be discovered experimentally, and we would like to show that all coupling constants of the theory can be derived using the sum rules enumerated in Sec. 2.6.1. The boson sector consists of massive W and Z vectors and a massless photon γ. Also, in order to obtain meaningful solutions of the STIs we need an electrically neutral scalar boson, h. We also consider only two types of fermions denoted by f_1 and f_2, and it is assumed that they are chiral fermions and $W-$boson talks only to left-handed fermions. The electric charges of fermions and W boson are denoted by $Q_{f_i} = eq_{f_i}$ and $Q_W = eq_W$, respectively, and in our convention $q_{f_1} = +1$ and $q_W = +1$. In addition the masses of W and Z bosons are fixed by the following relation

$$c = \frac{M_W}{M_Z}, \tag{2.82}$$

for now the nature of the constant 'c' is unknown. The first observation is that there are no three-point couplings of the same-type vector bosons such as $g_{ZZ\bar{Z}}$. This statement can be proved by using the Eq. (2.78) for the case $v_1 = v_2 \to V \in \{\gamma, Z, W\}$,

$$\sum_{v_3=V} g^{\sigma}_{v_3 \bar{f}_1 f_2} g_{VV\bar{v}_3} = 0 \implies g_{VV\bar{V}} = 0. \tag{2.83}$$

The same sum rule, (2.78) gives another four more relations by choosing different combinations of gauge bosons for v_i,

$$g^L_{W^+\bar{f}_1 f_2} g_{\gamma W^+ W^-} = g^L_{\gamma \bar{f}_1 f_1} g^L_{W^+\bar{f}_1 f_2} - g^L_{W^+\bar{f}_1 f_2} g^L_{\gamma \bar{f}_2 f_2} \qquad \text{and} \qquad \gamma \to Z, \tag{2.84}$$

$$\sum_{Z,\gamma} g^L_{Z\bar{f}_1 f_1} g_{ZW^+W^-} = g^L_{W^+\bar{f}_1 f_2} g^L_{W^-\bar{f}_2 f_1} \qquad \text{and} \qquad f_1 \leftrightarrow f_2. \tag{2.85}$$

The conservation of the electric charge is expressed in (2.84), *i.e.*, $Q_W = Q_{f_1} - Q_{f_2}$. Next, we consider Eq. (2.80) for three different cases. In the first case v_1 will represent the neutral vectors,

$$\begin{aligned} 0 = \frac{M_Z^2 - M_W^2}{M_W^2} g_{ZW^+W^-} g^L_{W^+\bar{f}_1 f_2} - g^L_{W^+\bar{f}_1 f_2} g^L_{Z\bar{f}_2 f_2} \\ - g^L_{Z\bar{f}_1 f_1} g^L_{W^+\bar{f}_1 f_2} + 2 g^R_{Z\bar{f}_1 f_1} g^L_{W^+\bar{f}_1 f_2} \quad \text{and} \quad (Z \to \gamma) \,. \end{aligned} \tag{2.86}$$

In the remaining two cases s_1 is taken to be the physical scalar, h, that couples to fermions only diagonally,

$$g_{hW^+W^-} y^L_{h\bar{f}_1 f_1} = -m_{f_1} g^L_{W^+\bar{f}_1 f_2} g^L_{W^-\bar{f}_2 f_1} \qquad \text{and} \qquad (f_1 \leftrightarrow f_2)\,, \tag{2.87}$$

$$g_{hZZ} y^{\sigma}_{h\bar{f}_1 f_1} = -2 m_{f_1} \left(g^{\sigma}_{Z\bar{f}_1 f_1} - g^{\bar{\sigma}}_{Z\bar{f}_1 f_1} \right)^2 \qquad \text{and} \qquad (f_1 \leftrightarrow f_2)\,, \tag{2.88}$$

where $\sigma \in \{L, R\}$ and $\bar{\sigma}$ denotes flipped chirality. From Eq. (2.80) we can see that if s_1 is the SM Higgs, h, and $f_{1/2}$ were vector-like fermions, which have interactions with vector bosons as $g^{\sigma}_{v\bar{f}_i f_i} = g^{\bar{\sigma}}_{v\bar{f}_i f_i}$ ($i = 1, 2$), we get the following relation

$$g_{hv_1v_2} y^{\sigma}_{h\bar{f}_i f_i} = 0$$

which implies either $g_{hv_1v_2} = 0$ or $y^{\sigma}_{h\bar{f}_i f_i} = 0$, or both of them zero. Thus, vector-like fermions have Dirac masses straightforwardly, and no need to implement the Higgs mechanism.

Furthermore, we use Eq. (2.81) with the choice $s_2 = h$ and obtain the following relations,

$$0 = -\frac{m_{f_1}}{2M_W^2} g_{hW^+W^-} g^L_{W^+\bar{f}_1 f_2} - y^L_{h\bar{f}_1 f_1} g^L_{W^+\bar{f}_1 f_2}, \tag{2.89}$$

$$0=-\frac{m_{f_1}}{2M_Z^2}g_{hZZ}\left(g^{\sigma}_{Z\bar{f}_1f_1}-g^{\bar{\sigma}}_{Z\bar{f}_1f_1}\right)-y^{\sigma}_{h\bar{f}_1f_1}\left(g^{\sigma}_{Z\bar{f}_1f_1}-g^{\bar{\sigma}}_{Z\bar{f}_1f_1}\right)\quad\text{and}\quad(f_1\leftrightarrow f_2)\,. \tag{2.90}$$

From these relations it is possible to see that the masses of vector bosons are proportional to their interactions with the scalar h,

$$\frac{g_{hW^+W^-}}{g_{hZZ}}\propto\frac{M_W^2}{M_Z^2}\equiv c^2. \tag{2.91}$$

Finally, we would like to use equations (2.75)-(2.76) to write four-point vector boson interactions in terms of triple vector couplings,

$$g_{W^+W^+W^-W^-}=g^2_{AW^+W^-}+g^2_{ZW^+W^-},\qquad g_{W^+W^-ZZ}=-g^2_{ZW^+W^-}, \tag{2.92}$$

$$g_{W^+W^-AZ}=-g_{AW^+W^-}g_{ZW^+W^-},\qquad g_{W^+W^-AA}=-g^2_{AW^+W^-}. \tag{2.93}$$

In a similar way, the vector-scalar couplings can be expressed as

$$g_{ZZhh}=\frac{1}{2M_Z^2}g^2_{hZZ},\quad g_{W^+W^-hh}=\frac{1}{2M_W^2}g^2_{hW^+W^-}. \tag{2.94}$$

Further, using the above relations between coupling constants, we would like to give explicit forms of each coupling constant. Particularly, couplings in the fermion sector are

$$g^L_{Z\bar{f}_1f_1}=-\frac{1}{c\sqrt{1-c^2}}\left(\left(1-c^2\right)Q_{f_1}-\frac{1}{2}Q_W\right),\qquad g^R_{Z\bar{f}_1f_1}=-\frac{\sqrt{1-c^2}}{c}Q_{f_1}, \tag{2.95}$$

$$g^L_{Z\bar{f}_2f_2}=-\frac{1}{c\sqrt{1-c^2}}\left(\left(1-c^2\right)Q_{f_2}+\frac{1}{2}Q_W\right),\qquad g^R_{Z\bar{f}_2f_2}=-\frac{\sqrt{1-c^2}}{c}Q_{f_2}, \tag{2.96}$$

$$g^L_{W^+\bar{f}_1f_2}g^L_{W^-\bar{f}_2f_1}=\frac{1}{2\left(1-c^2\right)}Q_W^2, \tag{2.97}$$

$$y^{\sigma}_{h\bar{f}_1f_1}=-\frac{Q_W}{2\sqrt{1-c^2}}\frac{m_{f_1}}{M_W}\qquad\text{and}\qquad(f_1\to f_2)\,, \tag{2.98}$$

where left- and right-handed Yukawa-type couplings are the same. The left-handed $Z-$boson couplings to f_1 and f_2 fermions come with different signs of $\frac{1}{2}Q_W$ which indicates that the $f_{1/2}$ fermions might be the components of a certain multiplet.

In the boson sector the scalar h coupling with massive vectors are

$$g_{hW^+W^-} = \frac{Q_W}{\sqrt{1-c^2}} M_W, \qquad g_{hZZ} = \frac{Q_W}{c^2\sqrt{1-c^2}} M_W, \tag{2.99}$$

and the three- and four-point vector couplings are

$$g_{ZW^+W^-} = \frac{c}{\sqrt{1-c^2}} Q_W, \tag{2.100}$$

$$g_{W^+W^+W^-W^-} = \frac{1}{1-c^2} Q_W^2, \qquad g_{W^+W^-ZZ} = -\frac{c^2}{1-c^2} Q_W^2, \tag{2.101}$$

$$g_{W^+W^-AZ} = -\frac{c}{\sqrt{1-c^2}} Q_W^2, \qquad g_{W^+W^-AA} = -Q_W^2, \tag{2.102}$$

and four-point scalar-vector couplings are

$$g_{ZZhh} = \frac{1}{2}\frac{Q_W^2}{c^2(1-c^2)}, \qquad g_{W^+W^-hh} = \frac{1}{2}\frac{Q_W^2}{1-c^2}. \tag{2.103}$$

Comparing these coupling constants with the corresponding standard model couplings, it is obvious to see that the constant defined in Eq.(2.82) is identified as a cosine of the weak mixing angle, $c = \cos\theta_W$.

3 FLAVOUR VIOLATING EFFECTIVE INTERACTIONS

In particle physics a low-energy approximation of interactions can be formulated using the effective field theory (EFT) approach. An intuitive idea behind the EFT is that one can calculate experimentally measurable quantities at a certain energy scale without having detailed knowledge about a more fundamental theory, often referred as '*full theory*'. In EFT all modes above the scale of consideration can be excluded by means of the '*integrating out*'[3] [62]. For example, quantum electrodynamics is an effective theory obtained from the standard model, where all standard model particles are integrated out except a photon and an electron.

Another classical example of an effective field theory is the Fermi theory for the $\beta-$decay of a neutron, $n \to pe\bar{\nu}_e$, which had been formulated even when the scales of weak interaction mediators were unknown [63]. The quark-level decay amplitude in the 't Hooft-Feynman gauge is

$$\mathcal{A} = \left(\frac{-ig}{\sqrt{2}}\right)^2 V_{ud} \left(\bar{u}\gamma^\mu P_L d\right) \left(\bar{e}\gamma^\nu P_L \nu_e\right) \left(\frac{-ig_{\mu\nu}}{p^2 - M_W^2}\right), \tag{3.1}$$

where $g/\sqrt{2}$ is $W-$ boson coupling constant and V_{ud} is an element of the CKM matrix. In this transition the maximum momentum transfer is much smaller than the mass of $W-$boson ($M_W = 80.425(38)$ GeV), because the mass difference between neutron and proton is $m_n - m_p \simeq 1.29$ MeV.

Therefore, in the limit $p \ll M_W$ the W propagator can be Taylor expanded with an expansion parameter p/M_W,

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \dots\right). \tag{3.2}$$

At leading order the above amplitude is

$$\mathcal{A} = \frac{i}{M_W^2}\left(\frac{-ig}{\sqrt{2}}\right)^2 V_{ud} \left(\bar{u}\gamma^\mu P_L d\right) \left(\bar{e}\gamma_\mu P_L \nu_e\right) + \mathcal{O}\left(\frac{1}{M_W^4}\right), \tag{3.3}$$

which is the same amplitude that can be obtained from the following local Lagrangian

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} V_{ud} \left(\bar{u}\gamma^\mu P_L d\right) \left(\bar{e}\gamma_\mu P_L \nu_e\right), \qquad \text{where} \qquad \frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8M_W^2} = \frac{1}{2v^2}. \tag{3.4}$$

G_F denotes the Fermi constant and $v \sim 246$ GeV is the scale of electroweak symmetry

[3] The term 'integrating out' is used in the path integral language, alternatively in the canonical operator formalism it is said 'contracted out' through the application of Wick's theorem.

breaking. It is obvious that in this Lagrangian W boson is not a dynamical variable anymore and it is integrated out. In general higher order terms in the expansion (3.2) can be included which give higher dimensional effective operators containing derivative terms. However they are suppressed by powers of $1/M_W^2$.

In this chapter we will discuss some key concepts and methods which will be used in the calculations of amplitudes for flavour changing neutral current transitions. Particularly, we give the weak effective Lagrangian and relevant operators in Sec. 3.1. After that we will discuss the GIM mechanism in Sec. 3.2 and the procedure how to convert tensor integrals into scalar integrals is outlined in Sec. 3.3. The relevant mathematical prescriptions that deal with infinities arising in the evaluation of loop integrals are reviewed in the last two sections. Namely, dimensional regularisation is overviewed in Sec. 3.4 and in Sec 3.5 we will discuss fermion field renormalisation that has direct relevance to our one-loop calculations.

3.1 Weak effective theory

The weak decays of particles with masses in the range between a few hundred MeV and a few GeV, for example the muons, the pions, the kaons, the neutron, charmed mesons like the D^0, etc., can be studied in the effective field theory framework. Similarly, the scattering processes that occur via exchanges of weak bosons can be effectively approximated. For instance, the electron-neutrino scattering $e^-\nu_e \to e^-\nu_e$ at centre of mass energies well below the mass of $W-$boson.

In general processes mediated by heavy particles such as electroweak gauge bosons in the low momentum transfer limit can be described by the effective Lagrangian[4]

$$\mathcal{L}_{\text{eff}} = \sum_i C_i\left(\mu\right)\mathcal{O}_i\left(\mu\right). \tag{3.5}$$

Here $\mathcal{O}_i$ are the relevant local operators which govern the processes in question and C_i denote the Wilson coefficients which describe the strength with which a given operator enters the Lagrangian and summarise the physics contributions from scales higher than the scale of the interest μ.

The decay amplitude that is produced by the effective Lagrangian can be viewed as a product of two distinct parts that are separated by the scale μ: the effects of the *short-distance/high-energy* physics are given with effective couplings $C_i(\mu)$, while the matrix elements $\langle\mathcal{O}_i(\mu)\rangle$ comprise effects of the *long-distance/low-energy* physics. The scale indepen-

[4]Note that we use an effective Lagrangian, as opposed to an effective Hamiltonian as in Ref. [24].

dence of the amplitude implies that the $\mu-$dependence of the couplings $C_i(\mu)$ must cancel the $\mu-$dependence of the matrix elements [64].

The evaluation of the matrix elements are theoretical challenging in the sense that they involve low energy strong interactions where the perturbation theory cannot be applied. Thereby non-perturbative methods such as lattice calculations, the $1/N$ expansion (N is the number of colours), QCD sum rules, hadronic sum rules, or chiral perturbation theory can be used. In the case of certain $B-$meson decays the Heavy Quark Effective Theory (HQEFT) and Heavy Quark Expansion (HQE) also turn out to be useful tools. However, there are some exceptions, such as semi-leptonic rare decays, $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$, $B \to X_s \nu\bar{\nu}$, where the relavant matrix elements can be obtained from well measured leading decays or calculated perturbatively, or as in the case of $B_s \to \mu\bar{\mu}$ expressed fully in terms of meson decay constants [24].

In the study of effective decays of hadrons the theoretical calculations of the experimental observables such as branching ratios or decay rates can be proceeded using the so-called *spectator model.* According to the spectator model a decay of the given meson can be approximated by the decay of its constituting partons [64]. For instance, the branching fraction of the $(M \to F)$ transition can be modelled as

$$Br\left(M \to F\right) \simeq Br\left(q_M \to q_F\right), \tag{3.6}$$

where q_M and q_F denote constituent quarks of the initial and final state mesons, respectively. Indeed this is a naive approximation, since the hadronic uncertainties can be significant. In order to obtain theoretically clean quantities, it is customary to take ratios of branching fractions.

The decays where the final states are specified are known as *exclusive* decays. However, decays of heavy mesons like B meson are realised to be easier to study as *inclusive* decays where a final state is taken as a sum of all accessible final states. The branching ratio of the inclusive $B-$meson decay can be computed in the expansion in inverse powers of m_b with the leading term described by the spectator model in which the $B-$meson decay is oriented by the decay of the $b-$quark, the HQE [65, 66],

$$Br\left(B \to X\right) = Br\left(b \to q\right) + \mathcal{O}\left(\frac{1}{m_b^2}\right). \tag{3.7}$$

3.1.1 Matching

Analytic expressions for Wilson coefficients can be obtained by performing the *matching* of the full theory onto the effective theory. Specifically, it is required that the amplitude calculated in the full theory A_{full} to be reproduced by the corresponding amplitude in the effective theory A_{eff},

$$A_{full} = A_{eff} \equiv \sum_i C_i \langle \mathcal{O}_i \rangle. \tag{3.8}$$

The matching procedure is conducted by following the three main steps: *i)* evaluation of the A_{full} where all particles including the heavy ones appear as dynamical degrees of freedom; *ii)* calculation of matrix elements of effective operators $\langle \mathcal{O}_i \rangle$; *iii)* extracting the Wilson coefficients in accordance with the relation (3.8).

In this work we consider decays of mesons mediated by neutral currents. Specifically, we will focus on transitions of down-type quarks that occur by exchanges of the SM photon and $Z-$boson. The operators we will focus on are the magnetic moment of the photon,

$$\mathcal{O}_7^{bs} = m_b \, \bar{s} \sigma^{\mu\nu} P_R b \, F_{\mu\nu}, \tag{3.9}$$

where $\sigma^{\mu\nu} = i/2 \, [\gamma^\mu, \gamma^\nu]$ and $F_{\mu\nu}$ is electromagnetic field strength tensor, and four-fermion operators

$$\mathcal{O}_9^{\ell} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell), \tag{3.10}$$

$$\mathcal{O}_{10}^{\ell} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell). \tag{3.11}$$

It is well known that the leading order diagrams for flavour changing neutral currents start at one-loop order due to the GIM mechanism (see Sec. 3.2). In order to perform the matching procedure we calculate one loop flavour-violating photon and $Z-$penguin amplitudes in the full theory with extended particle content (2.68). For the evaluation of loop-integrals it is assumed that the masses and momentas of the external particles are very small compared to masses and momenta of the loop particles. Therefore, the Taylor expansion of propagators in the external momenta is applied,

$$\frac{1}{(q+p)^2 - m^2} = \frac{1}{q^2 - m^2} - \frac{p^2 + 2q.p}{(q^2 - m^2)^2} + \ldots, \tag{3.12}$$

where q is the integration momentum and p denotes the momentum of the external particle and m is the mass of the propagating particle. Expanding propagators in this way allows us to deal with only massive, one-scale tadpole integrals which are known up to the four-loop level [67–70].

After Taylor expansion we are left with massive tensor tadpole integrals. To clean the integration momenta from indices we used tensor decomposition discussed in Sec. 3.3. The obtained scalar integrals are regularised dimensionally, Sec. 3.4, and to remove divergencies renormalisation of the external fermion legs is conducted, Sec 3.5.

3.2 GIM mechanism

In the decays of hadrons and leptons that are mediated by charged currents, the strength of the interaction is almost the same in both cases, and is approximately equal to the universal Fermi coupling constant, G_F. This is known as the *quark-lepton universality* (QLU) of the weak interaction. Historically, there were experimental indications for the violation of the QLU, specifically it was observed that the strangeness conserving, $\Delta S = 0$, hadronic currents are slightly weaker than the purely leptonic currents. Also, it was noticed that the strangeness changing, $\Delta S = 1$ decays such as $\Lambda^0 \to pe^-\bar{\nu}_e$ have weaker currents in comparison with $\Delta S = 0$ processes, *e.g.*, $n \to pe^-\bar{\nu}_e$ [71].

In order to address the question of the QLU breakdown in 1963 Cabibbo suggested to redefine the quark iso-doublets so that the weak eigenstates become linear superpositions of the strong interaction eigenstates [72],

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d \times \cos\theta_C + s \times \sin\theta_C \end{pmatrix}, \tag{3.13}$$

where θ_C is Cabibbo angle. Thus, the quark involved in the weak interactions is the Cabibbo rotated quark d', instead of d.

This formalism enabled to interpret differences between $\Delta S = 0$ and $\Delta S = 1$ decays and $\Delta S = 0$ and pure leptonic decays for specific values of θ_C. With the changed basis the decay rates read

$$\Gamma\left(\mu^+ \to e^+\bar{\nu}_\mu\nu_e\right) \propto g^4 \qquad \text{purely leptonic,} \tag{3.14}$$

$$\Gamma\left(n \to pe^-\bar{\nu}_e\right) \propto g^4\cos^2\theta_C \qquad \Delta S = 0 \text{ semi-leptonic,} \tag{3.15}$$

$$\Gamma\left(\Lambda \to pe^-\bar{\nu}_e\right) \propto g^4\sin^2\theta_C \qquad \Delta S = 1 \text{ semi-leptonic.} \tag{3.16}$$

A drawback of the Cabibbo formalism was the experimental observation of the absence of the $\Delta S = 1$ neutral currents. Namely, the measured branching fractions for $\Gamma\left(K^+ \to \mu^+\nu_\mu\right)/\Gamma = (63.51 \pm 0.18)\%$ and $\Gamma\left(K_L^0 \to \mu^+\mu^-\right)/\Gamma = (7.2 \pm 0.5) \times 10^{-9}$ [71]. This issue was resolved by Glashow, Iliopoulos and Maiani (GIM) who proposed the existence of an another quark flavour, the *c*-quark, in 1970 [61]. In the GIM scheme there is a second quark doublet, consisting of the left-handed *c* quark and a linear combination of *s* and *d* quarks that is orthogonal to d'. Particularly,

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d \times \cos\theta_C + s \times \sin\theta_C \end{pmatrix}, \quad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s \times \cos\theta_C - d \times \sin\theta_C \end{pmatrix}. \tag{3.17}$$

The neutral currents are defined for d' and s' quarks, not for d and s ones and the matrix elements for the two families that contribute are

$$\begin{aligned}\left\langle d' \left| J^{\mathrm{NC}} \right| d' \right\rangle + \left\langle s' \left| J^{\mathrm{NC}} \right| s' \right\rangle &= \left\langle d \left| J^{\mathrm{NC}} \right| d \right\rangle \left(\cos^2\theta_C + \sin^2\theta_C\right) + (d \to s) \\ &+ \left\langle s \left| J^{\mathrm{NC}} \right| d \right\rangle (\cos\theta_C \sin\theta_C - \cos\theta_C \sin\theta_C) + (d \leftrightarrow s) \\ &= \left\langle d \left| J^{\mathrm{NC}} \right| d \right\rangle + \left\langle s \left| J^{\mathrm{NC}} \right| s \right\rangle .\end{aligned} \tag{3.18}$$

Therefore, there are no leading order transitions involving down and strange quarks and the neutral weak current couples only diagonally in the fermion types. Accordingly, flavour changing neutral current transitions receive only loop contributions.

Thereby, it can be concluded that the electroweak interaction eigenstates d' and s' are orthogonal combinations of the quark mass eigenstates of definite flavour, d and s. The mixing of quarks are characterized by a single parameter θ_C,

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} . \tag{3.19}$$

In an analogous way, the mixing for the three generations of quarks can be presented by the complex Cabibbo-Kobayashi-Maskawa (CKM) matrix whose elements link mass eigenstates, d, with weak eigenstates, d',

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \boldsymbol{U}_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{3.20}$$

The CKM matrix is unitary, *i.e.*, $\boldsymbol{U}^\dagger_{\mathrm{CKM}} \boldsymbol{U}_{\mathrm{CKM}} = \boldsymbol{U}_{\mathrm{CKM}} \boldsymbol{U}^\dagger_{\mathrm{CKM}} = 1$ with $\boldsymbol{U}^\dagger_{\mathrm{CKM}} = \boldsymbol{U}^{-1}_{\mathrm{CKM}}$. For an $n \times n$ unitary matrix there are $n(n-1)/2$ real angles and $(n-1)(n-2)/2$ phases. Therefore, the CKM matrix can be expressed by the three generalised Cabibbo angles θ_i with $i = 1, 2, 3$ and a phase factor δ. The presence of a phase in the mixing matrix is related to *CP* violation. There are several representations of the CKM matrix and the best-known one among them is the Wolfenstein parametrisation [73].

The practical application of the GIM mechanism in flavour-changing neutral current (FCNC) processes is related with the unitarity of the CKM matrix and masses of particles in the loops. The CKM factors in any FCNC process can be expressed as

$$C_k \propto \sum_{i=u,c,t} \lambda_i F(x_i) \quad \text{or} \quad \sum_{i,j=u,c,t} \lambda_i \lambda_j \tilde{F}(x_i, x_j), \tag{3.21}$$

where the loop functions $F, \tilde{F}$ can be defined as a function of the mass ratios $x_i = m_i^2/M_W^2$, and the λ_i are given in the case of K and B meson decays and particle-antiparticle mixing as

$$\lambda_i = \begin{cases} V_{is}^* V_{id} & K\text{-decays, } K^0 - \bar{K}^0, \\ V_{ib}^* V_{id} & B\text{-decays, } B_d^0 - \bar{B}_d^0, \\ V_{ib}^* V_{is} & B\text{-decays, } B_s^0 - \bar{B}_s^0. \end{cases} \tag{3.22}$$

The unitarity property of the CKM matrix gives a relation

$$\lambda_u + \lambda_c + \lambda_t = 0, \tag{3.23}$$

and in the limit $x_u = x_c = x_t = 0$ it implies $C_k = 0$, and consequently FCNC decays and transitions are absent. However, it is experimentally known that in nature $x_i \neq 0$, $i \in (u, c, t)$, thus the GIM mechanism is broken at the one-loop level generating FCNC processes. The size of this breakdown, and respectively the size of FCNC transitions, depends on the disparity of masses and the behaviour of loop functions [64].

In general in the calculation of decay amplitudes the unitarity relation (3.23) can be used to omit mass-independent terms when the sum over loop fermions is taken.

3.3 Tensor decomposition

The tadpole tensor integrals can be reduced to scalar integrals with the help of the $D-$ dimensional metric tensor [74]. In our calculations, loop integrals contain propagators with different masses, but in this section we consider integrals with the same masses for the demonstration purpose. A general form of one-loop tensor integrals can be written as

$$T^{a_1 a_2 \ldots a_l}_{n_1 n_2 \ldots n_l n_{11} \ldots n_{1l} \ldots n_{l-1l}} = \\ m^{-l \cdot D - \sum a_i + 2\sum n_i} \pi^{-lD/2} \int \frac{\prod_i d^D q_i q_{i\mu_{i,1}} \ldots q_{i\mu_{i,a_i}}}{\prod_i (q_i^2 + m^2)^{n_i} \prod_{i<j} \left((q_i - q_j)^2 + m^2\right)^{n_{ij}}}. \tag{3.24}$$

The integral is symmetric under an exchange of indices, $\mu_{i,j} \leftrightarrow \mu_{i,k}$, and it will be proportional to a sum of symmetrised products of metric tensors. This symmetrised product of metric tensors can be denoted by the number of metric tensors which contain indices of two given loop momenta. For instance, if two metric tensors have an index of the first and second loop momenta, their symmetrised product can be expressed as

$$g_{\mu_{1,1}\mu_{2,1}} g_{\mu_{1,2}\mu_{2,2}} + g_{\mu_{1,1}\mu_{2,2}} g_{\mu_{1,2}\mu_{2,1}} \equiv \\ g\left[b_1 = 0, \ldots, b_l = 0, b_{12} = 2, \ldots, b_{1l} = 0, \ldots, b_{l-1l}\right]. \tag{3.25}$$

Consequently, the general tensor integral can be written as a sum of the metric tensor products times some constant

$$T^{a_1 \ldots a_l}_{n_1 \ldots n_{l-1l}} = \sum_{b_i} F_{b_i} g\left[b_1, \ldots, b_{l-1l}\right]. \tag{3.26}$$

The constant F_{b_i} is determined by contracting (3.26) with all products of metric tensors. The same result can be obtained by a contraction with products of metric tensors which can be related by $\mu_{i,j} \leftrightarrow \mu_{i,k}$. By denoting $g^{(1)}\left[c_1, \ldots, c_{l-1l}\right]$ the fist term of the sum of the products of the metric tensor as a representative of the corresponding symmetry, the following set of equations is obtained,

$$T^{a_1 \ldots a_l}_{n_1 \ldots n_{l-1l}} g^{(1)}\left[c_1, \ldots, c_{l-1l}\right] = \\ m^{-l \cdot D - 2\sum c_i + 2\sum n_i} \pi^{-lD/2} \int \frac{\prod_i d^D q_i \prod_i (q_i, q_i)^{c_i} \prod_{i<j} (q_i, q_j)^{c_{ij}}}{\prod_i (q_i^2 + m^2)^{n_i} \prod_{i<j} \left((q_i - q_j)^2 + m^2\right)^{n_{ij}}} \equiv \\ S^{c_1, \ldots, c_{l-1l}}_{n_1 \ldots n_{l-1l}} = \sum_{b_i} g^{(1)}\left[c_1, \ldots, c_{l-1l}\right] g\left[b_1, \ldots, b_{l-1l}\right] F_{b_i}. \tag{3.27}$$

Thus the constant F_{b_i} can be expressed by the inverse of the matrix

$$M^{a_i}_{c_i,b_i} = g^{(1)}\left[c_1, \ldots, c_{l-1l}\right] g\left[b_1, \ldots, b_{l-1l}\right], \tag{3.28}$$

where the indices b_i and c_i should satisfy the following condition:

$$2b_i + \sum_{j>i} b_{ij} = 2c_i + \sum_{j>i} c_{ij} = a_i. \tag{3.29}$$

The above discussed procedure is independent on the specific form of the denominator, therefore one can apply the tensor decomposition by the following replacement of loop momenta in the numerator of (3.24),

$$\prod_i q_{i\mu_{i,1}} \ldots q_{i\mu_{i,a_i}} \to \sum_{b_i,c_i} \prod_i (q_i, q_i)^{c_i} \prod_{i<j} (q_i, q_j)^{c_{ij}} \left(M^{a_i}_{c_i,b_i}\right)^{-1} g\left[b_1, \ldots, b_{l-1l}\right]. \tag{3.30}$$

3.4 Dimensional regularisation

In the evaluation of radiative corrections to Green's functions, to deal with the divergent integrals one must cut off, or regularise, the momentum integration to have a precise parametrisation of the singularities. The cut-off scale Λ will then appear in the divergent part, while the finite part will be cut-off independent in the limit $\Lambda \to \infty$. The requirement for the choice of the cut-off procedure is that it should not destroy the Lorentz invariance and symmetry of the theory. There are two mostly used regularisation schemes: the covariant cut-off and dimensional regularisation [34]. Here, we will discuss the dimensional regularisation which is employed in our loop calculations.

