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DOCTORAL THESIS

Essays in Group Contests

This thesis is submitted in accordance with the requirements of the University of Liverpool for the degree of *Doctor of Philosophy*.

by
Zining Huang

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Supervisors:
Dr Olga Gorelkina
Dr Alessandro Marchesiani
Dr Ritesh Jain

Abstract

This thesis employs the Tullock contest success function to model behaviour and evaluate the strategic decisions made by interacting individuals within the framework of group contests. Its objective is to offer theoretical predictions concerning individual behaviours and group contest outcomes by incorporating psychological elements into the dynamics of group contests.

In the first chapter of this thesis, a broad review of theoretical literature on group contests is provided, with special emphasis on studies employing the Tullock contest success function. These theoretical frameworks, crucial for modelling strategic interactions and decisions among participants, have a wide array of applications, extending from economics and political science to biology and sports. The reviewed theoretical works are categorized along two dimensions: firstly, the model, including an examination of both standard and alternative models; and secondly, three key research topics—group formation, optimal group design, and the impact of interdependent preferences—related to group contests. These topics correspond to the three essential elements of a strategic game. By adopting this categorization approach, the first chapter aims to provide a comprehensive overview of the theoretical foundation in the field of group contests.

Building upon the theoretical findings detailed in the first chapter, the second chapter investigates the impact of ambiguity on players' equilibrium strategies in a group contest, assuming that players perceive ambiguity about the strategies of their teammates and opponents. In this chapter, I define ambiguity as a situation where players have beliefs about other players' strategies but may not be fully confident in them. By comparing the level of equilibrium effort under ambiguity with that of Nash equilibrium, I aim to provide an explanation for the commonly observed over-expenditure behaviour of subjects in previous experimental studies. Additionally, this chapter examines the influence of group structure on the overall effort, an aspect that is generally of primary concern to the game organizer. My findings offer an explanation for the commonly observed limitation on the number of groups in real-world contexts, such as sports games and R&D contests.

Moving from the exploration of ambiguity in Chapter 3, Chapter 4 proposes a new framework - the group size paradox and its relationship with interdependent preferences. In group contests, it is common that participants exhibit spite or altruism, which can affect their choices of action. Therefore, spite and altruism may serve as determinants of the Group Size Paradox (GSP). In this chapter, I present a model in which two groups of differing sizes compete against each other for a prize, where players may exhibit *intra*-group altruism or spite, combined with *inter*-group spite. My findings show that if the aggregate level of spite is relatively high, the larger group has a greater chance of winning the prize. Additionally, if players display *intra*-group altruism, the smaller group is more likely to emerge victorious. This research provides sufficient conditions for the invalidity of the GSP.

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Contents

Chapter 1

Introduction

Group contests, ranging from political activities to sports games, play a critical role in shaping the course of societies and economies. The expansive and multifaceted field of group contests has progressively emerged as a fundamental component in economic research. This thesis offers a comprehensive review of the literature on the Tullock group contests, conducts a theoretical analysis of the impact of ambiguity on group contests, and presents a new explanation for over-expenditure behaviour as well as a fresh perspective on the group size paradox.

Numerous real-world activities can be represented as group contests. Take sports games, for instance, team sports like basketball or American football naturally lend themselves to a group contest framework. In these scenarios, individuals form two competing teams. By investing in skilled coaches and dedicating more time and effort to training, they can enhance their team's probability of winning. The prize in such sports contests could be perceived as the victory itself. Additionally, this victory often comes bundled with tangible benefits such as progressing in a tournament, improving league standings, receiving a trophy, and potentially even monetary rewards. On the intangible front, benefits might include increased team morale, enhanced reputation, and stronger fan support.

This thesis is structured into five chapters. The first chapter serves as a general introduction, providing a broad overview of the thesis. In the second chapter, my focus is on identifying and summarizing the previously developed models, offering a structured review of the existing theoretical research. Chapter three presents an engaging analysis of players' behaviour, specifically, examining how their best responses are affected by the perceived ambiguity regarding the effort levels expended by other players. In the fourth chapter, we shift our focus to the group size paradox, a fascinating puzzle where larger groups paradoxically experience a disadvantage in terms of their winning potential due to their larger size. While chapters 2, 3, and 4 each could stand alone as individual papers, they share a number of common characteristics. Firstly, all three chapters centre around Tullock group contests. Secondly, all of them are grounded in theoretical frameworks. Thirdly, chapter 2 serves as a theoretical foundation for chapters 3 and 4. Moreover, both chapters 3 and 4 are dedicated to integrating ideas from behavioural economics into theoretical models of group contests. Finally, the fifth chapter concisely wraps up the thesis, providing a conclusion that brings together the key points explored in the previous chapters.

While significant progress has been made in understanding the dynamics of group contests, incorporating concepts from behavioural economics into theoretical models of group contests may provide novel insights into prevalent questions in the field. Because theoretical predictions based on the assumption of self-interested players occasionally fall short in explaining certain behaviours observed in experimental studies. This includes over-expenditure in comparison to the Nash equilibrium, as indicated by Sheremeta and Zhang (2010) and Bhattacharya (2016). Experimental data also challenge the group size paradox (e.g., Oliver and Marwell, 1988; Kugler et al., 2010). Thus, there is a growing necessity to broaden the conventional theoretical assumptions, incorporating concepts like ambiguity and interdependent preferences into the theoretical model. This approach would potentially offer a more accurate prediction of individual behaviours.

The concept of ambiguity, initially introduced by Ellsberg (1961), has inspired a wealth of theoretical and experimental literature on the economics of ambiguity and ambiguity aversion in recent decades, including Fox and Tversky (1995), Maccheroni et al. (2006), and Baillon et al. (2018), among others.[1] Ambiguity is a ubiquitous phenomenon in various fields, ranging from professional actuaries and insurance underwriters (Kunreuther, 1989; Hogarth and Kunreuther, 1992) to medical decision-making by both patients and doctors (Ritov and Baron, 1990), as well as in financial markets (Bossaerts et al., 2010; Ahn et al., 2014). Further, even in a casino game like Roulette, where the objective probabilities are clearly defined - with each number carrying a $\frac{1}{37}$ chance of winning - gamblers may hold doubts regarding these established probabilities. This uncertainty can arise from several sources. For instance, casinos typically display the previous games' winning numbers. Coupled with cultural biases, such as in Chinese culture where the number 4 is often associated with bad luck, while numbers 6 and 8 are typically linked to good fortune, players can potentially misinterpret these factors as having an influence

[1] For a survey of economic papers on ambiguity, refer to Al-Najjar and Weinstein (2009), Etner et al. (2012) and Machina and Siniscalchi (2014).

on the game's outcomes.[2] If players perceive these past results and cultural beliefs as consequential, they might perceive ambiguity in what are, in fact, well-defined and fixed probabilities. Generally speaking, ambiguity is prevalent, even in activities with well-defined probabilities where certain factors may still induce ambiguity. Thus, it is reasonable to assume that individuals perceive ambiguity about other players' strategies in group contests. A range of factors can contribute to this ambiguity, including incomplete information, the complexity of the contests, as well as cultural and psychological influences.

Compared to ambiguity, there exists an extensive body of studies on group contests exploring the roles of spite and altruism and their impacts, such as those by Konrad (2004), Abbink et al. (2012) and Hu and Treich (2014). Research has revealed that identity can indeed boost contribution due to *inter*-group altruism (Chowdhury et al., 2016a). In scenarios where players face a threat from an out-group, it can trigger *inter*-group spite (Weisel and B0̈hm, 2015). Additionally, as suggested by Sheremeta (2018), parochial altruism could explain the observed over-expenditure of effort in group contests. Thus, it is natural to assume that a player may exhibit altruism towards *intra*-group members and show spite towards *inter*-group players. *Intra*-group spite, although seemingly counterintuitive, is more prevalent than one might assume. For instance, within a sports team, a player might deliberately withhold the ball from a teammate to prevent them from scoring and outperforming them, even if it could result in the team's loss.

This thesis presents several key findings that contribute to the field of group contests and the understanding of player behaviours. Firstly, the research reaffirms two widely acknowledged observations in literature: a tendency among individuals to avoid forming groups with other players, and the extensive investigation of the grand coalition. Secondly, it highlights the growing popularity of studies focusing on optimal group structure and contest design. This thesis highlights and summarizes the understanding of how interdependent preferences, a recent addition in contest models, help clarify certain behaviours observed in experiments. These behaviours include over-expenditure, cooperative punishment, and reciprocity. Importantly, this research uncovers the role of ambiguity aversion, *intra*-group altruism, and *inter*-group spite in effort expenditure: individuals tend to expend more effort when they exhibit a higher degree of ambiguity aversion, a stronger sense of *intra*-group altruism, or a higher degree of *inter*-group spite. From a policy standpoint, this thesis finds that group designers can reduce social waste by controlling the level of ambiguity perceived by players. Furthermore, with

[2] According to Marmurek et al. (2015), the belief that past results can influence future outcomes is known as the gambler's fallacy. Situations where certain numbers are considered 'lucky' and thought to have a higher probability of winning fall under the illusion of control.

the presence of ambiguity, the aggregate effort might decrease as the number of groups increases. Finally, a key discovery of this research is related to the group size paradox (henceforth, the GSP). The results suggest that this paradox can be overturned if the combined degree of *intra*- and *inter*-group preferences is relatively high.

This thesis primarily contributes to the literature in eight areas: i) it identifies and summarizes the existing theoretical literature on Tullock group contests, creating a comprehensive understanding of this area of study; ii) it adds value to the body of existing theoretical literature by critically examining the agreements and disagreements among researchers in the Tullock group contests. Through this comparative analysis, it illuminates the robustness of specific findings; iii) it enriches the discourse surrounding the phenomenon of over-expenditure in group contests, shedding light on its underlying causes; iv) it also contributes to the literature exploring mechanisms to mitigate the occurrence of over-expenditure in group contests; v) this research provides insights into why participant expenditures decrease over time, as observed in experimental studies; vi) it explains the reasons behind the common structure of certain sports games, which typically involve two teams instead of a larger number; vii) it provides conditions that could influence the validity of the GSP.

Chapter 2 identifies current trends in group contest research, including investigations into asymmetric players, group structure, the incorporation of interdependent preferences into contests, the impact of group structure and size, and the role of information sharing. Furthermore, it aims to present a thorough understanding of the present state of theoretical research on group contests, identify existing models for potential future development, and pinpoint research questions that remain unanswered or require a broader range of interpretations. The reviewed papers are categorized along two dimensions: the type of theoretical models employed and the research topics addressed. The first dimension focuses on the classification of explored models, where I introduce the conventional model of group contests and overview studies that employ alternative models. These alternative approaches may involve an array of elements, such as group impact functions that steer away from using perfect-substitute functions, extended contest success functions, diverse distribution rules, or non-linear cost functions. The second dimension groups papers according to three research topics: group formation, optimal group design, and interdependent preferences. Each of these topics corresponds to a crucial component of strategy games, as proposed by Osborne (2009).

Chapter 3 introduces the Choquet expected utility function as a tool to model group contests, offering a deviation from the subjective expected utility approach typically employed in the existing literature. This approach addresses scenarios where players perceive ambiguity in their teammates' strategies, as well as those of their opponents. In this context, a player's utility can be interpreted as a weighted average of payoffs drawn from three distinct scenarios: a standard Tullock group contest, an optimal scenario in which a player's teammates exert maximum effort and rival teams exert minimum effort, and a worst-case scenario wherein a player's teammates exert minimal effort and rival teams exert maximum effort. This chapter investigates how factors such as the degree of ambiguity, the extent of ambiguity aversion, the number of groups, and group size influence equilibrium effort and optimal group structure, particularly in relation to aggregate expenditure.

Chapter 4 offers a new theoretical explanation for the contrast between the GSP and experimental research findings. The model examines a group contest where two groups of different sizes compete for a prize, which is equally divided among all the members of the winning group. The model is based on four additional key assumptions: players are homogeneous; the sharing rule applied for prize allocation is exogenous; one of the most popular instances of the Tullock contest success function, known as the lottery contest, is adopted; and the group impact function used in the model is the most commonly discussed function, namely, the perfect-substitutes function.

Chapter 5 brings the thesis to a close, presenting a thorough discussion of its contributions and proposing potential directions for future research that can expand upon the groundwork laid in this thesis.

Chapter 2

A Survey of Theoretical Research on Group Contests: the Tullock Contests

2.1 Introduction

Group contests represent a type of activity where groups, composed of at least two individuals, compete against each other for one or more prizes. In such contests, individuals in each group pool their resources, including effort, time, intellect, and money, to compete as a unit. Group contests hold significant importance due to their pervasiveness and consequential nature in numerous real-world scenarios, such as sports matches, political activities, and research and development (R&D) contests. For example, the political process of dividing the Gross National Product (GNP) can be modelled as a multi-stage group contest (Wärneryd, 1998; Risse, 2011), and lobbying activities directed towards government entities can also be understood as a form of group contest (Riaz et al., 1995; Cheikbossian, 2008). Moreover, group contests include a wider range of decision-making processes, compared to contests that solely involve individual participants. An individual in a group contest may engage in numerous decisions, such as choosing a team to join based on its size or other factors, selecting other players to form a group, determining the rules for prize sharing if their group wins, deciding on the method of aggregating resources from within the group, and choosing whether to share certain information with *inter*-and *intra*-group players.

The purpose of this paper on group contests is multifaceted, primarily aimed at providing a comprehensive understanding of the current state of theoretical research and identifying the models that have been developed. In section 2.2, I present the standard model of group contests and review papers that adopt alternative models. These alternative models may adopt a variety of elements, such as group impact functions that do not employ the perfect-substitute function, generalized contest success functions, different sharing rules, or non-linear cost functions. Additionally, the articles reviewed in sections 2.3, 2.4, and 2.5 are all grounded on the models outlined in section 2.2.

This paper provides an identification of the trend of focus in group contest research, including the investigation of asymmetric players, group structure, the adoption of interdependent preferences into contests, the effects of group structure and size, and the role of information sharing. Additionally, by compiling previous papers around three topics, this paper is able to compare disagreements and agreements among researchers, thereby shedding light on the robustness of certain findings. For instance, the optimal group structure, which is aimed at maximizing aggregate expenditure, may vary depending on the specificities of the contest's setting. A balanced sorting of players may not consistently prove to be the optimal strategy. In regards to the agreement, one commonly accepted result is that *inter*-group altruism often leads players to spend more, while *intra*-group altruism tends to have the opposite effect.

This survey paper reviews an expansive range of theoretical papers on group contests. The studies discussed in this survey cover cases where elements such as the sharing rule, prize allocation, group size, and the number of groups are considered exogenously given. It also includes papers where these factors are considered endogenous. Research that deals with situations involving symmetric players, as well as those involving asymmetric players, is also reviewed. Papers exploring scenarios where information is fully disclosed, alongside those where some information is private, are reviewed. Additionally, investigations into contest success functions, illustrating decreasing, increasing, or constant returns to scale, are also reviewed. Moreover, the research surveyed spans a spectrum of player preferences, ranging from those displaying self-interest to those with interdependent preferences. Studies examining the impact of the cost functions, whether they are concave or convex, are also included in this paper. To ensure clarity and focus, this review does not include papers on group auctions, even though they could be regarded as fully discriminatory contests.

Papers are grouped and discussed according to three themes, rather than chronologically. The three thematic sections in this paper - group formation (Section 2.3), optimal group design (Section 2.4), and interdependent preferences (Section 2.5) - have been chosen to align with the fundamental components of strategy games as outlined by Osborne (2009). These components are a set of players, a set of actions for each player, and players' preferences regarding various action profiles. In the context of group contests, research on group formation can be viewed as corresponding to the first element, focusing on the players. From studies on group formation, we can obtain valuable insights into the composition of group members and the decision-making process of individuals who choose either to form part of a group or to remain as singletons. This exploration helps in our understanding of the mechanisms and motivations behind group formation and illuminates the endogenous dynamics of group organization.

The second theme, optimal group design, corresponds to the second element of a strategic game: the set of actions. Regardless of the contest designers' objective — whether they aim to maximize or minimize the aggregate effort, to increase the expected social surplus generated by the contest, to increase the likelihood of selecting the highest-performing participant, or to achieve other goals — these objectives are actually tied to players' actions. Therefore, by reviewing papers related to optimal group design, we can gain insights into how players make decisions regarding their strategies and how various factors influence their choice of actions, and thus affect the designers' objective.

The third theme concerns interdependent preferences, corresponding to the last component of a strategic game: preferences. Research focusing on interdependent preferences explores how the interdependent preferences of players can influence choices and outcomes in group contests, thereby deviating from scenarios where players are purely self-interested. Therefore, when taken in conjunction with Section 2.2, these three themes collectively cover the key elements of strategic games, thereby enabling a comprehensive understanding of group contests from various critical angles. In the final section, this paper concludes by drawing together insights from the reviewed papers and discussing their relevance to real-world scenarios. Furthermore, it highlights unresolved questions related to the study of group contests, which suggests potential directions for future investigation.

2.2 The model of group contests

Consider a Tullock group contest in its most general form where n groups compete against each other to win a prize. The prize is valued at v_{ij} by player i in group j. Group j consists of m_j risk neutral players, for $j = 1, 2, ..., n$. All players choose their irreversible and costly efforts simultaneously and independently. Player ij's effort is denoted by x_{ij}, for all $i = 1, 2, ..., m_j$ and $j = 1, 2, ..., n$. The group performance of group j, X_j, is determined by the combined efforts of all its individual members:

$$X_j = f_j(x_{1j}, x_{2j}, ..., x_{m_j j}) \tag{2.1}$$

The function f_j demonstrates how individual efforts are combined within the group, and it is also as known as the group impact function (e.g., Münster, 2009). There are four common group impact functions in the literature. The perfect-substitutes function $f_j(x_{1j}, x_{2j}, ..., x_{m_j j}) = \sum_{j=1}^{m_j} x_{ij}$ is the most commonly used one (e.g., Baik, 1993, 2008). The perfect-substitutes function's inherent characteristic of efforts being addable makes it suitable to model group contests where individuals' efforts are monetary. Many real-life activities, such as relay races, military conflicts, business projects, and football matches, rely on the total efforts of all team members for success. Specifically, when a crew is rowing, the success of their team relies on the coordinated effort of all rowers working together to propel the boat forward. Therefore, the group performance is dependent on the joint effort of all members in the boat.

The weakest-link function, $f_j(x_{1j}, x_{2j}, ..., x_{m_j j}) = min\{x_{1j}, x_{2j}, ..., x_{m_j j}\}$, describes a group contest in which the group performance is determined by the individual with the lowest effort within the group (e.g., Cornes and Hartley, 2007; Lee, 2012; Chowdhury and Topolyan, 2016b; Brookins et al., 2018). The weakest-link function is not an uncommon occurrence in group contests, and system reliability is a fitting example (Hausken, 2008). To be more specific, a system of software operations can be considered to follow a weakest-link function, because if any of them is infected, then the entire system will be susceptible to infection. Conversely, contests in which the group performance depends on the individual with the highest effort within a group can be characterized by the best-shot function, $f_j(x_{1j}, x_{2j}, ..., x_{m_j j}) = max\{x_{1j}, x_{2j}, ..., x_{m_j j}\}$ (e.g., Chowdhury et al., 2013; Barbieri and Malueg, 2016; Kelsey and Leroux, 2017). Arce et al. (2012) provide a prominent example of these two group impact functions, suggesting that in the context of counter-terrorism, the group of agencies who are responsible for protecting the potential target(s) from a terrorist group may adopt the weakest-link impact function, given that the attainment of complete safety from all possible attacks is a critical benchmark for successful counter-terrorism efforts. On the other hand, for terrorists, any successful terrorist attack by any member of the group is considered a desirable outcome, thus following a best-shot structure.

