What the Solitaire Illusion tells us about perception of numerosity

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Abstract

In four experiments we investigated the Solitaire illusion. In this illusion, most observers see as more numerous a set of dots that forms a single central group, compared to dots on the outside of that group. We confirmed and extended the effect to configurations with much higher numerosity than the original and of various colours. Contrary to prediction, separating the two groups, so that they are presented side by side, reduced but did not abolish or reverse the illusion. In this illusion, therefore, neither total size of the region (area), not average distance of the elements has the expected effect. In Experiments 3 and 4 we eliminated the regularity of the pattern, by sampling 50% (Exp 3) or only a 10% (Exp 4) of the elements. These produces quasi-random configurations. For these configurations the bias for the inner groups was still present, and it was only eliminated when the groups were shown as separate. However, the effect never reversed (no bias for the outer group, despite its larger area). We conclude that the Solitaire illusion is evidence of a strong bias in favour of centrally located elements, a bias that can overcome other factors.

Keywords: numerosity, Solitaire illusion, visual perception, grouping

Data availability: Raw data from our studies as well as example images are available online: https://osf.io/9utzx/.

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Keywords: numerosity, Solitaire illusion, visual perception, grouping

Human observers can estimate the numerosity of a set of visual elements by means of a process that is fast and does not rely on symbols or counting. This type of numerosity estimation is shared with other non-human species (e.g., Vallortigara, 2014) and is present already in infants (e.g., Xu et al., 2004). The mechanism of estimation has been called the approximate number system (ANS) (Dehaene, 2011; Anobile et al., 2016). Studies of the ANS have shown a number of interesting properties, for example the fact that it is subject to Weber's law (Anobile et al., 2014; Cantlon & Brannon, 2006; Moyer & Landauer, 1967; but see Testolin & McClelland, 2020).

The estimation process may not be relevant for small numerosity or very high-density displays. This is because below 5 the numerosity is available directly, a process that has been called subitisation (Kaufman et al., 1949). At the other extreme, configurations with high density become texturised, and in that case observers may estimate numerosity on the basis of density, or spatial frequencies (Anobile et al., 2015; Cicchini et al., 2016; Dakin, et al., 2011).

It is known that some irrelevant features of the stimulus can bias judgments of numerosity. Perceived numerosity is higher for smaller elements (Ginsburg & Nicholls, 1988; Shuman & Spelke, 2006), more regular patterns (Ginsburg, 1976, 1991), and for larger areas of the configuration (Dakin, et al., 2011; Krueger, 1972; Poom et al., 2019; Tokita & Ishiguchi, 2010; Vos et al., 1988). When two configurations are presented in sequence, the second tends to be perceived as more numerous (van den Berg et al., 2017). With respect to the spatial arrangement of the elements, its role is illustrated by two illusions. The Regular-Random numerosity illusion (Ginsburg, 1976; 1980), and the Solitaire illusion (Frith & Frith, 1972). They are presented in Figure 1 and we will refer to them as RRI and SI.

The RRI shows that as dots cluster together, their contribution to the overall numerosity of a dot pattern starts to diminish. The issue is not regularity but clustering. When two or more dots came close together, the overlap in their area of influence could explain the reduced contribution to numerosity. This concept has been developed and led to the occupancy model (Vos et al., 1988; Allïk and Tuulmets, 1991). Although it is difficult to know the precise size of the area of influence, the occupancy model or more generally the importance of clustering have been supported by empirical studies (Allik & Raidvee, 2021; Bertamini et al., 2016; Valsecchi et al., 2013).

There is also evidence that spatial configuration, regularity and clustering have similar effects in humans and other animals. Beran (2006) specifically used the RRI with chimpanzees and macaques, and Bertamini et al. (2018) manipulated inter dot distances, and therefore clustering, with chicks.

In this study we focus on the SI, introduced in 1972 by Frith and Frith (1972). The name comes from a tabletop game played with pegs. The configuration shown in Figure 1 has 16 black elements and 16 white elements. For most observers it appears that the inner group of elements is more numerous. This illusion is very robust to changes in stimuli and paradigm. For example, Agrillo et al. (2016) used blue and yellow items. They found that the inner configuration was selected as more numerous in 82% of the trials. None of the 16 observers showed a reversed pattern. They also used a direct estimation task and found an overestimation of inner elements and a larger underestimation of elements on the outer perimeter. In this study we systematically investigate which mechanisms govern the SI by eliminating some of the contextual factors one at a time. To do so, in four different stimulus manipulations we tried to destroy the illusion on perceived numerosity.

Valsecchi et al. (2013) found a strong effect of clustering on perceived numerosity, in line with the RRI and as predicted by the occupancy model. The methodology used constrained the minimum distance between randomly located elements. In addition, they also discovered another strong effect, a reduced perceived numerosity for patterns presented in the periphery compared to central vision. In theory this may be relevant for the SI as the outside group is underestimated. However, the SI works for extended presentation, allowing the participant to inspect every region with multiple fixations.

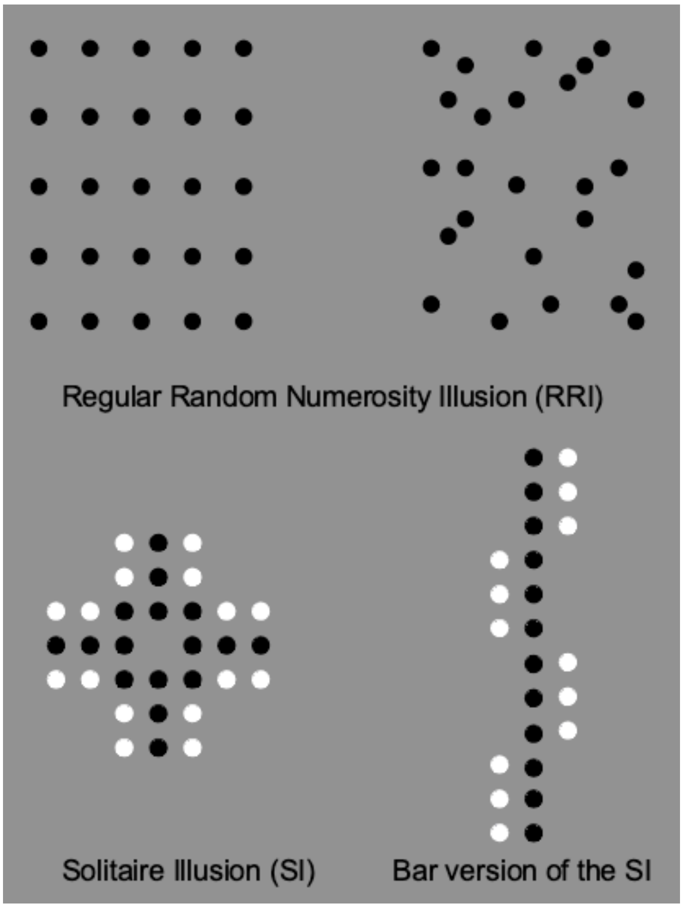

Figure 1. *Top row: In the Regular-Random Numerosity Illusion the elements that form a regular array (left) appear to be more numerous (Ginsburg, 1976). Bottom row: In the original Solitaire illusion the inner set of elements (black) appear more numerous than the outer set. Frith and Frith (1972) tested also a version of the SI with elements placed along a line (bar version). People tend to perceive more elements for the bar (shown in black).*

It is important to study the SI because the effect is strong and easy to demonstrate, and because we do not currently have a good explanation for it. There is also some evidence that it is an illusion specific to adult humans, because it does not work in chimpanzees, rhesus or capuchin monkeys (Agrillo et al., 2014, Parrish et al., 2019) in dogs ((Lõoke et al., 2020), and in children under the age of four (Parrish et a., 2016). Although a recent report found some evidence of the SI in fish (*Poecilia reticulata*, Miletto Petrazzini et al., 2018). We provide a summary of the findings using the two versions of the illusion and different populations in Table 1.

Table 1: A summary of the experimental studies on the Solitaire illusion, listed in chronological order.

Authors	Year	Version	Participants	Result

Frith & Frith	1972	S B	Ha Hc	yes
Agrillo et al.	2014	S	Ha Ch Rh Ca	only in Ha
Parrish et al.	2016	S	Hc Ca	weak
Agrillo et al.	2016	S	Ha	yes
Miletto Petrazzini et al.	2018	S	Gu	weak
Parrish et al.	2019	S B	Ha Ca Rh	only in Ha
Lõoke et al.	2020	B	Do	no
Pecunioso & Agrillo	2021	S	Ha	yes

Version: S=Standard B=Bar
Participants: Ha= Human adults Hc= Human children Ch=Chimpanzees Rh=Rhesus Ca=Capuchin Gu=Guppies Do=Dogs

Frith and Frith (1972) described the SI as a result of grouping and Gestalt factors. The inner group is stronger because all its elements are contiguous, and they form a better Gestalt. More recently, Poom et al. (2019) confirmed that perceived numerosity decreases with number of groups, independently by how the groups were created (e.g., colour, motion). The SI may therefore depend on the fact that the outer elements are separated into subsets.