The Feynman integrals can be dealt with conveniently by the Wick rotation which transforms the Minkowski momentum space to the Euclidean momentum space, *i.e.*, the time component of the momentum is $q_0 = iq_4$ with $q_4 \in \mathbb{R}$.

In order to demonstrate the dimensional regularisation technique, let us first consider the following four-dimensional integral defined over the Euclidean momentum space

$$I(D=4) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2+m^2)^2},$$

which is logarithmically divergent when $q^2 \to +\infty$, but the integral is finite for $D < 4$. Therefore by lowering the dimensionality of the space, it is possible to make the above integral convergent.

In general there are no restrictions on D being an integer number, and the dimensionality of space can be generalised as $D = 4 - \epsilon$. The concept of analytic continuation in the number of space-time dimensions is the central idea in the technique of dimensional regularisation. After changing the dimension, the above integral is evaluated as

$$I(D) \equiv m^{-D+4}\pi^{-\frac{D}{2}} \int d^D q \frac{1}{(q^2+m^2)^2} = \frac{\Gamma(2-\frac{D}{2})}{\Gamma(2)}, \tag{3.31}$$

where Γ is *Euler's Gamma* function, $\Gamma(z) = \int\limits_0^\infty t^{z-1}e^{-t}dt$. Returning back to four dimensions $D = 4 - \epsilon$, divergences are identified as poles at $\epsilon \to 0$.

However an application of the dimensional regularisation for amplitudes containing γ_5, or, equivalently, the totally antisymmetric tensor $\epsilon_{\alpha\beta\mu\nu}$ $\left(\gamma_5 = 1/4!\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu\right)$ is problematic, since the commutation and trace rules for γ_5 cannot be defined properly in D dimensions. This algebraic inconsistency can be handled by adopting one of the properly defined schemes discussed in Ref. [75]. For example, the naive dimensional regularisation (NDR) scheme where the $D-$dimensional metric tensor $g_{\mu\nu}$ is introduced which satisfies $g_{\mu\nu} = g_{\nu\mu}$, $g_{\mu\rho}g^\rho_\nu = g_{\mu\nu}$, and $g^\mu_\mu = D$. Consequently, the anti-commutation rules for the Dirac matrices γ_μ and γ_5 are defined as

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \qquad \{\gamma_\mu, \gamma_5\} = 0. \tag{3.32}$$

Although in the NDR scheme the trace $\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma)$ is not evaluated unambiguously, it is the most widely used scheme in computations, because it is easy to implement into computer programs.

In order to absorb divergent terms in the regularised integrals, we need to define a *renormalisation program.* Since renormalisation is a very broad subject, we discuss only the field renormalisation of the external fermion legs that has direct relevance to our calculation.

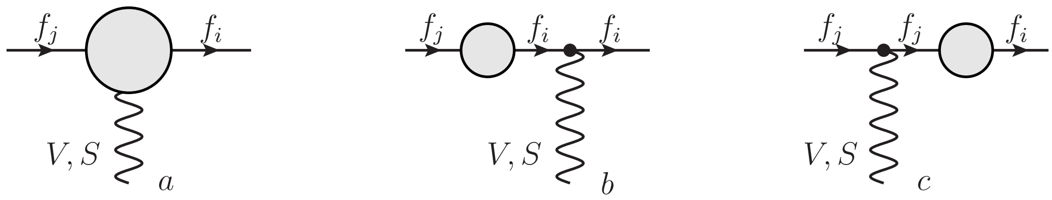


Figure 1: *A general form of flavour-changing neutral current transitions. The grey blobs contain all possible one loop corrections and V and S stands for vector and scalar bosons, respectively. The diagram a is for proper vertex corrections, while b and c diagrams depict couplings of V and S bosons with external fermion legs.*

3.5 Renormalisation

A general form of penguin diagrams responsible for FCNC transitions of fermions (quarks) can be visualised as in Fig. 1. In our case, the grey blobs correspond to one-loop corrections. Since our framework is formulated for the spontaneously broken theories, the renormalisation of the above diagrams can be conveniently done in the *on-shell renormalisation* program, where renormalised parameters of the physical particles are directly linked to the physical quantities [37, 76]. In the on-shell renormalisation, there are no contributions from b and c diagrams because they vanish when quarks are on the mass shell. Consequently, one needs to renormalise diagrams corresponding to the proper vertex, diagram a.

By adapting the counter-term approach we will discuss the derivation of the three-point effective vertex flavour-changing counter-term, which absorbs all infinites arising in the calculation of flavour-violating penguin diagrams.

Let us first write the *renormalisation transformation* of fermion wave functions in the first-order

$$f_{i,0}^{\sigma} = Z_{ij}^{1/2,f,\sigma} f_j^{\sigma} = \left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}^{f,\sigma}\right) f_j^{\sigma}, \tag{3.33}$$

where $\sigma \in \{L, R\}$ denotes the chirality of fermions, where the fermion fields f_j^{σ} are in mass eigenstates. The on-shell renormalisation conditions are imposed to fix the renormalisation constants, requiring that the renormalised mass parameters of the physical particles are equal to the physical masses, *i.e.*, to the real parts of the poles of the corresponding propagators. Therefore, renormalisation conditions for the two-point functions for on-shell external physical fields can be defined as

$$\begin{aligned} \widetilde{\mathrm{Re}}\hat{\Gamma}_{ij}^{f\bar{f}}(p)u_j(p)\Big|_{p^2=m_{f,j}^2} &= 0, \\ \lim_{p^2\to m_{f,i}^2} \frac{\not{p} + m_{f,i}}{p^2 - m_{f,i}^2}\widetilde{\mathrm{Re}}\hat{\Gamma}_{ii}^{f\bar{f}}(p)u_i(p) &= u_i(p), \end{aligned} \tag{3.34}$$

where $u(p)$ is a spinor of the external fields, and the superscript 'hat' denotes the renormalised quantities. Here $\widetilde{\mathrm{Re}}$ takes the real part of the loop integrals appearing in the self-energies but not of the elements of the quark-mixing matrix appearing there. The first condition fixes off-diagonal counter-terms, while the second condition is for the diagonal counter-terms. The two-point fermionic functions can be decomposed into covariants,

$$\hat{\Gamma}^{f\bar{f}}_{ij}(p) = \not{p}P_L\Gamma^{f,L}_{ij}(p^2) + \not{p}P_R\Gamma^{f,R}_{ij}(p^2) + P_L\Gamma^{f,l}_{ij}(p^2) + P_R\Gamma^{f,r}_{ij}(p^2), \tag{3.35}$$

where $P_{L/R} = (1 \mp \gamma_5)/2$ is a chiral projection operator and lowercase l, r letters denote left- and right-chiral scalar parts of the two-point functions, respectively. As a consequence of the hermiticity of the Lagrangian, the fermionic two-point functions must satisfy the following symmetry $\hat{\Gamma}^{f\bar{f}}_{ij}(p) = \gamma^0\hat{\Gamma}^{f\bar{f}\dagger}_{ij}(p)\gamma^0$, which implies $\Gamma^{f,\sigma}_{ij}(p^2) = \Gamma^{f,\sigma*}_{ji}(p^2)$ and $\Gamma^{f,l}_{ij}(p^2) = \Gamma^{f,r*}_{ji}(p^2)$.

In general the renormalised two-point functions entering in Eq. (3.34) may be written as a sum of tree-level contributions, unrenormalised self-energies (Σ) and counter-terms (δZ). At the one-loop level the renormalised two-point functions relevant to the off-diagonal field renormalisation are

$$\hat{\Gamma}^{f,\sigma}_{ij}(p^2) = \delta_{ij} + \Sigma^{f,\sigma}_{ij}(p^2) + \frac{1}{2}\left(\delta Z^{f,\sigma}_{ij} + \delta Z^{f,\sigma\dagger}_{ij}\right), \tag{3.36}$$

where the $\Sigma^{f,\sigma}_{ij}(p^2)$ function describes the $\not{p}$ dependent part of the off-diagonal self-energy. The explicit forms of renormalisation constants for $i \neq j$ are obtained by inserting Eq. (3.36) into the renormalisation conditions in Eq. (3.34),

$$\begin{aligned}\delta Z^{f,\sigma}_{ij} = \frac{2}{m^2_{f,i} - m^2_{f,j}}\widetilde{\mathrm{Re}}\Big[&m^2_{f,j}\Sigma^{f,\sigma}_{ij}(m^2_{f,j}) + m_{f,i}m_{f,j}\Sigma^{f,\bar{\sigma}}_{ij}(m^2_{f,j}) \\ &+ m_{f,i}\Sigma^{f,\varsigma}_{ij}(m^2_{f,j}) + m_{f,j}\Sigma^{f,\bar{\varsigma}}_{ij}(m^2_{f,j})\Big],\end{aligned} \tag{3.37}$$

where $\Sigma^{f,\varsigma}_{ij}$ ($\varsigma = l/r$) corresponds to the scalar part of the self-energy and the lines over σ/ς indicate opposite chirality. The hermiticity of the Lagrangian implies that

$$\delta Z^{\dagger}_{ij} = \delta Z_{ij}\left(m^2_i \leftrightarrow m^2_j\right). \tag{3.38}$$

Finally, knowing the explicit form of the non-diagonal counter-terms, it is straightforward to write the three-point effective vertex counter-term for $i \neq j$ in the following way

$$\begin{aligned} \delta Z^{\sigma}_{V\bar{f}_i f_j} &= \frac{1}{2}\left(g^{\sigma}_{V\bar{f}_i f_i}\,\delta Z^{f,\sigma}_{ij} + g^{\sigma}_{V\bar{f}_j f_j}\,\delta Z^{f,\sigma\,\dagger}_{ij}\right)\,, \\ \delta Z^{\sigma}_{S\bar{f}_i f_j} &= \frac{1}{2}\left(y^{\sigma}_{S\bar{f}_i f_i}\,\delta Z^{f,\sigma}_{ij} + y^{\sigma}_{S\bar{f}_j f_j}\,\delta Z^{f,\bar{\sigma}\,\dagger}_{ij}\right)\,, \end{aligned} \tag{3.39}$$

where the first line is relevant to vector-penguins such as a photon or $Z-$boson penguins, and the second line for scalar penguins such as the SM Higgs, $S-$penguin.

Note that since extensions of the SM could generically contain new sources of chiral symmetry breaking interactions, we have allowed the fermion masses not be proportional to the Yukawa couplings, *i.e.*, $y_f \not\propto m_f$. Using the expressions for the off-diagonal two-point counter-terms in Eq. (3.37) the effective vertex counter-terms become

$$\begin{aligned} \delta Z^{\sigma}_{V\bar{f}_i f_j} &= \frac{1}{m_i^2 - m_j^2}\widetilde{\mathrm{Re}}\left\{\left(m_j^2\, g^{\sigma}_{V\bar{f}_i f_i} - m_i^2\, g^{\sigma}_{V\bar{f}_j f_j}\right)\Sigma^{\sigma}_{ij}(\mu^2)\right. \\ &\qquad \left. + \left(g^{\sigma}_{V\bar{f}_i f_i} - g^{\sigma}_{V\bar{f}_j f_j}\right)\left[m_i m_j \Sigma^{\bar{\sigma}}_{ij}(\mu^2) + m_i \Sigma^{\varsigma}_{ij}(\mu^2) + m_j \Sigma^{\bar{\varsigma}}_{ij}(\mu^2)\right]\right\}\,, \\ \delta Z^{\sigma}_{S\bar{f}_i f_j} &= \frac{1}{m_i^2 - m_j^2}\widetilde{\mathrm{Re}}\left\{\left(m_j y^{\sigma}_{S\bar{f}_i f_i} - m_i y^{\sigma}_{S\bar{f}_j f_j}\right)\left[\Sigma^{\bar{\varsigma}}_{ij}(\mu^2) + m_j \Sigma^{\sigma}_{ij}(\mu^2) + m_i \Sigma^{\bar{\sigma}}_{ij}(\mu^2)\right]\right. \\ &\qquad \left. + \left(m_i y^{\sigma}_{S\bar{f}_i f_i} - m_j y^{\sigma}_{S\bar{f}_j f_j}\right)\Sigma^{\varsigma}_{ij}(\mu^2)\right\}\,. \end{aligned} \tag{3.40}$$

The universality of the vector boson couplings to the same-type quarks, *i.e.*, $g^{\sigma}_{V\bar{f}_i f_i} = g^{\sigma}_{V\bar{f}_j f_j}$ ensures that the effective vertex $\delta Z^{\sigma}_{V\bar{f}_i f_j}$ receives contributions only from momentum slashed terms of the off-diagonal self-energy. Similarly, if Yukawa couplings are $y_i \propto m_i$, the vertex $\delta Z^{\sigma}_{S\bar{f}_i f_j}$ is expressed by the scalar part of the off-diagonal self energy.

4 1-LOOP RESULTS FOR GENERIC MODELS OF NP

Recent experimental results on lepton flavour non-universality in rare B-meson decays [9] and on the anomalous magnetic moment of the muon [10] have reaffirmed and strengthened the pre-existing tensions with the corresponding standard model (SM) predictions. In the SM, both processes are loop-induced; hence, it is reasonable to expect that physics beyond the standard model (BSM) to also contribute at the one-loop level if present. The SM contribution to B-meson decays is well described by the weak effective Lagrangian [24]. The same is true for many of the SM extensions if they involve particles with masses above the electroweak scale. However, matching onto the effective theory is tedious and generally has to be repeated for every new model. The tediousness is exacerbated if one wants to, additionally, check that the result is gauge-independent and that all UV divergences properly cancel.

In this chapter, we consider generic extensions of the SM (see Sec. 2.6) with vectors, scalars, and fermions with the additional assumption that the theory is perturbatively unitary and, thus, renormalisable [17–19]. Once the particle content is specified, the resulting weak effective Lagrangian can immediately be read off. The Wilson coefficients depend on a minimal set of physical parameters and are guaranteed to be finite and gauge independent. These properties follow from coupling constant sum rules derived from Slavnov-Taylor identities as outlined in Sec. 2.6.1.

As an example, consider the SM contribution to the Wilson coefficient C_9 that is $C_9^\ell = 1/2(C_{LL}^{23\ell}+C_{LR}^{23\ell})$ in Eq. (4.3). Generically, the minimal field content in the loop that is required to obtain a non-zero, finite, result consists of two massive vector bosons, one charged and one neutral, two charged fermions, and one neutral fermion – see the left panel in Table 1. Once the couplings of these states are specified, and the sum rules among them are applied, Eq. (4.18) directly gives the finite and gauge-independent result,

$$C_9 = \frac{e^2 G_F V_{ts}^* V_{tb}}{\sqrt{2}} \left[\frac{1}{s_W^2} F_V^{L,BZ} - 4 F_V^{\gamma Z} \right] , \tag{4.1}$$

where $F_V^{L,BZ}$ and $F_V^{\gamma Z}$ are loop functions that, in the SM, only depend on m_t^2/m_W^2, see Eq. (4.45). Here, G_F is the Fermi constant, e is the positron charge, $s_W \equiv \sin\theta_W$ is the sine of the weak mixing angle, and V_{ij} are the elements of the Cabibbo-Kobayashi-Maskawa matrix. The procedure is exactly the same for any extension of the SM.

There are two important points to note here. First, the unitarity of the quark-mixing matrix is guaranteed by the sum rule in Eq. (4.6). Furthermore, in the absence of tree-level flavour-changing neutral currents (FCNCs), at least two fermion generations in the loop are required

Field	Mass	$U(1)_Q$ Charge
W	m_W	$+1$
Z	m_Z	0
ν	0	0
$\{u, t\}$	$\{0, m_t\}$	$+2/3$

Coupling	Value
$\{W,\, \bar{t},\, b\}$	$-1/\sqrt{2}\, g\, U_{tb}$
$\{W^*,\, \bar{s},\, t\}$	$-1/\sqrt{2}\, g\, U^*_{ts}$
$\{W,\, \bar{\nu},\, \mu\}$	$-1/\sqrt{2}\, g$

Table 1: *The loop field content (left table) and the couplings of those fields (right table). The matrix U_{ij} is the two-generation quark-mixing matrix. Note that since we only consider two fermion generations inside the loop, the charged vector couplings need to be specified only for one generation.*

to give a non-zero contribution. Second, and more remarkable, the same sum rule, Eq. (4.6), fixes the couplings of the Z boson to the internal and external fermions and, consequently, it is not necessary to specify them in the first place. In this way, the Z penguin, photon penguin, and boxes are combined into gauge-independent loop functions that generalise the penguin-box expansion of Ref. [77]. The penguin-box functions X, Y, and Z of Ref. [77] are

$$\begin{aligned} X(x_t) &= C(x_t, \xi) - 4B\big(x_t, \xi, \frac{1}{2}\big) = C(x_t) - 4B(x_t), \\ Y(x_t) &= C(x_t, \xi) - B\big(x_t, \xi, -\frac{1}{2}\big) = C(x_t) - B(x_t), \\ Z(x_t) &= C(x_t, \xi) + \frac{1}{4}D(x_t, \xi) = C(x_t) + \frac{1}{4}D(x_t), \end{aligned} \tag{4.2}$$

where C and D are Z and photon penguin loop functions, respectively, and box loop functions are denoted by B whose last argument defines final lepton states with the third component of the isospin. The functions X, Y, and Z are directly related to our functions $F_V^{L,B'Z}$, $F_V^{L,BZ}$ and $F_V^{\gamma Z}$ in the SM limit. Apart from an overall normalisation, the only difference is that $F_V^{\gamma Z}$ also incorporates the light particle contribution in the matching procedure. Our functions generalise X, Y, and Z to extensions of the SM with an arbitrary number of massive vectors, scalars, and fermions while remaining gauge independent.

General expressions for the photon dipole have already been presented in Refs. [78–81], while contributions of heavy new scalars and fermions to the $b \to s\ell\ell$ transition were considered in Refs. [82–84]. Here, we extend the discussion to the contributions of the photon and $Z-$penguins to the semileptonic current-current operators, with a special focus on proving gauge invariance in the presence of heavy vectors, and eliminating couplings to unphysical scalars such as would-be Goldstone bosons. Moreover, we provide easy-to-use code to obtain the Wilson coefficients in general perturbatively unitary models, it is available at https://wellput.github.io.

The chapter is organized as follows. The effective $\Delta F = 1$ Lagrangian and relevant sum rules are discussed in Sec. 4.1 along with the dipole and current-current Wilson coefficients. There, we also explain the cancellation of the gauge dependent terms. In Sec 4.2 we give the effective Lagrangian for particle-antiparticle mixings. The gauge invariance of the flavour changing scalar current is outlined in Sec. 4.3. In Sec. 4.4, we apply our setup to different models taken from the literature to illustrate how the one-loop matching contributions can be easily obtained. The explicit expressions for the loop functions in App. B.

4.1 The effective $\Delta F = 1$ Lagrangian

This section provides the explicit form of the effective Lagrangian relevant for leptonic, semileptonic, and radiative B, B_s, and K meson decays for a generic renormalisable model. We write the five-flavour effective Lagrangian that describes the $d_j \to d_i$ transition, obtained by integrating out the W and Z bosons, the top quark, as well as all heavy new particles at the electroweak scale, as

$$\begin{aligned}\delta\mathcal{L}_{\Delta F=1} = &\frac{1}{16\pi^2}\sum_{\substack{\ell\in\{e,\mu,\tau\}\\ \sigma,\sigma'\in\{L,R\}}} C^{ij\ell}_{\sigma\sigma'}\left(\bar{d}_i\gamma^\mu P_\sigma d_j\right)\left(\bar{\ell}\gamma_\mu P_{\sigma'}\ell\right)\\ &+\frac{1}{16\pi^2}\sum_{\sigma\in\{L,R\}} D^{ij}_\sigma \bar{d}_i\sigma^{\mu\nu}P_\sigma d_j F_{\mu\nu} + \text{h.c.}\,.\end{aligned} \tag{4.3}$$

The operators in the first sum have the form of a product of a leptonic current and a FCNC. The second sum contains the photon dipole operators. Here, $d_i = d, s, b$ denote the down-type quark fields and ℓ the lepton fields. $P_L \equiv (1-\gamma_5)/2$ and $P_R \equiv (1+\gamma_5)/2$ are the chirality projection operators, and σ and σ' denote the chiralities of the incoming quarks and leptons. We neglect all operators with mass dimension larger than six. The explicit results for the Wilson coefficients are given below in Eqs. (4.14) and (4.18) - (4.20).

The Wilson coefficients of the effective Lagrangian are functions of the couplings of the generic Lagrangian and the associated masses. They are determined by calculating suitable Green's functions: The photon penguin diagrams (Fig. 2) contribute in part to the dipole coefficients D^{ij}_σ, and in part to the current-current coefficients $C^{ij\ell}_{\sigma\sigma'}$ via the equations of motion of the photon field which is given by the operator

$$\mathcal{O}_{36} = \frac{e}{g_s^2}\bar{d}_i\gamma^\mu P_\sigma d_j\partial^\nu F_{\mu\nu} + \frac{e^2}{g_s^2}\left(\bar{d}_i\gamma^\mu P_\sigma d_j\right)\sum_f Q_f\left(\bar{f}\gamma^\mu f\right). \tag{4.4}$$

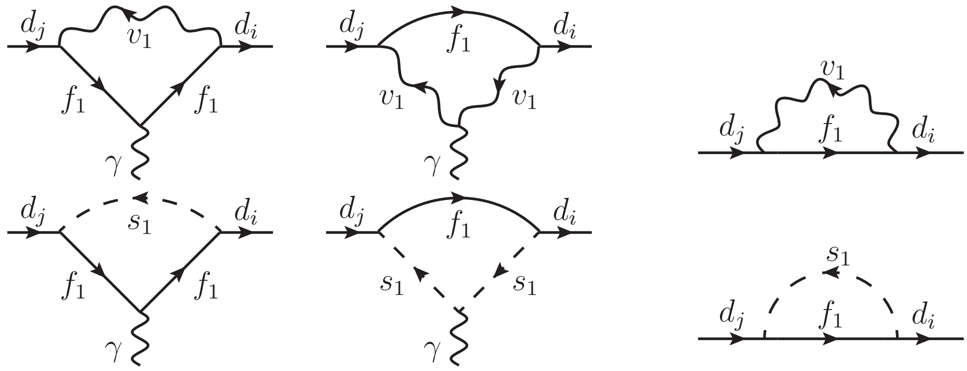


Figure 2: *Diagrams for the off-shell $d_j \to d_i\gamma$ Green's function. The third column diagrams correspond to the off-diagonal self-energy corrections. Physical scalars are denoted by dashed lines, while the contribution of massive vector bosons and their related Goldstone bosons are denoted by a wavy line.*

In order to extract expressions for the D^{ij}_{σ} and $C^{ij\ell}_{\sigma\sigma'}$ coefficients, the full set of operators which vanish by the equation of motion of the photon field need to be considered in the matching procedure. These operators can be found in App. A.

The $Z-$penguin and box diagrams (Fig. 5) contribute to the current-current coefficients $C^{ij\ell}_{\sigma\sigma'}$. In the remainder of this section, we spell out the details of this calculation, with a focus on obtaining a finite and gauge-independent result. We incorporate the constraints from the STIs by repeatedly applying the sum rules on the one-loop amplitudes.

4.1.1 $d_j \to d_i\gamma$

The relevant diagrams for the $d_j \to d_i\gamma$ process is depicted in Fig. 2, where off-diagonal self-energy corrections are needed for the renormalisation (see Sec. 3.5). These diagrams and transition amplitudes are generated using the `MATHEMATICA` package `FeynArts` [58] with a model file (.mod file) that is adjusted for our generic model. Particularly, new particles are added and coupling constants are defined generically. In addition all unphysical couplings of would-be-Goldstone bosons associated with loop vectors are replaced by relations in Eq. (2.74). The amplitudes are generated in the R_ξ gauge with arbitrary gauge-fixing parameter ξ. Propagators of the would-be-Goldstone bosons are decomposed as

$$\frac{1}{q^2-\xi m_V^2} = \frac{1}{q^2-m_V^2} - (1-\xi)\frac{m_V^2}{q^2-m_V^2}\frac{1}{q^2-\xi m_V^2}, \tag{4.5}$$

where m_V denotes mass of associated gauge bosons. This decomposition allows us to separate gauge-dependent and gauge-independent parts of the amplitude conveniently. The gauge dependent part will vanish in the Feynman gauge where $\xi = 1$. Moreover, the loop integrals are Taylor expanded (3.12) to the second order in the external momenta. Furthermore, we used the "unitarity sum rule" in Eq. (2.78)

$$\sum_{v_3} g^{\sigma}_{v_3\bar{d}_i d_j} g_{v_1\bar{v}_2\bar{v}_3} = \sum_{f_1} g^{\sigma}_{v_1\bar{d}_i f_1} g^{\sigma}_{\bar{v}_2\bar{f}_1 d_j} - \sum_{f_1} g^{\sigma}_{\bar{v}_2\bar{d}_i f_1} g^{\sigma}_{v_1\bar{f}_1 d_j} \,, \tag{4.6}$$

where summation over v_3 is taken over neutral vectors, while f_1 summation runs over fermions that satisfy the charge conservation conditions. Setting $Q_{d_i} = Q_{d_j} \equiv Q_d$ implies

$$Q_{v_3} = 0 \qquad \text{and} \qquad Q_v \equiv Q_{v_1} = -Q_{\bar{v}_2}. \tag{4.7}$$

Additionally, this implies that the charges of the fermions f_1 can either be

$$Q_{f_1} = Q_d - Q_v \qquad \text{or} \qquad Q_{f_1} = Q_d + Q_v, \tag{4.8}$$

which respectively contribute to the first or second sum on the right hand side. Since we assume that there is no flavour changing neutral currents at the tree level, $g_{v_3\bar{d}_i d_j} = 0$ for any neutral vector v_3, we find the following generalisation of the Glashow-Iliopoulos-Maiani (GIM) relation

$$g^{\sigma}_{\bar{v}_2\bar{d}_i f_0} g^{\sigma}_{v_1\bar{f}_0 d_j} = -\sum_{f_1 \neq f_0} g^{\sigma}_{\bar{v}_2\bar{d}_i f_1} g^{\sigma}_{v_1\bar{f}_1 d_j} + \sum_{f_1} g^{\sigma}_{v_1\bar{d}_i f_1} g^{\sigma}_{\bar{v}_2\bar{f}_1 d_j} \,. \tag{4.9}$$

This relation can be used to eliminate the couplings of any one member of the set of fermions of charge Q_{f_1} that generate flavour changing neutral currents through charged vector interactions. For definiteness, we always choose to eliminate the lightest of such fermions. This choice is just a convention which may have some phenomenological significance. For example, in the $b \to s$ transition, the CKM factor of up quark is usually written in terms of CKM factors of charm and top quarks, this can be due to the fact that the CKM matrix is very close to a unit matrix with off diagonal terms that are small.

We remark that we can project the off-shell photon Green's function onto the off-shell basis, including physical, equation-of-motion-vanishing, and BRST-exact operators (see App. A), only after applying the sum rule (4.9). However, if the photon is taken on-shell then it is possible to do the matching without doing the GIM mechanism and this allows to obtain only the dipole coefficients, D^{ij}_{σ}. In this way we can simultaneously determine the Wilson coefficients of the dipole operators and the photon penguin contribution to the $\Delta F = 1$ current-current operators. The Wilson coefficients of the dipole operators are independent of the gauge fixing parameters, while the photon penguin contribution is not.

The Wilson coefficients then depend on the mass of the lightest fermion that can contribute in the loop, denoted by the index f_0. This particle could be either a light standard-model fermion, such as an up quark, or a heavy fermion, such as a chargino. Here, the notion of light and heavy is defined via the characteristic scale of the matching calculation, which is determined by the masses of the heavy degrees of freedom in the UV theory. In this work we assume that this is the electroweak scale, even though the formalism could be easily applied to a matching to a different effective theory.

A fermion mass of f_0 that is considerably smaller than the matching scale requires an appropriate effective theory counterpart (second diagram in Fig 3) that will account for the infrared logarithm generated in the limit $m_{f_0} \to 0$ or $x_0 \to 0$ where $x_0 \equiv m_{f_0}^2/M_{v_1}^2$.

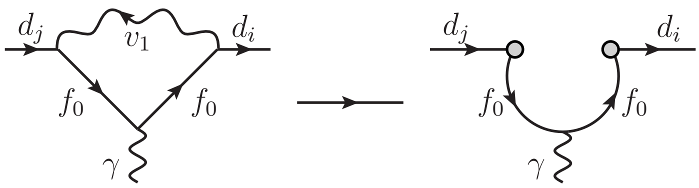


Figure 3: *Diagrams with internal heavy vectors and light fermions. The second diagram is obtained after integrating out the heavy vector.*

In the matching equation for the dipole operators the scalar loop functions that multiplies Yukawa couplings of the same chirality must contain an odd number of chirality flips as explained in Sec. 4.1.3. This implies that the infrared logarithm $\sqrt{x_0}\log(x_0)$ vanishes in the limit $x_0 \to 0$. Thus, the effective theory contribution is vanishing in this limit and we do not have to consider the scalar functions further. The vector contributions of the dipole operator have no infrared logarithm in the limit of the lightest internal fermion mass tending to zero. Since $F_{V'}^d$ is multiplied with the internal fermion mass we only need to consider the limit $x_0 \to 0$ for the F_V function.

