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Government subsidy and firm's cost sharing in sustainable agriculture supply chain

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Abstract: This paper studies an agriculture supply chain consisting of a government, a leading firm, and a group of farmers. The government encourages farmers to make sustainable technologies' input by providing cost-share or flat payment. The firm procures products from multiple farmers and shares a portion of input costs. The optimal sustainable technologies' input, wholesale price, and government subsidy are determined, and the impacts of different parameters on the decisions with the endogenous or exogenous subsidy are analysed. The results show that when competition intensifies to a certain threshold, farmers will not invest in sustainable technologies. When the subsidy is endogenous, the firm will obtain less profit if he shares more costs because the optimal subsidy decreases with the cost sharing proportion. When the subsidy is exogenous, the farmers' optimal sustainable technologies' input under the government's cost-share is higher, and the firm may obtain more profit with cost sharing if the subsidy is relatively high. The government provides a higher subsidy to motivate farmers to adopt sustainable technologies when the pollution is more environmentally damaging. The government's cost-share is superior to flat payment because it alleviates the government's financial burden with the same effect on motivating sustainable technologies' input.

Keywords: Agri-product supply chain; Sustainable technologies' input; Cost sharing; Government subsidy; Social welfare

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1. Introduction

Due to the overuse of fertilizers, herbicides, and insecticides in agricultural production, agriculture becomes the biggest source of water pollution (Younis *et al.*, 2021; Correa-Cano *et al.*, 2022). Other pollution, such as soil degradation and greenhouse gas pollution, is also serious (Ren *et al.*, 2021; Guo *et al.*, 2022; Huang *et al.*, 2022). In the era of Agri-Food 4.0, the concept of sustainable agriculture, which emphasizes reducing environmental degradation, maintaining agricultural productivity, and promoting economic viability, becomes more important. It has been listed as the second of the 17 sustainable development goals set by the United Nations in 2015 (Younis *et al.*, 2021). To reduce agricultural pollution, provide environmental and social benefits, and grow more food (for profit), novel sustainable agriculture technology becomes increasingly made in agri-businesses (Yadav *et al.*, 2022).

Sustainable technologies, including IoT technology, blockchain technology, big data analysis, and smart production techniques can increase production quantity and total supply chain's surplus (Latino *et al.*, 2022; Cao *et al.*, 2022). With the adoption of new sustainable technologies, greenhouse gas emissions will be mitigated up to an extent of 30% (Lezoche *et al.*, 2020; Adegbeye *et al.*, 2020). The use of big data can minimize food waste and carbon footprint, and make better resource use (Lezoche *et al.*, 2020). In agricultural practice, farmers can monitor and control the growth status and environment of produces through IoT and blockchain techniques, and take timely actions to ensure optimal yields (Yadav *et al.*, 2022; Achour *et al.*, 2021). More enterprises are adopting and benefiting from new sustainable technologies in the age of Agri-Food 4.0, too. For example, the FarmBeats project developed by Microsoft utilizes the latest digital tech including the IoT and artificial intelligence to develop agriculture with lower environmental impacts and track carbon in agriculture[1]. Walmart uses blockchain technology to reduce the impact of pollution and operation costs (Niu *et al.*, 2021). Agro-enterprises that use big data make their operations sustainable

[1] https://www.microsoft.com/en-us/research/project/farmbeats-iot-agriculture/news-and-awards/

(Yadav *et al.*, 2022).

However, more than 98% farmers in China are smallholder farmers with only around 0.7 hectares of farmland on average, who lack economic incentives for sustainable technologies' input because of the high operation and adoption costs (Kamble *et al.*, 2019； Guo *et al.*, 2022； Huang *et al.*, 2022). Therefore, leading enterprises drive farm households by providing technical advices (Smith *et al.*, 2015; Niu *et al.*, 2021). Thus, it is important to study whether cost sharing will benefit both the farmers and the firm. This paper studies the complete vertical integration where a leading firm contracts with a group of farmers. This setting widely exists in real practice and existing research, such as Nadal *et al.*, (2019); Nadal *et al.*, (2020); and Liu *et al.*, (2022). To support sustainable technology applications to the agriculture industry, the government also shares investments in smart and digital sustainable technologies[2,3].

Therefore, it is critical to investigate the impacts of government's regulations on technical input level and environmental damage. The joint influence of subsidy and the firm's cost sharing on the firm and the farmer's profits will also be examined. This paper studies the cooperative relationship between the firm (he) and the farmer (she), considering sustainable technologies' input costs. The decisions on wholesale price, technology investment, and government subsidy are optimized to maximize the profits of channel members and social welfare. To study the decision of government subsidy, we introduce the concepts of exogenous subsidy and endogenous subsidy. The exogenous subsidy is the subsidy provided by the government at a fixed value, and it will be considered as a fixed parameter in this research. Whereas endogenous subsidy is that the subsidy value is a decision variable, and it will be determined by a local government aiming at social welfare maximization. The solutions with an exogenous subsidy serve as intermediate variables in solving the model with the endogenous subsidy.

Different subsidy policies in motivating sustainable technologies' input are

[2] https://www.gov.uk/government/news/government-unveils-path-to-sustainable-farming-from-2021
[3] https://www.wri.org/blog/2020/07/redirecting-agricultural-subsidies-sustainable-food-future

compared to provide government policy implications. One subsidy policy is cost-share, where the government provides the farmer subsidy based on the farmer's sustainable technology investment. The other is the flat payment per unit area, where the government provides the farmer subsidy based on the farmer's production yield and sustainable technology investment. In this way, farmers have incentives to adopt novel technologies to increase their yield.

When considering the exogenous government subsidy, the leading firm serving as a retailer procures from multiple farmers. The farmer determines the technologies investment, and the government provides the farmer with subsidy according to the farmer's sustainable technologies' input or production quantity. The firm determines the wholesale price and shares a portion of the sustainable technologies' input costs. The local government determines the subsidy by social welfare maximization. We study the sustainable effort investment, pricing, and subsidy decisions, and compare optimal decisions in the situations with exogenous subsidy and endogenous subsidy, respectively. To inform the government strategies for motivating sustainable technologies' input, we also compare two subsidy policies, namely, the government's cost-share which is based on the effort investment, and flat payment per unit area, which is based on the production quantity and technology investment.

This paper contributes to the literature in several ways. Most of the research on the application of sustainable technologies such as the IoT and blockchain in agriculture focuses on technical challenges, and few works in the literature focus on economic or social problems (Maroli *et al.*, 2021). Therefore, this paper attempts to fill this gap. To be specific, first, this paper studies the effect of sustainable technologies' input on vertically integrated operations for the first time. Although Liu *et al.* (2022) researched the sustainable supply chain's decision-making analysis problem under the vertical integration background, they didn't consider the cost-sharing strategy. Second, we combine firms' cost sharing and government subsidy and study the interaction of the above two members on technologies' input. Our research results provide insights for the government to subsidize sustainable technology inputs. Third, we compare flat payment per unit area and cost-share from the perspective of the government policy. Chen *et al.* (2017) compared these two different government subsidies, but they didn't consider the vertical integration and the sustainable technologies' input in an agricultural product supply chain.

To sum up, focusing on sustainable technologies' input in the agricultural sector, we combine the problems of collaborative innovation in supply chains' sustainable operations with government subsidy policy on mitigating agricultural pollution. Our paper aims to discover the impact of the subsidy on sustainable technologies' input, profits of supply chain members, and the environment (See Figure 1). Considering the complex game relationship between the government, the firm, and the farmers, we conduct the analytical model based on the game theory.

2. Model settings and notations

We assume that the farmer (she) produces agricultural products at a basic unit production cost of c, such as the cost of seeds. The farmer invests additional effort q in sustainable technologies which can increase input in an environmentally friendly way and reduce environmental impacts. Similar to Niu *et al.* (2016) and Hsu *et al.* (2019), the quantity of production linearly increases with the sustainable technologies' input. Then the actual output from the ith farmer can be defined as $Q_i = a_i(1 + \delta q_i)$ $(0 \leq q_i \leq \bar{q} < 1, i = 1,2, \dots n)$, where a_i is the lower-bound quantity farmers can produce given their initial production skills. We assume that farmers are regionally homogeneous, they have equal lower-bound quantity; thus, $a_i = a$ for $i = 1,2, \dots n$. According to Niu *et al.* (2016), a corresponding input cost can be defined as $\frac{\lambda q^2}{2}$, where λ is a coefficient representing the efficiency of the farmer's sustainable technologies' input, and a larger λ means lower efficiency. The quadratic cost function of sustainable technologies' input is based on the Law of Diminishing Returns, meaning that to achieve a higher level of sustainability, the cost increase will be increasingly accelerated (Keating *et al.*, 2010). $a(1+\delta\bar{q})$ stands for the maximal production quantity the farmer can produce under certain constraints, which may be attributed to the land capacity and technologies etc. The total production quantity from multiple homogenous farmers can be denoted as $Q=\sum_{i=1}^{n} a_i(1+q_i)=a\cdot n(1+\delta q)$, and the assumption of the same production quantity from identical farmers can be found in An *et al.* (2015).

Assuming that the original unit emission without sustainable technologies' input is e and emission after such input is $e(1-q)$ (Chen *et al.*, 2017; Yi *et al.*, 2018). The measure of the environmental effect of pollution can be calculated by the environmental damage factor, overall production quantity, and unit emission, which can be denoted by $EI=a\cdot ner(1+\delta q)(1-q)$; where r represents the environmental damage factor. To ensure a positive environmental effect (*i.e.,* $\frac{\partial EI}{\partial q}<0,\ 0\leq q\leq\bar{q}<1$), as suggested by Xia *et al.* (2017) and Cui *et al.* (2018), we assume $\delta\in(0,1)$.

The firm (he) procures the products from farmers at a wholesale price w and sells the products to customers at a selling price p. Customers are heterogeneous in the perceived valuation of products. We assume that customers value the product at v, which is uniformly distributed on $[0,1]$ (Chiang *et al.*, 2003; Yu *et al.*, 2022). With a widely adopted definition, a consumer's utility from purchasing a product with valuation v and price p can be defined as $U=v-p$. The customer will buy the product if his utility is not less than zero, *i.e.*, $U=v-p\geq 0$. Thus, with market size of 1, the consumer demand is formulated as $Q_t=1-p$, and the inverse demand function is $p=1-Q_t$, where Q_t is the quantity of the product for sale (Chiang *et al.*, 2003). As assumed above, the overall demand Q_t $(0<Q_t<1)$ can be divided equally by multiple farmers, we have $0<a<<1$. If $a\geq 1$, the firm only needs to choose one farmer, and this is outside the scope of this research. Since the overall production quantity from n homogenous farmers in this paper is assumed as $a\cdot n(1+\delta q)$, the inverse demand function can be described as $p=1-a\cdot n(1+\delta q)$. A similar assumption on the market price in the agricultural industry can be found in Tang *et al.*

(2015).

3. Government's cost-share

The subsidy for sustainable technologies' input in this research is similar to the innovation subsidy in Chen *et al.* (2019), where the government provides the farmer unit subsidy based on the farmer's sustainable technologies investment. We study the effects of such a policy but in a collaborative innovation context in the following four-step scenario: (1) the local government sets the subsidy s before the production season begins; (2) the firm sets the wholesale price w and a fixed cost-sharing proportion t by considering the subsidy and anticipating the farmer's sustainable technologies' input; (3) the farmer who has observed s, w, and t decides her sustainable technologies' input q; and (4) the production quantity and retail price of the agricultural products are then realized according to the optimal s, w, and q. Let subscripts e and q represent the situation with the government's cost-share and flat payment per unit area, respectively.