However, there are reasons that make the SI effect counterintuitive. As we have seen in the case of the RRI, strong Gestalt (a regular grid) can lead to overestimation. Moreover, elements that form clusters (grouping by proximity) lead to underestimation, as shown in many studies (e.g., Bertamini et al., 2016; Valsecchi et al., 2013). This is the opposite of what happens in the SI. Because elements are placed on a grid with fixed cells, if we take the outer configuration on its own, these elements are farther apart than the elements in the inner group. Average distance is also higher for the outer group of elements. Similarly, if we were to apply the occupancy model, if the region of influence is large enough to create overlap despite the grid, then more overlap will be present for the inner set of elements (more contiguity). Again, this leads to a prediction that is the opposite of what is observed.

Grouping can also be manipulated by connecting elements. He et al. (2009) and Franconeri, Bemis, and Alvarez (2009) used randomly distributed dots, but some pairs were joined. Connected patterns are perceived as less numerous than unconnected patterns. Even symmetry may increase strength of grouping between elements and reduce numerosity (Apthorp & Bell, 2015). In general, the SI seems inconsistent with these findings because the elements grouped in a regular and compact region appear more numerous.

Equally counterintuitive is the role of area, as we have seen larger areas lead to a bias towards larger numerosity (e.g., Poom et al., 2019). One simple way to measure the area of a set of elements in the plane is by computing the area of the convex hull. In the case of the SI the convex hull is larger for the outer elements than for the inner elements in the SI. Despite this it is the inner group that is perceived as more numerous. Note that the convex hull measures objective size; it is also known that perceived (subjective) size can affect numerosity, as illustrated in the context of the Ponzo illusion (Ponzo, 1928), the horizontal vertical illusion (Pecunioso et al., 2020) or in terms of changes of perceived size after adaptation (Zimmermann & Fink, 2016).

In summary, if we accept that both spatial proximity of the elements and total area bias responses in the direction of greater numerosity, than some other factor must operate in the case of the SI that is strong enough to overpower these and lead to the opposite outcome.

In our study we started with the original SI configuration and manipulated two properties: overall numerosity, and enclosure. First, we note that each of the outer subsets of the original SI pattern has only two elements, or four if we consider the quadrants (Figure 1). These values are within the subitizing range. The comparison is therefore between a value that has to be estimated (inner set), and the sum of values each of which could be subitized (outer sets). To test the hypothesis that the SI is specific to the original configuration of 32 elements (16 in the inner and 16 in the outer sets) we increased the number of elements while keeping the overall structure. We do this by treating each original element as a cell and filling the cell with either 1 element (original version), 4 or 9 elements. Therefore, we have Solitaire configurations with 16+16, 64+64, and 144+144 elements. A similar manipulation for the Bar version of the SI creates configurations with 12+12, 48+48, and 108+108 elements.

The second manipulation is a direct test of the importance of having one set enclosed within the other set. Therefore, we took the two configurations and we displaced them horizontally, so that there is no overlap, as shown in Figures 2 (original Solitaire version) and 3 (Bar version). We report results of four experiments in which the basic design was the same, but the appearance of the stimuli changed as follows: Exp 1) change in total numerosity and separation; Exp 2) same as Exp 1 but with lines that form closed polygons; Exp 3) degradation of the groups (50%reduction in dot density/numerosity); Exp 4) degradation of the groups (90%reduction in dot density/numerosity). To anticipate the results, the basic effect of a bias in favour the inner group of elements turned out to be remarkably strong and general.

During the review process, we became aware of a recent study, published in 2021, that is very relevant. Starting from the observation that the SI is absent in children, Pecunioso and Agrillo (2021) were interested in the role of expertise, and compared musicians to non musicians. They predicted that musical expertise would reduce the illusion. They found no effect of expertise in experiment 1 (forced choice), and some evidence in favour of their hypothesis in experiment 2 (absolute number estimation). What is most relevant here is that in their experiment 2 they presented the dots from the SI in the standard configuration, or as isolated patterns. The outside set of elements were overestimated compared to the inner set even when presented in isolation. This supports the hypothesis that the SI is a robust effect that does not requires enclosure. However, this study relied on absolute numerosity judgments, which can be biased by many factors. Indeed, only when using absolute judgments there was a difference between musicians and non musicians. Moreover, observers were in general closer to the correct estimation for the outer configuration of the dots. Here we have subgroups of just two dots, which may be perceived by subitization.

Experiment 1.

This experiment tests the role of numerosity and of enclosure of the Solitaire illusion. We use the original version (16+16 elements) as well as versions of the illusions with much larger total number of elements by increasing the number of elements in each cell. We also use both the original configuration that took the name from the Solitaire game, and a version with a line of elements that had already been introduced by Frith and Frith (1972), and used also by Parrish et al. (2019).

Given the novelty of the experimental design, it was not possible to conduct an a priori power analysis based on the size of similar effects in the literature. We chose to test 20 subjects (for a total of 1920 trials). The main hypotheses concerned the factors Separation and Numerosity and their interaction (2 x 3). However, for subsequent Experiments we report a power analysis based on the data collected in Experiment 1.

Methods

Participants. Twenty individuals participated (age range 19 to 50, with 7 males). All participants had normal or corrected-to-normal vision, and none reported colour blindness. The study was approved by the Health and Life Sciences Committee on Research Ethics (Psychology, Health and Society) and conducted in accordance with the Declaration of Helsinki (revised 2008). Participants were naive with respect to the hypotheses.

Design. The factors were the Illusion version (the original Solitaire illusion or the Bar version), Separation (whether the inner and outer patterns were separated on the screen), Colour (red/green or blue/yellow) for the inner and outer pattern (for example if the colours were red/green the inner elements could be red or green) and Numerosity (1, 4, 9 cell size). This 2 x 2 x 4 x 3 design has 48 unique stimuli. Each observer was shown each stimulus twice, giving a total of 96 trials, which were split into two blocks for the two colours (red/green and blue/yellow). Block order was counter-balanced between participants.

Stimuli and Procedure. The experiment was conducted in a dark room, using a mac (Intel i5 processor, 8 Gb of RAM running Mac OS version 10.11.16). All stimuli were generated using PsychoPy version 1.84.2 (Peirce, 2009), and presented using an Apple studio 20-inch monitor, with resolution 1152x870 (75Hz). Each colour was adjusted to have similar luminance (25.20 cd/m^2). The standard RGB (sRGB) values were as follows. Yellow: 0.7, 0.7, 0.5; blue: 0.565, 0.565, 1.0; red: 1.0, 0.5375, 0.5375; green: 0.5, 0.77, 0.5. Distance from the screen was 57cm, and it was controlled with a chinrest.

We treated each dot in the original illusion as a cell, and replaced it with either 4 dots, or 9 dots. The size and spacing between dots remained the same. Hence, we turned the Solitaire illusion containing 16 dots for the inner and outer pattern, into two larger versions containing 64 and 144 dots. We label these numerosities as 1, 4 and 9 because each cell had either 1, 4 or 9 elements. We repeated the same procedure to increase the numerosity for the bar version of the illusion.

