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RESEARCH ARTICLE

The Hutong neighbourhood grammar: A procedural modelling approach to unravel the rationale of historical Beijing urban structure

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Abstract Hutong neighbourhoods, composed of Chinese courtyard dwellings (Siheyuan), are historically and socially significant urban spaces that embody the traditional Chinese way of life and philosophy. As part of the national heritage, there is an increasing research interest in Hutong neighbourhoods, many of which are facing oblivion. This study presents a formal grammar for Hutong neighbourhood generation. This research investigates traditional principles of urban planning of ancient Beijing, based on examples on the historical map Qianlong Jingcheng Quantu, to derive the lost design rules. These rules are used to build up a procedural modelling framework, which reveals the development of Beijing's urban structure from the Yuan (1271—1368) to the Qing (1644—1911) dynasty. Our findings present a grammar incorporated into the procedural modelling framework to parametrically generate Hutong neighbourhoods, which replicates the morphological characteristics of historic cases. It contributes to the understanding of the generation of Hutong neighbourhoods. In support of heritage sustainability, this grammar can be implemented in a computational environment by visual scripting that enables the generation of new instances of Hutong neighbourhoods, both real and virtual.

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1. Background

When the government of the Yuan dynasty (1271—1368) decided to locate its primary capital to Beijing, the urban planner, Binzhong Liu (1216—1274) designed a new Beijing city on a new site, called Dadu (The Great Capital), which eventually became contemporary Beijing. Although Beijing has been developed during the Ming (1368—1644) and Qing (1644—1911) dynasties, the initial consideration of its urban planning, which resulted in an orthogonal grid system, has remained until today. It is believed that the planning of Beijing followed the rules recorded in *Kao Gong Ji* (*Record of Trades*), in which Confucianism was embedded (Steinhardt, 1990). Being is not only an important part of ancient Beijing but also the socio-cultural artefact of traditional Chinese urban planning, Hutong neighbourhoods are historically and socially significant urban heritage that embodies traditional Chinese philosophy. Except for the imperial and governmental sectors of ancient Beijing, the urban area was divided into four-sided sectors as residential neighbourhoods under the constraint of an orthogonal grid system. During the Qing dynasty, these neighbourhoods were reformed due to population increase, after which they were composed of variants of a vernacular dwelling type—Chinese courtyard housing, namely Siheyuan. Since these neighbourhoods usually existed between two Hutongs (alleys), they were called Hutong neighbourhoods. The Hutong neighbourhoods are rectangular on plan, whose basic units are Siheyuan. An ideal Siheyuan consists of axially aligned courtyards symmetrically surrounded by individual buildings, connected by orthogonally located corridors, walls, and gates, as illustrated in Fig. 1.

As the carriers of traditional Chinese culture, both Hutong neighbourhoods and Siheyuan have interested historians. However, despite their cultural significance, after many historians and urbanists' proposals to preserve historical Beijing city were rejected by the Beijing government around 1950, vast areas of traditional Beijing Hutong neighbourhoods were cleared to give way for modern construction (Yu, 2017). Ni (2009) estimated that more than eighty percent of Beijing Hutong neighbourhoods and Siheyuan houses have been destroyed up to 2009. Meanwhile, although historical literature such as *Kao Gong Ji* recorded the ideal pattern of a Chinese city, the information is fragmentally scattered in different chronicles, documents, and journals and lacks clarity. Moreover, the ancient language used in historical literature is difficult to understand for current people. The demolition of Hutong neighbourhoods and our decreasing knowledge about how to design them indicate that they are facing oblivion.

As Li (2016) pointed out, a large number of contemporary architectural and urban planning projects attempting to retrieve the Chinese style have failed over the past decades, one of whose reasons may be the lack of design knowledge of traditional Chinese architecture and urban planning. Although many urban renewal projects in Hutong neighbourhoods successfully transformed this old urban structure into a modern style to satisfy the needs of contemporary life, most of them are not based on any original design concepts and do not embody traditional Chinese philosophy. One famous precedent is the Ju Er Hutong Renovation Project held by the academician Liangyong Wu. The planning of this project complied with

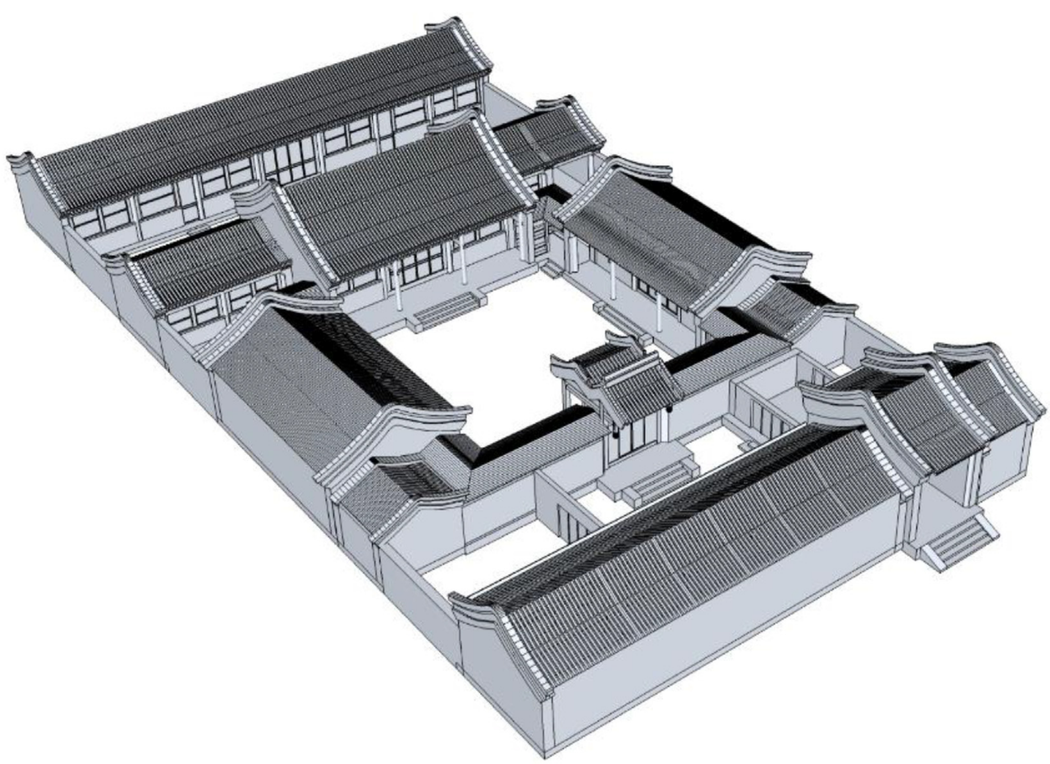

Fig. 1 A three-courtyard Beijing Siheyuan.

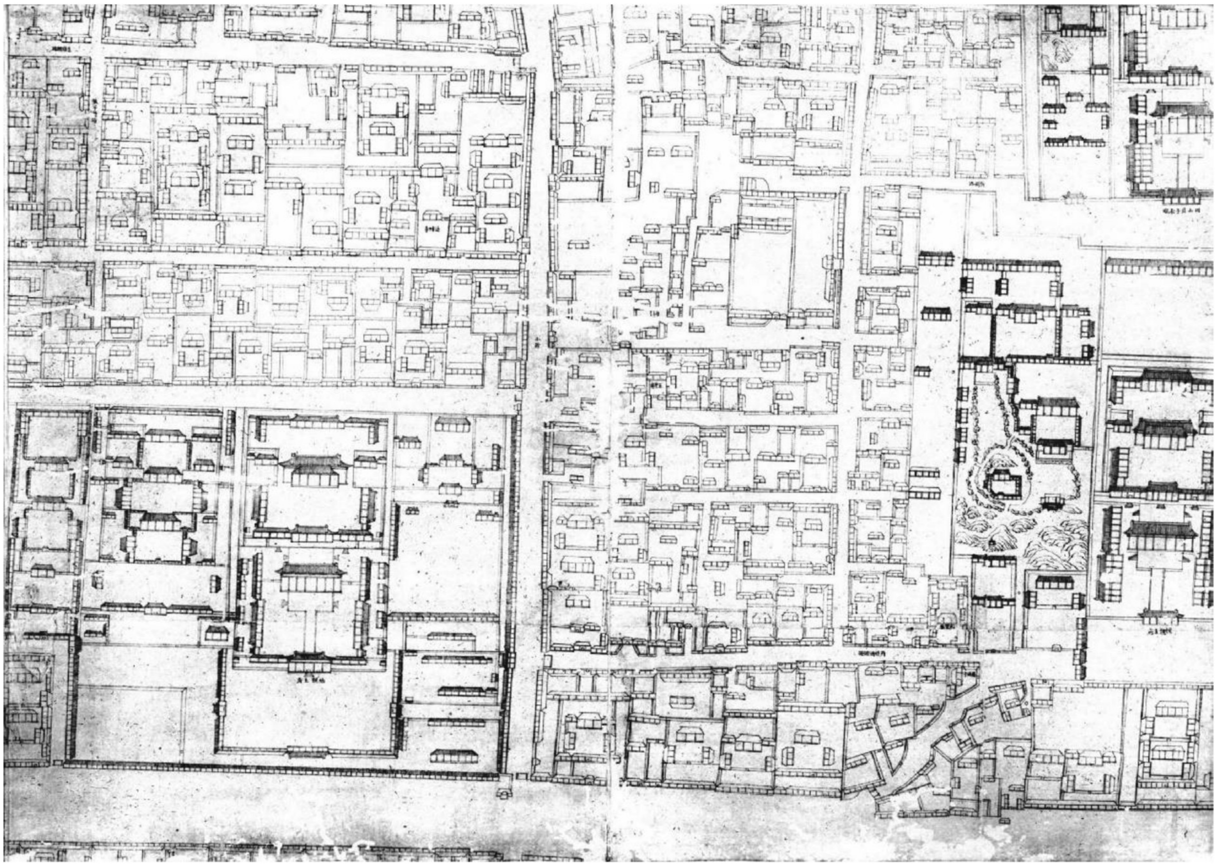

Fig. 2 A section of Qianlong Jingcheng Quantu (Qianlong Capital Map, 1750) indicating the urban planning of historical Beijing is based on a grid system.

Beijing's urban texture in a grid composition to make harmony with the surrounding urban fabric but re-designed the Siheyuan in new forms with multiple storeys, which failed to inherit the traditional culture of national spirit (Zhang, 2016), despite that this project is widely considered a successful regeneration project as it won many awards. Considering the Siheyuan to be a cultural artefact (Zhang, 2015), Zhang (2013) emphasized the importance of studying its morphology to enable cultural sustainability. To achieve this, it is necessary to provide a new approach capable of conveying the design knowledge of Hutong neighbourhood morphology to current architects and urban planners.

