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RESEARCH ARTICLE

Multiobjective Atomic Orbital Search (MOAOS) for Global and Engineering Design Optimization

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ABSTRACT In the real world, many optimization problems have such levels of uncertainty and complexity that a single objective function cannot represent all the characteristics of the considered system. Hence, multi-objective optimization algorithms are needed to account for multiple aspects of the problem, represented by multiple objective functions, to achieve reasonable and useful results through the optimization procedure. In this paper, we introduce the multi-objective version of a recently-developed single-objective metaheuristic algorithm known as Atomic Orbital Search (AOS), which will be called Multi-Objective Atomic Orbital Search (MOAOS). To this end, the general aspects and main searching loop of the AOS algorithm are modified to make it capable of dealing with problems with multiple objectives. For the performance evaluation of this algorithm, the mathematical benchmark problems ZDT and DTLZ, alongside several real-world engineering design problems and the CEC- 2020 MMO test problems, are utilized. Based on the results obtained in this study, we can conclude that MOAOS is capable of producing either superior or closely comparable results when evaluated in competition with alternative state-of-the-art metaheuristic methods.

INDEX TERMS Atomic orbital search (AOS), multi-objective optimization, metaheuristic, mathematical benchmark, real-world engineering problems.

I. INTRODUCTION

One of the main capabilities of the human brain is to recognize the ‘best’ choice and achieve it accordingly. However, we are also aware that identifying all the conditions which lead to the best solution to a problem is extremely difficult or impossible in many cases. Therefore, rather than ‘globally optimal’ solutions to a problem, in many scenarios, ‘sufficiently satisfactory’ solutions can be practically and effectively used. Optimization seeks to improve the overall performance of a system in reaching a point (or points) at which the optimal behavior of the system is achieved. Optimization has two main aspects: (1) the considered minimization (or maximization) problem, and (2) the utilized

search method. In many cases, optimization procedures focus on convergence (i.e., reaching an optimal point) while their overall behavior and the role of utilized algorithms are generally less investigated. In this regard, developing novel optimization algorithms, implementing methodical improvements to the existing ones, and hybridizations enable us to achieve algorithms with a better performance which can lead to considerable enhancements in the overall behavior of the optimization procedure. Metaheuristic algorithms are high-level procedures in which the minimization (or maximization) of an objective function is considered for reaching the best optimal solution in dealing with complicated problems by means of upper-level searching methods. Genetic Algorithm (GA) [1], Differential Evolution (DE) [2], [3], Stochastic Paint Optimizer (SPO) [4], Chaos Game Optimization (CGO) [5], Ant colony Optimization (ACO) [6],

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Particle Swarm Optimizer (PSO) [7], Atomic Orbital Search (AOS) [8], Dynamic Water Strider Algorithm (DWSA) [9], Flow Direction Algorithm (FDA) [10], Crystal Structure Algorithm (CryStAl) [11], and Material Generation Algorithm (MGA) [12] are some of the well-formulated meta-heuristic optimization algorithms which have contributed to solving a wide range of problems across different fields of science and technology [13]–[25].

In general, depending on the total number of objective functions, optimization problems can be categorized into two main types, namely ‘single-objective’ and ‘multi-objective’ problems. In many of the previously-discussed optimization problems, only one aspect of the problem was to be minimized (or maximized), subject to specific constraints given in the problem. In dealing with real-world problems, relying on one specific aspect of an optimization problem cannot guarantee the optimal behavior of the considered system. Considering multiple aspects of the system for incorporation into the optimization procedure can enhance the system behavior. Therefore, in recent decades, many multi-objective optimization algorithms have been proposed by researchers in the communities of computer science and applied mathematics. Examples include the Non-Dominated Sorting Genetic Algorithm, known as NSGA and NSGAI [26] developed based on the Genetic Algorithm (GA) [27], Multi-Objective PSO called MOPSO [28], Multi-Objective Strength Pareto Evolutionary Algorithm (SPEA2) [29], Multi-Objective ACO called MOACO [30], and Multi-Objective Simulated Annealing called MOSA [31]. Mirjalili, *et al.* [32] proposed the Multi-Objective Ant Lion Optimizer (MOALO), in which a repository is used to hold non-dominated Pareto optimum solutions. Based on Multi-Verse Optimizer (MVO), Mirjalili, *et al.* [33] introduced Multi-Objective Multi-Verse Optimizer (MOMVO); then, the findings were compared quantitatively and qualitatively to those of previously-introduced algorithms, demonstrating the utility of MOMVO in handling a variety of problems. Mirjalili, *et al.* [34] proposed the Multi-objective Salp Swarm Algorithm (MSSA), revealing that it can approach Pareto optimum solutions with a high level of convergence and coverage. Chen, *et al.* [35] developed Multi-Objective Ant Colony System (MOACO) based on a co-evolutionary multiple populations model that optimizes execution time and cost. In other studies, the authors utilized MOACO to solve the multi-objective airline crew rostering problem and the economic emission dispatch (EED) problem, respectively [36], [37]. Zhan, *et al.* [38] proposed a novel method of co-evolution named Multiple Populations for Multiple Objectives (MPMO). The uniqueness of MPMO is that it enables us to solve multi-objective optimization problems simply by associating each population with a single objective. In this research, PSO was used for each population, and a co-evolutionary multi-swarm PSO, known as CMPSO, based on the MPMO approach, was developed. Subsequently, for many-objective optimization, the authors combined co-evolutionary particle swarm

optimization with a bottleneck objective learning (BOL) technique. To increase the quality of archived solutions, a solution reproduction technique comprising both an elitist learning strategy (ELS) and a juncture learning strategy (JLS) was created [39].

To solve the job-shop scheduling problem (JSSP), Liu, *et al.* [40] introduced a new multiple population for multiple objectives (MPMO) framework based on GA called MPMOGA. Based on their results, the authors asserted that MPMOGA outperformed comparable algorithms such as NSGA-III, Stochastic ranking algorithm (SRA), and green GA (GGA). Liu, *et al.* [41] proposed a Multi-Objective framework for Many-Objective (Mo4Ma) optimization, transforming the many-objective space into a multi-objective space. Tejani, *et al.* [42] proposed Multi-Objective Heat Transfer Search (MOHTS) for structural optimization problems. After a number of runs, obtained results were compared to those of other optimizers existing in the literature, such as the multi-objective ant system, multi-objective ant colony system, and multi-objective symbiotic organism search, demonstrating that the proposed algorithm outperformed its competitors. Subsequently, Kumar, *et al.* [43] suggested a modified version of MOHTS for truss mass minimization and nodal displacement maximization. Tejani, *et al.* [44] developed Multi-Objective Adaptive Symbiotic Organisms Search (MOASOS) for solving truss optimization problems.

Multi-Objective Teaching–Learning-based Optimization (MOTLBO) [45], Multi-Objective Thermal Exchange Optimization (MOTEO) [46], Multi-Objective Hybrid Heat Transfer Search and Passing Vehicle Search optimizer (MOHTS–PVS) [47], Multi-Objective Heat Transfer Search with modified Binomial Crossover (MOHTS–BX) [48], Multi-Objective Plasma Generation Optimization (MOPGO) [49], Multi-Objective Crystal Structure Algorithm (MOCryStAl) [50], Multi-objective Forest Optimization Algorithm (MOFOA) [51] and Competitive Mechanism Integrated Multi-objective Whale Optimization Algorithm with Differential Evolution (CMWOA) [52] are some well-known algorithms which have been developed for dealing with multiple objective functions.

Given that the recently-introduced single-objective optimization algorithm Atomic Orbital Search (AOS) has turned out to be a promising metaheuristic by outperforming many existing algorithms [8], here we introduce its multi-objective version called MOAOS. Similar to AOS, the new method MOAOS is inspired by the principles of quantum mechanics, particularly the circulation of electrons around the nucleus of atoms. Here, to evaluate the capability of MOAOS in dealing with multi-objective optimization problems, the mathematical benchmarks (ZDT and DTLZ) are considered, followed by examining its performance in dealing with real-world engineering problems.

The remainder of this paper is organized as follows. Section II presents the mathematical details of the proposed Multi-Objective AOS optimization algorithm (MOAOS).

Section III describes the results and discussion of the study, including performance metrics, experimental setup, discussion of the ZDT and DTLZ test functions, and a set of different engineering design problems including the four-bar truss, welded beam, disk brake, and speed reducer design problems, as well as the CEC-2020 MMO test problems [53]. Finally, Section IV concludes this research.

II. THE PROPOSED MULTI-OBJECTIVE ATOMIC ORBITAL SEARCH (MOAOS) OPTIMIZATION ALGORITHM

A. ATOMIC ORBITAL SEARCH (AOS)

In an atom, an orbital is an area around the nucleus where electrons are likely to be found. There is a certain probability for each of the electrons around the nucleus to be found at a certain distance from the nucleus. In other words, the atomic orbital is a mathematical function that determines the position as well as the wave-like behavior of an electron or a pair of electrons in an atom. This function can be used to calculate the probability of the presence of an electron in an atom in certain areas around the nucleus. Furthermore, it can be used to plot a three-dimensional diagram of the probability of the presence of electrons in such regions. In particular, atomic orbitals may be explained by the orbital function, especially in the case of a single electron in a set of electrons around a single atom. As a recently-proposed metaheuristic algorithm, AOS was developed based on the atomic orbital concept in quantum mechanics in which the movement of electrons around the nucleus of an atom is utilized for formulating a search algorithm. In the first step of AOS, an initialization process is conducted in which the initial position of the candidate solutions, which mimics the electrons around the nucleus, are determined as follows:

$$X = [X_1 \ X_2 \ \dots \ X_i \ \dots \ X_m]^T$$

$$= \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^j & \dots & x_1^d \\ x_2^1 & x_2^2 & \dots & x_2^j & \dots & x_2^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_i^1 & x_i^2 & \dots & x_i^j & \dots & x_i^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_m^1 & x_m^2 & \dots & x_m^j & \dots & x_m^d \end{bmatrix}, \quad \begin{cases} i = 1, 2, \dots, m. \\ j = 1, 2, \dots, d. \end{cases} \quad (1)$$

$$x_i^j(0) = x_{i,min}^j + \mathcal{R}(x_{i,max}^j - x_{i,min}^j), \quad \begin{cases} i = 1, 2, \dots, m. \\ j = 1, 2, \dots, d. \end{cases} \quad (2)$$

where m denotes the initial number of electrons as candidate solutions; $x_i^j(0)$ is the initial values of decision variables; d represents the dimension of the optimization problem; $x_{i,min}^j$ and $x_{i,max}^j$ represent the lower and upper bounds of the j^{th} decision variable concerning the i^{th} electron (candidate

solution); and \mathcal{R} is a random number distributed uniformly in the range $[0, 1]$.

The objective function values for each candidate solution are calculated which represent the energy levels of electrons around the nucleus as follows:

$$E = [E_1 \ E_2 \ \dots \ E_i \ \dots \ E_m]^T, \quad i = 1, 2, \dots, m. \quad (3)$$

where E represents the energy level of the atom which includes the objective function values of candidate solutions (electrons), and E_i denotes the specific energy level of the i^{th} electron (candidate solution).

In the AOS algorithm, the well-known quantum staircase analogy [54] is utilized as the critical characteristic of the main searching loop where a number of n imaginary layers are generated for positioning electrons around the nucleus of the atom. A Probability Density Function (PDF) based on Gaussian distribution is formulated for dispersing the electrons (candidate solutions) around the nucleus and positioning these candidates at each of the imaginarily-created layers based on their energy levels. Electrons with lower energy levels (better objective function values) are positioned in the inner layers, while those with higher energy levels (worse objective function values) are positioned in the outer layers. The schematics of these theoretical concepts are presented in Fig. 1(a) and (b); the mathematical representation of dispersing electrons in the imaginary layers is given below:

$$X^k = [X_1^k \ X_2^k \ \dots \ X_i^k \ \dots \ X_p^k]^T$$

$$= \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^j & \dots & x_1^d \\ x_2^1 & x_2^2 & \dots & x_2^j & \dots & x_2^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_i^1 & x_i^2 & \dots & x_i^j & \dots & x_i^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_p^1 & x_p^2 & \dots & x_p^j & \dots & x_p^d \end{bmatrix}, \quad \begin{cases} i = 1, 2, \dots, p. \\ j = 1, 2, \dots, d. \\ k = 1, 2, \dots, n. \end{cases} \quad (4)$$

$$E^k = [E_1^k \ E_2^k \ \dots \ E_i^k \ \dots \ E_p^k]^T, \quad \begin{cases} i = 1, 2, \dots, p. \\ k = 1, 2, \dots, n. \end{cases} \quad (5)$$

where X_i^k represents the i^{th} electron positioned in the k^{th} layer; p denotes the overall number of candidates positioned in the k^{th} layer; n represents the total number of imaginary layers; and E_i^k is the energy level of the i^{th} candidate in the k^{th} layer.

