Reliability evaluation of RC columns with wind-dominated combination considering random biaxial eccentricity

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Abstract: Reinforced concrete (RC) columns in frame structures are often subjected to biaxial bending and axial compression under both horizontal loads (e.g., wind load in a given direction) and vertical loads (e.g., gravity). Owing to the random properties of loads, it is important to consider the uncertainties of biaxial eccentricity. However, the fixed eccentricity criterion used in the conventional design methods cannot capture the effects of random biaxial eccentricity on reliability. Based on the reciprocal load method, the reliability is analyzed for columns with both the fixed eccentricity and random eccentricity criterion by Monte Carlo simulation. It is demonstrated that random biaxial eccentricity has a significant influence on the reliability of RC columns with wind-dominated combination.

Keywords: RC column; Biaxial bending and axial compression; Wind-dominated combination; Random biaxial eccentricity; Reliability index;

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Introduction

The reliability of RC columns has attracted extensive attention. Frangopol et al. (1996) stated that the load path and load correlation have considerable effects on the reliability of RC columns. Castaldo et al. (2022) discussed the influence of aleatory uncertainties (e.g., material and geometric uncertainties) on the resistance of slender columns. Milner et al. (2001) proposed a new method to evaluate the safety of RC short and slender columns with varying degrees of correlation between axial loads and bending moments. Moreover, Castaldo et al. (2019) considered epistemic uncertainties in their evaluation of the design ultimate strength of RC structures and proposed a failure-mode-based safety factor to assess the design global resistance.

For RC columns, the fixed eccentricity criterion can be applied well by assigning less importance to uncertainty of eccentricity (e.g., under vertical load only), and usually, an appropriate column reliability level is obtained (Breccolotti and Materazzi, 2010). However, the reliability of RC columns can be overestimated when using the fixed eccentricity criterion, especially as the uncertainty of eccentricity increases. For example, Jiang et al. (2015) pointed out that with consideration of the random properties of eccentricity, the design reliability may be considerably lower than the targeted reliability level for columns in the case of tension failure. Moreover, Milner et al. (2001) and Jiang (2013) found that the current fixed criterion in the American code ACI 318-14 and the Chinese code GB 50010-2010 would lead to an insecure design because the randomness of eccentricity is not considered.

These above mentioned studies focus mainly on columns subjected to uniaxial bending and axial compression. However, in frame structures with vertical and horizontal loads (e.g., earthquake load and wind load), RC columns are usually subjected to biaxial bending and axial compression, and the uncertainty of the biaxial eccentricity is often so large that practitioners and researchers need to attach importance to it. As for the members subjected to earthquake loads, the main problem is not bearing capacity but ductility and energy consumption. Nevertheless, wind loads are usually considered static loads when the RC frame structure is short and its rigidity is high. Therefore, to analyze the effects of random biaxial eccentricity on RC columns, we focus on the reliability evaluation of RC columns with a wind-dominated combination by conducting a parametric analysis while considering random biaxial eccentricity.

Limit State Function with Random Biaxial Eccentricity

According to ACI 318-14, the capacity of a rectangular section of an RC column subjected to biaxial bending and axial compression can be specified as follows by using the reciprocal load method:

$$\frac{1}{P_n}=\frac{1}{P_{nx}}+\frac{1}{P_{ny}}-\frac{1}{P_0} \tag{1}$$

where P_n denotes the nominal load strength of the section with biaxial eccentricities e_x and e_y; P_{nx} (P_{ny}) denotes the nominal load strength with eccentricity e_x (e_y) only; and P_0 denotes the nominal axial load strength without any eccentricity. These variables can be calculated as follows:

$$P_0 = 0.85 f_c'(A_g - A_{st}) + f_s A_{st} \tag{2}$$

$$P_{nx} = 0.85 f_c' ab + A_{sx}' f_s' - A_{sx} f_s \tag{3a}$$

$$P_{ny} = 0.85 f_c' ab + A_{sy}' f_s' - A_{sy} f_s \tag{3b}$$

where f_c' denotes the specified compressive strength of concrete; A_g denotes the gross section area; A_{st} denotes the total area of longitudinal reinforcement; f_s denotes the specified yield strength of reinforcement; A_{sx} and A_{sy} denote the area of compressive reinforcement in each direction, separately; A'_{sx} and A'_{sy} denote the area of tensile reinforcement in each direction, separately; a denotes the depth of an equivalent rectangular stress block; and b denotes the width of the compression face of the member.

