Efficient system reliability analysis for layered soil slopes with multiple failure modes using sequential compounding method

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Abstract

Evaluating the system reliability of layered soil slopes is a challenging issue because multiple failure modes may be included along the slip surfaces, which makes the overall failure probability greater than any individual slip surface. In this paper, an efficient system reliability analysis concerning the layered soil slopes is conducted based on the sequential compounding method (SCM) that has the ability to compound multiple failure events into an equivalent event sequentially. First, the first order reliability method (FORM) is employed to quantify initial reliability indices and correlation coefficients among these failure modes. Subsequently, the SCM is used to calculate the equivalent reliability indices and correlation coefficients until the multiple failure events are reduced to a compound event, and then the system reliability of the slope is obtained accordingly. The application of the approach to probabilistic evaluation of layered slopes is illustrated by two typical examples, and the correctness is verified by the Monte Carlo simulation (MCS). The results show that the SCM can deliver accurate system failure probability and greatly improve the computational efficiency compared with the MCS, which is an advantageous and promising strategy in evaluating the system reliability of layered soil slopes.

Keywords: System reliability; Layered soil slopes; Multiple failure modes; First order reliability method; Sequential compounding method; Monte Carlo simulation

Introduction

There is an ever-increasing interest in evaluating the performance of the layered soil slopes from the perspective of reliability analysis, because it can take into account the uncertainties that exist in engineering geology widely (Cho 2009; Phoon and Ching 2015; Li and Wang 2021; Zai et al. 2021; Liao et al. 2021a). Compared with conventional stability analyses assuming that the geotechnical properties are constant and result in a single factor of safety, reliability analysis considers the random variation of geotechnical properties and quantifies the probability of failure to reflect the hazard level of slopes, which can be further applied in risk management and decision-making (Zhang et al. 2013b; Hicks et al. 2014; Vardanega and Bolton 2016; Liao et al. 2021b; Jiang et al. 2022). However, calculating the failure probability of individual slip surface mechanically, even the most critical one, may underestimate the failure probability when the slope involves multiple slip surfaces (Zhang et al. 2013a; Kang et al. 2015; Zeng et al. 2018). In this case, the instability of the slope could be triggered by any mobilization of the potential slip surfaces. The slope can be therefore considered as a series system.

In recent decades, several methods have been introduced to calculate the system reliability of slopes with the rapid advance of fundamental theory and computer science. Initially, Cornell's bounds method, which considers the sign of the correlation coefficient among multiple failure surfaces, provides a rough range of the system failure probability (Cornell 1967). However, limited by ignoring the value of the correlation coefficient, the results obtained are usually overestimated. Further, Ditlevsen's bounds method solves this issue by using approximation algorithm, such as first order reliability method (FORM), giving a relatively narrow probability range (Ditlevsen 1979; Chowdhury and Xu 1995; Low et al. 2011; Liao et al. 2022; Low and Bathurst 2022). Although this technique yields seemingly good results and is widely applied in the practical analysis, the bounds can be broad when the single-mode failure probabilities are all high and the modes of failure are numerous (Ang and Tang 1984). Hence, some other methods have emerged in search of more precise solutions. The most representative one is the Monte Carlo simulation (MCS), which is a straightforward tool to calculate system reliability and serves as an unbiased way for verifying the accuracy of other methods as well (Ji and Low 2012; Jiang et al. 2015). But due to its disadvantages of computational effort and time consumption, more efficient simulation methods represented by importance sampling (IS), subset simulation (SS), and Latin hypercube sampling (LHS), have also been implemented in this framework (Ching et al. 2009; Li et al. 2017; Guardiani et al. 2021). To date, although a series of efforts have been made to calculate system reliability, it is still perplexing to balance the efficiency and accuracy, especially when the system is complex.

Sequential compounding method (SCM) proposed by Kang and Song (2010) is developed to answer the system reliability of general events through solving multivariate normal integrals, which has the ability to cover the system with a variety of correlation properties even for those with numerous components (Chun et al. 2015). The principle of SCM is to sequentially compound two components that are coupled by a logical operation, such as union or intersection, until the components of the system are reduced to a single compound event. In this way, the seemingly difficulty of evaluating the comprehensive performance of a complicated event is simplified into solutions of calculating equivalent reliability indices and correlation coefficients, in which the initial failure probabilities of the individual events and the corresponding correlation coefficients can be calculated by the FORM (Haldar and Mahadevan 2000; Cho 2013). The proposed method has shown that it has major advantages in terms of computational accuracy and efficiency (Chun 2021). Despite the superior properties of this method, its application in the field of geotechnical engineering, including slopes, is still rarely reported except for a few studies (Johari et al. 2020; Johari and Kalantari 2021).