Lastly, another common group performance is represented by the constant elasticity of substitution (henceforth, CES) function (e.g., Choi et al., 2016; Kolmar and Rommeswinkel, 2013; Lee and Song, 2019; Konishi and Pan, 2020, 2021),

$$f_j(x_{1j}, x_{2j}, ..., x_{m_j j}) = \left(\sum_{j=1}^{m_j} a_{ij} * (x_{ij})^{\gamma_j} \right)^{\frac{1}{\gamma_j}}, \ \gamma_j \in \{(-\infty, 0), (0, 1)\}, \ a_{ij} \geq 0. \quad (2.2)$$

Kolmar (2013) adopts a_{ij} to measure an individual's effort efficiency. However, with assumptions that $a_{ij} \leq 1$ and $\sum_{i=1}^{m_j} a_{ij} = 1$, a_{ij} is utilized to indicate the weight of player i's expenditure in group j (Kobayashi, 2021).

This CES function is viewed as a generalized form to map individuals' efforts onto group impact. The variable elasticity of substitution of efforts within the group is $\frac{1}{1-\gamma_j}$, and γ_j is a parameter that describes the degree of complementarity. The best-shot impact function is the result for $\gamma_j \to \infty$ in the above function. In the limiting case of γ_j approaching $-\infty$, we have perfect complements impact function and in the limit, we have the weakest-link function. If γ_j approaches 0 in the limit, then a Cobb–Douglas aggregation function is obtained. As the value of γ_j approaches 1, the degree of complementarity between individuals' efforts decreases. In the limit $\gamma_j = 1$, the CES aggregator function is linear and we have perfect substitutability (i.e., $f_j(x_{1j}, x_{2j}, ..., x_{m_j j}) = \sum_{j=1}^{m_j} x_{ij}$) .[1] This form of impact function allows for intermediate degrees of complementarity between players for different valuations of γ_j.

Considering the more realistic scenarios, the perfect substitution between efforts is not the most appropriate benchmark case. According to Alchian and Demsetz (1972), effort complementarities within a group are one of the primary drivers for the existence of group production. This is unsurprising given that the existence of a group is motivated by synergy and cooperation between individuals. This is certainly true in the case of a team in an architectural company consisting of administrative staff, architects, and engineers. In a team in which workers have diverse abilities, group performance is determined by different degrees of complementarity between individuals (Brookins et al., 2015b).

A group's chance of winning is determined by its performance. Group j wins the contest with a probability

$$p_j(X_1, X_2, ..., X_n) = \begin{cases} \frac{(X_j)^r}{\sum_{k=1}^{n}(X_k)^r}, & \text{if } \exists X_k > 0 \\ \frac{1}{n}, & \text{if } X_k = 0 \ \forall \ k \end{cases}, j = 1, 2, .., n, \tag{2.3}$$

where $p_j \in [0, 1]$ is referred to as the Tullock contest success function (henceforth, CSF). [2] Through expending a greater effort, an individual can enhance the winning probability of their group. This Tullock CSF was first introduced by Tullock (1980) to study individual contests. Katz et al. (1990) were among the first to study group contests using the

[1] The detailed discussion of convergence results of these limit cases can be found in Kolmar and Rommeswinkel (2013).

[2] Münster (2009) provides an axiomatization for group contest success functions.

Tullock contest success function. r is the parameter that measures the degree to which the probability of winning is influenced by the group performance and the degree to which is left to chance. For example, when $r \to 0$, the probability in eq. (2.3) of all groups becomes equal and it approaches $\frac{1}{n}$. This implies that players' effort will not affect their chances of winning the contest. A higher value of r implies that group performance has a more significant impact on the contest outcome. Also, r determines the type of returns to scale, whether the contest is decreasing $(r < 1)$, constant $(r = 1)$, or increasing $(r > 1)$ returns to scale. However, the majority of research up to now has excluded the case of increasing returns to scale technology, because a pure strategy Nash equilibrium is not guaranteed to exist (e.g., Epstein and Mealem, 2009; Risse, 2011; Boosey et al., 2019; Send, 2020). Lee (2015) investigates this overlooked case and provides the characterization of the Nash equilibrium in this case.

There are two prominent CSFs, the lottery contest (e.g., Ueda, 2002; Kolmar and Wagener, 2013; Brookins et al., 2018; Dasgupta and Neogi, 2018; Nitzan and Ueda, 2018; Trevisan, 2020; Fallucchi et al., 2021) and the auction case (e.g., Barbieri et al., 2014; Chowdhury et al., 2016b; Chowdhury and Topolyan, 2016a; Andreoni and Brownback, 2017). In the former scenario in which $r = 1$, the group's probability of winning increases in its group performance. In general, this kind of contest is interpreted as a lottery where each group receives the number of tickets that is equivalent to their group performance. After all players have made their choices of decision, one ticket is drawn at random from the total number of tickets $\sum_{k=1}^{n} X_k$, and the group that holds the winning ticket is announced to be the winning group of the prize (Baik, 2008; Faravelli and Stanca, 2012). While in the latter case where $r \to \infty$, the contest becomes perfectly discriminating,[3] leaving no room for chance. This auction case, while important, falls outside the scope of this survey paper.

Once the winning group is selected, the next natural question to ask is what sharing rule will be adopted. Firstly, there is a special case, group rent-seeking for pure public goods such as public parks, lighting in the public road, and clean air. The common properties shared by these examples are that they are non-rival and non-excludable. A good being non-rival means that more people consuming the good will not diminish the benefits of another person's consumption. Non-excludability describes the case that once the good is provided, an individual cannot practically be excluded from consuming it (Deneulin and Townsend, 2007). The existing literature on group rent-seeking for pure public goods is extensive and focuses on various areas including how individuals' attitudes towards risk affect their behaviour (Katz et al., 1990; Nitzan, 1994; Loehman et al., 1996; Brookins

[3]The group with the highest group performance wins the contests with certainty.

and Jindapon, 2021), total rent dissipation (Ursprung, 1990; Gradstein, 1993; Epstein and Mealem, 2012), the asymmetric contests (Nti, 1998; Epstein and Mealem, 2009; Nitzan and Ueda, 2014; Nupia, 2013; Fallucchi et al., 2021), contests with players having interdependent preferences (Konrad, 2004; Konrad and Morath, 2012; Cheikbossian, 2021b).

Secondly, in the case in which the public good is rival, a sharing rule is necessary to determine how the collective good will be distributed among members of the winning group. Money is a good example of the prize being fully rival, meaning that when one person receives more money, there is less available for others. Nitzan (1991) firstly defines a general form of sharing rule,

$$s_{ij} = \frac{\alpha_j}{m_j} + \frac{(1-\alpha_j) \cdot x_{ij}}{X_j}, \tag{2.4}$$

where a proportion of the prize (denoted as α_j) is distributed according to the egalitarian rule, while the residual proportion $(1 - \alpha_j)$ is divided based on the proportional rule. A higher weight on the proportional rule provides incentives for higher efforts among members of the group, thereby fostering *intra*-group competition. Additionally, There is a large body of literature adopting and extending upon the original setting proposed by Nitzan (1991), such as Baik and Shogren (1995), Ueda (2002), Baik and Lee (2007), Ursprung (2012), Flamand and Troumpounis (2015), and Balart et al. (2016). The sharing rules could either be endogenous or exogenous. Prominent examples of the former case include Lee (1995), Noh (1999), and Nitzan and Ueda (2011). There are two extreme cases, $\alpha_j = 0$ and $\alpha_j = 1$. When $\alpha_j = 1$, the collective good is shared equally among all players, regardless of their relative efforts (e.g., Ahn et al., 2011; Cheikbossian, 2012; Nitzan and Ueda, 2018); whereas when $\alpha_j = 0$, it is divided completely based on the proportional rule (e.g., Hausken, 2005; Gunnthorsdottir and Rapoport, 2006; Hoffmann and Thommes, 2022).

Before presenting the expected payoff of a player, there is one last essential ingredient, the cost function $c(x_{ij})$. The most common form is the linear effort cost function, $c(x_{ij}) = c_{ij} \cdot x_{ij}$ where $c_{ij} > 0$ is an individual-specific cost parameter(e.g., Baik, 1993; Lim et al., 2014; Brookins and Ryvkin, 2016; Boosey et al., 2019; Chang et al., 2022). In this case, the cost of effort is deducted directly from the payoff. It is applicable in various situations, such as a competition between sales teams for a bonus, in which the cost of a salesperson's effort is measured by the time they spend on making calls or scheduling meetings with clients. Each salesperson who spends an additional unit of time incurs the same amount of cost, for this reason, the cost of effort is linear. However, it is not a surprise that the application of the linear cost function may not be universal. An alternative form of the cost function which is commonly adopted is a strictly convex function (e.g., Moldovanu and Sela, 2001; Ryvkin, 2011; Cubel and Sanchez-Pages, 2022). The group contest in which individuals contribute by expending physical effort provides a good illustration of how the marginal cost of an individual's effort can increase. A relevant example is football games in which the marginal cost of a football player's performance may increase rapidly. This is because as they exert more effort, the risk of injury or burnout also increases.

Given eqs.(2.1), (2.3), (2.4) and the cost function $c(x_{ij})$, with the assumption that players are risk neutral, the expected utility function of player i in group j can be written as,

$$\pi_{ij}(x_{1j}, ..., x_{m_j j}, \mathbf{X}_{k/j}) = v_{ij}(m_j) \cdot p_j(X_1, ..., X_n) - c(x_{ij}), \tag{2.5}$$

where $\mathbf{X}_{k/j}$ refers to the vector $X_1, ..., X_n$ without X_j, and $v_{ij}(m_j) = v_{ij} \cdot s_{ij}$ is the amount of the prize that player i will receive in the case group j wins, which could be viewed as the effective value of the prize for player ij, or the valuation of player ij on group j winning the contest. For a rational and self-interested individual, their objective is to maximize the expected utility. This can be achieved through strategically making decisions, such as determining the level of expenditure, deciding whether to participate in the contest and choosing whether to punish the free rider.[4] Therefore, a decision maker's behaviour is crucially dependent on several factors, including the group size, the heterogeneity of players, players' preferences, the sharing rule, the cost function, the group impact function, and the CSF.

2.3 Group formation in contests

Before delving into more details about group contests, a brief overview of the features of group formation will be provided in this section. Despite the potential occurrence of the alliance paradox (see, Esteban and Sakovics, 2003; Konrad et al., 2009) and the possibility that a larger group may suffer from a group size disadvantage (namely, the group size paradox) (Esteban and Ray, 2001; Epstein and Mealem, 2009; Balart et al., 2016), individuals frequently form groups and combine their efforts for a variety of reasons. These reasons include combing their complementary abilities to achieve a

[4]A free rider refers to a participant who abstains from contributing any effort, or contributes a smaller amount of effort compared to other members within the group during a contest, yet still benefits from the group's success if it wins. According to Sheremeta (2011b), there is no free-riding in a group if all the players within that group exert an equal amount of effort.

shared objective, achieving synergy, and facilitating the division of labour. For example, companies often form strategic partnerships, and these partnerships enable them to share resources, exchange information, and enhance their overall competitive advantage in the market (Jamali et al., 2011).

The existing literature on group formation is extensive, and a significant portion of studies examine the case in which players are asymmetric. Skaperdas (1998) addresses the questions on the characteristic of the group alliance in the model where three players decide whether to form a group and which player among the other two they should choose as their teammate to fight together against the third person. By adopting a generalized CSF, they find that the sufficient and necessary condition for the formation of a group is that the contest exhibits increasing returns in scale. This paper provides a thorough and insightful analysis of three cases with different assumptions regarding the prize. First, it examines the scenario where the prize is indivisible and the final winner of the alliance is exogenously determined if two players form an alliance and win. Second, it considers the case where the size of the prize depends on which two players form a group. Finally, the paper investigates the scenario in which players can choose between either a lower guaranteed prize or participating in a contest for a chance to win a higher prize. This paper demonstrates a deep understanding of the impact of the prize structure on group formation. Tan and Wang (2010) expand Skaperdas's (1998) work by relaxing the limit on the number of players. A shared characteristic of these two papers is that a player's probability of winning is determined by their endowments, such as resources and skills. Therefore, players in both studies do not need to expend effort to increase their chances of winning. This kind of contest can be viewed as one in which the cost parameter is zero for every player.

To date, numerous studies have investigated the case where players are identical, and many scholars have attempted to investigate group formation in a standard rent-seeking contest where players increase their chances of winning by expanding efforts. For example, Baik (1994) provides a formal analysis of a special case of group formation in which the winner is an individual player, not a group. Specifically, during the contest, all players act as if they are singletons - that is, each player's winning probability depends on their own effort, rather than on the group performance. If a player from a group happens to win, other members of this group can share a portion of the prize, according to a prior agreement. Building on this, Baik and Shogren (1995) examine how group formation is affected by players' decisions on sharing rules instead of their strengths, but in their study, the member players aggregate their effort during the contest. Both studies conclude that the size of the alliance is dependent on the total number of players in the model, and that forming a group might be beneficial for member players relative to the individual contest. Additionally, the rent dissipation is lower when a group is formed. One of their settings, where the number of the group is exogenously given, is challenged by Baik and Lee (2001) who eliminate the restriction on the number of groups from the model, and allow for endogenous determination of it. They point out that when more than one group is formed, the benefit of reduced social waste from forming only one group will not hold. Moreover, rent will not be fully dissipated, regardless of how many groups are formed.

Some of the current literature on group formation pays particular attention to the case in which the standard assumption is that the sharing rule of each group is private information. With unobservable sharing rules, Baik (2016) examines group structures and the number of singletons in equilibrium. Moreover, a comparison has been made between the contest outcomes from unobservable and observable sharing rules. Group contests with unobservable sharing rules can indeed occur in real-world scenarios. For example, within a company, the sharing rule for the prize from a business competition, might be based on trade secrets and, therefore, not disclosed to individuals in other companies.

There is a common characteristic of Baik (1994), Baik and Shogren (1995), Baik and Lee (2001), and Baik (2016), in that their contests can be viewed as a three-stage game. In the first stage, players decide whether to become a member of a group; in the second stage, member players decide on the sharing rules; and finally, in the third stage, all players independently and simultaneously choose their expenditure levels. With a similar activity in the first stage, Garfinkel (2004), Konishi and Pan (2020) and Konishi and Pan (2021) consider alliance formation in a three-stage game where players participate in group conflict in the second stage. Instead of having a pre-agreed sharing rule before the group conflict, only players in the winning group can enter the third stage and engage in an *intra*-group contest to determine each player's share of the prize.

The models presented in Garfinkel (2004) and Konishi and Pan (2021) differ in a few settings. For example, the former one adopts the perfect substitute impact function, while the CES impact function is employed in the latter paper. By examining three specific values of the degree of effort complementarity, it has been demonstrated that a nontrivial alliance[5] will not be formed if players' efforts are either too complementary (resulting in all players forming a single grand alliance in equilibrium) or too substitutable (with players acting as singletons in equilibrium). A potential limitation of this finding is its

[5] In the event that a nontrivial alliance is formed, then the partition of the set of individuals is neither characterized by a single grand alliance structure nor by alliances consisting solely of individual players.

reliance on the assumption of individuals having free mobility (i.e., open membership). Konishi and Pan (2020) therefore limit players from freely choosing their group, and analyse the impact of exclusive membership[6] on group structure in equilibrium. As expected, when alliances are permitted to restrict their memberships, a meaningful alliance could be formed even when the degree of effort complementarity is high.

Pursuing ways to achieve universal peace is an important social issue. Under the exclusive membership rule, Bloch et al. (2006) provide an explanation for why singletons form alliances and participate in contests instead of reaching agreements. Their research centres around understanding the situations in which deviating from the grand group structure in equilibrium will be profitable, and they attempt to identify the motivations that might lead to the establishment of universal peace. Following Bloch et al. (2006), a potential area for future research is to expand upon their work by investigating the possibility of group members transferring from one group to another. Sáchez-Pagés (2007a) also explores universal peace in their research on group formation. In another 2007 study, Sáchez-Pagés (2007b) delves into several stability concepts in a model where players need to make decisions on how to divide their endowment between productive activities and rent-seeking activities. Yi and Shin (2000) examine the impact of changes in research joint venture structures on the profits of firms, consumer surplus, and investment in R&D contests. They also explore the structure of research joint ventures with open membership and exclusive membership.

Group formation in rent-seeking contests is not exclusively focused on identical players in the literature. The current body of research on group formation also extensively investigates such contests involving asymmetric players. Noh (2002) explores a model featuring three asymmetric players, each of them assigned a different amount of endowment, who need to carefully decide how to allocate their resources between productive and rent-seeking activities. The prize is endogenous, and its value is dependent on the allocation decisions made by the players. Additionally, in the event that a group is formed, the sharing rules that govern the distribution of the prize are also endogenous. They conclude that an alliance between any two players could be stable, depending on their respective endowments, as well as the effectiveness of the group performance when compared to the expenditure of the singleton player. It is worth noting that when member players are not more effective in the rent-seeking contest as a group, compared to when they act as individuals, no group will be formed, regardless of players' endowments. A feature of the model discussed in Noh (2002) is that it requires players to utilize all of their endowment, not allowing for the retention of any portion as a

[6]The establishment of a group needs its potential members reaching a mutual agreement.

private good. This may offer a somewhat simplified representation when compared to the multifaceted nature of real-world scenarios. In the theoretical part of the work of Herbst et al. (2015), the group formation does not coincide with Noh's(2002) findings. Their findings suggest that the two less powerful players have a greater tendency to form a group, while it is more advantageous for the strongest individual to remain as a singleton. The rationale behind this is that weaker players may benefit from being free-riders in a group, if their teammate is strong. Conversely, the strong player has a greater chance of winning without joining a group, while they may be exploited by their teammate who is weaker. In their study on an endogenous group contest with asymmetric players, Lee and Kim (2022) examine both types of memberships: exclusive membership and open membership. In the former game, there are multiple equilibrium outcomes which include every structure of groups. In contrast, in the latter game, one of the equilibrium group structures emerges where the two strongest players form a group, and the remaining players do not form groups with each other. In both games, there exists an equilibrium group structure where all players, except for the weakest one, form a coalition and compete against the weakest player. One area for further exploration in their study could be distinguishing between the strength of players based on their level of motivation and their efficiency in allocating effort.

Although the papers in this body of literature tend to concentrate on an array of various settings of the game and procedures for alliance formation, there are two general intuitions that can be extracted from the literature. First, as it has been comprehensively demonstrated and clarified by the theoretical study conducted by Esteban and Sakovics (2003), it appears that individuals may exhibit a greater inclination towards engaging in competition as singletons, as opposed to their tendency towards forming group(s). This observed lack of motivation towards forming alliances can be intuitively reasoned by taking into account two factors. Firstly, there is a distinct possibility that some member players may be free-riders, since the expenditure is costly and often irreversible. Secondly, it is possible that conflicting interests may arise within the group, such as each member in the group desiring to obtain a larger share of the prize, which could potentially lead to another *intra*-group contest which will incur additional expenditure. Thus, a persistent question that still needs to be addressed and understood by economists is the reason why individuals continue to choose to form groups in the complex and dynamic real-life world. The other general observation from the literature is that the significance of the grand coalition has been widely emphasized. This leads to other important and thought-provoking questions for economists to address, which concern how to achieve universal peace, the factors that trigger the breakdown of agreements, and what motivates players to deviate from the grand coalition.[7]

2.4 Optimal contest design

When we refer to the concept of an "optimal group contest," we are discussing the design of a group contest wherein the contest designer aims to minimize or maximize a specific objective function. For contest organizers, it is a vain attempt to look for a universal optimal contest design, since the optimal contest structure may vary significantly, depending on the specific objectives and constraints of the situation. The objective function may include various elements, such as the total expenditure spent by participants on rent-seeking, the expected social surplus generated by the contest, and the expected chance of selecting the highest-performing participant. To accomplish their objectives, the designers need to establish specific rules or restrictions.

For a game organizer whose objective is to maximize the total expenditure, the design of the game should focus on stimulating players to expend more effort. For instance, in a contest involving two groups competing for a purely private good shared equally within the winning group, the total effort decreases as the size of each group increases(Ahn et al., 2011). While in the models of endogenous sharing rules, the total effort increases as the size of the smaller group increases, provided that the sharing rule is constrained to a weighted average of the proportional and egalitarian sharing rules; if each group is allowed to assign any weights to the proportional and egalitarian sharing rules, the total rent-seeking increases with the total number of participants (Baik and Lee, 1997). Additionally, their results imply that the total effort expended in the former scenario is less than that in the latter scenario.

