

Scalable risk assessment of large infrastructure systems with spatially correlated components

Diqi Zeng^{a,1}, Hao Zhang^{*,a,2}, Hongzhe Dai^{b,3}, Michael Beer^{c,4}

^a*School of Civil Engineering, The University of Sydney, NSW 2006, Australia*

^b*School of Civil Engineering, Harbin Institute of Technology, Harbin 150090, China*

^c*Institute for Risk and Reliability, Leibniz Univ. Hannover, 30167 Hannover, Germany;*

Institute for Risk and Uncertainty, Univ. of Liverpool, Liverpool L69 3BX, United Kingdom; International Joint Research Center for Engineering Reliability and Stochastic Mechanics, Tongji Univ., Shanghai 200092, China

Abstract

Risk assessment of spatially distributed infrastructure systems under natural hazards shall treat the performance of individual components as stochastically correlated due to the common engineering practice in the community including similarities in building design code, regulatory practices, construction materials, construction technologies, and the practices of local contractors. Modelling the spatially correlated damages of an infrastructure system with many components can be computationally expensive. This study addresses the scalability issue of risk analysis of large-scale systems by developing an interpolation technique. The basic idea is to sample a portion of components in the systems and evaluate their correlated damages accurately, while the damages of remaining components are interpolated from the sampled

*Corresponding author

¹E-mail address: dzen5770@uni.sydney.edu.au

²E-mail address: hao.zhang@sydney.edu.au

³E-mail address: hzdai@hit.edu.cn

⁴E-mail address: beer@irz.uni-hannover.de

components. The new method can handle not only linear systems, but also systems with complex connectivity such as utility networks. Two examples are presented to demonstrate the proposed method, including cyclone loss assessment of the building portfolios in a virtual community, and connectivity analysis of an electric power system under a scenario cyclone event.

Key words: Probabilistic risk assessment, Community resilience, Random field, Structural reliability

1 **1. Introduction**

2 Civil infrastructure systems such as building portfolios, transportation
3 systems and utility networks provide essential support to the well-being of a
4 community, and are susceptible to natural hazards such as tropical cyclones
5 and earthquakes. The damages and failures of infrastructure systems lead to
6 not only direct economic loss resulting from repair and reconstruction, but
7 also indirect economic loss such as population dislocation and employment
8 loss. To advocate a whole-of-community approach of hazard mitigation, com-
9 munity resilience assessment has become internationally an imperative [1].
10 The resilience of a community is defined by the ability of its physical and
11 non-physical infrastructure, which includes built environment, social institu-
12 tions, and its people, to return to a level of normalcy within a reasonable time
13 following the occurrence of an event [1]. A fundamental task for community
14 resilience assessment is to conduct damage assessment of distributed infras-
15 tructure systems under a large-scale natural hazard. The results of damage
16 assessment provide the initial conditions of a community right after a haz-
17 ard, which can be used to further assess the social and economic impact of

the hazard during post-hazard recovery period. Infrastructure system losses 18
are uncertain in nature due to the stochastic variability in hazard demands 19
and system capacities. The uncertainties must be thoroughly understood to 20
facilitate risk-informed decision making. 21

The state of the art in risk assessment of individual infrastructure facil- 22
ities is reasonably mature. The capacities of infrastructure facilities against 23
a specific hazard demand are modelled by their fragility functions. The 24
fragility function of a structure provides its conditional probability of reach- 25
ing a particular damage state given a specific hazard demand. In previous 26
studies, fragility functions were typically developed independently for differ- 27
ent structural types such as Lee and Rosowsky [2] and Li and Ellingwood 28
[3]. However, in risk assessment of distributed infrastructure systems, the 29
fragility functions of individual components should be modelled as stochas- 30
tically correlated. The spatial correlation arises due to the similarities of 31
individual components in construction materials, regulatory practices, struc- 32
tural design, construction technologies, and construction practices of local 33
contractors over a community [4]. 34

Significant progress has been made in risk assessment of distributed infras- 35
tructure systems considering the spatial correlation of damages to individual 36
components. The spatial correlation of infrastructure components may arise 37
due to the spatial correlation of hazard demands placed by a natural hazard 38
with a large geographic footprint and the spatial correlation of structural 39
fragilities in a community. Most previous studies focused on the impact of 40
the spatial correlation of hazard demands placed by an earthquake event in 41
system risk assessment, such as building portfolio risk [5–8] and lifeline sys- 42

43 tem risk [9–11]. An initial effort has also been made to consider the spatial
44 correlation of wind speeds from a tropical cyclone event in system risk as-
45 sessment [12]. Compared with studies on the spatial correlation of hazard
46 demands, the studies on the spatial correlation of structural fragilities are
47 relatively limited [4, 13–16]. Lee and Kiremidjian [13] examined the effect of
48 the fragility correlation between bridges on the total repair cost of a bridge
49 network subjected to a scenario earthquake. A simple equi-correlation as-
50 sumption was applied to the fragility correlation and the sensitivity of the
51 repair cost to the correlation was investigated. Wang et al. [16] also adopted
52 an equi-correlation assumption for the fragility correlation between buildings
53 in evaluating cyclone damage cost to a community’s residual buildings. Vi-
54 toontus and Ellingwood [4] were among the first to mathematically model
55 the fragility correlation between buildings in an urban area as a function of
56 material, structural type, building code, and workmanship in construction.
57 The correlation of fragilities was included in the evaluation of repair cost to
58 building portfolios under a scenario earthquake. In general, the impact of the
59 correlation between individual structures in system risk assessment depends
60 on the system loss metric selected. Many previous studies evaluated the lin-
61 ear loss metrics of infrastructure systems, such as the summed economic
62 losses of individual buildings [4–6, 16]. In this case, ignoring the correlation
63 does not change the mean loss, but would underestimate the uncertainty of
64 the loss. However, for nonlinear loss metrics such as the power outage ratio
65 of an electric power grid (a connectivity problem), ignoring the correlation
66 would affect both the mean and variance of the loss estimate. This point was
67 also observed in other studies [12, 17]

Previous system risk studies typically modelled an infrastructure system 68
in an individual component basis, e.g., bridge networks [13, 18] and building 69
portfolios [5, 19]. Considering the performance of each component as cor- 70
related, the analysis needs to handle a correlation matrix of size of $N \times N$, 71
which N denoted the number of the components in the systems. Modelling 72
the correlated components can become computationally infeasible if the sys- 73
tem has a large number of components. To overcome the computational cost 74
issue, a random sampling technique has been proposed [4] for evaluating the 75
repair cost of building portfolios. In this technique, a small number of build- 76
ings are randomly sampled. The loss of the sampled buildings is computed 77
and scaled by the ratio between the total building number and the sampled 78
building number to approximate the total loss of the building portfolio. The 79
random sampling technique was also used by Lin and Wang [20] to evaluate 80
the resilience of building portfolios. The accuracy of the random sampling 81
technique deteriorates if the building portfolio becomes more heterogeneous. 82
Most importantly, the random sampling technique cannot capture the con- 83
nectivity of lifeline networks. For example, in functional loss assessment of 84
an electric power system, individual components, such as distribution sub- 85
stations and transmission line support structures, are interconnected in such 86
a way that a structurally undamaged facility may lose its function due to the 87
failures of other facilities. To capture the failures of undamaged facilities, the 88
damage states of all the interconnected components are required. Since the 89
random sampling technique only captures damages of sampled components, 90
it cannot be used to evaluate the risk of lifeline networks. New approaches 91
are needed to address the scalability issue of risk assessment. 92

93 In this paper, an interpolation technique is developed to evaluate the
94 correlated damages of a large-scale infrastructure system. The basic idea is
95 to sample a portion of components in the system and evaluate their corre-
96 lated damages accurately, while the damages of remaining components are
97 interpolated from the sampled components. The idea of interpolation origi-
98 nates from random field discretization techniques in stochastic finite element
99 methods. The optimal linear estimation (OLE) method, originally developed
100 for continuous random fields [21], is improved to simulate the random vector
101 of component damages. Due to the interpolation, the size of the correla-
102 tion matrix required to analyze the system is reduced remarkably, making
103 the proposed method suitable to systems of large size. Two examples are
104 provided to demonstrate the proposed method, including cyclone loss assess-
105 ments of building portfolios in a virtual community, and an electric power
106 system with interconnected infrastructure components. In the first example,
107 cyclone-induced damages to individual buildings are considered. Focused
108 samples are selected uniformly at random from each building type in each
109 building zone. In Example 2 (power grid), damages to transmission support
110 structures are considered. Focused transmission structures include all the
111 structures located at the intersections of the transmission lines, and the re-
112 maining focused structures are randomly sampled along each transmission
113 line.

