

Research Article

Dark Matter Garden: A case study in algorithmic modelling and digital fabrication of complex steel structures

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Abstract This paper is focusing on the application of algorithmic modelling techniques to represent, design and fabricate gravitational lens effects (as described by the astrophysical theory of the 'Dark Matter') in form of a garden pavilion for the Royal Horticultural Society's, Chelsea Flower Show. In addition, this research-led project is exploring the challenges occurring in the use of three-dimensional CNC bending technologies. This is a research by design project and its method is based on a design framework, which incorporates a generative algorithm linked to feedback loops related to parameters such as laws of gravity, plot dimensions, materiality, positioning of the plants, construction and fabrication, requirements and cost as well as the overall aesthetics. Its findings are highlighting accomplishments and failures of a file to factory design and fabrication process, which incorporates algorithmic modelling and digital manufacturing techniques in a collaborative environment. The 'Dark Matter Garden' installation was awarded the gold medal for 'Best Fresh Garden' by the Royal Horticultural Society in 2015.

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1. Introduction, materials, and methods

The Dark Matter Garden is a collaborative, research-led, design project developed and supported by the National Schools Observatory, the Science and Technologies Facilities Council, Liverpool John Moors University, Dori Miller,

David Binks, and the author, aiming to educate and inspire schoolchildren in science and technology. In particular, it is a garden design installation, which was exhibited at the Chelsea Flower Show hosted by the Royal Horticultural Society, aiming to represent gravitational lens effects, caused by rays of light deformed by Dark Matter. It was conceived as a three-dimensional grid of horizontally arrayed steel pipes, punctually deformed in three dimensions. In order to visualize, design and fabricate the

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garden's steel grid, we re-developed and extended the algorithm applied on the 'Deceptive Landscape' installation (Agkathidis, Kocaturk, 2014), based on multiple point attractors and point clouds. The finalized design was awarded the prestigious RHS gold medal in the category 'best fresh garden', in July 2015. After the completion of the Flower Show, the garden was relocated to the National Innovation Campus in Cheshire in early 2016, where it remains until today.

The design brief was developed by the project team and tried to accommodate the various requirements set by all stakeholders, as well as the constraints defined by the organisers, the limited budget and the site, a 10 m × 3 m strip of land, within the Royal Horticultural Society's premises in Chelsea, London. Our main aim was the visual representation of gravitational lens effects as caused by the 'Dark Matter' phenomenon (Fig. 1) by designing and constructing a garden installation.

According to CERN (2018), the behaviour of Galaxies in our universe could not be explained by the gravity generated by their rotating speed. It would not be enough to hold them together but tear them apart instead. This leads scientist to believe, that there must be something invisible, creating the additional gravity necessary to keep galaxies together. This unknown, still undetected matter is called 'dark matter', as it is invisible. The only way scientists were able to infer its existence was the gravitational effect it seems to have on the visible matter. Massive galaxy clusters like MACS 1206 allow us to study the dark matter. Due to their enormous heft, they are able to act as gravitational lenses, magnifying, distorting and bending light that passes through them (NASA, 2018).

Attractor algorithms simulate gravitational or magnetic attraction effects, causing a similar distortion when applied on point clouds. Consequently, the development of a linear system in interaction with point attractors could potentially result in a similar visual condition, whereby the linear elements would represent the rays of light and the point attractors act as dark matter. In addition to the gravitational lens effect visualization, there was the necessity to integrate horticultural elements, as the installation was part of a flower show competition. Furthermore, as the

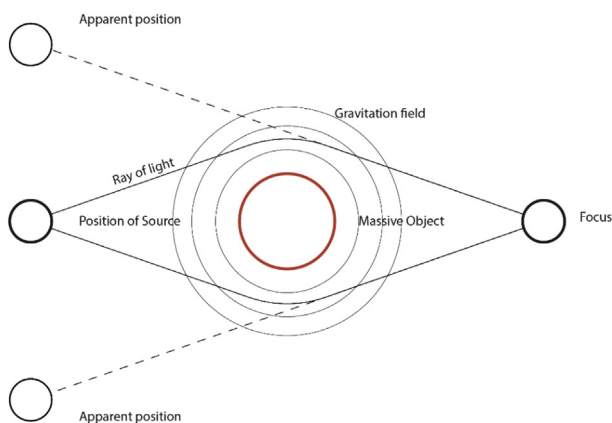


Fig. 1 Gravity from a foreground object bends light from a more distant object.