The positive externalities of R&D contests include product innovations and technological progress, which could lead to lower production costs, improved effectiveness of resources, and increased consumer welfare (Kräkel, 2004). For this reason, maximizing total effort in R&D competition may have a positive impact on other firms, entire industries, or the whole society. On the other hand, effort spent on rent-seeking activities, which do not contribute to any physical or material social production, is often considered as a societal waste that needs to be minimized (Dompere, 2014). In this case, the optimal contest structure is one that minimizes the total effort.

Numerous studies have focused on examining the relationship between aggregate effort

[7] A comprehensive survey of the various forms of group formation in all-pay auctions can be found in the work of Bloch (2012).

and various critical factors that influence group performance in contests, such as the number of players, the number of groups, and the asymmetry of players. Based on the study conducted by Baik (1993), it has been found that the total effort expended is dependent on the number of groups participating in the contest and the highest valuation in each group. Instead, it is independent of both the group size of each group and the distribution of valuations in each group. The group sizes themselves may not always be a critical factor in determining the level of total effort in group contests, however, the exposure of such information to players can be a different story. Boosey et al. (2019) demonstrate that, the total effort is higher when the group sizes are made public information, compared to the scenario where sizes are uncertain for all players.

Ryvkin (2011) provides the first exploration of the optimal group contests design with players who may have asymmetric individual-specific cost parameters, and they find that when the costs of effort are sufficiently steep, the optimal sorting of players to maximize the aggregate effort is the least "balanced" one, in which the variation in abilities of players is maximized. In other words, when the effort costs are steep enough, grouping the strongest individuals together instead of allocating them into different groups will lead to a higher level of aggregate effort. One claim in Ryvkin (2011) that could be problematic is that, a cost function is considered steep as long as it takes the form of a convex function, which is over-generalized and may lead to a misleading interpretation if readers assume that "steep" implies a certain degree of convexity. Instead of assuming that players have different values of cost parameters, Cubel and Sanchez-Pages (2022) study how heterogeneous *inter*-group valuations of the prize affect the aggregate effort. They conclude that an increase in heterogeneity between groups leads to a higher level of the aggregate effort. Distinct from Ryvkin (2011), this positive correlation holds regardless of the level of convexity of the cost function.

According to Nitzan and Ueda (2014), if players in a group have different valuations for the prize and all other groups are replicas of that group, then increasing the number of replica groups leads to a higher level of total effort. One feature of this setting is its focus on the asymmetry among *intra*-group members, while that positive correlation could potentially change if they were to include the possibility of asymmetry among *inter*-group players. They also investigate a particular scenario in which there are two groups that are completely identical to each other, except for one single factor: one of the groups has an advantage in terms of the cost associated with expending effort. This is conducted to examine how the existence of a superior group affects the total expenditure. Additionally, research on group contests, where players are allowed to exhibit varying attitudes towards risk, is quite recent. It was first examined by Brookins and Jindapon in 2021, they explore the optimal sorting of individuals who have varying degrees of risk aversion, risk-loving tendencies, and who are risk-neutral, under the framework of linear and strictly convex cost functions.

In a follow-up study, Brookins et al. (2015b) question the realism of applications of perfect substitutes technology which is adopted in Ryvkin (2011), and expand the impact function to a CES aggregation function. They find that the optimal structure is also affected by the degree of complementarity among efforts. Given that steepness of the effort costs is not too high, if the degree of complementarity is low, then the optimal design for maximizing the total effort is to form groups consisting of players with mixed ability; with a higher degree of complementarity, that is to increase the difference in ability between groups. Thus, there exists a cut-off level of complementarity, beyond which the latter sorting strategy becomes the one that maximizes the total effort. By using a set of specific values of parameters, Brookins et al. (2018) shows that when the group performance is given by the weakest-link function, a sorting strategy in which two players with higher ability are grouped together and the other group consists of two weaker players leads to higher total effort compared to a strategy where each group has a mix of a strong and a weak player.

Upon considering the key elements of group contests, one can recognize that employing various rules concerning the prize can serve as a powerful tool for contest designers, enabling them to manage contests effectively in accordance with their desired outcomes. For example, the case in which the method of allocating prizes among players with different levels of skill is determined by the manager of each group if their group wins, has been examined by Trevisan (2020). Consequently, the contest organizers can select specific players to act as managers for each group in order to maximize or minimize the aggregate expenditure, or to ensure that a particular group wins the contest.

Unlike conventional contests, Chang et al. (2022) introduce the possibility of a draw into group contests to investigate its impact on the decisions of the players (e.g., the endogenous sharing rule) and the contest organizers. In the symmetric case, where players value the prize equally regardless of which group they belong to, the presence of the possibility of a draw decreases players' efforts. Thus, from the perspective of the contest designer who aims to induce a higher aggregate effort, it is optimal not to introduce the possibility of a draw into the contest. But in the case where players have heterogeneous valuations of the prize, introducing the possibility of a draw always maximizes the total effort if the contest designer places more weight on the expenditure of the strong group. It is also important to note that selective incentives play a significant role in inducing players to contribute to the group's collective objective (Olson, 1965;

Nitzan and Ueda, 2011). For example, research has shown that requiring that each and every player has an obligation to share part of the cost of other *intra*-group members has been shown to induce players to contribute towards their group Nitzan and Ueda (2018). Accordingly, contest designers could consider implementing various devices of selective incentives to achieve their goals relating to effort expenditure. According to Nitzan and Ueda (2011), the proportional sharing rule can also serve as a selective incentive, in addition to its function as a mechanism for allocating prizes. Thus, to maximize the overall effort of participants, a contest designer may set a rule that requires all groups to use the proportional sharing rule.

In recent years, scholars have paid their attention towards examining models in which the contest designer's objective is to maximize the weighted sum of players' efforts, rather than simply the sum of all players' expenditures. As such, Lee and Song (2019) examine the scenario in which the contest designer's objective is to maximize the weighted sum of players' efforts by choosing the optimal group impact function, and provide an explanation for the fact that different sports organized in the form of groups use different methods of selecting the winning team. By adopting the generalized CES impact function, it has been shown that the optimal choice of the impact function is influenced by the weighting of the efforts of the strong and the weak player. The strategy for the designer is to choose an impact function with an optimal degree of complementarity. As fans of the sport put more care about the performance of the strong player (such as star players or famous players) and less about the weak player, the optimal degree of complementary decreases. Finding the optimal way to sort high-ability and low-ability players into groups is not only useful in sports, but according to Xiao (2023), it is also a significant question in the economic literature on education. This is because the way in which high-performance and low-performance students are sorted into classrooms can affect their achievement, and in the long run, even the fairness of society.

It is important to note that not all contest designers prioritize solely the aggregate effort or a weighted sum of efforts, some contests may be designed with alternative objectives in mind. For instance, there are certain types of contests in which the objective of designers is to maximize the collective profits of all participating groups. Winfree (2021) provides an analysis of this type of contest, where designers can achieve their objectives through choosing the CSF, using a sports league as an illustrative example.

One popular theme in the literature on optimal group structure is maximizing aggregate effort, which means finding the most efficient way to incentivize players to invest more effort. For contest organizers, they might need to consider various factors, including how to allocate the prize, how to manage the disclosure or concealment of certain information, the optimal value of some parameters, and how to sort individuals into groups. The valuable insights gained from exploring the effort in the literature on optimal contests can be applied to a wide range of contexts, such as sports, education, and military conflicts. In future investigations, it might be possible to consider the application of alternative subjective expected utility functions, such as maxmin expected utility and Choquet expected utility, to examine the dynamics of optimal group contests within different contexts.

2.5 Interdependent preferences in group contests

In this section, studies that challenge and reassess the prevalent assumption of purely self-interested preferences within the context of group contests are reviewed. In these studies, the focus is not on players whose preferences are solely based on their own material payoff, but rather on investigating interdependent preferences, which can be characterized by different forms, including envy (or, spite, hostility), altruism (or, favouritism), and relative preferences (or, relative rank[8]) within diverse contexts. In experimental research on group contests, certain behaviours among subjects are unable to be explained by theoretical results based on independent preferences alone. For example, overbidding compared to the Nash prediction is a common observation in experimental literature (e.g., Erev et al., 1993; Sheremeta, 2010; Chowdhury et al., 2016a). Kolmar (2013) proposes a hypothesis that, the self-interest assumption on subjects may not be empirically accurate, which could explain why overbidding often occurs in group contests.

Cheikbossian (2021a) investigates the model where a player's utility is interdependent on their teammates' material payoff. Their analysis leads them to conclude that the *intra*-group favouritism is more likely to be evolutionarily stable, when the degree of effort complementarity is greater, or when there is an increase in the number of groups. The impact of an exogenously given degree of *intra*-group altruism on players' behaviour has been studied in Hu and Treich (2018). In their analysis, it is discovered that the efforts within a group increase and are positively affected by a rise in altruism among its members. Additionally, if altruism in a group exceeds a certain threshold, the larger group may be more likely to succeed. However, since the prize comprises both pure public goods and private goods, this cut-off degree of altruism depends on the size of the pure public goods and the private goods, as well as the weights placed on them. In light

[8]For a more detailed and non-mathematical discussion on how and why such preferences are formed, see Postlewaite (1998)

of this, some might suggest exploring the composition of the prize as a potentially more fitting explanation for the invalidity of the group size paradox, rather than focusing primarily on *intra*-group altruism.

Apart from *intra*-group altruism, *inter*-group spite has also been examined in various contexts. Chopra et al. (2020) offer a discussion on the impact of *intra*-group altruism and the effect of *inter*-group spite on the Nash prediction in the context of incomplete information regarding the cost function, valuation, and the group size of the rival group. This analysis is conducted when these two preferences are included in the model separately, without the presence of the other one.

In group contests, envy and altruism often come in a pair, that is, players exhibit *inter*-group envy and *intra*-group altruism. This is also known as parochial altruism[9, 10] (Choi and Bowles, 2007). Konrad and Morath (2012) clarify that, this phenomenon is considered evolutionarily stable within a framework where groups engage in conflict with one another. They also demonstrate that for a pair of degrees of *intra*-group altruism and *inter*-group spite consisting of an evolutionarily stable preference, these two parameters act as substitutes for one another; while for significantly large groups, the levels of spite and favouritism that are evolutionarily stable tend to approach zero. One area their argument touches upon less extensively is the explanation for the existence of negative degrees of *intra*-group altruism within certain evolutionarily stable preferences. This seems to imply that *intra*-group spite could also be evolutionarily stable, although the assumption is that players mainly exhibit favouritism towards *intra*-group members. In a model encompassing four scenarios, where each of the two groups of players can either be purely self-interested or parochially altruistic, Kolmar and Wagener (2019) demonstrate that, whether a player exhibits parochial altruism is an outcome of Nash equilibrium, rather than an exogenous choice made by individuals or groups. By adopting a similar setting, the theoretical part in the work of March and Sahm (2021) demonstrates that an increase in the degree of *intra*-group altruism or an increase in *inter*-group spite can motivate players to exert more effort, when the prize is shared equally within the winning group. This correlation remains valid even in cases where the prize consists of pure public goods(Eaton et al., 2011).

Unlike studies on parochial altruism, Cheikbossian (2021b) have developed a model in

[9] Bowles (2008) as well as Lehmann and Feldman (2008) provide a suggestion that, during the early history of humanity, violent interaction between groups might have contributed to the development of parochial altruism. However, in another psychology paper, Yamagishi and Mifune (2016) criticize the legacy of parochial altruism within modern human psychology.

[10] The economic experiment was carried out by Chowdhury et al. (2016a) shows that a real group identity (race) has an impact on parochial altruism.

which individuals are not restricted to exhibiting spite or altruism towards other players' material payoffs based on their group affiliation. In contrast to Konrad and Morath (2012), they demonstrate that parochial altruism may not be an evolutionarily stable preference type when the elasticity of the endogenous prize function is considerably large. Furthermore, they prove that as the group size grows, there is a higher likelihood of *intra*-group spite emerging in evolutionarily stable preference.

As mentioned earlier, interdependent preferences can be expressed in the form of maximizing the relative payoff, which can be interpreted as players aiming to achieve a superior rank compared to their peers. Risse (2011) analyzes a two-stage group contests model where players aim to maximize the relative payoff. They prove that, in equilibrium, the total rent-seeking is always greater when the relative preferences are present compared to when the pure self-interested preferences are considered.

2.6 Conclusions

This survey paper has sought to provide a comprehensive review of theoretical papers on group contests, with a particular focus on three research topics: group formation, optimal group design, and interdependent preferences. Models that have been utilized in previous literature are presented in section 2.2. As illustrated in section 2.2, the objective functions for players are influenced by fundamental elements such as the group impact function, the contest success function, the cost function, the sharing rule, and the symmetry or asymmetry of the contest. In addition to these elements, various other factors also shape contest outcomes, including players' preferences, attitudes towards risk, access to information, and endogenous entry. Despite the extensive body of literature on group contests, constraints related to article length prevent us from discussing all studies in detail. Consequently, not every factor influencing the outcome of group contests is included, which constitutes a limitation of this paper. For various scenarios, the model can be appropriately modified, and corresponding assumptions and contest designs can be selected. In doing so, this theoretical framework can accommodate a diversity of real-life group contest scenarios. For example, the effect of identity on group contests, which has been widely investigated in experimental studies such as those by Chen and Li (2009) and Benistant and Villeval (2019), could be represented in theoretical research as a model where players display interdependent preferences.

In the realm of group formation, the surveyed literature provided numerous insights into how groups formed and the variables that influence these dynamics. Researchers have found that factors such as the number of players, the sharing rules, and the types of membership can significantly impact group formation. Two general findings emerge: individuals tend to avoid forming groups with other players, and the grand coalition has been widely investigated. Research on group formation can be employed to examine business strategies, such as companies forming alliances to gain a competitive edge or enter new markets. Additionally, it can guide policymakers in formulating public policies aimed at discouraging antitrust activities or promoting collaboration in research and development.

Regarding optimal group design, we reviewed various studies that examine how group structures influence contest outcomes. This body of work suggests that the optimal structure of a group heavily depends on the specific context and objectives of the contest. The literature indicates that various factors, such as increased heterogeneity between groups and a decrease in group size, may increase the aggregate expenditure. The relevance of investigating optimal group structure in the real world, as discussed in Section 2.4, lies in its potential to enhance various real-world outcomes. For instance, in education, it can help maximize educational results for diverse groups of students. Similarly, in sports, optimizing players' roles, team size, and the balance of skills can enhance the audience's experience. This, in turn, can lead to increased profitability and potentially greater popularity for the sport.

In the realm of papers focusing on interdependent preferences — a fundamental component of many socio-economic phenomena such as altruism, inequality aversion, and social comparison — these preferences in group contests can serve a crucial role in explaining contradictions between theoretical predictions (assuming purely self-interested players) and experimental observations. Consequently, research concerning interdependent preferences highlights the necessity of recognizing interdependent dynamics and how these dynamics can influence the outcome of group contests.

Yet, there is still much to explore about how to model real-world applications under different contexts into contests and how different factors or assumptions shape group contests' outcomes. For instance, experimental studies commonly observe heterogeneous behaviour within and between groups, in the absence of observable heterogeneity within those groups. On the contrary, theoretical studies mainly concentrate on analyzing the symmetric equilibrium when players are identical. There are, however, a few exceptions, one notable example being Baik (1993). Accordingly, the equilibrium considering free riders could be an interesting perspective for future research.

Chapter 3

Ambiguity with Group Contests

3.1 Introduction

Not only can economic situations be modelled as group contests, but an increasing number of social situations can also be explained in this way. For example, productive activities (Hausken, 2005), lobbying the government for the provision of public goods (Riaz et al., 1995), competition for publication in academic journals (Baik and Lee, 2000), R&D race (Flamand and Troumpounis, 2015), as well as a myriad of other situations, have been modelled as group contests and examined theoretically. This highlights the flexibility of group contests as a framework for understanding competitive behaviour in a wide range of contexts. A contest can be regarded as an interaction in which the players, also referred to as contestants, expend effort, time, or other types of resources in an attempt to win a prize (Konrad, 2009).

In decision-making under ambiguity, uncertainty or ambiguity exists regarding the probability of various outcomes. Conversely, in decision-making under risk, the probabilities associated with different outcomes are known (Evren, 2019). In the context of group contests, ambiguity can be understood as a situation where players hold beliefs about the actions of both *intra*-group and *inter*-group participants. These beliefs may be formed from prior experiences or based on available information. However, players are not fully confident in these beliefs, leading to a degree of ambiguity regarding the strategies of others in the contest. Accordingly, ambiguity may affect decision-making and overall contest dynamics. Given the inherent nature of various contests, such as R&D competitions, political campaigns, and conflicts or wars, the presence of ambiguity seems likely.

The groundbreaking work of Ellsberg (1961) first introduced the notion of ambiguity.[1] Subjects' preferences in the Ellsberg Urn experiment violate the independence axiom in Savage's (1954) subjective expected utility (henceforth, SEU) theory. After Ellsberg (1961) demonstrated this phenomenon, a large number of subsequent studies have also investigated the case where agents struggle to assign subjective probabilities to ambiguous situations (e.g., Gilboa, 1987; Eichberger and Kelsey, 2002; Bose and Renou, 2014). Furthermore, evidence from the experimental results presented in the work of Georgalos (2021) demonstrates that the subjective utility of the majority of subjects is not adequately represented by the SEU. It follows, therefore, that utility theories of decision-making under ambiguity need to be sought to describe players' behaviour more accurately. Apart from the SEU, agents' beliefs or preferences can be represented in various models, such as Choquet expected utility Zhang (2002), Maximin (Gilboa and Schmeidler, 1989), and Maximax (Troffaes, 2007).

Under the Maxmin expected utility theory, decision-makers demonstrate complete pessimism when faced with ambiguity. They are characterized by their focus on the worst-case outcome of each potential strategy, and then choosing the one that provides the most beneficial outcome amongst these worst-case scenarios. Conversely, under the Maximax expected utility theory, decision-makers exhibit optimism towards ambiguity. Here, decision-makers identify the best potential outcome for each strategy and select the one that provides the greatest benefit among these outcomes. Unlike the prior two theories, the Choquet expected utility theory positions decision-makers to maximize a weighted average that considers three elements, the expected utility when the decision-maker is confident in their subjective probabilistic assessment, the maximal outcome and the minimal outcome (Chateauneuf et al., 2007). In other words, the Choquet expected utility preference incorporates characteristics from the preference of SEU, Maximin preference, and Maximax preference. Additionally, Hey et al. (2010) was the first to provide experimental evidence for determining the best-performing preference function in terms of predicting subjects' behaviour. Their results revealed that, overall, the Choquet expected utility model outperformed the others.

In light of these findings, this paper adopts the Choquet expected utility function to model group contests in which players perceive ambiguity regarding both their teammates' strategies and those of their opponents, deviating from the use of SEU employed in previous literature. In this game, a player's utility can be understood as a weighted average of payoffs derived from three distinct scenarios: the standard Tullock group contest; the best possible scenario, in which a player's teammates exert the highest

[1] Ellsberg's thesis has since been published as Ellsberg (2001).

possible effort while players from rival teams exert the lowest possible effort; and the worst possible scenario, where a player's teammates put forth the lowest possible effort while players from rival teams exert the highest possible effort. Consequently, it seems reasonable to propose that a player's best response can be viewed as the outcome of considering these three scenarios in a weighted manner. In this paper, I investigate the impact of several factors on equilibrium effort and optimal group structure in relation to aggregate expenditure. These factors include the degree of ambiguity, the extent of ambiguity aversion, the number of groups, and group size.

The findings of this paper offer contributions to four primary areas of research: i) this study enriches the discourse surrounding the phenomenon of over-bidding in contests, shedding light on its underlying causes; ii) it also contributes to the literature exploring mechanisms to mitigate the occurrence of over-bidding in contests; iii) this research provides insights into the discourse on why there is a marked decrease in participant expenditures over time, as observed in experimental studies; iv) additionally, it illuminates the reasons behind the common structure of certain sports games, which typically involve two teams instead of a larger number.

