ELECTRON DIFFUSION IN GASES

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A THESIS SUBMITTED FOR THE DEGREE OF Ph. D.

AT THE UNIVERSITY OF LIVERPOOL

DECEMBER, 1965

INDEX.

SUMMARY.

The aim of the present work was to repeat D/μ measurements made
by other workers and to extend them into higher regions of E/p. Thus
the diffusion of medium energy electrons in H_2, N_2 and O_2 has been
investigated by swarm experiments in a Townsend-type apparatus. The
E/p ranges considered are 5 to 120, 3 to 30 and 5 to 40 (volts/cm.mm.Hg.)
respectively. Pressures vary approximately from 0.1 to 1.5 mm.Hg.
Agreement with the results of other workers is in general good, but a
pressure dependence was found in N_2 which has been attributed to the
appropriate diffusion equation being non-linear. Results in H_2 and O_2
agree well with other modern values, and values of attachment coefficients
in O_2 are also presented.

Theoretical investigations have yielded solutions of the diffusion
equation in the presence of the secondary effects, electron attachment,
resonance radiation and non-resonance radiation. Townsend's and
Huxley's solutions of the diffusion equation are shown to be similar.

CHAPTER 1.   HISTORICAL DEVELOPMENT OF SWARM EXPERIMENTS.

Diffusion theory originated a century ago with Clerk Maxwell (1867) when he gave a mathematical solution for the interdiffusion of two gases. In 1899, J.S.E. Townsend, one of J.J. Thomson's first students, expanded Maxwell's work to explain the diffusion of charged particles through a gas under the influence of an applied electric field. Townsend devised an apparatus (1908) where electrons passed through a circular aperture and were collected by an anode composed of an inner disc surrounded by an outer ring.

He later devised an ingenious experiment, using a magnetic field to deflect the diffusing electron stream, for measuring drift velocities. These values, together with D/μ values from the radial diffusion measurements, enabled electron mean energy, electron mean free path, electron mobility and electron energy loss in collisions with gas molecules to be readily estimated (Townsend and Tizard 1913). At this time Townsend wrote his classic text-book 'Electricity in Gases', which has served as the foundation for subsequent studies in this, and allied, fields.

During the First World War Townsend was involved in important work on radio communications and it was some years before Townsend and Bailey (1921, 1922, 1923) and Skinker (1922) published their comprehensive sets of results in various gases. Bailey was appointed a professor at Sydney University in 1924

and, after studying Neon (1924), he became involved in attachment studies (1925) and drift velocity measurements (1930).

In 1930, Townsend published a summary of known work entitled 'Energies of Electrons in Gases'. He thereafter became interested in electron energy distributions (Townsend 1936) and allied subjects. 'Electrons in Gases', written by Townsend, appeared in 1948. An account of the early measurements and others is given by Healey and Reed (1941), and Llewellyn-Jones (1957) has written an appreciation of Townsend's work.

A much simpler solution of the diffusion equation than that given by Townsend has been given by L. G. H. Huxley (1940). It may be stated that this new solution originated a new interest in swarm experiments, although mathematical mistakes were made in the derivation (Huxley and Crompton 1955) and the correct formula resulted by chance. Huxley and Zaazou (1949) were next to give a set of results, in air, but their apparatus is open to some criticism. Adelaide University, Australia appointed Huxley as a professor and he began to establish a group equipped with apparatuses capable of making accurate measurements on electron swarms. Members of the group, notabl R. W. Crompton, have since published the results of many accurate and consistent experiments in the low E/p range. The group moved from Adelaide in 1960 to form, under Crompton, the Ion Diffusion Unit at Canberra University, Australia.

Amongst Huxley's contributions since then are a general theory of electron and ion motion in gases (1960) and a solution of the diffusion equation when electron ionization and attachment are

Author(s)	Year	Gas	E/p_{min}	E/p_{max}
Townsend	1908	Air	0.09	0.27
Townsend and Tizard	1913	Air	0.5	100
Townsend and Bailey	1921	N_2	0.25	60
		H_2	0.25	50
		O_2	2	50
		Air	0.5	100
Skinker	1922	CO_2	0.25	50
Townsend and Bailey	1922	A	0.125	15
Townsend and Bailey	1923	He	0.013	5
Bailey	1924	Ne	0.06	6
Brose	1925	O_2	0.4	50
Bailey	1925	Air	0.5	2.5
Bailey and Rudd	1932	CO_2	2	16
Healey and Kirkpatrick	1939	O_2	0.25	50
Huxley and Zaazou	1949	Air	0.5	25
Crompton and Sutton	1952	H_2	0.05	20
		N_2	0.05	20
Crompton, Huxley and Sutton	1953	Air	0.1	20
Huxley and Crompton	1955	H_2	0.5	5
Huxley, Crompton and Bagot	1959	O_2	5	20
Cochran and Forester	1962	H_2	0.2	5
		N_2	0.2	5
		CO_2	0.2	5

Table 1.   Published D/μ values (continued over).

Author(s)	Year	Gas	E/p_{min}	E/p_{max}
Warren and Parker	1962	He	0.00015	0.4
		A	0.00015	0.04
		N_2	0.0002	3
		H_2	0.00015	5
		CO_2	0.005	15

When the present work began, no modern results were available for $E/p > 20$ in H_2, N_2 and O_2. Since then some further results have appeared in the literature.

Author(s)	Year	Gas	E/p_{min}	E/p_{max}
Crompton and Elford	1963	N_2	0.006	5
		H_2	0.006	0.1
Rees	1964	CO_2	0.1	50
Rees	1965	O_2	0.4	6
Lawson and Lucas	1965a	H_2	15	100
Crompton, Liley, McIntosh and Hurst	1965	H_2	10	70
Lawson and Lucas	1965b	H_2	60	350

Table 1.   Published D/μ values, continued.

CHAPTER 2.   THEORY.

2.1. DIFFUSION COEFFICIENT.

When a number of particles having agitational energy occupy a
space and have an uneven spatial distribution then in general they
will tend to distribute themselves uniformly throughout this space.
This process is known as 'diffusion', and the rate at which the process
occurs is governed by the 'diffusion coefficient', D.   If there is a
particle concentration gradient in a particular dierection then D is
defined as being the net number of particles passing in unit time
through unit area perpendicular to the gradient direction and for unit
concentration gradient.   In three dimensions this may be written

$$\frac{\partial n}{\partial t} = - D \, grad \, n$$

where n is the particle concentration.

By kinetic theory, $D = \overline{Lc}/3$ where L is electron free path and c
is electron velocity (Huxley 1960).

2.2. DRIFT VELOCITY.

.   When an electron exists in a region between two parallel plates
together with gas molecules and an electric field is applied, the
electron receives energy and begins to collide with the gas molecules.
Most of its motion is in random directions but it also has a relatively

small velocity in the field direction. This velocity is known as the 'drift velocity'. i.e. drift velocity W, is given by

$$W = \frac{\text{gap length}}{\text{electron transit time}}$$

Kinetic theory gives W in terms of microscopic parameters as

$$W = \frac{Ee}{3m} \cdot \overline{c^{-2} \frac{d}{dc}(Lc^2)} \qquad \text{(Huxley 1960)}.$$

Electron mobility is defined as drift velocity per unit electric field. i.e. $\mu = W/E$.

2.3.   TOWNSEND'S ENERGY FACTOR AND MEAN ENERGY.

The ratio of drift velocity and diffusion coefficient in terms of microscopic parameters (Huxley 1960) is written

$$\frac{W}{D} = \frac{Ee}{m} \cdot \overline{c^{-2} \frac{d}{dc}(Lc^2)} \cdot \frac{1}{Lc} \qquad \text{———(1)}$$

$$= \frac{3}{2} \cdot \frac{Ee}{\frac{1}{2}m\overline{c^2}} \qquad \text{(for a Maxwellian distribution)}$$

$$= \frac{3}{2} \cdot \frac{Ee}{\overline{u}} \text{ , where } \overline{u} \text{ is the mean electron energy.}$$

A more convenient form is $D/\mu = \frac{2}{3}\overline{u}$.

When positive or negative ions diffuse and drift through a gas at low E/p they remain in thermal equilibrium with the gas molecules and it may be written

$$\overline{u} = \tfrac{1}{2}\,m\overline{c^2} = \tfrac{3}{2}\,kT$$

where k = Boltzmann's constant, T = gas temperature (absolute)

= ion temperature.

When electrons drift through a gas, in general their energies are higher than those of the gas molecules and Townsend introduced an energy factor, k_T, which is defined as the number of times that electron mean energy is greater than thermal energy,

i.e. $\overline{u} = k_T\,(3/2\ kT)$,

and $D/\mu = k_T\,(kT/e)$. ————(2)

From (2), $k_T = D/\mu \times$ constant, for a fixed temperature. The values of the constant are 40.3 at 15°C., 39.6 at 20°c., and 39.0 at 25°C., when the units of D/μ are electron-volts.

(2) is derived from (1) assuming a Maxwellian velocity distribution. If a Druyvesteyn distribution is assumed, (2) becomes

$$D/\mu = 1.14\ k_T\,(kT/e).$$

2.4. OTHER PARAMETERS OF ELECTRON COLLISIONS.

Once D/μ values have been obtained, a knowledge of drift velocity, W, either from other workers' results or from a magnetic deflection experiment on the swarm, uncovers a variety of information (Huxley and Zaazou 1949).

The root mean square electron velocity is given immediately by

$$\overline{c} = 7.24 \times 10^7\ (D/\mu)^{\frac{1}{2}}\ \text{cm/sec}\quad \text{(Maxwell)}$$

where D/μ is in electron-volts,

$$\text{or } \bar{c} = 6.30 \times 10^7 \ (D/\mu)^{\frac{1}{2}} \ \text{cm/sec} \quad \text{(Druyvesteyn)}.$$

The mean free path at unit pressure (1 mm.Hg.) is given by

$$L = 4.41 \times 10^{-8} \ \frac{W(D/\mu)^{\frac{1}{2}}}{E/p} \ \text{cm} \quad \text{(Maxwell)}$$

$$L = 4.59 \times 10^{-8} \ \frac{W(D/\mu)^{\frac{1}{2}}}{E/p} \ \text{cm} \quad \text{(Druyvesteyn)}.$$

The mean proportion of energy lost by an electron per collision is given by

$$f = 4.51 \times 10^{-16} \ W^2/(D/\mu) \quad \text{(Maxwell)}$$

$$f = 4.23 \times 10^{-16} \ W^2/(D/\mu) \quad \text{(Druyvesteyn)}.$$

The above equations assume a gas temperature of 20°C.

2.5. RELATION BETWEEN DRIFT VELOCITY AND DIFFUSION COEFFICIENT.

If a stream of electrons enters a parallel plate gap through a small hole in the cathode, then the electrons, under the influence of an electric field, drift and diffuse through the gap until they are collected by the anode. Between collisions, an electron will gain energy from the electric field, and at a collision it will lose a small proportion of its energy to the gas molecule concerned. Equilibrium is established between these two processes, and the electrons in the swarm attain a characteristic mean energy. The spreading of the electron stream is due to the electron diffusion, and since the time during which this process is acting is inversely proportional to the drift velocity W, then the radial concentration

distribution of the electrons at the anode (the only plane of interest) is dependent on the ratio, W/D.

Thus W/D, or λ (=W/2D) as is commonly used in the literature, is a direct measure of the spread of the electron stream (and indirectly a measure of the electron mean energy).

2.6. THE DIFFUSION EQUATION.

The diffusion equation, representing electron movement through a gas, in its simplest form may be formulated by considering the electron currents passing through the two planes, z and z+dz. The electrons are assumed to have a fixed energy distribution. Considering firstly the currents due to electron drift, the number passing through unit area of the z plane in unit time is nW, and the corresponding number through the z+dz plane is $nW + \frac{d}{dz}(nW)dz$. Similarly the diffusion currents are, through the z plane, $-D\, dn/dz$, and through the z+dz plane, $-D\frac{dn}{dz} - \frac{d}{dz}\left[D\frac{dn}{dz}\right]dz$.

There can be no accumulation of particles between the planes so that, equating the number passing through each plane,

$$nW - D\frac{dn}{dz} = nW + \frac{d}{dz}(nW)dz - D\frac{dn}{dz} - \frac{d}{dz}\left[D\frac{dn}{dz}\right]dz$$

or, $\quad \frac{d^2n}{dz^2} = \frac{W}{D}\frac{dn}{dz}.$

This is the diffusion equation in one dimension. Extension of the above to three dimensions (remembering that drift is only in the

z direction and diffusion is 3-dimensional) yields the equation

$$\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} + \frac{\partial^2 n}{\partial z^2} = \frac{W}{D} \frac{\partial n}{\partial z}$$

or, $$\nabla^2 n = \frac{W}{D} \frac{\partial n}{\partial z} = 2\lambda \frac{\partial n}{\partial z} . \qquad \text{——(3)}$$

It remains to solve this equation for the usual form of discharge gap for diffusion studies. That is, a gap consisting of circular parallel plane electrodes with the electron current entering through an aperture at the cathode centre and being collected by an anode composed of concentric circular annuli. The gap may be surrounded by a series of uniformly spaced guard rings.

2.7. TOWNSEND'S SOLUTION (1915) OF THE DIFFUSION EQUATION IN CYLINDRICAL COORDINATES.

Consider a parallel plate gap, length d, with metal walls at a radius b from the axis of the gap, the axis passing through the centre of an aperture in the cathode. Suppose this aperture to be of radius a, and its centre to be the origin of the cylindrical coordinate system.

The diffusion equation (3) of the last section in cylindrical coordinate is

$$\frac{1}{r} \frac{\partial}{\partial r}\left[r \frac{\partial n}{\partial r}\right] + \frac{\partial^2 n}{\partial z^2} = 2\lambda \frac{\partial n}{\partial z} \qquad \text{——(4)}$$

where $\lambda = W/2D$.

Let $n = \phi e^{-uz}$; where $\phi = \phi(r,z)$ and $u = u(r)$. Then (4) becomes

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} = -(u^2 + 2\lambda u)\phi = -\theta^2 \phi .$$

where $u = -\lambda \pm (\lambda^2 + \theta^2)^{\frac{1}{2}}$. ————(5)

This is Bessel's equation of order zero, and its solution is written

$$\phi = A J_o(\theta r) + BY_o(\theta r)$$

where A and B are arbitrary constants. Since n and thus ϕ must be finite at $r = 0$, then B=0, and a particular solution for the electron concentration n may be written, $n = AJ_o(\theta r) e^{-uz}$. Summing all possible solutions gives a general solution

$$n = \sum_{s=1}^{\infty} A_s J_o(\theta_s r) \exp(-u_s z) .$$ ————(6)

The unknown constants A_s and θ_s are found by making the above equation satisfy the boundary conditions. These are (according to Townsend):

(i) n is zero at the walls,

i.e. $n = 0$ for $r = b$, all z.

(ii) n is constant across the cathode aperture and zero over the rest of the cathode,

i.e. $n = n_o$ for $0 \leqslant r \leqslant a$, $z = 0$

and $n = 0$ for $a < r \leqslant b$, $z = 0$.

Condition (i) is satisfied by taking $J_o(\theta_s b) = 0$. i.e. the $\theta_s b$, which are always positive, are roots of the equation $J_o(x) = 0$. The corresponding values of u_s are given from (5) by $u_s = (\lambda^2 + \theta_s^2)^{\frac{1}{2}} - \lambda$,

the positive root of the quadratic equation being taken so that n
remains finite as $z \rightarrow \infty$.

