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Bi-objective optimization of last-train timetabling with multimodal coordination in urban transportation ☆

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ABSTRACT

When urban rail transit (URT) does not provide 24-hour services, passengers who travel at late night may not be able to reach their destinations with only URT trains. As a result, passengers have to find alternative transport means, or combine URT trains with other transport services to fulfill their journeys. This paper investigates the integrated optimization of last train timetabling and bridging service design with consideration of passenger path choices. Two bridging services are considered: taxis and buses. Based on pre-constructed path sets, a bi-objective mixed-integer nonlinear programming (MINLP) model is developed, aiming at minimizing total passenger travel time and total passenger travel cost. To reduce the model scale and improve solution efficiency, three path dominance principles are proposed to remove redundant passenger paths without loss of optimality. An adaptive iterative algorithm is designed to obtain the Pareto frontier curve. The proposed model and solution methods are demonstrated on the Chengdu URT network. Results indicate that passenger travel costs and travel times can be significantly reduced by the integrated optimization. It also provides passengers with a safer night travel environment due to the reduction in passenger travel times in taxis.

1. Introduction

Urban rail transit (URT) system plays a pivotal role in urban passenger transportation to alleviate road congestion and reduce environmental pollution. Due to the benefits of large capacity, wide accessibility, high security, and low carbon emissions, etc., brought by URT systems, urban residents increasingly rely on it for their daily travels. Nowadays, more and more URT networks have been constructed and subsequently expanded in megacities. As of the end of 2020, a total of 538 cities in 77 countries have operated urban rail transit, with an operating mileage of more than 33,346 km (Han et al., 2021). However, the URT systems in most countries (e.g., China, Singapore, Japan, Denmark, Italy, etc.) do not provide 24-hour services. They are closed late at night and/or early morning for system maintenance and operation cost savings purposes. In this sense, the last train for each URT line every day would be the final chance for passengers to reach their destinations via URT trains. However, since URT trains cannot traverse different URT lines due to facility restrictions, passengers can sometimes miss the connecting train due to late arrivals at the transfer station. When

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Table 1
Comparison of bridging taxis and bridging buses.

	Type	Safety	Flexibility	Fixed route	Speed	Price	Capacity	Operating costs
Taxi	Private	Low	High	No	Fast	High	Small	No
Bus	Public	High	Low	Yes	Slow	Low	Large	Yes

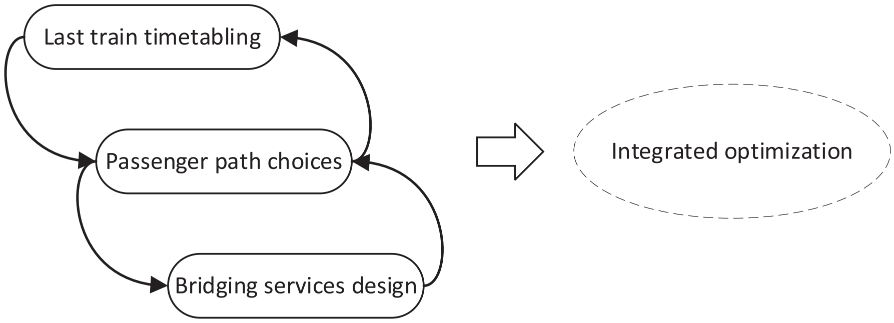

Fig. 1. The hierarchical planning process.

the connecting train is the last train, passengers will not be able to reach their intended destinations via URT trains only.

For URT operators, they try to transport as many passengers as possible through effective last-train timetable design. Nevertheless, limitations of URT network topology and large variation in individual passenger behavior make it impossible for URT operators to transport all passengers to their intended destinations via last trains, especially when it comes to large-scale URT networks. In some scenarios, passengers may need to make long detours through the URT network to reach their destinations, meaning a relatively long travel time for passengers and it is not desirable. Instead, passengers may change their minds at any point on a URT train and switch to a more reliable transportation mode for the remainder of their journeys, especially when passengers have realized that they have little chance to reach their intended destinations via URT trains only. Therefore, in cities without 24-hour URT services, multimodal paths can be easily developed as a norm for late-night passengers, and effective coordination between last URT trains and alternative transportation modes becomes exclusively important for URT operators to maintain their operational performance, although it is very challenging.

The term ‘multimodal path’ refers to the use of more than one transport mode to complete a journey. During late-night, passengers traveling by URT trains can choose alternative transport means to help complete their travels, especially if certain destinations cannot be reached by the last URT trains. Particularly, taxi services and bus services as parts of road transport are not uncommon to be provided for passengers as bridging services to address reachability issues in late night. The characteristics of taxis and buses are compared in Table 1. In contrast, a passenger can transfer to a bridging taxi at any URT station to reach his/her destination directly, but he/she can only transfer to a specific bridging bus at a designated station to reach the destination. This is because it is unrealistic to have a bridging bus between any two stations. Due to high operating costs, it is not economical to deploy bridging buses when passenger demand is quite low. Dispatchers need to design operating routes and schedules for bridging buses in advance according to the passenger demand and the timetables of the last URT trains.

This paper aims to improve the service quality of passengers travelling at night by optimizing the timetable of last URT trains and designing bridging services. An intuitive approach is to address this problem in a step-by-step manner, as shown in Fig. 1. First, the last train timetabling is executed to determine the arrival and departure times of each train at each station and the feasibility of each transfer. Second, passengers are routed based on the feasibility of each transfer. Third, bridging services are designed to serve the passengers who cannot reach their destinations within acceptable travel times. However, the solution obtained by this process is hardly the optimal solution. This is because when the bridging services are determined, passengers may change to more favorable paths (e.g. lower travel costs or/and shorter travel times). However, some favorable paths may not be accessible, prompting adjustments to train timetables. Therefore, it is necessary to integrate last train timetable optimization and bridging service design under consideration of passenger path choices.

The integrated optimization of last train timetabling and bridging service design brings the following difficulties and challenges:

First, there would be a huge number of candidate paths for passengers. On the one hand, this is due to the complexity of the URT network; on the other hand, it is because passengers might transfer to bridging services at any URT stations. Since the most favorable paths for passengers may vary with the train timetable and bridging services, it is a challenge to select paths for passengers based on variable train timetables and bridging services among the numerous candidate paths.

Second, there would be a huge number of candidate routes for bridging services. On the one hand, there will be a bridging taxi route between any two URT stations; on the other hand, a combination of some URT stations can form a bridging bus route. Taxi bridging and bus bridging have their pros and cons. The optimal design of bridging services needs to consider the path choices of passengers. It is challenging to design bridging services based on variable path choices for passengers among the numerous options.

Third, the last train timetable is required to be optimized based on the path choices of passengers. Besides the constraints on train safety operation, the established train timetable needs to ensure that all transfers included in the selected passenger paths are feasible. It is challenging to optimize the last train timetable based on variable path choices for passengers.

Fourth, since passengers may transfer between non-last trains and last trains, appropriate adjustments to the timetable of non-last trains may further improve service quality. The increase of trains with variable arrival and departure times increases the difficulty of solving the integrated optimization problem.

With respect to the above-discussed challenges, this study focuses on the integrated optimization of the last train timetabling problem and bridging service design problem under consideration of passenger path choices. Through a bi-objective mixed-integer nonlinear programming (MINLP) model, this study aims at: 1) finding an optimal last-train timetable and bridging services that allows passengers to reach their destination timely and cost-effectively; 2) developing an effective passenger path generation method that assures the practicability of this model in handling large network size problems; 3) revealing managerial insights to support URT operators' decision-making. The remainder of the paper is organized as follows. Section 2 presents a comprehensive literature review of the last train timetabling problem, identifies research gaps, and articulates novelties. Section 3 presents the bi-objective MINLP model for the last train timetable optimization incorporating bridging service design. Section 4 introduces how to construct candidate path sets for passengers and emphatically expounds three path dominance principles. Section 5 presents an adaptive iterative algorithm to obtain the Pareto frontier curve of the bi-objective optimization model. A case study of a real-world URT network is conducted in Section 6 to evaluate the proposed model and solution methods with practical applications. Finally, Section 8 concludes the paper and provides suggestions for future research.

2. Literature review

As we aim to investigate the integrated optimization of last train timetabling problem and bridging design problem with consideration of passenger path choices, we organize this part from three aspects: the last train timetabling (LTT) problem without considering passenger path choices, the LTT problem with considering passenger path choices, and the bridging bus design problem. Based on the literature review, the contributions of this paper are given at the end.

2.1. The LTT problem without considering passenger path choices

Many studies for an LTT problem do not take passenger path choices into account. These studies assume that the number of passengers who need to transfer at each transfer station is fixed. Then, the last train timetable is optimized to provide passengers with more feasible transfers, or to reduce the passenger transfer waiting time. For example, Xu et al. (2008) propose a heuristic multi-layer coordination algorithm to calculate the desired departure time periods of the last trains with the aim of minimizing total passenger transfer waiting time. Kang et al. (2015a) propose a non-linear programming model as well as a genetic algorithm for the LTT problem to maximize the number of feasible transfers and minimize total transfer waiting times at the same time. To solve large-scale problems, Kang and Meng (2017) develop a mixed-integer linear programming (MILP) model and a two-phase decomposition method to address the last train departure time choice problem. Guo et al. (2020) develop a bi-objective MILP model to determine the last train timetable in such a way that the number of transfer synchronization events can be maximized and meanwhile the longest transfer synchronization time can be minimized. The existing researches that focus on the LTT problem but do not consider passenger path choices also include Chen et al. (2019a), Dou and Guo (2017), Guo et al. (2017), Huang et al. (2021), Kang et al. (2015b, 2019, 2020), Kang and Zhu (2017), Nie et al. (2021), Yin et al. (2019), Zhang et al. (2021a,b), Zhou et al. (2018), Yang et al. (2017a,b, 2018, 2021).

Few studies incorporate LTT problems with other transport modes coordination. For example, Kang et al. (2019) study the design of bus bridging services for the passengers who fail to transfer between the last trains. They develop a last train and bus bridging coordination MILP model to maximize the number of feasible train-to-train transfers and minimize the total waiting times for train-to-bus passengers. Huang et al. (2021) progressively develop three MILP models to optimize the last train timetable incorporating intermodal coordination (i.e., intercity trains and airplanes). They classify the URT stations and lines into different levels. Based on the refined classification of stations and lines, they aim to maximize the intermodal coordination, considering the space–time distribution of the arrivals and departures of the connecting modes.

The advantage of not considering passenger path choices is that it can effectively reduce the scale and the complexity of the optimization, so that other influencing factors of an LTT problem can be further considered. For example, Yang et al. (2017a,b, 2018, 2021) study the LTT problem under transfer passenger demand uncertainty. Chen et al. (2019a) optimize the last train timetable considering heterogeneous transfer walking times. However, since passengers can reach their destinations only if all transfers included in the selected paths are feasible, more feasible transfers do not guarantee that more passengers can reach their destinations (Chen et al., 2019b; Zhou et al., 2019). Additionally, it is not realistic to assume that passenger path choices are always fixed, especially when the LTT problem is embedded in cooperation with flexible transport modes (e.g. taxis).

2.2. The LTT problem with considering passenger path choices

Differently, some studies for LTT problems include passenger path choices considerations (e.g., Chen et al., 2019b; Li et al., 2016; Long et al., 2020; Yang et al., 2020; Yao et al., 2019; Zhou et al., 2013, 2019). They optimize the last train timetable and passenger path choices at the same time, with the aim of maximizing the total number of destination-reachable passengers or origin–destination (OD) pairs. To the best of our knowledge, all the existing studies on LTT problems considering passenger path choices only develop their optimization models based on pre-generated passenger candidate paths. For instance, Zhou et al. (2013) study the last-train departure time choice problem and formulate passenger route choices with a Logit model. Li et al. (2016) propose a method to calculate the last boarding time (i.e., the latest time for a passenger to board the last train at his/her origin station in order to reach his/her destination successfully). Based on this, they propose a non-linear programming model and a genetic algorithm for the LTT problem to maximize the number of destination-reachable passengers. Chen et al. (2019b) and Zhou et al. (2019) propose two different MILP models to optimize the last train timetable with the aim of maximizing the number of destination-reachable passengers. Specifically, Chen et al. (2019b) model passengers' destination reachability with passengers' actual arrival times at their destinations. To solve large-scale instances, a genetic algorithm is designed to solve the model. Differently, Zhou et al. (2019) use the logical relations between the transfer feasibility and destination reachability to formulate passengers' destination reachability. In addition, they propose some network simplification strategies to reduce the model scale, so that this model can be solved by the commercial optimization software. Besides, Yang et al. (2020) formulate the LTT problem as a 0–1 linear programming model using a space–time network framework. Yao et al. (2019) develop a bi-level last train timetable optimization model. The upper level aims to maximize the number of destination-reachable passengers and minimize their transfer waiting times. The lower level aims at determining passenger path choices considering a detour routing strategy.

For those papers addressing LTT problems with passenger path selection, the coordination with other transport modes is barely considered. For example, Long et al. (2020) study the problem of synchronizing the last URT trains to better serve passengers from the last high-speed railway (HSR) trains. They develop a bi-objective MILP model in which only the passenger demand from the last HSR trains is considered. The two objectives are to maximize the number of destination-reachable passengers and minimize total service closure time of the last URT trains.

The advantage of considering passenger path choices is that the optimized last train timetable can better serve passengers by considering their actual needs. However, it also leads to a significant increase in the scale of optimization models. Most of the existing studies focusing on LTT problems adopt heuristic algorithms (e.g., genetic algorithm, Lagrangian relaxation) as the solution strategy. These heuristic algorithms can significantly shorten the computational time but have compromise in the quality of their optimal solutions. In addition, almost all the studies about LTT problems with considering passenger path choices ignore these passengers who cannot reach their destinations only via URT trains. Alternatively, the multimodal path combining URT and bridging services resolves the unreachable destination issue, but it complicates the optimization process for the LTT problem.

2.3. The bridging service design problem

Bridging services are usually used as a temporary replacement for rail transit to deal with rail transit operation interruption issues. Two bridging services are considered in this study. In the following, we will review the research on these two bridging services respectively.

(1) **Bus bridging**

In recent years, there has been a lot of research on bus bridging, especially in designing bus bridging services to cope with disruptions to rail transit systems. Zhang et al. (2021b) and Kuo et al. (2022) give comprehensive literature reviews on bus bridging design issues. Kepaptsoglou and Karlaftis (2009) is the first paper to deal with bus bridging in response to metro disruptions. They propose a framework for designing bridging buses: determining the extent of disruption and the metro stations that need to be bridged, designing bridging routes and frequencies, and assigning buses to routes. Codina et al. (2013) propose a nonlinear integer programming model and a heuristic algorithm to determine the frequency of predefined bus bridging routes under congested conditions. Jin et al. (2016) develop a column generation procedure and a path-based multicommodity network flow model to respectively generate the most efficient bus bridging routes and allocate available bus resources to these candidate routes. Considering uncertainties in passenger demand and spare capacities of existing rail and bus lines, Luo and Xu (2021) develop a two-stage stochastic programming model to formulate the optimal routes and frequencies of bridging buses. In this model, they take into account that passengers can transfer between different transportation modes and lines and provide multiple alternative paths for passengers to choose from. Chen and An (2021) propose a MILP model to simultaneously solve the bus bridging route selection, bus deployment, and bus timetabling problem. Wang et al. (2022) consider two sources of bridging buses (i.e. spare buses and running buses) and develop an integer programming model to address the problem of bus bridging route generation and bus deployment. Zheng et al. (2022) establish a bi-level programming model to handle the bus bridging service design problem considering the reliability of transportation system.

In addition to addressing the disruptions of rail transit systems, bus bridging services are also used in other scenarios. For example, Yang et al. (2017a) integrate passenger flow control and bus bridging service design to tackle the congestion issue of metro system. Kang et al. (2021) incorporate bus bridging service design into first train timetable optimization to reduce excessive transfer waiting times. Kang et al. (2019) design bus bridging services for the passengers who fail to transfer between the last trains.

It is worth noting that when applying bridging buses in rail transit disruption, rail transit overcrowding, or the first train period, bridging buses are generally used for short-distance replacement of interrupted or congested rail transit or transfer stations with long waiting times. Therefore, after transferring from a train to a bridging bus, passengers need to return to the rail transit system to continue their subsequent journey. In contrast, when applying bridging buses to address transfer failures between last trains, passengers need to go directly to their destinations through the bridging bus after transferring from the last train to the bridging bus. Due to the wide distribution of passengers' destinations, it is difficult and even impossible to meet the needs of all passengers by designing bridging buses, especially considering the high operating costs of operating a shuttle bus. In this case, ride-hailing (or taxis) can be used as a supplement to bridging buses to meet the needs of passengers who cannot reach their destinations due to transfer failures.

Table 2
Comparisons with the most relevant studies.

Paper	Scenario	Train timetabling	Passenger path choices	Bridging services	Solution method
Zhou et al. (2019)	Last train	√	√	×	Commercial solver
Kang et al. (2019)	Last train	√	×	bus	Decomposition method
Kang et al. (2021)	First train	√	×	bus	Decomposition method
Luo and Xu (2021)	Rail transit disruption	×	√	bus	Benders decomposition
This paper	Last train	√	√	Taxi and bus	Commercial solver

(2) **Ride-hailing (taxi) bridging**

Ride-hailing is a form of bridging that can be constantly deployed and highly flexible, but with quite limited in capacity. Due to its characteristics, research on ride-hailing (or ride-sharing) focuses on the impact of ride-hailing services on rail transit disruption management, the impact on road congestion, and the extent to which ride-hailing services can be used as a supplement to disrupted rail transit (Zhang et al., 2021b). For example, based on the Munich case, Zeng et al. (2012) investigate whether to cooperate with taxi companies and how to price compensation services in response to short-term disruptions in public tram systems. Based on the Vancouver case, Tyndall (2019) investigates the extent to which carsharing services can replace disrupted public transit rail systems. Yang and Chen (2019) propose a model of cooperation between URT operators and ride-hailing platforms to deal with URT disruptions. They also propose a pricing strategy for ride-hailing platforms in the event of URT disruption. Liu et al. (2020) consider using in-service buses to provide bus bridging services during subway disruptions, resulting in lower service levels for other bus passengers. They investigate bus bridging policies and the affordability of ride-hailing services from a social equity perspective.

To the best of our knowledge, there is currently no literature on taking taxis as the bridging services to deal with transfer failures between last trains. Unlike bridging buses, the difficulty in the cooperation between last trains and taxis is that passengers can transfer to taxis at any URT station to reach their destinations directly. Where these transfers will happen depends on the timetable of URT trains and also how passengers perceive the travel costs and travel times. For example, when reaching the destination by URT is not possible passenger may leave the URT system at an intermodal transfer station where they take a taxi to reach the destination. This intermodal transfer station is better to be as close as possible to passengers' destinations, as passengers wish to reduce the travel costs for taxis. However, passengers may need to take a long detour within the URT network to reach the intermodal transfer station that is close to the destination. A long detour means long travel time, which is usually not preferred by passengers particularly during late night. Hence, it is challenging to design a coordinated last-train timetable with taxi bridging to minimize passenger travel time and travel cost.