For a basic combination with a dead load, live load, and wind load, the design load effect S_d (i.e., design axial force P_d and design moments M_{dx} and M_{dy}) is given as follows:

$$S_d = \gamma_D S_{D\mathrm{n}} + \gamma_L S_{L\mathrm{n}} + \gamma_W S_{W\mathrm{n}} \tag{4}$$

where γ_D, γ_L, and γ_W are partial coefficients of the dead load, live load, and wind load, respectively. According to the code (ASCE 7-16 [26]), γ_D = 0.9, 1.2, or 1.4, γ_L = 1.0 or 1.6, and γ_W = 1.0 for different load combinations.

Considering the complex loading conditions, it is important to consider the uncertainty of the resistance model in the reciprocal load method. Let the uncertainty of the resistance model be denoted by Ω and expressed as follows:

$$\Omega = P_t / P_p \tag{5}$$

where P_t is the ultimate load capacity during the test, and P_p is the load capacity predicted using the reciprocal load method.

As reported by Castaldo et al. (2019), epistemic uncertainty has a significant on the resistance of concrete structures. Epistemic uncertainty has been analyzed and discussed for columns as well. Herein, the experimental results of 103 specimens were collected from relevant literatures (Mavichak and Furlong, 1976; Heimdahl and Bianchini, 1975; Hsu, 1975; Ramamurthy, 1966; Bresker, 1960; Anderson and Lee, 1951), as summarized in Table 1. To find the most appropriate probability model, the experimental results of Ω were subjected to the Kolmogorov–Smirnov test by using multiple probability models. The distribution with highest p-value 0.76 is selected, which is a normal distribution. Thus, it is assumed that Ω follows the normal distribution, and a normal probability plot comparing Ω to the normal distribution is depicted in Fig. 1. According to statistical calculations, the mean value of the uncertain variable Ω is 1.09, and its coefficient of variation (COV) is 0.103.

The limit state function of columns subjected to biaxial bending and axial compression is often considered in terms of axial force. For the case of random biaxial eccentricity, the limit state function Z_1 can be expressed as follows:

$$Z_1 = \Omega / [\frac{1}{P_x(e_x,\ f_c,\ f_y)} + \frac{1}{P_y(e_y,\ f_c,\ f_y)} - \frac{1}{P_0}] - P_D - P_L - P_W = 0 \tag{6}$$

For the case of fixed biaxial eccentricity, the limit state function Z_2 can be expressed as follows:

$$Z_2 = \Omega / [\frac{1}{P_x(e_{dx},\ f_c,\ f_y)} + \frac{1}{P_y(e_{dy},\ f_c,\ f_y)} - \frac{1}{P_0}] - P_D - P_L - P_W = 0 \tag{7}$$

Probability analysis of eccentricities with different parameters

For columns with a certain section size and concrete and steel strengths, the design information can be described in terms of the ratio of moments in two directions (M_x, M_y) and axial force (P), which are defined as follows:

$$\rho_{Mx} = M_{Wnx} / (M_{Dnx} + M_{Lnx}) \tag{8a}$$

$$\rho_{My} = M_{Wny} / (M_{Dny} + M_{Lny}) \tag{8b}$$

$$\rho_P = P_{Wn} / (P_{Dn} + P_{Ln}) \tag{9}$$

where D, L, and W denote dead load, live load, and wind load, respectively.