The present paper intends to estimate the system reliability of layered soil slopes with multiple failure modes using the SCM. The FORM is first employed to calculate the initial probability information of the slope, including the failure probabilities and correlation coefficients among these failure modes. Then the system reliability is evaluated by the SCM. Further, the application of the approach is illustrated with two typical layered soil slopes, and the accuracy of the system failure probability is verified by the MCS. It is anticipated that this study can provide a feasible strategy to evaluate the comprehensive performance of the slopes even for those with a large number of slip surfaces from a probabilistic point of view.

Methodology

Sequential compounding method

It is unpractical to determine the reliability of a complex engineering system containing multiple components by direct numerical integration because of absence of closed-form solution for its multi-fold integration. The SCM proposed by Kang and Song (2010) has advantages in dealing with multivariate normal integrals, which is capable of yielding a satisfactory result.

In general, the failure of any one component will cause the system to fail as a whole for the linear structure, which can therefore be regarded as a series system. Supposing a series system contains n components, the two components E_1 and E_2 coupled by union can be compounded into a single equivalent event $E_{1\mathrm{or}2}$ as:

$$P\left(E_1 \cup E_2 \cup E_3 \ldots \cup E_n\right) = P\left(E_{1\mathrm{or}2} \cup E_3 \cup \ldots \cup E_n\right) \quad (1)$$

where E_i, $i = 1, 2, 3, \cdots, n$, denotes the event of the ith component failure, and $P\left(E_i\right)$ denotes the probability of the ith component failure.

Considering De Morgan’s rule and the symmetry of the standard normal distribution, the reliability index $\beta_{1\mathrm{or}2}$ of the compound event $E_{1\mathrm{or}2}$ can be calculated as:

$$\beta_{1\mathrm{or}2} = -\Phi^{-1}\left[P\left(E_1 \cup E_2\right)\right] = -\Phi^{-1}\left[1 - P\left(\bar{E}_1 \cap \bar{E}_2\right)\right] = \Phi^{-1}\left[P\left(\bar{E}_1 \cap \bar{E}_2\right)\right] \quad (2)$$

where Φ denotes the standard normal cumulative distribution function (CDF).

Let the correlation coefficient between E_1 and E_2 be $\rho_{1,2}$, which can be quantified by employing the results of FORM. The FORM can not only enable the evaluation of structural performance from a probabilistic perspective, but also provide additional valuable information that can be used as the medium for further analysis, such as the design point in the standard normal space (Low et al. 2011). Hence, the $\rho_{1,2}$ can be acquired as:

$$\rho_{1,2} = \frac{\boldsymbol{n}_1^{*T} \boldsymbol{R}^{-1} \boldsymbol{n}_2^{*}}{\beta_1 \beta_2} \tag{3}$$

where $\boldsymbol{n}_1^*$ and $\boldsymbol{n}_2^*$ denote the design points corresponding to components 1 and 2, $\boldsymbol{R}$ denotes the correlation matrix of the random variables, β_1 and β_2 denotes the reliability indexes corresponding to components 1 and 2, respectively.

All of the terms in Eq. (3) can be given by FORM. Then Eq. (2) can be expressed as:

$$P\left(\bar{E}_1 \cap \bar{E}_2\right) = \Phi_2\left(\beta_1, \beta_2; \rho_{1,2}\right) \tag{4}$$

where Φ_2 denotes the joint CDF of the bivariate standard normal distribution and can be computed by a single-fold numerical integral as:

$$\Phi_2\left(\beta_1, \beta_2; \rho_{1,2}\right) = \Phi\left(\beta_1\right)\Phi\left(\beta_2\right) + \int_0^{\rho_{1,2}} \varphi_2\left(\beta_1, \beta_2; \rho\right) d\rho \tag{5}$$

where φ_2 denotes the bi-variate joint probability density function (PDF) of standard normal distribution, which can be evaluated by means of improved algorithm introduced by Genz (2004).

By this way, $\beta_{1\mathrm{or}2}$ can be simplified to solve:

$$\beta_{1\mathrm{or}2} = \Phi^{-1}\left[\Phi\left(\beta_1\right)\Phi\left(\beta_2\right) + \int_0^{\rho_{1,2}} \varphi_2\left(\beta_1, \beta_2; \rho\right) d\rho\right] \tag{6}$$

Once $\beta_{1\mathrm{or}2}$ is successfully calculated by the above steps, the goal turns to calculate the correlation coefficients between the equivalent event $E_{1\mathrm{or}2}$ and each of the other remaining components in the system.