2.4. The contributions of this paper

This paper contributes to the literature with the integrated optimization of last train timetabling and bridging service design with the consideration of passenger path choices. For clarify, the comparison with the most relevant research is shown in Table 2. Based on pre-constructed path sets of passengers, we develop a bi-objective MINLP model, aiming to minimize total passenger travel time and minimize total passenger travel cost. To reduce the model scale and improve the solution efficiency, we propose three path dominance principles to eliminate redundant passenger paths without loss of optimality. To explore the trade-off between the two objectives, we design an adaptive iterative algorithm based on an augmented ε-constraint model. The key contributions of this paper are summarized as follows.

- We fill the gap of research in the last train timetabling field with the integrated optimization of last train timetabling and bridging service design with the consideration of passenger path choices. We consider the bridging services of both buses and taxis.
- A bi-objective optimization model is proposed to minimize both the travel time and travel cost of passengers. The model can be solved efficiently by a commercial solver for a real-life URT network.
- Three path dominance principles are proposed to effectively eliminate numerous redundant passenger paths without loss of optimality. These principles help to speed up the computation of our optimization model further. The proposed path dominance principles are not limited to be used in our work, but can also be used in other path-based last-train timetable optimization models in general.
- We demonstrate the effectiveness of our model and methods with extensive numerical experiments on a real-world URT network, and derive insightful findings and suggestions for URT operators.

3. Mathematical formulation

This paper focuses on the last-train timetable optimization within multimodal urban transport networks. We integrate the last-train timetable optimization and bridging service design with the consideration of passenger path choices. We consider two bridging services: taxis and buses. *Bridging taxis* are provided when passengers cannot reach their destinations via last URT trains or when passengers need to take long detours within the URT network to reach their destinations. *Bridging buses* are further provided when a large number of passengers will take bridging taxis from the same station to the same/close destination. Passengers may transfer to bridging

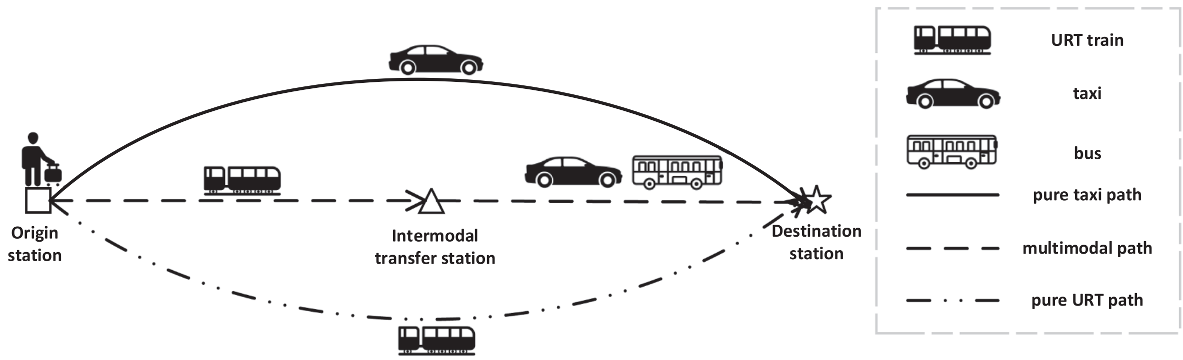

Fig. 2. Passenger candidate path types.

taxis at any station with available taxis, and to a bridging bus at any station with a bridging bus. Thus, within the multimodal urban transport network, there are the following types of candidate paths for passengers (as illustrated in Fig. 2):

1) ***The pure taxi path***: a passenger takes a taxi from the origin directly to the destination.
2) ***The pure URT path***: a passenger takes URT trains from the origin to the destination, and may need multiple transfers between URT trains during the journey.
3) ***The multimodal path***: a passenger first takes URT trains from the origin to a non-destination station, and then transfers to a bridging taxi/bus to the destination. Before reaching the intermodal transfer station, this passenger may transfer between URT trains.

The last train timetable determines the transfer feasibility between URT trains, and further determines the accessibility of the pure URT path to the destination and the accessibility of the multimodal path to the intermodal transfer station. In this paper, we consider that for passengers, when both bridging taxis and buses are available, the main difference between using taxi bridging and bus bridging lies in travel cost and travel time. Specifically, bus bridging is much cheaper than taxi bridging; however, the travel time for taxi bridging is shorter than using bus bridging, because the bus may stop at multiple stops along the way. To control the operational cost of bridging buses, a bus bridging route will only be deployed when there is sufficient passenger demand. We consider that each bridging bus only allows passengers to board at its starting station, and stops at multiple stations along the way for passengers to get off. Additionally, the number of taxis available are also took into account. Under the above considerations, we aim to provide passengers with advantageous paths (i.e., lower travel costs and/or shorter travel times) by integrating the last train timetable optimization and bridging service design.

In this section, we formulate the integrated optimization model of last train timetabling problem and bridging service design problem as a bi-objective mixed integer nonlinear programming model. The structure of this section is organized as follows. Section 3.1 introduces the assumptions used throughout this paper. Section 3.2 gives all the sets, parameters, and decision variables defined for the MINLP model. Sections 3.3 and 3.4 describe all the constraints and objective functions established in the bi-objective MINLP model, respectively. Section 3.5 analyzes the complexity of the model (i.e. the number of decision variables and constraints) and compares it with the existing advanced models.

3.1. Assumptions

Assumption 1. *The capacity of the last URT train is sufficient to carry all passengers.*

This assumption is common in the literature on the LTT problem (e.g., Kang et al., 2015a; Yang et al., 2020; Zhou et al., 2019) and is explained as follows. In general, the late-night passenger demand is relatively low. If the capacity of the last URT train is not enough to carry all passengers (e.g., on Friday night), URT operators should consider extending the operating hours and/or adding more URT trains in operation, as it is one of the most important night transportations for passengers. However, this problem is out of the scope of this paper.

Assumption 2. *The transfer walking time between two URT lines at a transfer station is known and fixed.*

This assumption is also common in the literature on the LTT problem (e.g., Chen et al., 2019b; Kang et al., 2015a; Zhou et al., 2019) and is used to reduce the problem complexity. Additionally, when the passenger demand is low, the transfer walking time can be obtained through on-the-spot investigations, with slight fluctuation. We set the transfer walking time a little larger than the average transfer walking time for most passengers as in Chen et al. (2019b) and Huang et al. (2021).

Assumption 3. *The travel time and travel cost along each passenger candidate path are known and fixed when the bridging service has been determined.*

This assumption is a simplification of the problem and is used to reduce the problem complexity. In reality, the travel time of

Table 3
Definition of decision variables.

Variable	Type	Description
$t^{\text{arr}}_{l,s,i}$	Continuous	The arrival time of train $i \in I(l)$ at station $s \in S(l)$ on line $l \in L$.
$t^{\text{dep}}_{l,s,i}$	Continuous	The departure time of train $i \in I(l)$ at station $s \in S(l)$ on line $l \in L$.
x_{tf}	Binary	$= 1$, if transfer $tf \in TF$ is feasible; $= 0$, otherwise.
$y_{od,p}$	Binary	$= 1$, if passengers in group $od \in U$ choose path $p \in P_{od}$; $= 0$, otherwise.
z_{s_1,s_2}	Binary	$=1$, if a bus bridging route $(s_1, s_2) \in BR$ is deployed; $= 0$, otherwise.
tb_{s_1,s_2}	Continuous	The earliest departure time of each bridging bus along route $(s_1, s_2) \in BR$ at its starting station s_1.
$TC_{od,p}$	Continuous	The travel cost of all passengers in group $od \in U$ traveling along path $p \in P_{od}$.
$TT_{od,p}$	Continuous	The travel time of all passengers in group $od \in U$ traveling along path $p \in P_{od}$.

passengers depends on travel distance, the speed of URT trains and bridging services, real-time road conditions, and waiting times for bridging services, etc. The travel cost of passengers is mainly determined by travel distance. Since the focus of this study is to explore how last URT trains cooperate with bridging services and to reveal the benefits of the cooperation between URT trains and bridging services, how to design a robust train timetable under the uncertainty of travel time and travel cost is out of the scope of this study.

Assumption 4. *Passengers are assumed to transfer to bridging taxis at any URT station, but can only transfer to bridging buses at train-to-train transfer stations rather than at any URT station along the way.*

This assumption has no impact on the model formulation, but only affects the input of the proposed optimization model. It helps to reduce the size of candidate paths. This assumption is justified as follows. On the one hand, this assumption helps to bundle the demand so that the operators do not need to set up a bridging bus at every location where there is demand but small. On the other hand, the accessibility of all URT stations that are located on the same URT line between two adjacent train-to-train transfer stations is the same. That means passengers who are travelling in the same line and need to transfer from this line to another will gather at the train-train transfer stations. If bridging buses are offered at locations before these passengers reaching the train-train transfer stations, there will be overlaps between the train lines and bus routes, while this is not necessary and not cost-effective.

Assumption 5. *As for the path type of multimodal paths, we only consider paths along which passengers first take URT trains and then transfer to taxis/bus. In other words, we exclude paths along which passengers first take taxis/bus and then transfer to URT trains.*

We exclude such multimodal paths (i.e. taxi/bus first and then URT trains) because such paths are uncommon in practice. This is because owing to uncertain road conditions (e.g. congestion), there is great uncertainty about whether passengers taking taxis/buses can arrive at the intermodal transfer station on time to catch the last URT train. Consequently, such paths are not welcomed by passengers.

Assumption 6. *Passengers will not go through the same URT line more than once.*

This assumption is used when constructing the passenger path set. It is reasonable because redundant transfers will increase the risk of failed transfers.

3.2. Sets, parameters, and variables

Table 3 gives all decision variables defined for the bi-objective MINLP model, including train timetabling variables (i.e. $t^{\text{arr}}_{l,s,i}$ and $t^{\text{dep}}_{l,s,i}$), transfer feasibility variables (i.e. x_{tf}), passenger routing variables (i.e. $y_{od,p}$), bridging bus design variables (i.e. z_{s_1,s_2}), and service evaluation variables(i.e. $TC_{od,p}$ and $TT_{od,p}$). All the sets and parameters used in this model are summarized in Appendix A.

3.3. Constraints

The constraints formulated in this model include train timetabling constraints, passenger path choice constraints, constraints on the relationship between train timetabling and passenger routing, and bridging bus route design constraints.

3.3.1. Train timetabling

The URT network in a city consists of numerous bi-directional URT lines. Each bi-directional URT line is a double-track railway line and consists of two different directional URT lines (i.e., down direction and up direction). We define L as the set of all directional URT lines. For each directional URT line $l \in L$, a sequence of stations is denoted by $S(l) = \{s_l^1, s_l^2, \cdots, s_l^n\}$, where station s_l^1 and s_l^n represent the start and terminal stations, respectively. Trains on each directional URT line run from the start station to the terminal station and stop at each station along the way for passengers to get on and off. Trains do not overtake because each station has only one dedicated track to serve the trains of a specific directional URT line.

This study aims at optimizing the timetables of last trains. However, since passengers may transfer between non-last trains and last trains, optimizing the timetable of both last trains and several related non-last trains (e.g. the penultimate train and the antepenultimate train) may be helpful to further improve passenger service (e.g. to transport more passengers to reach their destinations via URT

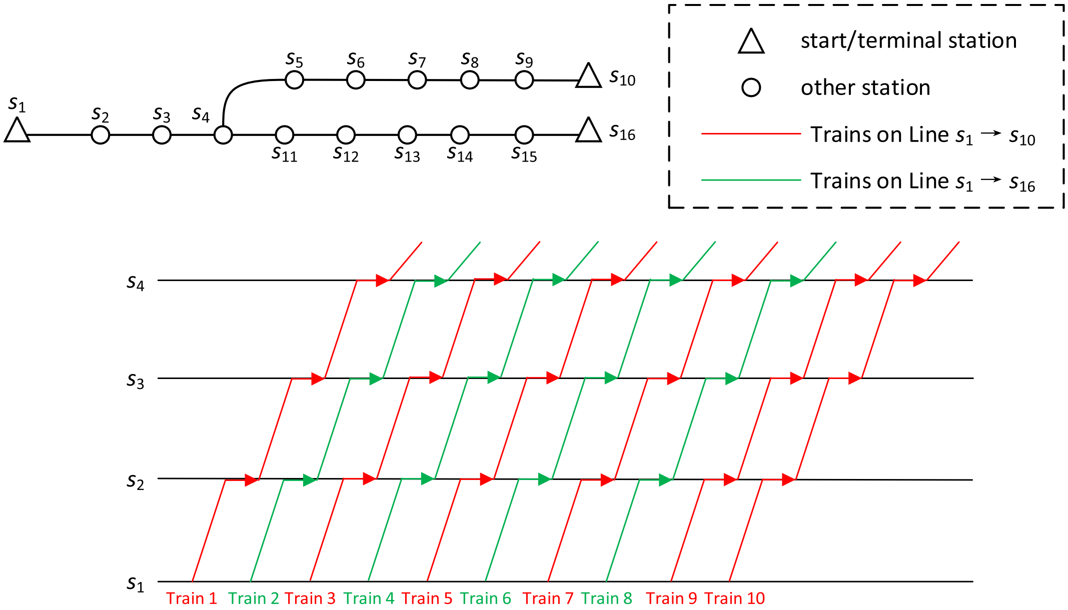

Fig. 3. An example of two URT lines sharing the same tracks at/between specific stations.

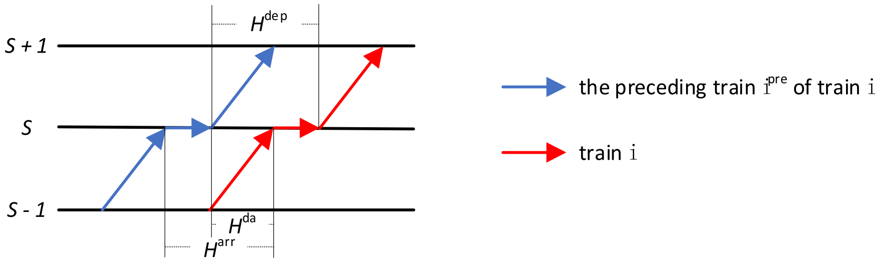

Fig. 4. Illustration of safety headways.

trains). In addition, when two directional URT lines share the same tracks at/between specific stations, the non-last trains operating between the two last trains of the two directional lines should also be adjusted in order to ensure sufficient optimization space for the last train timetable. For example, as illustrated in Fig. 3, two directional URT lines (i.e., Line $s_1 \rightarrow s_{10}$ and Line $s_1 \rightarrow s_{16}$) share the same tracks from station s_1 to station s_4. The last train of Line $s_1 \rightarrow s_{10}$ is train 10, and the last train of Line $s_1 \rightarrow s_{16}$ is train 8. In this case, the timetable of train 8, train 9, and train 10 will all be adjusted, although train 9 is a non-last train. Additionally, this study considers that the sequence of trains running along each directional URT line is fixed. For each directional URT line $l \in L$, we define $I(l) = \{i_l^1, i_l^2, \cdots, i_l^{n-1}, i_l^n\}$ as the sequence of trains running on the line l, where i_l^n denotes the last train, and i_l^{n-1} denotes the penultimate train. Then, we define $Iv(l)$ as the set of trains (including the last trains and several designated non-last trains) of which the timetable will be optimized, and $Iv(l) \subseteq I(l)$.

We define two decision variables $t_{l,s,i}^{arr}$ and $t_{l,s,i}^{dep}$ representing the arrival and departure times of train $i \in I(l)$ at station $s \in S(l)$ on line $l \in L$, respectively. The constraints on train timetabling are established as follows:

The dwell time at each station should be within an appropriate range to ensure the quality of passenger services. That is,

$$\underline{DT}_{l,s} \leq t_{l,s,i}^{dep} - t_{l,s,i}^{arr} \leq \overline{DT}_{l,s}, \forall l \in L, i \in Iv(l), s \in S(l) \tag{1}$$

where $\underline{DT}_{l,s}$ and $\overline{DT}_{l,s}$ represent the minimum and maximum dwell times at station s on line l, respectively.

The running time between two adjacent stations should be within an appropriate range to ensure the safety of train operation and the quality of passenger services. That is,

$$\underline{RT}_{l,s,s+1} \leq t_{l,s+1,i}^{arr} - t_{l,s,i}^{dep} \leq \overline{RT}_{l,s,s+1}, \forall l \in L, i \in Iv(l), s, s+1 \in S(l) \tag{2}$$

where $\underline{RT}_{l,s,s+1}$ and $\overline{RT}_{l,s,s+1}$ represent the minimum and maximum running times from station s to station $(s+1)$ on line l, respectively.

As depicted in Fig. 4, safety headways (i.e., headway between successive arrivals H_l^{arr}, headway between successive departures H_l^{dep}, and headway between the departure and arrival of different trains H_l^{da}) should be satisfied between every two adjacent trains (i.e., train i and its preceding train i^{pre}) at the same station. This is expressed by Constraints (3) – (5). It is worth noting that if two directional URT lines share the same tracks at/between some specific stations, the preceding train of the same train can be different at different stations. For example, in Fig. 3, the preceding train of train 8 at station s_4 is train 7, while at station s_{11} is train 6.

$$t_{l,s,i}^{\mathrm{arr}} - t_{l^{\mathrm{pre}},s,i^{\mathrm{pre}}}^{\mathrm{arr}} \geq H_l^{\mathrm{arr}}, \forall l \in L, i \in Iv(l), s \in S(l) \tag{3}$$

$$t_{l,s,i}^{\mathrm{dep}} - t_{l^{\mathrm{pre}},s,i^{\mathrm{pre}}}^{\mathrm{dep}} \geq H_l^{\mathrm{dep}}, \forall l \in L, i \in Iv(l), s \in S(l) \tag{4}$$

$$t_{l,s,i}^{\mathrm{arr}} - t_{l^{\mathrm{pre}},s,i^{\mathrm{pre}}}^{\mathrm{dep}} \geq H_l^{\mathrm{da}}, \forall l \in L, i \in Iv(l), s \in S(l) \tag{5}$$

where l^{pre} represents the directional URT line where train i^{pre} serves. That is, $i^{\mathrm{pre}} \in I(l^{\mathrm{pre}})$.

The service closure time of each directional URT line should not be later than the latest allowed closure time, otherwise it would delay vehicle cleaning and system maintenance. This is expressed by

$$t_{l,s_l^n,i_l^n}^{\mathrm{arr}} \leq T_l, \forall l \in L \tag{6}$$

where i_l^n represents the last train of line l, and s_l^n represents the terminal station of line l. T_l represents the latest allowed service closure time of line l, which varies from line to line.