3.1. Optimal solutions

When solving this game by backward induction, the farmer chooses her sustainable technologies' input q_e to maximize the profit:

$$\pi_{fe} = (w_e - c)a(1 + \delta q_e) - \frac{\lambda(1-t){q_e}^2}{2} + s_e q_e,\ s.t.,\ 0 \le q_e \le \bar{q}. \tag{1}$$

By setting $\frac{\partial \pi_{fe}}{\partial q_e} = 0$, we generate the optimal sustainable technologies' input as the following:

$$q_e = \begin{cases} 0, & \frac{-s_e + a\delta(c-w_e)}{(-1+t)\lambda} < 0 \\ \frac{-s_e + a\delta(c-w_e)}{(-1+t)\lambda}, & 0 \le \frac{-s_e + a\delta(c-w_e)}{(-1+t)\lambda} \le \bar{q}. \\ \bar{q}, & \frac{-s_e + a\delta(c-w_e)}{(-1+t)\lambda} > \bar{q} \end{cases} \tag{2}$$

Therefore, anticipating the farmer's optimal reaction to the sustainable technologies' input, the firm decides his wholesale price so as to maximize his profit:

$$\pi_{re} = (1 - w_e)an(1 + \delta q_e) - a^2 n^2 (1 + \delta q_e)^2 - \frac{\lambda n t {q_e}^2}{2} = -a^2 n^2 (1 +$$

$$\frac{\delta(-s_e+a\delta(c-w_e))}{(-1+t)\lambda})^2 + an(1+\frac{\delta(-s_e+a\delta(c-w_e))}{(-1+t)\lambda})(1-w_e) - \frac{nt(s_e+a\delta(-c+w_e))^2}{2(-1+t)^2\lambda},$$

$$s.t.,\ c-\frac{s_e}{a\delta} \le w_e \le \frac{ac\delta+(1-t)\bar{q}\lambda-s_e}{a\delta}. \quad (3)$$

Substituting (2) into (3) and setting $\frac{\partial \pi_{re}}{\partial w_e}=0$, we can attain the equilibrium wholesale price:

$$w_1 \triangleq \frac{2a^3cn\delta^4+a(c+(-1+2an)(-1+t))\delta^2\lambda-(-1+t)^2\lambda^2-\delta(2a^2n\delta^2+\lambda)s_e}{2a^3n\delta^4-a(-2+t)\delta^2\lambda},$$

$$w_e = \begin{cases} c-\frac{s_e}{a\delta}, & \frac{-s_e+a\delta(c-w_1)}{(-1+t)\lambda} < 0 \\ w_1, & 0 \le \frac{-s_e+a\delta(c-w_1)}{(-1+t)\lambda} \le \bar{q}. \\ \frac{ac\delta+(1-t)\bar{q}\lambda-s_e}{a\delta}, & \frac{-s_e+a\delta(c-w_1)}{(-1+t)\lambda} > \bar{q} \end{cases} \quad (4)$$

Substituting (4) into (2), we can obtain the equilibrium sustainable technologies' input:

$$q_e = \begin{cases} 0, & \frac{-s_e+a\delta(c-w_1)}{(-1+t)\lambda} < 0 \\ \frac{-a(-1+c+2an)\delta^2+(-1+t)\lambda+\delta s_e}{2a^2n\delta^3-(-2+t)\delta\lambda}, & 0 \le \frac{-s_e+a\delta(c-w_1)}{(-1+t)\lambda} \le \bar{q}, \\ \bar{q}, & \frac{-s_e+a\delta(c-w_1)}{(-1+t)\lambda} > \bar{q} \end{cases} \quad (5)$$

The profits of the chain members, consumer surplus, and social welfare (as defined below) can be seen, respectively, in Appendix A. We here assume that the government decision can be year by year, and differ in products (see the US Federal Government subsidy policy)[4]. We finally determine the optimal subsidy of the government by means of social welfare maximization. Social welfare function $SW(s_e)$ consists of four components:

I. Consumer surplus $CS(s_e)$, following Yenipazarli *et al.* (2017), the function of consumer surplus can be shown as:

$$E(CS) = E\left(\int_p^1 (v-p)dv\right) = \frac{(1-p)^2}{2} = \frac{a^2n^2(1+\delta q_e)^2}{2}. \quad (6)$$

II. The profit of supply chain members

III. Subsidy expense

IV. The environmental effect, which can be denoted as $a \cdot ner(1+\delta q_e)(1-q_e)$

[4] https://usafacts.org/ articles/federal-farm-subsidies-what-data-says/

Combining the components I, II, III, and IV, the total social welfare for a given s_e is as follows:

$$SW(s_e) = -\frac{1}{2}n(a(-2+2c+an+2er)+2a(er(-1+\delta)+(-1+c+an)\delta)q_e+(a\delta(-2er+an\delta)+\lambda)q_e^2),$$

$$\textit{s.t.},\ max\left\{0, a(-1+c+2an)\delta+\frac{\lambda-t\lambda}{\delta}\right\} \le s_e \le \frac{a\delta^2(-1+c+2a(n+n\bar{q}\delta))+(1-t+(2-t)\bar{q}\delta)\lambda}{\delta}, \quad (7)$$

and we can get the optimal subsidy as shown in Lemma 1.1.

To simplify the exposition, we let

$$\bar{s_1} = \frac{a^2\delta^3\left(a(1-c)n\delta+2er\left(1-c-an(1+\delta)\right)\right)+a\delta\left((1+an-c(1-t)-t)\delta+er(t-(2-t)\delta)\right)\lambda+(1-t)\lambda^2}{\delta(a\delta(-2er+an\delta)+\lambda)}.$$

Lemma 1.1. *When the government provides cost-share, the farmers' optimal sustainable technologies' input* $q_e^* = \frac{aer+a(1-c-an-er)\delta}{a\delta(-2er+an\delta)+\lambda}$, *the optimal subsidy is:* $s_e = \bar{s_1}$.

All proofs are in the Appendix B.

Lemma 1.2. *When the government provides cost-share, the firm will hire* n *farmers, which satisfies:* $max\left\{r_{11}, \frac{(-1+c+an)\delta}{e(1-\delta)}\right\} \le r \le \frac{a\delta(-1+c+a(n+n\bar{q}\delta))+\bar{q}\lambda}{ae(1-(1-2\bar{q})\delta)}$ and $n > \frac{2aer\delta-\lambda}{a^2\delta^2}$, where $r_{11} = \frac{(a(1-c)\delta^2+\lambda)(a^2n\delta^2+\lambda-t\lambda)}{ae\delta(2a\delta^2(-1+c+an(1+\delta))-(t-(2-t)\delta)\lambda)}$.

Lemma 1.2 shows that when the competition intensity is high ($n > \frac{2aer\delta-\lambda}{a^2\delta^2}$), the government is more willing to compensate the farmers to maximize social welfare. Since $\frac{\partial r_{11}}{\partial t} = \frac{(a(-1+c)\delta^2-\lambda)\lambda(a\delta^2(-2+2c+an(1+\delta))+(-1+\delta)\lambda)}{ae\delta(-2a\delta^2(-1+c+an(1+\delta))+(t+(-2+t)\delta)\lambda)^2} > 0$, it shows that if the firm shares more costs, the government is less likely to provide subsidies given a fixed environmental damage factor. This implies that the firm can help the government reduce expenses. If the firm shares more costs, the government will provide subsidies only when the pollution is more damaging.

The descriptions of wholesale price, profits of the chain members, consumer surplus, and social welfare can be seen in Appendix A.

3.2. The influence of parameters

The influence of parameters on the decisions is analysed in this section, including fixed subsidy ($s_e = s_q = s$), the proportion of cost sharing, the environmental damage factor, and the intensity of the competition.

3.2.1. The influence of subsidy

Proposition 1. *When the subsidy is fixed, if the government provides cost-share, the farmer's optimal profit decreases with the subsidy, and the farmer's optimal sustainable technologies' input increases with the subsidy; while the firm's optimal profit increases with the subsidy; if the government provides flat payment per unit area, the profits of the firm and the farmer both increase with the subsidy.*

Proposition 1 proves that the farmer will have more incentives for sustainable input when the government provides more subsidies. However, higher production quantity caused by sustainable technologies' input will result in a lower selling price and wholesale price, *i.e.,* higher subsidy helps reduce the wholesale price ($\frac{\partial w_e^*}{\partial s} = -\frac{2a^2n\delta^2+\lambda}{2a^3n\delta^3-a(-2+t)\delta\lambda} < 0$, $\frac{\partial w_q^*}{\partial s} = \frac{2a^2n\delta^2+\lambda}{-2a^2n\delta^2+(-2+t)\lambda} < 0$). For the farmer, the profit loss from more investment efforts and a lower wholesale price dominates the profit increase from a higher subsidy, and the whole profit will be lower when the investment effort subsidy is higher. However, if the government provides a flat payment per unit area, as seen in Section 4, the farmer can still benefit from such subsidy. For the firm, we can confirm that the marginal profit (per unit profit of the product) will be as follows: $m_e^* = \frac{a^3(1-c)n\delta^4+a(1-c+an-2ant)\delta^2\lambda+(1-t)^2\lambda^2+\delta(a^2n\delta^2+\lambda)s_e}{2a^3n\delta^4+a(2-t)\delta^2\lambda}$ and $m_q^* = \frac{a^3(1-c)n\delta^4+a(1-c+an-2ant)\delta^2\lambda+(1-t)^2\lambda^2+a\delta^2(a^2n\delta^2+\lambda)s_q}{2a^3n\delta^4+a(2-t)\delta^2\lambda}$, respectively, which increases with s ($\frac{\partial m_e^*}{\partial s} = \frac{a^2n\delta^2+\lambda}{2a^3n\delta^3+a(2-t)\delta\lambda} > 0$, $\frac{\partial m_q^*}{\partial s} = \frac{a^2n\delta^2+\lambda}{2a^2n\delta^2+2\lambda-t\lambda} > 0$). We also note that the whole production quantity (Q_t) increases with s. These two forces result in the firm's profit increasing with s.

3.2.2. The influence of cost-sharing proportion

3.2.2.1. The influence of cost-sharing proportion on the wholesale price

To simplify exposition, we let $s_2 = a(-1+c)\delta + \frac{3\lambda}{\delta} + \frac{4a^4n^2\delta^3}{2a^2n\delta^2+\lambda}$.

Corollary 1.

(1) *When the subsidy is exogenous, if* $s > s_2$, $\frac{\partial w_e^*}{\partial t} < 0$*; if* $s < s_2$*, there exists* $t_{1e}^* = 2 + \frac{2a^2n\delta^2\lambda - \sqrt{\lambda^2(2a^2n\delta^2+\lambda)(a(1-c)\delta^2+\lambda+\delta s_e)}}{\lambda^2}$*; if* $t < t_{1e}^*$, $\frac{\partial w_e^*}{\partial t} > 0$*, if* $t > t_{1e}^*$, $\frac{\partial w_e^*}{\partial t} < 0$.

(2) *When the subsidy is endogenous,* w_e^* *increases with* t.

