# Probabilistic analysis of resistance for RC columns with wind-dominated combination considering random biaxial eccentricity

Youbao Jiang[1]; Junlin Zheng[2]; Kailin Yang[3]; Hao Zhou[4*]; Michael Beer[5]

**Abstract:** For reinforced concrete (RC) column with biaxial eccentricity, the conventional design methods usually use the fixed eccentricity criterion to check its resistance, which may underestimate the variations of column resistance. Based on the load statistics compatible with the codes, the random properties of biaxial eccentricity are analyzed with Monte Carlo simulation (MCS) for representative columns in regular frame structures under both vertical load and wind load. Then, the tested capacity results of 103 relevant column specimen are collected from literatures. The uncertainty of the resistance model is analyzed for the reciprocal load method in code ACI 318-14. Based on the criterion of both random eccentricity and fixed eccentricity, the probability regarding load bearing capacity exceedance is analyzed for columns by MCS with different design parameters (e.g., axial compression ratio, etc.). The results indicate that based on the prescribed load statistics, the random properties of eccentricities along two principal directions are mainly controlled by the stochastic wind load, leading to that the eccentricities along two principal directions show an approximate perfect correlation; the random biaxial eccentricity has a significant influence on resistance variations and the maximum coefficient of variation is as large as 0.73.

**keywords**: RC column; Biaxial bending and axial compression; Wind-dominated combination; Random biaxial eccentricity; Resistance Statistics.

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1. **Introduction**

Hurricanes are one of the costliest natural hazards, and it is reported that Hurricane Katrina (2006), one of the most destructive hurricanes in the US history, caused about $160 billion losses and destructive damages to constructions. Under the attack of hurricanes, RC frames designed with old codes could collapsed due to column failures (e.g., Meyer-Kiser Bank Building destroyed by hurricane in Miami, 1926), which indicates that the reliability regarding wind-resistance needs to be improved for columns supporting the whole buildings.

Earlier, many scholars (e.g., Hong and Zhou, 1999; Zou and Hong, 2011) reported the significant studies on reliability evaluation of RC columns under uniaxial bending and axial compression. With these developments, the RC columns design methods in codes have been mainly established with the fixed eccentricity criterion by reliability calibrations. The fixed eccentricity criterion can be applied well for RC columns with less important uncertainty of eccentricity (e.g., under vertical load only), and usually leads to an appropriate column reliability level (Stewart and Attard, 1999; Mirza, 1996; Breccolotti and Materazzi, 2010). However, it is known that the reliability of RC columns could be overestimated with the fixed eccentricity criterion when the uncertainty of the eccentricity becomes larger. For example, Frangopol et al. (1996) and Jiang et al. (2015) pointed out that the design reliability with consideration of random properties of eccentricity may be much lower than the targeted reliability level for columns in the case of tension failure. Moreover, Milner et al. (2001) and Jiang and Yang (2013) found that the current fixed criterion in the American code ACI 318-14 and in the Chinese code GB 50010-2010 would lead to an unsafe design, due to neglecting the randomness of eccentricity.

Actually, in frame structures with both vertical load and horizontal load (e.g., earthquakes or wind load with a certain direction), the RC columns are usually subjected to biaxial bending and axial compression rather than uniaxial bending and axial compression. Many methods have been proposed for computing the capacity of RC columns with biaxial bending and axial compression. Furlong (1961) proposed the concept of three-dimensional failure surface. Bresler (1960) followed the failure surface concept and proposed two methods: the reciprocal load method and the load contour method, which can approximate the failure surface well and have been generally accepted by researchers later. These two methods are also suggested in the corresponding specifications and standards for design (e.g., ACI 318-14; AASHTO LRFDUS-2012; ACI SP-17(14)). Subsequently, many scholars have made improvements on the basis of Bresler's work and put forward some new design equations. For example, Hsu (1998) provided a simplified mathematical equation that can describe both the failure surface and strength interaction diagrams for columns under biaxial bending and axial compression. Bonet et al. (2014) presented an analytical method to calculate the failure surfaces for columns with rectangular sections and symmetrical reinforcements. Furlong (1979), Hartley (1985) and Hong (2000) also improved the form of the equations proposing relevant calculation formulas. Galvis et al. (2020) Proposed a design method and corresponding simplified calculation formula under biaxial bending for rectangular or circular shallow foundations with or without an opening in the middle. Ma et al. (2021) Studied the bearing capacity of concrete-encased concrete-filled steel tube (CECFST) stub columns under biaxial eccentric compression through experiments and numerical simulation, and evaluated the applicability of American specification ANSI / AISC 360-10 and European Code EN 1994-1-1 for the design method (load contour method) of biaxial eccentric compression composite section. In addition, to calculate the strength of RC columns more accurately, computer-based iterative methods have been established with the development of computing techniques (e.g., the computer-aided iterative procedures provided byWang, 1988; Furlong, 2004).

However, it is worth noting that the current practice and codes for column design are less appropriate for ensuring reliability of RC columns with biaxial bending and axial compression. The reliability analyses reported by Wang and Hong (2002) and Kim and Lee (2017) were still based upon the fixed eccentricity criterion, and the loading conditions of columns were considered as simple (e.g., only vertical loads applied) rather than complex (e.g., both vertical loads and wind load applied, with random biaxial eccentricity). Although the randomness of eccentricity and the sensitivity of the column load carrying capacity to load eccentricity are obvious, the influence of random biaxial eccentricity on column resistance has not yet been studied.

In this paper, the calculations of the load carrying capacity according to the reciprocal load method specified in the ACI code (ACI 318-14) against experimental results are verified for columns, and the effects of random biaxial eccentricity on resistance are analyzed through Monte Carlo Simulation (MCS). Taking columns in three typical regular frame structures under both vertical load and wind load as examples, the random properties of biaxial eccentricity is analyzed. Then, the probabilistic analysis of column resistance is carried out with different related design parameters, specifically, using the moments ratio and the axial compression ratio. The results show that the random characteristics of biaxial eccentricity are important for RC frame columns. By comparing the resistance variation with random and fixed biaxial eccentricity, the fixed biaxial eccentricity criterion can lead to an underestimation for column resistance variation.

## 2. Capacity Design Method of Columns

### *2.1. Code-based Capacity Model of RC Column*

Biaxial bending of columns occurs when the loading causes bending simultaneously about both principal axes. According to the ACI 318-14, Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load. For a rectangular section which is shown in Figure 1, the capacity of RC columns with biaxial bending and axial compression is specified with the reciprocal load method as

$$\frac{1}{P_n} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_0} \tag{1}$$

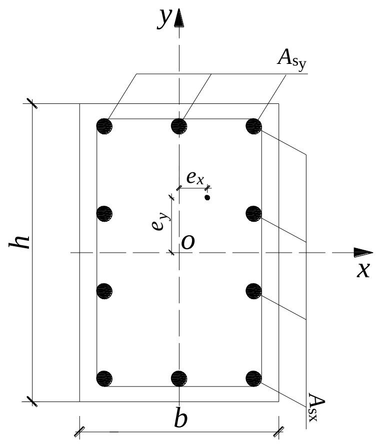



**Figure 1.** Cross section with biaxial eccentricity

where $P_n$ denotes the nominal load strength of the section under biaxial eccentricities $e_x$ and $e_y$; $P_{nx}$ ($P_{ny}$) denotes the nominal load strength with only eccentricity $e_x$ ($e_y$) presented; and $P_0$ denotes the nominal axial load strength without any eccentricities as follows:

$$P_0 = 0.85 f_c' \left( A_g - A_{st} \right) + f_y A_{st} \tag{2}$$

where $f_c'$ denotes the specified compressive strength of concrete; $A_g$ denotes the gross area of the section; and $A_{st}$ denotes the total area of the longitudinal reinforcement; $f_y$ denotes the specified yield strength of reinforcement. Experimental results have shown the Equation (1) to be reasonably accurate when flexure does not govern design. The equation should only be used when:

$$P_n \geq 0.1 \times f_c' \times A_g \tag{3}$$

If this condition is not satisfied, it would be more accurate to neglect the axial force entirely and to calculate the section for biaxial bending only. To account for the effect of accidental eccentricity, ACI 318-14 Code Section 22.4.2.1 specifies that the maximum load on a column must not exceed 0.85 times the load from Equation (2) for spiral columns and 0.8 times Equation (2) for tied columns.

