**Supplementary Information**

# **Regression packages with splines in R**

With the progress on computer technology and statistical software, nonparametric method has become an established tool in statistical regression analysis. The most popular nonparametric method is splines regression, which are obtained by joining smoothed polynomial functions separated by a sequence of knots. In recent years, spline-based regression models have been also used in hydrology frequently. An important prerequisite for spline modelling is the availability of user friendly, well documented software packages. Here, we list the most widely used regression packages with spline techniques in R software to provide a reference for practitioners/ hydrologists.

Table S-1 Popular regression packages that can fit models using some sort of splines in R

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Package name | Description | Author | References | Website |
| gam | Generalized additive models | T. Hastie | Hastie and Tibshirani (1990)  Chambers and Hastie (1991) | <https://cran.r-project.org/web/packages/gam/index.html> |
| mgcv | Generalized additive mixed models | S. Wood | Wood (2017) | <https://cran.r-project.org/web/packages/mgcv/index.html> |
| VGLM/VGAM | Vector generalized linear and additive models | T.W. Yee | Yee (2015) | <https://www.stat.auckland.ac.nz/~yee/VGAM/>  <https://cran.r-project.org/web/packages/VGAM/index.html> |
| gamlss | Generalised additive  models for location scale and shape | M. Stasinopoulos | Stasinopoulos et al (2017)  Rigby and Stasinopoulos (2005) | <https://www.gamlss.com/>  <https://cran.r-project.org/web/packages/gamlss/index.html> |

The gam library is one of the main packages that can be used for fitting and working with Generalized additive models, as described in Chapter 7 of Chambers and Hastie (1991), and Hastie and Tibshirani (1990). The package contains code that fits several different generalized regression models, with several different types of responses. The mgcv package is particularly useful for fitting spline models, and it includes many functions that perform smoothness estimation, fit generalized additive and mixed models. Another popular package VGAM was created for fitting vector generalized additive and linear models. The package is quite powerful, in the sense that can fit a range of complicated statistical methods, including multivariable GLMs, non-linear and reduced rank models amongst other (Perperoglou et al., 2019). Package gamlss is currently the most commonly used approach in nonstationary flood frequency analysis implementation. GAMLSS models are flexible enough to accommodate the presence of abrupt changes in the mean and/or variance as well as temporal trends in the variable of interest.