Condition (ii) gives the values for A_s. Putting $z = 0$,
multiplying both sides of (6) by $J_0(\theta_s r)r\,dr$ and integrating from
$r = 0$ to $r = b$, A_s is given by

$$\int_0^b n\,J_0(\theta_s r)r\,dr = A_s \int_0^b J_0^2(\theta_s r)r\,dr.$$

Use is made of the relation

$$\int_0^b J_0(\theta_m r)\,J_0(\theta_n r)r\,dr = 0, \qquad m \neq n,$$

$$= \frac{b^2}{2}\,J_1^2(\theta_m b), \qquad m = n,$$

when $\theta_m b$ and $\theta_n b$ are roots of $J_0(x) = 0$. Simplifying the right
hand side now gives

$$\frac{an_0}{\theta_s}\,J_1(\theta_s a) = A_s\,\frac{b^2}{2}\,J_1^2(\theta_s b)$$

where $J_1(x)$ is a Bessel function of the first kind. Thus,

$$A_s = \frac{2an_0}{\theta_s b^2}\,\frac{J_1(\theta_s a)}{J_1^2(\theta_s b)}$$

and the spatial electron concentration satisfying the boundary
conditions (i) and (ii) and the diffusion equation is given by

$$n = \sum_{s=1}^{\infty} \frac{2an_0}{b^2}\,\frac{J_1(\theta_s a)}{\theta_s\,J_1^2(\theta_s b)}\,J_0(\theta_s r).\exp(-u_s z)$$

$$\text{-----(7)}$$

The ratio, R, of current collected by an anode area of radius p to that collected by an area of radius q is found by integrating n over the anode surface.

$$\text{i.e.} \quad R = \frac{\int_q^p 2\pi r \, n \, dr}{\int_0^q 2\pi r \, n \, dr} \, .$$

$$\frac{\displaystyle\sum_{s=1}^{\infty} \frac{J_1(\theta_s a)}{\theta_s^2 J_1^2(\theta_s b)} \, J_1(\theta_s p) \, p \, \exp(-u_s d)}{\displaystyle\sum_{s=1}^{\infty} \frac{J_1(\theta_s a)}{\theta_s^2 J_1^2(\theta_s b)} \, J_1(\theta_s q) \, q \, \exp(-u_s d)} \qquad\qquad \text{-- (8).}$$

Townsend, in his original apparatus, made the centre collecting disc and the cathode aperture the same diameter (0.75 cm, compared with a gap length of 7 cm). This facilitates computation.

2.8. HUXLEY'S SOLUTION (1959) OF THE DIFFUSION EQUATION.

If in the diffusion equation (3)

$$\nabla^2 n = 2\lambda \frac{\partial n}{\partial z}$$

it is written

$$n = V \, e^{\lambda z}$$

then the equation satisfied by V is

$$\nabla^2 V = \lambda^2 V \, .$$

This equation is similar to an equation in wave-propagation theory, and its solution may be represented as a simple pole source,

i.e. $V = \text{constant} \times e^{-\lambda r}/r,$

where, in this case, r is the distance from the source.

The solution of the diffusion equation for electron concentration (Huxley 1940, 1959) is obtained by placing a positive electron source at the centre of the cathode. The walls are assumed to be at a large distance from the axis of the discharge gap. In order to satisfy the anode boundary condition of zero concentration over its surface (z = d), a negative image source at z = 2d of suitable strength is added . The equation for n then becomes

$$n = Ae^{\lambda z}(e^{-\lambda r}/r - e^{-\lambda r'}/r')$$

where r' is the distance measured from the negative image source. At the anode r = r', and n = 0. The currents at the anode will be diffusion currents, and the anode current distribution is found by making the integral

$$i = \int \frac{\partial n}{\partial z} \rho \, d\rho ,$$

over the anode surface.

The current to a disc of radius ρ from the anode centre is then given by

$$i = i_o (1 - \frac{d}{r} e^{\lambda(d-r)})$$

where $r^2 = \rho^2 + d^2$.

The ratio R of the current to radius ρ to the total current collected by the anode is thus given by

$$R = 1 - \frac{d}{r} e^{\lambda(d-r)} . \qquad \text{———(9)}$$

2.9.   EQUIVALENCE OF TOWNSEND'S AND HUXLEY'S SOLUTION.

The analysis of the two previous sections (2.7 and 2.8) shows that the solution of the diffusion equation may be obtained in two dissimilar ways. Each solution satisfies only one electrode boundary condition, the anode in Townsend's solution and the cathode in Huxley's solution. The analysis given below shows the two solutions to be, in fact, identical. This is a satisfying result not previously appreciated by other workers in this field.

Townsend's solution for the electron concentration at the anode is

$$n = \sum_{s=1}^{\infty} \frac{2an_o}{b^2} \frac{J_1(\theta_s a)}{\theta_s J_1^2(\theta_s b)} \quad J_0(\theta_s r) \; e^{-u_s d}$$

where $u_s = (\lambda^2 + \theta_s^2)^{1/2} - \lambda$, and $J_0(\theta_s b) = 0$.

It is desired to simplify this solution for a point source and infinite walls i.e. $a \rightarrow 0$, $b \rightarrow \infty$. As $b \rightarrow \infty$, $\theta_s \rightarrow 0$, $u_s \rightarrow 0$, and $J_0(\theta_s r) \rightarrow 1$. At small s, the terms in the above series tend to zero. The terms only become significant when θ_s is large enough to make $J_1(\theta_s b)$ small. Thus only the later terms in the series need be considered.

As $a \rightarrow 0$, $J_1(\theta_s a) \rightarrow \dfrac{\theta_s a}{2}$ and the concentration becomes

$$n = \sum_{s=1}^{\infty} \frac{a^2 n_o}{b^2} \frac{J_0(\theta_s r)}{J_1^2(\theta_s b)} \; e^{-u_s d} \quad .$$

When s is large, $J_1^2(\theta_s b) \simeq 2/\pi \, \theta_s b$. In the limit, the above series may be written as an infinite integral,

$$\text{i.e.} \quad n = \int_0^\infty \frac{a^2 n_0}{b} \, \theta_s \, J_0(\theta_s r) \, e^{\left[\lambda - (\lambda^2 + \theta_s^2)^{1/2}\right] d} \, d\theta_s \ .$$

The total current, I, to a ring of radius r is found by multiplying by the drift velocity W and integrating over the anode surface.

The total current at the pinhole is given by

$$i_0 = n_0 W \pi a^2 \ .$$

$$I = e^{\lambda d} \int_0^r \int_0^\infty i_0 \, \theta_s r \, J_0(\theta_s r) \, e^{-(\lambda^2 + \theta_s^2)^{1/2} d} \, d\theta_s \, dr \ .$$

$$\frac{d}{d\lambda}\left[\frac{I}{e^{\lambda d}}\right] = \int_0^r \int_0^\infty i_0 \, \theta_s r \, J_0(\theta_s r) \frac{\lambda d \, e^{-(\lambda^2 + \theta_s^2)^{1/2} d}}{(\lambda^2 + \theta_s^2)^{1/2}} \, d\theta_s \, dr \ .$$

Use is now made of a standard solution

$$\text{i.e.} \int_0^\infty J_0(\theta_s r) \frac{\theta_s \, e^{-(\lambda^2 + \theta_s^2)^{1/2} d}}{(\lambda^2 + \theta_s^2)^{1/2}} \, d\theta_s = \frac{e^{-\lambda(r^2 + d^2)^{1/2}}}{(r^2 + d^2)^{1/2}}$$

$$= e^{-\lambda R'}/R'$$

where $R' = (r^2 + d^2)^{1/2}$.

This formula was widely used by A. Sommerfeld and is often called the Sommerfeld relation. Then,

$$\frac{d}{d\lambda}\left[\frac{I}{d \, e^{\lambda d}}\right] = \int_0^r i_0 r \lambda \frac{e^{-\lambda R'}}{R'} \, dr$$

$$= i_0 \left[-e^{-\lambda R'}\right]_0^r$$

$$= i_0 (e^{-\lambda d} - e^{-\lambda R'}) \ .$$

Integrating with respect to λ,

$$I = i_0 \left(1 - \frac{d}{R'} e^{\lambda(d - R')}\right) \ . \qquad\qquad \text{——(10)}$$

This is Huxley's solution (equation (3)) relating the current received by a disc of radius r to the total current received by the anode

2.10. MODIFICATIONS OF HUXLEY'S SOLUTION.

Although the Townsend and Huxley solutions seem correct experimentally, they do not satisfy the electrode boundary conditions exactly. Other workers have attempted to obtain exact solutions.

It is possible to obtain Huxley's relationship (9) by a different procedure (Crompton and Jory 1962). Another solution to the diffusion equation is then

$$n = A \, e^{\lambda z} \, \frac{z}{r} \, \frac{d}{dr} \left[\frac{e^{-\lambda r}}{r} \right] .$$

The current collected by the anode is taken to be equal to the current passing through a plane at $z = d$. This procedure gives exactly Huxley's relation between R and λ but, in common with Townsend's solution, it does not satisfy the anode boundary condition of $n = 0$ over the anode surface. But, also like Townsend's , it does satisfy the cathode boundary condition of zero over the whole cathode surface, except at the electron source. Huxley's solution of the previous section results from a satisfied anode condition but an unsatisfied cathode condition.

Thus the curious situation arises whereby the same formula results from satisfying independently the two boundary conditions. When the two are satisfied together the formula obtained is

$$R = 1 - \left[\frac{z}{r} - \frac{1}{\lambda d} + \frac{d}{r^2 \lambda} \right] \frac{d}{r} \exp(\lambda (d-r)) .$$

Over large experimental regions this formula yields the same results (within experimental error) as Huxley's formula, but at low λd and at relatively large distances from the axis (i.e. small d/r)

there are discrepancies. Huxley and Crompton (1955) and Crompton and Jory (1962) have shown empirically however that the so-called approximate solution

$$R = 1 - \frac{d}{r} e^{\lambda(d-r)}$$

is the more correct solution, whilst the solution satisfying both boundary conditions does not explain experimental results over an adequate range.

Lawson and Lucas (1965) succeeded in solving the diffusion equation and both boundary conditions by considering the time-resolved equation in the presence of primary ionization,

i.e.
$$\frac{1}{D} \frac{\partial n}{\partial t} = \nabla^2 n - \frac{\mu E}{D} \frac{\partial n}{\partial z} + \alpha \mu \frac{E}{D} n ,$$

where the field acts in the z direction.

A set of image charges are considered, of strengths n_s, velocities v_s and positions d_s (Lucas 1964). These values are chosen specifically to satisfy the anode and cathode boundary conditions. The time-resolved electron concentration is then given by

$$n(x,y,z,t) = \sum_{s=1}^{\infty} \frac{n_s e^{\alpha v t}}{(4\pi Dt)^{3/2}} \exp -\left\{ \frac{x^2+y^2+(z-d_s-v_s t)}{4Dt} \right\}$$

The total number of electrons collected by an anode disc of radius b is

$$N(d,b) = \int_0^{\infty} \int_0^{b} \left\{ e(nW) - eD \frac{\partial n}{\partial z} \right\} 2\pi \rho \, d\rho \, dt$$

where $\rho = (x^2+y^2)^{1/2}$ and $W = \mu E$.

Integrating,

$$1 - R = 1 - N(d,b)/N(d, \infty)$$

$$= e^{-u(r-d)} \left\{ \frac{d/r + (\lambda/u - d/r) e^{-2\lambda d} + \ldots}{1 + (\lambda/u - 1) e^{-2\lambda d} + \ldots} \right\}$$

where $\lambda = \mu E/2D$, $u^2 = \lambda^2 - 2\alpha\lambda$ and $r = (d^2+b^2)^{\frac{1}{2}}$. When $\lambda d > 2$, the equation reduces to

$$1 - R = \frac{d}{r} e^{-(r-d)u} . \qquad \text{———(11)}$$

Equation (11) above shows that an experimental determination of R will give a value for u, but the coefficient λ is required to find values of D/μ. In the absence of primary ionization, $u=\lambda$, and (11) reduces to Huxley's equation, (9). When ionization is present $u \neq \lambda$, but the term $(\lambda - u)$ can be determined by measuring the variation of the current amplification $A(z)$ with electrode separation z (Huxley 1959, Lucas 1964). In this case

$$\lambda - u = \frac{1}{a} \log_e \left[\frac{A(x+a)}{A(x)} \right]$$

and

$$\frac{D}{\mu} = \frac{E}{2[u + (\lambda - u)]} . \qquad \text{———(12)}$$

In all the above equations it has been assumed that no current is lost to the walls of the diffusion gap. Comparing ratios found from Townsend's and Huxley's solutions it may be shown that the current collected by a wall, even at quite a small distance from the outer collector, is small except at low values of λd. Fig.12 shows the magnitude of this effect.

2.11.   PARKER'S   CONCENTRATION   DISTRIBUTION.

Parker (1963) has shown theoretically, that the accepted
distribution of electron concentration at the collecting anode is
incorrect.   He arrives at this postulation by considering exactly the
terms in the Boltzmann transport equation involving spatial derivatives
of the electron distribution.   These terms are normally neglected, the
electron energy distribution being taken to be position independent.
The concentration distribution at the anode is then given by

$$n_{approx} = (1+x)^{-1/2} \ exp \left\{ - \lambda d \left[(1+x)^{1/2} - 1 \right] \right\} \ ,$$

where $x = b^2/z^2$, (b = distance from anode centre, z = field direction).
The position dependent distribution as given by Parker is

$$n_{exact} = (1 + x/4)^{-3/2} \ exp(- \lambda dx/2).$$

This expression has been integrated over the anode surface to find
the current distribution.

The ratio, R, of currents to a central ring of radius b, and an
annulus of inner and outer radii b_1 and b_2 is given by

$$R = \frac{f_o - f_1}{f_1 - if_2} \ , \qquad\qquad \text{———(13)}$$

where $\quad f_o = \dfrac{1}{k} - \dfrac{0.375}{k^2} + \dfrac{0.234}{k^3} - \dfrac{0.204}{k^4} + \ \ldots \ ,$

$$f_{1,2} = e^{-p} \left[\frac{1}{k} - 0.375\frac{(p+1)}{k^2} + 0.117\frac{(p^2+2p+2)}{k^3} - 0.034\frac{(p^3+3p^2+6p+6)}{k^4} \right.$$
$$\left. + \ \ldots\ldots \right] \ ,$$

$$k = \lambda d/2, \text{ and } p(b_1,b_2) = kb^2/z^2 = kx \ .$$

The diffusion equation has been extended to cover the cases when electron attachment and secondary ionization by photons at the cathode are present. A knowledge of the quantitative effects of secondary ionization will enable further gases to be investigated and the range of E/p to be extended to include higher values. Photons may travel through a gas in two ways, as resonance radiation or as non-resonance radiation. Both have been considered in the following analysis.

3.1. DIFFUSION IN THE PRESENCE OF ELECTRON ATTACHMENT.

Huxley (1959) has given an account of a theoretical investigation of a stream of electrons and ions drifting and diffusing in a gas when ionization by collision and electron attachment are present. His solution contains definite integrals and does not lend itself to rapid calculation, even with the use of a computer.

A simplified solution, following from Townsend's solution and amenable to quick calculation, is given below.

A steady state is assumed and the electron and negative and positive ion currents will be calculated separately.

(a)   Electron current.

The spatial electron concentration $n_e(x,y,z)$ for electrons

drifting and diffusing under the action of a uniform electric field (direction z) is found from solving the basic diffusion equation

$$\frac{\partial n_e}{\partial z} = (\alpha - \eta)n_e - \frac{D_e \nabla^2 n_e}{\mu_e E} \qquad\qquad (14)$$

where α = primary ionization coefficient,

   η = attachment coefficient,

and μ_e = electron mobility.

The solution of (14) has been given by Lucas (1965). It assumes point emission of electrons from the centre of the cathode and zero electron concentration at the electrodes (z=0 and z=d) and at the walls (r=b) : the sum of three sources is sufficient when $\lambda d > 2$, giving

$$n_e(z,r) = \sum_{s=1}^{\infty} A_s J_0(\theta_s r) \left\{ \exp(\lambda - u_s)z - \exp\left[(\lambda + u_s)z - 2u_s d\right] \right. $$
$$\left. - \exp\left[-(\lambda + u_s)z\right] \right\} \qquad\qquad (15)$$

where $A_s = \dfrac{j_0}{2\pi \mu_e E} \; \dfrac{2}{b^2 J_1^2(\theta_s b)} \dfrac{u}{u_s}$,

$$u^2 = \lambda^2 - 2(\alpha - \eta)\lambda \; ,$$

$$u_s^2 = \lambda^2 - 2(\alpha - \eta)\lambda + \theta_s^2 \; ,$$

$$\lambda = \frac{\mu_e E}{2D_e} \; , \text{ and } J_0(\theta_s b) = 0 \; .$$

The electron current density at any point is then given by the

vector equation

$$j_e(x,r) = j_{ez} + j_{er} \quad . \qquad \text{------(16)}$$

(i) Axial electron current at anode.

The current to a radius r at the anode is given by

$$J_{ez}(d,r) = \int_o^r j_{ez}(d,r)2\pi r dr \quad .$$

Now

$$j_{ez} = n_e \mu_e E - D_e \frac{\partial n_e}{\partial z}$$

and substitution from (15) gives

$$J_{ez}(d,r) = \sum_{S=1}^{\infty} 2\pi \mu_e EA_s \frac{r}{\theta_s} J_1(\theta_s r) \frac{u_s}{\lambda} \exp(\lambda - u_s)d \quad \text{----(17)}$$

for $\lambda d > 2$.

(ii) Axial electron current at cathode.

