Agent Risk Management in Electronic Markets Using Option Derivatives

Thesis submitted in accordance with the requirements of
the University of Liverpool for the degree of Doctor in Philosophy
by
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July, 2008

Dedicated to my parents, my brother and Marcela.

Abstract

In this thesis I present a framework for intelligent software agents to manage risk in electronic marketplaces using Option Derivatives. To compare the performance of agents that trade Option Derivatives with agents not using them, I create a simulation of a financial marketplace in which software agents are vested with decision rules for buying and selling assets and Options. The motivation of my work is the need of risk management mechanisms for those Multi–Agent Systems where resources are allocated according to a market mechanism. Autonomous agents participating in such markets need to consider the risks to which they are exposed when trading in them, and to take actions to manage those risks. This thesis considers the hypothesis that software agents can benefit from trading Option Derivatives, using them as a tool to manage their exposure to uncertainty in the market.

The main contributions of this thesis are: First, an abstract framework of an Option trading market is developed. This framework serves as a foundation for the implementation of computational Option trading mechanisms in systems using Market–Based resource allocation. The framework can be incorporated into existing Market–Based systems using the traded resources as the underlying assets for the Option market. Within the framework, four basic Option trading strategies are introduced, some of which reason about the risks exposed by their actions. These strategies are provided as a foundation for the development of more complex strategies that maximise the utility of the trading agents by the use of Options. The second contribution of this thesis is the analysis of the results from simulation experiments performed with the implementation of a software Multi–Agent System based on the developed Option trading framework. The system was developed in Java using the Repast simulation platform. The experiments were used to test the performance of the developed trading strategies.

This research shows that agents which traded Options by choosing actions aiming to minimize their risk performed significantly better than agents using other trading strategies, in the majority of the experiments. Agents using this risk–minimizing strategy also observed a lower correlation between the asset price and their returns, for the majority of the experimented scenarios. Agents which traded Options aiming to maximize their returns performed better than their peers in the scenarios where the asset price volatility was high. Finally, it was also observed that the performance differential of the strategies increased as the uncertainty about the future price of the asset was increased.

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Acknowledgement

My research could not have been possible without the advice and support of many people. To begin with, I want to deeply thank my supervisors Wiebe van der Hoek and Peter McBurney for their unlimited patience and advice. A special thanks goes to Rafael Bordini at the University of Durham for his insightful contributions. Similarly I want to thank the people at IIIA, specially Juan Antonio Rodriguez-Aguilar, for their hospitality and their contributed feedback to my research. I want to also thank Ullrich Hustadt for allowing me unlimited access to the department computer cluster. I'm grateful to the members of the Market Based Control project group at the Universities of Southampton and Birmingham for letting me participate in the group. Also, the people at BAE Systems, BT and specially Andrew Byde at HP. I would like to thank Michael Wooldridge and Pablo Noriega for their insigthful comments and feedback to my work. Several people have contributed in some way or another to my research, some of them, in no particular order are: Alexei Lisitsa, Edmundo Sánchez Nuñez, Tim Miller, Leonid Sheremetov, Miguel Conteras Montoya, Ernesto German Soto, Raul Herrera Avelar, Meritxell Vinyals, Steve Phelps, Thierry Moyaux, Yanbo (Justin) Wang and Mehdi Dastani.

Finally, my research was fully supported by the grant number 187564 of the Mexican Council of Science and Technology CONACYT.

Chapter 1

Introduction

This thesis presents the development of a Multi–Agent Option contracts trading framework which allows software agents to execute some of the actions performed by traders in real financial Option markets. The framework contains the definition of an underlying asset; agents trade this type of asset as the main commodity, and can trade Option contracts generated from the asset. The Option trading framework is extended with several Option trading strategies that can be used by the agents to trade assets and Option contracts.

The objective of this framework is to show that autonomous software agents can manage risk using Option Derivatives in software marketplaces with similar features to those composing real Option markets. To compare agents using Options with agents not using them, I create a simulation of a financial marketplace, in which software agents are vested with decision rules for buying and selling goods and Options.

1.1 Motivation and Objectives

The motivation of this work arises from the need for risk management in Market–Based Multi–Agent systems with limited resources, such as resource allocation in Grid computing [39]. Agents trading in such markets face the possibility of resources not being available when needed. Agents also face the possibility of not being able to acquire the resources even when they are available, due to high prices. As computational resource allocation systems become increasingly common, participants will require agents able to reserve future resources on their behalf, and hedge against future risks. As these computational systems become more common, the need for autonomous traders which consider the risks in their decisions will also be required. For the agents to be able to act given their perceived risks in such markets, mechanisms for the management of those risks must be available.

Financial Derivatives are economic securities whose values depend on the performance of another security or asset [52]. In real world markets, Derivatives are used to manage the risk inherent from the participation in the market. Option contracts are one type of Derivatives in which the buyer of the Option obtains the right to buy or sell an asset at a future price and time established in the contract. Options allow agents to prevent high losses due to the variability of the price and also allow them to profit from this variability even when they do not own the assets. In [3], Arnoldi proposes that derivatives can be seen as virtual assets which take their value from other assets. This derived value allows the valuation of the intangible concept of risk and its trading in markets similar to other asset markets. It is this property of Options (and Derivatives in general) that provides agents with the possibility of quantitatively analysing those risks and hedge them by the use of the provided instruments.

The main issue under analysis in my research is to find out if it is possible to create a software market with a structure similar to a real Option trading market; where intelligent software agents trade Option contracts with similar elements to those in real world Option markets. To achieve this this, an abstract framework of an Option market is defined and an implementation is developed. Using this Option market framework, I aim to create software agents that can trade both Option contracts and the underlying asset in the market; to detect the scenarios where these agents have better performance than software agents that are able to trade only the underlying asset; and to test whether agents can use Option contracts to decrease their losses caused by the exposure to uncertain future states of the market (i.e., caused by risk).

To address those issues, the following research questions are then considered:

1. Can software agents benefit from the exchange of Options in the software market?

2. Is it possible to characterise specific cases where software agents trading Options have a better performance than those agents not using them?

3. Are agents trading Options less susceptible to price variations than those not using Options?

4. What is the difference in the performance among the developed Option trading strategies?

The answers to these questions will be constrained to the created model. That is, I do not aim find the answers to the stated questions for all possible cases (which may not be feasible with a simulation study) but to identify and study cases where the answer to such questions would benefit a Market–Based system. This study is done to identify the benefits that trading Options would provide to the agents; and to establish a basis of Option trading mechanisms for the management of risk in Multi–Agent Systems, where resources are allocated with a Market–Based mechanism.

1.2 Thesis Outline

In this thesis I present the developed Option trading framework and a set of Option trading strategies; a computational implementation of this framework; and a set of experiments designed to test the performance of the strategies under different proposed scenarios. The work is separated in the following chapters:

- Chapter 1 *Introduction:* This chapter, where the general objectives of the research are described.
- Chapter 2 *Background:* Where the relevant background research is presented and discussed.
- Chapter 3 *Option Trading Model:* Presents the abstract Option trading framework describing the Option and asset trading market; the trading strategies and the forecasting functions.
- Chapter 4 *Implementation:* Describing the computational implementation of Option market framework, including the verification and validation of the system.
- Chapter 5 *Design of Experiments:* Presents the parameters used to setup the simulation experiments for the study.
- Chapter 6 *Experimental Results:* Presenting the outcome of the simulation study. Discussing the results obtained from the resulting data.
- Chapter 7 *Future Work and Conclusion:* Presenting the conclusions of my work and the possible ways how it can be extended.

Additionally, three appendices are provided:

- Appendix A *Statistics Background:* Containing an introduction to some of the statistical concepts used along the thesis.
- Appendix B *Normality Statistical Analysis Data Tables:* Containing the detailed values from the Normality test performed to some data used in Chapter 6.

- Appendix C *Trading Volume Analysis Data Tables:* Containing the detailed values of the trading volume analysis used in Chapter 6.

The following sections provide a summary of the work described throughout the chapters of this thesis.

1.2.1 Background

The research starts on Chapter 2 where the background of the different fields related to this work is presented. The work in this thesis is based on the field of Financial Economics. Option Derivatives and the used concepts related to them are presented in this chapter. Similarly background on Multi–Agent Systems is described, as the developed framework is modelled as a set of agents interacting in the market environment. The fields of Agent–Based Computational Economics and Market Based Control are also visited; they are two related fields relevant for this work which share several research lines but aim at different objectives. Much of the current research done on computational models of Derivatives can be classified as Agent–Based Computational Economics. In contrast, the research developed in my thesis is aimed to the Market–Based Control field; such a difference will be analyzed on Chapter 2.

1.2.2 Option Trading Model

After presenting the relevant background research, Chapter 3 presents the abstract Option trading framework. The elements that comprise a market where agents are capable of trading Options is described, presenting the model of the Exchange (the place where the agents gather to trade) and of the trading agents. A definition of agents that are capable of trading Options is described; providing them with the minimal properties which will allow them to trade in the defined Option market. The agents are provided with different strategies; some of the strategies allow the agents to trade Options using information from the market and reasoning about the risk of their possible actions. Other defined strategies allow the agents to only trade assets. Finally, two forecasting mechanisms are defined to allow the agents to obtain information about the future state of the market. The obtained information may be used by the agents to formulate a model of the future and obtain a measurement of the risk that each of the available actions will expose. Some of the work presented in Chapter 3 is published in [10].

1.2.3 Implementation

Chapter 4 describes the implementation of the Option market model into a computer program. This implementation is developed to run a series of simulation experiments to answer the thesis research questions. The software engineering considerations related to the implementation are discussed, and the development methodologies adopted for the development of the system are described. The chapter also shows the verification and validation tests that were carried out to achieve the accreditation of the implementation. Part of the work presented in Chapter 4 is published in [9].

1.2.4 Design of Experiments

The description of the simulation experiments executed with the implementation are described in Chapter 5. The values of the different parameters to be used in the defined test cases are presented. Of special relevance are the values used for the price series of the asset. These price series represent different market conditions in which the agents will be trading, defining the changes occurring in each scenario through time. The experiments are focused on the comparison of the performance between the developed strategies. Therefore, sets of agents using the same strategy are created and a summary of their performance will be compared to the performance of other strategies. Also described in Chapter 5 are the analyses that will be performed on the data obtained from the simulation. These analyses will define the performance metrics used to compare the outcomes among the strategies. Some of the work presented in Chapter 5 is published in [9].

1.2.5 Experimental Results

The results obtained from the experiments are presented in Chapter 6. The outcomes of the strategies are compared using the performance metrics defined in Chapter 5 after analysing the experimental data. The tests are based on different statistical analyses on the returns and quantity of offers (both cleared or non–cleared by the market) made by the use of the different strategies. The results are compared to the established research questions and the conclusions obtained from the analysis are presented.

1.2.6 Future Work and Conclusion

The research work in this thesis ends with Chapter 7 where I present the the concluding remarks of the developed research. The chapter then presents a discussion of the possible domains in which my developed Option trading framework can be applied to provide the systems with a risk management mechanism. The chapter concludes with a description of the improvements that can be performed to the model.

1.2.7 Statistics Background

Appendix A presents an introduction to the statistical concepts that are used throughout the thesis. This statistics background is included because the presented work makes use of probability and statistics concepts.

1.2.8 Tables

Tables with detailed results obtained form the analysis of the experimental data in Chapter 6 is presented in Appendix B and Appendix C. A summary of the values in the tables is used for the presentation of the results throughout Chapter 6.

1.3 Contributions

The contributions of this thesis are the following: First, an abstract model of an Option trading market is developed. This aims to serve as foundation for the implementation of computational Option trading markets for systems that are controlled with a Market–Based mechanism. The Option Trading model is created in such a way to allow a market designer to *attach* the created Option trading market into existing Market–Based software systems without much difficulty, using the resource (or resources) traded in such market as the underlying asset for the Option market. Thus, for example, the Option trading modules could be readily added to software agents active in a computational Grid marketplace. In addition to the model, four basic Option trading strategies are introduced: the *OTMinR* strategy, which chooses its actions aiming to minimize the magnitude and probability of its possible losses; the *OTMaxW* strategy, which chooses its actions aiming to maximize the magnitude and probability of its possible profits; the *OTMix* strategy, which chooses between the *OTMinR* and *OTMaxW* depending on a detected trend in the market price; and the *OTRnd* strategy which chooses actions randomly. The developed strategies provide the foundational elements for the development of more complex strategies that maximise the utility of the trading agents by the combined use of Options and assets.

The second contribution of this thesis comprises the results from simulation experiments performed with the implementation of a software Multi–Agent System based on the developed Option trading framework. The results from these experiments show that it is possible to create an Option trading software market with a structure similar to real Option Exchanges. This is an advance on the current published work where Option trading agents are always guaranteed to have a matching offer for every offer they submit. Using this market, it is shown that it is possible to develop agents that trade Option contracts and which also outperform other agents which do not trade Options. A relevant contribution is the development of Option trading strategies that reason about their measurement of *risk* and uncertainty to choose the best offer to submit to the market. The use of strategies that evaluate the risks of their actions should be considered when aiming to implement software agents that make use of Options, given that Option derivatives were developed as an instrument to quantify such risks.

Therefore, the work presented in my thesis is aimed at the field of Market–Based Control of distributed systems with the objective of using Option Derivatives as tools for risk management. It is not my objective to demonstrate the theoretical utility of Option contracts, as this has been shown elsewhere, as in [16, 63, 64]; instead, I make use of the properties of Option contracts which are already known. This is achieved by developing a framework where autonomous agents can trade Option contracts, using the Option contracts' properties to manage risk in software markets which implement the presented framework.

Part of the work presented in this thesis is published in the following peer–reviewed articles:

1. Baqueiro Espinosa, O., van der Hoek, W. and McBurney, P., Designing Agents for Derivatives Markets. A preliminary Framework, in P. Gmytrasiewicz and S. Parsons, ed., *IJCAI-05 Workshop on Game-Theoretic and Decision-Theoretic Agents*. 2005.

2. Baqueiro Espinosa, O., McBurney, P. and van der Hoek, W., The Performance of Option-Trading Software Agents: Initial Results, in Andrea Consiglio, ed., *Artificial Markets Modeling. Methods and Applications*, Lecture Notes in Economics and Mathematical Systems, Vol. 599, Springer. 2007.

Chapter 2

Background

This chapter presents a review of the concepts that serve as the theoretical foundation for the research developed in this thesis. The chapter discusses the previous research that has been done related to the research subject and explains the concepts that need to be considered in order to address the research problem.

Because of the interdisciplinary nature of the research problem tackled in this thesis, it is necessary to draw concepts and definitions from a number of different fields. My work is based on research from fields such as Multi-Agent Systems, Finance, Econometrics and Software Engineering with the aim of making a contribution mainly aimed at the Computer Science field with a focus on Market Based Control of computational systems.

The present chapter first addresses the definition and background of Agent–Based modelling of complex Systems used in the thesis in Section 2.1. In Section 2.2, the description of the relevant background concepts in the field of Financial Economics used in this research is detailed. Afterwards, a discussion of the relevant research from the field of Agent-based Computational Economics is reviewed in Section 2.3. Similarly, Section 2.4 discusses the relevant research in the field of Market Based Control. Finally, Section 2.5 presents a summary of this chapter.

Although this chapter discusses the background research on which the theoretical model developed in this thesis is based, the description of the background related to Software Engineering used in this research is left for Chapter 4 where the computational implementation of the model is described.

2.1 Agent–Based Modelling

Computer based modelling and simulation of complex systems has been one of the driving forces in the development of computer systems. A general definition of a simulation is the imitation of the operation of a process or a real world system through time [8]. A computational model is the representation of a real system through a computer program, represented by a set of algorithms and mathematical formulas implemented as code in a programming language.

In contrast with pure mathematical models, the objective of computational models is not usually to obtain analytical solutions to specific questions. Instead, computational models allow the design of experiments to test the developed models under different scenarios. These experiments are carried out with the objective of testing the behaviour of the modelled systems under a certain set of assumptions [8]. This experimentation allows the designer to obtain insight of certain aspects of a complex system which would not be possible to detect using mathematical analysis, or for problems for which there is no tractable mathematical representation.

As a complex system modelling paradigm, Agent Based Modelling (ABM) and simulation have been used to model real systems in a diversity of domains such as Biology [22], Manufacturing [82], Computing [21] and Economics [12] among others. This variety of uses demonstrates the acceptance of ABM as a useful system modelling approach to gain knowledge about complex systems in such domains.

In [7], Banks cites three reasons for the importance of Agent Based Modelling for social sciences: first, that other approaches have been proved not suitable for the modelling of these systems; second, that the agent based approach is a natural representation of many social systems and third that the emergence property in agent based models is not easily achieved with other approaches.

In ABM, a system is modelled as a set of autonomous entities which are named Agents. Each of these agents is positioned in an environment (either virtual, or real) from which the agent obtains information by the use of *sensors* and makes decisions based on its perceived state of the environment and his objectives. These decisions are then reflected as actions performed to modify the state of the environment (either direct actions to the environment, communication with other agents or further reasoning).

The behaviour of the agent may be modelled by sets of rules representing its reasoning engine. The model of such behaviour is often done by representing the agents' mental state by Beliefs, Desires and Intentions (known as BDI software agents) [77]. According to [77], in a BDI model, Beliefs represent the information known by the agent, Desires (also called Goals) are the objectives that the agents want to fulfil and Intentions are a subset of those Desires which the agent has chosen to achieve (that is, to execute a plan in order to achieve the goals).

An agent can have different behaviours according to the system it populates [98]. Agents also have three basic properties such as reactivity (the ability to respond to events in the environment), pro-activity (the ability to demonstrate some behaviour determined by its particular objectives, taking the initiative to satisfy its necessities) and sociability (the ability to interact with other agents and humans to fulfil its objectives) [100]. These properties give Agent Based Systems a great versatility in comparison with typical object based systems by providing a new type of abstraction for the representation of problem domains.

2.2 Financial Economics Background

Derivatives are economic securities whose values depend on the performance of another security or asset. There are different types of Derivatives which vary in the terms of the agreement between the parties negotiating the contracts. Future contracts (or Futures) and Option contracts (or Options) are two of the most common Derivatives; they are used by investors to *hedge* the risk of an investment and to increase their wealth [52]. To hedge is defined by the Encyclopedia Britannica as:

> *To protect oneself financially: as a) to buy or sell commodity futures as a protection against loss due to price fluctuation b) to minimise the risk of a bet.*

In both Futures and Options the traders engage in a contract to trade some commodity which is called the *underlying asset.* The use of Derivatives has increased since its creation, currently there are Derivative markets that allow to trade contracts for diverse kinds of assets ranging from physical goods like animals and grains to intangible assets like market stock shares and even other Derivative contracts.

In [3] it is proposed that Derivatives can be seen as virtual assets which take their value from other assets. This derived value is a function of the perceived volatility of the asset price and allows the valuation of the intangible concept of risk and its trading in markets similar to other asset markets.

2.2.1 Futures Contracts

A Future contract is an agreement to buy or sell an asset for delivery in a specific place and time in the future[52]. Some futures do not lead to delivery of goods because the traders close their positions (selling the previously acquired contract) before the time of expiration. Future contracts are usually traded in an exchange. The first established exchange was Chicago Board Of Trade (CBOT),[1] which was established in 1848. Futures markets are

[1]The web site of the CBOT is accessible via the URL `http://www.cbot.com/` (January 20, 2008).

widely known by economists today, they are centralised markets for buyers and sellers from around the world. Future contracts are the securities traded in these markets. Futures markets are characterised by a central regulatory body who states policies in which the Futures can be traded by the participants. These are policies such as the prices and dates of the futures contracts [23].

The most important difference between Future and Option contracts is that when trading a Future contract, both traders become obliged to fulfil their part of the contract. In contrast, when trading Option contracts one of the traders gets the ability to decide if the exchange of the underlying assets in the contract will be executed. The other participant has the obligation of fulfilling the contract if it is executed.

2.2.2 Option Contracts

In Economics, Option contracts are one of the most widely used financial instruments for risk management. The definition of an Option is a contract between two agents, a holder and a writer. The terms holder and writer are used when referring to the agent that buys the Option and the agent that sells it, respectively. The terms buyer and seller refer to the agent that buys and sells goods accordingly. This differentiation is done in order to avoid confusion between transactions of Options and assets. The Option contract gives the holder the right, but not the obligation, to buy or sell an asset at a future time at a price (the exercise price) agreed at the time of the contract [23].[2] An Option that gives the holder the possibility to buy goods at a later time is called a *call* Option, likewise an Option that gives the agent that holds the Option the possibility to sell goods at a later time is called a *put* Option. The quantity of goods to be traded in an Option defines the *volume* of the Option.

The action of choosing to trade the assets as established in the Option contract for the corresponding prices is called to *exercise* an Option; thus, an agent that holds an Option can choose whether or not to exercise it. If exercised, the writer of the Option has the obligation to trade the assets at the agreed prices (i.e., at the exercise price), even if such action incurs a loss for this agent.

Similarly to Future contracts, Options contracts are usually traded in an Exchange which establishes the trading rules as well as the Option exercise prices. Each Option is composed of the *exercise price*, indicating the price at which the underlying assets will be traded; the *expiration time*, which defines at what time in the future the Option has to be exercised; the *volume*, which indicates the number of assets that are traded in each Option contract; and

[2] The terms *asset* and *good* are used interchangeably to refer to the underlying commodity that is exchanged in the market.

Element	Description
Holder	The trader that acquires the *right* to exercise the Option.
Writer	The trader that acquires the *obligation* to trade the assets if the holder chooses to exercise the Option.
Underlying assets	The assets that are traded when the Option is exercised.
Volume	The quantity of assets that are traded when the Option is exercised.
Expiration time	Time when the Option life ends and has to be exercised or discarded.
Exercise price	Price to pay for the assets if the Option is exercised.
Call Option	An Option where the holder buys the underlying assets when exercising the Option.
Put Option	An Option where the holder sells the underlying assets when exercising the Option.
Option premium	The price that the holder must pay to the writer in order to acquire the Option.

Table 2.1: Main elements of an Option contract.

the type of the Option, which can be can be *call* if the underlying assets are to be bought by the holder of the Option or *put*, indicating that the underlying assets will be sold by the holder of the Option.

The ability to choose whether or not to exercise a Option comes at a price to the Option holder. This holding price, also called the Option *premium*, is the price that must be paid to hold the Option. This amount will be paid to the writer of the Option who will acquire the obligation of honouring the contract if the holder decides to exercise it. The main elements of an Option contract are summarised in Table 2.1.

Option contracts can be classified according to the time when they can be exercised. The two most common types are American Options and European Options. American Options can be exercised at any time after they have been exchanged and until their expiration time arrives. European Options can only be exercised at their expiration time [52]. European Options are the type of Options used in my research in order to simplify the model of the Option contracts and the agents' strategies. Option Exchanges usually set the expiration time of Options with a standard time separation, usually of 3 months, between the Options expiration date. Therefore, an Option created to be traded in the month of January, it would either expire on April, July or October. Similarly, an Option created to be traded in April, would have an expiration time in the month of July, October or January of the following year. A similar approach to this standardisation of the separation time is used in my implementation of the Option trading model to generate Option contracts.

As Option contracts derive their value from the value of some underlying asset, it is necessary to consider the market of the underlying asset when trading Options. The choices whether or not to hold or write Options are performed considering the information available from the underlying asset, such as the history of the asset price and the variance of the price. It is this analysis what provides traders with information to choose whether to trade Options or trade the underlying assets directly in the market.

2.2.2.1 Option Trading Analysis

In [93], Options are visualised as separators of the *good* and the *bad* parts of owning an asset. For example, a call Option (gives the right to buy assets), will allow the holder to limit its potential loss if the price of the asset increases in the future, therefore the *bad* part of the market is cut from an unlimited loss to a limited loss defined by the cost of the Option (the premium). The following analysis of the outcomes of the different asset and Option trading outcomes is based on the essential speculative behaviour of the trading agents. Elements such as dividends and interest rates are not considered for the sake of simplicity. Also, external agent constraints such as the need to own specific numbers of assets or specific amounts of money at any time point is not considered in this analysis. If agents are faced with such constraints, the outcome of some of the actions may be preferred even when they lead to incurring in a loss.

The analysis is started by looking at the scenarios where agents buy or sell the underlying asset in the market. Figure 2.1 shows the two possible outcomes when an agent buys an asset in the market. If the price increases after the agent buys the asset, the agent will obtain a profit from the buy. If the price decreases, then the agent will lose wealth after buying the asset. When the agent sells an asset (shown in Figure 2.2), it will obtain a profit if the price of the asset decreases after the trade. However, if the price of the asset increases then the agent will make a loss from the sell of the asset.

Thus, the theoretical potential profit and loss from executing the simple buy and sell actions is unlimited. In the case of buying an asset, a loss will be incurred if the price of the asset decreases after buying it. In the case of selling an asset, a loss will happen if the price of the asset increases after selling it.