On the other hand the combination of Z and photon penguin loop functions (see Sec. 4.1.4) exhibits an infrared logarithm $\log x_0$ in the limit of a light internal fermion. This logarithm is cancelled through the effective theory contribution of the light fermion. A tree-level matching of the vector boson contributions will generate a four-fermion Wilson-coefficient that has a non-vanishing one-loop matrix element whose projection $\delta r_{\sigma\sigma'}^{ij\ell}$ onto the tree-level matrix element of the neutral current operator of Eq. (4.3) reads

$$\delta r_{\sigma\sigma'}^{ij\ell} = \sum_{v_1 f_1} \frac{g_{\bar{v}_1 \bar{d}_i f_1}^{\sigma} g_{v_1 \bar{f}_1 d_j}^{\sigma}}{M_{v_1}^2} \frac{e^2 Q_\ell Q_f}{16\pi^2} \left(\frac{2}{3} - \frac{2}{3} \log \frac{\mu^2}{m^2} \right) , \tag{4.10}$$

if we keep the dependence on the light fermion mass to regularise the infrared divergence and include the operator mixing of the tree-level operator to renormalise the ultraviolet pole. Subtracting this effective theory contribution from our full theory result, the light mass dependence will cancel out and we obtain the matching corrections in the limit of light internal fermion masses.

4.1.2 $d_j \to d_i Z$

The calculation of the $d_j \to d_i Z$ Green's function in the 't Hooft-Feynman gauge is performed in [20] and we recalculated it with arbitrary gauge-fixing parameter, ξ. The results are in agreement in the limit $\xi \to 1$. Here, we just discuss about sum rules which are used for the renormalisation. The relevant diagrams are the ones depicted in Fig. 2, where the photon leg is replaced by the $Z-$boson, plus non-diagonal diagrams in loop particles. In addition, there are mixed diagrams involving internal scalars, vectors, and fermions, simultaneously which are shown in Fig. 4.

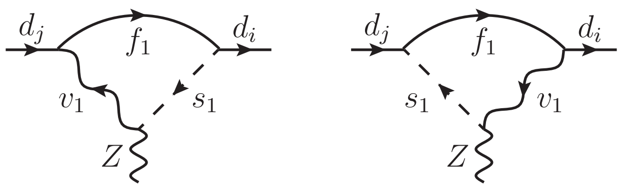


Figure 4: *Diagrams for the $d_j \to d_i Z$ Green's function which involve mixes of vectors and scalars.*

In order to deal with divergent parts, the sum rules in Eqs. (2.78), (2.80) and (2.81) are utilised. Particularly, they are solved for the case when $\sigma = L$ and particles are chosen such that $v_1 = Z$, $f_2 = d_j$ and f_1 being loop fermions. Also, the universality of the $Z-$boson couplings to external fermions is supposed, *i.e.*, $g^{\sigma}_{Z\bar{d}_i d_i} = g^{\sigma}_{Z\bar{d}_j d_j}$.

With this choice, $Z-$ boson couplings with right-handed loops fermions can be expressed in terms of other couplings. First, we solve Eq. (2.80) for a product of $g^R_{Z\bar{f}_1 f_1} g^L_{v_1 \bar{f}_1 d_j}$,

$$\begin{aligned} \sum_{s_1} g_{Z v_1 \bar{s}_1} y^L_{s_1 \bar{f}_1 d_j} = &\sum_{v_2} \frac{M_Z^2 - M_{v_1}^2}{M_{v_2}^2} g_{Z v_1 \bar{v}_2} g^L_{v_2 \bar{f}_1 d_j} m_{f_1} \\ &- m_{f_1} \left(g^L_{v_1 \bar{f}_1 d_j} g^L_{Z \bar{d}_j d_j} + g^L_{Z \bar{f}_1 f_1} g^L_{v_1 \bar{f}_1 d_j} + \sum_{f_2} g^L_{Z \bar{f}_1 f_2} g^L_{v_1 \bar{f}_2 d_j} \right) \\ &+ 2 m_{f_1} \boldsymbol{g^R_{Z\bar{f}_1 f_1} g^L_{v_1 \bar{f}_1 d_j}} + \sum_{f_2} 2 m_{f_2} g^R_{Z \bar{f}_1 f_2} g^L_{v_1 \bar{f}_2 d_j}. \end{aligned} \tag{4.11}$$

If there are no flavour-changing scalars in a theory, we will use only this relation with Eq. (2.78). In that case the left-hand side of the relation would be zero.

Since there are internal scalars in our generic model, we need to consider the Eq. (2.81) as well. More precisely, this equation is solved for the product of $g^{R}_{Z\bar{f}_1 f_1} y^{L}_{s_1 \bar{f}_1 d_j}$ couplings,

$$\sum_{s_2} g_{Z s_1 \bar{s}_2} y^{L}_{s_2 \bar{f}_1 d_j} = -\sum_{v_2} \frac{1}{2M_{v_2}^2} g_{Z\bar{v}_2 s_1} g^{L}_{v_2 \bar{f}_1 d_j} m_{f_1} + \boldsymbol{g^{R}_{Z\bar{f}_1 f_1} y^{L}_{s_1 \bar{f}_1 d_j}} + \sum_{f_2} g^{R}_{Z\bar{f}_1 f_2} y^{L}_{s_1 \bar{f}_2 d_j} - y^{L}_{s_1 \bar{f}_1 d_j} g^{L}_{Z\bar{d}_j d_j}. \tag{4.12}$$

Using these two relations we express $Z-$boson couplings with right-handed diagonal fermions by other couplings.

Furthermore, Eq. (2.78) is applied to combine self-energy diagrams with proper vertex diagrams in the loops with vectors and fermions. Namely, it is solved for $g^{\sigma}_{v_1 \bar{f}_1 d_j} g^{\sigma}_{Z\bar{d}_j d_j}$,

$$\sum_{v_2} g^{\sigma}_{v_2 \bar{f}_1 d_j} g_{Z v_1 \bar{v}_2} = g^{\sigma}_{Z\bar{f}_1 f_1} g^{\sigma}_{v_1 \bar{f}_1 d_j} + \sum_{f_2} \left(g^{\sigma}_{Z\bar{f}_1 f_2} g^{\sigma}_{v_1 \bar{f}_2 d_j} \right) - \boldsymbol{g^{\sigma}_{v_1 \bar{f}_1 d_j} g^{\sigma}_{Z\bar{d}_j d_j}}. \tag{4.13}$$

After using these three relations we are left with only divergent parts which are mass independent constants. Their cancellation is realised by performing the generalised GIM mechanism that is derived in Eq. (4.9).

4.1.3 Dipole Operator Coefficients

Here and in the following, we write the Wilson coefficient of the five-flavour effective Lagrangian as a product of the coupling constants and gauge-independent loop functions that depend on various mass ratios defined by $x_b^a \equiv m_a^2/m_b^2$. The matching coefficients of the dipole operators are immediately gauge independent. We find

$$\begin{aligned} D_R^{ij} = &\sum_{s_1 f_1} \frac{y^R_{\bar{s}_1 \bar{d}_i f_1}}{M^2_{s_1}} \Big(m_{f_1} y^R_{s_1 \bar{f}_1 d_j} F_S^d(x^{f_1}_{s_1}) + m_{d_j} y^L_{s_1 \bar{f}_1 d_j} F_{S'}^d(x^{f_1}_{s_1}) \Big) \\ &+ \sum_{v_1 f_1} \frac{g^L_{\bar{v}_1 \bar{d}_i f_1}}{M^2_{v_1}} \Big(m_{f_1} g^R_{v_1 \bar{f}_1 d_j} F_{V'}^d(x^{f_1}_{v_1}) + m_{d_j} g^L_{v_1 \bar{f}_1 d_j} F_V^d(x^{f_0}_{v_1}, x^{f_1}_{v_1}) \Big) \ , \end{aligned} \tag{4.14}$$

where here and in all analogous equations below the sums run over all combinations of indices that are allowed by charge and colour conservation. The explicit form of the loop functions is given in App. B.1. The first line represents the contribution of internal fermions and scalars. The appearance of two right-handed Yukawa couplings in the first term requires an odd number of mass insertions in the fermion line, hence the loop function F_S^d is multiplied with the internal fermion mass m_{f_1}. The mass factor in the second term is supplied by the Dirac equation acting on the external d_j spinor (we neglect the lighter m_{d_i} mass). The second line represents the effects of internal charged massive vector bosons and fermions. Now, the first term proportional to two vector couplings of opposite chirality requires an odd number of mass insertions, resulting in the factor m_{f_1}. The second term, proportional to the function F_V^d, involves two vector couplings of the same chirality and receives a factor m_{d_j} from the Dirac equation. Moreover, we have used Eq. (4.9), generating the explicit dependence on the mass of the fermion f_0. If there are fermions of charge $Q_{f_1'} = Q_d - Q_v$ we have to add their contribution through the sum

$$D_R^{ij} \to D_R^{ij} + \sum_{\bar{v}_1 f_1'} \frac{m_{d_j}}{M^2_{v_1}} g^L_{v_1 \bar{d}_i f_1'} g^L_{\bar{v}_1 \bar{f}_1' d_j} F^d_{\bar{V}}(x^{f_0}_{v_1}, x^{f_1'}_{\bar{v}_1}) \tag{4.15}$$

if the generalised GIM mechanism of Eq. (4.9) has already been applied to the sum of fermions of charge $Q_{f_1} = Q_d + Q_v$. The modified loop function $F^d_{\bar{V}}$ is obtained from F^d_V by the simple replacement $Q_{v_1} \to -Q_{v_1}$. Finally, let us note that we could further simplify the function $F^d_{V'}$ using the sum rule Eq. (2.80) if tree-level neutral current and scalar interactions are absent.

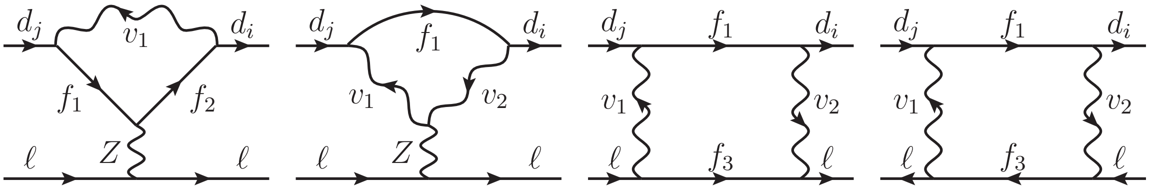


Figure 5: *Diagrams that directly match onto the $\Delta F = 1$ current-current operators. Here only contributions of internal massive vector bosons and fermions are shown. In addition, there is a finite contribution from the off-diagonal fermion self-energy diagram. The contribution of massive vector bosons and their related Goldstone bosons are denoted by a wavy line.*

In this limit we have

$$m_{f_0} g^R_{v_1 \bar{f}_0 d_j} g^L_{\bar{v}_2 \bar{d}_i f_0} = - \sum_{f_1 \neq f_0} m_{f_1} g^R_{v_1 \bar{f}_1 d_j} g^L_{\bar{v}_2 \bar{d}_i f_1} \,, \tag{4.16}$$

when we set $m_{d_j} = m_{d_i} = 0$. Our results agree with Ref. [80] if we apply our generalised GIM mechanism to their results. Here we note that it is only possible to project the off-shell Green's function after using the GIM mechanism. The coefficients D^{ij}_L can be recovered from D^{ij}_R by simply interchanging the chirality of all coupling constants, *i.e.*, by replacing $y^L_{\dots} \leftrightarrow y^R_{\dots}$ and $g^L_{\dots} \leftrightarrow g^R_{\dots}$ in Eq. (4.14).

4.1.4 Neutral-Current Wilson Coefficient

Both the photon penguin diagrams of Fig. 2 and the Z penguin and box diagrams of Fig. 5 contribute to the matching conditions for the current-current Wilson coefficients. The analytic expression of each of the three diagram classes depends on the gauge fixing parameters of the massive vector bosons in the loop. A renormalised result for the $Z-$penguin was derived in Ref. [20] in 't Hooft-Feynman gauge using sum-rules derived from Slavnov-Taylor identities. Here we will show how to apply these same sum rules to combine the amplitudes of all three diagram classes into a finite and gauge-parameter independent result for the Wilson coefficients. To this end, we split our final expression into three parts,

$$\tilde{C}^{ij\ell}_{L\sigma} = v^{ij\ell}_{L\sigma} + m^{ij\ell}_{L\sigma} + s^{ij\ell}_{L\sigma} \,, \tag{4.17}$$

as a sum of diagrams that in the loop contain only massive vectors and fermions, denoted by $v^{ij\ell}_{L\sigma}$, massive vectors, massive scalars and fermions, denoted by $m^{ij\ell}_{L\sigma}$, and massive scalars and fermions, denoted by $s^{ij\ell}_{L\sigma}$. The index L denotes the left chirality of the external quarks, while $\sigma = L, R$ stands for the chirality of the external field ℓ. Again, the expressions for $\tilde{C}^{ij\ell}_{R\sigma}$

can be recovered from $\tilde{C}^{ij\ell}_{L\sigma}$ by simply swapping the chirality of all coupling constants, *i.e.*, by replacing $y^L_{\dots} \leftrightarrow y^R_{\dots}$, $g^L_{\dots} \leftrightarrow g^R_{\dots}$ and $\sigma \leftrightarrow \bar{\sigma}$, where $\bar{L} = R$ and vice versa.

The contribution of massive vectors and fermions,

$$\begin{aligned} v^{ij\ell}_{L\sigma} = \sum_{v_1 v_2 f_1} \frac{g^L_{\bar{v}_2 \bar{d}_i f_1} g^L_{v_1 \bar{f}_1 d_j}}{M^2_{v_1}} \Bigg[& e^2 Q_\ell \delta_{v_1 v_2} F^{\gamma Z}_V (x^{f_0}_{v_1}, x^{f_1}_{v_1}) \\ & + \sum_{f_3} \Big(g^\sigma_{\bar{v}_1 \bar{\ell} f_3} g^\sigma_{v_2 \bar{f}_3 \ell} F^{\sigma, BZ}_V (x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_3}_{v_1}) \\ & \qquad + g^\sigma_{v_2 \bar{\ell} f_3} g^\sigma_{\bar{v}_1 \bar{f}_3 \ell} F^{\sigma, B'Z}_V (x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_3}_{v_1}) \Big) \Bigg] \\ + \sum_{Z v_1 v_2 f_1 f_2} \frac{g^\sigma_{Z \bar{\ell} \ell} g^L_{v_1 \bar{f}_1 d_j} g^L_{\bar{v}_2 \bar{d}_i f_2}}{M^2_Z} \Bigg\{ & \delta_{f_1 f_2} g_{Z \bar{v}_1 v_2} F^Z_{V''} (x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}) \\ & + \delta_{v_1 v_2} \Big[g^L_{Z \bar{f}_2 f_1} F^Z_V (x^{f_1}_{v_1}, x^{f_2}_{v_1}) + g^R_{Z \bar{f}_2 f_1} F^Z_{V'} (x^{f_1}_{v_1}, x^{f_2}_{v_1}) \Big] \Bigg\}, \end{aligned} \tag{4.18}$$

contains several gauge-independent loop functions. The functions $F^{\gamma Z}_V$ and $F^{\sigma, B^{(\prime)} Z}_V$ are the gauge-independent combinations of the photon penguin with the $Z-$penguin and the $Z-$penguin with the box-diagrams, correspondingly. While all of the above functions involve contributions from the lightest fermionic particle in the loop through our generalised GIM mechanism, only $F^{\gamma Z}_V$ will contain an infrared logarithm in the limit $x^{f_0}_{v_1} \to 0$. This logarithm is reproduced by a light-quark loop involving f_0 in the effective theory. In the standard model this corresponds to the leading logarithm associated with the mixing of the operator Q_2 into Q_9 of Ref. [77]. The loop function $F^{\gamma Z}_V$ reproduces this leading logarithm if the considered model of new physics has the same light-particle content as the standard model. It will then drop out in the difference of the standard model and the new-physics contribution and we can consider the resulting difference the leading new-physics contribution.

There are two gauge-independent combinations for the Z-penguin and box diagram that are distinguished by their fermion flow. Charge conservation implies that the left box diagram in Fig. 5 contributes if

$$Q_{f_3} = Q_\ell + Q_{d_j} - Q_{f_1},$$

while the right box diagram contributes if

$$Q_{f_3} = Q_\ell - Q_{d_j} + Q_{f_1}.$$

In the SM, $F^{\sigma, BZ}_V$ and $F^{\sigma, B'Z}_V$ will then contribute to $b \to s \mu^+ \mu^-$ and $s \to d \bar{\nu} \nu$, respectively.

The loop functions $F^Z_{V^{(\prime/\prime\prime)}}$ are the M_Z-independent parts of the functions evaluated in Ref. [20] and are only non-zero in physics beyond the standard model. In particular, we note that all contributions with diagonal Z couplings vanish since $F^Z_{V^{(\prime)}}(x,x) = F^Z_{V''}(x,y,1) = 0$.

Finally, we give the contributions involving internal scalars, vectors, and fermions,

$$\begin{aligned} m^{ij\ell}_{L\sigma} =& \sum_{s_1 v_1 f_1 f_3} \frac{1}{M^2_{v_1}} \left(g^L_{\bar{v}_1 \bar{d}_i f_1} y^L_{s_1 \bar{f}_1 d_j} + y^R_{\bar{s}_1 \bar{d}_i f_1} g^L_{v_1 \bar{f}_1 d_j} \right) \\ & \qquad \times \left(y^{\bar{\sigma}}_{\bar{s}_1 \bar{\ell} f_3} g^{\sigma}_{v_1 \bar{f}_3 \ell} + g^{\sigma}_{\bar{v}_1 \bar{\ell} f_3} y^{\sigma}_{s_1 \bar{f}_3 \ell} \right) F^B_{VS}(x^{f_1}_{s_1}, x^{s_1}_{v_1}, x^{f_3}_{s_1}) \\ & + \sum_{s_1 v_1 f_1 Z} \frac{g^{\sigma}_{Z\bar{\ell}\ell}}{M^2_Z} \Big[g^L_{\bar{v}_1 \bar{d}_i f_1} y^L_{s_1 \bar{f}_1 d_j} g_{Z v_1 \bar{s}_1} F^Z_{VS}(x^{f_1}_{s_1}, x^{f_1}_{v_1}) \\ & \qquad + y^R_{\bar{s}_1 \bar{d}_i f_1} g^L_{v_1 \bar{f}_1 d_j} g_{Z \bar{v}_1 s_1} F^Z_{VS'}(x^{f_1}_{s_1}, x^{f_1}_{v_1}) \Big] \,, \end{aligned} \tag{4.19}$$

and only scalars and fermions,

$$\begin{aligned} s^{ij\ell}_{L\sigma} =& \sum_{s_1 s_2 f_1} \frac{1}{M^2_{s_1}} y^L_{s_1 \bar{f}_1 d_j} y^R_{\bar{s}_2 \bar{d}_i f_1} \\ & \times \left\{ \delta_{s_1 s_2} e^2 Q_\ell F^{\gamma}_S(x^{f_1}_{s_1}) + \sum_{f_3} \left(y^{\bar{\sigma}}_{\bar{s}_1 \bar{\ell} f_3} y^{\sigma}_{s_2 \bar{f}_3 \ell} - y^{\bar{\sigma}}_{s_2 \bar{\ell} f_3} y^{\sigma}_{\bar{s}_1 \bar{f}_3 \ell} \right) F^B_S(x^{f_1}_{s_1}, x^{s_1}_{s_2}, x^{f_3}_{s_1}) \right\} \\ & + \sum_{s_1 s_2 f_1 Z} \frac{g^{\sigma}_{Z\bar{\ell}\ell}}{M^2_Z} y^L_{s_2 \bar{f}_1 d_j} y^R_{\bar{s}_1 \bar{d}_i f_1} \left(\delta_{s_1 s_2} g^L_{Z \bar{d}_j d_j} + g_{Z s_1 \bar{s}_2} \right) F^Z_S(x^{f_1}_{s_1}, x^{f_1}_{s_2}) \\ & + \sum_{f_1 f_2 s_1 Z} \frac{g^{\sigma}_{Z\bar{\ell}\ell}}{M^2_Z} y^L_{s_1 \bar{f}_1 d_j} y^R_{\bar{s}_1 \bar{d}_i f_2} \left(g^L_{Z \bar{f}_2 f_1} F^Z_{S'}(x^{f_1}_{s_1}, x^{f_2}_{s_1}) + g^R_{Z \bar{f}_2 f_1} F^Z_{S''}(x^{f_1}_{s_1}, x^{f_2}_{s_1}) \right) \,, \end{aligned} \tag{4.20}$$

where in both cases we have a single box function that covers both fermion flow directions, albeit with a sign difference.

Derivation of the pure vector part

In the following we will show how the combination of the results of Ref. [20] with our calculation of the photon penguin will lead to gauge independent results for the Wilson coefficients. Denoting the contribution of the photon penguin that involves a photon coupling to the internal fermion and vector boson by F_γ and $F_{\gamma'}$, respectively, we write[5]

$$
\begin{aligned}
v^{ij\ell}_{L\sigma} =& \sum_{Zf_1f_2v_1v_2} \frac{g^L_{\bar v_2 \bar d_i f_2} g^L_{v_1 \bar f_1 d_j} g^\sigma_{Z\bar\ell\ell}}{M_Z^2} \Big\{ \delta_{v_1v_2} \left[g^L_{Z\bar f_2 f_1} F^Z_V\left(x^{f_1}_{v_1}, x^{f_2}_{v_1}\right) + g^R_{Z\bar f_2 f_1} F^Z_{V'}\left(x^{f_1}_{v_1}, x^{f_2}_{v_1}\right)\right] \\
&\qquad + \delta_{f_1f_2} g_{Zv_2\bar v_1} \left[F^Z_{V''}\left(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}\right) + \frac{M_Z^2}{M_{v_1}^2} F^{(2)}_{V''}\left(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}, \xi\right)\right] \Big\} \\
&+ \sum_{f_1v_1} g^L_{\bar v_1 \bar d_i f_1} g^L_{v_1 \bar f_1 d_j} g_{\gamma\bar\ell\ell} \frac{1}{M_{v_1}^2} \left[g_{\gamma \bar f_1 f_1} F_\gamma\left(x^{f_0}_{v_1}, x^{f_1}_{v_1}\right) + g_{\gamma v_1 \bar v_1} F_{\gamma'}\left(x^{f_0}_{v_1}, x^{f_1}_{v_1}, \xi\right)\right] \\
&+ \sum_{f_1f_3v_1v_2} g^L_{\bar v_2 \bar d_i f_1} g^L_{v_1 \bar f_1 d_j} \frac{1}{M_{v_1}^2} \Big[g^\sigma_{\bar v_1 \bar\ell f_3} g^\sigma_{v_2 \bar f_3 \ell} F^{L\sigma}_B\left(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_3}_{v_1}, \xi\right) \\
&\qquad\qquad - g^\sigma_{v_2 \bar\ell f_3} g^\sigma_{\bar v_1 \bar f_3 \ell} F^{L\sigma}_{B'}\left(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_3}_{v_1}, \xi\right)\Big]
\end{aligned}
\tag{4.21}
$$

where all functions are independent of the masses M_Z arising from one light particle irreducible diagrams involving neutral massive vector-particle propagators. The functions $F^Z_{V^{(\prime,\prime\prime)}}$ have already been combined with the terms that originate from the off-diagonal field renormalisation, as described in Ref. [20]. This combination is essential to arrive at a gauge-independent result. In this context it is interesting to note that we can further use the sum rules to write F^Z_V in a simpler and more symmetric form. The combination $F^Z_{V''} + (M_Z^2/M_{v_1}^2)F^{(2)}_{V''}$ agrees with the $F_{V''}$ of Ref. [20] in the limit of 't Hooft-Feynman gauge; here the gauge-parameter dependent part has been split off into the loop function $F^{(2)}_{V''}$. The dependence on the mass of the lightest fermion f_0 originates from the application of the generalised GIM mechanism, Eq. (4.9), to our result. It implies that the functions $F^{(2)}_{V''}$ approach zero in the limit $m_{f_1} \to m_{f_0}$. The functions F_γ and $F_{\gamma'}$ have been calculated here for the first time, while the box functions $F^{L\sigma}_{B^{(\prime)}}$ and $F^{L\sigma}_{B^{(\prime)}}$ are related to the expressions of Ref. [20] in the limit $\xi_v = 1$ in the following manner:

$$
F^{LL}_B(\cdot) = -f_d(\cdot) - f_{\tilde d}(\cdot)\,, \qquad F^{LL}_{B'}(\cdot) = -f_d(\cdot) - 4 f_{\tilde d}(\cdot)\,, \tag{4.22}
$$

$$
F^{LR}_B(\cdot) = -f_d(\cdot) - 4 f_{\tilde d}(\cdot)\,, \qquad F^{LR}_{B'}(\cdot) = -f_d(\cdot) - f_{\tilde d}(\cdot)\,, \tag{4.23}
$$

[5] The additional function argument ξ indicates that the loop function is gauge dependent. In the actual calculation, we kept the full dependence on the gauge parameters ξ_v for each heavy vector boson, as defined in Eq. (2.49).

where

$$
\begin{aligned}
f_d(x_{v_1}^{f_0}, x_{v_1}^{f_1}, x_{v_2}^{v_1}, x_{v_1}^{f_3}, \xi) = & \frac{m_{f_1}^2 m_{f_3}^2}{M_{v_2}^2}\bigg\{\frac{1}{4}\tilde{D}_0\big(m_{f_1}, m_{f_3}, m_{v_1}, m_{v_2}\big) \\
& - (M_{v_1}^2 + M_{v_2}^2) D_0\big(m_{f_1}, m_{f_3}, m_{v_1}, m_{v_2}, \xi\big)\bigg\} - (m_{f_1} \to m_{f_0})\,, \\
f_{\tilde{d}}(x_{v_1}^{f_0}, x_{v_1}^{f_1}, x_{v_2}^{v_1}, x_{v_1}^{f_3}) = & M_{v_1}^2 \tilde{D}_0\big(m_{f_1}, m_{f_3}, m_{v_1}, m_{v_2}\big) - (m_{f_1} \to m_{f_0})\,,
\end{aligned}
\tag{4.24}
$$

where D_0 and $\tilde{D}_0$ functions are given in [20]. For an arbitrary gauge-fixing parameters ξ_v, only the f_d function contains ξ_v-dependent terms. To combine the penguin and box contributions of (4.21) we specify the sum rule (4.6) to the interaction of leptons with vector bosons,

$$
\sum_Z g^\sigma_{Z\bar{\ell}\ell} g_{Zv_2\bar{v}_1} = -\delta_{\bar{v}_1 v_2} g^\sigma_{\gamma\bar{\ell}\ell} g_{\gamma v_2 \bar{v}_1} - \sum_{f_3} \left(g^\sigma_{\bar{v}_1\bar{\ell}f_3} g^\sigma_{v_2\bar{f}_3\ell} - g^\sigma_{v_2\bar{\ell}f_3} g^\sigma_{\bar{v}_1\bar{f}_3\ell}\right), \tag{4.25}
$$

which allows us to identify

$$
F_V^{\gamma Z}(x_{v_1}^{f_1}) = g_{\gamma\bar{f}_1 f_1} F_\gamma\left(x_{v_1}^{f_1}, x_{v_1}^{f_2}\right) + g_{\gamma v_1 \bar{v}_1}\left[F_{\gamma'}\left(x_{v_1}^{f_0}, x_{v_1}^{f_1}, \xi\right) - F_{V''}^{(2)}\left(x_{v_1}^{f_0}, x_{v_1}^{f_1}, 1, \xi\right)\right] \tag{4.26}
$$

and

$$
F_V^{\sigma, B^{(\prime)} Z}(x_{v_1}^{f_0}, x_{v_1}^{f_1}, x_{v_2}^{v_1}, x_{v_1}^{f_3}) = F^{L\sigma}_{B^{(\prime)}}(x_{v_1}^{f_0}, x_{v_1}^{f_1}, x_{v_2}^{v_1}, x_{v_1}^{f_3}, \xi) - F^{(2)}_{V''}(x_{v_1}^{f_0}, x_{v_1}^{f_1}, x_{v_2}^{v_1}, x_{v_1}^{f_3}, \xi)\,. \tag{4.27}
$$

Using the explicit form of the loop functions, it can then be shown that the resulting expressions are independent of the gauge-fixing parameter.

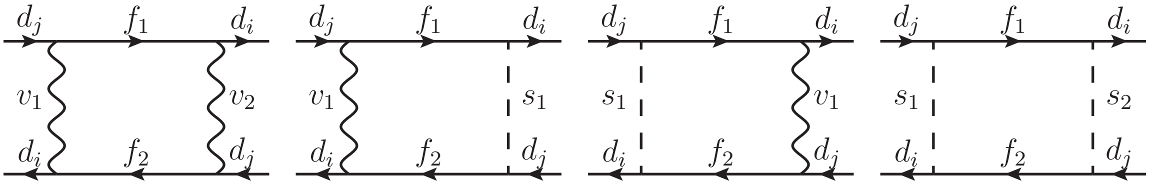


Figure 6: *Diagrams that correspond to $\Delta F = 2$ current-current operators.*

4.2 The effective $\Delta F = 2$ Lagrangian

The particle-antiparticle mixings of neutral mesons proceeds to an excellent approximation only through box diagrams as depicted in Fig. 6. These diagrams are finite by themselves and in this section we do not use any sum rules. In general the effective Lagrangian for the $K - \bar{K}$ and $B_d^0 - \bar{B}_d^0$, $B_s^0 - \bar{B}_s^0$ mixings in the weak effective field theory can be given by

$$\begin{aligned}\delta\mathcal{L}_{\Delta F=2} = \frac{1}{16\pi^2} \sum_{\sigma\in\{L,R\}} C^{ij}_{\sigma\sigma} \left(\bar{d}_j \gamma^\mu P_\sigma d_i\right)\left(\bar{d}_j \gamma_\mu P_\sigma d_i\right) \\ + \frac{1}{16\pi^2} \sum_{\substack{\sigma\neq\sigma' \\ \sigma,\sigma'\in\{L,R\}}} C^{ij}_{\sigma\sigma'} \left(\bar{d}_j \gamma^\mu P_\sigma d_i\right)\left(\bar{d}_j \gamma_\mu P_{\sigma'} d_i\right) \ ,\end{aligned} \tag{4.28}$$

where the operator on the first line is for $P_L \times P_L$ or $P_R \times P_R$ structures, while the second operator is for the mixed chiralities.