In combination with Eq. 2, the nominal values of axial force and moment in the x direction can be obtained as follows:

$$S_{Dnx} = S_d / [\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{Mx}(1+\frac{L_n}{D_n})] \tag{10a}$$

$$P_{Dn} = P_d / [\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_w \rho_P (1+\frac{L_n}{D_n})] \tag{10b}$$

$$S_{Lnx} = S_d L_n / [(\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{Mx}(1+\frac{L_n}{D_n}))D_n] \tag{11a}$$

$$P_{Ln} = P_d L_n / [\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_w \rho_P (1+\frac{L_n}{D_n})]D_n \tag{11b}$$

$$S_{Wnx} = [S_d \rho_{Mx}(1+\frac{L_n}{D_n})] / [\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{Mx}(1+\frac{L_n}{D_n})] \tag{12a}$$

$$P_{Wn} = [P_d \rho_P (1+\frac{L_n}{D_n})]/[\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_w \rho_P (1+\frac{L_n}{D_n})] \tag{12b}$$

Then, the designed fixed eccentricity e_{dx} and random eccentricity e_x can be expressed as follows:

$$e_{dx} = M_{dx} / P_d \tag{13}$$

$$e_x = (M_{Dnx}\frac{D}{D_n} + M_{Lnx}\frac{L}{L_n} + M_{Wnx}\frac{W}{W_n}) / (P_{Dn}\frac{D}{D_n} + P_{Ln}\frac{L}{L_n} + P_{Wn}\frac{W}{W_n}) \tag{14}$$

Therefore, the random normalized eccentricity η_x corresponding to e_x can be calculated as follows:

$$\eta_x = e_x / e_{dx} = \frac{[\frac{D}{D_n} + \frac{L_n}{D_n}\frac{L}{L_n} + \rho_{Mx}(1+\frac{L_n}{D_n})\frac{W}{W_n}][\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_P (1+\frac{L_n}{D_n})]}{[\frac{D}{D_n} + \frac{L_n}{D_n}\frac{L}{L_n} + \rho_P (1+\frac{L_n}{D_n})\frac{W}{W_n}][\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_{Mx}(1+\frac{L_n}{D_n})]} \tag{15}$$

In the same way, the relevant parameters (S_{Dny}, S_{Lny}, S_{Wny}, e_{dy}, e_y, η_y) can be obtained. After combining with the internal force results of RC columns reported by Jiang et al. (2015), the values of ρ_{Mx} and ρ_{My} ranged from 1.0 to 4.0 and those of ρ_P ranged from -0.15 to 0.15. The typical value of L_n/D_n for RC frame structures was reported by Ellingwood et al. (1980) as 1.0, and we used the same value for simplicity.

In this study, three, three, and four different representative numbers were selected from their ranges as the values of ρ_{Mx}, ρ_{My}, and ρ_P respectively, and they were combined for No.1–No.36, as summarized in Table 2. For the load variables in Table 3, the probability distributions of random normalized eccentricity are illustrated in Fig. 2.

From Eq. (15) and Fig. 2, it is known that the range of random values of η_x is related to ρ_{Mx} and ρ_P. However, the probability of the event $\eta_x \leqslant 1$ is certain for different ρ_{Mx} and ρ_P at approximately 0.36. It can be expressed as follows:

$$P\left(\eta_x \le 1\right) = P\{[(\gamma_D + \gamma_L \frac{L_n}{D_n})W / (\frac{D}{D_n} + \frac{L_n}{D_n}\frac{L}{L_n})\gamma_W W_n] \le 1\} \tag{16}$$

where ρ_{Mx} and ρ_P are canceled out in comparison to Eq. (15). In the other direction, P($\eta_y \leqslant 1$) is the same because ρ_{My} and ρ_P are not involved.

Parametric reliability analysis

To capture the effects of the ratio of moments along two principles and the compression force, two other parameters, namely the angle of moments in two directions θ and axial compression λ_P, are defined as follows:

$$\theta = \arctan(M_{dy} / M_{dx}) \tag{17}$$

$$\lambda_P = P_d / P_{cr} \tag{18}$$

where P_{cr} denotes the design force at balanced failure; value of λ_P usually ranges from 0.5 to 3.0 for RC columns, as reported by Jiang et al. (2015); and the value of θ usually ranges from 0° to 90°. Herein, 3 and 4 different typical values are selected for θ and λ_P, respectively. Thus, 432 cases are considered in total.