Since the Hutong neighbourhoods were composed of a large number of Siheyuan variants, morphological studies of the neighbourhoods were linked to the typological analysis of Siheyuan. Ni (2009) investigated the forms of extant Siheyuan and Hutong separately using statistical analysis, which failed to summarize the common features between Siheyuan variants but proved the flexibility underlying Siheyuan variants design in Hutong neighbourhood contexts. Li (2009) chose several sections of a historical map Qianlong Jingcheng Quantu (Qianlong Capital Map, 1750, a section of this map is shown in Fig. 2. to study how the Siheyuan variants clustered in a Hutong neighbourhood interacted with each other to iterate their forms in history. Li's work revealed the principles explaining how the archetype of Siheyuan evolved variants that composed Hutong neighbourhoods. Similarly, Liu (2019) investigated several types of Siheyuan precedents within five Hutong neighbourhoods drawn on this map by classifying the courtyards of Siheyuan into five categories to understand how the different Siheyuan layouts were deformed to adapt themselves to the various site contexts within the Hutong neighbourhoods.

One common conclusion of the above studies is that the rules underlying the evolution of Hutong neighbourhoods are too complicated to understand in conventional ways. However, computational generative approaches are powerful enough to explicate design rules and develop design iterations. One famous example is A Pattern Language by Alexander et al. (1977), which listed architectural tropes and examined how they can be composed to shape buildings, communities, and towns. The shape grammar invented by Stiny (1980) to interpret the generative rules of objects graphically is another innovative approach. A large number of Stiny's followers employed shape grammar to investigate principles of architectural design and urban planning in specific case studies, among which the most famous ones are Li (2001) and Duarte (2005). One step further, Stouffs (2018) and Li (2018) dedicated themselves to implementing shape grammar as a generative tool in Rhino/Grasshopper environment (Stouffs and Li, 2020). By using the Python programming language to code, Stouffs created the Sortal Grammar Interpreter, a plug-in of Rhino/Grasshopper, to support shape computations and their visual enumeration combining parametric modelling with rule-based generation. Hou and Stouffs (2018, 2019) employed algorithmic design grammars to solve design problems. Despite the algorithmic difficulty of shape grammar implementation in computers (Wortmann and Stouffs, 2018), analytical grammar has its advantage in straightforwardly demonstrating the morphological iteration of shapes, which was employed to explain historical styles or languages of designs. Many projects are using the grammar-based approach defined in specific case study contexts of procedural modelling of buildings and cities. In the context of urban planning, Muller et al. (2006) created a Computer Generated Architecture (CGA) Shape Grammar to efficiently generate massive urban building models. Another famous example is the "Rome Reborn 2.0" project (Dylla et al., 2008), which imported grammar rules of historical buildings to virtually regenerate three-dimensional models of Rome in 320 AD in CityEngine. Following these procedural modelling approaches, Flora et al. (2018) regenerated virtual three-dimensional models of urban blocks of nineteenth-century Camden London in CityEngine, by deriving the generative rules of Victorian houses, an iconic dwelling type in London, from historical building regulations. Verniz and Duarte (2020) developed a shape grammar to examine the urban planning of Santa Marta, an informal settlement in Brazil, in a procedural modelling process. Contrary to conventional top-down approaches in urban planning, their work introduced a bottom-up framework capable of generating housing variants. Taking contextual forces such as topography, urban context, and functional organization into consideration, their grammar clarified how the multiple factors simultaneously forced the development of favelas, which caused the complex urban forms. Costa et al. (2019) implemented this grammar in Grasshopper as a pedagogical tool to convey urban design knowledge of Santa Marta.

2. Research aims and methodology

Given that current architects and urban planners find it difficult to acquire an understanding of Hutong neighbourhood design principles, we aim to develop a digital tool capable of generating Hutong neighbourhoods and exhibiting their design rules. To create the parametric modelling tool with pedagogical functions, we intended to develop a shape grammar interpreter in Grasshopper by scripting, which can both generate design models parametrically and illustrate the iterative process graphically. To achieve this aim, the following questions are addressed.

(1) What are the rules underlying the morphological development of Hutong neighbourhoods?
(2) How could the rules be translated into a shape grammar to simulate the procedural generation of Hutong neighbourhoods?
(3) How could such a shape grammar be transformed into an algorithm and implemented by Grasshopper scripting as an interactive tool for parametric modelling of Hutong neighbourhood variants replicating their original morphological characteristics?

To answer the above research questions, the research method consists of three stages correspondingly. This paper is focusing on the first and second questions, corresponding to the second stage. The third question corresponding to the third stage is briefly discussed in the concluding section and is the subject of subsequent research.

Stage 1 Generation of representational models.

Literary records and extant buildings are the two crucial references for studying Chinese architecture (Li, 2016). Although some Siheyuan houses were preserved today individually, most traditional Hutong neighbourhoods were demolished from the 1950s to the 2000s (Yu, 2017). The historical research on Siheyuan is limited due to the lack of literary records. Surviving historical literature on Chinese city planning such as *Kao Gong Ji* and studies of them could be utilized to study Hutong neighbourhoods. In our project, data are derived from the Qianlong Capital Map, which recorded the urban plan of Beijing around 1750 at a scale of 1:650, including all the Hutong neighbourhoods and Siheyuans they contained. It is believed that this map offers an accurate representation of each Siheyuan plot in the top view and each Siheyuan building in the elevation view. By observing the map, it is noted that there were a large number of Hutong neighbourhood variants, which resulted in the diverse urban structure of ancient Beijing within its orthogonal grid system. We chose 27 east-west oriented Hutong neighbourhoods to study. Wang et al. (2020) revealed the parametric generative rules of Siheyuan building design from the ancient manual *Gongcheng Zuofa Zeli* (*Structural Regulations*, Qing Department of Qing Dynasty, 1733). Referring to these rules and Liu's (2019) method, we transformed the plans and elevations into digital three-dimensional models in Rhino 3D. The digital models provide a visualization of Hutong neighbourhoods for observation.

Stage 2 Development of a model encoded into a shape grammar.

The formalization of the grammar follows the three steps outlined by Li (2001): 1) Corpus analysis. The digital three-dimensional models supported a thorough analysis of the Hutong neighbourhoods. The models include information on the dimensions and locations of Siheyuan plots, courtyards, rooms, and walls. In this grammar, the corpus includes the shapes of the above four Siheyuan elements. There is a top-down containment relationship between these elements: Hutong neighbourhood > Siheyuan > courtyard > room and wall. A Hutong neighbourhood plan was composed of Siheyuan plots. Siheyuan plots could be classified considering their different iterations in history, which are used to construct different types of Siheyuan. The Siheyuan type defines the courtyard number and type of each courtyard the Siheyuan contains, and the courtyard type constraints the number and the type of rooms the courtyard contains. This analysis focused on the shapes, dimensions, and locations of plots, courtyards, and rooms, as they are the key features of identifying a Hutong neighbourhood. The location of a plot influences its iteration mode, and the location of the courtyard determines its type. The location analysis is based on room locations in a courtyard plus the gate location. In addition, the spatial relations between plots, courtyards, and rooms are considered, since they are essential factors that caused the neighbourhood variants. 2) Inference of rules. After the identification of the corpus, the relationships between shapes could be identified. In parallel to the observation of the three-dimensional models shown in the Qianlong Capital Map, historical literature that records Beijing urban planning (for example, Song, 1369, and E, 1739) and other studies relevant to the morphology of Hutong neighbourhoods, such as the studies by Li (2009), Liu (2019), Ni (2009), and Deng and Mao (2003), were employed as complementary sources to infer the rules. 3) Verification. Once the rules are created, we used them to test whether we can recreate Hutong neighbourhoods recorded on the Qianglong Capital Map.

Stage 3 Development of an algorithm by Grasshopper scripting to implement the shape grammar.

We will algorithmically implement the grammar in Rhino/Grasshopper based on a procedural modelling process. Then an interactive tool will be produced in the form of a Grasshopper script, enabling parametric modelling of Hutong neighbourhoods. Finally, we will verify the tool by inputting the parameters to generate new instances and compare them with the historical examples to examine whether the new instances replicate the original morphological characteristics.

3. Creation of the Hutong neighbourhood grammar

The plots, courtyards, and rooms include many variants. To be able to generate the variants, in the generation process the application of some rules could be altered by each other. The Hutong Neighbourhood Grammar is a parametric shape grammar. Focusing on the plan view, this grammar is defined in the Cartesian product of algebras:

$U12 \times (V02 \times V12 \times V22)$.

Shapes in algebras $U12$ include lines to represent the boundaries of Siheyuan plots and alleys. Labels are alphanumeric characters attached to a shape to represent additional information about the shape, which are employed to identify the available rules to be applied to the shape (Stiny, 1980). Shapes in algebras $V02$ include labelled points to represent the central point of a room plan and the midpoint of an edge of a room plan or a courtyard plan; shapes in algebras $V12$ include labelled line segments to represent the edge of Siheyuan plot, courtyard, and room; and shapes in algebras $V22$ include labelled plane segments to represent the plan of Siheyuan plot, courtyard, and room. In the rules, variables are introduced to control the generation of variants. For clarity, labels are used in the form of points, lines, or planes to mark the geometries in assistance in the derivation process, which are deleted once the generation is finished.

The grammar describes the iteration of plots from Yuan to Qing dynasties and the generation of Siheyuan on the Qianlong Capital Map in a top-down fashion by dividing and iterating shapes representing the plots and courtyards and adding shapes representing rooms and gates. Corresponding to the historical planning of Hutong neighbourhoods, the generation process followed the steps: 1) generating the Hutong neighbourhood and dividing the neighbourhood into Siheyuan plots, 2) iterating Siheyuan plots, 3) dividing plots into courtyards and defining their types, 4) iterating courtyard, 5) defining room layout pattern and locating rooms and walls. Fig. 3 demonstrates the workflow of the grammar, the rules that could be applied in each step, and the shapes for this grammar. It is noted there are branches in some steps, which case variants of Siheyuan. In the grammar, the rectangles represent neighbourhoods, plots, sub-plots, courtyards, and rooms. The letters in the rectangles are employed to distinguish types of sub-plots and courtyards, and colours of rectangles are used to distinguish types of rooms. The black solid line segments represent the Siheyuan walls and the black dash line segments represent the courtyard walls.