Finally, for practicality, timetable modification to trains in set $Iv(l)$ cannot raise conflicts with trains in set $I(l)\backslash Iv(l)$. Therefore, trains of which the timetables will not be adjusted, i.e., trains in set $I(l)\backslash Iv(l)$, should also be considered in the optimization model and have their timetables fixed as the original timetables. That is,

$$t_{l,s,i}^{\mathrm{dep}} = \tau_{l,s,i}^{\mathrm{dep}}, \forall l \in L, i \in I(l)\backslash Iv(l), s \in S(l) \tag{7}$$

$$t_{l,s,i}^{\mathrm{arr}} = \tau_{l,s,i}^{\mathrm{arr}}, \forall l \in L, i \in I(l)\backslash Iv(l), s \in S(l) \tag{8}$$

where $\tau_{l,s,i}^{\mathrm{arr}}$ and $\tau_{l,s,i}^{\mathrm{dep}}$ represent the original arrival and departure times of train i at station s on line l, respectively.

3.3.2. Passenger path choices

We define U as the set of passenger groups. Each passenger group in U is indexed by od, where o and d indicate the origin and destination stations, respectively. For each passenger group $od \in U$, we define P_{od} as the set of candidate paths. The paths in each set P_{od} include the pure taxi paths, the pure URT paths, and the multimodal paths that combine URT and bridging services. The construction of passenger path sets will be introduced in Section 4. Here we wish to emphasize that passenger candidate paths are generated before timetable optimization but include all possible paths that could appear in any optimized timetables.

We define a binary decision variable $y_{od,p}$, which is equal to 1 if passengers in group $od \in U$ choose path $p \in P_{od}$. For simplicity, we assume that passengers in the same group choose the same path. Then, Constraint (9) is established, which requires each passenger group to choose only one path from the candidate path set.

$$\sum_{p \in P_{od}} y_{od,p} = 1, \forall od \in U \tag{9}$$

3.3.3. Relationship between train timetabling and passenger routing

Transfer feasibility is the key to linking the train timetable to passenger path choices. Specifically, passengers can only choose paths that are reachable to their destinations. A passenger path can be reachable only when all transfers contained in this path are feasible. The last train timetable determines the transfer feasibility between URT trains, and further determines the accessibility of pure URT paths and multimodal paths.

We define TF as the set of all train-to-train transfers in the given URT network. Each transfer in the set TF is denoted by $tf = (l, i, l^{'}, i^{'}, s)$, which means that passengers transfer from train $i \in I(l)$ to train $i^{'} \in I(l^{'})$ at station $s \in S(l) \cap S(l^{'})$. We define a binary variable x_{tf}, which is equal to 1 if transfer $tf \in TF$ is feasible. Then, Constraints (10) and (11) are developed to formulate the relationships between train timetabling and passenger path choices. To be specific, Constraint (10) indicates that a transfer must be feasible if it is included in a selected passenger path. Constraint (11) indicates that a transfer is feasible only if the departure time of the connecting train is later than the arrival time of the feeder train plus the transfer walking time at this transfer station.

$$x_{tf} \geq \frac{\sum_{od \in U} \sum_{p \in P_{od}} y_{od,p} \cdot \theta_{tf,p}}{|U|}, \forall tf \in TF, \tag{10}$$

where $\theta_{tf,p}$ is a given binary parameter, which is set to 1 if transfer tf is contained in path p. $|U|$ represents the number of passenger groups, so that the value of the right-hand side of this constraint is between 0 and 1.

$$t_{l^{'},s,i^{'}}^{\mathrm{dep}} - t_{l,s,i}^{\mathrm{arr}} - W_{tf} \geq M \cdot \left(x_{tf} - 1\right), \forall tf \in TF, tf = (l, i, l^{'}, i^{'}, s) \tag{11}$$

where W_{tf} represents the transfer walking time associated with transfer tf. M is a sufficiently large positive value, which can be determined by the method proposed by Kang and Meng (2017).

3.3.4. Bridging services design

Bridging services are typically used when passengers travel from one URT line to another (but fail to transfer or require a long detour). We consider two bridging services: taxis and buses. As mentioned, it is not economically viable to deploy a bus bridging route between any two URT stations where passenger demand is quite low. We therefore only deploy bridging buses for routes with sufficient passenger demand. We consider that each bridging bus only picks up passengers at its starting station, but stops at multiple stations for passengers to get off. In a pre-processing step, we design all possible bridging bus routes. We specify for each route its starting station and destination, and the intermediate stops between. We generate all candidate bus bridging routes based on passenger candidate path sets, which will be detailed in Section 4.5. We define BR as the set of all candidate bus bridging routes, which are the input to our optimization model.

In the model, we decide whether a bus bridging route will be used, as well as the departure times of bridging buses at their starting stations. We define a binary variable z_{s_1,s_2}, which is equal to 1 if bus bridging route $(s_1,s_2) \in BR$ is deployed. Then, Constraint (12) is established, which indicates that a bus bridging route is deployed between two given stations when the number of passengers requiring bridging service is greater than a certain value.

$$\sum_{od\in U} Q_{od} \cdot \left(\sum_{p\in P_{od}:s_p^{\mathrm{dep}}\neq d, s_p^{\mathrm{dep}}=s_1,\ \mathrm{and} d=s_2} y_{od,p} \right) \geq MQ \cdot z_{s_1,s_2}, \forall (s_1,s_2) \in BR \tag{12}$$

where MQ represents the minimum passenger demand requirement for deploying a bus bridging route. Q_{od} represents the number of passengers in group od. s_p^{dep} represents the station where passengers leave the URT network along path $p \in P_{od}$. When path p is a pure taxi path or a multimodal path (i.e., $s_p^{\mathrm{dep}} \neq d$), passengers need to transfer to bridging services.

Each bridging bus only picks up passengers at its starting station, and stops at multiple stations for passengers to get off. Thus, to ensure the feasibility of the train-to-bus transfer, it is only required to collaboratively optimize the departure times of bridging buses at their starting stations and last train timetables. We define a decision variable tb_{s_1,s_2} to represent the earliest departure time of each bridging bus along route $(s_1,s_2) \in BR$ at its starting station s_1. Then, Constraint (13) is established to ensure the feasibility of the train-to-bus transfer.

$$tb_{s_1,s_2} \geq \max\left\{ W_s^{\mathrm{TB}} + t_{l,s,i}^{\mathrm{arr}} \middle| \forall l \in L, i \in I(l), s \in S(l), s = s_1 \right\}, \forall (s_1,s_2) \in BR \tag{13}$$

where W_s^{TB} represents the transfer walking time associated with the train-to-bus transfer at station s.

When choosing to taking bridging taxis, passengers may transfer to bridging taxis at any URT stations, provided that the station has sufficient taxis available. Therefore, Constraint (14) is established, requiring that the number of passengers transferring to bridging taxis at each station is less than the number of available taxis.

$$\sum_{od\in U} \left(Q_{od} \cdot \sum_{p\in P_{od}:s_p^{\mathrm{dep}}=s \mathrm{and} s_p^{\mathrm{dep}}\neq d} y_{od,p} \cdot (1 - z_{s,d}) \right) \leq MN_s, \forall s \in \bigcup\nolimits_{l\in L} S(l) \tag{14}$$

where MN_s represents the number of taxis available at station s. Since $y_{od,p}$ and $z_{s,d}$ are both binary decision variables, this constraint is nonlinear and then linearized by the method introduced in Appendix B.

3.4. Objective functions

Two objectives are considered in this model: one is minimizing total travel cost of passengers, and the other is minimizing total travel time of passengers. Travel times and travel costs vary with different paths and different bridging services. Generally speaking, bridging buses have lower travel costs than bridging taxis, but longer travel times than bridging taxis. The calculation formulas of travel cost and travel time are given in Sections 3.4.1 and 3.4.2, respectively, and then the two objective functions are given in Section 3.4.3.

3.4.1. Travel cost calculation

We define $TC_{od,p}^{\mathrm{taxi}}$ to represent the travel cost of all passengers in group $od \in U$ traveling along path $p \in P_{od}$, where taxis are used as bridging services. We define $TC_{od,p}^{\mathrm{bus}}$ to represent the travel cost of all passengers in group $od \in U$ traveling along path $p \in P_{od}$, where buses are used as bridging services. Then, the travel cost along path $p \in P_{od}$ can be calculated by:

$$TC_{od,p} = \begin{cases} TC_{od,p}^{taxi} & (s,d) \notin BR \\ TC_{od,p}^{bus} \cdot z_{s,d} + TC_{od,p}^{taxi} \cdot (1 - z_{s,d}) & (s,d) \in BR \end{cases}, \quad \forall od \in U, \quad \forall p \in P_{od} \tag{15}$$

Table 4
Number of decision variables and constraints in our proposed model.

Variables	Maximum amount	Constraints	Maximum amount
$t^{\text{arr}}_{l,s,i}$ and $t^{\text{dep}}_{l,s,i}$	$2 \cdot \sum_{l\in L}(\lvert I(l)\rvert \cdot \lvert S(l)\rvert)$	Constraints (1) – (8)	$\lvert L\rvert + 5 \cdot \sum_{l\in L}(\lvert I(l)\rvert \cdot \lvert S(l)\rvert)$
x_{tf}	$\lvert TF\rvert$	Constraints (9)	$\lvert U\rvert$
$y_{od,p}$	$\sum_{od\in U}\lvert P_{od}\rvert$	Constraints (10) and (11)	$2 \cdot \lvert TF\rvert$
z_{s_1,s_2}	$\lvert BR\rvert$	Constraints (12) and (13)	$2 \cdot \lvert BR\rvert$
$TC_{od,p}$ and $TT_{od,p}$	$2 \cdot \sum_{od\in U}\lvert P_{od}\rvert$	Constraints (14)	$\lvert \bigcup_{l\in L} S(l)\rvert$

where d represents passengers' destination station. The value of s depends on the type of path p. s represents passengers' origin station if path p is a pure taxi path. s represents passengers' destination station if path p is a pure URT path. And, s represents the intermodal transfer station where passengers transfer from URT trains to bridging services if path p is a multimodal path.

3.4.2. Travel time calculation

Similar to the travel cost, the travel time along path $p \in P_{od}$ can be calculated by:

$$TT_{od,p} = \begin{cases} TT^{taxi}_{od,p} & (s,d) \notin BR \\ TT^{bus}_{od,p} \cdot z_{s,d} + TT^{taxi}_{od,p} \cdot (1 - z_{s,d}) & (s,d) \in BR \end{cases}, \forall od \in U, \forall p \in P_{od} \tag{16}$$

where $TT^{\text{taxi}}_{od,p}$ and $TT^{\text{bus}}_{od,p}$ represent the travel cost along path $p \in P_{od}$ by using taxis and buses as the bridging services, respectively. d and s have the same meaning as in formula (14).

3.4.3. Objectives

Finally, the two objective functions, i.e., minimizing total travel cost of passengers and minimizing total travel time of passengers, are given by:

$$\min z_1 = \sum_{od\in U}\sum_{p\in P_{od}} \left(TC_{od,p} \cdot y_{od,p}\right) \tag{17}$$

$$\min z_2 = \sum_{od\in U}\sum_{p\in P_{od}} \left(TT_{od,p} \cdot y_{od,p}\right) \tag{18}$$

where $TC_{od,p}$, $TT_{od,p}$, and $y_{od,p}$ are all decision variables. Therefore, the objective functions are nonlinear and can be linearized by the method introduced in Appendix B.

3.5. Model complexity analysis

We analyze the complexity of the proposed optimization model (i.e., the numbers of decision variables and constraints) and compare it with the existing state-of-the-art model proposed by Zhou et al. (2019).

3.5.1. Analysis of the number of decision variables and constraints

Table 4 lists the number of decision variables and constraints in our proposed model. It can be observed that the number of decision variables in our model depends on the size of the URT network (i.e., the number of URT stations, and the number of transfers), the total number of passenger candidate paths, and the number of candidate bus bridging routes. The number of constraints in our model depends on the size of the URT network (i.e., the number of directional URT lines, the number of URT stations, and the number of transfers), the number of passenger groups, and the number of candidate bus bridging routes.

3.5.2. Comparison with the existing advanced models

To highlight the advantages of our model in scale and solution efficiency, we compare our model with the existing state-of-the-art model proposed by Zhou et al. (2019), where they stated that their model can be solved directly with the commercial solver with satisfactory computation time and solution quality. The model in Zhou et al. (2019) was developed to solve the last train timetabling, taking into account passenger path choices but not any bridging services.

Apart from bridging service design, the biggest difference between our proposed model and the one proposed in Zhou et al. (2019) is how to formulate the relationship between train timetabling and passenger routing, which is why our model has advantages in model scale and solution efficiency. Specifically, Zhou et al. (2019) define a series of binary variables for passenger path choices, transfer feasibility, path destination reachability, and passenger destination reachability. Then, they establish numerous big-M constraints to formulate the logical relations between these binary variables. Therefore, their model has a huge number of constraints and decision variables. Differently, in this paper, we only define binary variables for passenger path choices and transfer feasibility, and establish a set of linear constraints (i.e. Constraints (10)) to formulate the logical relations between these binary variables. By comparison, our proposed model has far fewer decision variables and constraints than the model in Zhou et al. (2019). Table 5 reports the number of

Table 5
Number of decision variables and constraints in the two models.

Model	Number of variables	Number of constraints
Zhou et al. (2019)	$\lvert TF\rvert + \lvert U\rvert + \lvert U\rvert \cdot \lvert TF\rvert + \sum_{od \in U} \lvert P_{od}\rvert$	$2 \cdot (\lvert U\rvert + \lvert U\rvert \cdot \lvert TF\rvert)$
This paper	$\lvert TF\rvert + \sum_{od \in U} \lvert P_{od}\rvert$	$\lvert TF\rvert$

All decision variables and constraints counted here are those related to the relationship between train timetabling and passenger routing.

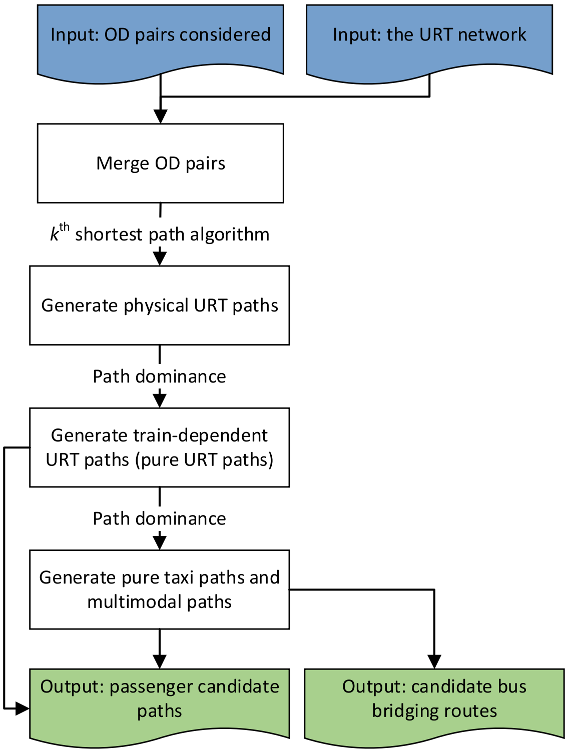

Fig. 5. Framework for generating passenger candidate paths and candidate bus bridging routes.

decision variables and constraints related to the relationship between train timetabling and passenger routing in the two models. In addition, in terms of solution efficiency, our proposed model also outperforms the model in Zhou et al. (2019), which we will present in Section 6.3.

4. Generation of passenger candidate paths and candidate bus bridging routes

In this section, we introduce the method of constructing the set of passenger candidate paths. Recall that there are three types of candidate paths for passengers: pure taxi paths, pure URT paths, and multimodal paths. Candidate bus bridging routes are then generated based on the passenger candidate paths.

There are the following three difficulties in generating passenger candidate paths:

(1) Passenger candidate paths are generated before timetable optimization. Since the reachability of each path may vary with the train timetable, the candidate path set should cover all possible favorable paths that could exist in any optimized timetables. For this purpose, when we decide on connecting trains at a transfer station, we take all trains whose latest departure times are greater than the earliest arrival time of the feeder train as candidate connecting trains for passengers. For trains with fixed timetables, their latest departure times and earliest arrival times can be obtained from their original timetables. For trains with variable timetables, their latest departure times and earliest arrival times can be calculated based on the constraints of train operations (i.e. Constraints (1) – (8)) and the earliest arrival times of passengers at the boarding station. See Section 4.2 for more details.

(2) There may be a huge number of candidate paths for passengers, which increases the scale of the optimization model. On the one hand, there will be a large number of pure URT paths due to the complexity of the URT network and the possibility of passengers

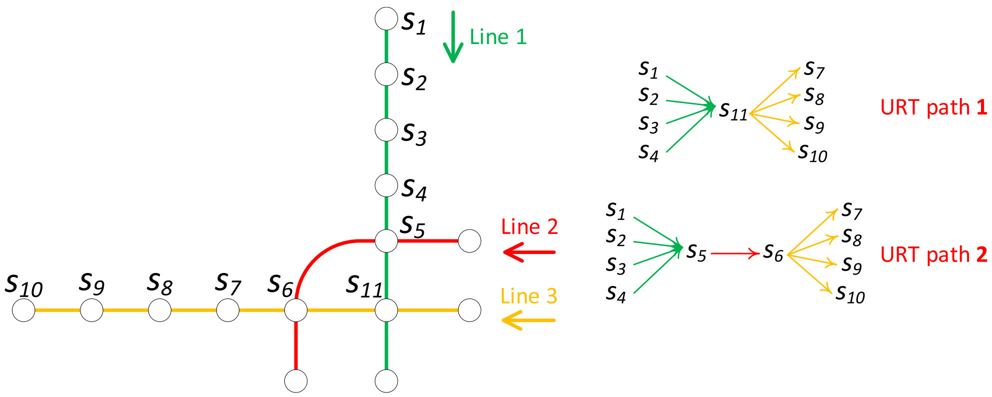

Fig. 6. An example of merging OD pairs.

transferring between last trains and non-last trains. On the other hand, since passengers might transfer to bridging taxis at any URT stations with available taxis, and to a bridging bus at any URT transfer station with a bridging bus (according to Assumption 4), there will be a large number of multimodal paths. Therefore, we propose three path dominance principles to eliminate redundant passenger paths without loss of optimality. Such redundant passenger paths could be either pure URT paths or multimodal paths, which do not contribute to improving the optimal objective value (i.e., minimizing passenger travel time and travel cost). These three path dominance principles are presented in Section 4.3, and their effects are demonstrated in Section 6.2.3.

(3) Due to the large number of passenger candidate paths, path generation can be quite time consuming. Hence, before path generation, we merge some OD pairs if their relevant candidate paths will be the same in terms of included transfers. The method of merging OD pairs is introduced in Section 4.1, and its effects are demonstrated in Section 6.2.1.