The design parameters of a typical symmetrical RC column (e.g., section dimensions and materials) are listed in Table 4, and probabilistic models of load and resistance are summarized in Table 3. Based on the results of a Monte Carlo simulation (run 1e6 times), different combinations of design parameters are adopted in the reliability index calculations by considering fixed eccentricity and random eccentricity, and Fig. 3 shows the calculation results.

As shown in Fig. 3, the reliability indices vary strongly when random biaxial eccentricity is considered. For example, the maximum value is 4.75, but the minimum value is only 1.43. Moreover, the reliability indices determined by considering a fixed eccentricity are higher than those determined by considering random eccentricity, which indicates that the fixed eccentricity criterion in the design code may lead to an unsafe design of RC columns.

Conclusions

In this paper, the uncertainty of the resistance model is analyzed for the reciprocal load method, and a parametric analysis of reliability is performed for columns considering random biaxial eccentricity. The main conclusions are as follows:

(1) Based on 103 sets of column results collected from the literature, the uncertainty of the resistance computation model is analyzed for the reciprocal load method, and the mean value of uncertainty and its coefficient of variation are 1.09 and 0.103, respectively.

(2) For a certain RC column with a wind-dominated load combination, considering random biaxial eccentricity, the guarantee probability of the design value of eccentricity is independent of load effect ratio, and it remains constant in each direction when the load statistics are given.

(3) The reliability indices of RC columns with random biaxial eccentricity are lower than those of columns with fixed biaxial eccentricity. That is, the use of design methods that follow the fixed biaxial eccentricity criterion can lead to unsafe designs.

Notably, the reliability of RC columns is affected by slenderness and geometric uncertainties as well. Therefore, further investigation is warranted.

Data Available Statement

All data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

Acknowledgment

The research is supported by National Key R&D Program of China (2021YFB2600900)，the Natural Science Foundation of Hunan province (Grant No. 2021JJ3716), and this support is gratefully acknowledgment.

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Table 1. The results of tested and calculated biaxial bending and axial compression columns

Sources	Numbers	f_c/MPa	h/b	P_t/kN	P_p/kN	Ω
Anderson and Lee (1951)	2	37.47	1	60.8~64.1	72.67~72.67	0.84~0.89
Bresler (1960)	4	19.18~47.86	1	214.94~238.00	206.96~223.90	0.96~1.15
Ramamurty (1966)	29	21.47~34.13	1~2	331.36~369.33	305.23~385.30	0.86~1.21
Hsu (1975)	35	22.06~29.15	1	96.79~110.31	103.09~109.99	0.88~1.26
Heimdahl and Bianchini (1975)	15	23.99~35.61	1	339.36~347.83	265.52~357.23	0.95~1.31
Mavichak and Furlong (1976)	18	19~20.5	1.5	280.50~303.80	233.69~304.89	0.92~1.30

Table 2. Design parameters for No.1-No.36

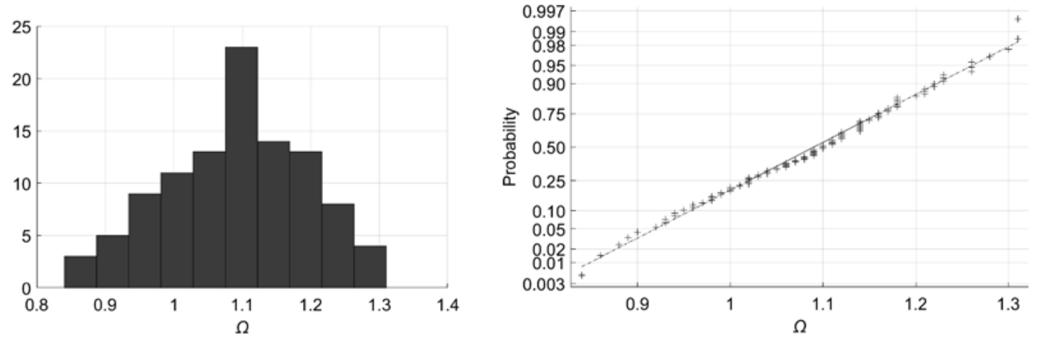
No.	ρ_{Mx}	ρ_{My}	ρ_P	No.	ρ_{Mx}	ρ_{My}	ρ_P	No.	ρ_{Mx}	ρ_{My}	ρ_P	No.	ρ_{Mx}	ρ_{My}	ρ_P
1	1	1	-0.15	10	1	4	-0.05	19	2.5	2.5	0.05	28	4	1	0.15
2	1	1	-0.05	11	1	4	0.05	20	2.5	2.5	0.15	29	4	2.5	-0.15
3	1	1	0.05	12	1	4	0.15	21	2.5	4	-0.15	30	4	2.5	-0.05
4	1	1	0.15	13	2.5	1	-0.15	22	2.5	4	-0.05	31	4	2.5	0.05
5	1	2.5	-0.15	14	2.5	1	-0.05	23	2.5	4	0.05	32	4	2.5	0.15
6	1	2.5	-0.05	15	2.5	1	0.05	24	2.5	4	0.15	33	4	4	-0.15
7	1	2.5	0.05	16	2.5	1	0.15	25	4	1	-0.15	34	4	4	-0.05
8	1	2.5	0.15	17	2.5	2.5	-0.15	26	4	1	-0.05	35	4	4	0.05
9	1	4	-0.15	18	2.5	2.5	-0.05	27	4	1	0.05	36	4	4	0.15