First, our results indicate that when the degree of ambiguity aversion among players surpasses a certain threshold, the level of equilibrium effort under ambiguity (henceforth, EUA) is always greater than that of Nash equilibrium.[2] This can be attributed to the scenario where a player's best response, in the most unfavourable circumstances and given the upper bound value of the effort, may surpass the level of Nash equilibrium. As such, when players exhibit a sufficiently high degree of ambiguity aversion, the worst-case scenario is given substantial weight, significantly influencing the level of EUA. This observation provides a novel explanation for the over-expenditure behaviour observed in experimental studies, a phenomenon that contradicts theoretical predictions.

The second main finding reveals that the level of EUA decreases with a reduction in the degree of ambiguity, provided that the degree of ambiguity aversion is sufficiently high. This is because a lower degree of ambiguity implies that less weight is given to the worst-case scenario, resulting in diminished influence on the level of EUA. As a result, the level of EUA decreases. This finding could be employed to explain the observed downward trend in subjects' expenditures over time in experiments, suggesting that players expend less effort in the later stages compared to the beginning of the experiment. As these experiments progress into later stages, subjects may gain insights into other players' strategies, leading to a boost in confidence regarding their beliefs.

[2]In this paper, the term 'Nash equilibrium' refers to the equilibrium effort in group contests without ambiguity.

The third main result of this paper is that the aggregate effort decreases as the number of groups decreases, which contradicts the findings of previous theoretical research (e.g., Nitzan, 1991; Baik and Lee, 2001; Münster, 2007). This finding offers an economic explanation for why certain real-life group contests often impose limitations on the number of groups and their sizes. To the best of our knowledge, this area has not been extensively researched before. Using American football games as an example, it is generally taken for granted that there are two teams competing in each game, rather than involving multiple teams. Indeed, one might think that having two teams is simply a matter of tradition or a way to simplify the rules, ensuring that the game is easy to follow for both players and spectators. However, this paper offers another explanation, suggesting that game designers aim to maximize the total effort in a game, and having two teams is the optimal choice to achieve this goal.

The remainder of this paper is organized as follows: the next subsection provides an overview of the relevant literature. Sections 3.2 and 3.3 present the baseline model and the model with ambiguity, respectively. Section 3.4 offers a comparison of the level of Nash equilibrium and that of EUA. Section 3.5 examines how the group structure impacts individual and total effort in the presence of ambiguity. Finally, section 3.6 summarizes the key findings of this study.

Related Literature

Beyond the Ellsberg urn, numerous experimental studies have explored ambiguity across various contexts. Within these studies, it is a common finding that the majority of subjects display an aversion to ambiguity (e.g., Becker and Brownson, 1964; Charness and Gneezy, 2010). Borghans et al. (2009) draw conclusions from their experimental data suggesting that, on average, participants exhibit ambiguity aversion. Additionally, their findings indicate that females tend to display a higher level of risk aversion compared to males. Using a modified Ellsberg urn choice, Liu and Colman (2009) concluded that subjects demonstrated a significant aversion to ambiguity in the one-time choice treatment. This aversion attitude, however, was not observed in the other treatment, where subjects were required to make decisions repeatedly. Contrary to the prevailing view in the literature, which suggests a prevalence of ambiguity-averse subjects, Maccrimmon's(1968) data shows that only 10% of subjects display ambiguity aversion. Charness et al. (2013) also argue that a vast majority of subjects actually exhibit ambiguity-neutral attitudes. Notably, Charness et al. (2013) find that the combined number of subjects who are ambiguity-averse or ambiguity-seeking is not even greater than the number of subjects who neither display ambiguity-neutral attitudes nor are ambiguity-averse or seeking. However, this directly contradicts the findings in the experiment presented in Halevy (2007). Additionally, Kocher et al. (2018) find that subjects' attitude towards ambiguity depends on the degree of ambiguity and the range of possible outcomes, and suggested that the majority of their subjects exhibit ambiguity neutrality.

Many studies have simulated real-life scenarios to analyze the prevalence and impact of ambiguity. These research efforts have consistently concluded that ambiguity is a common phenomenon and they further investigate its influence across various contexts. For instance, studies conducted by Hogarth and Kunreuther (1992) and Kunreuther (1989) reveal that ambiguity aversion is prevalent among individuals, including professional actuaries and insurance underwriters. Additional experiments show that not only consumers but also insurance companies also display an ambiguity aversion towards low likelihood losses (e.g., Hogarth and Kunreuther, 1989; Kahn and Sarin, 1988). By its very nature, medical decisions made by both patients and doctors also entail elements of ambiguity (Ritov and Baron, 1990). Bossaerts et al. (2010) reported a significant prevalence and impact of ambiguity aversion in financial markets, a finding that is corroborated by the work of Ahn et al. (2014).

A correlation between risk aversion and ambiguity aversion is proposed by Koch and Schunk (2013) and Shupp et al. (2013), with the tendency being that individuals who exhibit risk aversion often also display ambiguity aversion. However, the impact of ambiguity aversion outlined in this paper does not align with the effects of risk aversion presented in earlier literature. For example, my research suggests that the total effort expended increases with an increase in ambiguity aversion. Conversely, Katz et al. (1990) provide a theoretical demonstration that an increase in the coefficient of absolute risk aversion results in a decrease in aggregate rent-seeking. Further, according to their experimental findings, Abbink et al. (2010) assert that equilibrium effort diminishes with increasing risk aversion. The behaviour of subjects in groups is not significantly affected by risk aversion, as demonstrated in another experimental study conducted by Sheremeta and Zhang (2010).

The frequent observation of ambiguity in various contexts naturally leads us to consider the possibility of introducing ambiguity into contests. This could potentially be employed to investigate significant issues in group contests, such as the over-expenditure of effort compared to theoretical predictions and the determination of optimal group structure. In terms of the over-expenditure of effort, experimental studies of group contests consistently reveal its prevalence. Sheremeta (2018) states that over-expenditure is observed in almost all existing experimental literature on group contests. For instance, the results of Abbink et al. (2010) demonstrate that, on average, the total expenditure of a team in group contests with two groups, exceeds the theoretical prediction by more than fourfold. There is a downward trend in the average individual bid over time; however, during the later stages of the experiments, the efforts exerted by participants are still significantly higher than the Nash predictions (Abbink et al., 2010; Leibbrandt and Sääksvuori, 2012; Bhattacharya, 2016). In the experiment carried out by Leibbrandt and Sääksvuori (2012), participants were permitted to communicate during certain treatments. The findings reveal that, expenditures decrease when participants can communicate with individuals from both conflict parties.

In the literature, over-expenditure of effort is linked to various factors including incomplete information (Chopra et al., 2020), inequality aversion (Bolton and Ockenfels, 2000), individual preferences Hu and Treich (2014), bounded rationality (Sheremeta, 2013), attitudes towards risk (Hillman and Katz, 1984; Sheremeta, 2011a), non-monetary incentives (Sheremeta, 2015), and the heterogeneity of players (Brookins et al., 2015a). However, these factors cannot explain the downward trend in subjects' expenditures over time. Additionally, Sheremeta (2011b) asserts that the over-expenditure behaviour observed in their experiment, which involved asymmetric players, cannot be accounted for by either risk aversion or inequality aversion. In the context of my research, the findings indicate that as the degree of ambiguity decreases — potentially facilitated by communication and the accrual of experience in later periods, which in turn could reduce the ambiguity of others' strategies — there is a corresponding decrease in equilibrium effort.

Concerning the optimal structure, this paper addresses it in terms of maximizing or minimizing the total effort, which is a primary focus for game designers. The findings from Nitzan (1994) suggest that rent dissipation increases with the number of groups, assuming that each group has the same size. Similarly, Hartley (2017) discovers that the positive correlation between the number of groups and rent dissipation holds true in group contests where the size of each group may vary. Flamand and Troumpounis (2015) show that the total expenditure on rent-seeking activities increases in the number of groups when the prize is shared under the egalitarian sharing rule. With a constraint on the total number of players, or while maintaining the group size constant, Münster (2007) points out that in both scenarios, an increase in the number of groups consistently leads to an increase in the total effort. In a study conducted by Brookins et al. (2015b), contests are examined in which players' efforts are aggregated within groups using the function with a constant elasticity of substitution, under the condition of a fixed total population of participants. The findings suggest that the complementarity of efforts does not alter the established positive correlation between the number of groups and total effort. Moreover, in modified group contests where groups compete with each other as well as with individuals, Baik (2016) finds that total effort also increases with the number of groups. To the best of my knowledge, this paper is the first to present differing findings regarding the effect of the number of groups on aggregate effort. As such, it provides a theoretical rationale, from an economic standpoint, for the common practice of having two teams in most sports games, rather than more.

The study most closely related to my work is Kelsey and Melkonyan (2018), which is the pioneering research paper to introduce ambiguity into contest theory. Both my paper and theirs conclude that the level of equilibrium effort may exceed that of Nash equilibrium. Additionally, my paper provides an explicit function that indicates when over-investment occurs. However, there are clear distinctions between my study and theirs. Our paper focuses on group contests and further investigates the optimal group structure, whereas Kelsey and Melkonyan (2018) focus on individual contests.

3.2 The baseline model

Assume that there are m groups competing against each other to win a contest and receive a commonly valued prize, denoted by V. Each group consists of n risk-neutral players. To simplify the model and make it more tractable, players are assumed to be identical. All players simultaneously and independently expend irreversible and costly efforts. The effort level chosen by player j from group i is denoted by x_{ij}. The cost function of expending x_{ij} is assumed to be linear, $c(x_{ij}) = x_{ij}$. Players in group i aggregate their efforts using the most prevalent function in the literature on group contests, which is the perfect-substitutes function, $X_i = \sum_{j=1}^{n} x_{ij}$ for $i = 1, ..., m$ (Katz et al., 1990). The winning probability of group i is determined by its group performance relative to the total effort exerted by all groups. Formally, the contests success function is defined by (Tullock, 1980; Skaperdas, 1996; Münster, 2009):

$$p_i(X_i, X_{-i}) = \begin{cases} \frac{1}{m} & \text{if } X_1 = X_2 = ... = X_m = 0 \\ \frac{X_i}{\sum_{g=1}^{m} X_g} & \text{otherwise,} \end{cases} \tag{3.1}$$

where X_{-i} represents the total effort exerted by all groups except group i. After the winning group has been determined, the prize will be divided equally among the players in the group that wins.

Thus, the expected payoff of player j in group i is given by:

$$u_{ij} = p_i \frac{V}{n} - x_{ij}. \tag{3.2}$$

The first-order condition for player ij to maximize their expected payoff is:

$$\frac{V}{n} \frac{\sum_{g \neq i}^{m} X_g}{(\sum_{g=1}^{m} X_g)^2} - 1 = 0, \tag{3.3}$$

According to the assumption that the players have identical preferences, a symmetric equilibrium can be obtained, $x_{ne} = x_{11} = ... = x_{1n} = ... = x_{m1} = ... = x_{mn}$. Substituting this into eq.(3.3), we can get the symmetric Nash equilibrium:

$$x_{ne} = \frac{V(m-1)}{m^2 n^2}.^{3} \tag{3.4}$$

3.3 The model with ambiguity

Now, consider the case where individuals perceive ambiguity about the choice of effort level made by other players. In other words, players are unable to assign precise probabilities to their teammates' strategies and to the strategies of their opponents from other teams. Following Kelsey and Melkonyan (2018), this paper adopts a neo-additive capacity to be the representation of ambiguity. The neo-additive capacity function is employed to capture decision-making under ambiguity, in which players may have an additive belief over other players' strategies, but may not be fully confident in their beliefs. For player ij, v_{ij} is defined on the set of all the players' strategies (except player ij's) $\mathbf{X}_{-ij}$(Chateauneuf et al., 2007; Eichberger and Kelsey, 2014; Kelsey and Melkonyan, 2018):

$$v_{ij}(\emptyset) = 0, v_{ij}(\mathbf{X}_{-ij}) = 1, \text{and } v_{ij}(A) = \delta_{ij}(1-\alpha_{ij}) + (1-\delta_{ij})\pi_{ij}(A) \text{ for all } \emptyset \subsetneqq A \subsetneqq \mathbf{X}_{-ij}, \tag{3.5}$$

where $\alpha_{ij}, \delta_{ij} \in [0, 1]$. According to Chateauneuf et al. (2007), player ij's attitude towards ambiguity is characterized by the parameter α_{ij}. A player ij with a smaller value of the parameter α_{ij} is considered an ambiguity-seeking participant, while a player with a larger value of α_{ij} is considered an ambiguity-averse participant. The parameter α_{ij} reflects the degree of ambiguity aversion of player ij, whereas $(1-\alpha_{ij})$ reflects their

[3] The analysis of x_{ne}, which includes the optimal group structure that can lead to the maximum total effort in the baseline model if the total number of players (mn) is fixed, and how changes in m or n affect x_{ne}, is presented in Section 3.5.

degree of ambiguity seeking. In the second term, π_{ij} represents an additive probability distribution that reflects the subjective belief of player ij over their teammates' and opponents' strategies. The parameter δ_{ij} reflects the degree of ambiguity that player ij perceives. In other words, $(1 - \delta_{ij})$ stands for the degree of confidence that the decision-maker holds in the belief π_{ij}. If the players are fully confident in their beliefs, i.e., there is no ambiguity, then $\delta_{ij} = 0$ for all players.

With a neo-additive capacity, the Choquet expected utility can be written as (Kelsey and Melkonyan, 2018)

$$W_{ij} = \int u_{ij} dv_{ij} = \delta_{ij}(1-\alpha_{ij})M_{ij}(x_{ij}) + \delta_{ij}\alpha_{ij}m_{ij}(x_{ij}) + (1-\delta_{ij})\int u_{ij}(x_{ij})d\pi_{ij}, \tag{3.6}$$

where M_{ij} refers to the best possible scenario for player ij, while m_{ij} corresponds to the worst possible scenario. The third term is the expected payoff under the belief π_{ij}. Under the assumption that players can choose their effort level from a range $x_{ij} \in X = [\underline{x}, \overline{x}]$ where $0 = \underline{x} < x^{ne} < \overline{x} < \infty$, in the former case (M_{ij}), all of player ij's teammates select the upper bound of the range, while all their opponents from other teams select the lower bound. Conversely, in the latter case (m_{ij}), player ij's teammates choose the lower bound and all players from other teams choose the upper bound. Formally,

$$M_{ij}(x_{ij}) = \frac{V}{n} - x_{ij}, \tag{3.7}$$

$$m_{ij}(x_{ij}) = \frac{x_{ij}}{x_{ij} + (m-1)n\overline{x}} \cdot \frac{V}{n} - x_{ij}. \tag{3.8}$$

From eq.(3.6), the best and the worst possible payoffs will have an effect on the expected utility only when δ_{ij} is positive. As the degree of perception of ambiguity increases, the weights of these two extreme scenarios become greater.

In this paper, I restrict attention to the symmetric case where

$$\delta_{11} = ... = \delta_{mn} = \delta, \alpha_{11} = ... = \alpha_{mn} = \alpha, \tag{3.9}$$

and thus, the objective function that players want to maximize is given by:

$$Z_{ij}(x_{ij}, \mathbf{x}_{-ij}) = \delta(1-\alpha)(\frac{V}{n} - x_{ij}) + \delta\alpha(\frac{x_{ij}}{x_{ij} + (m-1)n\overline{x}}\frac{V}{n} - x_{ij}) + (1-\delta)(\frac{X_i}{\sum_{g=1}^{m} X_g}\frac{V}{n} - x_{ij}) \tag{3.10}$$

This utility gained by player ij can be interpreted as the weighted average of benefits resulting from participating in three simultaneous games, each with a distinct objective function: M_{ij}, m_{ij}, and u_{ij}, while selecting a single level of effort. Thus, the weight assigned to them significantly determines their impact on the level of EUA. Since the game is symmetric, I concentrate on the equilibrium where players within each group exert the same level of effort.

Proposition 1. In this group contest game, a unique pure strategy Nash equilibrium exists in which players within each group exert the same level of effort. That is, $x^*_{11} = ... = x^*_{1n} = ... = x^*_{ij} = ... = x^*_{mn} = x^*$. The symmetric EUA can be written as

$$x^* = \begin{cases} 0, & \text{if } \frac{\partial Z_{ij}(0, \mathbf{x}_{-ij})}{\partial x_{ij}} < 0 \\ \overline{x}, & \text{if } \frac{\partial Z_{ij}(\overline{x}, \mathbf{x}_{-ij})}{\partial x_{ij}} > 0 \\ \text{unique positive solution of} & \\ \text{the implicit function } F(x^*, \alpha, \delta, m, n) = 0, & \text{otherwise,} \end{cases} \tag{3.11}$$

where

$$F(x^*, \alpha, \delta, m, n) = \delta\alpha \frac{(m-1)\overline{x}}{(x^* + (m-1)n\overline{x})^2} V + (1-\delta) \frac{m-1}{x^* n^2 m^2} V - 1. \tag{3.12}$$

Proof: See Appendix 3.7.1.

If the weight given to M_{ij} nears 1, $\frac{\partial Z_{ij}(0, \mathbf{x}_{-ij})}{\partial x_{ij}}$ tends towards -1. Intuitively, as the weight assigned to M_{ij} approaches 1, x^* diminishes until it cannot decrease further, reaching 0. If $\frac{\partial Z_{ij}(0, \mathbf{x}_{-ij})}{\partial x_{ij}} > 0$, it implies that the weight on m_{ij} is high enough and the upper limit on effort is sufficiently small. For the interior equilibrium, there is no explicit analytical solution; we only have an implicit one. After obtaining the interior EUA, defined by an implicit equation, the following section of this paper will explore the impact of several factors on the level of EUA, including the degree of ambiguity, the degree of pessimism about ambiguity, the number of groups, and the group size. Additionally, the level of Nash equilibrium and that of EUA are compared.

3.4 The Level of Nash Equilibrium and The Level of EUA

Recalling the utility function of player ij in the contest with ambiguity, it becomes crucial to identify the best response for each of the three scenarios individually, before discussing the level of EUA.

Lemma 1. The best response of player ij is

1) to not expend any effort in M_{ij};

2) to expend x_m^* in m_{ij}, which can be either 0, $\sqrt{V(m-1)\overline{x}} - (m-1)n\overline{x}$, or equal to $\overline{x}$, depending on the values of $\overline{x}$.

Proof see appendix 3.7.3

It is intuitive that expending no effort is the best response in M_{ij}, as the prize is guaranteed to be won by group i, and effort is irreversible and costly. Recalling the payoff function for player ij in the worst-case scenario (refer to Eq.(3.8)), when the choice of effort made by their opponents (in this case, represented by $\overline{x}$) increases, the marginal benefit of player ij's expenditure experiences a decline when $\overline{x}$ is substantially high. Conversely, it will increase when $\overline{x}$ is relatively small. Intuitively, when $\overline{x}$ is relatively small, resulting in a reasonably high chance of winning for group i, player ij will be more inclined to exert greater effort. On the contrary, player ij will choose to reduce their efforts, when the total effort of other groups is substantially high, as this significantly decreases their group's winning probability, while efforts are costly. With the assumption that $\overline{x} > x_{ne}$, the actions of player ij and those of other players (i.e., $\overline{x}$) are strategic substitutes. Accordingly, when $\overline{x}$ is sufficiently high, the best response is to refrain from exerting any effort.