Taking the three components, E_1, E_2, and E_i into account, the equivalent correlation coefficient can be defined as $\rho_{(1\mathrm{or}2),i}$, which provides the same result on the probability of the event, $\Omega_s = \left[\left(E_1 \le -\beta_1\right) \cup \left(E_2 \le -\beta_2\right)\right] \cap \left(E_i \le -\beta_i\right)$, after compounding E_1 and E_2.

$$\int_{\Omega_s} \varphi_3\left(E_1, E_2, E_i; \rho_{1,2}, \rho_{1,i}, \rho_{2i}\right) dE = \Phi_2\left(-\beta_{1\mathrm{or}2}, -\beta_i; \rho_{(1\mathrm{or}2),i}\right) \tag{7}$$

Except for $\rho_{(1\text{or}2),i}$, all the terms in Eq. (7) are already addressed after the foregoing steps. Next, decomposing the CDFs via the conditional probabilities and dividing both terms by $\Phi(-\beta_i)$, the Eq. (7) is approximated as:

$$1-\Phi_2\left(\beta_{1|i},\beta_{2|i};\rho_{1,2|i}\right)=\Phi\left(-\beta_{(1\text{or}2)|i}\right) \tag{8}$$

The conditional reliability indices and correlation coefficients shown in Eq. (8) can be figured out by the following formulas:

$$\begin{cases}\beta_{1|i}=\left(\beta_1-\rho_{1,i}A\right)\Big/\sqrt{1-\rho_{1,i}^2B}\\ \beta_{2|i}=\left(\beta_2-\rho_{2,i}A\right)\Big/\sqrt{1-\rho_{2,i}^2B}\\ \rho_{1,2|i}=\left(\rho_{1,2}-\rho_{1,i}\rho_{2,i}B\right)\Big/\left(\sqrt{1-\rho_{1,i}^2B}\sqrt{1-\rho_{2,i}^2B}\right)\\ \beta_{(1\text{or}2)|i}=\left(\beta_{1\text{or}2}-\rho_{(1\text{or}2),i}A\right)\Big/\sqrt{1-\rho_{(1\text{or}2),i}^2B}\end{cases} \tag{9}$$

where A and B are defined as:

$$\begin{cases}A=\varphi\left(-\beta_i\right)\big/\Phi\left(-\beta_i\right)\\ B=A\left(-\beta_i+A\right)\end{cases} \tag{10}$$

By this way, the unknown $\rho_{(1\text{or}2),i}$ in Eq. (7) is determined avoiding possible multi-fold numerical integration in conventional methods. Consequently, $\beta_{1\text{or}2}$ and $\rho_{(1\text{or}2),i}$ are evaluated seriatim. Even if a given system has complex logical description among the components, each calculation of SCM involves only two components, so the compounding process can be implemented sequentially without being affected by other logical operation.

System reliability analysis for layered soil slopes

Usually, a layered slope contains more than one potential slip surface. When there are several potential failure modes inside the slope, the mechanical evaluation of the failure probability for each mode cannot meet the actual demand, even the most dangerous one, so the goal turns to system reliability analysis.

The factor of safety refers to the ratio of sliding resistance *R* to sliding force *T* along a given slip surface for the problems of slope stability, which can be denote as F_s. The limit state function Z can be therefore outlined as follows:

$$Z = g(\boldsymbol{X}) = R(\boldsymbol{X})/T(\boldsymbol{X}) - 1 = F_s(\boldsymbol{X}) - 1 \tag{11}$$

Considering the uncertainty of the material properties, such as strength parameters (cohesion c and internal friction angle φ), the system reliability analysis which can be also described by system failure probability P_{fs} is required for evaluating the comprehensive performance of the slope. The SCM is adopted here to achieve this goal. The implementation process of SCM with regards to a layered soil slope is shown in Fig. 1.

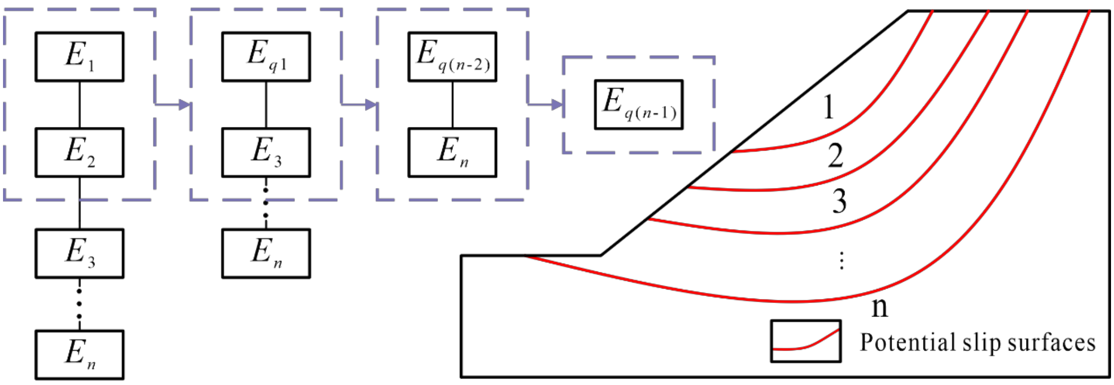

Fig. 1. Illustration of SCM to compute a layered soil slope.