114 **2. Monte Carlo simulation of spatially correlated damages**

115 Let D denote the damage state of a structure. D is a discrete random
116 variable and typically taken as different numerical values for different damage

states, e.g., $D = 0$ for no damage, $D = 1$ for minor damage, and $D = 2$ for 117
moderate damage. In a large infrastructure system such as a building portfo- 118
lio or utility network, the damage states of individual components are invari- 119
ably positively correlated due to common engineering practices [4]. In order 120
to compute the probabilistic characteristics of the collective loss/damage of 121
an infrastructure system, the joint probabilities of individual components' 122
damage states are required. However, only knowing the fragility functions of 123
the components and their correlations is insufficient to determine the joint 124
probabilities. In general, using copulas to approximate the stochastic depen- 125
dence of structural damages is required. To the best of our knowledge, all 126
previous studies [4, 14–16] regarding modelling spatially correlated damages 127
of multiple structures explicitly or implicitly adopted Gaussian copulas to 128
model the stochastic dependence of the structures, where correlated damage 129
states are converted into correlated Gaussian random variables. Currently, 130
there is no data available to justify if the stochastic dependence of struc- 131
tural damages is Gaussian or not. Using Gaussian copulas is mainly due 132
to its computational feasibility. Using non-Gaussian copulas to capture the 133
correlation of random variables is computationally feasible only if the num- 134
ber of the variables is relatively small, e.g., 2 correlated random variables 135
[22]. A recent work [23] developed a method to construct the joint probabil- 136
ity density function (PDF) of multiple random variables using non-Gaussian 137
copulas, when marginal distributions and correlations are specified. In this 138
method, a joint PDF is expressed in terms of pairwise bivariate copula den- 139
sity functions and the pair copulas are determined based on a particular vine 140
structure. The method was demonstrated by the examples of system relia- 141

142 bility analysis with up to 8 random variables. However, risk assessment of
 143 infrastructure systems typically involves a large number of correlated random
 144 variables and constructing the joint PDF of these variables by non-Gaussian
 145 copulas is still computationally difficult.

146 In this study, a Gaussian copula is used to model the stochastic depen-
 147 dence among the components [4, 15]. Let q_{ij} denote the correlation coefficient
 148 between the damages of components i and j . Consider an infrastructure sys-
 149 tem with N components. Let U_i denote the hazard intensity for component
 150 i . For a set of given hazard intensities (U_1, \dots, U_N) , the procedures to gen-
 151 erate the correlated damage states (D_1, \dots, D_N) by Monte Carlo simulation
 152 (MCS) and copula are as follows [4, 15],

153 **Step 1:** Generate N correlated standard Gaussian random variables, s_1, \dots, s_N ,
 154 with a correlation coefficient matrix $\mathbf{W} = [q_{ij}]$.

155 **Step 2:** Transform s_1, \dots, s_N into the samples of standard uniformly dis-
 156 tributed random variables x_1, \dots, x_N , by $x_i = \Phi(s_i)$, in which $\Phi(\cdot)$
 157 represents the cumulative distribution function of a standard normal.

158 **Step 3:** Map the samples of damage states D_1, \dots, D_N from x_1, \dots, x_N by
 159 $D_i = v$, if $P(D_i \leq v - 1 | U_i = u_i) < x_i \leq P(D_i \leq v | U_i = u_i)$.

160 In Step 3, $P(\cdot)$ represents the probability of the event in the bracket; $D_i =$
 161 $v - 1$ represents the damage state that is one-level less severe than the damage
 162 state of v ; if v is already the lowest damage state, $P(D_i \leq v - 1 | U_i = u_i) = 0$.
 163 To specify q_{ij} in Step 1, the exponential form of fragility correlation model
 164 is taken from the literature [4]. More discussion about the correlation model
 165 will be given in the next section. It should be noted that this paper will

develop a general method to simulate the correlated damages of a large-scale distributed infrastructure system. The correlation model itself is not the focus. In simulating the correlated damages of all components, the main computational cost comes from Step 1. The correlation coefficient matrix \mathbf{W} has a size of $N \times N$ and it would become computationally costly for large-scale systems.

3. A new interpolation-based method for simulating correlated damages

3.1. Optimal linear estimation method for random field discretization

As discussed in Section 2, a key step to simulate the correlated damage of a large-scale infrastructure system is to simulate a high-dimension vector of correlated standard Gaussian random variables. One idea to reduce the computational cost is to apply interpolation techniques of random field discretization. In this method, only some components are sampled, and based on which the remaining components are interpolated. Random field discretization with interpolation techniques has been used in stochastic finite element methods to represent continuous random fields such as structural material properties with spatial variability.

One of the interpolation technique for random field discretization is the Optimal linear estimation (OLE) method [21]. We start with the original OLE method for generating N correlated standard Gaussian random variables. Among the N components, assume that M components have been generated and denoted as $\mathbf{s}_o = \{S_k\}$, $k = 1, \dots, M$. The remaining $N - M$ components are represented by $\mathbf{s}_* = \{S_i\}$ where $i = M + 1, \dots, N$. S_i can be

190 interpolated from \mathbf{s}_o using OLE as follows [21],

$$S'_i = \bar{S}_i + \mathbf{d}_i^T \mathbf{G}^{-1}(\mathbf{s}_o - \boldsymbol{\mu}), \quad (1)$$

191 in which S'_i is the interpolated value of S_i ; \bar{S}_i represents the mean of S_i ;
 192 \mathbf{d}_i is an M -dimension vector containing the covariance between S_i and the
 193 elements of \mathbf{s}_o ; \mathbf{G} is the covariance matrix of \mathbf{s}_o ; $\boldsymbol{\mu}$ is an M -dimension vector
 194 containing the mean values of \mathbf{s}_o , i.e., $\boldsymbol{\mu} = \{\bar{S}_1, \dots, \bar{S}_M\}$. If \mathbf{G} is empiri-
 195 cally formed to represent the correlation of real infrastructure items, it may
 196 not satisfy the requirements of covariance matrices, which is not permissi-
 197 ble in simulating correlated random variables. Thus, empirically estimated
 198 correlation coefficients are typically fitted to a mathematically feasible cor-
 199 relation model, such as the exponential model later discussed in this study.
 200 Alternatively, an empirically formed correlation matrix can be adjusted to
 201 a mathematically feasible correlation matrix using the algorithm of Higham
 202 [24]. Then \mathbf{G} is invertible. Eq. (1) requires the inverse of matrix \mathbf{G} . It is
 203 noted that for the generation of \mathbf{s}_o , the spectral decomposition of \mathbf{G} would
 204 be determined [25]. Given the spectral decomposition, \mathbf{G}^{-1} can be obtained
 205 by simply inverting the diagonal matrix which includes the eigen-values of
 206 \mathbf{G} .

207 Since S'_i is a linear transformation of Gaussian random vector \mathbf{s}_o , S'_i is
 208 also a Gaussian random variable. The mean of S'_i is given as

$$\begin{aligned} \text{E}(S'_i) &= \text{E}(\bar{S}_i + \mathbf{d}_i^T \mathbf{G}^{-1}(\mathbf{s}_o - \boldsymbol{\mu})) \\ &= \bar{S}_i + \mathbf{d}_i^T \mathbf{G}^{-1} \text{E}(\mathbf{s}_o - \boldsymbol{\mu}) \\ &= \bar{S}_i, \end{aligned} \quad (2)$$

in which $E(\cdot)$ represents the expectation. Eq. (2) shows that S'_i and S_i have the same mean value, indicating that OLE is unbiased.

The covariance between two interpolated variables S'_i and S'_j ($i, j = M + 1, \dots, N$), denoted as δ_{ij} , is given by

$$\delta_{ij} = \mathbf{d}_i^T \mathbf{G}^{-1} \mathbf{d}_j. \quad (3)$$

The variance of S'_i , denoted by $\sigma_i'^2$, is obtained from Eq. (3) by taking $i = j$. The correlation between S'_i and S'_j , denoted as ρ_{ij} , is given as

$$\rho_{ij} = \frac{\delta_{ij}}{\sigma_i' \sigma_j'}. \quad (4)$$

The variance of $\epsilon_i = S_i - S'_i$, denoted as $\sigma_{\epsilon_i}^2$, is as follows,

$$\begin{aligned} \sigma_{\epsilon_i}^2 &= E \left[(S_i - S'_i)^2 \right] \\ &= E \left\{ [S_i - \mu_i - \mathbf{d}_i^T \mathbf{G}^{-1} (\mathbf{s}_o - \boldsymbol{\mu})]^2 \right\} \\ &= \sigma_i^2 - \mathbf{d}_i^T \mathbf{G}^{-1} \mathbf{d}_i \\ &= \sigma_i^2 - \sigma_i'^2 \end{aligned} \quad (5)$$

where σ_i^2 and $\sigma_i'^2$ are the variance of S_i and S'_i , respectively. It can be seen that $\sigma_i'^2$ is always smaller than σ_i^2 .

The OLE method was initially developed to simulate continuous Gaussian random fields. Li and Der Kiureghian [21] suggested that the accuracy of OLE deteriorates for a nondifferentiable random field whose correlation function $q(\mathbf{x}, \mathbf{x}')$ for sites \mathbf{x} and \mathbf{x}' does not have a zero slope at $\mathbf{x} = \mathbf{x}'$. The damage of individual components of a large system is a random vector, and may not be differentiable. For example, a common function to represent

224 the correlations of structural fragility is taken from the literature [4, 15, 20]
 225 as follows,

$$q(\mathbf{x}, \mathbf{x}') = \begin{cases} a \cdot \exp\left(-\frac{|\mathbf{x}-\mathbf{x}'|}{b}\right) + r & \text{if } \mathbf{x} \neq \mathbf{x}', \\ 1 & \text{if } \mathbf{x} = \mathbf{x}', \end{cases} \quad (6)$$

226 where a , b and r are model parameters. In the study of Vitoontus and
 227 Ellingwood [4], the correlation length b is a constant, a and r depend on two
 228 buildings' similarities in construction material, structural type, storey range
 229 and design code. Random fields with a correlation function of Eq. (6) are
 230 generally non-differentiable.