In Equation (1), $P_{nx}$, $P_{ny}$ can be obtained from capacity formulas of uniaxial bending and axial compression by adopting an equivalent rectangular stress block assumption, which are introduced in ACI 318-14, as shown in Figure 2. According to the code, tension in concrete is usually neglected, and the formulas are specified as

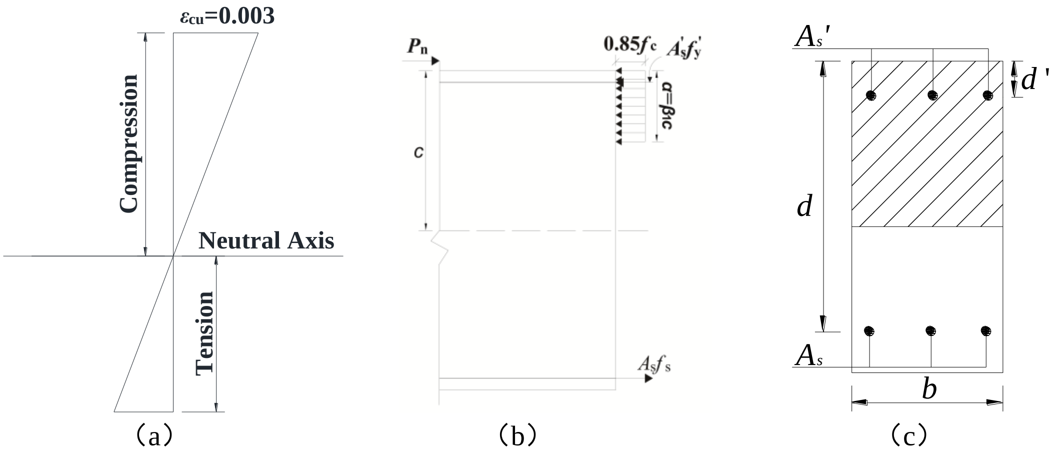



**Figure 2.** Capacity model of RC columns with uniaxial bending and axial compression: (a) strain distribution; (b) capacity model; (c) symmetrical section

$$M_n = 0.85 f_c' ab(\frac{h}{2} - \frac{a}{2}) + A_s' f_y' \left(\frac{h}{2} - d'\right) + A_s f_s (d - \frac{h}{2}) \tag{4}$$

$$P_n = 0.85 f_c' ab + A_s' f_y' - A_s f_s \tag{5}$$

$$-f_y \leq f_s = E_s \varepsilon_{cu} \left(d - c\right) / c \leq f_y \tag{6}$$

$$a = \beta_1 c \tag{7}$$

where $M_n$ and $P_n$ denote the nominal moment and the compressive force, respectively; $a$ denotes the depth of equivalent rectangular stress block; $b$ denotes the width of compression face of member; $d$ denotes distance from compression face to tension reinforcement; $d'$ denotes Concrete cover to center of reinforcing; $A_s$ and $A'_s$ denote the area of compressive and tensile reinforcement, respectively; $f_s$ denotes tensile stress in reinforcement at service loads; $f_y'$ denote specified yield strength of compressive reinforcement, and $f_y$=$f_y'$ is assumed; $E_s$ denotes the elastic modulus of steel; and $\varepsilon_{cu}$ denotes the assumed ultimate strain of concrete, with a value of 0.003; $c$ denotes the distance from extreme compression fiber to neutral axis; $\beta_1$ denotes the depth of rectangular stress distribution to the depth to the neutral axis. It is noted that for $P_{nx}$, $P_{ny}$ calculation, $A_s$ and $A_s'$ in Equations (4) - (5) need to be replaced by $A_{sx}$, $A_{sy}$ and $A_{sx}'$, $A_{sy}'$ respectively.

### *2.2. Load resistance factors design of RC Column*

For RC columns under the combination of dead load, live load and wind load, the axial force $P$ and moment

$M_x$ and $M_y$ along both section principal directions can be expressed as

$$P = P_D + P_L + P_W \tag{8}$$

$$M_x = M_{Dx} + M_{Lx} + M_{Wx} \tag{9}$$

$$M_y = M_{Dy} + M_{Ly} + M_{Wy} \tag{10}$$

Where $P_D$, $P_L$ and $P_W$ are the axial forces generated by dead load, live load and wind load respectively; $M_{Dx}$, $M_{Lx}$ and $M_{Wx}$ are moments along $x$ direction generated by dead load, live load and wind load respectively; $M_{Dy}$, $M_{Ly}$ and $M_{Wy}$ are moments along y direction generated by dead load, live load and wind load respectively.

For a basic combination with dead load, live load and wind load, the design axial force $P_d$ and design moment $M_{dx}$ and $M_{dy}$ can be the sum of factored nominal values, and given by

$$P_d = \gamma_D P_{Dn} + \gamma_L P_{Ln} + \gamma_W P_{Wn} \tag{11}$$

$$M_{dx} = \gamma_D M_{Dnx} + \gamma_L M_{Lnx} + \gamma_W M_{Wnx} \tag{12}$$

$$M_{dy} = \gamma_D M_{Dny} + \gamma_L M_{Lny} + \gamma_W M_{Wny} \tag{13}$$

where $\gamma_D$, $\gamma_L$ and $\gamma_W$ are factors for dead load, live load and wind load, respectively, in the code (ASCE 7-16), $\gamma_D$ =0.9 or 1.2 or 1.4, and $\gamma_L$ = 1.0 or 1.6, and $\gamma_W$ =1.0 for different load combinations; $P_{Dn}$, $P_{Ln}$ and $P_{Wn}$ are the nominal axial forces generated by dead load, live load and wind load respectively; $M_{Dnx}$, $M_{Lnx}$ and $M_{Wnx}$ are the nominal moments along $x$ direction generated by dead load, live load and wind load respectively; $M_{Dny}$, $M_{Lny}$ and $M_{Wny}$ are the nominal moments along $y$ direction generated by dead load, live load and wind load respectively.