The specific steps for implementing this approach are summarized as below, and the corresponding flowchart is shown in Fig. 2.

Step 1: *Set parameters*. Define the geotechnical and geometrical input parameters, including but not limited to strength parameters, unit weights, and model configuration of slope. Identify the random variables and determine its statistical characteristics, such as distribution types, means, coefficients of variation, and correlation coefficients.

Step 2: *Search for representative slip surfaces*. Establish the deterministic stability analysis model using the means of random variables. An in-house software and existing procedures are implemented for generating the potential slip surfaces and further locating the representative slip surfaces with small indices (Krahn 2004; Zhang et al. 2011).

Step 3: *Calculate reliability indices and correlation coefficients*. Introduce the statistical characteristics of random variables for probability analysis of the representative slip surfaces. The FORM is adopted in this procedure to determine the reliability indices and correlation coefficients along different representative slip surfaces, which serves as basis for subsequent analysis.

Step 4: *Obtain equivalent reliability indices and correlation coefficients*. Compound adjacent reliability indices resulted in step 3 to obtain the reliability of a single equivalent failure event and the correlation coefficients between this new compound event and each of the other remaining failure models in the slope by using SCM.

Step 5: *Evaluate the system failure probability*. Repeat step 4 until multiple failure events are simplified into an equivalent event. The system failure probability of the slope can therefore be evaluated.

Step 6: *Check accuracy*. Verify the correctness of the system failure probability produced by the above steps. The MCS which serves as an unbiased calculation method is conducted to achieve this goal.

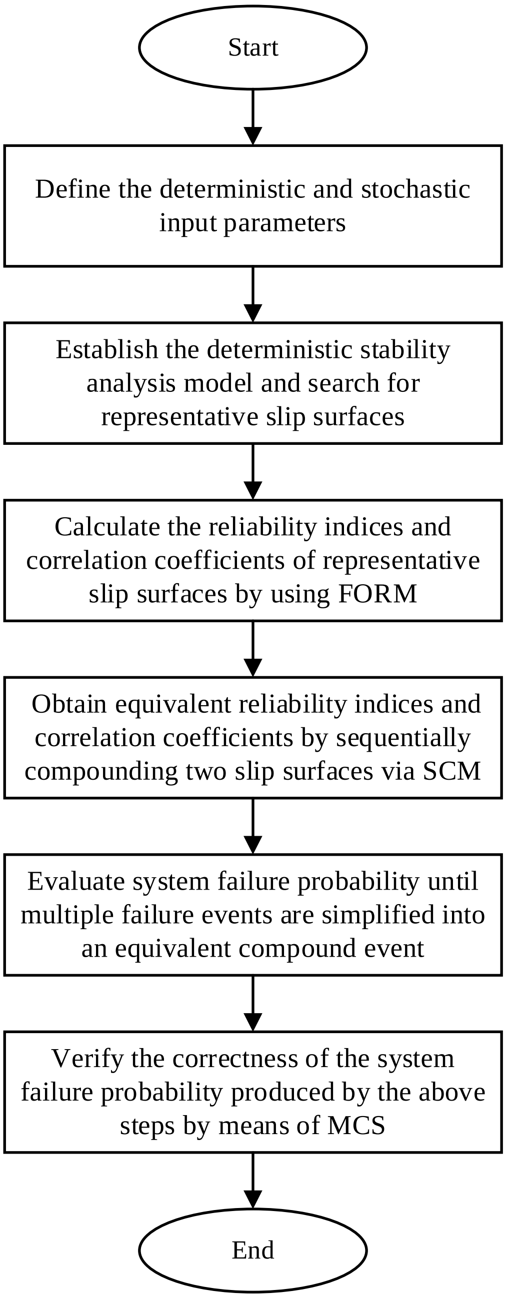

Fig. 2. Flowchart for evaluating the system reliability of the slope.