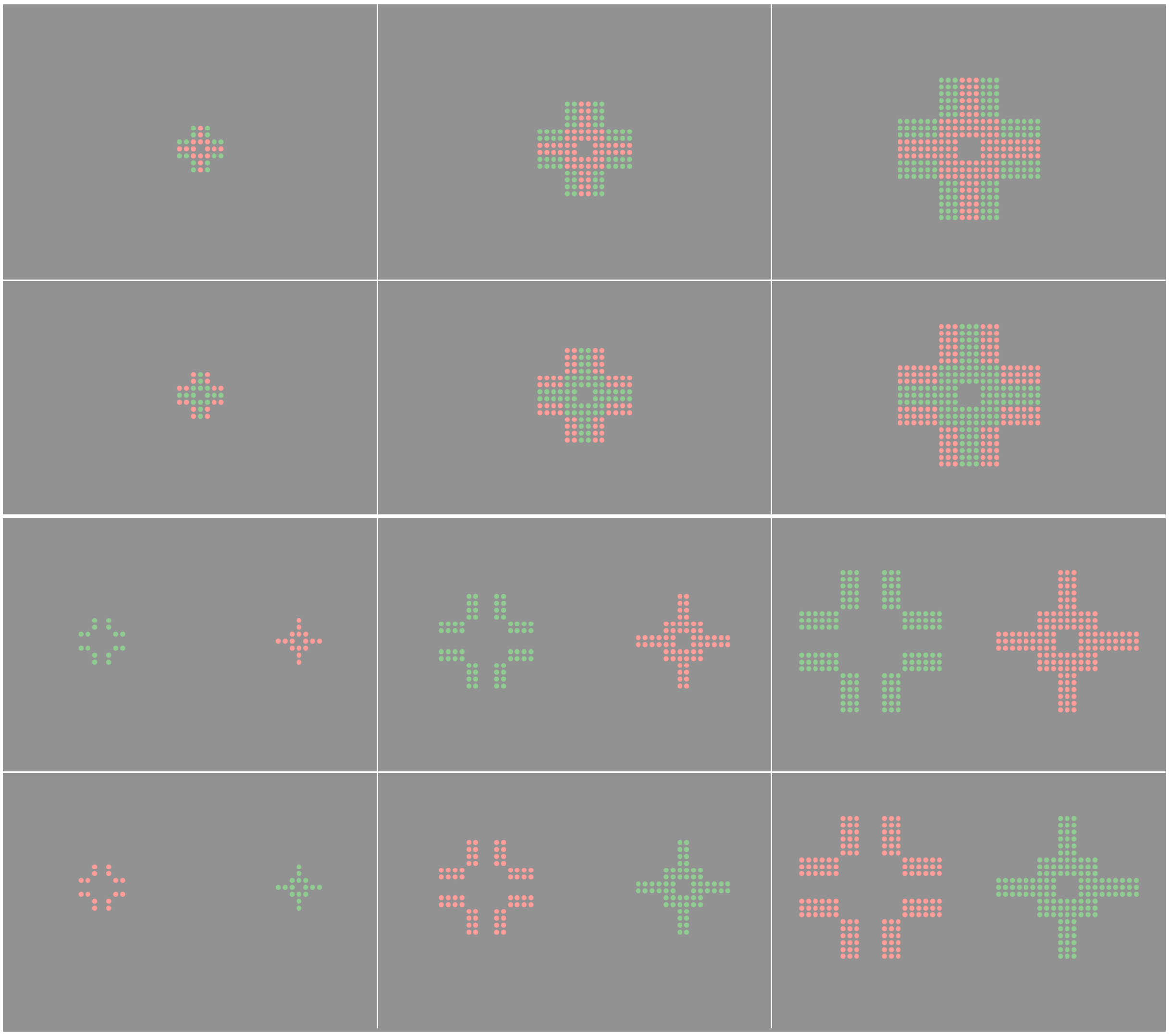

Figure 2: *Top half: The original Solitaire illusion has 16 elements for the inner and outer sets, the middle column has 64 for each set and the last column has 144 elements per set. Bottom half: the same stimuli presented as separate groups. The examples shown have green-inner and red-outer colours, and the opposite arrangement underneath. For another set of stimuli, not shown here, the colours were blue and yellow.*

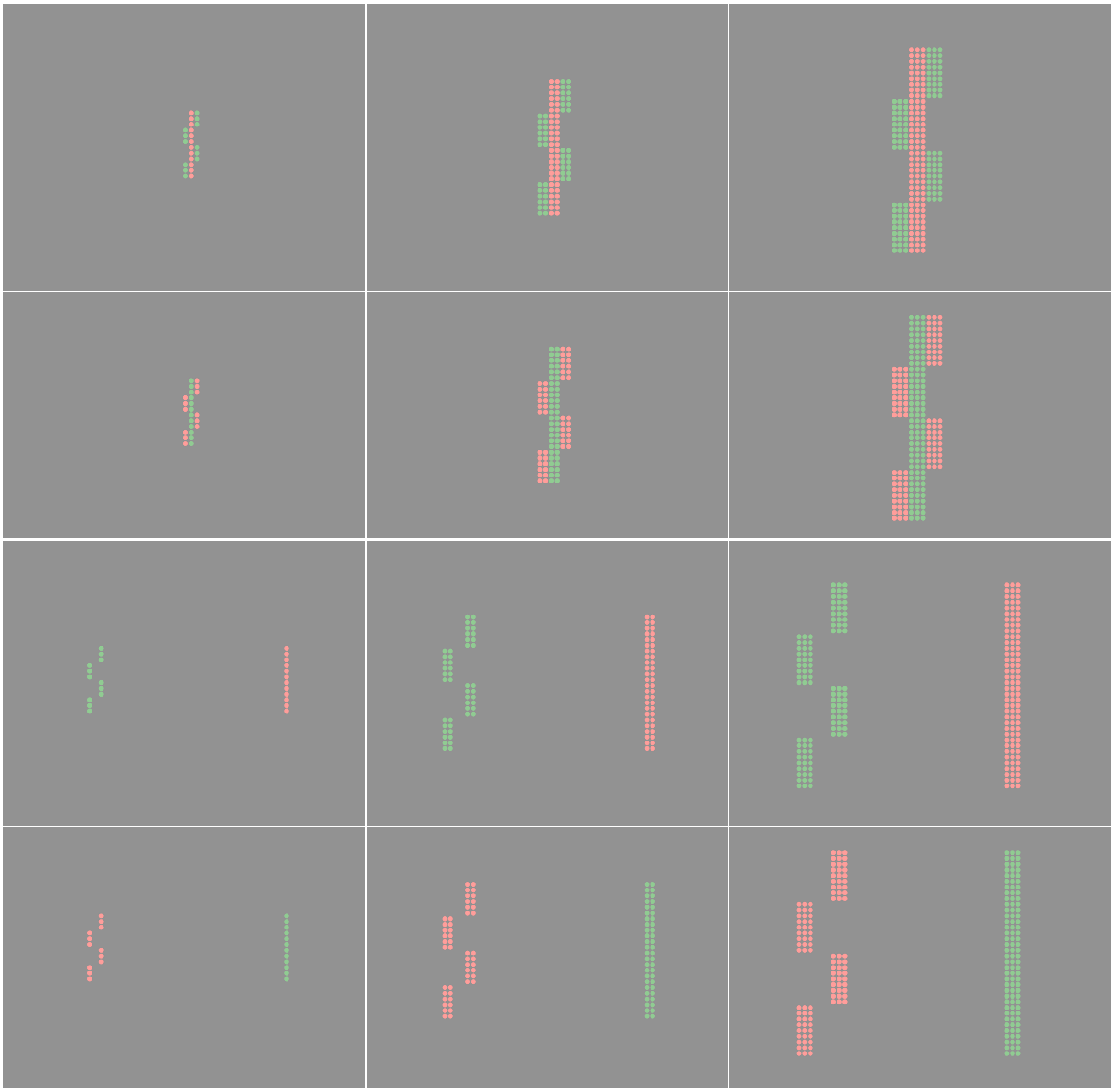

Figure 3: *Top half: For the bar version of the illusion the numbers of elements in each of the three columns are 12, 48 and 108. Bottom half: stimuli for the bar version of the Solitaire illusion presented as separate groups. The examples shown have green-inner and red-outer colours, and the opposite arrangement. For another set of stimuli, not shown here, the colours were blue and yellow.*

Participants were told that they would see two sets of dots with two different colours. Sometimes the sets would be in the centre of the screen, sometimes that would be separated. The task was to choose which colour set appeared to have more dots.

Stimuli were presented on the screen for two seconds. Only after the stimulus disappeared the participants could enter a response. The keys 'a' and 'l' were mapped to the two colours in such a way that participants could press 'a' to judge that the red colour dots were more numerous and the 'l' to judge that the green colour dots were more numerous. Similarly, the same two keys were used for the blue/yellow stimuli. Raw data from our studies as well as example images are available online: https://osf.io/9utzx/.

Data analysis

Data analysis was performed using R Version 3.5.3 (R Core Team, 2013). Data were analysed by means of a generalized logit-linear mixed model for binomially distributed outcomes (GLMM) suitable for analysing complex datasets with repeated or grouped observations (Bolker, 2009; Schielzeth, 2020). We performed an omnibus test based on type-III Wald chi-square with the *anova* function from the *car* package (Fox & Weisberg, 2014).

For all experiments we included in the model the Illusion version (Solitaire vs Bar), Separation (original or separated inner and outer patterns), Numerosity (1, 4, 9 cell size) and we had Colour (red inner/green outer, blue inner/yellow outer, red outer/green inner, blue outer/yellow inner) as a within subject factors. Moreover, block Order was included as a between subject factors. The dependent variable was the number of times the inner pattern was selected as having more dots. The participant, the Colour pairing and the block Order were entered as random effects. By doing so, we assumed a by-subject variation in the intercept for each colour pair and block order combination.

As an estimate of the effect size, we calculated the semi-partial coefficients of determination, also known as part R^2, by means of the partR2 package (Stoffel et al., 2021). As suggested by Stoffel, part R^2 for main effects and interactions were calculated separately and part R^2 for the main effects were estimated after excluding the interaction from the model. The package does not calculate the part R^2 for effects who are included in the random effects, so the coefficient for Colour pairing and the block Order was not reported.

Results and discussion

The analysis of deviance with the Type III Wald chi-square tests showed a significant main effect of Separation (χ^2= 64.748, df= 1, p< 0.001, part R^2= 0.036, C.I.= 0.031 – 0.069), and Numerosity (χ^2= 6.437, df= 2, p= 0.040, part R^2= 0.001, C.I.= 0.001 – 0.033). Among the interactions, Separation:Illusion (χ^2= 20.631, df= 1 , p< 0.001, part R^2= 0.012, C.I.= 0.007 – 0.047), and Separation:Numerosity (χ^2= 6.627, df= 2, p= 0.036, part R^2= 0.008, C.I.= 0.004 – 0.043) were also significant. All other effects were not significant. Table 2 summarizes the results of the Wald test for experiment 1.

Table 2: Analysis of Deviance Table (Type III Wald chi-square tests) for Experiment 1 with part R^2 for each term in the model. Model marginal R^2 = 0.067, C.I = [0.057 – 0.149].