According to the implicit function theorem, in order to explore how the level of EUA is affected by various factors, it is necessary to first determine the sign of $\frac{\partial F(x^*,\alpha,\delta,m,n)}{\partial x^*}$. In Appendix 3.7.2 I prove that $sgn(\frac{\partial F(x^*,\alpha,\delta,m,n)}{\partial x^*}) = -1$. This means that given the values of α, δ, m, n, if x^* increases to $x^{'}$, it will cause $F(\cdot)$ to decrease and become negative ($F(x^{'}) < F(x^*) = 0$). Thus, by comparing $F(x_{ne}, \alpha, \delta, m, n)$ with $F(x^*, \alpha, \delta, m, n)$, we can determine whether the level of Nash equilibrium or the level of EUA is larger. More specifically, if $x_{ne} \leq x^*$, then we have $F(x_{ne}, \alpha, \delta, m, n) \geq F(x^*, \alpha, \delta, m, n) = 0$, of which, the equivalent condition can be obtained by rearranging $F(x_{ne}, \alpha, \delta, m, n)$:

$$g(\alpha) = \alpha \frac{(m-1)\overline{x}}{(x_{ne} + (m-1)n\overline{x})^2} V - 1 \geq 0 \tag{3.13}$$

Two main conclusions can be drawn from (3.13). Firstly, $g(\alpha)$ is a monotonically increasing function of α. This implies that, if α is sufficiently low, the level of EUA effort level is unlikely to exceed that of Nash equilibrium effort level. This is due to the fact that the impact of changes in α on player behaviour is directly reflected through the weight assigned to both the best and worst possible scenarios. A lower value of α suggests that the utility of player ij is influenced less by m_{ij} and more by M_{ij}. This means there exists a cut-off value for the degree of pessimism, denoted as α^* such that $g(\alpha^*) = 0$, given the exogenous parameters $\overline{x}, V, m$ and n. Secondly, whether the value of α^* falls within the interval $[0, 1]$ depends on four factors: group size, number of groups, upper bound on effort, and the size of the prize. These findings can be summarized and presented as Lemma 2.

Lemma 2. i) For $\alpha^* \in [0, 1]$, we have $g(\alpha) < 0$ in the interval $[0, \alpha^*)$, which implies that $x^* < x_{ne}$; meanwhile, $g(\alpha) \geq 0$ in the interval $[\alpha^*, 1]$, leading to the conclusion that $x^* \geq x_{ne}$.

ii) α^* consistently falls within the range $[0, 1]$, when $\overline{x}$ lies within the interval $(x_{ne}, \overline{x}_1]$, where $\overline{x}_1 = \frac{m^2n-2m+2+\sqrt{m^2n(m^2n-4m+4)}}{2(m-1)n^2m^2}\frac{V}{n}$.

Proof: See Appendix 3.7.4.

The intuition behind part (i) of the proposition was explained above. The result in part (ii) is due to the fact that, according to Lemma 1, the occurrence of $x^* > x_{ne}$, which is equivalent to $g(\alpha) > 0$, will only arise if player ij expends more effort than the level of Nash equilibrium in m_{ij}. Thus, for α^* to fall within the interval $[0, 1]$, a necessary and sufficient condition is that $x^*_m > x_{ne}$. It has been demonstrated in the appendix that when $\overline{x} \in (x_{ne}, \overline{x}_1]$, $x^*_m \geq x_{ne}$ always holds.

Proposition 2. The level of EUA remains constant with respect to α when $\overline{x}$ is sufficiently high; otherwise, it increases in α.

Proof see appendix 3.7.5.

The second part of Proposition 2 is consistent with intuition. As previously discussed, the weight allocated to m_{ij} in the objective function increases as α increases, while the weight on u_{ij} remains unchanged. This, in turn, increases the significance of the optimal response in m_{ij}, while reducing the relevance of the best possible scenario where player ij expends no effort. Recalling Lemma 1 and its proof, the best response of player ij in m_{ij} is greater than 0 when $\overline{x} < \overline{x}_z$. Therefore, the level of EUA increases in α in this scenario. Using similar logic, the first part of Proposition 2 can be interpreted. When $\overline{x} \geq \overline{x}_z$, the best response for a player in the worst possible scenario is to not expend any effort. Therefore, an increase in α will not affect the weight assigned to the scenario where a player's best response is to refrain from exerting effort. That is, the level of EUA remains constant regardless of any change in α.

Thus, for group contest organizers who aim to minimize the total effort, as in some games the expenditure spent on rent-seeking is viewed as social waste (e.g., the political contests, contests for licensing and permits, and some labour unions' activities), they can reduce this waste by involving ambiguity-loving participants, or by setting a relatively high upper bound $\overline{x}$. On the other hand, considering the possible scenarios where the level of EUA is larger than that of Nash equilibrium, this offers a new explanation for the commonly observed phenomenon in experimental studies where subjects often expend more effort than predicted by the level of Nash equilibrium.

Besides studying individuals' attitudes towards ambiguity, it is also crucial to analyze how the degree of ambiguity influences players' behaviour, to gain a comprehensive understanding of the role of ambiguity in group contests.

Proposition 3. the level of EUA decreases in the degree of ambiguity, denoted as δ, if and only if $x^* < x_{ne}$; conversely, it increases in the degree of ambiguity δ if and only if $x^* > x_{ne}$.

Proof see appendix 3.7.6.

This proposition, in conjunction with Lemma 2, allows us to draw the following conclusions: if the given α lies within the interval $(\alpha^*, 1]$, then x^* will always increase with respect to δ, and x^* is greater than the level of Nash equilibrium even for a sufficiently small value of δ, provided that δ is greater than 0. This phenomenon arises because, as δ increases, the encouraging effect on effort associated with assigning a sufficiently high weight to the worst possible scenario becomes more prominent. Conversely, following a similar rationale, if α is in the interval $[0, \alpha^*)$, then x^* will consistently decrease with respect to δ. In the special event that α^* is not in the interval $[0, 1]$, x^* will invariably decrease with respect to δ, and x^* will consistently be smaller than the level of Nash equilibrium.

Now, incorporate the specific values from the experimental study of Chowdhury et al. (2021) into our model to assess whether the level of EUA provides a better prediction of players' effort, compared to the Nash prediction. If we are given $m = 2$, $n = 3$, and $V = 180$, these are the same parameter values used in Chowdhury et al.'s(2021) experiment. The level of Nash equilibrium for individual effort is calculated to be 5, whereas the average expenditure of subjects in the experiment is 14.9. Assuming that $\overline{x} = 12$, we obtain the function $g(\alpha) = \frac{2160}{1681}\alpha - 1$. The graph of $g(\alpha)$ has been plotted and

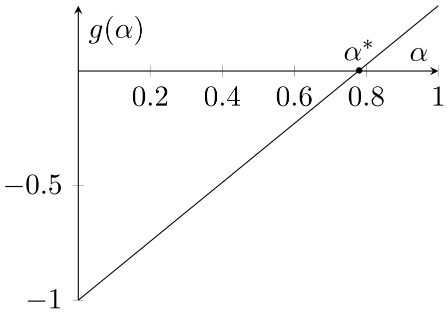

Figure 3.1: The cut-off value of α

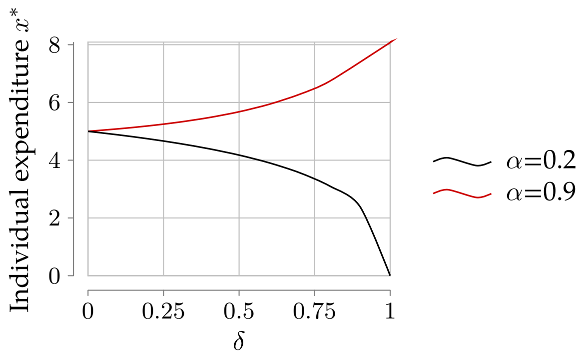

Figure 3.2: Degree of ambiguity

is displayed in Figure 3.1, where $\alpha^* = \frac{1681}{2160} \approx 0.778$ represents the threshold level of the degree of participants' pessimistic attitude towards ambiguity, such that $g(\alpha) = 0$.

Figure 3.2 illustrates the level of EUA as a function of the degree of ambiguity, comparing the case where participants are assumed to be ambiguity-loving ($\alpha = 0.2 < \alpha^*$) with the case where participants are assumed to be ambiguity-averse ($\alpha = 0.9 > \alpha^*$). At $\delta = 0$, there is no ambiguity, and the equilibrium depicted at both the black and red curves is equal to the level of Nash equilibrium. As I have previously discussed, as δ increases, the red curve demonstrates over-expenditure and indicates an increase in the value of x^*. Given $\alpha = 0.9$, the highest possible level of EUA is over 8, which is a better prediction for the experimental data than the level of Nash equilibrium. However, for an experiment like in Chateauneuf et al. (2007), assuming a value of α between 0 and 1 may not accurately predict the behaviour of all subjects. Therefore, the restriction on α may need to be relaxed to better accommodate a wider range of subjects. Moreover, Chateauneuf et al. (2007) report a significant decrease in subjects' effort expenditure over time. This suggests that as subjects gain more experience in later periods of the experiment, they expend less effort. One reason for this might be the subjects' growing familiarity with other players' strategies, which reduces the degree of ambiguity they face. Figure 3.2 reinforces this observation, illustrating that when $\alpha = 0.9$, the level of EUA decreases as the degree of ambiguity diminishes. The theoretical prediction is consistent with the empirical findings.

3.5 The optimal group structure

Moving on, let us now consider the relationship between x^* and the group structure in terms of both m and n.

Proposition 4. the level of EUA always decreases with an increase in the number of groups (m), and it also always decreases with the group size, denoted as n.

Proof: See Appendix 3.7.7.

These relationships outlined in Proposition 4 remain robust regardless of the presence of ambiguity.[4] Intuitively, in M_{ij}, m_{ij} and u_{ij}, when the number of groups is larger, the contest between groups would be more fierce. This is due to the fact that each group is competing with more opponents, which makes it harder to secure a dominant position in the contest. This suggests that, as the number of groups increases, the likelihood of any particular group winning declines. This, in turn, encourages each player to be more cautious in making decisions and, thus, reduces their willingness to expend effort in the competition. Moreover, as the number of rival players increases, the influence of a player's effort over the outcome of the contest reduces, which makes players less motivated to expend more effort. As a result, x^* decreases as m increases. This effect of the number of groups on the level of EUA is consistent with the effect observed in group contests without considering ambiguity.

An increase in the group size means that the group performance is determined by more players. Therefore, even if a player exerts a high level of effort, it may not have a significant impact on the group performance, which can have a discouraging effect on effort. Additionally, as the group size increases, the share of the prize that each player will receive decreases. This can make winning the prize less attractive to the player, leading to a decrease in their willingness to expend effort. This result is consistent with the findings of the group contest in which ambiguity is absent.

Frequently, contest designers concentrate on maximizing or minimizing the collective effort.

Proposition 5. In the presence of ambiguity,

i) the total effort decrease as the number of groups increases when a sufficiently high weight is assigned to m_{ij} and $\overline{x}$ is suitably large,

ii) the total effort decreases as the group size increases.

Proof: See Appendix 3.7.8

[4]Proof see Appendix 3.7.7.

The first finding in this proposition differs from those observed in a conventional group contest. In the absence of ambiguity, the total effort consistently increases with the number of groups.[5] The rationale for this phenomenon is that the increased effort derived from the inclusion of more players outweighs the discouraging effect of extra groups on effort. In contests with ambiguity, increasing m does not impact the outcome of the term M_{ij}. However, for the term m_{ij}, mnx_m^* decrease as m increases when $m \geq 4$. Consequently, the overall effect of increasing the number of groups could be either positive or negative, depending on the values of other parameters. More specifically, when the weight assigned to the worst possible scenario is sufficiently high, the aggregate EUA decreases as the number of groups increases. This finding can help explain why certain sports games, such as football or basketball, limit the number of teams in contests to two, as this structure may stimulate greater aggregate effort, making the games more intense and engaging for players and spectators alike.

The second finding suggests that the negative correlation between group size and total effort in contests with ambiguity is consistent with that observed in games without ambiguity. This is because, in all three terms of the utility equation (3.10), the discouraging effect of a larger group size on effort outweighs the potential increase in effort from additional players. In other words, the negative impact of competition within larger groups is stronger than any potential benefits of having more players. This result sheds light on the phenomenon of group contests, such as sports competition and R&D contests, typically limiting the size of each group. It suggests that by restricting the number of players in each group, the aggregate expenditure in the contest increases as the negative impact of competition within larger groups is minimized.

3.6 Conclusion

In conclusion, this research has provided insights into the group contests with ambiguity. My primary objective was to understand how the presence of ambiguity in group contests influences equilibrium strategies. The findings emphasize the substantial role that ambiguity plays in group contests. One of the main results indicates that the level of EUA is likely to exceed that of Nash equilibrium when the degree of ambiguity aversion is high. The impact of the degree of ambiguity aversion on the level of EUA is reflected through its influence on the best and worst possible outcomes. As the degree of ambiguity aversion increases, players assign more weight to the worst possible scenario. Moreover, as players face increased ambiguity in the contests, the level of EUA may also

[5] Proof see appendix 3.7.8.

rise. This result offers a new explanation for the over-expenditure behaviour observed in experimental studies, which deviates from the Nash predictions. Additionally, it provides insight into the downward trend in expenditures observed in such studies.

Counter-intuitively, another key finding is that the aggregate EUA may decrease with an increase in the group size, particularly when the degree of ambiguity and the level of ambiguity aversion are both sufficiently high. This suggests that when more groups participate in the contest, the overall effort levels might potentially decrease. This decrease is due to the large weight placed on the worst possible scenario. These insights bear significant implications for the design of group contests in a variety of real-world settings, including sports.

Despite the insights from this research, it is crucial to acknowledge its limitations. One limitation of this study is that ambiguity aversion is assumed to be constant across all players. In reality, individual levels of ambiguity aversion can significantly vary, potentially affecting the conclusions drawn from the model. For future research, one possibility is to empirically test the findings presented in this paper. Future experimental studies could be conducted to empirically validate and expand upon the insights from the theoretical model proposed in this paper. These studies could investigate whether subjects' behaviours align with the theoretical predictions. The degree of ambiguity could be controlled by various methods, such as allowing players to communicate with other participants, regulating the extent of information disclosure, or implementing complex scoring systems. Regarding the subjects' attitudes towards ambiguity, these can be tested by conducting pre-experiment measures.

In summary, this research contributes to the ongoing discourse on group contests by highlighting the role of ambiguity aversion in shaping equilibrium strategies, providing a new perspective on over-expenditure, and offering a rationale for why it is optimal to maintain two teams in sports games like American football, rather than accommodating a larger number of teams, from an economic standpoint. These results add to the rapidly expanding field of the contest theory. By incorporating ambiguity into the analysis of group contests, I hope to stimulate further research in this area, and to enhance the understanding of strategic behaviour under ambiguity.

3.7 Appendix

The proofs of those results not already provided in the main text are presented in this appendix.

3.7.1 Proof of Proposition 1

Proof: The procedure for determining the level of EUA proceeds in the following manner. Initially, I identify the first-order conditions for the individual player ij, which leads to a unique equilibrium. Upon applying within-group symmetry, the candidate equilibrium level of individual effort is revealed. In the final step, I investigate the second-order conditions at this candidate equilibrium and confirm that all players employ the best response to other players' equilibrium strategies. This process confirms the uniqueness of such a symmetric EUA.

i) By taking first order derivative of player ij's utility function (as per equation (3.10)) with respect to x_{ij}, it yields that

$$\frac{\partial Z_{ij}(x_{ij},\ \mathbf{x}_{-ij})}{\partial x_{ij}} = \delta\alpha \frac{(m-1)n\overline{x}}{(x_{ij}+(m-1)n\overline{x})^2}\frac{V}{n} + (1-\delta)\frac{\sum_{g\neq i}^{m} X_g}{(\sum_{g=1}^{m} X_g)^2}\frac{V}{n} - 1. \tag{3.14}$$

ii) Given that players are required to select their effort level within the range $[0, \overline{x}]$, two corner cases may occur, depending on the values of the parameters. If $\frac{\partial Z_{ij}(0,\ \mathbf{x}_{-ij})}{\partial x_{ij}}$ is negative, then we have $x^* = 0$; if $\frac{\partial Z_{ij}(\overline{x},\ \mathbf{x}_{-ij})}{\partial x_{ij}}$ is positive, then $x^* = \overline{x}$. In other cases, where neither of the above situations holds true, an interior equilibrium exists.

By applying symmetry that $X_g = mx_{ij} = mx^*$ for $i = 1, ..., m$, the implicit expression representing the unique interior symmetric EUA (provided it exists) is given by $F(x^*, \alpha, \delta, m, n) = 0$, where

$$F(x^*, \alpha, \delta, m, n) = \delta\alpha \frac{(m-1)\overline{x}}{(x^*+(m-1)n\overline{x})^2}V + (1-\delta)\frac{m-1}{x^* n^2 m^2}V - 1. \tag{3.15}$$

iii) The second-order derivative for player ij can be derived by taking another derivative of equation (3.14) with respect to x_i:

$$\frac{\partial^2 Z_{ij}}{\partial x_{ij}^2} = -\delta\alpha \frac{2V(m-1)\overline{x}}{(x+(m-1)n\overline{x})^3} - (1-\delta)\frac{2V\sum_{g\neq i}^{m} X_g}{(\sum_{g=1}^{m} X_g)^3 n} < 0 \tag{3.16}$$

Consequently, considering that other players are adhering to their equilibrium strategy, all players respond optimally. Furthermore, as these utility functions display global concavity in relation to the players' own strategies, there is no other symmetric pure strategy equilibrium exists.

Q.E.D.

3.7.2 Proof of $sgn(\frac{\partial F(x^*,\alpha,\delta,m,n)}{\partial x^*}) = -1$

$sgn(\frac{\partial F(x^*,\alpha,\delta,m,n)}{\partial x^*}) = -1$ **Proof**:

$$\frac{\partial F(x^*,\alpha,\delta,m,n)}{\partial x^*} = -\delta\alpha\frac{2V(m-1)\overline{x}}{(x+(m-1)n\overline{x})^3} - (1-\delta)\frac{V(m-1)}{(mnx^*)^3} < 0. \tag{3.17}$$

Accordingly, it obtained that $sgn(\frac{\partial F(x^*,\alpha,\delta,m,n)}{\partial x^*}) = -1$. *Q.E.D.*

3.7.3 Proof of Lemma 1

Proof: The impact of α on the level of EUA is evident through its effects on M_{ij} and m_{ij}. To explore how the level of EUA changes with α, it is necessary to ascertain the best response of player ij in two distinct scenarios, M_{ij} and m_{ij}, independently.

i) The best response of player ij in M_{ij}
Consider the best possible case in isolation, without taking m_{ij} and u_{ij} into account. Recall that $M_{ij} = \frac{V}{n} - x_{ij}$. Given that the effort is irreversible and costly, the optimal response for player ij is to expend 0.

ii) The best response of player ij in m_{ij}
Recall the payoff from the worst possible case m_{ij} in equation (3.8). If we consider it in isolation, without taking into account M_{ij} and u_{ij}, and then take the derivative of m_{ij} with respect to x_{ij} to find the best response of player ij (given the values of other parameters), we can find the equilibrium effort x^*_m in m_{ij}. The first-order condition, along with the equilibrium effort, are given by

$$\frac{\partial m_{ij}}{\partial x_{ij}} = \frac{V(m-1)\overline{x}}{(x+(m-1)n\overline{x})^2} - 1 = 0, \tag{3.18}$$

$$x_m = \{\sqrt{V(m-1)\overline{x}} - (m-1)n\overline{x}, -\sqrt{V(m-1)\overline{x}} - (m-1)n\overline{x}\}. \tag{3.19}$$

Consider that player's effort have to be non-negative, thus, $x^*_m = \sqrt{V(m-1)\overline{x}} - (m-1)n\overline{x}$.

iii) x^*_m does not monotonically increase or decrease globally with respect to $\overline{x}$.

$$\frac{\partial x^*_m}{\partial \overline{x}} = -mn + n + \frac{V(m-1)}{2\sqrt{\overline{x}V(m-1)}} = 0 \text{ at } \overline{x}^* = \frac{V}{4n^2(m-1)} \tag{3.20}$$

so we have that x^*_m increases in $(0, \overline{x}^*)$, and decreases in $[\overline{x}^*, \infty)$. Given that $\overline{x} > x_{ne}$, it is necessary to compare x_{ne} with $\overline{x}^*$ to determine whether all possible levels of $\overline{x}$

fall within the same interval or span across both intervals,

$$\begin{aligned}\overline{x}^* - x_{ne} &= \frac{V}{4n^2(m-1)} - \frac{V(m-1)}{m^2n^2} \\ &= -\frac{V(3m-2)(m-2)}{4n^2(m-1)m^2} \\ &\leq 0.\end{aligned} \tag{3.21}$$

Consequently, x_m^* decreases in $\overline{x} \in (x_{ne}, \infty)$. To identify the range of x_m^* within the given range of $\overline{x}$, it is necessary to find the value of x_m^* at $\overline{x} = x_{ne}$ and x^*m lies on the 45-degree line, as well as when x_m^* is 0 and equal to the level of Nash equilibrium, separately,

$$\begin{aligned}&\text{when } \overline{x} = x_{ne}, x_m^* = \frac{V(m-1)}{m^2n} > \frac{V(m-1)}{m^2n^2} = x_{ne}; \\ &\text{when } x_m^* \text{ lies on the 45-degree line, } \overline{x}_d = \frac{V(m-1)}{((m-1)n+1)^2}; \\ &\text{when } x_m^* = x_{ne}, \overline{x}_{ne} = \frac{m^2n - 2m + 2 + m\sqrt{n(m^2n - 4m + 4)}}{2(m-1)n^2m^2}\frac{V}{n} > x_{ne}; \\ &\text{when } x_m^* = 0, \overline{x}_z = \frac{V}{(m-1)n^2}.\end{aligned} \tag{3.22}$$

Now, the best response in m_{ij}, which is constrained within the interval $[0, \overline{x}]$, can be concluded as the following,

$$x_m^* = \begin{cases} \overline{x} > x_{ne} & \text{when } \overline{x} \in (x_{ne}, \overline{x}_d] \\ \sqrt{V(m-1)\overline{x}} - (m-1)n\overline{x} \geq x_{ne}, & \text{when } \overline{x} \in (\overline{x}_d, \overline{x}_{ne}] \\ \sqrt{V(m-1)\overline{x}} - (m-1)n\overline{x} < x_{ne}, & \text{when } \overline{x} \in (\overline{x}_{ne}, \overline{x}_z) \\ 0, & \text{when } \overline{x} \in [\overline{x}_z, \infty) \end{cases} \tag{3.23}$$

Q.E.D.