Following Ref. [20] we can write a generalised form of the Wilson coefficient for a generic renormalisable model as

$$C^{ij}_{\sigma\sigma'} = v^{ij}_{\sigma\sigma'} + m^{ij}_{\sigma\sigma'} + s^{ij}_{\sigma\sigma'}, \tag{4.29}$$

where $v^{ij}_{\sigma\sigma'}$ and $s^{ij}_{\sigma\sigma'}$ denote contributions coming from vectors and scalars, respectively. $m^{ij}_{\sigma\sigma'}$ contains mixed contributions from both vectors and scalars. Each of these contributions can be presented by products of gauge invariant loop functions with coupling constants. Particularly, the part with vectors and fermions in the loop is

$$\begin{aligned}v^{ij}_{\sigma\sigma'} = \sum_{v_1 v_2 f_1 f_2} \frac{1}{M^2_{v_1}} \Big\{ F^B_{V'}(x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_2}_{v_1}) g^{\sigma}_{\bar{v}_2 \bar{d}_i f_1} g^{\sigma}_{v_1 \bar{f}_1 d_j} \left(g^{\sigma'}_{\bar{v}_1 \bar{d}_i f_2} g^{\sigma'}_{v_2 \bar{f}_2 d_j} - g^{\sigma'}_{\bar{v}_1 \bar{f}_2 d_j} g^{\sigma'}_{v_2 \bar{d}_i f_2} \right) \\ + F^B_{V''}(x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_2}_{v_1}) g^{\sigma}_{\bar{v}_2 \bar{d}_i f_1} g^{\sigma}_{v_1 \bar{f}_1 d_j} \left(g^{\sigma'}_{\bar{v}_1 \bar{d}_i f_2} g^{\sigma'}_{v_2 \bar{f}_2 d_j} - 4 g^{\sigma'}_{\bar{v}_1 \bar{f}_2 d_j} g^{\sigma'}_{v_2 \bar{d}_i f_2} \right) \Big\}.\end{aligned} \tag{4.30}$$

For the case when $\sigma = \sigma'$, the function $F^B_{V''} = -F^B_{V''}$. The mixed contributions of scalars and vectors with fermions are

$$m^{ij}_{\sigma\sigma'} = \sum_{v_1 s_1 f_1 f_2} \frac{1}{M^2_{v_1}} F^B_{VS'}(x^{f_1}_{s_1}, x^{s_1}_{v_1}, x^{f_2}_{s_1}) \times \left(g^{\sigma}_{\bar{v}_1 \bar{d}_i f_1} y^{\sigma}_{s_1 \bar{f}_1 d_j} + g^{\sigma}_{\bar{v}_1 \bar{f}_1 d_j} y^{\bar{\sigma}}_{s_1 \bar{d}_i f_1} \right) \left(y^{\bar{\sigma}'}_{\bar{s}_1 \bar{d}_i f_2} g^{\sigma'}_{v_1 \bar{f}_2 d_j} + y^{\sigma'}_{\bar{s}_1 \bar{f}_2 d_j} g^{\sigma'}_{v_1 \bar{d}_i f_2} \right). \tag{4.31}$$

The scalar part is

$$s^{ij}_{\sigma\sigma'} = \sum_{s_1 s_2 f_1 f_2} \frac{1}{M^2_{s_1}} F^B_{S'}(x^{f_1}_{s_1}, x^{s_1}_{s_2}, x^{f_2}_{s_1}) y^{\bar{\sigma}}_{\bar{s}_2 \bar{d}_i f_1} y^{\sigma}_{s_1 \bar{f}_1 d_j} \left(y^{\bar{\sigma}'}_{\bar{s}_1 \bar{d}_i f_2} y^{\sigma'}_{s_2 \bar{f}_2 d_j} - y^{\sigma'}_{\bar{s}_1 \bar{f}_2 d_j} y^{\bar{\sigma}'}_{s_2 \bar{d}_i f_1} \right). \tag{4.32}$$

The loop functions for $C^{ij}_{\sigma\sigma}$ can be obtained by changing $\sigma' \to \sigma$ in $C^{ij}_{\sigma\sigma'}$ and defining

$$f^{ij}_{\sigma\sigma} = \frac{1}{2} f^{ij}_{\sigma\sigma'} \Big|_{\sigma' \to \sigma}, \qquad \text{where} \qquad f \in \{v, m, s\}. \tag{4.33}$$

Here, the loop functions are given without performing the GIM mechanism. The explicit expressions for them can be found in App. B.2.

For example, in the SM only the $P_L \times P_L$ structure is present and the Wilson coefficient C^{23}_{LL} for the $B_s - \bar{B}_s$ mixing after the GIM subtraction can be expressed as

$$C^{23}_{LL} = \frac{g^4}{8M^2_W} V^2_{tb} V^{*2}_{ts} \times \Big\{ F^B_{V'}\big(x^{m_t}_{M_W}, 1, x^{m_t}_{M_W}\big) - F^B_{V''}\big(x^{m_t}_{M_W}, 1, x^{m_t}_{M_W}\big) + F^B_{V'}(0,1,0) - F^B_{V''}(0,1,0) - F^B_{V'}\big(0, 1, x^{m_t}_{M_W}\big) + F^B_{V''}\big(0, 1, x^{m_t}_{M_W}\big) - F^B_{V'}\big(x^{m_t}_{M_W}, 1, 0\big) + F^B_{V''}\big(x^{m_t}_{M_W}, 1, 0\big) \Big\}. \tag{4.34}$$

In this derivation the masses of up and charm quarks are assumed to be zero and only the top quark contribution is considered. This results agrees with the definition of the $S_0(x_t)$ function given in [64].

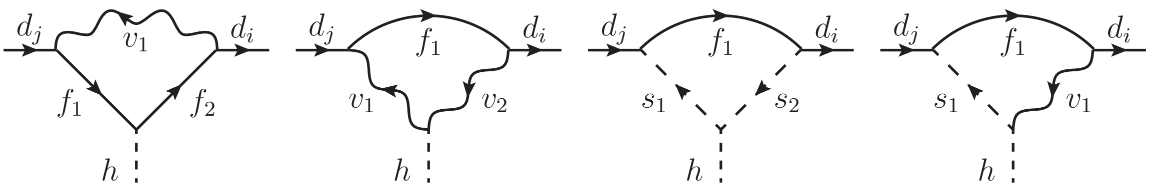


Figure 7: *Diagrams that contribute to the flavour changing neutral scalar current processes. In addition, there is a diagram similar to the last one where internal vectors* $\leftrightarrow$ *scalars.*

4.3 Gauge invariance of scalar-penguin diagrams

In this section we will discuss the gauge invariance of the flavour changing processes mediated by neutral scalars such as the SM Higgs. The effective Lagrangian can be written as

$$\mathcal{L}_{\Delta F=1} = \frac{1}{16\pi^2} \sum_{\sigma,\sigma' \in \{L,R\}} C^{s,ij}_{\sigma\sigma'} \left(\bar{d}_i P_\sigma d_j\right) \left(\bar{\ell}_k P_{\sigma'} \ell_l\right), \tag{4.35}$$

where the index s in the Wilson coefficient stands for 'scalar'. Here, we do not give explicit expressions for the $C^{s,ij}_{\sigma\sigma'}$, because the renormalisation of the $h-$penguin diagrams has not been completed yet. The relevant diagrams are depicted in Fig. 7. Feynman amplitudes that contain the gauge fixing parameter ξ when evaluated in an arbitrary R_ξ gauge come from the loops with internal vector-fermion and mixed vector-scalar-fermion diagrams.

First, we would like to discuss the GIM-like mechanism that is needed for the cancellation of mass independent terms in the amplitude. Considering that there are no tree level flavour changing neutral scalar vertices, *i.e.*, $y^\sigma_{h\bar{d}_i d_j} = 0$, we can derive generalised unitarity relation by solving Eq. (2.80) with the following choice of particles: $s_1 \to h$ and $f_{1,2}$ being the external fermions $d_{i,j}$, respectively. The sum over v_3 in the first line of the expression vanishes since by the conservation of the electric charge v_3 can only be a neutral vector in the coupling $g^{\sigma/\bar{\sigma}}_{v_3 \bar{d}_i d_j}$. But in our formalism there is no tree level FCNC. The remaining part with non-zero masses of external fermions m_{d_i/d_j} reads

$$\begin{aligned} 0 = \sum_{f_1} \Bigg[&- m_{d_i} \left(g^\sigma_{v_2 \bar{d}_i f_1} g^\sigma_{v_1 \bar{f}_1 d_j} + g^\sigma_{v_1 \bar{d}_i f_1} g^\sigma_{v_2 \bar{f}_1 d_j} \right) \\ &- m_{d_j} \left(g^{\bar{\sigma}}_{v_2 \bar{d}_i f_1} g^{\bar{\sigma}}_{v_1 \bar{f}_1 d_j} + g^{\bar{\sigma}}_{v_1 \bar{d}_i f_1} g^{\bar{\sigma}}_{v_1 \bar{f}_2 d_j} \right) \\ &+ 2 m_{f_1} \left(g^{\bar{\sigma}}_{v_2 \bar{d}_i f_1} g^\sigma_{v_1 \bar{f}_1 d_j} + g^{\bar{\sigma}}_{v_1 \bar{d}_i f_1} g^\sigma_{v_2 \bar{f}_1 d_j} \right) \Bigg], \end{aligned} \tag{4.36}$$

where the sum over internal fermions is applied. The first two lines in this equation correspond to the usual unitarity relation derived from the unitarity of the quark-mixing matrix in the SM for $\sigma/\bar{\sigma} = L$. The last line comes with a mass insertion of the loop fermion and it can be used in theories where both L and R chiralities of the fermions are allowed. Furthermore, with the same choice of fields and following the same steps as above, it is possible to obtain the following expression from Eq. (2.81),

$$\sum_{f_1} \left(g^{\bar{\sigma}}_{v_1 \bar{d}_i f_1} y^{\sigma}_{s_1 \bar{f}_1 d_j} - y^{\sigma}_{s_1 \bar{d}_i f_1} g^{\sigma}_{v_1 \bar{f}_1 d_j} \right) = 0, \tag{4.37}$$

which could be used to remove mass independent terms from the amplitudes when both vectors and scalars with fermions run in the loop.

In general, the gauge dependency of $h-$penguin diagrams can be decomposed into two parts,

$$F(\xi, x^i_j, \dots) = g(\xi, x^i_j, \dots) + M_h^2 \times f(\xi, x^i_j \dots), \tag{4.38}$$

where x^i_j denotes mass ratios of loop particles and dots for other mass parameters, and M_h is the mass of the external scalar boson. Only diagrams that include triple boson vertices contribute to the $f(\xi, x^i_j \dots)$ function, while the $g(\xi, x^i_j, \dots)$ function gets contributions from all the $h-$penguin diagrams, including the self-energy corrections.

In order to achieve cancellation of $g(\xi, x^i_j, \dots)$, we can utilise Eq. (2.81) for the case $s_2 \to h$ and $f_2 \to d_j$,

$$\begin{aligned} \sum_{s_1} g_{h v_1 \bar{s}_1} y^{\sigma}_{s_1 \bar{f}_1 \bar{d}_j} = &- \sum_{v_2} \frac{1}{2M_{v_2}^2} g_{h v_1 \bar{v}_2} \left(m_{f_1} g^{\sigma}_{v_2 \bar{f}_1 d_j} - m_{d_j} g^{\bar{\sigma}}_{v_2 \bar{f}_1 d_j} \right) \\ &+ g^{\bar{\sigma}}_{v_1 \bar{f}_1 d_j} y^{\sigma}_{h d_j d_j} - y^{\sigma}_{h \bar{f}_1 f_1} g^{\sigma}_{v_1 \bar{f}_1 d_j}, \end{aligned} \tag{4.39}$$

where the sums over internal scalars s_1 and vectors v_2 are taken. Or, analogously $s_2 \to h$, $f_1 \to d_i$ which involves $y^{\sigma}_{h d_i d_i}$ coupling constants.

The deletion of the $f(\xi, x^i_j \dots)$ function occurs only after including the box diagrams such as the last diagram in Fig. 5. Firstly, the function $f(\xi, x^i_j \dots)$ should be multiplied by the propagator of the $h-$boson, $1/M_h^2$, and by the $h-$coupling with the external lepton lines, $y^{\sigma}_{h \bar{\ell} \ell}$. For the case when two vectors run in the loop Eq. (2.80) is utilised with $s_1 \to h$, $f_{1/2} \to \ell$ and $m_\ell = 0$,

$$\sum_{h} g_{h v_1 \bar{v}_2} y^{\sigma}_{h \bar{\ell} \ell} = \sum_{f_2} 2 m_{f_2} g^{\bar{\sigma}}_{\bar{v}_2 \bar{\ell} f_2} g^{\sigma}_{v_1 \bar{f}_2 \ell}. \tag{4.40}$$

The sum over h could be appropriate if a theory consists of more than one neutral scalar. On the other hand, for diagrams with mixed bosons such as a vector-scalar-fermion mixture, the expression extracted from Eq. (2.81) is applied,

$$\sum_h g_{hv_1\bar{s}_1} y^{\sigma}_{h\bar{\ell}\ell} = \sum_{f_2} \left(g^{\bar{\sigma}}_{v_1\bar{\ell}f_2} y^{\sigma}_{\bar{s}_1\bar{f}_2\ell} - y^{\sigma}_{s_1\bar{\ell}f_2} g^{\sigma}_{\bar{v}_1\bar{f}_2\ell} \right) . \tag{4.41}$$

To conclude, by using the above combination of coupling constants it is possible to achieve full gauge invariance of the amplitudes for the flavour changing transitions mediated by neutral scalars.

4.4 Applications to Beyond the Standard Model Phenomenology

To exemplify our formalism we will apply it to models of new physics that address the current rare B-decay anomalies. In this context, it is standard to write vector and axial-vector current operators; the Wilson coefficients of this effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \frac{1}{16\pi^2}\left\{C_9^\ell(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) + C_{10}^\ell(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\gamma_5 \ell)\right\}\,, \tag{4.42}$$

are related to our coefficients of Eq. (4.17) via the linear transformation

$$C_{9/10}^\ell = \frac{1}{2}\left(\tilde{C}_{LR}^{23\ell} \pm \tilde{C}_{LL}^{23\ell}\right)\,. \tag{4.43}$$

The relation for the operators involving right-handed quarks can be inferred from the above relation by replacing $C_{L\sigma}^{23\ell} \to C_{R\sigma}^{23\ell}$.

If we are interested in deviations from the standard model background, we have to subtract the standard model one-loop contribution from our complete new-physics calculation; hence, we define

$$C_{9/10}^{\ell\,\text{NP}} = C_{9/10}^\ell - C_{9/10}^{\ell\,\text{SM}}\,. \tag{4.44}$$

The standard model contribution follows directly from the vector contribution of Eq. (4.18) and reads

$$\begin{aligned} C_9^{\ell\,\text{SM}} &= \frac{e^2 G_F V_{ts}^* V_{tb}}{\sqrt{2}}\left\{\frac{1}{s_W^2}F_V^{L,BZ}(0,x_W^t,1,0) - 4F_V^{\gamma Z}(0,x_W^t)\right\}\,,\\ C_{10}^{\ell\,\text{SM}} &= -\frac{e^2 G_F V_{ts}^* V_{tb}}{\sqrt{2}s_W^2}F_V^{L,BZ}(0,x_W^t,1,0)\,, \end{aligned} \tag{4.45}$$

where we have used the fact that $F_{V^{(\prime)}}^Z(x,x) = 0 = F_{V^{\prime\prime}}^Z(x,y,1)$.

4.4.1 A Z'-Model with flavour off-diagonal couplings

To demonstrate the utility of the expressions derived in Sec. 4.1 we begin by applying them to a simple model [85] developed to address the $b \to s\ell\ell$ lepton flavour non-universality anomaly.

In this model the standard model gauge group is enlarged as $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)'$, where a new $\mathrm{U}(1)'$ gauge symmetry is spontaneously broken by the vacuum expectation value (VEV) of a scalar field Φ, transforming as $\Phi \sim (1, 1, 0, q')$ under the above gauge symmetry. The model consists of a vector-like, colored Dirac fermion $T' \sim (3, 1, 2/3, q')$, whose charge under the $\mathrm{U}(1)'$ gauge group is denoted by q'.

The part of the Lagrangian relevant to $\mathrm{U}(1)'$ can be written as

$$\mathcal{L}_{\mathrm{U}(1)'} = |(D_\mu \Phi)|^2 - \frac{m_\phi^2}{2\tilde{v}^2}\left(\Phi^2 - \frac{\tilde{v}^2}{2}\right)^2 + \bar{T}'\left(i\not{D} - M_T\right)T' - \frac{1}{4}F'^2_{\mu\nu}, \tag{4.46}$$

where the covariant derivative is $D_\mu = D_\mu^{\mathrm{SM}} + i\tilde{g}q'Z'_\mu$ and $\tilde{g}$ is the gauge coupling of the $\mathrm{U}(1)'$. The field strength for the $\mathrm{U}(1)'$ gauge boson Z' is $F'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$. The scalar field is $\Phi = (\phi + \tilde{v})/\sqrt{2}$, where ϕ corresponds to the physical scalar boson that acquires mass m_ϕ after breaking of $\mathrm{U}(1)'$ by the VEV $\tilde{v}$. The mass of the Z' boson can be derived from the kinetic term for Φ, *i.e.*, $m_{Z'} = \tilde{g}q'\tilde{v}$.

All the SM fields are singlets under $\mathrm{U}(1)'$. There are only three renormalisable interactions between the SM and the $\mathrm{U}(1)'$ sector: the SM Higgs H coupling with Φ; the kinetic mixing between $\mathrm{U}(1)'$ and SM hypercharge, $B_{\mu\nu}$; the Yukawa-type coupling of T' and Φ with the SM right-handed up quarks u_R^i,

$$\mathcal{L}_{\mathrm{mix}} = -\lambda' |\Phi|^2 |H|^2 - \epsilon B^{\mu\nu} F'_{\mu\nu} - \left(y_T^i \bar{T}' \Phi u_R^i + \mathrm{h.c.}\right), \tag{4.47}$$

where $i = 1, 2, 3$ is a generation index. Considering the fact that the masses of up and charm quarks are significantly small in comparison with the top mass, the Yukawa couplings of T' with the up and charm quarks can be neglected. Moreover, in order to keep the model simple $|y_T^t| \gg \lambda', \epsilon$ is assumed.

After the electroweak symmetry breaking, by diagonalising the $t - T'$ part of the mass matrix the basis is changed from the interaction eigenstates t, T' to the mass eigenstates t, T, respectively.

The relevant couplings of the mass eigenstates to the gauge bosons are given by

$$\begin{aligned}\mathcal{L}_{\text{int}} \supset &- \frac{e}{\sqrt{2}s_W} V_{ti}\left[\left(c_L\bar{t}+s_L\bar{T}\right) \not{W}^+ P_L d_i\right] + \text{h.c.}\\ &- \frac{e}{2c_W s_W}\left[\left(c_L\bar{t}+s_L\bar{T}\right)\not{Z} P_L\left(c_L t + s_L T\right) - \frac{4}{3}s_W^2\left(\bar{t}\not{Z}t+\bar{T}\not{Z}T\right)\right]\\ &- \tilde{g}q'\left[\left(s_L\bar{t}-c_L\bar{T}\right)\not{Z}' P_L\left(s_L t - c_L T\right) + \left(s_R\bar{t}-c_R\bar{T}\right)\not{Z}' P_R\left(s_R t - c_R T\right)\right]\\ &- \tilde{g}\bar{\mu}\not{Z}'\left(q'_{\mu,V}+q'_{\mu,A}\gamma_5\right)\mu\,,\end{aligned} \tag{4.48}$$

where $s_{L/R}$ and $c_{L/R}$ are the sine and cosine of the left-/right-handed $t-T$ mixing angles. The last line in $\mathcal{L}_{\text{int}}$ describes a coupling of Z' with the lepton sector, specifically vectorial/axial couplings $q'_{\mu,V/A}$ with muon. With these couplings, Eq. (4.18) directly gives the contribution to the Wilson coefficients which are

$$\begin{aligned}C_9^{\mu\,\text{NP}} =\; & s_L^2\left(C_9^{\mu\,\text{SM}}(x_W^t\to x_W^T) - C_9^{\mu\,\text{SM}}\right) - \frac{e^2 G_F V_{ts}^* V_{tb}}{\sqrt{2}} s_L^2 c_L^2 \left\{\frac{1-4s_W^2}{s_W^2}F_V^Z(x_W^t,x_W^T)\right.\\ &\left. + 2\,\tilde{g}^2 q' q'_{\mu,V}\frac{M_W^2}{M_{Z'}^2}\left(2F_V^Z(x_W^t,x_W^T) + \frac{s_R c_R}{s_L c_L}\left[F_{V'}^Z(x_W^t,x_W^T)+F_{V'}^Z(x_W^T,x_W^t)\right]\right)\right\}\end{aligned} \tag{4.49}$$

and

$$\begin{aligned}C_{10}^{\mu\,\text{NP}} =\; & s_L^2\left(C_{10}^{\mu\,\text{SM}}(x_W^t\to x_W^T) - C_{10}^{\mu\,\text{SM}}\right) + \frac{e^2 G_F V_{ts}^* V_{tb}}{\sqrt{2}} s_L^2 c_L^2 \left\{\frac{1}{s_W^2}F_V^Z(x_W^t,x_W^T)\right.\\ &\left. - 2\,\tilde{g}^2 q' q'_{\mu,A}\frac{M_W^2}{M_{Z'}^2}\left(2F_V^Z(x_W^t,x_W^T) + \frac{s_R c_R}{s_L c_L}\left[F_{V'}^Z(x_W^t,x_W^T)+F_{V'}^Z(x_W^T,x_W^t)\right]\right)\right\}\,,\end{aligned} \tag{4.50}$$

where we have subtracted the SM contribution. To evade collider constraints, one furthermore assumes that $m_T \gg m_t$. In this limit we find:

$$C_{9/10}^{\mu,\text{NP}} = \frac{s_R^2}{2}q' q'_{\mu,V/A}\frac{m_t^2}{M_{Z'}^2}\frac{\tilde{g}^2}{e^2}\left\{\frac{1}{2}\log\left(x_W^T\right) + \frac{1}{c_R^2} + \frac{3}{2(x_t-1)} - 1 - \frac{1}{2}\left(\frac{3}{(x_t-1)^2}+1\right)\log(x_W^t)\right\} \tag{4.51}$$

where the $\log(x_W^T)$ agrees with the result in Ref. [85], while the remaining terms are new and reduce the contribution to both C_9 and C_{10} by 13(7)% for $m_T = 1(10)$ TeV.

4.4.2 A $\mathrm{U}(1)_{L_\mu-L_\tau}$ model with Majorana fermions

The gauged $\mathrm{U}(1)_{L_\mu-L_\tau}$ model was originally proposed in Refs. [86, 87] and has been studied extensively in the context of lepton universality violation. In the lepton sector, the second (third) generation left-handed doublet and right-handed singlet, ℓ_L^μ, μ_R (ℓ_L^τ, τ_R) are charged under the local $\mathrm{U}(1)_{L_\mu-L_\tau}$ symmetry with charge 1 (-1). Here we focus on the model of Ref. [88] where an additional Dirac fermion N and a coloured $\mathrm{SU}(2)_L$-doublet scalar $\tilde{q} \equiv (\tilde{u}, \tilde{d})^T$, a singlet-scalar S are introduced. The transformation properties of the new fields under the (SM gauge group)$\times\mathrm{U}(1)_{L_\mu-L_\tau}$ are $N \sim (1,1,0,Q)$ and $\tilde{q} \sim (3,2,1/6,-Q)$, $S \sim (1,1,0,2Q)$, where Q is the charge of $\mathrm{U}(1)_{L_\mu-L_\tau}$.

The Lagrangian in the interaction basis can be expressed as

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \bar{N}\left(i\not{D} - M_N\right)N + \left(D_\mu\tilde{q}^\dagger\right)\left(D^\mu\tilde{q}\right) - m_{\tilde{q}}^2\tilde{q}^\dagger\tilde{q} + \left(D_\mu S^\dagger\right)\left(D^\mu S\right) \\ - \sum_{i=s,b}\left(y_L^i\bar{q}_L^i\tilde{q}N + \text{h.c.}\right) - \left(\frac{f}{2}\overline{N^c}NS^\dagger + \text{h.c.}\right) - V\left(H,S,\tilde{q}\right),\end{aligned} \tag{4.52}$$

where $F'_{\mu\nu} \equiv \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$ is the field strength tensor for the $\mathrm{U}(1)_{L_\mu-L_\tau}$ gauge boson Z'. The charge conjugate state of N is denoted by N^c. The down-type quarks in the Lagrangian are assumed to be in the mass eigenstates. The scalar potential $V\left(H,S,\tilde{q}\right)$ contains the SM Higgs doublet H and new scalars of the model, $\tilde{q}$ and S, and mixing terms between these scalar fields.

When the scalar S gets a non-zero VEV (v_s), it mixes N and N^c states which are not mass eigenstates. By diagonalising the mass matrix of N and N^c, the mass eigenstates are defined as admixture of N and N^c, *i.e.*, $N_\pm = \left(N \pm N^c\right)/\sqrt{2}$. The $N_\mp$ are two Majorana fermions with masses $m_\mp = M_N \mp f v_s/\sqrt{2}$. The N_- state has Majorana phase π so that $N_-^c = -N_-$, but $N_+^c = N_+$.

After spontaneous symmetry breaking the relevant interactions in terms of the mass eigenstates read

$$\begin{aligned}\mathcal{L}_{\text{int}} \supset -\frac{g_X Q}{2}\left(\overline{N}_- + \overline{N}_+\right)\not{Z}'\left(N_- + N_+\right) - \frac{1}{\sqrt{2}}\left[\left(y_L^b\bar{b}_L + y_L^s\bar{s}_L\right)\tilde{d}\left(N_- + N_+\right) + \text{h.c.}\right] \\ - i\left(g_X Q Z'_\mu + g\frac{3-2s_W^2}{6c_W}Z_\mu\right)\left[\tilde{d}\partial_\mu\tilde{d}^c - \left(\partial^\mu\tilde{d}\right)\tilde{d}^c\right] - g_X\bar{\mu}\not{Z}\mu\,,\end{aligned} \tag{4.53}$$

where g_X is the $\mathrm{U}(1)_{L_\mu-L_\tau}$ gauge coupling constant, and $y_L^{s/b}$ are the Yukawa couplings of the SM bottom and strange quarks to $\tilde{d}$.

The Z'−penguin does not involve any SM particles and is lepton universality violating by construction. The complete one-loop new physics contributions to C_9^μ can be read off from Eq. (4.20). Noting that the charge conjugated scalar $\tilde{d}^c$ contributes in the sum of (4.20), we find

$$\begin{aligned} C_9^{\mu\,\mathrm{NP}} = &\frac{g_X^2 Q y_L^b y_L^s}{4M_{Z'}^2}\left[\sum_{f_1,f_2=N_\pm}\left\{F_{S'}^Z(x_{\tilde{d}}^{f_1},x_{\tilde{d}}^{f_2})+F_{S''}^Z(x_{\tilde{d}}^{f_1},x_{\tilde{d}}^{f_2})\right\}-2\sum_{f_1=N_\pm}F_S^Z(x_{\tilde{d}}^{f_1})\right] \\ &-\frac{e^2 y_L^b y_L^s}{m_{\tilde{d}}^2}\sum_{f_1=N_\pm}F_S^\gamma(x_{\tilde{d}}^{f_1})\,, \end{aligned} \tag{4.54}$$

where the first line represents the Z'−penguin contribution and agrees with the results of Ref. [88]. The terms in the second line represent the lepton flavour universal new physics contribution to C_9 from the photon penguin which is new. Note that the photon penguin decouples faster than the Z'−penguin in the limit of large scalar mass $m_{\tilde{d}}$. The Z coupling to the down quarks cancels with the Z couplings to $\tilde{d}^c$ in (4.20) so that the Z-penguin contribution cancels. The contribution to $C_{7,bs}^{\mathrm{NP}}$ can be calculated from the general formula (4.14) and is given by

$$C_{7,bs}^{\mathrm{NP}} = \frac{1}{m_b}\frac{y_L^b y_L^s}{2m_{\tilde{d}}^2}\sum_{f_1=N_\pm}F_{S'}^Z(x_{\tilde{d}}^{f_1})\,, \tag{4.55}$$

where the operator O_7^{bs} is defined in Eq. (3.9). Note that only one of the terms is present since N is electrically neutral and therefore only the charged scalar, $\tilde{d}^c$, contributes.

Moreover, this model can be studied to explain the dark matter physics. After breaking the $\mathrm{U}(1)_{L_\mu - L_\tau}$ symmetry via the vacuum expectation value of the singlet-scalar field S, v_s, the Lagrangian in Eq. (4.52) still has a discrete $\mathbb{Z}_2$ symmetry due to the $\left(\frac{f}{2}\overline{N^c}NS^\dagger + \text{h.c.}\right)$ terms. The $\mathbb{Z}_2$ symmetry stabilises the lightest neutral $\mathbb{Z}_2$ odd particle, which could be N_- in this case. The stable N_- fermion can be viewed as a candidate for dark matter and its interaction with the SM particles can occur through the Higgs portal or Z' boson exchange.