The strength design in the code (ACI 318-14) can be expressed by

$$R_d = \varphi R_n \geq U \tag{14}$$

where $R_d$ and $R_n$ denote the design strength and nominal strength, respectively; $U$ denotes the factored load and moment applied to the column; and $\varphi$ denotes the strength reduction factor, and given by

$$
\varphi = \begin{cases} \varphi_c & \text{if } \varepsilon_t \le \varepsilon_{ty} \\ \varphi_c + (\varphi_t - \varphi_c)\dfrac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})} & \text{if } \varepsilon_{ty} < \varepsilon_t \le 0.005 \\ \varphi_t & \text{if } \varepsilon_t \ge 0.005 \end{cases} \tag{15}
$$

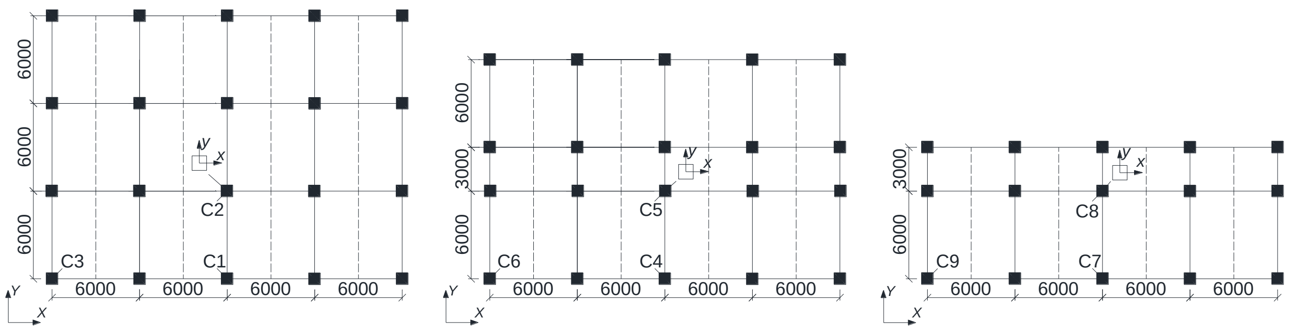
Where $\varphi_t$ is the strength reduction factor of tension-controlled sections, which is 0.90; $\varphi_c$ is the strength reduction factor of compression controlled sections, which is 0.75 for column sections with spiral reinforcement, and 0.65 for column sections with tied reinforcement; $\varepsilon_t$ is net tensile strain in extreme layer of longitudinal tension reinforcement at nominal strength; $\varepsilon_{ty}$ is net tensile strain in the extreme layer of longitudinal tension reinforcement of compression-controlled section.

Following the reciprocal load method, the nominal compressive strength $P_n$ with biaxial eccentricities $e_x$ and $e_y$ can be expressed by

$$
\frac{1}{\varphi P_n} = \frac{1}{\varphi P_{nx}} + \frac{1}{\varphi P_{ny}} - \frac{1}{\varphi P_0} \tag{16}
$$

***2.3 Reinforcement Design of Typical RC Frame Columns***

Three typical RC frame structures: models A, B, C are considered as shown in Figure 3, where the directions X and Y are set for the applied wind load, and x and y are set for the sectional principal directions. Model A, as shown in Figure 3(a), has 8 stories, where the first story height is 4.8 meters, and the other stories are 3.6 meters high. Model B, as shown in Figure 3(b), is assumed to have the same story height as Model A. Model C, as shown in Figure 3(c), has 5 stories, where the first story height is 4.5 meters, and other story heights are all 3.6 meters .




(a) Model A (b) Model B (c) Model C

**Figure 3.** The plane views of RC frame models (unit: mm)

For these models, assume that the nominal values of dead load and live load are 3kN/m$^2$ and 2kN/m$^2$, and a uniformly distributed load of 10 kN/m is applied along the frame beams of each floor to consider the gravity of the infill wall. These models are exposed to a wind speed of 54 m/s, and wind load is calculated by ETABS based on US code (ASCE 7-16). The other design information is shown in Table 1. Three representative columns are selected for each model, as shown in Figure 3. With ETABS software, the internal force and the reinforcement information are obtained for these representative columns and shown in Appendix 1 and Table 2, respectively.

**Table 1.** The design information of RC frame structure models

| Design Parameters | Model A | Model B | Model C |
|---|---|---|---|
| Main beam(mm) | 300×800 | 300×700 | 300×700 |
| Secondary beam(mm) | 250×500 | 250×500 | 250×500 |
| Concrete strength(MPa) | 27.58 | 27.58 | 27.58 |
| Rebar grade | A615Gr60 | A615Gr60 | A615Gr60 |
| Slab thickness(mm) | 100 | 100 | 100 |
| Inner columns size(mm) | 500×500 | 500×500 | 450×450 |
| Edge columns size(mm) | 500×500 | 500×500 | 450×450 |
| Corner columns size(mm) | 450×450 | 450×450 | 450×450 |

**Table 2.** The reinforcement information of representative columns

| No | $A_{sx}$/(mm$^2$) | $A_{sy}$/(mm$^2$) | $A_s$/(mm$^2$) | Controlled Combination |
|---|---|---|---|---|
| C1 | 1299 | 1299 | 2908 | $1.2D+1.0L+1.0W$ |
| C2 | 2226 | 2226 | 6331 | $1.2D+1.0L+1.0W$ |
| C3 | 675 | 675 | 2512 | $1.2D+1.0L+1.0W$ |
| C4 | 1136 | 1136 | 2713 | $1.2D+1.0L+1.0W$ |
| C5 | 1432 | 1432 | 4296 | $1.2D+1.0L+1.0W$ |
| C6 | 675 | 675 | 2029 | $1.2D+1.0L+1.0W$ |
| C7 | 995 | 995 | 2544 | $1.2D+1.0L+1.0W$ |
| C8 | 1186 | 1186 | 3945 | $1.2D+1.0L+1.0W$ |
| C9 | 675 | 675 | 1646 | $1.2D+1.0L+1.0W$ |

## 3. Random Properties of Biaxial Eccentricity for Typical Columns

### *3.1. Computational Model of Biaxial Eccentricity*

For a given column, the eccentricity $e_x$ and $e_y$ in both directions can be expressed by

$$e_x = M_x / P \tag{17}$$

$$e_y = M_y / P \tag{18}$$

Combining Equation (9) with Equation (10), the following formulas can be obtained further,

$$e_x = (P_D \cdot e_{Dx} + P_L \cdot e_{Lx} + P_W \cdot e_{Wx}) / P \tag{19}$$

$$e_y = (P_D \cdot e_{Dy} + P_L \cdot e_{Ly} + P_W \cdot e_{Wy}) / P \tag{20}$$

where $e_{Dx}$ and $e_{Dy}$, $e_{Lx}$ and $e_{Ly}$, $e_{Wx}$ and $e_{Wy}$ are the eccentricities caused by dead load, live load and wind load, respectively.

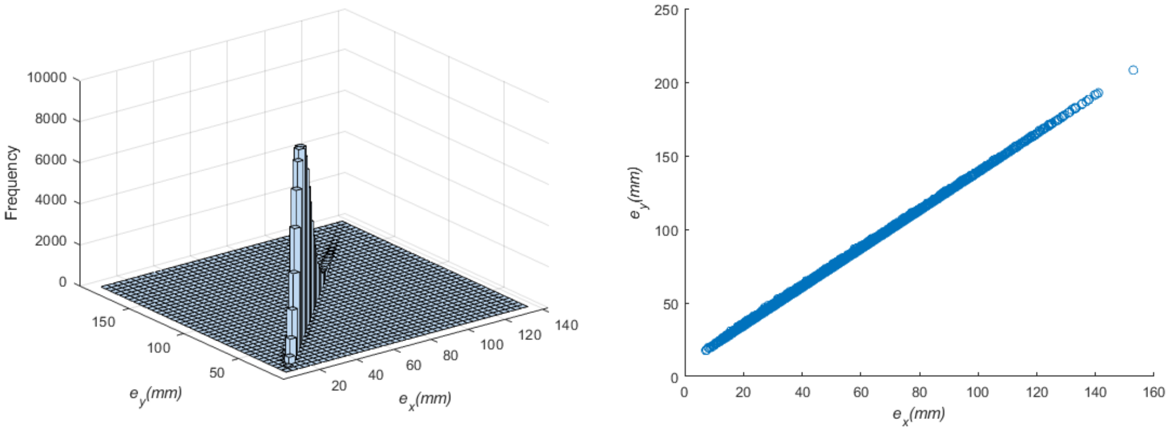
### *3.2 Random Distributions of Biaxial Eccentricity for Typical Columns*

The dead load, the live load and the wind load are usually random variables and follow different distributions. Thus, the associated eccentricities show random characteristics. The statistics of the load variables are derived from literatures as shown in Table 3, which are obtained by Nowak and szerszen (2003). Nowak and szerszen (2003) summarized the resistance models for calibration of the ACI 318 Code and provided statistical parameters of load and material strength.