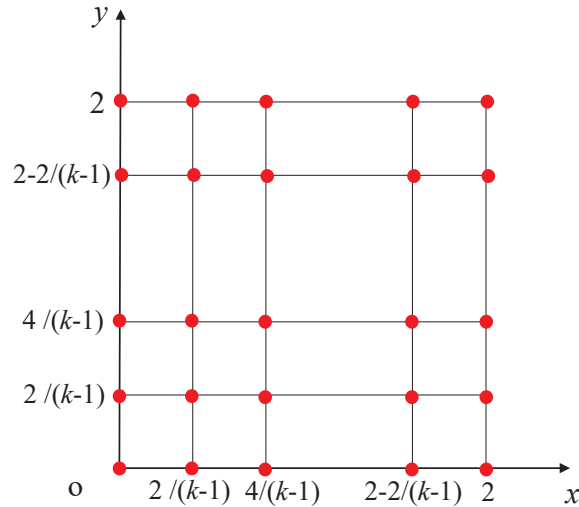


Figure 1: The distribution of sampled points in the random field.

231 To investigate OLE's accuracy for non-differentiable random fields, con-
 232 sider a standard Gaussian random field $S(\mathbf{x})$ with a correlation function of
 233 Eq. (6), in which the parameters a , r , b are taken as 0.7, 0 and 1. The ran-
 234 dom field is defined in a Cartesian coordinate system (x, y) with $0 \leq x \leq 2$

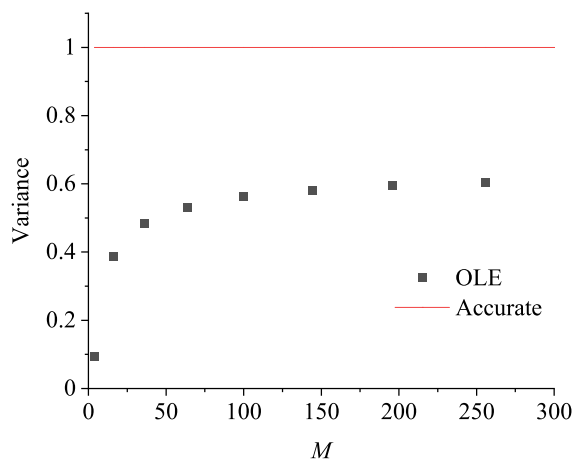


Figure 2: The variance of point A=(1,1) from OLE, interpolated from M sample points.

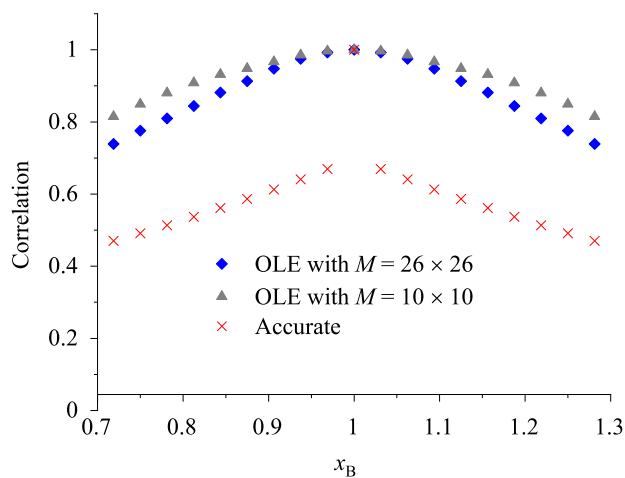


Figure 3: The accuracy of OLE in solving correlation between interpolated points A = (1, 1) and B = (x_B, x_B) .

and $0 \leq y \leq 2$. A total of M points are selected and they are arranged 235
such that in each orthogonal direction there are k equally spaced points 236
($M = k \times k$), as shown in Fig. 1. Consider point A, located in (1,1). Point 237

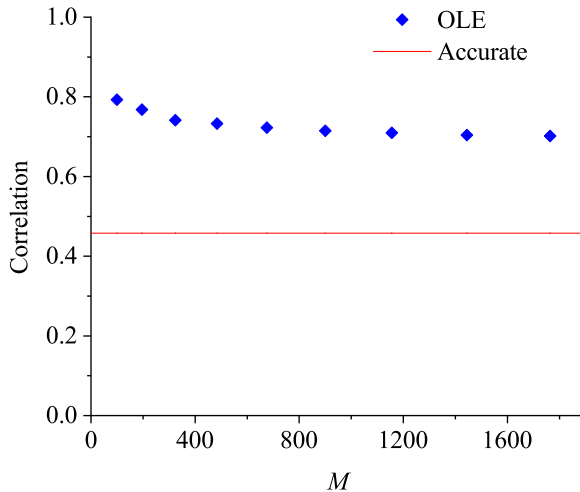


Figure 4: The accuracy of OLE in solving the correlation of $A = (1, 1)$ and $C = (0.7, 0.7)$, given different number of sampled points.

238 A is interpolated using the M sampled points. The variances of point A
 239 from OLE with different M are plotted in Fig. 2. It can be seen that the
 240 variance of OLE is always smaller than the true variance and does not con-
 241 verge with increasing M . Next, consider another interpolation point B, with
 242 a coordinate of (x_B, x_B) . The correlation coefficient of points A and B
 243 plotted in Fig. 3 for two cases, i.e., $M = 10 \times 10$ and $M = 26 \times 26$. In
 244 both cases, significant error is observed. Also, the correlation coefficients of
 245 point A and point $C = (0.7, 0.7)$ given different M are plotted in Fig. 4. The
 246 correlation of OLE is always higher than the true correlation and does not
 247 converge with increasing M . These results demonstrate that in simulating
 248 a non-differentiable Gaussian random field, the original OLE underestimates
 249 the variance of an interpolation point, and overestimates the correlation of

two interpolation points. 250

3.2. Improved OLE technique 251

This study proposes an improved OLE technique, referred to as IOLE, 252
 suitable for non-differentiable Gaussian random fields. In the IOLE, the 253
 interpolated value of S_i , denoted by \tilde{S}_i , is given by 254

$$\tilde{S}_i = S'_i + \zeta_i \cdot (\sigma_i^2 - \sigma_i'^2)^{0.5}, \quad (7)$$

in which S'_i and $\sigma_i'^2$ are obtained from the original OLE, i.e., Eq. (1) and 255
 Eq. (3) respectively; σ_i^2 is the accurate variance at point i ; and ζ_i is an 256
 independent standard Gaussian random variable. Since the mean of the 257
 second term in Eq. (7) is zero, the mean of \tilde{S}_i is equal to the mean of S'_i , 258
 thus \tilde{S}_i is also unbiased. The covariance of two interpolated points \tilde{S}_i and 259
 \tilde{S}_j , denoted as $\tilde{\delta}_{ij}$, is equal to 260

$$\begin{aligned} \tilde{\delta}_{ij} &= \text{E} \left(\left(\tilde{S}_i - \text{E}(\tilde{S}_i) \right) \left(\tilde{S}_j - \text{E}(\tilde{S}_j) \right) \right) \\ &= \text{E}(S'_i S'_j) - \text{E}(S'_i) \text{E}(S'_j) + \text{E}(\zeta_i \zeta_j) \sqrt{(\sigma_i^2 - \sigma_i'^2)(\sigma_j^2 - \sigma_j'^2)}, \end{aligned} \quad (8)$$

and 261

$$\tilde{\delta}_{ij} = \begin{cases} \mathbf{d}_i^T \mathbf{G}^{-1} \mathbf{d}_j & \text{if } i \neq j \\ \sigma_i^2 & \text{if } i = j. \end{cases} \quad (9)$$

Eqs. (8) and (9) show that if $i \neq j$, $\tilde{\delta}_{ij}$ is the same as the covariance obtained 262
 by the original OLE. If $i = j$, $\tilde{\delta}_{ij}$ represents the variance of \tilde{S}_i and is equal 263
 to the accurate variance of point i (σ_i^2). 264

To demonstrate the accuracy of IOLE, the example of standard Gaussian 265
 random field presented in the last section is repeated using the IOLE. The 266
 variances of interpolated point $\mathbf{A} = (1, 1)$ using OLE and IOLE, as well as the 267

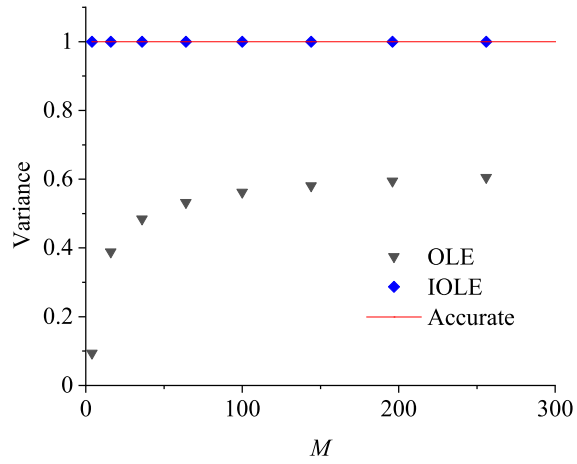


Figure 5: Comparison of IOLE and OLE: variance of point $A=(1,1)$ interpolated from M sample points.

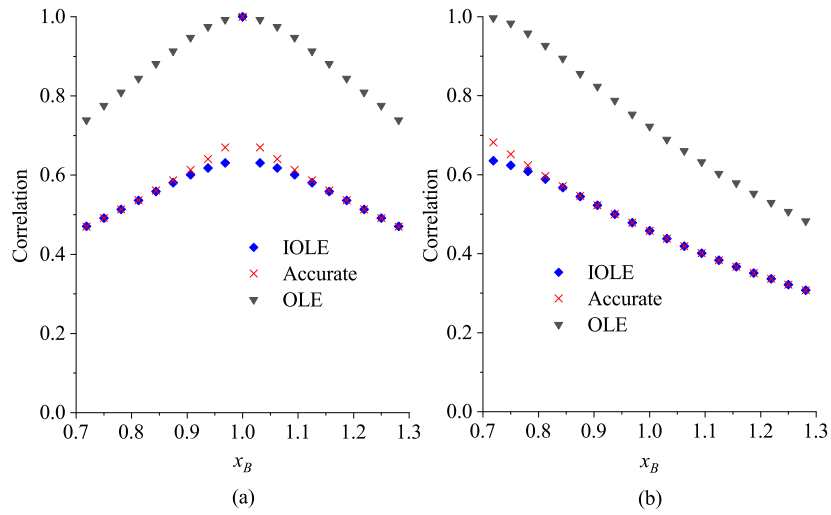


Figure 6: Comparison of IOLE and OLE: correlation coefficient of (a) points $A=(1,1)$ and $B = (x_B, x_B)$ and (b) points $C=(0.7,0.7)$ and $B = (x_B, x_B)$.