Illustrative examples

Example 1: a two-layered slope

The first example, a fill embankment resting on a clay layer, is adopted from Chowdhury and Xu (1995). The geometry of the slope is illustrated in Fig. 3 and the material properties, including deterministic and stochastic soil parameters, are given in Table 1. Among these parameters, the shear strength of the embankment and the undrained shear strength of the foundation are recognized as random variables, normally distributed and statistically independent. The unit weights of the embankment and foundation are constant.

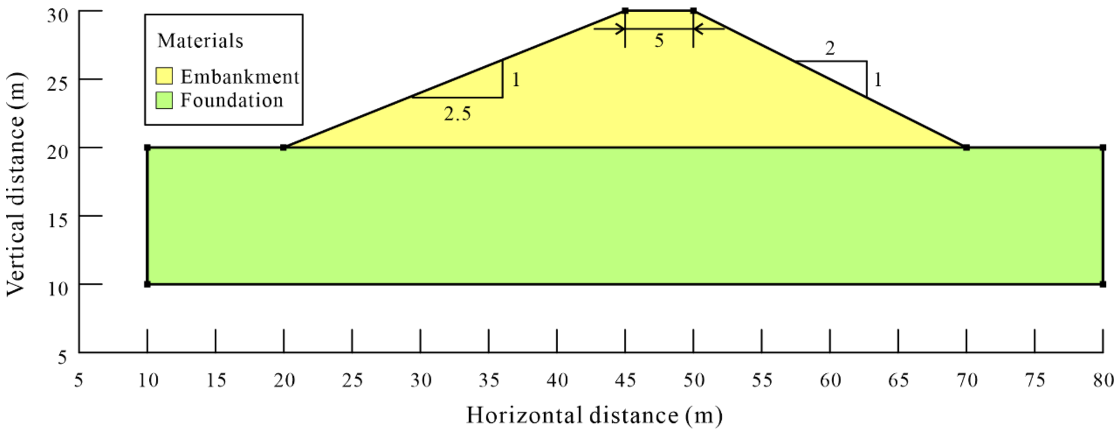

Fig. 3. Geometry of a two-layered slope considered in example 1.

Table 1. Statistical properties of soil parameters in example 1

Layers	Unit weight (kN/m3)	Cohesion (kPa)			Internal friction angle (°)		
		Mean	COV	Distribution	Mean	COV	Distribution
Embankment	20	10	0.2	Normal	12	0.25	Normal
Foundation	18	40	0.2	Normal	0	-	-

At first, the deterministic stability analysis is conducted to determine the number of potential slip surfaces using Morgenstern-Price method. The slip surfaces which are set to be circular but with different radii or centers of rotation are produced through the option of “Entry and Exit” method in Slope/W (Krahn 2004). According to Zhang et al. (2011), it is anticipated that two modes of failure with the smallest β values can occur either in the embankment only or traversing both the embankment and foundation. Figure 4 shows that the two reliability-based representative slip surfaces are tangent to the bottoms of the upper and lower clay layers, respectively.

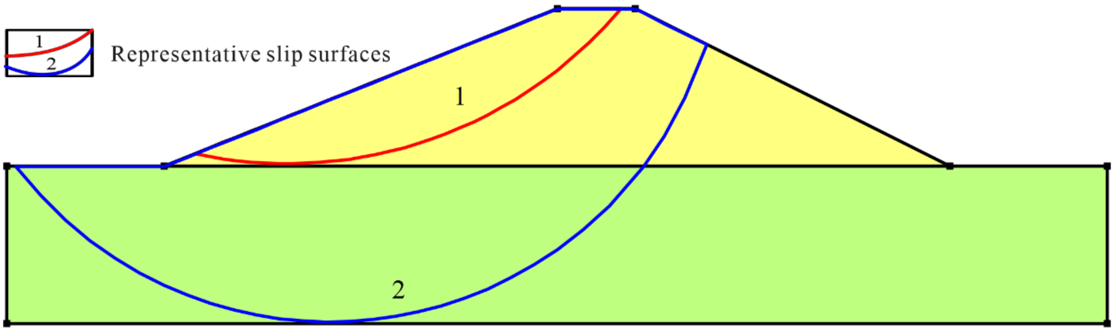

Fig. 4. Representative slip surfaces of example 1.

Once representative slip surfaces have been identified successfully, FORM can be used to estimate the failure probability of a single failure mode. Table 2 presents the FORM results, including reliability index, failure probability, design point, and correlation coefficient.