Fig. 5 shows the overall framework for generating passenger candidate paths and candidate bus bridging routes. *First*, OD pairs are merged by the method proposed in Section 4.1; *Second*, physical URT paths, which do not include train information (i.e., the trains that passengers will take along the path), are generated based on the given URT network by the k shortest path algorithm (Eppstein, 1998); *Third*, train-dependent URT paths (i.e., pure URT paths) are generated based on physical URT paths by the method proposed in Section 4.2, and further filtered by the three path dominance principles proposed in Section 4.3; *Fourth*, pure taxi paths and multimodal paths are generated based on the pure URT paths by the method proposed in 4.4, and further filtered by the three path dominance principles; *Fifth*, all candidate bus bridging bus routes are generated based on the multimodal paths by the method proposed in Section 4.5.

4.1. Merging OD pairs

If two OD pairs will have the same candidate paths in terms of included transfers, then these two OD pairs can be merged. An example is given to illustrate the idea of merging OD pairs. Fig. 6 shows a simplified URT network with three bi-directional URT lines. We consider 16 different OD pairs with origins at station s_1, s_2, s_3, or s_4 and destinations at station s_7, s_8, s_9, or s_{10}. As can be seen, they have the same candidate URT paths (when they take the same train at their origin stations) in terms of included transfers. In this case, the 16 OD pairs can be merged so that we only perform the path generation process once for the 16 OD pairs instead of 16 times without OD pairs merging.

We propose the following method of merging OD pairs. Given two OD pairs, denoted by o_1d_1 and o_2d_2, they can be merged if their origin stations o_1 and o_2 satisfy at least one of Conditions (1) – (3), and their destination stations d_1 and d_2 also satisfy at least one of Conditions (1) – (3).

Condition 1. Stations $\boldsymbol{o}_1$ and $\boldsymbol{o}_2$ ($\boldsymbol{d}_1$ and $\boldsymbol{d}_2$) are the same station.
Condition 2. Stations $\boldsymbol{o}_1$ and $\boldsymbol{o}_2$ ($\boldsymbol{d}_1$ and $\boldsymbol{d}_2$) are not transfer stations and on the same URT line, and there is no transfer station between $\boldsymbol{o}_1$ and $\boldsymbol{o}_2$ ($\boldsymbol{d}_1$ and $\boldsymbol{d}_2$) along this line.
Condition 3. The four stations $\boldsymbol{o}_1$, $\boldsymbol{o}_2$, $\boldsymbol{d}_1$, and $\boldsymbol{d}_2$ are all on the same URT line.

4.2. Generating pure URT paths

The pure URT path contained in the path set P_{od} refers to the train-dependent URT path that contains train information. Each pure URT path $p \in P_{od}$ is denoted by three sets: set $L_{od,p}$ contains a sequence of URT lines that path p will go through, set $S_{od,p}$ contains a sequence of stations that path p will go through (including the origin station, the transfer stations, and the destination station), and set $I_{od,p}$ contains a sequence of URT trains that passengers will take along path p. To generate train-dependent URT paths, first, the k shortest path algorithm (Eppstein, 1998) is adopted to generate at most k physical URT paths that connect station o to station d within

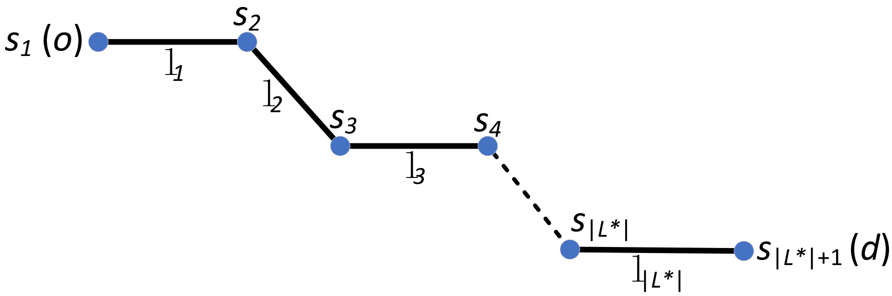

Fig. 7. A physical URT path between OD pair ***od***.

the URT network. A physical URT path does not include train information. Then, we add train information for each physical URT path according to passengers' earliest arrival times at transfer stations. For the train-dependent URT paths generated based on the same physical URT path, they have the same line set $L_{od,p}$ and the same station set $S_{od,p}$ but different train sets $I_{od,p}$. The procedure to generate multiple URT train-dependent paths based on a given URT physical path is described as follows:

Step 1. **Initialization.**

Step 1.1. Construct the line set L^* and the station set S^* based on the physical URT path (as illustrated in Fig. 7), i.e., $L^* = \{l_1, l_2, \cdots, l_{|L^*|}\}$, $S^* = \{s_1, s_2, \cdots, s_{|L^*|}, s_{|L^*|+1}\}$, where s_1 indicates passengers' origin station, s_m $(m = 2, \cdots, |L^*|)$ indicates the transfer station where passengers will transfer from the train of line l_{m-1} to the train of line l_m, and $s_{|L^*|+1}$ indicates passengers' destination station.

Step 1.2. Define variables t^{boa} and t^{EA}. t^{boa} indicates the earliest time when passengers board a designated train, and t^{EA} indicates the earliest time when passengers arrive at a designated station on a designated train.

Step 1.3. Define and initialize the train set $I^{\text{new}} = \varnothing$ and the set of train sets $\Omega = \varnothing$. Proceed to Step 2.

Step 2. **Designate trains for line l_1 traveling from station s_1 to s_2.**

Step 2.1. Passengers are assumed to take the last train at the origin station. Thus, we construct a train set containing the last train $i^n_{l_1}$ of line l_1, i.e., $I^{\text{new}} = \left\{ i^n_{l_1} \right\}$, $\Omega = \Omega \bigcup \{I^{\text{new}}\}$.

Step 2.2. Calculate the earliest time t^{boa} for passengers to board train $i^n_{l_1}$ (i.e., the earliest arrival time of train $i^n_{l_1}$ at station s_1) under the constraints of train operations (i.e., Constraints (1) – (8)). Proceed to Step 3.

Step 3. **Designate trains for line $l_{|I^*|+1}$ traveling from station $s_{|I^*|+1}$ to station $s_{|I^*|+2}$.**

For each train set in Ω, denoted by $I^* = \{i_1, i_2, \cdots, i_{|I^*|}\}$, we calculate t^{EA} (i.e., the earliest time when passengers arrive at station $s_{|I^*|+1}$ by train $i_{|I^*|}$) and t^{boa} (i.e., the earliest time when passengers board the connecting train at station $s_{|I^*|+1}$). Then, we designate trains for the line $l_{|I^*|+1}$ according to t^{boa}. Specifically, if the timetable of train $i_{|I^*|}$ is fixed, then perform Steps 3.1 and 3.2; otherwise, perform Steps 3.3 and 3.4. After processing all train sets in Ω, proceed to Step 4.

Step 3.1. t^{EA} is equal to the original arrival time of train $i_{|I^*|}$ at station $s_{|I^*|+1}$. That is, $t^{\text{EA}} = \tau^{\text{arr}}_{l_{|I^*|}, s_{|I^*|+1}, i_{|I^*|}}$. Then, t^{boa} is determined by $t^{\text{boa}} = t^{\text{EA}} + W_{tf}$, where W_{tf} is the corresponding transfer walking time.

Step 3.2. If there is at least a train in the train set $I(l_{|I^*|+1}) \backslash Iv(l_{|I^*|+1})$ that departs from station $s_{|I^*|+1}$ later than t^{boa}, then the train with the smallest departure time (denoted by i^*) is selected to add the train set I^*, that is, $I^* = I^* \bigcup \{i^*\}$. If such train cannot be found, then for each train i in the train set $Iv(l_{|I^*|+1})$, construct a new train set $I^{\text{new}} = I^* \bigcup \{i\}$, $\Omega = \Omega \bigcup \{I^{\text{new}}\}$, and remove I^* from Ω. Proceed to consider the next train set in Ω.

Step 3.3. t^{EA} is calculated under the constraints of train operations (i.e., Constraints (1) – (8)) and an additional constraint requiring that the departure time of train $i_{|I^*|}$ from station $s_{|I^*|}$ should be greater than passengers' earliest boarding time t^{boa}, i.e., $t^{\text{dep}}_{l_{|I^*|}, s_{|I^*|}, i_{|I^*|}} \geq t^{\text{boa}}$. Then, t^{boa} is determined by $t^{\text{boa}} = t^{\text{EA}} + W_{tf}$, where W_{tf} is the corresponding transfer walking time.

Step 3.4. If there is at least a train in the train set $I(l_{|I^*|+1}) \backslash Iv(l_{|I^*|+1})$ that departs from station $s_{|I^*|+1}$ later than t^{boa}, then for each train i that satisfying this condition, construct a new train set $I^{\text{new}} = I^* \bigcup \{i\}$ and $\Omega = \Omega \bigcup \{I^{\text{new}}\}$. Then for each train i in the train set $Iv(l_{|I^*|+1})$, construct a new train set $I^{\text{new}} = I^* \bigcup \{i\}$, and $\Omega = \Omega \bigcup \{I^{\text{new}}\}$. Remove I^* from Ω, and proceed to consider the next train set in Ω.

Step 4. **Termination check.**

If the length of every train set in Ω is equal to the length of the line set L^*, then for each train set I^* in Ω, we generate a train-dependent URT path $p = \{L^*, S^*, I^*\}$; otherwise, proceed to Step 3.

4.3. Path dominance principles

In order to reduce the model scale without loss of optimality, we propose three path dominance principles to eliminate redundant passenger paths that do not contribute to improving the optimal objective value (i.e., minimizing passenger travel time and travel cost). Note that redundant passenger paths could be either pure URT paths or multimodal paths.

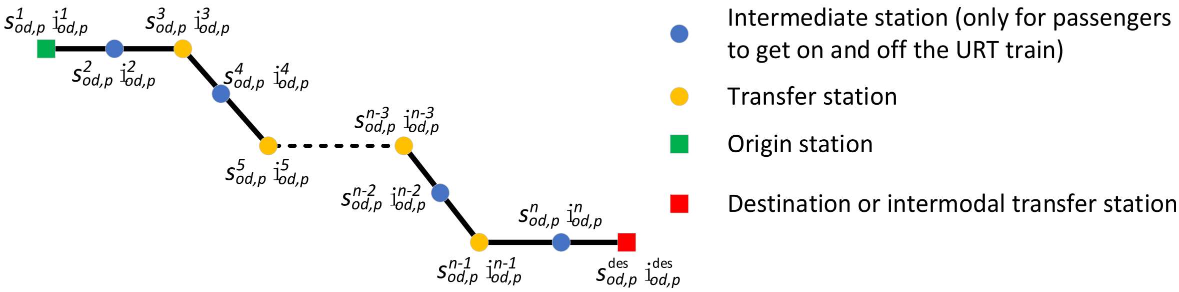

Fig. 8. A pure URT/multimodal path between OD pair ***od***.

To this end, each path p is represented by a sequence of station-train pairs (as illustrated in Fig. 8), that is, $SI_{od,p}^{\text{all}} = \{(s_{od,p}^1, i_{od,p}^1), (s_{od,p}^2, i_{od,p}^2), \cdots, (s_{od,p}^{\text{des}}, i_{od,p}^{\text{des}})\}$, which records *all* the stations (not just transfer stations) that path p goes through and the trains that passengers take when they *depart* from each station. However, for the station-train pair $(s_{od,p}^{\text{des}}, i_{od,p}^{\text{des}})$, $s_{od,p}^{\text{des}}$ represents passengers' destination station or intermodal transfer station, and $i_{od,p}^{\text{des}}$ represents the train from which passengers get off at station $s_{od,p}^{\text{des}}$.

The development of the three path dominance principles is inspired by Florian (2004) and Otto Anker Nielsen & Frederiksen (2006, 2009), where path dominance principles are proposed to reduce the number of alternative paths in schedule-based transit assignment problems. However, their proposed principles cannot be directly applied to this paper, because the last train timetable considered in this paper is unfixed, while the bus/train timetable in their papers is fixed. The three path dominance principles proposed in this paper are introduced as follows. The optimality proofs for these three principles are given in Appendix C, and their effects are demonstrated in Section 6.2.3.

Principle 1. *Given two paths $\boldsymbol{p}_1 \in \boldsymbol{P}_{od}$ and $\boldsymbol{p}_2 \in \boldsymbol{P}_{od}$ $(\boldsymbol{p}_1 \neq \boldsymbol{p}_2)$, if the following conditions are all satisfied, then path $\boldsymbol{p}_2$ can be removed from set $\boldsymbol{P}_{od}$ without loss of optimality.*

(1) The station-train pair set of path p_1 is a subset of the station-train pair set of path p_2, i.e., $\boldsymbol{SI}_{od,p_1}^{\text{all}} \subseteq \boldsymbol{SI}_{od,p_2}^{\text{all}}$.
(2) The travel cost and travel time of path p_1 are both smaller than path p_2, i.e., $\max(TC_{od,p_1}^{\text{taxi}}, TC_{od,p_1}^{\text{bus}}) \leq \min(TC_{od,p_2}^{\text{taxi}}, TC_{od,p_2}^{\text{bus}})$, and $\max(TT_{od,p_1}^{\text{taxi}}, TT_{od,p_1}^{\text{bus}}) \leq \min(TT_{od,p_2}^{\text{taxi}}, TT_{od,p_2}^{\text{bus}})$.

Principle 2. *Given two paths $p_1 \in P_{od}$ and $p_2 \in P_{od}$ $(p_1 \neq p_2)$ that connect to the same destination station or the same intermodal transfer station, if the following conditions are all satisfied, path p_2 can be removed from set P_{od} without loss of optimality.*

(1) The two paths go through the same station s.
(2) The station-train pairs after station s (including station s) are the same for the two paths.
(3) If the trains boarding at the origin station along the two paths are the same, then there is only one transfer along path p_1 before station s (including station s); otherwise, there is no transfer along path p_1 before station s (including station s).
(4) The *longest* total travel time to station s along path p_1 is *shorter* than the *shortest* total travel time to station s along path p_2.
(5) The travel cost and travel time of path p_1 are both smaller than path p_2, i.e., $TC_{od,p_1}^{\text{taxi}} \leq TC_{od,p_2}^{\text{taxi}}$, $TC_{od,p_1}^{\text{bus}} \leq TC_{od,p_2}^{\text{bus}}$, $TT_{od,p_1}^{\text{taxi}} \leq TT_{od,p_2}^{\text{taxi}}$, and $TT_{od,p_1}^{\text{bus}} \leq TT_{od,p_2}^{\text{bus}}$.

Principle 3. *Given two paths $\boldsymbol{p}_1 \in \boldsymbol{P}_{od}$ and $\boldsymbol{p}_2 \in \boldsymbol{P}_{od}$ $(\boldsymbol{p}_1 \neq \boldsymbol{p}_2)$ that connect to the same destination station or the same intermodal transfer station, if the following conditions are all satisfied, path $\boldsymbol{p}_2$ can be removed from set $\boldsymbol{P}_{od}$ without loss of optimality.*

(1) The two paths go through the same stations in the same order.
(2) Passengers take the same trains along these two paths, except for the trains (denoted by i_{od,p_1}^{des} and i_{od,p_2}^{des}) that take passengers to their destination or the intermodal transfer station from the last train-to-train transfer station.
(3) Train i_{od,p_2}^{des} precedes train i_{od,p_1}^{des} in terms of running order.
(4) The travel cost and travel time of path p_1 are both smaller than path p_2, i.e., $TC_{od,p_1}^{\text{taxi}} \leq TC_{od,p_2}^{\text{taxi}}$, $TC_{od,p_1}^{\text{bus}} \leq TC_{od,p_2}^{\text{bus}}$, $TT_{od,p_1}^{\text{taxi}} \leq TT_{od,p_2}^{\text{taxi}}$, and $TT_{od,p_1}^{\text{bus}} \leq TT_{od,p_2}^{\text{bus}}$.

4.4. Generating multimodal paths and pure taxi paths

Along each generated pure URT path, passengers may encounter transfer failures at any transfer stations and fail to reach their destinations, or passengers may not be satisfied with the travel time or travel cost of the pure URT path. Under the above circumstances, passengers would choose pure taxi paths or multimodal paths instead. As mentioned, passengers can transfer to bridging taxis at any stations that have available taxis, and to a bridging bus at any train-to-train transfer station with a bridging bus. Therefore, for each station s traversed along a pure URT path, we generate a multimodal path along which passengers take URT trains from the origin station o to station s and then transfer to the bridging service to the destination station d.

Any stations that are located on the same URT line between two adjacent transfer stations have the same accessibility. Among these stations, only the ones that dominate in terms of travel time and/or travel cost and have sufficient number of available taxis need to be considered when generating the multimodal paths that involve taxi bridging services. The method proposed in our previous work (i.e. Ning et al. (2022)) is used here to eliminate the taxi bridging stations that have no advantage in improving our optimal objective value. When generating the multimodal paths that involve bus bridging services, only train-to-train transfer stations are considered as the starting stations of bus bridging routes. Furthermore, to reduce the number of candidate multimodal paths, the three path dominance principles proposed in Section 4.3 are applied to eliminate redundant multimodal paths without loss of optimality.

In addition, passengers may also choose to take bridging taxis from their origin stations directly to their destinations. Therefore, a pure taxi path is generated for the passengers of each OD pair. We consider only one pure taxi path for each OD pair which is set as the shortest path from station o to station d within the road network.

4.5. Generating candidate bus bridging routes

To control the operating cost of bridging buses, a bus bridging route is only deployed when there is a large amount of passenger demand. We consider the bus bridging routes that only pick up passengers at buses' starting stations and stop at multiple stations for passengers to get off. As introduced before, OD pairs are merged into groups and thus passengers in the same group may have different destinations but the same intermodal transfer station where they will transfer to the same bridging service. Therefore, the process of generating candidate bus bridging routes is designed as follows:

Step 1. For each candidate multimodal path $p \in P_{od}$ where its intermodal transfer station is a train-to-train transfer station (denoted by s), a candidate bus bridging route is generated that starts at station s and ends at a passenger's destination station d.

Step 2. For each candidate bus bridging route, denoted by (s_1, s_2), calculate its maximum passenger demand (denoted by MQ_{s_1,s_2}) based on the OD pairs it can cover. If MQ_{s_1,s_2} is less than MQ (i.e., the minimum passenger demand requirement for deploying a bus bridging route), this candidate bus bridging route is removed from the candidate bus bridging route set.

5. Solution method

To explore the trade-off between the two objectives (i.e., minimizing total passenger travel cost and total passenger travel time), we design an *adaptive iterative algorithm* based on the augmented ε-constraint method proposed by Mavrotas (2009) to solve the Pareto frontier curve. The augmented ε-constraint method can generate Pareto optimal solutions and avoids generating weak Pareto solutions (Mavrotas, 2009). The adaptive iterative algorithm is designed to iteratively implement the augmented ε-constraint method based on different settings of ε. By adaptively adjusting the value of ε, the proposed iterative algorithm can achieve a good balance between the density of Pareto frontier points and the computation efficiency.