gravitational lens deformation is related to curved/deformed rays of light, it became evident from the early stages, that we would have to use linear building components able to be shaped and fabricated accordingly (steel pipes in particular). In that spectrum the research questions which were incorporated into our design brief were the following:

- How can algorithmic design tools be applied in order to enable visualization and design of a gravitational lens effect using horizontally arrayed steel pipes?
- How can such a design scheme be fabricated by the use of three-dimensional CNC bending techniques?

As this was a collaborative design-research project, the method applied in order to investigate these questions includes the set-up of a collaborative working framework which would allow all stakeholders (astrophysics, landscape architect, engineer, designer, and fabricators) to contribute in the design and fabrication process (Fig. 2). Thus, all parameters relevant to the form-finding process, such as laws of gravity, size of the plot, materiality, positioning of the plants, construction and fabrication requirements and cost as well as the overall aesthetics, were defined as constraints of the generative algorithm which would become our primary design tool.

The algorithm would then be used for the generation of design iterations, which would be evaluated by the team members, allowing a feedback loop for optimizing the parametric values until the preferred iteration could be determined. In continuation, the pipe geometries would be translated into the Length-Radius-Angle coordination system and then test-fabricated by the CNC bending machine, forming a second feedback-loop chain. Once the results were satisfactory, the digital data would be passed on to the manufacturer who would complete the production of all the metal building components. Finally, the entire fabrication and assembly processes and the final output itself served as an assessment of the techniques and methods applied. Were they effective? Did they fulfil the design brief's aims and objectives? What were the challenges experienced during this process and how could it be improved?

The chosen operative software was Rhinoceros and Grasshopper as well as the Grasshopper plug-in panelling tools. In particular, attractor algorithms were chosen as means to simulate the gravitational lens effect, as described in the 'Dark Matter' theory. Steel was the chosen

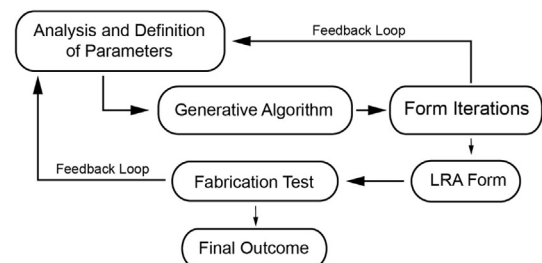


Fig. 2 Design Framework based on a generative algorithm.

construction material, as it guaranteed the best performance in relation to humidity incorporated in the various plants which would become parts of the installation as well as the weathering conditions at the site.

2. Theory and research context

2.1. Generative systems and application of attractor algorithms in architectural design

By looking back at the and late 80s early 90s, just before the boom of computational architecture, Peter Eisenman, starts applying a set of design techniques, such as scaling, fractals, overlay, and superposition, influenced by Derrida's deconstruction theory. He applies these techniques in relation to rules of order, developing several projects based on that method, such as the Biocentre in Frankfurt in 1987 and the Nunotani office building in Tokyo completed in 1992 (Eisenman, 2004). One could claim that his design method was the first contemporary generative design attempt. As software started to offer new possibilities, Eisenman introduces other techniques to his approach, such as morphing images, soon followed by UN studio and its reference to hybridization and the "Manimal" (Van Berkel B, Boss C, 1999). The technique of Folding appears in Eisenman's Rebstockbad in 1991 (Eisenman, 2004). With further advance of computational tools, Greg Lynn starts applying new tools such as animation, splines, NURBS and isomorphic poly-surfaces (Lynn, 1999) influencing a whole wave of architectural production, often described as blob architecture. As algorithms and scripting become more and more accessible and digital fabrication more affordable to architects and designers, parametric tools, penalization tools, simulation software, optimization and generative algorithms are dominating today's generative design techniques.

In their book 'Generative Gestaltung' (Bohnacker et al., 2009), the authors define generative design as a cyclical process based on a simple abstracted idea, which is applied to a rule or algorithm. It then translates into a source code, which produces serial output via a computer. The outputs return through a feedback loop, enabling the designer to re-inform the algorithm and the source code. It is an iterative operation, relying on the feedback exchange between the designer and the design system (Agkathidis, 2016).