There are four basic possible actions that an agent can execute when trading Options. Each action gives the agent the possibility to limit the potential magnitude of profits or losses, given an increment or decrement of the price of the asset in the future. The outcome of the scenarios is summarised in Table 2.2, where X is the exercise price (the price to pay for the assets according to the Option contract), t^o is the time when the Option expires

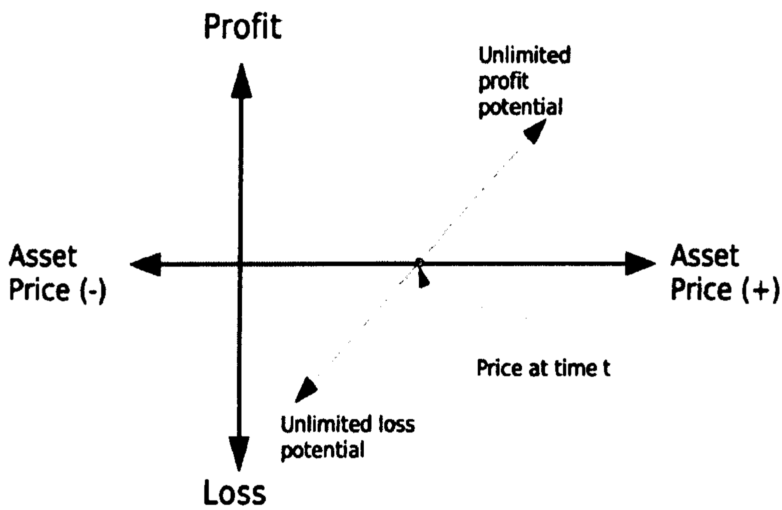

Figure 2.1: Profit and Loss potential when buying an asset directly in the market.

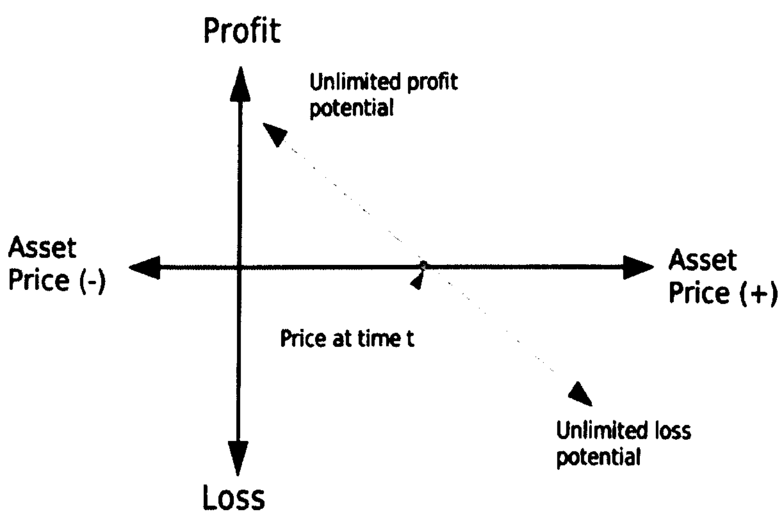

Figure 2.2: Profit and Loss potential when selling an asset directly in the market.

and has to be exercised, $p(t^o)$ is the market price of the assets at time of expiration and p_o is the price of the Option. These potential unlimited profits and losses come from the possibility (for the holder) of choosing to trade assets at the most convenient price (i.e., choosing between paying X and $p(t^o)$). Therefore, the higher the difference between the exercise price X and the price of the asset $p(t^o)$, the higher the profits or losses will be.

For each scenario, a profit-loss diagram is shown in Figures 2.3 to 2.6. Each figure presents the scenario when an agent executes an action (writing or holding, a call or put Option). The horizontal axis represents the price of the asset at expiration time $p(t^o)$. The vertical axis represents the profit or loss that will be incurred by the agent after executing the action being analysed.

For example, to read Figure 2.3 it is assumed that the agent chose to write a put Option. The figure then shows the two possible outcomes. The first outcome is when at expiration time, the price of the asset is lower than the exercise price (i.e., when $p(t^o) < X$) which is the segment of the horizontal axis to the left of the exercise price X point. The second outcome is when at expiration time, the price of the asset is higher than the exercise price (i.e., when $p(t^o) > X$) which is the segment of the horizontal axis to the right of exercise price X point. The combination of actions and type of Options provide different limits in the profits and losses for the agents.

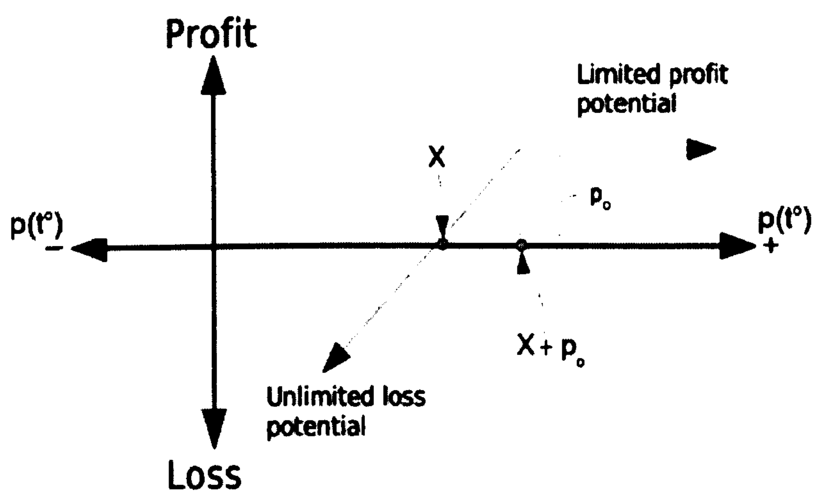

Figure 2.3: Profit and Loss potential writing a put Option. The writer *must* buy assets if the Option is exercised.

In the scenario when an agent writes a put Option (shown in Figure 2.3), the writer will acquire the obligation to buy assets at the exercise price. The first outcome of this scenario is when the price of the asset at expiration time is higher than the exercise price (i.e., when

$p(t^o) > X$). Under this outcome, the writer will be exposed to a maximum profit equal to the Option premium p_o, as the Option holder would prefer to sell the assets at the price of the market $p(t^o)$, choosing not to exercise the Option (losing the premium). The second outcome is when the price of the asset at expiration time is lower than the exercise price (i.e., when $p(t^o) < X$). Under this outcome the agent will be exposed to a potential unlimited loss given by $X - p(t^o) - p_o$, as the Option holder would prefer to sell the assets at the exercise price X.[3] The profit and loss of an agent choosing to write a put Option can be defined as:[4]

$$Profit = Min(0, p(t^o) - X) + p_o \tag{2.1}$$
$$Loss = Max(0, X - p(t^o)) - p_o \tag{2.2}$$

As the price of the asset at expiration time $p(t^o)$ gets lower than the agreed exercise price X, the writer will lose more wealth when buying the assets at the higher exercise price X.

The second scenario is when an agent writes a call Option (shown in Figure 2.4). In this scenario the writer will acquire the obligation to sell assets at the exercise price X. The first outcome of this scenario is when the price of the asset at expiration time is lower than the exercise price (i.e., $p(t^o) < X$). Under this outcome, the writer will be exposed to a maximum profit equal to the Option premium p_o, as the Option holder would prefer to buy the assets at the price of the market $p(t^o)$, choosing not to exercise the Option (losing the premium). The second outcome is when the price of the asset at expiration time is higher than the exercise price (i.e., $p(t^o) > X$). Under this outcome the agent will be exposed to a potential unlimited loss given by $p(t^o) - X - p_o$, as the Option holder would prefer to buy the assets at the exercise price X. The profit and loss of an agent choosing to write a call Option can be defined as:

$$Profit = Min(0, X - p(t^o)) + p_o \tag{2.3}$$
$$Loss = Max(0, p(t^o) - X) - p_o \tag{2.4}$$

As the price of the asset at expiration time $p(t^o)$ gets higher than the agreed exercise price X, the writer will lose more wealth, having to sell the assets at the lower exercise price X.

The third scenario is when an agent holds a call Option (shown if Figure 2.5), acquiring the right to buy an asset. The first outcome of this scenario is when the price of the asset at

[3] Although the loss is limited to the case when $p(t^o) = 0$, this type of loss is treated as unlimited throughout the finance literature (see [23] and [52]).

[4] In equations 2.1 to 2.8, a negative profit denotes a loss and a negative loss denotes a profit.

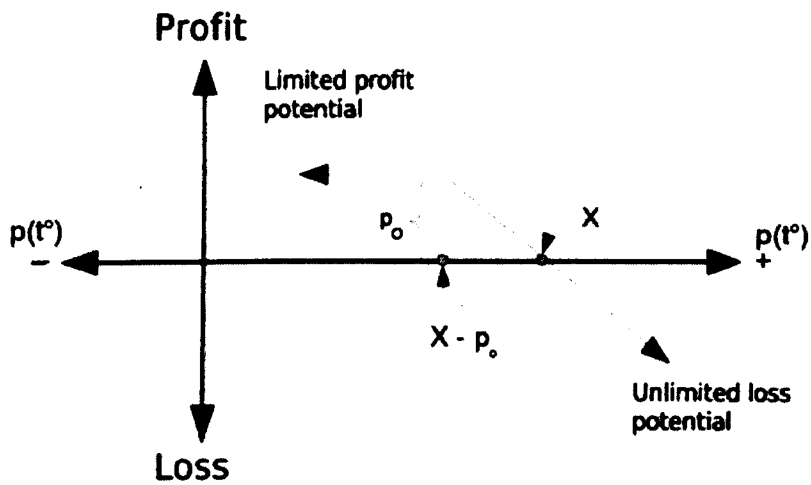

Figure 2.4: Profit and Loss potential when writing a call Option. The writer *must* sell assets if the Option is exercised.

expiration time is lower than the exercise price (i.e., when $p(t^o) < X$). Under this outcome, the agent will be exposed to a limited loss equal to the Option premium p_o, as it would prefer to buy the assets at the price of the market $p(t^o)$ and will choose not to exercise the Option (losing the premium). The second outcome is when the price of the asset at expiration time is higher than the exercise price (i.e., when $p(t^o) > X$). Under this outcome the agent has the possibility to obtain potential unlimited profits given by $p(t^o) - X - p_o$ by buying the assets at the exercise price X. The profit and loss of an agent choosing to hold a call Option can be defined as:

$$Profit = Max(0, p(t^o) - X) - p_o \quad (2.5)$$

$$Loss = Min(0, X - p(t^o)) + p_o \quad (2.6)$$

As the price of the asset at expiration time $p(t^o)$ gets higher than the exercise price X, the holder will gain more wealth buying the assets at the lower exercise price X.

The last scenario is when an agent holds a put Option (shown in Figure 2.6), acquiring the right to sell an asset. The first outcome of this scenario is when the price of the asset at expiration time is higher than the exercise price (i.e., when $p(t^o) > X$). Under this outcome, the agent will be exposed to a limited loss equal to the Option premium p_o, as it would prefer to sell the assets at the price of the market and will choose not to exercise the Option (losing the premium). The second outcome is when the price of the asset at expiration time is lower than the exercise price (i.e., $p(t^o) < X$). Under this outcome the

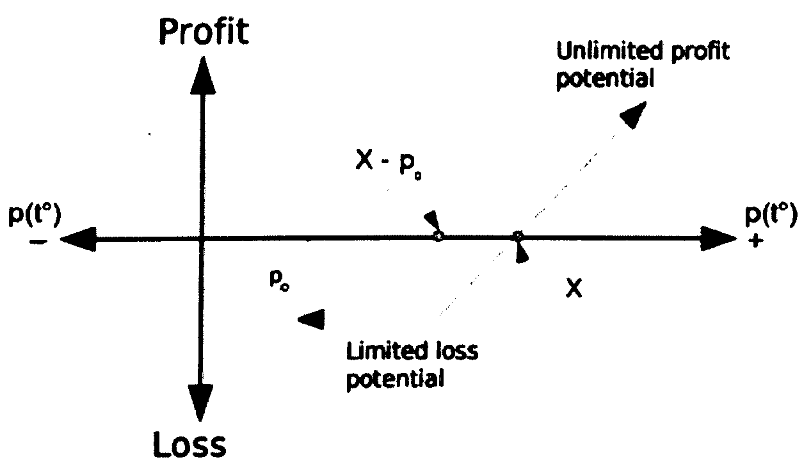

Figure 2.5: Profit and Loss potential when holding a call Option. The holder *may* buy assets if it chooses to exercise the Option.

agent has the possibility of obtaining potential unlimited profits given by $X - p(t) - p_o$ by selling the assets at the exercise price X. The profit and loss of an agent choosing to hold a put Option can be defined as:

$$Profit = Max(0, X - p(t^o)) - p_o \quad (2.7)$$

$$Loss = Min(0, p(t^o) - X) + p_o \quad (2.8)$$

As the price of the asset at expiration time $p(t^o)$ gets lower than the agreed exercise price X, the holder will gain more wealth being able to sell the assets at the higher exercise price X.

Action	Type of Option	
	Call Option	Put Option
Hold the Option	Limited loss when $X > p(t^o)$ Unlimited profit when $X < p(t^o)$	Limited loss when $X < p(t^o)$ Unlimited profit when $X > p(t^o)$
Write the Option	Limited profit when $X > p(t^o)$ Unlimited loss when $X < p(t^o)$	Limited profit $X < p(t^o)$ Unlimited loss when $X > p(t^o)$

Table 2.2: Option trading actions and their possible outcomes

As an illustrative example, consider a call Option with an exercise price of $X = 10$, a premium of $p_o = 1$, and an expiration time of $t^o = 2$. If an agent holds such Option at

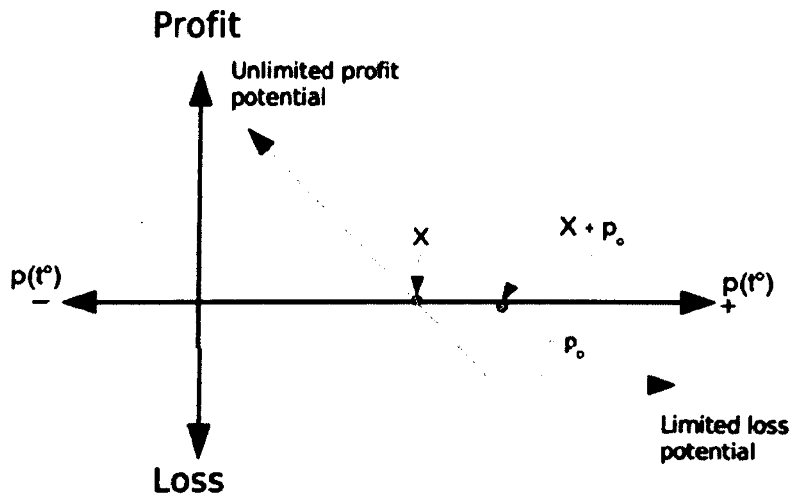

Figure 2.6: Profit and Loss potential when holding a put Option. The holder *may* sell assets if it chooses to exercise the Option.

time $t = 1$, it will have the opportunity of buying assets at time $t = 2$ for the exercise price $X = 10$. One possible scenario is when the price of the asset in the market at $t = 2$ is $p(2) = 15$ (higher than X). In this scenario the agent may choose to exercise the Option and buy the assets for the price X. From Equation 2.5, the profit obtained by the agent after exercising the Option is given by:

$$\begin{aligned} Profit &= Max(0, p(t^o) - X) - p_o \\ &= Max(0, 15 - 10) - 1 \\ &= 5 - 1 \\ &= 4 \end{aligned} \tag{2.9}$$

An alternative scenario is when the price of the asset in the market at $t = 2$ is $p(2) = 5$. In this scenario the agent may do better by not exercising the Option and instead, buying the asset at the market price of 5. According to Equation 2.6, the loss incurred by the agent in this case will be:

$$\begin{aligned} Loss &= Min(0, X - p(t^o)) + p_o \\ &= Min(0, 10 - 5)) + 1 \\ &= 0 + 1 \\ &= 1 \end{aligned} \tag{2.10}$$

which will be equal to the Option premium $p_o = 1$.

Consider now an agent writing a put Option with the same expiration time and same exercise price. In the scenario when the market price of the asset at expiration time is $p(2) = 15$, the writer of the put Option will not have to buy the goods at the exercise price X because the holder will sell them at the higher market price $p(2) = 15$. Therefore, from Equation 2.1 the writer will obtain a profit given by:

$$\begin{aligned} Profit &= Min(0, p(t^o) - X) + p_o \\ &= Min(0, 15 - 10) + 1 \\ &= 0 + 1 \\ &= 1 \end{aligned} \tag{2.11}$$

This profit will be equal to the Option premium $p_o = 1$. On an alternative scenario when the market price of the asset at time $t = 2$ is $p(2) = 5$, the writer will have to buy the goods from the holder at the exercise price $X = 15$ incurring in a loss given by (From Equation 2.2):

$$\begin{aligned} Loss &= Max(0, X - p(t^o)) - p_o \\ &= Max(0, 10 - 5)) - 1 \\ &= 5 - 1 \\ &= 4 \end{aligned} \tag{2.12}$$

2.2.2.2 Option Trading Strategies

Option trading strategies range from the most simple to a combination of trading actions with different objectives. The most simple strategies are the writing and holding of single

Option contracts in order to make use of their limiting properties. Other more complex strategies combine different types of Option contracts and asset trading to achieve specific market positions limiting different factors.

Option holding can be used to assure the availability of assets at a specific price in a future time. This is accomplished holding *call* Options and choosing to exercise it. Option writing can be used to obtain profits when aiming to buy assets at an undefined future time. Instead of directly buying the assets in the market, it is possible to write *put* Options. In such a scenario, there are two outcomes: the first outcome is that when the Option is exercised, the writer will obtain the assets it wanted; the second outcome is if the Option is not exercised in which case, the writer will profit from the premium of the Option and may still obtain the assets at the market price or continue writing *put* Options until one of them is exercised.

An Option writer will also benefit from a price which is persistently close to the exercise price. This can be seen from Figures 2.3 and 2.4: When the exercise price is slightly less than or equal to the asset price at the time of expiration and an agent wrote a *put* Option (depicted in Figure 2.3), then the writer will obtain obtain a profit equal to the value of the premium. Similarly, when the exercise price is slightly greather than or equal to the asset price at the time of expiration and an agent wrote a call Option (depicted in Figure 2.4), then the writer will obtain a profit equal to the value of the premium. The strategy of writing Options can thus be beneficial in a market with low price volatility (where the prices variations are small).

In this thesis I focus on the research of the use of simple Option trading strategies such as single hold and write offers to trade *call* or *put* Options, focusing on the performance obtained by the trading of these single Option contracts.

2.2.2.3 Option Pricing

In order to make an Option contract available for trading in the market, the premium or price to pay to hold the Option needs to be calculated. Before describing the Option pricing models the term *arbitrage* must be defined. According to [52], arbitrage is the ability to obtain a profit from a transaction in a market without being subject to any risk. That is, if an agent can trade an asset in two different markets which have different prices for the same asset, then the agent can make a direct profit without any risk by buying assets in the market with the lower price and selling them immediately in the market where the price of the asset is higher.

There are two main models used to calculate the price of an Option, the Binomial Option

Pricing model and the Black-Scholes Option pricing model. Both models determine the equilibrium price of an Option aiming to calculate a fair price that minimises arbitrage opportunities. To calculate the equilibrium price of an Option the models are based on the valuation of a theoretical portfolio containing an Option, and deriving the price of such Option from the model. Also, both models obtain the valuation of Options considering stock shares as the underlying asset.

The Black-Scholes model was firstly presented by Fischer Black and Myron Scholes in [15]. The model was developed by these two authors in cooperation with Robert Merton. A discussion of the model of Black and Scholes is given in [14]. For the creation of the Black-Scholes model, Myron Scholes and Robert Merton received the Nobel in Economics in 1997.[5]

The binomial Option pricing model was presented by John Cox and Mark Rubinstein in the article [28] in 1979. The main difference between the Binomial and the Black-Scholes model is that, to obtain the fair price of an Option, the binomial model assumes discrete finite time periods whereas the Black-Scholes model assumes a continuous time interval partitioned in to infinitely small periods. Hence, the binomial model is a discrete model that converges to the Black-Scholes model as the number of periods periods increase and the length of such periods is infinitesimally short [52]. Pricing an Option using the binomial Option pricing model is slower than with the Black-Scholes formula. The complexity of the calculation of an Option price using the binomial model increases with the number of periods used. However, using small number of periods would yield a wrongly priced Option.

I chose to use the Black-Scholes pricing model to calculate the Option prices because it is generally accepted as a good method for pricing European Options and because it has been used in other works that modelled Option contracts for markets simulation, specifically in [56], [87] and [36]. Although a complete derivation of the formula is out of the scope my thesis, following, I provide a brief description of the Black-Scholes model and its main properties are presented.

Black-Scholes Pricing Model The Black-Scholes pricing model is based on five main assumptions which are: The variance of the returns of the stock is constant through the life of the Option contract; the risk free interest rate is constant through the life of the Option; there are no dividends paid for the stock; the returns of the stock prices are log-Normally distributed, that is the natural logarithm of the returns are Normally distributed; finally, there

[5] Fischer Black was not eligible as recipient because he died in 1995.

are no transactions costs when trading stocks or Options (such as taxation). The model is affected by four factors: The current stock price; the exercise price; the expiration time and the risk free rate of return.

The value of a *call* (c) and a *put* (p) Option under such assumptions as described in [15] is:

$$\begin{aligned} c &= s\Phi(d_1) - Xe^{-rT}\Phi(d_2) \\ p &= Xe^{-rT}\Phi(-d_2) - s\Phi(-d_1) \end{aligned} \tag{2.13}$$

where c is the price of a call Option and p is the price of a put Option; s is the current price of the stock; X is the strike price of the Option; r is the interest rate; and T is the duration of the Option (time to expiration). $\Phi(\cdot)$ is the standard Normal cumulative distribution function (c.d.f.) and the terms d_1 and d_2 are defined as:

$$d_1 = \frac{\ln(s/X) + (r + 1/2\sigma^2)T}{\sigma\sqrt{T}} \tag{2.14}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{2.15}$$

finally, σ^2 is the variance of the historic return of the stock.

The returns of the stock at time step t are defined by the formula:

$$R(t-1, t) = ln(\frac{p(t)}{p(t-1)}) \tag{2.16}$$

This return is also called the *rate of return* and indicates a rate of profit or loss generated by the stock for the price movement observed in the period $[t-1, t]$.

It is possible to understand the model if the Equation 2.13 is split in two sections. The first section is $s\Phi(d_1)$ and represents the expected benefits obtained from buying the underlying asset in the market. The second part $Xe^{-rT}\Phi(d_2)$ derives the value of paying the exercise price of the Option at the expiration time. Thus, the fair market value of the Option is calculated by obtaining the difference between these two sections.

2.2.3 Option contracts as a risk management mechanism

The reasons for choosing Option contracts for risk management in markets are described now. As has been shown in previous sections, Option contracts can be used to guarantee the availability of a resource at a future time. Option contracts also allow traders to ensure that the resources will be obtained at a specific price. These two properties make the use of Option an attractive alternative for risk management.

However, one of the main issues arising from the management of risk in Market-Based distributed systems is that, the risk and uncertainty in market systems cannot be reduced, but only transferred [37]. Therefore, a mechanism that allows agents to manage their risk must provide the means to transfer such risks between the agents participating in the system.

Such risk transference can be achieved by the use of different Derivatives such as Future contracts, Option contracts and Swaps. However the trading of Swaps and Future contracts require that both parties trading the derivative have an external motive to enter into a contract (e.g. when trading a forward, one party must want to buy the assets at the predefined price and the other party must want to sell it at the same price). Whereas agents who trade Option contracts can obtain some benefit by using a speculative strategy (e.g. when trading a call Option, the holder may want to buy the assets at the predefined price while the writer may just want to obtain a profit from the premium). This property of Options may lead to a higher participation of agents in the market.

Another advantage of using Options for risk management is that the transference of risk can be performed among the agents that participate in the markets as peers. This is, in contrast to having a central authority (like a bank) which will try to absorb the risk of the agents. This type of pattern can be observed in companies that sell processing time which is consumed at different peak intervals. Such companies must maintain the availability of resources at all times (potentially spending in costs related to energy and administration of the infrastructure), even when resources are not being used. A solution to such a problem was proposed by the use of one type of Swing Options in [51].

2.2.4 Forecasting

In order to take advantage of the use of Options, traders need to formulate a belief about the future prices of the assets. Specifically, the forecasting of price series needs to be used. In Economics, price forecasting is generally performed by means of time series analysis [80]. Forecasting is performed by the analysis of the statistical properties of the historic prices (such as mean and variance) and then trying to limit the possible range of future prices based on the data obtained from the statistical analysis.

There are two main models used for statistical analysis of time series: auto-regressive models and moving averages. In both models the forecasted values are obtained as a function of a defined range of values obtained from previous values in the time series. In auto-regressive models each previous value is multiplied by a coefficient whereas in moving averages the average of the previous values is obtained [67].

2.2.4.1 Moving Average

A moving average is one technique used to analyse time series; it consists of calculating the average of the n last values of the time series in order to obtain a value that can indicate the direction of the trends that the data series observe. In some moving average formulas more weight is given to the most current values in order to detect trend changes more accurately. In economics, moving averages are often applied to stock prices, used to smooth out short-term fluctuations to highlight long-term trends. The threshold between short-term and long-term depends on the application.

The idea behind the use of moving averages is that they behave like trend lines that follow the movement of a stock. They are not restricted to being straight lines but may become rather curved and turn up or down, depending on the variation of the market price [80].

Simple Moving Average A Simple Moving Average (SMA) is the unweighted mean of the previous n data points. For a price series in which prices at time t are denoted by $p(t)$, the SMA can be obtained with the formula:

$$SMA(n,t) = \frac{\sum_{i=0}^{n} p(t-i)}{n} \tag{2.17}$$

assuming that $t - n > 0$.

An example of a 10, 50 and 100 period (where $n = 10, 50$ and 100 respectively) simple moving averages for a price time series is shown on Figure 2.7. In this Figure it can be seen that the smoothing of the curve increases as the number of periods in the moving average is increased.

Using the moving average of a price series it is possible to detect trends that the time series is carrying. These trends can then be used as information to obtain a forecast of the future value of the time series according to the observed trends.