For each imaginary layer around the nucleus, and also for the atom, the binding state and binding energy are calculated as follows (binding energy is the energy required to remove an electron from its shell, i.e. its binding state):

$$BS^k = \frac{\sum_{i=1}^p X_i^k}{p}, \quad \begin{cases} i = 1, 2, \dots, p. \\ k = 1, 2, \dots, n. \end{cases} \quad (6)$$

$$BE^k = \frac{\sum_{i=1}^p E_i^k}{p}, \quad \begin{cases} i = 1, 2, \dots, p. \\ k = 1, 2, \dots, n. \end{cases} \quad (7)$$

$$BS = \frac{\sum_{i=1}^m X_i}{m}, \quad i = 1, 2, \dots, m. \quad (8)$$

$$BE = \frac{\sum_{i=1}^m E_i}{m}, \quad i = 1, 2, \dots, m. \quad (9)$$

where BS^k and BE^k represent the k^{th} layer's binding state and binding energy, respectively; BS and BE represent the atom's binding state and the binding energy, respectively; X_i^k and E_i^k represent the position vector and energy level of the i^{th} candidate positioned in the k^{th} imaginary layer, respectively; and X_i and E_i represent the position vector and energy level of the i^{th} candidate, respectively.

Based on the principles of quantum mechanics, the electron with the best objective function value inside the k^{th} layer is determined as LE^k which denotes the electron with the lowest energy level in the layer. Similarly, the electron with the best objective function value in the atom is determined as LE which denotes the electron with the lowest energy level around the nucleus of the atom.

The position updating process for the candidate solutions in the imaginary layers around the nucleus of the atom is determined using the quantum staircase analogy in which photons, magnetic fields, and interaction with other particles can excite an electron by absorbing or emitting energy (see Figure 1(c)). Photon Rate (PR) is defined as a factor for determining the interaction of photons with electrons, and φ is a random number generated for each candidate solution, representing the probability of the interaction of photons with electrons (for $\varphi \geq PR$). For every electron inside the imaginary layers, if $E_i^k \geq BE^k$, the energy emission is determined and the position updating is performed as follows:

$$X_{i+1}^k = X_i^k + \frac{\alpha_i (\beta_i LE - \gamma_i BS)}{k}, \quad \begin{cases} i = 1, 2, \dots, p. \\ k = 1, 2, \dots, n. \end{cases} \quad (10)$$

where X_i^k and X_{i+1}^k represent the now and then position vectors of the i^{th} candidate inside the k^{th} imaginary layer, respectively; and α_i , β_i , and γ_i are random vectors in the range (0, 1) to determine the amount of emitted energy.

On the other hand, for every electron inside the imaginary layers, if $E_i^k < BE^k$, the energy absorption is determined and the position updating is conducted as follows:

$$X_{i+1}^k = X_i^k + \alpha_i (\beta_i LE^k - \gamma_i BS^k), \quad \begin{cases} i = 1, 2, \dots, p. \\ k = 1, 2, \dots, n. \end{cases} \quad (11)$$

where X_i^k and X_{i+1}^k represent the now and then position vectors of the i^{th} candidate inside the k^{th} imaginary layer; and α_i , β_i , and γ_i are random vectors in the range of (0, 1) to determine the amount of absorbed energy.

In situations that $\varphi < PR$, the movements of electrons are based on the magnetic fields of interactions with other particles, so the position updating process is expressed

as follows:

$$X_{i+1}^k = X_i^k + r_i, \quad \begin{cases} i = 1, 2, \dots, p. \\ k = 1, 2, \dots, n. \end{cases} \quad (12)$$

where X_i^k and X_{i+1}^k represent the now and then position vectors of the i^{th} candidate inside the k^{th} imaginary layer; and r_i is a random number in the range (0, 1). The pseudo-code of AOS is presented in Fig. 1(d); furthermore, the flowchart of this algorithm is shown in Fig. 2.

B. MULTI-OBJECTIVE ATOMIC ORBITAL SEARCH (MOAOS)

AOS was designed to solve single-objective optimization problems and cannot be directly used to tackle multi-objective challenges. As a result, a multi-objective variant of AOS, denoted by MOAOS, is presented in this study for solving multi-criterion optimization problems.

Traditionally, heuristic algorithms are used to find and store Pareto optimal solutions. However, such solutions are difficult to identify when there are significant variations. Hence, a range of alternative approaches to discovering and storing Pareto optimal solutions have been discussed in the literature. In order to overcome this difficulty, and inspired by the MOPSO algorithm, the MOAOS algorithm contains three multi-objective optimization mechanisms as follows:

- (1) **Archive Mechanism:** It serves as a storage module for storing or restoring derived Pareto optimal solutions. The archive is controlled by a single controller, which determines which solutions are added to the archive and when it is full. The number of solutions that can be saved in the archive is limited. The archive's occupants are compared to the non-dominated solutions developed so far during each iteration. It is not allowed to enter the archive if at least one member in the archive dominates the new solution. If the new solution dominates at least one existing solution in the archive by omitting the one already in the archive, it may be included in the archive. If the new and archive solutions do not dominate each other, the new solution is added to the archive.
- (2) **Grid mechanism:** It is an effective technique for enhancing non-dominated solutions in the archive. If the archive overflows, the grid technique will be used to rearrange the object space's division and discover the most occupied area, allowing one of the solutions to be eliminated. To increase the diversity of the final approximated Pareto optimal front, the additional member should be added to the least crowded segment. The potential of removing a solution increases as the number of possible solutions in the hypercube grows. If the archive is full, the busiest sections are chosen first, and a solution from one of them is removed randomly to make room for the new one. When a solution is placed outside the hypercubes, a special case arises. All segments in this scenario are expanded to fit the

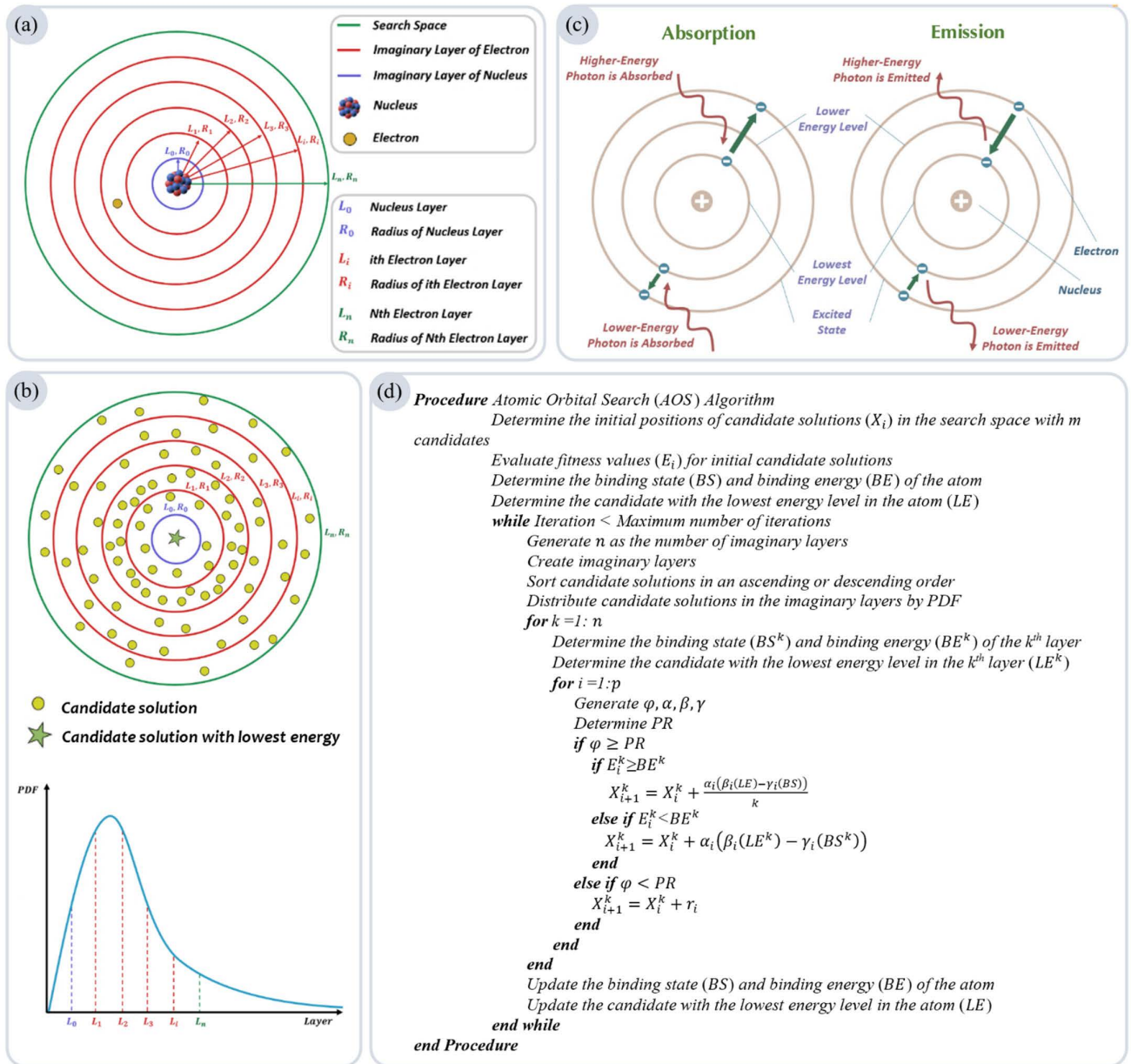


FIGURE 1. Atomic Orbital Search Algorithm (AOS). (a & b) Imaginary layers around the nucleus and the position of electrons based on Probability Density Function (PDF). (c) Absorption and emission of energy by electrons around the nucleus of an atom. (d) The Pseudo-code of the Atomic Orbital Search (AOS). (adapted from [8]).

most recent solutions. As a result, the segments of alternative solutions can also be changed.

- (3) **Leader Selection Mechanism:** Solutions in a multi-objective search space are compared with this mechanism. The search leaders guide the other search candidates to possible areas of the search space to obtain a solution that is close to the global optimum. As stated previously, the archive contains only the best non-dominated solutions, and the leader selection mechanism chooses the least crowded portions of the search space and presents the best non-dominated answers.

The selection for each hypercube is made using a roulette-wheel approach with the following probability:

$$P_i = \frac{C}{N_i} \quad (13)$$

where C is a constant number greater than one, and N is the variety of acquired Pareto optimal answers in the i^{th} section.

Less-crowded hypercubes have a larger probability of suggesting new leaders, as can be seen from Eq. (13). When the number of obtained solutions in the hypercube is reduced, the probability of selecting a hypercube to select leaders from increases. MOAOS's convergence is ensured by the fact

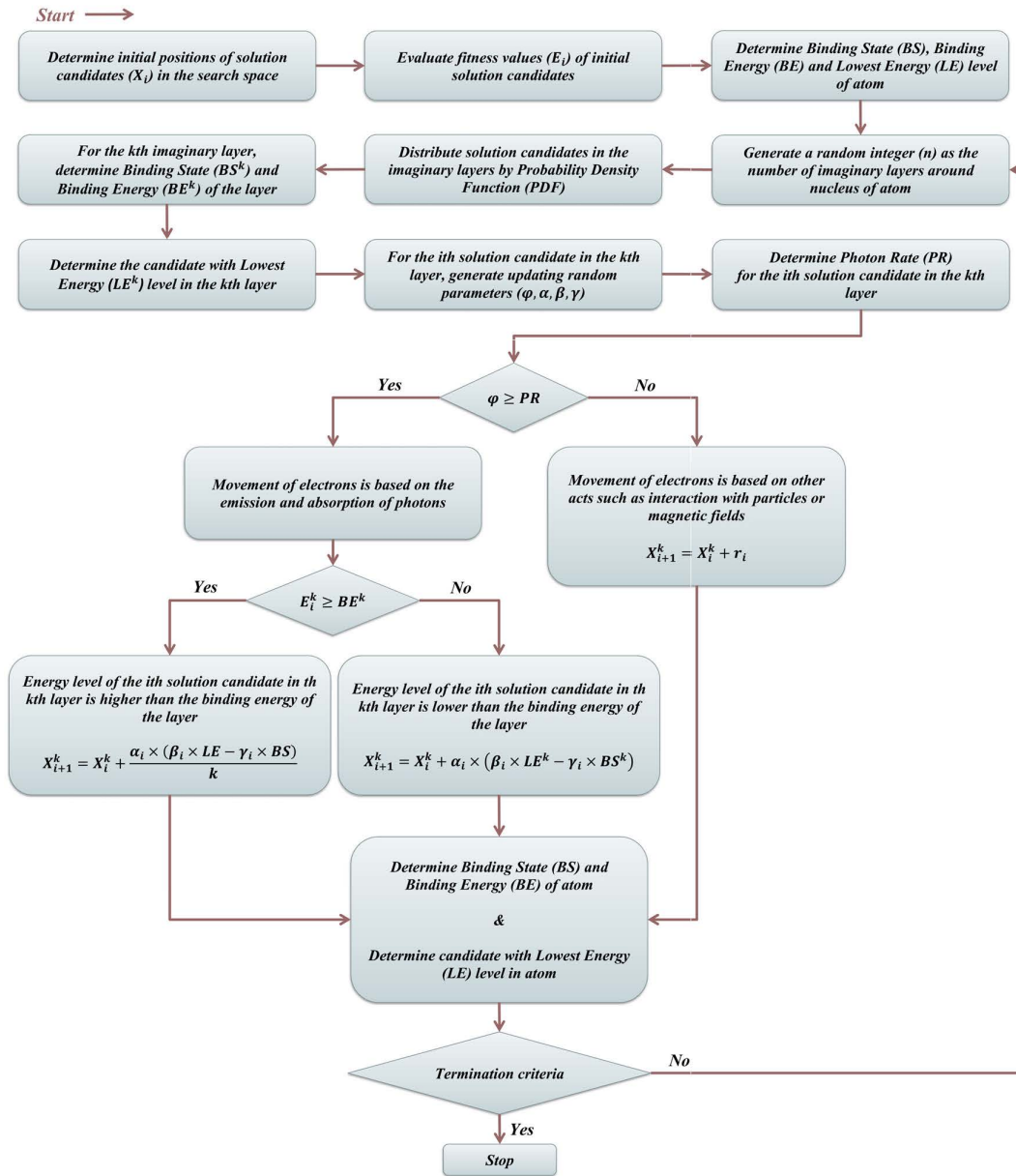


FIGURE 2. Flowchart of the Atomic Orbital Search (AOS) algorithm [8].

that its search for optimal solutions is based on the same mathematical model. In order to achieve optimal solutions using AOS, it has been demonstrated that search agents must change positions suddenly in the beginning stages of optimization and gradually in the end stages. According to Van den Bergh and Engelbrecht (2006), this tendency ensures that the algorithm will eventually reach its destination in the search space. The MOAOS algorithm incorporates all of AOS's features, thus all of the search agents operate in the same manner when exploring or exploiting the search space. The most significant distinction is that MOAOS searches around a group of archive members, whereas AOS stores and improves only the finest solutions available. The

computational complexity of MOAOS is of $O(MN^2)$ where N is the number of individuals in the population and M is the number of objectives. In comparison to NSGA-II, MOAOS and MOPSO consume more memory due to the use of archives to store the best non-dominated solutions. The pseudo-code of MOAOS is presented in Fig. 3.