**Table 3.** Statistics of load variables

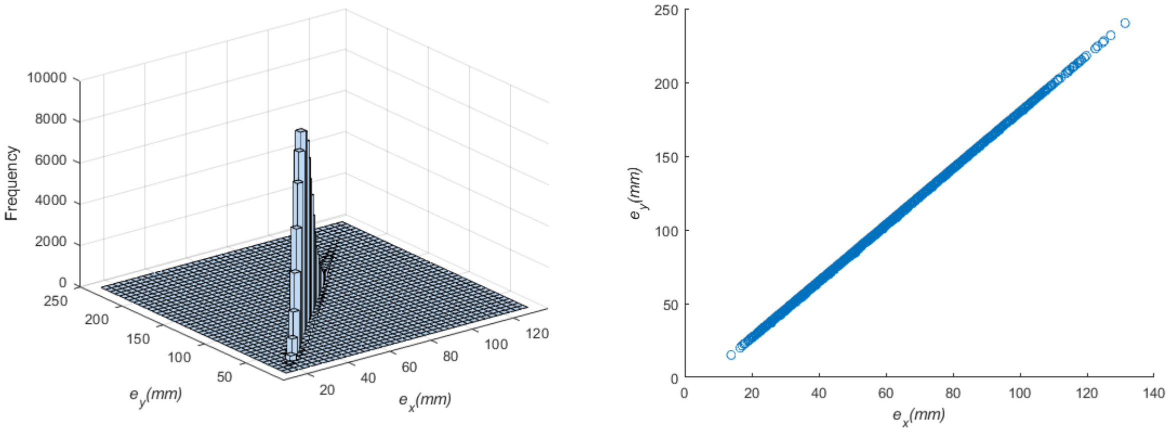
| Variable | Distribution | Mean | COV | Reference |
|---|---|---|---|---|
| $D/D_n$ | Normal | 1.05 | 0.10 | Nowak and Szerszen (2003) |
| $L/L_n$ | Gamma | 0.24 | 0.65 | Nowak and Szerszen (2003) |
| $W/W_n$ | Type-I-Largest | 0.78 | 0.37 | Nowak and Szerszen (2003) |

It is sampled $10^6$ times by MCS for statistical analysis of biaxial eccentricity of Columns C1 to C9. $\rho_{xy}$ (correlation between $e_x$ and $e_y$) of each column is particularly similar, with a value very close to 1. Thus, for simplifying, only C1 and C6 are selected as the representative columns. For a given column, the random values of internal forces (e.g.axial force and moment) caused by each applied load (e.g. dead load, live load and wind load) can be acquired by structural analysis with MCS. Then, the corresponding eccentricities ($e_{Dx}$ and $e_{Dy}$, $e_{Lx}$ and $e_{Ly}$, $e_{Wx}$ and $e_{Wy}$) are obtained based on the internal forces values. According to Eq. (19) and Eq. (20), the total random biaxial eccentricities could be obtained. By collecting the random eccentricities along two principal axis, the frequency of these data are calculated and plotted in Figures 4 and 5.




(a) histogram distribution (b) correlation

**Figure 4.** Frequency statistics of $e_x$ and $e_y$ for C1 column




(a) histogram distribution (b) correlation

**Figure 5.** Frequency statistics of $e_x$ and $e_y$ for C6 column

It can be seen that $e_x$ and $e_y$ in Figure 4 and Figure 5 present an approximately perfect correlation. As shown in Table 4, $\rho_{xy}$ are both very close to 1. $e_{Dx}$, $e_{Dy}$, $e_{Lx}$, $e_{Ly}$ are so small compared with $e_{Wx}$, $e_{Wy}$ that they can even be neglected as 0, and Equation (19) and Equation (20) can be expressed as Equation (21) and Equation (22). Thus, both $e_x$ and $e_y$ are dominated by wind load, and they present an approximately perfect correlation.

$$e_x = P_W \cdot e_{Wx} / P \tag{21}$$

$$e_y = P_W \cdot e_{Wy} / P \tag{22}$$

**Table 4.** Eccentricities produced by dead, live and wind load

| Columns | $e_{Dx}$ (mm) | $e_{Dy}$ (mm) | $e_{Lx}$ (mm) | $e_{Ly}$ (mm) | $e_{Wx}$ (mm) | $e_{Wy}$ (mm) | $\rho_{xy}$ |
|---|---|---|---|---|---|---|---|
| C1 | 0.1 | 8.1 | 0 | 15.6 | 431.9 | 572.2 | 0.999 |
| C6 | 14.4 | 12.9 | 24.1 | 20.1 | 302.1 | 567.1 | 1 |

## 4. Parametric Analysis of Resistance

### *4.1. Related Design Parameters*

The ultimate load capacity of column section is determined by the ratio of moments in two direction and axial force, which are defined as

$$\lambda_{\mu} = M_{dy} / M_{dx} \tag{23}$$

$$\theta = \arctan(\lambda_{\mu}) \tag{24}$$

$$\lambda_N = N_d / N_{cr} \tag{25}$$

Where $\lambda_{\mu}$ denotes the design moment ratio in $y$ direction to $x$ direction, $\theta$ denotes the angle of moments in two derection, and $\lambda_N$ denotes the load ratio of the design force and $N_{cr}$, $N_{cr}$ denotes the design force at balanced strain conditions, which is the condition that the concrete in the compression area just reaches the ultimate compressive strain while the tensile reinforcement yields, and given by

$$N_{cr} = 0.85 f_c' \beta_1 cb + A_s' f_y' - A_s f_y \tag{26}$$

$$c = \frac{d}{\varepsilon_y + \varepsilon_{cu}} \varepsilon_{cu} \tag{27}$$

Where $\varepsilon_y$ denotes the yield strain of reinforcement.

To distinguish the different load effect cases, other ratios are also introduced. They are given by

$$\rho_{Mx} = M_{Wnx} / (M_{Dnx} + M_{Lnx}) \tag{28}$$

$$\rho_{My} = M_{Wny} / (M_{Dny} + M_{Lny}) \tag{29}$$

$$\rho_N = N_{Wn} / (N_{Dn} + N_{Ln}) \tag{30}$$

The nominal values of axial force and moment in each direction are obtained as

$$M_{Dnx} = M_{dx} / [\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{Mx} (1 + \frac{L_n}{D_n})] \tag{31}$$

$$M_{Dny} = M_{dy} / [\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{My} (1 + \frac{L_n}{D_n})] \tag{32}$$

$$N_{Dn} = N_d / [\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_N (1 + \frac{L_n}{D_n})] \tag{33}$$

$$M_{Lnx} = \frac{M_{dx}}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{Mx} (1 + \frac{L_n}{D_n})} \frac{L_n}{D_n} \tag{34}$$

$$M_{Lny} = \frac{M_{dy}}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{My} (1 + \frac{L_n}{D_n})} \frac{L_n}{D_n} \tag{35}$$

$$N_{Ln} = \frac{N_d}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_N (1 + \frac{L_n}{D_n})} \frac{L_n}{D_n} \tag{36}$$

$$M_{Wnx} = \frac{M_{dx} \rho_M}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{Mx} (1 + \frac{L_n}{D_n})} (1 + \frac{L_n}{D_n}) \tag{37}$$

$$M_{Wny} = \frac{M_{dy} \rho_M}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{My} (1 + \frac{L_n}{D_n})} (1 + \frac{L_n}{D_n}) \tag{38}$$

$$N_{Wn} = \frac{N_d \rho_N}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_N (1 + \frac{L_n}{D_n})} (1 + \frac{L_n}{D_n}) \tag{39}$$