Since $\frac{\partial s_2}{\partial n} = \frac{8a^4n\delta^3(a^2n\delta^2+\lambda)}{(2a^2n\delta^2+\lambda)^2} > 0$, it can be seen that a stronger competition intensity increases the possibility of the wholesale price increasing with t. If the subsidy is fixed at value, since the increase of sustainable technologies' input can increase production quantity, therefore the selling price will be lower, and the firm's profit first increases and then decreases with the sustainable technologies' input. It can be found that a higher subsidy leads to higher sustainable technologies' input ($\frac{\partial q_e^*}{\partial s} = \frac{1}{2a^2n\delta^2+(2-t)\lambda} > 0$); thus, the sustainable technologies' input will be low if the subsidy is low ($s < s_2$). The firm can motivate farmers to invest more in sustainable technologies by increasing the wholesale price when $t < t_{1e}^*$. If the cost-sharing parameter becomes higher ($t > t_{1e}^*$), the firm will undertake too high costs; thus, the firm will lower the wholesale price. This is because the sustainable technologies' input will be high when the subsidy is high ($s > s_2$), which will be detrimental to the firm. The firm will therefore lower the wholesale price with a higher cost-sharing proportion. Since $\frac{\partial t_{1e}^*}{\partial s} < 0$, we also find that the firm will be less likely to increase the wholesale price when the subsidy is high.

If the subsidy is endogenous, the wholesale price becomes higher with more input costs shared by the firm. Since increasing the wholesale price can motivate farmers to invest more and Proposition 1 shows that the firm benefits more from the subsidy, the firm tends to motivate farmers to invest more so that the government will provide more subsidies.

3.2.2.2. The influence of cost-sharing proportion on members' profit and social welfare

Proposition 2. *When the firm shares a portion of the sustainable technologies' input cost, he can obtain more profit if the subsidy is exogenous and* $s > s_3^e$ *under the government's cost-share or* $s > s_3^q$ *under flat payment per unit area (otherwise, he will obtain less profit), while he can only obtain less profit if the subsidy is endogenous since the optimal subsidy decreases with the proportion of cost sharing, where* $s_3^e = a(-1 + c + 2an)\delta + \frac{\lambda - t\lambda}{\delta}$, $s_3^q = -1 + c + 2an + \frac{\lambda - t\lambda}{a\delta^2}$.

When the subsidy is high, the firm can obtain more sales profit given higher sales quantity, and the profit gain outweighs the profit loss from higher shared input costs. However, when the subsidy is low, he will obtain less profit if he shares the input costs. When the subsidy is endogenous, the optimal subsidy always decreases with the proportion of cost sharing. Furthermore, Proposition 1 shows that the firm will obtain more profit with the increase in subsidy. Therefore, sharing a portion of sustainable technologies' input costs becomes detrimental to the firm's profit.

In the farmer's decision process, the effect of the cost-sharing proportion on the profits is complex. Following Niu *et al.* (2016) and Cui *et al.* (2018), we assume $a = 0.028$, $c = 0.002$, $\delta = 0.8$. We also assume $\lambda = 0.02$, $r = 0.2$, $e = 0.3$, $n = 2$, $s = 0.001$.

Fig. 2 reflects that the farmer's profit will first increase and then decrease with the proportion of cost sharing. When the subsidy is exogenous and high, the firm may benefit from a higher cost-sharing proportion, while the farmer will suffer profit loss since we have confirmed in Corollary 1 that the wholesale price will decrease with t

when t is higher than a threshold level. Fig. 3 studies the effect of the cost-sharing proportion on social welfare, which shows that the firm's cost sharing can help the government increase social welfare.

When the subsidy is endogenous, the farmer's profit will be linearly dependent on t. If the sustainable technologies' input cost coefficient is relatively high, the farmer will obtain more profit by means of the firm's cost sharing under the cost-share; otherwise, he will obtain less profit. However, the farmer will gain less profit with cost sharing under endogenous flat payment per unit area (see Appendix B).

3.2.3. The influence of the environmental damage factor

Proposition 3. *When the subsidy is endogenous, the optimal subsidy, the sustainable technologies' input, and the firm's profit increase with* r. *The wholesale price and the social welfare decrease with* r. *The farmer's profit increases with* r *if* $r > r_{14}$*; otherwise, it decreases with* r *with cost-share. The farmer's profit always increases with* r *with a flat payment per unit area, where* $r_{14} = \frac{2a^4n^2\delta^4+a(-1+c+4an+t-(c+an)t)\delta^2\lambda+\lambda^2}{4a^3en\delta^3+ae(3+t(-1+\delta)-\delta)\delta\lambda}$.

The government raises subsidy when the emission is more environmentally damaging, encouraging the farmer to increase the sustainable technologies' input. Under the government's cost-share, higher production quantity will result in a lower wholesale price; if $r > r_{14}$, the profit gain from higher subsidy and higher production quantity dominates the profit loss from a lower wholesale price. Furthermore, since r_{14} increases with t, we find that the farmer's profit will be more likely to decrease with r if the firm shares more input costs. However, we have confirmed in Proposition 1 that the firm's optimal profit increases with the subsidy and that the subsidy will increase with r; thus, the firm will obtain more profit with the increase of the environmental damage factor. Since r_{14} increases with n, the farmer more easily obtains less profit with the increase of r facing stronger competition.

3.2.4. The influence of competition degree

Proposition 4.

(1) *The optimal sustainable technologies' input always decreases with the degree of competition.*

(2) *When the subsidy is endogenous, the optimal subsidy will increase with the degree of competition if* $r < \frac{t\lambda}{4ae\delta}$*; otherwise, it decreases with the degree of competition.*

(3) *When the subsidy is exogenous, the optimal wholesale price decreases with the degree of competition, while with an endogenous subsidy, it may increase with the degree of competition if* $r > \frac{\lambda}{4ae\delta}$.

When farmers are in a competitive environment since the final selling price decreases with the degree of the competition given the fixed input effort (*i.e.*, $p = 1 - an(1 + \delta q)$ and $\frac{\partial p}{\partial n} < 0$), farmers will lose the enthusiasm for sustainable investment if competition intensifies.

Proposition 4(2) reflects that the effect of competition degree on the optimal subsidy depends on the environmental damage factor. Since the optimal subsidy increases with r, when the environment damaging factor is relatively low ($r < \frac{t\lambda}{4ae\delta}$), the final subsidy may also be lower than that in a situation with more damaging pollution. Thus, when $r < \frac{t\lambda}{4ae\delta}$, the government can still have the economic power to alleviate the drop in sustainable technologies' input by providing more subsidies when the competition level increases. However, when $r > \frac{t\lambda}{4ae\delta}$, the optimal subsidy level ($\overline{s_1}$ and $\overline{s_2}$) tends to be high. If the government continues to raise the subsidy with the increase in competition degree, it will become detrimental to social welfare due to the expensive government expenses. Especially, if the firm does not share the input costs ($t = 0$), the received subsidy would always decrease with the degree of competition. As, from Proposition 2, the optimal subsidy decreases with the proportion of cost sharing, the firm is therefore encouraged to share the sustainable technologies' input costs and consequently help the government motivate sustainable practice.

Proposition 4(2) also shows that the effect of competition degree on the optimal subsidy depends on the rate of cost sharing. As t increases, the optimal subsidy will be easier to increase with the degree of competition. However, Proposition 2 shows that when the firm shares the sustainable technologies' input costs, it can only obtain less profit if the subsidy is endogenously given. Thus, when the subsidy is endogenous, cost sharing will be detrimental to the firm's profit, whereas it can help the government alleviate the economic pressure on promoting sustainable technologies' input.

Interestingly, Proposition 4(3) reflects that the optimal wholesale price may not always decrease with the degree of competition. The subsidy can be used to lower the price of the product (see $\frac{\partial w_e^*}{\partial s} < 0$, $\frac{\partial w_q^*}{\partial s} < 0$ in Proposition 1). When the subsidy decreases with the competition degree in a more damaging environment ($r > \frac{\lambda}{4ae\delta}$), the wholesale price may increase with the competition level because the farmer will obtain less subsidy.

Figs. 4 and 5 show the effect of competition intensity (n) on the social welfare with exogenous subsidy and endogenous subsidy, respectively. It can be seen that the social welfare under different subsidy policies increases as the competition intensity n increases, *i.e.*, stronger competition intensity can help reduce the price and, consequently, increase the customer surplus and contribute to improving social welfare.

When the subsidy is exogenous, the social welfare under the flat payment per unit area is higher than that under cost-share ($SW_q^* > SW_e^*$, see Fig. 4), and the social welfare gap between the two subsidy policies increases with competition intensity. However, $\pi_{re}^* > \pi_{rq}^*$ and $q_e^* > q_q^*$ can be confirmed if the subsidy is exogenous. Thus, the only factor contributing to $SW_q^* > SW_e^*$ is $\pi_{fq}^* > \pi_{fe}^*$, which means that the farmers can obtain more profits under flat payment per unit area if the subsidy is fixed at value.

4. Flat payment per unit area

4.1. Optimal solutions

In this section, we analyse the impact of the flat payment per unit area. The government provides the farmer unit subsidy based on the farmer's production quantity and technology investment supporting sustainability. The farmer chooses her sustainable technologies' input q to maximise the profit:

$$\pi_{fq} = \left(w_q - c + s_q\right)a\left(1 + \delta q_q\right) - \frac{\lambda(1-t){q_q}^2}{2},\ s.t.,\ \ 0 \le q_q \le \bar{q}. \qquad (8)$$

According to the first-order condition, we generate the optimal sustainable technologies' input as follows:

$$q_q = \begin{cases} 0, & \frac{a\delta(c-s_q-w_q)}{(-1+t)\lambda} < 0 \\ \frac{a\delta(c-s_q-w_q)}{(-1+t)\lambda}, & 0 \le \frac{a\delta(c-s_q-w_q)}{(-1+t)\lambda} \le \bar{q}. \\ \bar{q}, & \frac{a\delta(c-s_q-w_q)}{(-1+t)\lambda} > \bar{q} \end{cases} \qquad (9)$$

Therefore, anticipating the farmer's optimal reaction to the sustainable technologies' input, the firm decides his wholesale price so as to maximise the profit:

$$\pi_{rq} = \left(1 - w_q\right)an\left(1 + \delta q_q\right) - a^2n^2\left(1 + \delta q_q\right)^2 - \frac{\lambda nt{q_q}^2}{2} = \frac{1}{2}an(-2an\left(1 - \frac{a\delta^2(c-s_q-w_q)}{(1-t)\lambda}\right)^2 + 2(1 - \frac{a\delta^2(c-s_q-w_q)}{(1-t)\lambda})(1 - w_q) - \frac{at\delta^2(c-s_q-w_q)^2}{(1-t)^2\lambda}),$$

$$s.t.,\ \ c - s_q \le w_q \le c + \frac{(1-t)\bar{q}\lambda}{a\delta} - s_q, \qquad (10)$$

which renders

$$w_3 \triangleq \frac{2a^3cn\delta^4 + a(c+(1-2an)(1-t))\delta^2\lambda - (1-t)^2\lambda^2 - a\delta^2(2a^2n\delta^2+\lambda)s_q}{2a^3n\delta^4 + a(2-t)\delta^2\lambda},$$

$$w_q^* = \begin{cases} c - s_q, & w_3 < c - s_q \\ w_3, & c - s_q \le w_3 \le c + \frac{(1-t)\bar{q}\lambda}{a\delta} - s_q. \\ c + \frac{(1-t)\bar{q}\lambda}{a\delta} - s_q, & w_3 > c + \frac{(1-t)\bar{q}\lambda}{a\delta} - s_q \end{cases} \qquad (11)$$

Substituting (11) into (9), we can obtain the equilibrium sustainable technologies' input:

$$q_q^* = \begin{cases} 0, & w_3 < c - s_q \\ \frac{a(1-c-2an)\delta^2 - (1-t)\lambda + a\delta^2 s_q}{2a^2n\delta^3 + (2-t)\delta\lambda}, & c - s_q \le w_3 \le c + \frac{(1-t)\bar{q}\lambda}{a\delta} - s_q. \\ \bar{q}, & w_3 > c + \frac{(1-t)\bar{q}\lambda}{a\delta} - s_q \end{cases} \qquad (12)$$

the profits of the chain members, consumer surplus, and social welfare, respectively, can be seen in Appendix A.

