

Marcelo de Carvalho Borba, State University of São Paulo, Rio Claro, Brazil, [marcelo.c.borba@unesp.br](mailto:marcelo.c.borba@unesp.br)  
Kay L. O'Halloran, University of Liverpool, Liverpool, United Kingdom, [kay.ohalloran@liverpool.ac.uk](mailto:kay.ohalloran@liverpool.ac.uk)  
Liliane Xavier Neves, State University of Santa Cruz, Ilhéus, Brazil, [lxneves@uesc.br](mailto:lxneves@uesc.br)

#### **Abstract** (approximately 250 words)

*Dynamic media such as videos stand out as technologies that stimulate the senses in the production of mathematical knowledge, leading to new ways of knowing. Since the analysis of videos produced by students is a relatively new area in mathematics education, we will discuss some research on the field, but we will focus on one case study in order to illustrate how the approach can contribute to teaching and learning mathematics. In this context, we will discuss the results of research that investigates how the combination of semiotic resources to express mathematical ideas in digital videos can enhance mathematical understanding. The starting point of the discussion is the results of a survey in which students of the Distance Education Mathematics undergraduate course of Brazil, produced videos to express mathematical concepts and ideas. The applied methodology was qualitative in nature, and virtual participant observation was adopted to follow the video production process in forums which took place in the virtual learning environment for the course. Systemic Functional Multimodal Discourse Analysis was adopted as a theoretical approach for analysing the combinations of semiotic resources in the videos produced by the students. Detailed analysis of two of the videos produced by the research participants is presented in this article. The multimodal analysis of the videos shows that the multimodal nature of video made it possible for students to combine semiotic resources from traditional written forms of mathematical discourse (i.e. language, images and mathematical symbolism) with audio-visual resources to create dynamic representations of mathematical content, resulting in expanded knowledge and understanding of key concepts and mathematical ideas.*

#### **Keywords**

*Mathematical videos. Semantic expansion. Digital technologies. Distance education. Multimodal analysis.*

#### **Introduction**

The technological revolution of audiovisual communication in contemporary times highlights the educational role of digital media and places new demands on institutions and their teachers to develop methodologies that value dialogical learning and the joint production of knowledge. The process of producing videos with mathematical content enables interaction and sharing of theoretical and technical knowledge between students and between students and teachers. In addition, the mathematical content expressed through video can be constructed using a range of resources that extend beyond formal mathematical discourse, changing the way Mathematics is perceived by students. Research carried out in Brazil (Cetic.Br, 2019) shows that educational videos are already part of an educational routine, helping students to understand the content developed in classrooms and helping teachers to understand student learning and planning their classes. In this context, it is worth asking about the potential of video as a medium for digital mathematical discourse.

In this chapter, it will be reported how the theoretical approach Functional Systemic - Multimodal Discourse Analysis (SF-MDA) (e.g. O'Halloran 2007, 2015) was used to analyse the organization of mathematical discourse in videos produced by Brazilian students in a distance education Mathematics undergraduate course. This analysis was carried out in a research that sought to investigate how students in online mathematics teacher training courses combine semiotic resources to express mathematical ideas in videos. The number of students enrolled in online teacher training courses has grown significantly in Brazil. In this scenario, videos are frequently used in order to make the content of the subjects available to students. The aim of this study was to analyse the potential of videos for the communication of Mathematics and with that goal in mind, the choices and combinations of semiotic resources used in the Digital Mathematical Discourse were observed. Semiotic resources, such as verbal language, mathematical symbolism, images, body language, music, and sounds, are communicative means produced physiologically or technologically in order to generate meanings (van Leeuwen, 2005). It is through these resources that human beings interact with each other and with the world. In fact, according to Santaella (2012), humans use images, graphics, signs, arrows, numbers, lights, objects, sounds, music, gestures, expressions, smells and touch, through looking,

feeling and feeling, communicate and orient themselves in the world.

The combinations of semiotic resources, realized through processes called intersemiosis, allow different nuances to be considered in communication, such as facial expressions and gestures, in addition to the tone of voice in the expression through verbal language, for example. These combined resources produce a meaning different from the sum of the individual meanings of each resource involved in the communicative event. In fact, Lemke (2010, p. 462) explains that the possibilities of meaning resulting from intersemiosis are not merely additive and exemplifies that “text and figure together are not two ways of saying the same thing; the text means more when juxtaposed to the figure, and so does the figure when placed next to a text”.

In Mathematics Education, for a long time, verbal language, mathematical symbolism, and mathematical images were the central resources, both in scientific articles and textbooks, and even in lectures. These three resources have been developed throughout history in such a way that they allow us to describe the world in new ways which proved to be advantageous, when compared to other semiotic resources (O'Halloran, 2011). However, communication is multimodal (Kress, 2010) and digital technologies, especially videos, expand the possibilities of semiotic combinations due to its multisemiotic characteristics which include auditory and visual resources (O'Halloran, 2011). In this sense, it is asked, how can digital videos transform the communication of mathematics? When answering this question, it should be considered that the technology used as an educational methodology has a qualitative influence on mathematical learning (Oechsler & Borba 2020). In this sense, the role of digital videos in constructing mathematical knowledge has become a dimension of interest for the research described here.

According to Neves (2020), videos that express mathematical ideas stand out as a technology that stimulates hearing and vision in the production of knowledge, through combinations of verbal language, mathematical symbolism, and images, with other semiotic resources that are common in audiovisual language, such as sounds, music, scenarios, body expressions, camera movements, among others. These combinations, specifically designed for mathematical discourse, allow associations between the content contained in the video and the emotions of those who watch it. As exemplified by Neves and Borba (2020), a video that shows images referring to applications of Mathematics with the accompaniment of a breath-taking or joyful soundtrack, allows the practical results of this discipline to be associated with something engaging, in addition to modifying the formal scenario with which Mathematics is always associated, bringing it closer to people and thus transforming its public image (Scucuglia, 2014).

Videos have been used in mathematics education for time. However the idea of having students producing videos has only been documented for the past decade, with the approach steadily gaining interest (Borba et al, 2016; Engelbrecht et al, 2020a). With the COVID-19 pandemic, importance of video production has been extended to include assessment (Engelbrecht et al, 2020b; Borba, 2021). Videos have taken such an active role, that video technology has been conceived as a co-actor in productions of knowledge, collectively referred to as 'humans-with-media'. In such a view, humans and digital technology are both seen as having agency (Borba & Villarreal, 2005; Borba, 2021).

In this chapter we will present a case study which aims to understand the pedagogical implications of having students producing videos in online courses, with the help of teachers. We will not review the research on this domain, since this has been done in Oechsler and Borba (2020), neither we will develop further on the view of technology shortly described in the above paragraph. Instead we will present a research approach which has been carefully developed and strongly supported by Social Semiotics (e.g. van Leeuwen 2005).

We will first explain how mathematical digital videos were used in an online distance education course in Brazil. Following this, we describe the context in which the research was undertaken. The Systemic Functional - Multimodal Discourse Analysis (SF-MDA), one of the theoretical approaches which underpins the research, is introduced in the third section. This theoretical perspective has not been commonly adopted in the mathematics education literature. The methodology and procedures are presented in the fourth section, as well as a brief presentation of the data. The fifth section contains the analysis of two videos produced by the research participants. These two videos are representative of the two intersemiotic organizational models identified in the research data. Finally, in the last section, the conclusions are presented.

## **Video production in Distance Education**

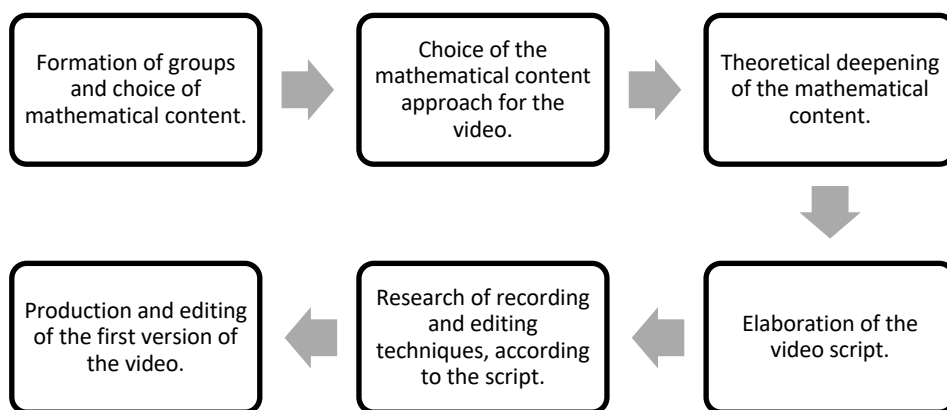
The empirical research reported in this chapter was developed with the participation of 86 students of the pre-service mathematics teacher education from the distance education at the State University of the Bahia, based on a proposal for an activity to produce videos about mathematical content. Distance Education has been spreading in Brazil, with an increased number of online courses for teacher training. With the expansion of such courses in Brazil, Higher Education, in particular, teacher training courses are reaching remote places in Brazil, allowing advances in the quality of teaching in those regions. However, there are many difficulties faced by Distance Education in Brazil, and most of the courses are offered by Federal and State Universities

in Brazil, with funding from the federal government, which also face difficulties due to financial constraints. In this scenario, initiatives around teacher training that allow experiences with methodologies involving the use of technology are urgently required due to social demands and the key role of educational institutions in providing opportunities for citizens to be able to develop to their potential in today's digital society.

Considering this context, students were invited to produce videos about mathematical concepts studied in two courses (i.e. Analytical Geometry and Informatics Applied to Mathematics Education), with support from the researchers. In addition, participants attended the video production forums, promoted by the researchers, together with the subject teacher, in the virtual learning environment of the course in order to obtain help and guidance for their productions. Thus, the virtual forums in the two courses in which the research data were produced were intended to offer students the theoretical and technical support for the production of videos, in addition to being a way of monitoring the production process by the researchers.