**Table 5.** Range of normalized design parameters

| Normalized design parameters | Value ranges |
|---|---|
| $\rho_{Mx}$ | (2.5,20) |
| $\rho_{My}$ | (2.5,20) |

| | |
|---|---|
| $\rho_N$ | (-0.15,0.15) |
| $\lambda_N$ | (0.5,3.0) |

**Table 6.** Design parameters for No.1-No.36

| No. | $\rho_{Mx}$ | $\rho_{My}$ | $\rho_N$ | No. | $\rho_{Mx}$ | $\rho_{My}$ | $\rho_N$ | No. | $\rho_{Mx}$ | $\rho_{My}$ | $\rho_N$ | No. | $\rho_{Mx}$ | $\rho_{My}$ | $\rho_N$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.5 | 2.5 | -0.15 | 10 | 2.5 | 20 | -0.05 | 19 | 5 | 5 | 0.05 | 28 | 20 | 2.5 | 0.15 |
| 2 | 2.5 | 2.5 | -0.05 | 11 | 2.5 | 20 | 0.05 | 20 | 5 | 5 | 0.15 | 29 | 20 | 5 | -0.15 |
| 3 | 2.5 | 2.5 | 0.05 | 12 | 2.5 | 20 | 0.15 | 21 | 5 | 20 | -0.15 | 30 | 20 | 5 | -0.05 |
| 4 | 2.5 | 2.5 | 0.15 | 13 | 5 | 2.5 | -0.15 | 22 | 5 | 20 | -0.05 | 31 | 20 | 5 | 0.05 |
| 5 | 2.5 | 5 | -0.15 | 14 | 5 | 2.5 | -0.05 | 23 | 5 | 20 | 0.05 | 32 | 20 | 5 | 0.15 |
| 6 | 2.5 | 5 | -0.05 | 15 | 5 | 2.5 | 0.05 | 24 | 5 | 20 | 0.15 | 33 | 20 | 20 | -0.15 |
| 7 | 2.5 | 5 | 0.05 | 16 | 5 | 2.5 | 0.15 | 25 | 20 | 2.5 | -0.15 | 34 | 20 | 20 | -0.05 |
| 8 | 2.5 | 5 | 0.15 | 17 | 5 | 5 | -0.15 | 26 | 20 | 2.5 | -0.05 | 35 | 20 | 20 | 0.05 |
| 9 | 2.5 | 20 | -0.15 | 18 | 5 | 5 | -0.05 | 27 | 20 | 2.5 | 0.05 | 36 | 20 | 20 | 0.15 |

For RC frame structures, a typical value of $L_n/D_n$ is 1.0 (Ellingwood et al., 1980). For simplification, $L_n/D_n$ in the following analysis is adopted as 1.0. By analyzing the internal force information of the models A, B and C and combining with the research results obtained by Jiang et al. (2017), the common ranges of these normalized design parameters are obtained and shown in Table 5.

In this study, the representative numbers for $\rho_{Mx}$, $\rho_{My}$, and $\rho_N$ are selected as 3, 3, and 4 respectively, which are combined for No.1-No.36, as shown in Table 6. Furthermore, the representative numbers for $\theta$, $\lambda_N$ are 3 and 4. Thus, 432 cases are considered in total.

### *4.2. Uncertainty of Resistance Model*

For columns under biaxial bending and axial compression, both random biaxial eccentricity and fixed biaxial eccentricity cases need to be considered. As mentioned earlier, due to complex loading conditions, it is important to consider the uncertainty of the resistance model for the reciprocal load method. Let the uncertainty of the resistance model be denoted as $\Omega$, which is the ratio of tested resistance to the predicted one, as expressed by

$$\Omega = P_t / P_p \tag{40}$$

where $P_t$ is the ultimate loads capacity of test; $P_p$ is the predicted loads capacity by the reciprocal load method.

Herein, a set of experimental results of 103 specimens is collected from the relevant literatures. In order to make the collected experimental data more representative, a larger range of values was considered for the design parameters. Among them, the concrete strength range from 19MPa to 48MPa, and the reinforcement ratio range from 0.89% to 5%. Their capacities are calculated by the reciprocal load method, and the uncertainty of the resistance model has been analyzed. The collected data are shown in the Appendix 2. It is shown that the uncertainty variable $\Omega$ is not constant around a certain value, but scattered within a large range from 0.86 to 1.31. Thus, the uncertainty of the variable $\Omega$ should not be neglected when assessing the capacity of a column under biaxial bending and axial compression by the reciprocal load method.

In order to find the most appropriate probability model, the experimental results of $\Omega$ have been tested with conventional probability models (e.g., normal distribution, lognormal distribution, gamma distribution, extreme values distribution, etc.) through Kolmogorov-Smirnov test. The p-value of the test result of normal distribution is 0.76, while the p-values of other distributions are less than 0.05. Thus, the variable $\Omega$ is assumed to follow a normal distribution. Through statistical calculations, the mean value of the uncertain variable $\Omega$ is 1.09 and its coefficient of variation (COV) is 0.103, as shown in Appendix 2.

### *4.3. Statistics of Resistance with Different Parameters*

For resistance evaluation, the randomness of $f_c$ and $f_y$ is usually considered. The uncertainty of the variable $\Omega$ is also considered due to its large COV. By contrast, the uncertainty of the remaining variables (e.g., dimensions of column section) can be neglected because their COV is small and has no significant impacts on resistance uncertainty. The statistics of resistance variables are given in Table 7. In the table, $f_{cs}$ and $f_{cn}$ are the sampling and nominal values of concrete strength respectively; $f_{ys}$ and $f_{yn}$ are the sampling and nominal values of steel strength respectively.

**Table 7.** Statistics of resistance variables

| Variable | Distribution | Mean | COV | Reference |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| $f_{cs}/f_{cn}$ | Normal | 1.35 | 0.10 | Nowak and Szerszen (2003) |
| $f_{ys}/f_{yn}$ | Normal | 1.145 | 0.05 | Nowak and Szerszen (2003) |
| $\Omega$ | Normal | 1.09 | 0.10 | Obtained from collected columns |

The conventional analysis methods for resistance uncertainty quantification usually neglect the effects of the randomness of eccentricity. To illustrate the importance of the randomness of eccentricity, the cases with random eccentricity and fixed eccentricity are compared under different parameter combinations. Let $N$ denotes the column resistance expressed as the axial force strength, which obviously varies with eccentricities $e_x$, $e_y$, concrete strength $f_c$ and steel strength $f_y$. Herein, a normalized resistance factor $R^{'}$ is introduced for the cases with random eccentricity and fixed eccentricity, which are respectively expressed as:

$$R^{'} = \frac{\Omega R}{R_n} = \frac{\Omega N(e_x, e_y, f_c, f_y)}{N(e_{dx}, e_{dy}, f_{cn}, f_{yn})} \tag{41}$$

$$R^{'} = \frac{\Omega R}{R_n} = \frac{\Omega N(e_{dx}, e_{dy}, f_c, f_y)}{N(e_{dx}, e_{dy}, f_{cn}, f_{yn})} \tag{42}$$

where $e_{dx}$ ,$e_{dy}$, $e_x$ and $e_y$ are respectively defined as:

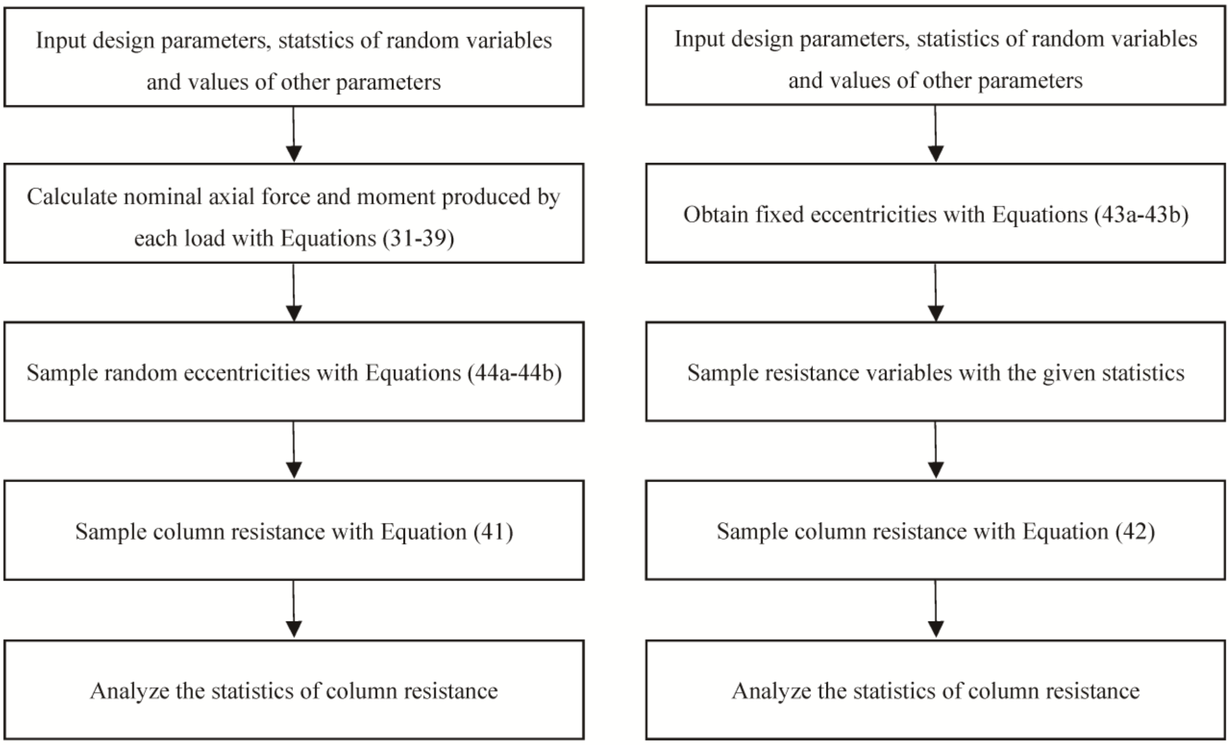
$$e_{dx} = \frac{M_{dx}}{P_d} \tag{43a}$$

$$e_{dy} = \frac{M_{dy}}{P_d} \tag{43b}$$

$$e_x = \frac{M_{Dnx}\dfrac{D}{D_n} + M_{Lnx}\dfrac{L}{L_n} + M_{Wnx}\dfrac{W}{W_n}}{P_D + P_L + P_W} \tag{44a}$$

$$e_y = \frac{M_{Dny}\dfrac{D}{D_n} + M_{Lny}\dfrac{L}{L_n} + M_{Wny}\dfrac{W}{W_n}}{P_D + P_L + P_W} \tag{44b}$$

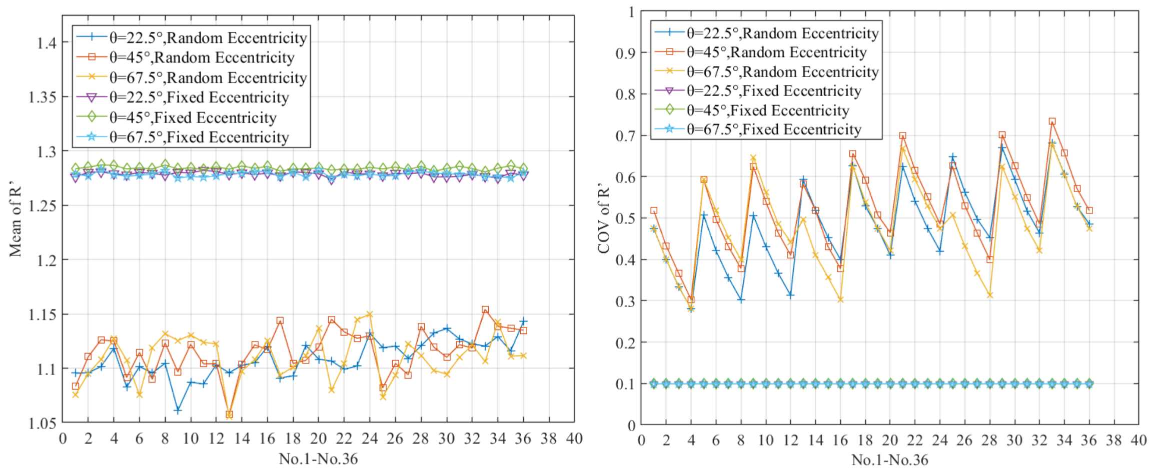
The flowchart for calculation of RC column resistance with random and fixed eccentricity is given in Figure 6.




(a) random eccentricity case (b) fixed eccentricity case

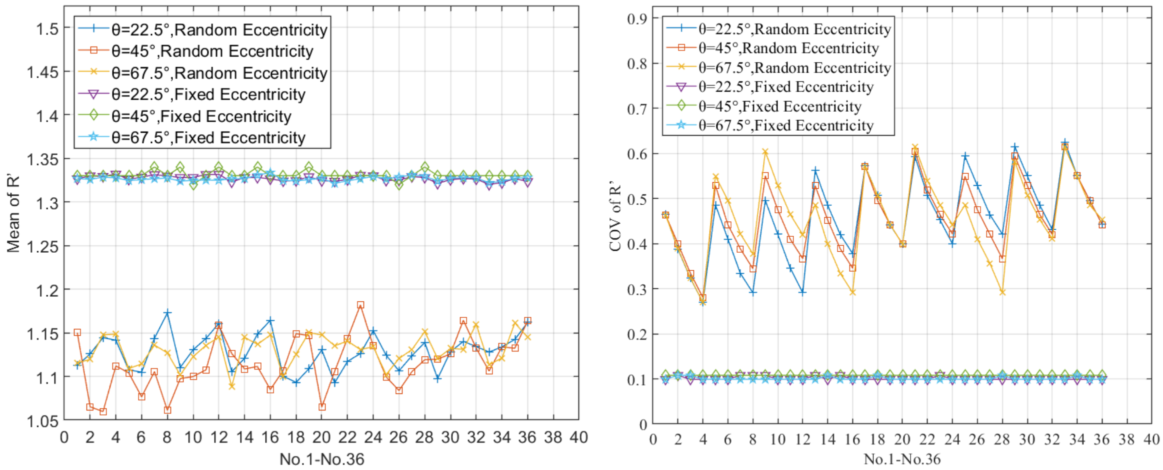
**Figure 6.** Flowchart for calculation of RC column resistance

Taking the representative column C3 as an example, the normalized resistance factor $R^{'}$ with both random eccentricity and fixed eccentricity under different parameter combinations is obtained with MCS (run $1\times10^5$ times), and the results are compared in Figure 7-10.