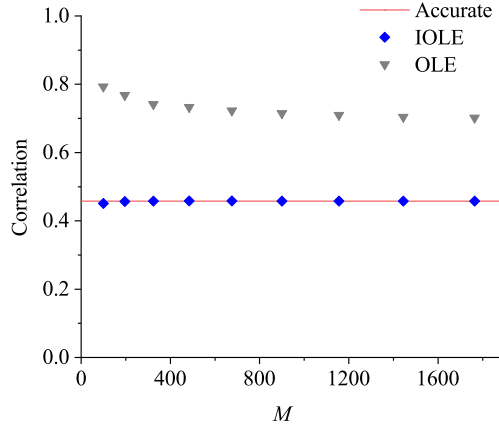


Figure 7: Comparison of IOLE and OLE in solving correlation given different number of sampled points.

true variance, are given in Fig. 5. It can be seen that compared to the original 268
OLE, the IOLE represents a significant improvement in approximating the 269
variance of an interpolated point. Fig. 6 gives the correlation coefficient 270
of two interpolation points $A = (1, 1)$ and $B = (x_B, x_B)$ by using $M =$ 271
 26×26 sampled points and that of two interpolation points $C = (0.7, 0.7)$ 272
and $B = (x_B, x_B)$. The correlation obtained by IOLE is considerably more 273
accurate than that obtained by OLE. For example, the accurate correlation 274
of $A=(1,1)$ and $B=(0.91,0.91)$ is 0.61. The correlation obtained by IOLE is 275
0.60, while the correlation obtained by OLE is 0.95. The correlation between 276
 $C = (0.7, 0.7)$ and $A = (1, 1)$ by using various numbers of sampled points, are 277
given in Fig. 7. The correlation obtained by IOLE becomes essentially the 278
same as the accurate correlation if M reaches 200. However, the correlation 279
obtained by OLE does not converge to the accurate correlation. 280

281 **4. Example 1: damage assessment of building portfolios in Center-**
 282 **ville under a scenario cyclone**

283 The first example is to evaluate the cyclone damage of the building port-
 284 folios in the Centerville Virtual Community Testbed [26]. A scenario-based
 285 risk assessment approach is adopted, in which a single postulated cyclone
 286 event is used as hazard input. The cyclone damage is measured by the cost
 287 ratio, Z , of the building portfolios. Z is defined as the ratio of total repair
 288 costs to the total replacement costs,

$$Z = \frac{\sum_{l=1}^L \sum_{i=1}^{n_l} R_{il} w_{il}}{\sum_{l=1}^L \sum_{i=1}^{n_l} w_{il}}, \quad (10)$$

289 in which L represents the number of occupancy classes; n_l represents the
 290 number of buildings of occupancy class l ; w_{il} is the replacement cost of
 291 building i of occupancy class l ; R_{il} is the cost ratio of building i of occupancy
 292 class l and is a function of the building's damage state. In this study, the
 293 replacement cost w_{il} of individual building is considered as deterministic,
 294 while the cost ratio R_{il} is treated as a random variable. The building portfolio
 295 cost ratio Z is the weighted average of the cost ratios to individual buildings.
 296 The weighting coefficients are deterministic, while the cost ratios are random.
 297 Thus, the expectation of Z is equal to the weighted average of the expected
 298 cost ratios to individual buildings, as follows

$$E(Z) = \frac{\sum_{l=1}^L \sum_{i=1}^{n_l} E(R_{il}) w_{il}}{\sum_{l=1}^L \sum_{i=1}^{n_l} w_{il}}. \quad (11)$$

299 Eq. (11) shows that $E(Z)$ is independent of the spatial correlation among
 300 individual buildings. However, the standard deviation of Z would depend on
 301 the spatial correlation of individual buildings. The standard deviation of Z ,

σ_Z , is given by

302

$$\sigma_Z = \frac{(\sum_{l=1}^L \sum_{i=1}^{n_l} \sigma_{il}^2 w_{il}^2 + \sum_{i_1 \neq i_2 \text{ or } l_1 \neq l_2} \rho_{i_1 l_1, i_2 l_2} \sigma_{i_1 l_1} \sigma_{i_2 l_2} w_{i_1 l_1} w_{i_2 l_2})^{0.5}}{\sum_{l=1}^L \sum_{i=1}^{n_l} w_{il}}, \quad (12)$$

where σ_{il} is the standard deviation of the cost ratio R_{il} to building i in

303

occupancy class l ; $\rho_{i_1 l_1, i_2 l_2}$ is the correlation coefficient of $R_{i_1 l_1}$ and $R_{i_2 l_2}$.

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4.1. Description of Centerville

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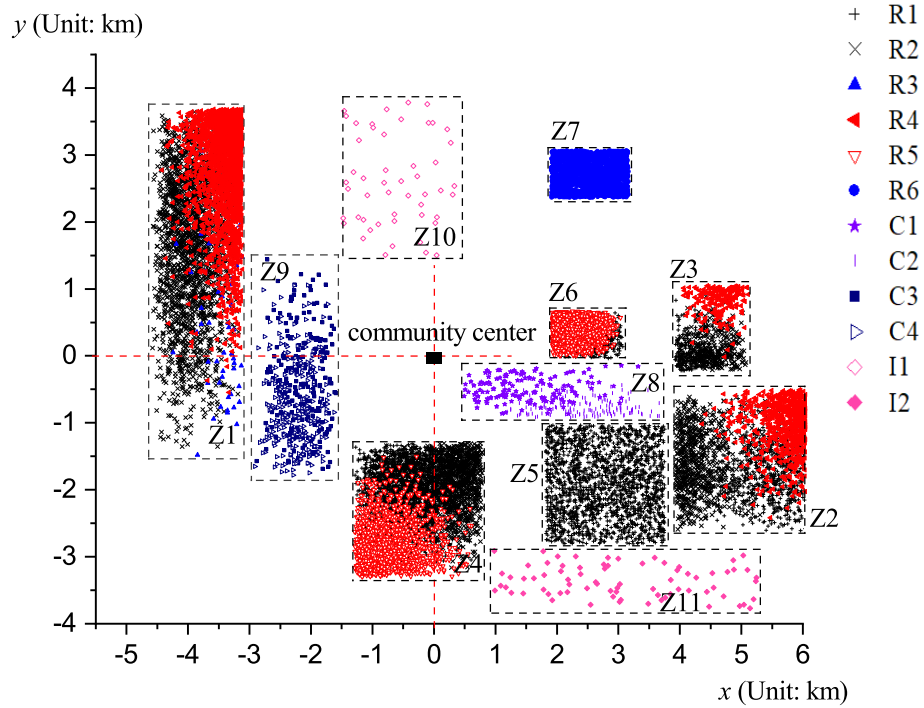


Figure 8: Example 1: building inventory of Centerville.

Centerville represents a typical middle-class city in the USA [26]. It has a

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size of roughly $13 \times 8 \text{ km}^2$. The building inventory includes 12 basic building

307

308 types, distributed in 11 zones, as shown in Fig. 8. Z1-Z7 are residential zones,
 309 while Z8 and Z9 are business zones and Z10 and Z11 are industrial zones.

Table 1: Example 1: descriptions of building types [26, 27].

Occupancy	ID	Description
Residential	R1	1-storey, Unreinforced masonry, Single-family, 1400 ft ² , 1945-1970
	R2	1-storey, Unreinforced masonry, Single-family, 2400 ft ² , 1985-2000
	R3	2-storey, Wood frame, Single-family, 3200 ft ² , 1985-2000
	R4	1-storey, Unreinforced masonry, Single-family, 2400 ft ² , 1970-1985
	R5	3-storey, Wood frame, Multi-family, 12,000 ft ² /floor, 1985
	R6	Single-family, Mobile home
Commercial	C1	1-storey, Steel, 50,000 ft ² , 1980
	C2	2-storey, Steel, 50,000 ft ² , 1980
	C3	2-storey, Steel, 25,000 ft ² , 1960
	C4	Steel, 125,000 ft ² , 1995
Industrial	I1	2-storey, Reinforced masonry, 100,000 ft ² , 1975
	I2	1-storey, Reinforced masonry, 500,000 ft ² , 1995

ft²= 0.0929 m²

310 The building information is adopted from Ellingwood et al. [26]. Table. 1
 311 summarizes the descriptions of 12 types of buildings considered in Ellingwood
 312 et al. [26]. In total, 20,609 buildings are distributed in 11 zones. The number
 313 of buildings of each type in each building zone is summarized in Table 2. In
 314 a given zone, buildings are located randomly; It is assumed that buildings

of the same type are more likely adjacent to each other, and their locations 315
 have a correlation coefficient of 0.6. Fig. 8 shows the assumed distribution 316
 of the 20,609 buildings, generated by a MCS run. 317

Table 2: The number of buildings of each type in each building zone [26].