Participating students were instructed to form groups to carry out the activity and they produced 30 videos in subsequent semesters for courses in Analytical Geometry in 2016 and Informatics Applied to Mathematics Education in 2017. The second course includes the revision of content such as Flat Geometry, Spatial Geometry, Analytical Geometry and Differential and Integral Calculus using mathematical software for the construction of objects and mathematical analyses. The proposed activity of developing videos with mathematical content was conceived in six stages, as shown in Figure 1. The stages involved: (1) forming groups and choosing the mathematical content; (2) developing an approach for the video; (3) exploring the mathematical content; (4) elaborating the video script; (5) exploring recording and video editing techniques; and (6) editing the first version of the video.

Figure 1 - Video production steps.



Source: Neves (2020)

After the completion of the first two stages (see Figure 1), the participants sent a report to researchers justifying the decisions made for the production of the video. The researchers monitored the video production process through open forums in the course's virtual learning environment. In these forums the students shared experiences about video production techniques and the mathematical content in the video, in addition to discussing the use of videos in Mathematics classes and the combinations of representations for understanding Mathematics. The production of a video reveals stages formed by cycles of theoretical and technical reasoning and a deepening of knowledge arising from the organization and synthesis of the ideas being expressed in the audiovisual format (Borba, Neves & Domingues, 2018). Significantly, the syntheses of ideas about the mathematical content include aesthetic factors: for example, which image best represents this theorem, property or mathematical concept? According to Neves and Borba (2020), semiotic choices for digital mathematical discourse qualitatively condition the production of mathematical knowledge, based on the combinations of semiotic resources which were made. This process enables the reorganization of thought for those who produce the videos and significantly impact those who watch mathematical content on video, leading to the metaphor of 'human beings-with-digital videos' (Domingues, 2014).

Thus, the natural locus of this investigation was the virtual environment in which participant observation was used in forums so that students participating in the research could be guided as to the video production process, in addition to interacting with them in order to share knowledge.

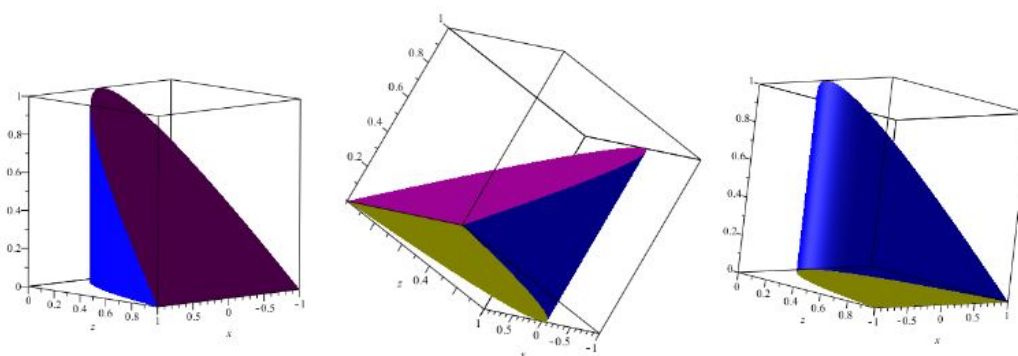
## Systemic Functional – Multimodal Discourse Analysis

Systemic Functional - Multimodal Discourse Analysis (SF-MDA) is a theoretical approach that analyses scenarios involving the production of meanings through the combination of socially constructed resources to

meet the communicational needs of individuals. According to O'Halloran and Lim (2014), SF-MDA aims to understand and describe the functions of these resources as systems of meanings, in addition to analysing the meanings that arise when semiotic choices are made from combinations of these resources. This theoretical approach involves the description of the systems of meanings for the different resources, so that the units of analysis for the meanings that arise through semiotic interactions are specified, considering the context. These resources are called semiotic resources, following Halliday's (1978) social semiotic approach. Semiotic resources are resources which have evolved over time through their use to produce meanings in socially and culturally organized communities (Jewitt, Bezemer & O'Halloran, 2016). Verbal language, gestures, facial expressions, music, sound, graphic images, photographs, paintings, mathematical symbolism, three-dimensional objects, moving images, clothing, scenery, framing, camera movement and space are examples of semiotic resources. With respect to phenomena or events in the semiotic field, O'Halloran (2011) call them monosemiotic if they use a single resource (e.g. a linguistic text), or multisemiotic, when they involve two or more semiotic resources (e.g. a mathematical text involving language, image and symbolism). O'Halloran (2011) also states that a semiotic resource is materialized through a modality, which can be visual, auditory, or somatic. Music and sounds, for example, are materialized by the auditory modality. A gesture or a graph of a function, by visual modality (or by touch in some cases). The somatic modality concerns the physical sensations of the human body in the materialization of semiotic resources, such as touch, smell, and taste, and can be considered, for example, in the production of meanings in Mathematics when activities that use concrete objects are performed. On the other hand, language can be written or spoken, and these are considered to be two different modes, following Halliday (1985). From this, a multimodal phenomenon or event is described as one that involves two or more modalities, although the term "mode" has various interpretations in the literature. The term multimodality is generally used in the field of semiotic studies to refer to the characteristic of social and communicational phenomena that produce meanings from combinations of semiotic resources. In multimodal practices, semiotic resources interact to perform specific functions, considering their different potential in terms of representation and meaning. In fact, as Jewitt, Bezemer and O'Halloran (2016, p. 34) exemplify, "images [...] do not structure and order the world in the same way that language does."

In mathematical phenomena specifically, Friedlander and Tabach (2001) state that the use of language, as well as numerical representation, images and symbolism, combine to make the learning process meaningful. Beyond this, mathematical discourse is constructed using the three semiotic resources: i.e. language, image and symbolism. For example, a type of exercise common in the Differential and Integral Calculation books used in higher education courses in the Exact Sciences area refers to the calculation of the volume of solids limited by surfaces, as illustrated in Figure 2.

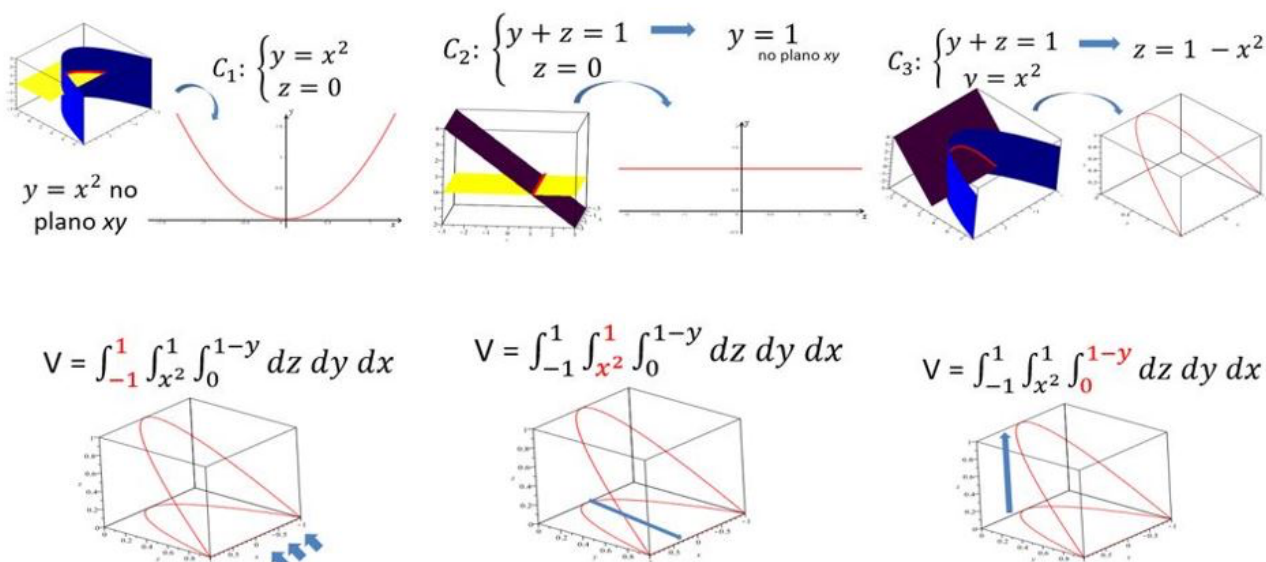
Figure 2 - Solid resulting from the intersection of surfaces.



Source: Neves (2020).

The solid shown in Figure 2 is bounded by the cylindrical surface  $y = x^2$  and plans  $z = 0$  e  $y + z = 1$ . For the development of the volume calculation, algebraic resources (mathematical symbolism) are used to obtain the curves resulting from the intersections of the surfaces, which are the integration limits necessary for calculating the integral. Using the iterated integrals and using the Fundamental Theorem of Calculus, the volume of that solid is calculated. However, for the composition of the integral, it is necessary to carry out a graphic analysis of the solid, which can be seen in Figure 3.

Figure 3 - Construction of the triple integral to calculate the volume of the solid.

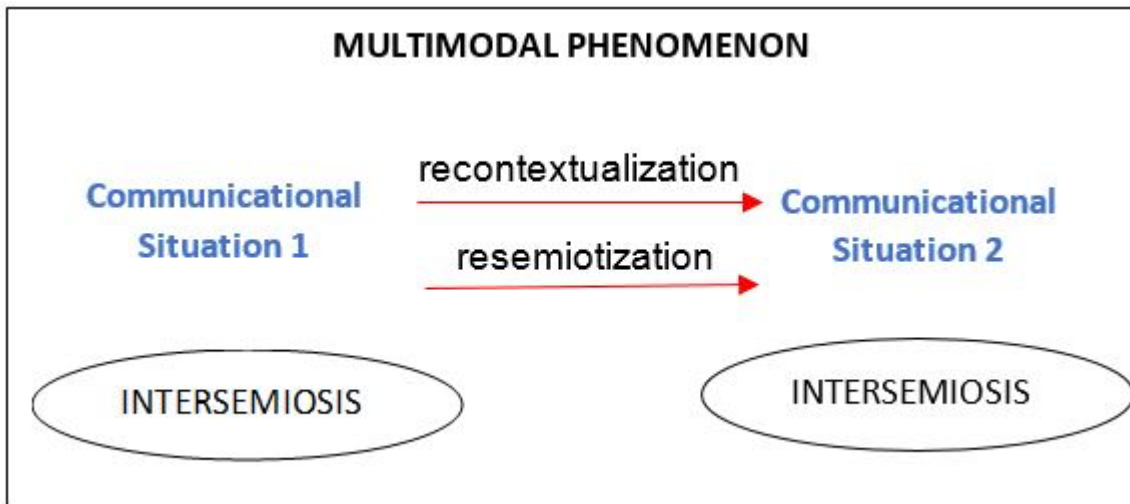


Source: Neves (2020).