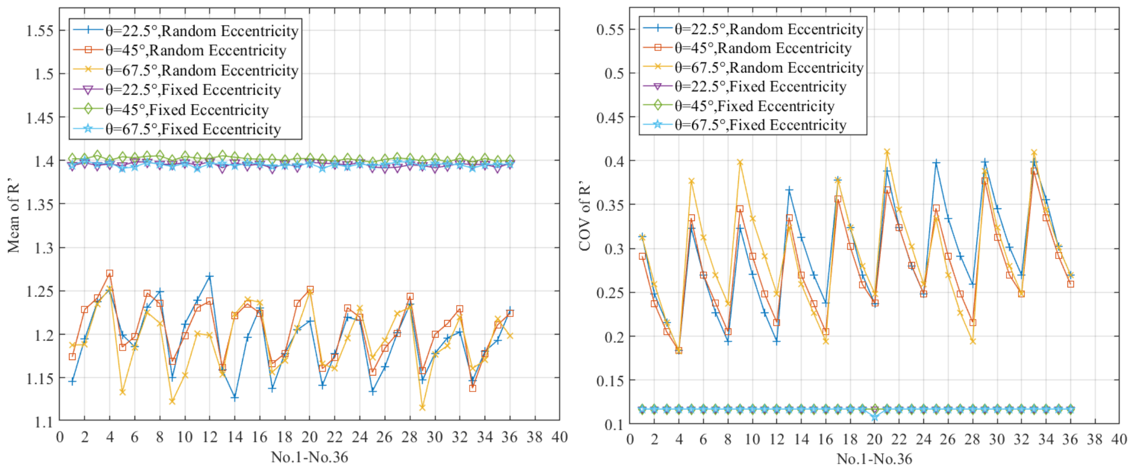

(a)Mean of $R^{'}$ (b)COV of $R^{'}$

**Figure 7.** Statistics of resistance with $\lambda_N$=0.5



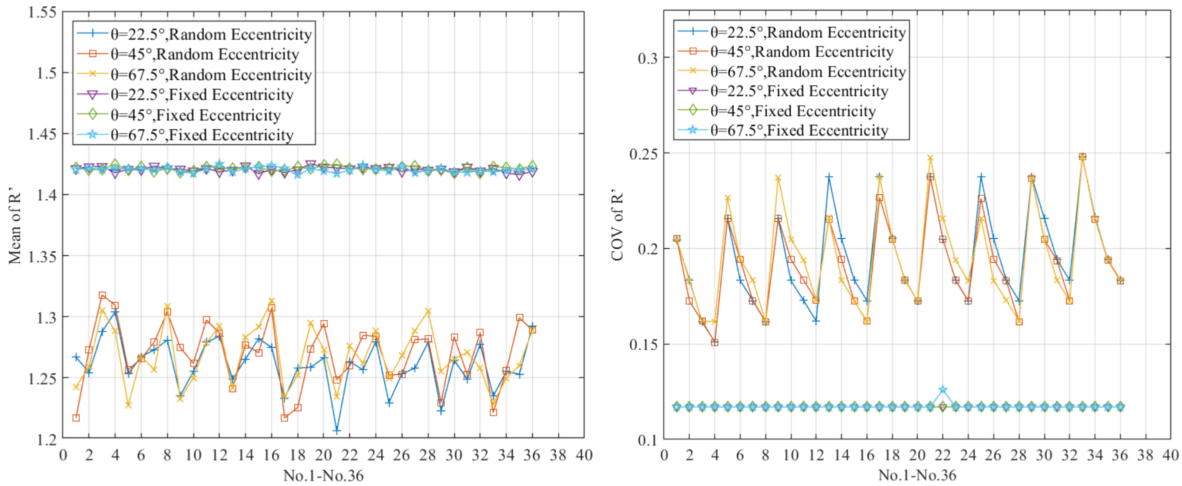

(a)Mean of $R'$ (b)COV of $R'$

**Figure 8.** Statistics of resistance with $\lambda_N$=1.0




(a)Mean of $R'$ (b)COV of $R'$

**Figure 9.** Statistics of resistance with $\lambda_N$=2.0




(a)Mean of $R'$ (b)COV of $R'$

**Figure 10.** Statistics of resistance with $\lambda_N$=3.0

It is shown that the cases with random eccentricity are quite different from the cases with fixed eccentricity. The mean values of $R^{'}$ with fixed eccentricity are higher than those with random eccentricity, and vary little among the cases. However, the mean values of $R^{'}$ with random eccentricity vary largely from 1.05 to 1.32. The COV values of $R^{'}$ with random eccentricity vary largely from 0.15 to 0.73, and those with fixed eccentricity nearly keep constant about 0.11. Generally, the mean values of $R^{'}$ with random eccentricity increase with the increase of $\lambda_N$, but the COV with random eccentricity decreases with the increase of $\lambda_N$.

**Table 8.** Differences between statistics of $R^{'}$ with fixed eccentricity and random eccentricity

| Statistics | | $R^{'}_{\mathrm{R}}$ (Combination case) | $R^{'}_{\mathrm{F}}$ (Combination case) | Error ($SI_F$-$SI_R$)/ $SI_R$ |
|---|---|---|---|---|
| Mean | Max | 1.32 ($\lambda_N$=3; $\theta$=45º; No.3) | 1.43 ($\lambda_N$=3; $\theta$=22.5º; No.19) | 8.3% |
| | Min | 1.05 ($\lambda_N$=0.5; $\theta$=67.5º; No.13) | 1.29 ($\lambda_N$=0.5; $\theta$=67.5º; No.9) | 22.9% |
| COV | Max | 0.73 ($\lambda_N$=0.5; $\theta$=45º; No.33) | 0.13 ($\lambda_N$=3; $\theta$=67.5º; No.22) | -82.2% |
| | Min | 0.15 ($\lambda_N$=3; $\theta$=45º; No.4) | 0.10 ($\lambda_N$=0.5;) | -33.3% |

Note: SI denotes the corresponding statistics item, and items with subscript 'R' and 'F' denote those with random and fixed eccentricity criterion, respectively. The following is the same.

The comparison of results for fixed eccentricity versus random eccentricity, see Table 8, shows a difference of 8.3% in the maximum mean values of *R*'. For the minimum mean values of *R*', the difference is 22.9%. Moreover, the difference in the maximum COVs of *R*' is calculated as large as 82.2% in absolute value. This indicates that the conventional methods following the fixed eccentricity concept could cause large errors in probabilistic evaluations of resistance. Thus, the random properties of eccentricity cannot be neglected for resistance evaluation and reliability calibration.

## 5. Conclusions

In this paper, taking typical RC frame columns as examples, the random characteristics of biaxial eccentricity were analyzed. With MCS, the column resistance with random biaxial eccentricity and with fixed biaxial eccentricity are calculated and compared in detail for a representative range of cases. The main conclusions are drawn as follows:

(1) Based on the prescribed load statistics, the random properties of biaxial eccentricity are found to be important for RC frame columns, and the eccentricities along two principal directions show an approximately perfect correlation with the bending moment caused by vertical load much smaller than that caused by wind load.

(2) Based on 103 sets of column results collected in literatures, the uncertainty of the resistance computation model is analyzed for the reciprocal load method, and can be assumed as a normal variable with the mean value and COV 1.09 and 0.103, respectively.