Table 2. FORM results of identified representative slip surfaces in example 1

Failure mode	Reliability index	Failure probability	Design point x'^*	Design point x^*	Correlation coefficient
1	0.844	19.93%	-0.42	9.16	$\rho_{1,2} = 0.0582$
			-0.732	9.803	
			0	40	
2	0.819	20.65%	-0.0227	9.955	
			-0.0419	11.874	

-0.817	33.462

The failure probabilities of the two circles at each design point are 19.93% and 20.65%, and the correlation coefficient between the two slip surfaces is 0.0582. Further, the SCM is introduced to perform system reliability analysis. Since there are only two representative slip surfaces in this example, the outcome can be obtained in one step without calculating an equivalent correlation coefficient. Besides, the Ditlevsen's bounds which gives the probability range of failure is calculated as well, and the correctness of the system failure probability is verified by MCS with 100,000 samplings. Table 3 lists the results of system reliability calculation.

Table 3. Results of system reliability analysis for example 1.

Method	System failure probability p_{fs}
SCM	35.78%
MCS	35.92%
Ditlevsen's bounds	31.8%-36.19%

As shown in Table 3, the system failure probabilities obtained by the SCM, MCS and Ditlevsen's bounds are 35.78%, 35.92%, and 31.8%-36.19%, respectively. The probability estimated by the SCM is almost the same compared with MCS. On the other hand, Ditlevsen's bounds method gets the system failure probability as well, but the probability range it gives is relatively extensive, which cannot meet the demands of engineering practice, especially when risk control requirements are stringent.

In addition, this typical example was also analyzed by Ji and Low (2012), Zhang et al. (2013a), and Cho (2013), and the results are listed in Table 4. Table 4 shows that the system failure probability obtained by the proposed method is close to the value that was calculated by the previous studies.

Table 4. Results of system reliability by different methods for example 1

Method	β_i	ρ_{ij}	p_{fs}	References
Ditlevsen's bounds	0.8471, 0.8067	0.0854	31.7-36.6%	Ji and Low (2012)
MCS with 50,000 samplings	-	-	35.75%	Ji and Low (2012)
Slide V6.0 with 2000 samplings	-	-	36.1%	Ji and Low (2012)
Ditlevsen's bounds	0.843, 0.821	0.063	31.74-36.15%	Cho (2013)
Multi-point FORM			35.94%	Cho (2013)
MCS with 20,000 samplings	-	-	36.56%	Cho (2013)
MCS with 10,000 samplings	-	-	35.6%	Zhang et al. (2013a)
SCM	0.844, 0.819	0.0582	35.78%	This study

Example 2: a multiple-layered slope

The second example is a well-known real case slope with a sand fill layer overlaying three clay layers, named Congress Street Cut, and has been well documented in several publications (Chowdhury and Xu 1995; Ching et al. 2009; Ji and Low 2012; Zhang et al. 2013b; Reale et al. 2016). The geometry of the slope is presented in Fig. 5. The material properties and associated uncertainties are listed in Table 5, in which the strength of the sand fill layer is defined as deterministic value while the undrained shear strengths of the three clay layers are modeled as uncertain variables, normally distributed and statistically independent.

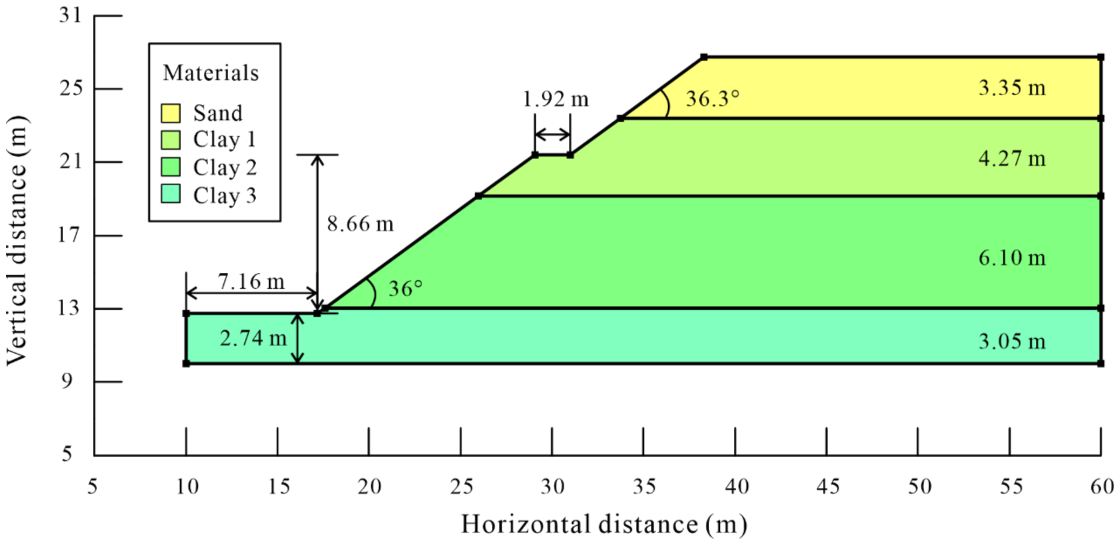

Fig. 5. Geometry of Congress Street Cut in example 2.