	Q_1	q_1	Q_2	q_2	Q_3	q_3	L_1	l_1	L_2	l_2	L_3	l_3	$\Psi_{Q,(L,R)}$	$\Psi_{\ell,(L,R)}$	Φ	H	χ
Q_F	3	-1	2	0	0	0	-4	1	2	1	-2	-2	2	4	-2	0	1

Table 2: *$U(1)_F$ family symmetry charges for the quarks and leptons, where Q_i/L_i denotes left-handed quark/lepton isospin doublets, q_i/l_i stands for right-handed quark/lepton isospin singlets and the family index $i = 1, 2, 3$. H indicates the SM Higgs doublet.*

4.4.3 A model with vector-like fermions and neutral scalars

To give another application of our results, we consider a model which is aimed to explain anomalies in both neutral and charged current processes by introducing a *family symmetry* [89]. The idea behind the family symmetry is that the origin of fermion masses and mixings is through a spontaneously broken family symmetry in which the order parameters for family symmetry breaking are the vacuum expectation values (vevs) of scalar (familon) fields, ϕ_i, that transform non-trivially under the family symmetry. Since in the SM masses and mixings of fermions are not universal, the violation of the lepton universality could be related to the observed fermion mass pattern.

The model introduces an abelian family symmetry, $\mathrm{U}(1)_F$ under which the three families of known fermions are charged as demonstrated in Table. 2. New particles of the model are $\mathrm{SU}(2)_L$ doublet vector-like quarks Ψ_Q and leptons Ψ_ℓ, with the SM quantum numbers $\Psi_{Q,(L,R)} \sim (3, 2, 1/6)$ and $\Psi_{\ell,(L,R)} \sim (1, 2, -1/2)$, respectively. Also there are additional two complex scalars: $\Phi \sim (1, 1, 0)$ and SM singlet scalar χ. The latter one gets a vev and is responsible for generating the hierarchical structure of the quark and charged lepton masses and mixings.

The $\mathrm{U}(1)_F$ symmetry dictates the masses and mixing parameters of the quarks and charged leptons when the symmetry is spontaneously broken. Particularly, fermion masses are generated via the terms consistent with the charge assignments shown in the Table. 2. The effective Lagrangian for the second and third family quark/lepton masses and mixings is

$$\begin{aligned}\mathcal{L}^m_{eff} \propto\ & \bar{Q}_{3,L}q_{3,R}H + \bar{Q}_{3,L}q_{2,R}H + \bar{Q}_{2,L}q_{2,R}H\left(\frac{\chi}{M_q}\right)^2 + \bar{Q}_{2,L}q_{3,R}H\left(\frac{\chi}{M_q}\right)^2 \\ & + \bar{L}_{3,L}l_{3,R}H + \bar{L}_{2,L}l_{2,R}H\frac{\chi}{M_q} + \bar{L}_{2,L}l_{3,R}H\left(\frac{\chi}{M_q}\right)^4 + \bar{L}_{3,L}l_{2,R}H\left(\frac{\chi^\dagger}{M_q}\right)^3 + \text{h.c.},\end{aligned} \tag{4.56}$$

where $O(1)$ coupling constants are suppressed, and M_q are mediator masses associated with additional vector-like states associated with the Froggatt-Nielsen mechanism [90]. Similarly, the masses of light quarks could be generated in accordance with the same $\mathrm{U}(1)_F$ family symmetry.

Moreover, the model generates the flavour charge in the scalar sector in addition to the quark sector where the flavour charge is originated by the quark mixing. In order to demonstrate this, another SM singlet scalar $\tilde{\Phi}$ is added to the model, thus the relevant interaction Lagrangian can be given as

$$\mathcal{L}_{int} = \Gamma_b \bar{Q}_3 P_R \Phi_Q \Phi + \tilde{\Gamma}_s \bar{Q}_2 P_R \Phi_Q \tilde{\Phi} + \Gamma_\mu \bar{L}_2 P_R \Phi_\ell \Phi + \text{h.c.}, \tag{4.57}$$

where the indices of the interaction strength constants $\Gamma_f/\tilde{\Gamma}_f$ indicate the types of fermions. Adding the scalar $\tilde{\Phi}$ has no impact on fermion masses and mixings in the model.

The scalar mass eigenstates used in our calculations come from the mixing of $\tilde{\Phi}$ with Φ, *i.e.*, $\Phi_{L,H} = \left(\Phi \pm \tilde{\Phi}\right)/\sqrt{2}$, here H/L stands for 'heavy/light'. The mixing occurs via the term $\Phi\tilde{\Phi}\chi^2$ in the scalar potential when χ acquires a vev.

The interaction Lagrangian of interest reads

$$\begin{aligned}\mathcal{L} \supset \frac{1}{\sqrt{2}} \Big\{ &\left[y^R_{\Phi_L \bar{b} \Psi_Q}\, \Phi_L + y^R_{\Phi_H \bar{b} \Psi_Q}\, \Phi_H \right] \bar{b} P_R \Psi_Q + \left[y^R_{\Phi_L \bar{s} \Psi_Q}\, \Phi_L + y^R_{\Phi_H \bar{s} \Psi_Q}\, \Phi_H \right] \bar{s} P_R \Psi_Q \\ &+ \left[y^R_{\Phi_L \bar{\ell} \Psi_\ell}\, \Phi_L + y^R_{\Phi_H \bar{\ell} \Psi_Q}\, \Phi_H \right] \bar{\ell} P_R \Psi_\ell \Big\} + \text{h.c.}\,.\end{aligned} \tag{4.58}$$

Hermitian conjugation gives the left-handed Yukawa couplings, y^L, which are related to the right-handed ones via

$$y^L_{\bar{\Phi}\bar{\Psi}f} = \left(y^R_{\Phi \bar{f} \Psi} \right)^*, \tag{4.59}$$

where $\Phi \in \{\Phi_L, \Phi_H\}$, $\Psi \in \{\Psi_Q, \Psi_\ell\}$, and $f \in \{b, s, \ell\}$ as applicable. The expressions for the Yukawa couplings can be read off from Ref. [89] and we omit writing them explicitly. The NP contribution to C_9 and C_{10} are, then,

$$C^{\mu,\text{NP}}_{9/10} = \frac{1}{2} \left(s^{23\mu}_{LR} \pm s^{23\mu}_{LL} \right) , \tag{4.60}$$

and, from Eq. (4.20), we have

$$
\begin{aligned}
s_{LR}^{23\mu}\big|_{\text{box}} &= 0\,, \\
s_{LL}^{23\mu}\big|_{\text{box}} &= \frac{1}{4M_{\Phi_L}^2}\, y^L_{\Phi_L^*\bar{\Psi}_Q b}\, y^R_{\Phi_L \bar{s}\Psi_Q} \left|y^R_{\Phi_L\bar{\mu}\Psi_\ell}\right|^2 F_S^B(x_{\Phi_H}^{\Psi_Q}, 1, x_{\Phi_L}^{\Psi_\ell}) \\
&+ \frac{1}{4M_{\Phi_L}^2}\, y^L_{\Phi_H^*\bar{\Psi}_Q b}\, y^R_{\Phi_H \bar{s}\Psi_Q} \left|y^R_{\Phi_H\bar{\mu}\Psi_\ell}\right|^2 F_S^B(x_{\Phi_H}^{\Psi_Q}, 1, x_{\Phi_H}^{\Psi_\ell}) \\
&+ \frac{1}{4M_{\Phi_L}^2}\, y^L_{\Phi_L^*\bar{\Psi}_Q b}\, y^R_{\Phi_H \bar{s}\Psi_Q} \left(y^R_{\Phi_L\bar{\mu}\Psi_\ell}\, y^L_{\Phi_H^*\bar{\Psi}_\ell \mu}\right) F_S^B(x_{\Phi_L}^{\Psi_Q}, x_{\Phi_H}^{\Phi_L}, x_{\Phi_L}^{\Psi_\ell}) \\
&+ \frac{1}{4M_{\Phi_H}^2}\, y^L_{\Phi_H^*\bar{\Psi}_Q b}\, y^R_{\Phi_L \bar{s}\Psi_Q} \left(y^R_{\Phi_H\bar{\mu}\Psi_\ell}\, y^L_{\Phi_L^*\bar{\Psi}_\ell \mu}\right) F_S^B(x_{\Phi_H}^{\Psi_Q}, x_{\Phi_L}^{\Phi_H}, x_{\Phi_H}^{\Psi_\ell})\,.
\end{aligned}
\tag{4.61}
$$

Note that C_9^ℓ receives a lepton-flavour-universal contribution from the photon penguin. This contribution breaks the relation $C_9 = -C_{10}$ but it is suppressed by fermion masses and is therefore subleading in the limit where the scalars are lighter. Substituting the couplings from Ref. [89] and translating the box functions, F_S^B, into their G functions gives perfect agreement with their result.

4.4.4 A littlest Higgs model with T-parity

The 'Little Higgs' models [91] have been introduced in order to address the gauge hierarchy problem, where the electroweak Higgs boson is a pseudo-Goldstone boson of a certain global symmetry that is broken spontaneously. The breaking scale $\Lambda \sim 4\pi f \sim \mathcal{O}(10$ TeV) is much higher than the vacuum expectation value of the standard model Higgs doublet. In these models the Higgs as a Goldstone boson has light mass and the approximate global symmetry protects its mass from the divergencies under radiative corrections that arise in the SM through the top quark and heavy weak bosons running in the loop.

A simple version of these models is the 'Littlest Higgs' (LH) model [92], where the number of new heavy degrees of freedom is limited. Particularly, in this section we will focus on the Littlest Higgs model with a discrete symmetry, $T-$parity (LHT). The LHT model is based on a non-linear sigma model describing the spontaneously breaking of a a global $G = \text{SU}(5)$ to a global $\text{SO}(5)$ via a vacuum expectation value of an $\text{SU}(5)$ symmetric matrix Σ. This symmetry breakdown generates 14 pseudo-Goldstone bosons, which include the SM Higgs doublet [93].

It is assumed that the group G contains a weakly gauged subgroup,

$$G \supset G_1 \times G_2 = [\text{SU}(2) \times \text{U}(1)]_1 \times [\text{SU}(2) \times \text{U}(1)]_2$$

that will be broken to the diagonal $G_{\text{EW}} = [\text{SU}(2) \times \text{U}(1)]$ subgroup which is identified as the electroweak gauge symmetry. The gauge bosons of the $G_{1/2}$ groups are symmetric under the $T-$parity, *i.e.*, $W_1^a \leftrightarrow W_2^a$ and $B_1 \leftrightarrow B_2$. Hence, the gauge boson $T-$parity eigenstates can be defined as

$$W_L^a = \frac{W_1^a + W_2^a}{\sqrt{2}}, \qquad B_L = \frac{B_1 + B_2}{\sqrt{2}} \qquad \text{(T-even)} \tag{4.62}$$

$$W_H^a = \frac{W_1^a - W_2^a}{\sqrt{2}}, \qquad B_H = \frac{B_1 - B_2}{\sqrt{2}} \qquad \text{(T-odd)}, \tag{4.63}$$

where the subindex 'H/L' stands for 'heavy/light'. The breaking of the gauge symmetry of the theory is $G \supset G_1 \times G_2 \xrightarrow{f} G_{\text{EW}} \xrightarrow{v} \text{U}(1)_{\text{EM}}$, where the electroweak symmetry breaking occurs through the usual vacuum expectation value of the Higgs doublet, v. After these breakings and changing the basis, there are three massive $T-$odd gauge bosons labeled as W_H, Z_H and a massive photon A_H. $T-$even gauge bosons, W_L, Z_L and massless photon A_L, correspond to the standard gauge fields. The implementation of the $T-$symmetry guarantees that $M_{W_L} = M_{Z_L} \cos\theta_W$ holds at tree level.

In the fermion sector of the model, it is required to duplicate the SM isospin doublet fields due to the $T-$symmetry. For each $\mathrm{SU}(2)_L$ doublet, a doublet under $\mathrm{SU}(2)_1$ and one under $\mathrm{SU}(2)_2$ are introduced. The $T-$even combination of these doublets is related to the SM electroweak quark and lepton doublets, while the $T-$odd combination is known as mirror fermions and they acquire a mass of order the breaking scale f via Yukawa interactions. In this work the mass eigenstates of these $T-$odd fermions are denoted by capital letters in accordance with their SM partner's names (for leptons they are denoted as ℓ_H).

Also, to eliminate the effect of the top quark to the Higgs mass an additional $T-$even heavy quark, T_+, is introduced. At the leading order in v/f, the T_+ transforms as a singlet under $\mathrm{SU}(2)_L$. The $T-$odd partner of this new fermion T_- is an exact singlet under $\mathrm{SU}(2)_1\times\mathrm{SU}(2)_2$ and it has only sizeable 'flavour changing' couplings with the SM top quark mass eigenstate and the A_H.

Moreover, there are flavour mixing interactions between SM fermions and mirror fermions mediated by the heavy gauge bosons W_H, Z_H and A_H. The mixing matrices satisfy the following conditions in the quark and lepton sectors, accordingly,

$$V_{Hu}^\dagger V_{Hd} = V_{\text{CKM}}, \qquad\qquad V_{H\nu}^\dagger V_{H\ell} = V_{\text{PMNS}}. \tag{4.64}$$

In this section we demonstrate the usage of our results for the LHT model. Particularly, we derive explicit expressions for three complex gauge invariant functions,

$$X = |X|\, e^{i\theta_X}, \qquad Y = |Y|\, e^{i\theta_Y}, \qquad Z = |Z|\, e^{i\theta_Z}, \tag{4.65}$$

which are used to describe the semi-leptonic decays of K, B_d and B_s mesons. The first two functions are combinations of one-loop functions resulting in Z_L-penguin and box diagram calculations. They contribute to decays with final state $I_3 = 1/2$ and $I_3 = -1/2$ lepton pairs, respectively. The last function is obtained by combining Z_L-penguin diagrams with off-shell photon penguin diagrams. It contributes to decays with final state charged leptons. Note that there are no Z_H or A_H mediated diagrams, because of $T-$symmetry their couplings to the SM leptons are forbidden.

In order to parametrise effects of the new physics in the framework of LHT, the above functions could be decomposed as

$$F = F_{\text{SM}} + \bar{F}_{\text{even}} + \frac{1}{\lambda_t}\bar{F}_{\text{odd}} \equiv |F|\, e^{i\theta_F}, \qquad \text{where} \qquad F \in \{X, Y, Z\}, \tag{4.66}$$

where $\lambda_t = V_{tb}V_{ts}^*$ is the CKM factor. F_{SM} are the SM contributions, and $\bar{F}_{\text{even}}$ are the contributions from the $T-$even part, that is the contributions of T_+ and t. The function $\bar{F}_{\text{odd}}$ represents the $T-$odd sector of the LHT model with mirror fermions.

Before proceeding to derive explicit expressions for the above functions, we discuss briefly the normalisation and compact notations for mixing matrices. The loop functions in our setup, Eqs. (4.18), (4.19) and (4.20), are defined such that they come with a $1/M^2_{\text{boson}}$ factor, where M_{boson} is the mass of the loop boson. In the LHT model there are three $T-$odd vector bosons which contribute to the functions of interest. In order to be consistent with the normalisation used in the literature, we adopt the following normalisation factor,

$$N_{\text{LHT}} = \frac{g^4}{M^2_{W_L}} \frac{v^2}{64 f^2}, \tag{4.67}$$

where M_{W_L} is the mass of the light $T-$even charged gauge boson. At the leading order the relation between light and heavy boson masses can be defined as

$$M_{W_H} \approx M_{Z_H} = \sqrt{a} M_{A_H} = 2 M_{W_L} \frac{f}{v} \frac{(1 - v^2/(8f^2))}{(1 - v^2/(12f^2))} \approx 2 M_{W_L} \frac{f}{v}, \tag{4.68}$$

where $a = \frac{M^2_{Z_H}}{M^2_{A_H}} = \frac{5g^2}{g'^2} \frac{1 - v^2/(8f^2)}{1 - 5v^2/(8f^2)}$, at the leading order $a = 5/\tan^2\theta_w$ which allows to write $g' = g\sqrt{5/a}$.

For mixing matrices the following short forms are used,

$$\xi^c_i = (V_{H\ell})_{i\mu} (V_{H\ell})^*_{i\mu}, \qquad \lambda^c_i = (V_{Hd})_{ib} (V_{Hd})^*_{is}, \tag{4.69}$$

where ξ_i and λ_i denote mixings in the $T-$odd lepton and quark sectors, respectively. The index i in ξ^c_i denotes $T-$odd loop-leptons, and for λ^c_i, $i = 1, 2, 3$ corresponds to the generation index. The upper index 'c' can be 0 or $\pm$ and indicates that what kind of loop bosons are used. For example, for neutral vectors ξ^0_i and λ^0_i and for charged vectors $\xi^\pm_i$ and $\lambda^\pm_i$.

In analogy to the SM CKM matrix, the unitarity of the mixing matrix of $T-$odd heavy quarks is assumed and the GIM-like mechanism was performed via the following expression,

$$\lambda^c_1 = -\lambda^c_2 - \lambda^c_3. \tag{4.70}$$

First, we consider the $T-$even sector where the relevant Feynman rules are [93]

$$g^L_{W^+_L \bar{t} b} = \frac{ig}{\sqrt{2}} (V_{\text{CKM}})_{tj} \left(1 - \frac{x_L}{2} \frac{v^2}{f^2}\right), \tag{4.71}$$

$$g^L_{W^+_L \bar{T}_+ b} = \frac{ig}{\sqrt{2}} (V_{\rm CKM})_{tj}\, x_L \frac{v}{f} \left(1 + \frac{v^2}{f^2} d_2\right), \tag{4.72}$$

$$g_{Z_L \bar{T}_+ t} = \frac{ig}{\cos\theta_w} \frac{x_L}{2} \frac{v}{f} \left[1 + \frac{v^2}{f^2}\left(d_2 - \frac{x_L^2}{2}\right)\right], \tag{4.73}$$

in addition to the usual SM couplings of leptons to $W^\pm$ and Z bosons. For all coupling constants we use bottom b and strange s quarks. In the above rules $x_L = \lambda_1^2/(\lambda_1^2 + \lambda_2^2)$ and $d_2 = -5/6 + 1/2 x_L^2 + 2x_L(1-x_L)$. In order to derive order $\mathcal{O}(v^2/f^2)$ corrections, one should take into account that the mass of the T_+ is of order f/v, particularly at the leading order $m_{T_+} = \frac{f}{v}\frac{m_t}{\sqrt{x_L(1-x_L)}}$. In the derivation of $X_{\rm even}$ or $Y_{\rm even}$, we substituted this expression for the mass of m_{T_+} and after performing the Taylor expansion in v/f at the second order, switched back to m_{T_+}. Below, we will consider the case for $X_{\rm even}$, for the $Y_{\rm even}$ function it can be done with in a similar manner.

For simplicity we take masses of the leptons and masses of the first two generation of quarks equal to zero. Thus the $X_{\rm even}$ function gets contributions from the following loop functions

$$\bar{X}_{\rm even} \propto \cdots \times \frac{1}{M^2_{W_L}} F_V^{L,B'Z}\left(x^t_{W_L}, x^{T_+}_{W_L}, 1, 0\right) + \ldots \frac{1}{M^2_{Z_L}} F_V^Z\left(x^t_{W_L}, x^{T_+}_{W_L}\right), \tag{4.74}$$

where dots represent a product of coupling constants. At this stage we have not used the explicit forms of couplings yet, because they contain a v/f dependence and are quite lengthy. By using $M_{Z_L} = M_{W_L}/\cos\theta_W$ and performing the second order Taylor expansion in v/f we found that

$$\bar{X}_{\rm even} = 64 N_{\rm LHT} x_L^2 \times \left(\frac{-2x_t^2 + 2x_t - 3}{8\,(x_t - 1)} - \frac{(2x_t^2 - x_t - 4)\, x_t}{8\,(x_t-1)^2} \log(x_t) + \frac{1}{8}(2x_t + 3) \log\left(x_{T_+}\right) + \frac{x_L}{8} x_t\right), \tag{4.75}$$

where $x_i = x^i_{W_L}$. This result is in agreement with the one given in Ref. [93].

Next, we work with $T-$odd part of the LHT model, where loop fermions are heavy mirror quarks U_i/D_i and leptons ℓ_H. We adopt the following set of rules from the model

$$g^L_{W_H \bar{U}_i b} = \frac{ig}{\sqrt{2}} (V_{Hd})_{ib}, \tag{4.76}$$

$$g^L_{A_H \bar{D}_i b} = i\left(-\frac{g'}{10} + \frac{g}{2} x_H \frac{v^2}{f^2}\right)(V_{Hd})_{ib}, \quad g^L_{Z_H \bar{D}_i b} = i\left(-\frac{g}{2} - \frac{g'}{10} x_H \frac{v^2}{f^2}\right)(V_{Hd})_{ib}, \tag{4.77}$$

where $x_H = 5gg'/(4(5g^2 - g'^2))$. The couplings for the leptons are similar, where $(V_{Hd})_{ij} \to (V_{H\ell})_{ij}$ and fields are changed accordingly. Also, signs of gauge couplings constants, g and g', should be altered in the neutral boson couplings, *i.e.,* for charged leptons the sign of $g' \to -g'$ and for neutrinos $g \to -g$. Since we consider loop functions at $\mathcal{O}(v^2/f^2)$ order, the correction terms in $g^L_{A_H/Z_H\dots}$ are redundant.

The result for the $\bar{Y}_{\text{odd}}$ function reads

$$
\begin{aligned}
\bar{Y}_{\text{odd}} = N_{\text{LHT}} \times \Bigg\{ & \xi^{\pm}_{\nu_H} \times \sum_{i=2,3} 4\lambda^{\pm}_i F^{L,BZ}_V\big(x^{U_1}_{W_H}, x^{U_i}_{W_H}, 1, x^{\nu_H}_{W_H}\big) \\
& + \xi^0_{\mu_H} \times \sum_{i=2,3} \lambda^0_i \Big[F^{L,BZ}_V\big(x^{D_1}_{Z_H}, x^{D_i}_{Z_H}, 1, x^{\mu_H}_{Z_H}\big) - F^{L,B'Z}_V\big(x^{D_1}_{Z_H}, x^{D_i}_{Z_H}, 1, x^{\mu_H}_{Z_H}\big) \\
& + \frac{1}{25a}\Big(F^{L,BZ}_V\big(x^{D_1}_{A_H}, x^{D_i}_{A_H}, 1, x^{\mu_H}_{A_H}\big) - F^{L,B'Z}_V\big(x^{D_1}_{A_H}, x^{D_i}_{A_H}, 1, x^{\mu_H}_{A_H}\big)\Big) \\
& + \frac{1}{5a}\Big(F^{L,BZ}_V\big(x^{D_1}_{Z_H}, x^{D_i}_{Z_H}, x^{Z_H}_{A_H}, x^{\mu_H}_{Z_H}\big) - F^{L,B'Z}_V\big(x^{D_1}_{Z_H}, x^{D_i}_{Z_H}, x^{Z_H}_{A_H}, x^{\mu_H}_{Z_H}\big)\Big) \\
& + \frac{1}{5}\Big(F^{L,BZ}_V\big(x^{D_1}_{A_H}, x^{D_i}_{A_H}, x^{A_H}_{Z_H}, x^{\mu_H}_{Z_H}\big) - F^{L,B'Z}_V\big(x^{D_1}_{A_H}, x^{D_i}_{A_H}, x^{A_H}_{Z_H}, x^{\mu_H}_{Z_H}\big)\Big) \Big] \Bigg\},
\end{aligned}
\tag{4.78}
$$

where the argument 1 in functions means that x^V_V with $V \in \{W_H, Z_H, A_H\}$. The first three lines come from the contributions where W_H, Z_H and A_H run in the loops, correspondingly. The last two lines are contributions from the diagrams where Z_H and A_H are included simultaneously in the loops. The similar expression for X_{odd} can be derived by exchanging external charged leptons for neutrinos.

Furthermore, our result for the $\bar{Z}_{\text{odd}}$ function is given as

$$
\begin{aligned}
\bar{Z}_{\text{odd}} = -N_{\text{LHT}} \times \sin\theta^2_w \Bigg\{ & \sum_{i=2,3} 8\lambda^{\pm}_i F^{\gamma Z}_V\big(x^{U_1}_{W_H}, x^{U_i}_{W_H}\big) \\
& \sum_{i=2,3} \lambda^0_i \left(4F^{\gamma Z}_V\big(x^{D_1}_{Z_H}, x^{D_i}_{Z_H}\big) + \frac{4}{5} F^{\gamma Z}_V\big(x^{D_1}_{A_H}, x^{D_i}_{A_H}\big)\right) \Bigg\},
\end{aligned}
\tag{4.79}
$$

where the overall minus sign comes from the photon coupling with external lepton legs, i.e., $Q_\ell = -1$. These expressions are consistent with the results presented in Ref. [93] (with corrections to some functions [94, 95]).

The result for the $\Delta F = 2$ Wilson coefficients in the $T-$odd sector is,

$$\begin{aligned} C_{LL}^{bs} = N_{\text{LHT}} \times \sum_{ij} \Bigg\{ &\lambda_i^c \lambda_j^c \Big[2F_{V'}^{B}\big(x_{W_H}^{U_i}, 1, x_{W_H}^{U_j}\big) - 2F_{V''}^{B}\big(x_{W_H}^{U_i}, 1, x_{W_H}^{U_j}\big) \Big] \\ &+ \lambda_i^0 \lambda_j^0 \Big[\frac{3}{50a} F_{V''}^{B}\big(x_{A_H}^{D_i}, 1, x_{A_H}^{D_j}\big) + \frac{3}{2} F_{V''}^{B}\big(x_{Z_H}^{D_i}, 1, x_{Z_H}^{D_j}\big) \\ &+ \frac{3}{10} F_{V''}^{B}\big(x_{A_H}^{D_i}, x_{Z_H}^{A_H}, x_{A_H}^{D_j}\big) + \frac{3}{10a} F_{V''}^{B}\big(x_{Z_H}^{D_i}, x_{A_H}^{Z_H}, x_{Z_H}^{D_j}\big) \Big] \Bigg\}, \end{aligned} \tag{4.80}$$

and is in agreement with results calculated in Ref. [96]. In this result the unitarity relation (4.70) has not been applied yet.

4.4.5 A loop induced dark matter model

In this last example for an application of our results we consider a dark matter (DM) model where the dark matter candidate is a singlet Dirac fermion and its interactions with the standard model sector take place at loop level [97]. In analogy to the proton stability that is guaranteed by an accidental global $\mathrm{U}(1)$ symmetry (baryon number), to maintain the stability of the DM the model introduces an additional global $\mathrm{U}(1)_D$ symmetry under which the DM has non-zero charge.

In the model the Dirac fermion DM χ is composed of two Weyl fermions, ξ_χ and η_χ which are singlets under the SM gauge group. The $\mathrm{U}(1)_D$ charges of these fermions are +1 and -1, accordingly. All SM fields are non-charged under the $\mathrm{U}(1)_D$ symmetry.

Since the DM is neutral under the SM gauge symmetry, to generate renormalisable interactions between DM and SM particles new mediators are necessitated. Particularly, in this model additional vector-like fermions, ψ_{f_i}, and complex scalars, $\widetilde{f}_i$, are introduced. These new fields participate in SM gauge interactions and they mediate DM couplings with the SM particles through quantum corrections. The new vector-like fermions have zero $\mathrm{U}(1)_D$ charges, while new scalars are $\mathrm{U}(1)_D$-charged and they are supposed to be heavier than the DM to ensure the stability of DM particle.

The relevant part of the interaction Lagrangian for the SM gauge boson couplings with new fields in the mass eigenbasis is

$$\begin{aligned}\mathcal{L}_{\text{gauge}} = -eA_\mu \sum_{f,i} Q_f \bar{\psi}_{f_i}\gamma^\mu \psi_{f_i} - g_Z Z_\mu \sum_{f,i,j} \bar{\psi}_{f_i}\gamma^\mu \left(C^{ij}_{fZL}P_L + C^{ij}_{fZR}P_R\right)\psi_{f_j} \\ -ieA_\mu \sum_{f,i} Q_f \widetilde{f}^*_i \overleftrightarrow{\partial^\mu} \widetilde{f}_i - ig_Z Z_\mu \sum_{f,i,j} \widetilde{C}^{ij}_{fZ} \widetilde{f}^*_i \overleftrightarrow{\partial^\mu} \widetilde{f}_j + \ldots,\end{aligned} \tag{4.81}$$

with

$$C^{ij}_{fZL/R} = \left(V_{fL/R}\right)^*_{1i}\left(V_{fL/R}\right)_{1j} T_{3f} - Q_f \sin^2\theta_W \delta_{ij}, \tag{4.82}$$

$$\widetilde{C}^{ij}_{fZ} = \left(\widetilde{V}_f\right)^*_{1i}\left(\widetilde{V}_f\right)_{1j} T_{3f} - Q_f \sin^2\theta_W \delta_{ij}, \tag{4.83}$$

where e denotes the elementary charge, $g_Z \equiv \sqrt{g'^2 + g^2}$, g' and g are the gauge coupling constants of $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$, respectively. The mixing matrices for new fermions and scalars are $V_{fL/R}$ and $\widetilde{V}_f$, correspondingly. The left-right derivative is defined as $A\overleftrightarrow{\partial_\mu}B \equiv A(\partial_\mu B) - (\partial_\mu A)B$.