III. RESULTS AND DISCUSSION

The efficiency of the suggested approach was evaluated using various performance measures and case studies, including unconstrained and constrained bi- and tri-objective mathematical problems and real-world engineering design problems. The ability of a range of popular multi-objective

Procedure Multi-objective Atomic Orbital Search (MOAOS) Algorithm
Initialize the MOAOS parameters (α , n_{Grid} , β , and γ)
Determine initial positions of candidate solutions (X_i) in the search space with m candidates
Evaluate objective values for initial candidate solutions
Obtain the non-dominated solutions and create Archive
Find the leaders
Determine the binding state (BS) and binding energy (BE) of the atom
while Iteration < Maximum number of iterations
Generate n as the number of imaginary layers
Create imaginary layers
Distribute candidate solutions in the imaginary layers by PDF
for $k=1:n$
Determine the binding state (BS^k) and binding energy (BE^k) of the k^{th} layer
Determine the candidate with the lowest energy level in the k^{th} layer (LE^k)
for $i=1:p$
Generate $\varphi, \alpha, \beta, \gamma$
Determine PR
if $\varphi \geq PR$
if $E_i^k \geq BE^k$
 $X_{i+1}^k = X_i^k + \frac{\alpha_i(\beta_i(LE^k) - \gamma_i(BS^k))}{k}$
else if $E_i^k < BE^k$
 $X_{i+1}^k = X_i^k + \alpha_i(\beta_i(LE^k) - \gamma_i(BS^k))$
end
else if $\varphi < PR$
 $X_{i+1}^k = X_i^k + r_i$
end
end
end
Evaluate the objective values for all populations
Obtain the non-dominated solutions
Update Archive according to non-dominated solutions
found
if Archive = full
Use the grid mechanism to omit the current Archives
Add the new solutions to the Archive
if any of the newly added solutions to Archive is placed outside the hypercubes
Update the grids
End if
end while
Return Archive
end Procedure

FIGURE 3. Pseudo-code of the Multi-Objective Atomic Orbital Search (MOAOS) algorithm.

optimizers to handle problems with non-convexity and non-linearity was tested using these problems and mathematical functions.

A. PERFORMANCE METRICS

To evaluate the results of the algorithms, the following four metrics are used:

1. Generational Distance (GD) represents the overall sum of the adjacent distances of candidate solutions associated with different achieved sets using multiple algorithms. It is considered an intelligent indicator for evaluating the convergence characteristics of metaheuristic algorithms with

multiple objectives.

$$GD = \left(\frac{1}{n_{pf}} \sum_{i=1}^{n_{pf}} dis_i^2 \right)^{\frac{1}{2}} \quad (14)$$

2. Spacing (S) is a measure of the total distance between candidates associated with different achieved sets by means of multiple algorithms.

$$S = \left(\frac{1}{n_{pf}} \sum_{i=1}^{n_{pf}} (d_i - \bar{d})^2 \right)^{\frac{1}{2}}, \quad \bar{d} = \frac{1}{n_{pf}} \sum_{i=1}^{n_{pf}} d_i \quad (15)$$

3. Maximum Spread (MS) represents the spread of candidates among other achieved sets by considering distinct optimal choices.

$$MS = \sqrt{\frac{1}{m} \sum_{i=1}^m \left[\frac{\min(f_i^{max}, F_i^{max}) - \max(f_i^{min}, F_i^{min})}{F_i^{max} - F_i^{min}} \right]^2} \quad (16)$$

4. Inverted Generational Distance (IGD) is a precise measure for the performance estimation of Pareto front approximations utilizing the results of multiple many-objective optimization algorithms.

$$IGD = \left(\sqrt{\sum_{i=1}^n d_i^2} \right) / n \quad (17)$$

5. Hypervolume (HV) is an indicator which has been used by the community since 2003. The hypervolume of a set of solutions measures the size of the portion of the objective space that is dominated by those solutions as a group. In general, hypervolume is favored because it captures in a single scalar both the closeness of the solutions to the optimal set and, to some extent, the distribution of solutions across the objective space.

B. EXPERIMENTAL SETUP

MOPSO, MOALO, MOMVO, MOSPEA2, and MONSGA2 were compared to MOAOS in order to identify the best figure of a collection of Pareto optimal solutions. The initial parameters of all described algorithms are summarized in Table 1. It is worth mentioning that each experiment employed 100 populations and a maximum of 1000 iterations. As illustrated in Table 2 and Appendix, the proposed algorithm was tested in 25 diverse case studies, including ten unconstrained and constrained mathematical problems, eight real-world engineering design problems, and ten CEC-2020 benchmark tests.

C. DISCUSSION OF THE ZDT AND DTLZ TEST FUNCTIONS

In this section, the numerical results of the proposed algorithm, namely MOAOS, alongside those of the alternative algorithms are presented. Regarding performance metric *GD* which is presented in Table 3, it can be concluded that the

TABLE 1. Parameters setting of all algorithms.

Function	Mathematical formulation	D	Range
ZDT1	$F_1 = x_1, F_2 = g \left(1 - \sqrt{F_1/g} \right),$ $g = 1 + \frac{9}{D-1} \sum_{i=2}^D x_i.$	30	$x_i \in [0,1]$
ZDT2	$F_1 = x_1, F_2 = g \left(1 - \left(F_1/g \right)^2 \right),$ $g = 1 + \frac{9}{D-1} \sum_{i=2}^D x_i.$	30	$x_i \in [0,1]$
ZDT3	$F_1 = x_1,$ $F_2 = g \left(1 - \sqrt{F_1/g} - F_1/g \sin(10\pi F_1) \right),$ $g = 1 + \frac{9}{D-1} \sum_{i=2}^D x_i.$	30	$x_i \in [0,1]$
ZDT4	$F_1 = x_1, F_2 = g \left(1 - \sqrt{F_1/g} \right),$ $g = 1 + 10(D-1) + \sum_{i=2}^D (x_i^2 - 10\cos(4\pi x_i)).$	10	$x_1 \in [0,1]$ $x_i \in [-5,5]$ $i = 1, \dots, D$
ZDT6	$F_1 = 1 - \exp(-4x_1) \sin^6(6\pi x_1),$ $F_2 = g \left(1 - \left(F_1/g \right)^2 \right), g = 1 + 9 \left(\frac{\sum_{i=2}^D x_i}{D-1} \right)^{0.25}.$	10	$x_i \in [0,1]$
DTLZ2	$F_1 = (1+g) \cos \left(x_1 \left(\frac{\pi}{2} \right) \right) \cos \left(x_2 \left(\frac{\pi}{2} \right) \right),$ $F_2 = (1+g) \cos \left(x_1 \left(\frac{\pi}{2} \right) \right) \sin \left(x_2 \left(\frac{\pi}{2} \right) \right),$ $F_3 = (1+g) \sin \left(x_1 \left(\frac{\pi}{2} \right) \right), g = \sum_{i=3}^D (x_i - 0.5)^2.$	12	$x_i \in [0,1]$
DTLZ4	$F_1 = (1+g) \cos \left(x_1^\pi \left(\frac{\pi}{2} \right) \right) \cos \left(x_2^\pi \left(\frac{\pi}{2} \right) \right),$ $F_2 = (1+g) \cos \left(x_1^\pi \left(\frac{\pi}{2} \right) \right) \sin \left(x_2^\pi \left(\frac{\pi}{2} \right) \right),$ $F_3 = (1+g) \sin \left(x_1^\pi \left(\frac{\pi}{2} \right) \right), g = \sum_{i=3}^D (x_i - 0.5)^2.$	12	$x_i \in [0,1]$

TABLE 2. Multimodal benchmark functions with fixed dimensions.

Parameters	MOPSO	MALO	MOMVO	MOAOS
Mutation Probability (P_w , or pro)	0.5	-	-	-
Population Size (N_{pop})	100	100	100	100
Archive Size (N_{rep} , or TM)	100	100	100	100
Number of Adaptive Grid (N_{grid})	30	30	30	30
Personal Learning Coefficient (C_1)	1	-	-	-
Global Learning Coefficient (C_2)	2	-	-	-
Inertia weight (w)	0.4	-	-	-
Beta	4	4	4	4
Gamma	2	2	2	2

proposed algorithm is capable of outranking the other meta-heuristic algorithms in five of the considered 7 benchmark problems.

Tables 4 and 5 present the results of different methods considering performance metrics *IGD* and *MS*, respectively. The proposed MOAOS algorithm can outrank the

TABLE 3. The statistical results of the ZDT and DTLZ benchmark functions for performance metric *GD*.

Function		Algorithm			
		MOPSO	MOALO	MOMVO	MOAOS
ZDT1	Ave	1.7932E-03	1.3644E-04	7.9682E-03	1.3627E-04
	SD	5.0496E-03	1.7358E-04	1.2073E-02	3.8037E-05
ZDT2	Ave	1.4749E-01	4.0621E-05	5.3988E-03	3.2872E-05
	SD	6.5211E-02	1.9757E-05	2.9609E-04	1.2908E-05
ZDT3	Ave	2.0489E-04	1.7253E-03	1.0954E-02	1.4610E-04
	SD	2.7322E-05	1.1636E-03	1.5715E-02	1.8895E-05
ZDT4	Ave	4.1299E+00	2.0045E+00	1.2363E+00	2.4935E+01
	SD	3.9519E+00	9.6098E-01	7.0133E-01	9.4153E+00
ZDT6	Ave	2.5805E-02	3.3034E-02	1.0183E-02	1.0058E-02
	SD	5.8008E-02	2.0337E-02	8.4995E-03	7.0214E-03
DTLZ2	Ave	6.8600E-03	2.1134E-02	3.1626E-03	1.1758E-01
	SD	9.0878E-04	1.0861E-02	5.5846E-04	4.4579E-04
DTLZ4	Ave	1.0097E-02	3.6328E-02	4.6253E-03	1.2635E-01
	SD	2.4570E-03	1.9865E-02	6.4434E-04	5.3910E-04

TABLE 4. The statistical results of the ZDT and DTLZ benchmark functions for performance metric *IGD*.

Function	Algorithm				
		MOPSO	MOALO	MOMVO	MOAOS
ZDT1	Ave	8.0879E-04	1.5144E-02	1.5496E-03	2.9535E-04
	SD	1.4764E-03	2.1178E-03	1.0733E-04	4.3465E-05
ZDT2	Ave	5.2023E-02	2.1623E-02	1.9238E-03	1.5651E-03
	SD	9.5007E-03	1.7638E-03	5.1549E-04	7.2199E-03
ZDT3	Ave	2.6241E-04	3.7118E-03	1.6411E-03	2.3471E-04
	SD	3.9884E-05	1.6638E-03	2.1512E-04	2.2085E-05
ZDT4	Ave	7.0747E-01	6.1546E-01	2.4965E-01	1.2622E+00
	SD	3.1784E-01	2.4280E-01	1.3922E-01	1.0836E-01
ZDT6	Ave	8.2281E-03	3.4649E-03	5.0062E-04	4.9371E-04
	SD	2.4862E-02	1.8983E-03	1.9857E-04	1.0701E-04
DTLZ2	Ave	4.6940E-04	3.2518E-03	8.6496E-04	3.6407E-03
	SD	2.6098E-05	4.6848E-04	1.4210E-04	1.8141E-04
DTLZ4	Ave	1.6840E-03	1.1027E-02	4.0415E-03	1.3279E-02
	SD	9.4057E-05	2.9491E-03	8.4951E-04	6.3320E-04

other methods in four of the seven cases regarding metric *IGD* while this algorithm demonstrates better performance considering metric *MS* in which the MOAOS outranks the other algorithms in most of the investigated cases. True and obtained Pareto fronts for the ZDT problems are shown in Fig. 4.