ID	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	Z11
R1	0	767	300	2567	1856	700	0				
R2	2000	700	300	1000	0	0	0				
R3	50	0	0	0	0	0	0				
R4	2196	800	200	0	0	0	0				
R5	0	0	0	1200	0	3696	0				
R6	0	0	0	0	0	0	1352				
C1								150	0		
C2								150	0		
C3								0	250		
C4								0	250		
I1										50	0
I2										0	75

The wind fragility functions of the 12 building types have been studied by 318
 Wang [27], based on HAZUS-MH [28]. The fragility functions consider the 319
 damages of roof cover, roof sheathing panels, windows, doors, and wall sec- 320
 tions. Four damage states are considered, with $D = 0, 1, 2,$ and 3 represent- 321
 ing insignificant, moderate, severe and complete damage states, respectively. 322

323 The fragility functions have a form of lognormal function as follows,

$$P(D \geq v|U = u) = \Phi \left(\frac{\ln(u) - \lambda_v}{\xi_v} \right), \quad (13)$$

324 in which U represents the wind speed; the two parameters λ_v and ξ_v of each
 325 building type are taken from Wang [27] and summarized in Table 3.

326 The replacement costs w for a given type of building are assumed de-
 327 terministic [27], and the values are summarized in Table 3. The functional
 328 relation between the damage state D and cost ratio R is taken from Wang
 329 [27], i.e., $R = 0$ for $D = 0$, $R = 0.2$ for $D = 1$, $R = 0.4$ for $D = 2$, and
 330 $R = 0.8$ for $D = 3$. In general, the (functionality/economic) loss is a func-
 331 tion of structural damage state and the damage value conditioned on the
 332 damage state. In reality, uncertainties exist in the loss for a given damage
 333 state, thus the conditional damage value shall also be modelled as a random
 334 variable. The present study only considers the uncertainty in damage states,
 335 while treats the conditional damage value as deterministic. It is because: 1)
 336 there is a lack of data to estimate the damage value uncertainty, and 2) in
 337 some previous studies, uncertainty in conditional damage value was ignored
 338 (e.g., Goda and Hong [5]; Vitoontus and Ellingwood [4]). The present study
 339 follows the same assumption.

340 The fragility correlation q_{ij} for buildings i and j ($i \neq j$) is modelled as [4]

$$q_{ij} = a \cdot \exp\left(-\frac{h_{ij}}{b}\right) + r \quad (14)$$

341 where h_{ij} is the separation distance between buildings i and j ; a , r and b are
 342 model parameters, with b also known as correlation length. The parameters
 343 a and r are considered dependent of the similarity between two buildings
 344 in material and design code, and the correlation length b is considered as

Table 3: Example 1: wind fragility functions (wind speed unit: m/s) and replacement costs w [27].

ID	w (\$US)	λ_1	λ_2	λ_3	ξ_v ($v = 1, 2, 3$)
R1	80,430	3.8906	4.0217	4.1375	0.1097
R2	137,880	4.1137	4.2448	4.3606	0.1097
R3	183,840	3.9510	4.0732	4.1661	0.0998
R4	137,880	4.0084	4.1394	4.2552	0.1097
R5	2,068,200	3.9582	4.1405	4.1814	0.0998
R6	63,000	4.2460	4.3770	4.4200	0.1295
C1	2,872,500	3.9776	4.0729	4.6035	0.0799
C2	2,872,500	3.8822	3.9775	4.5082	0.0799
C3	1,436,250	3.7769	3.8722	4.4028	0.0799
C4	7,181,250	3.8822	3.9775	4.5082	0.0799
I1	5,745,000	3.6624	3.7486	3.9014	0.1393
I2	28,725,000	3.8765	3.9627	4.1156	0.1393

345 constant for the whole community [4]. For demonstration purpose, the cor-
 346 relation length b is taken as 2 km, roughly equal the average dimension of each
 347 building zone. The values of a , b and r used in this example are summarized
 348 in Table. 4. It is assumed that R2 and R3, C1 and C2 were designed using
 349 the same generations of building codes. Sensitivity analysis on correlation
 350 length b will be conducted in the next section.

Table 4: Example 1: model parameters of fragility correlation.

Building description	a	r	b (Unit: km)
Same materials; Same design codes	0.5	0.2	2
Same materials; Different design codes	0.35	0.14	2
Different materials; Same design codes	0.35	0.14	2
Different materials; Different design codes	0.245	0.098	2

351 Hurricane Andrew is a destructive Category 5 Atlantic hurricane that hit
 352 Florida, US, in August 1992. Its surface wind field at landfall (1992/8/24
 353 9:05 according to HURDAT database [29]) is used as hazard input. It is
 354 assumed that the community center is 12.4-km west of the storm center at
 355 landfall and the y direction of the community shown in Fig. 8 is parallel
 356 to the nearby coastline of South Florida. To determine surface gust wind
 357 speeds at buildings, the surface sustained wind speeds are computed using
 358 the gradient wind field model and the gradient-surface conversion factor in
 359 Georgiou [30], and the gust factor model in Vickery and Skerlj [31] is used
 360 to convert sustained wind speeds to gust wind speeds. To apply the wind
 361 field model, hurricane key parameters are required as model input. Transla-

tion speed and direction, the location of storm center, central pressure and
maximum surface wind speed, are collected or derived from HURDAT [29].
The radius to maximum wind is collected from Landsea et al. [32]. Holland
parameter in the wind field model was determined such that the computed
maximum surface wind speed matches the recorded value [33]. To consider
the uncertainty of wind speeds, the computed wind speeds are multiplied by
a lognormal random variable with a mean of 1 and a COV of 0.1 [34]. It is
assumed that the bias terms at different sites are statistically independent.

4.2. The comparison between IOLE and RS

The statistics of building portfolio cost ratio Z are evaluated using three
methods, i.e., the accurate method, the random sampling (RS) method [4],
and the improved OLE (IOLE) method. This example has more than 20,000
buildings. It is computationally difficult to accurately simulate the correlated
damages of all the buildings by constructing the correlation coefficient matrix
of all the buildings. Herein, 80% of buildings are uniformly sampled from
each building type in each building zone. All the sampled buildings are
considered in analyzing the building portfolio loss and the estimated loss is
treated as “accurate”. In each building zone, for any building type, the RS
method samples η percent, and the IOLE samples η percent and interpolates
additional c percent. η is referred to as the sampling ratio, and c is the
interpolation ratio.

For each method, MCS is applied to estimate the statistics of Z including
mean loss \bar{Z} , standard deviation σ_Z , and the probable maximum loss (PML)
of cost ratio. The PML is the loss value with a small exceedance probability
(10^{-3} in this example unless specified otherwise), which is a common decision

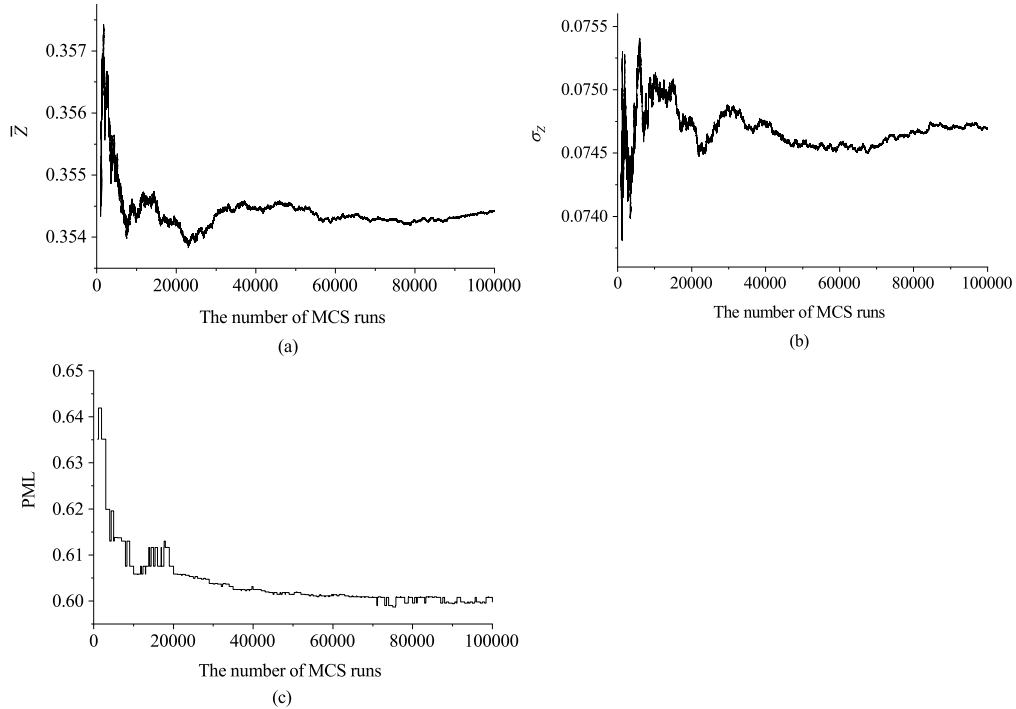


Figure 9: Example 1: The impact of MCS run in estimating (a) \bar{Z} , (b) σ_Z , (c) PML, for Centerville building portfolios.

387 metric for decision-making when financial consequences are severe [35]. To
 388 determine the suitable number of MCS runs, different numbers of MCS runs
 389 were checked when the “accurate” method was applied. Estimated statistics
 390 are given in Fig. 9. It was found that the variation of estimated statistics
 391 is negligible if more than 50,000 MCS runs are used. For example, as the
 392 number of MCS run increases from 50,000 to 100,000, the variation of the
 393 \bar{Z} estimate is about 0.089%. Thus, 50,000 MCS runs were used in all the
 394 computation of this example.