5.1. Augmented ε-constraint method

The augmented ε-constraint (AUGMECON) model of the bi-objective optimization model is shown below, where the objective $\min z_1$ is used as an extra constraint with the value of ε as the upper bound. The positive slack variable π is introduced to represent the difference between ε and z_1 and is forced to be maximized so that the AUGMECON model can generate a Pareto optimal solution. Readers can refer to Mavrotas (2009) for the proof of Pareto optimal solutions.

Augmented **ε**-constraint (AUGMECON) model:

$$
\begin{aligned}
&\min z_2 - eps \cdot \pi \\
&s.t. \\
&z_1 + \pi = \varepsilon \\
&Constraints (1) - (16) \\
&z_1 = \sum_{od \in U} \sum_{p \in P_{od}} \left(TC_{od,p} \cdot y_{od,p} \right) \\
&z_2 = \sum_{od \in U} \sum_{p \in P_{od}} \left(TT_{od,p} \cdot y_{od,p} \right) \\
&\pi \in R^+
\end{aligned}
\tag{19}
$$

where eps is a sufficiently small number.

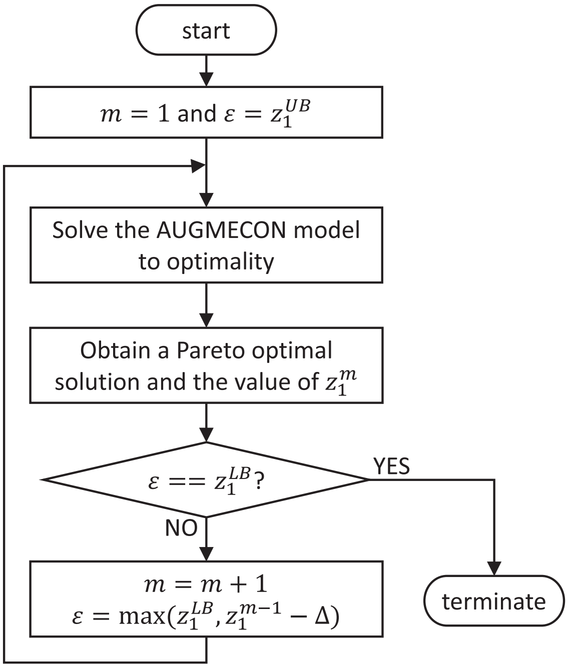

Fig. 9. Flowchart of the proposed adaptive iterative algorithm.

5.2. Adaptive iterative algorithm

We first calculate the upper and lower bounds (denoted by z_1^{UB} and z_1^{LB}) of the objective z_1. Specifically, z_1^{UB} is set as the sum of the maximum travel costs in each passenger's candidate paths. z_1^{LB} is determined by solving the proposed integrated optimization model with only the objective $\min z_1$. The upper and lower bounds are input to our adaptive iterative algorithm.

The process of the adaptive iterative algorithm is depicted in Fig. 9. In each iteration, the AUGMECON model with a specific value of ε is solved to generate a Pareto optimal solution. In the first iteration, the value of ε is set to z_1^{UB}. In the m $^{\text{th}}$ iteration ($m > 1$), the value of ε is set to $z_1^{m-1} - \Delta$, where z_1^{m-1} represents the value of z_1 in the Pareto optimal solution obtained in the $(m-1)$ $^{\text{th}}$ iteration, and Δ represents the minimum difference in z_1 between two adjacent Pareto optimal solutions obtained. By adjusting the setting of Δ, a trade-off between the density of Pareto frontier points and computation time is advisable. The smaller the Δ is, the denser the Pareto frontier points will be, and the longer the computation time is. The iteration stops when the value of ε is less than or equal to z_1^{LB}.

6. Case study

In this section, a series of numerical experiments were implemented on the Chengdu URT network to illustrate the application of the proposed integrated optimization model. The model was solved by GUROBI 9.5.2 on a laptop with AMD Ryzen 7 5800H @ 3.20 GHz and 13.9 GB RAM. The structure of this section is organized as follows. Section 6.1 describes the basic information about the Chengdu URT network and bridging services. Section 6.2 reports the effectiveness of the proposed passenger path generation method. Section 6.3 reports the solution efficiency of the proposed integrated optimization model. Section 6.4 reports the effect of cooperation between URT trains and taxis, and Section 6.5 reports the effect of cooperation between URT trains, taxis, and bridging buses further. The analyses in Sections 6.4 and 6.5 are based on single-objective optimization, while Section 6.6 is designed to explore the trade-off between the two objectives (i.e., to minimize total passenger travel cost, and to minimize total passenger travel time). Finally, while only the last train timetable is considered to be variable in Sections 6.4 – 6.6, in Section 6.7 we further study the effect of simultaneously adjusting the last train and non-last train timetable together.

6.1. Network description

Fig. 10 depicts the map of the Chengdu URT network as of April 2019. It consists of 7 bi-directional URT lines and 156 stations. The down direction of each URT line is denoted by the arrow next to the line name. The start and terminal stations of each bi-directional line are marked with triangles. The original train timetable, the maximum and minimum train running/dwell times, and the minimum safety headways were collected from the Chengdu URT corporation. According to the original timetable, the earliest closure time among all directional lines was 23:14:09 (i.e., the down direction of Line 10), while the latest closure time among all directional lines was 00:12:46 (i.e., the down direction of Line 3). In the numerical experiments, the closure time of each URT directional line is allowed

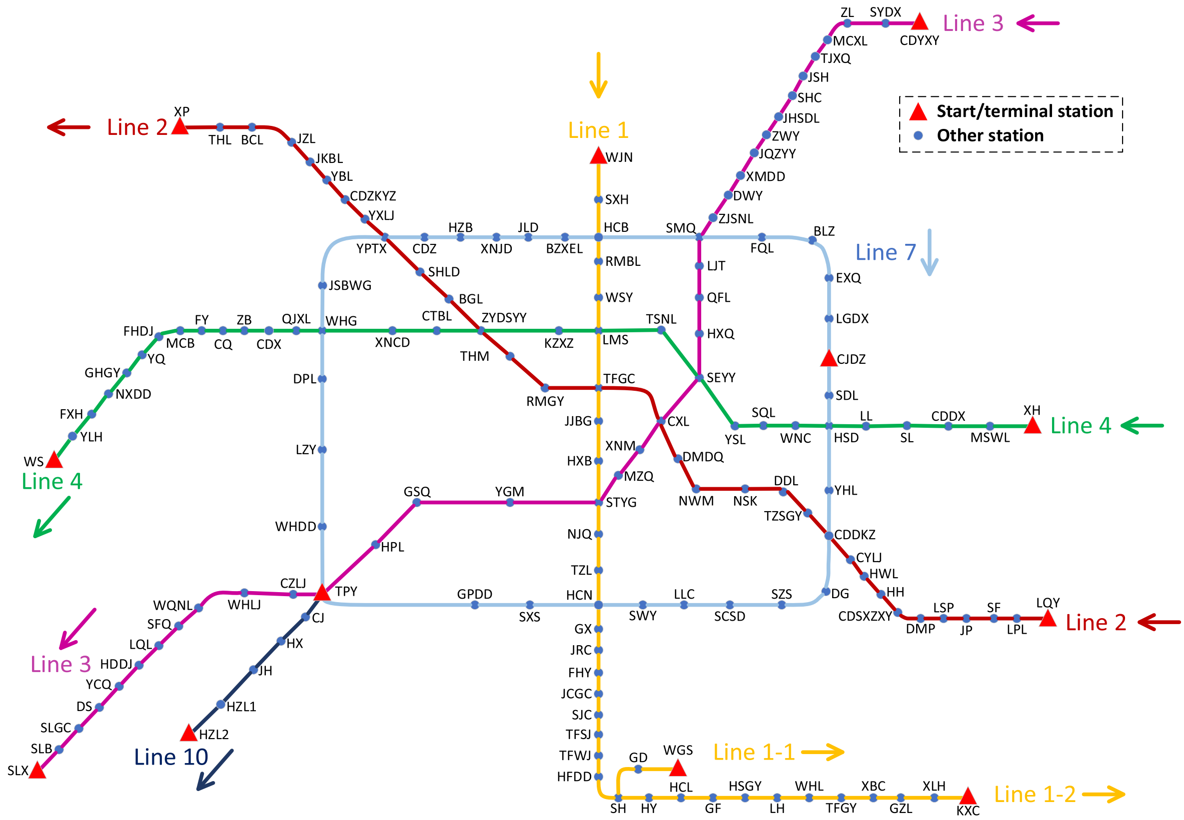

Fig. 10. Chengdu URT network.

to be postponed by at most 10 min than in the original timetable, which coincides the setting in Chen et al. (2019b).

The 156 stations form a total of 24,180 OD pairs with different origin stations and/or different destination stations. To comprehensively analyze the performance of the Chengdu URT network, these 24,180 OD pairs were all considered in the numerical experiments. The number of passengers for each OD pair was set as 1 to consider equal OD importance. Each passenger was assumed to arrive at the origin station just after the departure time of the penultimate train but before the departure time of the last train.

The travel time and travel cost in URT trains and taxis were obtained from the website (https://map.baidu.com/). The minimum passenger demand requirement for deploying a bus bridging route (i.e. *MQ*) was set to 200 passengers. The travel cost for a bridging bus per passenger was set to ¥ 2, which coincides with the bus price in most cities in China. Buses are cheaper than both taxis and URT trains in the considered real-life cases of Chengdu city. The travel times of bridging buses were assumed to be longer than bridging taxis, and were set as the travel times of bridging taxis plus the additional dwell times at each stop along the way. In all cases of Section 6, the number of taxis available at each train-to-train transfer station is set to infinity, and to zero at other URT stations. In Section 7, we performed cases where limited numbers of available taxis at all stations are considered.

6.2. *The effectiveness of passenger path generation methods*

In this section, we present the scale and computation time for generating passenger candidate paths, and demonstrate the effectiveness of merging OD pairs and path dominance principles.

6.2.1. *The results of merging OD pairs*

The considered 24,180 OD pairs were merged into 1,296 groups by the method introduced in Section 4.1. Since the OD pairs in the same group have the same candidate paths in terms of contained transfers, the path generation process only needs to be performed 1,296 times instead of 24,180 times without merging, which saved a lot of computation time for generating passenger paths.

We illustrated the distribution of the number of OD pairs in each group in Fig. 11. It can be seen that most of the groups contained less than 10 OD pairs, while a few groups contained more than 100 OD pairs. On average, each group contained 19 OD pairs.

6.2.2. *The results of passenger path generation*

For the considered 1,296 passenger groups, different passenger path sets were generated based on different values of k (i.e. the maximum number of physical URT paths generated per OD pair), the number of trains with variable timetables, and whether path

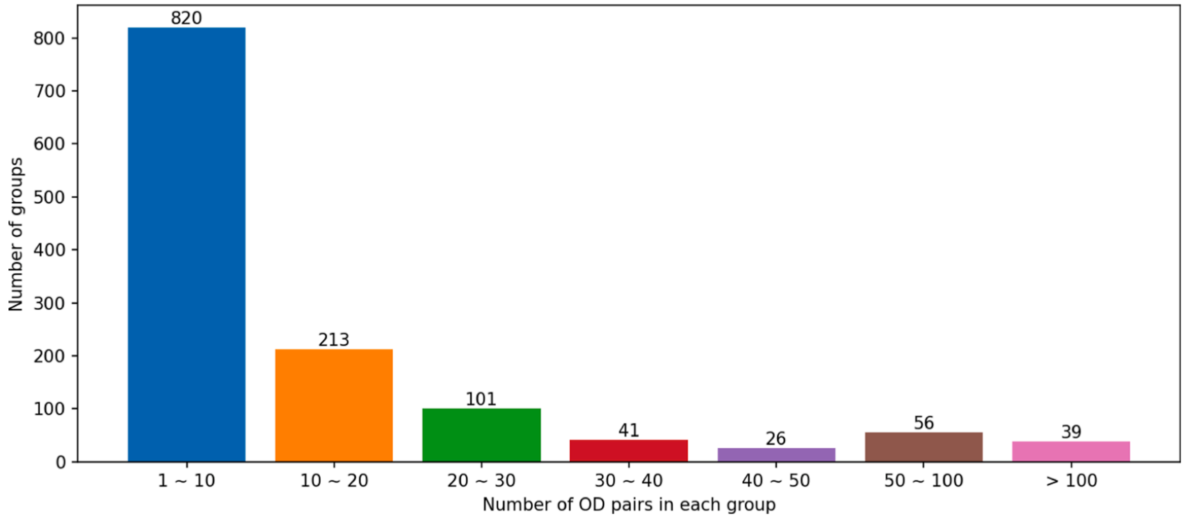

Fig. 11. Distribution of the number of OD pairs in each group.

Table 6
Results of passenger path generation.

# train(s) per line	K	time [min]	# all paths			# pure URT paths			# multimodal paths		
			N	Y	RED	N	Y	RED	N	Y	RED
1	3	2.4	15,221	11,370	25.3%	8,719	4,956	43.2%	5,206	5,118	1.7%
1	5	5.4	27,238	19,123	29.8%	16,402	8,722	46.8%	9,387	8,952	4.6%
1	10	12.6	46,401	29,124	37.2%	29,591	13,903	53.0%	15,323	13,734	10.4%
1	inf	41.1	72,031	38,347	46.8%	47,984	19,018	60.4%	22,553	17,835	20.9%
2	3	10.2	39,352	20,936	46.8%	28,335	10,235	63.9%	9,721	9,405	3.3%
3	3	30.6	94,550	37,615	60.2%	75,557	19,413	74.3%	17,697	16,906	4.5%

 # trains: 1 – only last train timetables are variable; **2** –the timetables of last trains and penultimate trains are variable; **3** – the timetables of last trains, penultimate trains, and antepenultimate trains are variable.
N/Y: The path set was constructed **without/with** path dominance principles.
RED: The reduction of the number of paths by path dominance principles.

dominance principles were applied or not. The computational results are summarized in Table 6. Note that the computation time is the total time of merging OD pairs, generating each type of candidate paths (including pure URT paths, pure taxi paths, and multimodal paths), and with applying path dominance principles to remove redundant paths.

From Table 6, we can observe that when only considering the last train of each URT line, the larger the k was, the more candidate paths for passengers will be generated, and the longer the computation time was. When k was the same, the more the trains with variable timetables we have, the more candidate paths for passengers will be generated, and the longer the computation time was. However, thanks to merging OD pairs, the total computation time for generating passenger paths was not long. In our numerical experiments, the longest computation time for path generation was 41 min, and the corresponding number of passenger candidate paths was 38,347 (after applying path dominance principles). In addition, by our method, the path generation of each passenger group is independent of each other, parallel computing can thus be adopted to further reduce the computation time of path generation.

6.2.3. The effect of path dominance principles

Results in Table 6 show that the proposed path dominance principles can effectively eliminate numerous paths. The larger the value of k, the more paths were generated, and more paths were eliminated by the path dominance principles. The more trains with variable timetables were, the more paths will be generated, and more paths were eliminated by the path dominance principles. In particular, when k was set to 3 and the timetables of last trains, penultimate trains, and antepenultimate trains were all variable, up to 60.2% of the paths were eliminated. In addition, among the removed paths, there are far more pure URT paths than multimodal paths.

To further demonstrate the effectiveness of path dominance principles, we solved our proposed optimization model on the passenger path sets constructed with and without path dominance principles. To illustrate the generality of these path dominance principles, we here adopted the objective function that is most commonly used in the existing literature on the last train timetabling problem. That is, to maximize the number of passengers who can take pure URT paths to their destinations. The model solution results are reported in Table 7.

Results in Table 7 show that although numerous paths were removed by path dominance principles, the models can still obtain the same optimal solutions as the ones without path reductions. This indicates that our path dominance principles effectively eliminated

Table 7
Model solution results with and without path dominance principles.

# train	*K*	# variables			# constraints	time [sec]			obj
		N	Y	RED		N	Y	RED	
1	3	10,012	6,185	38.2%	5,201	31	11	64.5%	16,439
1	5	17,698	9,961	43.7%	5,374	68	30	55.9%	16,954
1	10	30,887	15,143	51.0%	5,414	113	72	36.3%	17,150
1	inf	49,280	20,258	58.9%	5,421	130	60	53.8%	17,178
2	3	30,919	12,602	59.2%	8,771	53,527	7,587	85.8%	17,988

N/Y: The path set was constructed without/with path dominance principles.
RED: The reduction of the number of variables or the solution time of the optimization model due to path dominance principles.

Table 8
Comparison of the scale and efficiency of our model with an existing advanced model.

# trains	*K*	# paths	This paper					Zhou et al. (2019)				
			# VAR	# CON	obj	gap [%]	time [sec]	# VAR	# CON	obj	gap [%]	time [sec]
1	3	4,956	6,185	5,201	16,439	0	11	496,075	984,357	16,439	0	149
1	5	8,722	9,961	5,374	16,954	0	30	599,141	1,183,146	16,954	0	2,278
1	10	13,903	15,143	5,414	17,150	0	72	628,967	1,232,489	17,150	0	8,083
1	inf	19,018	20,258	5,421	17,178	0	60	637,961	1,240,256	17,178	0	37,610
2	3	10,235	12,602	8,771	17,988	0	7,587	956,690	1,896,040	17,988	0	76,696

VAR: The number of variables.
CON: The number of constraints.

the paths that do not contribute to improving the optimal objective value. Besides, owing to the reduction of paths, the number of decision variables in models was significantly reduced, especially when k was set to 3 and the timetables of last train and penultimate trains were both variable, i.e. 59.2% of decision variables were reduced. The number of constraints is independent of the number of passenger paths, which has been analyzed in Section 3.5.1. This indicates that our model has good scalability. Furthermore, the computation time was also reduced dramatically by applying path dominance principles. Particularly, when k was set to 3 and the timetables of last train and penultimate trains were both variable, the computation time was reduced by 85.8% (from 15 h to 2 h) by applying path dominance principles.

6.3. The efficiency of our proposed model

In this section, we demonstrate the efficiency of our proposed model and compare it with the existing state-of-the-art model proposed by Zhou et al. (2019), where they stated that their proposed model can be solved directly with the commercial solver with satisfactory computation time and solution quality. For consistency purpose, the two models adopted the same objective function (i.e., maximizing the number of passengers who can take pure URT paths to their destinations, the one used in Zhou et al. (2019)), same passenger input (i.e., 1,296 passenger groups) and same set of passenger candidate paths (i.e., only include pure URT paths). The computational results of the two models are summarized in Table 8.