3.1. Generation of Hutong neighbourhoods plan and Siheyuan plots

As recorded in ancient literature such as *Kao Gong Ji*, an ideal Chinese city should be planned in a square shape to conform to the concept of "round sky and square earth (tian yuan di fang)". When rebuilt by the Mongols in 1264, the scheme of Beijing was planned in a rectangular shape, which was close to the set forth by the Chinese philosophers. On this rectangular city plan, the urban neighbourhoods were also rectangular, enclosed by the south-north and east-west oriented streets, which caused an orthogonal grid system. This urban system passed through Yuan, Ming, and Qing dynasties and survived into the last century (Steinhardt, 1990). Zhao (1972) pointed out that, when Beijing was rebuilt in the Yuan dynasty, each neighbourhood's depth (length in south—north orientation) was planned to be 67.76 m and its width (length in east-west orientation) was 677.6 m. The Hutong alley between two adjacent neighbourhoods was 9.24 m wide. These key urban elements and dimensions are determining the initial shapes of the Hutong grammar. Each neighbourhood was averagely divided into ten plots aligned in an east—west orientation. We defined these neighbourhoods as "Type A Neighbourhood" (TAN). It is noted that some TANs were replanned in the Qing dynasty. Specifically, in the south-north orientation, two adjacent neighbourhoods were combined as a whole and were divided into three new neighbourhoods, with an alley between each of the two adjacent new neighbourhoods. We defined these new neighbourhoods as "Type B Neighborhoods" (TBN). The neighbourhoods planned in the Yuan dynasty (TAN) were divided into 67.67 m $\times$ 67.67 m Siheyuan plots. Although the development of neighbourhoods in the Qing dynasty changed the depth of both Siheyuan plots and neighbourhoods, the width remained.

Two 67.76 $\times$ 677.6 m^2 neighbourhood plans, represented by two rectangles are placed on a two-dimensional (XY) coordinate system, which is defined as the initial shape. On the XY coordinate system, the X-axis is defined as the east-

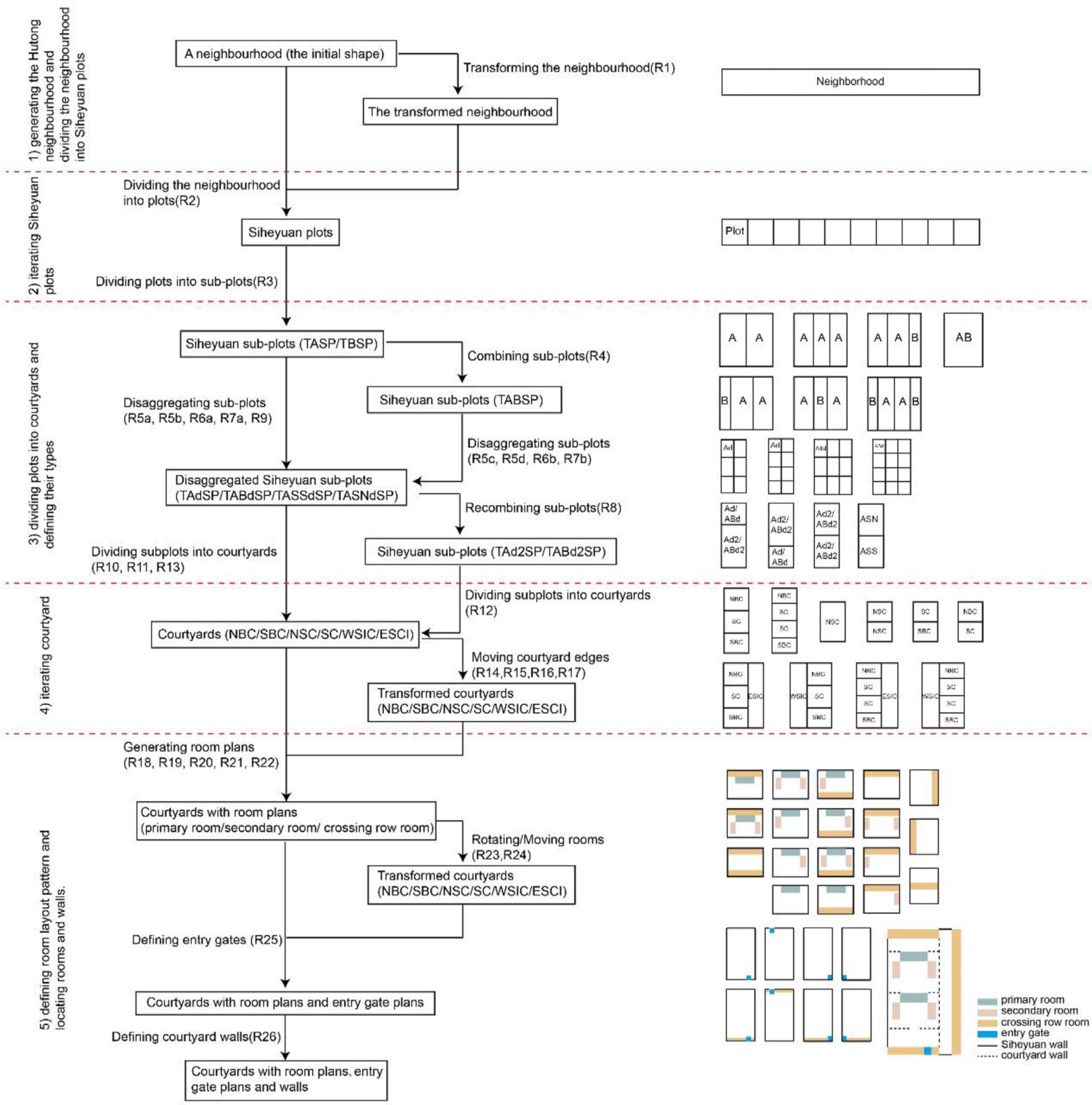

Fig. 3 The workflow and the shapes of the grammar.

west orientation and the *Y*-axis is defined as the north-south orientation, and the origin (0, 0) is on the south-west vertex of the south neighbourhood plan. Rule R1 describes the transformation of neighbourhoods from two TANs with one alley to three TBNs with two alleys. Variables associated with the rules are the neighbourhood depth (Nd) and the alley width (Aw), whose values are smaller than the original neighbourhood depth and alley width. Rules R2 describe the division of a TAN or a TBN neighbourhood into Siheyuan plots, represented by squares. Rules R1—R2 are shown in Fig. 4.

3.2. Siheyuan plots iteration

It is noted the plots experienced iterations in various modes in Yuan, Ming, and Qing dynasties, which result in Siheyuan variants shown on the Qianlong Capital Map.

3.2.1. Division of plots into sub-plots

A Siheyuan plot in the Yuan dynasty could be divided into two to four sub-plots in the east-west orientation. Each sub-plot could include several courtyards aligned in south-north orientation (Type A sub-plot, TASP) or empty space (Type B sub-plot, TBSP). We categorized six dividing patterns of a plot, by which the plot is divided into one or two sub-plot types. We found that the width of a TASP is normally around 12—35 m, while most TBSPs are close to or narrower than TASPs but wider than 5 m. Therefore, for simplification, we define the value of the parameter TASPw, the width of a TASP, which ranges from 12 to 35 m, and the value of the parameter TASPw, the width of a TBSP, from 5 to 12 m.

The six division patterns of the plots are defined as Rules R3a to R3f respectively. In Fig. 5, Rule R3a divides a plot into two TASPs, and R3b divides a plot into three TASPs.

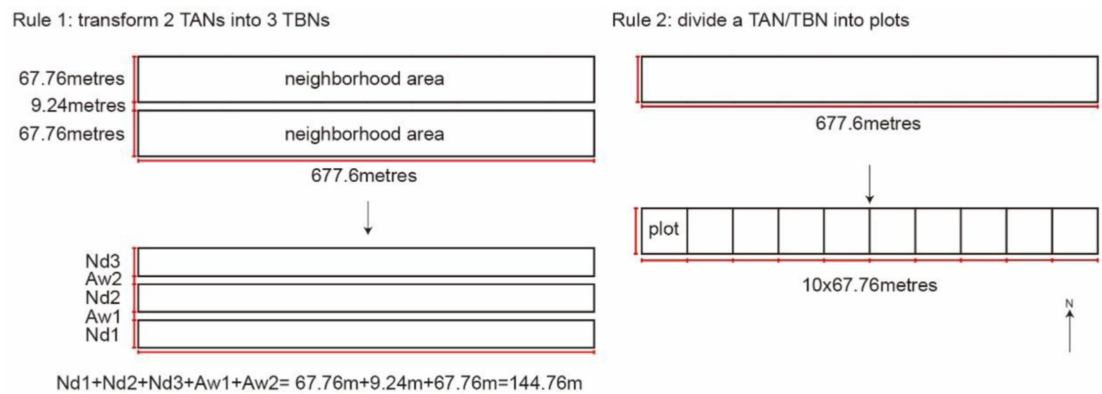

Fig. 4 Rule R1 transforms TAN to TBN and Rule R2 divides a TAN or TBN into plots.

Rule R3c and R3e describe three modes of dividing a plot into two TASPs and one TBSP. The difference between them is the location of the TBSP, which are in the east, west, and mid respectively. Rule 3f describes that a plot is divided into four sub-plots: two TASPs in the mid and a TBSP in the east and west respectively. In these rules, north-south orientated line segments are inserted into the plots to generate new rectangles on the *XY* plane to represent sub-plots. To locate these line segments, the variables, width of TASP (TASPw) and width of TBSP (TBSPw), are used for description.

3.2.2. Sub-plot combination

Some TASPs and TBSPs were combined to recreate new sub-plots. It is noted some sub-plots were additionally developed into new forms. As the historical material (E, 1739) recorded, to relieve the stress of the increase of the population of Beijing in the Qing dynasty, the government advocated using empty spaces to construct dwellings. It is noted that, since the width of TBSPs was usually too narrow to construct Siheyuan and many Siheyuans on TASPs were dilapidated or derelict due to the change of dynasty, some TASPs and TBSPs were combined to recreate new sub-plots. According to Liu's (2019) investigation of selected Siheyuan examples shown on the Qianlong Capital Map, we inferred the principles that one TASP could be combined with one adjacent TBSP to become a combined sub-plot, called Type AB sub-plot (TABSP). Then the TABSP could be used to construct a Siheyuan with a side courtyard. Since the TABSPs are wider than other types, it enables a Siheyuan to be built on the plot with two courtyards in the east-west orientation. Normally, on a TABSP, a set of courtyards are aligned in a south-north orientation, with a side courtyard next to these courtyards in parallel and connecting the south and north sub-plot boundaries.