The SMA is used in this thesis as the basis for one forecasting mechanism. There exist other more accurate models for price forecasting (such as auto-regression models) [45]. However, the SMA is used to create simulation scenarios where agents trade using a forecasting mechanism used in real markets with low accuracy to have agents with different views of the future price. An alternative forecasting mechanism is developed in this thesis, allowing the experimentation with specific levels of forecasting accuracy including per-

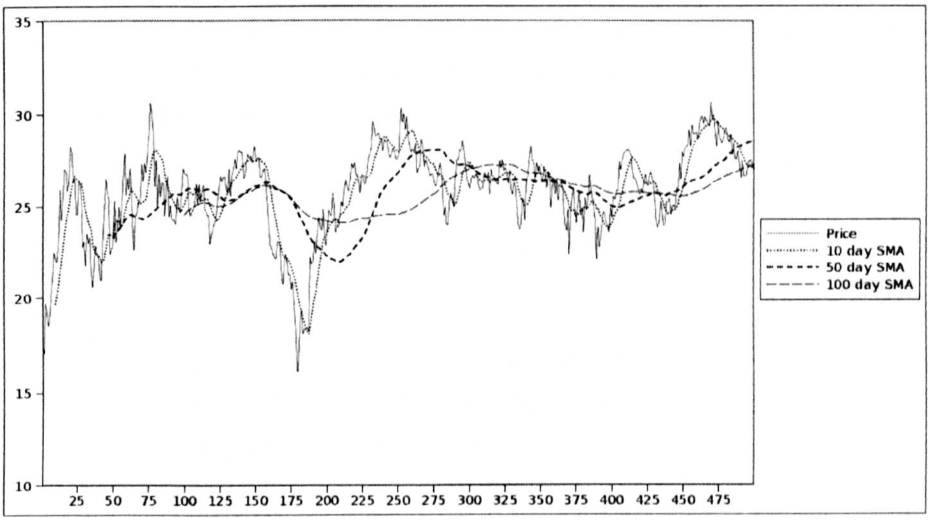

Figure 2.7: Example of the 10, 50 and 100 period simple moving averages of a price series.

fect forecasting. This mechanism is called α-Perfect forecasting and is covered in Section 3.2.2.2.

2.2.5 Risk and Uncertainty

In order to be able to deal with risk first a definition of it has to be adopted. As discussed in [50], there are many published efforts in establishing a definition of risk. One of the most important works on the definition of risks was carried out by Frank Knight in his work presented in [57] on 1921 where he establishes a difference between measurable and unmeasurable uncertainty:

> *It will appear that a measurable uncertainty, or "risk" proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an uncertainty at all. We ... accordingly restrict the term "uncertainty" to cases of the non-quantitative type.* (Paragraph 26, Chapter I, Part I)

defining the presence of risk when a probability can be assigned to future events, and uncertainty to those events for which no probability can be calculated. However, it was until 1944 that John von Neumann and Oskar Morgenstern work in [95] that uncertainty and risk was formally incorporated to economic decision theory. In this work, the authors model risk as probability values assigned to the different choices that are available for an agent.

Risk is usually defined in terms of uncertainty, which is the state of not knowing if a proposition is true or false. However, even if an agent observes some level of uncertainty against a specific proposition, that does not make the agent exposed to some risk. Hence, a second component, that of *exposure* needs to be considered when defining risk. Therefore, the definition of risk I use in this thesis is the one discussed in [50]:

> *If someone has a personal interest in what transpires, that person is exposed. Second, People don't know what will happen. ... the outcome is uncertain. ... Risk, then, is exposure to a proposition of which one is uncertain.*

Risk and uncertainty are important concepts to address to support decision making [78]. Managing the risks arising from exposure to unwanted outcomes can be achieved either by hedging or insurance. This involves guaranteeing future exchange rates or ensuring that losses from adverse movements are compensated by other gains [78]. However what these actions achieve is the transmission of certain risks to other entities which are willing to take such risks. This thesis deals with the risk arising from the fluctuations of the price of the asset (called market risk).

Market risk, is defined as the risk of losing wealth from the fluctuations in the price of a resource [78]. When an agent is exposed to a loss due to market fluctuations, there is a probability that the agent will lose large amounts of money. In order to reduce the magnitude and probability of such loss, agents can make use of financial instruments such as Derivatives.

Agents are exposed to this type of risk every time they trade in a market. However, agents are also exposed to risks derived from the specific system application. For example, in a Grid Multi-Agent System where agents bid and ask for specific resources, they face the risk of not obtaining some required resource at the time they need it. Or, as the demand and supply of resources varies, agents face the risk of having to pay very high prices in order to acquire a resource at the required time.

2.3 Agent Based Computational Economics

One of the many applications of Agent Based Modelling is the modelling of economic systems and the processes that develop in them. The study of such complex systems falls under the term Agent Based Computational Economics (ACE) [91], a methodology which is used to research the behaviour of economic systems modelled by systems composed of autonomous agents which develop through time.

One of the main objectives of ACE is the study of the self organisation phenomenon that arises from the interaction of the different agents participating in markets. The main concern is to realise how such patterns arise from the apparent non organised interaction that happens in such markets [90].

As with other uses of computational modelling, ACE allows the investigation of different phenomena in the modelled systems by permitting the experimentation of controlled scenarios which are relevant to the established hypotheses. Using this kind of experimentation, researchers can design experiments to see the behaviour of the system under certain scenarios that could not easily be tested in real markets (for example, it would be not practical if possible at all, to fix the number of traders in the real New York Stock exchange to test other factors in the market). Furthermore, it has been argued [4] that using ACE enables the investigation of the behaviour of markets without requiring the assumption of *market equilibrium* , or the assumption that agent choices will be rational.

There have been major research advances in the processes of Auctions and Stock Markets; specially in modelling the dynamics among the different entities that participate in those markets. Modelling these systems as Multi-Agent Systems is a sound approach given the complexity of the trading markets. The interaction among the individual entities can be represented as the interaction of different agents within their environment in accordance with a set of protocols.

2.3.1 Market Modelling

In order to develop an Option market, a market where the underlying asset is traded must be implemented first. There have been several efforts to model agent based stock markets, some of them are reviewed by Hakman, *et al.* in [44] which presents a survey of numerous approaches to model stock markets with Multi-Agent Systems. The authors give a characterisation of a stock market agent model. The proposed parts of the model consist of a set of independent agents that trade *stocks* or assets and *cash*. The model requires some mechanism which allows the agents to trade among them. The trading mechanism can be implement through a central clearing mechanism or using specialised clearing agents. In the same work, the authors characterise the agents participating in the market as having some *decision process* used to determine the actions of the agents. These agents base their decision on *information* they obtain. The information can be obtained from the market itself (like the historical prices of the assets) or from some type of estimation about the future states of the market (a forecasting). The majority of the models surveyed in the work of Hakman observe such properties, differing in the specific implementations of the components in the model.

One of the most famous models of artificial stock markets is the Santa Fe artificial stock market developed by Brian Arthur *et al.*, which is described in [5, 59, 71]. Their work studies the success of technical traders which attempt to predict the future price of the stock by analysing past market trends. To achieve this, the work relies on learning techniques such as genetic algorithms to improve the agents forecasting ability. Their work however, does not address the factors of risk management and hedging. The work I present in this thesis is partially based on the Santa Fe stock market. Specifically, my model of the underlying asset is a modification from the model proposed in their work described in [71]. However, in this thesis I do not use any kind of evolving algorithm for the decision process of the agents. Instead, I develop strategies that base their decisions in the expected utility obtained from executing an action, considering the probability of losing wealth by executing such action (i.e., risk).

2.4 Market Based Control

In contrast to ACE, research in Computer Science has been trying to adopt the mechanisms used by markets to control aspects of distributed computational systems such as resource allocation. This area of research has been coined Market Based Control (MBC) [26]. The idea of using market mechanisms for the control of computational systems has been researched since 1968 [88]. The use of distributed systems was first addressed by Miller *et al.* [66], where they argue that markets can promote efficient and cooperative interactions among agents with diverse knowledge, capabilities and goals. Moreover, they theorize that it is possible to use economic markets as a paradigm for the modelling of computer systems, exploiting the advantages of such a paradigm.

Market Based Control of distributed systems offers advantages over standard systems like self emergent behaviour [26]. Moreover if a distributed system problem is modelled as a Market Based resource allocation problem, it will be possible to use economic theory for the maximisation of resource allocation efficiency (among agents or traders participating in the markets) [26].

However, it has not been until recently that the processing power of computers has allowed the simulation of such systems to experiment with diverse market mechanisms. One of the most researched market mechanisms for the use in MBC is the auction. Auctions have played a very important role in free market models. The auction model dates back to 500B.C., when men used to bid for women, whom they wanted to marry on the streets of Babylon.[6] Auctions have changed constantly over years. The arrival of computers and the

[6]Described by Herodotus of Halicarnassos in the first book of his work named *The Histories* [48].

Internet bring new possibilities to auctions, as online auctioning services like *e-bay* and *Yahoo! Auctions* have had major impact on the trading of goods and services.

The auctioning model of trading resources has been applied to a wide range of computing problems such as grid resource allocation [21], adaptive sensor networks [96] and supply chain management [68]. Such works have demonstrated that there are advantages in the use of MBC mechanisms for the control of the modelled systems. It is argued in [84] that one of the main issues faced by Market Based resource allocation systems is the establishment of virtual or imaginary currencies, which are difficult to relate to real costs. The use of closed monetary systems allow the market traders to prevent real losses in the events of system errors. However, as systems become increasingly used to trade real world resources, there will be a need to map the traded resources to real money. This in consequence will bring the necessity of risk control over such monetary systems.

2.4.1 Software Models of Derivatives Markets

There has been little published efforts in the use of Financial Derivatives as a tool for MBC or in the computational modeling and simulation of those markets. One of the first research to use Derivatives to control computational systems is the work by Sutherland, published in [88]. In this work the author presents an experiment where users of a PDP computer system reserved slots of processing time in the future using an auction. This approach can be compared to *Futures* contracts. However, the work presented in [88] deals only with the model of the process–time allocation (i.e., the market mechanism) and it did not made any attempt to create software components which automatically participated in the market.

The work presented in [31] describes a series of experiments with a market that simulates a Futures Markets with non-rational trading agents. In this work they use genetic algorithms to evolve simple strategies to trade Future contracts. One point of interest for my thesis is that the authors provide a 10 day moving average as part of the information available for the trading agents. This moving average is used by the agents to calculate the *fair* price of the Future contract in order to make a bid in the market. The market model deals with Future contracts which expire one day after they are traded. The agents will choose to buy if their calculated fair price is higher than an equilibrium price (calculated from the agents fair prices, to balance the number of buyers and sellers); if the calculated fair price is lower than the equilibrium price then the agents will sell.

In [56], Alan King *et al.* describe a model of Derivatives markets. The work in [87] describes those experiments with more detail. The work presented by King *et al.* comprises a Multi-Agent model of a Derivatives market comprised of two types of agents which trade

Option contracts. The first type of agents are called *brokers* which are created specifically to fulfil the requests made by the *investors* which are the second type of traders. The *broker* agents are responsible for setting the price of the Options and have an unlimited number of assets to provide to the *investors*. The price is based on the Black-Scholes model for Option pricing. The *investor* agents are further divided in to two main sets that are differentiated by their position in the market according to some external constraints, the ones that aim to provide the underlying asset and others which will need to acquire the asset. The investors' actions are obtained as the result of a portfolio optimisation process where agents aim to maximise their utility measured by the present and possible future values of their portfolios.

The work by King *et al.* is relevant to my research because they make an approach to Option modelling. However, the work I present differs in several points. First, unlike King's work, the research I present in this thesis bases the model of the stock market in real Option Exchanges; thus, instead of using *broker* agents, the price of the Options are provided by the central market (similarly to how it is done in real world Exchanges). Also in my presented model, the number of assets are limited; for this reason, not all of the offers made by agents may be fulfilled. This constraint is implemented because I argue that, in Market Based distributed systems, resources are limited. Thus an agent trading in such markets must consider the availability of the resources when making a decision. Another difference is that, one of the objectives of the work I present is to compare the performance of Option trading agents against asset only trading agents in order to characterize scenarios where trading Options make sense; whereas in the work by King, *et al.*, there is no comparison of the agents' performance, but instead, the comparison is made among the difference in the positions taken by the trading agents.

The work presented by Sabrina Ecca *et al.*, in [36] investigates the impact that the use of Option Derivatives has in the agents' wealth and in the behaviour of the underlying market. In their approach, the market model consists of an underlying market in which four sets of agents trade with different strategies; these traders are: *random traders*, which submit offers to buy or sell assets randomly to the market; *fundamentalist traders* which establish their position depending on the difference between an equilibrium price and the market price of the asset; *momentum traders* are speculators that follow the trend of the markets directly (if the asset price is increasing then the agents will believe that it will continue to do so in the future); finally, *contrarian traders* which speculate that the direction of the price movement will change (the opposite of the momentum traders). Some of these agents are allowed to trade Options in addition to trading assets. This allows the authors to test different strategies used in real markets such as *covered positions* (trading an Option each time an asset is traded in order to "cover" their risk) or *straddles* (an Option trading strategy consisting of obtaining a put and a call Option contract at the same time).

In the work by Ecca *et al.*, the model makes use of a central bank which is in charge of fulfilling all the offers submitted by the agents. Thus, similarly to the work by King *et al.*[56], all the agents' offers will be matched and cleared. One of the objectives of their research is to test the difference in the performance of the agents in a market where prices are characterised by a *mean reverting* behaviour where the price varies among a predefined mean. Their results show that when using the strategies which they consider appropriate for the asset price behaviour (contrarian and fundamentalist traders), the use of Options does not benefit the traders, whereas agents that use strategies which are not adequate for the price behaviour will increase their performance when using Options. Their results also suggest that the use of Options decreases the volatility of the price of the underlying asset.

The work by Ecca *et al.* shares several concepts with the work presented in my thesis like the use of random trading agents and the use of speculators. However in my work the reasoning mechanism of the speculator strategy uses a different mechanism to choose the action to execute. Their work also intends to compare the difference between Option trading and asset only trading, but as mentioned before, the fact that all the submitted offers are cleared by a central bank makes their model fundamentally different from the work that I present in this thesis.

2.4.2 Relation With Previous Work

The work by Ecca *et al.* is focused on the study of the interaction between their stock Option market and the underlying stock market, whereas in my research the main interest lies in the comparison between the performance of agents that trade Options and agents that do not trade Options. Although they make use of agents with different strategies, these strategies are *hard coded* in the sense that they are constrained to execute a specific action (like, buy an asset and a call Option at each step) without performing any reasoning in their decision mechanism.

In contrast to my work, the work by King *et al.* [56] focuses on finding the performance of Option trading agents which have external constraints. Their experiments focuses on the comparison of the *positions* taken by Option trading agents that participate in the market with the different predefined external constraints. Their work does engage in the valuation of the agents portfolios with a combination of assets and goods. However, due to the complexity of the simulation of the agents decision mechanism, their results are based on the information obtained from 2 or 3 periods in the market.

The majority of the models surveyed focus in the modelling of Derivatives markets with the objective of researching their financial properties. Therefore, they can be classified as part of the ACE research effort. There has been very little effort in investigating the use of Derivatives as tools to manage the trading of resources in distributed computational systems (which is the aim of MBC). Moreover, none of the surveyed models attempt to represent the agents' uncertainty or risk, and therefore do not provide the agents with a mechanism to make their decisions based on the agents' *perception* of the future states of the market, considering probability. In this thesis, the decision of the agents are based on their perceived probability over different outcomes and a valuation of the utility of the possible actions (including trading Options) considering the uncertainty in the market. It is claimed that in order to make a successful use of Options, agents must consider the market risk in their strategies. The work described in my thesis focuses on the modelling of an Option trading Market and the development of strategies which reason about their risk in order to trade Option contracts. I do this research with the aim of looking for the possible applications of Option contracts as a tool for the management of risk in distributed computational systems.

2.4.3 Option Trading and Multi–Agent Resource Allocation

Multi–Agent Resource Allocation (MARA) is the field that studies the process of distributing items amongst a number of agents [24]. One of the most important issues when addressing a MARA problem is the mechanisms used to distribute the resources among the agents in a system. Auctioning is a type of mechanism that deals with the allocation of the items. In the last few years, auction mechanisms have been sucessfuly used as Market–Based control mechanisms for resource allocation in Multi–Agent Systems [42, 79].

Several of such auction mechanisms focus on finding the *equilibrium* or fair price of a resource at which agents will agree to trade it, according to a predetermined set of rules (the auction protocol) and to the agents' preferences.

Such auctioning frameworks can be extended with Derivatives trading mechanisms where agents trade contracts according their preferences. Using parallel Derivative markets such as Options will provide the agents a richer set of strategies for the maximization of their utility. This would allow the use of Derivatives such as Options in order to ensure the availability of the resources at future times and, secondly, in order to hedge the risk given by the variations of the fair price of the resources traded in the auction market.

The basic properties that need to be available in order to run an Option market model is an underlying market where some resource is traded. Such resource must have an equilibrium price, as this is the price from which the Option price will be obtained. There must be a measure of risk–free interest rate. Evidently, the market for the underlying asset must provide a definition of cash, or a common manner to identify (between the participants) the general value of the resource being traded. Although different agents may obtain different utility values for a specific resource, they must be able to specify such values in terms of an amount of cash, in order to exchange the resource in the market.

Under these assumptions, the development of the parallel Option market will provide agents with a value of the guarantee of trading assets at a future time, in terms of the same generic unit (i.e. the cash). This quantification is convenient because the agents will then be able to include in their decision mechanisms the possibilities provided by the Option market, comparing it with the possibilities available in the market of the resource.

2.5 Summary

In this Chapter, a survey of the concepts of the fields related to the current thesis were discussed. The research presented in this thesis makes use of advances in Financial Economics, Agent-Based Computational Economics and Market Based Control to extend the field of Computer Science with the development of a Market Based Control mechanism that will allow the management, through hedging or insurance, of the risks that are generated in artificial markets.

This work borrows heavily from the Financial field but does not pretend to be a deep study of the subjects it adopts, instead, the research tries to investigate what are the advantages and disadvantages that the proposed approach will bring to Market Based Control systems which implement the proposed framework.

The research presented in this thesis makes a contribution to Market Based Control in providing an Option market framework. This framework allows autonomous agents participating in an electronic market to trade Options in order to hedge the risk inherent to trading in such electronic markets. My work also contributes to MBC by providing the strategies that allow autonomous agents to consider Option trading reasoning about the market risk exposed by their possible actions.

Although this thesis is not particularly aimed at Agent Based Computational Economics, my work also provides ACE with a model of an Option market which is comprised by sufficient properties to mimic a real Exchange market. My provided model can be extended to experiment with scenarios in which different types of Options are traded (such as American or Swing Options), by integrating a suitable Option pricing model.

Chapter 3

Option Trading Model

This chapter describes the theoretical model of the Option trading framework. In Section 3.1, the chapter first describes the asset and Option market mechanisms where agents will be able to submit offers to trade. The definition of the agents as well as the developed strategies and forecasting mechanisms used by the agents are described on Section 3.2. Finally, Section 3.3 presents a summary of the chapter.

3.1 The Option Exchange Market

In order to define the Option trading market, the market for the underlying asset must be defined first. The market of the underlying asset is the basis of the Option market. It is from the information in the market of the underlying asset that the Option contracts are defined.

My model is a simplified version of the market created by Palmer, *et al.* described in [71]. My model considers *goods* or assets instead of *stock*; the goods cannot be divided and I only consider one type of good. It is also assumed that no dividends are distributed among agents, given that the distribution of dividends is a particular property of stocks and the present model is meant to be used for computing resources. These simplifications are established in order to use the minimal properties of a simple market that allows trading. As will be shown later in this chapter, the used definition of the market is rich enough to be the basis for the Option trading framework.

The market is composed by a set of agents $A = \{A_1, A_2, A_3, .., A_N\}$. The sub-index i is used to refer to a specific agent in the set (i.e., $A_i \in A$ denotes an agent from the set), therefore the variables that include a sub-index i correspond to values that are specific for one agent. A is composed of two subsets. The set of agents that can only trade goods is denoted by A_g. The set of agents that can trade assets as well as Option contracts is denoted by A_o.

The life of the market is defined by discrete time steps $t = \{0, 1, 2, 3, ..., T\}$, the minimum unit of time in the market is a period, which is defined as:

Definition 1. *Period: The events that happen in the market between two consecutive time steps* $[t, t+1]$.

At the beginning of each step t, each agent A_i has a number of goods $g_i(t)$ and an amount of cash $c_i(t)$. The total number of goods in the model is fixed, being $\sum_i g_i(t) = G$ for all t. Each agent also has an Option portfolio $\mathcal{O}_i(t)$ which is composed by the Option contracts the agent holds $O_i^h(t)$ and the Options it wrote $O_i^w(t)$.

An Option o is defined by the tuple:

$$\langle X^o, t^o, v^o, \tau^o \rangle \tag{3.1}$$

Where X^o is the exercise price of that Option (the price agreed to pay for each good at expiration time); t^o is the expiration time (the time when the Option has to be exercised); v^o is the volume (the quantity of goods to trade when exercising that Option) and τ^o is the type of Option (either call or put), specifying if the holder is going to buy or sell the assets traded in the Option.

For every Option o there is a a corresponding premium p_o. This is the price an agent has to pay to hold the Option. This premium is calculated at the time when the Option is defined.

At each step in time, the market provides a set of Option templates to trade. These templates, denoted by $O(t)$, define the Option contracts available to trade at step t. Each template provides the values of X^o, t^o, v^o and τ^o available for trading. Agents are only allowed to submit offers to hold or write Options which match any of the Option templates. Given these assumptions, there can be more than one identical offer submitted by different agents in the same step.

Finally, there is an interest rate of return associated to the market, this rate is defined as r and is applied to the cash owned by the agents at the end of a period. The rate r has the range $[0, 1]$.

3.1.1 Pricing Mechanism

The price of the goods is defined as a set P where $p(t) \in P$ is the price of one good at time t. This price is obtained form an exogenous source. The model does not define a specific mechanism to calculate the price of the asset, and it is treated as a random time series. The only assumption about the asset price made by the model is that the market and the agents are capable of accessing the history of past prices of the asset for a defined time horizon. The historic asset prices are required by the market and the agents to calculate statistical properties of the asset prices. The specific used statistical information will be described later. Treating the asset price series as an exogenous variable allows the asset market mechanism to be independent from the Option market.

The price of the Option contracts p_o is calculated each step using the Black-Scholes model for Option pricing (described in Section 2.2.2.3). To calculate the price of an Option the formula takes the form:

$$p_o = F(p(t), \sigma_R^2(t), X^o, \tau^o, r) \tag{3.2}$$

The parameters used as input for the Black-Scholes formula are: The price of the good at the current step $p(t)$; the value of the strike price of the asset (X^o) and type of Option defined by τ^o; the risk free interest rate of return r; and the variance of the price returns for the elapsed steps in the market defined as σ_R^2(t). Specifically, if t_{ini} is the first step in the market and t is the current step, then the variance is defined as:

$$\sigma_R^2(t) = \frac{\sum_{k=t_{ini}}^{t} (R(k) - \mu)^2}{t - t_{ini}} \tag{3.3}$$

where μ is the mean of the returns for the specified steps defined by:

$$\mu = \frac{\sum_{k=t_{ini}}^{t} R(k)}{t - t_{ini}} \tag{3.4}$$

$R(k)$ is the rate of returns (defined in Section 2.2.2.3) of the asset at time k.

3.1.2 Market Time line

Each period of time starts when the market *publishes* the new price for the asset. Then the *Option clearing phase* starts, where the market receives instructions from the agents to exercise any Option that expires at this time. The agents holding any expiring Option must either decide to exercise the Option or choose to let it expire without exercising. All the Options that expire at this time are removed from the set of the agents' held Options O_i^h and the corresponding written Options are removed from the sets O_i^w of the agents. When an Option is exercised, the agents clear the Option by immediately trading the corresponding assets at the Option established exercise price X^o. Next, the market publishes the new Option templates to trade on the present period and the *trading phase* begins, where the agents submit their offers to buy and sell assets or hold and write Options. Next, the market tries to randomly match the buy and sell offers. To do this, the market will first randomize the order of the set of all the offers to buy goods $V_b(t)$ and also randomize the order of the offers to sell goods $V_s(t)$. if $|V_b(t)| < |V_s(t)|$ then the market will clear the offers to sell assets $V_s(t)$ until there are no more offers to buy assets $V_b(t)$. Likewise, if $|V_b(t)| > |V_s(t)|$, the market will clear all the offers to buy assets until there are no more offers to sell.

Similarly, the market will try to match the hold and write offers. This is done by randomizing the order of the offers to write $V_w(t)$ and the offers to hold $V_h(t)$ Options, and subsequently traversing the list of offers to hold Options ($V_h(t)$) until it is not possible to match any offer. The cleared Options are added to the agents' Option set ($O_i(t)$) and the Options not matched will be ignored. A graphical representation of the time line is shown in Figure 3.1. Using this clearing mechanism not all the agents may be able to clear their offers but the market will ensure a fair (although inefficient) allocation of the goods.