From Table 8, we can observe that the solution efficiency of our proposed model is greatly affected by the number of trains with variable schedules. This may be due to the inclusion of a set of big M constraints in our model to describe the relationship between transfer feasibility and train timetables (i.e. Constraints (11)). When the number of trains with variable timetables increases, the number of transfers that need to be decided increases, and the number of the big M constraints also increases, thus affecting the solution efficiency of the model.

Compared with the model in Zhou et al. (2019), our proposed model outperforms in terms of model scale and solution efficiency. Specifically, the numbers of decision variables and constraints of our model were much less than those of the model in Zhou et al. (2019). The computation time of our model was also much less than that of the model in Zhou et al. (2019). In particular, when the timetables of last trains and penultimate trains were both variable, the number of decision variables in our model was only 1.3% of the model in Zhou et al. (2019), and the number of constraints in our model was only 0.5% of the model in Zhou et al. (2019). The model in Zhou et al. (2019) took more than 21 h to solve to the optimum, but our model only took 2 h.

6.4. The effect of cooperation between URT trains and taxis

To reveal the performance of the cooperation between URT trains and taxis, we optimized the last train timetable under two different passenger path sets using our proposed integrated optimization model: 1) one path set only includes pure URT paths and pure taxi paths (U + T); and 2) another set includes pure URT paths, pure taxi paths, and multimodal paths (U + T + M). The two optimization objectives (i.e., minimizing total passenger travel cost and minimizing total passenger travel time) were considered separately. In this section, the timetable of non-last trains was considered to be fixed to the original timetable. We considered generating

Table 9
Results under URT and taxi cooperation.

Case	Path types	# paths	Solution time [sec]	Total travel cost [¥]	Total travel time [hour]	Average travel cost [¥]	Average travel time [min]
	min *total travel cost*						
1	U + T	6,252	11	481,287	13,078	19.9	32.5
2	U + T + M	11,370	8	374,414	12,523	15.5	31.1
	min *total travel time*						
3	U + T	6,252	3	744,222	12,273	30.8	30.5
4	U + T + M	11,370	1	638,078	9,445	26.4	23.4

T/U/M: respectively represent three types of passenger paths, i.e., T (pure taxi path), U (pure URT path) and M (multimodal path).

Table 10
The effect of the multimodal path under different optimization objectives.

Case	Path types	# passengers			URT revenue [¥]	Travel time in taxis [hour]
		T	U	M		
	min *total travel cost*					
1	U + T	7,861 (32.5%)	16,319 (67.5%)	0	87,006	4,716
2	U + T + M	3,093 (12.8%)	16,291 (67.4%)	4,796 (19.8%)	100,534	3,556
	min *total travel time*					
3	U + T	14,235 (58.9%)	9,945 (41.1%)	0	47,970	8,079
4	U + T + M	5,205 (21.5%)	4,477 (18.5%)	14,498 (60.0%)	53,270	7,316

T/U/M: respectively represent the number of passengers who choose pure taxi paths, pure URT paths, or multimodal paths to reach their destinations.

three physical URT paths for each passenger, therefore, a total of 11,370 candidate paths were generated by applying path dominance principles. The computational results are summarized in Table 9.

As shown in Table 9, the total travel time and total travel cost of passengers were significantly reduced with the addition of the multimodal paths (i.e., with the cooperation between URT trains and taxis). Specifically, when considering minimizing total passenger travel cost, the total passenger travel cost in case 2 (U + T + M) was reduced by 22.2% compared to case 1 (U + T). When considering minimizing total passenger travel time, the total passenger travel time in case 4 (U + T + M) was reduced by 23.0% compared to case 3 (U + T).

We next analyze the change in the number of passengers who choose each type of paths after adding multimodal paths. As shown in Table 10, when considering minimizing total passenger travel cost in cases 1 (U + T) and 2 (U + T + M), the number of passengers choosing pure URT paths barely changed after adding multimodal paths. This is because multimodal paths include the use of taxi services, which are generally more expensive than pure URT paths. In addition, about 20% of passengers moved on from pure taxi paths to multimodal paths in pursuit of lower travel costs. The average travel time for passengers dropped from 32.5 min (in case 1, U + T) to 31.1 min (in case 2, U + T + M) as shown in Table 9. When considering minimizing total passenger travel time in cases 3 (U + T) and 4 (U + T + M), as shown in Table 10, 37.4% (=58.9% – 21.5%) of passengers moved on from pure taxi paths to multimodal paths, and 22.6% (=41.1% – 18.5%) of passengers moved on from pure URT paths to multimodal paths. Consequently, up to 60.0% of passengers chose multimodal paths when minimizing total passenger travel time. This implies that multimodal paths may take shorter travel times than pure URT paths or/and pure taxi paths.

In addition, multimodal paths can also bring in the following two significant benefits. The first benefit is that the revenue of the URT company (i.e., the total travel cost of passengers for URT trains) increased despite URT services sharing passengers with taxi services in multimodal paths. Such increase is demonstrated in Table 10, for a 15.6% higher revenue in case 2 (U + T + M) than in case 1 (U + T) when considering minimizing total travel cost. When considering minimizing total travel time, the URT revenue in case 4 (U + T + M) was also increased by 11.0% compared to case 3 (U + T). The reason behind this increase is that although multimodal paths require passengers sharing between URT and taxi services, they also help URT services attract more passengers.

Another benefit of introducing multimodal paths is that the total passenger travel time in taxis was reduced. Specifically, as shown in Table 10, when considering minimizing total passenger travel cost, the total passenger travel time in taxis in case 2 (U + T + M) was reduced by 24.6% compared to case 1 (U + T). When considering minimizing the total passenger travel time, the total passenger travel time in taxis in case 4 (U + T + M) was reduced by 9.4% compared to case 3 (U + T). The reason for such reduction is because the number of passengers choosing pure taxi paths decreased. Although such passengers may turn to multimodal paths which also include the use of taxis, the increased use of URT services can still significantly reduce the total travel time in taxis. This also implies that multimodal paths could be a safer transport mode for passengers during night traveling, particularly for female travelers who may find safer with public transportation over private ones.

Finally, we report the changes in the optimized last train timetable after adding the multimodal paths in Table 11. It can be seen that the number of feasible transfers included in passengers' selected paths (i.e. # FT) was reduced after adding the multimodal paths. On the one hand, this is beneficial for passengers, because it can reduce the possibility of passenger transfer failure caused by uncertain

Table 11
Comparison of the optimized timetables with/without URT and taxi cooperation.

Case	Path types	# FT	# SA	# SD	TotalA [min]	TotalD [min]	MaxA [min]	MaxD [min]
	min *total travel cost*							
1	U + T	178	167	280	1525.8	3621.6	8.7	12.6
2	U + T + M	170	174	272	1527.2	3193.8	8.7	12.1
	min *total travel time*							
3	U + T	160	205	242	1191.6	2694.1	6.3	12.6
4	U + T + M	127	252	193	1655.5	2525.6	8.3	10.8

FT: The number of feasible transfers included in passengers' selected paths.
SA/SD: The number of stations at which the last train timetable was advanced/delayed compared to the original timetable.
TotalA/TotalD: Total advance/delay time.
MaxA/MaxD: Maximum advance/delay time.

Table 12
Results under URT, taxi, bridging bus cooperation.

Case	Bridging services	Solution time [sec]	# bus routes	Total travel cost [¥]	Total travel time [hour]	Average travel cost [¥]	Average travel time [min]
2	Taxi	8	0	374,414	12,523	15.5	31.1
5	Taxi, bus*	/	4	347,286	12,540	14.4	31.1
6	Taxi, bus (opt)	37	31	301,043	11,629	12.5	28.9

 # bus routes: Number of bus bridging routes deployed in the solution.

factors in the actual operation. On the other hand, this is also beneficial for URT operations. There are fewer feasible transfers that need to be maintained in actual operation, while the URT revenue still increased after adding the multimodal paths (as shown in Table 10). In addition, Table 11 indicates that the total delay time (TotalD) was decreased and the total advance time (TotalA) was increased after adding the multimodal paths. It implies that passenger may be able to reach their destinations or intermodal transfer stations earlier (i.e. shorter travel time), which helps to increase passenger satisfaction with URT services.

6.5. *The effect of URT train, taxi, and bridging bus cooperation*

In this section, we analyze the effect of incorporating bridging buses based on the collaboration between URT and taxis. Compared to bridging taxis, bridging buses offer lower travel costs but longer travel times for passengers. This means that when we take minimizing total passenger travel time as the objective function, the number of bus bridging routes deployed will be zero. Therefore, we only considered the objective function of minimizing total passenger travel cost. We only adjusted the timetable of last trains and considered the non-last train timetable to be fixed to the original timetable. The passenger candidate paths were the same as in Section 6.4 (i.e. 11,370 paths). The minimum passenger demand requirement for deploying a bus bridging route (i.e. *MQ*) was set to 200. Based on this, a total of 52 candidate bus bridging routes were generated. Two new scenarios (cases 5 and 6) were designed to verify the effect of bus bridging services, as well as the scenario (case 2) that has been investigated for another purpose in Section 6.4.

(1) **Case 2**: Bridging buses were not considered. The last train timetable was optimized under the cooperation between URT trains and taxis. (i.e. case 2 in Section 6.4)
(2) **Case 5**: The last train timetable was optimized under the cooperation between URT trains and taxis. Then, based on passenger path choices, bus bridging routes were deployed to replace bridging taxis where passenger demand was greater than the minimum value of *MQ*.
(3) **Case 6**: The last train timetable was optimized under URT train, taxi, and bridging bus cooperation.

We first compare the optimization results of cases 2 and 5. As shown in Table 12, replacing some bridging taxis with four bridging buses on the basis of URT and taxi cooperation (in case 5) can reduce passenger travel costs by 7.2%, while only causing a slight increase (i.e. 0.1%) in travel times for passengers. As shown in Table 13, 4.7% of passengers chose the bridging buses after adding bridging buses (in case 5), so that the total passenger travel time in taxis was reduced by 14.8% compared to case 2.

Further, we investigate the effect of integrating bridging bus route design into the last train timetable optimization (i.e. case 6). Compared to case 5 (without integration), in case 6 the total travel cost of passengers was further reduced by 13.3%, and the total travel time of passengers was also unexpectedly reduced by 7.3% (as shown in Table 12). The reasons for the reduction in passenger travel time are explained below. When bridging buses were not available in case 5, in pursuit of lower travel costs, passengers may need to take long detours in the URT network to reach their destinations or intermodal transfer stations (that are close to their destinations) to reduce the travel cost by taxis. Since buses are cheaper than both taxis and URT trains, when bridging buses were available in case 6, rather than detouring within the URT network, these passengers would move on to multimodal paths and transfer to a

Table 13
The effect of the integrated optimization of the last train timetable and bus bridging routes.

Case	Bridging services	# passengers				URT revenue [¥]	Travel time in taxis [hour]
		T	U	M	Bus		
2	Taxi	3,093 (12.8%)	16,291 (67.4%)	4,796 (19.8%)	0	100,534	3,556
5	Taxi, bus*	3,093 (12.8%)	16,291 (67.4%)	4,796 (19.8%)	1,146 (4.7%)	100,534	3,029
6	Taxi, bus (opt)	3,051 (12.6%)	10,650 (44.1%)	10,479 (43.3%)	8,051 (33.3%)	79,442	2,557

T/U/M: respectively represent the number of passengers who choose pure taxi paths, pure URT paths, or multimodal paths to reach their destinations.
Bus: the number of passengers who choose bridging buses as bridging services.

Table 14
Comparison of the optimized timetables with/without bridging buses.

Case	Bridging services	# FT	# SA	# SD	TotalA [min]	TotalD [min]	MaxA [min]	MaxD [min]
2	Taxi	170	174	272	1527.2	3193.8	8.7	12.1
5	Taxi, bus*	170	174	272	1527.2	3193.8	8.7	12.1
6	Taxi, bus (opt)	195	190	257	1299.1	3675.5	6.5	12.3

FT: The number of feasible transfers included in passengers' selected paths.
SA/SD: The number of stations at which the last train timetable was advanced/delayed compared to the original timetable.
TotalA/TotalD: Total advance/delay time.
MaxA/MaxD: Maximum advance/delay time.

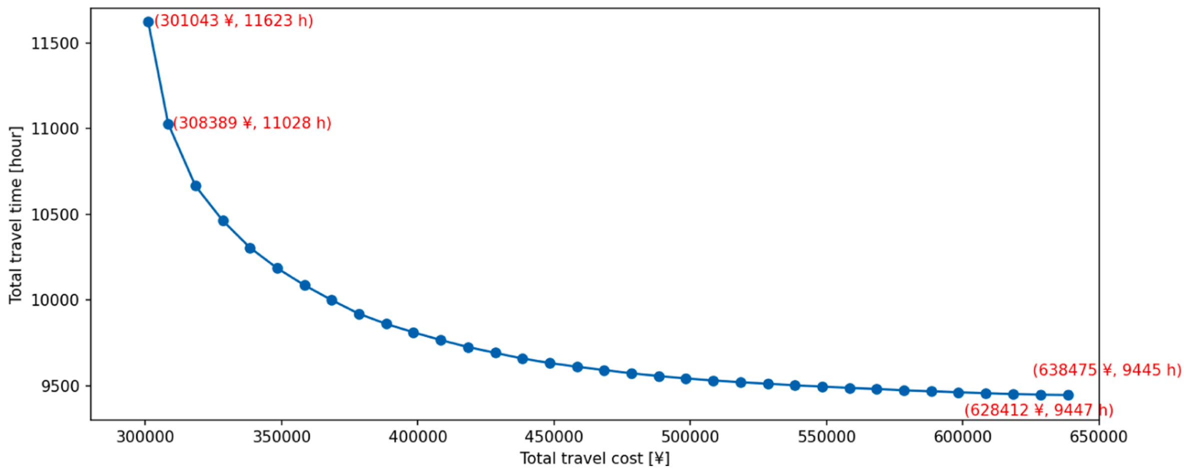

Fig. 12. Pareto frontier curve under the last train timetable optimization.

bridging bus as early as possible. This is verified by that 23.3% (=67.4% – 44.1%) of passengers moved on from pure URT paths to multimodal paths, and the number of passengers choosing bridging buses were increased by 28.6% (=33.3% – 4.7%), as shown in Table 13. Consequently, the passenger travel time in URT trains were greatly reduced in case 6 compared to case 5, which was far greater than the increase in travel time caused by the use of bridging buses. This is verified by the 21.0% reduction in URT revenue in case 6 (as shown in Table 13). Therefore, the total passenger travel time was reduced when bridging bus route design was integrated into the last train timetable optimization. Additionally, the total passenger travel time in taxis was further reduced by 15.6% (as shown in Table 13) through the integrated optimization of the last train timetable and bus bridging routes.

Finally, we report the changes in the optimized last train timetable after incorporating the bus bridging route design into the last train timetable optimization in Table 14. It can be seen that the number of feasible transfers included in passengers' selected paths (i.e. # FT) was increased after incorporating collaborative optimization of bridging bus routes. This implies that after the introduction of bridging buses, although passengers will reduce the use of URT trains, higher requirements are put forward on the transfer feasibility between URT trains, which deserves the attention of URT operators.

6.6. The trade-off between travel cost and travel time

In this section, we analyze the trade-off between the total passenger travel cost and the total passenger travel time by generating the Pareto frontier curve using the adaptive iterative algorithm described in section 5. Here we only adjusted the timetable of last trains and considered the non-last train timetable to be fixed to the original timetable. To balance the density of Pareto frontier points and

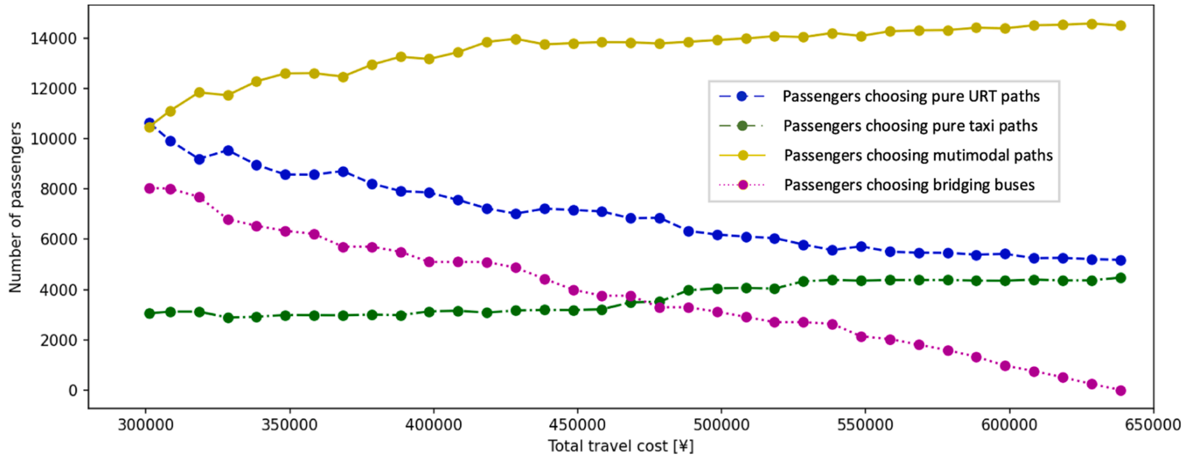

Fig. 13. Numbers of passengers choosing different types of paths under different Pareto-optimal solutions.

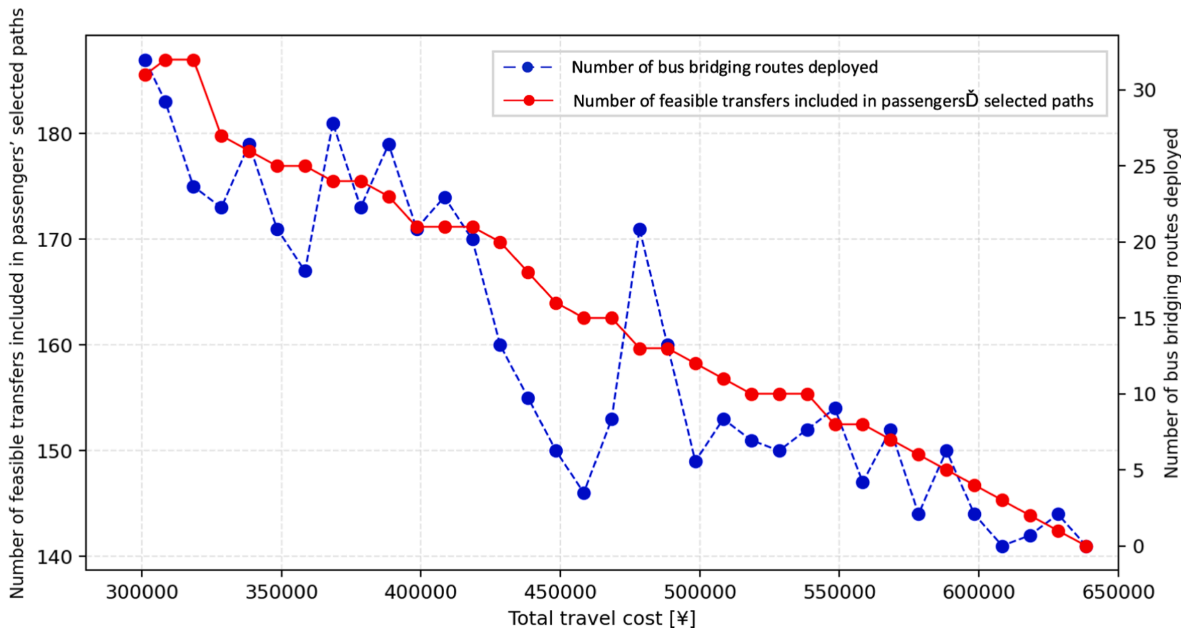

Fig. 14. Numbers of bus bridging routes and numbers of feasible transfers under different Pareto solutions.

computation time, Δ (i.e., the minimum difference in z_1 between two adjacent Pareto optimal solutions obtained) was set to 10,000. Recall that z_1 refers to the total passenger travel time. The algorithm carried out a total of 35 iterations, and the average calculation time of each iteration was only 51 s. The obtained Pareto frontier curve is illustrated in Fig. 12.