Attractor algorithms can function as generative algorithms. Attractors have been described in mathematics as part of dynamic systems, where an attractor are used to describe the asymptotic behaviour of physical systems or the long-term behaviour of all kinds of natural phenomena (Ruelle, 1981). An attractor can be a point, a finite set of point, a curve a manifold or a complicated set with a fractal structure, also known as a 'strange attractor'.

In architectural design, attractor algorithms have been widely applied, aiming to respond to complex architectural design problems. They can be developed within various software platforms and scripting languages, such as 3D studio max, Maya, C++, or Grasshopper for Rhinoceros. In his book Parametric Architecture with Grasshopper, Tedeschi (2011) describes the use of point attractor algorithms, mainly as two-dimensional patterning strategy,

allowing variable density on rectangular point cloud grids, Voronoi patterns or other types of tessellations. Points or vectors are used as variable attractor entities, attracting or repulse point grids, often related to variable units in width and height. Similar approaches are described by Tang (2014) using Autodesk Maya scripting code. Their use and outputs are comparable and remain in a strictly geometrical approach with no directly related architectural application. Jabi (2014) presents equivalent techniques in his book Parametric Design for Architecture using 3D studio max scripting code.

Ajlouni (2012) developed octagonal, quasi-periodic formations using C++ based on point attractors, exploring tessellation techniques inspired by Islamic geometric patterns. In the 'Emphatic Lines' paper (Baerlecken and, Gokmen, 2014), the authors propose a comparable method to Ajlouni, using Rhinoceros and Grasshopper, in order to distribute ornamental patterning on octagonal tiles. Dimcic and Knippers (2011) have developed point attractors genetic algorithms in combination with FEM software in order to optimize stress and deformation of Voronoi, triangular or polygonal grid shell structures. Their method applies a C++ based Rhinoceros plug-in in combination with NURBS as host surfaces. Pe (2012) uses algorithmic point attractor techniques aiming to develop urban planning organisational models. In the 'Deceptive Landscape' installation project (Agkathidis and Kocaturk, 2014), point attractors are applied via the panelling tools plug-in for Grasshopper in combination with the DIVA solar radiation plug-in, in order to distribute perpendicular pipes achieving a desired, solar radiation related effect. However, all the above precedents are mainly focusing on two-dimensional, patterning solutions and don't explore three-dimensional deformation of linear systems. How could such a technique be applied in order to deform a three-dimensional point cloud, forming a linear system?

As CNC bending is the key fabrication technology applied for the fabrication of the metal rods and part of the second feedback loop of the design framework used in the 'Dark Matte Garden', we will now cast our attention into literature sources investigating it.

2.2. CNC bending fabrication techniques

CNC bending is part of the formative digital fabrication techniques, applied to a material in order to form it into the desired shape by reshaping or deforming it (Afify and Abd Elghaffar, 2007). It is mainly applied to metals, plastics, and glass, achieving deformation by bending beyond the elastic limit, often after preheating (Kolarevic, 2005). CNC bending is being used widely in the automotive and product design industry but started being applied in the fabrication of building components, during the last decade as well. It can be applied to two or three dimensions, deforming tubes, pipes, sheets, and rods. The BMW Pavilion, designed by Bernhard Franken in 2000, is among the first, characteristic precedents, where this technique was used widely (Corser, 2010). Many others, such as the Beijing National Stadium build in 2008 by Herzog & De Meuron and the Kings Cross roof structure build in 2012 by John McAslan and Partners did follow.

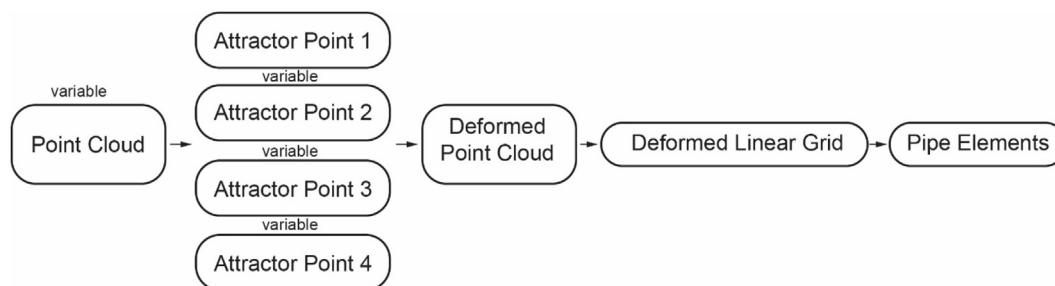


Fig. 3 Core of generative algorithm developed and applied for the 'Dark Matter' garden design.