3.7.4 Proof of Lemma 2

Proof: By solving $g(\alpha) = 0$, we can obtain,

$$\alpha^* = \frac{(m-1)(\overline{x}m^2n^3 + V)^2}{n^4m^4\overline{x}V} > 0 \tag{3.24}$$

The subsequent step is to determine the condition that ensures $\alpha^* \leq 1$. To examine how α^* varies with $\overline{x}$, it is necessary to take the derivative of α^* with respect to $\overline{x}$,

$$\frac{\partial \alpha^*}{\partial \overline{x}} = \frac{(m-1)(\overline{x}^2 m^4 n^6 - V^2)}{\overline{x}^2 V m^4 n^4}. \tag{3.25}$$

According to eq.(3.25), α^* exhibits a decreasing trend with respect to $\overline{x}$ within the interval $(0, \frac{V}{n^3 m^2}]$, and shows an increasing trend when $\overline{x}$ lies within the interval $(\frac{V}{n^3 m^2}, \infty)$. Since $\frac{V}{n^3 m^2} < \frac{V(m-1)}{m^2 n^2} = x_{ne} < \overline{x}$, α^* increases in $\overline{x} \in (x_{ne}, \infty)$. By solving $\alpha^*(\overline{x}; \overline{x} > x_{ne}) = 1$, we have,

$$\overline{x}_1 = \frac{m^2 n - 2m + 2 + m\sqrt{n(m^2 n - 4m + 4)}}{2(m-1)n^2 m^2} \frac{V}{n}. \tag{3.26}$$

Recalling eq.(3.22), we can confirm that $\overline{x}_1 = \overline{x}_{ne}$. Consequently, we can deduce that α^* resides within the interval $[0, 1]$ if and only if $x_m^* \geq x_{ne}$. *Q.E.D.*

3.7.5 Proof of Proposition 2

Proof: According to the implicit function theorem (de Oliveira, 2018), we have $\frac{\partial x^*}{\partial \alpha} = -\frac{\frac{\partial F(\cdot)}{\partial \alpha}}{\frac{\partial F(\cdot)}{\partial x^*}}$.

$$\frac{\partial F(\cdot)}{\partial \alpha} = \frac{V\delta(m-1)\overline{x}}{(x^* + (m-1)n\overline{x})^2} > 0 \tag{3.27}$$

It has been proved that $\frac{\partial F(\cdot)}{\partial x^*} < 0$, thus, $\frac{\partial x^*}{\partial \alpha} > 0$. That is, the level of EUA increases with the degree of ambiguity aversion. *Q.E.D.*

3.7.6 Proof of Proposition 3

Proof:

$$\frac{\partial F(\cdot)}{\partial \delta} = \frac{\alpha V(m-1)\overline{x}}{(x^* + (m-1)n\overline{x})^2} - \frac{V(m-1)}{m^2 n^2 x^*} \tag{3.28}$$

From eq.(3.12), we have

$$\frac{\alpha V(m-1)\overline{x}}{(x^* + (m-1)n\overline{x})^2} = \frac{1}{\delta}(1 - (1-\delta)\frac{V(m-1)}{m^2 n^2 x^*}). \tag{3.29}$$

Accordingly,

$$\begin{aligned} \frac{\partial F(\cdot)}{\partial \delta} &= \frac{1}{\delta}(1 - (1-\delta)\frac{V(m-1)}{m^2 n^2 x^*}) - \frac{V(m-1)}{m^2 n^2 x^*} \\ &= \frac{1}{\delta}(1 - \frac{V(m-1)}{\delta m^2 n^2 x^*}). \end{aligned} \tag{3.30}$$

The paraphrase of eq.(3.30) is that $\frac{\partial F(\cdot)}{\partial \delta} \leq (>)0$ holds if and only if $x_{ne} \geq (<)x^*$. Combining eq. (3.17), it reveals that the level of EUA decreases in δ if and only if the level of Nash equilibrium effort surpasses that of EUA, while it increases in δ if and only if the level of Nash equilibrium is less than that of EUA. *Q.E.D.*

3.7.7 Proof of Proposition 4

Proof: Taking derivatives of $F(\cdot)$ with respect to m and n separately, we obtain,

$$\frac{\partial F(\cdot)}{\partial m} = -\alpha\delta\frac{((m-1)n\overline{x} - x^*)V\overline{x}}{(x^* + (m-1)n\overline{x})^3} - (1-\delta)\frac{V(m-2)}{n^2m^3x^*} < 0, \tag{3.31}$$

and

$$\frac{\partial F(\cdot)}{\partial n} = -2\alpha\delta\frac{V\overline{x}^2(m-1)^2}{(x^* + (m-1)n\overline{x})^3} - 2(1-\delta)\frac{V(m-1)}{n^3m^2x^*} < 0. \tag{3.32}$$

Hence, the level of EUA decreases with respect to both m and n.

In the group contest without ambiguity, the level of equilibrium effort is equal to x_{ne}. Taking the derivative of x_{ne} with respect to m and n separately, we obtain

$$\frac{dx_{ne}}{dm} = -\frac{V(m-2)}{m^3n^2}, \ \frac{dx_{ne}}{dn} = -\frac{2V(m-1)}{m^2n^3}. \tag{3.33}$$

Thus, the level of Nash equilibrium decreases with respect to both m and n. *Q.E.D.*

3.7.8 Proof of Proposition 5

Proof: The first part of Proposition 5

i) First, let's consider the group contest without ambiguity, focusing on u_{ij}. mnx_{ne} decreases in m because

$$\frac{d}{dm}(mnx_{ne}) = \frac{V}{m^2n} > 0 \tag{3.34}$$

ii) Now, consider m_{ij} in isolation. According to eq.(3.23), x_m^* is non-zero only when $\overline{x}$ lies within the interval $(x_{ne}, \overline{x}_z)$. Additionally, when $\overline{x}$ falls within the interval $(x_{ne}, \overline{x}_d]$, the value of x_m^* remains constant, equating to $\overline{x}$. Consequently, the total effort escalates as the number of groups increases.

If $\overline{x} \in (\overline{x}_d, \overline{x}_z)$, the total effort mnx_m^* may decreases in m, because

$$\frac{\partial}{\partial m}(mnx_m^*) = n(-\overline{x}mn + \overline{x}n + \sqrt{\overline{x}Vm - \overline{x}V}) + mn(\frac{\overline{x}V}{2\sqrt{\overline{x}Vm - \overline{x}V}}), \tag{3.35}$$

where $\frac{\partial}{\partial m}(mnx_m^*)$ decreases in $\overline{x} \in (\overline{x}_d, \overline{x}_z)$ because,

$$\frac{\partial^2}{\partial m \partial \overline{x}}(mnx_m^*) = n(-mn+n+\frac{V(m-1)}{2\sqrt{\overline{x}Vm - \overline{x}V}})+mn(-n+\frac{V}{4\sqrt{V(m-1)\overline{x}}}), \quad (3.36)$$

and at $\overline{x}_m = \frac{(3m-2)^2V}{16(m-1)(2m-1)^2n^2} < \overline{x}_d$, we have

$$\frac{\partial^2}{\partial m \partial \overline{x}}(mnx_m^*) = 0. \quad (3.37)$$

Thus, for $\overline{x} \in (\overline{x}_d, \overline{x}_z)$, we have $\frac{\partial}{\partial m}(mnx_m^*)$ decreases in $\overline{x}$. To confirm the sign of $\frac{\partial}{\partial m}(mnx_m^*)$, it is necessary to calculate it at $\overline{x} = \overline{x}_d$ and $\overline{x} = \overline{x}_z$ separately. When $\overline{x} = \overline{x}_z$,

$$\frac{\partial}{\partial m}(mnx_m^*) = -\frac{Vm}{2(m-1)} < 0, \quad (3.38)$$

while when $\overline{x} = \overline{x}_d$,

$$\frac{\partial}{\partial m}(mnx_m^*) = -\frac{(m^2-m)n - 3m + 2}{2((m-1)n+1)^2}Vn < 0 \quad (3.39)$$

Thus, when a sufficiently high weight is assigned to the worst possible scenario and $\overline{x}$ is suitably large compared to the Nash equilibrium, there is a decrease in the aggregate EUA as the number of groups increases.

iii) Let's consider a specific example from American football, where there are two teams, each consisting of 11 players. Assuming that $\alpha = 0.8,\ \delta = 0.7,\ V = 100$ and $\overline{x} = 1$, we have

$$22 \cdot x^*(m=2) > 33 \cdot x^*(m=3) \quad (3.40)$$

Next, let's examine the impact of group size on the total effort exerted.

i) mnx_{ne} decreases in n because,

$$\frac{d}{dn}(mnx_{ne}) = -\frac{V(m-1)}{mn^2} < 0. \quad (3.41)$$

ii) In the context of m_{ij}, we focus on the situation where $x_m > 0$ and $\overline{x}$ falls within the interval $(x_{ne}, \overline{x}_z)$. x_m^* decreases in $\overline{x}$, thus we have

$$\frac{\partial}{\partial n}(mnx_m^*) < \frac{\partial}{\partial n}(mnx_m^*(\overline{x} = x_{ne})) = -\frac{V(m-1)(m-2)}{mn} < 0 \quad (3.42)$$

A change in n does not modify players' behaviour in M_{ij}. However, as I have demonstrated, in both u_{ij} and m_{ij}, the total effort decreases with n. Given that the level of EUA is a product of the weighted impacts of M_{ij}, m_{ij}, and u_{ij}, the level of EUA also declines as n increases.

Q.E.D.

Chapter 4

Group Contests with Interdependent Preferences: Equilibrium Behaviour and the Group Size Paradox

4.1 Introduction

In his seminal study, Olson described the tendency of larger groups to do worse than small groups in achieving a common goal that has become known as the group size paradox (henceforth, GSP). He writes: "The larger a group is, the farther it will fall short of obtaining an optimal supply of any collective good, and the less likely that it will act to obtain even a minimal amount of such a good. In short, the larger the group, the less it will further its common interests" (Olson, 1965, p. 36). In group contests, where group members expend resources such as time, money, and effort to increase the likelihood of their group winning, the GSP implies that the winning probability of a smaller group is higher than it of a larger group.

The GSP has been found theoretically in different contexts. For example, Katz and Tokatlidu (1996) proves the GSP theoretically in a two-stage group rent-seeking contest; Baik and Lee (1997) proves it in the context of group contests with an endogenous sharing rule, where groups are permitted to impose fines or rewards on their members. However, experimental literature does not align with the theory of GSP in contests. According to a survey by Sheremeta (2018), nearly all existing studies on the effect of group size in group contests disprove the GSP experimentally (e.g., Oliver and Marwell, 1988; Abbink et al., 2010; Kugler et al., 2010; Ahn et al., 2011). One exception to this rule is Ahn et al. (2011), where in one treatment, a group of five players has a higher winning probability (60%) than a single player (40%) that they compete against.

In this paper, we provide a new theoretical explanation for the contradiction between the GSP and the results of experimental research. We extend the Tullock (1980) contest to the case where players have interdependent preferences; this distorts the equilibrium effort compared to the standard setting with independent preferences. This area has yet to be fully explored. In our examination, we categorize these interdependent preferences into two types. One of the types of participants is altruistic and gains some utility even if other players win the prize. The other type is spiteful or envious, and experiences a negative external effect from other players' payoffs.[1]

Previous literature has explored various conditions that could influence the validity of the GSP, including factors related to the group impact function, cost function, and heterogeneity. However, these conditions may not be consistent with the assumptions found in theoretical papers. As a result, the applicability of these theories to elucidate the GSP in experimental results could be limited. Nonetheless, these theories remain suitable for explaining a wide range of real-life GSP scenarios.

We consider a group contest in which two groups of different sizes compete for a prize against each other, and the prize is divided equally among all group members in the winning group. To our knowledge, most research on public goods has been carried out in these two cases, a non-rival[2] and non-excludable[3] public good (i.e., a pure public good) and a fully rival public good.[4] Theoretically, the GSP holds true when public goods exhibit rivalry among users (e.g., Nitzan and Ueda, 2009; Pecorino, 2009). However, this is in contradiction with experimental findings, such as those presented by Ahn et al. (2011). Thus, this paper concentrates on fully rival public goods, aiming to bridge this gap between theoretical predictions and observed behaviour. It is a common phenomenon for many publicly provided goods to exhibit a level of competition, for example, the medical assistance provided by a public hospital will diminish as more people use its services; and a public park can become overcrowded with an increase in foot traffic.

This research is built upon four additional key assumptions. First, we assume that players

[1]Following Bester and Güth (1998), we distinguish the material payoff and the utility function. The material payoff is the reflection of the objective prosperity of the player (Schmidt, 2009), while the utility function represents the player's interdependent preference (Konrad, 2004). In addition, if a player is narrowly self-interested, their utility function and the material payoff are the same (Konrad, 2004).

[2]A non-rival good means that one player consuming a good does not reduce the availability to others.

[3]Non-excludability refers to the case that, once the good is provided, one player cannot prevent other players from accessing it.

[4]Such a good is fully divided among group members.

are homogeneous. This is because the literature on group contests with heterogeneous players has highlighted the existence of a correlation between heterogeneity and players' behaviour (e.g., Chowdhury et al., 2016a). By assuming player homogeneity, we aim to eliminate any potential equilibrium disruptions caused by heterogeneity. Second, the exogenous egalitarian sharing rule is employed for prize allocation in our model. This allows us to independently examine the effects of interdependent preferences on the GSP, as certain sharing rules, like the proportional sharing rule, tend to favor larger groups, as established by Ke (2011). Third, we adopt one of the most popular cases of the Tullock contest success function, known as the lottery contest. Given that the existence of a pure-strategy Nash equilibrium is not guaranteed when the technology parameter in the group contest success function exceeds 1 (e.g., Malueg and Yates, 2006), we have excluded such scenarios from our paper. Finally, the group impact function in our model is the most commonly discussed function, the perfect-substitutes function. These assumptions in this paper are standard and consistent with the majority of experimental papers (e.g., Abbink et al., 2012; Ke et al., 2015; Chowdhury et al., 2021; Song and Houser, 2021).

Our study differs from the existing literature in that our model incorporates the case where players have a spiteful preference towards other players from their own group. By doing this, we provide sufficient conditions of the GSP being valid and invalidated. In our analysis, interdependent preferences are reflected by putting weight on the average material payoff of one's teammates and that of players in the rival group. When a player expends extra effort, it increases the winning probability of their group and thus the material payoffs of players within the same group. Therefore, a player who is spiteful towards their fellow group members will receive less benefit, compared to a purely self-interested player. Through our analysis of players' utility functions, we find that the effective valuation of the prize decreases for both the larger and smaller groups as *intra*-group spite increases. In other words, as players become more spiteful towards their teammates, their desire to win the group contest decreases, leading to a decrease in effort expenditure. The discouraging effect on effort is more pronounced in smaller groups compared to larger groups. This is because winning in a smaller group leads to a more significant gain for the average player, given the smaller group size.

Using similar reasoning, a player who shows altruism towards their fellow group members receives more benefit by spending extra effort than a self-interested player. Accordingly, when there is an increase in *intra*-group altruism, it leads to an increase in the effective value of the prize. The effects of *intra*-group altruism that motivate individual effort are more concentrated in a smaller group, resulting in an increased probability of winning for the smaller group when players exhibit more altruism towards their group members.

On the other hand, as a player's level of spite towards the rival group increases, they gain more benefit from the same amount of extra effort compared to a purely self-interested player. This is because the reduction of the rival group's winning probability, in addition to the increase in one's own group's winning probability, generates additional utility. As a result, individual effort increases in *inter*-group spite. Because this effect of *inter*-group spite on players' decision-making works between groups, the players in the larger group are more sensitive to it than players in the smaller group. Therefore, as the level of *inter*-group spite increases, the winning probability of the larger group increases. With only two groups in the group contest, an increase in the winning probability of one group will inevitably lead to a corresponding decrease in the winning probability of the other group. When the degree of *intra*-group altruism and the degree of *inter*-group spite sum up to 1, the negative effect and positive effect on players' effort expenditure balance out. Thus, the two groups have equal winning probabilities. However, when the sum of levels of *intra*- and *inter*-group interdependent preferences favours the larger group, that is, it is greater than 1, the GSP becomes invalid.

The rest of the paper is organized as follows. Section 2 presents the related literature. Section 3 sets up the basic model and presents the analysis. Section 4 concludes the paper.

4.2 Literature review

The research on the GSP has gained momentum. Previous studies have demonstrated an inverse relationship between the winning probability of a group and its size in group contests (e.g., Nitzan, 1991; Cheikbossian, 2008). These studies have explored various factors, including heterogeneity, cost function, attitudes towards risk, sharing rules, group impact function, returns to scale of effort, and the nature of the prize.

The group expenditure is influenced by heterogeneity among players. Olson (1965) states that asymmetric valuation may make a change in the GSP, while Baik (1993) proves that the winning probabilities are independent of any changes in group sizes and the composition of players' valuations of the prize, if the highest valuation of the group stays unchanged. In situations where *inter*-group players have differing valuations of the prize, Kolmar and Rommeswinkel (2020) prove that factors such as returns to scale, the impact function, and the degree of rivalry for the prize determine the occurrence of the GSP. The higher the rivalry for the prize, the more likely it is for the GSP to occur. However, if the increase in the group impact is sufficiently large, given that the total effort of the group members remains constant but is distributed among more members, and if the returns to scale are sufficiently low, the GSP is unlikely to occur. In cases where *intra*-group players are also permitted to have asymmetrical valuations of the prize, the composition of individual valuations will affect the GSP. Chowdhury and Topolyan (2016b) explore a scenario where one group adopts the weakest-link impact function and the other adopts the best-shot function. In this case, where players may value the prize differently, the equilibrium demonstrates that winning probability is not dependent on group size but rather on players' valuations. Chowdhury et al. (2016a) prove that in contests under the shadow of internal conflict, a high level of *intra*-group inequality can enhance a group's effort in external conflict if the degree of complementarity of efforts is moderately low.