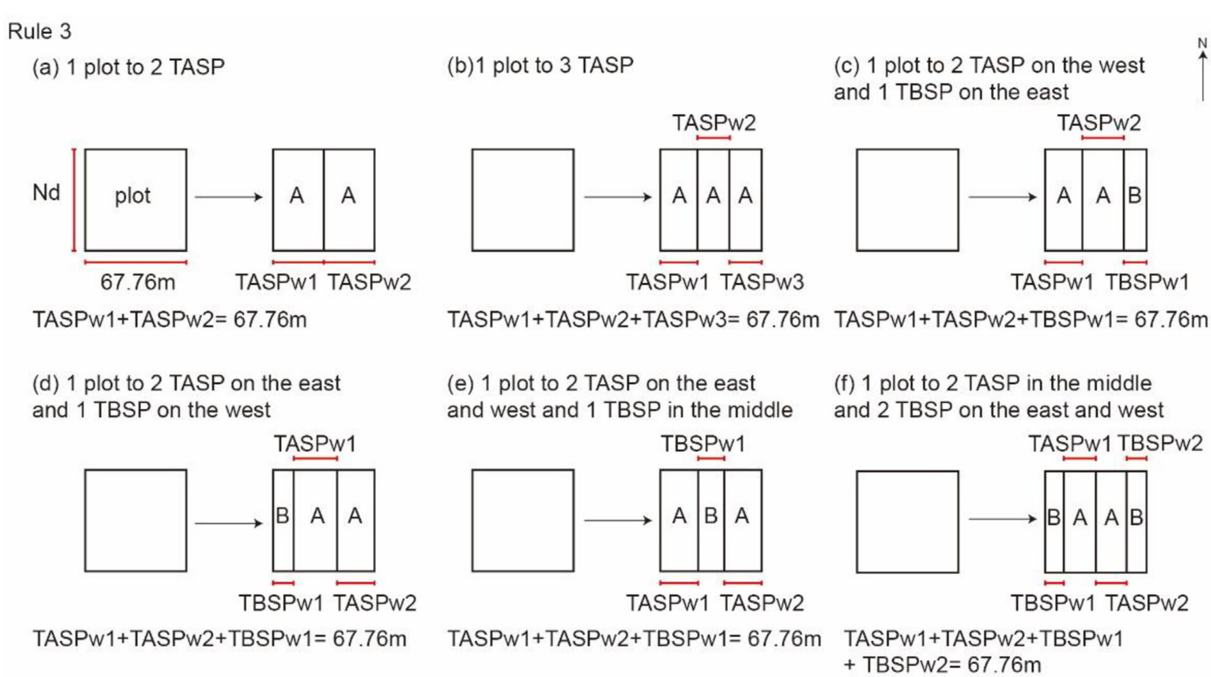

Fig. 5 Rule R3 divides a plot into sub-plots.

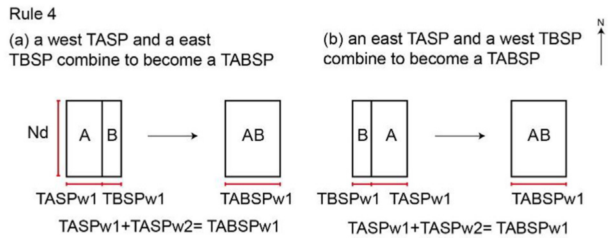

Fig. 6 Rule R4 combines sub-plots to become a new sub-plot.

Rule R4a and R4b describe how a TASP and a TBSP combine to become a TABSP, as shown in Fig. 6. The difference between them is the locations of the TASP and the TBSP. On the right side of the rule, the line segment between the two sub-plots is removed and two rectangles are merged to become a new rectangle, representing the TABSP. The summation of the variables TASPw and TBSPw is the width of TABSP (TABSPw).

3.2.3. Sub-plot disaggregation and recombination

The Siheyuan housings with a side courtyard were rare and usually belonged to upper-class owners in ancient times. In many cases, the TABSP disaggregates into many smaller sub-plots, called a Type ABd sub-plot (TABdSP), on which a one-courtyard Siheyuan is constructed. The disaggregation may also happen to some TASPs, whose disaggregated sub-plot is called a Type Ad sub-plot (TAdSP). Two TABdSPs or two TAdSPs adjacent in north-south orientation may be recombined to become a new sub-plot, called Type Ad2 sub-plot (TAd2SP) and Type ABd2 sub-plot (TABd2SP) respectively. In the disaggregation, the number of generated TAdSPs or TABdSPs in a north-south orientation, constrained by the depth of the neighbourhood, is 3 or 4 in most cases. For simplification, we assume it is 4 in TANs and 3 in TBNs. The number in the east-west orientation, constrained by the width of the sub-plot, is 2 or 3 mostly. We define it as 3 if the width is more than 20 m, otherwise 2. These Siheyuans built on disaggregated sub-plots are clustered, in which some Siheyuans are not on street.

To create access to these Siheyuans, the west and east edges of disaggregated sub-plots that are adjacent in the east-west orientation, shrink to give space to generate a small south-north oriented alley. The alley might, or might not, cross the neighbourhood to connect the alleys on the south and north side of the neighbourhood. To simplify, we assume that an alley crosses the neighbourhood in this grammar. For 2×3 and 2×4 disaggregated sub-plots, an alley is generated by shrinking the sub-plot edges. 3×3 and 3×4 cases could be considered subplots in three rows aligned in parallel in the north-south orientation. For the pattern of three rows, it is unusual to generate an alley between every two adjacent rows. Alternatively, one small alley is generated between two rows in the way the same as the rules of 2×3 or 2×4 disaggregated sub-plots. For the third row of sub-plots that are not adjacent to the alley, if there are four sub-plots in the rest row, they will recombine to become two TAd2SPs/TABd2SPs, and if there are three sub-plots, two adjacent sub-plots will recombine to become a TAd2SPs/TABd2SP, both of which ensure each sub-plot to have access to the urban fabric. In the shrinkage, the movement distance of each sub-plot edge is slightly different, which makes the alley geometrically irregular. Except for the above disaggregation, there was another mode that a TASP separates into two sub-plots. Historically, a TASP was usually used to construct a three-courtyard Siheyuan or a four-courtyard Siheyuan, whose courtyards were aligned in a row in the north-south orientation. In this mode, the TASP separates by dividing the boundary of the second and the third courtyard, in which the original type of each courtyard remains. We call the separated sub-plot on the south side as the Type ASS sub-plot (TASSSP) and the one on the north as the Type ASN sub-plot (TASNSP).

Rule R5 defines the disaggregation of a TASP and a TABSP. Considering the variants caused by neighbourhood depth and width, the rule includes variants Rule R5a-d. Line segments are introduced to indicate the edges of new sub-plots after the disaggregation of a TASP or a TABSP. In Rule R5a, two east-west orientated line segments and one north-south line segment are inserted to divide the TASP into 2×3 TAdSPs. In Rule R5b, there is one more east-west orientated line segment, enabling the generation of 2×4 TAdSPs. In Rule R5c, two east-west segments and two north-south segments are inserted to divide the plot into 3×3 TABdSPs. And in Rule R5d, one more east-west orientated segment than in R5c is introduced to divide the plot into 3×4 TABdSPs. The location of each line segment is important since they determine the size of each generated subplot. To describe them, including the inserted north-south orientated line segments and the line segments representing the east and west edges of the plot, the distances between each two of them are defined as w1, w2, and w3 from west to east. And the distances between each two east-west orientated line segments, including the ones representing the north and south edges of the plot and the inserted ones, are defined as d1, d2, d3, and d4 from south to north. These distances are variables to control the disaggregation. Rule R6 describes the generation of a small alley between two north-south orientated rows, which includes two variations.

In Rule R6a, six TAdSPs or TABdSPs are clustered in two north-south orientated rows (2×3 mode) on the left of the rule. The shared edges of each two adjacent sub-plots in the west and east are labelled as thick line segments. On the right of the rule, for each segment, two copies of it are generated by offsetting itself in the east and west in the mirror using the segment as an axis, which are the boundaries between sub-plots and the small alley edge fragment. The distances describing the width of the small alley fragments (SAw1, SAw2, and SAw3) are variables in this rule. In Rule R6b, there are eight TAdSPs or TABdSPs clustered in two north-south orientated rows (2 × 4 mode). The generation of each small alley fragment is the same. The small alley fragments may move in the east-west orientation. Rule R7 is introduced to simulate the movement. In Rule R7, each pair of line segments moves in the east-west orientation using its mirror axis as an indicator. The displacement is a variable, (SAm1, SAm2, SAm3, and SAm4) in the east-west orientation, and the movement toward the east is positive (+x) and toward the west is negative (-x). The same as Rule R6, Rule R7 has two variations corresponding to the two clustered sub-plot modes.

Rule R8 introduces the recombination of two TABdSPs or two TAdSPs, which normally happen to sub-plots that have no access to the urban fabric. In this rule, the shared edge of the two sub-plots adjacent in north-south orientation is removed and the two sub-plots are merged. Considering the disaggregation patterns and sub-plots locations, it includes three variations. Rule R8a-R8c.

Rule R9 describes the division of a TASP into a TASNSP and a TASSSP. In the rule, an east-west line segment is inserted into the rectangle, whose variable is the depth of the TASSSP. Rules R5-R9 are shown in Fig. 7.

3.3. Courtyard division and type definition

After the sub-plot type is defined, it is divided into courtyards depending on the Siheyuan type to be built on the sub-plot. When describing the word 'courtyard' in the Siheyuan context, it usually means the space enclosed by walls, which includes both the open courtyard space and all the buildings surrounding that space. In many cases, some parts of the "courtyard" are not completely enclosed by walls. Instead, the rear boundary of a building is extended in the east-west orientation to define the boundary of a courtyard. Borrowing the approach of categorizing courtyards into types based on the courtyard location and the room type that a courtyard contains (Wang et al., 2020), we classified courtyard types as northern boundary courtyard (NBC), southern boundary courtyard (SBC), east side courtyard (ESIC), west side courtyard (WSIC), standard courtyard (SC), and non-standard courtyard (NSC). A TASP is used to build a Siheyuan with three or four courtyards aligned in a north-south orientation, in which the southernmost and northernmost ones are non-standard courtyards while the middle one or two are standard courtyards. The only difference between a TASP and a TABSP is that a side courtyard is additionally aligned to the east or west of a TABSP.