The social welfare function can be shown as:

$$SW(s_q) = -\frac{\lambda n {q_q}^2}{2} + (1-c)an(1+\delta q_q) - aner(1+\delta q_q)(1-q_q) - \frac{a^2n^2}{2}(1+\delta q_q)^2, \text{ s.t., } \max\left\{0, -1+c+2an+\frac{\lambda - t\lambda}{a\delta^2}\right\} \leq s_q \leq -1+c+2a(n+n\bar{q}\delta)+\frac{(1-t+(2-t)\bar{q}\delta)\lambda}{a\delta^2}. \tag{13}$$

To simplify the exposition, we let

$$\overline{s_2} = \frac{a^2\delta^3\big(a(1-c)n\delta+2er(1-c-an(1+\delta))\big)+a\delta\big((1+an-c(1-t)-t)\delta+er(t-(2-t)\delta)\big)\lambda+(1-t)\lambda^2}{a\delta^2(a\delta(-2er+an\delta)+\lambda)},$$

then we can obtain the optimal subsidy as shown in Lemma 2.1.

Lemma 2.1. *When the government provides flat payment per unit area, the farmers' optimal sustainable technologies' input* $q_q^* = \frac{aer-a(-1+c+an+er)\delta}{a\delta(-2er+an\delta)+\lambda}$, *the optimal subsidy is:* $s_q^* = \overline{s_2}$.

Lemma 2.2. *When the government provides a flat payment per unit area, the firm will hire* n *farmers which satisfies:* $\max\left\{r_{11}, \frac{(-1+c+an)\delta}{e(1-\delta)}\right\} \leq r \leq \frac{a\delta(-1+c+a(n+n\bar{q}\delta))+\bar{q}\lambda}{ae(1-(1-2\bar{q})\delta)}$ *and* $n > \frac{2aer\delta - \lambda}{a^2\delta^2}$.

The explanation here is similar to that in Lemma 1.2, we omit it here. The descriptions of the wholesale price, profits of the chain members, consumer surplus, and social welfare can be seen in Appendix A.

4.2. Comparing analyses

Proposition 5.

(1) *When the subsidy is endogenous, the government's cost-share surpasses the flat payment per unit area because it alleviates the government's financial burden while achieving the same sustainable technologies' input as those under the flat payment per unit area.*

(2) *When keeping the subsidy fixed under two subsidy policies, the government's cost-share can result in higher sustainable investment and higher consumer surplus, and*

it can also help reduce the selling price and the wholesale price of the product.

(3) *When keeping the subsidy fixed under two subsidy policies, the optimal social welfare under the flat payment per unit area is larger (or smaller) than that under the government's cost share if* $s > s_4$ $(or\ \ if\ \ s < s_4)$, *where*

$$s_4 = \frac{2a^2\delta^3\left(a(1-c)n\delta+2er\left(1-c-an(1+\delta)\right)\right)+2a\delta\left((1+an-c(1-t)-t)\delta+er(t-(2-t)\delta)\right)\lambda+2(1-t)\lambda^2}{\delta(1+a\delta)(a\delta(-2er+an\delta)+\lambda)}.$$

From the proof in Appendix B, it is easy to find that if the government intervenes in the subsidy decision, the optimal wholesale price and sustainable technologies' input under the two subsidies are equal, which means that the government can obtain the same results on pollution abatement and social welfare. However, the overall government expenditures under cost-share will be lower than those under flat payment per unit area; thus, cost-share will be a better choice for the government. This finding is similar to the conclusion of Chen *et al.* (2017), who found that emission-reduction innovation subsidy is better than quantity subsidy due to less pollution and higher profit. However, we investigate the situation wherein the local government determines the subsidy for social welfare maximization. It is found that the government can reduce expenditures but achieve the same results in motivating sustainable technologies' input under the government's cost-share as with flat payment per unit area. When the subsidy is fixed at value, the cost-share can also motivate the farmer to invest in more sustainable technologies' input. When the firm shares more input costs, the government is suggested to adopt the flat payment per unit area given the fixed subsidy due to the fact that $s > (<) s_4$, $SW_q^* > (<) SW_e^*$ and $\frac{\partial s_4}{\partial t} = \frac{2(a\delta((-1+c)\delta+er(1+\delta))-\lambda)\lambda}{\delta(1+a\delta)(a\delta(-2er+an\delta)+\lambda)} < 0$.

5. Conclusions

This paper studies a three-stage game model made up of one local government and an agricultural product supply chain. The firm serves as a retailer who may share sustainable technologies' input costs with multiple farmers investing in sustainable technologies, and a local government that provides cost-share or flat payment per unit area for farmers to motivate sustainable technologies' input. The interplay between the firm's cost sharing and the government subsidy is analysed with exogenous subsidy and endogenous subsidy, respectively. Two subsidy policies are also comparatively studied to inform policies for regulating agricultural pollution and motivating the application of sustainable technologies.

The main findings are listed as follows:

(1) The government subsidy can incentivize farmers to invest in sustainable technologies. When the subsidy is exogenous, the optimal sustainable technologies' input in the situation with government's cost-share is higher than flat payment per unit area. When the subsidy is endogenous, the optimal sustainable technologies' input is the same under two different subsidy policies. Besides, when the competition becomes more intense, farmers will invest less in sustainable technologies.

(2) When the subsidy is exogenous, the firm's optimal profit increases with the subsidy under the two subsidy policies; the farmer's optimal profit increases with the subsidy under flat payment per unit area and decreases with the subsidy under cost-share. This result suggests the firm to increase the wholesale prices to compensate the farmers to achieve the goal of motivating the farmers to adopt sustainable technologies when the subsidy is exogenous.

(3) When the subsidy is exogenous, the firm can obtain more profit by sharing the sustainable technologies' input costs with a relatively high subsidy. However, the optimal subsidy decreases with the proportion of cost sharing. Thus, the firm's profit will decrease with the proportion of cost sharing when the subsidy is endogenous.

(4) When more environmentally damaging pollution exists, the government will provide a higher subsidy, motivating farmers to invest in greater sustainable technologies. However, social welfare also decreases with the environmental damage factor due to the increasing government expenditure.

(5) When the subsidy is endogenous, the government's cost-share surpasses flat payment per unit area because it alleviates the government's financial burden while achieving the same technical input results as those under flat payment per unit area. When the subsidy is exogenous, if the firm shares more input costs, the government is suggested to adopt a flat payment per unit area.

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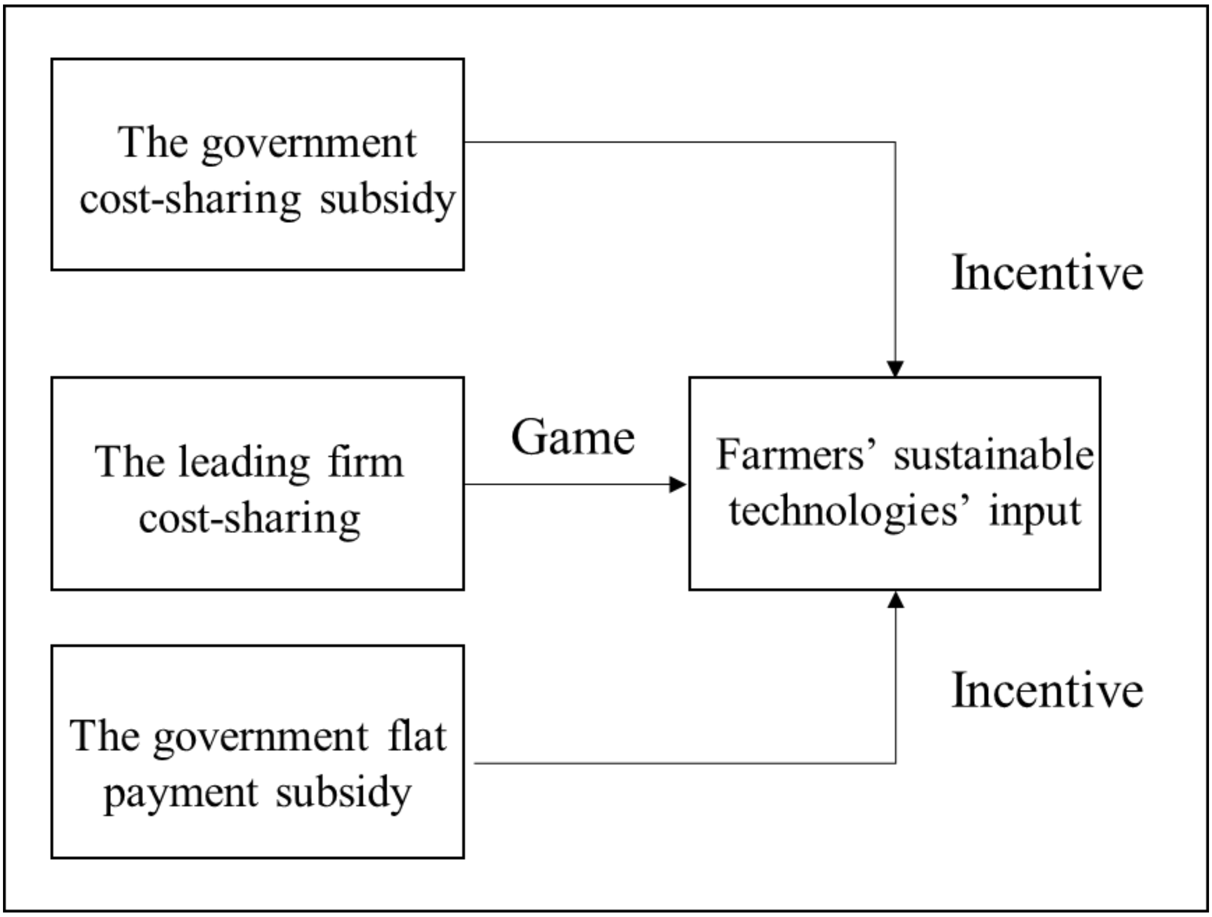