(3) The normalized resistance factors of RC columns with random biaxial eccentricity are largely different from those with fixed biaxial eccentricity. The mean values of the factors with fixed biaxial eccentricity are larger than those with random biaxial eccentricity. However, the COV values of the factors with fixed biaxial eccentricity appearing in the range between 0.10 and 0.13 are much smaller than those with random biaxial eccentricity spanning from 0.15 to 0.73.

It should be noted that this paper mainly discusses the influence of random biaxial eccentricity on the resistance of RC columns, without considering the load correlation. The influence of load correlation on the reliability of RC columns under uniaxial compression and bending has been deeply discussed in references (e.g., Frangopol et al. 1996; Hong and Zhou, 1999), but the influence of load correlation on the reliability of RC columns under random biaxial eccentricity needs to be further studied.

**Data Available Statement**

All data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

**Acknowledgment**

The research is supported by the National Natural Science Foundation of China (Grant No. 51678072), HunanProvincial Natural Science Foundation of China (2021JJ30716), Key Discipline Foundation of Civil Engineering of Changsha University of Science and Technology (18ZDXK01), and Open Fund of Bridge Engineering of Changsha University of Science and Technology (19KC01). This support is gratefully acknowledged.

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**Appendix 1.** The internal forces of representative frame columns under different load

| No. | Internal Forces | Dead | Live | $W_X$ | $W_Y$ |
|---|---|---|---|---|---|
| C1 | $P$/kN | -1762 | -269.2 | 0 | 283.8 |
| | $M_x$/ (kN·m) | 0.02 | 0 | 0 | 122.6 |
| | $M_y$/ (kN·m) | 14.2 | 4.2 | 162.4 | 0 |
| C2 | $P$/kN | -2652.5 | -520.2 | 0 | 254.8 |
| | $M_x$/ (kN·m) | 0.02 | 0 | 0 | 180.7 |
| | $M_y$/ (kN·m) | 1.22 | 0.05 | 255.9 | 0 |
| C3 | $P$/kN | -1202.7 | -143.3 | 136.8 | 268.7 |
| | $M_x$/ (kN·m) | 14.3 | 2.9 | 1.2 | 113.5 |
| | $M_y$/ (kN·m) | 12.5 | 2.3 | 168.0 | 1.5 |
| C4 | $P$/kN | -1602.0 | -255.2 | 0 | -301.6 |
| | $M_x$/ (kN·m) | 0 | 0 | 0 | 103.7 |
| | $M_y$/ (kN·m) | 16.2 | 5.0 | 165.5 | 0 |
| C5 | $P$/kN | -1984.5 | -401.0 | 0 | 242.1 |
| | $M_x$/ (kN·m) | 0 | 0 | 0 | 151.0 |
| | $M_y$/ (kN·m) | -13.0 | -3.2 | 267.0 | 0 |
| C6 | $P$/kN | -1091.9 | -135 | 180.6 | 314.8 |
| | $M_x$/ (kN·m) | 15.8 | 3.25 | 0.7 | 95.1 |
| | $M_y$/ (kN·m) | 14.1 | 2.71 | 178.5 | 0.9 |
| C7 | $P$/kN | -1446.5 | -157.9 | 0 | 244.4 |
| | $M_x$/ (kN·m) | 0 | 0 | 0 | 67.2 |
| | $M_y$/ (kN·m) | 26.75 | 8.1 | 176.6 | 0 |
| C8 | $P$/kN | -1723.9 | -216.8 | 0 | 191.3 |
| | $M_x$/ (kN·m) | 0 | 0 | 0 | 61.2 |
| | $M_y$/ (kN·m) | -17.7 | -4.0 | 200.8 | 0 |
| C9 | $P$/kN | -895.5 | -78.2 | 80.9 | 220.6 |
| | $M_x$/ (kN·m) | 18.7 | 3.0 | 0.1 | 61.4 |
| | $M_y$/ (kN·m) | 18.7 | 3.0 | 194.0 | 0.3 |

**Appendix 2.** Resistance uncertainty based on collected column tests

| Source | Specimen No. | $\Omega$ | Source | Specimen No. | $\Omega$ | Source | Specimen No. | $\Omega$ |
|---|---|---|---|---|---|---|---|---|
| Anderson and Lee (1951) | SC-4 | 0.84 | Hsu (1975) | D-1 | 1.11 | Heimdahl and Bianchini (1975) | BR-2 | 0.98 |
| | SC-9 | 0.89 | | D-2 | 1.18 | | BR-3 | 1.02 |
| Bresler (1960) | B-5 | 0.96 | | D-3 | 1.23 | | BR-4 | 0.96 |
| | B-6 | 1.15 | | D-4 | 1.16 | | BR-5 | 0.95 |
| | B-7 | 1.05 | | D-5 | 1.18 | | BR-6 | 0.90 |
| | B-8 | 1.12 | | D-6 | 1.21 | | CR-1 | 1.02 |
| Ramamurthy (1966) | A-1 | 0.99 | | E-1 | 1.26 | | CR-2 | 0.98 |
| | A-2 | 1.09 | | E-2 | 1.12 | | CR-3 | 1.02 |
| | A-3 | 1.05 | | E-3 | 1.21 | | CR-4 | 1.00 |
| | A-4 | 1.06 | | E-4 | 1.11 | | CR-5 | 0.99 |
| | A-5 | 1.15 | | F-1 | 1.08 | | CR-6 | 1.00 |
| | A-6 | 1.02 | | F-2 | 0.94 | | ER-1 | 1.31 |
| | A-7 | 1.14 | | F-3 | 1.09 | | ER-2 | 1.09 |
| | A-8 | 1.02 | | F-4 | 0.88 | | FR-1 | 1.23 |
| | A-9 | 1.12 | | F-5 | 1.06 | | FR-2 | 1.12 |
| | A-10 | 0.93 | | G-1 | 1.14 | Mavichak and Furlong (1976) | C-5 | 0.92 |
| | A-11 | 0.86 | | G-2 | 1.07 | | C-6 | 1.18 |
| | A-12 | 0.97 | | G-3 | 1.09 | | C-7 | 1.31 |
| | A-13 | 1.11 | | G-4 | 1.01 | | C-8 | 1.16 |
| | A-14 | 1.21 | | G-5 | 0.94 | | C-9 | 1.14 |
| | A-15 | 1.10 | | R-138 | 1.17 | | C-10 | 1.30 |
| | B-1 | 1.10 | | R-238 | 1.18 | | C-11 | 1.26 |
| | B-2 | 1.09 | | R-338 | 1.17 | | C-12 | 1.22 |
| | B-3 | 1.10 | | S-1 | 1.03 | | C-13 | 1.23 |
| | B-4 | 1.04 | | S-2 | 1.04 | | RC-1 | 1.28 |
| | B-5 | 1.04 | | U-1 | 1.14 | | RC-2 | 1.22 |
| | B-6 | 1.09 | | U-2 | 1.14 | | RC-3 | 1.11 |
| | B-7 | 1.18 | | U-3 | 1.12 | | RC-4 | 1.26 |
| | B-8 | 1.07 | | U-4 | 1.12 | | RC-5 | 1.06 |
| | C-1(a) | 1.14 | | U-5 | 1.03 | | RC-6 | 1.16 |
| | C-2(a) | 1.00 | | U-6 | 1.06 | | RC-7 | 1.14 |
| | C-3 | 1.11 | | H-1 | 1.08 | | RC-8 | 1.14 |
| | C-4 | 1.17 | | H-2 | 0.98 | | RC-9 | 1.16 |
| | C-5 | 1.20 | | H-3 | 1.08 | | Mean | 1.09 |
| | C-6 | 1.18 | | BR-1 | 0.93 | | COV | 0.13 |