The generation of a three-courtyard Siheyuan on a TASP is defined as Rule R10a and a four-courtyard one as Rule

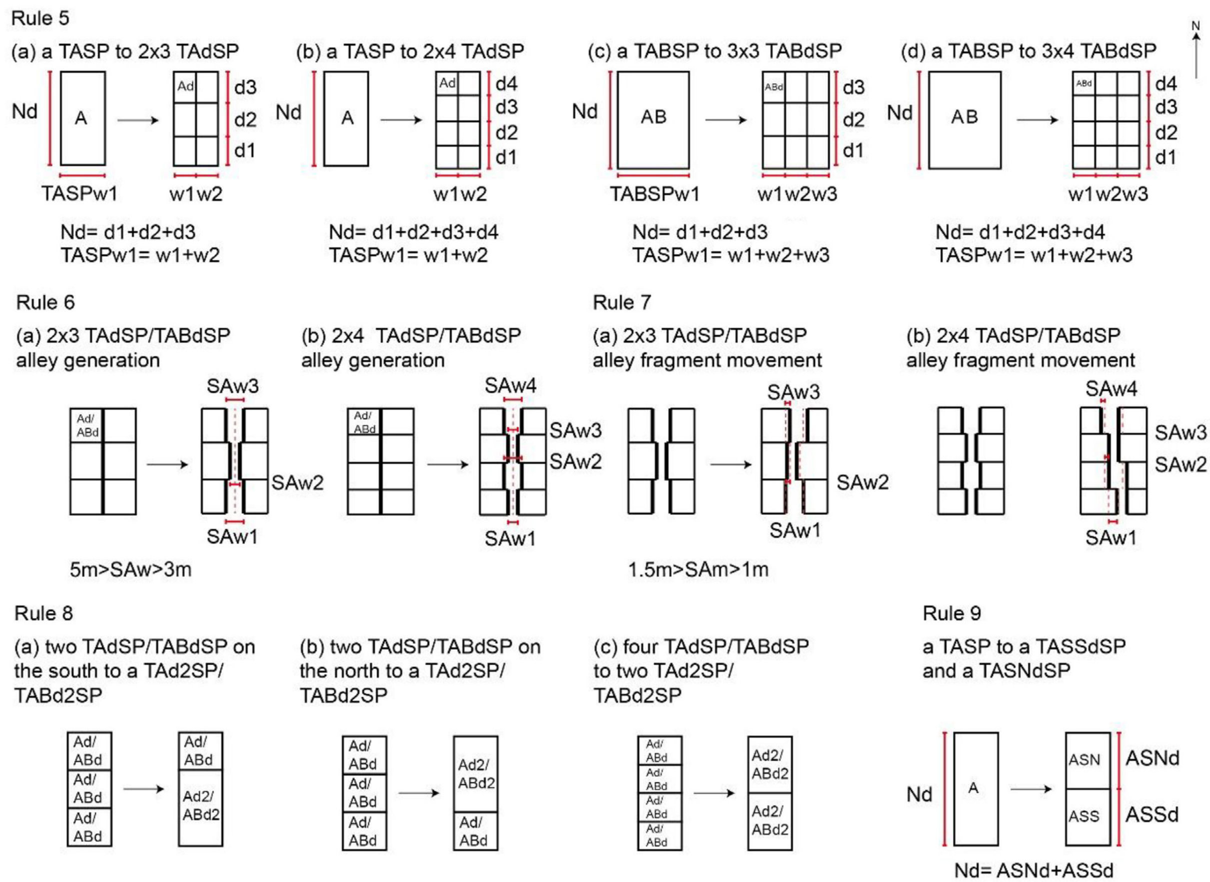

Fig. 7 Rules R5-R9 define the disaggregation and recombination of sub-plots.

R10b. The generation of three courtyards with an east side courtyard on a TABSP is defined as Rule R11a, of three courtyards with a west side courtyard as Rule R11b, of four courtyards with an east side courtyard as Rule R11c, and of four courtyards with a west side courtyard as Rule R11d. A Siheyuan built on a TAdSP or TABdSP containing one courtyard is defined as Rule R12a, in which no shape iteration happens on the plan view, while on a TAd2SP or a TABd2SP two non-standard courtyards connected in a north-south orientation, is defined as Rule R12b. Siheyuan built on a TASSSP always divides the site into two courtyards: a standard one on the north and a non-standard one on the south. This iteration is defined as Rule R13a. On the contrary, on a TASNSP the division may allocate a non-standard courtyard on the north and a standard one on the south, which is defined as Rule R13b. The type of sub-plot determines the courtyard division by defining courtyard types and numbers, which consequently determines the type of the Siheyuan. In Rule R10, R12, and R13, east-west orientated line segments are inserted to divide the sub-plot into courtyards, and each courtyard depth (Cd1, Cd2, Cd3, and Cd4), defined by the location of these line segments, is variable in the rules. In Rule R11, the north-south orientated line segment defining the side courtyard is inserted to break the rectangle. After the side courtyard width (SCw), a variable, is decided, the rest east-west orientated line segments are inserted to define the remaining courtyards. Rules R11, Rule R12, Rule R13, and Rule R14 are illustrated in Fig. 8.

3.4. Movement of courtyard edges

In the neighbourhoods planned in the Yuan dynasty, the south and north edges of each Siheyuan plot were ideally divided into line segments from the corresponding edges of the neighbourhoods, which are straight line segments, enabling the elevations of Siheyuans to be arrayed smoothly. After the reconstruction of Siheyuans in the Qing dynasty, the edges of many Siheyuans have been moved in the south-north orientation, which caused the extension or shrinkage of courtyards adjacent to the urban fabric in the south-north orientation. Rules R14 and R15 define the shrinkage and extension respectively of a courtyard adjacent to a Hutong alley to its south and north, in which the south or north edge of the courtyard moves in south-north orientation and the east and west edges correspondingly extend or shrink. In the rules, the line segments represent the edges of the courtyard and the boundaries of the Hutong alley. And the red dash line segments represent the courtyard edges before the movement. The variable is the moving distance of the courtyard south/north edge (Emd), whose value is positive when moving to the north and negative when to the south.

The movement of edges also took place to the east and west edges of non-standard courtyards, boundary

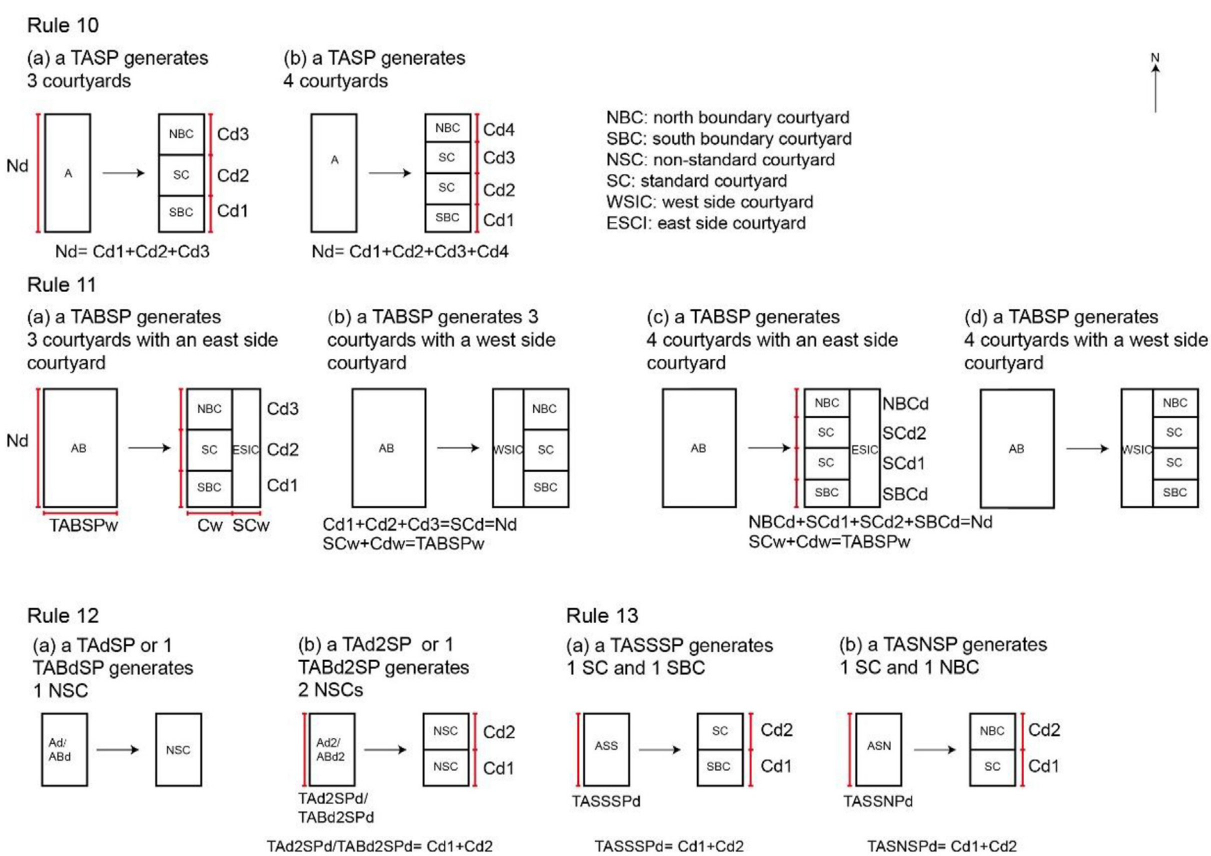

Fig. 8 Rules R10-R13 define the division of sub-plots into Siheyuan courtyards.

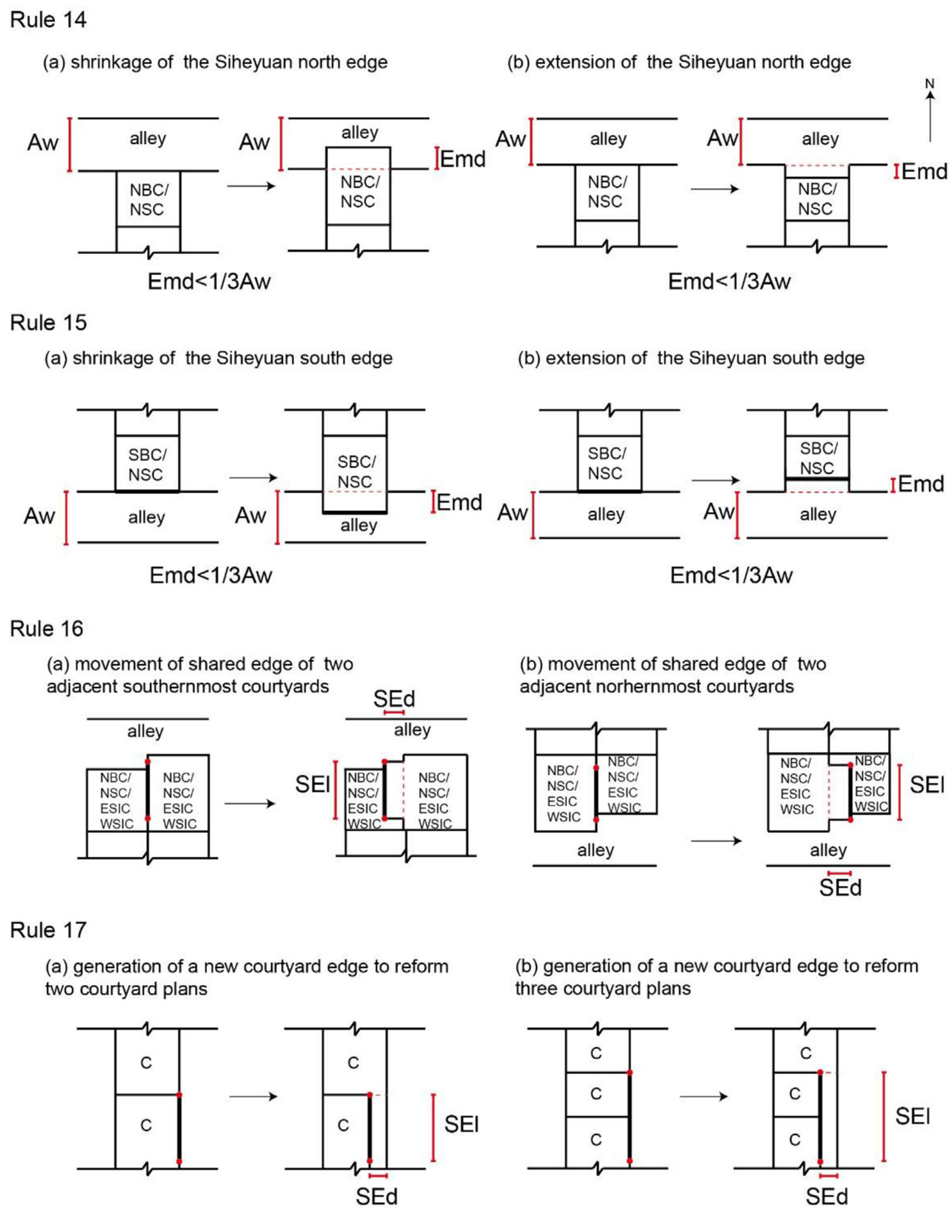

Fig. 9 Rules R14-17 define the movement or generation of courtyard edges.