# **Catchment characteristics**

Table S-2 Catchment information and descriptors

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Gauge ID | Mean flow | Maximum flow | AREA | ALTBAR | BFIHOST | FARL | PROPWET | SPRHOST |
| 2001 | 13.245 | 187.6 | 551.4 | 213 | 0.324 | 0.858 | 0.65 | 52.88 |
| 3003 | 16.292 | 404.9 | 330.7 | 297 | 0.359 | 0.915 | 0.81 | 53.57 |
| 4003 | 6.236 | 123.1 | 201 | 396 | 0.385 | 0.908 | 0.63 | 48.11 |
| 6007 | 90.4 | 768.8 | 1839.1 | 375 | 0.418 | 0.679 | 0.71 | 49.23 |
| 7001 | 13.955 | 270.8 | 415.6 | 560 | 0.451 | 0.982 | 0.68 | 55.84 |
| 7002 | 19.648 | 612 | 781.9 | 442 | 0.434 | 0.973 | 0.56 | 51.62 |
| 8002 | 22.914 | 285.8 | 1011.7 | 534 | 0.452 | 0.927 | 0.71 | 49.67 |
| 8004 | 14.658 | 259.4 | 542.8 | 525 | 0.451 | 0.989 | 0.63 | 43.44 |
| 8006 | 65.386 | 1089 | 2861.2 | 460 | 0.485 | 0.959 | 0.63 | 43.98 |
| 9002 | 16.839 | 387.7 | 954.9 | 244 | 0.511 | 0.997 | 0.46 | 35.27 |
| 10002 | 4.92 | 131 | 325 | 87 | 0.522 | 0.984 | 0.4 | 33 |
| 11002 | 14.397 | 264.6 | 787 | 332 | 0.573 | 0.997 | 0.55 | 32.44 |
| 12001 | 37.234 | 860.8 | 1370 | 512 | 0.506 | 0.976 | 0.62 | 40.27 |
| 12002 | 47.224 | 803.7 | 1844 | 447 | 0.507 | 0.98 | 0.58 | 39.69 |
| 12005 | 3.809 | 87.92 | 110 | 592 | 0.512 | 0.896 | 0.68 | 43 |
| 14001 | 4.027 | 68.85 | 307.4 | 109 | 0.609 | 0.992 | 0.4 | 29.71 |
| 15003 | 141.631 | 1503 | 3210 | 460 | 0.437 | 0.807 | 0.69 | 45.36 |
| 15006 | 171.924 | 1965 | 4587.1 | 411 | 0.473 | 0.847 | 0.58 | 42.87 |
| 15007 | 59.046 | 601.6 | 1149.4 | 467 | 0.442 | 0.836 | 0.7 | 44.6 |
| 15011 | 12.645 | 365.7 | 391.1 | 552 | 0.439 | 0.907 | 0.7 | 47.57 |
| 15012 | 74.419 | 706.5 | 1670 | 485 | 0.418 | 0.758 | 0.72 | 47.93 |
| 15016 | 46.952 | 361.6 | 600.9 | 446 | 0.423 | 0.76 | 0.71 | 44.79 |
| 16003 | 5.256 | 123.9 | 99.5 | 395 | 0.433 | 1 | 0.59 | 44.13 |
| 17005 | 4.158 | 130.8 | 195.3 | 161 | 0.411 | 0.964 | 0.56 | 40.3 |
| 18001 | 5.371 | 88.23 | 161 | 245 | 0.507 | 0.974 | 0.59 | 41.33 |
| 19007 | 4.404 | 107.3 | 330 | 239 | 0.567 | 0.944 | 0.49 | 34.09 |
| 21003 | 16.036 | 306.7 | 694 | 355 | 0.517 | 0.974 | 0.56 | 37.16 |
| 21006 | 37.482 | 647.6 | 1500 | 358 | 0.496 | 0.963 | 0.58 | 38.34 |
| 21009 | 81.306 | 1335 | 4390 | 264 | 0.495 | 0.981 | 0.49 | 38.88 |