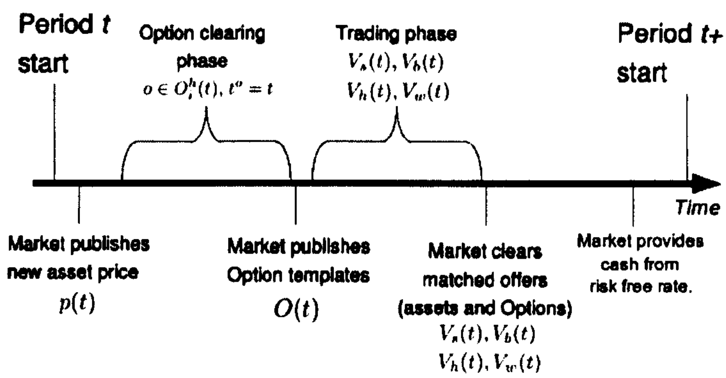

Figure 3.1: Timeline for one time step of the market

3.2 Trading Agents

The definition of an agent used in my model is a general definition described in [99] which defines an agent as a software computer program with the four basic properties of autonomy, social ability, reactivity and pro-activeness. The agents in my model are said to be autonomous because they will choose their course of action depending only on their strategies and their perceived state of their environment (the market). The agents have social ability given that they interact with other agents by trading Options and assets through the market. Agents are reactive as they use the changes in the state of their environment to select their next actions. Finally, the created agents are pro-active given that their actions are guided by their specific objective which depends on the strategy they use.

An agent A_i trading in the market is formally defined by the tuple:

$$\langle g_i, c_i, w_i, \mathcal{O}_i, \mathcal{S}_i, \mathcal{F}_i \rangle \tag{3.5}$$

At time t, the term $g_i(t)$ is the number of goods the agent owns; $c_i(t)$ denotes the quantity of cash the agent has. The term $w_i(t)$ denotes the *wealth* of the agent which is obtained by the equation:

$$w_i(t) = p(t) \times g_i(t) + c_i(t) \tag{3.6}$$

The agent also owns a set of Option contracts $\mathcal{O}_i(t)$ either written or held at previous time steps and that will expire at a future time. The term $\mathcal{S}_i$ is the agent's strategy used to select actions to perform at each step. Finally, $\mathcal{F}_i$ is the forecast function used by the agents to obtain a forecast of the price of the asset at future times.

For each agent A_i with a wealth $w_i(t)$ at time t, the *wealth return* is also defined as:

$$R_i(t) = ln(\frac{w_i(t+1)}{w_i(t)}) \tag{3.7}$$

Indicating a rate of profit or loss obtained by the agent in the period $[t, t+1]$. The wealth return is similar to the stock returns described in Section 2.2.2.3. However, while the stock returns represent the rate of profit or loss observed in the stock or asset price, the returns on the agents measure the particular profit or loss observed by a specific agent.

3.2.1 Actions

At each step t, the agent has to decide on what Options to exercise during the *Option clearing phase* and what action to execute in the *trading phase* . The action is chosen from a set of actions, and the Options to exercise are the subset of the agent's held Options $O_i^h(t)$ that expire at t.

3.2.1.1 Option Exercise

After obtaining the price of the asset for the new period, the agent will choose which Options to exercise from the set of held Options $O_i^h(t)$. The rules used to choose to exercise an Option are:

$$\begin{aligned}(\tau^o = call \wedge t^o = t) &\Rightarrow (exercise(o,t) \Leftrightarrow X^o < p(t)) \\ (\tau^o = put \wedge t^o = t) &\Rightarrow (exercise(o,t) \Leftrightarrow X^o > p(t))\end{aligned} \tag{3.8}$$

That is, an agent will only exercise a *call* Option if the price it will pay for the assets by trading the Option is lower than the current price of the assets in the market (i.e., if $X^o < p(t)$). Doing this, the agent will buy assets from the Option writer at a price which is lower than the current market price, making a profit of $p(t) - X^o$. Similarly, the agent will only exercise a *put* Option if the exercise price is higher than the current asset price in the market (i.e., when $X^o > p(t)$). Doing this, the agent will sell assets to the Option writer at a price which is higher than the current market price, making a profit of $X^o - p(t)$.

3.2.1.2 Market Trading

After the *Option clearing phase* is finished, agents enter the *trading phase*. There are five basic actions that an agent can perform during the *trading phase*. These actions denote offers submitted to the market. The set of actions an agent can execute are listed in Table 3.1.

Action	Description
$buy(g,t)$	Make an offer to buy an asset at time t.
$sell(g,t)$	Make an offer to sell an asset at time t
$hold(o,t)$	Make an offer to hold an Option o at time t
$write(o,t)$	Make an offer to write an Option o at time t
$pass(t)$	Do not make any offer

Table 3.1: Available actions for the agents during the *trading phase* at time t.

The $buy(g,t)$ and $sell(g,t)$ actions denote an offer to buy g number of assets in the market at step t accordingly. The $hold(o,t)$ and $write(o,t)$ actions denote an offer to hold or write an Option o in the market at time t. Finally, the $pass(t)$ action denotes that the agent will not submit any offer to the market at time t.

The $buy(g,t)$ and $sell(g,t)$ actions are not strictly part of the Option market. These actions can be omitted from the model when the market of the underlying asset is modelled separately from the Option market. However, the actions are added in the current model to allow agents to trade assets without the use of Options.

3.2.2 Forecasting

Agents are provided with a forecasting function $\mathcal{F}_i$ which they use in order to formulate a belief about the future state of the market. Two forecasting mechanisms are developed, the SMA_n forecasting function and the α–Perfect forecasting function. The agents make use of these functions in some of the trading strategies described in Section 3.2.4.

3.2.2.1 Simple Moving Average Forecasting

The first forecasting function is based on the Simple Moving Average function described in Section 2.2.4.1. Prices at future times are obtained by first calculating the SMA for an interval window of $[t - n, t]$ as $SMA(n, t)$, where t is the present time step. Then the forecasted price of agent A_i at future time step $t + m$ is obtained by extrapolating the price at current time using the formula:

$$SMA_n(t + m) = p(t) + m \times (p(t) - SMA(n, t)) \tag{3.9}$$

Where $SMA_n(t + m)$ is the agent's forecasted price for time $t + m$ and $p(t)$ is the market price at time t. The Simple Moving Average forecasting function is then denoted by $SMA_n(m)$.

3.2.2.2 α–Perfect Forecasting

In the second forecasting function, the α–Perfect forecasting, agents obtain the future prices from the actual price series. This forecasting function can only be used when the price of the asset is known for all future time steps. The real asset price is modified multiplying it by a noise factor bounded by the parameter α. The forecasted price is calculated as:

$$p_i(t + m) = p(t + m) \times (1 + k_\alpha) \tag{3.10}$$

Where $p_i(t + m)$ is the resulting agent's forecasted price at $t + m$, $p(t + m)$ is the real market asset price at time $t + m$ and k_α is a uniformly distributed pseudo–random number within the range $[\alpha - 1, 1 - \alpha]$, being α within the range of $[0, 1]$. Using this function it is possible to provide an agent with complete knowledge of future asset prices making $\alpha = 1$.

As it is clearly seen, it may not be possible to use the α–Perfect forecasting function in real electronic markets, given that the future price of the asset is usually unknown to the market. However, the α–Perfect forecasting function is developed to test the performance of the trading strategies under different known degrees of uncertainty. By varying the value of α it is possible to provide the trading agents with different degrees of uncertainty about the future price.

3.2.3 Perceived Risk

Following the definition described in 2.2.5, risk is modelled as the probability that the agent loses wealth when it carries out a specific action. To calculate this probability, it is assumed that price changes are distributed according to a Normal distribution $N(\mu, \sigma)$. Under this assupmtion, an agent defines the probability distribution of the prices using a forecasted price $p_i(t')$ (obtained using one of the previously described forecasting functions) for some future time t' as the mean of the distribution μ. The standard deviation of the Normal distribution is then given by the standard deviation of the historic asset price series σ_P, which is formally defined as:

$$\sigma_P = \sqrt{\frac{\sum_{k=t_{ini}}^{t}(p(k) - \mu_P)^2}{t - t_{ini}}} \tag{3.11}$$

Where μ_P is the mean of the historic asset prices within the range $[t_{ini}, t]$, t_{ini} is the first price in the history of the asset prices and t is the current price of the asset. As a result, the agent will formulate a model of the future of the market price with with the form $N(p_i(t'), \sigma_P)$.

The probability of losing wealth after executing an action a at a step t (i.e., the risk that comes from the exposure to the uncertainty by executing action a) is then defined as:

$$P\left[w_i(t') < w_i(t) | a; N(p_i(t'), \sigma_P)\right] \tag{3.12}$$

Denoting the probability that the agent's wealth at time t' is less than the wealth at time t (represented by $w(t') < w(t)$) given that the agent executes action a, and given the distribution $N(p_i(t'), \sigma_P)$. The probability of losing wealth is then calculated using the standard Normal cumulative distribution function (c.d.f.) $\Phi(z)$,[1] and because the probability of losing wealth depends on the action the agent executes, the value of z will depend on the action for which the probability is being calculated. In the case of asset trading, z will be the standardized value of the asset price at the current time step t. Alternatively, in the case of Option trading, the value of z will be the standardized value of the exercise price and the Option premium $X^o \pm p_o$, depending on the type of the Option (the specific cases will be described later in this chapter).

With this model an agent is able to calculate, with basis on its model of the volatility of the asset price (given by the Normal distribution), the probability that the execution of an action will lead to a loss of wealth.

Using the calculated probability, an agent then calculates the *risk loss factor* ρ_L which is a combination of the probability of losing wealth and the magnitude of the loss that the agent can incur after executing an action. The general definition of ρ_L is:

Definition 2. *The risk loss factor ρ_L is the product of the probability that after executing an action, an agent loses wealth due to a change of the asset price multiplied by the maximum*

[1] The definition of the standard Normal c.d.f. is shown in Appendix A.

possible magnitude of the loss. That is:

$$\rho_L = P\left[w_i(t') < w_i(t)|a; N(p_i(t'), \sigma_P)\right] \times \textit{Maximum Possible Magnitude of the Loss} \tag{3.13}$$

In a similar way to ρ_L, a measure of the probability of gaining wealth in each action is calculated. This is used to obtain the *risk gain factor*[2] ρ_G, which is defined as:

Definition 3. *The risk gain factor ρ_G is the product of the probability that after executing an action, an agent gains wealth due to a change of the asset price, multiplied by the magnitude of the possible gain. That is generally:*

$$\rho_G = P\left[w_i(t') > w_i(t)|a; N(p_i(t'), \sigma_P)\right] \times \textit{Maximum Possible Magnitude of the Profit} \tag{3.14}$$

To obtain the *Maximum Possible Magnitude of Loss* and *Maximum Possible Magnitude of Profit*, an approximation is used in the cases where the real magnitude would be either positive of negative infinity; considering that although the maximum wealth an agent can lose or gain may tend to infinity, the probability of obtaining such wealth is very low.

The maximum magnitude of the profit or loss is then bounded considering that according to the properties of the Normal distribution, 99.7% of the values in the distribution will lie within $3\sigma_P$ (at three standard deviations of the mean) in the Normal probability density function (p.d.f). Therefore, it is considered that the *Maximum Possible Magnitude of Loss* and from *Maximum Possible Magnitude of Profits* Equations 3.13 and 3.14 is equal to $3\sigma_P$.

As stated before, the value of the parameters for the standard Normal c.d.f. $\Phi(z)$ depend on the evaluated action. For the action $pass(t)$, the ρ_L and ρ_G values are both equal to 0, given that the valuation of the risk is based on the change in the amount of cash and assets of the agent between two steps in the market relative to doing nothing. The definitions of the parameters used to calculate the risk factors for the rest of the actions are described next.

3.2.3.1 Buying An Asset

When the action a is $buy(g, t)$, the c.d.f. parameter z is defined as:

$$z = \frac{p(t) - p_i(t+1)}{\sigma_P} \tag{3.15}$$

And ρ_L is calculated with the function:

$$\rho_L = \Phi(z) \times 3\sigma_P \tag{3.16}$$

[2]Although my used definition of risk is the probability of losing wealth, I chose to use the name *risk gain factor* to make clear that there is a relation between ρ_L and ρ_G as it will be shown later.

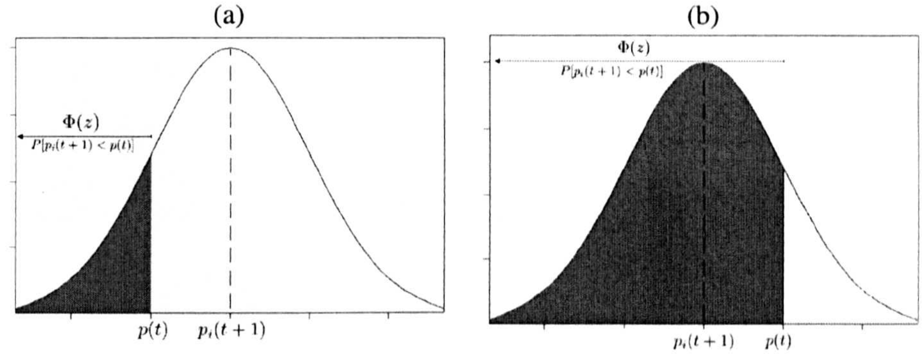

Figure 3.2: Graphical representation of the probability of wealth loss when buying an asset given a forecasted price $p_i(t+1)$ for (a) $p_i(t+1) > p(t)$ and (b) $p_i(t+1) < p(t)$.

where σ_P is the standard deviation of the historic prices of the asset; $p(t)$ is the current price of the asset and $p_i(t+1)$ is the agent's forecasted price for time $t+1$.

The term $\Phi(z)$ in Equation 3.16 is used to obtain the probability that $p_i(t+1) < p(t)$, that is, the cumulative probability from $[-\infty, p(t)]$. The value of z is the standardized value of $p(t)$ in order to obtain the equivalent probability $P[p_i(t+1) < p(t)]$ using the standard Normal c.d.f $\Phi(z)$.

Figure 3.2 shows a graphical representation of two possible outcomes of $\Phi(z)$ when executing the action $buy(g,t)$. The grey area under the curve in Figure 3.2a represents the probability of wealth loss in the case when, according to the agent's forecast, the state of the world at the next step will be $p_i(t+1) > p(t)$. The grey area under the curve in Figure 3.2b represents the probability of wealth loss when the agent's forecast is such that $p_i(t+1) < p(t)$.

The term $3\sigma_P$ in Equation 3.16 represents the maximum amount of cash that an agent can lose if its forecasting is wrong. In the case of buying an asset, the agent can theoreticaly lose an infinite amount of wealth, for this reason the approximated magnitude of wealth loss is used.

To obtain the value of ρ_G for the action of buying an asset, the value of $1 - \Phi(z)$ is used as the probability of gaining wealth due to buying an asset. The value of ρ_G is calculated as:

$$\rho_G = (1 - \Phi(z)) \times 3\sigma_P \tag{3.17}$$

The value of z will be the same as the one used to calculate ρ_L for the same action. The probability of gaining wealth will be the complement of the probability of losing wealth. This can be seen in Figure 3.2, where the white area under the curve represents the probability of gaining wealth.

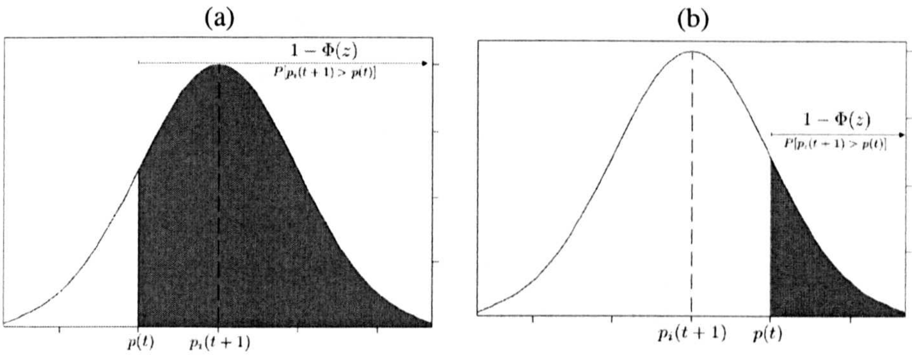

Figure 3.3: Graphical representation of the probability of wealth loss when selling an asset given a forecasted price $p_i(t+1)$ for (a) $p_i(t+1) > p(t)$ and (b) $p_i(t+1) < p(t)$.

3.2.3.2 Selling An Asset

When the action a is $sell(g, t)$, the c.d.f. parameter z is defined as:

$$z = \frac{p(t) - p_i(t+1)}{\sigma_P} \tag{3.18}$$

And ρ_L is the calculated with the function:

$$\rho_L = (1 - \Phi(z)) \times 3\sigma_P \tag{3.19}$$

where σ_P is the standard deviation of the asset price series; $p(t)$ is the current price of the asset and $p_i(t+1)$ is the agent's forecasted price for time $t+1$. In this case, $1 - \Phi(z)$ is used to obtain the probability that $p_i(t+1) > p(t)$, that is, the cumulative probability within the range $[p(t), \infty]$.

Figure 3.3 shows the graphical representation of two possible outcomes of $1 - \Phi(z)$ when executing the action $sell(g, t)$. The grey area under the curve in Figure 3.3a represents the probability of wealth loss in the case when according the agent's forecast, it believes that the state of the world at the next step will be such that $p_i(t+1) > p(t)$. Alternatively, the grey area under the curve in Figure 3.3b represents the probability of wealth loss when the believes that $p_i(t+1) < p(t)$.

Similarly to the function to calculate ρ_L when buying an asset, $3\sigma_P$ represents the magnitude of wealth that an agent can lose if its forecasting is wrong; as well as the magnitude of wealth that an agent can gain.

To obtain the value of ρ_G, $\Phi(z)$ is used as the probability of gaining wealth due to selling an asset. The value of ρ_G is calculated as:

$$\rho_G = \Phi(z) \times 3\sigma_P \tag{3.20}$$

3.2.3.3 Holding An Option

When the action a is $hold(o,t)$, the value of ρ_L depends on the type of the Option τ^o. When $\tau^o = call$, the value of ρ_L is calculated as:

$$\rho_L = \Phi(z) \times p_o \tag{3.21}$$

where p_o is the price of the Option o. The value probability of losing wealth can be derived from Equation 2.6, from this equation we know that the loss of an agent holding a call Option is given by $Loss = Min(0, X - p(t^o)) + p_o$. Therefore, an agent will loss wealth when:

$$\begin{aligned} X - p_i(t^o) + p_o &> 0 \text{ or} \\ p_i(t^o) &< X + p_o \end{aligned} \tag{3.22}$$

Then, to obtain the probability that the agents incurs in a loss it is necessary to obtain $P[p_i(t^o) < X + p_o]$. Under this assumptions, the variable z is defined as:

$$z = \frac{(X^o + p_o) - p_i(t^o)}{\sigma_P} \tag{3.23}$$

where X^o is the Option strike price and $p_i(t^o)$ is the agent's forecasted price for time t^o (the time when the Option expires) and σ_P is the standard deviation of the asset price series.

Equation 3.21 may be analysed in two separated sections. First, $\Phi(z)$, is used to obtain the probability that $p_i(t^o) < X^o + p_o$, that is, the cumulative probability in the range $[-\infty, X + p_o]$, assuming that the price follows a Normal distribution with a mean of $p(t^o)$ and a standard deviation of σ_P. The value of z is the standardized value of $X^o + p_o$ in order to obtain the equivalent probability $P[p_i(t^o) < X^o + p_o]$ using the standard Normal c.d.f. $\Phi(z)$.

Figure 3.4 shows the graphical representation of two possible outcomes of $\Phi(z)$ when executing the action $hold(o,t)$, with $\tau^o = call$. The grey area under the curve represents the probability of losing wealth on each scenario. Figure 3.4a presents the case when according to the agent's forecasted price, it believes that $p_i(t^o) > X^o + p_o$. Similarly, Figure 3.4b presents the case when according to the agent's forecasted price, it believes that $p_i(t^o) < X^o + p_o$.

The second part of Equation 3.21, which is p_o, represents the maximum amount of cash that an agent can lose if its forecasting is wrong. In contrast to the actions of buying and selling assets, the maximum loss an agent can incur when holding an Option is the Option premium p_o.

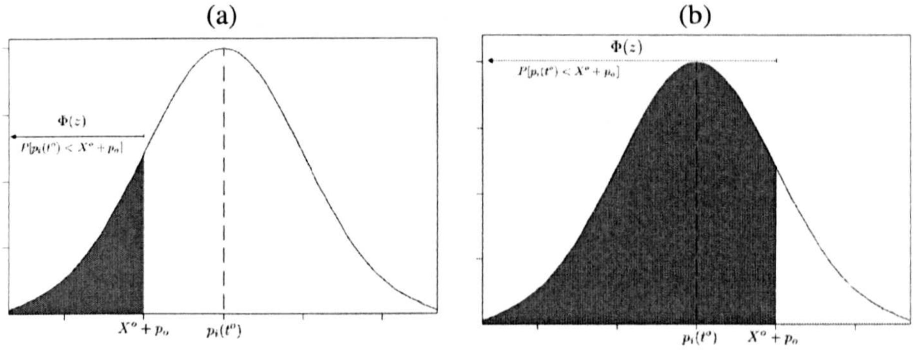

Figure 3.4: Graphical representation of the probability of wealth loss when holding a call Option given a forecasted price $p_i(t^o)$ for (a) $p_i(t^o) > X^o + p_o$ and (b) $p_i(t^o) < X^o + p_o$.

To calculate the value of ρ_G, the value of $1 - \Phi(z)$ is used as the probability of gaining wealth. In the case of holding an Option, the value of ρ_G is obtained using the equation:

$$\rho_G = (1 - \Phi(z)) \times 3\sigma_P \tag{3.24}$$

In the case of ρ_G, the magnitude of gaining wealth is bounded to $3\sigma_P$ given that the agent's maximum wealth gain is theoretically infinite.

If the type of the Option to hold is $\tau^o = put$ then it is possible to calculate the value of ρ_L from Equation 2.8 which defines the loss of holding a put Option as $Loss = Min(0, p(t^o) - X) + p_o$. Therefore, an agent will incur in a loss if:

$$\begin{aligned} p_i(t^o) - X + p_o &> 0 \text{ or} \\ p_i(t^o) &> X - p_o \end{aligned} \tag{3.25}$$

And the probability of losing wealth in this scenario is given by $P[p_i(t^o) > X - p_o]$ with z defined as:

$$z = \frac{(X^o - p_o) - p_i(t^o)}{\sigma_P} \tag{3.26}$$

Under this assumptions, the value of the *risk loss factor* ρ_L is calculated as:

$$\rho_L = (1 - \Phi(z)) \times p_o \tag{3.27}$$

and the value of ρ_G for the put Option will be:

$$\rho_G = \Phi(z) \times 3\sigma_P \tag{3.28}$$

The difference between the magnitudes of wealth gain ($3\sigma_p$) and wealth loss (p_o) arising from holding an Option exposes the possibility of an agent to obtain high profits while limiting its losses (thus, having low risk).

3.2.3.4 Writing An Option

When the action a is $write(o, t)$, the value of ρ_L depends on the type of the Option τ^o. When $\tau^o = call$, the value of ρ_L is calculated as:

$$\rho_L = (1 - \Phi(z)) \times 3\sigma_P \tag{3.29}$$

where σ_P is the standard deviation of the asset price series; p_o is the price of the Option o. The probability of losing wealth when writing a call Option can be derived from Equation 2.4 which defines a loss as $Loss = Max(0, p(t^o) - X) - p_o$. Therefore, an agent will incur in a loss if:

$$\begin{aligned} p(t^o) - X - p_o &> 0 \text{ or} \\ p(t^o) &> X + p_o \end{aligned} \tag{3.30}$$

The probability of losing wealth in this scenario is then given by $P[p(t^o) > X + p_o]$ and z will be defined as:

$$z = \frac{(X^o + p_o) - p_i(t^o)}{\sigma_P} \tag{3.31}$$

where X^o is the Option strike price and $p_i(t^o)$ is the agent's forecasted price for time t^o (the time when the Option expires).

Figure 3.5 shows the graphical representation of two possible outcomes of $1 - \Phi(z)$ when executing the action $write(o, t)$, with $\tau^o = call$. The grey area under the curve represents the probability of losing wealth on each scenario. Figure 3.5a presents the case when according to the agent's forecasted price, it believes that $p_i(t^o) > X^o + p_o$. Similarly, Figure 3.5b presents the case when according to the agent's forecasted price, it believes that $p_i(t^o) < X^o + p_o$.

To calculate the value of ρ_G, the value of $\Phi(z)$ is used as the probability of gaining wealth. In the case of holding an Option, the value of ρ_G is obtained using the equation:

$$\rho_G = \Phi(z) \times p_o \tag{3.32}$$

If the type of the Option to write is $\tau^o = put$ then it is possible to calculate the value of ρ_L from Equation 2.2 which defines the loss of holding a put Option as $Loss = Max(0, X - p(t^o)) - p_o$. Therefore, an agent will incur in a loss if:

$$\begin{aligned} X - p(t^o) - p_o &> 0 \text{ or} \\ p(t^o) &< X - p_o \end{aligned} \tag{3.33}$$

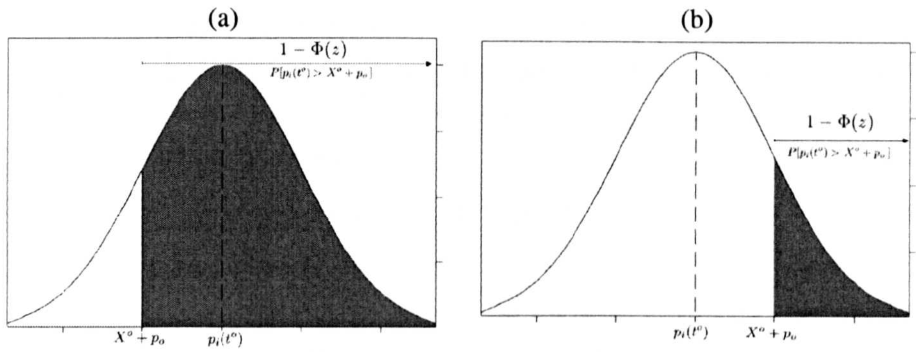

Figure 3.5: Graphical representation of the probability of wealth loss when writing a call Option given a forecasted price $p_i(t^o)$ for (a) $p_i(t^o) > X^o + p_o$ and (b) $p_i(t^o) < X^o + p_o$.