In fact, it is necessary to visualize how the surfaces are graphically arranged in three-dimensional space to understand how they intersect two by two, determining the regions that are at the boundaries of the solid and, thus, establish the integral that will result in the volume. In this process, the use of graphic and algebraic resources is complementary, as can be seen in Figure 2, which describes the steps for the composition of the triple integral. Numerical representation is also used in combination with the other two resources to obtain the solution. It is noted that the composition of the integral without the graphic analysis of the solid could lead to a mistake in the order of the integration limits, resulting in an incorrect volume value. In addition, a problem such as this would be accompanied by some linguistic text in the mathematical textbooks. As such, mathematics is a multisemiotic construct, involving the production of meaning resulting from the combination of the different metafunctional meanings of the semiotic resources involved. This combination of resources results in a new sense, which is different from the sum of individual meanings.

Other important concepts in the SF-MDA approach are the ideas of intersemiosis, resemiotization and semantic expansion. Intersemiosis is the processes by which semiotic choices interact and combine to produce meaning, considering the context surrounding the multimodal phenomenon. Resemiotization refers to the reconstruction of semiotic choices within and through multimodal phenomena. According to Iedema (2001, p. 24), "it is through this resemiotization process that the community transposes and reifies its knowledge, techniques and technologies, as well as its interpersonal, social and cultural practices and positions.". According to O'Halloran (2011), the recontextualization process is related to the idea that the meaning of objects and events are not fixed, rather they are modified according to the situation and the cultural context in which they appear. That is, resemiotization, as a possible result of the recontextualization process, enables semantic expansions in Mathematics, as concrete activities being resemiotized in abstract symbolic forms. Semantic expansions are contextualized meanings resulting from intersemiotic processes arising from semiotic choices from different semiotic resources. This notion refers to the multiplier meaning (Lemke, 2010), since it is about the expansion of the semantic power obtained by the integrated use of semiotic resources. Thus, as shown in Figure 4, in multimodal phenomena, recontextualization suggests resemiotization, a change in the choices of semiotic resources that are used intersemiotically in communicational situations.

Figure 4 - Process involving recontextualization and resemiotization.



Source: Produced by the authors.

Multimodal mathematical phenomena involve the traditional semiotic resources of mathematical discourse, such as verbal language, mathematical symbolism and mathematical images, especially graphic images, in addition to other resources, according to the context. The intersemiosis of these resources, according to O'Halloran (2007), results in a semantic expansion in Mathematics which was responsible for its success in rewriting the physical world. Friedlander and Tabach (2001) state that for effective mathematical thinking, one must be aware of the possibilities and limitations of each resource so that the combinations can fill possible gaps in the constitution of mathematical knowledge.

### Semiotic resources in digital mathematical discourse

Jewitt, Bezemer and O'Halloran (2016) state that to understand the functions of a particular semiotic resource in a multimodal phenomenon, it is necessary to pay attention to the functions of the other resources as well. In Mathematics, as already highlighted, three resources are often combined, namely, mathematical language, mathematical symbolism, and mathematical images. O'Halloran (2015) states that these three resources are accessed to construct mathematical reality, each with their own grammatical characteristics which have evolved to fulfill specific functions. According to O'Halloran (2015), Mathematical language is used to contextualize and interpret mathematics in an argumentation-type discourse; mathematical symbolism establishes relationships between mathematical concepts and operations, obtaining results through a grammatical organization that maintains variables and configurations of operations through the use of symbols and conventions specific to that science; and mathematical images, in turn, present themselves as resources that allow the visualization of the relationships established between variables and mathematical operations, making it possible to visualize the mathematical phenomenon as a whole and its parts.

Mathematical language (spoken and written), mathematical symbolism and mathematical images can manifest themselves through various auditory and visual representations in a multisemiotic composition such as videos. Semantic expansions are made both intrasemiotically (e.g. the symbolic derivation of a problem) and intersemiotically with movements between the three semiotic resources (language, image and symbolism). That is, the SF-MDA approach considers the analysis of the meaning that arises from integration of language, symbolism, mathematical visual image, and other resources in multimodal phenomena. These phenomena can present themselves in written, printed, or digital form (O'Halloran, 2007). When presented in written or printed formats, the semiotic resources that make up the multimodal mathematical phenomenon become more restricted, mainly, to mathematical language, mathematical symbolism, and mathematical image. In face-to-face interactions, such as those that occur in classrooms, the possibilities for using semiotic resources are expanded, making it possible to articulate other means to produce meanings in mathematics, such as body movements, music, and sounds. Digital video is characterized by the possibility of combining all of these resources, in addition to other resources of cinematographic language. In this composition, the digital mathematical discourse may become a didactic resource, influencing qualitatively differently in educational actions.

Semiotic resources linked to cinematographic language can be used, just as it is done in cinema, combining logical thinking and emotion, in order to make mathematical ideas more impactful and meaningful. In fact, the types of semiotic choices, such as camera movements, lighting, music, sounds, scenarios, costumes, and other resources are responsible for directing attention and arousing diverse feelings from the content being watched. For example, when the camera distance shows a long shot of the entire environment in which the

object of the action is situated, according to Moletta (2009), this suggests loneliness, isolation or a challenge to be overcome by the character. In contrast, the zoom effect serves to draw attention to a specific object in the image, while the camera movement from top to bottom suggests feeling of oppression and inferiority of the character, who stands as helpless before the universe and from bottom to top denotes superiority of the character. However, these systemic choices must be interpreted in relation other semiotic choices which combine with sounds, music, stage objects, costumes and lighting produce meanings for the viewer.

Considering the recent technological revolution in audiovisual communication, as advocated by Setton (2015) and the use of videos with mathematical content as teaching material by teachers and students in their learning (Borba & Oechsler, 2018), the analysis of the production of meanings in digital mathematical phenomena reveals the potential and limitations of the different resources. The point of view defended here is that the semiotic resources of verbal language, mathematical symbolism and mathematical images combine with the proper resources of cinematographic language enhance mathematical discourse, enabling Mathematics to be presented as a science that is close to people and their daily lives, thus modifying the public image of mathematics as a result (Scucuglia, 2014).

Among the semiotic resources involved in the analyses undertaken in the research described here, body language, specifically, deictic gestures and music, stand out for being present in all videos produced by the participants. Body language is a form of non-verbal communication, in which the individual expresses him/herself through signs such as the look, facial expressions, gestures and body positions. According to Gois, Nogueira and Vieira (2011), body language is loaded with meanings, which are evaluated together, making it possible to reinforce ideas, in addition to favouring or hindering understandings. Facial expressions, as resources that are part of body language, include tensions in the musculature of the face, such as frowning, in addition to variations in the look and smile. This set of resources can emphasize, confirm, or deny points cited in the speech. In multimodal phenomena such as mathematics classes, the teacher's facial expressions have a considerable impact on the students' attitudes, who reshape their response by frowning, or follow a certain line of reasoning after a discrete smile from the teacher approving the argument. These facial expressions can also expose doubts of those who deliver the speech, such as the fact that the interlocutor does not look directly at the camera when making statements, and this also applies to mathematical speech. Gestures help reinforce affirmations, adding visual elements to the multimodal discourse, such as the gesture with the hands parallel and apart to give the idea of the size of an object to which it refers or the use of the index finger to show something for which one wants to reference and associate with a name or concept, or gestures to express the shape of an object referred to in speech.

Farsani (2016) relates the gestures to the movements of the hands or arms that are synchronized with the oral verbal language, enabling the occurrence of a verbal - visual connection. An integral part of mathematical thinking, gestures constitute a mode of communication in the mathematics class, which, according to McNeill (1992, p. 132),

[...] the gesture lets us actually observe thoughts as they occur. Iconic gestures have this power precisely because they are unconstrained by systems of rules and standards. They are not forced, as is speech, to include features solely to meet standards of form. Thus, they can limit themselves to what stands out. Not only are gestures free in this way to incorporate the relevant dimensions in thought, but they also cannot avoid incorporating these dimensions. We ask what forms the gesture, and it is the speaker's construction of meaning at the moment of speaking. The gesture does not manifest kinesic form of its own accord. It cannot help but expose the relevant dimensions of the speaker's thought.

Alibali et al (2013) present two types of evidence of the incorporation of mathematical cognition by gestures, namely, deictic gestures, which establish a reference based on the foundation of mathematical thinking in the physical environment and representational gestures, which reflect simulations of perceptual actions and states. For the case of deictic gestures, the authors exemplify by saying that “[...] if a teacher point to the 3 in  $x^3$  while saying the word “exponent”, a student who is not certain what an exponent is may be more likely to understand the teacher’s utterance.” (Alibali et al, 2013, p. 437). Representational gestures, according to these authors, can be seen when, for example, the teacher uses his hands to portray characteristics of a cube that he had in mind and helps, in this way, students to visualize this cube. The speech-gesture combination is highly effective, according to Goldin-Meadow, Kim, and Singer (1999). These authors state that the additional information presented in the form of gestures can be more easily captured, making the notion described in an abstract way in the speech more concrete, such as the circular movement with the hands while discussing the circular nature of the spheres. The gestures, then, provide a second representation for the notions that are at stake in the discourse.

With regard to music, according to Jewitt, Bezemer and O’Halloran (2016), this resource does not constitute a reality based on easily identifiable actions or even events that are logically connected. On the other hand, when music or sound are combined with other semiotic resources, such as language and images, they result

in the production of meanings (O'Halloran & Lim, 2014). The music and the image, when linked, originate a new sense that establishes a meaning in the audiovisual context from the auditory - visual interaction, which are combined in multiple variables building a new reality, qualitatively different from that defined by the parties. Likewise, verbal language combined with music produces meaning. The intersemiotic expansions obtained through the combinations of music, sound, image, and verbal language contribute with their potential to raise emotions and build a reality through memory.