Concerning performance metric *S*, which is demonstrated in Table 6, MOAOS can provide better results in dealing with ZDT1, ZDT2, ZDT3, and ZDT6, while for the other problems, its results are very competitive.

In Figs. 4 and 5, the true and obtained Pareto fronts for the DTLZ problems using MOAOS are presented in which the capability of the algorithm in providing better solutions with closer distances to the Pareto front is observable.

D. ENGINEERING DESIGN PROBLEMS

In this section, the capability of the proposed multi-objective approach MOAOS is evaluated in dealing with several real-world engineering design problems. These problems include the four-bar truss design, welded beam design, disk brake design, and speed reducer design problems.

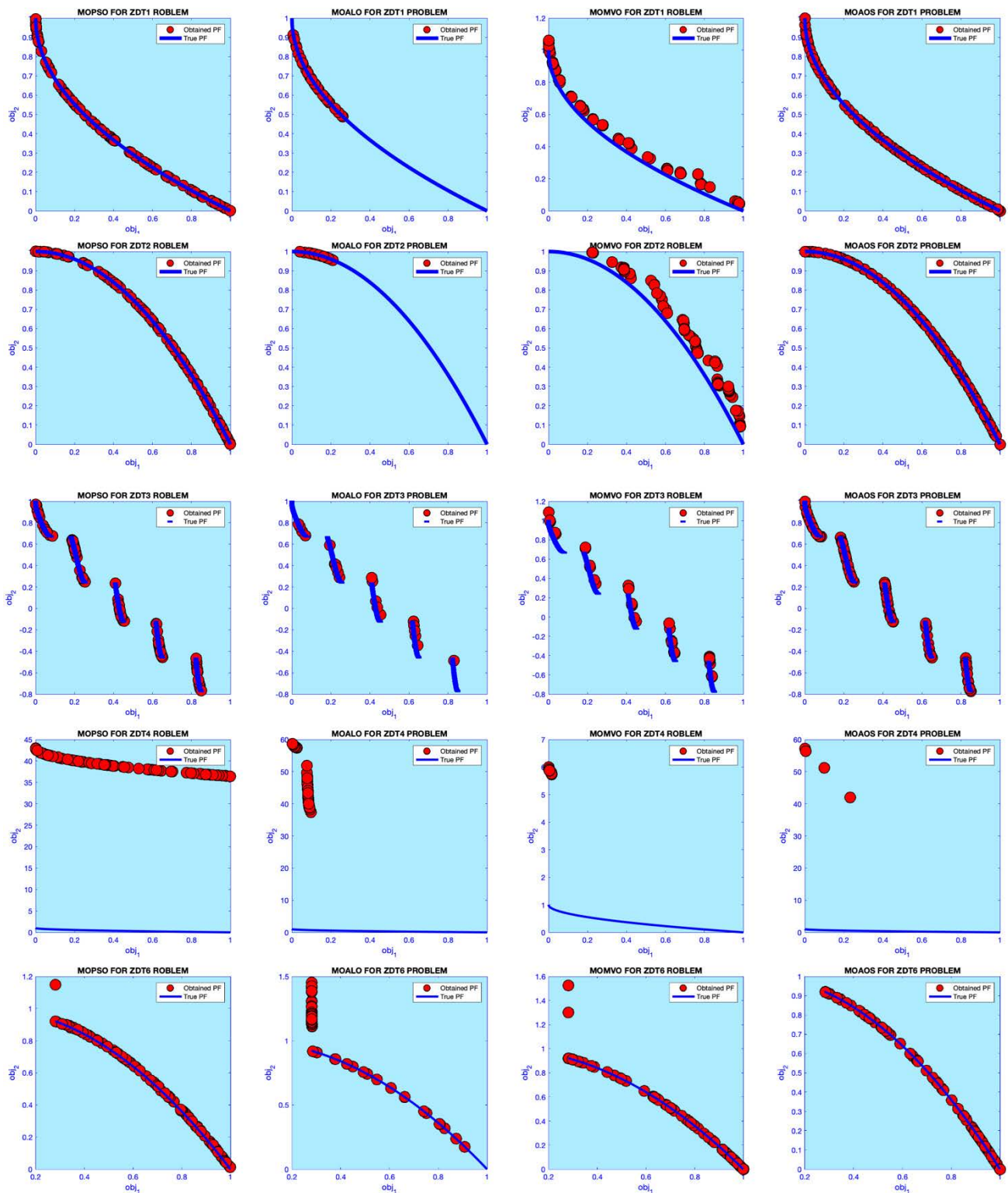


FIGURE 4. True and obtained Pareto fronts for the DTLZ problems.

1) FOUR-BAR TRUSS DESIGN PROBLEM

In the 4-bar truss design [55], two objectives (structural volume f_1 and displacement f_2) are considered to be minimized. As illustrated in Fig. 6, this problem has four design variables

x_1 to x_4 corresponding to the cross-sectional area of members 1 to 4. The equations of this example are given below:

$$\text{Minimize: } f_1(x) = 200(2x_1 + \sqrt{2x_2} + \sqrt{x_3} + x_4) \quad (18)$$

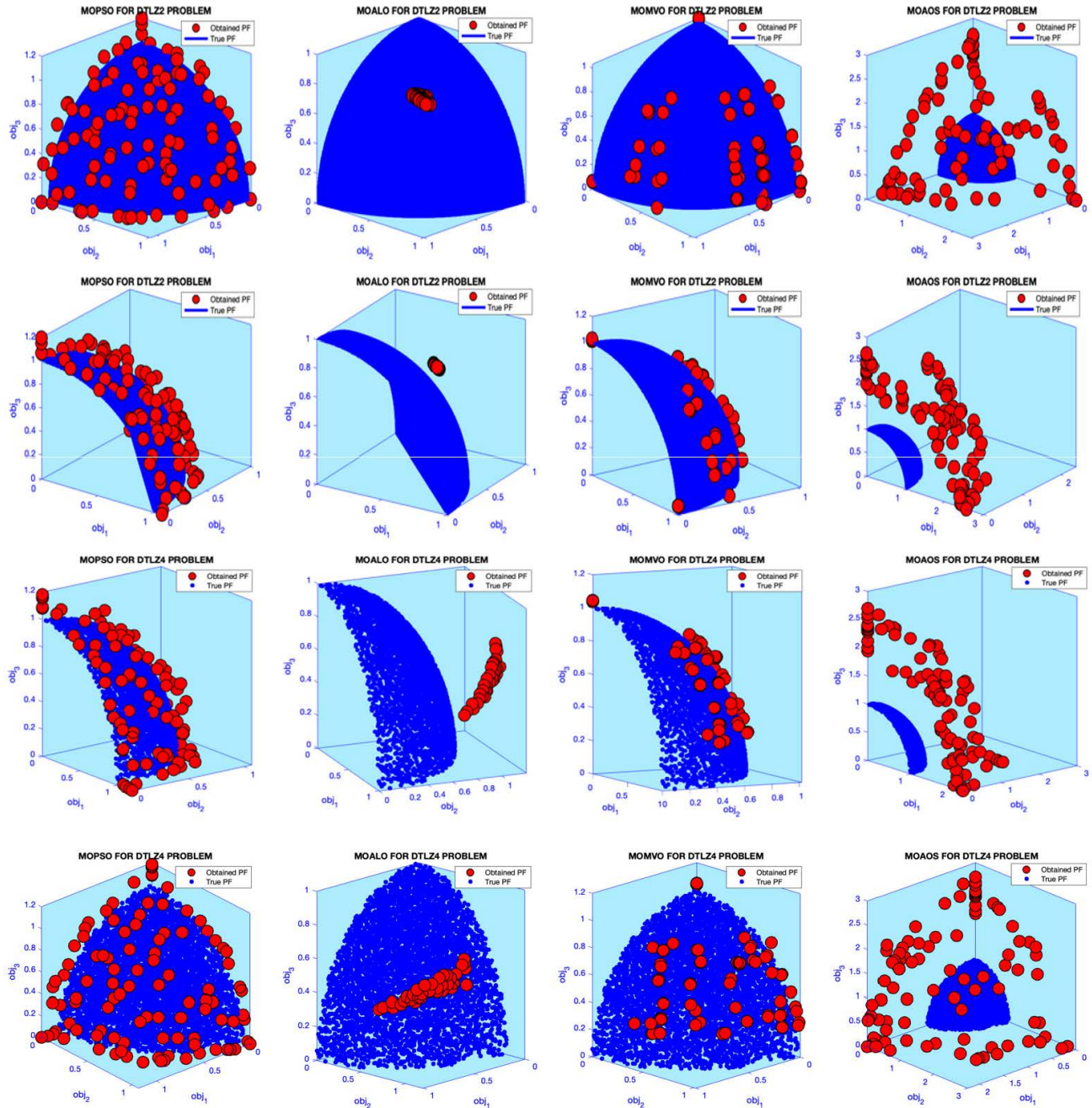


FIGURE 5. True and obtained Pareto fronts for the DTL problems (continued from the previous figure).

$$\begin{aligned}
 \text{Minimize: } f_2(x) &= 0.01 \left(\frac{2}{x_1} \right) + \left(\frac{2\sqrt{2}}{x_2} \right) \\
 &\quad - (2\sqrt{2}/x_3) + (2/x_1)) \\
 1 \leq x_1 \leq 3, \quad 1.4142 \leq x_2 \leq 3 \\
 1.4142 \leq x_3 \leq 3, \quad 1 \leq x_4 \leq 3
 \end{aligned} \quad (19)$$

2) WELDED BEAM DESIGN PROBLEM

The welded beam design problem was first introduced by Ray and Liew [56] with four constraints and two objectives, namely the fabrication cost f_1 and beam deflection f_2 of a

welded beam. As shown in Fig. 7, this problem has four design variables: the thickness of the weld x_1 , the length of the clamped bar x_2 , the height of the bar x_3 , and the thickness of the bar x_4 .

$$\text{Minimize: } f_1(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) \quad (20)$$

$$\text{Minimize: } f_2(x) = 65856000/(30 \times 10^6 x_4 x_3^3) \quad (21)$$

$$\text{where: } g_1(x) = \tau - 13600 \quad (22)$$

$$g_2(x) = \sigma - 30000 \quad (23)$$

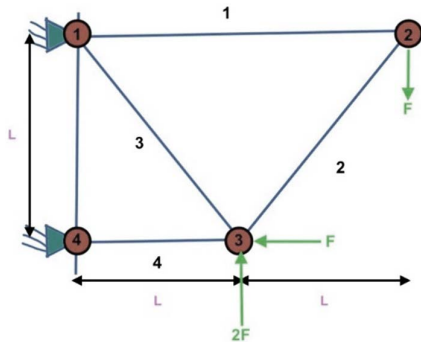
$$g_3(x) = x_1 - x_4 \quad (24)$$

TABLE 5. The statistical results of the ZDT and DTLZ benchmark functions for performance metric *MS*.

Function		Algorithm			
		MOPSO	MOALO	MOMVO	MOAOS
ZDT1	Ave	9.9870E-01	3.0049E-01	9.2574E-01	1.0000E+00
	SD	2.3779E-02	5.1645E-02	8.6425E-01	0.0000E+00
ZDT2	Ave	7.4311E-02	3.6133E-02	9.4126E-01	1.0000E+00
	SD	2.3499E-01	4.3001E-02	1.9721E-02	0.0000E+00
ZDT3	Ave	9.9865E-01	7.1037E-01	1.2699E+00	9.9671E-01
	SD	1.7888E-03	1.1021E-01	5.8528E-01	1.0020E-03
ZDT4	Ave	4.6863E-01	2.3408E+00	1.6088E-01	1.1529E+00
	SD	1.4819E+00	4.8105E+00	7.1679E-02	6.5499E-02
ZDT6	Ave	1.1170E+00	1.4199E+00	1.5084E+00	5.3501E+00
	SD	5.3983E-01	5.8815E-01	2.6233E-01	1.2843E+00
DTLZ2	Ave	1.0976E+00	1.5348E-01	9.9553E-01	1.7522E+00
	SD	9.0869E-02	6.5189E-02	8.1926E-02	5.4922E-02
DTLZ4	Ave	1.2437E+00	3.3046E-01	8.2379E-01	1.7613E+00
	SD	1.4187E-01	1.8953E-01	5.5562E-02	5.9416E-01

TABLE 6. The statistical results of the ZDT and DTLZ benchmark functions for performance metric *S*.