395 IOLE and RS are first compared in estimating \bar{Z} , σ_Z , and PML (for an

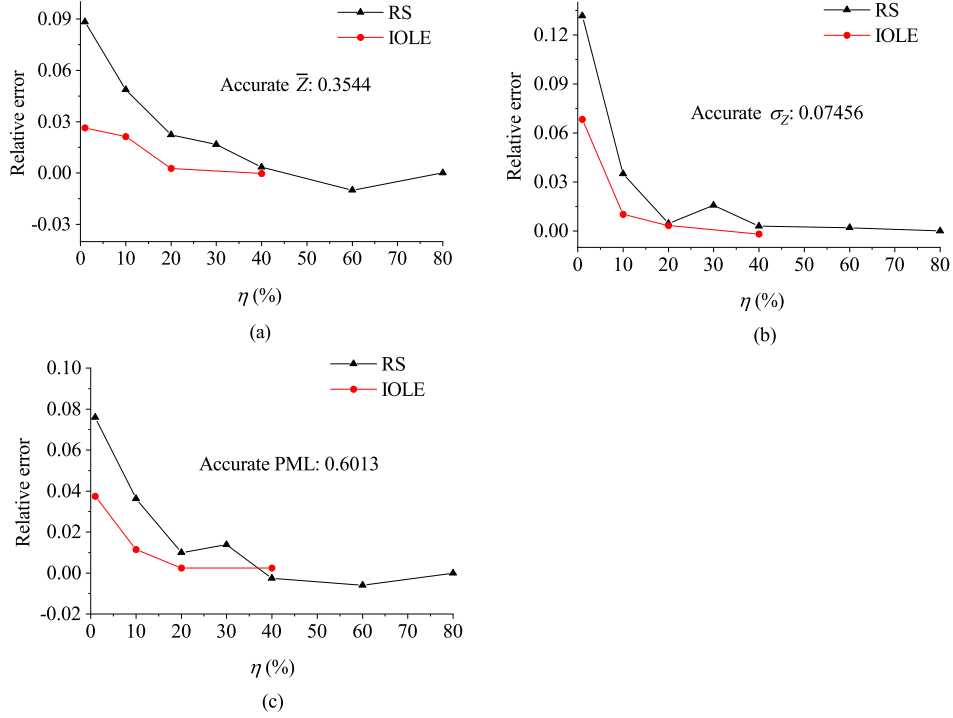


Figure 10: Example 1: comparison of RS and IOLE given different sampling ratio η , in estimating (a) \bar{Z} , (b) σ_Z , (c) PML, for Centerville building portfolios.

exceedance probability of 10^{-3}), given different sampling ratio η . For IOLE, 396
the interpolation ratio c is set equal to η . Relevant results are given in Fig. 10. 397
The advantage of the IOLE over the RS is most obvious when the number 398
of samples are limited (i.e., $\eta < 10\%$). IOLE can generally achieve the same 399
accuracy of RS by sampling 50% less buildings. 400

For further sensitivity analysis, the η of RS is set as 10%, and the η and c 401
of IOLE are both set as 10% unless specified otherwise. First, RS and IOLE 402
are compared in estimating the PML of different exceedance probabilities. 403
Fig. 11 gives the complementary cumulative distribution function (CCDF) 404

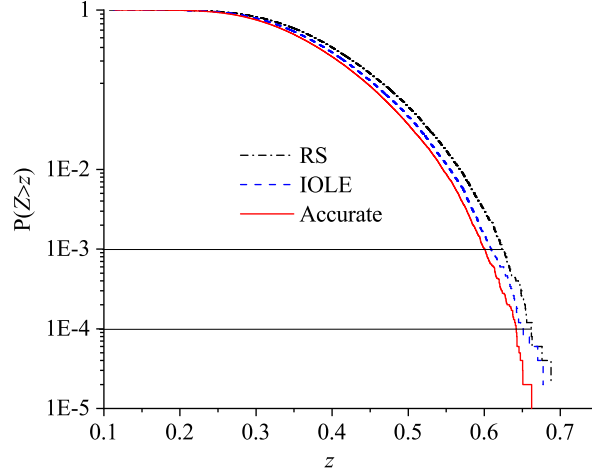


Figure 11: Example 1: CCDF of Z estimated by different methods, for Centerville building portfolios.

405 of Z solved by various methods. It is found that PML solved by IOLE is
 406 consistently more accurate than that solved by RS. Then different correlation
 407 lengths are considered. The loss statistics estimated by different methods
 408 are given in Fig. 12. It was found that the curves of IOLE, RS and the
 409 accurate method are roughly parallel to each other. The improved accuracy
 410 of IOLE, compared with RS, is not influenced by the change of correlation
 411 length. Finally, the impact of interpolation ratio c in the accuracy of IOLE
 412 is examined, with relevant results shown in Fig. 13. Again, the relative error
 413 of IOLE is always lower than that of RS. The relative error of IOLE can be
 414 further reduced by increasing c .

415 The above analyses adopt a simplified deterministic cost ratio for each
 416 building damage state. In practice, uncertainties exist in damage/loss values.

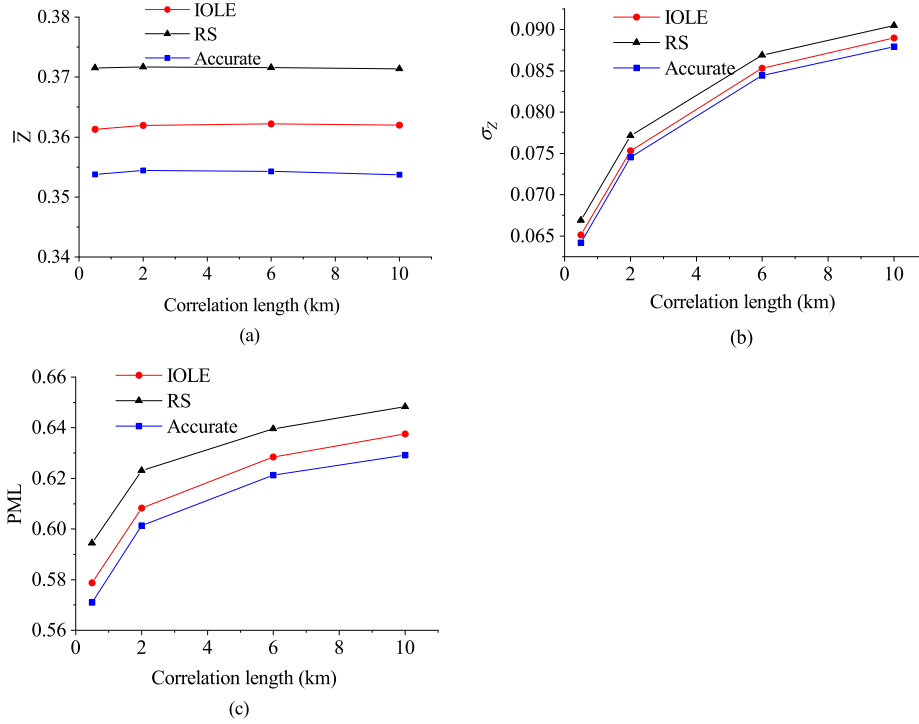


Figure 12: Example 1: comparison of RS and IOLE given different correlation length, in estimating (a) \bar{Z} , (b) σ_Z , (c) PML, for Centerville building portfolios.

To examine the impact of loss value uncertainty, it is assumed that the cost 417
ratio R is a lognormal, with a mean value of 0.2 for the damage state $D = 1$, 418
a mean of 0.4 for the damage state $D = 2$, and a mean of 0.8 for the damage 419
state $D = 3$. The COV of the cost ratio R is assumed to be the same for 420
each damage state, varying between 0.05 to 0.3. It was found that the COV 421
of the cost ratio R does not affect the mean value of the building portfolio 422
loss, and has an insignificant effect on the standard deviation of the building 423
portfolio loss as shown in Figure 14. Figure 14 also shows that the proposed 424
method is more accurate than the random sampling method. 425

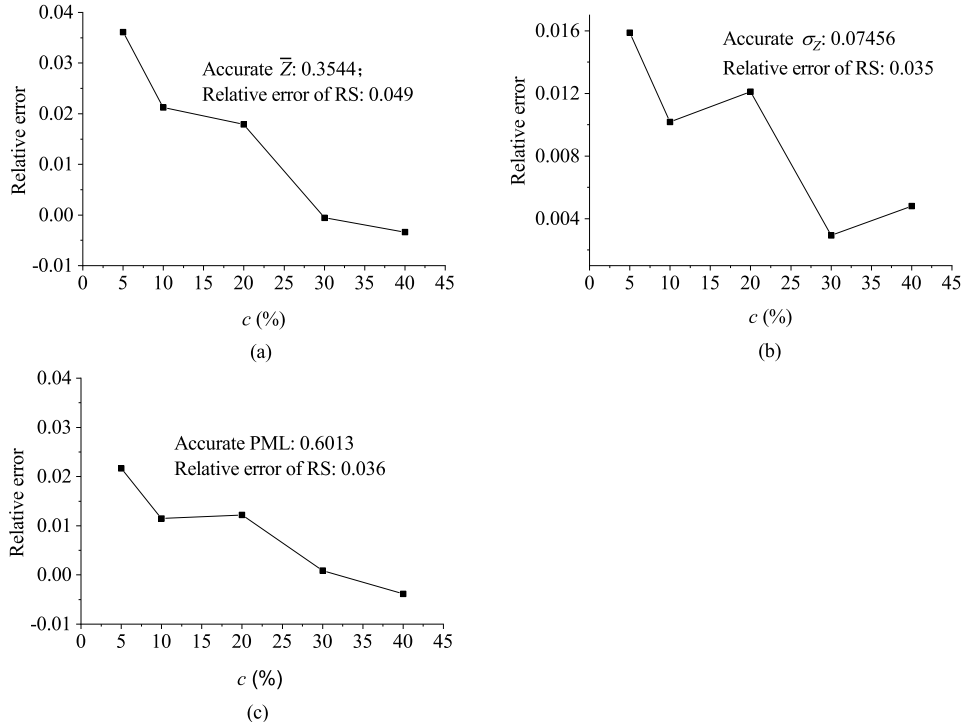


Figure 13: Example 1: accuracy of IOLE given different interpolation ratio c , in estimating (a) \bar{Z} , (b) σ_Z , (c) PML, for Centerville building portfolios.