Moletta (2009) states that music is responsible for the expression of emotions in the audiovisual. In an action scene, music with a sound impact, strong beats and grand chords awakens adrenaline in those who watch. Likewise, melancholy music causes sadness. On the other hand, sound effects, adds the author, bring audiovisuals closer to reality, when in a film, for example, you see the door closing, but the certainty that it is really closed is only due to the sound effect. In this case, the sound effect awakens the memory of something that is real and habitual, and this is part of the cinema's way of operating, which works with the mechanism of apprehending by the senses, of using the memory and concepts already formed to build meaning in accordance with directions provided in scripts.

For Sekeff (2007), music helps perception and stimulates memory and intelligence, and can also assist in the training of linguistic and logical-mathematical skills. In multisemiotic phenomena, the intersemiotic processes that combine image, movement and sound in cinema can seek inferences of sound to reinforce intended meanings and these combinations lead to new productions of meaning, even in films that deal with didactic content, reinforcing the message that can be presented through contextualization.

Audiovisual content that deals with topics related to Mathematics can also benefit from music and sound in the composition of a script. Music can be presented as a backdrop in scenes that aim to evoke emotions, feelings, or states, such as joy, curiosity, commotion, concentration, expectation, or mystery, arousing the attention and interest of those who assist with the production of the video, especially students. Music can also appear in the foreground with its sound resources and lyrics that deal with mathematical content. To participate in the 1st Festival of Digital Videos and Mathematical Education<sup>1</sup>, an event held at São Paulo State University in 2017 that promotes the production of videos that express mathematical ideas by teachers and students as a collaborative action, a high school student produced the rap *Infinito*<sup>2</sup> whose lyrics deal with numerical sets. It is worth noting that for a video produced to express a mathematical idea, there are a variety of possibilities for production with different combinations of semiotic resources in a variety of video formats, such as video lessons, staged videos, stopmotion videos, artistic videos, videos made with manipulative materials and animation with drawings. Considering these possibilities, many semiotic resources can compose different video genres, influencing the meaning that is constructed to express mathematical ideas. O'Halloran (2000) states that each semiotic system is functionally unique in its contribution to the construction of meaning in Mathematics, which means that each resource presents something important about the mathematical element or concept but cannot fully characterize it. With intersemiosis, however, the mathematical discourse can be enhanced, and the understanding of the concept can be constructed so that the features of the resources are complementary. The technologies contribute so that the functionalities of the resources are explored in an effective way, mainly with regard to the functions of the mathematical images and accompanying spoken language (monologue, dialogue or voice-over narration).

In the video, other semiotic resources can be integrated into the analysis presented in the aforementioned research, in addition to those already mentioned, namely, mathematical images (graphs of functions), numerical representation (table with numerical data) and mathematical symbolism (algebraic expression of functions). A particularly interesting video resource that can add a lot of value to this discussion is the dynamic image. The movement of functions guided by mathematical transformations visually formulates the mathematical relationship that describes each function in the family of functions presented. This effect can be achieved with software animation resources, but the video stands out as a resource that can explore mathematical ideas like this by allowing elements such as music, verbal language, and various types of visual effects to be integrated, enabling semantic expansions.

In the current scenario of Mathematics Education, videos appear as a multimodal methodology for expanding the realm of mathematical discourse. The possibility of joining language, image, gestures, costumes, music, sound, space, and action, among others, manifesting them in visual and auditory formats, allows intersemiosis to occur in order to enhance the message that promotes a mathematical idea. Borba, Scucuglia and Gadanidis (2018) argue that videos should be taken into the classroom because this is the way in which the new generation communicates, in addition to allowing mathematical discourse, specifically, to present itself in aesthetic ways that would be impossible using only verbal language. O'Halloran (2004), agrees with these ideas and states that the technological and social configuration resulting from the popularization of technological resources stimulates new patterns of discourse characterized by the construction of meanings through the combination of different semiotic resources, influencing the modes of communication through linguistic and cultural diversity. With respect to the influence of technological development, O'Halloran (2011)

---

<sup>1</sup>Available in <[www.festivalvideomat.com](http://www.festivalvideomat.com)>

<sup>2</sup> Available in <<https://youtu.be/QF2RQQu666A>>



states that the new reality created by computational technologies promises an expansion in the semiotic field, beyond the limits of the scientific revolution, which rewrote the world in mathematical terms. The new technologies allow different interactions of semiotic resources that are beyond combinations in written texts and oral discourse.

In the video production process to express mathematical ideas, semiotic choices condition the production of knowledge (Borba & Villarreal, 2005) by the structure of the message presented. Video, a technological artifact, allows different forms of intersemiosis, producing meanings and enabling the transformation of mathematical knowledge. The media producers are co-authors of the meanings produced as a result of intersemiosis. These potential meanings are understood from the functionalities of the semiotic resources involved in the multimodal phenomenon, which characterizes, in this context, video producers as co-authors in the collective that produces knowledge, human beings-with-digital videos (Domingues, 2014). Thus, digital technologies contribute to multimodal studies by expanding the possibilities of intersemiosis, in this case for the construction of mathematical knowledge. In SF-MDA, the aim is to elucidate how meaning can be constructed through the combination of multiple semiotic resources in multimodal phenomenon, such as those presented on video. This analysis leads to a systematization of knowledge related to the study of educational videos, contributing to this theoretical approach.

From the analysis of the 30 videos produced in the research, two types of intersemiotic processes involving the organisation of the resources of the cinematographic language and the resources of verbal language, mathematical symbolism, and mathematical images were identified. Semiotic choices for intersemiosis presented clear objectives in the mathematical narratives produced in the videos. In the next sections, analysis of two representative videos of each of the identified intersemiosis organization models is presented.

## **Methodology and Procedures for video research**

The analysis aimed to show how the research participants combined semiotic resources when producing videos about mathematical content, taking into account the individual (i.e. 'intrasemiotic') functionalities of the semiotic resources chosen in the video production. In the next step, the analysis was based on the potential for producing meanings from the intersemiotic combinations of these semiotic resources. The objective is to understand "how it is done", and thus follows a more descriptive response based on the subjects' actions, which also defined the procedures to be adopted, leading to a qualitative research design (Bogdan & Biklen, 2006; Creswell, 2014).

This investigation revolved around the question: How do distance education graduates combine semiotic resources when using videos to express mathematical ideas? This question involves an analysis of the nature of the relationships between the usual mathematical representations, namely, language, symbolism, and mathematical images, with other semiotic resources present in the videos produced by the subjects when they express mathematical knowledge.

The research was undertaken in the virtual learning environment provided by the distance education mathematics course in the Distance Education programme in which the participants were enrolled. The production of the videos was monitored in the forums provided in this virtual environment. The records of interactions in these forums were constituted as research data. The researchers observed and dialogued with the subjects about technical issues relating to the video production process, the mathematical content, and the use of videos in the teaching of Mathematics so that they could reflect on that, since they were teachers in training. During these interactions, participants shared experiences about video production. The data resulting from these interactions provided the basis for investigating the research questions in this study, addition to analysis of the videos themselves.

The procedures adopted considered the limitations and potential of communication in the virtual learning environment in the distance education course. In fact, the temporal and spatial limits could be mitigated, given that other instruments were involved in the data collection. For example, the students held face-to-face meetings while producing the videos and, in these meetings, planned by the researchers, the video scripts were prepared. Detailed reports of these meetings were made, providing important sources of information about the design choices in the videos. The reports, scripts, videos produced and interactions in the forums of the virtual learning environment were the data used in this research. In summary, the procedures used resulted in three types of data to be compared in the analysis, namely, thirty videos with mathematical content, records of students' posts on video production forums, content of scripts and reports with the description and justification of the decisions taken regarding the production of videos by groups formed by participants.

In the data analysis, the three sources of information (i.e. the videos, student posts and the student interaction data), were collated in order to identify and interpret the semiotic choices made by the students participating in the research. Assuming that some videos could bring more elements for analysis than others when being examined under the theoretical lens adopted in the research, it was decided to perform a first level analysis of the videos produced in the research. As a result of this analysis at the first level, a smaller number of videos were selected for a more detailed analysis of the data, which are representative in relation to the total videos

produced.

Among the advantages offered by data in video format, Powell, Francisco, and Maher (2004) point out the possibility of researchers to view recorded events as often as necessary, in addition to considering different formats of images for this observation, such as the format in slow motion or frame by frame, making subtle nuances in speech as well as non-verbal behaviors visible, provides a rich data source. The detailed analysis of the video images also makes it possible to perform interpretations from multiple perspectives.

The first level analysis of the videos followed some of the steps of the video analysis model presented by Scucuglia (2012) in an adaptation of the model by Powell, Francisco, and Maher (2004). The steps for video analysis proposed by Scucuglia (2012) are (a) visualization and description, which refers to the repeated visualization of the videos considering multiple points of view for familiarization with the data, followed by the elaboration of a descriptive table of the videos containing information considered important by the researcher; (b) coding, which is related to the identification of themes that help the researcher to interpret the data; (c) critical events, referring to the moments that confirm or disprove research hypotheses and, in this case, directed to the moments when key intersemiotic processes occur in the videos; (d) transcription of critical events; and (e) construction of the plot and composition of the narrative, relative to the contrast of the result of the codification and interpretation of critical events with other research data.

Thus, the videos were viewed repeatedly, resulting in a descriptive table containing information about all videos produced in this study. The different semiotic resources involved in the multimodal phenomenon and the way they were combined were the two main criteria for choosing those that would be selected for the second analysis. In addition, technical information present in the descriptive table highlighted factors that led to some videos not being considered in the second part of the analysis involving the SF-MDA approach, such as poor visibility, sound interference and exceptionally long video time. Another strategy was to eliminate videos that used semiotic resources already present in other videos which were considered more comprehensive in relation to the combinations of these resources.