Another significant factor affecting the GSP is the cost function. Esteban and Ray (2001) demonstrate that the winning probability of the larger group is always higher, if the elasticity of the marginal cost of players' effort is at least as large as 1, regardless of how private the prize is. Cheikbossian and Fayat (2018) prove that in a contest where the prize is an impure public good and individual efforts are perfect substitutes, the GSP does not hold if the degree of convexity of the cost of the player's effort is sufficiently high relative to the degree of rivalry for the prize. Similarly, Kobayashi and Konishi (2021) point out that when the constant elasticity of the cost function is significantly larger than the elasticity of substitution of efforts, the GSP will not occur. On the other hand, the linearity of the cost function and the GSP occurring appear to be closely linked. Flamand and Troumpounis (2015) simplify Esteban and Ray's (2001) study and summarize that the GSP always occurs when the cost of effort is linear, with the exception being when the prize is a pure public good. Cheikbossian and Fayat (2018) also investigate the case where the cost function is linear, finding that the GSP will not occur if players' efforts exhibit a degree of substitution that is both positive and less than 1, provided the prize is a pure public good. Moreover, the GSP will not occur if the degree of complementarity of efforts is relatively high, given the prize is fully rival.

The third factor that influences the conditions for the validity of the GSP is the sharing rule. Nitzan (1991) was the first to prove that under the proportional sharing rule, each player expends the same amount of effort regardless of the size of their group. Hence, the aggregate effort of the larger group is higher, making it more likely to win the contest. Conversely, the larger group is at a disadvantage when the egalitarian sharing rule is in effect. For mixed sharing rules, Flamand and Troumpounis (2015) demonstrate that the GSP will not occur if the group sharing rules assign more weight to the proportional sharing rule than to egalitarianism; otherwise, the GSP emerges. Intuitively, sharing rules that place weight on the proportional distribution suggest that members who contribute more will receive a larger share of the prize, thereby offering a high degree of selective incentives (Ueda, 2002). Kugler et al. (2010) show that under the proportional sharing rule, the larger group always has a higher probability of winning, given the absence of any endogenous group-specific public good.[5] However, the GSP could still arise under the proportional rule if endogenous group-specific public goods exist. In the presence of such endogenous group-specific public goods, the GSP may not occur, even under the egalitarian sharing rule.

Moving beyond the constraints of exogenous sharing rules, studies have explored endogenous sharing rules. Lee (1995) finds that larger groups tend to employ a mixed sharing rule, while smaller groups favour a purely proportional rule. Interestingly, despite these differing approaches, both groups retain an equal probability of winning. This equality in winning probabilities between larger and smaller groups is also confirmed by Flamand and Troumpounis (2015). However, Lee and Hyeong Kang (1998) present a contrasting view, suggesting that larger groups exhibit higher winning probabilities when there are positive externalities [6] linked with rent-seeking activity. When both groups ultimately decide on the proportional rule, Gupta (2023) concludes that the occurrence of the GSP can be avoided. Conversely, if neither group places all weight on the proportional sharing rule, the probabilities of winning even out. If one group opts for the proportional rule while the other leans towards either the egalitarian or a mixed sharing rule, the latter group is more likely to win, irrespective of the sizes of the two groups. Baik and Lee (1997) further expand on this discussion by easing the restrictions[7] on mixed sharing rules, demonstrating that the GSP can occur under these conditions. Similarly, Lee and Hyeong Kang (1998) reach a parallel conclusion when they loosen the constraints on mixed sharing rules. Moreover, they highlight that the disadvantage faced by larger groups is somewhat mitigated in scenarios with externalities, compared to those without.

The influence of the group impact function employed in group contests is another vital aspect in analyzing GSP. Lee (2012) discusses a special case of perfect complementarity. Their findings suggest that the winning probabilities of groups depend on the player

[5]This endogenous group-specific public good, the size of which depends on a prize generated from players exerting effort in *inter*-group contests, is supplementary to the contest prize. Hence, even if a group doesn't win the contest, its players can still derive benefits from this group-specific public good.

[6]In their model, the positive externality is represented by each player obtaining a certain proportion of the aggregate effort as an externality effect, regardless of whether they emerge as the winner.

[7]The lack of restriction allows the group to take away resources expended by its members and then distribute them among group members, either equally or proportionately.

with the lowest valuation within each group, and these probabilities are irrespective of the group size. Chowdhury et al. (2013) draw attention to another distinct case, referred to as a 'best-shot' group contest. They discover that the probabilities of winning are independent of group sizes but are notably influenced by the composition of players' valuations of the prize. The degree of complementarities in *intra*-group individuals' efforts, has been proven to be one of the determining factors of the validity of the GSP. For example, Kolmar and Rommeswinkel (2010) research further into the topic and demonstrate that a higher degree of complementarity can potentially provide an advantage for larger groups. Kolmar (2013) reveals that larger groups are more likely to win if the elasticity of substitution of the group impact function is positive, the prize is pure public, and first-best institutions - those in which each group member strives to maximize the welfare of the entire group - are accessible to groups. For a fully rival prize when the first-best institutions are not available, the GSP occurs when the elasticity of substitution of efforts is either negative or significantly large; otherwise, it will not occur.

The GSP has been studied in the context of pure public goods, as well as prizes exhibiting varying degrees of rivalry. Researchers like Chamberlin (1974), McGuire (1974) and Oliver and Marwell (1988) have argued, though without formal justification, that the GSP is invalid without the assumption that the prize is fully rival. This is because the initial argument on the GSP is predicated on the premise that the prize is purely private and not a public good. Conversely, Katz et al. (1990) analyzed a model where players are risk averse, the prize is a pure public good, and the coefficient of absolute risk aversion is constant with respect to wealth, the winning probability negatively depends on the group size. However, they argue, without presenting a formal analysis, that this result may not hold if the utility function exhibits decreasing absolute risk aversion. Riaz et al. (1995) provide evidence for a positive correlation between group size and group effort within a general model where players display risk aversion. Furthermore, Cheikbossian (2008) demonstrates that the GSP is invalid under within-group cooperation, as the winning probabilities for larger and smaller groups are equal. However, under conditions of within-group non-cooperation, the GSP remains valid. In contests where the prize comprises both a pure public component and a fully rival component, Hu and Treich (2014) further prove that the smaller group is at a disadvantage in terms of winning probability when there is full cooperation among individuals within the same group.

The free-riding problem is also viewed as an explanation for the GSP (Hartley, 2017; Sheremeta, 2018). Various studies have suggested that the free-riding problem is more prevalent in the larger group (e.g., Levine and Palfrey, 2007; Rapoport and Bornstein, 1989; Kugler et al., 2010). This can be attributed to the fact that players in larger groups have a heightened incentive to shirk given the effort is costly and irreversible. Moreover, the perceived impact of a group member deciding to either abstain from participation or contribute less seems to be less significant. According to Pareto (1972), the free-riding problem makes the larger group less effective. Moreover, Levine and Palfrey (2007) provide evidence that there is less free riding in smaller groups in their laboratory study. One question that needs to be asked, however, is whether this inherent disadvantage present in larger groups can be overcome. Rapoport and Bornstein (1989) show that a larger group is more likely to win, despite having greater incentives to free ride. In their study of *intra*-group efficient incentive schemes, Kolmar and Wagener (2013) argue that a larger group might adopt a more efficient incentive scheme under the higher free-riding problem. Therefore, with a better organizational structure, the GSP could be invalidated.

Spite and altruism are explored across a wide range of economic areas, and an extensive body of published research underscores their crucial roles in various aspects of social and economic life (e.g., Schoeck, 1966; Mui, 1995). In contests where the prize includes two components, a purely public element and a fully rival element, Hu and Treich (2014) establish that when two groups exhibit an equal degree of altruism, the GSP might be invalidated if the degree of altruism surpasses a critical threshold. This threshold depends on the group sizes, the degree of the rivalry of the prize, and the sizes of the pure public and private components of the prize. However, this critical degree could be too large to be exceeded within the given range of altruism, thereby ensuring the persistent occurrence of the GSP. Expanding on this by relaxing the identical degree of altruism assumption, Hu and Treich (2018) demonstrate that an increase in the degree of *intra*-group altruism in either the larger or smaller group results in increased total effort from that group. They further disclose that the GSP can be overturned if the size of the larger group is sufficiently large or if the degree of *intra*-group altruism is high enough, potentially requiring a degree of *intra*-group altruism so large that a player values the aggregate material payoff of their *intra*-group members more than their own material payoff. Spite and altruism are often introduced together in models, leading to a concept known as parochial altruism (e.g., Chowdhury et al., 2016a; Kolmar and Wagener, 2019; Chowdhury, 2021).[8] Moreover, Abbink et al. (2012) analyze their experimental data, indicating that subjects generally exhibit parochial altruism, neither purely altruistic nor purely hostile. March and Sahm (2021) broaden the scope of parochial altruism, allowing for both *intra*-group spite and altruism, as well as *inter*-group spite and altruism. However, they restrict their model by ensuring no differences exist in

[8] An individual exhibiting parochial altruism means being altruistic to fellow group members while being hostile toward *inter*-group individuals (Bernhard et al., 2006).

intra-group and *inter*-group attitudes between the two groups. They conclude that the winning probability of the larger group increases with *intra*-group spite and decreases with *inter*-group spite. However, it could increase to the point where it surpasses the winning probability of the smaller group.

4.3 The Model

Consider a contest where two groups, A and B, compete to win a prize $V > 0$. Group A consists of n_s risk-neutral players and group B n_l risk-neutral players. Without loss of generality, let $n_l \geq n_s \geq 2$. Accordingly, group A(B) is the smaller(larger) group. With a constant marginal cost of 1, all players simultaneously and independently expend irreversible and costly efforts denoted x_i, $i = 1, \ldots, n_s$, for players in group A and y_j, $j = 1, \ldots, n_l$ for players in group B.

Group performance is determined by a group's total effort, i.e., the sum of all individual efforts within a group. Group performances are thus $X = \sum_{i=1}^{n_s} x_i$ and $Y = \sum_{j=1}^{n_l} y_i$ for A and B respectively. A group's winning probability is determined by the ratio of the group's performance to the total effort exerted by all players. Group A hence wins with probability

$$p_A(X, Y) = \frac{X}{X+Y}. \tag{4.1}$$

Accordingly, the winning probability of group B is $p_B(X, Y) = {}^{Y}/_{(X+Y)} = 1 - p_A$. In the event $X = Y = 0$, $p_A = p_B = {}^{1}/_{2}$.

The prize is evenly distributed among members of the winning group. The material payoff of player i in group A is thus

$$\pi_i(x_1, \ldots, x_{n_s}, y_1, \ldots, y_{n_l}) = p_A \frac{V}{n_s} - x_i = \frac{x_i + X_{-i}}{x_i + X_{-i} + Y} \frac{V}{n_s} - x_i, \tag{4.2}$$

where X_{-i} is the total effort of all players in group A except i. For group B players it is

$$\pi_j(x_1, \ldots, x_{n_s}, y_1, \ldots, y_{n_l}) = p_B \frac{V}{n_l} - y_j = \frac{y_i + Y_{-j}}{X + y_j + Y_{-j}} \frac{V}{n_l} - y_j, \tag{4.3}$$

where Y_{-i} is the total effort of all players in group B except j.

We allow the players to have either negatively or positively interdependent preferences towards players within their own group, and negatively interdependent preferences towards players in the opponent group. In other words, players can be either spiteful or altruistic towards their fellow group members but can only potentially be spiteful towards players in the competing group. Besides matching intuition, this specification also conveniently guarantees an interior solution for equilibrium.

Let $s_1 \in (-1, 1)$[9] and $s_2 \in [0, 1)$ measure the levels of *intra*- and *inter*-group interdependent preferences, respectively. Then, the utility of player i in group A is

$$U_i = p_A \frac{V}{n_s} - x_i - s_1 \frac{p_A \frac{V}{n_s}(n_s - 1) - X_{-i}}{n_s - 1} - s_2 \frac{p_B V - Y}{n_l}. \tag{4.4}$$

We note that what associate with s_1 and s_2 in (4.4) are the average material payoff of fellow group members and that of players in the competing group, respectively. Similarly, the utility function of player j in group B is

$$U_j = p_B \frac{V}{n_l} - y_j - s_1 \frac{p_B \frac{V}{n_l}(n_l - 1) - Y_{-j}}{n_l - 1} - s_2 \frac{p_A V - X}{n_s}. \tag{4.5}$$

When $s_1 > 0$, players are spiteful towards fellow group members. On the other hand, if $s_1 < 0$, players are actually altruistic towards members of their own group. As $s_2 > 0$, players are always at least weakly spiteful towards players of the rival group. In the limiting case of s_1 and s_2 approaching 0 respectively, all players are purely self-interested, and the game becomes a standard Tullock contest between two groups of varying sizes. Finally, if we were to allow $s_2 < 0$, the competitive nature of a group contest would simply disappear. This scenario is outside the scope of what the model is equipped to analyze.

4.4 Equilibrium Analysis

We proceed to solve for the Nash equilibrium of the contest game. As the game is symmetric except for group size, we focus on the Nash equilibrium where players in each group exert the same amount of effort. In other words, we look for a Nash equilibrium that is *within-group symmetric*. The following proposition summarizes the finding.

Proposition 6. There exists a unique within-group symmetric pure strategy Nash equi-

[9]Establishing a lower limit is sensible, as it helps prevent the occurrence of paradoxes associated with altruism. Within economic settings, it may result in inefficiencies and sub-optimal outcomes Bolle (2000). A well-known instance of an altruism paradox in a broader context can be seen in "The Gift of the Magi" by Henry (2021).

librium in this group contest game where each player in group A exerts effort

$$x^* = \frac{(n_s s_2 + (1 - s_1)n_l)^2 (n_l s_2 + (1 - s_1)n_s)V}{(1 - s_1 + s_2)^2 (n_l + n_s)^2 n_s^2 n_l}, \tag{4.6}$$

and each player in group B exerts effort

$$y^* = \frac{(n_l s_2 + (1 - s_1)n_s)^2 (n_s s_2 + (1 - s_1)n_l)V}{(1 - s_1 + s_2)^2 (n_l + n_s)^2 n_s n_l^2}. \tag{4.7}$$

Proof: See Appendix 4.7.

The process of solving for this equilibrium is standard and hence the details have been relegated to the Appendix. However, it's noteworthy that while the equilibrium pair of group performances (X^*, Y^*) is unique, there exist other equilibria where individual players within each group may contribute differently, so long as (X^*, Y^*).

It is instructive to see how interdependent preferences affect a player's equilibrium effort. First, to understand the effects of *intra*-group interdependent preferences, let's shut down the channel of *inter*-group spite by setting s_2 to the limit 0. Taking a group A player for an example, in this case equilibrium effort x^* as in (4.6) approaches $\frac{(1-s_1)n_l V}{(n_l+n_s)^2 n_s}$. It follows that in the absence of concerns of *inter*-group relative payoff, spite towards fellow group members reduces a player's equilibrium effort. On the other hand, altruistic preferences towards fellow group members ($s_1 < 0$) increase equilibrium effort. This result is rather intuitive. When a player increases their effort, the winning probability of their group increases. Then, compared to a purely self-interested player, for the same amount of additional effort, an *intra*-group spiteful player receives less benefit because their fellow group members' material payoffs also increase. In contrast, an *intra*-group altruistic player receives more benefit precisely for the same reason. Thus, different from how spite usually affects effort in individual contests (Hehenkamp et al., 2004), *intra*-group spite (altruism) decreases (increases) equilibrium individual effort.

Second, we isolate the effects of *inter*-group spite by shutting down the channel of *intra*-group interdependent preferences. As s_1 approaches 0, x^* arrives at $\frac{(n_l s_2+n_s)(s_2 n_s+n_l)^2 V}{n_s^2 n_l (1+s_2)^2 (n_l+n_s)^2}$ which can be demonstrated to be increasing in s_2. That is, when a player becomes more spiteful of rival group players, they obtain more benefit than a purely self-interested player does from the same amount of extra effort. In other words, in addition to the increase in one's own group's winning probability, the concurrent reduction of the rival group's winning probability brings about additional utility. Equilibrium individual effort thus increases in *inter*-group spite. A similar analysis applies to individual efforts in the larger group, y^*.

Proposition 7. i) In the absence of *inter*-group interdependent preferences, *intra*-group spite (altruism) decreases (increases) equilibrium individual effort.

ii) In the absence of *intra*-group interdependent preferences, *inter*-group spite increases equilibrium individual effort.

To gain further intuition, we decompose the players' utility functions (4.4) and (4.5):

$$U_i = \frac{x_i + X_{-i}}{x_i + X_{-i} + Y}\left(\frac{1-s_1}{n_s} + \frac{s_2}{n_l}\right)V - x_i + s_1\frac{X_{-i}}{n_s - 1} + s_2\frac{Y - V}{n_l}, \tag{4.8}$$

$$U_j = \frac{y_i + Y_{-j}}{X + y_j + Y_{-j}}\left(\frac{1-s_1}{n_l} + \frac{s_2}{n_s}\right)V - y_j + s_1\frac{Y_{-j}}{n_l - 1} + s_2\frac{X - V}{n_s}. \tag{4.9}$$

It is evident from (4.8) and (4.9) that interdependent preferences affect a player's decision situation only through their "effective" valuation of the prize. For example, in the case of group A this is $\hat{V}_i := \left({}^{(1-s_1)}/_{n_s} + {}^{s_2}/_{n_l}\right)V$. The last two terms in both equations are independent of an individual's own effort. Therefore, for group A players, their decision is effectively the same as in a group contest where winning entails a prize of $\hat{V}_i$ for each member. Likewise, for group B players, their decision is effectively the same as in a group contest where winning entails a prize of $\hat{V}_j := \left({}^{(1-s_1)}/_{n_l} + {}^{s_2}/_{n_s}\right)V$ for each.

An increase in *intra*-group spite s_1 then effectively decrease both $\hat{V}_i$ and $\hat{V}_j$. In a similar vein, an increase in *intra*-group altruism, which is measured by $-s_1$ for $s_1 < 0$, increases the effective value of the prize. As far as *inter*-group interdependent preferences are concerned, spite increases both $\hat{V}_i$ and $\hat{V}_j$ because winning it for a player's own group has the extra benefit of preventing players in the other group from gaining higher material payoffs. Finally, due to the usual positive relationship between prize and effort, Proposition 7 follows.

A notable and important corollary of Proposition 7 speaks to the well-documented overbidding phenomenon in lab experiments (see, e.g., Sheremeta, 2013; Dechenaux et al., 2015). Compared to theoretical Nash equilibrium predictions, subjects in the lab often spend significantly more resources in contests. This observation holds for contests in general and group contests in particular (Sheremeta, 2018).

A direct consequence of Proposition 7 however is that *intra*-group altruism and *inter*-group spite encourage players to exert more effort compared to the Nash equilibrium benchmark with standard purely self-interested players. While spite or negatively interdependent preferences has been proposed to explain overbidding in the lab (Herrmann and Orzen, 2008), *notably* we demonstrate that *intra*-group altruistic preferences can also lead to overbidding.

Corollary 1 (overbidding)**.** All else being equal, individual equilibrium effort with either *intra*-group altruism or *inter*-group spite is higher than that with purely self-interested preferences.

4.5 Winning Probabilities and the GSP

Following Proposition 6, the winning probabilities of the two groups in equilibrium are respectively

$$p_A^* = \frac{n_s x^*}{n_s x^* + n_l y^*} = \frac{n_s s_2 + (1-s_1)n_l}{(1-s_1+s_2)(n_l+n_s)}, \tag{4.10}$$

$$p_B^* = \frac{n_l y^*}{n_s x^* + n_l y^*} = \frac{n_l s_2 + (1-s_1)n_s}{(1-s_1+s_2)(n_l+n_s)}. \tag{4.11}$$

Before investigating which group is more likely to win the contest, it helps to understand how a group's winning probability changes in *intra*-group and *inter*-group preferences. Take group A's winning probability as an example. The first order derivative with respect to *intra*-group preferences s_1 is

$$\frac{\partial p_A^*}{\partial s_1} = -\frac{s_2(n_l-n_s)}{(1-s_1+s_2)^2(n_l+n_s)} < 0.$$

That is, as *intra*-group spite increases, the smaller group's equilibrium winning probability decreases. Since the two groups' winning probabilities sum up to 1, the larger group's winning probability increases. Intuitively, as players become more spiteful toward fellow group members, their utilitarian value of winning the contest decreases, and are likely to exert less effort (Proposition 7). However, this effort discouraging effect is *more concentrated* in a *smaller* group than that in a larger group because an average player gains more in a smaller group when winning due to its smaller group size. Likewise, *intra*-group altruistic effects, which encourage individual effort, are also more concentrated in a smaller group, and hence, a smaller group's winning probability increases when players are becoming more altruistic toward fellow group members (i.e., $s_1 < 0$ and decreases).