Figure 1. Conceptual framework

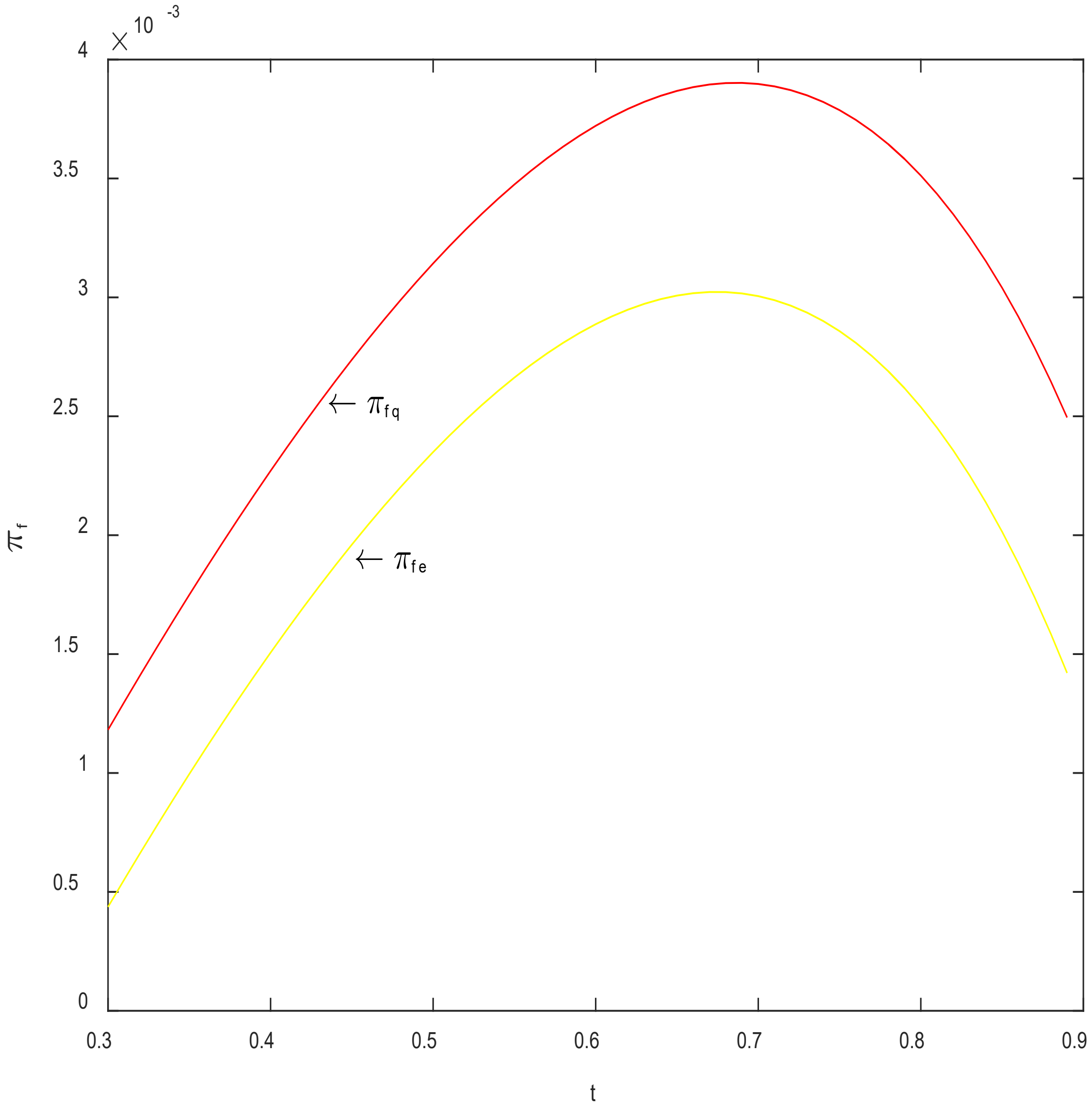

Figure 2. The effect of t on the farmer's profit with an exogenous subsidy.

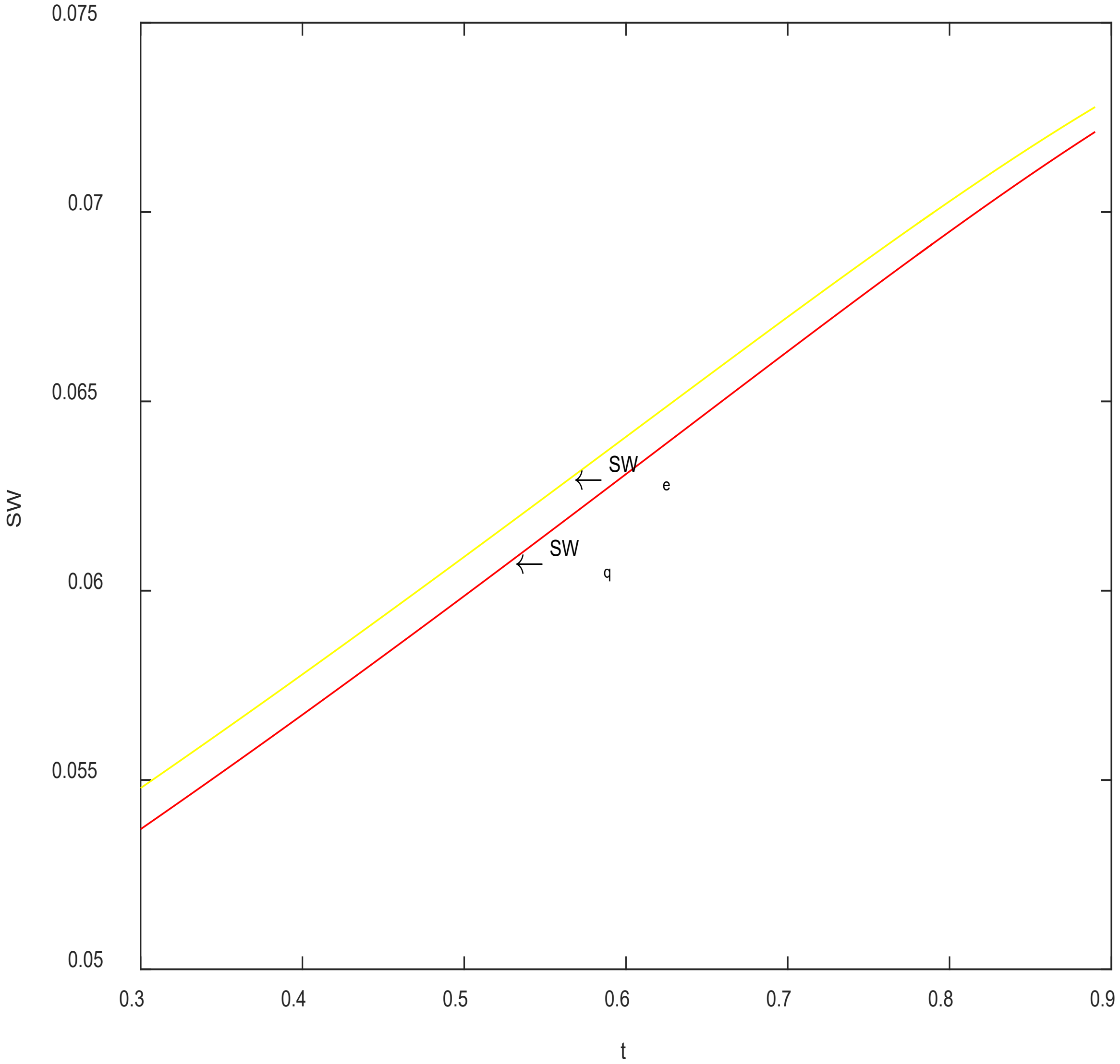

Figure 3. The effect of t on social welfare with an exogenous subsidy.

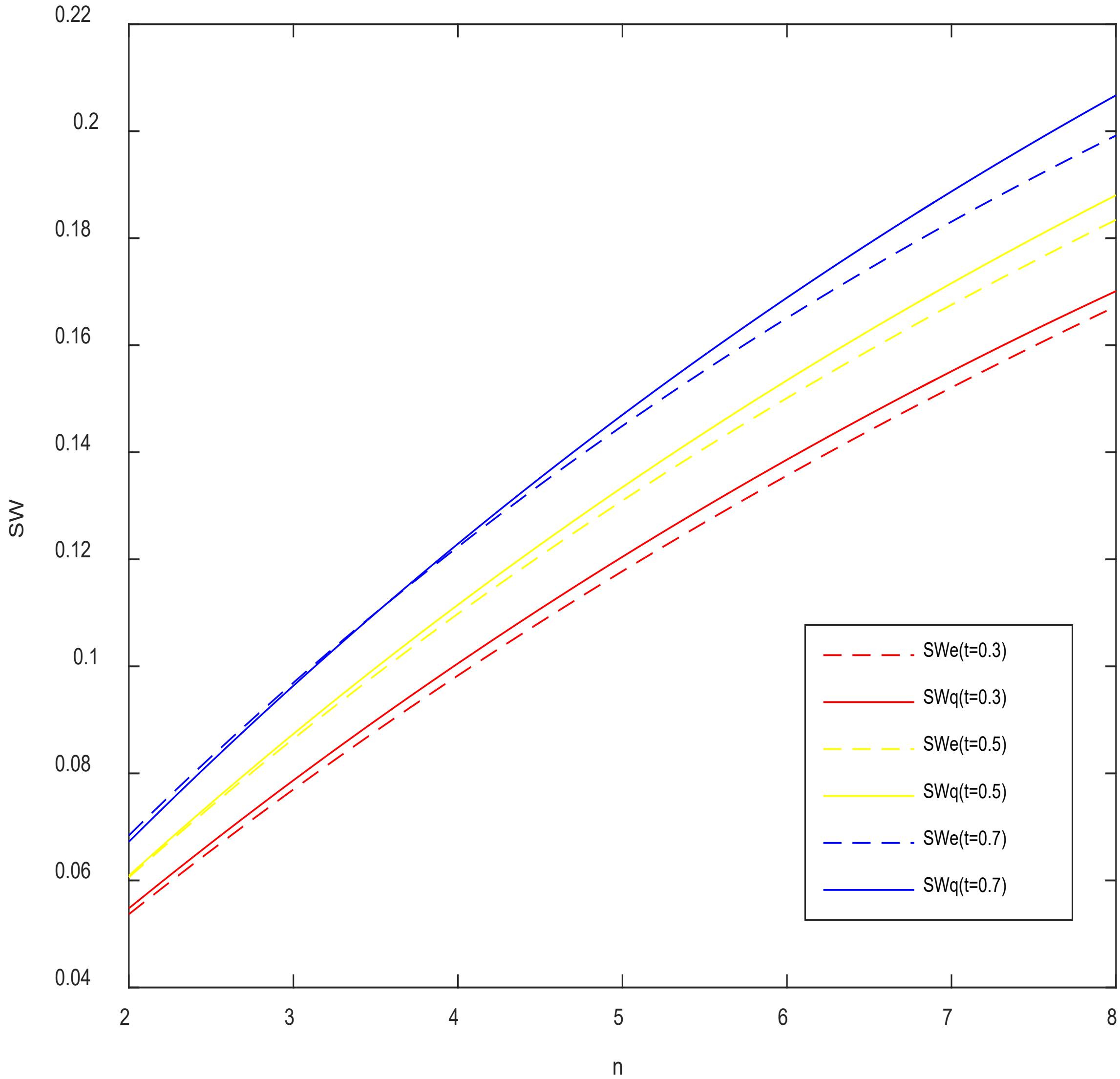

Figure 4. The effect of *n* on social welfare with an exogenous subsidy.