Table 3. Statistics of load variables

Variable	Distribution	Mean	COV	Reference
D/D_n	Normal	1.05	0.10	Szerzen and Nowak (2003)
L/L_n	Gamma	0.24	0.65	Szerzen and Nowak (2003)
W/W_n	Type-I-Largest	0.78	0.37	Szerzen and Nowak (2003)
f_c/f_{cn}	Normal	1.35	0.10	Szerzen and Nowak (2003)
f_y/f_{yn}	Normal	1.145	0.05	Szerzen and Nowak (2003
Ω	Normal	1.09	0.10	Obtained from collected columns

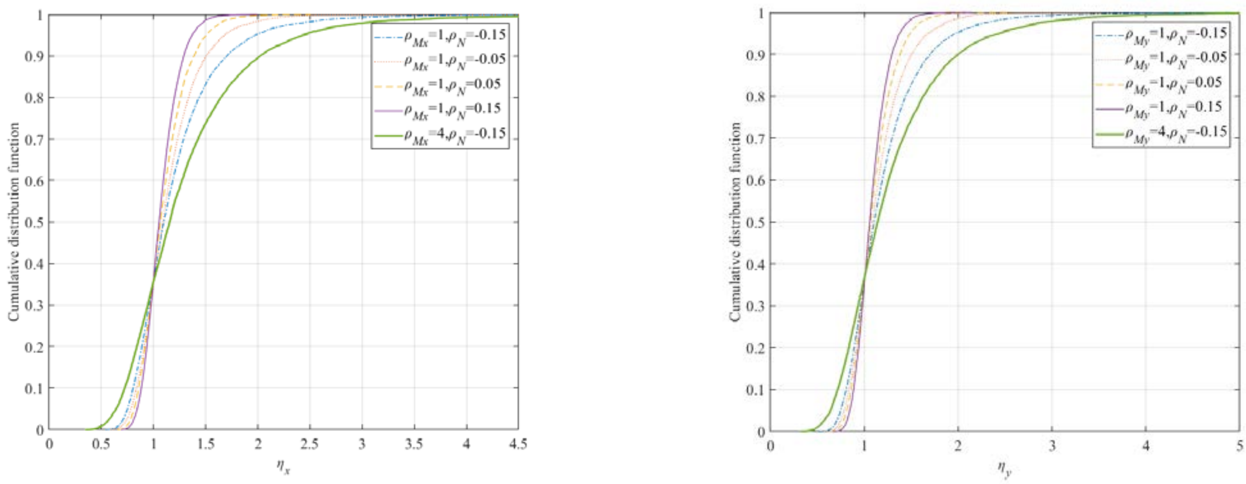
Table 4. Design parameters of RC column

b (mm)	h (mm)	a_s (mm)	A_s (mm^2)	Concrete strength(MPa)	Rebar strength(MPa)
450	450	50	2512	27.58	413.8