courtyards, and side courtyards. These edges could move in east or west directions to extend or shrink the courtyard area. Specifically, an east edge of the courtyard moves in the east direction or a west edge moves in the west direction to extend the courtyard area, and the corresponding adjacent Siheyuan's edge moves to shrink its area. This only took place in courtyards located adjacent to an alley. The movement is defined as Rule R16. On the left of Rule R16, the edges of courtyards, represented by line segments, are marked by two dots. The thick line segment generated by connecting the two dots represents the section of the courtyard edge to be moved. The locations of the two dots is freely defined on the initial courtyard edge. On the right of the rule, the thick line segment moves in the east or west direction and the track of the movements of the two dots are generated as two-line segments, representing new courtyard edges. A variable is the distance between the two dots (SEl). Another variable is the moving distance of the thick line segment (SEd), whose value is recorded as positive when the movement is in the east, and negative when in the west.

In addition, new courtyard edges may be generated in a Siheyuan to reform the courtyard plans. This happened to the courtyards whose south-north edges are the Siheyuan boundaries. A part of the courtyard edge is moved in the east—west orientation in the direction that could shrink the courtyard to generate a new edge. This could also happen to two adjacent courtyards in south-north orientation, whose edges are in line with each other. Rule R17 describes the generation of the new courtyard edges. Similar to Rule R16, on the left of the rule, the courtyard(s) edges, represented by line segments, are marked by two dots, and the thick line segment generated by connecting the two dots represents the section of the courtyard edge to be moved.

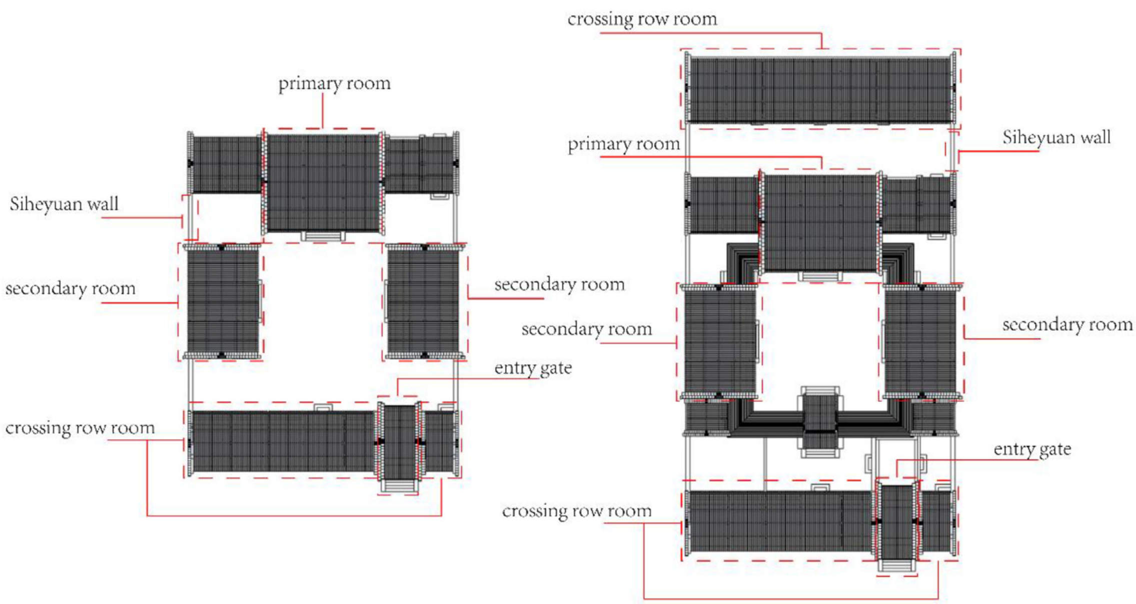

Fig. 10 A one-courtyard Siheyuan and a three-courtyard Siheyuan (left) and a three-courtyard Siheyuan in plan view (right).

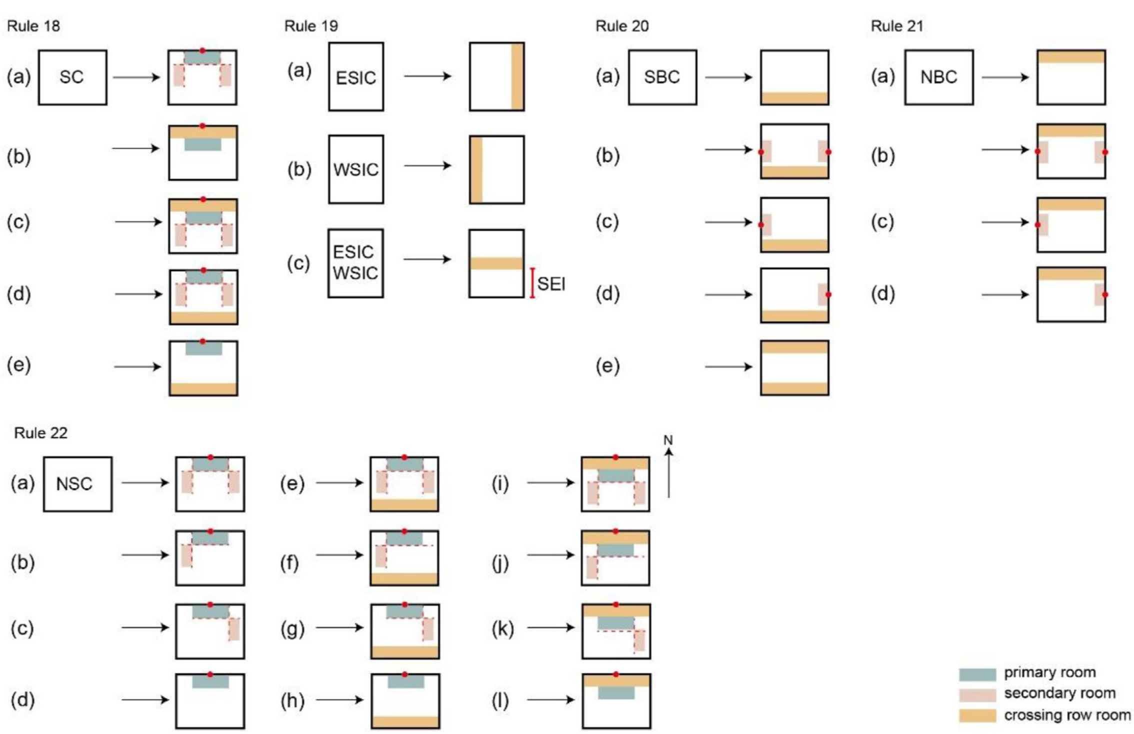

Fig. 11 Rules R18-R22 define the generation of rooms.

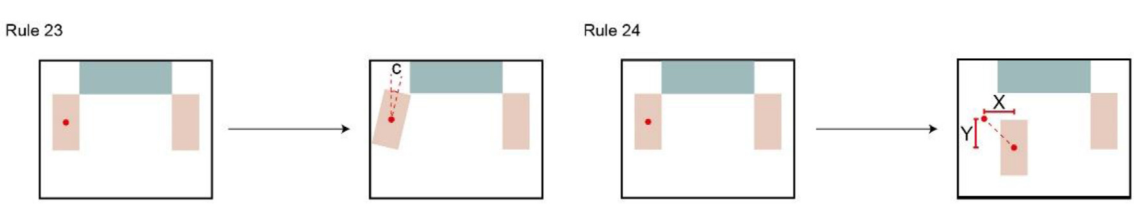

Fig. 12 Rule R23 defines the rotation of a room and Rule R24 defines its movement.

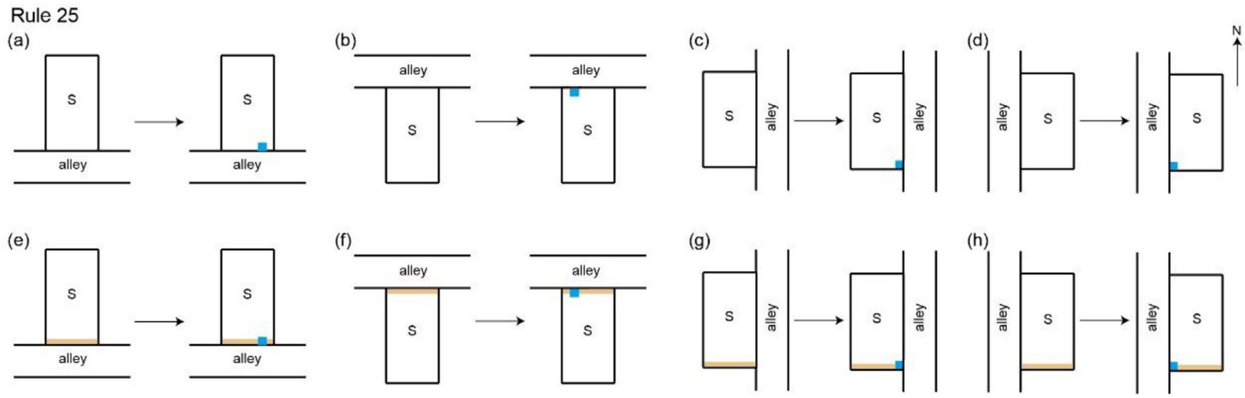

Fig. 13 Rule R25a-h defines the generation of an entry gate.

On the right of the rule, the thick line segment moves to become the new courtyard edge. Then, the Siheyuan plan changes as one or two courtyards shrink and the rest extend. Also similarly, the variables are the distance between the two dots (SEl) and the moving distance of the thick line segment (SEd). Rules R14, Rule R15, Rule R16, and Rule R17 are demonstrated in Fig. 9.