Function		Algorithm			
		MOPSO	MOALO	MOMVO	MOAOS
ZDT1	Ave	1.1319E-02	5.1688E-03	5.1442E-02	4.0592E-03
	SD	1.0762E-03	1.9499E-03	1.2884E-01	9.7899E-04
ZDT2	Ave	7.3295E-04	8.6826E-04	1.4373E-02	6.0334E-04
	SD	2.3178E-03	1.1868E-03	6.5898E-03	1.1380E-03
ZDT3	Ave	1.3270E-02	1.4655E-02	9.1613E-02	1.2581E-02
	SD	2.3561E-03	9.2716E-03	1.6310E-01	1.7040E-03
ZDT4	Ave	4.3804E-03	1.4977E-02	4.5664E-03	1.3679E+00
	SD	1.3852E-02	2.5415E-02	2.2788E-03	1.9593E+00
ZDT6	Ave	1.8109E-02	2.4910E-02	6.1993E-02	1.7239E-02
	SD	3.0071E-02	1.2571E-02	3.7249E-02	1.2670E-02
DTLZ2	Ave	5.7741E-02	1.1726E-02	6.1739E-02	8.0189E-02
	SD	4.9507E-03	5.3284E-03	1.0072E-02	1.5451E-02
DTLZ4	Ave	6.1316E-02	1.8314E-02	4.5515E-02	1.7155E-01
	SD	5.3260E-03	1.0509E-02	5.4484E-03	1.7547E-02

**FIGURE 6.** Schematic view of the four-bar truss.

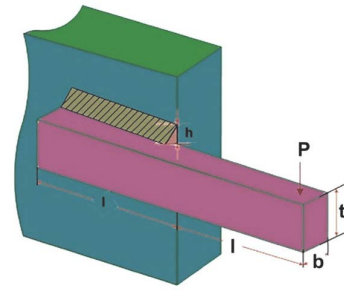
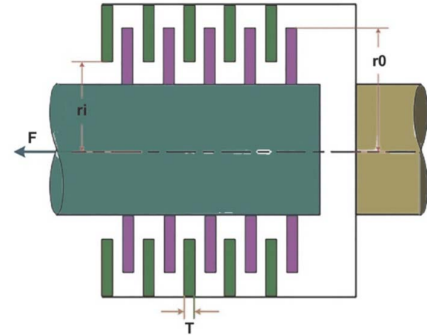
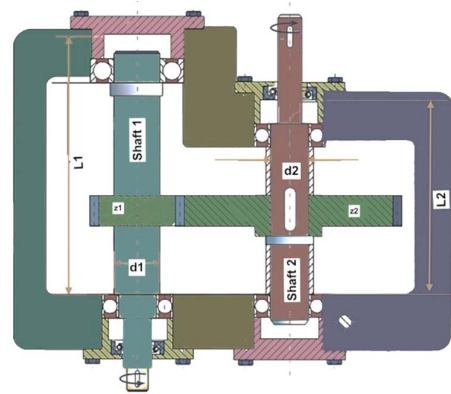
$$g_4(x) = 6000 - P$$

$$0.125 \leq x_1 \leq 5, \quad 0.1 \leq x_2 \leq 10 \quad (25)$$

$$0.1 \leq x_3 \leq 10, \quad 0.125 \leq x_4 \leq 5 \quad (25)$$

$$\text{where: } q = 6000 \left(14 + \frac{x_2}{2} \right) \quad (26)$$

$$D = \sqrt{\frac{x_2^2}{4} + \frac{x_1 + x_3}{4}}$$

**FIGURE 7.** Geometric specifications of the welded beam.**FIGURE 8.** Schematic view of the disk brake.**FIGURE 9.** Schematic view of the speed reducer.

$$J = 2 \left(x_1 x_2 \sqrt{2} \left(\frac{x_2^2}{12} + \frac{x_1 + x_3}{4} \right) \right) \quad (27)$$

$$\alpha = \frac{6000}{\sqrt{2} x_1 x_2} \quad (28)$$

$$\beta = Q \left(\frac{D}{J} \right) \quad (29)$$

3) DISK BRAKE DESIGN PROBLEM

Ray and Liew [56] proposed the disc brake design problem, which has five constraints and two objectives to be minimized, namely the stopping time f_1 and brake mass f_2 of the disc brake. As illustrated in Fig. 8, this problem has five design variables: the inner radius of the disc x_1 , the outer

TABLE 7. The statistical results of the engineering design problems for performance metric *GD*.

Function		Algorithm			
		MOPSO	MOALO	MOMVO	MOAOS
P1: BNH	Ave	3.3894E-02	5.3758E-02	8.5875E-02	3.0077E-02
	SD	2.1392E-03	1.7132E-02	4.0751E-02	2.0987E-03
P2: CONSTR	Ave	8.3098E-04	1.5711E-03	9.2588E-04	6.5993E-04
	SD	3.3369E-05	7.7806E-04	2.4850E-04	2.8834E-05
P3: DISK BRAKE	Ave	2.3045E-03	4.1887E-02	1.5855E-01	2.0314E-03
	SD	5.6273E-03	5.4951E-03	2.4372E-01	1.7673E-04
P4: 4-BAR TRUSS	Ave	1.4095E+01	2.5901E+01	1.1017E+01	1.0506E+01
	SD	5.1580E-01	2.2161E+00	4.6552E+00	1.1502E+00
P5: WELDED BEAM	Ave	1.1946E-02	4.4399E-03	1.5031E-02	4.4274E-03
	SD	1.9956E-03	6.9956E-04	4.7477E-03	8.8654E-03
P6: OSY	Ave	3.5765E+00	7.0760E-01	8.4098E-01	3.4486E+00
	SD	2.5250E+00	2.4932E-01	1.4591E-01	1.4434E+00
P7: SPEED REDUCER	Ave	8.2516E+01	7.6817E+00	9.2613E+00	6.6481E+00
	SD	9.8695E+01	3.4697E+00	1.6724E+00	1.0654E+00
P8: SRN	Ave	3.1617E-02	1.5798E-02	3.3373E-01	1.2143E-02
	SD	1.0695E-02	3.2892E-03	1.4098E-01	1.6906E-03

TABLE 8. The statistical results of the engineering design problems for performance metric *IGD*.

Function		Algorithm			
		MOPSO	MOALO	MOMVO	MOAOS
P1: BNH	Ave	9.6868E-04	1.2198E-02	3.2191E-03	7.1596E-04
	SD	1.7147E-04	3.6455E-03	3.3695E-04	6.6771E-05
P2: CONSTR	Ave	5.1838E-04	2.5041E-03	1.8516E-03	5.0894E-04
	SD	5.5618E-05	9.6528E-04	3.6526E-04	4.3545E-05
P3: DISK BRAKE	Ave	5.8831E-04	1.7399E-03	1.3515E-03	4.8101E-04
	SD	5.5995E-05	7.7421E-04	2.1737E-04	3.6266E-05
P4: 4-BAR TRUSS	Ave	2.0010E-02	2.2004E-02	2.1020E-02	2.0004E-02
	SD	3.9632E-05	1.1024E-03	4.6879E-04	2.5172E-05
P5: WELDED BEAM	Ave	5.9705E-04	6.1770E-03	2.1075E-03	5.8465E-04
	SD	4.6341E-05	2.8159E-03	4.2017E-04	1.0097E-05
P6: OSY	Ave	1.4663E-02	8.1016E-03	5.8358E-03	5.0063E-03
	SD	8.6917E-03	2.2613E-04	7.8106E-04	6.0080E-04
P7: SPEED REDUCER	Ave	6.0305E-02	1.7737E-02	8.7559E-03	1.5280E-02
	SD	7.2130E-02	3.3878E-03	2.1149E-03	4.4627E-03
P8: SRN	Ave	4.5146E-04	6.2137E-03	1.1274E-03	2.4041E-04
	SD	1.1656E-04	1.8810E-03	2.5526E-04	1.8863E-05

TABLE 9. The statistical results of the engineering design problems for performance metric *MS*.

Function		Algorithm			
		MOPSO	MOALO	MOMVO	MOAOS
P1: BNH	Ave	1.0000E+00	5.4090E-01	9.6908E-01	1.0000E+00
	SD	0.0000E+00	1.0692E-01	2.7128E-02	0.0000E+00
P2: CONSTR	Ave	9.9384E-01	7.7822E-01	9.7543E-01	9.9485E-01
	SD	6.8603E-03	7.6343E-02	1.7403E-02	2.8503E-02
P3: DISK BRAKE	Ave	9.9928E-01	9.1779E-01	1.3230E+00	9.9937E-01
	SD	1.1417E-03	2.0904E-01	4.6277E-01	6.5402E-04
P4: 4-BAR TRUSS	Ave	1.4876E+00	8.4793E-01	1.4014E+00	1.4891E+00
	SD	5.4502E-04	1.2454E-01	9.2764E-02	5.0405E-04
P5: WELDED BEAM	Ave	1.0072E+00	6.2463E-01	1.0552E+00	1.0701E+00
	SD	6.1053E-02	6.9354E-02	9.0721E-02	6.0974E-02
P6: OSY	Ave	3.2390E-01	5.7530E-01	7.1746E-01	8.8144E-01
	SD	3.4284E-01	2.6351E-02	3.8964E-02	1.7061E-01
P7: SPEED REDUCER	Ave	2.3456E-01	6.6868E-01	8.0454E-01	8.4518E-01
	SD	2.9012E-02	6.0939E-02	3.9588E-02	1.4373E-02
P8: SRN	Ave	0.9000E-01	3.9177E-01	9.2751E-01	9.8360E-01
	SD	4.5463E-02	8.2165E-02	4.4769E-02	1.9377E-02

radius of the disc x_2 , engaging force x_3 , and the number of friction surfaces x_4 . The equations of this example are given

TABLE 10. The statistical results of the engineering design problems for performance metric *S*.

Function		Algorithm			
		MOPSO	MOALO	MOMVO	MOAOS
P1: BNH	Ave	1.0901E+00	8.8402E-01	1.0487E+00	8.0847E-01
	SD	1.3631E-01	4.4648E-01	7.7663E-01	1.1128E-01
P2: CONSTR	Ave	5.8740E-02	6.9071E-02	5.0134E-02	4.3725E-02
	SD	7.8936E-03	1.8623E-02	2.7727E-02	1.3034E-02
P3: DISK BRAKE	Ave	1.1452E-01	1.4457E-01	2.7320E-01	1.0103E-01
	SD	1.3022E-02	1.1754E-02	3.9542E-01	7.9666E-03
P4: 4-BAR TRUSS	Ave	5.3605E+00	4.6988E+00	4.8245E+00	4.5530E+00
	SD	2.6169E-01	1.1721E+00	3.3581E+00	5.3213E-01
P5: WELDED BEAM	Ave	2.3432E-01	2.2431E-01	2.0967E-01	2.0302E-01
	SD	2.5702E-02	1.3595E-01	1.1261E-01	1.2009E-02
P6: OSY	Ave	1.1278E+00	1.4382E+00	1.7991E+00	7.3144E+00
	SD	1.4675E+00	4.8498E-01	3.4639E-01	2.2376E+00
P7: SPEED REDUCER	Ave	3.4532E+01	3.6722E+01	2.2500E+01	5.9626E+01
	SD	4.4378E+00	9.3623E+00	4.7232E+00	1.9261E+01
P8: SRN	Ave	2.2396E+00	1.7386E+00	2.7418E+00	1.5170E+00
	SD	4.7324E-01	8.6872E-01	1.0405E+00	2.0609E-01

below:

$$\text{Minimize: } f_1(x) = 4.9(10)^{(-5)}(x_2^2 - x_1^2)(x_4 - 1) \quad (30)$$

$$\text{Minimize: } f_2(x) = (9.82(10)^{(6)})(x_2^2 - x_1^2)/(x_2^3 - x_1^3)x_4x_3 \quad (31)$$

$$g_1(x) = 20 + x_1 - x_2 \quad (32)$$

$$g_2(x) = 2.5 + (x_4 + 1) - 30 \quad (33)$$

$$g_3(x) = x_3/(3.14(x_2^2 - x_1^2)^2) - 0.4 \quad (34)$$

$$g_4(x) = (2.22(10)^{-3}x_3(x_2^3 - x_1^3)) / (x_2^2 - x_1^2)^2 - 1 \quad (35)$$

$$g_5(x) = 900 - 2.66(0.01x_3x_4(x_2^3 - x_1^3)) / (x_2^2 - x_1^2)^2$$

$$55 \leq x_1 \leq 80, \quad 75 \leq x_2 \leq 110$$

$$1000 \leq x_3 \leq 3000, \quad 2 \leq x_4 \leq 20 \quad (36)$$

4) SPEED REDUCER DESIGN PROBLEM

In the speed reducer design problem [57] (see Fig. 9), weight f_1 and stress f_2 are the two objectives to be minimized. There are seven design variables: gear face width x_1 , teeth module x_2 , the number of teeth of a pinion x_3 (integer variable), distance between bearings 1 (x_4), the distance between bearings 2 (x_5), the diameter of shaft 1 (x_6), and the diameter of shaft 2 (x_7) as well as eleven constraints. The equations of this example are presented below:

$$\text{Minimize: } f_1(x) = 0.7854x_1x_2^2(3.3333(x_3^2 + 14.9334x_3) \dots - 43.0934) - 1.508x_1(x_6^2 + x_7^2) \quad (37)$$

$$\text{Minimize: } f_2(x) = (\sqrt{(745x_4/x_2x_3)^2 + 19.9e6})/(0.1x_6^3) \quad (38)$$

$$\text{where: } g_1(x) = 27/(x_1x_2^2x_3) - 1 \quad (39)$$

$$g_2(x) = 397.5/(x_1x_2^2x_3^2) - 1 \quad (40)$$

$$g_3(x) = (1.93x_4^3)/(x_2x_3x_6^4) - 1 \quad (41)$$

$$g_4(x) = (1.93x_5^3)/(x_2x_3x_7^4) - 1 \quad (42)$$

$$g_5(x) = \left(\sqrt{(745x_4/x_2x_3)^2} + 16.9e6 \right) / (110x_6^3) - 1 \quad (43)$$

$$g_6(x) = \left(\sqrt{(745x_4/x_2x_3)^2} + 157.5e6 \right) / (85x_7^3) - 1 \quad (44)$$

$$g_7(x) = (x_2x_3/40)1 \quad (45)$$

$$\tau = \sqrt{\alpha^2 + 2\alpha\beta \left(\frac{x_2}{2D} \right) + \beta^2} \quad (46)$$

$$\sigma = \frac{504000}{x_4x_3^2} \quad (47)$$

$$tmpf = 4.013 \left(\frac{30(10^6)}{196} \right) \quad (48)$$

$$P = tmpf \sqrt{\left(x_3^2 \left(\frac{x_4^6}{36} \right) \right) \left(1 - x_3 \left(\frac{\sqrt{(30/48)}}{28} \right) \right)} \quad (49)$$