This current may be written as an expansion of Bessel functions

$$j_o = J_{ez}(0,r) = \sum_{S=1}^{\infty} 2\pi \mu_e EA_s \frac{r}{\theta_s} J_1(\theta_s r) \quad . \qquad \text{------(18)}$$

(iii) Total radial electron current to walls.

The total wall current is written

$$J_{er}(d,r) = \int_o^d j_{er} 2\pi b \, dz \quad .$$

Making use of $\quad j_{er} = D_e \frac{\partial n_e}{\partial r} \quad ,$

equation (15) gives

$$J_{er}(d,r) = \sum_{s=1}^{\infty} 2\pi \mu_e E A_s \frac{r}{\theta_s} J_1(\theta_s r) \frac{\theta_s^2}{\lambda^2 - u_s^2} \left\{ \frac{u_s}{\lambda} e^{(\lambda - u_s)d} - 1 \right\} \cdot$$

$$\text{———————(19)}$$

(b) Ion currents.

Equations (14) and (15) may be combined to give the electron current as

$$\text{div } j_e(z) = (\alpha - \eta)n_e \mu_e E \cdot$$

Likewise similar equations may be obtained for the positive and negative ions,

$$\text{div } j_+(z) = \alpha n_e \mu_e E,$$

and

$$\text{div } j_-(z) = \eta n_e \mu_e E \cdot$$

These equations combine to give

$$\text{div } \left[\frac{j_+}{\alpha} - \frac{j_-}{\eta} \right] = 0 \qquad \text{————————(20)}$$

and

$$\text{div } (j_e - j_+ + j_-) = 0 \cdot \qquad \text{————————(21)}$$

Generally, ions have much lower energies than electrons in the present experiments, and it is justified to assume that their diffusion in a radial direction is negligible. Applying Stokes' theorem to a region bounded by the walls, the anode and the cathode gives

$$J_{ez}(d,r) + J_{er}(d,r) - J_{ez}(0,r) + J_{-}(d,r) + J_{+}(0,r) = 0 . \text{————}(22)$$

electron	total	electron	negative	positive
current	electron	current	ion	ion
at anode	wall	at	current	current
	current	cathode	at anode	at cathode

This condition gives the anode negative ion current and the cathode positive ion current in terms of the electron currents.

From equation (20),

$$J_{-}(d,r) = - \frac{\eta}{\alpha} J_{+}(0,r) .$$

Substitution into (22) gives

$$J_{-}(d,r) \frac{(\alpha - \eta)}{\eta} = J_{ez}(d,r) + J_{er}(d,r) - J_{ez}(0,r)$$

and from equations (17) and (19),

$$J_{-}(d,r) = \frac{\eta}{\alpha - \eta} \sum_{s=1}^{\infty} 2\pi \mu_e E A_s \frac{r}{\theta_s} J_1(\theta_s r) \cdot \left\{ \left[\frac{u_s}{\lambda} \left[e^{(\lambda - u_s)d} \right] - 1 \right] \right.$$
$$\left. + \frac{\theta_s^2}{(\lambda^2 - u_s^2)} \left[\frac{u_s}{\lambda} e^{(\lambda - u_s)d} - 1 \right] \right\} ,$$

or

$$J_{-}(d,r) = \sum_{s=1}^{\infty} 2\pi \mu_e E A_s \frac{r}{\theta_s} J_1(\theta_s r) \frac{2\eta\alpha}{\left[2(\alpha - \eta) - \theta_s^2 \right]} \left\{ \frac{u_s}{\lambda} e^{(\lambda - u_s)d} - 1 \right\}$$

$$\text{————}(23)$$

The total electron and negative ion current at the anode is

$$J(d,r) = J_{ez}(d,r) + J_-(d,r).$$ ———————————(24)

Substituting the results of equations (17) and (23) into equation (24) gives

$$J(d,r) = \sum_{s=1}^{\infty} 2\pi \mu_e E \, A_s \, \frac{r}{\theta_s} \, J_1(\theta_s r) \left\{ \frac{u_s}{\lambda} e^{(\lambda - u_s)d} \right.$$

$$\left. + \frac{2\eta\lambda}{2(\alpha - \eta)\lambda - \theta_s^2} \left[\frac{u_s}{\lambda} e^{(\lambda - u_s)d} - 1 \right] \right\}$$

$$= \sum_{s=1}^{\infty} \frac{2j_0 J_1(\theta_s r)}{b^2 J_1^2(\theta_s b)} \, \frac{u}{u_s} \, \frac{r}{\theta_s} \left\{ \frac{u_s}{\lambda} e^{(\lambda - u_s)d} \right.$$

$$\left. + \frac{2\eta\lambda}{2(\alpha - \eta)\lambda - \theta_s^2} \left[\frac{u_s}{\lambda} e^{(\lambda - u_s)d} - 1 \right] \right\}$$ ———(25)

where $\lambda = \mu_e E / 2D_e$,

$$u_s^2 = \lambda^2 - 2(\alpha - \eta)\lambda + \theta_s^2 ,$$

$$u^2 = \lambda^2 - 2(\alpha - \eta)\lambda ,$$

b = wall radius,

and $J_0(\theta_s b) = 0$.

Thus the current distribution at the anode is given in the presence of primary ionization and electron attachment. If allowance is to be made for a finite diameter cathode aperture, then each term in the series is multiplied by a factor, $2J_1(\theta_s a)/\theta_s a$, where a = aperture radius. This assumes a constant electron concentration over the cathode aperture.

3.2. RESONANCE RADIATION.

When electrons drift through a gas they will be continuously colliding with gas molecules. Occasionally an electron in collision with a gas molecule will excite one of the atoms of the molecule and radiation will be emitted of a frequency appropriate to the difference in electron energy levels between the excited state and the ground state in the particular atom concerned. The emitted radiation travels through the gas in a random direction before contacting another molecule, and again excites one of its atoms before re-emitting. Thus the resonance radiation process consists of a continual emission and absorption of radiation by the gas molecules, and, by virtue of the random direction of emission, it may be considered as a diffusion process (Little 1959). The time taken for a photon to be absorbed and re-emit is of the order of 10^{-9} seconds, whilst the time spent between emission and absorption may be neglected. In the present apparatus the photon transit time between electrodes is about 10^{-5} seconds, so the absorption process occurs about 10^4 times in one transit.

The movement of resonance radiation, having been described as a diffusion process, may now be represented by a diffusion equation. If n is the photon concentration and D the photon diffusion coefficient, then

$$D \nabla^2 n - n/\tau + \alpha_p v_e n_e = 0 , \qquad\qquad (26)$$

where n/τ represents the sum of the rate at which photons are lost

by absorption with foreign gases and impurities, and the rate at which the emitting atoms may be transferred to some other radiating state. α_p is the number of photons produced by electron collision per cm. electron travel in the field direction, and v_e is electron drift velocity.

The electron concentration n_e is given by equation (15),

$$\text{i.e.} \qquad n_e = \sum_s A_s \, J_o(\theta_s r) \, e^{(\lambda - u_s)z} . \qquad \text{---------}(27)$$

When resonance radiation is present photons are emitted which may liberate electrons from the cathode surface. Townsend determined the A_s of the above equation by consideration of the relevant cathode boundary condition. In the present case the cathode boundary condition is modified and to avoid confusion, B_s will be used instead of A_s.

Substitution for n_e into equation (26) gives

$$D \nabla^2 n - n/\tau = - \sum_s \alpha_p v_e B_s \, J_o(\theta_s r) \, e^{(\lambda - u_s)z} . \qquad \text{---------}(28)$$

Let the photon concentration be written as a series solution,

$$n = \sum_s f_s(z) \, J_\bullet(\theta_s r) .$$

Substitution into equation (28) gives the relation

$$D \, \frac{d^2 f_s}{dz^2} - \theta_s^2 f_s - \frac{f_s}{\tau} = - \alpha_p v_e B_s \, e^{(\lambda - u_s)z}$$

$$\text{or} \quad \frac{d^2 f_s}{dz^2} - \left[\frac{1}{D\tau} + \theta_s^2 \right] f_s = - \frac{\alpha_p v_e B_s}{D} \, e^{(\lambda - u_s)z} . \qquad \text{---------}(29)$$

This is a linear 2nd-order differential equation and may be solved by observing that $1/D\tau = h^2$, where h = photon absorption coefficient = fractional number of photons absorbed /cm.path/ photon, and letting $h_s{}^2 = h^2 + \theta_s{}^2$. Its solution is

$$f_s = X_s \exp(h_s z) + Y_s \exp(-h_s z) - \frac{\alpha_p v_e B_s}{D} \frac{e^{(\lambda - u_s)z}}{(\lambda - u_s)^2 - h_s{}^2} \qquad\qquad (30)$$

where the X_s and Y_s are unknown constants. The photon concentration is then given by

$$n = \sum_s \left\{ X_s \exp(h_s z) + Y_s \exp(-h_s z) - C_s e^{(\lambda - u_s)z} \right\} J_0(\theta_s r) \qquad\qquad (31)$$

where

$$C_s = \frac{\alpha_p v_e B_s}{D} \frac{1}{(\lambda - u_s)^2 - h_s{}^2} \quad . \qquad\qquad (32)$$

X_s and Y_s are determined from the boundary conditions of zero concentration at the electrodes. The cathode boundary condition gives

$$X_s + Y_s = C_s \quad ,$$

and the anode boundary condition

$$X_s \exp(h_s d) + Y_s \exp(-h_s d) = C_s e^{(\lambda - u_s)d} \quad .$$

X_s and Y_s are then

$$X_s = C_s \frac{(e^{(\lambda - u_s)d} - \exp(-h_s d))}{(\exp(h_s d) - \exp(-h_s d))} \quad , \qquad\qquad (33)$$

$$Y_s = -C_s \frac{(e^{(\lambda-u_s)d} - \exp(h_s d))}{(\exp(h_s d) - \exp(-h_s d))} \cdot \frac{\quad\quad}{\quad\quad} \tag{34}$$

The constants B_s are found from consideration of the electron currents at the cathode.

The photon current density, j_p, at the cathode is written

$$j_p = +D \left[\frac{\partial n}{\partial z}\right]_{z=0}$$

$$= D \sum_s \left[h_s(X_s - Y_s) - C_s(\lambda - u_s) \right] J_0(\theta_s r)$$

$$= D \sum_s C_s \left\{ \frac{2h_s e^{(\lambda-u_s)d} - \exp(-h_s d) - \exp(h_s d)}{\exp(h_s d) - \exp(-h_s d)} - (\lambda - u_s) \right\} J_0(\theta_s r)$$

$$= D \sum_s C_s \left\{ \frac{h_s e^{(\lambda-u_s)d} - \cosh(h_s d)}{\sinh(h_s d)} - \lambda + u_s \right\} J_0(\theta_s r) \tag{35}$$

The photon current to a region radius r of the cathode is written

$$J_p = \int_0^r 2\pi r \, j_p \, dr \, .$$

Integrating and substituting for C_s from (32),

$$J_p = \sum_s \frac{2\pi r}{\theta_s} \frac{J_1(\theta_s r) \alpha_p v_e B_s}{(\lambda-u_s)^2 - h_s^2} \frac{h_s e^{(\lambda-u_s)d} - \cosh(h_s d)}{\sinh(h_s d)} - \lambda + u_s$$

$$= \sum_s \frac{2\pi r}{\theta_s} J_1(\theta_s r) \alpha_p v_e B_s Q_s \tag{36}$$

letting Q_s replace some of the expression. The above equation neglects

the effect of electrodes on the photon distribution. But from

equations (15) and (17) it may be seen that if electrodes are

allowed for, the gap length is effectively decreased from d to $(d-1/\lambda)$.

The term $(d-1/\lambda)$ may replace d in the above expression for Q_s.

Having found the photon current at the cathode, the B_s are found

by applying the equation

$$J_e(0) = j_0 + \gamma_p J_p(0) \qquad \text{———————(37)}$$

i.e. total electron current at cathode = electron current injected

through pinhole + electron current due to photon bombardment of the

cathode.

The total electron current at the cathode is (see equation(18))

$$J_e(0) = \sum_s \frac{2\pi r}{\theta_s} J_1(\theta_s r) v_e B_s , \qquad \text{———————(38)}$$

and the current through the pin-hole, j_0, is

$$j_0 = \sum_s \frac{2\pi r}{\theta_s} J_1(\theta_s r) v_e A_s . \qquad \text{———————(39)}$$

Substituting the results of equations (36), (38), (39) into

equation (37), we get

$$B_s = A_s + \alpha_p \gamma_p Q_s B_s$$

so that

$$B_s = \frac{A_s}{1 - \alpha_p \gamma_p Q_s} \quad .$$

It remains to substitute for B_s (and A_s) in the equation for the electron current at the anode;

i.e. $J_e(d,r) = \sum_s \frac{2\pi r}{\theta_s} J_1(\theta_s r) v_e B_s \frac{u_s}{\lambda} e^{(\lambda - u_s)d}$.

Thus, the final expression for the anode current distribution is

$$J_e(d,r) = \sum_{s=1}^{\infty} \frac{2j_o J_1(\theta_s r)}{b^2 J_1^2(\theta_s b)} \frac{u}{\lambda} \frac{r}{\theta_s} \frac{e^{(\lambda - u_s)d}}{(1 - \alpha_p \gamma_p Q_s)}$$

where $Q_s = \frac{1}{(\lambda - u_s)^2 - h_s^2} \left\{ \frac{h_s e^{(\lambda - u_s)d} - \cosh(h_s d)}{\sinh(h_s d)} - \lambda + u_s \right\}$. ——(40)

As in the case of the distribution in the presence of attachment, allowance may be made for a finite diameter cathode aperture. Each term in the above series is multiplied by a factor $2J_1(\theta_s a)/\theta_s a$, where a = aperture radius. Care must be taken not to calculate for too large an aperture, because allowance has not been made for photons falling on the aperture itself. These cannot liberate electrons.

3.3.   NON-RESONANCE   RADIATION.

A gas atom, after a collision with an electron, may become excited into a normal excited state and, after a further collision, pass into an adjacent 'metastable' state. Metastable atoms may have long life-times, of the order of 10^{-3} seconds, before they de-exite to the ground state. The de-excitation is accompanied by radiation emission

of a frequency corresponding to the difference in characteristic energies of the ground and metastable states. Atoms in the ground state cannot be excited directly into a metastable state without first passing through a normal excited state; and the radiation emitted in the de-excitation process is not of the correct frequency for raising atoms to normal excited states. The result is that the emitted radiation, known as non-resonance radiation, enjoys a virtually uninterrupted path through the gas.

The emission of photons from a point source may be thought of as analogous to electric lines of force leaving a point charge. In each case the flux density obeys the same laws governing its intensity.

The equation governing voltage distribution is Laplace's equation

$$\nabla^2 V = 0 \quad\text{------------------(41)}$$

whose general solution in cylindrical coordinates (r,z), after separating the variables and solving two ordinary linear differential equations, may be written

$$V = \sum_s P_s\, J_0(\theta_s r)\, \exp(-\theta_s z) \quad\text{------------(42)}$$

where P_s and θ_s are unknown constants.

If photon absorption is to be allowed for, the analogous electrical equation becomes

$$\nabla^2 V = h^2 V$$

and its general solution is

$$V = \sum_s P_s J_0(\theta_s r)\, \exp(-h_s z) \quad\text{------------(43)}$$

where $h_s = (h^2 + \theta_s^2)^{1/2}$.

The flux density, q, in the field direction is

$$q = -\mathcal{E}_o \frac{\partial V}{\partial z}$$

$$= \sum_s \mathcal{E}_o P_s h_s \ J_o(\theta_s r) \exp(-h_s z) \quad .$$

The total flux onto a disc radius r and distance z from a source is given by

$$Q = \int_0^r q \ 2\pi r \ dr$$

$$= \sum_s \mathcal{E}_o P_s h_s \frac{2\pi r}{\theta_s} \ J_1(\theta_s r) \exp(-h_s z) \quad . \quad \text{———(44)}$$

If more than one source, or even a charge distribution, exists at a distance z from the measuring plane, the flux density distribution still obeys the above formula, the source distribution being incorporated into the factors, P_s. The source distribution must of course be symmetrical about the axis, r = 0. Equation (44) shows by analogy that the distribution of photons striking the cathode from a symmetrical plane source distance z from the cathode will depend on z as $\exp(-h_s z)$.

The number of photons produced in unit time in a region between the planes z and z+dz and of radius r is given by

$$dJ_p(z) = \int_0^r \alpha_p v_e n_e \ dz \ 2\pi r \ dr \ . \quad \text{———(45)}$$

When $\lambda d > 2$, the sum effect of three sources is sufficient to determine the electron concentration and satisfy the electrode boundary

conditions (Lucas and Lawson 1965, and equation (15)).