| 21012 | 9.058 | 285.3 | 323 | 283 | 0.429 | 0.993 | 0.59 | 43.58 |
| 21017 | 1.935 | 54.68 | 37.5 | 470 | 0.421 | 1 | 0.72 | 43.7 |
| 21022 | 6.631 | 223.2 | 503 | 236 | 0.518 | 0.981 | 0.35 | 37.54 |
| 21023 | 1.022 | 71.79 | 113 | 88 | 0.388 | 0.999 | 0.3 | 37.88 |
| 21024 | 2.402 | 84.84 | 139 | 253 | 0.437 | 0.997 | 0.57 | 43.99 |
| 21034 | 3.731 | 91.34 | 116 | 460 | 0.39 | 0.767 | 0.72 | 46.92 |
| 22007 | 3.274 | 225 | 287.3 | 182 | 0.347 | 0.973 | 0.45 | 41.66 |
| 23004 | 20.928 | 693 | 751.1 | 350 | 0.298 | 0.989 | 0.6 | 49.22 |
| 25001 | 18.298 | 439 | 818.4 | 361 | 0.354 | 0.945 | 0.54 | 46.76 |
| 25006 | 2.313 | 59.2 | 86.1 | 402 | 0.241 | 0.999 | 0.62 | 55.22 |
| 25012 | 0.992 | 28.8 | 25.1 | 539 | 0.261 | 1 | 0.59 | 53.45 |
| 27006 | 5.035 | 185 | 373 | 262 | 0.416 | 0.884 | 0.37 | 38.36 |
| 27034 | 16.383 | 333 | 510.2 | 366 | 0.386 | 0.99 | 0.63 | 46.93 |
| 27035 | 6.558 | 137 | 282.3 | 230 | 0.385 | 0.977 | 0.62 | 42.48 |
| 27040 | 0.563 | 20.4 | 67.9 | 114 | 0.433 | 0.968 | 0.38 | 32.6 |
| 27041 | 16.779 | 140 | 1586 | 128 | 0.608 | 0.994 | 0.34 | 28.21 |
| 27042 | 1.042 | 18.13 | 59.2 | 221 | 0.495 | 1 | 0.4 | 36.84 |
| 27053 | 4.995 | 112 | 217.6 | 320 | 0.357 | 0.913 | 0.53 | 45.34 |
| 28001 | 2.091 | 99.71 | 126 | 416 | 0.365 | 0.786 | 0.48 | 48.42 |
| 28008 | 7.497 | 101 | 399 | 270 | 0.555 | 0.991 | 0.41 | 24.77 |
| 28009 | 84.923 | 982 | 7486 | 142 | 0.505 | 0.944 | 0.31 | 34.23 |
| 28018 | 13.799 | 168 | 883.2 | 217 | 0.528 | 0.976 | 0.42 | 30.17 |
| 28040 | 0.624 | 29.7 | 53.2 | 185 | 0.403 | 0.969 | 0.44 | 33.94 |
| 28043 | 6.367 | 134 | 335 | 335 | 0.461 | 0.909 | 0.41 | 37.49 |
| 28046 | 1.923 | 13.8 | 83 | 317 | 0.651 | 1 | 0.46 | 15.67 |
| 28056 | 0.731 | 15.5 | 94 | 114 | 0.353 | 0.962 | 0.3 | 43.22 |
| 28072 | 0.303 | 10.7 | 46.2 | 68 | 0.623 | 0.98 | 0.27 | 32.4 |
| 28080 | 13.774 | 174 | 799 | 132 | 0.472 | 0.945 | 0.3 | 36.22 |
| 28082 | 1.341 | 21.6 | 183.9 | 103 | 0.445 | 0.982 | 0.3 | 39.95 |
| 29003 | 0.448 | 5.64 | 55.2 | 91 | 0.82 | 0.958 | 0.29 | 14.11 |
| 29004 | 0.532 | 14.5 | 54.7 | 28 | 0.558 | 0.996 | 0.26 | 29.37 |
| 30001 | 1.858 | 31.6 | 297.9 | 86 | 0.592 | 0.975 | 0.27 | 28.49 |
| 30015 | 0.274 | 2.48 | 50.5 | 129 | 0.847 | 0.931 | 0.27 | 11.78 |