Table 7 presents the statistical results of MOAOS and the other algorithms in dealing with the engineering design problems using performance metric *GD*. It can be seen that MOAOS can outrank the alternatives in most of the cases,

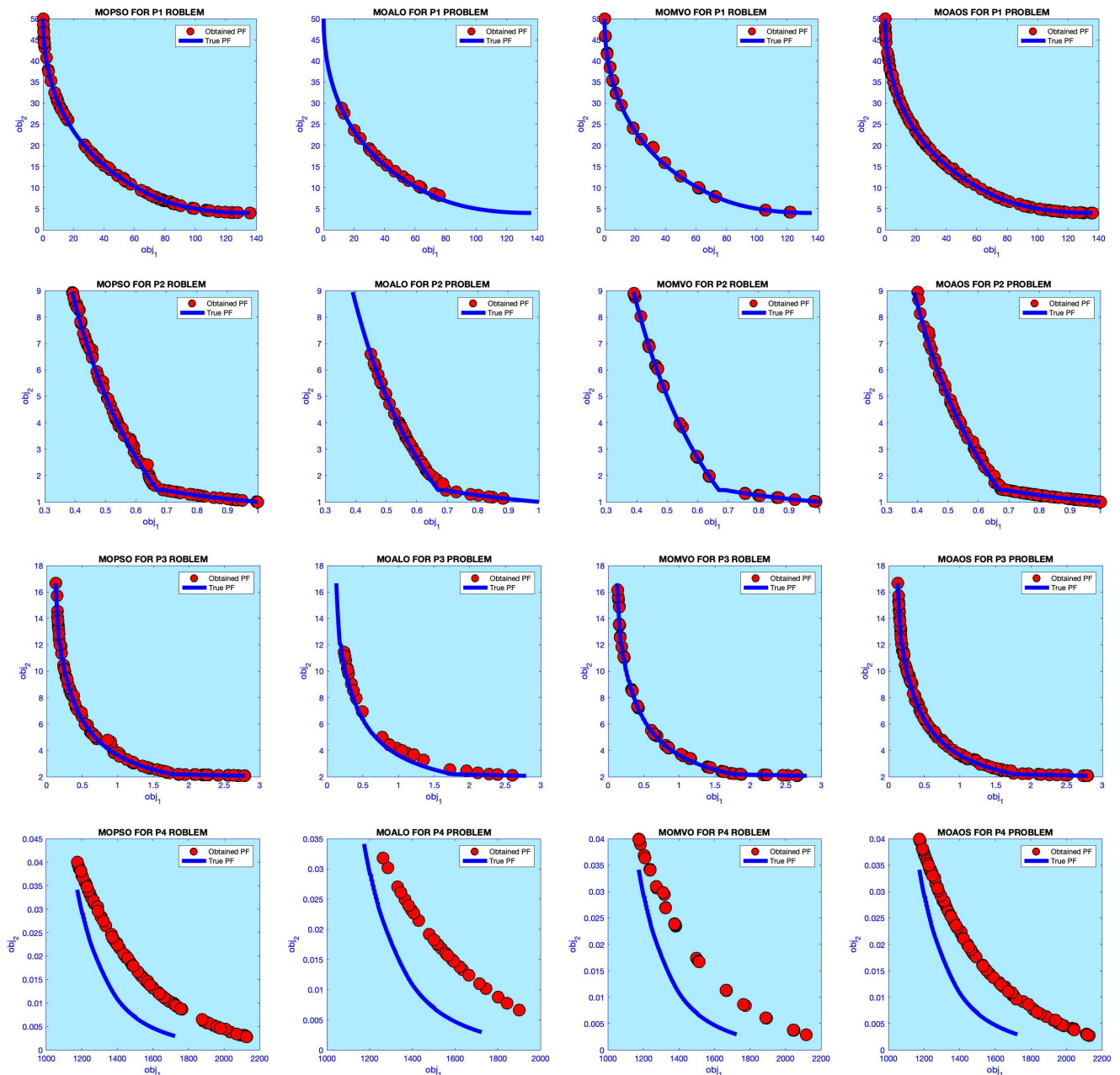


FIGURE 10. True and obtained Pareto fronts for the engineering design problems (BNH, CONSTR, DISK BRAKE, and 4-BAR TRUSS).

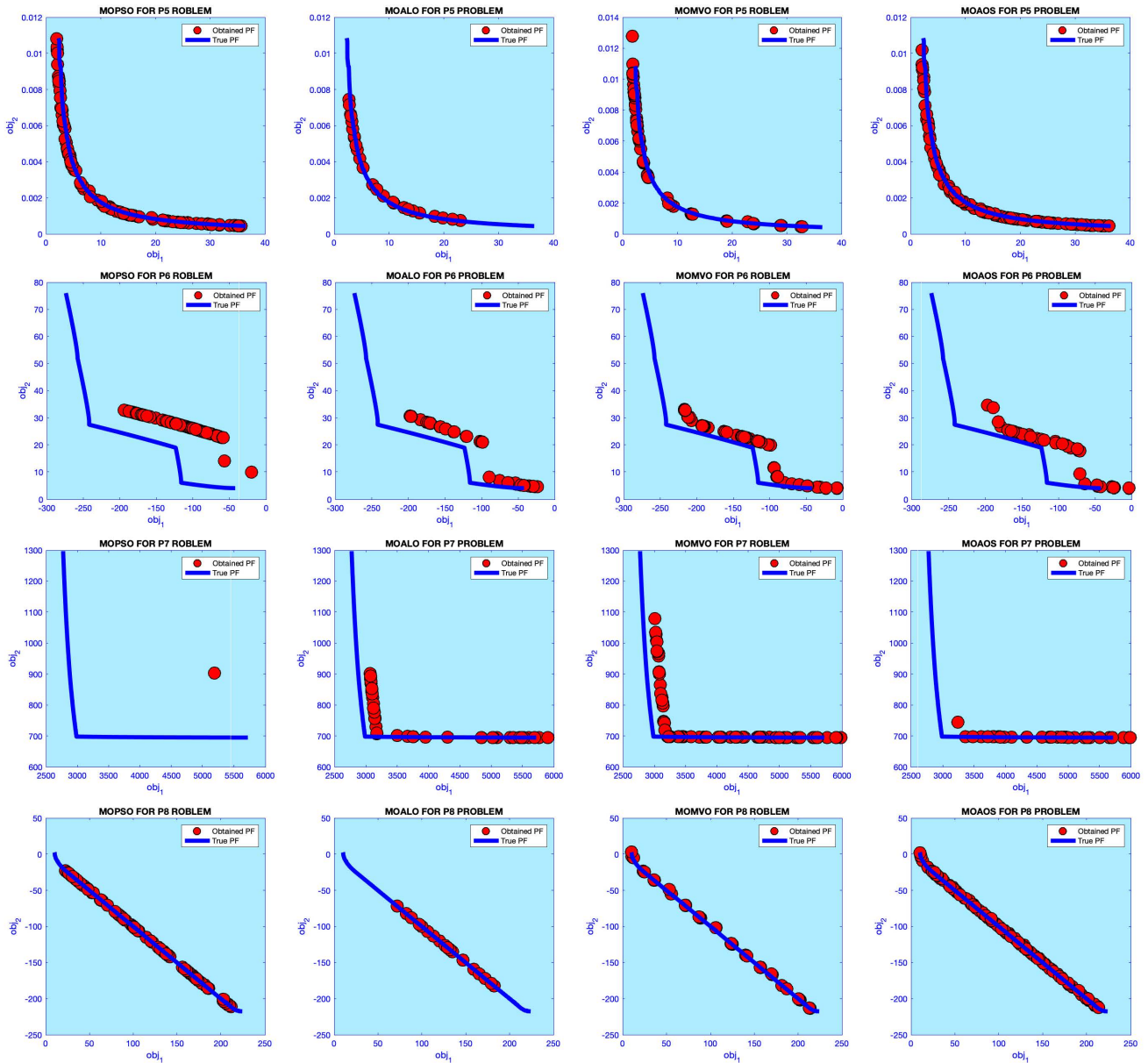


FIGURE 11. True and obtained Pareto fronts for the engineering design problems (WELDED BEAM, OSY, SPEED REDUCER, and SRN).

except for the P6: OSY, for which the result of this method is still competitive.

Considering performance metric *IGD* in dealing with the engineering design problems (Table 8), MOAOS is capable of calculating better results in comparison with MOMVO, MOALO, and MOPSO in 7 of the 8 problems, while for problem P7: SPEED REDUCER, the results of MOAOS are very close to those of the other algorithms.

Tables 9 and 10 present the results of the competing methods concerning performance metrics *MS* and *S*, respectively, for the considered engineering design problems. According to the results, the proposed method MOAOS can outrank the other algorithms in 7 of the 8 cases regarding performance metric *MS*. Furthermore, this algorithm demonstrates better

performance considering metric *S*, that is MOAOS outranks the other algorithms in most cases.

In Figs. 10 and 11, true and obtained Pareto fronts for the considered engineering design problems are presented in which the capability of the algorithm in providing better solutions with a closer distance to the Pareto fronts is noticeable.

E. DISCUSSION OF THE CEC-2020 MMO TEST PROBLEMS

The potential of the suggested multi-objective method, MOAOS, to deal with a variety of CEC-2020 problems is evaluated in this section. The details of the MMO test functions are presented in [53]. The features of the MMO test problems are linear, non-linear, convex, and concave

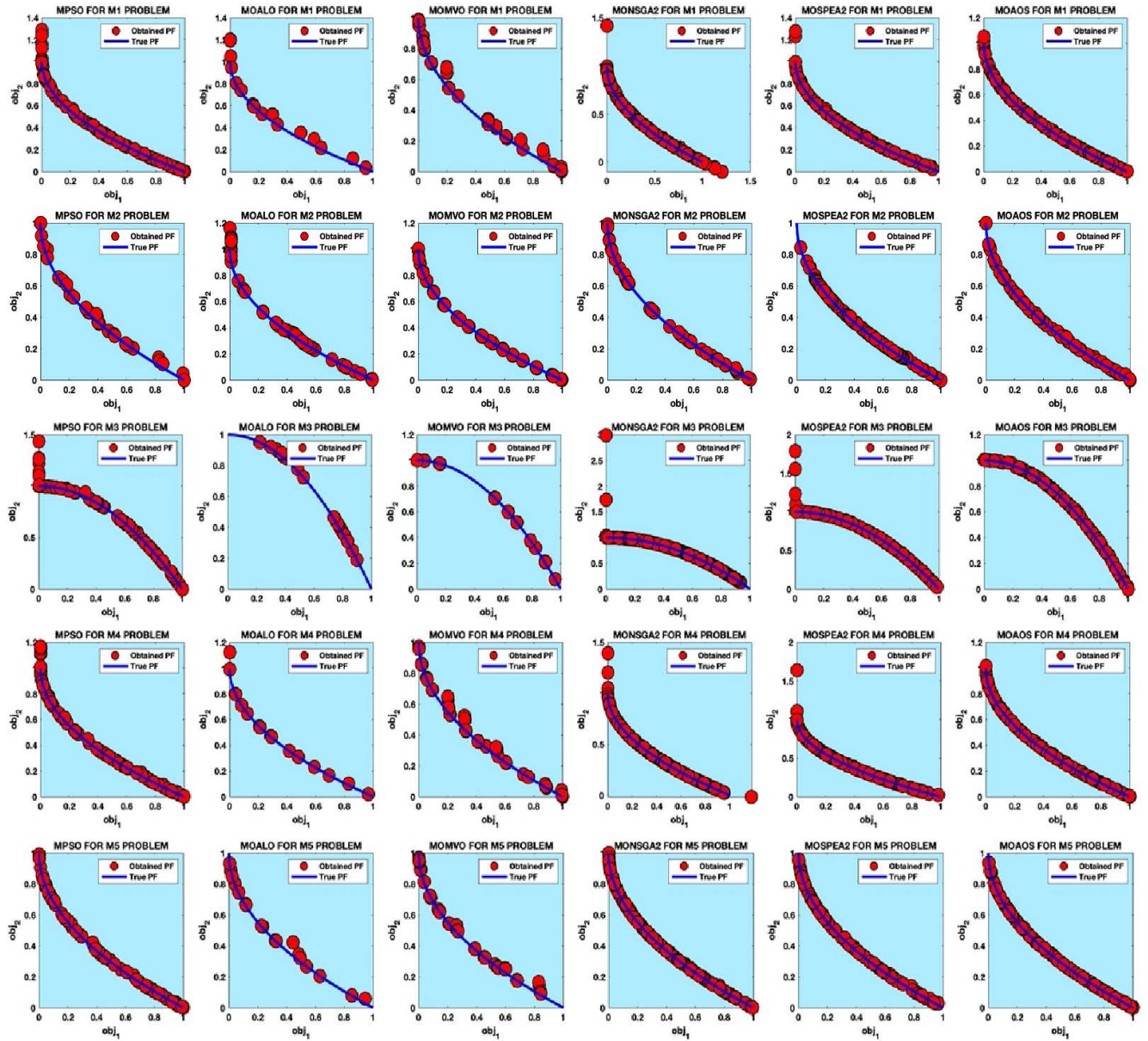


FIGURE 12. True and obtained Pareto fronts for the CEC-2020 problems M1-M5.

problems. The true and obtained Pareto fronts for the CEC-2020 problems M1-M5, as well as problems M6-M10, are presented in Figs. 12 and 13, respectively. As can be seen from these figures, the differences between the true and obtained Pareto fronts are very small. For all the given examples, the final solutions obtained by the present algorithm are along the curve of true results, while for the other methods, there are some differences between these results for all of the ten examples.