The repeated viewings of the videos functioned as a magnifying glass that raised expectations regarding the semiotic potential of resource combinations, but a more in-depth analysis, based on SF-MDA, was necessary to confirm the preliminary assumptions. The videos that stood out, according to the analysis at the first level, were five: Practical Use of Analytical Geometry; Displacement made in a wheel; Civil Construction; Sine and Cosine Functions in GeoGebra; and Rosacea. According to Marshall (1996), this was a representative sample which was considered the most productive sample to answer the research question.

The five selected videos were analyzed according to the SF-MDA approach, with the aim of investigating the functions of mathematical language, mathematical symbolism, mathematical images, and other resources used in the videos, as well as their interactions in order to produce meanings. The videos present different types of images, body language, music, sound, scenarios, camera movements, moving images and different types of plans as resources used by students in the videos. The in-depth analysis revolved around the ways in which these resources are integrated, based on the functions they exercise in the specific contexts in the videos. In addition, the possibilities of semantic expansion from intersemiotic processes were analysed. The analysis was carried out from the comparison of the videos with the reports obtained from the discussions held in the forums, the reports prepared regarding the face-to-face meetings of the groups of students and the scripts of the videos. This allowed for a more rigorous analysis, based on the triangulation of the data produced.

The SF-MDA theoretically supported the analysis of the videos produced. This approach involves the analysis of semiotic resources based on the description of the systems through which meanings are made (e.g. linguistically, visually and aurally) so that the units of analysis for the meanings that arise through semiotic interactions are specified according to the context. As a result of the in-depth analysis, two forms of organization of the mathematical discourse in videos were identified by the participants, called the 'Intersemiosis Model 1' and 'Intersemiosis Model 2'. In the next section, these two models are discussed, based on two examples of videos produced in the research.

## **Analysis of the “Civil Construction<sup>3</sup>” and “Practical Use of Analytical Geometry<sup>4</sup>” Videos**

The Civil Construction video presents mathematical content about the distance between points, focusing on the discussion of a problem in which one wants to calculate the height of the vertex of a triangular roof in relation to the slab line (Figure 5). The participants used notions of vectors, such as the formula of the angle between vectors, to determine the height. The preliminary analysis of the Civil Construction video revealed the use of oral and written verbal language resources, mathematical symbolism, numerical representation,

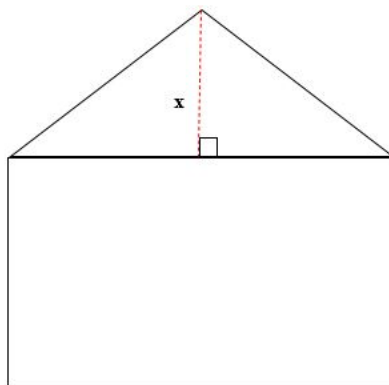
---

<sup>3</sup> Available in <<https://youtu.be/2wjTnFpv3oM>>

<sup>4</sup> Available in <[https://youtu.be/Rbl\\_CN1YdCM](https://youtu.be/Rbl_CN1YdCM)>

mathematical images, moving images, deictic gestures, and music.

Figure 5 – Image representing the problem proposed in the video.



Source: The authors.

The applicability of Analytical Geometry translates as the motivation for discussing the mathematical content in the video, showing how Mathematics is connected to elements of everyday life. In the introduction, students cite aeronautics, graphic design, mechanics, and civil construction as areas in which Analytical Geometry can be applied and state that the video content focuses on its application in civil construction. To introduce the theme, the semiotic resources of music, verbal language, different images and moving image were combined in such a way that the idea that something dynamic would be presented in order to modify the view of those who watch on Mathematics, making it more accessible. The first part of the video combines images recorded in a building with slide shows, while the second part focuses on recording with a cell phone the resolution of the problem about distance between points on a sheet of paper (Figure 6).

Figure 6 – Contextualizing the problem proposed in the video.



Source: Research data.

The mathematical speech in the video involves a problem that deals with the calculation of the distance between two points and to solve the proposed problem, verbal language resources, mathematical symbolism, and different types of images were combined in such a way that the information inherent to each one, together, produced a broader meaning around the mathematical concept in question. The deictic gestures were used intensively to associate the calculations developed from the mathematical symbolism, using the concept of vectors, with the image that represented the house with the triangular roof (Figure 7). From this image, the vertices, angles, and sides of the triangle that represents the roof are identified by the use of deictic gestures.

Figure 7 – Combination of deictic gestures, image, and mathematical symbolism.



Source: Research data.

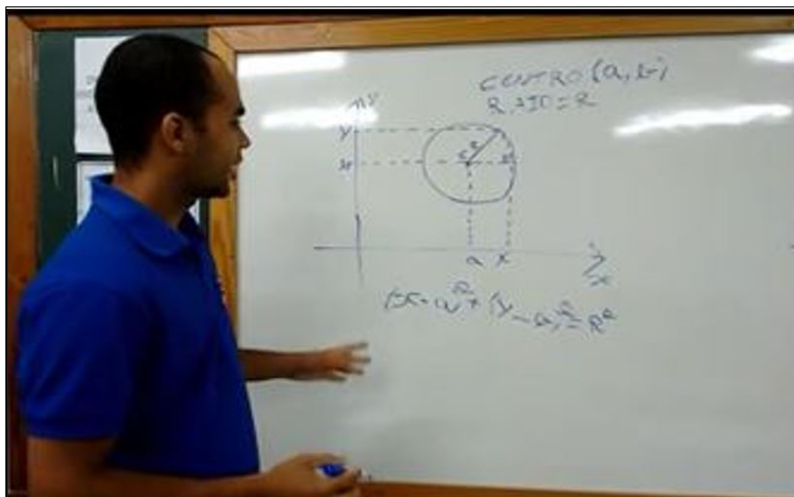
In order to relate the Vectors content to practical everyday problems, the participants of the group responsible

for the video used the resources of oral verbal language, static images, moving images and music. These semiotic choices were made so that the features of the resources could complement each other. In the video introduction, the verbal language had its functionality aimed at explaining the applicability of Analytical Geometry in order to make the topic interesting. According to the reports of the participants, the objective was to show how mathematics is close to people's everyday reality and can assist in solving practical problems in their daily lives. The static images used in the introduction can contribute with visual elements that represent the existence of activities outside of mathematics itself. Music and moving images contribute to making the environment for mathematical speech more engaging. The instrumental music chosen harmonizes with verbal language in an informal tone, bringing emotion as a variable in the promotion of mathematical discourse. Moving images reaffirm the context in which the discourse is inserted, as it introduces real images related to the theme of the problem proposed in the video into the discourse. The aesthetics resulting from the care with the images add value to the introduction of the video, the suspense caused by the door that opens and the image of a slab without the roof shows that there is a real problem that can be solved with the manipulation of concepts of Analytical Geometry. The possibilities around the production of meanings in the introduction of the video are characterized by the idea of the specific applicability of the context that the video deals with. Analytical Geometry, which is generally associated with Conics and Surfaces, forms another mental image, linked to other areas, but specifically, to civil construction. In this analysis, the multisemiotic characteristics of video as a technology that allows moving images to be mixed with static images, thus enabling semantic expansions by combining these resources connecting the mathematical theme with real images that represent situations experienced by the participants, is evident. Oral verbal language, mathematical symbolism, numerical representation, and mathematical image had their semantic potential optimized when used in conjunction with deictic gestures in the second part of the video, which involves solving the proposed mathematical problem. In the video, the usual functions of the first resources were used: that is, the language was used to present the context of the proposed mathematical problem and the image of the house represents the scenario in which the problem is immersed, thus contributing to the development of mathematical relationships expressed through symbolism. Numerical representation was prioritized in solving the problem, and the formula for the angle between vectors used was deduced from the interaction of oral verbal language with the mathematical image and deictic gestures. The combination of gestures in this interaction helped to clarify and complement the speech expressed by oral language. The combination of these semiotic resources can help to understand the relationships established between the variables and expressed in the formula used to solve the problem. The intersemiotic relations resulting from the combination of oral language with mathematical images and deictic gestures promoted the materialization of vector notions, such as the subtraction operation between vectors, the modulus of a vector, and the angle between two vectors. The use of gestures was essential for the expression of mathematical ideas, allowing the semantic expansion from the concepts that involve the variables represented in the formula to be used in combination with the image and oral verbal language to solve the problem.

In summary, the three main resources of mathematical discourse, when combined from organized semiotic choices, operated intersemiotically so that the predilections between visual and auditory resources led to a new meaning for ideas related to vector content. Gestures had an intense contribution to the formation of concepts, in addition to promoting the idea of their applicability by expressing mathematical concepts implicit in the context. In this video, it is clear that different semiotic resources were combined to enhance the mathematical discourse. The gestures reinforce and complement the oral verbal language and contribute to the construction of the formula with the use of mathematical symbolism. The mathematical image presents the context of the problem and highlights elements that form the formula for solving the problem. There is an effort to show the practical utility of the angle between vectors formula. The music invites those who watch for this experience, in which a relationship is established between Mathematics and civil construction.

The purpose of the video Practical Use of Analytical Geometry, according to the participants who produced it, is to show that concepts of Analytical Geometry can be used in practical activities. Specifically, the research participants justify that the use of the equation of the circumference helps in the study of large structures with circular shape (Figure 8). In order to show this, they present a lesson on the equation of the circumference in the video. In the beginning, the importance of the content is justified when the student argues that the compass is used to draw circles, however it is not possible to use this instrument to draw large circles. To validate their argument, the participants present an image of the Guangzhou Building still under construction, which is in the province of Guangzhou, China. The Guangzhou Building is 138 meters high and has a circular shape.

Figure 8 – Intersemiosis in the deduction of the circumference equation.



Source: Research data.

When presenting a generalized circumference in the Cartesian plane constructed on the whiteboard, a brief reading of the image is performed with the indication of the center of the circumference  $(a, b)$  and any point belonging to it  $(x, y)$ . The student continues with the lesson by deducing the equation of the circumference from this image and then resolves two examples; in the first he deduces the equation of a circumference with center at point  $(2, 3)$  and radius  $R = 5$  and in the second he considers the circumference whose equation is  $(x - 2)^2 + (y - 3)^2 = 5^2$ , checking if the point  $(5, -1)$  belongs to it (Figure 9).