| 31010 | 0.513 | 12.7 | 68.9 | 113 | 0.529 | 0.998 | 0.3 | 33.1 |
| 32006 | 1.438 | 38.2 | 223 | 124 | 0.452 | 0.974 | 0.3 | 42.63 |
| 33012 | 0.609 | 21.9 | 137.5 | 61 | 0.309 | 0.992 | 0.24 | 49.43 |
| 33018 | 1.056 | 32.8 | 138.1 | 132 | 0.368 | 0.986 | 0.3 | 41.17 |
| 33019 | 1.875 | 15.1 | 316 | 39 | 0.707 | 0.932 | 0.31 | 23.94 |
| 33023 | 0.238 | 10.8 | 101.8 | 65 | 0.561 | 0.996 | 0.21 | 18.95 |
| 33029 | 0.502 | 4.7 | 98.8 | 25 | 0.864 | 0.991 | 0.23 | 12.39 |
| 33033 | 0.672 | 7.56 | 108 | 87 | 0.771 | 0.99 | 0.3 | 20.35 |
| 34003 | 1.145 | 11.8 | 164.7 | 51 | 0.778 | 0.974 | 0.31 | 20.83 |
| 34006 | 1.749 | 89.8 | 370 | 46 | 0.422 | 0.998 | 0.28 | 36.6 |
| 36003 | 0.22 | 6.79 | 53.9 | 60 | 0.555 | 0.993 | 0.26 | 37.7 |
| 36009 | 0.127 | 6.35 | 25.7 | 87 | 0.395 | 1 | 0.28 | 46.67 |
| 37005 | 1.06 | 30.3 | 238.2 | 66 | 0.537 | 0.97 | 0.25 | 38.13 |
| 37019 | 0.331 | 13.7 | 49.7 | 41 | 0.368 | 0.972 | 0.27 | 41.14 |
| 38011 | 0.207 | 2.44 | 98.7 | 130 | 0.74 | 0.981 | 0.3 | 25.98 |
| 38026 | 0.275 | 7.74 | 54.6 | 82 | 0.387 | 0.984 | 0.31 | 46.91 |
| 39019 | 1.753 | 9.529 | 234.1 | 163 | 0.839 | 0.979 | 0.32 | 16.08 |
| 39020 | 1.373 | 7.25 | 106.7 | 197 | 0.858 | 0.968 | 0.33 | 12.17 |
| 39025 | 1.315 | 26.5 | 147.6 | 120 | 0.5 | 0.978 | 0.32 | 32.78 |
| 39028 | 0.685 | 3.26 | 101.3 | 157 | 0.768 | 0.988 | 0.31 | 21.3 |
| 39034 | 3.784 | 51.4 | 430 | 142 | 0.699 | 0.965 | 0.32 | 24.1 |
| 40011 | 3.177 | 30.9 | 345 | 84 | 0.706 | 0.965 | 0.34 | 25.41 |
| 41001 | 0.22 | 9.53 | 16.9 | 49 | 0.377 | 0.999 | 0.34 | 44.84 |
| 41022 | 0.603 | 19.4 | 52 | 82 | 0.48 | 0.951 | 0.35 | 38.72 |
| 41027 | 0.533 | 21 | 37.2 | 109 | 0.658 | 0.972 | 0.35 | 26.96 |
| 42008 | 0.667 | 4.564 | 75.1 | 121 | 0.941 | 0.995 | 0.34 | 6.89 |
| 42010 | 5.506 | 21.87 | 360 | 111 | 0.949 | 0.949 | 0.34 | 6.09 |
| 43005 | 3.501 | 25.2 | 323.7 | 135 | 0.903 | 1 | 0.34 | 10.68 |
| 44006 | 0.194 | 1.13 | 12.4 | 188 | 0.879 | 0.944 | 0.38 | 13.35 |
| 45001 | 16.083 | 286 | 600.9 | 245 | 0.526 | 0.985 | 0.46 | 36 |
| 45005 | 3.205 | 86.4 | 202.5 | 144 | 0.549 | 0.996 | 0.4 | 34.34 |
| 46003 | 11.289 | 267 | 247.6 | 327 | 0.523 | 0.995 | 0.47 | 32.88 |
| 46005 | 1.264 | 29 | 21.5 | 459 | 0.363 | 1 | 0.46 | 47.42 |
| 47009 | 0.99 | 12.8 | 37.2 | 109 | 0.591 | 1 | 0.48 | 30.81 |