Substitution for this n_e in (45), and the inclusion of a factor 1/2 (because only 1/2 the photons are emitted towards the cathode; the rest being towards the anode) gives

$$dJ_p(z) = \sum_s \frac{\alpha_p v_e B_s}{2} \frac{2\pi r}{\theta_s} J_1(\theta_s r)\, dz \left[e^{(\lambda - u_s)z} - e^{-(\lambda + u_s)z} \right.$$

$$\left. - \exp((\lambda + u_s)z - 2u_s d) \right]. \quad\text{------(46)}$$

The result of equation (44) indicates that the number of these photons which fall on the cathode is found by including a term, $\exp(-h_s z)$, where $h_s^2 = h^2 + \theta_s^2$ and h is the photon absorption coefficient. The total number of photons falling on the cathode then becomes

$$J_p(0) = \int_0^d dJ_p\, dz$$

$$= \sum_s \frac{\alpha_p v_e B_s}{2} \frac{2\pi r}{\theta_s} J_1(\theta_s r) \left\{ \frac{e^{(\lambda - u_s - h_s)d} - 1}{\lambda - u_s - h_s} \right.$$

$$\left. + \frac{e^{-(\lambda + u_s + h_s)d} - 1}{\lambda + u_s + h_s} - \frac{e^{(\lambda - u_s - h_s)d} - \exp(-2u_s d)}{\lambda + u_s - h_s} \right\}.$$

The two terms of the order $e^{-2\lambda d}$ may be neglected to give

$$J_p(0) = \sum_s \frac{\alpha_p v_e B_s}{2} \frac{2\pi r}{\theta_s} J_1(\theta_s r) \left\{ \frac{2u_s\, e^{(\lambda - u_s - h_s)d}}{(\lambda - h_s)^2 - u_s^2} \right.$$

$$\left. - \frac{2\lambda}{\lambda^2 - (u_s + h_s)^2} \right\}. \quad\text{------(47)}$$

The relation between A_s and B_s is found from the cathode current condition as in the previous section on resonance radiation;

$$\text{i.e.} \quad J_e(0) = j_o + \gamma_p J_p(0) \quad . \quad \text{———(48)}$$

Now

$$J_e(0) = \sum_s v_e B_s \frac{2\pi r}{\theta_s} J_1(\theta_s r) , \quad \text{———(49)}$$

and

$$j_o = \sum_s v_e A_s \frac{2\pi r}{\theta_s} J_1(\theta_s r) . \quad \text{———(50)}$$

Substituting the results of equations (47), (49) and (50) into equation (48), we get

$$B_s = A_s + \alpha_p \gamma_p B_s T_s$$

so that

$$B_s = \frac{A_s}{1 - \alpha_p \gamma_p T_s} .$$

It remains to substitute for B_s (and A_s) in the equation for the anode electron current;

$$\text{i.e.} \quad J_e(d,r) = \sum_s v_e B_s \frac{2\pi r}{\theta_s} J_1(\theta_s r) \frac{u_s}{\lambda} e^{(\lambda - u_s)d} .$$

The final expression for the anode current distribution then becomes

$$J_e(d,r) = \sum_{s=1}^{\infty} \frac{2 j_o J_1(\theta_s r)}{b^2 \mathbb{I}_1{}^2(\theta_s b)} \frac{u}{\lambda} \frac{r}{\theta_s} \frac{e^{(\lambda - u_s)d}}{(1 - \alpha_p \gamma_p T_s)} \quad \text{———(51)}$$

where

$$T_s = \frac{u_s \, e^{(\lambda - u_s - h_s)d}}{(\lambda - h_s)^2 - u_s^2} - \frac{\lambda}{\lambda^2 - (u_s + h_s)^2} \, ,$$

b = wall radius,

$$J_o(\theta_s b) = 0 \, ,$$

$$u^2 = \lambda^2 - 2\alpha\lambda \, ,$$

$$u_s^2 = u^2 + \theta_s^2 \, ,$$

$$h_s^2 = h^2 + \theta_s^2 \, .$$

Each term in the series may be multiplied by $2J_1(\theta_s a)/\theta_s a$ to allow for a finite radius (=a) cathode aperture as in the formulae for attachment and resonance radiation.

CHAPTER 4.   APPARATUS.

4.1.   ELECTRODE STRUCTURE.

  (a) Construction.

    The final apparatus, on which all the present results were taken,
is shown in fig.1. The metal parts are of brass, except the steel
anode springs, the copper wiring, and the glow discharge cathode which
is of aluminium. The insulators are glass, PTFE and Perspex. The
electrode structure is mounted on three 12 mm. diameter glass rods:
brass collars fitted with screws keep the structure rigid. The whole
electrode assembly is contained in a glass chamber (fig.2.) fitted
with 3/8 in. brass endplates. O-rings are fitted to the plates and
the ends of the chamber are ground to make the joints vacuum-tight.

  (b) Diffusion gap.

    The diffusion gap, of fixed length, consists of anode (A,fig.1),
cathode (C) and guard rings (G) all made of brass. Thus the electrons,
after entering this region, are presented with only one type of
surface so eliminating contact potential and other awkward effects.
    (i) Anode.

    The anode consists of a central disk surrounded by three
concentric rings, and an outer portion always kept at zero potential.
These are mounted on a ¼ in. thick PTFE base and retained in position
by several 4BA screws for each ring. Each ring is separated by an
air gap 0.25 mm. wide at the anode surface. But the back of the

P.T.F.E.

To anode
end-plate

A

C

D

F

G

H

Tension for anode
springs

H.T.(glass covered)

Other leads

Fig. 1.   Electrode structure(Scale 1:2).

Fig.2. Chamber(Scale 1:4) and schematic gas-line equipment.

rings channel away to give a separation of about 2 mm. at the PTFE surface. This increases the leakage resistance between the rings at the PTFE layer and also much lessens the chance that, when assembling, a small particle of dust, or suchlike, might alight in the air gap and reduce the resistance. The ring radii, to the centre of each air gap and as measured by a travelling microscope, are 1,2,3, and 4 cm., all accurate to 0.001 cm. The currents collected by the rings are led out to the anode endplate by weak steel springs attached to the retaining screws at the rear of the anode.

(ii) Cathode.

The cathode of the diffusion gap is made of ⅛ in. brass with a 2 mm. diameter pin-hole at its centre.

A recess on the rear surface of 1/16 in. depth and 2 in. diameter is provided. The rear of the pin-hole is further recessed so that the actual final hole height is only about 0.5 mm.(compared with 2 mm. diameter). Then the electric field inside the pin-hole is not so much different from the fields either side of the cathode. The front surface of the cathode is precision turned to a mirror finish making it as flat and regular as possible.

Originally a pin-hole diameter of 1 mm. was used but calculation showed that the error involved in regarding a 2 mm. diameter hole as a point source was negligible. The larger hole allows at least four times as much current to pass as the smaller. This extra conductance was found to be necessary at the higher pressures.

(iii) Guard rings.

Five thick brass rings (G,Fig.1), of internal diameter 10.3 cm. are used as guard rings to keep the electric field in the diffusion gap linear and eliminate outside effects. Originally conventional flat guard rings with 2 cm. spacing were used, but after some time it became apparent that the glass walls of the chamber external to the electrode structure were charging up sufficiently to distort the field in the gap. An effect attributed by Crompton and Jory (1962) to surface effects at the anode was later attributed to the charging of glass spacers used to separate their conventional guard rings (Crompton, Elford and Gascoingne 1965). In the present apparatus, there was also a suspicion that some electrons, especially at low pressures, were reaching the anode through the open guard ring structure without passing through the pin-hole. Huxley and Zaazou (1949) could be criticised on this count.

The advantages of a thick guard ring structure are the virtually complete enclosure of the diffusion region with metal boundaries, and the greater mechanical strength and accuracy which give rise to a more linear electric field in the gap. The main errors arising from the use of a non-linear field in swarm experiments are due to field errors in the radial direction. Field errors in the longitudinal direction have less effect (Crompton, Elford and Gascoigne 1965). In the present apparatus changes in guard ring voltages have no dramatic effects on the observed current ratios, but it is not possible to estimate the effect of radial distortion.

It may seem surprising that such a severe step-function wall

Fig. 3. Calculated voltage distortion in diffusion gap.
Solid lines for 3 guard rings and broken lines
for 6 guard rings.

boundary condition as suggested would be acceptable. But calculation
shows that field distortion due to the non-linear wall condition becomes
negligible at a small distance from the wall. The mathematics of this
potential problem is given in the Appendix. At some points (near the
walls) the series solution has been summed to 150 terms before
attaining the required accuracy. Fig. 3 and Table II show the
calculated voltage disturbances.

In the present chamber, the voltage steps near the anode are
made finer giving a more linear field where it is most necessary.
Greater errors (though still small) are readily tolerated near the
cathode where the electron stream hasn't diffused sufficiently to
reach a region of significant field distortion. Fig. 4 shows the
guard ring arrangement diagramatically. The bold line gives the
theoretical wall condition with no spacers whilst the finer lines
give the ideal condition and the condition when spacers are included.
The dotted line shows the voltage deviations at some distance from
the walls.

The guard ring widths are nominally 1.0, 2.0, 1.5, 1.0 and 0.5 cm.
taken from cathode to anode. The middle three values are lessened by
1 mm. to allow for a 1 mm. gap between each ring. The outer two are
only made 0.5 mm. less because they are in direct contact with cathode
and anode. The spacers separating the rings are slightly less than
1 mm. thick (0.9 mm.) and, when in position, the actual gap length,
as measured by travelling microscope, was found to be 5.96 cm.
Field distortion due to incorrect spacer thickness in completely
negligible.

Fig.4. Effect of thick guard rings in present system.

In a guard ring system where all the rings are of equal width the field linearity improves with increasing spacer thickness, as may be seen by modifying fig.4 slightly. The results indicated in fig. 3 and Table 1 were calculated assuming zero spacer thickness; so the field distortion in the present system will be about 10% or 20% less than indicated.

(c) Equalising gap.

This is the region between C and D in fig.1. Its purpose is to provide a field to the rear of the pinhole equal to the field in the diffusion gap, allowing electrons to enter the diffusion gap with the correct energy. The distance between C and D is 4 cm. with a guard ring mid-way.

Compton (1923) has given an expression which gives an estimate of the distance travelled in an electric field by an electron with zero initial energy before it reaches a certain fraction of its terminal equilibrium energy. The expression is

$$d = \frac{u_t}{2E} \log_e \left\{ \frac{1+Q}{1-Q} \right\}$$

where u_t = terminal energy (electron volts),

        E = electric field strength (volts/cm.),

        Q = fraction of equilibrium energy attained,

        d = distance travelled by electron from zero energy position.

Calculations for d when Q is 90% are shown in fig.5. It is seen that with the present experimental conditions d is always less than

Fig. 5. Electron equilibrium in nitrogen and hydrogen. (Initial energy = 0).

1 cm. and making CD = 4 cm. gives the electrons ample time to find

their correct energy distribution before entering the diffusion gap.

The electric field between C and D is variable between 0 and 30

volts/cm. Two glass cylinders, 10.8 cm. diameter and 1 cm. height,

serve to separate the plates C and D. The 8 holes in plate D are 1 cm.

diameter on a 4 cm. p.c.d. Negative ions which may be produced in

the regions behind D and pass through these holes will not reach the

pinhole in C because of their low diffusion. They will merely drift

through and be collected by C. Originally a short negative ion gap was provided behind the control gap to stop any negative ions leaving the glow discharge. A small reverse field was intended to capture the low energy ions and allow most of the higher energy electrons to pass through. This was found to attenuate the electron current too much and the off-centre holes in D were included to ~~beat~~ stop the negative ions. Even so, in nitrogen and hydrogen these ions were only found to be present when the gas sample was impure or the electrodes relatively dirty.

(d) Control gap.

The control gap, DF, controls the electron current passing into the diffusion gap. The plates D and F are separated by another glass cylinder 10.8 cm. diameter and 3 cm. high. The plate F has a 4 cm. diameter hole with fine (28 SWG) wire threaded through 20 small holes around its perimeter. This gives an open structure for low electron attenuation but is sufficient to prevent the glow discharge extending into the control gap at the higher pressures when as much current as possible is needed. Continuously variable fields between $\pm$ 40 volts/ cm. are available using a potentiometer arrangement and reversing switch (fig.6). The strongest reverse field is sufficient to stop all electrons from the glow discharge source from passing into the equalising and diffusion gaps at all pressures but the lowest. Current cut-off is in this way conveniently gained for zeroing the current measuring units. At the lowest pressures the field in the equalising gap is also made zero or the glow discharge switched off

altogether. Also at the lowest pressures, the amount of electron

current allowed through is very sensitive with the control gap field

and for this reason a fine voltage control potentiometer was fitted.

(e) Glow discharge.

The glow discharge is contained in the region between electrodes

F and H in fig.1. The glow cathode H is made of aluminium. Brass

and copper have also been tried but aluminium gives the most stable

discharge. A nylon screw threaded into the centre knob holds the

cathode to the Perspex (for ease of drilling) backing plate. At low

pressures the glow discharge fills the whole region between F and H,

but at high pressures $(>1$ mm. Hg.) it becomes contracted and the knob

was fitted to keep it near the centre. A glass cylinder 9 cm. diameter

completely surrounds the glow discharge preventing stray electrons

from reaching the anode leads at the other end of the chamber. The

other cylinders separating the plates C, D and F and the guard rings

G also prevent stray currents from affecting results. The high

tension lead to the plate H is surrounded by glass tubing, thus

preventing a glow discharge from forming between this lead and the

other electrode leads, which are left uncovered.

4.2. DISCHARGE CHAMBER AND VACUUM SYSTEM.

The electrode structure is enclosed in a 6 in. internal diameter

glass cylinder with ground glass end surfaces (fig.2). Two brass

discs, 8½ in. diameter and 7/16 in. thick, fitted with O-ring grooves

make the end-plates. Glass lead-throughs (AEI-type 2201 S/109) are
fitted into these plates to carry the electric currents to and from
the electrodes. The anode currents are taken out through weak steel
springs: a retaining chain to the other end-plate holds these in
tension. Some method such as this must be used when leads are taken
out through both end-plates. Taking leads only from the anode of the
diffusion gap out through one plate means that a minimum of complicated
precautions have to be taken to prevent stray currents reaching these
leads; they themselves carrying minute currents ($\sim 10^{-10}$ amps.).

Leads to all other electrodes are taken to the other end-plate
through glass tubes (2 mm.bore) serving to insulate the leads from
the other electrodes they pass through. The copper wire is coiled in
the region behind the glow cathode to facilitate manoeuvering the
end-plate after soldering to the lead-throughs.

The copper piping, connected to the chamber through 1 in.
Housekeeper seals and Edwards' unions, is made as large a diameter
as is practical, 1 in., to keep pumping speeds high. A minimum amount
of vacuum rubber tubing is used.

Two evacuating pumps in series are included; an oil diffusion
pump (Speedivac-type 203 B-no. 26.139) and a rotary pump (Metrovac -
type DRI-K-no. 26.88). The former is only used for initial pumping,
after, say, the chamber has been opened for electrode alterations and
water vapour must be pumped away. Although the diffusion pump is
left in position, the rotary pump is adequate at other times, as
shown below.

Gas supply to the chamber is from a commercial high pressure cylinder through a multi-stage regulator (set at 10 psig constant output) and a fine-control needle valve (Edwards-type LB2A). As the gas enters it is also being pumped away by the rotary pump and equilibrium is established at an internal pressure determined by the needle valve setting. The gas inlet is near the gas outlet (fig.2) so the gas enters the glass chamber and McLeod gauge by a back-diffusion process. Apart from convenience of construction, it was felt that this method was the least likely to establish pressure gradients within the apparatus.

The cold traps were surrounded by liquid nitrogen when investigating nitrogen and hydrogen, and by a mixture of solid carbon dioxide and acetone for oxygen.

The impurity level in the chamber may be calculated as follows. Suppose,

$$Q_o = \text{pumping rate,}$$
$$Q_i = \text{input rate,}$$
$$\text{and } Q_L = \text{leak rate.}$$

Then, $Q_o = Q_i + Q_L$, and impurity level $= Q_L/Q_i \simeq Q_L/Q_o$.

Now $Q_L = A \dfrac{dp_L}{dt}$ and $Q_o = A \dfrac{dp_o}{dt}$

where A is a constant. $\dfrac{dp_L}{dt}$ is found by measuring the rate of pressure increase in the closed system (=L microns/hr.), and $\dfrac{dp_o}{dt}$ by measuring the rate of increase of chamber pressure when the pumps are switched off. Experimental measurement of $\dfrac{dp_o}{dt}$ proves it to be proportional to

pressure, yielding the relation

$$\text{impurity level} \simeq 5 \times 10^{-3} \, L/p$$

where p is in microns, and L in microns/hr.