The interaction Lagrangian in the dark sector can be given by

$$\mathcal{L}_{\chi f\widetilde{f}} = \bar{\chi}\left(C^{ij}_{f\chi L}P_L + C^{ij}_{f\chi R}P_R\right)\psi_{f_i}\widetilde{f}^*_j + \text{h.c.}, \tag{4.84}$$

with

$$C^{ij}_{f\chi L} = e^{-\frac{i}{2}\theta_\chi}\left[a_f\left(V_{fL}\right)_{1i}\left(\widetilde{V}_f\right)^*_{1j} + a_{\bar{f}}\left(V_{fL}\right)_{2i}\left(\widetilde{V}_f\right)^*_{2j}\right], \tag{4.85}$$

$$C^{ij}_{f\chi R} = e^{\frac{i}{2}\theta_\chi}\left[b^*_f\left(V_{fR}\right)_{1i}\left(\widetilde{V}_f\right)^*_{1j} + b^*_{\bar{f}}\left(V_{fR}\right)_{2i}\left(\widetilde{V}_f\right)^*_{2j}\right], \tag{4.86}$$

where $\theta_\chi \equiv \arg(\mu_\chi)$ that is in the DM mass term $-m_\chi\bar{\chi}\chi$ with $m_\chi \equiv |\mu_\chi|$. The coefficients a_f and b_f are coupling constants for the ξ_χ and η_χ fields, respectively.

The proposed DM candidate can be detected in dark matter direct detection experiments. In order to evaluate expected event rates one needs to calculate the relevant effective interactions of the DM with the SM sector. Particularly, we consider only the part when DM couples with the SM fields via neutral vector currents, *i.e.,* the SM photon and $Z-$boson mediated one loop interactions.

The effective interactions of DM with a photon, γ can be written as

$$\mathcal{L}_{\text{eff-}\gamma} = \frac{1}{2}C^\gamma_M\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu} - \frac{i}{2}C^\gamma_E\bar{\chi}\sigma^{\mu\nu}\gamma_5\chi F_{\mu\nu} + C^\gamma_R\bar{\chi}\gamma^\mu\chi\partial^\nu F_{\mu\nu}, \tag{4.87}$$

where effective couplings C^γ_i denote the DM magnetic moment C^γ_M, the DM electric moment C^γ_E and the DM charge radius C^γ_R.

After integrating out the $Z-$boson, the effective Lagrangian for the DM couplings with light quarks can be defined as

$$\mathcal{L}_{\text{eff-q}} = C^q_V\left(\bar{\chi}\gamma^\mu\chi\right)\left(\bar{q}\gamma_\mu q\right). \tag{4.88}$$

Furthermore, we derive analytic expressions for the above Wilson coefficients using our results presented in Sec. 4.1.3 and in Sec. 4.1.4. Particularly, the C^γ_M and C^γ_E coefficients will be obtained from Eq. (4.14) and for C^γ_R and C^q_V we use Eq. (4.20). For these couplings we need to take the sum of $\tilde{C}^{ij\ell}_{L\sigma}$ and its chirality flipped version, $\tilde{C}^{ij\ell}_{R\sigma}$, in Eq. (4.17).

Another point to consider is that in order to be consistent with the definition of the Yukawa-like couplings of the DM with internal scalars and fermions given in Ref. [97], we need to use the correct charge conjugation property for the DM-ψ-$\tilde{f}$ couplings. This is defined in Eq. (2.73),

$$y^\sigma_{\tilde{f}_j\bar{\psi}_{f_i}\chi} = \left(y^{\bar{\sigma}}_{\tilde{f}^*_j\bar{\chi}\psi_{f_i}}\right)^*. \tag{4.89}$$

In order to recover results of [97] we work in the limit that the mass of the DM particle is quite small in comparison with the masses of loop particles, $m_{\psi_{f_i}}, m_{f_i} \gg m_\chi$. Changing the notations for the external fermions, $d_{i,j} \to \chi$ and $\ell \to q$, the final results can be written in terms of our loop functions from Eq. (4.14) and the Eq. (4.20). Namely,

$$C_M^\gamma = \frac{1}{16\pi^2} \sum_{f,i,j} N_c \cdot eQ_f \frac{1}{M_{\tilde{f}_j}^2} \left\{ m_{f_i} \operatorname{Re}\left[C_{f\chi L}^{ij} C_{f\chi R}^{ij*}\right] F_S^d\left(x_{\tilde{f}_j}^{f_i}\right) + m_\chi C_{f\chi SS}^{ij} F_{S'}^d\left(x_{\tilde{f}_j}^{f_i}\right) \right\}, \quad (4.90)$$

$$C_E^\gamma = \frac{1}{16\pi^2} \sum_{f,i,j} N_c \cdot eQ_f \frac{m_{f_i}}{M_{\tilde{f}_j}^2} \operatorname{Im}\left[C_{f\chi L}^{ij} C_{f\chi R}^{ij*}\right] F_S^d\left(x_{\tilde{f}_j}^{f_i}\right), \quad (4.91)$$

$$C_R^\gamma = \frac{1}{16\pi^2} \sum_{f,i,j} N_c \cdot eQ_f \frac{1}{M_{\tilde{f}_j}^2} C_{f\chi SS}^{ij} F_S^\gamma\left(x_{\tilde{f}_j}^{f_i}\right), \quad (4.92)$$

where

$$C_{f\chi SS}^{ij} \equiv \frac{1}{2}\left\{ \left|C_{f\chi L}^{ij}\right|^2 + \left|C_{f\chi R}^{ij}\right|^2 \right\} \quad (4.93)$$

and N_c is the color factor relevant to the internal particles.

The Wilson coefficient for the $Z-$boson exchange can be written in terms of the effective DM-$Z-$vertex function, $C_{\chi Z}$,

$$C_V^q = \frac{1}{M_Z^2} \frac{e^2}{\sin^2\theta_W \cos^2\theta_W} \left(T_{3q} - 2Q_q \sin^2\theta_W\right) \times C_{\chi Z}, \quad (4.94)$$

where

$$C_{\chi Z} = \frac{1}{16\pi^2} \sum_{j,i,j,k} N_c \left\{ C_{fZ1}^{kij} F_{S'}^Z\left(x_{\tilde{f}_k}^{f_i}, x_{\tilde{f}_k}^{f_j}\right) + C_{fZ2}^{kij} F_{S''}^Z\left(x_{\tilde{f}_k}^{f_i}, x_{\tilde{f}_k}^{f_j}\right) + \widetilde{C}_{fZ1}^{jki} F_S^Z\left(x_{\tilde{f}_j}^{f_i}, x_{\tilde{f}_k}^{f_i}\right) \right\}. \quad (4.95)$$

The explicit forms for the couplings are

$$C_{fZ1}^{kij} = C_{fZR}^{ij} C_{f\chi L}^{ik} C_{f\chi L}^{jk*} + C_{fZL}^{ij} C_{f\chi R}^{ik} C_{f\chi R}^{jk*}, \quad (4.96)$$

$$C_{fZ2}^{kij} = C_{fZR}^{ij} C_{f\chi R}^{ik} C_{f\chi R}^{jk*} + C_{fZL}^{ij} C_{f\chi L}^{ik} C_{f\chi L}^{jk*}, \quad (4.97)$$

$$\widetilde{C}_{fZ1}^{jki} = \widetilde{C}_{fZ}^{jk} \left(C_{f\chi L}^{ik} C_{f\chi L}^{ij*} + C_{f\chi R}^{ik} C_{f\chi R}^{ij*}\right). \quad (4.98)$$

In Ref. [97] the functions multiplied by the C_{fZ1}^{kij} and $\widetilde{C}_{fZ1}^{jki}$ couplings contain a divergent constant part that will cancel in the sum in Eq. (4.95), since

$$\sum_{i,j,k} C_{fZ1}^{kij} = \sum_{i,j,k} \widetilde{C}_{fZ1}^{jki}. \quad (4.99)$$

In our case all functions in Eq. (4.94) are finite individually. This is a consequence of using the sum rule encoded in Eq. (2.81) when we renormalised the $Z-$penguin diagrams, Sec. 4.1.2.

In principle, the equality in Eq. (4.99) can be read from (2.81) when a theory contains only flavour-changing scalars,

$$g_{Z\tilde{f}_i\tilde{f}_j^*}\, y^{\sigma}_{\tilde{f}_j\bar{\psi}_{f_i}\chi} = g^{\bar{\sigma}}_{Z\bar{\psi}_{f_i}\psi_{f_j}}\, y^{\sigma}_{\tilde{f}_j\bar{\psi}_{f_i}\chi}. \tag{4.100}$$

5 DARK MATTER PHENOMENOLOGY

Weakly Interacting Massive Particles (WIMPs) are theoretically attractive candidates for a particle explanation of the Dark Matter (DM) content of the Universe. The interactions of WIMPs with light Standard Model (SM) particles can be tested experimentally via their elastic scattering off nuclei in ground-based direct detection experiments, in specific signatures at the Large Hadron Collider (LHC) involving visible SM objects and missing energy [26], or by observing their annihilation products with astrophysical experiments.

Up to date the leading liquid xenon based direct detection experiments LUX [27] and XENON1T [28] have not detected the DM, thereby severely constraining the interaction rates between WIMPs and the SM. This strongly constrains the possible parameter space of WIMP quark interactions – and with it also the production mechanisms in collider experiments – and requires a suppression mechanism for the direct detection rates. Isospin breaking interactions, where up and down quarks interact with different strength, can cancel the coherent spin independent contributions of protons and neutrons in direct detection experiments. The resulting suppression for vectorial couplings to dark matter was studied, for example, in Ref. [98]. This mechanism allows one to cancel the interaction of DM with a particular isotope and was used to reconcile the discrepancy between the claimed observation by DAMA/LIBRA [99] (using sodium iodide as a target material) and the null results from liquid xenon based experiments [6].

In this work, we focus on axial vector dark matter interactions, where the direct detection rate is naturally suppressed, cf. *e.g.*, Ref. [29]. The continuously tightening constraints led by XENON1T make this scenario phenomenologically relevant [29], and will require additional suppression if the null results persist in the future. For these type of interactions, similar to the discussion in Ref. [98], the DM could have suppressed couplings to specific isotopes, in particular to xenon.

The difference for axial vector dark matter interactions is, however, that both the axial vector and vector contributions on the quark side contribute equally to the scattering cross section [30]. Furthermore, the $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge invariance of the SM implies that the $V-A$ (vector minus axial) couplings of up and down quarks have equal strength, *i.e.*, that at dimension-6 effective interactions

$$(\bar{\chi}\chi)_A(\bar{u}u)_{V-A} \leftrightarrow (\bar{\chi}\chi)_A(\bar{d}d)_{V-A} \tag{5.1}$$

are directly related. Consequently, it is not clear if isospin suppression is even possible in the

[6] Recent new results from the COSINE-100 experiment did not confirm the DAMA result [100].

case of axial vector coupling DM.

To study the impact of the SM gauge invariance we use the effective theory above the electroweak scale from Ref. [30] that couples SU(2)×U(1) invariant SM fields with axial-vectorial DM currents. Such a theory naturally arises in models where the DM candidate is a Majorana fermion and the coupling of the dark and visible sectors is mediated by a tree-level neutral vector boson exchange, typically referred to as Z' and often related to an additional U(1) gauge symmetry. In such models, the vectorial couplings between the DM candidate and the Z' vanish because Majorana fermions are self-conjugate under charge conjugation while the vector current is odd. In addition, the extra U(1) gauge symmetry in these models results in constraints from anomaly cancellation, similar to those studied in Refs. [31] for Dirac dark matter. Experimentally, neutral gauge bosons that interact with the SM quark current can be searched for at the LHC, for instance via dijet final states [32]. Null results from these searches generally require mediator masses to be at the TeV scale. The large separation between the Z' mass and the momentum transfer in direct detection experiments justifies an effective field theory description of the interaction between the DM and the baryons and mesons. Matching the Z' exchange model to our effective field theory, we find that a significant suppression for the DM direct detection rate is only possible for very specific combinations of the effective theory parameters.

If we furthermore require the cancellation of gauge anomalies within one generation (see *Model-I*), we find that no suppression of the direct detection rate is possible. Yet, for generation dependent charges the direct detection rate can be suppressed by two orders of magnitude compared to the naive expectation for axial-vectorial couplings of the quarks and the dark matter candidate. We study this later case in benchmark scenario (*Model-II*) and compare current and future direct detection constraints with constraints coming from colliders.

The chapter is organised in the following way, in Sec 5.1 we discuss the EFT approach for DM direct detection experiments of axial-vectorial DM interactions along with suppression mechanism for the direct detection rate. Further, a possible UV completion of the EFT setup will be discussed in Sec. 5.2 where explicit constructions of anomaly free, UV complete models are considered. Sec. 5.3 is dedicated to the technical details for the experimental constraints. Particularly, in Sec. 5.3.1 we talk about the derivation of the exclusion curves for dark matter direct detection experiments. Then, DM searches at colliders will be outlined in Sec. 5.3.2. The complementarity between collider and direct detection searches is presented in Sec. 5.3.3.

5.1 Effective Field Theory for Direct Detection of Axial Vector DM

We consider a fermionic DM candidate coupled to the Standard Model (SM) via a spin-1 mediator arising from a gauged $\mathrm{U}(1)'$ dark sector. The gauge symmetry is spontaneously broken at a scale $M_* \gg M_W$. After electroweak symmetry breaking (EWSB), the Z boson is identified with the lighter of the two massive neutral eigenstates of the mixing of the electroweak (EW) and the $\mathrm{U}(1)'$ gauge eigenstates. We denote the heavier mass eigenstate by Z'. Consequently the DM candidate, χ, will inherit couplings to the Z boson. Thus, even with mixing angles of $\mathcal{O} \sim 10^{-3}$ [101], bounds set by direct-detection experiments rule out vectorial couplings. This scenario is naturally evaded if χ is a Majorana fermion, since a vectorial coupling is not allowed, while an axial-vectorial one is.

The dark matter effective field theory (DMEFT) Lagrangian can be written as

$$\mathcal{L}_{\text{DMEFT}} = \sum_{i,d} C_i^{(d)} Q_i^{(d)} \equiv \frac{\hat{c}_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \xrightarrow{\text{EWSB}} \sum_{i,d} \mathscr{C}_i^{(d)} \mathcal{Q}_i^{(d)} , \tag{5.2}$$

where we used a curly script notation for the operators and their coefficients below the EW scale to distinguish them from the ones above it. The lower case hatted coefficients are dimensionless while the uppercase un-hatted ones are dimensionful.

As a result of the pure axial-vectorial couplings on the dark-matter side, direct-detection rates are suppressed either due to the absence of coherent enhancement or because the coherent enhancement is suppressed by the DM velocity in the galactic halo. To be concrete, if χ is an $\mathrm{SU}(2)_L$-singlet Majorana fermion, the following three operators coupling χ to the SM quarks will be generated above the EW scale (following [30]),

$$\begin{aligned} Q_{6,i}^{(6)} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{Q}_L^i\gamma^\mu Q_L^i) , \\ Q_{7,i}^{(6)} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{u}_R^i\gamma^\mu u_R^i) , \\ Q_{8,i}^{(6)} &= (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{d}_R^i\gamma^\mu d_R^i) , \end{aligned} \tag{5.3}$$

where $i = 1, 2, 3$ denotes the quark generation. After EWSB, these three operators match onto the following two:

$$\mathcal{Q}_{2,q}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu q) , \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q) , \tag{5.4}$$

where $q \in \{u, d, s, c, b\}$. The operators involving the heavy quarks (c and b) are not relevant to our analysis. For example, in direct detection experiments, the operator $\mathcal{Q}_{4,q}^{(6)}$ leads to the cross section that depends on the nuclei spin which is non-zero only for the isotopes with unpaired nucleons (nuclear shell model). Thus, the matrix element evaluated at the scale Q is $\langle N|\bar{q}\gamma^\mu\gamma_5 q|N\rangle_Q \propto \Delta q_N$, where $\langle N|$ and $|N\rangle$ denote nucleon states at rest and Δq_N is the fraction of the nucleon spin carried by the quark q. For the proton, at $Q = 2$ GeV the values of the Δq with light quarks are [102],

$$\Delta u = 0.897(27) \qquad \Delta d = -0.376(27) \qquad \Delta s = -0.031(5),$$

while heavy quark contributions are negligible, $\Delta c \approx -5 \cdot 10^{-4}$ and $\Delta b \approx -5 \cdot 10^{-5}$, with probably at least a factor of two uncertainty on these estimates [30]. In the case of the $\mathcal{Q}_{2,q}^{(6)}$ operator, direct detection cross section is both momentum and velocity suppressed for $q = u, d, s$. While for $q = b, c$, the leading contribution comes from closing the heavy quark loop, exchanging a photon with the up- or down-quark vector current. Thus, the cross section is suppressed by an additional factor of $(\alpha/4\pi)^2$ [30].

The matching correction onto operators involving first-generation quarks is,

$$\begin{aligned} (A\otimes V)_u:&\quad \mathscr{C}_{2,u}^{(6)} = C_{7,1}^{(6)} + C_{6,1}^{(6)}\,, &\qquad (A\otimes V)_d:&\quad \mathscr{C}_{2,d}^{(6)} = C_{8,1}^{(6)} + C_{6,1}^{(6)}\,,\\ (A\otimes A)_u:&\quad \mathscr{C}_{4,u}^{(6)} = C_{7,1}^{(6)} - C_{6,1}^{(6)}\,, &\qquad (A\otimes A)_d:&\quad \mathscr{C}_{4,d}^{(6)} = C_{8,1}^{(6)} - C_{6,1}^{(6)}\,, \end{aligned} \tag{5.5}$$

where the shorthand notation on the left-hand-side of the colon gives the Lorentz structure of the operator as a product of dark matter and SM currents, respectively, where $(A)V$ means (axial-)vector current. The subscript denotes the quark-flavour of the operator.

The matching corrections in Eq. (5.5) present us with several possibilities:

1. We can entirely eliminate either the vectorial, V, or axial-vectorial, A, currents on the SM side (i.e. for all flavours). This can be accomplished via the choice

$$C_{6,1}^{(6)} = \mp C_{7,1}^{(6)} = \mp C_{8,1}^{(6)}\,. \tag{5.6}$$

   This automatically enforces isospin-symmetric coefficients in the EFT.

2. If isospin breaking is desired, the vectorial or axial-vectorial couplings cannot be eliminated, and one is left with a mixture of both.

Isospin breaking is typically invoked to suppress the direct detection rate especially for spin-independent DM–SM scattering in order to reconcile results (null and positive) from experiments with different target nuclei [98].

Unlike in the spin-independent case, however, when the coupling to the DM is purely axial, both the $A\otimes V$ and $A\otimes A$ operators can contribute equally to the direct-detection cross section. Consequently, it is not trivial to obtain a similar suppression as in the spin-independent case. To investigate the parameter space where the spin-dependent cross-section can be suppressed, it is useful to make the following transformation,

$$
\begin{aligned}
C_{6,1}^{(6)} &\to \frac{g'^2}{\Lambda^2}\cos\theta\,, \\
C_{7,1}^{(6)} &\to \frac{g'^2}{\Lambda^2}\sin\theta\cos\phi\,, \\
C_{8,1}^{(6)} &\to \frac{g'^2}{\Lambda^2}\sin\theta\sin\phi\,,
\end{aligned}
\tag{5.7}
$$

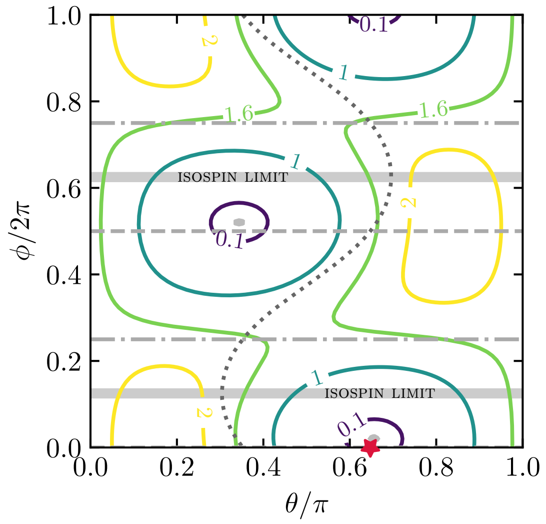
such that the sum of their squares equals g'^4/Λ^4 which removes one variable and allows us to show the relevant parameter space in one plot. This is shown in Fig. 8a where the contours show the relative size of the direct-detection rate in arbitrary units. The small solid gray ellipses have a relative rate of 10^{-2}. However, for a matter-field content consisting of the SM fermions and any number of dark Weyl fermions, the allowed regions or paths in the $\theta - \phi$ plane are strongly restricted by requiring that the mixed and pure $U(1)'$ anomalies cancel. This is discussed in more detail in Sec. 5.2 and Appendix C.1.

With only one dark Weyl fermion, different $U(1)'$ charges of the SM fermions restrict the parameters space to lie on the dashed, dotted, and dash-dotted gray lines in Fig. 8a. To study isospin breaking, we define the parameter,

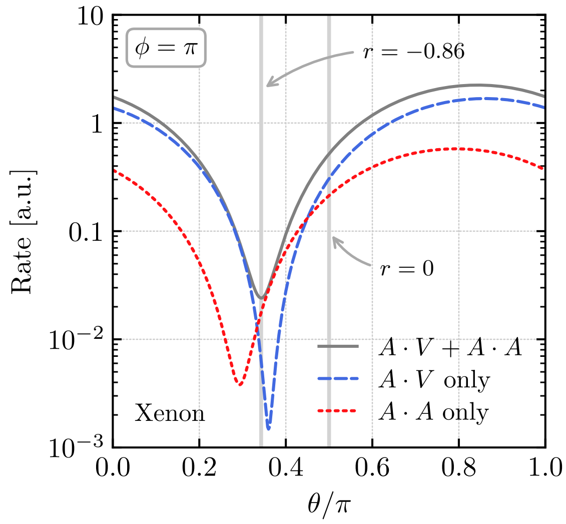
$$
r \equiv \mathscr{C}_{2,d}^{(6)}/\mathscr{C}_{2,u}^{(6)} = \frac{\sin\theta\sin\phi + \cos\theta}{\sin\theta\cos\phi + \cos\theta}\,, \tag{5.8}
$$

which takes the value $r = 1$ in the isospin limit, i.e. when the up- and down-flavoured coefficients are equal. This limit is realised on two horizontal lines in the $\theta - \phi$ plane with $\phi = \pi/4, 5\pi/4$. These lines are shown in Fig. 8a as thick grey lines that are labelled 'ISOSPIN LIMIT'. Fig. 8b shows the relative event rate using the same arbitrary normalization factor as in Fig. 8a, for comparison, along the $\phi = \pi$ constraint which arises in the *Model-II* that is discussed below. Similarly, the suppression curves of the event rate for the *Model-I* is depicted in Fig. 8c, where $\theta = \pi/2$. It can be easily seen that in the case of *Model-I*, the suppression of the event rate is much weaker. This is also true for $\theta = \arctan\left[2/(\sin\phi + \cos\phi)\right]$ where the $\mathrm{SU}(3)^2 \times \mathrm{U}(1)'$ anomaly vanishes and is presented in Fig. 8d.

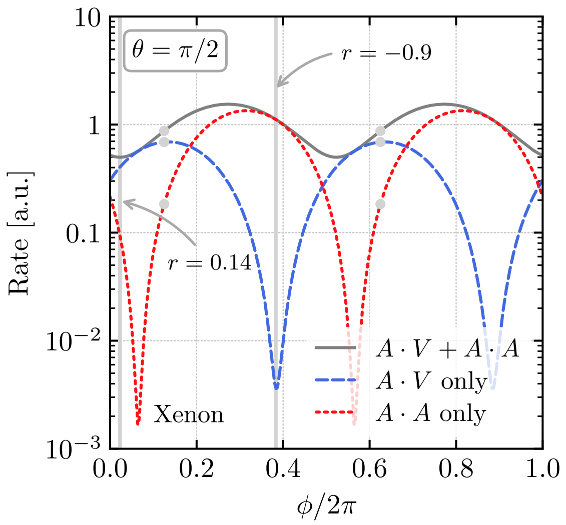
Note that $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge invariance forbids departures from the total (grey) curve in Figs. 8b, 8c, 8d. Furthermore, if isospin symmetry is respected, the relative rates cannot vary by more than an $\mathcal{O}(1)$ factor.



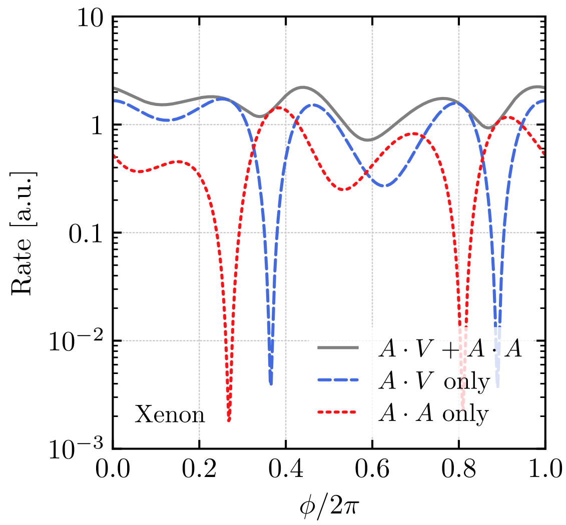
***(a)** Contours of the relative event rate [a.u.]*



***(b)** Relative event rate along $\phi = \pi$*



***(c)** Relative event rate along $\theta = \pi/2$*



***(d)** Rel. event rate along $\tan\theta = 2/(\sin\phi + \cos\phi)$*

***Figure 8:** The direct detection rate (arbitrary normalization) for $m_\chi = 100$ GeV. In the panel a: the dashed, dotted, or dash-dotted curves correspond to gauge anomaly freedom constraints. The thick horizontal lines correspond to the isospin limit where $r = 1$. In the panels b, c, d: the vertical grey lines correspond to different values of the isospin breaking ratio r as indicated. The solid grey curve shows the combined rate when both the $A \otimes V$ and the $A \otimes A$ contributions are included while the dashed (blue) and dotted (red) curves show the sole $A \otimes V$ and $A \otimes A$ respectively. They are plotted for the different values of θ and ϕ.*

Unitarity issue of the axial-vector couplings

In our setup, the dark matter particle is a Majorana fermion which leads to consider only the axial-vectorial current on the dark side. It is a well known fact that the purely axial-vectorial interactions violate the perturbative unitarity at large energies, pointing towards the presence of additional new physics to restore the unitarity. The simplest way to restore the unitarity is to assume that the spin-1 mediator is a gauge boson of an additional $U(1)'$ gauge symmetry, and its mass as well as the DM mass are generated by a new scalar field in the dark sector. The need to introduce this scalar field also can be seen from the sum rule in Eq. (2.80). Namely, it can be solved for the case when $v_1 = v_2 = v_3 \to Z'$ and $f_1 = f_2 = \chi$,

$$\sum_{s_1} g_{Z'Z'\bar{s}_1} y^{\sigma}_{s_1\bar{\chi}\chi} = -2m_\chi \left(g^{\sigma}_{Z\bar{\chi}\chi} g^{\sigma}_{Z\bar{\chi}\chi} + g^{\bar{\sigma}}_{Z\bar{\chi}\chi} g^{\bar{\sigma}}_{Z\bar{\chi}\chi} \right) + 4m_\chi g^{\sigma}_{Z\bar{\chi}\chi} g^{\bar{\sigma}}_{Z\bar{\chi}\chi}. \tag{5.9}$$

For the vector interactions, *i.e.,* where left- and right-handed fermion couplings to the Z' are the same, $g^{\sigma}_{Z\bar{\chi}\chi} = g^{\bar{\sigma}}_{Z\bar{\chi}\chi}$, the above equation gives $\sum_{s_1} g_{Z'Z'\bar{s}_1} y^{\sigma}_{s_1\bar{\chi}\chi} = 0$. Hence, in this case no additional new physics is needed to guarantee perturbative unitarity and the mass of the Z' can be generated via the Stueckelberg mechanism [103], for example.

On the other hand for the axial-vector interactions with the coupling $g^A_\chi = g^{\sigma}_{Z\bar{\chi}\chi} - g^{\bar{\sigma}}_{Z\bar{\chi}\chi}$, Eq. (5.9) reduces to

$$\sum_{s_1} g_{Z'Z'\bar{s}_1} y^{\sigma}_{s_1\bar{\chi}\chi} = -2(g^A_\chi)^2. \tag{5.10}$$

Therefore, neither of $g_{Z'Z'\bar{s}_1}$ nor $y^{\sigma}_{s_1\bar{\chi}\chi}$ couplings can be equal to zero.

In general, for the dark matter interactions with $g^A_\chi \neq 0$, bounds on the dark matter and scalar masses can be derived from unitarity, [104],

$$m_\chi \lesssim \sqrt{\frac{\pi}{2}} \frac{m_{Z'}}{g^A_\chi}, \qquad\qquad m_s < \frac{\pi\, m^2_{Z'}}{(g^A_\chi)^2\, m_\chi}. \tag{5.11}$$

$Q^c_{L,i}$	$u_{R,i}$	$d_{R,i}$	$L^c_{L,i}$	$\ell_{R,i}$	χ_R	$d_{R,j}$ or $u_{R,j}$
a	b	c	d	e	x	z

Table 3: *The $U(1)'$ charge assignments of the DM candidate and the SM fermions. i and j denote the generation index and the upper-index c means complex conjugation.*

5.2 Considerations from a UV perspective

A conspicuous path to UV completing the EFT setup in Sec. 5.1 is to augment the SM gauge group by a $\mathrm{U}(1)'$ gauge sector which couples to SM fermions and a DM candidate via a heavy Z', i.e. $m_{Z'} \gg m_Z$. Requiring the Z' to couple axial-vectorially to the DM means that the DM must be chiral under the $\mathrm{U}(1)'$. Apart from the need to Higgs this gauge symmetry, we must assign $\mathrm{U}(1)'$ charges to the DM and SM fermions that make the pure and mixed gauge anomalies cancel (see Sec. 2.5). In the following, we briefly discuss this and other general aspects that arise from considering possible UV completions of the EFT setup and we relegate explicit construction of UV complete models.