| 48004 | 0.865 | 12.5 | 25.3 | 219 | 0.499 | 0.978 | 0.45 | 35.7 |
| 49004 | 0.723 | 11.4 | 41 | 78 | 0.617 | 0.999 | 0.45 | 32.08 |
| 50001 | 18.037 | 358 | 826.2 | 182 | 0.472 | 0.997 | 0.48 | 37.82 |
| 52016 | 0.211 | 3.314 | 15.7 | 181 | 0.586 | 1 | 0.35 | 29.2 |
| 53006 | 1.731 | 53.5 | 148.9 | 73 | 0.362 | 0.993 | 0.35 | 43.42 |
| 53008 | 3.304 | 61.7 | 303 | 119 | 0.622 | 0.988 | 0.34 | 27.97 |
| 53009 | 1.291 | 13.8 | 72.6 | 135 | 0.643 | 0.983 | 0.37 | 27.31 |
| 53017 | 0.554 | 11.9 | 47.9 | 111 | 0.496 | 0.998 | 0.35 | 37.6 |
| 54005 | 43.384 | 462 | 2025 | 249 | 0.47 | 0.977 | 0.5 | 38.49 |
| 54006 | 2.772 | 41.63 | 324 | 114 | 0.655 | 0.978 | 0.3 | 26.95 |
| 54008 | 14.533 | 251 | 1134.4 | 231 | 0.612 | 0.994 | 0.36 | 28.53 |
| 54014 | 14.679 | 282.3 | 580 | 288 | 0.449 | 0.97 | 0.52 | 39.67 |
| 54020 | 1.597 | 15.74 | 180.8 | 102 | 0.654 | 0.954 | 0.4 | 26.5 |
| 54025 | 1.394 | 26.3 | 52.7 | 342 | 0.439 | 1 | 0.59 | 40.26 |
| 54034 | 0.369 | 25.1 | 40.8 | 127 | 0.634 | 0.997 | 0.32 | 18.84 |
| 54040 | 1.14 | 17.8 | 167.8 | 98 | 0.588 | 0.931 | 0.34 | 30.18 |
| 54044 | 0.846 | 13.16 | 92.6 | 125 | 0.698 | 0.96 | 0.34 | 25.06 |
| 55013 | 2.375 | 39.6 | 126.4 | 303 | 0.553 | 0.999 | 0.49 | 34.31 |
| 55014 | 3.908 | 58.69 | 203.3 | 299 | 0.593 | 0.996 | 0.49 | 31.41 |
| 55023 | 73.028 | 893.5 | 4010 | 227 | 0.542 | 0.979 | 0.38 | 35.6 |
| 56001 | 28.639 | 585 | 911.7 | 312 | 0.597 | 0.98 | 0.56 | 28.86 |
| 57005 | 20.892 | 397 | 454.8 | 320 | 0.409 | 0.949 | 0.5 | 43.27 |
| 57008 | 5.714 | 115.5 | 178.7 | 224 | 0.521 | 0.981 | 0.48 | 33.17 |
| 58001 | 6.728 | 139 | 158 | 211 | 0.478 | 0.998 | 0.52 | 36.37 |
| 60003 | 7.521 | 81.1 | 217.3 | 125 | 0.553 | 0.999 | 0.46 | 33.96 |
| 64006 | 1.36 | 42.5 | 47.2 | 262 | 0.504 | 0.983 | 0.66 | 38.47 |
| 65001 | 5.813 | 87.2 | 68.6 | 338 | 0.406 | 0.895 | 0.62 | 45.09 |
| 65005 | 0.611 | 28 | 18.1 | 175 | 0.439 | 0.991 | 0.56 | 39.31 |
| 66001 | 6.318 | 88.8 | 404 | 207 | 0.588 | 0.993 | 0.41 | 32.15 |
| 67003 | 0.58 | 11.87 | 20.2 | 419 | 0.319 | 0.587 | 0.7 | 52.98 |
| 67008 | 2.334 | 48 | 227.1 | 233 | 0.591 | 0.99 | 0.41 | 29.84 |
| 67018 | 3.158 | 66.23 | 53.9 | 387 | 0.312 | 1 | 0.71 | 51.9 |
| 70004 | 1.881 | 53.54 | 74.4 | 93 | 0.464 | 0.939 | 0.51 | 38.37 |