The statistical results for performance metric S are presented in Table 11. The best average result for 7 examples belongs to MOAOS while the next place belongs to MOPSO with just two best average values. MONSGA2 could find only

one best average while the other methods could not find even one. Regarding standard deviation, the present method can find 4 best results while it is two for MOPSO which ranked it in the second place.

The statistical results for performance metric MS are summarized in Table 12. As can be seen, 9 out of 10 best average results are found by the MOAOS method. This is 5 when the standard deviation is considered. All other methods find the worse values for both indices. The statistical results for metric IGD are presented in Table 13 where the new method can find 10 and 8 best average and standard deviation values, respectively. Two other best standard deviation values are obtained by MOSPEA2. Considering metric GD ,

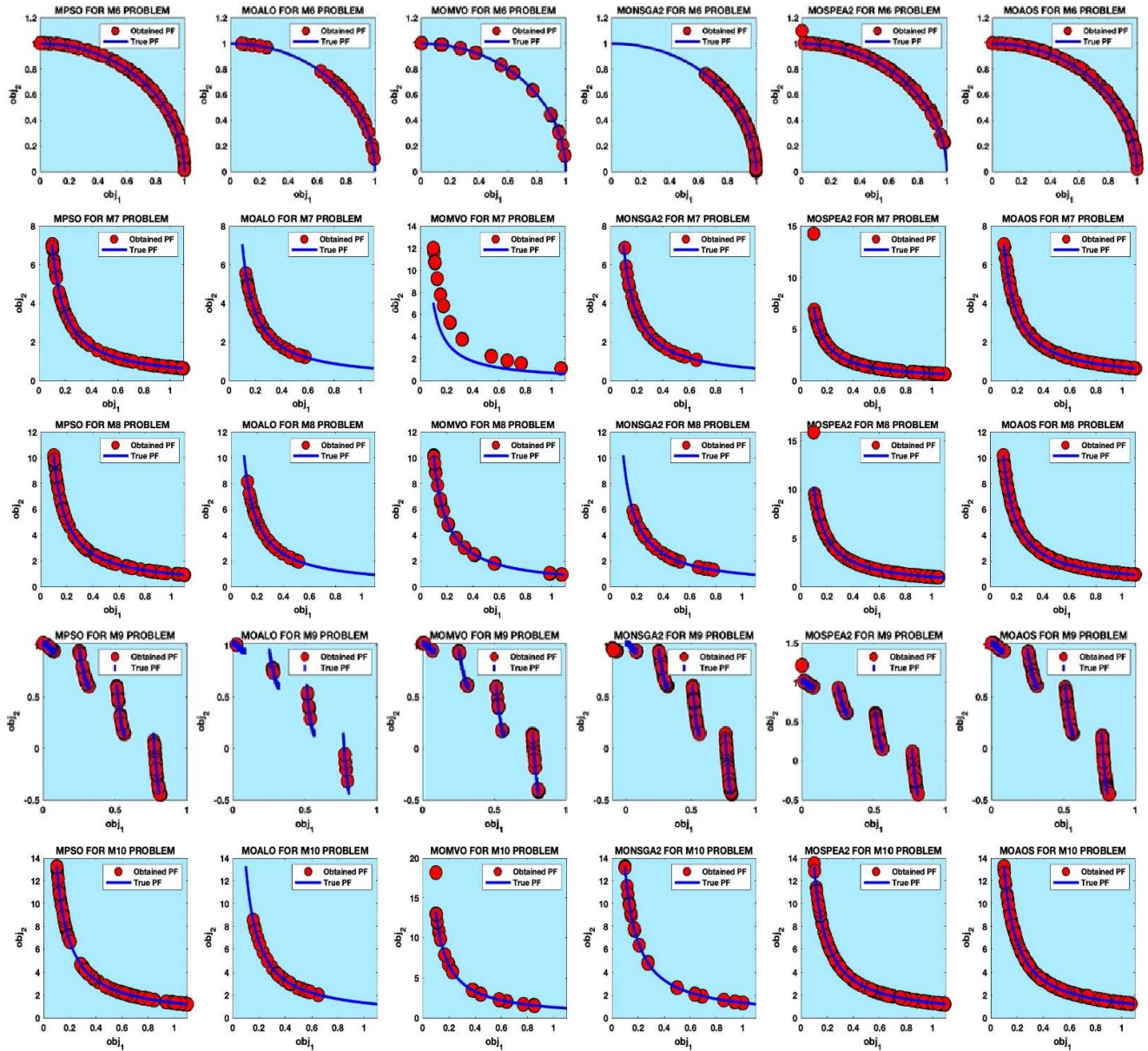


FIGURE 13. True and obtained Pareto fronts for the CEC-2020 problems M6-M10.

TABLE 11. The statistical results of the CEC-2020 problems for performance metric S .

Func.	Algorithm					
	MOPSO	MOALO	MOMVO	MONSGA2	MOSPEA2	MOAOS
M1:	Ave 1.2830E-02	1.1352E-02	2.6955E-02	1.6055E-02	1.6519E-02	1.1205E-02
MMF1	SD 1.9235E-03	6.3031E-03	3.7987E-02	1.5257E-02	1.3467E-02	1.1514E-03
M2:	Ave 1.7036E-02	6.9578E-02	9.3971E-02	4.4992E-03	2.3183E-02	1.5513E-02
MMF2	SD 3.3086E-03	1.0175E-01	2.8142E-01	4.2255E-03	6.4667E-03	1.3019E-03
M3:	Ave 1.4077E-02	6.8876E-02	8.1111E-02	4.7839E-02	6.0674E-02	1.2320E-02
MMF4	SD 6.6480E-03	1.1607E-02	1.9966E-02	6.1151E-02	4.9963E-02	4.9329E-02
M4:	Ave 1.6273E-02	1.7506E-02	2.4273E-02	2.8559E-02	2.0893E-02	1.4700E-02
MMF5	SD 1.7876E-02	1.5490E-02	3.1868E-02	1.8686E-02	1.8981E-02	1.0961E-02
M5:	Ave 1.1702E-02	1.2284E-02	1.0999E-02	6.5578E-03	7.4733E-03	1.9584E-02
MMF7	SD 2.0455E-03	6.2680E-03	4.7781E-03	7.7016E-04	8.1092E-04	1.4480E-02
M6:	Ave 1.0178E-02	3.5913E-02	1.6707E-01	5.4256E-02	1.0503E-02	1.0098E-02
MMF8	SD 8.8466E-04	7.0482E-02	4.4251E-01	4.8431E-02	2.4579E-03	4.9841E-04
M7:	Ave 4.5689E-02	7.2484E-02	1.3529E-01	6.0149E-01	1.8320E-01	7.4594E-02
MMF10	SD 5.8686E-03	4.2726E-02	1.7111E-01	9.2756E-01	3.0986E-01	6.2792E-02
M8:	Ave 6.7758E-02	6.0438E-02	1.6776E-01	8.2669E-01	1.7328E-01	6.2446E-02
MMF11	SD 8.1796E-03	3.5242E-02	2.9591E-01	9.9658E-01	2.5540E-01	1.1090E-02
M9:	Ave 1.1037E-02	7.1968E-02	2.9630E-02	2.4150E-02	1.1110E-02	8.6448E-03
MMF12	SD 1.4052E-03	1.0270E-02	3.4525E-02	2.0733E-02	6.6059E-03	1.2554E-03
M10:	Ave 8.4519E-02	8.2493E-02	2.7118E-01	7.2539E-01	8.2507E-02	9.6835E-02
MMF13	SD 1.0802E-02	3.8042E-02	2.8914E-01	1.5878E-01	1.8679E-02	1.7779E-02

TABLE 12. The statistical results of the CEC-2020 problems for performance metric MS .

Func.	Algorithm					
	MOPSO	MOALO	MOMVO	MONSGA2	MOSPEA2	MOAOS
M1:	Ave 1.1010E+00	9.8744E-01	1.1676E+00	1.0743E+00	1.0765E+00	1.2683E+00
MMF1	SD 7.7624E-02	5.7649E-02	3.7172E-01	1.7495E-01	9.3733E-02	3.4522E-02
M2:	Ave 1.0000E+00	7.3182E-01	5.6873E-01	8.7429E-01	9.0045E-01	1.0000E+00
MMF2	SD 0.0000E+00	1.6203E-01	1.9509E-01	1.6259E-01	6.5429E-02	0.0000E+00
M3:	Ave 1.1371E+00	1.1626E+00	1.0165E+00	1.3113E+00	1.4612E+00	1.5779E+00
MMF4	SD 1.2218E-01	4.9183E-01	1.4625E-01	5.1665E-01	2.8249E-01	1.0639E-01
M4:	Ave 1.1101E+00	1.1096E+00	1.0418E+00	1.2356E+00	1.1029E+00	1.1223E+00
MMF5	SD 2.6860E-01	2.9093E-01	2.2198E-01	3.0648E-01	1.4230E-01	4.3749E-01
M5:	Ave 1.0437E+00	7.7803E-01	9.5502E-01	1.0636E+00	9.8322E-01	1.0854E+00
MMF7	SD 6.6811E-02	1.2102E-01	3.1771E-02	9.2728E-02	2.3865E-02	1.1615E-01
M6:	Ave 9.9484E-01	8.1225E-01	8.0116E-01	9.2884E-01	9.6267E-01	1.0013E+00
MMF8	SD 3.7934E-03	4.7979E-01	3.0368E-01	2.2066E-01	1.6048E-02	3.6781E-02
M7:	Ave 9.9962E-01	7.7244E-01	7.0023E-01	7.0001E-01	1.1202E+00	1.0135E+00
MMF10	SD 1.4328E-03	2.1374E-01	2.6215E-01	5.5872E-01	2.9994E-01	6.2076E-02
M8:	Ave 9.9978E-01	5.5842E-01	4.0655E-01	5.1478E-01	1.0514E+00	9.9957E-01
MMF11	SD 5.5357E-04	1.3715E-01	2.1975E-01	1.5058E-01	1.5243E-01	6.2846E-05
M9:	Ave 1.0143E+00	8.1452E-01	1.0021E+00	1.0082E+00	1.0302E+00	1.3130E+00
MMF12	SD 2.7381E-03	1.1459E-01	1.7556E-01	2.0454E-01	2.8457E-02	1.8365E-03
M10:	Ave 9.9879E-01	5.4678E-01	1.0695E-01	3.1534E-01	9.9402E-01	9.9896E-01
MMF13	SD 1.6175E-03	9.7077E-02	1.4647E-01	9.3616E-01	1.8923E-02	1.4328E-02

TABLE 13. The statistical results of the CEC-2020 problems for performance metric *IGD*.