	χ^2	DF	p(>χ^2)	part R^2	R^2CI
(Intercept)	9.893	1	0.002		
Separation	64.748	1	0.000	0.036	0.031 – 0.069
Illusion	0.143	1	0.706	0.001	0.001 – 0-032
Numerosity	6.437	2	0.040	0.001	0.001 – 0.033
Colour	0.561	3	0.905		
Order	0.204	1	0.652		
Separation:Illusion	20.631	1	0.000	0.012	0.007 – 0.047
Separation:Numerosity	6.627	2	0.036	0.008	0.004 – 0.043
Illusion:Numerosity	3.675	2	0.159	0.003	0.001 – 0-038
Separation:Illusion: Numerosity	3.302	2	0.192	0.007	0.004 – 0.094

The analysis above does not directly test whether the response level is above chance. Although this may seem obvious from Figure 4, we decided to add an exact binomial test on the proportions. To avoid multiple tests, we run and report only the test for the Separate condition. Number of responses inner was 734 out of a total of 960 trials, p < 0.001. The alternative hypothesis was that the probability of success is not equal to 0.5 (95% confidence interval: 0.736 to 0.791). On an individual basis, proportions were above chance for 19 out of 20 subjects.

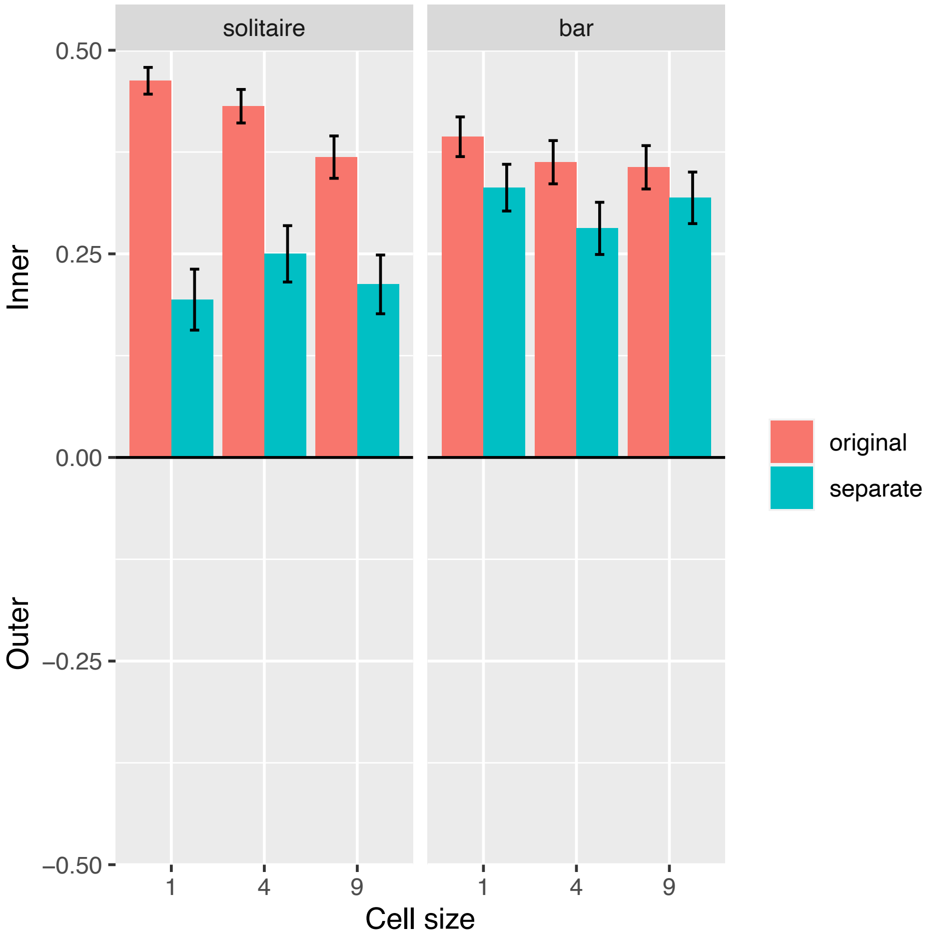

Figure 4: *Proportion of responses for the two versions of the illusion (Solitaire and Bar). The chance level (50%) has been shifted to correspond to zero. Cell size is the number of elements in each of the cells, leading to a total of 16+16, 64+64, and 144+144 elements (Solitaire version) and 12+12, 48+48, and 108+108 (Bar version). The two colours are for the original combined configuration and the presentation of the two sets as separate groups. Error bars are ± 1 SE of the mean.*

The results confirm the presence of the illusion, the inner set of elements tends to appear more numerous in the original configuration and in the bar version of the SI. The key feature of the SI is the arrangement of the two sets, one inside the other. Therefore, we expected that a separation of the two would have a large effect. Indeed, based on the literature one could predict a reversal of the effect given that the outer configuration occupies a larger area. This did not happen in our data. The illusion survived the separation of the two groups, but the effect was reduced at least for the original Solitaire version (an interaction illusion and separation).

The main motivation for Experiment 1 was a test of the role of numerosity. We wanted to test whether the SI would still be present with large sets of elements, and in particular when none of the subsets are within subitizing range. The results demonstrate that the illusion is not specific to the set size of the original configuration. In the original configuration the outer elements formed groups of two or three, which is within the subitization range, however the effect is present also when the smaller groups include sets of elements well above subitazation range. Overall, the SI reveals itself as robust to various manipulations (differences in colour, numerosity, separation of the groups).

Experiment 2

Frith and Frith (1972) pointed to the importance of perceptual grouping in the SI. We reasoned that adding information about grouping may therefore directly affect the illusion. In Experiment 2 we used the same stimuli and the same procedure as in Experiment 1, with the only addition of two lines. These lines connected the elements of a group as shown in Figure 5. The lines corresponded to the concave hull of each of the two colour sets and were of the same colour.

We predicted that by emphasising the area we would reduce the SI and produce instead an effect in line with the role of area: greater perceived numerosity for the larger area (outer group).

Power analysis. To define the sample size of Experiment 2, we performed a power analysis based on Experiment 1. The simulation-based power analyses was performed with the packages mixedpower (Kumle, 2021) and simr (Green & MacLeod, 2016) in R. The estimation of effects was conducted based on the data from Experiment 1. We performed 1,000 simulations for sample numerosity from 14 to 20 participants. Already with 14 participants the power for the Separation factor was close to 100% while the Numerosity reached 80% and their interaction 50%. Since we expected similar effects in this second experiment than in the first one, we collected a sample size of 14 participants.

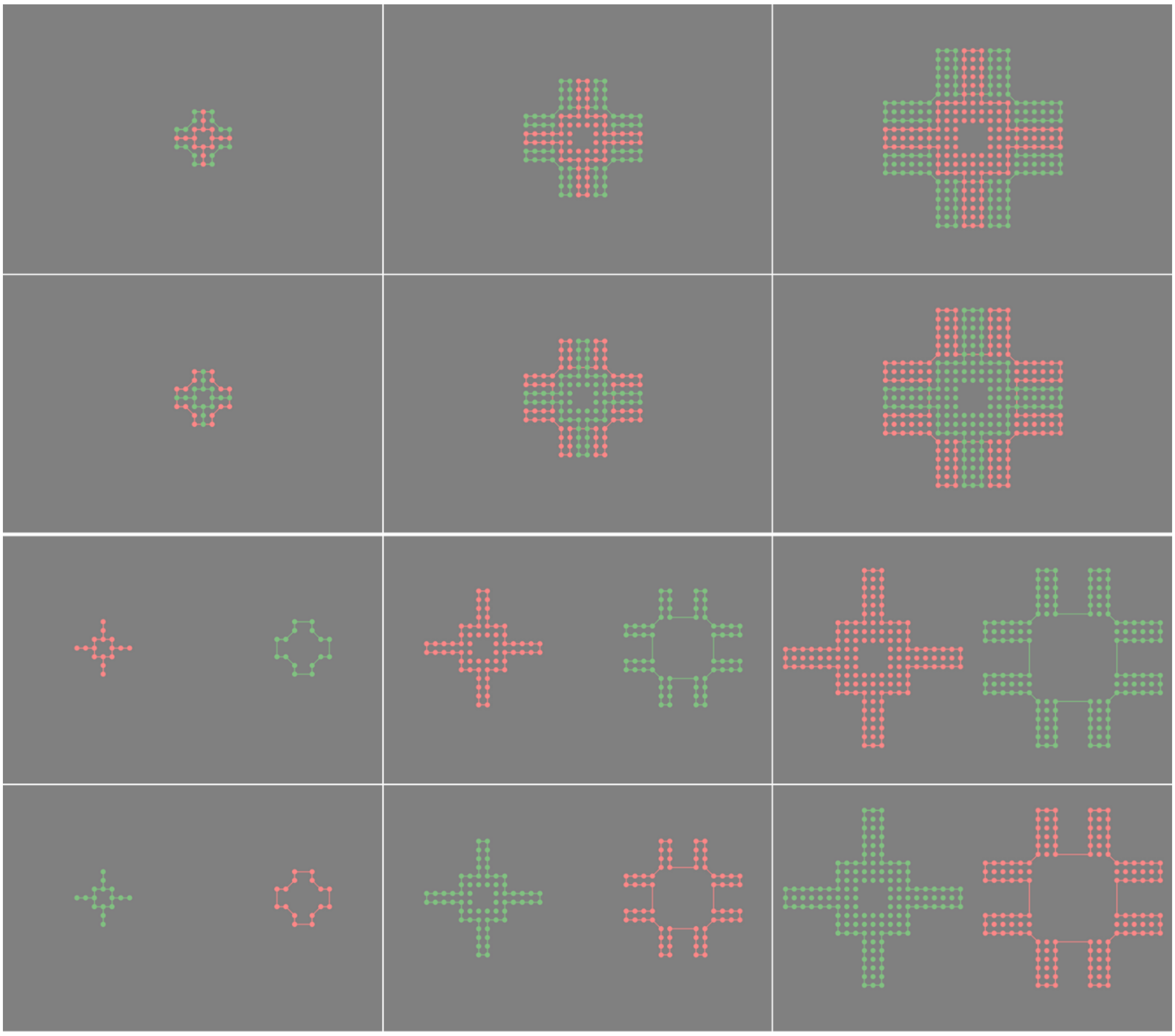

Figure 5: Stimuli for Experiment 2 were the same as those of Experiment 1 except for the thin lines that create closed polygons (we increased thickness by a factor of three to make them more visible in the figure). Here the stimuli can be compared to those in Figure 2.