| 71001 | 34.085 | 765 | 1145 | 220 | 0.371 | 0.974 | 0.56 | 42.13 |
| 71006 | 13.711 | 282.3 | 456 | 238 | 0.367 | 0.997 | 0.61 | 43.43 |
| 71010 | 2.769 | 73.3 | 108 | 252 | 0.388 | 0.948 | 0.58 | 39.31 |
| 73005 | 9.396 | 391 | 209 | 234 | 0.514 | 0.976 | 0.71 | 38.11 |
| 73010 | 14.179 | 191 | 247 | 229 | 0.44 | 0.694 | 0.71 | 46.57 |
| 74001 | 5.14 | 162 | 85.66 | 315 | 0.337 | 0.985 | 0.71 | 53.69 |
| 75004 | 5.436 | 166.1 | 116.6 | 300 | 0.483 | 0.83 | 0.63 | 40.13 |
| 75017 | 2.408 | 46.8 | 96 | 164 | 0.487 | 0.982 | 0.62 | 34.38 |
| 77002 | 17.952 | 385.5 | 495 | 279 | 0.405 | 0.994 | 0.61 | 44.23 |
| 79002 | 28.24 | 467.1 | 799 | 294 | 0.433 | 0.991 | 0.67 | 42.18 |
| 79003 | 5.865 | 138.2 | 155 | 330 | 0.358 | 0.972 | 0.63 | 45.45 |
| 79004 | 5.82 | 111.5 | 142 | 319 | 0.446 | 0.999 | 0.66 | 41.97 |
| 80001 | 6.056 | 113 | 199 | 155 | 0.376 | 0.963 | 0.64 | 48.39 |
| 81002 | 15.975 | 262 | 368 | 238 | 0.341 | 0.932 | 0.69 | 50.86 |
| 81004 | 10.062 | 128.8 | 334 | 105 | 0.291 | 0.946 | 0.62 | 53.24 |
| 82003 | 10.938 | 169.3 | 341 | 201 | 0.392 | 0.987 | 0.63 | 45.1 |
| 83006 | 16.142 | 310.6 | 574 | 219 | 0.33 | 0.992 | 0.62 | 46.09 |
| 84004 | 19.779 | 299.3 | 741.8 | 343 | 0.458 | 0.964 | 0.6 | 42.58 |
| 84017 | 4.74 | 95.8 | 103.1 | 175 | 0.445 | 0.786 | 0.61 | 42.39 |
| 94001 | 29.72 | 227.1 | 441.1 | 311 | 0.365 | 0.665 | 0.83 | 50.08 |
| 95001 | 8.335 | 58.71 | 137.5 | 292 | 0.399 | 0.67 | 0.77 | 45.87 |
| 96002 | 15.854 | 313.5 | 477 | 224 | 0.338 | 0.822 | 0.73 | 52.34 |
| 201005 | 6.777 | 138.3 | 276.6 | 159 | 0.514 | 0.989 | 0.64 | 33.87 |
| 201006 | 8.29 | 166.9 | 320 | 134 | 0.441 | 0.998 | 0.6 | 35.83 |
| 201008 | 14.553 | 242.9 | 335.4 | 177 | 0.504 | 0.914 | 0.62 | 35.43 |
| 203012 | 9.225 | 134.3 | 430.2 | 119 | 0.523 | 0.996 | 0.56 | 34.32 |
| 203018 | 6.1 | 115.5 | 277.6 | 146 | 0.425 | 0.993 | 0.52 | 38.03 |
| 203024 | 3.716 | 60.1 | 170.7 | 132 | 0.365 | 0.992 | 0.53 | 38.23 |
| 203025 | 2.809 | 39.58 | 166.9 | 128 | 0.385 | 0.958 | 0.53 | 38.22 |
| 203028 | 2.797 | 70.5 | 100.5 | 179 | 0.404 | 0.999 | 0.61 | 41.8 |
| 205004 | 8.239 | 146.6 | 491.6 | 102 | 0.459 | 0.983 | 0.52 | 38.83 |
| 206001 | 1.999 | 35.8 | 120.3 | 97 | 0.569 | 0.972 | 0.53 | 23.69 |