The probability of losing wealth in this scenario is given by $P[p(t^o) < X - p_o]$ and z will be defined as:

$$z = \frac{(X^o - p_o) - p_i(t^o)}{\sigma_P} \tag{3.34}$$

Under this assumptions, the *risk loss factor* ρ_L is calculated with the formula:

$$\rho_L = \Phi(z) \times 3\sigma_P \tag{3.35}$$

and the value of the *risk gain factor* ρ_G is:

$$\rho_G = (1 - \Phi(z)) \times p_o \tag{3.36}$$

3.2.4 Trading Strategies

To select the offer to submit to the market at one time step, an agent generates a set with the possible actions available for the present time step. The available actions at time t are the offers to buy and sell goods and the offers to write and hold the Option contracts that are published by the market for that time. Figure 3.6 shows an example of the possible actions available in the set at a time t.

Each element in the set is composed by the action a that the agent will execute, the new state of the agent after executing the action (the number of goods and cash as well as the set of Options) and the risk factor ρ_L and ρ_G corresponding to that action.

The available actions in the list are constrained by the following formulas:

$$\begin{aligned} g(t+1) &\geq 0 \\ c(t+1) &\geq 0 \end{aligned} \tag{3.37}$$

If an action does not fulfill any of these constraints then it is not added to the set of actions. These constraints make sure the agent has enough resources to execute the actions in the action set. The set of possible actions is used by different strategies to select the next offer to submit to the market.

$buy(g,t)$	$hold(o_1,t)$	$hold(o_2,t)$	$pass(t)$
$\rho_L = 6.5$	$\rho_L = .35$	$\rho_L = 1.2$	$\rho_L = 0$
$\rho_G = 3.5$	$\rho_G = 4.3$	$\rho_G = 2.3$	$\rho_G = 0$
$g(t+1) = 16$	$g(t+1) = 15$	$g(t+1) = 15$	$g(t+1) = 15$
$c(t+1) = 430$	$c(t+1) = 439$	$c(t+1) = 438.3$	$c(t+1) = 440$
$sell(g,t)$	$write(o_1,t)$	$write(o_1,t)$	
$\rho_L = 3.5$	$\rho_L = 4.3$	$\rho_L = 2.3$	
$\rho_G = 6.5$	$\rho_G = .35$	$\rho_G = 1.2$	
$g(t+1) = 14$	$g(t+1) = 15$	$g(t+1) = 15$	
$c(t+1) = 450$	$c(t+1) = 441$	$c(t+1) = 441.7$	

Figure 3.6: Example of possible action choices for an agent for a time step t.

There are six trading strategies developed for the market. Two strategies allow agents to trade only assets and are called Asset Trading Strategies (ATS) and four strategies allow agents to trade assets as well as Options; these strategies are named Option Trading Strategies (OTS). The ATS are the *ATSpec* strategy and the *ATNoise* strategy; the OTS are are *OTMinR*, *OTMaxW*, *OTMix* and *OTRnd*.

3.2.4.1 Asset trading strategies

Agents using the *ATNoise* strategy will select to buy assets, sell assets or pass according to a uniform random distribution function. The strategy will select one action at every step which will be the offer submitted to the market. The strategy is formally defined as:

$$\mathcal{S}_i(t) = RND(\{buy(g,t), sell(g,t), pass(t)\}) \tag{3.38}$$

where $RND(\cdot)$ is a uniform random distribution function that returns one of the actions from the set of possible actions selecting randomly. The number of assets in each offer is $g = 1$.

Using the *ATSpec* trading strategy, agents can select among buying an asset, selling an asset and passing depending on their forecast of the next time step. In this way they are able to *speculate* about the price of the asset and make an offer to the market trying to profit from their forecast. The strategy is formally defined as:

$$\mathcal{S}_i(p(t), p_i(t+1)) = \begin{cases} buy(g,t) & \text{if } p(t) < p_i(t+1) \\ sell(g,t) & \text{if } p(t) > p_i(t+1) \\ pass(t) & \text{if } p(t) = p_i(t+1) \end{cases} \tag{3.39}$$

where $p(t)$ is the current price of the asset in the market and $p_i(t+1)$ is the agent's forecasted price.

3.2.4.2 Option Trading Strategies

With the *OTRnd* trading strategy, an agent will select one of of the actions available at a time t in the actions set using a uniform random function. The actions can be to trade assets as well as trade any of the Option contracts available at time t. The selection function is formally defined as:

$$\begin{aligned} \mathcal{S}_i(t) &= RND\big(\{buy(g,t), sell(g,t), pass(t), \\ &\quad hold(o_1,t), write(o_1,t), ...hold(o_k,t), write(o_k,t)\}\big) \end{aligned} \tag{3.40}$$

where $RND(\cdot)$ is a random uniform distribution function and k is the number of Option templates $(O(t))$ available for trading at time t. Similarly to the *ATSpec* strategy, the number of assets in each offer is equal to 1.

With the *OTMinR* strategy, an agent will choose the action that has the minimum value of ρ_L from the actions set. The selection function is defined as:

$$\mathcal{S}_i(t) = \{a \in K | \forall k \in K, \rho_L(a) \leq \rho_L(k)\} \tag{3.41}$$

where K is the set of all the possible actions to execute at step t, excepting the action $pass(t)$. This strategy is based on the premise of choosing actions in which the agent is less exposed to the risk of losing wealth, minimising its risk. If there is more than one action with the same $\rho_L(\cdot)$ value then an action is chosen randomly from the subset of actions from that set using the function in Equation 3.40.

When using the *OTMaxW* strategy, an agent will choose the action which has the maximum value of ρ_G from the actions set. The selection function is defined as:

$$\mathcal{S}_i(t) = \{a \in K | \forall k \in K, \rho_G(a) \geq \rho_L(k),\} \tag{3.42}$$

where K is the set of all the possible actions to execute at step t, expecting the action $pass(t)$. Contrarily to the *OTMinR* strategy, the objective of the *OTMaxW* strategy is to choose the actions in which there is a higher probability of obtaining higher profits. If there is more than one action with the same $\rho_G(\cdot)$ value then an action is chosen randomly from the subset of actions from that set using the function in Equation 3.40.

The *OTMix* strategy is a combination of the *OTMinR* and *OTMaxW* strategies. An agent using this strategy is able to choose which of the two strategies to use to select the next action. To choose the strategy the agent will use the equation:

$$\mathcal{S}_i = \begin{cases} \textit{OTMinR} & \text{if } SMA(10,t) > p(t) \\ \textit{OTMaxW} & \text{if } SMA(10,t) < p(t) \end{cases} \tag{3.43}$$

where $SMA(10,t)$ is the 10 period Simple Moving Average of the asset price calculated using the asset prices in the range $[t - 10, t]$. This function is used to select the *OTMinR* strategy when the agent detects that the price of the asset is decreasing and will use the *OTMaxW* strategy if it detects that the price of the asset is increasing. Table 3.2 provides a summary of the designed strategies.

Strategy	Used Information	Objectives
ATSpec	$p(t), p_i(t+1)$	Buy if believes that asset price will increase. Sell if believes that asset price will decrease. Pass if believes that the price will not change.
ATNoise	None	Select to buy or sell assets or pass randomly.
OTMinR	ρ_L	Choose the action that presents the minimum risk loss factor.
OTMaxW	ρ_G	Choose the action that presents the maximum risk gain factor
OTMix	$[p(t-10), p(t)]$	Choose to use the *OTMinR* if the asset price is decreasing and *OTMaxW* if the asset price is increasing.
OTRnd	None	Select any action randomly.

Table 3.2: Summary of strategies developed to trade in the Option market model.

3.3 Summary

This chapter described the elements of the developed market where agents are able to trade goods and Option contracts for those goods. To model a market where agents are able to trade Option contracts, a market where the underlying asset is traded was first defined. After defining the market mechanism, a description of the agents that populate the Option trading market was provided. these agents are grouped in two main sets, agents that can trade Options and assets and agents that can only trade assets. This division is established to compare the performance between agents able to trade Options against agents that can only trade assets. To trade in the market agents were equipped with forecasting functions and trading strategies. The strategies available to trade in the market were presented and the agents' decision making process was explained. The SMA_n and α–Perfect forecasting functions were defined as well as the ρ_G and ρ_L factors used to make decisions by some of the strategies.

The elements of the defined model are considered sufficent to create a basic mechanism that enables software agents to trade Option contracts. Several considerations such as dividends and transaction costs were omitted from the model for simplicity. However I believe that even without the inclusion of those elements, the proposed model is sufficient to demonstrate the possibility of Option trading for Multi–Agent systems. The next chapter describes the details of the computational implementation of the model.

Chapter 4

Implementation

The present chapter details the implementation of the Option trading market model as a computer program. This implementation is performed with the objective of performing experiments doing several market simulations. The experiments are done to test the hypotheses established in Chapter 1. To implement the created model into a computer program, a Multi–Agent modelling framework is used as the basis of the implementation. There are several available frameworks to perform Multi–Agent Simulations. To select a framework, a survey of some available frameworks was carried out. The result of this survey is presented in Section 4.1, evaluating the frameworks on a set of predefined properties which were desired for the implementation of the model.

After selecting a suitable framework, the details of the implementation of the model are described in Section 4.2. In this section, the description of the design approach used and software development process are shown. Section 4.3 details the processes carried out to verify and validate the implementation. A discussion of the issues arising from the validation and verification of Agent-Based simulations is also presented. Finally, Section 4.4 presents a summary of the work described in this chapter, discussing some of the issues that arose during the implementation of the model.

4.1 A Survey of ABM Frameworks

There are currently several frameworks available which allow the development of Multi–Agent Systems; In order to implement the developed model, a survey of some of the available frameworks needed to be performed. Most of the commonly used Agent–Based Modeling (ABM) platforms follow the "framework and library" paradigm, providing a set of standard guidelines for designing and describing agent based models along with a software library implementing the framework and providing simulation tools [75].

To select a suitable MAS framework, a set of required properties defining the criteria for the selection of the model is first defined. For the implementation of the Option trading market model, the following are the properties relevant for the selection of the development framework. A suitable framework to implement the model needs to satisfy certain specific properties:

P1 Robustness: The simulation framework must be mature in its implementation and have been sufficiently tested to have a firm base to build the framework.

P2 Documentation availability: The framework should have good documentation available describing the framework functionality to facilitate the implementation of the model.

P3 Validation Tools: The framework should provide tools (or allow independent tools to interact) to test the validity of the model.

P4 Extensibility: The framework should allow the implementation and use of external libraries to extend the model or if required the framework itself.

P5 Programming Language: The framework must allow the development of the Multi–Agent System in either Java or C++ language.

P6 Focus on Simulation: The framework should be tailored to the simulation of Multi–Agent systems and should provide tools for the simulation of discrete event simulations and markets.

P6 Access to source code: The source code of the framework should be freely available for analysis.

Another property considered in the selection of the framework, was the popularity of the framework in the simulation of Agent Computational Economics and Market Based Control systems. The frameworks considered for evaluation were JADE, NetLogo, Jason, 3APL, Swarm and Repast.

4.1.1 The JADE Framework

Created by the Telecom Italia Lab, Jade[1] (Java Agent DEvelopment Framework) is a software framework implemented in the Java programming language. It allows the implemen-

[1] Available at `http://jade.tilab.com/` February 10, 2008.

tation of Multi–Agent systems by providing a FIPA[2] compliant middleware that provides the definition of the communication protocols between the agents according to the FIPA standards. It includes a set of graphical tools that support the debugging and deployment phases. The agent platform can be distributed across machines and the configuration can be controlled via a remote interface. The source code of Jade is freely available and is distributed under the GNU LGPL[3] License [13].

Being a MAS development framework, Jade contains a full set of tools that help through the development of the application. It provides tools to analyze the state of the agents when the application is running as well as tools to analyze the messages exchanged by the agents. Being implemented in Java it also enables the use of the extensive set of Java libraries available.

Jade is a robust framework which has been used to implement several full scale applications; it contains a very good collection of online documentation and an active mailing list. However it is not aimed at *simulation* building but rather at the development of operational Multi–Agent Systems; hence it lacks mechanisms for coordination (like clock timing) or visualization of the environment, which must be implemented by the developer.

4.1.2 The NetLogo Framework

NetLogo[4] was conceived as an educational tool; its primary design objective is ease of use. Its programming language is based in the Logo language and includes many high level structures and primitives aimed to reduce programming complexity. For this reason, the programming language provided for the agents interactions does not provide all the control and structuring capabilities of a standard programming language.

Netlogo also provides an extensive amount of documentation. NetLogo was designed to model concurrently–acting mobile agents based on a grid space; this is achieved using a parallel processing paradigm. Given such properties, modelling a discrete event simulation may be difficult [76]. Finally, NetLogo does not provide an application programming interface (API) to interact with other third party software libraries or programs developed in other languages [94]. The software is released as freeware but does not provide any source code.

[2]FIPA (Foundation for Intelligent Physical Agents is an IEEE standard committee that maintains a specification of Multi–Agent systems. Its website is accessible at `http://www.fipa.org/`, February 2, 2008.

[3]LGPL is the GNU Lesser General Public License. The text of the license is available at `http://www.gnu.org/licenses/why-not-lgpl.html`, February 10, 2008.

[4]NetLogo is available at `http://ccl.northwestern.edu/netlogo/`, February 10, 2008.

4.1.3 The Jason framework

Developed by R. Bordini and J. Hbner, Jason[5] is a Java-based interpreter for an extended version of AgentSpeak. AgentSpeak is a programming language aimed at the specification of agents defining their Belief, Desires and Intentions. AgentSpeak is itself a simplification of PRS and dMARS [34] reasoning systems. Jason provides an interpreter for a improved version of AgentSpeak, including speech-act based agent communication [18].

Jason allows for the development of BDI-reasoning agents providing a reasoning engine which allows the agents to make decisions based on sets of predefined rules. As a MAS framework Jason provides almost no development tools. The development environment offered is as a plugin of the JEdit[6] text editor. Given the nature of the BDI AgentSpeak language, the agents decision rules and their set of beliefs must be specified before running the program. As the program is running, the agents will reason using such rules until they fulfill their objectives. This feature require that each decision rule must be known and hard coded beforehand. The Jason framework documentation is heavily lacking; it consists of one small manual and a *Frequently Asked Questions* information page. The documentation leaves the developer with no choice but to learn from trial and error. There is also a user mailing list with almost no activity. At the time of surveying, a book on the use of Jason [19] was being written by the developers of the framework.

A preliminary implementation of the Option market model was done in Jason. The trading strategies were modelled as Beliefs, Desires and Intention rules. However, there were several setbacks that prevented the completion of the implementation. The issues encountered were: firstly, the difficulty to mix the calculus of the probability for the agents' risk assessment; secondly, the lack of the possibility to modify the agents decision rules once the program is running; finally, the lack of documentation complicated the implementation of the model.[7] The Jason source code is released under the LGPL Open Source license.

4.1.4 The 3APL Framework

3APL[8] was developed and maintained at the University of Utrecht in the Netherlands. It allows the specification of cognitive agent behaviour using actions, beliefs, goals, plans, and rules. It also permits agent communication using FIPA-like semantics [49].

[5]Jason is available at `http://jason.sourceforge.net/`, February 10, 2008.

[6]JEdit text editor is available at `http://www.jedit.org/` February 10, 2008

[7]I must thank Rafael Bordini for his patience and the information about Jason that he kindly provided me. It was partly from this information that I could make an informed choice of a suitable framework.

[8]3APL is available at `http://www.cs.uu.nl/3apl/`, February 10, 2008.

The 3APL platform has a visual interface for the monitoring and debugging of agents being run, and a source code editor. It has been released as Java-based software, which comes with some Java interfaces that can be used to develop Java-based plug-ins and libraries. A 3APL platform has the ability to be used in client–server architectures, allowing connections of client or server roles to other 3APL instances running across a network. This allows communication among 3APL agents on each platform. The language to implement agents under 3APL is BDI–oriented and consists of a combination of Prolog facts which acts as beliefs of the agents along with other 3APL specific keywords that allow the definition the agents plans, goals and beliefs. The 3APL platform has very little documentation available, consisting of an incomplete programming manual. At the time of the survey, the development and maintenance of the 3APL platform has been abandoned in favour of the development of a new framework named 2APL [30]. The source code of the 3APL framework is not available.

As with Jason, a preliminary implementation of the Option trading market was attempted in 3APL; however, due to the same reasons as in the Jason implementation (the difficulty in mixing the probability based reasoning in these logic based frameworks), the implementation of the market was not continued using 3APL. In the case of 3APL, another problem was the difficulty of implementing the *Market* section of the model in the provided language.[9]

4.1.5 The Swarm Framework

Swarm[10] was designed as a general language and toolbox for Agent Based Models, intended for widespread use across scientific domains. Swarm was designed before Java was a mature language and is developed in Objective-C. It has an extensive amount of documentation available online [92]. Even though there is a library which allows the usage of some of the functions of Swarm in the Java language, it does not permit to use the complete set of properties available as when using the Objective-C version [75]. Given that the framework programming language is Objective-C, the framework was not considered further.

4.1.6 The Repast framework

The initial objective of Repast[11] was to implement a similar functionality to Swarm using the Java language. Repast is intended to support the social science domain in particular

[9] I have to thank Mehdi Dastani, one of the creators of 3APL, for his insight in the use of 3APL. It was he who informed me that 3APL was being superseded by the new 2APL framework.

[10] Swarm is available at `http://www.swarm.org`, February 10, 2008.

[11] Repast is available at `repast.sourceforge.net/`, February 10, 2008

and includes tools specific to this domain [70]. Repast provides tools to aid in the development of the simulations, such as a built-in simple model which can be used as the basis for the implementation of other models. Menu driven interfaces and Python code can be used to simplify the creation of the model. The developing language used in Repast is the Java language and the source code is released in the Open Source BSD license.[12] Repast has been regarded as one of the best frameworks for Agent Based Modelling [92]. And Finally, Repast has been used for several Market Based Control simulations such as the ones described in [68] and [72].

4.1.7 Selection of Framework

Table 4.1 summarises the results of the survey for the different frameworks; the columns in the table indicate to what degree (low, medium or high) each of the surveyed frameworks fulfilled the established criteria.

After looking at the different frameworks and after developing some preliminary implementations in some of them, the Repast framework was selected for the implementation of the Option Market model. Repast was selected because it provides the flexibility for the implementation of the model and because it fulfills the established criteria better than the other surveyed frameworks.

Property	JADE	Netlogo	Jason	Swarm	3APL	Repast
P1 Robustness	High	High	Medium	High	Medium	High
P2 Documentation	High	High	Low	High	Low	Medium
P3 Validation	High	Low	Low	High	Low	Medium
P4 Extensibility	High	Low	Medium	Medium	Medium	High
P5 Language	High	Medium	Medium	Low	Medium	High
P6 Simulation	Low	Medium	Medium	High	Medium	High
P7 Source Code	High	Low	High	High	Low	High

Table 4.1: Resulting scores for the surveyed Agent Modelling frameworks.

4.2 Model Implementation

The implementation of the Option market model was done in the Java programming language using the Repast Java framework for agent based simulation. A copy of the Java source code of the developed model along with the required libraries and parameter files to replicate the presented experiments are available at `http://www.csc.liv.ac.uk/~omar/optionmarket/`.

[12] BSD is the Berkeley Software Distribution license. The text of the license is available at `http://www.opensource.org/licenses/bsd-license.php`, February 10, 2008.

4.2.1 Design Overview

As in any non-trivial software development project, it is important to use a specific software development process. A software development process defines the actions performed to develop the software and specifies the order in which such actions are executed [86].

One of the most accepted development processes is the Spiral Development Process in which several iterations of the steps of analysis, design implementation and testing are performed during the life of the software development project until the application is finished[86]. The Spiral Development Process was chosen for the computational implementation of the model. With the Spiral Development Process, software is developed in an iterative approach; starting with a basic prototype of the system and extending it until all the desired requirements are implemented. For each prototype, the analysis, design, coding and and testing steps are performed. Figure 4.1 shows the different steps performed for each iteration in the development.

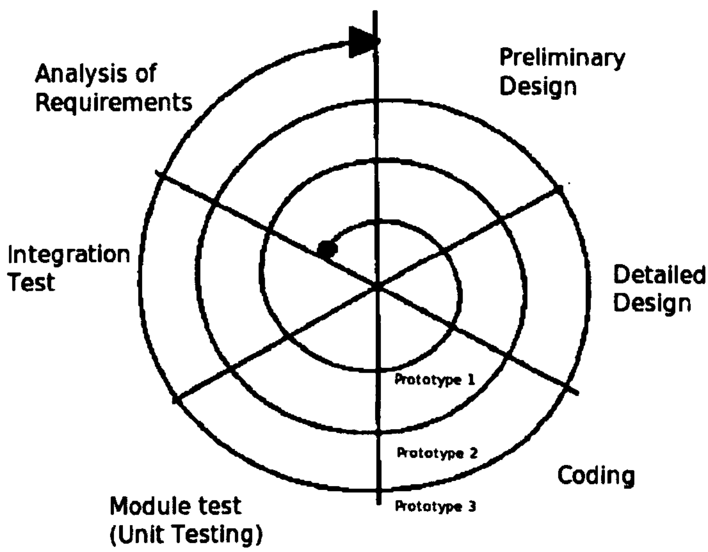

Figure 4.1: The Spiral Software Development Process, figure adapted from [32].

The Spiral Software Development process is suitable for the development of research software given that this type of software is never *finished* in the same way as commercial software. Software created for research is constantly being updated with new requirements. In the case of the Option market model, some first strategies were developed along with the minimal market mechanism. Afterwards, more strategies were implemented and then the logging and parameter configuration facilities. It was at the the final iteration of the development process that the graphical interface of the system was created.

Complementary to the development process, the Gaia methodology described in [101, 103] was used as the design approach given that Gaia is specifically aimed at the design of Agent-Based Systems.

Gaia comprises several engineering techniques suitable for hierarchical systems of heterogeneous agents. It offers techniques for the analysis of the application. Gaia also assumes that the properties of the agents are fixed during the system run time and separates the definition of the environment model from the agents. Usually the environment is represented as a set of variables which can be sensed (i.e., can be read during the life of the system) by the agents. For the description of the entities that interact with the environment Gaia defines *organisations* and *roles*. An organisation is a set of roles which are associated among them. The roles define the different tasks that every entity will adopt according to a set of responsibilities, permissions, activities and protocols. The roles defined for the system are Option Trader, Asset Trader and Market.

For the implementation of the model, the market is designed as the environment of the agents. Figure 4.2 shows a graphical representation of the market; different types of agents are able to *sense* a distinct number of variables from their environment, such as the asset price information and the Option contracts templates.

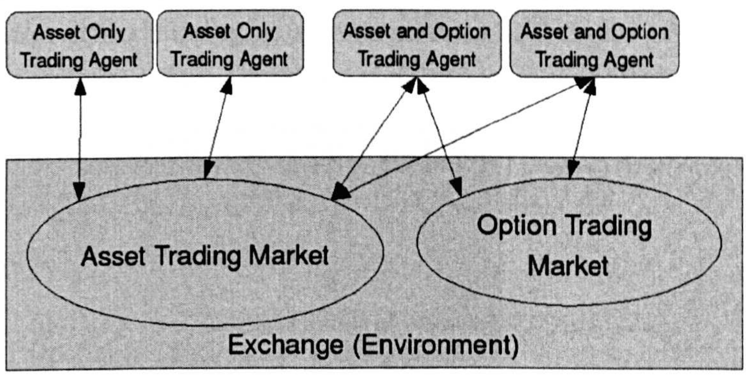

Figure 4.2: A graphical representation of the asset and Option trading market.

4.2.2 Market

The market is the controller of the simulation; it is defined as a module which contains the information that is needed to run the simulation. The market is in charge of the control of the flow of the overall system; and is in charge of querying for the agents offers and for updating the state of the environment at each step. Specifically, the market has the following responsibilities:

- Update and publish the available Option templates and the asset price to be traded by the agents for each time step.
- Register valid assets and Option trading offers from the agents.
- Match and clear Option and asset trading offers made by the agents.
- Clear the exercised Option contracts.

4.2.3 Trading Agents

Agents are represented by a main abstract interface which is adopted by specific modules for the Option trading agents and for the Asset only trading agents. The agents are composed of four main modules: the *Sensing module*, used to obtain the market state at each step; the *Actuator module*, used to execute the chosen actions according to the agent reasoning process; the *Forecasting module* which uses the information about the market to generate a forecast of future market states; and the *Decision support module*, which uses the strategy of the agent to select the actions to perform in the market. The Decision support and Forecasting module together conform the agent reasoning mechanism. Figure 4.3 shows the representation of these modules and their respective high level interaction relationships.

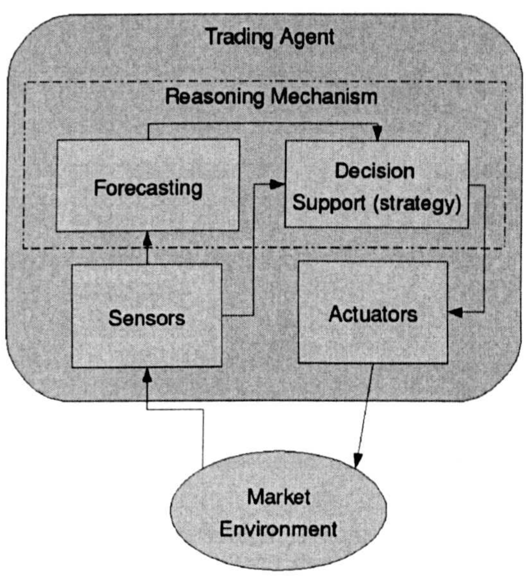

Figure 4.3: Diagram of modules composing a trading agent and their interaction.