Methods

Participants. Fourteen individuals participated (age range 21 to 38, 3 males). All participants had normal or corrected-to-normal vision, and none reported any colour blindness. The study was approved by the Health and Life Sciences Committee on Research Ethics (Psychology, Health and Society) and conducted in accordance with the Declaration of Helsinki (revised 2008). Participants were naive with respect to the hypotheses

Design. The design was the same as that of Experiment 1. The factors were the configuration (the original Solitaire illusion or the Bar version), separation (whether the inner and outer patterns were separated on the screen), colour (red/green or blue/yellow), colour for the inner and outer pattern (for example if the colours were red/green the inner elements could be red or green) and numerosity (1, 4, 9 cell size). Each observer was shown a total of 96 trials.

Stimuli and Procedure. The stimuli were the same as in Experiment 1 except for the addition of lines that created two polygons, one for each colour. The procedure was the same as in Experiment 1.

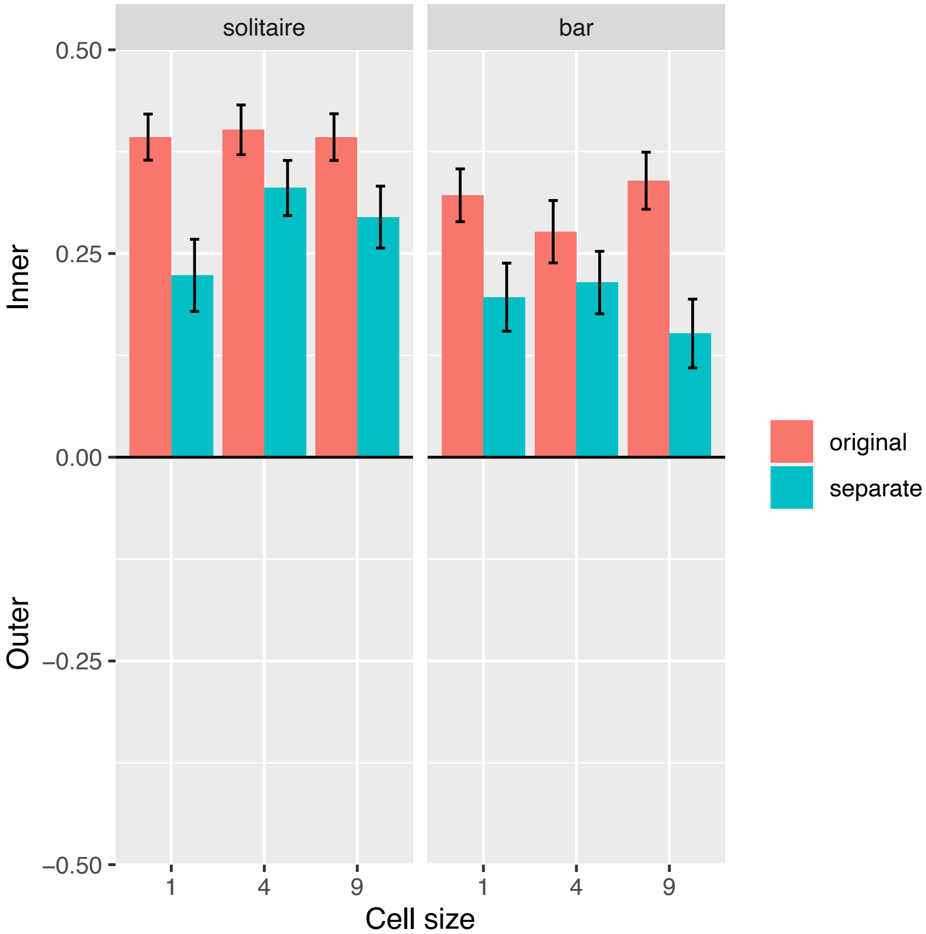

Figure 6*: Proportion of responses for the two versions of the illusion (Solitaire and Bar). The chance level (50%) has been shifted to correspond to zero. Cell size is the number of elements in each of the cells, leading to a total of 16+16, 64+64, and 144+144 elements (Solitaire version) and 12+12, 48+48, and 108+108 (Bar version). The two colours are for the original combined configuration and the presentation of the two sets as separate groups. Error bars are ± 1 SE of the mean.*

Results and discussion.

The results of Experiment 2 are consistent with those of Experiment 1. We confirmed the existence of the illusion for both the original and the bar version. We confirmed that the illusion persists for larger numerosities, and that it is reduced but not eliminated when the two sets of elements are shown as separate groups.

The novelty of Experiment 2 was the presence of lines that would surround the elements and highlight the two groups of different colours. We hypothesised that this may increase the effect of overall area, and also bind together the outer elements. If these lines had changed the strength of the grouping, making them more similar to each other, this should reduce the effect, and if it highlighted the areas, at least in the separate condition, it should reverse the effect. This did not happen. Overall, a preference for the inner pattern was present in all conditions.

The analysis of deviance with the Type III Wald chi-square tests showed a significant main effect of Separation (χ^2= 33.719, df= 1, p< 0.001, part R^2= 0.031, C.I.= 0.015 – 0.093), Illusion (χ^2= 20.056, df= 1, p< 0.001, part R^2= 0.018, C.I.= 0.002 – 0.082), and Order (χ^2= 6.693, df= 1, p= 0.01). All interactions were not significant. Table 3 summarizes the results of the Wald test for experiment 2.

Table 3: Analysis of Deviance Table (Type III Wald chi-square tests) for Experiment 2 with part R^2 *for each term in the model. Model marginal* R^2 *= 0.120, C.I = [0.088 – 0.206].*

	$\chi2$	DF	$p(>\chi^2)$	part R^2	R^2CI
(Intercept)	0.089	1	0.765		
Separation	33.719	1	0.000	0.031	0.015 – 0.093
Illusion	20.056	1	0.000	0.018	0.002 – 0-082
Numerosity	0.716	2	0.699	0.001	0.000 – 0.067
Colour	1.787	3	0.618		
Order	6.693	1	0.010		
Separation:Illusion	0.212	1	0.645	0.001	0.000 – 0.042
Separation:Numerosity	2.535	2	0.282	0.002	0.000 – 0.042
Illusion:Numerosity	2.049	2	0.359	0.002	0.000 – 0-042
Separation:Illusion: Numerosity	1.496	2	0.473	0.001	0.000 – 0.100

As for Experiment 1, we report an exact binomial test for the Separate condition. Number of responses inner was 494 out of a total of 672 trials, $p < 0.001$. The alternative hypothesis was that the probability of success is not equal to 0.5 (95% confidence interval: 0.735 to 0.768). On an individual basis, proportions were above chance for 9 out of 14 subjects.

Experiment 2 confirmed an illusion, but now it is stronger in the original than the bar version. This may be because the lines joining the elements interfered more with the Bar stimuli. Separation was again significant, confirming that this manipulation reduces (but does not eliminate) the illusion.

Experiment 3

We have seen in Experiments 1 and 2 that dots placed within a regular array form a configuration that leads to a perceived difference in numerosity in favour of the inner pattern. This illusion is robust and extends to large numerosities and even to cases when the two sets (the inner and the outer) are presented side by side.

The critical factor seems to be that one group form a central more compact set, either a cross or a bar, and this is the group that appears more numerous. In Experiment 3 we tested the role of the regularity of the array. To destroy the perception of a regular matrix we took the configurations generated according to the same process as in Experiment1, but we sampled only a proportion of the elements to keep (50%) and deleted the others. Examples of stimuli are shown in Figure 7, there is a degree of randomness in the configurations although it is still easy to see that one group is more central and one more peripheral.

Power analysis. Since the rationale of the experiment was to interfere with the illusion by degrading the stimulus, we expected a reduction in the strength of the illusion and thus an overall reduction in the effects of the factors. The estimation was conducted based on the data from Experiment 1. To ensure that we had sufficient power to test the effect of the factors of interest (Separation and Numerosity) the effects sizes of Separation, Numerosity and their interaction were reduced by 25%. We performed 1,000 simulations for sample numerosity from 14 to 20 participants. Already with 14 participants the power for the Separation factor was over 90%. The Numerosity reached 90% for n = 17, while their interaction for n = 20. Thus, we chose the same sample size as Experiment 1 (20).

Methods

Participants. Twenty individuals participated (age range 18 to 38, 9 males). All participants had normal or corrected-to-normal vision, and none reported any colour blindness. The study was approved by the Health and Life Sciences Committee on Research Ethics (Psychology, Health and Society) and conducted in accordance with the Declaration of Helsinki (revised 2008). Participants were naive with respect to the hypotheses.