A leak rate of 1.3 microns/hour has been obtained in the present apparatus, and remained consistently for the greater part of the experiments recorded. This gives an impurity level of about 35 p.p.m. at 200 microns, and 3.5 p.p.m. at 2 mm.Hg. The worst leak rate at which results have been taken is 4 microns/hour and the above impurity levels are then trebled.

The gases used are all from gas cylinders provided by the British Oxygen Co. The hydrogen is highest cylinder grade (100 p.p.m. impurity), the nitrogen is the 'oxy-free' grade (100 p.p.m.) and the oxygen used was the normal (5000 p.p.m.) grade. The largest source of impurity is therefore from the gas sample and not from apparatus leaks and out-gassing. Swarm experiments for measuring D/μ values are not as sensitive to impurities as some other experiments in gas discharge work, so it was felt that the above purities were adequate (Crompton, Elford and Gascoigne 1965).

4.3. ELECTRODE CIRCUITRY.

A general circuit diagram is shown in fig.6.

(a) Power sources.

A 500 volt power pack (A.P.T.--Model 509--no. 11.92) is used for

Fig. 6.   Electrode circuitry.

(Transformer--1:8 step-up,60mA-----Rectifier bridge--3 off 1S107 per arm,
800 volt peak inverse voltage, 750mA steady forward current, 40A peak
forward current-----Capacitors--4μF, 2kV working-----L.F.Choke--30H,
50mA d.c.)

Fig. 7.   D.C.Power-pack(20mA, 2kV)

the diffusion gap. The gap voltage under operation is limited to 150 volts to prevent significant secondary ionization although 200 volts has been used at isolated times. Potentials for the guard rings are taken from a resistance chain made of two 10KΩ and two 5KΩ resistors as shown. These were nominally accurate to 1% but actual voltage measurements showed that the field on the axis of the diffusion gap was not in error by more than 0.5% from uniformity at any point.

120 volt dry batteries and potentiometers are used to control the fields in the equalising and control gaps.

Two interchangeable power packs with floating outputs are used to drive the glow discharge,

(i) A Dynatron 4KV unit (Type P200A--no.11.94). This provides a stabilised output current of up to 1mA, and was used with a series resistance of 2MΩ(2 watts),

(ii) A simple 2kV power pack shown in fig.7. This provides an unstabilised output current of up to 20mA through a series resistance of 82KΩ(20 watts). It was found necessary to use these larger currents at high pressures in order to have sufficient electrons enter the diffusion gap.

(b) Meters.

The diffusion gap voltage is measured on an accurate multi-range voltmeter (no.3.S.118) of impedance 200Ω/volt. This was checked against a sub-standard meter and readings were found to be accurate to 0.4% at 25 volts (the lowest gap voltage used) and better than

0.1% at 150 volts.

A Weston voltmeter (0-150 volts) is used for the equalising gap and a Sangamo-Weston multimeter (no. 6.45) for measuring the glow discharge current. This is placed in the lead to the anode of the discharge for insulation reasons.

(c) Cathode ray oscilloscope.

A cathode ray oscilloscope (Cossor 1076--no.10.78) is connected, as shown in Fig.6, to determine glow discharge stability. The glow current, under certain conditions, was found to oscillate at a frequency of the order of 10 kc/s. Generally, the amplitude of the oscillations was found to be a function of glow discharge current and the frequency to be very sensitive to gas pressure. There is evidence to suggest that these observed oscillations are due to standing waves occurring in the discharge. These waves may also be the cause of positive column striations in glow discharges in long narrow tubes (Labrum and Bigg 1952).

The oscillation phenomenon has been used on a number of occasions as the basis for a leak-detection device. The voltage generated at the CRO input is fed into an audio amplifier and loudspeaker, giving a whistle output. External application of a gas or vapour, such as coal gas, hydrogen, or carbon tetrachloride, to a leak source results in a noticeable change in pitch of the whistle. The technique is only of use for large leaks in the present apparatus, because pressures lower than about 0.08 mm. will not support an oscillating glow discharge.

4.4. PRESSURE MEASUREMENT.

A McLeod gauge (fig.8) constructed in this Department is used to measure pressure.

(a) Operation.

The gauge is divided into two pressure regions; one (including bulb A) at chamber pressure and the other, B, at a pressure varying from virtually zero to atmospheric pressure. These regions are separated by the mercury pool and the tap T1, which is normally closed. In the diagram the pressures in A and B are approximately equal as shown by the equal levels of mercury.

Readings are taken by opening the needle valve, V1, thus allowing air from the atmosphere to pass into B through the two-way tap, T2. The mercury then rises into regions A, C, D, and E at a speed governed by the needle valve setting, trapping the chamber gas in A and D. The inequality of the heights of mercury in the capillary tubes D and E, due to the excess pressure in D of the compressed chamber gas, is a measure of the original chamber pressure. Care must be taken not to raise the mercury too fast to the neck of A (where the gas cut-off occurs) otherwise the pressures in A and C will not be equal and the resulting pressure reading will be too high. V2 is a valve to regulate the rate at which mercury returns to the reservoir. A cold trap prevents mercury vapour reaching the chamber and tap T1 may be opened in an emergency to equalise the pressures in A and B. The tubes D and E are placed close together to facilitate height measurements. The maximum obtainable difference in mercury level is conveniently made greater than 76 cm. to prevent

T3
→ To chamber

Cold
trap

Atmos-
phere

V1= needle-valve

T1

T2

V2

To
pump

E D C

A B →

Hg.reservoir

Fig. 8.   McLeod gauge.

possible overflow at the top of the gauge.

(b) Calibration.

The total volume, V, of A and D was measured by filling with mercury up to the trapping point and weighing this mass. The bore of the special capillary tube D is 1 mm. $\pm$ 0.01 mm. The tube E also has a nominal bore of 1 mm. to eliminate the surface tension capillary effects.

Let   x = length of chamber gas trapped in D (cm.),

        h = difference in height of columns in D and E (cm.)

          = pressure of trapped gas (cm.Hg.),

        V = total volume of A and D to trapping point (cm.^{3}),

and   d = capillary tube diameter (cm.).

Then the chamber pressure, p, assuming Boyle's Law to hold, is given by

$$ p = \frac{h}{\text{volume compression ratio}} $$

$$ = \frac{h\pi d^2 x}{4V} \quad \text{cm.Hg.} \quad = Khx \text{ mm.Hg.} $$

where K is the gauge constant. K was found to be 1.273×10^{-3} at 20°C., with negligible temperature correction for $\pm$ 5°C.

Thus,   p = 1.273 hx microns.

The absolute error of the gauge at a pressure of 0.5 mm.Hg. was estimated to be less than 2%. Most of this is an initial systematic error in finding K. The random error in reading h and x is considered to be only about 0.5%. The maximum pressure

measureable is 2 mm.Hg., and the gauge will also give sensible

readings around 1 micron.

In a perfect gas, the quantity hx should remain constant for

different values of h if the mass of gas in A and D remains the same.

Experiment proves this so. Variations in the product of less than

0.5% were consistently observed over a wide variation of h (a factor

of 2 and more). If any condensible vapours not obeying Boyle's Law

(such as water vapour) are present in the chamber this product ceases

to be constant, thus affording a useful check for gas purity.

4.5.   CURRENT MEASUREMENT.

(a) Electrometer amplifier.

The anode currents involved in these experiments are of the

order of 10^{-10} amps. They are measured simultaneously using two

electrometer amplifiers. The detailed amplifier circuit diagram

is shown in fig.9. The mode of operation may be shown by referring

to the schematic diagram (fig.10).

The circuit equations are

$$V_i = iR_x + IR_z ,$$

$$\text{and} \quad QV_i = I(R_y + R_z) ,$$

assuming the source impedance and the amplifier input impedance

both to be infinitely large. Q is the voltage amplification factor

of the d.c. amplifier.

Fig. 9. Electrometer amplifier circuit diagram.

Fig. 10. Electrometer amplifier -- schematic diagram.

Fig. 11. Circuit for current ratio measurement.

Thus, $\qquad I = -\dfrac{iR_x}{(R_z - R_y/Q - R_z/Q)}$.

Q is approximately 4000 and the equation reduces to

$$I = -iR_x/R_z \qquad \text{for } R_z > 100\,\Omega \text{ .}$$

Thus the output current (and hence the voltage at T) is proportional to the voltage drop across the high resistance, R_x, and inversely proportional to the feedback resistance, R_z.

When the circuit is earthed at point B, it may be seen that the input terminal is a virtual earth. This is necessary in the present experiments as it avoids field distortion at the receiving electrode and eliminates large time constants by reducing the effective input capacitance to near zero.

Special anti-microphonic co-axial cables feed the electrometer amplifiers. Conventional co-axial cable was found to affect the amplifiers, currents being induced in the cable by vibrations. Apparently this is due to friction between the centre conductor and the surrounding insulator.

Temperature instabilities, etc., are discussed by Leck and Austin (1960).

(b) <u>Ratio measurement</u>.

Because of instabilities, drift, and meter errors, it is difficult to measure two currents on two amplifiers independently and obtain an accurate ratio. To overcome this difficulty a method was devised whereby the ratio is read continuously. The arrangement

is shown in fig.11. One of the electrometers (El.1) has its fixed
feedback chain replaced by an external high-precision decade
resistance box (total resistance of 10KΩ). Jack-plugs and sockets
let into the amplifier unit facilitate the substitution. The other
amplifier (El.2.) operates normally having a fixed feedback resistance
of 10KΩ.

Suppose the currents i_1 and i_2 in fig.11. are unequal and it is
desired to find their ratio. The circuit equations may be written

$$V_1 = Ki_1R_1/r_1 \qquad \text{and} \qquad V_2 = Ki_2R_2/r_2 \ .$$

r_1 may be varied by altering the decade box setting, and V_1 made
equal to V_2 as determined by the null-detecting galvanometer
arrangement. In this case the equations reduce to

$$\frac{i_1}{i_2} = \frac{R_2}{R_1} \cdot \frac{r_1}{r_2} = f \times R \qquad \text{(where } f \text{ is constant).}$$

The resistances R_1 and R_2 have been measured separately on a
high resistance bridge every few months to make sure no changes in
their resistances occur. An example of the results obtained is
given below.

Date	R_1	R_2	$f = R_1/R_2$
January, 1964.	$0.9469 \times 10^{10} \Omega$	$0.9758 \times 10^{10} \ \Omega$	0.970
March, 1965.	$0.9478 \times 10^{10} \Omega$	$0.9747 \times 10^{10} \Omega$	0.973

These show the ratio to be almost constant over long periods of time.

In all the calculations associated with these experiments f has been taken to be 0.970.

The ratio R is read directly from the decade resistance box under balanced conditions. The constant ratio f is incorporated into all the calculations associated with diffusion; giving values of R directly in terms of the gas and chamber parameters. If $i_1/i_2 > 1$ (or 1.03 to be exact) the input leads must be reversed, because $R < 1$ for the system to function. A resistance of about 100KΩ between the two amplifiers prevents interaction.

By this method, current ratios are measured very much more quickly and simply, all the errors associated with meter reading and current instabilities being eliminated.

(c) Current switching.

Only two electrometer amplifiers are available to measure the three current ratios in the present experiments, so a multirange switch must be used. The switch is of the conventional Yaxley type specially constructed with PTFE wafers to keep the leakage resistances high. The anode electron currents pass through the multirange switch to the two co-axial cables carrying them to the electrometer amplifiers.

The region between the anode and end-plate is surrounded by earthed copper foil external to the glass chamber. Pick-up between collection and measurement is thus eliminated by enclosing the whole region with metal at zero potential.

The multirange switch has four positions. The first three

positions direct the currents i_1 and i_2, i_3 and i_2, i_4 and i_3 respectively to the two electrometer amplifiers. The ratios i_1/i_2, i_3/i_2, i_4/i_3, rather than their reciprocals, are those which remain at less than unity over the widest range of experiment parameters. The fourth position connects no electrode to either electrometer amplifier and may be used for zeroing the units.

CHAPTER 5.   OPERATION AND DESIGN OF EXPERIMENT.

5.1. PROCEDURE.

(a) Gas Pressure.

Gas pressure is set by adjusting the needle valve to an appropriate position, equilibrium being established between inflow and outflow of gas. Particular pressures cannot be set but the actual pressure set is measured accurately with the McLeod gauge (Section 4.4). At least three readings are taken at each pressure.

(b) Temperature.

Room temperature is taken after each pressure reading. The thermometer is situated close to the McLeod gauge, and the temperature inside the diffusion gap taken to be equal to room temperature. Justification for this assumption is experimental. No variations in ratios with temperature were ever witnessed. A possible source of error would have been heating of the gas by the glow discharge, but, although pressure rises in the closed system when a strong ($\sim$10mA) glow discharge is switched on, in the open system (back-diffusion) there is no change in pressure or in current ratios. The heat developed by the glow discharge must be absorbed into the parts of the electrode structure around the glow discharge and between glow and diffusion gap cathode.

(c) Glow discharge.

At low pressures a glow current, i_g, of about 300μA is sufficient

but at high pressures near 2 mm.Hg. currents of about 10mA must be
used to have enough current pass through the pinhole. Increasing
the pinhole diameter from 1 mm. to 2 mm. increased this transmission
by a factor between 5 and 10, enabling lower glow currents to be used.
Oscillations in the discharge are also checked for with a CRO. If
present, they are removed by adjusting the current, although the
effect of quite large a.c. signals on the electrometer amplifiers
is small.

(d) Measurement procedure.

When the electrometer amplifiers are zeroed, the diffusion gap
voltage, V_1, is set at a value and the control gap voltage, V_3, and
i_g adjusted to give reasonable diffusion gap current. In general,
when a ratio of two currents is being measured, the one collecting
the larger current is allowed to receive 2×10^{-10} A.

In principle the equalising gap voltage V_2 should obey the
relation

$$V_2 = \frac{2}{3} V_1 \qquad\qquad i.e. \quad E_2 = E_1$$

but in practice unequal fields were found to have little or no effect
on the current ratios, and V_2 was often used to adjust the pinhole
current. The currents and three ratios are measured as described in
Section 4.5.

Up to 15 separate values of V_1 are taken at each pressure, and
three independent current ratios recorded at each voltage. Values
of V_1 are limited to the range of 25 to 145 volts in general, to

prevent the effects of contact potentials and secondary ionization by positive ions and photons becoming important. The values of V_1 in this range at a particular pressure are chosen in a random manner. This prevents any human errors forming.

(e) <u>Analysis of current ratios.</u>

The dimensions of the diffusion region have been measured with a travelling microscope and found to be: gap length = 5.960 om., ring radii = 1.000, 2.000, 3.000, 4.000 om., guard ring radius = 5.150 om. and cathode aperture radius = 0.10 om.

These dimensions are fed into a general computer program (28.TOWNS,,Appendix) of Townsend's solution of the diffusion equation. Table 2 shows a sample output of this program giving three current ratios for each value of λ. Extensive calculations have been made with the aid of this program to determine the effect on results of varying guard ring radius and cathode aperture radius.

It is to be expected from physical considerations, that guard rings close to the outer measuring annulus will have the greatest effect on this annulus and hence on the ratio R_3. This is confirmed by comparing program outputs for b = 5 and b = 4.5 as shown in fig.12. In this diagram curves are not shown for the changes in λ due to the wall effects on R_1, because the change is undetectable for $\lambda > 1$ (when current ratios are output to only 3 significant figures). Also, a comparison with another set of results for b = 15.0 (i.e. a large distance from the diffusion gap axis) shows that a change in current ratios of 0.001, when b = 5.150 as in the present apparatus, occurs

Input

a = 0.100 = cathode aperture radius.

b = 5.150 = guard ring radius.

l_1, l_2, l_3, l_4 = 1.000, 2.000, 3.000, 4.000 = ring radii.

d = 5.960 = gap length.

A = 1.0, B = 0.5, C = 7.01 (λ steps).

f = 0.970 = electrometer resistance factor.

Output

λ	$R_1 = i_1/i_2$	$R_2 = i_2/i_3$	$R_3 = i_3/i_4$	Terms of series
1.00	404	898	-704	6
1.50	435	-915	-595	6
2.00	469	-791	-490	7
2.50	506	-683	-399	7
3.00	545	-590	-325	8
3.50	587	-511	-264	8
4.00	631	-443	-215	9
4.50	678	-385	-175	9
5.00	728	-334	-143	9
5.50	781	-291	-117	10
6.00	837	-254	-96	10
6.50	896	-222	-78	10
7.00	958	-194	-64	10

Table 2. Sample input and output of program 28.TOWNS. Each ratio
is multiplied by 1000 to eliminate decimal points, and by
the electrometer factor 0.970. Negative signs=inverse ratios.