Anomaly cancellation. Anomaly-free DM models were discussed in Ref. [31] where, however, the minimal models we consider were not discussed. The general anomaly equations that must be satisfied are given in Eq. (2.67). We denote the charge of the $\mathrm{U}(1)'$ symmetry by Y'. Arbitrary values of the Y' for the SM light fermions and the dark matter are given in Table 3, where $Q^c_{L_i}$ and $L^c_{L_i}$ denote left-handed quark and lepton doublets. As we see below, to achieve the full anomaly cancelation it is enough to charge either the right-handed strange or the right-handed charm quark. The possible solutions of the system of equations in (2.67) can be categorised as following:

1. *Model-I*, where we can charge only the first generation of fermions and the dark matter,

$$z = 0\,, d = \frac{1}{2}(-e-x), a = \frac{e+x}{6}\,, b = \frac{1}{3}(x-2e)\,, c = \frac{1}{3}(e-2x). \tag{5.12}$$

   For example, by setting $e = -x$ we can give non-zero charges only to the right-handed fields, while the left-handed doublets remain neutral. This scenario can be viewed as a simplified version of the model proposed in Ref. [105], where the right-handed neutrinos are introduced and the $\mathrm{U}(1)'$ is defined to be $\mathrm{U}(1)_R$ under which only right-handed SM fields are charged.

	$Q^c_{L,1}$	$u_{R,1}$	$d_{R,1}$	$d_{R,2}$	$L^c_{L,3}$	$\ell_{R,3}$	χ_R
Model-I	0	-1	$+1$	0	0	$+1$	-1
Model-II	$+\frac{1}{2}$	$+1$	0	-2	$-\frac{3}{2}$	0	3

Table 4: *The $U(1)'$ charge assignments to the Model-I and Model-II.*

2. *Model-II* corresponds to the solution with $Y'_{s_R} \neq 0$ and $Y'_{c_R} = 0$,

$$\begin{aligned} & z \neq 0 \,, e = 2x + 3z \,, \\ & d = \frac{1}{2}(-e - x) \,, a = \frac{e + x}{6} \,, b = \frac{1}{3}(x - 2e) \,, c = \frac{1}{3}(e - 2x - 3z). \end{aligned} \tag{5.13}$$

3. *Model-III* corresponds to the solution with $Y'_{s_R} = 0$ and $Y'_{c_R} \neq 0$,

$$\begin{aligned} & z \neq 0 \,, e = \frac{1}{2}(x - 3z) \,, \\ & d = \frac{1}{2}(-e - x) \,, a = \frac{e + x}{6} \,, b = \frac{1}{3}(-2e + x - 3z) \,, c = \frac{1}{3}(e - 2x). \end{aligned} \tag{5.14}$$

In our analysis, we consider only *Model-I* with $e = -x$ and $e = +1$, and *Model-II* with $z = -2$ and $x = 3$. The explicit $U(1)'$ charge assignments for these two models are represented in Table 4. It can be noticed that the structure of the *Model-I* is quite simple and it is discussed in App. C.1 in detail.

Couplings of the Z' to leptons. A feature of the mixed anomaly equations is that charges of the SM fermions are, in general, a linear combination of their hypercharge Y and $B - L$ where $B(L)$ are the baryon(lepton) numbers which are $\pm\frac{1}{3}(\pm 1)$. This general statement has a big consequence, namely, that coupling the Z' to the SM leptons is unavoidable. We note here that charging the right-handed electron under the U(1)$'$ leads to strong bounds from dilepton resonance searches [106] which places a bound on the Z' mass of $m_{Z'} \gtrsim 4.5$ TeV. One way to alleviate this constraint is to charge the τ instead. In this case ditau searches [107] put a bound of $m_{Z'} \gtrsim 2.1$ TeV. This bound was obtained with only 2.2 fb^{-1} and is expected to be stronger given the full dataset, however, this analysis is not yet available at the time of writing.

SM Yukawa couplings. The U(1)$'$ gauge invariance forbids some SM Yukawa couplings. If we only charge the right-handed fermions, then dimension-four Yukawa couplings are forbidden. This can be remedied by inserting powers of S/f and S^*/f, for example, where S

is the dark Higgs and f is the scale where this non-renormalizable term is generated. Upon spontaneous breaking of the $\mathrm{U}(1)'$ symmetry, the factor of $\langle S\rangle/f$ can be absorbed into the redefinition of the Yukawa couplings. Another option is to add Higgs doublets that also carry a $\mathrm{U}(1)'$ charge.

Loop-induced spin-independent cross-section. While spin-independent scattering is absent at tree-level, it can be induced at one loop via two insertions of the axial-vector coupling or by a potential tree-level exchange of a scalar that contributes to the extra $\mathrm{U}(1)'$ symmetry breaking. A heavy scalar mass, which is naturally generated for our choice of gauge coupling, will suppress the latter spin-independent contribution. Furthermore, one could tune the tree-level contribution of the scalar to zero, by extending the scalar sector, such that no scalar couples to both the quarks and the dark matter sector. Loop contributions could mix the different scalars, but the resulting loop suppression should be sufficient to suppress the spin independent contributions to the cross-section for heavy scalar masses. Since these corrections are highly model dependent, we neglecte them in our analysis.

In the following we estimate the rate here using naive dimensional analysis by taking the ratio of spin independent (SI) and spin dependent (SD) cross-sections induced by the scalar-scalar interaction, which is loop induced via internal vector bosons, and the current-current one

$$\frac{\sigma_{\text{SI}}}{\sigma_{\text{SD}}} \sim A^2 \frac{g'^4}{(4\pi)^4} \frac{m_N^2 m\chi^2}{m_{Z'}^4} \sim \mathcal{O}(10^{-11})\,, \tag{5.15}$$

where we used $g' = 0.1$, $m_{Z'} = 1$ TeV, $m_\chi = m_{Z'}/2$, and $m_N = 1$ GeV is the nucleon mass. We also took the mass number of the atomic nucleus (xenon, *e.g.*) to be $A = 100$. The dependence on the dark matter and nucleon masses arises from the required chirality flips in the scalar-scalar operator. Whether one should insert the nucleon mass or Λ_{QCD} is irrelevant to the estimate, but we note that the respective form factors would suppress the spin-independent contribution even further.

5.3 Experimental constraints

5.3.1 Dark matter direct detection search

The DM direct detection experiments search for signals from DM scattering off a nucleus in underground detectors. Since DM particles move in the Galactic halo with $v/c \sim 10^{-3}$, this scattering can be formulated in the non-relativistic effective theory (NREFT) framework. The momentum transfer of the scattering depends on the reduced mass of the DM-nucleus system and on the range of recoil energies, E_R, that the experiments are measuring. E_R is typically in the range of a few keV to a few tens of keV and the heaviest nuclei have masses of $m_A \sim 100$ GeV and this implies that the maximum amount of the momentum exchange

$$q_{\rm max} \lesssim 200 \text{ MeV}.$$

The inverse $1/q_{\rm max}$ can be the same order as nuclear radii $R \sim A^{1/3}$ fm [7]. This implies that the nuclei are not point-like from the perspective of DM and DM-nucleus interactions can not be considered as a contact interaction, and the interaction cross-section should involve momentum dependent form factors [108]. Usually, DM-nucleus interactions are studied under the assumption that a nucleus is just a collection of interacting nucleons, so that the nuclear matrix element of any current operator can be obtained from its one-nucleon matrix element and a nuclear wave function [109]. The momentum exchanged between the constituent nucleons bound inside the nucleus is the same order as the momentum transfer in the DM-nucleus interactions which is much less than the masses of protons/neutrons. Thus, the nucleons remain non-relativistic also after scattering and the nucleus does not break apart. Therefore, the theory of DM interacting with NR nucleons can be described by the Lagrangian

$$\mathcal{L}_{\rm NR} = \sum_{i,N} c_i^N(q^2)\mathcal{O}_i^N, \tag{5.16}$$

where $N = p, n$ denotes either proton or neutron. The possible NR interaction operators can be built by different combinations of Galilean invariant hermitian quantities [110],

$$i\vec{q}, \quad \vec{v}^{\perp}, \quad \vec{S}_\chi, \quad \vec{S}_N, \tag{5.17}$$

where the relative velocity of DM-nucleus system $\vec{v}_\perp = \vec{v} + \vec{q}/(2\mu_{A\chi})$, and $\vec{S}_N$, $\vec{S}_\chi$ are spins of nucleons and dark matter, respectively [8]. The reduced mass is $\mu_{A\chi} = m_A \times m_\chi/(m_A + m_\chi)$.

[7] femtometer (fm) 1 fm^{-1} $\approx$ 200 MeV

[8] Operators involving $\vec{S}_N$ are named as spin-dependent operators.

In principle each operator can have different couplings to protons and neutrons. One could factorise the space-spin and proton/neutron components of Eq. (5.16) by implementing the isospin symmetry, and an equivalent form for the above Lagrangian can be written as

$$\mathcal{L}_{\text{NR}} = \sum_i \left(c_i^0 \mathbb{1} + c_i^1 \tau_3\right) \mathcal{O}_i = \sum_{\tau=0,1} \sum_i c_i^\tau \mathcal{O}_i t^\tau , \tag{5.18}$$

where the isospin state vectors $|p\rangle = (1,0)^T$ and $|n\rangle = (0,1)^T$ and couplings are

$$c_i^0 = \frac{1}{2}\left(c_i^p + c_i^n\right), \quad c_i^1 = \frac{1}{2}\left(c_i^p - c_i^n\right). \tag{5.19}$$

The isospin operators are defined by $t^0 \equiv \mathbb{1}$ and $t^1 \equiv \tau_3$, where τ_3 is third Pauli matrix.
In the NR limit, the DMEFT operators in Eq. (5.4) will match onto combinations of the following operators

$$\begin{aligned} \mathcal{O}_4 &= \vec{S}_\chi \cdot \vec{S}_N \,, & \mathcal{O}_8 &= \vec{S}_\chi \cdot \vec{v}^\perp \,, \\ \mathcal{O}_6 &= \left(\vec{S}_\chi \cdot \vec{q}\right)\left(\vec{S}_N \cdot \vec{q}\right) , & \mathcal{O}_9 &= i\vec{S}_\chi \cdot \left(\vec{S}_N \times \vec{q}\right) , \end{aligned} \tag{5.20}$$

where $\mathcal{O}_4$ and $\mathcal{O}_6$ are even under the parity P, while $\mathcal{O}_8$ and $\mathcal{O}_9$ are $P-$odd operators. Since $\vec{q}$ is proportional to the velocity, the operators $\mathcal{O}_6, \mathcal{O}_8, \mathcal{O}_9$ are suppressed by the velocity.

UV→ NR matching. The matching procedure from $\mathcal{L}_{\text{DMEFT}}$ in Eq. (5.2) onto $\mathcal{L}_{\text{NR}}$ in Eq. (5.18) can be done by using the chiral EFT (ChEFT) approach [111] with an expansion parameter $q/\Lambda_{\text{ChEFT}} \sim m_\pi/\Lambda_{\text{ChEFT}} \sim 0.3$. In the ChEFT framework the hadronisation of the quark-level currents to a single nucleonic current at the chirally leading order can be expressed as

$$\begin{aligned} \langle N' \left|\bar{q}\gamma^\mu q\right| N\rangle &= \bar{u}'_N \left[F_1^{q/N}(q^2)\gamma^\mu + \frac{i}{2m_N} F_2^{q/N}(q^2)\sigma^{\mu\nu}q_\nu\right] u_N \,, \\ \langle N' \left|\bar{q}\gamma^\mu\gamma_5 q\right| N\rangle &= \bar{u}'_N \left[F_A^{q/N}(q^2)\gamma^\mu\gamma_5 + \frac{i}{2m_N} F_{P'}^{q/N}(q^2)\gamma_5 q^\mu\right] u_N \,, \end{aligned} \tag{5.21}$$

where nucleon states $|N\rangle$ and nucleon spinors u_N depend on the incoming/outgoing momenta of the nucleons. The different form factors $F_i^{q/N}$ are defined according to the type of quarks and nucleons. The axial-vector current contains a pseudoscalar hadronic current which corresponds to the light pseudoscalar meson exchanges, particularly the effects of pion and eta poles in $F_{P'}^{q/N}(q^2)$ is

$$F_{P'}^{q/N}(q^2) = \frac{m_N^2}{m_\pi^2 - q^2} a_{P',\pi}^{q/N} + \frac{m_N^2}{m_\eta^2 - q^2} a_{P',\eta}^{q/N} + \dots, \tag{5.22}$$

where the coefficients $a_{P',m}^{q/N}$ are momentum-independent constants. In most cases the form factors are evaluated in the long-wavelength limit, $q \to 0$, but the corrections due to non-zero momentum can be significant in some cases and effects of the light meson poles can not be ignored [102].

Furthermore, in order to extract explicit expressions for the NR Wilson coefficients c_i^N, one should take non-relativistic limits of both DM currents and nucleonic currents. This matching procedure which gives explicit expressions of c_i for protons and neutrons in terms of momentum dependent form factors are implemented in the publicly available `Mathematica` package `DirectDM` [112].

Direct detection constraints. Here, we will discuss derivation of the DD exclusion curves in Fig. 9. For concreteness, we consider the XENON1T experiment with an exposure of 278.8 days $\times$ 1300 kg [28], as well as its future projection with 20 ton$\times$year exposure [113].

The scattering rate $\mathcal{R}$, *i.e.,* the expected number of events per detector mass per unit time, can be expressed as

$$\frac{d\mathcal{R}}{dE_R} = N_T \frac{\rho_\chi m_A}{2\pi m_\chi} \left\langle \frac{1}{v} P_{\text{tot}}(v^2, q^2) \right\rangle, \tag{5.23}$$

where E_R denotes the nuclear recoil energy and $\langle \dots \rangle$ indicates averaging over the halo velocity distribution. In general, the halo average integral should include a lower-bound on the magnitude of the velocity at $v_{\text{min}} = \sqrt{2m_A E_R}/(2\mu_{A\chi})$. N_T is number of target nuclei per detector mass, ρ_χ is local dark matter density, m_χ is the dark matter mass, and m_A is mass of the target nuclei. The transition probability which is defined by the Galilean invariant amplitude, can be organised in a way that factorises the particle and nuclear physics,

$$P_{\text{tot}} = \frac{1}{2J_\chi + 1} \frac{1}{2J_A + 1} \sum_{\text{spins}} |\mathcal{M}|^2_{\text{NR}} \equiv \frac{4\pi}{2J_A + 1} \sum_{\tau,\tau'=0,1} \sum_{\alpha} R_\alpha^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2} \right) W_\alpha^{\tau\tau'}(q), \tag{5.24}$$

where J_χ, J_A are spin quantum numbers of DM and nucleus, respectively. $R_\alpha^{\tau\tau'}$ is the DM particle response functions which are expressed in terms of NR Wilson coefficients, $c_i^{\tau/\tau'}$. All effects of nuclear physics are included in the $W_\alpha^{\tau\tau'}(q)$ function. The summation index $\alpha = M, \Sigma', \Sigma'', \Delta, \Delta\Sigma'$ indicates different contributions from nuclear responses, for example, the spin-independent scattering is encoded in W_M which is at long wave-length $q \to 0$ and just counts the number of nucleons, $W_M(0) \propto A^2$. The response functions $W_{\Sigma''}$ and $W_{\Sigma'}$ measure the nucleon spin content of the nucleus. W_Δ is for the nucleon angular momentum content of the nucleus, while $W_{\Delta\Sigma'}$ is the interference term. The full form of these response functions are implemented in the `MATHEMATICA` code `dmformfactor` [114].

The relevant DM response functions to our case are given in Eq. (C.5). Furthermore, we define a four dimensional column-vector of non-relativistic Wilson coefficients,

$$(\vec{c}^{\,\tau})^T = (c_4^\tau, c_6^\tau, c_8^\tau, c_9^\tau)\,, \tag{5.25}$$

and the transition probability in term of this vector can be rewritten as

$$P_{\text{tot}} = \frac{4\pi}{2J_A+1}\frac{j_\chi(j_\chi+1)}{3}\sum_{\tau,\tau'=0,1}\sum_{ij}\left(M_W^{\tau\tau'}(q)\right)_{ij} c_i^\tau c_j^{\tau'}\,, \tag{5.26}$$

where $M_W^{\tau\tau'}$ is a matrix of nuclear functions and it is given by Eq. (C.6). The Eq. (5.26) can be translated for the individual proton/neutron by relations in (5.19),

$$P_{\text{tot}} = \frac{4\pi}{2J_A+1}\frac{j_\chi(j_\chi+1)}{3}\sum_{r,r'=p,n}\sum_{ij}\left(A_W^{rr'}(q)\right)_{ij} c_i^r c_j^{r'}\,, \tag{5.27}$$

where the $A_W^{rr'}(q)$ matrix is related to the $M_W^{\tau\tau'}(q)$ via Eq. (C.7). The effects of the high energy physics through the matching in Eq. (5.21) are

$$c_4^p = -4\sum_q F_A^{q/p}\mathscr{C}_{4,q}^{(6)}\,, \qquad c_8^p = 2\sum_q F_1^{q/p}\mathscr{C}_{2,q}^{(6)}\,, \tag{5.28}$$

$$c_6^p = \sum_q F_{P'}^{q/p}\mathscr{C}_{4,q}^{(6)}\,, \qquad c_9^p = 2\sum_q\left(F_1^{q/p}+F_2^{q/p}\right)\mathscr{C}_{2,q}^{(6)}\,. \tag{5.29}$$

Considering the matching corrections in Eq. (5.5), we can make the transformation of $\vec{c}^{\,\tau}$ vector in Eq. (5.25) into a vector whose components consist of the above electroweak Wilson coefficients $C_{k,j}^{(6)}$. For example, the transformation matrix K^r for the case of only first-generation quarks is expressed in Eq. (C.9). Finally, the transition probability in terms of $C_{k,j}^{(6)}$ reads

$$P_{\text{tot}} = \frac{4\pi}{2J_A+1}\frac{j_\chi(j_\chi+1)}{3}\sum_{\substack{r,r'=\\p,n}}\sum_{\substack{k,l=\\6,7,8}}\sum_{i,j}\left(\mathcal{K}^{rr'}\right)_{kl} C_{k,i}^{(6)}C_{l,j}^{(6)}\,, \tag{5.30}$$

where $\mathcal{K}^{rr'} = (K^r)^T . A_W^{rr'} . K^{r'}$, and k, l indicate operator number, i, j – generation indices.

Furthermore, the expected number of events with differential rate as in Eq. (5.23) is

$$\mathcal{N} = \int dE_R \frac{d\mathcal{R}}{dE_R} \times (\text{detector mass} \times \text{run time})\,, \tag{5.31}$$

where the integral is taken over recoil energies in the range $E_R \in [3, 40]$ keV [28] to approximate the detector efficiency. Using Eq. (5.30), a generic expression for $\mathcal{N}$ can be written as

$$\mathcal{N} = \sum_{i,j} \sum_{n,k=6,7,8} a_{kl}(m_\chi) C^{(6)}_{k,i} C^{(6)}_{l,j}\,, \tag{5.32}$$

where coefficients $a_{kl}(m_\chi)$ are non-trivial functions of the DM mass and to relate them with elements of the $\mathcal{K}^{rr'}$ matrix one needs to specify velocity distribution function in Eq. (5.23) and perform the integrations over velocity and recoil energy.

In our case, to compute $\mathcal{N}$ we wrote a code in the C programming language which is integrated with the `DirectDM` [112] and `dmformfactor` [114] packages. We used the Maxwell-Boltzmann (MB) function for the DM velocity distribution. The integrations over the velocity and recoil energies were performed numerically by employing the Monte Carlo integration technique. In C, this can be achieved using the GNU Scientific Library (GSL) with *gsl_monte.h*, *gsl_monte_plain.h* and *gsl_monte_miser.h*. In the end, the total rate is obtained by averaging over the natural isotopes of Xenon, which are 1.910 %, 26.401 %, 4.071 %, 21.232 %, 26.909 %, 10.436 %, 8.857 % for ^{128}Xe – ^{132}Xe and ^{134}Xe, ^{136}Xe, respectively.

For the large dark matter mass, $m_\chi > 100$ GeV, when $\mu_{A\chi} = m_A \times m_\chi/(m_A + m_\chi) \to m_A$, the behaviour of the $a_{nk}(m_\chi)$ is quite simple, *i.e.,*

$$a_{nk}(m_\chi > 100 \text{ GeV}) \sim \frac{1}{m_\chi}. \tag{5.33}$$

On the other hand, for the light dark matter mass, the a_{nk} coefficients depend on the dark matter velocity distribution. For example, for the MB velocity distribution,

$$a_{nk}(m_\chi < 100\text{GeV}) \sim x * (m_\chi)^{\sqrt{2}} * \text{Erf}\left[\frac{(m_\chi)^{-2}}{y}\right], \tag{5.34}$$

where x and y are some constants, Erf$[\dots]$ is the 'error function', Erf$[z] \equiv 2/\sqrt{\pi}\int_0^z e^{-t^2} dt$.

In order to generate the exclusion curves in Fig. 9, we used (naive) Poisson statistics assuming zero events in the signal region. This corresponds to signal mean $\mu = 2.44$ at the 90% C.L. [115]. Hence, the condition $\mathcal{N} < 2.44$ defines the excluded area.

5.3.2 Monojet and dijet searches at the LHC

At the LHC DM is searched for via monojets [116, 117] and monophotons analyses [118, 119], which can be interpreted in the context of minimal DM models, cf. [120–122], with the monojet final state typically providing the stronger limits. We consider the recent DM analysis by the ATLAS collaboration [116], where monojet events with large missing energy were used to constrain a minimal model with a vector mediator V, coupling to the DM χ, and the quark q, excluding V with masses around 2 TeV. The quoted limits on axial mediators are very similar.

We imported the simplified DM model from Ref. [123] into MadGraph5_aMC@NLO [124] to generate the WIMP s-channel process $pp \to jV \to j\bar{\chi}\chi$ where j is a jet from initial state radiation, and the DM particle pair $\bar{\chi}\chi$ gives rise to missing transverse energy E_T^{miss} in the detector. The process is implemented at LO in the strong coupling constant. We adopted the NNPDF3.0_LO PDF set [125], and for each event the factorization and renormalisation scales were set to $H_T/2$, with the total hadronic transverse energy $H_T = \sqrt{m_{\chi\chi}^2 + p_{T,j}^2} + p_{T,j}$ where $m_{\chi\chi}$ is the invariant mass of the DM pair, and $p_{T,j}$ is the transverse momentum of the parton-level jet. Events are hadronised using Pythia8 [126], and a fast detector simulation is carried out using Delphes [127]. We considered first the *Model-I* and simulated samples with $20k$ and $50k$ events for the cases with $r = -0.9$ and $r = 0.14$, respectively, for each combination of DM and Z' masses. Similarly, we consider the *Model-II* with $50k$ events .

We apply the kinematic cuts from Ref. [116], which are as follows: $E_T^{\text{miss}} > 200$ GeV; leading jet with $p_T > 150$ GeV and $|\eta| < 2.4$; no more than three additional jets with $p_T > 30$ GeV and $|\eta| < 2.8$; separation between missing transverse momentum and each of the jets $\Delta\phi(\text{jet}, p_T^{\text{miss}}) > 0.4$ (0.6) for events with $E_T^{\text{miss}} < 250$ GeV (200 GeV $< E_T^{\text{miss}} < 250$ GeV). The remaining simulated events were binned in thirteen exclusive signal regions as in Ref. [116] according to their missing transverse energy.

We exclude a parameter space point when the fiducial cross section of the signal in any bin is bigger than its uncertainty at 95% confidence, which is evaluated by adding the total systematic uncertainty of the signal quadratically to the statistical uncertainty of the signal and the overall uncertainty of the background (statistical and systematic[9]) from Ref. [116].

In this way we were able to reproduce with good approximation the exclusion limits for the simplified DM model with couplings $g_\chi = 1$ and $g_q = 0.25$ scanning over several configurations

[9] The systematic uncertainty of the signal is obtained by combining the relative uncertainties from Ref. [116]: luminosity uncertainty 1.7%; cross section scale uncertainty 10%; a PDF uncertainty 5%; PDF choice 10%; 1% to 7% for the jet E_T^{miss} reconstruction, energy scale and resolution; modelling initial and final state radiation 3% to 6%. The scale uncertainty of the signal are neglected. The systematic uncertainties are added linearly, overall systematic and statistical uncertainties are added in quadrature.

of DM and mediator masses [10].

The exclusion limits for the *Model-I* include purely right-handed couplings to quarks and axial-vectorial coupling to DM with $g_\chi = 1$. The couplings to quarks have been chosen in order to keep the overall magnitude of the interaction fixed: the coupling of the left-handed quark doublets is set to zero, $g_L^Q = 0$ ($|C_{6,1}^{(6)}| = 0$), and the couplings to right-handed quarks are defined according to the parameter r while keeping $(g_R^u)^2 + (g_R^d)^2 = (0.25)^2$. With this choice the comparison with the standard benchmarks with $g_q = 0.25$ is straightforward.

Heavy mediators that couple to quarks can be detected at the LHC via their decays into quarks. The most recent analysis by the ATLAS collaboration searching for heavy resonances in dijet final states, uses 139 fb^{-1} data for state-of-the art constraints [32] for mediator masses above 2 TeV. For lower masses we use the results presented in Ref. [128] that are based on 29.3fb^{-1} of data. Here we recast these limits into the considered model, and we estimate a lower limit on the Z' mass of about 2 TeV using the code developed in Ref. [129] except that we use the PDF, NNPDF23_lo_as_0130_qed.

The decay into leptons provides typically stronger constraints. The benchmark scenario of *Model-II* has a non-zero charge for the third generation leptons. Using 19.5-20.3fb^{-1} data of Ref. [130] we can constrain the decay of the heavy mediator into a $\tau^+\tau^-$ pair using the same setup as we use for the dijet constraints.

[10] Here we follow the benchmark values in [116]. In the *Model-I* the couplings g_χ and g_q are related by the corresponding charges of the DM and the quarks. Changing the couplings does not significantly affect the limits we obtain.

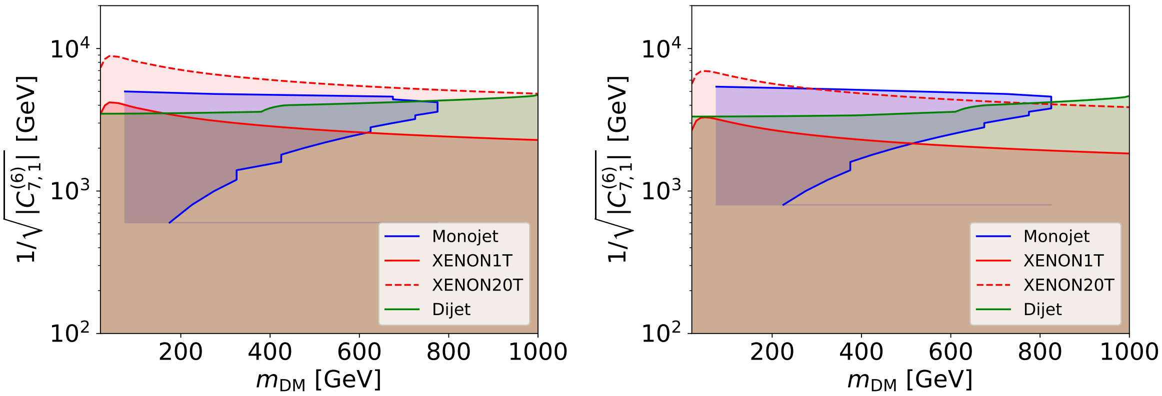


Figure 9: *Exclusion limits for axial mediator couplings to DM from the XENON experiment for 1 ton$\times$year exposure (red solid) and for 20 ton$\times$year exposure (red dashed), from LHC monojet (blue) and dijet (green) analyses with 139 fb^{-1} integrated luminosity. The results are shown in the plane with the DM mass on the x-axis and inverse of the square root of the Wilson coefficient $|C^{(6)}_{7,1}|$on the y-axis for the Model-I with r = -0.9 (left) and r = +0.14 (right), defined as in Eq. (5.8).*

5.3.3 Results

The exclusion curves in Fig. 9 and in Fig. 10 present complementarity between DM direct detection experiments and LHC searches. The inverse of the square root of the Wilson coefficient on the y axis gives direct information on the mediator mass for fixed couplings, and at the same time it provides a convenient model-independent interpretation. The choice of $C^{(6)}_{7,1}$ on the y axis is arbitrary, our reasoning for this choice is that both in the *Model-I* and *Model-II*, the right-handed up quark has non-zero charge. In Fig. 9 we show the exclusion curves for *Model-I* with modified right–handed up–and down–type quark charges. In the model, isospin limit is defined by $r = -1$ in Eq. (5.8), here, however, we take $r = -0.9$ that is very close to the anomaly free solution. This point also coincides to the minimum of the $A \cdot V$ operator in Fig. 8c. On the other hand, the choice $r = +0.14$ is taken because it corresponds to the (approximate) minimum of the grey curve in Fig. 8c where the event rate of direct detection experiments could be suppressed. For this model, we find that collider and direct detection experiments have comparable sensitivity if we do not sit in the singular points of suppressed direct detection rate. Moreover, we find that future direct detection constraints will exclude large parts of the parameter space.

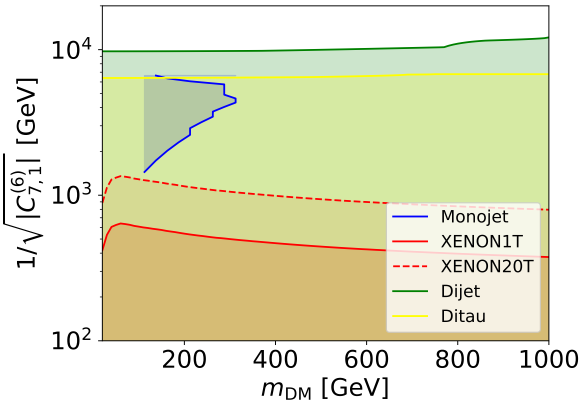


Figure 10: *Exclusion limits for Model-II from XENON experiment for 1 ton$\times$year exposure (red solid) and for 20 ton$\times$year exposure (red dashed), from LHC monojet (blue), dijet (green) and ditau (yellow) analyses with 139 fb^{-1} integrated luminosity. The results are shown in the plane with the DM mass on the x-axis and the inverse of the square root of the Wilson coefficient $|C^{(6)}_{7,1}|$ on the y-axis.*

In Fig. 10 we show the exclusion limits for *Model-II* with gauge coupling $g' = 0.1$ and charges as depicted in Tab. 4. This model has been chosen to minimise the event rate in the XENON experiment, thus resulting in a low direct detection sensitivity. Also the monojet sensitivity gets reduced by the smaller couplings to quarks and to DM. On the other hand, other collider constrains such as the ones from dijet (green curve) and ditau (yellow) searches are here relevant.