Similarly to the market, agents are also modelled according to a set of responsibilities, each type of agent is defined by a set of responsibilities. The responsibilities of each asset trading agent are:

- Obtain the price of the asset for the current time step from the market.
- Maintain its portfolio of goods and cash.
- Choosing the action to take at each step depending on its objectives (defined by its strategy).
- Submit its selected offers to the market.
- Maintain a model for the price of the asset to obtain forecasted prices.

The Option Trading agents have the same responsibilities as the asset trading agents. In addition, each agent that trades Options has the following Option–trading related responsibilities:

- Obtain the set of available Option templates for the current time step from the market.
- Choose whether to exercise expiring Options.

Figure 4.4 shows the main modules related to the trading agents, including the strategies and forecasting functions.

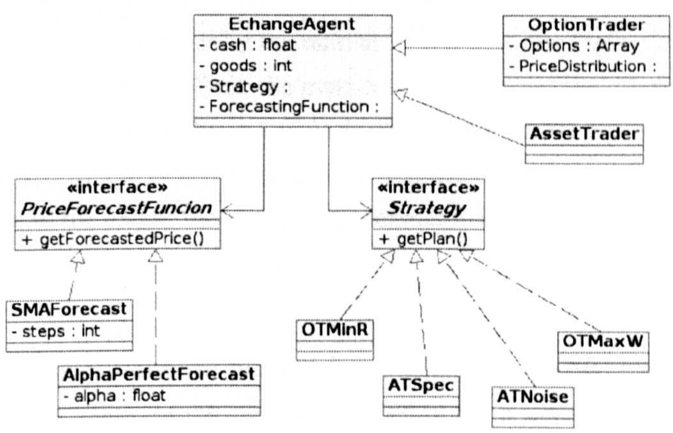

Figure 4.4: Class diagram showing the main operations and properties of the Trading Agents.

The agent reasoning mechanism is simulated using a step function to execute the agent's actions at each time step. This function is executed by each agent on each time step and it is inside this function where the agent makes its decision to participate in the market.

4.2.4 Strategies

Strategies are defined as independent modules that can be attached to the market traders. The six trading strategies defined in Section 3.2.4 were implemented.

4.2.5 Forecasting Functions

Forecasting functions are implemented as modules that can be attached to the market traders. The two forecasting functions defined in Section 3.2.2 were implemented.

4.2.6 Market Life

The flow of the market time line is controlled from the market module. At the beginning of each simulation the market is configured according to the parameters specified in a parameter file. The parameter file defines the initial state of the market including the name of the file containing the price series for the asset. Figure 4.5 presents a high level sequence diagram depicting the actions performed by the market and the agents at each time step.

The life of the market begins on the first step of the simulation with the market publishing the current price of the asset and generating the Options templates that will be available in that step. The market then queries the agents for any expiring Options to be exercised. Agents choose whether to exercise any expiring Options and submit their instructions to the market. The market then proceeds to clear any exercised Options. Afterwards, the market publishes the available Option templates and queries each agent for trading offers. Each agent then submits an offer which is collected by the market in two lists, the Option offers and the asset offers. After obtaining all the offers, the market will randomize the offers lists and try to find matching offers to clear. At the end of each period, the market will provide the agents with the cash earned by the risk free rate. This sequence of activities will be repeated for each time step until the end of the simulation (defined in the parameters).

4.2.7 Parameters

Simulations are configured using a parameter file which is created as an XML file. The parameter file contains the initial values of the variables needed to run a simulation. There are two types of parameters: the first type is the market parameters that describe the configuration of the asset and Option market; the second type of parameters are the agents parameters that define the initial conditions and behaviour of the agents. Agents are defined in sets, allowing to create several agents with identical starting properties; the properties defining each agent set depend on the type of agent (asset only trader or Option and asset

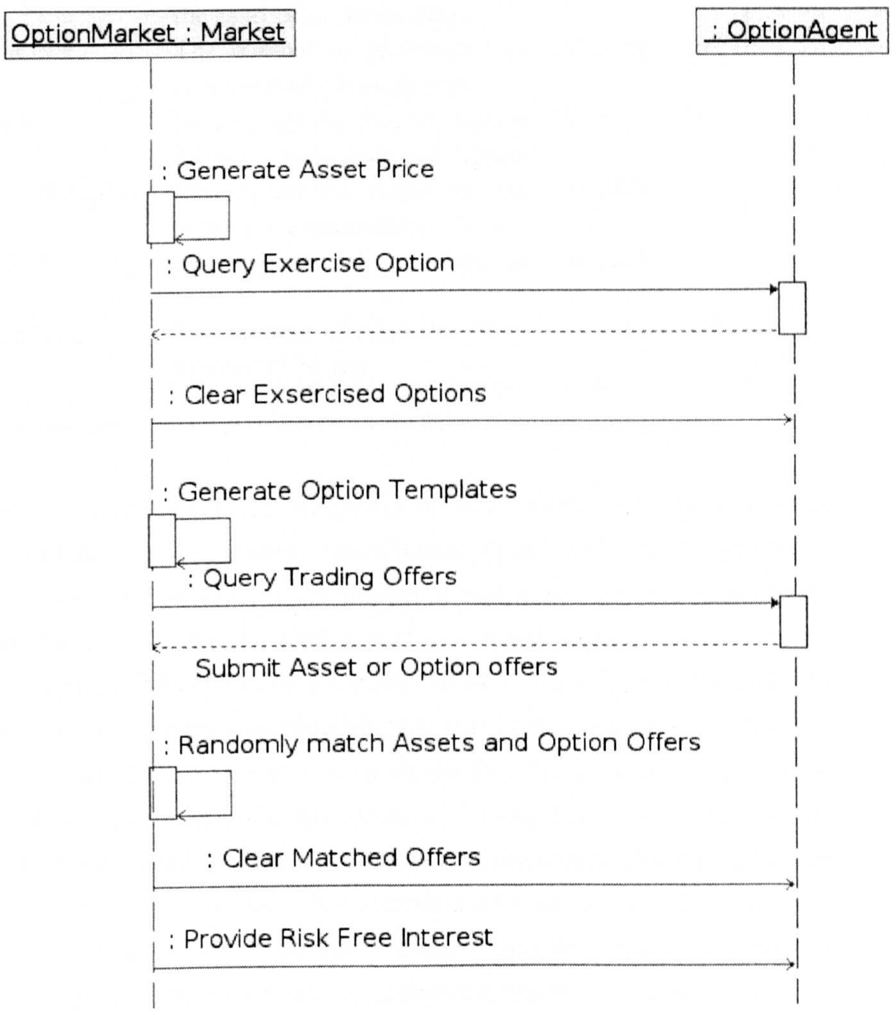

Figure 4.5: Sequence diagram for one step in the trading market.

trader) and the selected strategy for the agent set. Table 4.2 contains the list of available market parameters and their description; similarly, Table 4.3 describes the parameters used for the agents.

Parameter	Description	Range
PriceSeriesFile	Determines the source file for the asset price series data.	Text string
OptionNumber	The number of generated Option contracts at each step.	$[0, 2.14 \times 10^9]$
OptionStep	Determines the time separation between each generated Option	$[0, 2.14 \times 10^9]$
StrikePriceMultiplier	Determines the range for the strike price generation	$[0, 100]$
RiskFreeRate	The market risk free interest rate	$[0, 100]$
SimulationTime	The number of steps the simulation will be run	$[0, 2.14 \times 10^9]$

Table 4.2: Description of market parameters available for the simulation configuration.

The *PriceSeriesFile* parameter is used to specify the file containing the time series to be used for the price of the asset. Parameter *OptionNumber* specifies the number of Option contract templates that will be available in the market at each time step. For each Option contract a template for a *call* and a *put* Option will be created sharing the strike price, expiration date and volume. The *OptionStep* parameter is used to define the expiration time of each Option; the *OptionStep* parameter specifies how many time steps of separation will be between the expiration of each Option (i.e., if *OptionStep* has a value of 2, then the first generated Option will expire at $t + 2$, the second $t + 4$, the third at $t + 6$ and so on). The parameter *StrikePriceMultiplier* defines the maximum percentage that will be added or subtracted from the asset price in order to establish the Option strike price; its range is $[0, 100]$. When creating an Option, a number between 0 and the *StrikePriceMultiplier* parameter is randomly obtained (using a Uniform random distribution function), the current price of the asset is then multiplied by the obtained value to generate the Option strike price. The *RiskFreeRate* parameter establishes the market risk free interest rate, its range is $[0, 100]$. Finally, the parameter *SimulationTime* determines the number of steps that the simulation will be run. The simulation will stop either when the last asset price has been read from the price series file or when $t = SimulationTime$.

For the agent parameters, *MinCash* and *MaxCash* specify the limits used when providing agents with the initial amount of cash for the simulation. If these two values are

Parameter	Description	Range
MinCash	Minimum amount of cash at the start of the simulation	$[0, 1.7 \times 10^{308}]$
MaxCash	Maximum amount of cash at the start of the simulation	$[0, 1.7 \times 10^{308}]$
MinGoods	Minimum amount of goods at the start of the simulation	$[0, 2.14 \times 10^{9}]$
MaxGoods	Maximum amount of goods at the start of the simulation	$[0, 2.14 \times 10^{9}]$
Type	The type of the agents in the set (asset-only or Option and asset traders)	Text string
Number	The number of agents in the set	$[0, 2.14 \times 10^{9}]$
PlanStrategy	Defines the strategy used by the agents in the set	Text string
PlanStrategy	The strategy used by the agents in the set	Text string
ForecastFunction	The forecasting function used by the agents in the set	Text string
AlphaFactor	The value of α for the α–Perfect forecasting function	$[0, 100]$
MA-steps	The value of n for the SMA_n forecasting function	$0, 500]$

Table 4.3: Description of agent parameters available for the simulation configuration.

different, then a random value between the two will be chosen for each agent. If the value of *MinCash* is equal to *MaxCash* then every agent in the set will have the same specific initial amount of cash. The parameters *MinGoods* and *MaxGoods* function in the same way as the *MinCash* and *MaxCash* parameters, specifying the number of initial goods for the agents. This makes possible to provide all the agents in one set with different initial wealth. The *Type* parameter is used to specify whether agents in the set will be trading assets only or assets and Options. Parameter *Number* defines the quantity of agents that are created in the corresponding set. The *PlanStratregy* parameter is used to specify the strategy that will be used by the set of agents. The parameter *ForecastFunction* specifies the forecasting mechanism that will be used by the agents in the set; either the SMA_n forecasting function, defined in Section 3.2.2.1 or the α–Perfect forecasting function, defined in Section 3.2.2.2. When the chosen forecasting function is α–Perfect, the *AlphaFactor* parameter is used to specify the certainty value α for the function. Similarly when the SMA_n function is used, the $MA - steps$ parameter is used to specify the size of the window for the Simple moving average (parameter n of the $SMA(n, t)$ formula.

4.2.8 Auxiliary Functions

In addition to implementing the general functionality of the Option trading market model, three packages providing auxiliary functionality for the simulation were created.: The *user interface*, *unit testing* and *logging* packages. Each package provides a functionality which is external to the developed model but nevertheless is useful for the simulation experiments.

4.2.8.1 User Interface

The *user interface* package comprises the modules created to allow the graphical visualization of the simulation in real time. A sample screen shot of a running simulation is shown in Figure 4.6. This kind of visualization is provided to allow the user to see the development of the market, however for the purpose of experimentation it is possible to run the market without any graphical output to increase the speed of the simulation.

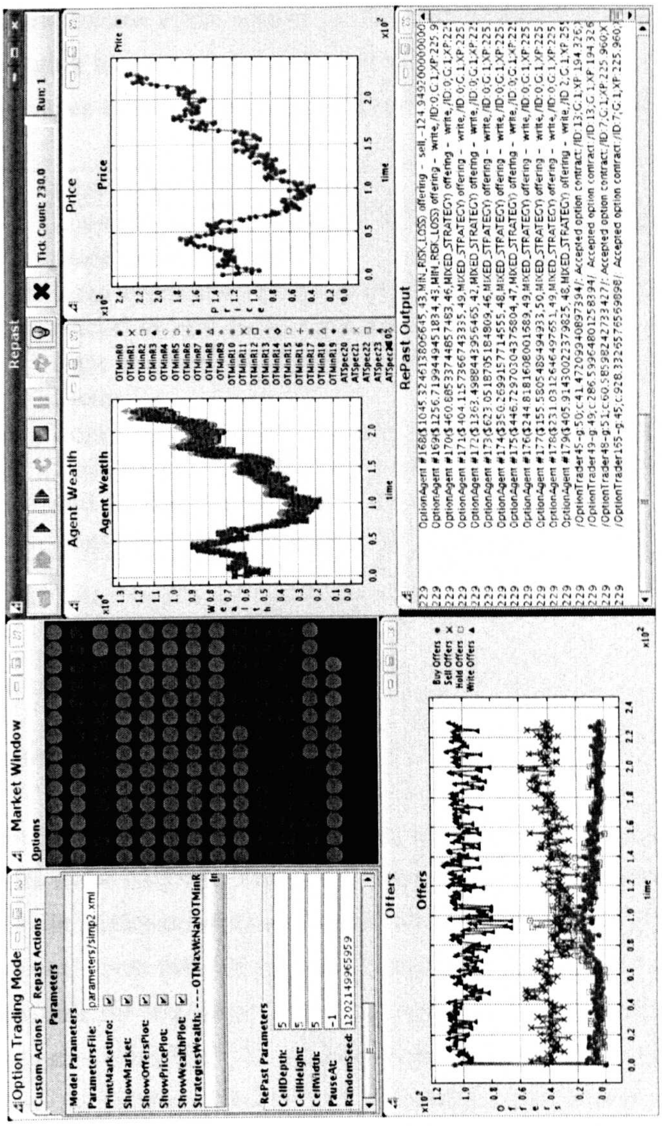

Figure 4.6: Sample Screen shot of a running simulation of the developed Option market simulator interface.

4.2.8.2 Logging files

Whenever the simulation is executed a set of six files are created to log the information about the state of the model throughout the life of the simulation. There are two types of logging files, the agent logging files and the strategy logging files. The agent logging files are used to gather the detailed information of the actions and the state of each agent during the simulation. The strategy logging files are used to summarize the state and actions for each strategy. Table 4.4 lists the names of the logging files and a summary of their description.

File Name	Description
ActionCount	Logs the actions executed by the agents (the offers made to the market).
MarketCount	Logs information of the cleared offers performed by the market as well as the exercised Options.
AgentWealth	Logs the number of goods and amount of cash of the agents.
StrategyActionCount	Logs the sum of actions executed by a specific strategy.
StrategyMarketCount	Logs the sum of cleared offers performed by the market and the sum of exercised Options for each strategy.
StrategyWealth	Logs the average wealth for each strategy.

Table 4.4: List of log files generated during a simulation.

4.2.8.3 External Libraries

In addition to the Repast Framework, other external libraries were used for the implementation of some functionalities in the program. The use of preexisting libraries decreases the possibility of errors in the code implementation as the used libraries are already tested and verified by several third parties; given that, for libraries which are provided with an Open Source license it is also possible for anyone to inspect the source code to detect possible errors. The used external libraries were JSci[13] for the implementation of the Normal distribution models and functions; the RngPack library[14], to generate pseudo-random numbers using an implementation of the the Merssene Twister generator presented in [61]; and the JDom[15] library for the manipulation of the XML configuration files.

[13] Developed by Mark Hale, available at `http://jsci.sf.net` (5-February-2008).
[14] Available at `http://www.honeylocust.com/RngPack/` (5-February-2008)
[15] Available at `http://www.jdom.org/` (2-February-2008)

4.3 Verification and Validation

One of the main issues inhibiting researches from fields outside Computer Science to accept ABM as a tool for the modelling of systems is the lack of verification and validation methodologies available. In fact, verification and validation of Multi–Agent simulations is a concept which has been investigated only in conjunction with the development of specific models. It has only been in recent times that researchers have engaged in independent development of techniques for verification and validation [65, 102, 97].

Verification and validation are two independent actions that need to be performed in order to achieve the accreditation of a simulation [6]. Verification aims to test whether the implementation of the model is an accurate representation of the abstract model. Hence, the *accuracy* of transforming a created model into the computer program is tested in the verification phase. Model validation is used to check that the implemented model can achieve the proposed objectives of the simulation experiments. That is, to ensure that the built model is the right representation of the modelled phenomena to simulate. In contrast with models which use analytical equations, there is still no consensus among the scientific community on the appropriate methods for verifying and validating an Agent–Based simulation [65]. However, part of the reason for the lack of formalisms which validate Agent–Based simulations is the inherent complexity that these systems try to represent. Some of the verification methods discussed in [65] are source code analysis, automatic theoretic verification and finite state verification. However, there is still some debate in the Agent–Based Modeling community on whether formal proofs of systems are useful [43]. Similarly, there is some debate on whether the verification of complex models with many parameters is possible [83].

In this section, some of the validation and verification techniques commonly used are applied to the implementation of the model. These techniques are applied with the intention of accrediting the simulation as an acceptable representation of the model defined in Chapter 3.

4.3.1 Model Verification

As described before, the verification process is used to ensure that the implementation of the model is accurate, therefore it is necessary to test the application looking for malfunctions or errors that could make the model behave differently than desired.

In order to verify the implementation of the model different Software Engineering testing techniques were used. First, static code analysis was performed to verify the complexity of the program; secondly unit testing and debugging was performed to verify that each of the modules in the implementation of the model was working as expected.

It must be noted that the process of verification was performed at different times as the program was being developed (required by the spiral development process), given that the implementation of new functionality can lead to new bugs. The process of verification ends when the warnings obtained from the analysis are verified and corrected in the case of actual errors.

4.3.1.1 General Static Code Analysis

Static code analysis is the analysis of the application source code in order to find sections of code with potential errors. Two kinds of static code analysis were performed, general static code analysis and software metrics analysis.

The general static code analysis was performed using the FindBugs[16] and JLint[17] applications. Table 4.5 lists some of the different tests performed by such applications[18].

Test Name	Test Description
Bad Use Of Return Value	Method checks to see if result of String.IndexOf is positive.
Badly Overridden Adapter	Class overrides a method implemented in super class Adapter wrongly
Check Immutable Annotation	Check that the fields of immutable classes are final.
Clone Idiom	Class implements cloneable but does not define or use clone method.
Comparator Idiom	Comparator does not implement serializable.
Confusion Between Inherited And Outer Method	Ambiguous invocation of either an inherited or outer method.
Do Inside DoPrivileged	Method invoked that should only be invoked inside a doPrivileged block.
Don't Catch Illegal Monitor Exception	Dubious Catching of IllegalMonitorStateException.
Dropped Exception	Method might drop or ignore exception.
Dumb Method Invocation	Code contains hard coded reference to absolute path name.
Find Bad Cast	Questionable cast to concrete collection, abstract collection or unconfirmed cast.
Infinite Loop	Apparent infinite loop.
Infinite Recursive	An apparent infinite recursive loop.

Table 4.5: Tests performed for the static test analysis.

[16]Version 1.2, available at `http://findbugs.sourceforge.net/` . February 20, 2008.

[17]Version 3.1, available at `http://jlint.sourceforge.net/`. February 20, 2008.

[18]For a complete list and description of the 276 tests refer to the application documentation page at `http://findbugs.sourceforge.net/bugDescriptions.html`. February 20, 2008.

4.3.1.2 Software Metrics Analysis.

The software metrics analysis was performed using the Eclipse Metrics plug-in[19]. The test output in Table 4.6 shows different measured metrics and the score value of them. The first column contains the name of the tested metric; the second column contains the average of the metric for all the classes in the model; the third column contains its standard deviation and the fourth column contains the value of the method or class with the highest score; the last column contains the the totals of the sum for the NOM and NORM metrics. The definition of the metrics are taken from [47].

Metric Name	Mean	Std.Dev.	Max.	Total
Number of Overridden Methods (NORM)	1.476	2.481	10	31
Number of Methods (NOM)	9.571	8.556	37	201
Depth of Inheritance Tree (DIT)	1.476	1.139	6	N/A
Specialization Index (SIX)	0.202	0.344	1.25	N/A
McCabe Cyclomatic Complexity	1.888	1.946	14	N/A
Lack of Cohesion of Methods	0.456	0.352	0.978	N/A
Afferent coupling (CA)	4.714	3.692	11	N/A
Efferent coupling (CE)	2.286	1.030	4	N/A
Instability (RMI)	0.369	0.130	0.6	N/A

Table 4.6: Results of source code metrics analysis.

The NORM metric specifies the number of redefined operations, which plays a role in the specialization of the class. Too many overridden operations implies too big a difference with the parent class and inheritance then makes less sense. The NOM metric indicates the number of methods on each class. The DIT metric indicates the number of base classes for a specific class. The SIX provides an the Specialization Index average which specifies the overall specialization of a class; very specialized classes are undesirable because of complexity and increased maintenance, this metric is calculated as Equation 4.1.

$$SIX = \frac{NORM \times DIT}{NOM} \tag{4.1}$$

The *McCabe Cyclomatic Complexity* measures the number of linear independent paths through a method. This provides a measure of the number of possible paths that the application can take when the simulation is running. There are a set of predefined threshold values to evaluate the model implementation using this score.[20] An acceptable score implying a

[19]Available at `http://metrics.sourceforge.net/`. February 20, 2008.

[20]See `http://www.sei.cmu.edu/str/descriptions/cyclomatic_body.html#table4`

low complexity and risk of errors is in the range of $[1, 10]$, an intermediate score implying moderate complexity and risk of errors is within the range of $[11, 20]$. A Cyclomatic complexity value of more than 20 may indicate a highly unstable application with high risk of errors.

In the case of the implemented model the average complexity is within the low complexity range. The maximum complexity value is obtained from the `step` method inside the `OptionMarketModel` class which is in charge of controlling each step on the overall simulation. Even so, the score for this maximum value is still within the moderate risk range with a value of 14.

The *Lack of Cohesion of Methods* metric is a measure for the cohesiveness of a class. According to [33] every object should have a single role and its services should be aligned with that responsibility. A score greater than one is considered to be above the accepted threshold (see [47]), the maximum score in the implementation of the model was below this threshold.

The *Afferent Coupling* metric measures the number of classes outside a package that depend on classes inside the package, similarly the Efferent coupling metric measures the number of classes inside a package that depend on classes outside the package. These metrics are used to obtain the Instability (RMI) metric which indicates the model's tolerance to change; it is defined in Equation 4.2. The range for this metric is within $[0, 1]$ with 0 indicating a completely stable program and 1 indicating a completely unstable program. In the case of the implemented model the average metric was below the medium of the range, indicating that the model implementation is highly stable. However the maximum score was obtained by the logging mechanism. This score seems reasonable as almost all of the classes in the model depend on the logging classes to log the simulation results.

$$RMI = \frac{CE}{CE + CA} \tag{4.2}$$

4.3.1.3 Unit Testing

To verify that the modules implemented in the system behaved as defined in the theoretical model a set of unit testing modules were implemented. Unit testing is a software quality assurance technique used to test the correct functionality of an isolated module [53]. Each module in the system is tested using independent test cases with a range of input values. The output values or behaviour of the tested modules is then analyzed to verify that they correspond to a set of expected values or an expected behaviour.

Unit tests were developed and performed for the forecasting function implementations

(SMA_n and $alpha$–Perfect), for the implemented strategies and for the Option contract pricing mechanisms.

4.3.2 Model Validation

Model validation is performed to make sure that the implementation of the model is a correct representation of the phenomena that is being modelled. In [102], validation is separated into two different tasks, *conceptual validation* and *operational validation*. Conceptual validation tries to assure that the *fidelity* of the model is enough to achieve the desired objectives with the simulation. Operational validation is used to test whether the simulation output data is accurate enough and sufficient to yield the desired objectives of the simulation. The three main techniques used for operational validation are graphical comparisons, confidence intervals and hypothesis testing [102].

4.3.2.1 Conceptual Validation

In order to validate the model, conceptual validation tests needed to be performed. For the purposes of this research the objective of the simulations is to test the behaviour of the Option trading agents against the behaviour of non–Option trading agents as defined in Chapter 1. Therefore, the developed model should consist of an abstraction of the processes carried out in the real financial Option markets allowing the trading of Options and assets in the market.

Hypothesis 1. *The model of the market will be valid for our purposes if it provides enough instruments to let the Agents trade assets and Option contracts on the assets.*

Claim 1. *The market model contains the four main features necessary for an Option market as defined in [52].*

These features are:

- An underlying asset.
- A specification (policy) of the Option contracts to trade.
- A risk free interest rate for the underlying asset.
- An Option pricing formula to provide a fair price.

In addition to those features, the market model contains also an Exchange which is in charge specifying the valid configurations (templates) for the Options at each step in time and is also in charge of setting the Options' strike price.

However, the model does not include features such as market makers, offsetting orders, commissions, taxation among others which were seen unnecessary to replicate the basic properties of an Option market.

Hypothesis 2. *The agents participating in the market will be valid for our purposes if they are capable of trading assets and Option contracts using some reasoning mechanism to make a decision to adopt a position in the market.*

Claim 2. *The agents are provided with a basic mechanism to trade assets and Options. Some agents also have a reasoning mechanism which allows them to discriminate among the different possible actions based on their perceived risk using a strategy.*

Thereupon, the agents are able to trade Options in the market to reason about the relevant properties that make the use of Options important in the model.