Fig. 12. Effect of wall radius 'b', cathode aperture radius 'a', and Parker's solution of the diffusion equation on λ. Reference conditions are:- a=0.01, b=5.15, d=5.96 and ring radii=1,2,3,4. Figures refer to ratios, i.e. $2 \equiv R_2$.

for $\lambda_1 = 0.8$, $\lambda_2 = 1.2$ and $\lambda_3 = 2.1$. That is, the walls have greatest effect on R_3.

The effect of increasing the size of the cathode aperture is also indicated in fig.12, where curves are drawn giving errors in λ for radii of 3mm. and 5mm. The effect on λ_1, λ_2, λ_3 (the λ values derived from R_1, R_2, R_3) increases as the value of λ increases. This is to be expected because the less divergent the electron stream (i.e. the larger its λ value) the more is the source size likely to affect the distribution at the anode. The greatest effect, at a particular λ, is generally on the current ratio R_1, the large diameter source having the greater effect on the current reaching the central disc. The errors in λ due to a source hole of diameter 0.1 cm. are not shown in fig.12: the λ values obtained with a 0.01 diameter hole being negligibly different when measured current ratios are considered to be accurate to 0.001 only. A change in ratio R_1 of 0.002 was indicated at λ values above 6, but the percentage error of less than 0.4% is considered negligible.

To determine λ values from current ratios in the present experiments an output from program 28.TOWNS was printed in λ steps of 0.1 (5 times finer than that of Table 2), for a = 0.10 and b = 5.15. Another computer program giving λ v.R tables from Huxley's solution has been written and could have been used because the effects of cathode aperture and guard rings (for $\lambda > 2$) are negligible. Both programs give virtually identical sets of tables for $\lambda > 2$: the program of Townsend's solution is used simply because it is available

on magnetic tape and is relatively fast.

A third program has been written which takes current ratios as input and outputs the corresponding λ values. Huxley's solution is used and the method of calculation involves the use of iteration procedures.

However, it was generally found to be more efficient to output a straightforward λ v.R table in fine λ steps. λ values are then read off this table from the measured current ratios, interpolating linearly to give values of λ accurate to 0.01.

5.2. DIFFICULTIES ENCOUNTERED.

(a) Negative ions.

If negative ions are formed then there is a possibility that they may enter the diffusion gap and affect the results. They have masses of the same order as the gas molecules and have near-thermal energies. This means that they will diffuse little and, if formed behind the diffusion gap cathode, will all be collected by the centre ring. Effectively, they upset only the ratio measured on the centre two rings. Their presence is then readily detected by observing a λ_1 inconsistent with λ_2 and λ_3. Also the number of negative ions reaching the pinhole is a function of V_2, V_3 and i_g, so R_1 will not remain constant when these parameters are varied.

It is known that electrons do not attach significantly to nitrogen or hydrogen molecules, so if negative ions are formed there must be impurities present. In oxygen, electrons attach freely

and theory cannot allow for negative ions formed behind the pinhole.
These are collected by the centre ring making the inner ratio useless
for obtaining D/μ values.

The cure is to make the holes in plate D (fig.1) in a circle
around the centre. Any negative ions passing through these holes
are collected by the diffusion gap cathode. This does not help in
the oxygen case because a significant number of negative ions are
formed in the equalising gap.

(b) Photons.

The early testing experiments in the present apparatus were
made in nitrogen which is strongly photon producing. Currents were
observed at the anode on initiating the glow discharge and, although
these were small ($\sim 10^{-11}$A) and could have been compensated for, it
was decided to eliminate them. Their magnitude was approximately
proportional to the glow current, i_g, but unaffected by gap voltages
V_2 or V_3. A larger V_1 would increase them slightly but, if V_1 was
reversed, large positive currents were registered.

These observations point to the existence of photons or meta-
stable atoms. They are not affected by electric fields, except in
the diffusion gap. Photons produced in the discharge passed directly
through to the pinhole and liberated electrons there, to be collected
at the anode. The large currents resulting from reversal of V_1 were
due to photons passing directly through the pinhole, striking the
anode and liberating electrons there, to be collected at the cathode.

The experimental technique described in (a) also solved this

problem. The hole positions in plates D and F were so adjusted that no direct line could be made from any point in the glow discharge region (FH) to any point in the diffusion gap. The spurious currents were then completely eliminated.

(c) Charging of glass walls.

The early method of using conventional flat guard rings separated by glass spacers allowed glass surfaces to be visible to the electron swarm. It was noticed that current ratios were altering with time. The use of a thick guard ring structure completely eradicated this source of error.

(d) Leakage resistances.

It is essential that leakage resistances from the current path between anode collector and electrometer valve input should be greater than about $5 \times 10^{12} \Omega$. Low leakage resistances will give erroneous measures of current. The resistances are especially liable to be low just after dismantling and cleaning when, for example, there might be a thin film of solvent over the glass surface of the lead-throughs.

The measurement of these resistances is carried out simply, using one electrometer amplifier unit and the diffusion gap power pack. The circuit is shown in fig. 13. All rings not under test are short circuited to the screening can, which itself is connected to the power pack output. The ring under test is connected directly to the electrometer amplifier input. When the applied voltage V is increased slowly a small current flows through the leakage resistance, R_L, and

Fig.13.   Circuit for measuring leakage resistances.

is recorded as a voltage drop across the amplifier resistance, R_a.

If v is the potential drop across R_a then the leakage resistance is given by $R_L = \frac{V}{v} R_a$, since $R_L \gg R_a$ normally. Typical values give $R_L = \frac{100}{0.01} \times 10^{10} = 10^{14} \Omega$. No results were taken in the present apparatus whilst any leakage resistance was less than $5 \times 10^{13} \Omega$. The leakage resistances measured in the above manner do not include the resistances of the anti-microphonic input cable and the remaining current path to the grid of the electrometer valve. This total was measured separately and was found to be greater than $10^{16} \Omega$.

5.3. RUNNING CHECKS.

The various experimental checks made whilst taking results to ensure correct operating conditions are listed below.

(a) Current ratios may be measured over a time interval of a few minutes occasionally, to make sure there is no steady drift.

(b) The current ratios should be independent of the magnitude of the currents to each ring. Space charge effects should be absent at currents smaller than 10^{-9}A. The ratio constancy may be observed as the current is raised from zero to its maximum value, and any change indicates the presence of unwanted effects.

(c) Ratios should be independent of the parameters V_3 and i_g, which may be varied together to give the same anode current. Also it has been experimentally found that ratios are generally independent of V_2.

(d) Generally, three current ratios are measured for each particular combination of field and pressure. Each ratio yields a corresponding value for λ. In the absence of effects such as attachment or secondary ionization, the three λ values should be equal. Their agreement, or otherwise, provides a useful guide to correct operation.

(e) It has been found empirically under the best operating conditions, that the removal of cold traps during operation affects neither the pressure reading nor the observed current ratios. The mercury vapour pressure at room temperature of 1 micron is apparently

not high enough to affect the electron swarm. A change in current ratios would indicate the presence in the cold traps of impurities which should be removed by a period of pumping.

(f) An accurate value for the ratio of electrometer amplifier resistances has been found using a special high-resistance electrometer bridge , and found to be 0.970. In practice the smaller current when measuring a ratio is passed through the smaller resistance. If the measured current ratio, R_1, is adjusted to be between 0.940 and unity, it may also be measured by reversing the leads from the rings to the electrometer amplifiers. This gives a ratio R_2. The electrometer resistance factor f is then given by

$$f^2 = R_1 \times R_2 ,$$

or more simply

$$f = (R_1 + R_2)/2.$$

The value of f thus obtained should be 0.970 and in practice never differed from this figure by more than 0.003.

CHAPTER 6.   RESULTS AND DISCUSSION.

6.1. HYDROGEN.

The present results in hydrogen are shown in figs.14 and 15.
It may be seen that the values of D/μ are functions of E/p only,
i.e. they show no pressure dependency. The ranges of E/p covered
at each pressure are indicated.

At each combination of field and pressure, 3 current ratios
are measured, R_1, R_2, and R_3, which yield the three values, λ_1, λ_2,
and λ_3. In general these values are in good agreement with each
other (theoretically, they should be all the same). The mean
deviation in the three values is always less than 3% of their mean
in the results used. The deviations appear to be random and the
value (or more strictly the value of $u = (\lambda^2 - 2\alpha\lambda)^{1/2}$) used in
calculating D/μ is taken as the mean of λ_1, λ_2 and λ_3.

The effect of primary ionization (α) in the gas must be corrected
for when extending the results to high E/p_o . Equation (12) gives
D/μ as

$$D/\mu = \frac{E}{2\left[u + (\lambda - u)\right]}, \quad \text{where } u = (\lambda^2 - 2\alpha\lambda)^{1/2} .$$

That is,

$$D/\mu = E/2\lambda = \frac{E/2u}{1 + \dfrac{E/2u.(\lambda - u)/p}{E/p}} .$$

Fig. 14. E/2u values in hydrogen.

Fig. 15. $D/\mu (=E/2u)$ values in hydrogen at low E/p_0.

It may be seen that the above expression is a function of E/p only. The effect of applying this formula is seen in fig.16, where the (λ -u)/p values used are those of Rose (1956): the effect of $\propto$ becomes significant for $E/p_o > 27$.

The results are compared with those of other workers in fig.16 and Table 3. The present corrected values for D/μ agree quite well at $E/p_o < 70$ with those of Townsend and Bailey (1921) and Crompton, Liley, McIntosh and Hurst (1965), but they are consistently smaller than those of Lawson and Lucas' first set (1965a). It is believed that secondary ionization at the cathode (γ) in the latter set is responsible for the discrepancy. Lawson and Lucas (1965b) have since allowed for γ, and their D/μ values fall below the present values for $E/p_o > 70$, although at 60 and 70 the agreement is good. Secondary ionization has not been allowed for in the present results. This may account for the discrepancies with the results of Lawson and Lucas (1965b) for $E/p_o > 70$.

In Table 4 are shown parameters relating to collisions between electrons and hydrogen molecules. The root mean square electron velocity $\bar{c}$, electron mean free path L, and mean fractional energy lost by an electron in a collision have been calculated (see Section 2.4.) from the present experimental values for D/μ and the drift velocity measurements of Townsend (1948).

Fig. 16. Comparison of D/μ values in hydrogen.

E/p_0	TB	T	CS	HC	CF	CJ	LLa	Pr	CLMH	LLb
1	.213	.221	.213	.215	.247	.224				
2		.360	.347	.345	.372	.358				
3	.446		.456		.477	.462				
4			.543		.566	.552				
5	.608	.630	.620	.617	.651	.631		0.67		
10	1.04	1.04	1.01					1.03	1.03	
15			1.46				1.58	1.48	1.49	
20	1.84	1.84	1.86				2.00	1.90	1.92	
30	2.48						2.62	2.48	2.46	
40	2.95	3.05					3.09	2.89	2.85	
50	3.32	3.55					3.46	3.23	3.17	
60							3.79	3.56	3.49	3.57
70							4.12	3.89	3.81	3.86
80							4.39	4.18		4.07
90							4.65	4.46		4.20
100							4.93	4.73		4.38
110								5.00		4.56
120								5.27		4.71
200										5.78
250										6.18
300										6.58
350										7.19

TB = Townsend and Bailey(1921)   T = Townsend(1948)

CS = Crompton and Sutton(1952)   HC = Huxley and Crompton(1955)

CF = Cochran and Forester(1962)   CJ = Crompton and Jory(1962)

LLa = Lawson and Lucas(1965a)   Pr = Present results

CLMH = Crompton, Liley, McIntosh and Hurst(1965)

LLb = Lawson and Lucas(1965b)

Table 3.   Comparison of D/μ values in hydrogen. (The results have all been presented to 3 significant figures although in some cases this degree of accuracy is greater than indicated). D/μ involts.

E/p_0 (volts/cm/mm.Hg)	D/ν (volts)	$\bar{c}$ ($\times 10^6$ cm/sec)	W ($\times 10^6$ cm/sec)	L ($\times 10^{-2}$ cm)	f ($\times 10^{-4}$)
50	3.23	130	19.5	3.09	530
40	2.89	123	14.5	2.72	329
30	2.48	114	10.0	2.32	182
20	1.90	100	6.5	1.98	85
10	1.03	75	3.6	1.61	57
5	0.67	59	2.4	1.73	39

Table 4a. Quantities determined from D/μ values in hydrogen.

Drift velocity values, W, are taken from Townsend (1948).

A Maxwellian distribution is assumed. Formulae are given

in Section 2.4.

$d = 5.96$ cm. $b_1, b_2, b_3, b_4 = 1, 2, 3, 4$ cm. $f = 0.970$. N_2.

wall radius = 5.15 cm. $p = 0.588$ mm.Hg. $T = 22.4°C$. $p_o = 0.543$ mm.Hg.

V_1	R_1	R_2	R_3	λ_1	λ_2	λ_3	$(D/\mu)_{12}$	E/p_o
145	.718	.348	.143	5.19	5.12	5.23	2.26	42.7
60	.571	.539	.266	3.52	3.51	3.66	1.37	17.7
15	.409	-.911	.687	1.20	1.15	1.17	1.02	4.4
100	.678	.383	.163	4.76	4.76	4.90	1.68	29.4
35	.479	.769	.446	2.29	2.23	2.36	1.24	10.3
50	.534	.620	.325	3.04	3.00	3.17	1.33	14.7
115	.699	.366	.153	4.99	4.93	5.07	1.86	33.9
40	.497	.713	.399	2.54	2.50	2.65	1.27	11.8
20	.426	.970	.621	1.48	1.39	1.48	1.12	5.9
10	.394	-.852	.762	0.90	0.86	0.78	0.91	2.9
130	.710	.355	.146	5.10	5.05	5.18	2.05	38.3
70	.606	.480	.223	3.95	3.93	4.11	1.42	20.6
45	.514	.665	.361	2.78	2.75	2.89	1.30	13.2
175	.737	.331	.134	5.38	5.31	5.42	2.62	51.5
200	.752	.317	.126	5.53	5.47	5.58	2.91	58.9
25	.442	.899	.558	1.73	1.67	1.78	1.18	7.4
85	.651	.418	.183	4.47	4.44	4.61	1.53	25.0
30	.459	.831	.499	2.00	1.95	2.07	1.22	8.8

Table 4. Sample set of data in nitrogen. (The negative signs indicate the reciprocal of normal ratio. Townsend's unmodified formula is used for converting ratios into λ values. The mean of λ_1 and λ_2 is used to determine D/μ, or $E/2\lambda$).

Fig. 17. D/μ values in nitrogen, showing pressure dependency.

Fig. 18. Comparison of D/μ values in nitrogen.

E/p_o	TB	CS	CF	CE	Pr
1	.515	.596	.610		
2	.736	.802	.790		
3	.862	.911	.886	.873	.905
4		.990	.964	.954	.98
5	1.01	1.05	1.02	1.02	1.04
7					1.115
10	1.19	1.23			1.21
15		1.34			1.31
20	1.45	1.45			1.42
30	1.76				1.73
40	2.11				
50	2.55				
60	2.97				
100	4.21				

Table 5. D/μ values in nitrogen (eV).

TB = Townsend and Bailey (1921)

CS = Crompton and Sutton (1952)

CF = Cochran and Forester (1962)

CE = Crompton and Elford (1963)

Pr = Present mean values.

looked up in a R v. λ table composed for a different gap length, they will yield three quite different values of λ. It has been shown however, using Townsend's formula, that the agreement between the three new λ values can still be within experimental errors for changes in gap length of about $\pm$ 10%. Thus, a phenomenon which alters the effective gap length with changing pressure could explain the pressure dependency.

An attempt has been made to attribute the pressure dependency to photon secondary effects. The theory is given in section 3.2. In order to explain the results a phenomenon must exist which will increase the D/μ value at high pressures and/or decrease it at lower pressures. That is the λ value should be decreased and increased respectively. Tables 6 and 7 show the effect of resonance radiation on observed current ratios. Each table applies at one value of E/p only. The values of α, α_p, h_p and γ_p are 'large' values (α = Townsend's primary ionization coefficient, α_p = photon primary ionization coefficient, h_p = photon absorption coefficient, γ_p = photon secondary ionization coefficient). In order to keep consistency as λ is increased, pressure must be increased in proportion; and in consequence α_p, h_p, and γ_p are also altered in proportion. The quoted values of the parameters are at 1 mm.Hg. The 'effective λ' values shown are obtained by inserting the ratios shown (which are already multiplied by the resistance factor, 0.97) into the unmodified R v. λ table (similar to that of Table 2).