3.5. Generation of room plans

Rooms are the most essential elements in Siheyuan. Courtyards of different types have different room layout patterns and contain rooms of different types. Fig. 10 illustrates a one-courtyard Siheyuan in plan view and a three-courtyard Siheyuan in perspective view, which explains the layout of rooms and walls.

To generate room plans in courtyards, there are two essential aspects: room location and room dimensions. A room plan is a rectangle, whose parameters to define them are room depth and width. Wang et al. (2020) have revealed the parametric dependence between them. For simplification, the definition of room dimensions is ignored in this grammar due to their less importance to shape the Siheyuan plan principles. The room location is initially defined by the room layout pattern of a courtyard, which is constrained by the courtyard type, as categorized above. Each courtyard type includes one or more room layout patterns. Specifically, a boundary courtyard normally contains an east-west crossing-row room, which is located on the side of the courtyard adjacent to the urban fabric. A boundary courtyard may also contain one or two secondary rooms. A side courtyard contains a south-north crossing-row room. The crossing-row room is located on the side of the courtyard which is the boundary of the Siheyuan plan. A standard courtyard normally contains a primary room on the north and two secondary rooms pairwise symmetric about the courtyard's central axis. This layout contains more variants regarding the criteria of the existence of wing rooms and secondary wing rooms, which have not been considered in this research due to their low importance and rareness in Siheyuans. For non-standard courtyards, the determination of the room layout was improvised by craftsmen. We concluded eight variants by observing examples on the Qianlong Capital Map.

We locate the midpoint of the north edge of the primary room to the midpoint of the north edge of the courtyard. We locate the north edge of a secondary room in line with the south edge of the primary room in a courtyard. And if the secondary room is on the east side of the primary room, the west edge of the secondary room is in line with the east edge of the primary room. Similarly, if the secondary room is on the west side of the primary room, the east edge of the secondary room is in line with the west edge of the primary room. If there is no primary room in a courtyard, the secondary room is located on the courtyard east edge with the midpoint of the room east edge overlapping with the courtyard edge midpoint or, similarly, located on the courtyard west edge with the midpoint of the room west

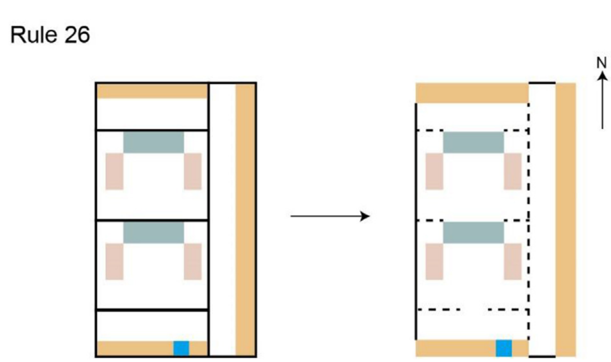

Fig. 14 Rule R26 defines the courtyard walls.

Fig. 15 The procedural modelling process of two Hutong neighbourhood examples.

edge overlapping with the courtyard edge midpoint. For a crossing-row room, we locate one of its edges to one of the courtyard edges to make them overlap. The choice of crossing-row room edges and courtyard edges depends on the courtyard types and their location as mentioned above.

The ancient construction manual *Gongcheng Zuofa Zeli* defined a parametric system that generates variants of single buildings, many of which are the rooms in a Siheyuan. Wang et al. (2020) created a parametric model algorithm based on that work that can generate the massing and the structural frame geometry of Siheyuan. Here, to generate the room geometry, that algorithm is used to calculate the height, depth, and width of a room and other parameters such as the number of bays in front. The generation of room on the plan view is associated with the courtyard layout patterns, which are defined as Rules R18- R22, as shown in

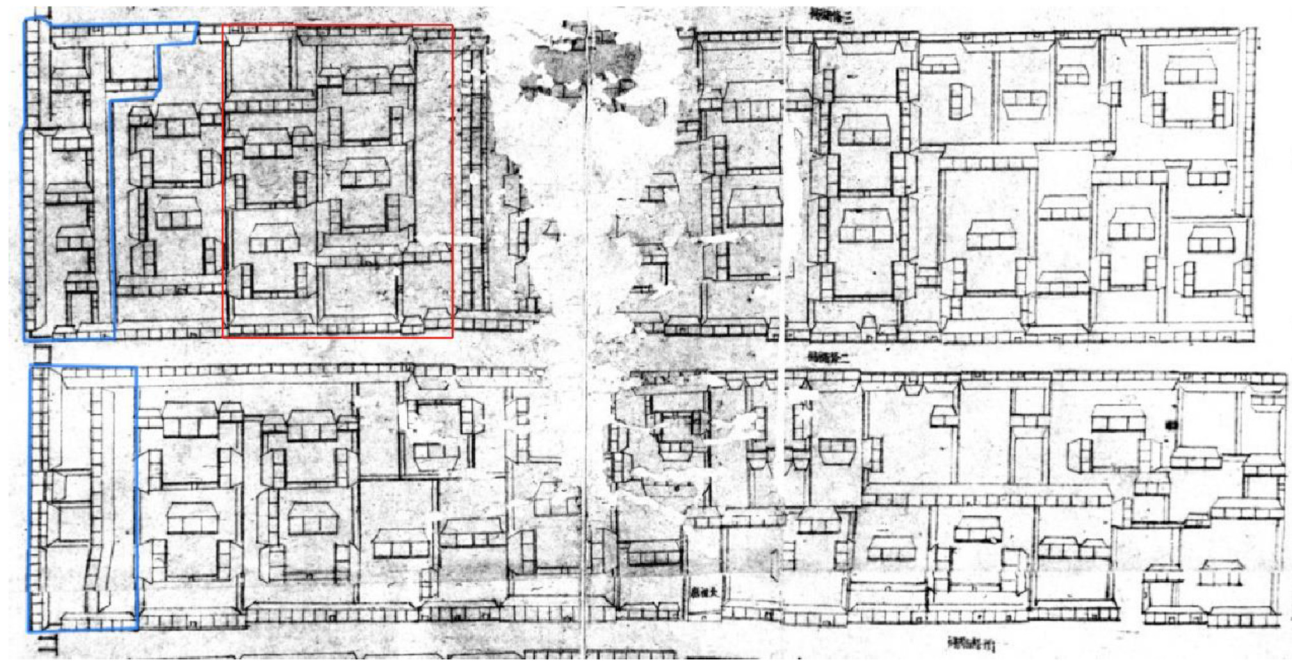

Fig. 16 Two Hutong neighbourhoods on the Qianlong Capital Map (Chen and Wang, 2005, p. 61).

Fig. 11. In these rules, the referred edges and points are labelled as line segments and dots respectively, and the room plans are labelled as plane segments. Variables are the five parameters determining the geometry of a room. The rooms could be slightly rotated and moved within the courtyard. The rotation is defined as Rule R23 and the movement is defined as Rule R24, as shown in Fig. 12. The rotation and the movement are described as a line vector by a coordinate point (x, y) and a rotation vector by a clockwise intersection angle (a) on the XY coordinate respectively, whose values are variables of the rules.

The location of the Siheyuan entry gate is determined by the urban fabric. For most Siheyuans, it is located southeast of the site, which is also the edge of the first courtyard in most cases. If the south edge of the site is not connected to urban space such as an alley or a street, the entry gate is located elsewhere. There is a sequence to rank the priority of four patterns of entry gate location regarding the urban fabric context: east of south edge > south of east edge > south of west edge > west of north edge. A secondary entry gate is normally located in the last courtyard. To simplify, we ignore the secondary gate since it is infrequent. The form of an entry gate is similar to rooms. However, it doesn't have an enclosed partition for defining the interior, but a single partition defining the outside and inside of a Siheyuan, which is a wall with a door on it. The central point of the entry gate plan is labelled to refer to its location. It is noted that, in some cases, the entry gate plan may be occupied by an east-west oriented crossing-row room. In these cases, the crossing-row room is cut off to give space for the entry gate. Considering the entry gate location patterns and the existence of a crossing-row room, Rule R25a-h defines the generation of an entry gate in plan view, as shown in Fig. 13. The algorithm to generate an entry gate is the same as other rooms, but one parameter, the number of bays in front is a constant, whose value is one. Therefore, the variables are the rest four parameters.

Walls usually exist on the edges of each courtyard, some of which are also the edges of the Siheyuan site. However, in many cases, parts of the edges are occupied by buildings, so no wall is needed. Two wall types are defined: a) Siheyuan wall: the wall on the site edges enclosing the Siheyuan space, and b) Courtyard wall: the wall on the edges between two neighbouring courtyards. The wall is defined as line segments on the plan view without considering its thickness. Rule R26 deletes the black line segments generated in previous steps to represent the courtyard edges and marks the courtyard/Siheyuan edges to be constructed by walls by using black dash line segments and black solid line segments for labelling on the plan view, representing courtyard walls and Siheyuan walls respectively. Fig. 14 illustrates an example of Rule R26 that marks the courtyard edges to be constructed by walls.

3.6. The procedural modelling of Hutong neighbourhoods

Once the rules are captured, the grammar for its procedural modelling is constructed. The procedures to implement the rules to generate the Hutong neighbourhoods could be concluded as 14 steps and each step has its corresponding rules that could be implemented. This grammar could be used as a tool to generate new Hutong neighbourhoods. Fig. 15 demonstrates the generating process of two pseudo examples.