Most CNC bending machines use the method of rotary draw bending (Dillenburger et al., 2007). They offer great freedom in deformation of tubes but have also restrictions such as minimum radius in relation to the tube's diameter as well as the minimum bending angle of 180° . In addition, there are other restrictions related to the length of each CNC bending machine can process. A three-axis CNC bending machine is controlled by three values, known as LRA: L stands for the length of straight between two bending points, R for the rotation of the tube around the axis of the tube and A for the bending angle. As a consequence, digital design components such as Splines or NURBS (Non-uniform rational B-spline), (McNeel, 2019), usually described by XYZ or UV coordinates have to be translated to approximate geometries described by LRA. Installations such as the 'configurable space loop' or 'particle intelligence' described by Dillenburger (2007), explore the different geometrical variations, which can be achieved by transforming a geometry from the one system into the other. Pigram and McGee (2011), describe robot-assisted CNC bending techniques as an alternative to rotary draw CNC bending. McDowell and Tomova (2011) present similar experimentations with robotic bending techniques, extending their assessment on branches of tubes.

3. Case study: designing the 'dark matter' garden by using algorithmic design techniques

The garden installation had to meet constraints regarding its size and construction cost, determined by the garden show organiser (Royal Horticultural Society) and funding bodies (National Schools Observatory, LJMU). The project was steered by a team of astrophysicists from LJMU. Their feedback informed the design solution, which derived as an array of horizontal tubes, within a footprint of ten to 3 m. The installation's height was determined by programmatic-functional requirements, into 3 m. All these constraints were embedded in a generative algorithm, based on four point-attractor described by the equation

$$f(t, (x, v)) = (x + tv, v)$$

taking effect in a point-cloud grid. Parameters affecting the grid's density, its height, width, and length could be numerically modified. In the equation formula, t represents time and $f(t, \bullet)$ a function which specifies the dynamics of the system.

The algorithm's core consists out of one subdivision component and three point-attractor components, parts

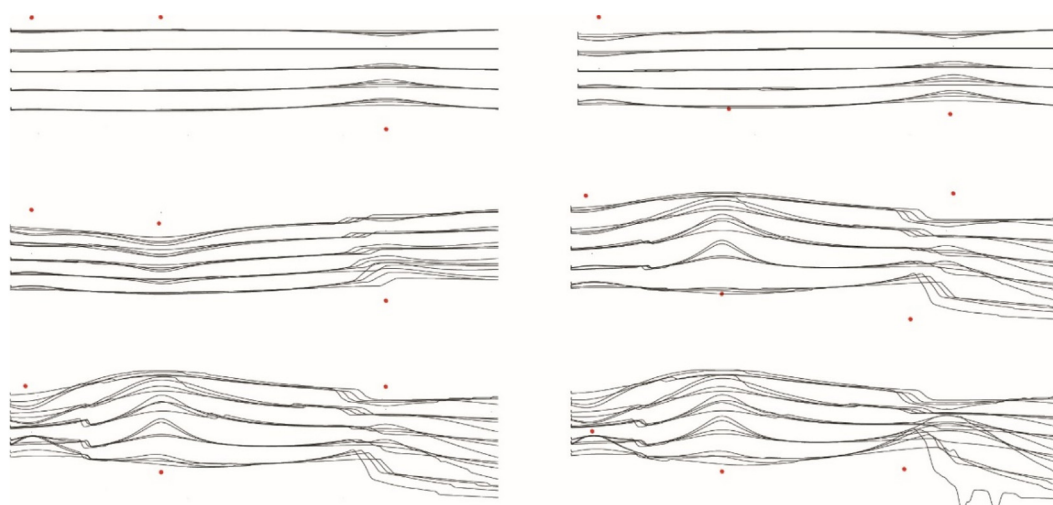


Fig. 4 Top view of tubular grid iterations, in relation to the variable magnitude of point attractors.