# **Fractional polynomials and their implementation in R**

Fractional polynomial models can provide an alternate approach to splines and conventional polynomials for modelling nonlinear relationships. Fractional polynomials (FP) were introduced by Royston and Altman (1994), the idea is that a very flexible base to fit a parametric curve can be achieved with only a few fractional polynomials. In our NFFA models, Fractional polynomials were used as an additive ‘smoother’ term to model the relationship between flood data and covariates. The FP can include up to three terms for fitting. For example, if the FP has three terms, the following function is fitted:

Where the power *p1, p2* and *p3* can take any value within the predetermined set (-2, -1, -0.5, 0, 0.5, 1, 2, 3), with denoting . If two powers ( happen to be identical then the two terms and are fitted instead. Similarly if three powers are identical, the terms fitted are and and .

Fractional polynomials can be fitted within gamlss library using the additive function fp( ). It takes as arguments the variable and npoly (the number of fractional polynomial terms) which takes the values 1, 2 or 3. Here is an example of using fp( ) within gamlss( ) to implement NFFA in catchment Tay at Caputh (station #15003). In our NFFA model, the response variable *y* is annual maximum flow series (amax), we used time (time-varying model) and annual rainfall amount (rainfall-informed model) as explanatory variables. Here we fit time-varying models using fractional polynomials with one and two terms, implemented with the following code:

*# Load the data*

Data1 <- read\_excel("a15003.xlsx")

*# time-varying models with fractional polynomials and log-normal distribution*

*# fractional polynomials with one term*

m1=gamlss(amax~fp(time,1), sigma.fo=~fp(time,1), data = Data1, family = LOGNO, trace=FALSE)

*# fractional polynomials with two terms*

m2=gamlss(amax~fp(time,2), sigma.fo=~fp(time,2), data = Data1, family = LOGNO, trace=FALSE)

*# comparing m1 and m2 using GAIC*

GAIC(m1,m2)

## df AIC

## m1 6 965.9736

## m2 10 968.3378

GAIC(m1,m2, k=log(length(Data1$time)))

## df AIC

## m1 6 979.4645

## m2 10 990.8227

*# Use a normality test for the residuals of m1*

shapiro.test(resid(m1)) *#Shapiro-Wilk test*

##

## Shapiro-Wilk normality test

##

## data: resid(m1)

## W = 0.9933, p-value = 0.973

*# to get the fitted GAMLSS model m1*

m1

##

## Family: c("LOGNO", "Log Normal")

## Fitting method: RS()

##

## Call: gamlss(formula = amax ~ fp(time, 1), sigma.formula = ~fp(time, 1), family = LOGNO, data =

## Data1, trace = FALSE)

##

## Mu Coefficients:

## (Intercept) fp(time, 1)

## 6.523 NA

## Sigma Coefficients:

## (Intercept) fp(time, 1)

## -1.153 NA

##

## Degrees of Freedom for the fit: 6 Residual Deg. of Freedom 64

## Global Deviance: 953.974

## AIC: 965.974

## SBC: 979.465

*# to get the fitted fractional polynomial*

getSmo(m1,what="mu")

##

## Call:

## lm(formula = y ~ x.fp, weights = w)

## Coefficients:

## (Intercept) x.fp

## -0.09356 0.20016

getSmo(m1,what="sigma")

##

## Call:

## lm(formula = y ~ x.fp, weights = w)

##

## Coefficients:

## (Intercept) x.fp

## 0.1349 -0.0579

*# to get the power parameters*

getSmo(m1,what="mu")$power

## [1] -0.5

getSmo(m1,what="sigma")$power

## [1] -2

For this case, both AIC and SBC favour the model m1 with a fractional polynomial with one term. The coefficients and the power transformations of the fractional polynomials can be obtained using the getSmo( ) function of the gamlss fitted object. For the flow data in station #15003, the power for parameter *μ* is -0.5, for *σ* is -2. Hence, the fitted model m1 can be written by

Where *y* is annual maximum flow (amax), t is time, the two parameters *μ* and *σ* vary with time. Since this catchment has 70 years available data, the maximum value of time is 70.

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