From Fig. 12, we can observe that the passenger travel cost can be reduced at the cost of increasing passenger travel time. When the total travel time was the minimum (i.e. 9,445 h), the total travel cost was the maximum (i.e. ¥ 638,475). At this point, to reduce the total travel cost by ¥ 10,063 (i.e. from ¥ 638,475 to ¥ 628,412), the total travel time only needs to be increased by 2 h (i.e. from 9,445 h to 9,447 h). However, as passenger travel costs decreased, the increase in passenger travel times accelerated. Particularly, to reduce the total travel cost from ¥ 308,389 to ¥ 301,043 (i.e. a reduction of ¥ 7,346), the total travel time has to be increased by 595 h (i.e. from 11,028 h to 11,623 h). When the total travel cost was the minimum (i.e. ¥ 301,043), the total travel time was the maximum (i.e. 11,623 h).

Fig. 13 illustrates the numbers of passengers choosing different types of paths under different Pareto optimal solutions. On the one hand, under each Pareto optimal solution, the number of passengers who chose multimodal paths was the largest, followed by the number of passengers who chose pure URT paths, and the least number of passengers who chose pure taxi paths. On the other hand, as passenger travel costs decreased, while the number of passengers who chose multimodal paths was decreased, the proportion of passengers who used bridging buses was increased. Specifically, when the total travel cost was the maximum, although the number of

Table 15
Results under the timetable optimization of last trains and non-last trains.

Case	# trains	# paths	# CBR	solution time [sec]	# SBR	Tot travel cost [¥]	Tot travel time [hour]	Avg travel cost [¥]	Avg travel time [min]
	min *total travel cost*								
6	1	11,370	52	37	31	301,043	11,629	12.5	28.9
7	2	20,936	76	1,260	42	186,561	11,370	7.7	28.2
8	3	37,615	81	52,135	44	159,198	11,352	6.6	28.2
	min *total travel time*								
9	1	11,370	52	1	0	638,078	9,445	26.4	23.4
10	2	20,936	76	11	0	587,488	8,921	24.3	22.1
11	3	37,615	81	51	0	573,474	8,748	23.7	21.7

trains: 1 – optimize the last train timetable; 2 – optimize the timetable of the last train and penultimate train; 3 – optimize the timetable of the last train, the penultimate train, and the antepenultimate train.
paths: The number of passenger candidate paths.
CBR/SBR: respectively represent the number of candidate bus bridging routes and the number of bus bridging routes deployed in the optimal solutions.

Table 16
The effect of collaborative optimization of non-last train timetables.

Case	# trains	# passengers				URT revenue [¥]	Travel time in taxis [hour]
		T	U	M	Bus		
	min *total travel cost*						
6	1	3,051 (12.6%)	10,650 (44.1%)	10,479 (43.3%)	8,051 (33.3%)	79,442	2,557
7	2	1,456 (6.0%)	10,969 (45.4%)	11,755 (48.6%)	10,334 (42.7%)	84,197	1,088
8	3	917 (3.8%)	11,626 (48.1%)	11,637 (48.1%)	10,319 (42.7%)	86,973	711
	min *total travel time*						
9	1	5,205 (21.5%)	4,477 (18.5%)	14,498 (60.0%)	0	53,270	7,316
10	2	3,176 (13.1%)	5,330 (22.1%)	15,674 (64.8%)	0	56,310	6,736
11	3	2,707 (11.2%)	5,351 (22.1%)	16,122 (66.7%)	0	57,296	6,560

T/U/M: respectively represent the number of passengers who choose pure taxi paths, pure URT paths, or multimodal paths to reach their destinations.
Bus: the number of passengers who choose bridging buses as bridging services.

passengers who chose multimodal paths is the largest among all Pareto optimal solutions, none of the passengers use bridging buses. When the total travel cost was the minimum, the number of passengers who chose multimodal paths was the least among all Pareto optimal solutions, but more than three-quarters of the passengers used bridging buses.

Finally, Fig. 14 illustrates the numbers of bridging buses deployed and the numbers of feasible transfers under different Pareto optimal solutions. It can be seen that as passenger travel costs decreased, the number of bus bridging routes deployed was increased, but the number of feasible transfers required by passengers was also increased. This indicates that when passengers pursue lower travel costs, more bus bridging routes can be deployed, but it will place higher requirements on the transfer feasibility between URT trains, which may increase the difficulty of daily operation. In actual operation, URT operators can determine the beneficial last-train timetable and bus bridging routes based on the above aspects.

6.7. The impact of collaborative timetable optimization of non-last trains and last trains

Since passengers may transfer between non-last trains and last trains, optimizing the timetable of both last trains and several related non-last trains may be helpful to further improve passenger services (e.g., transport more passengers to reach their destinations via URT trains, reduce passenger travel costs and travel times).

6.7.1. Single objective optimization

We first analyze the impact of collaborative timetable optimization of non-last trains and last trains under single objective. The two optimization objectives (i.e., minimizing total passenger travel cost and minimizing total passenger travel time) were considered separately. The results are summarized in Table 15 and Table 16 and compared with the results of optimizing only the last train timetable.

We first analyze the effect of optimizing the timetables of both last trains and penultimate trains. When considering minimizing total passenger travel cost, as shown in Table 15, optimizing the timetables of last trains and penultimate trains (in case 7) can significantly reduce the total passenger travel cost by 38.0% compared to optimizing only last train timetable (in case 6). Meanwhile, as shown in Table 16, the total travel time in taxis was also reduced by 57.5% through optimizing the timetable of penultimate trains.

Table 17
Comparison of the optimized timetables under collaborative optimization of non-last train timetables.

Case	# trains	# FT	# SA	# SD	TotalA [min]	TotalD [min]	MaxA [min]	MaxD [min]
	min *total travel cost*							
6	1	195	190	257	1299.1	3675.5	6.5	12.3
7	2	258	575	258	6872.5	2949.8	13.7	14.0
8	3	393	800	427	9870.9	3684.4	18.8	11.4
	min *total travel time*							
9	1	127	252	193	1655.5	2525.6	8.3	10.8
10	2	194	500	334	4844.1	3424.1	13.0	11.0
11	3	264	705	518	7157.0	5824.9	13.2	11.3

FT: The number of feasible transfers included in passengers' selected paths.
SA/SD: The number of stations at which the last train timetable has been advanced/delayed compared to the original timetable.
TotalA/TotalD: Total advance/delay time.
MaxA/MaxD: Maximum advance/delay time.

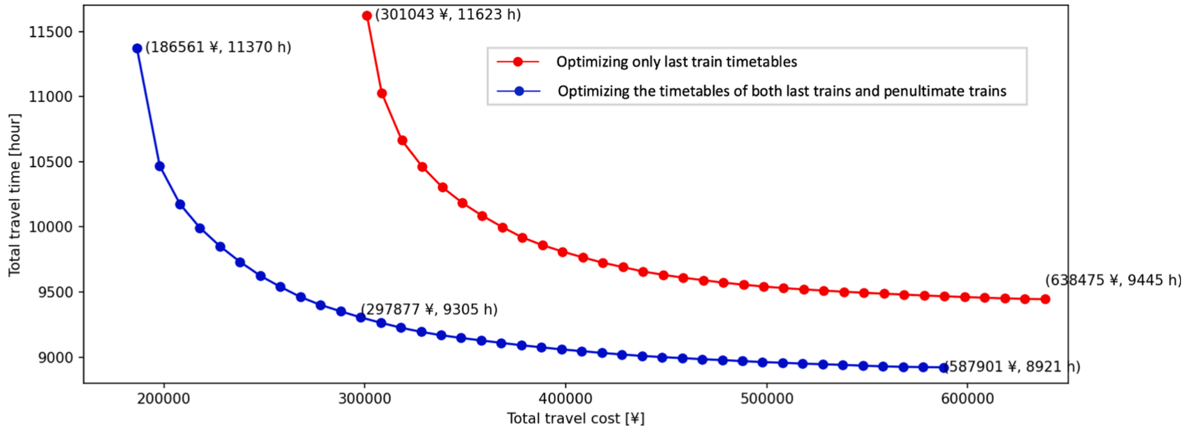

Fig. 15. Pareto frontier curve under the timetable optimization of different trains.

This is mainly due to the improved transfer feasibility between URT trains. On the one hand, the use of URT trains by passengers was increased, which is verified by the increase in URT revenue (shown in Table 16). On the other hand, there are eleven more bus bridging routes along which passenger demand was greater than *MQ* (shown in Table 15) and can therefore be deployed to serve passengers. When considering minimizing total passenger travel time, as shown in Table 15, optimizing the timetables of last trains and penultimate trains (in case 10) can reduce the total passenger travel time by 5.5% and reduce the total passenger travel cost by 7.9% compared to optimizing only last train timetable (in case 9). Meanwhile, as shown in Table 16, the URT revenue was increased by 5.7% through optimizing the timetable of penultimate trains. No bus bridging routes were deployed as the travel time by buses is generally longer than taxis. As for the computation time to optimize the timetables of both last trains and penultimate trains, the solution time was about 2 h when minimizing total passenger travel cost, and only 11 s when minimizing total passenger travel time. Both are acceptable because the last train timetabling problem is a tactical problem. In conclusion, it is worthwhile to optimize the timetables of both last trains and penultimate trains at the same time.

However, when simultaneously optimizing the timetables of last trains, penultimate trains, and antepenultimate trains, as shown in Table 15 and Table 16, there was only little improvement in passenger services quality (i.e., travel time and travel cost) and URT revenue in cases 8 and 11 compared to cases 7 and 10. Specifically, when minimizing total passenger travel cost in case 8, the average travel cost was reduced by ¥ 1.1, and the URT revenue was increased by 3.3%. When minimizing total passenger travel time in case 11, the average travel time was reduced by 0.4 min, and the URT revenue was increased by 1.8%. Nevertheless, the computation time when minimizing total passenger travel cost was quite long (i.e. about 14.5 h in case 8). From this aspect, it is not necessary to optimize the timetables of last trains, penultimate trains, and antepenultimate trains at the same time.

Finally, we report the changes in the optimized train timetable under the collaborative optimization of last train timetables and non-last train timetables in Table 17. It can be seen that the more trains that can be adjusted, the greater the changes to the original timetable will be. URT operators can decide which non-last train timetables to adjust taking into account the improvements discussed above.

6.7.2. Bi-objective optimization

We analyze the impact of collaborative timetable optimization of non-last trains and last trains under two objectives. The adaptive

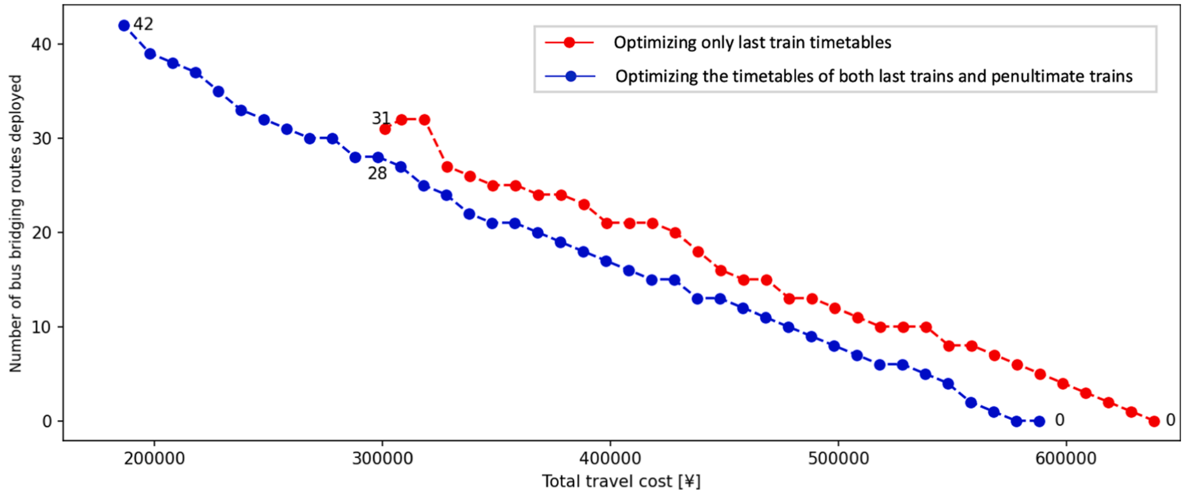

Fig. 16. Numbers of bus bridging routes under the timetable optimization of different trains.

Table 18
Results under different numbers of taxis available.

Case	# taxis	Solution time [sec]	# bus routes	Total travel cost [¥]	Total travel time [hour]	Average travel cost [¥]	Average travel time [min]
	min *total travel cost*						
6	Inf	37	31	301,043	11,629	12.5	28.9
12	Limited	116	31	312,602	11,856	12.9	19.4
	min *total travel time*						
4	Inf	1	0	638,078	9,445	26.4	23.4
13	Limited	331	26	524,360	10,778	21.7	26.7

taxis: Number of taxis available. “Inf” represents the situation where the number of available taxis was set to infinity at each train-to-train transfer station, and zero at other URT stations. “Limited” represents the situation where the number of taxis available was randomly generated based on historical taxi GPS data.
bus routes: Number of bus bridging routes deployed in the solution.

iterative algorithm was adopted to obtain the Pareto frontier curve. The algorithm carried out a total of 41 iterations, and the average calculation time of each iteration was 10.5 min. The value of Δ (i.e., the minimum difference in z_1 between two adjacent Pareto optimal solutions obtained) was set to 10,000. The collaborative optimization results were compared with the results where only last train timetables are optimized. See Fig. 15 and Fig. 16.

As shown in Fig. 15, similar to the optimization results that consider only last train timetables, when additionally optimizing the penultimate train timetables, the passenger travel cost can be reduced at the cost of increasing passenger travel time. As passenger travel costs decreased, the increase in passenger travel times accelerated. In addition, as shown in Fig. 16, as passenger travel costs decreased, the number of bus bridging routes deployed was increased.

Under the same total passenger travel cost, the minimum total travel time that can be achieved by optimizing the timetables of last trains and penultimate trains was much smaller than the minimum total travel time that can be achieved by optimizing only last train timetables. In particular, when the total passenger travel cost was ¥ 301,043 (i.e. the minimum travel cost that can be achieved by optimizing only last train timetables), the corresponding passenger travel time was 11,623 h under the optimization of only last train timetables, while the passenger travel time were reduced to 9,305 h (i.e. a 19.9% reduction) by coordinated timetable adjustment of penultimate trains. Meanwhile, the number of bus bridging routes was also reduced from 31 to 28 by coordinated timetable adjustment of penultimate trains, as shown in Fig. 16.

7. Further extensions

In the previous section, we set the number of taxis available at each train-to-train transfer station to infinity, and to zero at other URT stations. In this section, we verify the practicability and effectiveness of the proposed model and method in the case of a limited number of taxis available.

Based on historical taxi GPS data (mot.gov.cn), the spatial distribution of taxis in Chengdu during the last train operation period can be obtained. Based on this, the number of available taxis at each station was randomly generated. The last train timetable was optimized based on the two different objective functions (i.e., minimizing total passenger travel cost and minimizing total passenger

Table A1
Definition of sets.

Symbol	Description
L	Set of directional URT lines in the given URT network, indexed by l and l'. A bi-directional URT line includes two different directional URT lines.
$S(l)$	Set of stations on line l, $S(l) = \{s_l^1, s_l^2, \cdots, s_l^n\}$, where station s_l^1 and s_l^n represent the start and terminal stations, respectively. Each station is indexed by s.
$I(l)$	Set of trains on line l, $I(l) = \{i_l^1, i_l^2, \cdots, i_l^{n-1}, i_l^n\}$, where i_l^n and i_l^{n-1} represent the last and penultimate trains, respectively. Each train is indexed by i.
$Iv(l)$	Set of trains of which the timetable will be determined, $Iv(l) \subseteq I(l)$.
U	Set of passenger groups, indexed by od, where o and d indicate the origin and destination stations, respectively.
P_{od}	Set of candidate paths of passengers in group $od \in U$. Each path is indexed by p.
TF	Set of train-to-train transfers. Each transfer is indexed by $tf = (l, i, l', i', s)$, representing that passengers transfer from train $i \in I(l)$ to train $i' \in I(l')$ at station $s \in S(l) \cap S(l')$.
BR	Set of candidate bus bridging routes. Each bus route is denoted by (s_1, s_2), meaning that this bridging route will start at station s_1 and end at station s_2.

Table A2
Definition of parameters.