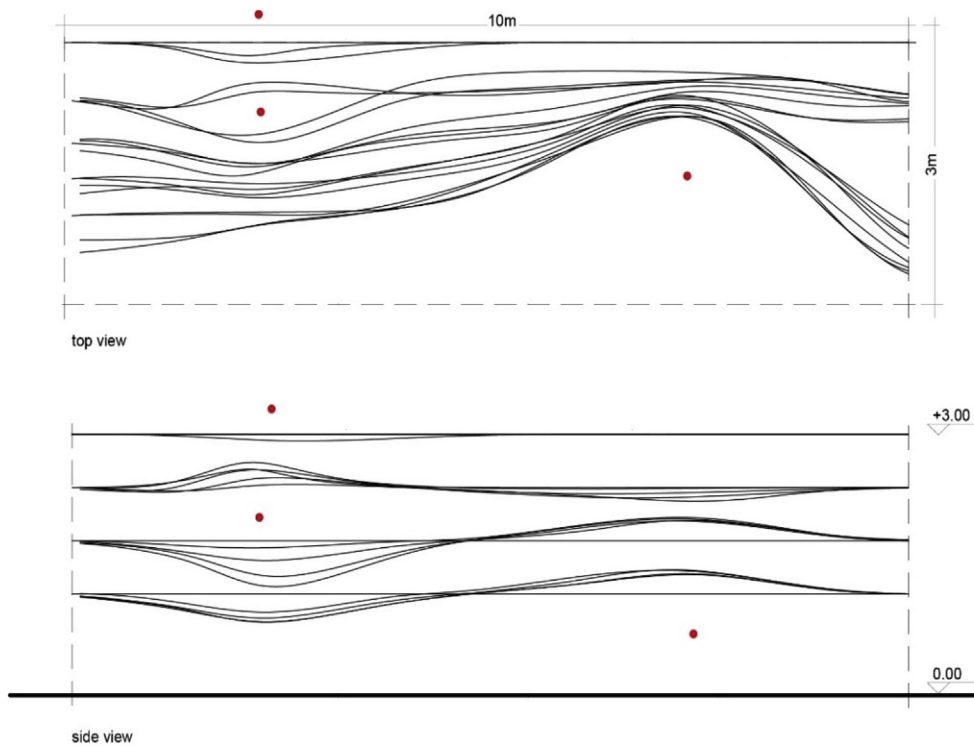


Fig. 5 Top and side views of the finalised installation design.

of the 'panelling tools' plug-in, in addition to all relevant numerical sliders, which can determine the number of applicable tubes as well as the magnitude of the attractor points (Fig. 3). These are connected to a NURB pipe component, which generates tubes with variable radius and curvature degree. Each point's magnitude can be increased or decreased, generating more or less

intense deformations. The final parametric model is able to generate infinite iterations (Fig. 4), allowing the designer to freeze the desired iteration according to the aesthetic and programmatic requirements set by the design brief.

The exact position of the four 'point attractors' was crucial for appropriate visualisation of the gravitational