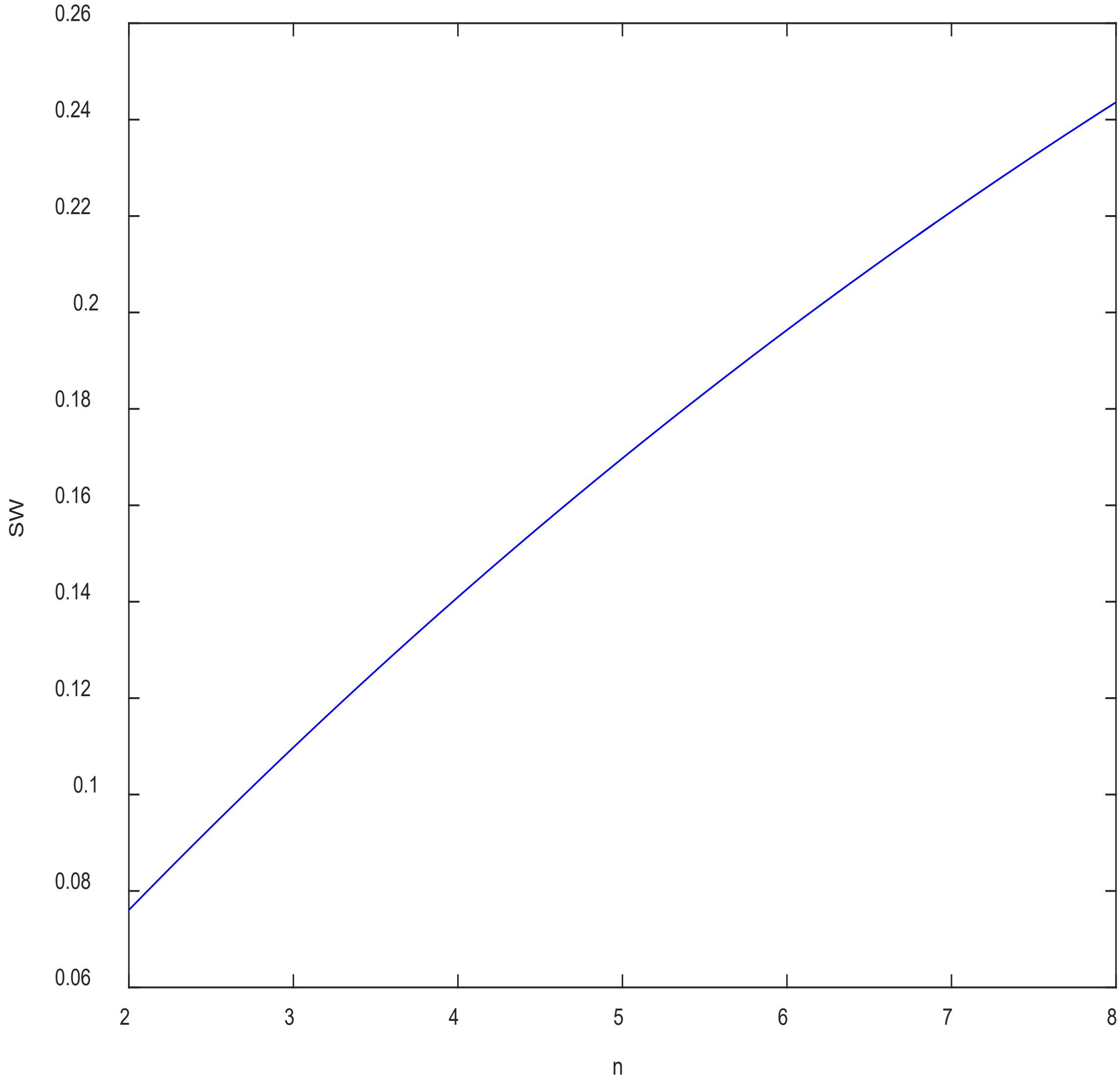

Figure 5. The effect of *n* on social welfare with an endogenous subsidy.

Appendix A. The optimal solutions under different scenarios

Section 4. Government's cost-share

When the subsidy is exogenous, the firm's profit can be denoted as:

$$\pi_{re}^{*} = \frac{n(a^2(1-c)^2\delta^4+2a(1-c-ant)\delta^2\lambda+(1-t)^2\lambda^2+\delta s_e(2a(1-c)\delta^2+2\lambda+\delta s_e))}{4a^2n\delta^4+2(2-t)\delta^2\lambda}. \quad \text{(A.1)}$$

The farmer's profit can be denoted as:

$$\pi_{fe}^{*} = \frac{1}{2(2a^2n\delta^3+(2-t)\delta\lambda)^2}((-1+t)\lambda(a(1-c+2an)\delta^2-(-3+t)\lambda)(a(-1+c+2an)\delta^2+\lambda-t\lambda)+\delta s_e(-8a^4n^2\delta^4+2a(1-4an(2-t)-c(1-t)-t)\delta^2\lambda-2(3-(3-t)t)\lambda^2+(1-t)\delta\lambda s_e)). \quad \text{(A.2)}$$

The consumer surplus is:

$$CS_e^{*} = \frac{a^2n^2(a(1-c)\delta^2+\lambda+\delta s_e)^2}{2(2a^2n\delta^2+(2-t)\lambda)^2}, \quad \text{(A.3)}$$

and the social welfare is:

$$SW_e^{*} = \frac{2a(er(-1+\delta)+(-1+c+an)\delta)(a(1-c-2an)\delta^2-(1-t)\lambda+\delta s_e)}{2a^2n\delta^3+(2-t)\delta\lambda} - \frac{1}{2}n(a(-2+2c+an+2er)+\frac{(a\delta(-2er+an\delta)+\lambda)(a(1-c-2an)\delta^2-(1-t)\lambda+\delta s_e)^2}{(2a^2n\delta^3+(2-t)\delta\lambda)^2}). \quad \text{(A.4)}$$

When the subsidy is endogenous, the corresponding wholesale price is as follows:

$$w_e^{*} = \frac{a^2\delta^3\big(a(-1+2c)n\delta+2er(-1+an(1+\delta))\big)+a\delta\big((c-an(2-t))\delta+er(1-2t+\delta)\big)\lambda-(1-t)\lambda^2}{a\delta^2(a\delta(-2er+an\delta)+\lambda)}. \quad \text{(A.5)}$$

The sustainable technologies' input is as follows:

$$q_e^{*} = \frac{aer+a(1-c-an-er)\delta}{a\delta(-2er+an\delta)+\lambda}. \quad \text{(A.6)}$$

The corresponding profits of the firm and the farmer can be denoted as follows:

$$\pi_{re}^{*} = -\frac{1}{(a\delta(-2er+an\delta)+\lambda)^2(4a^2n\delta^4+2(2-t)\delta^2\lambda)}n(2a^2n\delta^2+(2-t)\lambda)(4a^3n\delta^3\big((-1+c)\delta+er(1-t+\delta)\big)\lambda-2a\delta(er(-2+3t)+(1-c-er)(2-t)\delta)\lambda^2-2(1-t)\lambda^3+a^2\delta^2\lambda((1-c)^2(-2+t)\delta^2+2(1-c)er(2-t)\delta(1+\delta)+e^2r^2\big(t\big(5+\delta(2+\delta)\big)-2(1+\delta)^2\big)-2n(1-t)\lambda)+a^4n\delta^4(-2((-1+c)\delta+er(1+\delta))^2+nt\lambda)), \quad \text{(A.7)}$$

$$\pi_{fe}^{*}=\frac{1}{2\delta^2(a\delta(-2er+an\delta)+\lambda)^2}(2a^3\delta^4(-2er+an\delta)\Big(a(-1+c)n\delta+2er\big(-1+c+an(1+\delta)\big)\Big)+a^2\delta^2\Big(e^2r^2(-3+7t)+2er(-1+c+6an-3er+(-1+c-3an+er)t)\delta+\Big(-2c\big(an(-2+t)+(1-er)(1-t)\big)-a^2n^2(3-t)+c^2(1-t)+(1-er)^2(1-t)+2an(-2+er(4-t)+t)\Big)\delta^2\Big)\lambda+2a\delta\big(an(-3+2t)\delta+er(3-4t+\delta)\big)\lambda^2-2(1-t)\lambda^3). \quad \text{(A.8)}$$

The consumer surplus is:

$$CS_e^{*}=\frac{a^2n^2(-a\delta((-1+c)\delta+er(1+\delta))+\lambda)^2}{2(a\delta(-2er+an\delta)+\lambda)^2}, \quad \text{(A.9)}$$

and the social welfare is:

$$SW_e^{*}=\frac{an(2(1-c-er)\lambda+a(((-1+c)\delta+er(1+\delta))^2-n\lambda))}{2(a\delta(-2er+an\delta)+\lambda)}. \quad \text{(A.10)}$$

Section 5. Flat payment per unit area

When the subsidy is exogenous, the firm's profit can be denoted as:

$$\pi_{rq}^{*}=\frac{n(a^2(1-c)^2\delta^4+2a(1-c-ant)\delta^2\lambda+(1-t)^2\lambda^2+a\delta^2s_q(2a(1-c)\delta^2+2\lambda+a\delta^2s_q))}{4a^2n\delta^4+2(2-t)\delta^2\lambda}. \quad \text{(A.11)}$$

The farmer's profit can be denoted as:

$$\pi_{fq}^{*}=\frac{(-1+t)\lambda(a(-1+c-2an)\delta^2-(3-t)\lambda-a\delta^2s_q)(a(1-c-2an)\delta^2-(1-t)\lambda+a\delta^2s_q)}{2(2a^2n\delta^3+(2-t)\delta\lambda)^2}. \quad \text{(A.12)}$$

The consumer surplus is:

$$CS_q^{*}=\frac{a^2n^2(a(1-c)\delta^2+\lambda+a\delta^2s_q)^2}{2(2a^2n\delta^2+(2-t)\lambda)^2}, \quad \text{(A.13)}$$

and the social welfare is

$$SW_q^{*}=\frac{2a(er(-1+\delta)+(-1+c+an)\delta)(a(1-c-2an)\delta^2-(1-t)\lambda+a\delta^2s_q)}{2a^2n\delta^3+(2-t)\delta\lambda}-\frac{1}{2}n(a(-2+2c+an+2er)+\frac{(a\delta(-2er+an\delta)+\lambda)(a(1-c-2an)\delta^2-(1-t)\lambda+a\delta^2s_q)^2}{(2a^2n\delta^3+(2-t)\delta\lambda)^2}). \quad \text{(A.14)}$$

When the subsidy is endogenous, the corresponding wholesale price is as follows:

$$w_q^{*}=\frac{a^2\delta^3\Big(a(-1+2c)n\delta+2er\big(-1+an(1+\delta)\big)\Big)+a\delta\Big(\big(c-an(2-t)\big)\delta+er(1-2t+\delta)\Big)\lambda-(1-t)\lambda^2}{a\delta^2(a\delta(-2er+an\delta)+\lambda)}. \quad \text{(A.15)}$$

The sustainable technologies' input is as follows:

$$q_q^{*}=\frac{aer-a(-1+c+an+er)\delta}{a\delta(-2er+an\delta)+\lambda}. \quad \text{(A.16)}$$

The corresponding profits of the firm and the farmer can be denoted as follows:

$$\pi_{rq}^* = -\frac{1}{2\delta^2(a\delta(-2er+an\delta)+\lambda)^2} n(4a^3n\delta^3((-1+c)\delta + er(1-t+\delta))\lambda - 2a\delta(er(-2+3t) + (1-c-er)(2-t)\delta)\lambda^2 - 2(1-t)\lambda^3 + a^2\delta^2\lambda((1-c)^2(-2+t)\delta^2 + 2(1-c)er(2-t)\delta(1+\delta) + e^2r^2(t(5+\delta(2+\delta)) - 2(1+\delta)^2) - 2n(1-t)\lambda) + a^4n\delta^4(-2((-1+c)\delta + er(1+\delta))^2 + nt\lambda)), \quad \text{(A.17)}$$

$$\pi_{fq}^* = \frac{a(-1+t)(er(-1+\delta)+(-1+c+an)\delta)\lambda(a\delta((1-c+an)\delta-er(3+\delta))+2\lambda)}{2\delta(a\delta(-2er+an\delta)+\lambda)^2}. \quad \text{(A.18)}$$

The consumer surplus is

$$CS_q^* = \frac{a^2n^2(-a\delta((-1+c)\delta+er(1+\delta))+\lambda)^2}{2(a\delta(-2er+an\delta)+\lambda)^2}, \quad \text{(A.19)}$$

and the social welfare is

$$SW_q^* = \frac{an(2(1-c-er)\lambda+a(((-1+c)\delta+er(1+\delta))^2-n\lambda))}{2(a\delta(-2er+an\delta)+\lambda)}. \quad \text{(A.20)}$$

Appendix B.