Symbol	Description
i^{pre}	The preceding train of train i.
l^{pre}	The directional URT line where train i^{pre} is. That is, $i^{\text{pre}} \in I(l^{\text{pre}})$.
$\underline{DT}_{l,s}$	The minimum dwell time at station $s \in S(l)$ on line $l \in L$.
$\overline{DT}_{l,s}$	The maximum dwell time at station $s \in S(l)$ on line $l \in L$.
$\underline{RT}_{l,s,s+1}$	The minimum running time from station $s \in S(l)$ to station $s+1 \in S(l)$ on line $l \in L$.
$\overline{RT}_{l,s,s+1}$	The maximum running time from station $s \in S(l)$ to station $s+1 \in S(l)$ on line $l \in L$.
H_l^{arr}	The minimum safety headway between successive arrivals of trains on line $l \in L$.
H_l^{dep}	The minimum safety headway between successive departures of trains on line $l \in L$.
H_l^{da}	The minimum safety headway between the departure of the preceding train and the arrival of the following train on line $l \in L$.
T_l	The latest allowed service closure time for line $l \in L$.
$\tau_{l,s,i}^{\text{arr}}$	The original arrival time of train $i \in I(l)$ at station $s \in S(l)$ on line $l \in L$.
$\tau_{l,s,i}^{\text{dep}}$	The original departure time of train $i \in I(l)$ at station $s \in S(l)$ on line $l \in L$.
W_{tf}	The transfer walking time associated with transfer $tf \in TF$.
$W_{s_1}^{\text{TB}}$	The transfer walking time associated with the train-to-bus transfer at station s_1 where $(s_1, s_2) \in BR$.
Q_{od}	The number of passengers in group $od \in U$.
$\theta_{tf,p}$	$= 1$, if transfer $tf \in TF$ is contained in path $p \in P_{od}$; $= 0$, otherwise.
s_p^{dep}	The station where passengers leave the URT network along path $p \in P_{od}$. If path p is a pure taxi path, then station s_p^{dep} is passengers' origin station. If path p is a pure URT path, then station s_p^{dep} is passengers' destination station. If path p is a multimodal path, then station s_p^{dep} is the station where passengers transfer to bridging services.
MQ	The minimum passenger demand requirement for deploying a bus bridging route.
MN_s	The number of taxis available at station s.
$TC_{od,p}^{\text{taxi}}$	The travel cost of all passengers in group $od \in U$ traveling along path $p \in P_{od}$ where taxis are used as bridging transportation.
$TC_{od,p}^{\text{bus}}$	The travel cost of all passengers in group $od \in U$ traveling along path $p \in P_{od}$ where buses are used as bridging transportation.
$TT_{od,p}^{\text{taxi}}$	The travel time of all passengers in group $od \in U$ traveling along path $p \in P_{od}$ where taxis are used as bridging transportation.
$TT_{od,p}^{\text{bus}}$	The travel time of all passengers in group $od \in U$ traveling along path $p \in P_{od}$ where buses are used as bridging transportation.
M	A sufficiently large positive value.

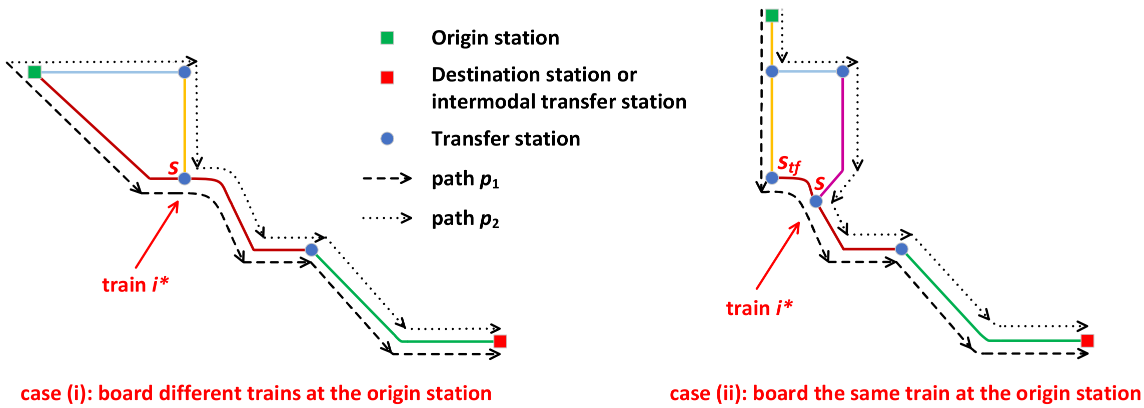

Fig. C1. Examples to illustrate path dominance principles.

travel time) respectively. The timetable of non-last trains was considered to be fixed to the original timetable. The computational results (i.e. cases 12 and 13) are compared with the solution results under the condition that the number of available taxis is infinite (i. e. case 4 in Section 6.4 and case 6 in Section 6.5), see Table 18.

The results show that the model and method proposed in this paper can also deal with the situation where the number of available taxis is limited and may even be smaller than passenger demand. The solution time was also satisfactory. Comparing to the case where the number of taxis was infinite, both travel cost and travel time increased slightly due to the limited number of available taxis. Interestingly, when minimizing total passenger travel time, due to the limitation of the number of available taxis, passengers would tend to concentrate on the designated transfer station to take the bridging bus, which reduced the total passenger travel cost.

8. Conclusions and future research

This paper focuses on the integrated optimization of last train timetabling and bridging service design within multimodal urban transport networks. Two bridging services are considered: taxis and buses. We developed a bi-objective MINLP model with low complexity and high efficiency. We considered two objectives – minimizing total passenger travel times and minimizing total passenger travel costs. An adaptive iterative algorithm was proposed to obtain the Pareto frontier curve of the bi-objective optimization model. The proposed algorithm can achieve a good balance between the density of Pareto frontier points and the computation efficiency. This model was established based on pre-generated passenger path sets. To improve solution efficiency of the model, we proposed three path dominance principles to eliminate redundant paths that do not contribute to improving the optimal objective value without loss of optimality. The proposed model and solution methods have been tested on the Chengdu URT network. According to the results of numerical experiments, we can draw the following conclusions:

- The proposed optimization model can be solved by commercial solvers with satisfactory computation time and solution quality for a real-life URT network.
- The proposed path dominance principles can effectively remove numerous redundant paths without loss of optimality and can reduce the computation time to solve the model.
- With the addition of the cooperation of bridging services (taxis and buses), passenger travel costs and travel times can be significantly reduced.
- By our model, URT companies can simultaneously optimize train timetables and design bridging services according to the trade-off between passenger travel times and travel costs. Multimodal paths can take shorter travel times than pure URT paths or/and pure taxi paths. With multimodal paths, the total passenger travel time in taxis was reduced, which provides passengers with a safer night travel environment particularly for female travelers. In addition, with the introduction of multimodal paths, fewer feasible transfers need to be maintained in the URT network while the revenues for URT companies still increase. For URT companies this means easier operation and more profits. When bridging buses were not available, in pursuit of lower travel costs, passengers may need to take long detours in the URT network to reach their destinations or leave from intermodal transfer stations (that are close to their destinations) to reduce the travel cost by taxis. When introducing bus bridging services, rather than detouring within the URT network passengers would choose multimodal paths to transfer to a bridging bus as early as possible, because buses are cheaper than both taxis and URT trains. However, when more bus bridging routes are deployed, it will place higher requirements on the transfer feasibility between URT trains, which may increase operation difficulty for URT companies.
- URT operators are suggested to simultaneously adjust the timetable of several non-last trains (e.g. penultimate trains are enough for Chengdu URT network) when determining the last train timetable, which can increase the revenue of URT companies and reduce the number of bus bridging routes deployed.

Further research can be extended in the following aspects. (1) The proposed model is a deterministic optimization model. In reality, on the one hand, the spatial–temporal distribution of passenger demand fluctuates daily. On the other hand, the travel times in taxis and buses fluctuate with stochastic road conditions, and the waiting time for taxis is affected by the number of available taxis. In addition, the number of available taxis fluctuates with stochastic road conditions. Therefore, it is of practical interest to design a robust train timetable considering uncertain passenger demand and uncertain road conditions. (2) For simplicity, in this paper, the train capacity is assumed to be sufficient to carry all passengers. Otherwise, URT companies should consider extending the operating hours and/or adding more URT trains to operation. An interesting topic could be to determine the service closure time and the number of trains running during the last-train period considering the train capacity and actual passenger demand. (3) In order to apply the integrated optimization model to larger URT networks, we can design a more efficient algorithm according to the structure and characteristics of the model in future research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. . Sets and parameters

See Table A1 and Table A2.

Appendix B. . Linearization

The Constraint (10) is the product of two binary variables. It can be linearized according to Equation (B.1).

$$y = x_1 \cdot x_2 \Leftrightarrow \begin{cases} y \leq x_1 \\ y \leq x_2 \\ y \geq x_1 + x_2 - 1 \\ y \in \{0, 1\} \end{cases} \tag{B.1}$$

where x_1 and x_2 are both binary variables.

The objective function is the product of a continuous variable and a binary variable. It can be linearized according to Equation (B.2).

$$y = x_3 \cdot x_4 \Leftrightarrow \begin{cases} y \leq x_4 \\ y \geq x_4 - ub \cdot (1 - x_3) \\ lb \cdot x_3 \leq y \leq ub \cdot x_3 \end{cases} \tag{B.2}$$

where x_3 is a binary variable, $x_4 \in [lb, ub]$ is a continuous variable.

Appendix C. . Optimality proofs of path dominance principles

Before illustrating the optimality proof of the three path dominance principles, we first introduce Lemma 1, which states the pivotal characteristics of redundant passenger paths.

Lemma 1. *Given two paths $\boldsymbol{p}_1 \in \boldsymbol{P}_{od}$ and $\boldsymbol{p}_2 \in \boldsymbol{P}_{od}$ ($p_1 \neq \boldsymbol{p}_2$) that connect to the same destination station or the same intermodal transfer station, if the trains boarding at the origin station along the two paths are the same, the two paths go through the same station s, and the station-train pairs after station s (including station s) are the same for the two paths, then, there must be at least one transfer along each path before station s (including station s). Additionally, under these circumstances, if there is only one transfer along path $\boldsymbol{p}_1$ (or path $\boldsymbol{p}_2$) before station s (including station s), this transfer cannot be located at station s.*

Proof of Lemma 1. Suppose that there is no transfer along path p_1 before station *s* (including station *s*), or there is only one transfer along path p_1 before station *s* (including station *s*) that is located at station *s*. The train boarding at the origin station *o* along the two paths is denoted by *i*. That is, passengers traveling along path p_1 take train *i* from the origin station *o* directly to station *s*. This indicates station *o* and station *s* are located on the same URT line. Since passengers will not go through the same URT line more than once (see Assumption 6), and passengers traveling along path p_2 also board train *i* at the origin station *o* and goes through station *s*, passengers traveling along path p_2 take train *i* from station *o* directly to station *s* without any transfer before station *s* (excluding station *s*). This indicates that paths p_1 and p_2 are exactly the same before station *s*. Besides, the station-train pairs after station *s* (including station *s*) are the same for the two paths. Consequently, paths p_1 and p_2 are exactly the same path, which contradicts $p_1 \neq p_2$. Therefore, there must be at least one transfer along path p_1 before station *s* (including station *s*), and if there is only one transfer along path p_1 before station *s* (including station *s*), this transfer cannot be located at station *s*. The proof is completed (see Fig. 17).

Proof of Principle 1. Since $SI^{\text{all}}_{od,p_1} \subseteq SI^{\text{all}}_{od,p_2}$, that is, all the transfers contained in path p_1 are included in path p_2. Therefore, if path p_2 is feasible, path p_1 must be feasible; whereas if path p_1 is feasible, path p_2 may not be feasible. Moreover, if the two paths are both feasible, path p_1 will be selected instead of path p_2 due to that $\max(TC^{\text{taxi}}_{od,p_1}, TC^{\text{bus}}_{od,p_1}) \leq \min(TC^{\text{taxi}}_{od,p_2}, TC^{\text{bus}}_{od,p_2})$, and $\max(TT^{\text{taxi}}_{od,p_1}, TT^{\text{bus}}_{od,p_1}) \leq \min(TT^{\text{taxi}}_{od,p_2}, TT^{\text{bus}}_{od,p_2})$. Thus, path p_2 can be removed from the path set without loss of optimality. The proof is completed.

Proof of Principle 2. Before the optimality proof, we wish to emphasize the fact that for a given train, the slower the train (i.e., the later the train arrives), or the earlier the passenger arrives, the more likely the passenger will catch up with the train. The following symbols are defined for the proof:

t: the arrival time of passengers at the origin station.

i^*: the train that passengers will take when they depart from station *s*.

STT_{s,p_2}: the shortest travel time from the origin station to station *s* along path p_2 (including the transfer walking time of passengers at station *s*).

LTT_{s,p_1}: the longest travel time from the origin station to station *s* along path p_1 (station *s* is not a transfer station along path p_1).

Since the station-train pairs after station *s* (including station *s*) are the same for the two paths, if passengers on both paths can reach station *s*, then the reachability of the two paths is the same. Therefore, the key reason for the different reachability of the two paths lies in whether passengers can reach station s (and successfully transfer at station s) along these two paths .

According to condition (3), Principle 2 includes two cases which will be discussed separately. For clarity, an example of each case is given in Figure C. 1, where different colors represent different URT lines.

(1) Passengers will board *different* trains at the origin station along the two paths.

In this case, one requires that there is no transfer along path p_1 before station s (including station s). Otherwise, when calculating the longest travel time of path p_1, the waiting time at the origin station cannot be ignored. In addition, since train i^* is defined as the train that passengers will take when they depart from station s, it indicates that passenger traveling along path p_1 will take train i^* at the origin station. Since the station-train pairs after station s (including station s) are the same for the two paths, station s is the transfer station where passengers will transfer to train i^* along path p_2. Note that there may be other transfers before station s (excluding station s) along path p_2.

We consider an extreme situation. That is, passengers travel along path p_2 with the shortest travel time (i.e., the trains run with the shortest running times and shortest dwell times, and passengers do not have any waiting time) and just catch train i^* at station s, and then train i^* departs from station s. Accordingly, the departure time of train i^* at station s is equal to $(t + STT_{s,p_2})$. Assume that train i^* runs with the longest running times and longest dwell times, and $(t + STT_{s,p_2})$ is the latest time for train i^* to depart from station s. Under this circumstance, we can obtain that the earliest departure time of train i^* at the origin station is equal to $(t + STT_{s,p_2} - LTT_{s,p_1})$. Since $STT_{s,p_2} > LTT_{s,p_1}$, $t + STT_{s,p_2} - LTT_{s,p_1} > t$. Therefore, passengers traveling along path p_1 can catch train i^* at the origin station. This indicates that if path p_2 is feasible, then path p_1 must be feasible.

However, consider another extreme situation, that is, suppose that passengers traveling along path p_1 can just catch train i^* at the origin station. Accordingly, the departure time of train i^* at the origin station is equal to t. Assume that train i^* runs with the longest running times and longest dwell times, then the departure time of train i^* at station s is equal to $(t + LTT_{s,p_1})$. Since $LTT_{s,p_1} < STT_{s,p_2}$, $t + LTT_{s,p_1} < t + STT_{s,p_2}$. This indicates that under this circumstance, even if passengers travel along path p_2 with the shortest travel time, they cannot catch train i^* at station s. Therefore, if path p_1 is feasible, path p_2 may not be feasible.

(2) passengers will board the *same* train at the origin station along the two paths.

In this case, one requires that there is only one transfer along path p_1 before station s (including station s). Otherwise, when calculating the longest travel time of path p_1, the waiting times (i.e., the waiting time at the origin station and the transfer waiting time) cannot be ignored. According to Lemma 1, this transfer is located before station s (excluding station s), denoted by s_{tf}. That is, passengers traveling along path p_1 will transfer to train i^* at station s_{tf}. Since the station-train pairs after station s (including station s) are the same for the two paths, passengers traveling along path p_2 will transfer to train i^* at station s. Note that there must be other transfers before station s (excluding station s) along path p_2, according to Lemma 1.

We consider an extreme situation. That is, passengers travel along path p_2 with the shortest travel time and just catch train i^* at station s, and then train i^* departs from station s. Accordingly, the departure time of train i^* at station s is equal to $(t + STT_{s,p_2})$. Assume that train i^* runs with the longest running times and longest dwell times, and $(t + STT_{s,p_2})$ is the latest time for train i^* to depart from station s. Under this circumstance, we can obtain that the earliest departure time of train i^* from the transfer station s_{tf} is equal to $(t + STT_{s,p_2} - \Delta LTT_i^{tf,s})$, where $\Delta LTT_i^{tf,s}$ represents the longest travel time from station s_{tf} to station s. In addition, the latest arrival time of passengers at station s_{tf} is equal to $(t + LTT_{s,p_1} - \Delta LTT_i^{tf,s})$. Since $STT_{s,p_2} > LTT_{s,p_1}$, $t + STT_{s,p_2} - \Delta LTT_i^{tf,s} > t + LTT_{s,p_1} - \Delta LTT_i^{tf,s}$. Therefore, passengers traveling along path p_1 can catch train i^* at station s_{tf}. This indicates that if path p_2 is feasible, then path p_1 must be feasible.

However, consider another extreme situation, that is, suppose that passengers traveling along path p_1 can just catch train i^* at station s_{tf}. Accordingly, the departure time of train i^* at station s_{tf} is equal to $(t + LTT_{s,p_1} - \Delta LTT_i^{tf,s})$. Assume that train i^* runs with the longest running times and longest dwell times, then the departure time of train i^* at station s is equal to $(t + LTT_{s,p_1})$. Since $LTT_{s,p_1} < STT_{s,p_2}$, $t + LTT_{s,p_1} < t + STT_{s,p_2}$. This indicates that under this circumstance, even if passengers travel along path p_2 with the shortest travel time, they cannot catch train i^* at station s. Therefore, if path p_1 is feasible, path p_2 may not be feasible.

The above discussion of the two cases shows that if path p_2 is feasible, path p_1 must be feasible; whereas if path p_1 is feasible, path p_2 may not be feasible. Moreover, condition (5) indicates that if the two paths are both feasible, path p_1 will be selected instead of path p_2 due to that $TC_{od,p_1}^{\text{taxi}} \le TC_{od,p_2}^{\text{taxi}}$, $TC_{od,p_1}^{\text{bus}} \le TC_{od,p_2}^{\text{bus}}$, $TT_{od,p_1}^{\text{taxi}} \le TT_{od,p_2}^{\text{taxi}}$, and $TT_{od,p_1}^{\text{bus}} \le TT_{od,p_2}^{\text{bus}}$. Thus, path p_2 can be removed from the path set without loss of optimality. The proof is completed.

Proof of Principle 3. According to conditions (1) and (2), the reachability of the two paths is the same before the transfer to trains i_{od,p_1}^{des} and i_{od,p_2}^{des}. Therefore, the key reason for the different reachability of the two paths lies in whether passengers can catch up with trains i_{od,p_1}^{des} and i_{od,p_2}^{des}. Condition (3) (i.e., train i_{od,p_2}^{des} precedes train i_{od,p_1}^{des}) indicates that if passengers can catch up with train i_{od,p_2}^{des}, then they must be able to catch up with train i_{od,p_1}^{des}; whereas if passengers can catch up with train i_{od,p_1}^{des}, then they may not be able to catch up with train i_{od,p_2}^{des}. Moreover, condition (4) indicates that if the two paths are both feasible, path p_1 will be selected instead of path p_2 due to that $TC_{od,p_1}^{\text{taxi}} \le TC_{od,p_2}^{\text{taxi}}$, $TC_{od,p_1}^{\text{bus}} \le TC_{od,p_2}^{\text{bus}}$, $TT_{od,p_1}^{\text{taxi}} \le TT_{od,p_2}^{\text{taxi}}$, and $TT_{od,p_1}^{\text{bus}} \le TT_{od,p_2}^{\text{bus}}$. Based on above, path p_2 can be removed from the path set without loss of optimality. The proof is completed.

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