Func.		Algorithm					
		MOPSO	MOALO	MOMVO	MONSGA2	MOSPEA2	MOAOS
M1:	Ave	9.5102E-04	3.5501E-03	3.0435E-03	1.2275E-03	6.0509E-04	6.0032E-04
MMF1	SD	1.5889E-04	6.4240E-04	1.0927E-03	7.9072E-04	8.6579E-05	4.6695E-05
M2:	Ave	1.7053E-03	2.2195E-03	1.8824E-03	8.4098E-02	1.1114E-03	1.0184E-03
MMF2	SD	5.2580E-04	6.0693E-04	2.1213E-04	3.6701E-02	1.5530E-04	1.1092E-04
M3:	Ave	8.8380E-04	4.3104E-03	4.1453E-03	9.2225E-04	6.2919E-04	6.1223E-04
MMF4	SD	1.3188E-04	2.3074E-03	1.4852E-03	4.7478E-04	7.8336E-05	4.5864E-05
M4:	Ave	6.2905E-04	2.2254E-03	1.9781E-03	9.3311E-04	4.2606E-04	4.0896E-04
MMF5	SD	1.2524E-04	3.7139E-04	2.9221E-04	8.1750E-04	4.7490E-05	4.2167E-05
M5:	Ave	5.9966E-04	4.1648E-03	2.2918E-03	4.1693E-04	4.4563E-04	4.0286E-04
MMF7	SD	8.5507E-05	2.2845E-03	4.2046E-04	1.3058E-04	4.5787E-05	3.0848E-05
M6:	Ave	4.8741E-04	2.7850E-03	2.3140E-03	9.7021E-03	5.5884E-04	4.8227E-04
MMF8	SD	4.6617E-05	1.4323E-03	3.1644E-04	6.0269E-03	2.4275E-04	4.4621E-05
M7:	Ave	8.6689E-04	1.0498E-02	9.8471E-03	1.2308E-01	5.5259E-04	6.1871E-04
MMF10	SD	1.1792E-04	4.6691E-03	2.3264E-03	6.3023E-02	4.6829E-05	4.8270E-05
M8:	Ave	8.3546E-04	1.3275E-02	4.1399E-03	9.0040E-02	6.5006E-04	6.0435E-04
MMF11	SD	1.3305E-04	3.7139E-04	1.3001E-04	5.4785E-03	8.0963E-05	7.4815E-05
M9:	Ave	5.7224E-04	7.9760E-03	3.2045E-03	4.5463E-03	5.1866E-04	4.0639E-04
MMF12	SD	7.3156E-05	4.7361E-03	8.4293E-04	4.0591E-03	8.2706E-05	2.2519E-05
M10:	Ave	4.7960E-04	8.5070E-03	2.7670E-03	5.8677E-02	3.7087E-04	3.5525E-04
MMF13	SD	7.4903E-05	2.3865E-03	8.6467E-04	1.8127E-02	2.8785E-05	4.6550E-05

TABLE 14. The statistical results of the CEC-2020 problems for performance metric *GD*.

Func.		Algorithm					
		MOPSO	MOALO	MOMVO	MONSGA2	MOSPEA2	MOAOS
M1:	Ave	4.3186E-03	3.4123E-03	1.0260E-02	1.9910E-03	2.0460E-03	4.8968E-03
MMF1	SD	3.4149E-03	2.2041E-03	1.9434E-02	1.8536E-03	1.6976E-03	5.6097E-03
M2:	Ave	9.1674E-04	4.5363E-02	1.4757E-02	1.8771E-02	8.4660E-04	5.0363E-04
MMF2	SD	4.0150E-04	5.9002E-02	3.3831E-02	4.1967E-02	5.5284E-04	1.3059E-04
M3:	Ave	5.0516E-03	1.2315E-02	2.5339E-03	6.6119E-03	8.9003E-03	1.3742E-03
MMF4	SD	3.5719E-03	1.9541E-02	5.6747E-03	9.1911E-03	4.5998E-03	2.5298E-03
M4:	Ave	3.5761E-03	7.1338E-03	2.7303E-03	5.6545E-03	2.3828E-03	2.2299E-03
MMF5	SD	7.0682E-03	1.1942E-02	3.1423E-03	6.7115E-03	2.6207E-03	7.3371E-03
M5:	Ave	2.1713E-03	1.5325E-03	1.2566E-03	2.1779E-03	4.1118E-04	1.8038E-03
MMF7	SD	2.7895E-03	7.6967E-04	6.8928E-04	2.9046E-03	3.7701E-05	2.0838E-03
M6:	Ave	6.1456E-04	8.2581E-01	1.5998E-02	7.4073E-03	4.4688E-04	3.5444E-04
MMF8	SD	6.2339E-05	1.7631E-01	4.4325E-02	5.3640E-03	3.2762E-04	3.6364E-04
M7:	Ave	3.7693E-03	1.9117E-02	1.9716E-01	8.3186E-02	1.6914E-02	2.0744E-03
MMF10	SD	3.1363E-04	4.9548E-02	9.5115E-02	1.3433E-01	3.0823E-02	6.7576E-05
M8:	Ave	5.4043E-03	2.7476E-03	2.9264E-02	1.7049E-02	1.5090E-02	1.3123E-03
MMF11	SD	6.4116E-04	1.2052E-03	5.9077E-02	1.9630E-02	2.3380E-02	6.1749E-04
M9:	Ave	5.3106E-04	1.1783E-03	3.8872E-03	5.7456E-03	8.9097E-04	5.2137E-04
MMF12	SD	7.4206E-05	3.2875E-03	4.1624E-03	5.3509E-03	6.4367E-04	5.3981E-05
M10:	Ave	2.3370E-03	1.4258E-03	3.7143E-02	7.8191E-02	3.1004E-02	2.3746E-03
MMF13	SD	2.5284E-04	5.2630E-04	4.8184E-02	1.5743E-01	1.7482E-03	2.1753E-03

TABLE 15. The statistical results of the CEC 2020 problems for performance metric *HV*.

Func.		Algorithm					
		MOPSO	MOALO	MOMVO	MONSGA2	MOSPEA2	MOAOS
M1:	Ave	8.6139E-01	8.0976E-01	1.7347E-01	8.6250E-01	8.6600E-01	8.6622E-01
MMF1	SD	2.2075E-03	1.5404E-02	3.1453E-01	4.8924E-03	1.4551E-03	1.1096E-03
M2:	Ave	8.4743E-01	8.1996E-01	1.7347E-01	2.5632E-01	8.5826E-01	8.5911E-01
MMF2	SD	8.4995E-03	3.9078E-02	3.1453E-01	1.1224E-01	3.3417E-03	1.3962E-03
M3:	Ave	5.2920E-01	4.6506E-01	1.7347E-01	5.2768E-01	5.2549E-01	5.3108E-01
MMF4	SD	1.6853E-03	2.2157E-02	3.1453E-01	5.5076E-03	8.2373E-03	2.9119E-03
M4:	Ave	8.6246E-01	8.1883E-01	1.7347E-01	8.6322E-01	8.6608E-01	8.6617E-01
MMF5	SD	2.3502E-03	9.4110E-03	3.1453E-01	5.7035E-03	1.1679E-03	9.8780E-04
M5:	Ave	8.6357E-01	8.0293E-01	1.7347E-01	8.6607E-01	8.6606E-01	8.6631E-01
MMF7	SD	1.7802E-03	1.8170E-02	3.1453E-01	5.2099E-03	1.4801E-03	7.4461E-04
M6:	Ave	4.1473E-01	2.4250E-01	1.7347E-01	5.9098E-01	4.0680E-01	4.1507E-01
MMF8	SD	9.8463E-04	4.5147E-01	3.1453E-01	1.1616E-01	1.0125E-02	1.3179E-03
M7:	Ave	1.2835E+01	1.2443E+01	1.7347E+01	1.7477E+01	1.2835E+01	1.9833E+01
MMF10	SD	6.4439E-03	2.1281E-01	3.1453E-01	6.1336E-01	1.4674E-02	4.9478E-03
M8:	Ave	1.4471E+01	1.3734E+01	1.2347E+01	1.3270E+01	1.4470E+01	1.4478E+01
MMF11	SD	8.4915E-03	4.5664E-01	3.1453E-01	2.3914E-01	2.0633E-02	4.8050E-03
M9:	Ave	1.5694E+00	1.2992E+00	1.0347E+00	1.3786E+00	1.5628E+00	1.6674E+00
MMF12	SD	2.3370E-03	6.8819E-02	3.1453E-01	1.8156E-01	1.0959E-02	1.1629E-03
M10:	Ave	1.8365E+01	1.7347E+01	1.7347E+01	1.5411E+01	1.8385E+01	1.9376E+01
MMF13	SD	1.6166E-02	3.1453E-01	3.1453E-01	2.5389E-02	5.7891E-03	1.1677E-02

according to Table 14, MOAOS finds seven best average values while in the cases of using MOPSO, MONSGA2, and MOSPEA2, each algorithm can find only one best average value. Similarly, for the best standard deviation, 5 best values

are obtained by MOAOS. In Table 15, the statistical results for performance metric *HV* are presented. The total number of best average and standard deviation values for the ten examples for MOAOS is 8 and 7, respectively, which is the best performance compared to the other methods.

IV. CONCLUSION

In this paper, the multi-objective version of a recently-proposed single-objective metaheuristic algorithm, namely Atomic Orbital Search (AOS), was developed. To develop this new algorithm, called Multi-Objective Atomic Orbital Search (MOAOS), the general aspects and the main searching loop of AOS were modified to make the new algorithm capable of dealing with problems with many objectives. For the performance evaluation of this algorithm, the mathematical benchmark problems ZDT and DTLZ, alongside a range of real-world engineering design problems were utilized. Concerning performance metric *GD* for the ZDT and DTLZ problems, it was shown that the proposed algorithm was capable of outranking the other metaheuristic algorithms in five of the seven benchmark problems. By considering performance metrics *IGD* and *MS* for the ZDT and DTLZ problems, the proposed MOAOS algorithm can outrank the other methods in four of the seven cases regarding metric *IGD*. MOAOS also demonstrated better performance considering metric *MS* by outranking its competitors in most cases.

Concerning the engineering design problems, the proposed algorithm, MOAOS, could outrank the other algorithms in seven of the eight cases regarding metric *MS*, while it demonstrated better performance considering metric *S* for which MOAOS outranked the other algorithms in most of the cases. Concerning the true and obtained Pareto fronts for the considered ZDT, DTLZ, and the engineering design problems, MOAOS was capable of providing better solutions with a closer distance to the Pareto front.

Concerning the CEC 2020-problems, various statistical indices were investigated. It was shown that for all of these indices, the maximum number of best average and best standard deviation values belonged to the proposed algorithm.

As future research, potential applications of MOAOS to solving problems across other fields of science and engineering will be explored where its capabilities in dealing with more challenging optimization problems will be examined.

APPENDIX

CONSTRAINED MULTI-OBJECTIVE TEST PROBLEMS

A. CONSTR

There are two constraints and two design variables in this problem, which have a convex Pareto front.

$$\text{Minimize } f_1(x) = x_1 \quad (\text{A.1})$$

$$\text{Minimize } f_2(x) = (1 + x_2)/x_1 \quad (\text{A.2})$$

$$\text{where } g_1(x) = 6 - (x_2 + 9x_1) \quad (\text{A.3})$$

$$g_2(x) = 1 + (x_2 - 9x_1)$$

$$0.1 \leq x_1 \leq 1, \quad 0 \leq x_2 \leq 5 \quad (\text{A.4})$$

B. SRN

Srinivas and Deb [47] suggested a continuous Pareto optimal front for the following problem:

$$\text{Minimize } f_1(x) = 2 + (x_1 - 2)^2 + (x_2 - 1)^2 \quad (\text{A.5})$$

$$\text{Minimize } f_2(x) = 9x_1(x_2 - 1)^2 \quad (\text{A.6})$$

$$\text{where } g_1(x) = x_1^2 + x_2^2 - 255 \quad (\text{A.7})$$

$$g_2(x) = x_1 - 3x_2 + 10 \\ -20 \leq x_1 \leq 20, \quad -20 \leq x_2 \leq 20 \quad (\text{A.8})$$

C. BNH

Binh and Korn [58] were the first to propose this problem as follows:

$$\text{Minimize } f_1(x) = 4x_1^2 + 4x_2^2 \quad (\text{A.9})$$

$$\text{Minimize } f_2(x) = (x_1 - 5)^2 + (x_2 - 5)^2 \quad (\text{A.10})$$

$$\text{where } g_1(x) = (x_1 - 5)^2 + x_2^2 - 25 \quad (\text{A.11})$$

$$g_2(x) = 7.7 - (x_1 - 8)^2 - (x_2 + 3)^2 \\ 0 \leq x_1 \leq 5, \quad 0 \leq x_2 \leq 3 \quad (\text{A.12})$$

D. OSY

Osyczka and Kundu [59] proposed five distinct regions for the OSY test problem. There are also six constraints and six design variables to consider as follows:

$$\text{Minimize } f_1(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 \quad (\text{A.13})$$

$$\text{Minimize } f_2(x) = [25(x_1 - 2)^2 + (x_2 - 1)^2 + (x_3 - 1) \\ + (x_4 - 4)^2 + (x_5 - 1)^2] \quad (\text{A.14})$$

$$\text{where } g_1(x) = 2 - x_1 - x_2 \quad (\text{A.15})$$

$$g_2(x) = -6 + x_1 + x_2 \quad (\text{A.16})$$

$$g_3(x) = -2 - x_1 + x_2 \quad (\text{A.17})$$

$$g_4(x) = -2 + x_1 - 3x_2 \quad (\text{A.18})$$

$$g_5(x) = -4 + x_4 + (x_3 - 3)^2 \quad (\text{A.19})$$

$$g_6(x) = 4 - x_6 - (x_5 - 3)^2 \quad (\text{A.20})$$

$$0 \leq x_1 \leq 10, \quad 0 \leq x_2 \leq 10, \quad 1 \leq x_3 \leq 5 \\ 0 \leq x_4 \leq 6, \quad 1 \leq x_5 \leq 5, \quad 0 \leq x_6 \leq 10 \quad (\text{A.21})$$

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