Proof of Lemma 1.1

The total social welfare for a given s_e is

$$SW_e = \frac{(a\delta(-2er+an\delta)+\lambda)(a(1-c-2an)\delta^2-(1-t)\lambda+\delta s_e)^2}{(2a^2n\delta^3+(2-t)\delta\lambda)^2}) - \frac{1}{2}n(a(-2+2c+an+2er) + \frac{2a(er(-1+\delta)+(-1+c+an)\delta)(a(1-c-2an)\delta^2-(1-t)\lambda+\delta s_e)}{2a^2n\delta^3+(2-t)\delta\lambda}, \ s.t., \ max\{0, a(-1+c+2an)\delta + \frac{\lambda-t\lambda}{\delta}\} \leq s_e \leq \frac{a\delta^2(-1+c+2a(n+n\bar{q}\delta))+(1-t+(2-t)\bar{q}\delta)\lambda}{\delta}.$$

$$\frac{\partial SW(s_e)}{\partial s_e} = -n\delta(\frac{a(er(-1+\delta)+(-1+c+an)\delta)}{2a^2n\delta^3+(2-t)\delta\lambda} + \frac{(a\delta(-2er+an\delta)+\lambda)(a(1-c-2an)\delta^2-(1-t)\lambda+\delta s_e)}{(2a^2n\delta^3+(2-t)\delta\lambda)^2}),$$

$$\frac{\partial^2 SW(s_e)}{\partial {s_e}^2} = -\frac{n(a\delta(-2er+an\delta)+\lambda)}{(2a^2n\delta^2+(2-t)\lambda)^2},$$

if $a\delta(-2er+an\delta)+\lambda > 0$, there exists $s = \bar{s_1}$, maximizing the social welfare.

Proof of Lemma 1.2

$$\frac{\partial s_e^*}{\partial r} = -\frac{ae(2a^2n\delta^2+(2-t)\lambda)(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2} > 0, \text{ since}$$

$max\left\{0, a(-1+c+2an)\delta+\frac{\lambda-t\lambda}{\delta}\right\} \le s_e \le \frac{a\delta^2\left(-1+c+2a(n+n\bar{q}\delta)\right)+(1-t+(2-t)\bar{q}\delta)\lambda}{\delta}$, when $r=r_{11}$, $s_e=0$; $r=\frac{(-1+c+an)\delta}{e(1-\delta)}$, $s_e=a(-1+c+2an)\delta+\frac{\lambda-t\lambda}{\delta}$; $r=\frac{a\delta(-1+c+a(n+n\bar{q}\delta))+\bar{q}\lambda}{ae(1+(-1+2\bar{q})\delta)}$, $s_e=\frac{a\delta^2\left(-1+c+2a(n+n\bar{q}\delta)\right)+(1-t+(2-t)\bar{q}\delta)\lambda}{\delta}$. Thus, we get $max\left\{r_{11}, \frac{(-1+c+an)\delta}{e(1-\delta)}\right\} \le r \le \frac{a\delta(-1+c+a(n+n\bar{q}\delta))+\bar{q}\lambda}{ae(1-(1-2\bar{q})\delta)}$.

Proof of Proposition 1

When the government provides fixed cost-share, $\frac{\partial q_e^*}{\partial s}=\frac{1}{2a^2n\delta^2+2\lambda-t\lambda}>0$, $\frac{\partial \pi_{re}^*}{\partial s_e}=\frac{n(a(1-c)\delta^2+\lambda+\delta s_e)}{2a^2n\delta^3+(2-t)\delta\lambda}>0$, $\frac{\partial \pi_{fe}^*}{\partial s_e}=\frac{F_2(t)}{((-2+t)\lambda-2a^2n)^2}$, $F_2(t)\triangleq -4a^4n^2\delta^4+a(1-c+4an(-2+t)-t+ct)\delta^2\lambda-(3-(3-t)t)\lambda^2+(1-t)\delta\lambda s_e$. Since $max\{0, a(-1+c+2an)\delta+\frac{\lambda-t\lambda}{\delta}\} \le s_e \le \frac{a\delta^2\left(-1+c+2a(n+n\bar{q}\delta)\right)+(1-t+(2-t)\bar{q}\delta)\lambda}{\delta}$, when $s_e=\frac{a\delta^2\left(-1+c+2a(n+n\bar{q}\delta)\right)+(1-t+(2-t)\bar{q}\delta)\lambda}{\delta}$, $F_2(t)max=-(2a^2n\delta^2+(2-t)\lambda)(\lambda+\delta(2a^2n\delta-(1-t)\bar{q}\lambda))$. Since $\lambda>a\delta(2er-an\delta)$, $\lambda+\delta(2a^2n\delta-(1-t)\bar{q}\lambda)>a\delta\left(2er+an\delta-(1-t)\bar{q}\delta(2er-an\delta)\right)>0$, thus, $F_2(t)max<0$, $\frac{\partial \pi_{fe}^*}{\partial s_e}<0$.

When the government provides fixed flat payment per unit area,

$\frac{\partial \pi_{rq}^*}{\partial s_q}=\frac{an(a(1-c)\delta^2+\lambda+a\delta^2 s_q)}{2a^2n\delta^2+(2-t)\lambda}>0$, $\frac{\partial \pi_{fq}^*}{\partial s_q}=\frac{a(-1+t)\lambda(a(-1+c)\delta^2-\lambda-a\delta^2 s_q)}{(2a^2n\delta^2+(2-t)\lambda)^2}>0$.

Proof of Corollary 1

When the subsidy is exogenous, $max\{0, a(-1+c+2an)\delta+\frac{\lambda-t\lambda}{\delta}\} \le s_e \le \frac{a\delta^2\left(-1+c+2a(n+n\bar{q}\delta)\right)+(1-t+(2-t)\bar{q}\delta)\lambda}{\delta}$, $\frac{\partial w_e^*}{\partial t}=\frac{\lambda F_1(t)}{a(2a^2n\delta^3+(2-t)\delta\lambda)^2}$, $F_1(t)\triangleq 2a^3n(-1+c+2an)\delta^4+a(-1+c+6an-4ant)\delta^2\lambda+(3-t)(1-t)\lambda^2-\delta(2a^2n\delta^2+\lambda)s_e$. $F_1(t=1)=\delta(2a^2n\delta^2+\lambda)(-a(1-c-2an)\delta-s_e)<-(1-t)\lambda(2a^2n\delta^2+\lambda)<0$, $F_1(t=0)=2a^3n(-1+c+2an)\delta^4+a(-1+c+6an)\delta^2\lambda+3\lambda^2-\delta(2a^2n\delta^2+\lambda)s_e$, if $s_e>a(-1+c)\delta+\frac{3\lambda}{\delta}+\frac{4a^4n^2\delta^3}{2a^2n\delta^2+\lambda}$, $F_1(t=0)<0$; else, $F_1(t=0)>0$.

When the subsidy is endogenous, $\frac{\partial w_e^*}{\partial t} = \frac{\lambda}{a\delta^2} > 0$.

Proof of Proposition 2

a. Government's cost-share

When the subsidy is exogenous,

$\frac{\partial \pi_{re}^*}{\partial t} = -\frac{n\lambda(a(-1+c+2an)\delta^2+\lambda-t\lambda-\delta s_e)(a(1-c+2an)\delta^2+(3-t)\lambda+\delta s_e)}{2(2a^2n\delta^3+(2-t)\delta\lambda)^2}$, $\frac{\partial \pi_{re}^*}{\partial t} > 0$. $\frac{\partial q_e^*}{\partial t} = \frac{\lambda(a(1-c)\delta^2+\lambda+\delta s_e)}{\delta(2a^2n\delta^2+(2-t)\lambda)^2} > 0$.

When the subsidy is endogenous, $\frac{\partial \pi_{re}^*}{\partial t} = \frac{n\lambda F_1(r)}{2\delta^2(a\delta(-2er+an\delta)+\lambda)^2}$, $F_1(r) \triangleq a^2\delta^2(-((1-c)^2+a^2n^2)\delta^2+2er\delta(1+2an+\delta-c(1+\delta))-e^2r^2(5+\delta(2+\delta)))+2a\delta((-1+c-an)\delta+er(3+\delta))\lambda-2\lambda^2$; when $r = \frac{a\delta^2(1+2an+\delta-c(1+\delta))+(3+\delta)\lambda}{ae\delta(5+\delta(2+\delta))}$, $F_1(r)max = -\frac{(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)^2}{5+\delta(2+\delta)} < 0$. Thus, $\frac{\partial \pi_{re}^*}{\partial t} < 0$; $\frac{\partial \pi_{fe}^*}{\partial t} = \frac{\lambda F_1(\lambda)}{2\delta^2(a\delta(-2er+an\delta)+\lambda)^2}$, $F_1(\lambda) = a^4n^2\delta^4 - 2a^3n\delta^3((-1+c)\delta+er(3+\delta)) - 8aer\delta\lambda + 2\lambda^2 - a^2\delta^2(-7e^2r^2+2er(1-c-er)\delta+(1-c-er)^2\delta^2-4n\lambda)$. Since $\frac{\partial F_1(\lambda)}{\partial \lambda} = 4(a\delta(-2er+an\delta)+\lambda) > 0$, if $\lambda > \frac{1}{2}(2a\delta(2er-an\delta)+\sqrt{2}\sqrt{a^2\delta^2(er(-1+\delta)+(-1+c+an)\delta)^2})$, $\frac{\partial \pi_{fe}^*}{\partial t} > 0$; $\frac{\partial s_e^*}{\partial t} = \frac{(a\delta((-1+c)\delta+er(1+\delta))-\lambda)\lambda}{\delta(a\delta(-2er+an\delta)+\lambda)} < 0$.

b. Flat payment per unit area

When the subsidy is exogenous,

$\frac{\partial \pi_{rq}^*}{\partial t} = -\frac{n\lambda(a(-1+c+2an)\delta^2+\lambda-t\lambda-a\delta^2 s_q)(a(1-c+2an)\delta^2+(3-t)\lambda+a\delta^2 s_q)}{2(2a^2n\delta^3+(2-t)\delta\lambda)^2}$; since $s_q > -1+c+2an+\frac{\lambda-t\lambda}{a\delta^2}$, $\frac{\partial \pi_{rq}^*}{\partial t} > 0$; $\frac{\partial q_q^*}{\partial t} = \frac{\lambda(a(1-c)\delta^2+\lambda+a\delta^2 s_q)}{\delta(2a^2n\delta^2+(2-t)\lambda)^2} > 0$.