426 In general, to simulate the correlated damages of N structures in an
 427 infrastructure system, the main computational demand is the orthogonal de-
 428 composition of the system's correlation matrix (which is $N \times N$). In the
 429 IOLE method with M focused samples, the size of the correlation matrix is
 430 reduced to $M \times M$. Consider the current example in which the building port-
 431 folio has a total of 20609 buildings. The building portfolio loss is estimated
 432 using two methods: the RS method with 40% sampling ratio, and the IOLE
 433 method with 10% sampling ratio and 30% interpolation ratio. The relative
 434 errors of both methods are below 1%. However, RS requires 1285 seconds to

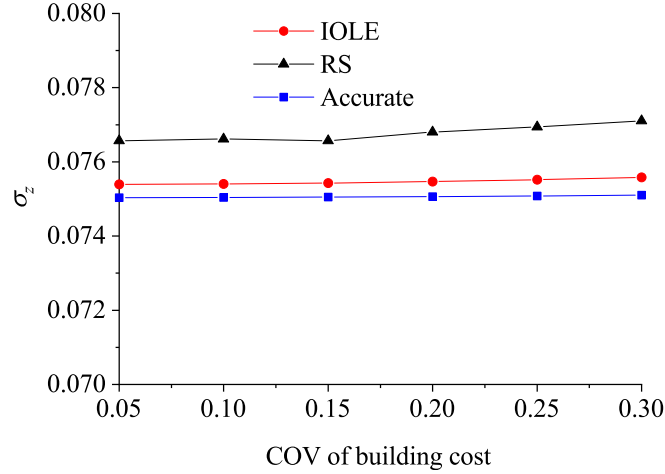


Figure 14: Example 1: comparison of RS and IOLE given different COV of cost ratio in estimating σ_Z for Centerville building portfolios.

estimate the loss, while IOLE requires 509 seconds. 435

It should be noted that in the RS method, only the damage/performance 436
of the sampled components are known. Thus, RS cannot be used for con- 437
nectivity analysis of lifeline networks while IOLE can, e.g. the power outage 438
ratio of an electric power network as will be demonstrated in the next section. 439
It is because the connectivity analysis of an infrastructure system requires 440
the information of all the infrastructure components to capture the interde- 441
pendent effects. 442

For practical applications, a suitable sampling ratio should be decided 443
in order to evaluate the loss of realistic building portfolios. In the seismic 444
loss analysis of community building portfolios in the studies of Vitoontus 445
[36], and Vitoontus and Ellingwood [4], a smaller representative region is 446
first modelled using different sampling ratios. For this smaller region, the 447

448 required sampling ratio can be determined by comparing the approximate
449 loss estimates with the accurate result. This sampling ratio is then used for
450 estimating the loss of the entire community. This idea of choosing a suitable
451 sampling ratio can be followed for application purposes.

452 **5. Example 2: connectivity analysis of an electric power system** 453 **under a scenario cyclone**

454 The second example evaluates the performance of an electric power sys-
455 tem under a scenario cyclone event, in which the connectivity between dif-
456 ferent infrastructure components need to be captured.

457 *5.1. Description of the electric power system*

458 The electric power system is based on the example of Salman and Li
459 [37]. The topology of the power transmission grid is for the power grid in
460 Shelby County, Tennessee, USA. In this paper, it is assumed located in a
461 coastal region of South Carolina to consider cyclone hazard. It covers an
462 area of about $50 \times 42 \text{ km}^2$. The topological structure of the power system
463 is shown in Fig. 15. It has 8 high-voltage gate stations, 17 medium-voltage
464 substations, 16 low-voltage substations and 12 intersections of transmission
465 lines, along with 66 transmission lines. Since there is no energy generating
466 plant in this system, the gate stations are assumed as supply nodes that
467 provide electricity for substations. It should be noted that the gate nodes are
468 typically boundary points to a larger power transmission grid whose energy
469 generation plants are located elsewhere.

470 The medium-voltage and low-voltage substations are demand nodes that
471 directly serve the customers in their neighbourhoods. The span of a transmis-

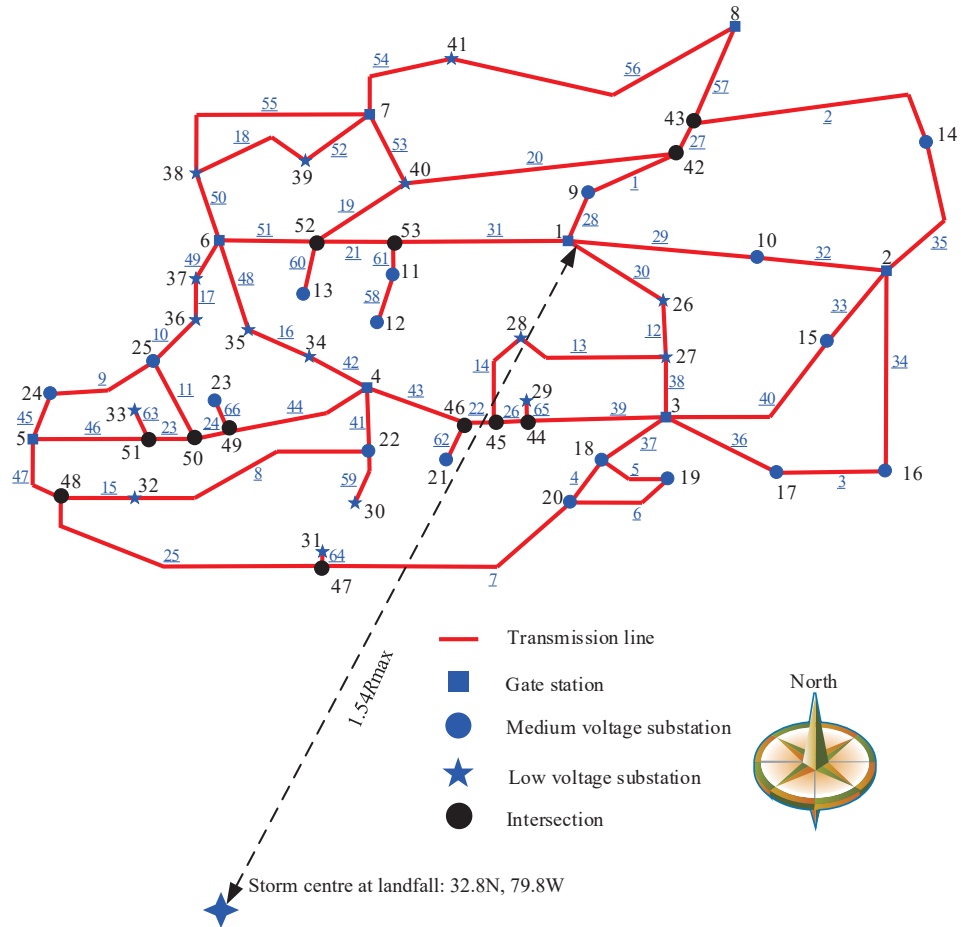


Figure 15: Example 2: an electric power system (modified from Salman and Li [37]) and the storm centre location of Hurricane Hugo (1989) (R_{\max} : radius to maximum wind).

sion line between two transmission support structures is assumed to be 244 472
m. There are 1767 transmission line-supporting structures in total. Wind- 473
induced damage to the support structures is considered, since they are the 474
most vulnerable structures under wind effects. The fragility function of the 475

476 transmission support structures is taken from Brown [38] as follows,

$$P(D = 1|U = u) = \min(2 \cdot 10^{-7} \exp(0.1866u), 1), \quad (15)$$

477 in which D is the binary damage state of a transmission support structure,
478 equal to 0 for no damage and 1 for failure, and U is the 3-s gust wind speed in
479 m/s. The fragility functions for any two transmission support structures i and
480 j are modelled as correlated, with a correlation function of Eq. (14), in which
481 $a = 0.5$, $r = 0.2$, and the correlation length $b = 15$ km. It should be noted
482 that the parameter values of correlation model are chosen for demonstration
483 purpose. Sensitivity analysis will be conducted in the next section.