With respect to *inter*-group spite s_2,

$$\frac{\partial p_A^*}{\partial s_2} = -\frac{(1-s_1)(n_l - n_s)}{(1-s_1+s_2)^2(n_l+n_s)} \leq 0.$$

It follows then *inter*-group spite reduces the smaller group's winning probability, while it increases that of the larger group. Again, the insight lies in Proposition 7. As players become more spiteful toward rival group players, the utilitarian value of winning the contest increases and players are likely to exert more effort. However, because this effect works *between* groups, it is stronger for players in the larger group than it is for players in the smaller group. Thus, the larger group becomes more likely to win, while the opposite holds for the smaller group. The following lemma summarizes the above discussion.

Lemma 3. i) *intra*-group spite (altruism) decreases (increases) the smaller group's winning probability. The opposite holds for the larger group.

ii) *inter*-group spite decreases (increases) the smaller (larger) group's winning probability.

Equilibrium analysis of the standard group contest game often leads to the puzzling result that smaller groups are more likely to win a contest, despite the intuitive collective strength associated with larger groups (see, e.g., Olson, 1965; Pecorino, 2015; Esteban and Ray, 2001). To understand how the groups' equilibrium winning probabilities compare in a richer environment that allows for interdependent preferences, let's start from the mathematical difference between their equilibrium winning probabilities (4.10) and (4.11):

$$p_A^* - p_B^* = \frac{(1-s_1-s_2)(n_l-n_s)}{(1-s_1+s_2)(n_l+n_s)}. \tag{4.12}$$

With the assumptions on s_1 and s_2, we have $1 - s_1 > 0$ and $s_2 \geq 0$, and therefore the denominator in (4.12) is positive. Given $n_l > n_s$, it follows that, if $s_1 + s_2 > 1$, then $p_A^* < p_B^*$; if $n_l > n_s$ and $s_1 + s_2 < 1$, $p_A^* > p_B^*$. If either $n_l = n_s$ or $s_1 + s_2 = 1$, the two groups win the contest equally likely. The next proposition follows.

Proposition 8 (GSP). i) Suppose $n_l > n_s$. The smaller group A is more likely to win the contest if $s_1 + s_2 < 1$. On the other hand, the larger group B is more likely to win the contest if $s_1 + s_2 > 1$.

ii) If either $n_l = n_s$ or $s_1 + s_2 = 1$, the two groups are equally likely to win the contest.

Proposition 8 presents the condition for the validity of the so-called GSP when interdependent preferences are considered. When the "combined" *intra*-group and *inter*-group spite is relatively moderate, i.e. $s_1 + s_2 < 1$, the usual result of the GSP holds where the smaller group is more likely to win the contest, despite the game being completely symmetric except for group size. Notably, if the players are purely self-interested (i.e., $s_1, s_2 \to 0$), this condition clearly holds and we then reproduce the GSP.

In contrast, when the "combined" *intra*-group and *inter*-group spite is relatively strong, i.e. $s_1 + s_2 > 1$, the larger group becomes the more likely winning group of the contest. This is a *novel* result to the literature as we demonstrate that interdependent preferences, or rather spiteful preferences, can overturn the GSP and result in the more intuitive perception that a group's strength grows in its size.

The main insights in Proposition 8 can be understood in the light of Lemma 3. Both *intra*-group and *inter*-group spite decrease the winning chances of the smaller group, while they increase that of the larger group. When the combined forces are strong enough, the larger group's winning probability overtakes that of the smaller group.

Proposition 8 sheds light on our understanding of group conflicts. Large groups often suffer the free-rider problem associated with group activities to greater extents than smaller groups. Moreover, as the same prize is divided by more members in the larger group, larger group players value the prize less than smaller group players do. Relatedly, larger groups are often less effective in achieving their objectives than smaller groups.[10] However, in an environment with spiteful preferences, larger groups can have an advantage. A larger group size makes the effort discouraging *intra*-group spite more diffused and at the same time reduces the intensity of effort encouraging *inter*-group spite for the rival group.

For small groups, however, an advantage is preserved when *intra*-group spite is sufficiently low, or indeed when there is *intra*-group altruism. For the parameters allowed in the current setup, $s_1 \leq 0$ is a sufficient condition for the smaller group to be the more likely winner. That is, *intra*-group altruism is an effective sentiment for the group to be united in effectively exerting collective effort even when *inter*-group spite may substantially motivate the larger group.[11]

Finally, to better understand the emergence of this "combined" spite condition in Proposition 8, recall first, the utility decomposition of (4.8) and (4.9), and second, the seminal

[10] Olson (1965) also discusses higher organizational costs that are often associated with larger groups.

[11] Note however, while *intra*-group altruism works in favour of smaller groups' winning probabilities, the specific condition of $s_1 \leq 0$ should not be read too literally. In an alternative model setup, being weakly *intra*-group altruistic might be not sufficient for offsetting strong *inter*-group spite.

group contest study by Katz et al. (1990) where the contested prize is a pure public good for the winning group, although valuation can be group specific. A well-received insight of Katz et al. (1990) is that a group's equilibrium winning probability is independent of group size and proportional to the group's valuation. Comparing the "effective" valuations of the two groups, $\left({}^{(1-s_1)}/_{n_s} + {}^{s_2}/_{n_l}\right) V$ and $\left({}^{(1-s_1)}/_{n_l} + {}^{s_2}/_{n_s}\right) V$, it is clear that when $1 - s_1 = s_2$ the two are equal and following Katz et al. (1990) the two groups are equally likely to win. If, however, $1 - s_1 > s_2$ the former is larger than the latter and hence group A is more likely to win the contest. Similarly, we can easily verify the rest of Proposition 8. Moreover, when both s_1 and s_2 approach 0, the game becomes a standard group contest where players are purely self-interested. In this case, as ${}^{1}/_{n_s} > {}^{1}/_{n_l}$, the smaller group is more likely to win the contest just as the GSP would predict.

4.6 Concluding Remarks

The present study was designed to provide a new explanation for the contradiction between the GSP and the results of experimental studies. In this paper, we have explored players' choices of action in group contests in which players are spiteful or altruistic towards other players. We build a model where the prize is shared in the egalitarian rule and the cost function is linear, this is consistent with the situation that Olson (1965) set and is the common setting in experimental studies. We find that the winning probability of the larger group is higher, if the aggregate weight of spite that a player places on the average material payoff among the other players in the same group with him and on the average material payoff among players in the rival group is higher than 1. This finding overturns the GSP. However, the smaller group is more likely to win even if players exhibit a very low level of *intra*-group altruism.

This project is the first comprehensive investigation of the effect of *inter*-group spite combined with *intra*-group spite/altruism on the occurrence of the GSP. Our findings have gone some way towards enhancing our understanding of why the GSP stands in contrast to the data from group contest experiments and provide a basis for further research on the GSP. However, the major limitation of this study is that we only examine symmetric contests. In addition, another limitation lies in the assumption that the values of s_1 and s_2 are treated as exogenous parameters, while it is more likely that they are endogenous in reality.

4.7 Appendix

Proof of Proposition 6

Proof: The process of solving for the Nash equilibrium goes as follows. First, we identify the first-order conditions for individual players in the two groups respectively. Second, these first-order conditions determine a unique pair of equilibrium *group performances*. After applying within-group symmetry, the candidate equilibrium levels of individual effort emerge. Finally, we investigate the second-order conditions at this candidate equilibrium and establish that all players play the best response to other players' equilibrium strategy. This process demonstrates the uniqueness of such a within-group symmetric Nash equilibrium.

i) Taking first order derivative of a group A player's utility function (4.4) with respect to x_i yields

$$\frac{\partial U_i}{\partial x_i} = \frac{n_l\left((1-s_1)YV - n_s(x_i+X_{-i}+Y)^2\right) + n_s s_2 YV}{n_s n_l\,(x_i+X_{-i}+Y)^2}. \tag{4.13}$$

ii) Simplify (4.13) we have

$$\frac{\partial U_i}{\partial x_i} = \frac{(n_s s_2 + (1-s_1)n_l)YV}{n_l n_s (x_i+X_{-i}+Y)^2} - 1. \tag{4.14}$$

Applying the same procedure to the utility function of a group B player, we have

$$\frac{\partial U_j}{\partial y_j} = \frac{(n_l s_2 + (1-s_1)n_s)XV}{n_l n_s (X+y_j+Y_{-j})^2} - 1. \tag{4.15}$$

Treating $x_i + X_{-i}$ as X and $y_j + Y_{-j}$ as Y, and solving (4.14) and (4.15) simultaneously, we obtain a unique pair of group performances for the candidate Nash equilibrium, X^* and Y^*:

$$X^* = \frac{(n_s s_2 + (1-s_1)n_l)^2 (n_l s_2 + (1-s_1)n_s)V}{(1-s_1+s_2)^2 (n_l+n_s)^2 n_s n_l}, \tag{4.16}$$

$$Y^* = \frac{(n_l s_2 + (1-s_1)n_s)^2 (n_s s_2 + (1-s_1)n_l)V}{(1-s_1+s_2)^2 (n_l+n_s)^2 n_s n_l}. \tag{4.17}$$

We note that as $s_1 \in (-1, 1)$ and $s_2 \geq 0$ both X^* and Y^* are positive. Essentially, the $n_s + n_l$ first order conditions characterized by (4.14) and (4.15) reduce to

$x_i^* = X^* - X_{-i}^*$ for $i = 1, \ldots, n_s$, $y_i^* = Y^* - Y_{-i}^*$ for $j = 1, \ldots, n_l$, with X^* and Y^* being given in (4.16) and (4.17).

By imposing within-group symmetry, we can divide (4.16) and (4.17) by n_s and n_l respectively. We then immediately have (4.6) and (4.7) in Proposition 6 which identify equilibrium individual effort levels in the two groups, respectively.

iii) The second order derivative facing a group A player can be obtained by taking the derivative of (4.13) with respect to x_i once more:

$$\frac{\partial^2 U_i}{\partial x_i^2} = -\frac{2\left(\left(1-s_1\right)n_l + n_s s_2\right)YV}{n_l n_s \left(x_i + X_{-i} + Y\right)^3} < 0.$$

Similarly, the second-order derivative facing a group B player is

$$\frac{\partial^2 U_j}{\partial y_j^2} = -\frac{2\left(n_l s_2 + (1-s_1)n_s\right)XV}{n_s n_l \left(y_j + Y_{-j} + X\right)^3} < 0.$$

Hence, given that other players are playing their equilibrium strategy, all players play the best response. Moreover, as these utility functions are globally concave in players' own strategies and the solution to (4.14) and (4.15) is unique, there exists no other within-group symmetric pure strategy equilibrium.

Q.E.D.

Chapter 5

Conclusions

This thesis investigates individuals' behaviour within Tullock group contests, placing particular emphasis on how behavioural economic concepts influence contest outcomes, such as the probability of winning. The findings presented in this thesis challenge traditional results, offering new insights into the reasons behind these discrepancies. More specifically, the research provides novel explanations for observed phenomena such as over-expenditure and the GSP. It also offers an understanding of why the number of groups in many real-life contests is typically restricted.

After providing a general introduction to the thesis, Chapter 2 critically reviews the existing theoretical papers on Tullock group contests. It presents a comprehensive review, specifically focusing on three significant research topics: group formation, optimal group design, and interdependent preferences. As illustrated in section 2.2, the objective functions for players are largely influenced by essential elements such as the group impact function, the contest success function, the cost function, the sharing rule, and the symmetry or asymmetry of the contest. Beyond these primary elements, an array of additional factors also shapes contest outcomes. These include player preferences, risk attitudes, access to information, and endogenous entry. Within the scope of group formation, the literature survey in chapter 2 furnished a multitude of insights into the processes through which groups form and the variables affecting these dynamics. Notable findings indicate that elements such as the number of players, the sharing rules, and the types of membership can considerably impact group formation. Two dominant trends emerged: a general tendency among individuals to avoid forming groups with other players, and a high frequency of investigation into grand coalitions. When discussing optimal group design, the literature suggests that the ideal structure of a group heavily relies on the specific context and goals of the contest. Factors such as increased heterogeneity between groups and a reduction in group size can potentially enhance aggregate expenditure. Lastly, in the realm of interdependent preferences — integral to numerous socio-economic phenomena such as altruism, inequality aversion, and social comparison — the role of these preferences in group contests is pivotal. They reconcile discrepancies between theoretical predictions and experimental observations. Theoretical predictions, assuming players are purely self-interested, often conflict with experimental observations of behaviours such as over-expenditure, cooperative punishment, and reciprocity.

In relation to over-expenditure behaviour compared to the Nash equilibrium, this thesis provides explanations from two perspectives: ambiguity and interdependent preferences. Firstly, when players perceive ambiguity regarding the strategies of other players, both within their team and from opposing teams, the equilibrium level is always greater than the Nash prediction if players exhibit sufficient aversion to ambiguity - that is, if the degree of ambiguity aversion exceeds a certain threshold. This is due to the direct influence of changes in the degree of ambiguity aversion on player behaviour, which is reflected through the weighting assigned to the best and worst possible scenarios. A higher degree of ambiguity aversion implies that a player's utility is more heavily influenced by the worst possible scenario and less so by the best possible scenario. In the worst possible scenario, the best response surpasses the Nash equilibrium when the degree of ambiguity aversion exceeds the particular threshold. Furthermore, when the degree of ambiguity aversion is high enough, the equilibrium effort increases with the degree of ambiguity. This is because the weight placed on the worst possible scenario increases, while the weight placed on the standard Tullock contests scenario decreases. Secondly, if players display either *intra*-group altruism or *inter*-group spite, they will exert more effort. An increase in effort from an *intra*-group altruistic player boosts their group's chance of winning, and compared to a purely self-interested player, an altruistic one derives greater benefits from the same increase in the effort because it increases the material gains of their fellow group members. Similarly, a player showing spite towards opponents receives more benefits from the same additional effort than would a purely self-interested player.

The concepts of ambiguity, spite, and altruism are established as factors capable of reducing social waste in the game. First, the results consistently reveal that individual effort decreases with a decrease in ambiguity aversion. This is due to the fact that as ambiguity aversion decreases, the weight allocated to the worst possible outcome in the objective function decreases, whereas the weight assigned to the best possible scenario correspondingly increases. This dynamic amplifies the importance of the optimal response in the best possible scenario where players expend no effort, while simultaneously diminishing the relevance of the worst possible scenario. Furthermore, when ambiguity aversion is already relatively high, the equilibrium effort level decreases with a decrease in the degree of ambiguity. Conversely, if ambiguity aversion is moderately low, the equilibrium effort level decreases when the degree of ambiguity increases. Additionally, this thesis establishes that the level of equilibrium effort invariably decreases with an increase in the number of groups and group size, irrespective of whether ambiguity is present or not. Secondly, a greater degree of *intra*-group spite or a lower degree of *inter*-group spite can reduce the equilibrium effort. This can be rationalized by understanding that a higher degree of *intra*-group spite or a lower degree of *inter*-group spite decreases the effective valuation of the prize.

A recurring theme in prior literature on optimal group design suggests that total expenditure in rent-seeking activities tends to rise with an increase in the number of participating groups, as outlined in Nitzan (1994) and Baik (2016). However, this dynamic may change in the presence of ambiguity. Depending on the upper limit of the effort, the aggregate effort could potentially decrease with a growing number of groups. If this upper limit of effort falls within a particular range, the total aggregate effort, in the worst-case scenario, reduces with an increase in the number of groups. Therefore, if the weight on the worst-case scenario is substantial — that is, if the degree of ambiguity and ambiguity aversion is high enough — the aggregate effort may decrease as the number of groups increases.

By incorporating the traditional framework of group contests as described in Lee (1993) and adding the additional factor of interdependent preferences, this thesis also concludes that the GSP can be overturned when the "combined" *intra*-group and *inter*-group spite is relatively strong. It is often observed that larger groups grapple with the free-rider problem to a greater extent than smaller groups due to the nature of group activities. Furthermore, as the rewards of victory are shared among more members in larger groups, individual players in these groups tend to value the prize less than those in smaller groups. This often results in larger groups being less effective at accomplishing their goals than their smaller counterparts. However, when spiteful preferences are factored in, the balance may tip in favour of larger groups. A larger group size can dilute the impact of effort-discouraging *intra*-group spite, while simultaneously diminishing the force of effort-encouraging *inter*-group spite exerted by competing groups.

The findings of this research have substantial implications for contest designers seeking to optimize group expenditure. First, contest designers can select individuals with specific attitudes towards ambiguity, according to Shupp et al. (2013), the preferences of subjects regarding ambiguity can be determined by presenting individuals with a choice between an ambiguous option and a safe option. It is designed in a way that subjects who are more averse to ambiguity would select the safe options more frequently, and would also switch to a safe option sooner than those less averse to ambiguity. The degree of perceived ambiguity can be controlled through strategic manipulation of factors such as information access, cultural elements, and communication channels. Additionally, by influencing players' social identity, interpersonal relationships, and reward structures, designers can manage levels of *intra*- and *inter*-group spite and *intra*-group altruism. Secondly, these insights can be used strategically by designers who wish to favour a specific group. Depending on the combined level of *inter*- and *intra*-group spite, they can manipulate the size of the rival group to achieve the desired outcome. Similarly, individuals looking to join a winning group can use these findings to make an informed decision about which group to join—either larger or smaller—based on the parameters established in this study. Lastly, the research also provides insights into determining optimal group structures to maximize aggregate effort. The study suggests that depending on factors like group sizes and the upper limit on the group, the most advantageous group structure could be either having only two groups or having as many groups as possible. Therefore, contest designers could potentially use these findings to structure their contests in ways that promote desired outcomes and behaviours.

While this paper contributes insights into the dynamics of group contests, it is necessary to acknowledge its limitations. Firstly, despite the rich body of literature available on group contests, the focus of this thesis was limited to a specific subset of these studies. As a consequence, not all factors that potentially influence the outcomes of group contests were incorporated into the discussion, representing a limitation of this work. Secondly, there is widespread evidence of heterogeneous behaviour within and between groups, as demonstrated in empirical studies, such as Abbink et al. (2010) and Sheremeta (2011b). Therefore, it may be essential to consider the existence of asymmetric equilibrium instead of concentrating exclusively on symmetric equilibrium as presented in this thesis. Thirdly, certain assumptions in the models used could be made more flexible. For instance, factors such as the degree of ambiguity, the degree of ambiguity aversion, the level of *intra*-group altruism, and the degree of *intra*- and *inter*-group spite could be treated as endogenous variables. They could also be allowed to vary both between groups and within the same group. By addressing these aspects, future research could potentially present a more nuanced view of group contests. By taking these aspects into consideration, future research could potentially offer a deeper exploration of group contests.

Future research should continue to explore the findings and implications of this thesis. Firstly, an exploration of the GSP auction within scenarios where players perceive ambiguity could be insightful. Specifically, examining how the presence of ambiguity influences the winning probabilities between larger and smaller groups could yield significant findings. Secondly, an examination of the relationship between interdependent preferences and ambiguity might reveal underlying dynamics, potentially identifying how these two factors mutually influence each other. In addition, it could empirically test the findings presented in this thesis. Experimental studies could be conducted to validate and expand upon the insights derived from the theoretical model proposed in this research. These investigations could examine whether subjects' behaviours align with the theoretical predictions in the presence of ambiguity or when players exhibit spite or altruism towards other players. As previously discussed, factors such as the degree of spite, aversion to ambiguity, and the level of perceived ambiguity can be effectively managed in a lab setting.

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