Table 5. Statistical properties of strength parameters of layers in example 2

Layers	Cohesion (kPa)			Internal friction angle (°)		
	Mean	COV	Distribution	Mean	COV	Distribution
Sand	-	-	-	30	-	-
Clay 1	55	0.37	Normal	0	-	-
Clay 2	43	0.19	Normal	0	-	-
Clay 3	56	0.24	Normal	0	-	-

Likewise, three representative slip surfaces shown in Fig. 6 are located in different clay layers since the circular failure can occur in any of them (Ji and Low 2012). The details of a reliability assessment based on preliminary FORM are listed in Table 6.

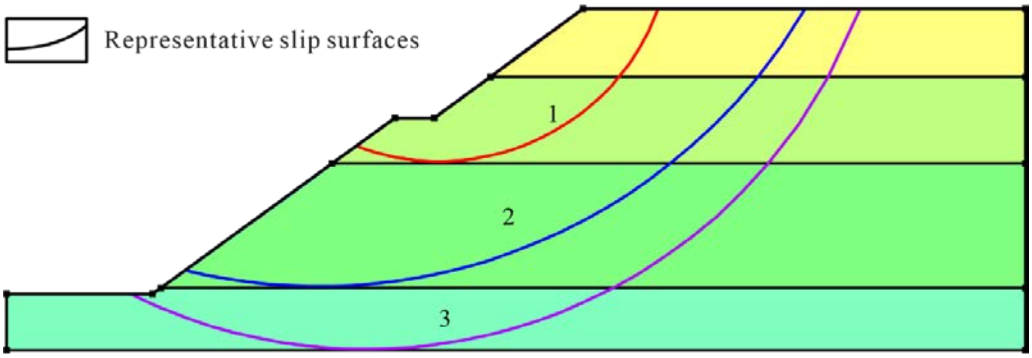

Fig. 6. Representative slip surfaces of example 2.

Table 6. FORM results of identified representative slip surfaces in example 2

Failure mode	Reliability index	Failure probability	Design point x'^*	Design point x^*	Correlation coefficient
1	1.636	5.09%	-1.636	21.709	$\rho_{1,2}=0.398$ $\rho_{1,3}=0.141$ $\rho_{2,3}=0.211$
			0	43	
			0	56	
2	0.669	25.17%	-0.266	49.587	
			-0.614	37.984	
			0	56	
3	0.712	23.83%	-0.100	52.963	
			-0.121	42.015	
			-0.694	46.670	

The failure probabilities of the three slip surfaces are 5.09%, 25.17%, and 23.83%, and the correlation coefficients among the failure modes are 0.398, 0.141, and 0.211, respectively. Then the

SCM is employed to calculate the system failure probability, and the Ditlevsen's bounds and MCS with 100,000 samplings are adopted here as well for comparative analysis and verification. Particularly, when the Ditlevsen's bounds method is implemented, sorting the slip surfaces in descending order of probability can often produce closer bounds (Haldar and Mahadevan 2000). The results are presented in Table 7.

Table 7. Results of system reliability analysis for example 2

Method	System failure probability p_{fs}
SCM	42.42%
MCS	42.18%
Ditlevsen's bounds	34.86%-44.49%

As shown in Table 7, the system failure probabilities obtained by the SCM, MCS and Ditlevsen's bounds are 42.42%, 42.18%, and 34.86%-44.49%, respectively. These indicators once again prove that the proposed method can provide a satisfactory result. Furthermore, comparing the results with previous studies, Table 8 is drawn below.

Table 8. Results of system reliability by different methods for example 2

Method	p_{fs}	References
Ditlevsen's bounds	27.39%-44.73%	Chowdhury and Xu (1995)
MCS with 50,000 samplings	39.11%	Ji and Low (2012)
Ditlevsen's bounds	32.84%-47.57%	Reale et al. (2016)

SCM	42.42%	This study

The system failure probability obtained in this study seems slightly higher, partly because all three surfaces are considered here, while some studies consider only two of them (Chowdhury and Xu 1995; Reale et al. 2016). Another reason behind this result is that this study specifies a denser increment when defining the slip surface in Slope/W, so more failure modes are generated and the representative circles with a higher probability of failure than before are located. For example, Ji and Low (2012) reported that the failure probabilities of the three critical slip surfaces were 4.79%, 24.27% and 23.18%, but 5.09%, 25.17%, and 23.83% in this study.