4. Verification of the grammar

To verify the grammar, we used it to regenerate many Hutong neighbourhoods recorded on the Qianlong Capital

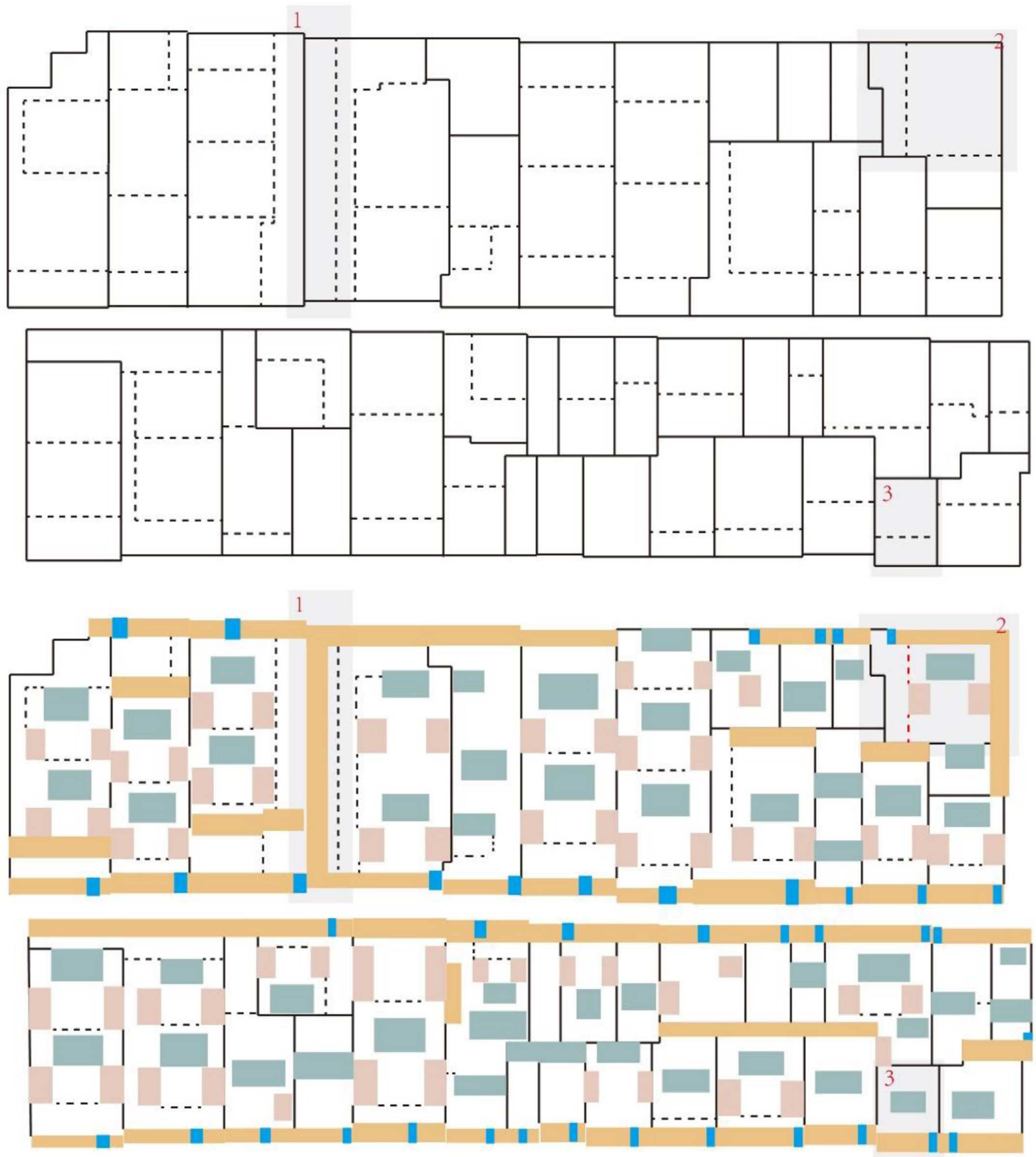

Fig. 17 The counterpart of the two Hutong neighbourhoods on the Qianlong Capital Map generated by using this grammar. The upper diagram demonstrates the generated Siheyuan edges and courtyard edges (Siheyuan edges in solid line segments and courtyard edges in dash lines to distinguish them) and the lower diagram shows the final generated result.

Map. For instance, the two Hutong neighbourhoods in Fig. 16. They have obvious traces indicating they followed the rules of historical urban planning since it was initially created in the Yuan dynasty (Hou, 2013). We generated it using our grammar, and the output is shown in Fig. 17. It is noted that the west two sub-plots (outlined in blue) were developed for commercial buildings in history, which is out of the scope of this research, therefore they are not involved in the rest steps after being divided into sub-plots. In addition, taking two Siheyuan cases on the map (outlined by red lines in Fig. 16) for example, their derivation is illustrated in Fig. 18, whose initial shape is a neighbourhood area. For simplification, only two plots are saved in the second step and only two subplots are saved in the fifth step, which corresponds to illustrate the derivation of the two Siheyuan cases on the map. Evidently, the algorithm can generate historically plausible neighbourhoods, as Fig. 17 shows. Although some areas on the map have been damaged, we can use the grammar to repair them by inferring the possible Siheyuan layouts in the damaged areas based on the rules in the grammar and the surviving information on the map. However, three discrepancies between the generated Hutong neighbourhoods and the real ones are noted in Fig. 17, as highlighted by the grey areas. Area 1 highlights a side courtyard, in which there are some east-west orientated walls to divide the side courtyard into smaller courtyards on the historical map, whose rules are not involved in our grammar. In Area 2, the south-north orientated courtyard edge marked in red is shortened and an east-west orientated edge connecting its south end

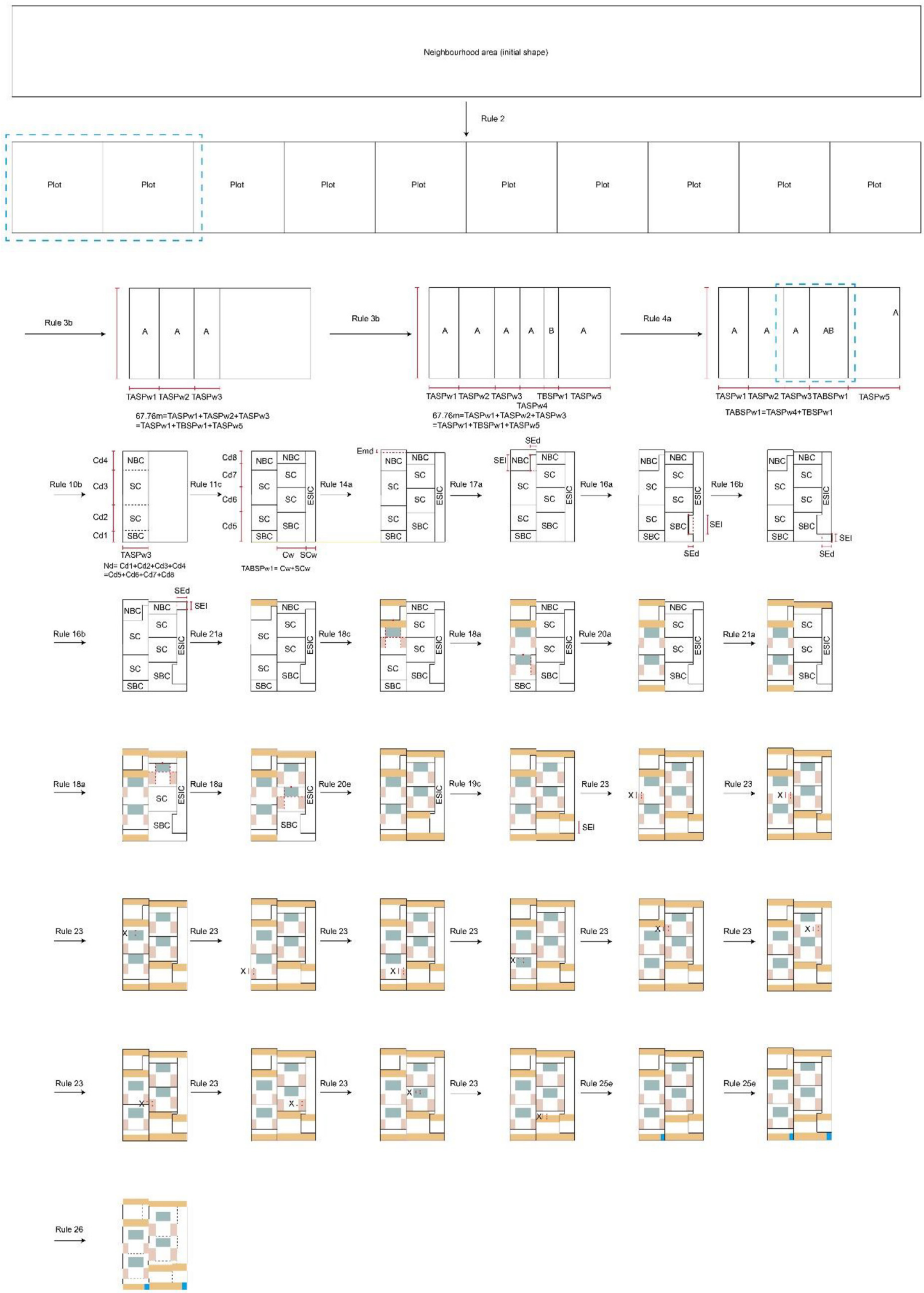

Fig. 18 The detailed process of applying the rules to generate two Siheyuan examples on the Qianlong Capital Map.

is generated to divide the northernmost courtyard into two courtyards on the map. This is opposite to our grammar since it caused the generation of five courtyards in the south-north orientation. There is a two-courtyard Siheyuan in Area 3 generated by the grammar. However, on the historical map, there is a one-courtyard that does not completely occupy the subplot but leaves a path and a square connecting to the urban fabric. Although all the three cases cannot be generated by this grammar, these situations are rare on the map.

5. Discussion and conclusion

Computational thinking in architecture is as old as architecture itself (Chiou, 1996). Re-visiting historical design from a computational generative perspective can inspire us to enhance our understanding of the knowledge underlying traditional architecture and urban planning. Formulation of a thing into an algorithm leads to a much deeper understanding than traditional ways (Knuth, 1973). However, comparing algorithms, a grammar can explicate design rules by graphic illustration, which is straightforward for architects and urban planners (Woodbury and Burrow, 2006, p. 69). This paper examined the details and showed the gradual process of establishing the rules necessary for the creation of shape grammar, based on the thorough analysis of the original historical map of Beijing neighbourhoods—the urban fabric which existed during governing of the two Chinese dynasties—the Yuan and the Qing, and supported by referent literature. The findings are highlighted as a grammar that revealed the mathematical considerations embedded in the historical generation of Hutong neighbourhoods. In this grammar, urban context, iteration pattern of the plot, and courtyard and room prototypes are the contextual forces shaping the Hutong neighbourhood forms. By encoding the implicit rules caused by these forces into this grammar, it explicated how the development of Hutong neighbourhoods evolved morphological variants.

According to this grammar, it is noted that the iteration of Hutong neighbourhoods is a spontaneous process in history, rather than the computational consideration initially embedded by the human in the design principles of traditional Chinese architecture (Chiou, 1996; Li, 2001, 2016; Wang, 2020), such as the rules in Fengshui and *Gongcheng Zuofa Zeli*.

Although the generation of Hutong neighbourhoods was uniformly constrained by the rules of urban morphology of traditional Chinese cities, it is noted that, in the developing process, diverse variations of objectives emerged to flexibly conform to the rules in various contexts.

The discrepancies indicate that this grammar has the potential to be improved, despite that the discrepancies are infrequent cases. We are alert that there might be other tacit rules that we are not aware of. However, these pathological cases could be considered as improvised products by craftsmen under multiple design constraints to approximate an ideal form (Wang et al., 2020). Serendipitously, it is noted this grammar could be used to repair the historical map.

Although this paper focused on the plans, we can easily employ Wang et al.'s (2020) tool to generate the three-dimensional models using the information derived from the plan generated by this grammar. As an application, the posted Hutong neighbourhood grammar could be employed as a guide to design Siheyuan dwelling neighbourhoods in traditional style based on the procedures mentioned above.

Declaration of competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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