When the subsidy is endogenously given, $\frac{\partial \pi_{rq}^*}{\partial t} = \frac{n\lambda F_2(r)}{2\delta^2(a\delta(-2er+an\delta)+\lambda)^2}$, $F_2(r) \triangleq a^2\delta^2(-((1-c)^2+a^2n^2)\delta^2+2er\delta(1+2an+\delta-c(1+\delta))-e^2r^2(5+\delta(2+\delta)))+2a\delta((-1+c-an)\delta+er(3+\delta))\lambda-2\lambda^2$. When $r =$

$\frac{a\delta^2(1+2an+\delta-c(1+\delta))+(3+\delta)\lambda}{ae\delta(5+\delta(2+\delta))}$, $F_2(r)max = -\frac{(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)^2}{5+\delta(2+\delta)} < 0$. Thus, $\frac{\partial \pi_{rq}^*}{\partial t} < 0$. $\frac{\partial \pi_{fq}^*}{\partial t} = \frac{a(er(-1+\delta)+(-1+c+an)\delta)\lambda(a\delta((1-c+an)\delta-er(3+\delta))+2\lambda)}{2\delta(a\delta(-2er+an\delta)+\lambda)^2} < 0$.

Since $a\delta(-2er + an\delta) + \lambda > 0$, we can get $a\delta\big((1 - c + an)\delta - er(3 + \delta)\big) + 2\lambda > 0$, thus, $\frac{\partial \pi_{fq}^*}{\partial t} < 0$.

Proof of proposition 3

When the government provides cost-share, $\frac{\partial s_e^*}{\partial r} = -\frac{ae(2a^2n\delta^2+(2-t)\lambda)(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2} > 0$, $\frac{\partial q_e^*}{\partial r} = \frac{ae(-a\delta^2(-2+2c+an(1+\delta))+\lambda-\delta\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2} > 0$,

$\frac{\partial \pi_{re}^*}{\partial r} = \frac{aen(a\delta((-1+c)\delta+er(1+\delta))-\lambda)(2a^2n\delta^2+(2-t)\lambda)(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)}{\delta(a\delta(-2er+an\delta)+\lambda)^3} > 0$;

$\frac{\partial w_e^*}{\partial r} = \frac{e(2a^2n\delta^2+\lambda)(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)}{\delta(a\delta(-2er+an\delta)+\lambda)^2} < 0$,

$$\frac{\partial \pi_{fe}^*}{\partial r} = \frac{ae(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)(2a^3n\delta^3(-2er+an\delta)+a\delta((-1+c+4an+t-(c+an)t)\delta-er(3-t-\delta+t\delta))\lambda+\lambda^2)}{\delta(a\delta(-2er+an\delta)+\lambda)^3}$$

; if $r > r_{14} = \frac{2a^4n^2\delta^4+a(-1+c+4an+t-(c+an)t)\delta^2\lambda+\lambda^2}{4a^3en\delta^3+ae(3+t(-1+\delta)-\delta)\delta\lambda}$, $\frac{\partial \pi_{fe}^*}{\partial r} > 0$; else, $\frac{\partial \pi_{fe}^*}{\partial r} < 0$.

$\frac{\partial SW_e^*}{\partial r} = \frac{aen(a\delta((-1+c)\delta+er(1+\delta))-\lambda)(a(-er(1+\delta)+\delta(-1+c+an(1+\delta)))+\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2}$.

Since $r < \frac{a\delta(-1+c+a(n+n\bar{q}\delta))+\bar{q}\lambda}{ae(1+(-1+2\bar{q})\delta)}$, we can get $a(-1 + c + 2re) - \lambda < \frac{(1+\bar{q}\delta)(a\delta^2(-2+2c+an(1+\delta))+(-1+\delta)\lambda)}{1-(1-2\bar{q})\delta} < 0$, $a(\delta(-1 + c + an(1 + \delta)) - er(1 + \delta)) + \lambda > \frac{(1-\bar{q})(a\delta^2(2-2c-an(1+\delta))+(1-\delta)\lambda)}{1-(1-2\bar{q})\delta} > 0$; thus, $\frac{\partial SW_e^*}{\partial r} < 0$.

$\frac{\partial r_{14}}{\partial t} = -\frac{\lambda(2a^2n\delta^2+\lambda)(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)}{ae\delta(4a^2n\delta^2+(3-t(1-\delta)-\delta)\lambda)^2} > 0$

When the government provides flat payment per unit area, the impacts of r on optimal sustainable technologies' input, the wholesale price, and the social welfare are the same as that under cost-share because $q_e^* = q_q^*$, $w_e^* = w_q^*$, $SW_e^* = SW_q^*$.

$a\delta^2\big(-2 + 2c + an(1 + \delta)\big) - (1 - \delta)\lambda < 2a\delta(er(-1 + \delta) + (-1 + c + an)\delta) <$

0. $a\delta\big((-1+c)\delta+er(1+\delta)\big)-\lambda<a\delta(er(-1+\delta)+(-1+c+an)\delta)<0$

$\frac{\partial s_q^*}{\partial r}=-\frac{e(2a^2n\delta^2+(2-t)\lambda)(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)}{\delta(a\delta(-2er+an\delta)+\lambda)^2}>0,$

$\frac{\partial \pi_{fq}^*}{\partial r}=\frac{ae(1-t)(a\delta((-1+c)\delta+er(1+\delta))-\lambda)\lambda(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)}{\delta(a\delta(-2er+an\delta)+\lambda)^3}>0,$

$\frac{\partial \pi_{rq}^*}{\partial r}=\frac{aen(a\delta((-1+c)\delta+er(1+\delta))-\lambda)(2a^2n\delta^2+(2-t)\lambda)(a\delta^2(-2+2c+an(1+\delta))-(1-\delta)\lambda)}{\delta(a\delta(-2er+an\delta)+\lambda)^3}>0.$

Proof of Proposition 4

a. Government's cost-share

When the subsidy is exogenous, $\frac{\partial q_e^*}{\partial n}=\frac{2a^2\delta(a(-1+c)\delta^2-\lambda-\delta s_e)}{(2a^2n\delta^2+(2-t)\lambda)^2}<0$, $\frac{\partial w_e^*}{\partial n}=\frac{2a(1-t)\lambda(a(-1+c)\delta^2-\lambda-\delta s_e)}{(2a^2n\delta^2+(2-t)\lambda)^2}<0.$

When the subsidy is endogenous,

$\frac{\partial w_e^*}{\partial n}=-\frac{a(4aer\delta-\lambda)(a\delta((-1+c)\delta+er(1+\delta))-\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2}$ (if $r<\frac{\lambda}{4ae\delta}$, $\frac{\partial w_e^*}{\partial n}<0$; else, $\frac{\partial w_e^*}{\partial n}>0$);

$\frac{\partial s_e^*}{\partial n}=\frac{a^2\delta(a\delta((-1+c)\delta+er(1+\delta))-\lambda)(4aer\delta-t\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2}$ (if $r<\frac{t\lambda}{4ae\delta}$, $\frac{\partial s_e^*}{\partial n}>0$; else, $\frac{\partial s_e^*}{\partial n}<0$);

$\frac{\partial q_e^*}{\partial n}=\frac{a^2\delta(a\delta((-1+c)\delta+er(1+\delta))-\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2}<0.$

b. Flat payment per unit area

When the subsidy is exogenous,

$\frac{\partial q_q^*}{\partial n}=\frac{2a^2\delta(a(-1+c)\delta^2-\lambda-a\delta^2 s_q)}{(2a^2n\delta^2+(2-t)\lambda)^2}<0$; $\frac{\partial w_q^*}{\partial n}=\frac{2a(1-t)\lambda(a(-1+c)\delta^2-\lambda-a\delta^2 s_q)}{(2a^2n\delta^2+(2-t)\lambda)^2}<0.$

When the subsidy is endogenously given,

$\frac{\partial w_q^*}{\partial n}=-\frac{a(4aer\delta-\lambda)(a\delta((-1+c)\delta+er(1+\delta))-\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2}$ (if $r<\frac{\lambda}{4ae\delta}$, $\frac{\partial w_q^*}{\partial n}<0$; else, $\frac{\partial w_q^*}{\partial n}>0$);

$\frac{\partial s_q^*}{\partial n}=\frac{a(a\delta((-1+c)\delta+er(1+\delta))-\lambda)(4aer\delta-t\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2}$ (if $r<\frac{t\lambda}{4ae\delta}$, $\frac{\partial s_q^*}{\partial n}>0$; else, $\frac{\partial s_q^*}{\partial n}<0$);

$\frac{\partial q_q^*}{\partial n}=\frac{a^2\delta(a\delta((-1+c)\delta+er(1+\delta))-\lambda)}{(a\delta(-2er+an\delta)+\lambda)^2}<0.$

Proof of Proposition 5

(1) When the subsidy is endogenous, $\frac{\overline{s_2}}{\overline{s_1}}=\frac{1}{a\delta}>1$, $\overline{s_2}>\overline{s_1}$, $q_q^*=q_e^*$, $w_q^*=w_e^*$, $SW_q^*=SW_e^*$. If the government provides cost-share, the overall government

expenditures can be $S_e = \overline{s_1} n q_e^*$. If the government provides flat payment per unit area, the overall government expenditures can be $S_q = \overline{s_2}\left(1 + \delta q_q^*\right)n$; thus, $\frac{S_e}{S_q} = \frac{\delta q_e^*}{1+\delta q_e^*} < 1$.

(2) When the subsidy is exogenous,

$q_q^* - q_e^* = \frac{s(-1+a\delta)}{2a^2n\delta^2+(2-t)\lambda} < 0$, $w_q^* - w_e^* = \frac{s(1-a\delta)\left(2a^2n\delta^2+\lambda\right)}{2a^3n\delta^3-a(-2+t)\delta\lambda} > 0$,

$CS_q^* - CS_e^* = \frac{s\delta a^2n^2(a\delta-1)\left(a(2-2c+s)\delta^2+\delta s+2\lambda\right)}{2(2a^2n\delta^2+(2-t)\lambda)^2} < 0$,

$SW_q^* - SW_e^* = \frac{ns(1-a\delta)F(s)}{2\delta(2a^2n\delta^2+(2-t)\lambda)^2}$, $F(s) = a\delta^2(an\delta(s + a(-2 + 2c + s)\delta) + 2er(-s(1 + a\delta) + 2a\delta(-1 + c + an(1 + \delta)))) + \delta(s - 2aert + a(-2 + 2c - 2an + 4er + s - 2(-1 + c + er)t)\delta)\lambda + 2(-1 + t)\lambda^2$, $\frac{\partial F(s)}{\partial s} = \delta(1 + a\delta)(a\delta(-2er + an\delta) + \lambda) > 0$, $F(s_4) = 0$, if $s > (<) s_4$, $SW_q^* > (<) SW_e^*$.