6 SUMMARY AND CONCLUSIONS

In this work, we considered a large class of renormalisable extensions of the standard model and derived analytical results that are needed for phenomenological analyses of new physics. In particular, we matched the class of perturbative unitary new physics models onto the $\Delta F = 1$ weak effective Hamiltonian at one loop level. Furthermore, we considered a Majorana dark matter model at tree level. For the one-loop results, we have presented finite and manifestly gauge-invariant matching contributions in generic extensions of the SM. That is, we add to its field content any number of massive vector bosons, physical scalars, and fermions. For a given field content, only a minimal number of couplings needs to be specified because perturbative unitarity of the S-matrix implies that not all couplings can be independent. The constraints on the couplings are codified in the sum rules that arise from Slavnov-Taylor identities which are in turn obtained from the invariance of appropriate Green's functions under BRST transformations.

The one-loop results of this work, the sum rules on the additional couplings and the finite and gauge-invariant one-loop contribution, are implemented in a `Mathematica` package available for download from

https://wellput.github.io.

The package contains generic Wilson coefficients for the $|\Delta F| = 1$ dipole and current-current operators. However, it is in our intension to include flavour-conserving magnetic and electric dipole operators and dimension-six scalar operators [11] as well in the near future.

The code can be integrated with other automated tools, which are designed for the evaluation of different experimental observables in flavour physics, such as `flavio` [25]. In fact, the derived one-loop results are not limited to flavour observables, and they are applicable to other NP models as well, such as DM models where DM interactions with the SM particles are one-loop induced. Furthermore, for the sake of simplicity, we did a phenomenological analysis for DM models where DM couples with the SM particles at tree level via exchange of a vector Z' that is gauge boson of an additional new $\mathrm{U}(1)'$ symmetry. It is assumed that the DM particle is a Majorana fermion which leads to consider only the axial-vector current on the dark side.

There are three $\mathrm{SU}(2) \times \mathrm{U}(1)$ invariant operators that couple the first generation of quarks to an axial vector DM current at dimension-6. Varying the relative magnitude of the three respective Wilson coefficients we only find singular points – as shown in Fig. 8a – where direct detection rate for a Xenon target is significantly suppressed.

[11] Renormalisation of dim-6 scalar operators has not been studied by me/us, see Sec. 4.3

In a model, where these Wilson coefficients are generated via Z'-exchange we find that collider and direct detection experiments have comparable sensitivity if we do not sit in the singular points of suppressed direct detection rate. For typical parameter points that are chosen to be comparable to current experimental benchmark scenarios we find that future direct detection constraints will exclude large parts of the parameter space, see *e.g.*, Fig. 9.

Yet, in a Z' model, anomaly conditions further constrain the allowed parameter space of the Wilson coefficients and the direct detection rate could only be suppressed by considering generation dependent charges. Charging the right-handed strange quark, we found an anomaly free charge assignment, that suppresses the direct detection rate for Xenon direct detection targets. If realised in nature, such a scenario could result in collider signals and direct detection signals for other target nuclei then Xenon.

To conclude, the results presented for the one-loop matchings are published in Ref. [1], while tree-level dark matter phenomenology is in preparation [2]. Also, in the future, we are planning to study loop induced dark matter phenomenology and implement its application into the `wellput` package.

A Photon operators

The unphysical operators, which are vanishing by applying equations of motion of the photon field, are

$$\begin{aligned}
\mathcal{O}_{32} &= \frac{1}{g_s^2} m_b \bar{s}_L \not{D}\not{D} b_R \,, \\
\mathcal{O}_{33} &= \frac{i}{g_s^2} \bar{s}_L \not{D}\not{D}\not{D} b_L \,, \\
\mathcal{O}_{35} &= \frac{ie}{g_s^2} \left[\bar{s}_L \overleftarrow{\not{D}} \sigma^{\mu\nu} b_L F_{\mu\nu} - F_{\mu\nu} \bar{s}_L \sigma^{\mu\nu} \not{D} b_L \right] + \mathcal{O}_7 \,, \\
\mathcal{O}_{36} &= \frac{e}{g_s^2} \bar{s}_L \gamma^\mu b_L \partial^\nu F_{\mu\nu} + \frac{e^2}{g_s^2} (\bar{s}_L \gamma_\mu b_L) \sum\nolimits_f Q_f (\bar{f} \gamma^\mu f) \,,
\end{aligned} \tag{A.1}$$

where $\mathcal{O}_7$ corresponds to the dipole operator in Eq. (4.3). When we use the R_ξ gauge we also have to consider the following set of equations of motion vanishing gauge variant operators,

$$\begin{aligned}
\mathcal{O}_{61} &= -\frac{i}{g_s} m_b \bar{s}_L \left[\overleftarrow{\not{D}} \not{A} - \not{A} \not{D} \right] b_R \,, \\
\mathcal{O}_{62} &= -\frac{1}{g_s} \left[\bar{s}_L \left(\overleftarrow{\not{D}}\overleftarrow{\not{D}} \not{A} + \not{A} \not{D}\not{D} \right) b_L + i m_b \bar{s}_L \not{A} \not{D} b_R \right] \,, \\
\mathcal{O}_{63} &= -\frac{1}{g_s} \left[\bar{s}_L \left(\overleftarrow{\not{D}} \overleftarrow{D}_\mu A^\mu + A_\mu D^\mu \not{D} \right) b_L + i m_b \bar{s}_L A_\mu D^\mu b_R \right] \,, \\
\mathcal{O}_{64} &= -\frac{1}{g_s} \left[\bar{s}_L \left(\overleftarrow{\not{D}} + \not{D} \right) b_L + i m_b \bar{s}_L b_R \right] \partial^\mu A_\mu \,, \\
\mathcal{O}_{65} &= \frac{1}{g_s} \left[\bar{s}_L \overleftarrow{\not{D}} \not{A} \not{D} b_L + i m_b \bar{s}_L \overleftarrow{\not{D}} \not{A} b_R \right] \,.
\end{aligned} \tag{A.2}$$

B Loop Functions

In this appendix we collect the analytical expressions of all loop functions that appear in the final results for the renormalised Wilson coefficients. These functions depend on the masses of the particles inside the respective loop diagrams and on their electromagnetic charges.

B.1 Loop Functions for the Dipole Coefficients

In the limit where no particles are much lighter than the matching scale, we find the functions involving scalars,

$$
\begin{aligned}
F_S^d(x) &= Q_{s_1}\left(\frac{x+1}{4(x-1)^2} - \frac{x\log(x)}{2(x-1)^3}\right) + Q_{f_1}\left(\frac{\log(x)}{2(x-1)^3} + \frac{x-3}{4(x-1)^2}\right) , \\
F_{S'}^d(x) &= Q_{s_1}\left(\frac{2x^2+5x-1}{24(x-1)^3} - \frac{x^2\log(x)}{4(x-1)^4}\right) + Q_{f_1}\left(\frac{x\log(x)}{4(x-1)^4} + \frac{x^2-5x-2}{24(x-1)^3}\right) ,
\end{aligned}
\tag{B.1}
$$

and vectors,

$$
\begin{aligned}
F_V^d(x_0,x) &= f_V^d(x) - f_V^d(x_0) , \\
f_V^d(x) &= Q_{v_1}\left(\frac{11x^2-7x+2}{8(x-1)^3} - \frac{3x^3\log(x)}{4(x-1)^4}\right) + Q_{f_1}\left(\frac{3x^2\log(x)}{4(x-1)^4} - \frac{2x^2+5x-1}{8(x-1)^3}\right) , \\
F_{V'}^d(x) &= Q_{v_1}\left(\frac{3x^2\log(x)}{2(x-1)^3} + \frac{x^2-11x+4}{4(x-1)^2}\right) + Q_{f_1}\left(\frac{x^2+x+4}{4(x-1)^2} - \frac{3x\log(x)}{2(x-1)^3}\right) ,
\end{aligned}
\tag{B.2}
$$

that contribute to the Wilson coefficient of the dipole operator in Eq. (4.14). As stated above, our results agree with Ref. [80] after employing the relevant unitarity sum rule.

In the limit of light fermion particles, $x_0 \to 0$ (see Sec. 4.1.1)

$$
F_V^d(0,x) = x\left\{Q_{v_1}\left(-\frac{3x^2\log(x)}{4(x-1)^4} + \frac{2x^2+5x-1}{8(x-1)^3}\right) + Q_{f_1}\left(\frac{3x\log(x)}{4(x-1)^4} + \frac{x^2-5x-2}{8(x-1)^3}\right)\right\} .
\tag{B.3}
$$

B.2 Loop Functions for the Neutral-Current Operators

We first give the functions that contribute to the Wilson coefficient of the neutral-current operators in the scenario where no light internal particles are in the loop. We start with the first term in Eq. (4.17) that comprises the contributions of internal vector bosons and fermions. We find the following gauge-invariant combination of the photon penguin and the $Z-$penguin

$$F_V^{\gamma Z}(x_0,x) = f_V^{\gamma Z}(x) - f_V^{\gamma Z}(x_0)\,, \tag{B.4}$$

where

$$\begin{aligned} f_V^{\gamma Z}(x) = Q_{f_1}&\left(\frac{14x^2-21x+1}{12(x-1)^3} + \frac{(-9x^2+16x-4)}{6(x-1)^4}\log(x)\right) \\ &+ Q_{v_1}\left(\frac{6x^4-18x^3-32x^2+87x-37}{12(x-1)^3} + \frac{x\left(8x^3-2x^2-15x+6\right)}{6(x-1)^4}\log(x)\right)\,. \end{aligned} \tag{B.5}$$

The terms proportional to Q_f and Q_v originate from the photon penguin and the combination of the $Z-$penguin and the photon penguin, respectively.

In the limit of light fermion particles, $x_0 \to 0$ (see Sec. 4.1.1)

$$\begin{aligned} F_V^{\gamma Z}(0,x) = Q_{f_1}&\left(\frac{2}{3}\log\left(\frac{\mu^2}{M_{v_1}^2}\right) + \frac{x\left(x^2+11x-18\right)}{(x-1)^3} + \frac{(-9x^2+16x-4)}{6(x-1)^4}\log(x)\right) \\ &+ Q_{v_1}\left(\frac{x\left(6x^3-41x^2+77x-48\right)}{12(x-1)^3} + \frac{x\left(10x^3-22x^2+9x+6\right)\log(x)}{6(x-1)^4}\right)\,. \end{aligned} \tag{B.6}$$

The remaining loop functions involving vector bosons are

$$F_{V''}^{Z}(x_0,x,y) \equiv f_{V''}^{Z}(x,y) - f_{V''}^{Z}(x_0,y)\,, \tag{B.7}$$

with

$$\begin{aligned} f_{V''}^{Z}(x,y) = &-\frac{x(y-1)\left(3x^2(y-1)y-10xy+4\right)}{4(x-1)(xy-1)^2}\log(x) + \frac{x\left(2xy^2-2xy+y+5\right)}{4xy-4} \\ &+ \frac{xy\left(x\left(-4y^2-5y+3\right)+y+5\right)}{4(y-1)(xy-1)^2}\log(y)\,, \end{aligned} \tag{B.8}$$

as well as

$$F_V^{L,BZ}(x_0,x,y,z) \equiv f_V^{L,BZ}(x,y,z) - f_V^{L,BZ}(x_0,y,z)\,, \tag{B.9}$$

with

$$
\begin{aligned}
f_V^{L,BZ}(x,y,z) = {} & \frac{xy\left(3x^2y(xy+x-2)-(x-1)z(xy(xy-2)+4)\right)}{4(x-1)(xy-1)^2(x-z)}\log(x) \\
& + \frac{3xy^2(x+(y-1)z-1)}{4(y-1)(xy-1)^2(yz-1)}\log(y) \\
& + \frac{xyz(z(y(z-4)-4)+4)}{4(z-1)(x-z)(yz-1)}\log(z) + \frac{xy(2xy-5)}{4xy-4}\,,
\end{aligned}
\tag{B.10}
$$

and

$$
F_V^{R,BZ}(x_0,x,y,z) \equiv f_V^{R,BZ}(x,y,z) - f_V^{R,BZ}(x_0,y,z)\,,
\tag{B.11}
$$

with

$$
\begin{aligned}
f_V^{R,BZ}(x,y,z) = {} & \frac{xy(3x(xy(xy+x-6)+4)-(x-1)z(xy(xy-2)+4))}{4(x-1)(xy-1)^2(x-z)}\log(x) \\
& + \frac{3xy^2(-4xy+x+(y-1)z+3)}{4(y-1)(xy-1)^2(yz-1)}\log(y) \\
& + \frac{xy(z-4)z(yz-4)\log(z)}{4(z-1)(x-z)(yz-1)} + \frac{xy(2xy-5)}{4xy-4}\,.
\end{aligned}
\tag{B.12}
$$

In addition, we have

$$
F_V^Z(x,y) = \frac{xy}{x-y}\log\left(\frac{x}{y}\right) - \frac{x+y}{2}
\tag{B.13}
$$

and

$$
\begin{aligned}
F_{V'}^Z(x,y) = \sqrt{xy}\Bigg[& \frac{y-4}{2\,(y-1)} - \frac{(x-4)\,x}{2\,(x-1)\,(x-y)}\log\,(x) \\
& + \frac{(x\,((y-2)\,y+4)-3y^2)}{2\,(x-y)\,(y-1)^2}\log\,(y)\Bigg]\,.
\end{aligned}
\tag{B.14}
$$

Moerover, we have the relations

$$
F_V^{L,B'Z}(x,y,z) = F_V^{R,BZ}(x,y,z), \qquad F_V^{R,B'Z}(x,y,z) = F_V^{L,BZ}(x,y,z).
\tag{B.15}
$$

The loop functions involving scalar particles are

$$\begin{aligned} F_S^\gamma(x) = Q_s &\left(\frac{11x^2 - 7x + 2}{36(x-1)^3} - \frac{x^3 \log(x)}{6(x-1)^4}\right) \\ &+ Q_{f_1}\left(\frac{7x^2 - 29x + 16}{36(x-1)^3} + \frac{(3x-2)\log(x)}{6(x-1)^4}\right), \end{aligned} \tag{B.16}$$

$$F_S^Z(x,y) = \frac{1-2y}{2(y-1)} - \frac{y\log(x)}{2(x-1)(x-y)} + \frac{(x-1)\,y\log(y)}{2(x-y)(y-1)^2}, \tag{B.17}$$

$$F_{S'}^Z(x,y) = \frac{\sqrt{xy}}{(x-y)}\left(\frac{x\log(x)}{x-1} - \frac{y\log(y)}{y-1}\right), \tag{B.18}$$

$$F_{S''}^Z(x,y) = \frac{y}{2(y-1)} - \frac{x^2\log(x)}{2(x-1)(x-y)} + \frac{y\,(x\,(y-2)+y)\log(y)}{2(x-y)(y-1)^2}, \tag{B.19}$$

$$\begin{aligned} F_S^B\,(x,y,z) = &\frac{x^2 y\log(x)}{4(x-1)(xy-1)(x-z)} \\ &+ \frac{yz^2\log(z)}{4(z-1)(z-x)(yz-1)} + \frac{y\log(y)}{4(y-1)(xy-1)(yz-1)}, \end{aligned} \tag{B.20}$$

while the loop functions with both vectors and scalars are

$$\begin{aligned} F_{VS}^B(x,y,z) = \sqrt{xz}\Bigg[&\frac{x(xy-4)\log(x)}{4(x-1)(xy-1)(x-z)} \\ &- \frac{3y\log(y)}{4(y-1)(xy-1)(yz-1)} - \frac{z(yz-4)\log(z)}{4(z-1)(x-z)(yz-1)}\Bigg], \end{aligned} \tag{B.21}$$

$$F_{VS}^Z(x,y) = \sqrt{y}\left[-\frac{(y-4x)\log(x)}{4(x-1)(x-y)} + \frac{y\,(x+2y-3)\log(y)}{4(y-1)^2(y-x)} + \frac{5-4y}{4(y-1)}\right], \tag{B.22}$$

$$F_{VS'}^Z(x,y) = \sqrt{y}\left[\frac{x\,(4x-y-3)\log(x)}{4(x-1)^2(x-y)} - \frac{3x\log(y)}{4(y-1)(x-y)} + \frac{1-2x}{4(x-1)}\right]. \tag{B.23}$$

$\Delta F = 2$ loop functions

$$\begin{aligned} F_{S'}^B(x,y,z) = &\frac{2x^2 y\log(x)}{(x-1)(xy-1)(x-z)} \\ &- \frac{2yz^2\log(z)}{(z-1)(x-z)(yz-1)} + \frac{2y\log(y)}{(y-1)(xy-1)(yz-1)} \end{aligned} \tag{B.24}$$

$$F^B_{VS'}(x,y,z) = \frac{2x^{3/2}\sqrt{z}(xy-4)\log(x)}{(x-1)(xy-1)(x-z)} - \frac{2\sqrt{x}z^{3/2}(yz-4)\log(z)}{(z-1)(x-z)(yz-1)} - \frac{6\sqrt{x}y\sqrt{z}\log(y)}{(y-1)(xy-1)(yz-1)} \tag{B.25}$$

$$F^B_{V'}(x,y,z) = \frac{2x^2yz((x-4)y-4)\log(x)}{(x-1)(xy-1)(x-z)} - \frac{2xy^2(4y+3)z\log(y)}{(y-1)(xy-1)(yz-1)} - \frac{2xyz^2(y(z-4)-4)\log(z)}{(z-1)(x-z)(yz-1)} \tag{B.26}$$

$$F^B_{V''}(x,y,z) = -\frac{8x^2y\log(x)}{(x-1)(xy-1)(x-z)} + \frac{8yz^2\log(z)}{(z-1)(x-z)(yz-1)} - \frac{8y\log(y)}{(y-1)(xy-1)(yz-1)} \tag{B.27}$$

u_R	d_R	e_R	χ_R
-1	+1	+1	-1

Table 5: *The charge assignments of the DM candidate and one generation of right-handed SM fermions under the additional $U(1)'$ gauge group. The DM candidate is neutral under the SM gauge group.*

C Dark matter

C.1 A possible UV completion

In our setup, we want a dark matter candidate with purely axial-vectorial coupling to a spin-1 mediator. Since the most minimal additional matter field content — one Weyl fermion charged under a spontaneously broken $\mathrm{U}(1)'$ gauge group — gives rise to a Majorana fermion after spontaneous symmetry breaking, the desired axial-vectorial coupling is automatically guaranteed. Hence, we extend the SM gauge group by an additional $\mathrm{U}(1)'$ which is spontaneously broken by the vacuum expectation value of a scalar field, S, and add one Weyl fermion, χ_R, that is charged under this $\mathrm{U}(1)'$ and is additionally odd under a $\mathbb{Z}_2$ symmetry that remains exact. This fermion is neutral under the SM gauge group.

Since the DM candidate χ_R is chiral under the $\mathrm{U}(1)'$, the gauge symmetry is anomalous (see Sec. 2.5). One simple solution to make it anomaly free is to also charge one generation of right-handed SM fermions under the $\mathrm{U}(1)'$. The assignment in Table 5 is sufficient to cancel all mixed and pure anomalies. However, charging the right-handed SM fermions under $\mathrm{U}(1)'$ forbids their Yukawa terms at dimension four. To write these terms, one needs to include powers of the $\mathrm{U}(1)'$ Higgs, S, suppressed by the same power of the scale M_* where the interaction is generated. Writing in terms of Weyl spinor fields transforming under the $(0, \frac{1}{2})$ representation of the Lorentz group and following the conventions of [131], the Lagrangian describing the fields charged under the $\mathrm{U}(1)'$ is given by

$$\mathcal{L}_{\mathrm{U}(1)'} = \sum_{f=u,d,e,\chi} f_R^\dagger\, iD_\mu\sigma^\mu\, f_R + (D_\mu S)^\dagger\, D^\mu S - \left[\frac{1}{2}y_\chi\, \chi_R\chi_R\, S + h.c.\right] , \tag{C.1}$$

where $D_\mu = \partial_\mu + i\, g\, q'\, Z'_\mu$ is the $\mathrm{U}(1)'$ covariant derivative and $\sigma_\mu \equiv (\mathbb{1}_{2\times 2}; \vec{\sigma})$ where $\sigma_i\ \forall\ i \in \{1, 2, 3\}$ are the Pauli matrices, see [131] and references therein. In order to be able to write the Yukawa term in the square bracket, the $\mathrm{U}(1)'$ charge of the Higgs field S must be $+2$. However, such a choice would require one additional $\mathrm{U}(1)'$ charged scalar with charge $+1$ to allow for the SM Yukawa terms given the charge assignment of Table 5. Thus, a solution that would allow all Yukakwa interactions with only one $\mathrm{U}(1)'$-charged scalar, S, forces us to assign it a charge $+1$. This choice forbids the Yukawa term in Eq. (C.1) at the renormalizable level and all Yukawa terms now arise at dimension-5 in the following way (again, writing in terms of fields that transform under the $(0, \frac{1}{2})$ representation of the Lorentz group as before)

$$\mathcal{L}_{U(1)'}^{\text{Yukawa}} = -y_u\, \widetilde{H}\cdot Q_L^\dagger u_R\, \frac{S}{M_*} - y_d\, H\cdot Q_L^\dagger d_R\, \frac{S^\dagger}{M_*} - y_e\, H\cdot L_L^\dagger e_R\, \frac{S^\dagger}{M_*} - y_\chi\, \chi_R\chi_R\, \frac{S^2}{M_*^2} + h.c.\,, \quad \text{(C.2)}$$

where $\widetilde{H}^a = \epsilon^{ab} H_b^\dagger$ and a, b are $\mathrm{SU}(2)_L$ indices which are made explicit here for clarity while they are suppressed in the equation above where their contraction via the δ_a^b invariant tensor is denoted by $X \cdot Y \equiv \delta_a^b\, X^a Y_b$ with X and Y transforming under (formally) conjugate representations. These higher dimensional operators can be generated at the scale M_* via vector-like fermions with masses of $\sim \mathcal{O}(M_*)$ as shown in Fig. 11. For each of the fermions in Table 5, we require one pair of vector-like Weyl fermions which are neutral under $\mathrm{U}(1)'$ but are otherwise charged under $\mathrm{SU}(3)_C$ or $\mathrm{U}(1)_Y$ as necessary. The vector-like fermion X corresponding to the dark matter candidate χ_R is completely neutral under the SM and the $\mathrm{U}(1)'$ gauge group. However, it must also be odd under $\mathbb{Z}_2$ in order for the DM Yukawa term to respect it.

After spontaneous symmetry breaking of the $\mathrm{U}(1)'$, the DM candidate χ_R only transforms under the $\mathbb{Z}_2$ symmetry as $\chi_R \to -\chi_R$ but does not carry any additional conserved charges. Thus, we can construct a left-handed, $(\frac{1}{2}, 0)$, fermion $\epsilon_{\alpha\beta}(\chi_R^\dagger)^\beta$ with the same quantum numbers as χ_R and, consequently, we can construct a 4-component Majorana spinor as

$$\chi_M = \begin{pmatrix} [\chi_R^\dagger]_\alpha \\ [\chi_R]^{\dot\alpha} \end{pmatrix}\,, \qquad \text{(C.3)}$$

which explicitly satisfies the Majorana "reality" condition $\chi_M^c = \chi_M$, though it is manifestly obvious that this must be so since the 4-component spinor is constructed from only one Weyl fermion. The Lagrangian of this Majorana DM is given by

$$\mathcal{L}_M = \frac{1}{2}\bar{\chi}_M\, i\partial\!\!\!/\, \chi_M + \frac{1}{2}\bar{\chi}_M\, \gamma^\mu\gamma_5\, \chi_M\, Z'_\mu - \frac{1}{2} m\, \bar{\chi}_M\, \chi_M\,. \qquad \text{(C.4)}$$

We note here that charging the right-handed electron under the $\mathrm{U}(1)'$ leads to strong bounds from dilepton resonance searches [106] which places a bound on the Z' mass of $m_Z' \gtrsim 4.5$ TeV. One way to alleviate this constraint is to charge the τ instead. In this case ditau searches [107] put a bound of $m_Z' \gtrsim 2.1$ TeV. This bound was obtained with only 2.2 fb^{-1}

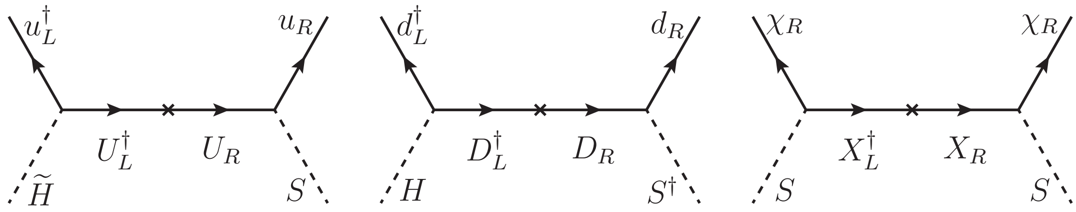


Figure 11: *Generating the dimension-5 Yukawa interactions via vector-like-fermions. The fermion flow reflects the fact that we work with (0, 12)-representation fermions, see [131]. The diagram for the electron is omitted but it can be obtained from the down-quark one by the replacement $d \to e$ and $D \to E$. The vector-like fermions, U, D, E, X are neutral under $U(1)'$.*

and it is expected to be stronger given the full dataset, however, this analysis is not yet available at the time of writing.

Finally, it is worthwhile to make contact again with the discussion of isospin breaking at the end of Sec. 5.1 in the context of the model presented here. As desired, the coupling of the DM to the Z' is axial-vectorial and matching this model onto the EFT of the previous subsection generates the operators $\mathcal{Q}_{2,u}^{(6)}$ and $\mathcal{Q}_{2,d}^{(6)}$ with the ratio of coefficients $r = \mathscr{C}_{2,d}^{(6)}/\mathscr{C}_{2,u}^{(6)} = -1$.

C.2 Formulae for DM direct detection constraints

In this appendix, we will give explicit formulae for some functions discussed in Sec. 5.3.1.

The relevant DM particle response functions [114] to our case are

$$\begin{aligned}
R_M^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\vec{v}_T^{\perp 2} c_8^\tau c_8^{\tau'} , \\
R_{\Sigma''}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{\vec{q}^2}{m_N^2}\left(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}\right) + \frac{\vec{q}^4}{m_N^4} c_6^\tau c_6^{\tau'}\right] , \\
R_{\Sigma'}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{\vec{q}^2}{m_N^2} c_9^\tau c_9^{\tau'}\right] , \\
R_{\Delta}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3} c_8^\tau c_8^{\tau'} , \\
R_{\Delta\Sigma'}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}\right) &= -\frac{j_\chi(j_\chi+1)}{3} c_8^\tau c_9^{\tau'} .
\end{aligned} \tag{C.5}$$

The matrix of nuclear functions is

$$M_W^{\tau\tau'}(q) = \begin{pmatrix} \frac{1}{4}\left(W_{\Sigma''}^{\tau\tau'} + W_{\Sigma'}^{\tau\tau'}\right) & \frac{\vec{q}^2}{m_N^2} W_{\Sigma''}^{\tau\tau'} & 0 & 0 \\ \frac{\vec{q}^2}{m_N^2} W_{\Sigma''}^{\tau\tau'} & \frac{\vec{q}^4}{m_N^4} W_{\Sigma''}^{\tau\tau'} & 0 & 0 \\ 0 & 0 & v_T^{\perp 2} W_M^{\tau\tau'} + W_\Delta^{\tau\tau'} & -W_{\Delta\Sigma'}^{\tau\tau'} \\ 0 & 0 & 0 & \frac{\vec{q}^2}{m_N^2} W_{\Sigma'}^{\tau\tau'} \end{pmatrix}. \tag{C.6}$$

For the individual proton/neutron, the nuclear functions are

$$\begin{pmatrix} A_W^{pp} \\ A_W^{pn} \\ A_W^{np} \\ A_W^{nn} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} M_W^{00} \\ M_W^{01} \\ M_W^{10} \\ M_W^{11} \end{pmatrix}. \tag{C.7}$$

A relation beween NR Wilson coefficients and the UV Wilson coefficients involving only the first generation quarks,

$$\vec{c}^{\,r} = \begin{pmatrix} c_4^r \\ c_6^r \\ c_8^r \\ c_9^r \end{pmatrix} = K^r \vec{C} = K^r \begin{pmatrix} C_{6,1}^{(6)} \\ C_{7,1}^{(6)} \\ C_{8,1}^{(6)} \\ 0 \end{pmatrix}, \tag{C.8}$$

where

$$K^r = \begin{pmatrix} 4F_A^{u/r} + 4F_A^{d/r} & -4F_A^{u/r} & -4F_A^{d/r} & 0 \\ -F_{P'}^{u/r} - F_{P'}^{d/r} & F_{P'}^{u/r} & F_{P'}^{d/r} & 0 \\ 2F_1^{u/r} + 2F_1^{d/r} & 2F_1^{u/r} & 2F_1^{d/r} & 0 \\ 2(F_1^{u/r} + F_2^{u/r} + F_1^{d/r} + F_2^{d/r}) & 2(F_1^{u/r} + F_2^{u/r}) & 2(F_1^{d/r} + F_2^{d/r}) & 0 \end{pmatrix}. \tag{C.9}$$

The numerical values of the form factors entering K^r matrix can be found in Ref. [102].

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