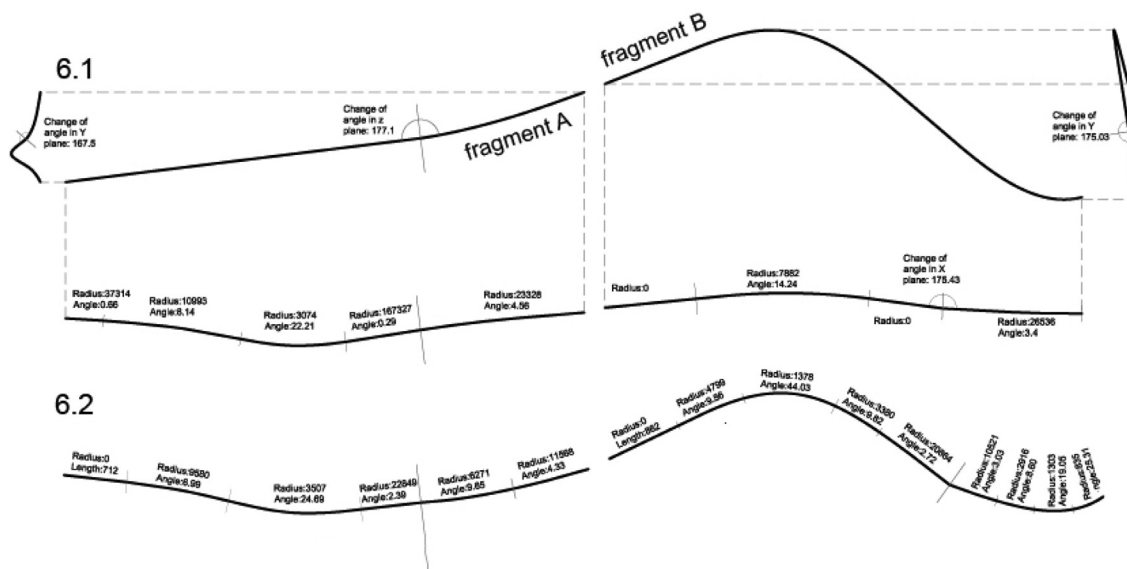


Fig. 6 Fabrication drawing of the singular tube. 6.1: top, side and perpendicular projection of 3D XYZ geometry. 6.2: top view of the planar LRA converted geometry.

lens deformation effect applied on the tubes, in plan and section. They embodied an abstract representation of rays of light, deformed by the dark matter. Increasing the tube's curvature degree value would have an immediate effect on their smooth visual appearance (Fig. 5). The design model could be modified in its grid density, height, deformation degree, and pipe radius, offering a large spectrum of variations, in relation to programmatic requirements, aesthetics and fabrication cost. The generative algorithm supported the collaborative approach, as all stakeholders could easily try out various solutions and optimise the outputs through the feedback loop until the preferred geometry was found.

The final design included a spatial grid, consisting of twenty deformed tubes, spanning between two, load-bearing screen walls, a sitting area, as well as various plants placed between and around the pipes. One of the screen walls was equipped with a lighting screen, while the other included an observation hole, allowing visitors a view of the pipes from the side perspective. The installation's load bearing structure is based on the two steel frame screens, placed on the edges, in addition to vertical steel lattices, which are distributed, in equal distances between the screens. The installation's structural system was optimised by finite element analysis techniques, in order to achieve minimum use of material and maximum stability.

Furthermore, the entire structure was based in a bed filled with earth planted mainly with bamboo in different heights and densities interacting with the steel structure. In the centre of the area with the most dramatic pipe deformation (Fig. 7) a ceramic pot, filled with tall bamboo plants was placed, acting symbolically as the 'dark matter', emphasising the gravitational lens effect deformation. A small sitting area, made of curved steel rods was placed around the pot, offering a resting area for visitors. Additional bamboo plants were planted along the 10-m long side edge, creating a green barrier towards the neighbouring garden.

4. Representation and fabrication process of double-curved, tubular structures

The main challenge in fabricating the double curved tubes was the translation of their geometry developed in the XYZ coordinate system, into another defined by LRA coordinates, as described by Dillenburger (2007). In addition, all tubes had to be fragmented into two pieces, in order to match the CNC fabrication constraints, which could process tubes with a maximum length of 5 m.

Since every pipe is represented as a double-curved, three-dimensional spline geometry, each of them had to be translated into LRA code, describing each fragment's length, its rotation around his own axis and its deformation angle on the Z plane. Other than the fabrication drawing technique described by Dillenburger (2007), based on two-dimensional representation of the tube's top view projection, we decided to include some additional drawings, including projections of the three-dimensional tube geometry, as well as the front, left and right views, in order to ensure the highest possible accuracy during data transfer between planning and fabrication teams (Fig. 6).

The geometry conversion process from the XYZ into the LRA coordinate system is directly linked to the form-finding process and our design framework, as described in Fig. 2. The converted LRA form is part of a feedback loop, which informs the geometry of the garden geometry before its final fabrication and assembly.

Other than the joint solution presented in the 'Tree-structure' canopy (Agkathidis and Brown, 2013), we have developed a simple, stick-in joint connection at the vertices of the two fragments. Due to their relatively lightweight, the stick-on joint would have no significant impact on the tube's structural performance. In addition, it would facilitate easy transportation and assembly. After the completion of the entire CNC bending process, the tubes were transported and assembled on site, where the horticultural works could finally take place.