Figure 9 - Intersemiosis with image, mathematical symbolism, and gestures.

$$(x - 2)^2 + (y - 3)^2 = 5^2$$

$$(x - 2)^2 + (y - 3)^2 = 25$$

The point  $(5, -1)$  belongs to the presented circumference:

$$(5 - 2)^2 + (-1 - 3)^2 = 25$$

$$(3)^2 + (-4)^2 = 25$$

$$9 + 16 = 25$$

$$25 = 25$$

Source: Research data.

After the first example, an image of the National Stadium of Brasília Mané Garrincha is presented. The video ends with the presentation of a problem that involves calculating the height of the columns that support a roof that has the shape of a semicircle. At the end of the video the image of a building whose roof is shaped like a semicircle is presented. The preliminary analysis of the video Practical Use of Analytical Geometry revealed the use of oral verbal and written verbal language resources, mathematical symbolism, images (graphics and photographs), music and body language (facial expressions, deictic gestures and representational gestures, posture).

Participants expressed in the forums of the virtual learning environment that one of the potentialities of using videos in Mathematics classes is the ease of relating the mathematical content to practical problems. The students use two types of images in the video: the images of real constructions with circular format presented at the beginning of the video, which were articulated with the written verbal language and the music for the production of meaning, and the mathematical images, which enable the production meaning by its articulation with oral verbal language, mathematical symbolism, and representational and deictic gestures.

The images of the buildings with the enunciated characteristics that served as problematization of the video's theme combine intersemiotically with the music and with the written verbal language, breaking the formal atmosphere that is dominant in the video due to its instructional characteristics (Figure 10). The music together with the images of the Guangzhou Building, still under construction, the National Stadium of Brasília Mané Garrincha and the building with a semicircle-shaped roof, transport the viewer out of that environment suggesting that using Mathematics in practical problems may be something entertaining. This sensation is induced by the melody of the music and complements a meaning produced also by the idea of the grandeur of the constructions of the images, given that they bring information regarding their sizes and capacities. In the last photo, which presents the building with a roof in the shape of an arc of circumference, the image does

not contain information in written verbal language, but the angle at which the photo of the building was taken expresses the grandeur of its size. In this analysis, the multisemiotic nature of video as a digital technology is again evident, allowing the introduction of images in cuts made in the instructional class scene. In turn, this enables semantic expansions by combining these resources to express the idea that Mathematics is engaging, in addition to being a formal and rigorous discourse.

Figure 10 – Practical context inserted in the formal environment of the video.



Source: Research data.

The semantic potential of language, mathematical symbolism, and images were optimized when used in conjunction with music and deictic and representational gestures. In the video, the usual functions of the first three resources were used: that is, the language was used to contextualize the proposed mathematical problem, the images on the whiteboard represented the circumference and the images of real constructions with a circumference format showed the mathematical relationships by referring to the content expressed through symbolism which was used to deduce the equation and the relationship of the content to practical problems. Mathematical symbolism was prioritized in solving the problem, and the equation was deduced from the interaction of the mathematical image of a generalized circumference, considering the center, the radius, and any point, with mathematical relations and operations. The combination of gestures in this interaction helped to clarify and complement the speech expressed by oral language, as when the student writes the equation  $(x - a)^2 + (y - b)^2 = R^2$  and confirms the meaning of the variables by pointing to the coordinates  $(a, b)$  from the center of the circle, to the radius  $R$  and to any point  $(x, y)$  on the image of the circle drawn on the board.

The image of the circumference with the inscribed right triangle helps to understand the relationships established and expressed in the equation. This is an important point in the analysis of the combination of semiotic resources, since the intersemiotic relations resulting from the combination of oral language with images mathematics and deictic gestures promoted a materialization of the abstract notions that were being treated in the discourse. The effect of intensifying the use of gestures for the concretization of mathematical ideas, together with the images of real constructions, enables the semantic expansion to solve a real problem. In the video, different semiotic resources were combined in the mathematical discourse. Gestures, music, oral and written verbal language, mathematical symbolism in algebraic form and graphic and photographic images complement each other in the video, in order to express a mathematical idea. There is an effort to show the practical utility of the circumference equation in the video and the music draws attention to the images of large constructions in the shape of circumferences, establishing a relationship between mathematics and civil construction demonstrating the utility of mathematics. The images assume their role, reinforced by the gestures, by establishing relations that define the equation of the circumference from the Pythagorean theorem.

## Discussion

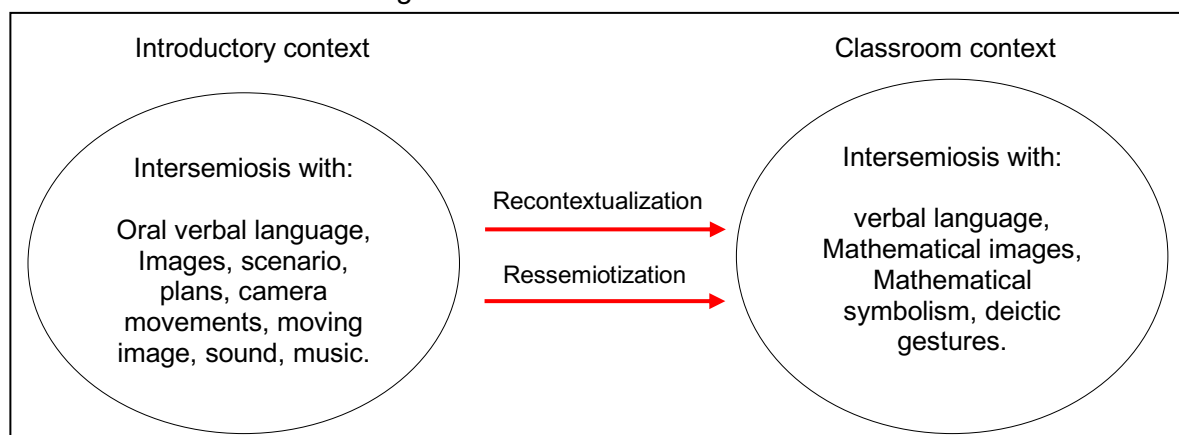
In the examples presented in this chapter, students used two different methods, or intersemiotic models, for representing the mathematical content: one that presents Mathematics in a situation of applicability and another that shows Mathematics in a classroom context. These two intersemiotic models are based on the choice of semiotic resources, as well as the way they recontextualize videos. In the first model of intersemiosis, the processes were carried out at two different moments separated by recontextualization. The Civil Construction video represents this model, in which there is an introductory context of applicability, followed by a moment of theoretical explanation, closer to what happens in the classroom. The second model of intersemiosis is configured by supporting, from the beginning to the end of the video, a classroom context

in which, at specific moments, contexts related to the applicability of mathematical content are inserted, momentarily changing the atmosphere created in the fixed context. The second model is represented by the video Practical Use of Analytical Geometry.

In the first model, the context is initially introduced in a format of an invitation to the viewer. In this invitation, the central idea is to arouse motivation and interest, presenting situations in which mathematics is close to people, being part of everyday life, as in the calculation for the construction of a roof in order to cover the slab of a house, as in the video Civil Construction. Specifically, in the first context the theme of the video is introduced with the presentation of the problem and cinematographic resources are used more intensively with well-defined features: the sound and the scenery, introduce elements that make the speech true, that is, closer to something real; the movements of the camera and planes, highlight visual elements considered important in the discourse at specific moments when oral verbal language is used; the moving image validates the context in which the mathematical problem is inserted, presenting real images that refer to the context in which the problem is inserted; oral verbal language explains the relationship between mathematics and the context created; representational gestures materialize elements of the speech represented by oral verbal language and music, which inserts the element of emotion into the multimodal phenomenon, allowing associations to be built between the mathematical content and the emotions suggested by the musical elements, such as rhythm and melody. These resources were present in the first part of the video Civil Construction, creating a context of the applicability of Mathematics in which intersemiosis produces meaning, different from the sum of the individual functions of each resource.

The second context that is formed from the recontextualization in the first model of intersemiosis, assumes a formal design and is close to what happens in the mathematics classrooms. In this approach, the mathematical content is explained with all its formality and the semiotic resources used in this part of the videos are, essentially, verbal language, mathematical images, mathematical symbolism, and deictic gestures. This reconstruction of semiotic choices based on recontextualization implies resemiotization. The oral verbal language, present in both contexts, undergoes changes in this process, being more informal in the introductory moment, when the participants strive to show the applications of Mathematics. The first model of intersemiosis is shown in Figure 11.

Figure 11 - Intersemiosis model 1.



Source: Research data.