Design. The design was the same as that if Experiment 3 except that the stimuli were different.

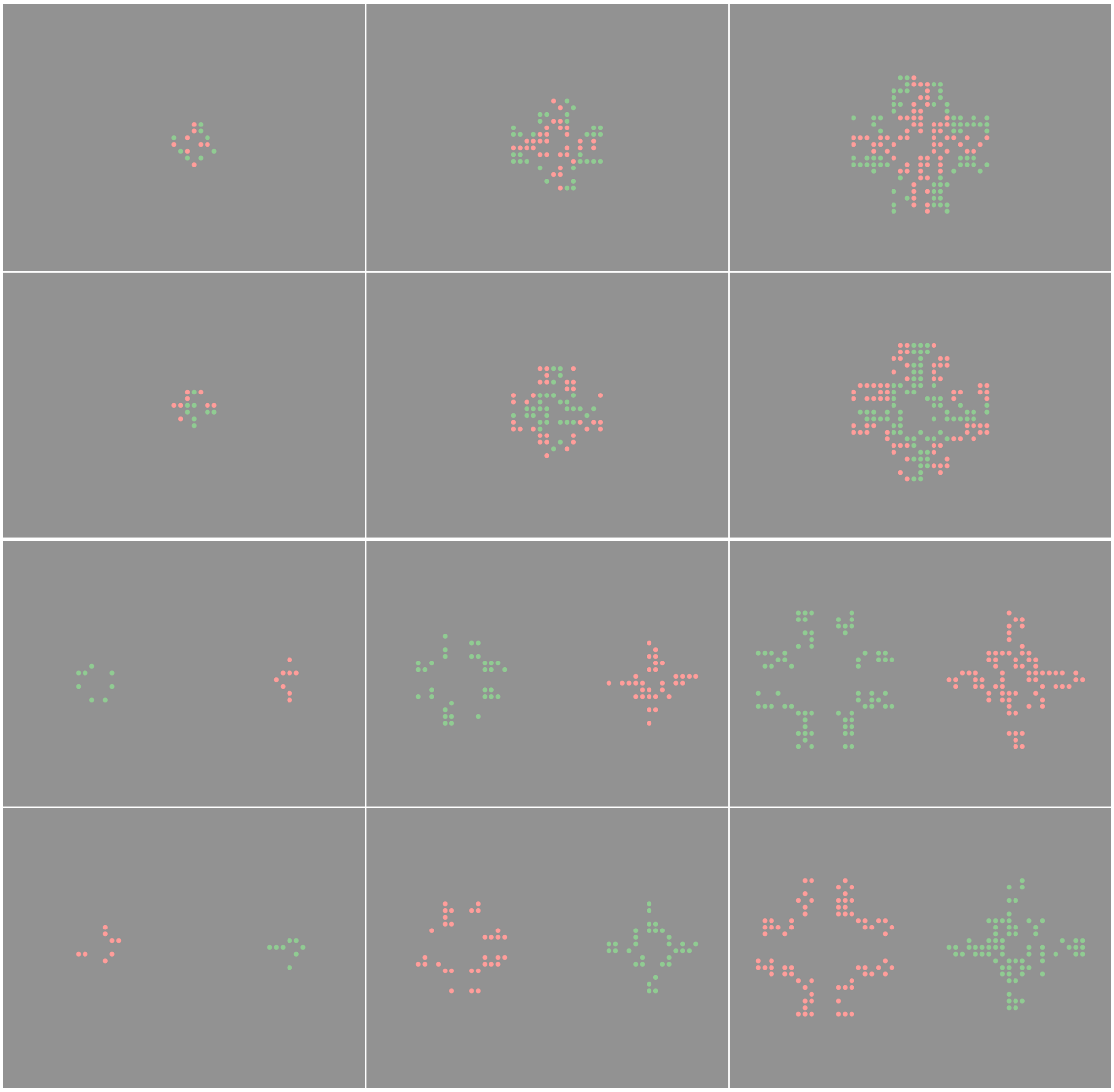

Figure 7*: Stimuli for Experiment 3 were generated starting from configurations like those of Experiment 1. Patterns were created, and then a random sample of 50% of the elements were kept and the rest deleted. Therefore, unlike Experiment 1, no exact stimulus was ever presented to the participants twice. Here examples of stimuli can be compared to those in Figure 2.*

Stimuli and Procedure. The procedure was the same as for the previous experiments.

Stimuli was the same as previously except that only some of the elements were visible. The regular configurations were reduced to just 50% of the full array. This sampling was done randomly using the computer random number generator.

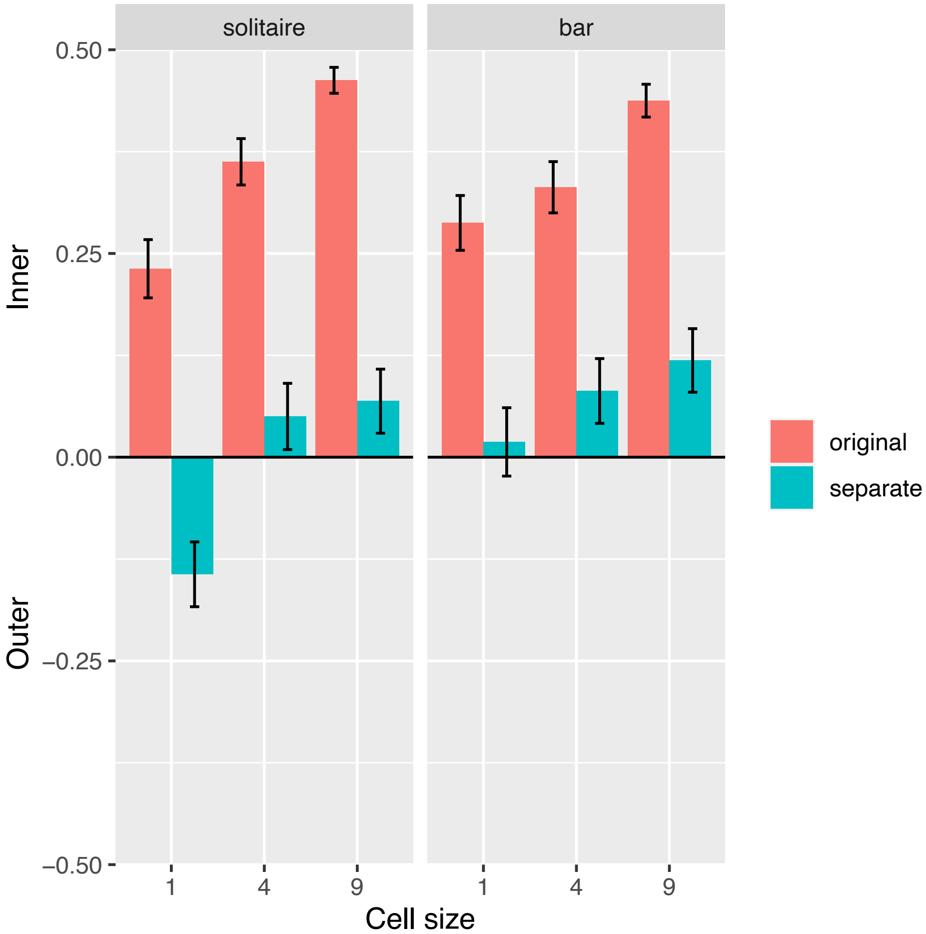

Figure 8: Proportion of responses for the two versions of the illusion (Solitaire and Bar). Cell size is the number of elements in each of the cells. The two colours are for the original combined configuration and the presentation of the two sets as separate groups. Error bars are ± 1 SE of the mean.

Results and discussion

The analysis of deviance with the Type III Wald chi-square tests for the memory score showed a significant main effect of Separation (χ^2= 203.860, df= 1, p< 0.001, part R^2= 0.138, C.I.= 0.111 – 0.187), and Numerosity (χ^2= 58.410, df= 2, p= 0.01, part R^2= 0.036, C.I.= 0.006 – 0.095). Among the interactions, Separation:Numerosity (χ^2= 13.769, df= 2 , p= 0.001, part R^2= 0.049, C.I.= 0.010 – 0.119), and Illusion:Numerosity (χ^2= 6.322, df= 2, p= 0.042, part R^2= 0.005, C.I.= 0.000 – 0.080) were also significant. All other effects were not significant. Table 4 summarizes the results of the Wald test for experiment 3.

Table 4: Analysis of Deviance Table (Type III Wald chi-square tests) for Experiment 3 with part R^2 *for each term in the model. Model marginal* R^2 *= 0.208, C.I = [0.188 – 0.306].*

	$\chi 2$	DF	$p(>\chi^2)$	part R^2	R^2CI
(Intercept)	7.227	1	0.007		
Separation	203.860	1	0.000	0.138	0.111 – 0.187
Illusion	0.546	1	0.460	0.003	0.000 – 0-066
Numerosity	58.410	2	0.000	0.036	0.006 – 0.095
Colour	0.779	3	0.854		
Order	0.611	1	0.434		
Separation:Illusion	3.761	1	0.052	0.003	0.000 – 0.078
Separation:Numerosity	13.769	2	0.001	0.049	0.010 – 0.187
Illusion:Numerosity	6.322	2	0.042	0.005	0.000 – 0-080
Separation:Illusion: Numerosity	0.358	2	0.836	0.004	0.000 – 0.126

As for Experiment1, we report an exact binomial test for the Separate condition. Number of responses inner was 511 out of a total of 960 trials, $p < 0.049$. The alternative hypothesis was that the probability of success is not equal to 0.5 (95% confidence interval: 0.500 to 0.564). Here the effect is extremely weak, and very different from Experiment 1 and 2. On an individual basis, proportions were above chance for 5 out of 20 subjects.