One of the features that the agents (and consequently the market) lack is the possibility of executing more than one type of trading at each time step. This would presumably allow for more complex behaviour of the agents and would make possible to model combination strategies.

Given that the market has the needed features to allow the trading of Options and assets and that the agents are capable of trading in the market, it is concluded that the conceptual validation test of the implementation is passed positively.

4.3.2.2 Operational Validation

The operational validation of the model is achieved in two steps: firstly by assuring that the data collected in the log files is enough to extract the information to achieve the experiments objectives; and secondly by testing the model with different formulated scenarios with hypotheses on the results and examining the results to verify the hypotheses.

In order to ensure that the collected data from the simulation is relevant for the achievement of the objectives, a comparison among the experiment research questions and the data must be done. This allows us to detect if there is certain data which should be added to the log files and also if certain data in the log files will not be used.

The main research questions (defined in Chapter 1 to answer from the information obtained by the experiments are:

1. Can software agents benefit from the exchange of Options in the software market?

2. Is it possible to characterise specific cases where software agents trading Options have a better performance than those not using them?

3. Are agents trading Options less susceptible to price variations than those not using Options?

4. What is the difference in the performance among the developed Option trading strategies?

Tables 4.8, 4.7 and 4.9 show a comparison between the different types of data gathered in the log files and the research questions for the experiments; the columns marked with a check mark indicate that the data can be used to answer the corresponding question.

	Time	Source Agent	Action	Destination Agent
Q.1				
Q.2	✓	✓	✓	✓
Q.3				
Q.4	✓	✓	✓	✓

Table 4.7: Comparison between research questions and *Market Trading* log file.

	Time	Price	Agent	Goods	Cash	Wealth
Q.1	✓	✓	✓			✓
Q.2	✓	✓	✓			✓
Q.3	✓	✓	✓	✓	✓	✓
Q.4	✓	✓	✓			✓

Table 4.8: Comparison between research questions and *Agents Wealth* log file.

	Time	Agent	Action
Q.1	✓	✓	✓
Q.2			
Q.3			
Q.4			

Table 4.9: Comparison between research questions and *Agents Actions* log file.

4.3.2.3 Option Pricing Validation

The implementation of the Black-Scholes Option pricing model was validated by comparing the generated Option prices to a different set of predefined prices. First, a set of Option contracts and its corresponding prices were obtained from Hull [52] and Cuthbertson [29]. These Option contracts were valuated with the implementation of the Black-Scholes model to compare both prices. Next, a set of Option contracts was defined and their prices were calculated using the Numa[21] Option pricing calculator and the Hoadley[22] Option pricing calculator. Table 4.10 shows a comparison between Option Prices obtained from Hull and Curtbertson and the prices obtained in the model for the same Options. In this tables, p denotes the price of the asset, X is the exercise price, r denotes the risk free interest rate, σ is the volatility, T is the time to expiration and τ is the type of the Option contract.

	Option 1	Option 2	Option 3	Option 4
p	45	45	42	42
X	43	43	40	40
r	0.1	0.1	0.1	0.1
σ	0.2	0.2	0.2	0.2
T	182 days	182 days	182 days	182 days
τ	call	put	call	put
Price from Hull [52] and Cuthbertson [29]	5.00	0.90	4.76	0.81
Implemented Black-Scholes function price	4.99	0.90	4.75	0.81

Table 4.10: Comparison between predefined Option contracts and prices from Hull [52] and Cuthbertson [29], and the prices obtained with the implemented Black-Scholes function

Similarly, Table 4.11 contains a comparison of the prices for different Option contracts. The table contrasts the prices obtained form the Numa calculator, the Hoadley calculator and used the implementation of the Black-Shcoles formula.

The data in the table shows that the prices of the Options obtained by the implementation of the Black-Scholes formula (last column in Table 4.11) is close to the prices obtained by the two independent Option price calculator. A statistical analysis between the sets of prices demonstrates that the differences in the prices are not statistically significant ($F = 1$ with $p = 0.99$ and $t = 0$ with $p = 0.99$).

4.3.2.4 Agent Behaviour Validation

Two tests were executed to validate that the data produced by the simulation and the behaviour of the agents was consistent. For each test, a specific scenario was defined and a hypothesis was formulated on that specific scenario. A range of expected results were also defined and compared to the results obtained after running the simulation.

Hypothesis 3. *In a market where all the agents trade with identical initial conditions and*

[21] Available online at `http://www.numa.com/derivs/ref/calculat/option/calc-opa.htm`. February, 2006.

[22] Available at `http://www.hoadley.net/options/optiongraphs.aspx?`. February, 2006.

p	X	r	σ	T	τ	Numa	Hadley	Model
100	150	0.05	0.6	50 days	call	0.38	0.39	0.38
100	150	0.05	0.6	50 days	put	49.39	49.36	49.36
100	150	0.05	0.6	70 days	call	0.91	0.91	0.91
100	150	0.05	0.6	70 days	put	49.52	49.48	49.48
100	150	0.05	0.6	90 days	call	1.56	1.57	0.91
100	150	0.05	0.6	90 days	put	49.77	49.73	49.48
150	100	0.05	0.6	50 days	call	51.00	51.01	51.01
150	100	0.05	0.6	50 days	put	0.33	0.33	0.33
150	100	0.05	0.6	70 days	call	51.70	51.73	51.73
150	100	0.05	0.6	70 days	put	0.77	0.77	0.77
150	100	0.05	0.6	90 days	call	52.51	52.53	52.53
150	100	0.05	0.6	90 days	put	1.31	1.31	1.31

Table 4.11: Comparison between the prices of predefined Option contracts calculated with the Numa calculator, the Hoadley calculator and with the implemented Black-Scholes function.

using any of the non-random strategies, no asset or Option trading will be performed as all the agents will make the same decisions.

To evaluate this hypothesis the following test simulation was performed. The market was initialized with the parameters (defined in Section 4.2) shown in Table 4.12. The *RandomA* price series was generated pseudo-randomly, more detail about the price series will be described in Section 5.3. The agents were initialized with the parameters as specified in Table 4.13 (defined in Section 4.2). The test was repeated providing the agents with each of the non-random trading strategies. For each test, 50 runs were made and the results were averaged.

Parameter	Initial value
Price series	RandomA
Simulation duration	1000
Number of available Option templates	6
Steps among available Options	10
Strike Price multiplier	15
Risk free rate	0.002
Initial price variance	1
Total number agents	50

Table 4.12: Initial parameters for the market in first validation test

The resulting data obtained form this test showed that the number of offers cleared by the market was 0. Given that all the agents in the market had the same strategy. Agents that have the same strategy and the same forecasting function will generate the same type

Parameter	Initial value
Planning strategy	*OTMaxW*
	OTMinR
	OTMix
	ATSpec
Initial cash	1000
Initial goods	100
Forecast strategy	Simple Moving Average
Forecast strategy steps	10

Table 4.13: Initial parameters for the agents in the first validation test

of offer at each time step. Table 4.14 summarizes the number of offers submitted by the agents and the cleared offers obtained for each of the experiments. Also, Figure 4.14 shows charts with the history of submitted offers for the tests using the four different strategies.

	Submitted Offers				Cleared Offers			
Strategy	Buy	Sell	Hold	Write	Buy	Sell	Hold	Write
OTMinR	100	0	14100	35850	0	0	0	0
OTMaxW	50	0	0	49950	0	0	0	0
OTMix	100	0	2200	47750	0	0	0	0
ATSpec	28150	21900	N/A	N/A	0	0	0	0

Table 4.14: Summary of the total number of submitted and cleared offers for the first validation test.

Hypothesis 4. *In a market where agents trade with the same initial conditions and using different trading strategies, the agents will submit different types of offers to the market and some of these will be matched and cleared, producing asset and Option trading.*

To evaluate this hypothesis another simulation test was conducted. The objective was to verify that by populating the modelled market with agents trading with the different Option trading strategies the agents would make different types of offers, some of which would be matched by the market to perform asset and Option trading.

For this test, the market was initialized with the parameters shown in Table 4.15 and six sets of agents were established in the market, each one using one of the designed trading strategies. The agents were initialized with the parameters shown on Tables 4.16. The experiment was run using the *RandomA* price series series as the price of the asset. The simulation was repeated 50 times and the results were averaged.

Table 4.17 summarises the total number offers submitted by the agents and the number of those offers that were successfully matched and cleared by the market for the different

Parameter	Initial value
Simulation duration	1000
Number of available Option templates	6
Steps among available Options	10
Strike Price multiplier	15
Risk free rate	0.02
Total number agents	300
Number of agents using *OTMaxW* strategy	50
Number of agents using *OTMinR*	50
Number of agents using *OTMix* strategy	50
Number of agents using *OTRnd* strategy	50
Number of agents using *ATSpec*	50
Number of agents using *ATNoise*	50

Table 4.15: Initial parameters for the market in second verification test.

Parameter	Initial value
Initial cash ($c_i(0)$)	1000
Initial goods ($g_i(0)$)	100
Forecast strategy (F_i)	Simple Moving Average
Forecast strategy steps (t_{F_i})	10

Table 4.16: Initial parameters for the different set of agents in the second validation test.

	Submitted Offers				Cleared Offers				Exercised
Strategy	Write	Hold	Buy	Sell	Write	Hold	Buy	Sell	Offers
OTMinR	21550	28400	100	0	655	706	28	0	385
OTMaxW	40450	9500	100	0	1195	151	27	0	110
OTMix	25529	24417	104	0	483	493	39	0	313
OTRnd	22215	21809	1970	2087	12557	13540	872	1354	4817
ATSpec	N/A	N/A	1866	23500	N/A	N/A	6175	6016	N/A
ATNoise	N/A	N/A	15710	17315	N/A	N/A	10837	10930	N/A

Table 4.17: Summary of the total number of submitted and cleared offers for the second validation test.

strategies participating in the market. The resulting data form this experiment is an heterogeneous market with agents making different types of offers. Some of these offers are matched and cleared by the market.

4.4 Summary

This chapter has presented the computational implementation of the Option Trading Market model developed to perform simulation experiments. Although the developed software is a simulation framework, its development requires the use of software development techniques. The developing process used during the implementation of the model was described as well as the adopted agent-based methodology.

To ensure the accreditation of the implementation and the accuracy of the simulation, verification and validation processes were carried out. There are no standard procedures for the verification and validation of agent based models; therefore, the verification and validation procedures carried out were borrowed from Software Engineering and object-oriented simulation systems.

Although Multi–Agent Systems share several similarities to typical object–oriented systems, MAS contain the element of complex and possibly dynamic interactions between the agents. On the one side, this complexity is a desired property of MAS, and is what characterize them from other modelling approaches; on the other side, there are no established methodologies to verify or validate such complex interactions between the agents participating in a Multi–Agent system.

Another characteristic of MAS simulations is the generation of high amounts of data. This data must be processed and analyzed in order to extract relevant information from it. Besides the necessity of the use of databases for the storage of the data (something which was wrongfully omitted from the present Option Market model implementation), data mining methodologies [58] could be to used to detect patterns that arise from the generated time series. Such patterns may be used to compare the behaviour of similar Multi-Agent systems at a higher level; allowing the validation process to focus on the comparison of generic patterns among different implementations, instead of focusing on specific data values which may vary between specific MAS implementations.

The next chapter presents the configuration of the simulation parameters to design the experiments that were run in order to test the hypotheses established on Chapter 1.

Chapter 5

Design of Experiments

This chapter describes the methodology used to run the experimental simulations to test the developed Option Trading model. The configuration of the different tested scenarios is also described. The experiments are designed to obtain information that allows the testing of the hypotheses stated in Chapter 1 using the Option trading framework described in Chapter 3 and implemented as shown in Chapter 4. While Chapter 4 is concerned with the details of the software implementation of the model, the present chapter focuses on the description of the configuration of the model as well as the techniques of data analysis used for the experimentation phase. Therefore, the present chapter addresses the issue of the designation of the values for the different parameters and variables described in Chapter 4 for the simulation experiments. This chapter also addresses the definition and description of the data analyses that are performed on the data resulting from the experiments.

The experiments are divided in two main sets according to the type of forecasting function used by the agents. The first set of experiments, using the α-Perfect forecasting function, is designed to observe the behaviour of the agents when they have different levels of uncertainty about the price. Testing under different levels of uncertainty allows the observation of the performance of the different strategies and the comparison of their behaviour under such conditions. The objective of the second set of experiments, using the SMA_n forecasting function, is to observe the behaviour of the agents with a forecasting mechanism often used in the analysis of real market price series.

The data obtained from the tests is analysed using three different performance metrics. First, the performance of each strategy is evaluated as the relative returns obtained among the different strategies; next an analysis of the correlation between the price and the agents' wealth is performed; and finally an analysis concerning the difference between the volumes of agents' offers and actual market cleared offers is executed.

A simulation run for the model requires the specification of parameters which are grouped in two sets, the global parameters and the agents parameters. The global parameters define the initial state of the underlying asset market as well as the properties of the Option market during each run. The global parameters also contain other initial properties of the market such as the risk free rate. The agent parameters correspond to the initial properties of the agents that will participate in the market such as the definition of their trading strategy and forecasting function. A detailed definition of each parameter can be found in Section 4.2.7.

The structure of this chapter is the following. Section 5.1 describes the initial value of the parameters that are used to configure the initial global conditions for each experiment. In Section 5.2 the values of the parameters used for the initialisation of the agents are described. The price series used as input for the underlying asset are described in Section 5.3. Section 5.4 describes the pre-processing performed on the data obtained from the simulations. This pre-processing is performed in order to obtain data with a format suitable for the subsequent analyses. Finally, Section 5.5 defines the metrics that are used in the analysis of the data obtained from the experiments to compare the performance of the strategies under the experimented scenarios.

5.1 Global Parameters

The global parameters are the values that define the properties of the exchange where the agents make their offers to buy or sell assets and hold or write Options. Table 5.1 lists the market parameters that remain fixed for all the experiments. Each run has 1000 time steps to allow the testing of the long run development of the market.

Initial parameters for the market	
Parameter	Initial value
Simulation duration (T)	1000
Number of available Option templates ($\|O\|$)	6
Steps between available Options (O_s)	1
Strike Price multiplier (SP_k)	15
Risk free rate (r)	0.001
Option volume (v^o)	1

Table 5.1: Initial market parameters for the experiments

The number of available Options *templates* ($|O|$) for each time step provides agents with a total of 12 possible Option contract choices, 6 call Option contracts and 6 put Option contracts. The time between Options expiration (O_s) is set to 1, this makes each generated Option expire one step later than the last one, starting at $t + 1$. The strike price multiplier

(SP_k) is set to 15, making the strike price of the goods fall in the range $[p(t) \times (1 - 0.15), p(t) \times (1 + 0.15)]$. This range is selected because the strike price of an Option is typically among the $\pm 15\%$ range from the price at the time when the Option is created [23].

The risk free interest rate (r) is set at 0.001 giving the agents an increase of 0.1% interest in their cash ($c_i(t)$) at each time step. This rate is also used throughout the life of the simulation as one of the parameters to calculate the price of the Option contracts (See Section 3.1.1).

The experimentation phase is split into two main sets of experiments. In experiments A, the agents are equipped with the α–Perfect forecasting function described in Section 3.2.2.2. The market is tested for six values of α as $\alpha = 0(0.2)1$. For each value of α, the market is tested using five different price series as the price of the underlying asset. The series will be described in Section 5.3.

In experiments B the agents are provided with the Simple Moving Average forecasting function SMA_n described in Section 3.2.2.1 with $n = 15(15)90$. The simulations are run using the same price series used in experiments A as the price of the underlying asset.

Table 5.2 summarises the complete range of values for the market parameters that defines the set of experiments. The first column lists the name given to the experiment set; the second column shows the name of the forecasting function used by the agents for each experiment; the third column shows the values of the forecasting function parameters used by the agents in each experiment. The group of the forecasting function parameters α and n is named ζ, making $\zeta = \{n, \alpha\}$. This notation is adopted for convenience and is used later on. The last column contains the name of the price series that will be used as input for the asset price in the market.

Experiment Set	Forecasting Function	Function Parameter (ζ)	Price series
A	α–Perfect	$\alpha \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$	$p \in \{Microsoft, Dell, IBM, RandomA, RandomB\}$
B	SMA_n	$n \in \{15, 30, 45, 60, 75, 90\}$	$p \in \{Microsoft, Dell, IBM, RandomA, RandomB\}$

Table 5.2: List of parameters modified for the different experiments.

After having defined the global parameters it is now possible to define the minimal unit of simulation that provides with the necessary data to extract information of interest for the thesis. This unit is called a *test case*.

Definition 4. *A **test case** is a simulation of the Option trading market comprising 50 runs of the market under the same initial conditions; fixing the **Foreacasting Function**, the value* *of the ζ parameter for the corresponding forecasting function (α or n) and* ***Price series*** *parameter P into specific values.*

The analysis that will be performed in the next chapter will be based in the comparison of the data obtained in the different test cases. For this analysis, the first 100 steps of the results obtained from each test case is removed in order to minimise any initialisation bias.

5.2 Agents Parameters

For every experiment the market is populated with six sets of agents. Each set contains 50 agents with their parameters initialised as the values shown in Table 5.3. Each set of agents is characterised by the use of one of the six trading strategies described in Section 3.2.4 (*OTMinR*, *OTMaxW*, *OTMix*, *OTRnd*, *ATSpec* and *ATNoise*).

Parameter	Values
Strategy	*OTMinR* *OTMaxW* *OTMix* *OTRnd* *ATSpec* *ATNoise*
Number of Agents	50 per strategy
Forecasting function (depending on the set of experiment)	SMA_n α–Perfect
Initial cash ($c_i(0)$)	1000
Initial goods ($g_i(0)$)	100

Table 5.3: Initial agents' parameters used in the experiments

To avoid any variability in the initial wealth conditions among the agents, the amount of cash $c_i(0)$ and number of goods $g_i(0)$ is set to be the same for all the agents at the beginning of each simulation run; thus making the wealth $w_i(0)$ equal among all the agents trading in the market. This is also done to facilitate the comparison of the performance among the agents. Each agent is provided with one of the two forecasting functions (SMA_n or α–Perfect) as described in Section 5.1. The initial wealth of the agent is established to provide them with enough resources to initially participate in the market.

5.3 Price Series

5.3.1 Description

Different price series are used for the experiments to test the performance of the agents under different market conditions. The price series are divided in two categories: stock prices series, which are obtained from the closing prices of different stocks on the NASDAQ

(Dell, Microsoft) and NYSE (IBM) stock markets for the period from January 2, 2001 to December 27, 2004; and random prices which are generated from a Normally distributed pseudo–randomly generated random walk process. Statistical information of the price series is summarised in Tables 5.4.[1]

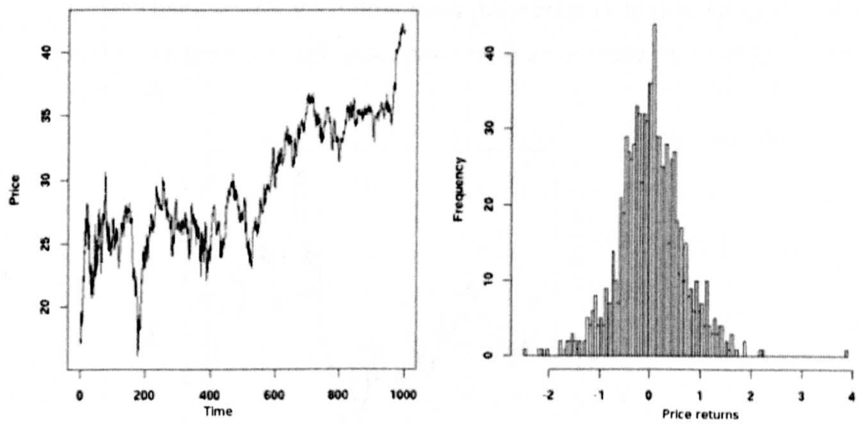

Figure 5.1: Time plot and frequency histogram of the *Dell* stock market price series used as price input for the experiments

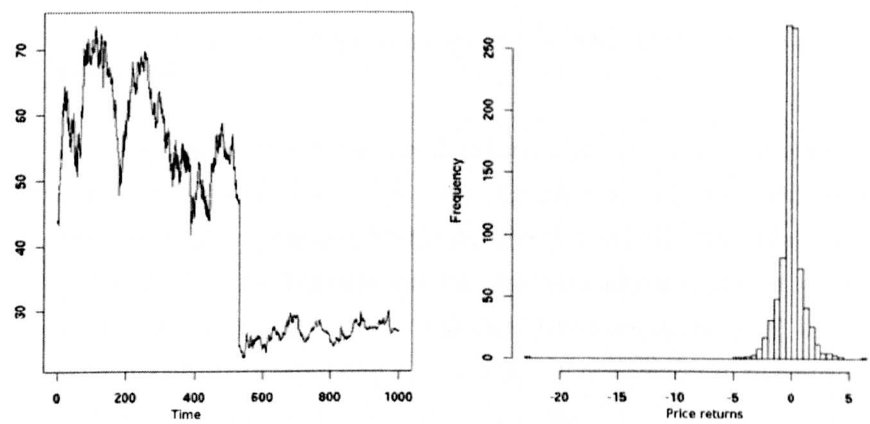

Figure 5.2: Time plot and frequency histogram of the *Microsoft* stock market price series used as price input for the experiments

The *Dell, Microsoft* and *IBM* price series is obtained from the stock prices of the corresponding companies. Time plots of these price series are shown in Figures 5.1, 5.2 and 5.3 along with the histogram of the frequencies of their returns.

[1]The used stock prices are freely available on line at `http://finance.yahoo.com/`, February 06, 2006.

The *Dell* price series is characterised by an average price of 29.58 and a standard deviation of 4.94. These values indicate that the *Dell* price series corresponds to a a low asset price in relation with the agents initial wealth (see Section 5.2).

With an average price of 43.34 and a standard deviation of 16.58, the *Microsoft* price series characterises a medium asset price relative to the agents initial wealth. The *IBM* price series represents a high asset price with an average price of 91.76 and a standard deviation of 13.48.

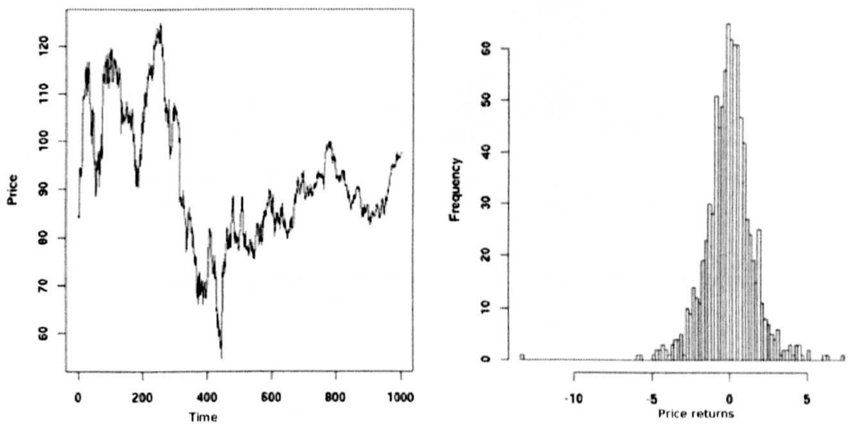

Figure 5.3: Time plot and frequency histogram of the *IBM* stock market price series used as price input for the experiments

The *Random A* and *Random B* price are pseudo–randomly generated as a random walk using a Mersenne twister algorithm and consecutively Normalising the obtained random numbers applying the Box-Muller transformation [20].[2] The *Random A* price series is composed of $T = 1000$ pseudo–random numbers with a seed of $s = 171,281$. The random walk starts with $p(t_0) = 130$ and the differences in the price at each step is within the range $[-10, 10]$ representing a high volatility price series in relation with the agents' initial wealth. The *Random B* price series is composed of $T = 1000$ pseudo–random numbers with a seed of $s = 171,281$. The random walk starts with $p(t_0) = 130$ and the differences in the price at each step is within the range $[-30, 30]$ to represent a price series with very high volatility.

The use of stochastic variables as prices for the underlying asset has been frequently applied in several Multi–Agent Models of derivatives markets such as [35] and [56]. Modelling the changes of the stock price as a stochastic process is an established procedure of

[2]The random sequences where generated using the RngPack library available at `http://www.honeylocust.com/RngPack/`, March 15, 2006.

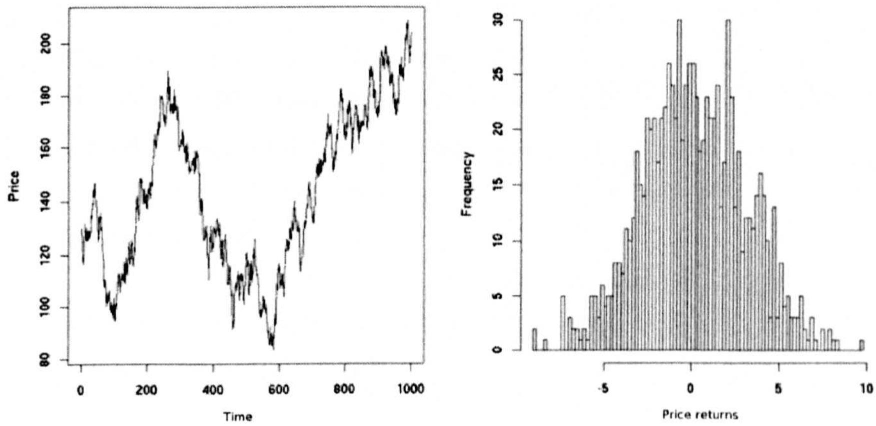

Figure 5.4: Time plot and frequency histogram of the *Random A* price series used as price input for the experiments.