Fig. 7 Artefact of the dark matter garden at the chelsea flower show.

5. Discussion and conclusions

Our research uncovered the complex relationship between algorithmic design, artistic expression, digital fabrication, and materiality. The algorithm applied, in order to visualize the 'Dark Matter' gravitation lens effect achieved its goal. Its final appearance and shape were approved by the steering astrophysicist team, involved in the project, as it matched their scientific expectations. The artefact's design fulfilled its educative purpose, balancing between artistic expression, horticultural requirements, and representation of an astrophysical phenomenon (Fig. 7). The garden installation, achieved its aim, to visualize and explain this complex phenomenon to thousands of visitors, of different ages and educational backgrounds. Our team managed to illustrate Dark Matter in an innovative way that would reach an entirely new audience at the Chelsea Flower Show. Having wind blow through the grass in the garden goes towards explaining the concept of Dark Matter; we see the grass move, but cannot see the wind itself. It became a successful president of how digital design and architecture, can become accessible to a broader audience and serve an educational purpose.

Our design and fabrication framework based on the point attractors based generative algorithm proved to be a functioning collaborative working tool, which was successfully applied for the garden's design. It allowed easy testing of design variants and made the communication and decision-making process between the stakeholders much more efficient. Its use went beyond a simple pattern distribution as described by Baerlecken (Baerlecken and Gokmen, 2014), or pattern generation as by Ajlouni (2012), into a three-dimensional operative system, which is able to deform a grid of linear elements according to the magnitude degree of each point. As evident in Fig. 4, the increasing point attractor magnitude value generates irregularity in the tube's curvature, visible through rough edges and distortions. This effect could be optimized through the addition of a curvature degree variable which was embedded in the algorithm. The slider allowed us to achieve the desired visual effect as shown in Fig. 5. The algorithm's main limitation is its dependence on host surfaces (NURBS), which define the positions of the tubes. In addition, the system becomes less controllable by increasing the number of points and magnitude values.

The artefact's fabrication process highlighted the challenges related to three-dimensional CNC bending technologies. As in the experimental installations, developed in the ETH Zurich (Dillenburger, Zaeh, 2007), the accurate reproduction of digital geometry into a physical, 1:1 object bears geometric difficulties. In addition, due to the machine's processing constraints, the desired 10 m' long tube could not be fabricated as one piece but had to be broken down into two, 5-m-long fragments. In order to avoid construction mistakes during their assembly, translation of XYZ into LAR coordinates had to be accomplished within the minimum possible spectrum of tolerances, which was defined by 5–6 mm' design and fabrication method as described in Fig. 6 made this accuracy possible. In order to achieve a clean and accurate assembly, both vertices of the

curved tube fragments had to meet the screen walls on one exactly defined point.

Using the representation method described in Fig. 6, each double-curved Spline geometry was flattened on the XY plane, then translated into arch fragments without distorting the original shape. Other than described in the 'Robotic rod bending' paper (McDowell and Tomova, 2011), where the rod geometry was consisting mainly out of straight linear and curved corner fragments, the garden's tubular components were far more complex, making the calculation of the Angle coordinate, particularly difficult. Representation of site views, alongside top view and flattened footpath of each tube (Fig. 5), proved very helpful and ensured avoidance of mistakes during data transmission between the designing and the fabricating team.

The garden's materiality and construction method, based on Corten steel elements, proved suitable for the required purpose. Corten rods and frames were easy to assemble and reacted well on the increased levels of humidity, linked to the horticultural elements and plants which were parts of the installation. This became particularly important as the garden was reassembled in STFC's Daresbury Laboratory after the completion of the Chelsea Flower Show to reach further audiences.

Overall, the combination of algorithmic modelling and the chosen file to factory fabrication path, achieved its aims and objectives, proving that it is a suitable method, which could be applied to other, similar design projects. The Installation representing deformed rays of light via three gravitational lens effects made the Dark Matter theory approachable to the wider public. The plants and horticultural elements interacted with the steel structure harmonically, emphasizing the areas where the gravitational lens effects were visualized. In conclusion, the Dark Matter Garden sets a successful precedent of algorithmic design methodology contributing to the NSO's outreach programme, engaging directly with over 160,000 visitors, as well as others through its coverage through the BBC and many national newspapers.