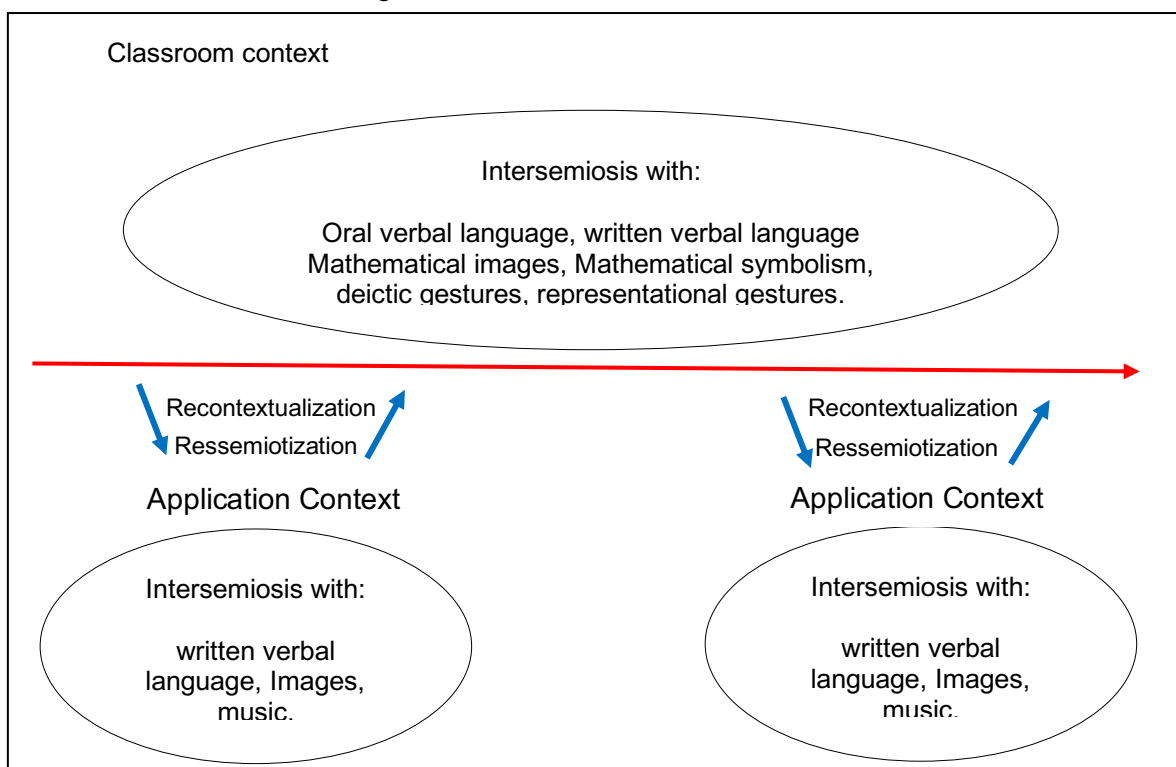
The intersemiotic relations arising from the semiotic choices in the first and second contexts presented in Figure 11 result in the production of meanings that constitute the message expressed in the video. The combination of semiotic resources enable the construction of the idea that Mathematics is involved in daily activities, enabling the construction of a new view of this discipline. Music plays an important role in the construction of this meaning by enabling associations between mathematical content and emotions. According to Sekeff (2007), musical elements, such as rhythm and melody, penetrate the psychological sphere, mobilizing emotional experiences that involve the subjectivity of the individual (Sekeff, 2007). The intersemiotic relations in mathematical discourse enabled by digital video, then produce and internalize through emotion meanings that relate specific concepts of Mathematics, in this case, the angle between vectors to specific practical applications in real life. The recontextualization of the mathematical content in the video, however, presents the idea that, in order to have access to the practical part of Mathematics, which is the part that justifies it, it is necessary to develop knowledge that depends on its formality, considering its interconnected concepts and the nature of mathematical knowledge itself.

In the second context, the mathematical idea is presented in a scenario close to that of the classroom with regard to its formality. In this context, some characteristics of the verbal language resources, mathematical symbolism and image used maintain the position of Mathematics as a dominant science. In addition, common strategies were used in formal mathematical discourse that make it semantically dense. The meaning is produced from the intersemiosis of the resources chosen in order to structure the presentation on the

resolution of the problem proposed in the video. The contextualized meaning resulting from the intersemiosis performed in the video, guides specific ideas, such as operations with vectors, were built in the videos from the intersemiotic relations between oral verbal language, mathematical images, mathematical symbolism and, with an essential role, the deictic gestures, which enabled connections between information provided by oral verbal language, images, and mathematical symbolism.

In the second model of intersemiosis presented in the video Practical Use of Analytical Geometry, a context is fixed, in this case, a classroom context, in which semiotic choices result in oral and written verbal language, mathematical symbolism, mathematical images, deictic and representational gestures. In this second model, recontextualizations happen with the insertion of elements that show the applicability of Mathematics, in specific moments of the mathematical discourse. The intersemiosis model of the video Practical Use of Analytical Geometry is illustrated in Figure 12.

Figure 12 - Intersemiosis model 2.



Source: Research data.

In the scheme presented in Figure 12, the intersemiotic relations in the classroom context combine oral verbal and written language, mathematical symbolism, mathematical images, the deictic gestures and representational and produce contextual meanings from the insertion of contexts that present the applied mathematical content. These contexts of applicability inserted into the mathematical discourse function intersemiotically with the usual semiotic resources of Mathematics. The images related to the applicability of the content, together with information that complement the context and the music, again, introduce an emotional factor into the mathematical discourse. When performing these recontextualizations - in the video Practical Use of Analytical Geometry, this occurs in two moments - the formal environment established in the classroom context is altered and the music provokes the manifestation of emotion due to its rhythm and melody, in addition to the state of attention.

The meanings resulting from these intersemiotic relations refer to the association of the equation of the circumference to its application in the construction of buildings with a circular shape. The verbal language was the main resource in the statement and in the resolution of the problems proposed in the video, highlighting its explanatory function in the production. With deictic and representational gestures, students highlighted and carried out the materialization of elements of speech represented by oral verbal language, which enabled the production of meanings by combining these semiotic resources with mathematical images. Thus, as a result of intersemiosis, the notions of radius of the circumference, as well as the constitution of the measurements of the sides of the circumscribed right triangle were used in the mathematical discourse, justifying the mathematical relations presented, without all the steps of the algebraic calculations being detailed by written verbal language. The deictic gestures played an important role in the constitution of meanings, enunciating implicit concepts such as the idea of radius and the measurements of the sides of the right triangle as a difference in the coordinates of the points belonging to the circumference, constructed intersemiotically with the mathematical image.



## Conclusion

Authors such as Engelbrech et al (2020a) claim that we are already in the fourth phase of the use of digital technology in mathematics education. From the first phase, “Logo phase”, in the 1980’s to the fourth phase, beginning around 2004, depending on the country, there was a quantitative and qualitative jump in relation to the use of the internet in education. With faster internet, many possibilities became available for mathematics education. The medium is the message (\*\*McLuhan\*\*)! Different technology, different media different mathematics (Borba, 2012; Borba, 2021). But how can we illustrate that media is the message, that media has agency and it transforms the way mathematics is? We believe that the way mathematics is expressed makes it difference, and this is why we believe that technology such as the digital video has agency. How to analyse and show this? This chapter is about students producing videos in a pedagogical context. This mathematical video, produced by students, may be seen from a perspective of a “test’ to be assessed by a teacher, or as an artefact with agency in the production of knowledge for those who seen it and so on. This chapter is about a research approach in which mathematical videos are carefully analysed using theoretical tools that came from Social Semiotics in order to understand ways in which mathematics teaching and learning can be enhanced to reveal the multi-faceted nature of mathematics itself.

Video presents a context in which mathematical discourse can integrate with other cinematographic devices, and in this case, the care aesthetics of the introductory part of the video is evident. In order to relate the mathematical content to the themes of the video, namely Vectors and Circumference equation, with practical problems of everyday life, the participating students gathered resources of oral verbal language, static images, moving images and music. These semiotic choices were made so that the functionalities of the resources would merge elements linked to the applicability of Mathematics.

For the videos, the verbal language had its functionality aimed at explaining the applicability of Analytical Geometry, thus making the topic interesting for people. The idea of the students producing the video was to show how mathematics is close to people's reality and can help solve practical problems in their daily lives. The static images used contribute with visual elements that represent the existence of activities outside of Mathematics, but that somehow depend on this science. Music and moving images contribute by making the environment for the construction of mathematical content more engaging through the use of instrumental music that harmonizes with the verbal language in an informal tone presented, bringing emotion as a variable in the promotion of mathematical content. Moving images reaffirm the context in which the discourse is inserted. The aesthetics resulting from the organization of the images add value to the introduction of the video Civil Construction, the suspense caused by the door that opens and the image of a slab without the roof show that there is a real problem that can be solved using concepts from Analytical Geometry.

In this analysis, the multisemiotic characteristic of video as a digital technology is apparent, allowing the introduction of moving images mixed with static images, which enables the semantic expansion due to the new meaning produced from the combination of these resources to express the idea that Mathematics can be entertaining, going beyond formal and rigorous speech which is typically associated with mathematics.

Oral verbal language, mathematical symbolism and mathematical image had their semantic potential optimized when used in conjunction with deictic gestures in the video. The usual functions of the first resources were used, that is, the language was used to present the context of the proposed mathematical problem, the image of the house represents the scenario in which the problem is immersed and contributed to the development of mathematical relationships referring to the content expressed through symbolism. Numerical representation was prioritized in solving the problem, and the formula of the angle between vectors used appeared in the discourse of the interaction of oral verbal language with the mathematical image and deictic gestures. The combination of gestures in this interaction helped to clarify and complement the speech expressed using oral language.

The three main resources of mathematical discourse, when combined from organized semiotic choices, operated intersemiotically so that the relations between visual and auditory resources led to a new meaning for notions related to mathematical content. In the videos, different semiotic resources were combined to enhance the mathematical discourse.

As a result of the intersemiotic relations between semiotic choices in the analyzed videos, there are possibilities of semantic expansion in specific concepts involved in the content treated in each video. The production of potential contextualized meanings calls attention to the fact that the combination of semiotic resources in the analysed videos allows the mathematical discourse to be understood by complementing the functionalities of each resource. This brings elements to the discussion about the potential of video as a digital resource for the expression of mathematical ideas.

The cinematographic resources used in the videos produced in the research, namely, images, moving images, scenarios, camera movements, plans, sound, and music, contribute to the aesthetic organization of the mathematical speech given and influence the message, being characteristic features of the videos. From the identified semantic expansion potentials, it is conjectured that the choice of semiotic resources can influence the way knowledge is constructed in mathematics, since the combination of semiotic resources

performed for the construction of the message can enhance digital mathematical discourse. In other words, there is a qualitative differential with regard to the semiotic resources involved in the multisemiotic and multimodal (video) phenomenon on which the mathematical discourse is built. This highlights the place of technologies in the construction of knowledge, considering the current scenario which reveals a technological revolution in audiovisual communication. Thus, the problem proposed in the research reported here has a dimension anchored in thinking-with-technology, since specific resources of digital videos conditioned, logically and aesthetically, the way in which mathematical discourse was organized for the purpose of producing contextualized meanings, which ensures the role of video as a potentializing factor in mathematical discourse.