Tables 6 and 7 show that as pressure (or λ) is increased the

λ	R_1	R_2	R_3	λ'_1	Effective values λ'_2	λ'_3
1.00	.404	-.900	.702	1.00	1.01	1.01
2.00	.472	.778	.474	2.04	2.05	2.08
3.00	.548	.580	.307	3.03	3.05	3.07
4.00	.626	.456	.235	3.93	3.89	3.78
5.00	.711	.368	.207	4.82	4.65	4.09
6.00	.809	.293	.172	5.74	5.46	4.54
7.00	.922	.229	.131	6.70	6.37	5.20
8.00	-.896	.177	.092	7.67	7.32	6.07

Table 6. Effect of resonance radiation.($\alpha = 0$, $\alpha_p=1$, $h_p=0.03$, $\gamma_p=0.3$).

λ	R_1	R_2	R_3	λ'_1	λ'_2	λ'_3
1.00	.404	-.899	.703	1.00	1.01	1.01
2.00	.467	.799	.500	1.98	1.96	1.95
3.00	.537	.616	.358	2.89	2.85	2.76
4.00	.620	.465	.242	3.87	3.82	3.71
5.00	.717	.349	.158	4.88	4.84	4.74
6.00	.827	.262	.103	5.90	5.87	5.80
7.00	.949	.198	.067	6.90	6.90	6.87
8.00	-.867	.151	.045	7.93	7.93	7.90

Table 7. As above.($\alpha=0$, $\alpha_p=1$, $h_p=0.1$, $\gamma_p=0.3$).

(Resistance factor, $f = 0.970$ for each current ratio

above).

true λ values that would be obtained, assuming resonance radiation to be effective, would be higher at large pressures, especially when absorption is low. In other words the pressure dependency due to resonance radiation would act in the opposite direction to that required to explain the results. If resonance radiation was present and effective, Table 6 shows also that the inequalities $\lambda_1 > \lambda_2 > \lambda_3$ should be present. The disagreements actually found in the values were not of this form ($\lambda \simeq \lambda_2$, $\lambda_2 < \lambda_3$).

The next approach to this problem is to assume that simple diffusion theory no longer holds. Parker (1963) has obtained an approximate solution to the problem at low E/p. His solution for electron concentration may be made to yield R v. λ curves (Section 2.11.). The changes in λ values from those derived from Huxley's or Townsend's solution are shown in fig.12. Application of Parker's result to experimental current ratios gives an agreement between the three λ values worse than that obtained from Huxley's or Townsend's formulae (Table 8).

Since October, 1965, the author's supervisor, Dr. J. Lucas, has been re-examining the diffusion equation. He has worked out a preliminary theory which is beginning to explain the nitrogen results.

The diffusion theory breaks down because of two factors,

(i) The electrons reaching the anode travel by different paths. Those reaching the outer part of the anode will have travelled an effective distance of $(6^2 + 4^2)^{\frac{1}{2}} = 7.2$ cm., compared with a 6 cm. path to the centre of the anode. The longer path means

d = 5.96 cm. p_o = 0.543 mm.Hg. N_2.

V_1	λ_{1P}	λ_{2P}	λ_{3P}	E/p_o
145	4.99	4.77	4.64	42.7
60	3.44	3.31	3.27	17.7
100	4.60	4.45	4.35	29.4
50	3.00	2.85	2.84	14.7
115	4.81	4.60	4.48	33.9
130	4.91	4.71	4.60	38.3
70	3.83	3.69	3.66	20.6
45	2.75	2.63	2.61	13.2
175	5.17	4.94	4.78	51.5
200	5.31	5.09	4.92	58.9
85	4.32	4.15	4.09	25.0

Table 8. Showing inconsistency of λ values interpreted from Parker's solution of the diffusion equation. (The ratios used are those in Table 5 for $\lambda > 2.5$. Parker's solution does not consider wall effects, but these are negligible for $\lambda > 2.5$).

more electron collisions and hence a lower mean electron energy.

(ii)   The pinhole injects electrons from the centre of the avalanche and these might have an energy which is higher than the mean electron energy for the collecting ring.

When these two factors are considered it is found that the value of λ is governed by the position of the collecting ring. For an inner ring of radius b and an infinite outer ring

$$\lambda = \lambda_M (r/d)^{1/n} + \frac{3}{4d} \log_e \left\{ \frac{1}{2} \left[(d/r)^{1/n} + 1 \right] \right\} \quad \text{------(52)}$$

where $r = (d^2 + b^2)^{1/2}$, and n governs the extent of non-linearity. For small variations

$$\frac{D/\mu}{(D/\mu)_M} = \left[\frac{(E/p)_M}{E/p} \right]^{1/n} \quad , \quad E/p \simeq (E/p)_M.$$

λ_M and $(D/\mu)_M$ correspond to the mean conditions over the swarm. These are what we want to measure. The position-dependent λ (equation(52)) has been inserted into the Townsend formula to give the three ratios of the current collected by the anode rings, namely i_1/i_2, i_3/i_2 and i_4/i_3. The results of the analysis for n = 1/3 and n = 1/2 are given in Table 9. Effective λ values have been obtained by substituting the obtained current ratios back into the output of Townsend's unmodified solution. It may be seen that the mean values for the three rings are too small at low λ, and too large at high λ. The range $2 < \lambda < 4$ is satisfactory and this corresponds

λ	R_1	R_2	R_3	λ_1'	λ_2'	λ_3'	Mean
				Effective values			
n = 1/3							
1.00	.411	-.910	.739	0.74	0.83	1.01	0.86
2.00	.478	.820	.494	1.79	2.05	2.10	1.98
3.00	.557	.606	.318	2.79	3.11	3.17	3.02
4.00	.646	.450	.205	3.80	4.12	4.27	4.06
n = 1/2							
1.00	.409	-.902	.744	0.69	0.91	0.99	0.86
2.00	.476	.822	.487	1.76	2.04	2.25	2.02
3.00	.555	.604	.309	2.76	3.09	3.23	3.03
4.00	.644	.447	.196	3.78	4.16	4.36	4.10

Table 9. Showing effect of using a non-linear diffusion equation.

(Resistance factor, $f = 0.970$ for each current ratio).

to selected parts of each of the curves as given in fig.17. A mean

curve (fig.18) has been obtained from these selected parts, and a

comparison made with the results of other workers. It may be

supposed that their results for variable pressures and values of

λ are in error. But the results of these workers lie within the

series of curves shown in fig.18.

The theory at this stage is sufficient to enable D/μ to be

determined within $\pm$ 5% for the range $4 < E/p_o < $ 40. The present

results in nitrogen have shown that the normal diffusion equation

does not hold and a non-linear diffusion equation must be used.

The use of the normal Townsend equation will only give experimental

values to $\pm$ 10% as indicated in fig.17.

6.3.   OXYGEN.

Oxygen is known to be strongly attaching in the E/p regions here investigated. Any negative ions formed in the apparatus behind the cathode and passing into the diffusion gap upset the calculated current distribution at the anode, and the centre ratio, R_1, then becomes of no use.

There are then left only two independent current ratios in the present system. It has been found that the effect of primary ionization in altering D/μ values is not as great as the effect of attachment; in fact, α moves the R v. λ curve about 1/5 as much as η but in the opposite direction. This may be seen (fig.19) by inserting values of λ, α and η into the 'attachment' equation described in Section 3.1. When α is neglected the two ratios, R_2 and R_3, are functions of the two remaining variables λ and η . Figs. 19 and 20 show the calculated dependences.

A combination of R_2 and R_3, measured under particular conditions, may be made to yield a pair of values, λ and η . Use is made of the calculated dependences shown in fig.20. In the results for nitrogen, a small systematic difference was obtained between λ_2 and λ_3, although λ_1 and λ_2 agreed perfectly well. It was decided that, in oxygen, the measured values of R_3 should be corrected for this effect; making λ_2 and λ_3 equal in the absence of attachment. The correction curve (obtained from N_2 results) is shown in the inset of fig.22.

Fig.19. Variation of R_2 v. λ_2 curve with attachment.

Fig. 20. R_2 v. R_3 as a function of λ and attachment coefficient, η. The primary ionization coefficient, α, is assumed zero.

The D/μ v. E/p_0 curve obtained by the above procedure is shown in fig. 22. Although the maximum scatter is about 0.2 electron-volts, in general the agreement with pressure is good. The results also agree very well with those of Huxley, Crompton and Bagot (1959) for $E/p_0 < 16$. It may be that insufficient account was taken of $\propto$ at higher values of E/p_0. This would bring the curve down in the higher E/p_0 region (as for hydrogen). But values of $\propto$ would have to exist far higher than those (fig.23) found by Prasad and Craggs (1961).

By-products of the above procedure are estimates of the attachment coefficient, η. A curve of η/p_0 v. E/p_0 is shown in fig.23. An experimental indeterminance of about 0.005 in η must be considered when assessing these results. At the lower pressures a remarkable consistency with pressure and E/p_0 is exhibited. The values do not agree well with those of other workers, but it is estimated that in order to obtain attachment coefficients to an accuracy of 5%, R_2 and R_3 would have to be consistent and accurate to better than 0.1%. In the present apparatus this figure is about 1.5%. When comparing with the results of Huxley, Crompton and Bagot (1959) it should be remembered that they used one pressure only ($p_0 = 1.86$ mm.Hg.), and their ratio-measurements are only slightly more accurate than in the present experiments.

In short, the present results for D/μ are in remarkably good agreement with those of Huxley, Crompton and Bagot although the values of attachment coefficient obtained are totally different. If their values of attachment coefficient are used, two values of λ

$(D/\mu)_2$ (volts)

E/p_o (volts/cm.mm.Hg.)

p_o (mm.Hg.)	
▽	1.747
▣	1.324
△	0.952
✕	0.686
⊙	0.494
+	0.325

● Huxley, Crompton and Bagot(1959).

O_2

Fig.21. $(D/\mu)_2$ values in oxygen, using HCB and BC's values for η.

Fig. 22. D/μ values in oxygen, and calibration curve for R_3.

Fig. 23. Attachment coefficients in oxygen.

Fig. 24. Comparison of D/μ values in oxygen.

Legend:
- ● Huxley, Crompton and Bagot (1959)
- ✕ Healey and Kirkpatrick (1939)
- ⊙ Townsend and Bailey (1921)
- + Brose (1925)
- —— Present

O_2

Interpolated

Axis labels: D/μ (volts) versus E/p_o (volts/cm.mm.Hg.)

result corresponding to each pair of current ratios, R_2 and R_3. The agreement of these values is poor, differences of about 20% being general. If their mean is used for calculating D/μ values, a much larger scatter (up to 0.45 volts) of points occurs (fig.21, cf. fig.22) The D/μ values are also up to 30% higher than those shown in fig.22; so they do not agree as well with the D/μ values of Huxley, Crompton and Bagot.

D/μ values obtained by Townsend and Bailey (1921), Brose (1925) and Healey and Kirkpatrick (1939) are consistently much lower than the present values (fig.24). But the first two sets were taken in apparatuses where the centre of the anode was used for making current measurements. Any negative ions formed by attachment behind the pinhole (or slit in the above two cases) are collected and give a higher apparent λ, and lower apparent D/μ, than if allowance had been made for the phenomenon.

Healey and Kirkpatrick used a different electrode configuration, due to Bailey (1925) and described in Healey and Reed (1941); but their low D/μ results cannot theoretically be ascribed to ion formation before the first aperture, because this has been allowed for. However, even negative ions have a small diffusion coefficient and the widths of successive apertures being exactly the same may have resulted in errors. In the present apparatus the finite size of the central collecting disc (1 cm.radius) allows a large safety margin for the diffusion of those negative ions formed outside the diffusion gap. Table 10 shows the available D/μ values in oxygen in the present range of E/p_o.

E/p_o	TB	B	HK	HCB	Pr
5	1.04	1.13	1.07	1.46	<u>1.52</u>
7.5	1.19	1.41	1.19	2.02	<u>1.93</u>
10	1.25	1.58	1.29	2.29	<u>2.23</u>
12.5	1.33	1.72	1.36	2.48	<u>2.47</u>
15	1.45	1.88	1.43	2.66	<u>2.68</u>
20	1.69	2.20	1.54	2.92	<u>3.05</u>
25	1.94	2.49	1.69		<u>3.35</u>
30	2.20	2.75	1.84		<u>3.63</u>
35	2.45	3.00	1.96		<u>3.90</u>
40	2.73	3.22	2.09		<u>4.14</u>

TB = Townsend and Bailey (1921)

B = Brose (1925)

HK = Healey and Kirkpatrick (1939)

HCB = Huxley, Crompton and Bagot (1959)

Pr = Present results

Table 10. D/μ values (eV) in oxygen.

APPENDIX 1.

POTENTIAL DISTRIBUTION IN A GAP WITH THICK GUARD RINGS.

The conventional method of fitting guard rings to parallel plane gaps is to build thin circular rings at fixed distances apart between the two electrodes, and to apply voltages to them depending on their position. Long insulators must be used to separate them, making alignment difficult and exposing the region between the plates to effects outside the gap which might influence field linearity (Section 4.1.b.iii). The use of thick guard rings reduces these errors, the chief disadvantage being the field distortion inherently introduced. The mathematical solution of the potential problem is given below.

The idea of using thick guard rings is initially Crompton's (Crompton and Elford 1963), and it is believed that the analysis below is due, in essence, to Dr. C.A. Hurst of Adelaide University, Australia.

It is desired to obtain the voltage at any point in an electrode system consisting of two parallel plates separated by several circular guard rings (fig.1). All guard rings are assumed to be the same size and to be separated by infinitely small air gaps. The effect of finite air gaps is to reduce the distortion at any point within the structure, and the values calculated using the formula derived below will be maximum values only. Each guard ring has a

potential corresponding to the position in the gap of the plane through its centre and perpendicular to its axis.

The best method of solving this problem is to find an expression for the voltage deviation from uniformity at each point.

It is required to solve Laplace's equation

$$\nabla^2 \phi = 0 \, ,$$

where ϕ is the voltage at any point. Consider a cathode at zero potential with the origin of cylindrical coordinates at its centre and an anode at potential V_o, with a distance d between them.

Now let $\phi = \dfrac{V_o z}{d} + V$ where V is the voltage deviation from uniformity at any point. Since ϕ and $\dfrac{V_o z}{d}$ both satisfy Laplace's equation, then so does V.

$$\text{i.e.} \quad \nabla^2 V = 0 \, . \quad \text{------------(53)}$$

By symmetry the voltages are independent of angle, and are a function of r and z only.

Then, $\dfrac{1}{r} \dfrac{\partial}{\partial r}\left[r \dfrac{\partial V}{\partial r}\right] + \dfrac{\partial^2 V}{\partial z^2} = 0 \, .$

Separating variables, let $V = R(r).Z(z)$. Then,

$$\frac{1}{Rr} \frac{d}{dr}\left[r \frac{dR}{dr}\right] + \frac{1}{Z} \frac{d^2 Z}{dz^2} = 0 \, .$$

Each term must be a constant, and two independent equations may be established,

$$\frac{d^2Z}{dz^2} = -a^2 Z$$

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} - a^2 R = 0 \ ,$$

the second of which is the modified Bessel equation of zero order. Thus a general solution for equation (53) may be written,

$$V = \left[A \sin(az) + B \cos(az) \right] \left[C \ I_0(ar) + D \ K_0(ar) \right]$$

where a, A,B,C and D are arbitrary constants.

Boundary conditions to be satisfied for a single guard ring at a potential $V_0/2$ are V = 0 at z = 0 and z = d, and V to be finite when r = 0 . Thus B = 0, a = $n\pi/d$ and D = 0 .

$$\text{i.e.} \quad V = K \ I_0\left(\frac{n\pi r}{d}\right) \ \sin\left(\frac{n\pi z}{d}\right) \ ,$$

where K is an arbitrary constant. Thus there are an infinite number of independent solutions, corresponding to n = 1,2,3,..., and a further solution would be the sum of all these,

$$\text{i.e.} \quad V = \sum_{n=1}^{\infty} K_n \ I_0\left(\frac{n\pi r}{d}\right) \ \sin\left(\frac{n\pi z}{d}\right).$$

The values K_n may be found from the wall condition using a property of Fourier-Bessel expansions giving

$$K_n = \frac{2}{d.I_0\left(\frac{n\pi b}{d}\right)} \int_0^d V_{wall} \ \sin\left(\frac{n\pi z}{d}\right) \ dz \quad .$$

When a single guard ring of potential $V_o/2$ is considered then

$$K_n = \frac{2V_o}{n\,\pi\,I_o\left(\frac{n\pi b}{d}\right)} \qquad \text{for n even}$$

and $\qquad K_n = 0 \qquad\qquad\qquad$ for n odd.