484 Hurricane Hugo was chosen as the scenario event. The hurricane made
485 landfall in South Carolina as a Category 4 storm at 4:00 AM on September
486 22, 1989. The location of the storm centre at landfall relative to the electric
487 power system is shown in Fig. 15. Note that gate station 1 is located 20 km
488 to the east of the storm centre and 60 km to the north of the storm centre.
489 The maximum surface gust wind speed of each transmission support struc-
490 ture during the hurricane passage was used to determine structural damage.
491 To this end, the time history of surface wind speed at each structure was first
492 sought in an interval of 30 minutes using the wind models same as those
493 of the building portfolio example, and then the maximum wind speed was
494 found. The key hurricane parameters, including storm translation direction
495 and speed, the latitude of storm center, central pressure and maximum sur-
496 face wind speed, were collected or derived from HURDAT database [29].
497 Since HURDAT only provides the records of hurricane key parameters in an
498 interval of 6 hours, the records in a 30-minute interval are obtained by lin-
499 ear interpolation [37]. The radius to maximum wind is estimated using the

empirical formula from Vickery and Wadhera [33]. The Holland parameter 500
in the wind field model was determined such that the computed maximum 501
surface wind speed matches the recorded value [33]. This study considered 502
the path of the hurricane from 0:00 to 12:00 on September 22, 1989 and 503
found that considering this path segment is enough to capture the maximum 504
wind speed of each structure during the storm passage. To consider wind 505
speed uncertainty, the lognormal distribution, same as that of the building 506
portfolio example, was used. 507

The performance of the power system is measured using the power out- 508
age ratio, Q , the ratio of the customers losing access to power to the total 509
customers, as follows 510

$$Q = \frac{\sum_{j=1}^{N'} r_j F_j}{\sum_{j=1}^{N'} r_j}, \quad (16)$$

in which r_j is the number of the customers served by demand node j , F_j 511
indicates the functional state of demand node j , 1 for failure and 0 for func- 512
tional, and N' is the total number of demand nodes. It is assumed that a 513
demand node fails if it loses connection to all supply nodes, and functions 514
if it is connected to at least one supply node. One low-voltage substation 515
serves 10,000 customers and one medium-voltage substation serves 14,000 516
customers. In total, there are 398,000 customers served by the power sys- 517
tem. Transmission lines connecting to a supply node are unidirectional and 518
electricity can only transmit from the supply nodes to the demand nodes. 519
Transmission lines connected to terminal substations such as nodes 12 and 520
13 in Fig. 15 are also unidirectional and electricity is transmitted to termi- 521
nal substations. Other transmission lines are bi-directional. A transmission 522
line connecting two nodes is modelled as a series system with multiple trans- 523

524 mission support structures. If any of the transmission support structures is
525 damaged, all transmission lines supported by the structure fail. The failed
526 lines are removed from the network and the status of the demand node F_j is
527 determined using a shortest path algorithm which searches for the available
528 path(s) from any supply node to the demand node.

529 *5.2. The performance of IOLE*

530 The performance of the IOLE method was examined in estimating the
531 power outage ratio Q of the system. To apply the IOLE method, η percent
532 of transmission support structures are sampled and their damage is obtained
533 accurately. Damages to the remaining $(100 - \eta)$ percent of the support
534 structures are interpolated. In the current example, the η percent of struc-
535 tures include two parts. The first part contains all the support structures
536 located at the intersections of transmission lines (marked as black solid circle
537 in Fig. 15), accounting for 0.68% of all the support structures. The other
538 $(\eta - 0.68)$ percent of structures are uniformly sampled along each transmis-
539 sion line. To demonstrate the accuracy of IOLE, the accurate method is
540 also used in which the full correlation coefficient matrix of all the support
541 structures is constructed and used to simulate the correlated damages to the
542 support structures. MCS is used to estimate the statistics of Q . Different
543 runs of MCS were checked, when the accurate method was applied. It is
544 found that 10^5 MCS runs are sufficient to stably estimate the mean \bar{Q} and
545 the standard deviation σ_Q of Q . In all the computation of this example, 10^5
546 MCS were used.

547 The IOLE was used to estimate the statistics of Q given different sampling
548 ratio η , in comparison with the accurate method. Results are shown in

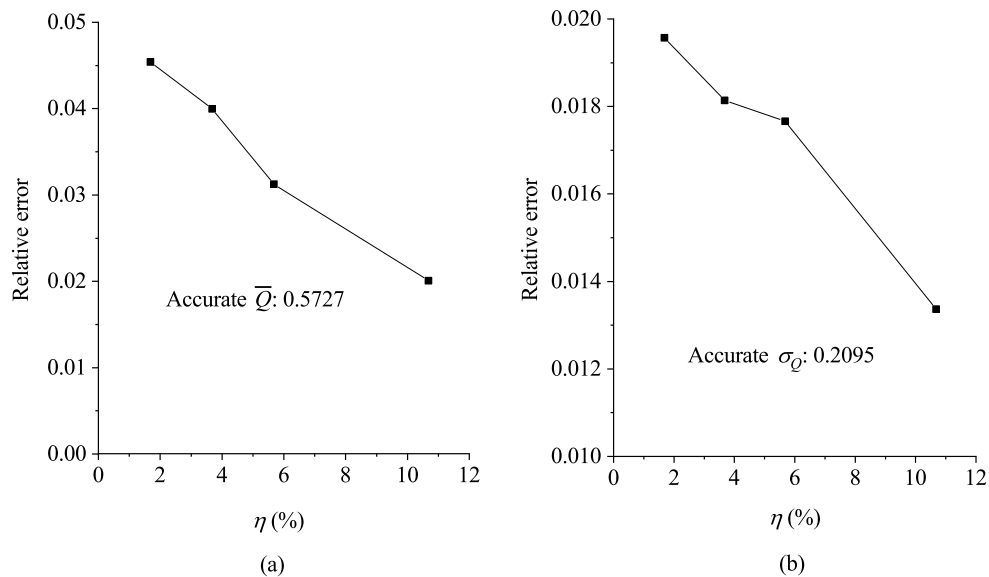


Figure 16: Example 2: relative errors of IOLE using different sampling ratio η , (a) \bar{Q} , (b) σ_Q .

Fig. 16. From the accurate method, the power outage ratio Q has a mean of 549
 0.57 with a standard deviation of 0.21, suggesting that on average 57% of the 550
 customers would lose access to power. The accuracy of IOLE improves as η 551
 increases. The relative errors in \bar{Q} and σ_Q are below 5% when η is just 2%. 552
 If the sampling ratio is further increased to 11%, the errors can be controlled 553
 below 2%. The results demonstrate that the IOLE can be used for utility 554
 networks with interconnected infrastructure components. 555

The accuracy of IOLE was further examined by considering different 556
 correlation lengths. Fig. 17 compares the accurate method, and the IOLE 557

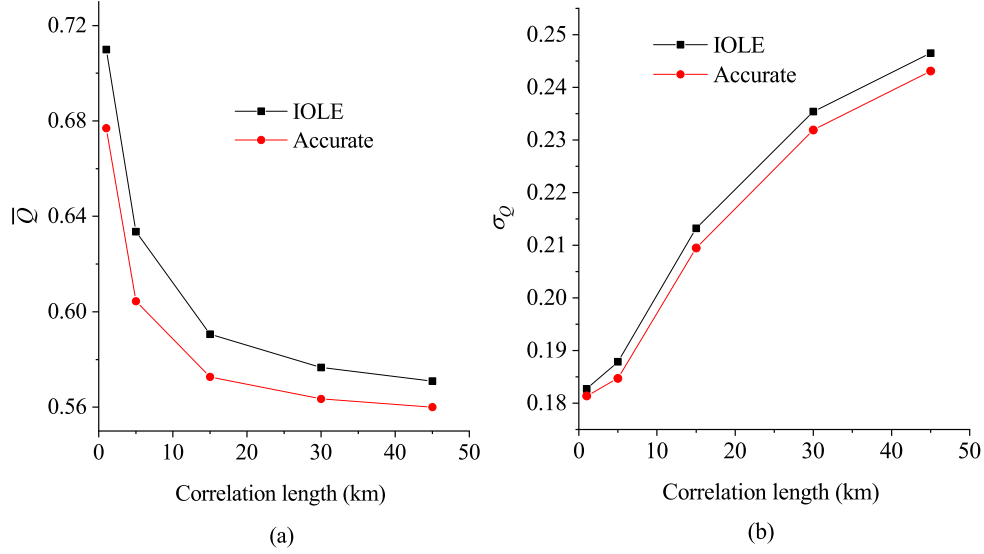


Figure 17: Example 2: Comparison between the IOLE and accurate methods given different correlation lengths, (a) \bar{Q} , (b) σ_Q .

558 method by sampling 5% structures in addition to the structures at the in-
 559 tersections of transmission lines. The discrepancy between the two methods
 560 roughly remains constant with an increasing correlation length, indicating
 561 that the accuracy of IOLE is insensitive to the change of correlation length.

562 6. Conclusion

563 The improved optimal linear estimation method can simulate non-differentiable
 564 random fields. The technique can be used for risk assessment of large-scale
 565 infrastructure systems with correlated components, as demonstrated by two

examples. 566

Example 1 evaluates the cyclone repair costs of building portfolios in 567
the virtual community Centerville. It was found that the accuracy of both 568
the IOLE and the RS methods increases as the sampling ratio increases. 569
However, for very small sampling ratios (i.e., $\eta \leq 10\%$), the accuracy of the 570
RS is rather poor, while the IOLE can still give reasonable results. Also, 571
sensitivity analysis on the threshold value of PML, correlation length and 572
the interpolation ratio of IOLE has been conducted. The accuracy of IOLE 573
was found consistently higher than that of RS. 574

The most significant advantage of the IOLE over the conventional random 575
sampling method is that it can handle the connectivity analysis of complex 576
systems. This point is demonstrated using the electric power distribution 577
system in Example 2. The conventional random sampling method cannot be 578
used for this example, as assessing the system requires the information of all 579
components in the system. For the IOLE, the relative errors of the mean 580
and standard deviation of the power outage ratio can be controlled below 5% 581
by using a sampling ratio of just 2% . The accuracy of IOLE remains given 582
different values of correlation length. 583

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