The research participants explored the potential of videos in their multisemiotic and multimodal character when producing videos in order to express mathematical ideas. The participants considered the specific functionalities of each resource, in particular, the use of cinematographic resources, characteristic of videos. Participating students see in the expression of mathematical ideas through the video a potential to relate the content to applied problems, allowing semantic expansions by associating the content with its applicability. The content-emotion association also appears as an element that deserves to be highlighted in the research, since it calls attention to a new way of knowing: a form that, through the use of digital technologies, raises emotion, in addition to sight and hearing, already used in the process of building mathematical knowledge. With the pandemic, students at different levels of education, mathematical videos may grow in importance. The digital video is already a symbol of the twenty first century, as it is of the fourth phase of digital technology in mathematics education. The intensive use during the pandemic already attests a quantitative use of digital mathematical videos in regular teaching and in this new classroom, let's call the video repository classroom, in which mathematic has also been created, shared, taught and learned. This evolving classroom (Borba et al., 2016), is now more important as we include homes and other things with agency in education (Borba, 2021). We may, with the pandemic living a new phase, the fifth phase, in which the cubic classroom will be left behind. Research on mathematical videos using this kind of framework may be a way of understanding the nature of this evolving mathematics education.

## References (should be restricted to 70 essential references compatible with good scientific practice)

- Alibali, M. W.; Nathan, M. J.; Church, R. B.; Wolfgram, M. S.; Kim, S.; Knuth, E. *Teachers' gestures and speech in Mathematics lessons: forging common ground by resolving trouble spots*. *ZDM Mathematics Education*, v. 45, p. 425 – 440. 2013.
- Bogdan, R. C.; Biklen, S. K. *Qualitative research for education: an introduction to theories and methods*. 5. ed. Boston: Pearson Education, 2006.
- Borba, M. C. *Humans-with-media and continuing education for mathematics teachers in online environments*. *ZDM (Berlin. Print) JCR*, v. 44, p. 802-814, 2012.
- Borba, M. C.; Askar, P.; Engelbrecht, J.; Gadavidis, G.; Llinares, S.; Aguilar, M. S. *Blended learning, e-learning and mobile learning in mathematics education*. *ZDM (Berlin. Print) JCR*, v. 48, p. 589-610, 2016.
- Borba, M. C.; Neves, L. X.; Domingues, N. S. *A atuação docente na quarta fase das tecnologias digitais Produção de vídeos como ação colaborativa nas aulas de Matemática*. *EM TEIA - Revista de Educação Matemática e Tecnológica Iberoamericana*. v.9, p.1 - 24, 2018.
- Borba, M. C.; Oechsler, V. *Tecnologias na educação: o uso dos vídeos em sala de aula*. *Revista brasileira de ensino de ciência e tecnologia*, v. 11, p. 181-213. 2018.
- Borba, M. C.; Scucuglia, R.; Gadavidis, G. *Fases das tecnologias digitais em Educação Matemática: sala de aula e internet em movimento*. 2.ed. Belo Horizonte: Autêntica, 2018.
- Borba, M. C., Villarreal, M. E. *Humans-with-Media and the reorganization of mathematical thinking: information and communication technologies, modeling, visualization and experimentation*. New York: Springer, 2005.
- Brasil. *Cetic.Br: relatório analítico da aprendizagem a distância no Brasil*. Curitiba: Intersaberes, 2019. Disponível em: < [http://abed.org.br/arquivos/CENSO\\_EAD\\_BR\\_2019\\_digital\\_completo.pdf](http://abed.org.br/arquivos/CENSO_EAD_BR_2019_digital_completo.pdf)>. Acesso em 22 de mai. 2019.
- Creswell, J. W. *Investigação qualitativa e projeto de pesquisa: escolhendo entre cinco abordagens*. Tradução de Sandra Mallmann da Rosa. 3. ed. Porto Alegre: Penso, 2014.
- Domingues, N. S. *O papel do vídeo nas aulas multimodais de Matemática Aplicada: uma análise do ponto de vista dos alunos*. 2014. 125 f. Dissertação (Mestrado em Educação Matemática) – Universidade Estadual Paulista “Júlio de Mesquita Filho”, Rio Claro, 2014.

Engelbrecht, J.; Llinares, S.; Borba, M. C. Transformation of the mathematics classroom with the internet. *ZDM - The International Journal on Mathematics Education JCR*, v. 52, p. 825-841, 2020a.

Engelbrecht, J.; Borba, M. C.; Llinares, S.; Kaiser, G. Will 2020 be remembered as the year in which education was changed?. *ZDM - The International Journal on Mathematics Education JCR*, v. 52.5, p. 821-824, 2020b.

Farsani, D. Deictic gestures as amplifiers in conveying aspects of Mathematics register. In: Congress of the European Society for Research in Mathematics Education, 9., 2015, Prague, Czech Republic. Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education. Prague: HAL archives – ouvertes, 2016, p. 1382 – 1386.

Friedlander, A.; Tabach, M. Promoting multiple representations in algebra. In: Cuoco, A. A.; Curcio, F.R. The roles of representation in schools Mathematics. 2001 Yearbook. Reston: NCTM, 2001. p. 173 – 185.

Gois, A. K.; Nogueira, M. F. M.; Vieira, N. V. A linguagem do corpo e a comunicação nas organizações. *Revista Anagrama*, p. 1 – 12. 2011.

Goldin-Meadow, S.; Kim, S.; Singer, M. What the teacher's hands tell the student's mind about Math. *Journal of Education Psychology*, v. 91, n. 4, p. 720 – 730. 1999.

Halliday, M. A. K. *Language as Social Semiotic: The Social Interpretation of Language and Meaning*. London: Edward Arnold, 1978.

Halliday, M. A. K. *Spoken and Written Language*. Geelong, Victoria: Deakin University Press [Republished by Oxford University Press 1989], 1985.

Iedema, R. Resemiotization. *Semiotica*, v. 1, n. 4, p. 23 – 39. 2001.

Jewitt, C.; Bezemer, J.; O'Halloran, K. *Introducing Multimodality*. New York: Routledge, 2016.

Kress, G. *Multimodality: a social semiotic approach to contemporary communication*. New York: Routledge, 2010.

Lemke, J. L. *Letramento metamidiático: transformando significados e mídias*. *Trabalhos em Linguística Aplicada*, v. 49, n. 2, p. 455 - 479. 2010.

Marshall, M. N. Sampling for qualitative research. *Family Practice*, v.13, n.6, p. 522-525. 1996.

McNeill, D. *Hand and mind: what gestures reveal about thought*. Chicago: University of Chicago Press, 1992.

Moletta, A. *Criação de curta-metragem em vídeo digital: uma proposta para produções de baixo custo*. 2.ed. São Paulo: Summus, 2009.

Neves, L. X. *Intersemioses em vídeos produzidos por licenciandos em Matemática da UAB*. 2020. 304 p. Tese (Doutorado em Educação Matemática) – Universidade Estadual Paulista “Júlio de Mesquita Filho”, Rio Claro, 2020.

Oechsler, V.; Borba, M. C. *Mathematical videos, social semiotics, and the changing classroom*. *ZDM - The International Journal on Mathematics Education*, v. 1, p. 1, 2020.

Neves, L. X.; Borba, M. C. *Vídeos em Educação Matemática sob a luz da Sistêmico Funcional – Análise do Discurso Multimodal*. *UNIÓN (San Cristobal de La Laguna)*, v.16, p.159 - 178, 2020.

O'Halloran, K. L. A multimodal approach for theorising and analysing mathematics textbooks. In: *International Conference on Mathematics Textbook Research and Development*, 2., 2017, Rio de Janeiro. Proceedings of the Second International Conference on Mathematics Textbook Research and Development. Rio de Janeiro: Instituto de Matemática da Universidade do Rio de Janeiro, 2018. p. 362 - 372.

O'Halloran, K. L. The language of learning Mathematics: a multimodal perspective. *The Journal of Mathematical Behavior*. Elsevier. vol.40, p. 63 – 74. 2015.

O'Halloran, K. L. Historical changes in the semiotic landscape: From calculation to computation. In: Jewitt, C. *The routledge handbook of multimodal analysis*. New York: Routledge, 2011. p. 98 – 113.

O'Halloran, K. L. *Systemic Functional Multimodal Discourse Analysis (SF-MDA) Approach to Mathematics, Grammar and Literacy*. In A. McCabe, M. O'Donnell, & R. Whittaker (eds.), *Advances in Language and Education*. London & New York: Continuum, 2007, p. 77 – 100.

O'Halloran, K. L. *Multimodal discourse analysis: systemic functional perspectives*. London: Continuum, 2004.

O'Halloran, K. L. *Classroom Discourse in Mathematics: A multisemiotic analysis*. *Linguistics and Education*, v. 10, n. 3, p. 359-388. 2000.

O'Halloran, K. L.; Lim Fei, V. *Systemic functional multimodal discourse analysis*. In: Norris, S.; Maier, C. D. *Interactions, images and texts: a reader in Multimodality*. Berlin: De Gruyter, 2014. p. 137 - 153.

Powell, A. B.; Francisco, J. M.; Maher, C. A. *Uma abordagem à Análise de Dados de Vídeo para Investigar o Desenvolvimento das Ideias Matemáticas e do Raciocínio de Estudantes*. *BOLEMA*, Rio Claro (SP), v. 17, n. 21, p. 81–140. 2004.

Santaella, L. *O que é semiótica*. São Paulo: Editora brasiliense, 2012.

Scucuglia, R. R. S. *On the nature of students' digital mathematical performances*. 2012. 273 p. Thesis (Doctor of Philosophy) – School of Graduate and Postdoctoral Studies, The University of Western Ontario, London- Ontario, 2012.

Scucuglia, R. R. S. *Narrativas Multimodais: a Imagem dos Matemáticos em Performances Matemáticas Digitais*. *Bolema. Boletim de Educação Matemática*, v. 28, n. 49, p. 950–973, 2014.

Sekeff, M. L. *Da música: seus usos e recursos*. 2. ed. São Paulo: editora UNESP, 2007.

Setton, M. G. *Mídia e educação*. São Paulo: Contexto, 2015.

Van Leeuwen, T. *Introducing social semiotics*. New York: Taylor & Francis E-Library, 2005.