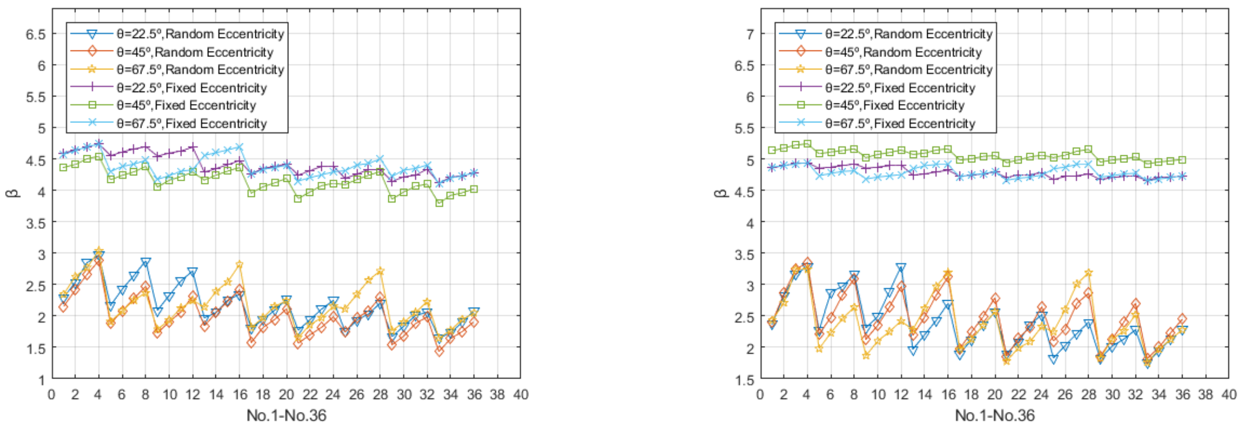
(a) Frequency histogram of *Ω* (b) Probability plot comparing to the normal distribution

Fig. 1. Normal probability plot comparing *Ω* to the normal distribution

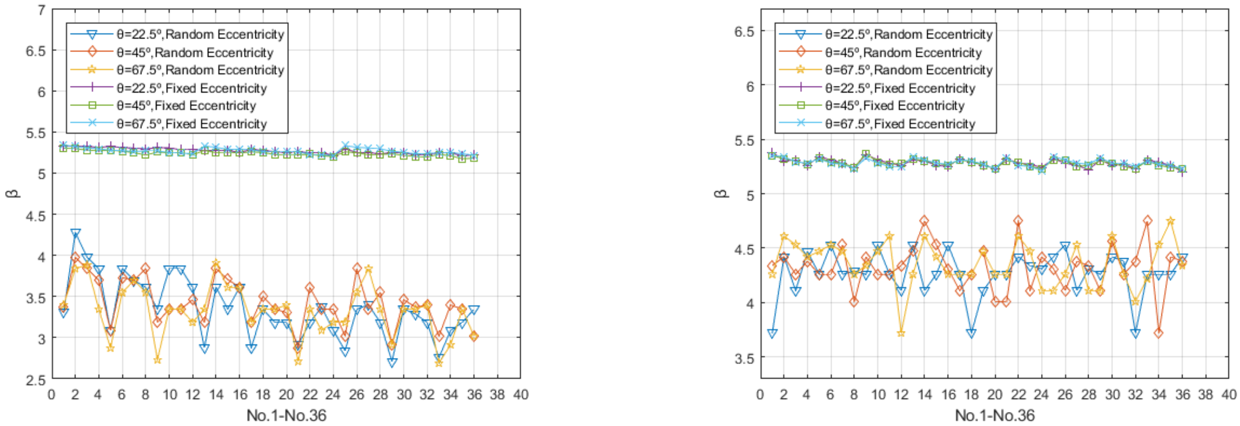


(a)η_x (b)η_y

Fig. 2. Probability distributions of random normalized eccentricity



(a)λ_N=0.5 (b)λ_N=1.0



(c)λ_N=2.0 (d)λ_N=3.0

Fig. 3. Reliability indices with random eccentricity and fixed eccentricity