Experiment 4

Experiment 3 tested the role of the regularity of the array. Removing 50% of the elements did not destroy the preference for the inner group. In Experiment 4 we went one step farther and kept only 10% of the elements. Examples of stimuli are shown in Figure 9. Because of the reduction in number of elements the starting configurations had larger numerosities, this was achieved by increasing the number of elements in each cell from the original 1, 4 & 9 to 9, 16 and 25. The 10% did not always produce a whole number, therefore each number was rounded up to the nearest whole number, hence the final numerosities were 15, 26, 40 for SI, and 11, 20, 30 for the Bar version of SI.

Note how we are now having stimuli much more similar to those used in the RRI (see Figure 1).

Power analysis. As in Experiment 3, we performed the a priori power calculation based on the data collected in Experiment 1. In this case, as we expected an even greater reduction in effect size, the observed effects sizes of Separation, Numerosity and their interaction were reduced by 50%. As before, we performed 1,000 simulations for sample numerosity from 14 to 20 participants. Once again, with 14 participants the power for the Separation factor was over 90%. The Numerosity reached 90% for n = 18, while their interaction for n = 20 reached a power above 80%. Therefore, we used the same sample size of Experiment 1.

Methods

Participants. Twenty individuals participated (age range 18 to 36, 3 males). All participants had normal or corrected-to-normal vision, and none reported any colour blindness. The study was approved by the Health and Life Sciences Committee on Research Ethics (Psychology, Health and Society) and conducted in accordance with the Declaration of Helsinki (revised 2008). Participants were naive with respect to the hypotheses.

Design. The design was the same as that if Experiment 4 except that the stimuli were different. The number of elements in each cell (before sampling) was increased from the original 1, 4 & 9 to 9, 16 and 25.

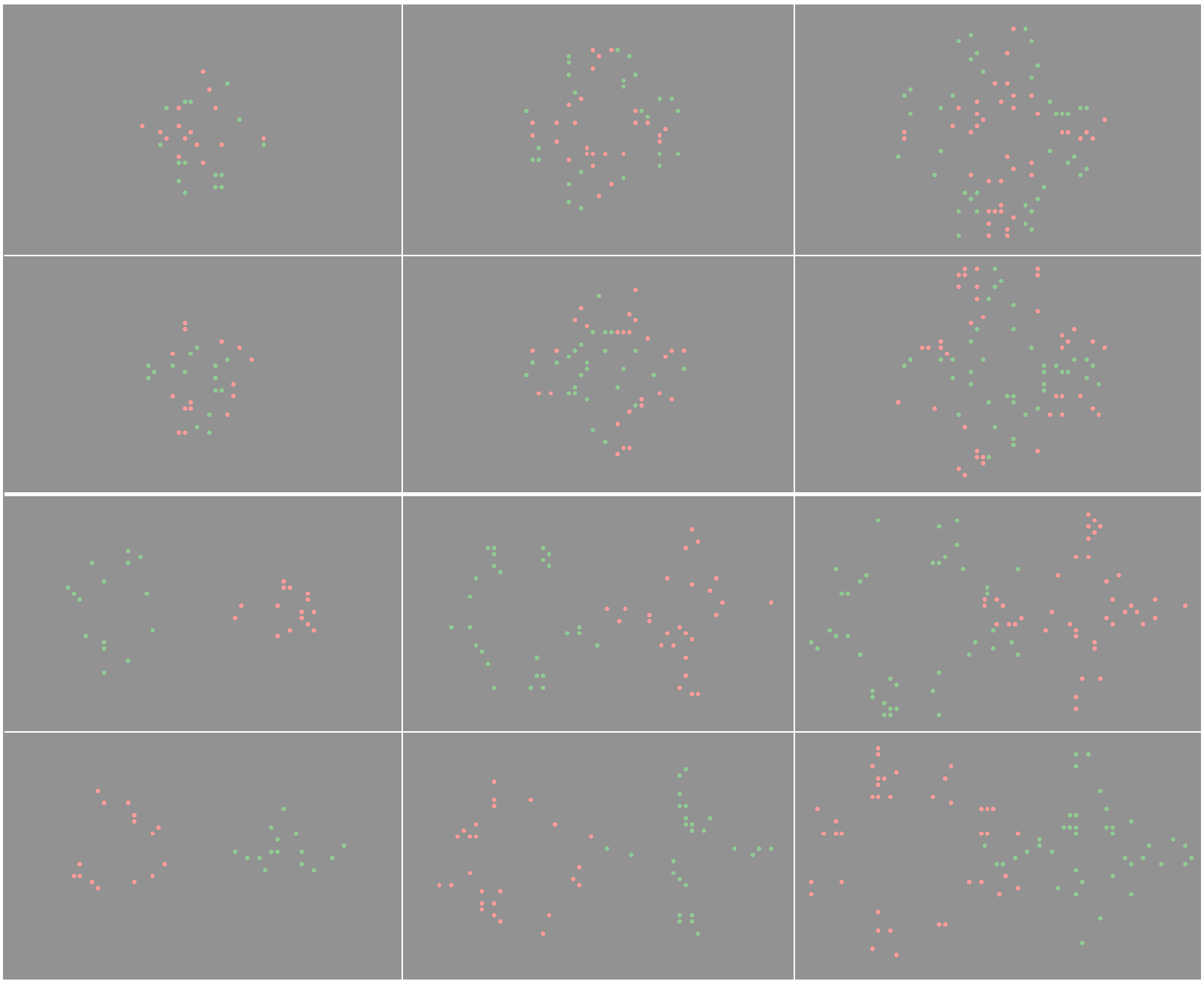

Figure 9*: Stimuli for Experiment 4. Configurations with high number of elements were created, and then a random sample of 10% of these elements were kept and the rest deleted. Here examples of stimuli can be compared to those in Figure 2 and in Figure 7.*

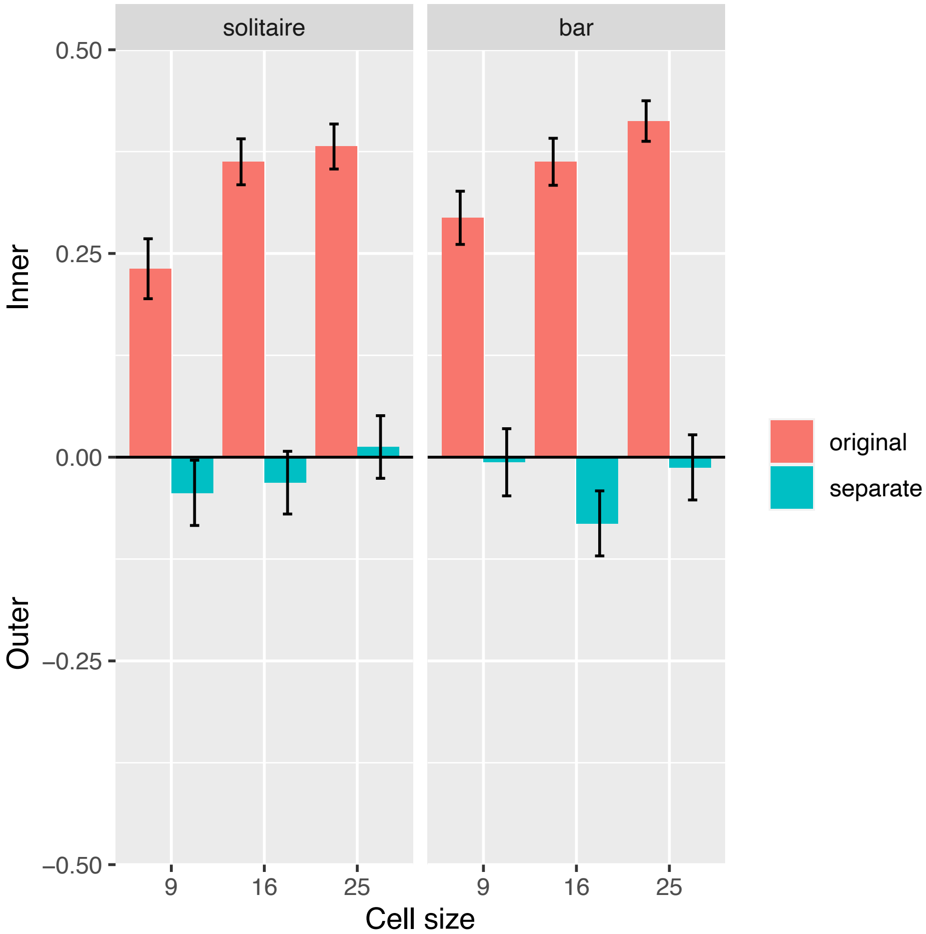

Figure 10*: Proportion of responses for the two versions of the illusion (Solitaire and Bar). Cell size is the number of elements in each of the cells. The two colours are for the original combined configuration and the presentation of the two sets as separate groups. Error bars are ± 1 SE of the mean.*

Results and discussion

The analysis of deviance with the Type III Wald chi-square tests showed a significant main effect of Separation (χ^2= 267.227, df= 1, p< 0.001, part R^2= 0.158, C.I.= 0.127 – 0.184), and Numerosity (χ^2= 16.317, df= 2, p= 0.01, part R^2= 0.008, C.I.= 0.000 – 0.042). Among the interactions, Separation:Numerosity (χ^2= 14.142, df= 2 , p= 0.001, part R^2= 0.019, C.I.= 0.009 – 0.071) was significant. All other effects were not significant. Table 5 summarizes the results of the Wald test for experiment 4.