Thus the final solution is

$$V = \sum_{n=2}^{\infty} \frac{2V_o}{n\pi} \frac{I_o\,(n\pi r/d)}{I_o\,(n\pi b/d)}\,\sin(n\pi z/d) \quad n = 2,4,6,8\ldots$$

The above solution applies only for a single guard ring but is readily extended to a system of p identical rings by writing $V_o{}' = V_o/p$ and $d' = d/p$. i.e. For p rings

$$V = \sum_{n=2}^{\infty} \frac{2V_o}{pn\pi} \frac{I_o(pn\pi r/d)}{I_o(pn\pi b/d)}\,\sin(pn\pi z/d) \text{ , n even.} \quad\text{———(54)}$$

Exact account may also be taken of the effect of including finite spacers. A 'spacer term' is merely added to the above expression; and then it reads

$$V = \sum_{n=2}^{\infty} \frac{2V_o}{pn\pi} \frac{I_o(pn\pi r/d)}{I_o(pn\pi b/d)}\,\sin(pn\pi z/d)\,\frac{\sin(pn\pi f/2d)}{pn\pi f/2d} \text{ ,n even} \text{——(55)}$$

where f is the spacing between guard rings.

If half-rings are used adjacent to cathode and anode then the solutions are identical to (54) and (55) but with the signs reversed.

The convergence of the series (54) depends on the ratio of Bessel functions and is very rapid away from the walls, but tends

to get slower as $r \rightarrow b$. The number of terms N needed to converge the series to an absolute error less than E is given approximately by

$$N = \frac{d \, \log_{10} E}{1.5(r-b)} \quad .$$

Before the existence of solution (54) was known, another solution was obtained by a different procedure. Two separate sets of boundary conditions were used,

1. anode $= V_o$, cathode $= 0$, walls $= 0$
2. anode $= 0$, cathode $= 0$, walls $=$ normal step function condition for three rings. The two solutions of Laplace's equation for these boundary conditions give two independent voltages at a particular point. Since the sum of the two boundary conditions considered yields the actual boundary condition, the actual voltage at a point in the region is the sum of the two voltages obtained. To find the voltage deviation from uniformity at a point, the voltage corresponding to the uniform field condition must be subtracted.

The solution obtained was

$$\phi = \sum_{s=1}^{\infty} \frac{2V_o}{\theta_s b} \frac{J_o(\theta_s r)}{J_1(\theta_s b)} \frac{\sinh(\theta_s z)}{\sinh(\theta_s d)}$$

$$+ \sum_{n=1}^{\infty} \frac{2V_o}{3n\pi} \left[\cos(t) + \cos(3t) + \cos(5t) - 3\cos(6t) \right] \frac{I_o(n\pi r/d)}{I_o(n\pi b/d)} \sin(n\pi z/d)$$

$$\text{———————(56)}$$

where $J_o(\theta_s b) = 0$ and $t = n\pi/6$. The voltage deviation V is given by

$$V(r,z) = \phi(r,z) - V_o z/d.$$

Solution (56) is only correct for three guard rings and is inferior to the solution (54) because of its greater complexity, lack of generality and slowness of convergence in certain regions. Voltage deviations calculated from both (54) and (56) have proved to be exactly the same.

*E12 28.TOWNS→

```
begin
real a,b,l1,l2,l3,l4,d,A,B,C,z,s1,s2,s3,s4,k,
     S1,S2,S3,S4,T1,T2,T3,p1,J,f;
integer i,n,e,Z,c,terms;
real array v[1:25],p[1:25],q1[1:25],q2[1:25],
           q3[1:25],q4[1:25],u[1:25];

real procedure bessel(y);real y;
        begin bessel:=if y<12 then bes one(y) else bes two(y) end;

real procedure bes one(y); value y;real y;
begin real f,g;integer w,X;
      X:=0;
      f:=g:=y/2;
      for w:=1,w+1 while abs(f)>abs(g×₁₀-6)
      do begin
              f:=f×(-y×y)/(4×(w+1)×w);
              g:=g+f;
              X:=X+1
         end;
      bes one :=g;
end;


real procedure bes two (y); value y;real y;
begin real a,b,f,g,h,j,q,l,m,r;
      integer t,Y;
      Y:=0;
      a:=sqrt(2/p1); b:=0.75×p1;
      f:=cos(y-b); g:=sin(y-b);
      h:=-64×y×y; j:=sqrt(y);
      q:=l:=1;
      m:=r:=3/8/y;
      for t:=2,t+2 while abs(l)>abs(q×₁₀-6) or abs(m)>abs(r×₁₀-6)
      do begin
              l:=l×(4-(2×t-3)↑2)×(4-(2×t-1)↑2)/(t×(t-1)×h);
              m:=m×(4-(2×t-1)↑2)×(4-(2×t+1)↑2)/(t×(t+1)×h);
              q:=q+l;
              r:=r+m;
              Y:=Y+1;
         end;
      bes two:=(q×f-r×g)×a/j;
end;
```

Appendix 2. Computer program in Liverpool University Algol for the
          calculation of R v. λ tables using Townsend's formula.

```
terms:=DATA;
pi:= 3.141 592 654;
v[1]:=2.4048256;
v[2]:=5.5200781;
v[3]:=8.6537279;
v[4]:=11.7915344;
v[5]:=14.9309177;
v[6]:=18.0710640;
v[7]:=21.2116366;
v[8]:=24.3524715;
for c:=9 step 1 until terms
do begin
        j:=c;
        v[c]:=pix(j-0.25+0.050661/(4xj-1));
    end;

n:=DATA; e:=0;
repeat:
a:=DATA; b:=DATA; l1:=DATA; l2:=DATA;
 l3:=DATA; l4:=DATA; d:=DATA;
A:=DATA; B:=DATA; C:=DATA; f:=DATA;
NEWLINE(3); PRINT(a,1,4);  PRINT(b,2,4);
PRINT(l1,1,4); PRINT(l2,1,4);
PRINT(l3,1,4); PRINT(l4,1,4); PRINT(d,2,4);
PRINT(f,1,4); PRINT(terms,3,0);
f:=fx1000;
for l:=1 step 1 until terms
do begin
        u[1]:=v[1]/b;
        p[1]:=bessel(u[1]xa)/bessel(u[1]xb)↑2/u[1]↑2;
        q1[1]:=p[1]xl1xbessel(u[1]xl1);
        q2[1]:=p[1]xl2xbessel(u[1]xl2);
        q3[1]:=p[1]xl3xbessel(u[1]xl3);
        q4[1]:=p[1]xl4xbessel(u[1]xl4);
    end;
```

```
for z:=A step B until C
do begin
     s1:=s2:=s3:=s4:=0; S1:=S2:=S3:=S4:=1; Z:=0;
     for 1:=1,1+1 while abs(S1)>abs(s1×₁₀-6)
                    or abs(S2)>abs(s2×₁₀-6)
                    or abs(S3)>abs(s3×₁₀-6)
                    or abs(S4)>abs(s4×₁₀-6)
     do begin
            k:=exp(d×(z-sqrt(z×z+u[1]×u[1])));
            S1:=k×q1[1];
            S2:=k×q2[1];
            S3:=k×q3[1];
            S4:=k×q4[1];
            s1:=s1+S1;
            s2:=s2+S2;
            s3:=s3+S3;
            s4:=s4+S4;
            Z:=Z+1;
            if Z>(terms-1) then goto out;
       end;
out:T1:=s1/(s2-s1);
    T2:=(s2-s1)/(s3-s2);
    T3:=(s3-s2)/(s4-s3);
   NEWLINE(2);
   PRINT(z,2,2);
   PRINT(if T1<1 then T1×f else -f/T1,4,0);
   PRINT(if T2<1 then T2×f else -f/T2,4,0);
   PRINT(if T3<1 then T3×f else -f/T3,4,0);
   PRINT(Z,3,0);
  end;
e:=e+1;
if e<n then goto repeat
end→


10;3;
.01;50;1;2;3;4;6.00;              (SAMPLE
0;1;10.1;1;                        DATA)
.1;50;1;2;3;4;6.00;
0;1;10.01;1;
1;50;1;2;3;4;6.00;
0;1;10.1;1;±
```

```
begin

comment  LUA 29  SEPT 1964;
         comment This procedure evaluates IO(x) for any
                  argument x>-8 by Chebyshev series;
         real procedure BIO(x,failure);
         value x;
         real x;
         label failure;
```

This is a standard library procedure and has been

omitted for space reasons. Exit is to 'failure' if $x<-8$.

```
real procedure IoR(W,R,B);value W,R,B; real W,R,B;
                IoR:=if W×R<15
                     then BIO(W×R,FAIL)/BIO(W×B,FAIL)
                     else sqrt(B/R)×exp(W×(R-B));

procedure P RINGS(Vo,R,B,Z,D,F,P); value Vo,R,B,Z,D,P;
                  real Vo,R,B,Z,D,F; integer P;
begin real p1,t,RATIO,term; integer T,s;
p1:=3.1415926536;
NEWLINE(1); PRINT(R,2,3);
PRINT(if TYPE=+1 then 2×Z×P/D
                  else 1-2×Z×P/D+2×entier(2×Z×P/D),2,3);
comment if half ring system used---output will give
        input transposed,i.e. the pt (3,2.12) becomes (3,2.88);
F:=0; T:=0;

for s:=2,s+2 while abs(RATIO)>abs(F×₁₀-7) and s≤ FIN×2
do begin
        t:=P×s×p1/D;
        RATIO:=IoR(t,R,B);
        term:= TYPE×2×Vo/P/s/p1×RATIO×sin(t×Z);
        F:=F+term;
        if s=FIN×2 then TEXT([TOO*SLOW]);
        T:=T+1;
    end;
PRINT(T,3,0); SPACE(4); PRINT(F,0,3);
if abs(F)>₁₀-15 then  PRINT(ln(abs(F))/2.302585,2,3)
                 else TEXT([[9s]-INF]);
SPACE(4);PRINT(term,0,4);
end;
```
                                              (CONTINUED OVER)

APPENDIX 3. Computer program in Liverpool University Algol for

voltage determination in a gap with thick guard rings.

```
real d,b,I,J,K,L,M,N,r,z,F; integer TYPE,FIN,p,X,Y,TIME,TURNS;
NEWLINE(2);
TEXT([VOLTAGE*****GENERAL*SOLUTION*FOR*P*RINGS]);

TURNS:=DATA;
for TIME:=1 step 1 until TURNS
do begin
TYPE:=DATA; p:=DATA; FIN:=DATA;
NEWLINE(3); TEXT([NO*OF*RINGS=]); PRINT(TYPE×p,2,0);
d:=DATA; b:=DATA;
NEWLINE(1); PRINT(d,2,3); PRINT(b,2,3);

NEWLINE(3);
I:=DATA; J:=DATA; K:=DATA; L:=DATA; M:=DATA; N:=DATA;
for r:=I step J until K
do for z:=L step M until N
    do P RINGS(100,r,b,z×d/2/p,d,F,p);
    end;

NEWLINE(2);
Y:=DATA;
for X:=1 step 1 until Y
do begin
        r:=DATA;
        z:=DATA;
        P RINGS(100,r,b,z×d/2/p,d,F,p);
    end;
FAIL:
end→



2;
-1;3;150;6;5.15;
0;1;5;.2;.2;.8;
-1;6;150;6;5.15;
0;1;5;.2;.2;.7;
2;
1;1.6;2;.6;
→
```

NO OF RINGS=   -6;
Gap= 6.000;   5.150;=guard ring radius

r	$\frac{6z}{d}$	No.of terms	δv	$\log_{10}(\delta v)$	Final term considered
0.000;	0.100;	2;	-2.061ₙ-13;	-12.686;	2.4575ₙ-27;
0.000;	0.300;	2;	-5.395ₙ-13;	-12.268;	3.9763ₙ-27;
0.000;	0.500;	2;	-6.668ₙ-13;	-12.176;	1.4335ₙ-37;
0.000;	0.700;	2;	-5.395ₙ-13;	-12.268;	-3.9763ₙ-27;
0.000;	0.900;	2;	-2.061ₙ-13;	-12.686;	-2.4575ₙ-27;
1.000;	0.100;	2;	-1.795ₙ-11;	-10.746;	8.0132ₙ-23;
1.000;	0.300;	2;	-4.699ₙ-11;	-10.328;	1.2966ₙ-22;
1.000;	0.500;	2;	-5.808ₙ-11;	-10.236;	4.6743ₙ-33;
1.000;	0.700;	3;	-4.699ₙ-11;	-10.328;	-1.2966ₙ-22;
1.000;	0.900;	2;	-1.795ₙ-11;	-10.746;	-8.0132ₙ-23;
2.000;	0.100;	2;	-6.719ₙ -9;	-8.173;	1.6111ₙ-17;
2.000;	0.300;	2;	-1.759ₙ -8;	-7.755;	2.6069ₙ-17;
2.000;	0.500;	2;	-2.174ₙ -8;	-7.663;	9.3982ₙ-28;
2.000;	0.700;	2;	-1.759ₙ -8;	-7.755;	-2.6069ₙ-17;
2.000;	0.900;	2;	-6.719ₙ -9;	-8.173;	-1.6111ₙ-17;
3.000;	0.100;	3;	-2.919ₙ -6;	-5.535;	-4.7035ₙ-18;
3.000;	0.300;	3;	-7.642ₙ -6;	-5.117;	-1.7966ₙ-18;
3.000;	0.500;	3;	-9.445ₙ -6;	-5.025;	5.8139ₙ-18;
3.000;	0.700;	3;	-7.642ₙ -6;	-5.117;	-1.7966ₙ-18;
3.000;	0.900;	3;	-2.919ₙ -6;	-5.535;	-4.7035ₙ-18;
4.000;	0.100;	4;	-1.353ₙ -3;	-2.869;	4.0129ₙ-13;
4.000;	0.300;	4;	-3.542ₙ -3;	-2.451;	-2.4801ₙ-13;
4.000;	0.500;	3;	-4.380ₙ -3;	-2.358;	7.7313ₙ-10;
4.000;	0.700;	4;	-3.545ₙ -3;	-2.450;	2.4801ₙ-13;
4.000;	0.900;	4;	-1.355ₙ -3;	-2.868;	-4.0129ₙ-13;
5.000;	0.100;	18;	-4.718ₙ -1;	-0.326;	-7.5410ₙ -9;
5.000;	0.300;	17;	-1.352ₙ +0;	0.131;	1.0773ₙ -8;
5.000;	0.500;	17;	-2.001ₙ +0;	0.301;	-3.4862ₙ -8;
5.000;	0.700;	17;	-2.090ₙ +0;	0.320;	1.0773ₙ -8;
5.000;	0.900;	18;	-1.018ₙ +0;	0.008;	7.5410ₙ -9;

Table 11.   Output of program '51 DIST ACC ANY', showing degree of voltage distortion (100 volts on anode).

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Gas supply to the chamber is from a commercial high pressure
cylinder through a multi-stage regulator (set at 10 psig constant
output) and a fine-control needle valve (Edwards-type LB2A). As the
gas enters it is also being pumped away by the rotary pump and
equilibrium is established at an internal pressure determined by the
needle valve setting. The gas inlet is near the gas outlet (fig.2)
so the gas enters the glass chamber and McLeod gauge by a back-diffusion
process. Apart from convenience of construction, it was felt that
this method was the least likely to establish pressure gradients
within the apparatus.

The cold traps were surrounded by liquid nitrogen when investigating
nitrogen and hydrogen, and by a mixture of solid carbon dioxide and
acetone for oxygen.

The impurity level in the chamber may be calculated as follows.
Suppose,

$$Q_o = \text{pumping rate,}$$
$$Q_i = \text{input rate,}$$
$$\text{and } Q_L = \text{leak rate.}$$

Then, $Q_o = Q_i + Q_L$, and impurity level $= Q_L/Q_i \simeq Q_L/Q_o$.

Now $\qquad Q_L = A \dfrac{dp_L}{dt} \qquad$ and $\qquad Q_o = A \dfrac{dp_o}{dt}$

where A is a constant. $\dfrac{dp_L}{dt}$ is found by measuring the rate of pressure
increase in the closed system ($=L$ microns/hr.), and $\dfrac{dp_o}{dt}$ by measuring
the rate of increase of chamber pressure when the pumps are switched
off. Experimental measurement of $\dfrac{dp_o}{dt}$ proves it to be proportional to