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References

- Agkathidis, A., 2016. *Generative Design: Form-Finding Techniques in Architecture*. Laurence King Publishing, London.
- Agkathidis, A., Brown, A., 2013. Tree-Structure Canopy: a case study in design and fabrication of complex steel structures using digital tools. *Int. J. Archit. Comput.* 01 (11), 87–104.
- Agkathidis, A., Kocaturk, T., 2014. Deceptive landscape installation, algorithmic patterning for a pavilion installation. *eCAAD Conference Proceedings 2*, 71–79. Newcastle upon Tyne.
- Afify, H., Abd Elghaffar, Z., 2007. Advanced digital manufacturing techniques (CAM) in architecture. *ASCAAD Conference Proceedings, Section 2*, Alexandria, pp. 67–80.

- Ajlouni, R., 2012. The Forbidden Symmetries. *ACADIA Proceedings*, San Francisco, pp. 391–397.
- Baerlecken, D., Gokmen, S., 2014. Emphatic Lines: surface structuring based on Walter Crane's pattern making methods. *eCAADe Conference Proceedings 2*, 107–114. Newcastle upon Tyne.
- Bohnacker, H., Gross, B., Laub, J., Lazzeroni, C., 2009. *Generative Design: Visualize, Program, and Create with Processing*. Princeton Architectural Press, New York.
- CERN, 2018. Dark Matter. Online: www.home.cern. (Accessed 11 July 2018).
- Corser, R., 2010. *Fabricating Architecture: Selected Readings in Digital Design and Manufacturing*. Princeton Architectural Press, Princeton.
- Dillenburger, B., Thesseling, F., Kotnik, T., Annen, M., Fuhr, C., Girod-Ifrah, Y., Tann, M., Shin, D., Markovic, S., Versteeg, M., Wendt, T., Záh, M., 2007. Architectural Use of Computer Controlled Deformation Techniques on the Example of CNC-Bent Tube Structures, vol. 1. *eCAADe Conference Proceedings, Session*, Frankfurt upon Main, pp. 21–26.
- Dimcic, M., Knippers, J., 2011. Free-Form Grid Shell Design Based on Genetic Algorithms. *ACADIA Conference Proceedings*, Calgary, pp. 391–400.
- Eisenman, P., 2004. *Eisenman inside Out: Selected Writings 1963-1988 (Theoretical Perspectives in Architectural History & Criticism)*. Yale University Press, Yale.
- Jabi, W., 2014. *Parametric Design for Architecture*. Laurence King Publishers, London.
- Kolarevic, B., 2005. *Architecture in the Digital Age: Design and Manufacturing*. Taylor and Francis, London.
- Lynn, G., 1999. *Animate Form*. Princeton Architectural Press, Princeton.
- McDowell, P., Tomova, D., 2011. Robotic Rod-Bending: Digital Drawing in Physical Space. *ACADIA conference proceedings*, Calgary, pp. 132–137.
- McNeel, R., 2019. What are NURBS? Online: www.rhino3d.com. (Accessed 19 March 2019).
- NASA, 2018. Dark Energy, Dark Matter. NASA Official Website.
- Pe, R., 2012. Hyper-localism and parametric mapping for collaborative urbanism. In: Buhmann, E., Ervin, S., Pietsch, M. (Eds.), *Peer Reviewed Proceedings of Digital Landscape Architecture 2012 at Anhalt University of Applied Sciences*. Wichmann, Berlin, pp. 389–396.
- Pigram, D., McGee, W., 2011. Formation Embedded Design: a Methodology for the Integration of Fabrication Constraints into Architectural Design. *ACADIA conference proceedings*, Calgary, pp. 122–130.
- Ruelle, D., 1981. Small random perturbations of dynamical systems and the definition of attractors. *Commun. Math. Phys.* 82 (01), 137–151.
- Tang, M., 2014. *Parametric Building Design Using Autodesk Maya*. Routledge, London.
- Tedeschi, A., 2011. *Parametric Architecture with Grasshopper*. Le Pensur, Brienza.
- Van Berkel, B., Bos, C., 1999. *UN Studio - Move: 3 Volume Set: Imagination/Techniques/Effects*. Groose Press, Amsterdam.