As before, separation reduces the effect, now the illusion has disappeared. When the illusion is present (original configuration) the strength of the illusion increases with numerosity for both Solitaire and Bar version.

Table 5: Analysis of Deviance Table (Type III Wald chi-square tests) for Experiment 4 with part R^2 *for each term in the model. Model marginal* R^2 *= 0.184, C.I = [0.159 – 0.214].*

	$\chi 2$	DF	$p(>\chi^2)$	part R^2	R^2CI
(Intercept)	7.227	1	0.007		
Separation	203.860	1	0.000	0.158	0.127 – 0.184
Illusion	0.546	1	0.460	0.000	0.000 – 0-036
Numerosity	58.410	2	0.000	0.008	0.000 – 0.042
Colour	0.779	3	0.854		
Order	0.611	1	0.434		
Separation:Illusion	3.761	1	0.052	0.001	0.000 – 0.055
Separation:Numerosity	13.769	2	0.001	0.019	0.000 – 0.071
Illusion:Numerosity	6.322	2	0.042	0.000	0.000 – 0-054
Separation:Illusion: Numerosity	0.358	2	0.836	0.001	0.000 – 0.038

As for all other experiments, we report an exact binomial test for the Separate condition. Number of responses inner was 454 out of a total of 960 trials, $p < 0.100$. The alternative hypothesis was that the probability of success is not equal to 0.5 (95% confidence interval: 0.441 to 0.505). Therefore, for this condition there is no evidence that people are more likely to select inner or outer. On an individual basis, proportions were above chance for 4 and below chance for 6 subjects out of 20.

General Discussion

The Solitaire illusion is an interesting and surprising effect. For most observers there is a strong bias to perceive a difference in numerosity between two sets of elements based on their configuration (Frith & Frith, 1972). In the original configuration the outer set, which is the one perceived as less numerous, is split into subsets of few elements. Specifically, the 16 elements are divided into 8 groups of two. We modified the pattern to include higher numerosities, to avoid that any of the subsets would fall within the subitizing range. In all our experiments we used both the original configuration (Solitaire) and the version with a line of dots (Bar) as illustrated in Figure 1.

We also used four colour combinations, red insider/green outside, green inside/red outside, blue insider/yellow outside, yellow inside/blue outside. We found no effect of colour. Note that we were careful to use colours similar in luminance and contrast. It is known that low contrast disks appear higher in numerosity when intermingled with the high contrast ones (Lei & Reeves, 2018).

Table 6 is a summary of the results from four experiments. In Experiment 1, we found that the illusion exists also at high numerosities, although it was slightly reduced for the Solitaire version as numerosity increased. The story was quite different in Experiment 3 and 4, where the elements appeared as clouds and not fixed within a rigid alignment grid. Here the illusion was still present (as long as the two groups were not separated) but the strength increased with numerosity. We believe this is due to the fact that at low numerosity the inner/outer separation is partly lost for sampled displays, while with high numerosity the inner/outer separation becomes easier to see (see Figures 7 and 9).

Table 6. A list of the four experiments and main findings.

Experiment	Main factors	Results
Exp 1	Version Numerosity Separation	The illusion generalises to large numerosities, well above subitazation. It also survives the separation of the two groups: Standard together: 92%, separate: 72% Bar together: 87%, separate: 81%
Exp 2	Version Numerosity Separation	As for Exp 1, there is an illusion in all conditions: Standard together: 89%, separate: 78% Bar together: 81%, separate: 69%
Exp 3	Version Numerosity Separation	Only 50% of the elements are shown. The illusion is weaker, especially in the separate condition, and it increases with numerosity: Standard together: 85%, separate: 57%

		Bar together: 85%, separate: 57%
Exp 4	Version Numerosity Separation	Only 10% of the elements are shown. The illusion is absent in the separate condition, when present it increases with numerosity: Standard together: 85%, separate: 57% Bar together: 85, separate: 57%

Another important feature of the SI is the enclosure of the inner set by the outer. This was already observed by Frith and Frith (1972) using the bar version of the illusion, but with limited data. Their observers made a single response and were then divided between those who had experienced the illusion and those who did not. For a configuration similar to the one we used (12+12 elements, as shown in Figure 1) 12 out of 13 adult observers chose the inner group as more numerous, but only 7 out of 13 did so for the divided version. Therefore, we expected that placing the two side by side (horizontal translation) would have a large effect.

The separation of the groups could eliminate or reverse the illusion because when separated it is clearer that one set spreads over a larger area (Dakin, 2011; Tokita & Ishiguchi, 2010). Contrary to this prediction, in our Experiments 1 and 2 the illusion survived the separation of the two groups, although the effect was reduced. This result is consistent with what recently reported by Pecunioso and Agrillo, (2021). They used the original version of the SI and presented it in the standard configuration and also as separate groups (inner and outer subsets, each with 16 elements). The task was different from what we have used, as observers had to report verbally an estimate of the number of dots. Despite the difference in tasks, it is clear in both studies that the SI is not completely eliminated by the separation, suggestion that the key factor is the distribution of the elements.

We were able to eliminate the SI only when the elements were made to appear as clouds of elements. We achieved this in Experiments 3 and 4 by starting with the original configurations and then removing randomly 50% (Experiment 3) or 90% (Experiment 4) of the elements. However, in no case did we find reversal of the effect. The manipulation necessary to destroy the illusion in Experiments 3 and 4 also weakened grouping and Gestalt factors such as continuity (Frith and Frith, 1972) and clustering

(Valsecchi, 2013). When enclosure is present, however, the illusory effect wins out over grouping (Experiment 1) and continuity (Experiment 2), and even when enclosure is weak (Experiments 3 and 4) dispersion over a larger area (causing overestimation) is countered by a possible effect of central location of the elements (causing underestimation). These two effects may then cancel each other out leading to no overall bias.

In the introduction we mentioned that at high densities, patterns behave differently because individual items are no longer discernible as separate items. The switching point can depend on various factors, but is around 1 dots/deg^2 (Anobile et al., 2016). In Experiment 1 and 2, relative to a square enclosing region, densities ranged between 1 and 1.79 dots/deg^2. The case is special because of the regularity of the pattern. It is a question that will require further work, regularity could increase the perception of a uniform texture, or do the opposite, and allow elements to remain separate items (for the role of regularity on crowding see Sayim et al., 2010). In Experiment 3 and 4 density was lower because of sampling (between 0.54 and 0.89 for Exp 3, between 0.16 and 0.26 for Exp 4).

We have seen that the central location is therefore the key factor in the SI, but why? One possibility was noted already in the introduction, a decrease in perceived numerosity in the visual periphery (Valsecchi et al., 2013). It has been suggested that in the periphery elements can become harder to identify and segment from each other, in line with the phenomenon of crowding. However, the relationship between numerosity and crowding is far from clear (Anobile et al., 2016; Chakravarthi & Bertamini, 2020). It should also be noted that in our experiments we did not control fixation and eye movements, we can therefore talk about central and peripheral vision only in terms of configuration, rather than in retinal coordinates. In other words, the central region remains a central region, and has higher density, because of the appearance, not position in the visual field.

Another factor may be more attentional, elements of one colour are located and found before those of the other colour. We can call this an availability bias, and it is similar to the availability heuristics in reasoning and problem solving (Tversky & Kahneman, 1973). This is also consistent with the fact that perceived numerosity is biased in favour of the second of two configurations (van den Berg et al., 2017).

In the original study, Frith and Frith (1972) pointed out that Gestalt factors are important for the illusion. Perhaps grouping is related to figure-ground organisation, and elements in the foreground become more influential in the numerosity judgment. The problem, as we have seen, is that other Gestalt factors, such as brightness (Ross & Burr, 2010), symmetry (Apthorp & Bell, 2015), or grouping by connectedness (Franconeri et al., 2009), lead to the opposite effect (a reduction in perceived numerosity).

These studies were conducted in a Department of Psychology, and the participants were mainly Psychology students. The basic illusion is strong, and readers can verify its existence by looking at Figure 1. Nevertheless, issues of generality could be addressed in future work. As already noted in the introduction, this illusion appears to be specific to human adults (Agrillo et al., 2014; Parrish et a., 2016), and there are factors that are known to affect perception of illusions, among them: culture (Doherty et al., 2008), autism (Happé, 1996) and schizotypy (Notredame et al. 2014).

We conclude that, despite the fact that the original illusion may have been noticed serendipitously, looking at a board game (Frith & Frith, 1972), the underlying phenomenon is strong and general. Alignment is not important, any time a set appears *inside* another, or is more centrally located, the estimation of numerosity is biased in its favour.

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