Discussion

In the analysis of the above examples, the strength parameters of the soil layers are related to c and φ, which are artificially assumed to be independent. In fact, the laboratory experiments have suggested that the c and φ are often correlated, and the correlation coefficient falls between -0.72 and 0.35 in most tests (Lumb 1970). The method can deal not only with variables that are independent but also with variables that are related. Taking the example 1 as the case study, the effect of the correlation coefficient of the c and φ on the system reliability is illustrated supposing that the correlation coefficient of embankment ranges from -0.8 and 0.8, and the results are shown in Fig. 7.

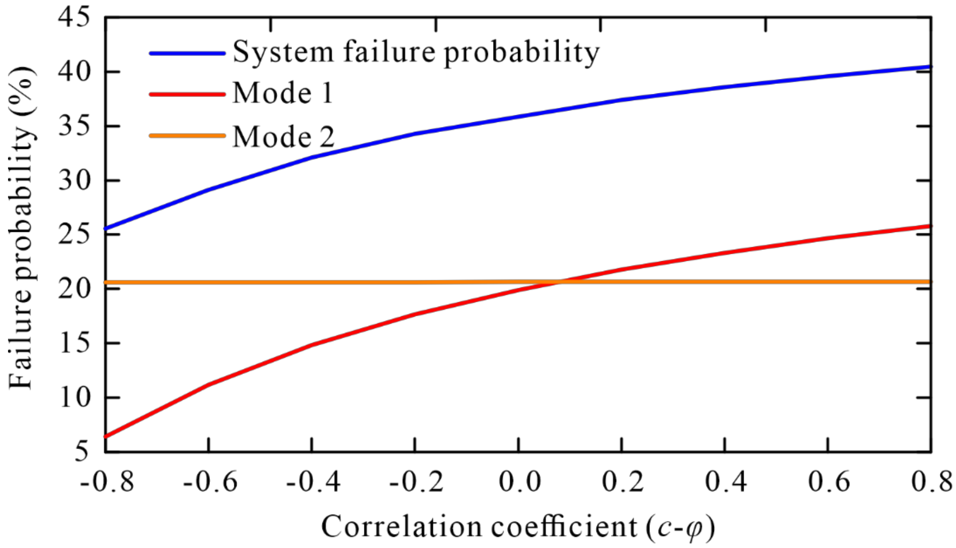

Fig. 7. Results of system reliability with different correlation coefficients for example 1.

It can be seen from Fig. 7 that the probability of the mode 1 and the slope system rises as the negative correlation weakens and the positive correlation increases except for mode 2 which is basically unchanged during the whole range. When the correlation coefficient is -0.8, the failure probabilities of mode1, mode 2, and slope system are 6.4%, 20.62%, and 25.58%; on the other hand, when the correlation coefficient is 0.8, the probabilities are 25.81%, 20.68%, and 40.44%. The reason behind this phenomenon is that the slip surface of mode 1 is fully located in the correlated soil layer, but only a small portion of mode 2 passes through the embankment. As a result, the change of correlation coefficient has a great influence on mode 1, thus improving the failure probability of the system.

In this study, two layered soil slopes that include two and three representative slip surfaces are analyzed respectively. In fact, even if the slope contains more representative slip surfaces, the SCM also works (Song et al. 2021). But as the number of failure modes increases, the computation time increases accordingly. In fact, in addition to the series system mentioned above, the SCM can be applied to parallel, cut-set and link-set systems as well by compounding two components in intersection and union flexibly (Kang and Song, 2010).

Conclusions

In this paper, the system reliability of layered soil slopes with multiple failure modes is evaluated in the framework of SCM. In which, the FORM is employed to obtain the initial reliability indices and correlation coefficients among these slip surfaces. Then the SCM is adopted to quantify the system failure probability by calculating the equivalent reliability indices and correlation coefficients until the slope system is simplified into a single compound event. Taking two typical layered soil slopes as the case study, the SCM is implemented for the analysis of the comprehensive performance of slopes, as well as the Ditlevsen's bounds method and MCS. Different from the Ditlevsen's bounds method resulting in a conservative probability range, the SCM can provide an exact value of the failure probability. The results from the SCM and MCS are very close, which verifies the accuracy of the SCM. But compared with MCS, the SCM greatly improves the calculation efficiency because it makes full use of the information resulted in FORM rather than a large number of mechanical samplings as MCS, which can be a promising tool for evaluating the system reliability of layered soil slopes with multiple failure modes.

Data availability statement

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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