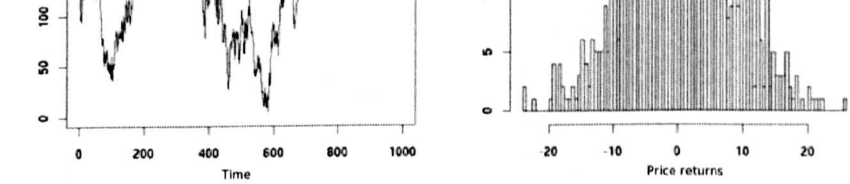

Figure 5.5: Time plot and frequency histogram for the *Random B* price series used as price input for the experiments.

computational finance [67].

5.3.2 Statistical Analysis of Price Series

The objective of using different price series as the input price for the underlying asset is to test how the differences in the underlying market conditions affect the performance of the Option Trading agents. In this section, the statistical properties of the used price series will be compared to assess the differences that using each of the price series represents to the the market in the experiments.

As stated before, it is generally accepted that the price of an asset in the stock market follows a random walk process. This means that at each step in time, the price of the asset will be given by the function:

$$p(t+1) = p(t) + \epsilon \tag{5.1}$$

where ϵ is a random disturbance term. It is also safe to assume that ϵ is distributed according a the Normal distribution function [67]. For the experiments, it is important to look at the characteristics of the random walk processes followed by the price series, that is, at the nature of its randomness. This is achieved by obtaining different statistical measures of the price returns at each time step. The results of the relevant statistical analysis are listed in Table 5.4 from where it can be seen that the mean and median of all the price series are approximately equal to 0. The differences in the series are the standard deviation, skewness and minimum and maximum statistics. The minimum and maximum statistics indicates how expensive or cheap will be the price in the price series compared to the initial wealth of the agents. As the standard deviation measures how varied are the price values around the mean, the price series with a high standard deviation (*Random A* and *Random B*) represent markets with high risk (according to the used definition of risk), whereas the price series with low standard deviation such as *Dell* represent markets with low risk. The *Microsoft* and *IBM* price series represent markets with moderate risk.

The skewness of a distribution (γ_1) is used to measure the asymmetry of a distribution around the mean. Specifically, it shows if the left tail of a distribution is significantly longer than the right tail (indicated by a negative skewness) or if the right tail is longer than the left tail (indicated by a positive skewness). This measures are in comparison with the Normal distribution. A test to verify if the skewness of a distribution is high enough to be relevant can be done by comparing the absolute value of skewness to a standard error of skewness *ses*; the absolute value of the skewness must be greater than two times the standard error of skewness which is defined as:

$$ses = \sqrt{\frac{6}{N}} \tag{5.2}$$

where N is the total number of samples [89]. In this case $N = 900$ and $2 \times ses = 0.163$.[3] Therefore if the absolute value of skewness of the returns is greater than 0.163 ($|\gamma_1| > 0.163$) the skewness is considered significant.

The analysis of the returns shows that the *Microsoft* prices series (second column of Table 5.4) has a skewness of -6.18 which absolute value is greater than 0.163, this means that the distribution of the returns has a long tail on the left of the mean. Specifically, the high negative skewness in the *Microsoft* series shows that there are certain periods when the magnitude of the decrement in the prices are very high compared to the other the steps in the price series. This can be seen in plot of the Microsoft series chart shown in Figure 5.2 where there is a sudden high decrement in the price. This decrement can be considered as a market crash, a very high decrement in the price of the asset in a small number of consecutive periods of time.

	Dell	Microsoft	IBM	Random A	Random B
Mean (μ)	-0.02	-0.02	0.01	0.07	0.2
Median	0	-0.01	0	-0.02	-0.05
Standard Deviation (σ)	0.73	1.34	1.78	2.95	7.86
Skewness (γ_1)	0.21	-6.18	-0.50	0.02	0.02
Minimum	-3.86	-22.63	-13.34	-8.82	-23.51
Maximum	3.07	6.49	11	9.64	25.71

Table 5.4: Descriptive statistics of the returns of the price series used as the price of the underlying asset.

Similarly, for *IBM* $|\gamma_1| = 0.5$, thus $|\gamma_1| > 0.163$ indicating that the price returns are significantly skewed to the left side of the normal Normal distribution. This pattern is caused by a decrease in the market price which is much higher than the average in the *IBM* price series. It is possible to observe in Figure 5.3 the decrease in the price at $t = 314$ which corresponds to the minimum price for the same price series listed in Table 5.4.

The *Dell* price series has a skewness of $\gamma_1 = 0.21$ indicating that as $|\gamma_1| > 0.163$, the returns are significantly right skewed. This can be observed in the histogram of the *Dell* prices depicted in Figure 5.1 where the maximum price drift corresponds to the maximum value listed in Table 5.4 for the same price series.

Both the *Random A* and *Random B* price series have a skewness value of $\gamma_1 = 0.02$ which is lower than 0.163 indicating that the returns of these series are not significantly

[3]Recall that the number of data samples is set to 900 after removing the data from the first 100 steps from the simulation.

skewed.

5.4 Output Data Pre–Processing

To obtain the relevant data for the different analyses the log files resulting from the experiments are pre-processed. In this section the different processing steps applied to the raw data log files are described.

5.4.1 Average of Wealth per Strategy

With the objective of calculating the relative returns for each strategy (described in Section 5.5.1) and the returns–price correlation (described in Section 5.5.2), the average of the returns at each time step for every set of agents is calculated.

Let A_S be the set of agents using strategy S. Let also $i \in A_S$ be one agent in A_S and let $w_i(t)$ be the wealth of agent i at time t. The average wealth for strategy S at time step t is:

$$w_S(t) = \frac{\sum_{i \in A_S} w_i(t)}{|A_S|} \tag{5.3}$$

Time	Option Traders				Asset Trader	
step	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise.
105	11757.1	10751.3	10854.8	10872.0	11159.8	11028.2
106	11958.7	11047.2	11144.2	11160.1	11442.8	11304.3
107	12119.3	11282.1	11374.3	11388.9	11667.2	11523.6
108	12269.6	11501.9	11589.8	11603.3	11877.2	11729.0
109	12319.0	11569.4	11656.4	11669.5	11942.5	11792.8

Table 5.5: Snapshot of a section of a resulting file after processing the strategy wealth average. Each cell contains the average wealth of all agents using the same trading strategy for each time step.

The obtained data then is averaged through the 50 runs executed per each test case. The result of this process is one table for each test case containing the time series with the mean wealth of each strategy. Each table contains information with a similar structure to the one shown on Table 5.5. Each row in this table represents one simulation step and each column contains the mean wealth for each strategy. Using this average, the results of the experiments are analysed considering the performance of each strategy. Through the rest of the thesis, the terms *strategy returns*, *strategy performance* and *strategy wealth* are used referring to the measures obtained from the average of the corresponding results across the set of agents using the strategies (i.e., *OTMinR* returns refers to the mean of the returns obtained by the set of agents that used the *OTMinR* strategy).

5.4.2 Agent Offers and trading Volume Count

To calculate the trading volume analysis ratios which are defined later in Section 5.5.3, first it is necessary to obtain the quantity of each type of offers submitted to the market. This is calculated by counting the types of offers submitted through each test case from the *ActionCount* and *MarketCount* log files (described in Section 4.2.8.2).

a) Agents Offers volume count data for *OTMinR* and *OTMaxW* strategies.

Time	OTMinR					OTMaxW				
step	Write	Hold	Buy	Sell	Pass	Write	Hold	Buy	Sell	Pass
19	1	14	12	15	8	14	23	0	13	0
20	1	19	18	10	2	16	16	0	18	0
21	2	17	17	5	9	12	22	0	16	0
22	2	21	7	15	5	19	17	0	14	0

b) Agents Offers volume count data for *OTMix* and *OTRnd* strategies.

Time	OTMix					OTRnd				
step	Write	Hold	Buy	Sell	Pass	Write	Hold	Buy	Sell	Pass
19	0	22	6	12	10	21	21	4	3	1
20	16	16	0	18	0	0	28	12	8	2
21	23	22	0	5	0	23	22	0	5	0
22	9	27	0	14	0	9	27	0	14	0

a) Agents Offers volume count data for *ATSpec* and *ATNoise* strategies.

Time	ATSpec			ATNoise		
step	Buy	Sell	Pass	Buy	Sell	Pass
19	20	30	0	17	17	16
20	23	26	1	13	21	16
21	21	27	2	16	20	14
22	30	19	1	13	16	21

Table 5.6: Snapshot of a section of a data file after processing the agents offers' volume count. Each cell contains the sum of offers made by each strategy for a specific type of offer and time step.

The result of this process is a table containing the different strategies and their corresponding offers for each step in one test case, as depicted in Tables 5.6.

The resulting data obtained after processing the *ActionCount* log files are the number of offers made by the agents while the results obtained from the *MarketCount* log files account for the number of those offers which where cleared by the market and actually traded by the agents.

Finally, the trading volumes for the different strategies are calculated by adding the offers made for all the steps in time. The obtained volumes are $vol_{OH}(S,p,\zeta)$, $vol_{OW}(S,p,\zeta)$, $vol_{CH}(S,p,\zeta)$, $vol_{CW}(S,p,\zeta)$ and $vol_{HE}(S,p,\zeta)$. These are formally defined defined as:

$$vol_{OH}(S,p,\zeta) = \sum_{t=100}^{1000} |O^H(t,S,p,\zeta)| \tag{5.4}$$

$$vol_{OW}(S,p,\zeta) = \sum_{t=100}^{1000} |O^W(t,S,p,\zeta)| \tag{5.5}$$

where $|O^H(t,S,p,\zeta)|$ and $|O^W(t,S,p,\zeta)|$ are respectively the number of submitted offers to hold and to write Options by the strategy S at each step t in the test case where the price of the asset is p and the forecasting function parameter is ζ.

$$vol_{CH}(S,p,\zeta) = \sum_{t=100}^{1000} |O^{CH}(t,S,p,\zeta)| \tag{5.6}$$

$$vol_{CW}(S,p,\zeta) = \sum_{t=100}^{1000} |O^{CW}(t,S,p,\zeta)| \tag{5.7}$$

where $|O^{CH}(t,S,p,\zeta)|$ and $|O^{CW}(t,S,p,\zeta)|$ are respectively the number of submitted offers to hold and to write Options which were cleared by the market. This volume is also calculated for strategy S at each step t in the test case where the price of the asset is p and the forecasting function parameter is ζ.

$$vol_{HE}(S,p,\zeta) = \sum_{t=100}^{1000} |O^{HE}(t,S,p,\zeta)| \tag{5.8}$$

where $|O^{HE}(t,S,p,\zeta)|$ is the number of cleared offers to hold Options which where exercised by the strategy S at each step t in the test case where the price of the asset is p and the forecasting function parameter is ζ.

5.5 Performance Analysis Description

To compare the performance of the different strategies three different analyses are done to the experimental data. The *relative returns*, *returns–price correlation* and *trading volume*. This section describes the different performance metrics used in these analyses and the process to obtain their values from the simulation data.

5.5.1 Relative Returns as a Performance Metric

Because we wish to compare the difference in performance among the strategies, a relative performance metric among the performance of all the strategies is used. This is achieved by measuring the strategies' relative returns. The relative returns are calculated obtaining the mean of the returns among all strategies and subtracting it from the returns of each strategy; thus obtaining a comparative measure of performance among the strategies.

Let $\mathcal{S}$ be the set of all strategies participating in a test case; according to [54] the returns (also called logarithmic rate of returns) obtained by a strategy $S \in \mathcal{S}$ for the step $[t-1, t]$ are calculated by the following formula:

$$R_S(t-1,t) = ln(\frac{w_S(t)}{w_S(t-1)}) \tag{5.9}$$

and the sum of returns $\mathcal{R}_S$ obtained by strategy S in one test case are calculated by the sum of the returns obtained through all the steps with:

$$\mathcal{R}_S = \sum_{t=100}^{1000} R_S(t-1,t) \tag{5.10}$$

The relative returns ΔR_S for strategy S are defined as:

$$\Delta R_S = \mathcal{R}_S - \frac{\sum_{k\in\mathcal{S}} \mathcal{R}_k}{|\mathcal{S}|} \tag{5.11}$$

Therefore, if the returns of a strategy ($\mathcal{R}_S$) are higher than the average returns obtained by all the strategies (i.e., $\frac{\sum_{k\in\mathcal{S}} \mathcal{R}_k}{|\mathcal{S}|}$) then it will have a positive value of ΔR_S. A higher ΔR_S value indicates that this strategy is performing comparatively better than the other strategies.

This performance metric allows for the comparison of the rate at which each strategy profits or loses wealth during the life of the market. In the modelled market, the wealth of each agent is the measure of their utility, therefore, strategy that obtains higher returns at the end of the simulations is considered better than the other strategies participating in the market.

5.5.2 Returns–Price Correlation as a Performance Metric

Another performance metric used is the comparison of the correlation between the returns of the asset price (obtained using Equation 2.16 from Section 2.2.2.3) and the wealth returns obtained by the use of each strategy through the simulation time steps (obtained using equation 5.9). This analysis allows the comparison of the difference in the correlations among the different strategies and the identification of the circumstances under which the difference is significant.

The correlation coefficient indicates the strength and direction of a linear relationship between two random variables [40]. To calculate the correlation between two values the

Pearson correlation coefficient can be used. The Pearson correlation (p) is obtained using the equation:

$$p = \frac{cov(X,Y)}{\sigma_X \sigma_Y} \tag{5.12}$$

where X and Y are the data sets to be tested which have the same number of items N, σ_X and σ_Y are the standard deviation of the X and Y data series and $cov(X,Y)$ is the covariance of the two data series. The resulting coefficient p is within the range of $[-1,1]$. A coefficient closer to $p = -1$ means that there is a strong negative linear relation between the variables; a coefficient closer to $p = 1$ indicates that there is a strong positive linear relation between the variables. A value close to 0 may indicate the absence of linear correlation among the variables, however the variables may still be correlated with a non linear function.

5.5.3 Trading Volume Analysis as a Performance Metric

With the objective of measuring the quantity of Option contracts being traded during each experiment, an analysis of the trading volumes (Option contracts or assets) is performed. This analysis measures the relation between the number of offers submitted to the market by agents using a strategy and number of offers from this agents that are cleared by the market at each step in time. This allows the measurement of the rate at which offers made by the agents are successfully matched and the amount of Options that are exercised.

Three different trading volume ratios are obtained; R_H^C, R_W^C and R_E^H. Each ratio is calculated for each strategy participating in each test case.

Definition 5. *$R_H^C(S,p,\zeta)$ is defined as the ratio of successful offers to hold Options for strategy S in the test case where the price of the asset is p and the forecasting function parameter is ζ:*

$$R_H^C(S,p,\zeta) = \frac{vol_{CH}(S,p,\zeta)}{vol_{OH}(S,p,\zeta)} \tag{5.13}$$

the R_H^C ratio allows the determination of the proportion of the number of offers to hold Option contracts that the agents make in relation with the number of those offers that are cleared by the market. If all the offers to hold Option contracts are matched by the market then $R_H^C = 1$, whereas $R_H^C = 0$ denotes that none of the offers were cleared in the market. If the agent does not make offers to hold Options then R_H^C is not defined.

Definition 6. *$R_W^C(S,p,\zeta)$ is defined as the ratio of successful offers to write Options for strategy S in the test case where the price of the asset is p and the forecasting function* *parameter is ζ:*

$$R_W^C(S,p,\zeta) = \frac{vol_{CW}(S,p,\zeta)}{vol_{OW}(S,p,\zeta)} \tag{5.14}$$

the R_W^C ratio allows to determine the proportion of the volume of offers to write Option contracts that the agents make in relation with the volume of those Options that are cleared by the market. If all the offers to write Option contracts are matched by the market then $R_W^C = 1$, whereas $R_W^C = 0$ denotes that none of the offers to write Options were cleared. If the agent does not make offers to write Options then R_W^C is not defined.

Definition 7. *$R_E^H(S,p,\zeta)$ is defined as the ratio of exercised hold Options for strategy S in the test case where the price of the asset is p and the forecasting function parameter is ζ:*

$$R_E^H(S,p,\zeta) = \frac{vol_{HE}(S,p,\zeta)}{vol_{CH}(S,p,\zeta)} \tag{5.15}$$

the R_E^H ratio represents the proportion of Options that agents hold in relation with the number of these Options that are exercised. If all the hold Option contracts are exercised by the agents then $R_E^H = 1$. Respectively, if none of the hold Options are exercised then $R_E^H = 0$. If the agent does not make offers to hold Options then R_E^H is not defined.

5.6 Experiments Platform Setup

The experiments were carried on a 15 node cluster which provided the possibility of performing each simulation in parallel over the 15 available nodes. All the nodes in the cluster shared similar hardware specifications. A summary of the hardware configuration in the cluster nodes is shown on Table 5.7.

Property	Value
Operating system	Fedora Core release 6
Kernel version	Linux Kernel 2.6.18-1.2798.fc6
Processor	Intel Core 2 CPU 6400
Processor speed	2.13 GHZ
Processor cache size	2048 KB
Installed RAM memory	3 GB

Table 5.7: Hardware configuration used for the simulation experiments.

5.7 Conclusion

In this chapter the methodology of the simulation experiments has been presented. The configuration of the different scenarios to be tested has been described and the methodology for the analysis of the obtained results has been defined.

The analysis of the performance of the agents trading in the Option market requires the use of performance metrics used for the evaluation of real financial markets such as the log-returns [54]. This type of analysis is adopted to perform a quantitative evaluation of the strategies participating in the market

The next chapter describes the results of the experiments presented in this chapter, detailing the results from the analysis of the simulation and comparing the performance of the different strategies under the proposed scenarios.

Chapter 6

Experimental Results

This chapter details the analysis of the data obtained from the experiments designed in Chapter 5. The chapter aims to compare the performance between the different strategies in the market and between the defined scenarios. The obtained data is analysed using the performance metrics specified in Section 5.5. This analysis is augmented with a discussion of the relation between results of the experimentation and the research objectives specified in Chapter 1 to demonstrate the relevance of the results.

The present chapter is structured in the following manner. First, in Section 6.1 a data analysis is performed to the data obtained from the experiments, comprising the test of the assumption of Normality and an analysis of the differences in variance among the data series. The test of Normality is used to know if the subsequent analyses can be done assuming that the data series follow a Normal distribution. The analysis of differences in variance is performed to ensure that the differences in the data series compared throughout the subsequent tests are statistically significant.

The results from the analysis of the relative returns for the different strategies are presented in Section 6.2. The results from the analysis are presented separately for each of the two defined sets of experiments. First, Section 6.2.1 presents the results for the set of experiments A (defined by the use of the α–Perfect forecasting function). Next, the results for the set of experiments B (defined by the use of the SMA_n forecasting function) are presented in Section 6.2.2. The results obtained from the variation of the respective forecast parameter ζ (α in the case of the α–Perfect forecasting and n in the case of SMA_n forecasting) are reported for each of these set of experiments. Subsequently, in Section 6.2.3, a summary of the results obtained from this analysis is provided with a discussion of the relation and significance of the results to the research objectives.

Section 6.3 contains the results from the analysis of correlation between the price returns and the returns obtained by each strategy. These results are obtained performing the statistical test described in Section 5.5.2. Results are presented grouped by the agents forecasting function; for α–Perfect forecasting in Section 6.3.1, and for SMA_n forecasting in Section 6.3.2. This analysis is used to compare the difference in the correlations between the strategies' returns and the returns of the asset price series used in the market. Later, a discussion of the results for this analysis is developed in Section 6.3.3 where the link between the results of the analysis of correlation and the research objectives is established and discussed.

Results of the trading volume analysis defined in Section 5.5.3 are presented in Section 6.4, where the three Option trading volume ratios (R_H^C, R_W^C and R_E^H) are obtained for each Option trading strategy on each test case. The section is split in three sub-sections; Sections 6.4.1 and 6.4.2, contain the trading volume analysis for the α–Perfect and SMA_n sets of experiments respectively, describing the results and comparison of the different ratios under the experimented scenarios. Section 6.4.3 contains the summary, a comparison of the results from the trading volume analysis and the relevance of such results in relation with the research objectives is discussed.

In Section 6.5, a summary comparison of the results obtained from the analysis performed in the previous sections from this chapter is presented and a discussion of the relevance of these results in relation to the research objectives is offered, contrasting the research objectives with the experimental results presented in the chapter. Finally, in Section 6.6 the conclusions drawn from the results presented in this chapter are provided.

6.1 Generic Data Analysis

This section focuses on the description of the general statistical analysis of the data series containing the wealth returns obtained by the strategies ($R_S(\cdot)$) from the experiments. The objective of this analysis is to verify that these returns can be considered as statistically different data among them. If the returns are statistically different among them then any comparison made between them will be significant. Also in this section, the test of Normality will be performed to the returns data series. After knowing if the data series follow a Normal distribution, the adequate statistical tests can be chosen in order to perform the analyses of the subsequent sections. The results of these tests are presented grouped into the two sets of experiments: set A for the experiments using α–Perfect and set B for the experiments using SMA_n forecasting.

The data to be analysed are the returns obtained by the use of each strategy for each test case (i.e., $R_S(t-1,t)$ from $t = 100$ to $t = 1000$ for each strategy S). The returns are first tested for Normality to select the test of correlation to use for the Analysis of Correlation performance test.[1] The Normality test results are also used to select a suitable variance test.[2]

Afterwards, the variance among the returns obtained by each strategy is analysed. For this, a test for heterogeneity of variances is performed among the returns obtained by the strategies in each test case. The results of this test are used to verify whether the differences among the data series are statistically significant.

6.1.1 Test of Assumption of Normality

To perform the assumption of Normality test, the Shapiro-Wilk [81] test is used. Let S stand for the data series containing the returns obtained by one strategy in one test case. The Shapiro-Wilk Test is performed with the null hypothesis H_0 and alternative hypothesis H_1 of:

$$\begin{aligned} H_0 &: S \text{ is Normally distributed (reject at } p < 0.01) \\ H_1 &: S \text{ is not Normally distributed,} \end{aligned} \tag{6.1}$$

therefore, if the null hypothesis is not rejected then the data S is assumed to be Normally distributed. Otherwise, H_0 is rejected in favour of the alternative hypothesis H_1, and it is assumed that S is not Normally distributed.

A summary of the results for the test of assumption of Normality is presented in Table 6.1. Each cell in the table shows the result of the test for the data series of one strategy, indicating whether the data is assumed to be Normally distributed or not.

6.1.1.1 α–Perfect Forecasting Experiments

The results from this analysis (shown in Table 6.1) indicate that for the majority the test cases, the resulting p-values are less than the predefined significance level established at 0.01. Consequently, the hypothesis H_0 is rejected in favour of H_1 and the returns of the agents are treated as non Normally distributed data series. Therefore subsequent statistical

[1] The Pearson's correlation coefficient [11] is used when the samples to be tested are Normally distributed. If this is not the case then another correlation test such as Spearman's rank correlation needs to be used as it does not require the assumption of Normality of sample data.

[2] If the returns are Normally distributed then it is possible to use a parametric test (which assumes Normality of the data), else a test that does not assumes Normality must be used. In the latter case, the Fligner-Killeen test will be used as it is one of the most robust tests against deviation from Normality according to [27].

tests applied to the data series avoid using methods that assume the Normality of the data. The p-values resulting from the application of the Shapiro-Wilk Test to the returns data series for the α–Perfect set of experiments are presented in Appendix B.

Only in the *Random A* price series was used, some strategies returns observed a p-value greater than the predefined significance level, indicating that the returns are Normally distributed. Given this outcome, statistical tests that do not assume the Normality of the data series will be used for the test of difference of variance performed later in this section and for the test of returns–price correlation with performed in Section 6.3.

6.1.1.2 SMA_n Forecasting Experiments

The results from this analysis also show that for the majority of the test cases, the p-values are less than the established significance level of 0.01. Therefore, the hypothesis H_0 is rejected in favour of H_1 and the returns of the agents are treated as non Normally distributed data series. The resulting p-values from the Shapiro-Wilk statistical test applied the returns data series for the SMA_n forecasting set of experiments are shown in Appendix B.

Only in the *Random A* price series was used, some strategies returns observed a p-value greater than the predefined significance level, indicating that the returns are Normally distributed. As with the α-Perfect forecasting experiments, statistical tests that do not assume the Normality of the data series will be used for the test for variance difference performed later in this section and for the test of correlation with price performed in Section 6.3.

a) Experiments using α–Perfect forecasting, for all values of α.						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATNoise	ATSpec
Microsoft	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal
Dell	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal
IBM	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal
Random A	Normal	Normal	Normal	Normal	Normal	Normal
Random B	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal
b) Experiments using SMA_n forecasting, for all values of n.						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATNoise	ATSpec
Microsoft	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal
Dell	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal
IBM	Non Normal	Normal	Normal	Normal	Normal	Normal
Random A	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal
Random B	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal	Non Normal

Table 6.1: Summary of results for the Shapiro-Wilk test of assumption of Normality for the returns obtained by the different strategies.

6.1.2 Test of Heterogeneity of Variance

The heterogeneity of variance test is performed using the Fligner-Killeen Test for Homogeneity of Variance [38] between the returns of the different strategies data series. This test is a distribution–free test which does not require the assumption that the data samples are Normally distributed. Because the majority of the tested data series were not Normally distributed (according to the results from the Normality test presented in the previous section), the Fligner-Killeen test is used to analyse all the data series including the returns from the *Random A* and *Random B* which are Normally distributed.

Let $S_1, S_2 ... S_n$ represent the data series containing the returns of the different strategies in one test case. Also, let $\sigma_1, \sigma_2, ... \sigma_n$ be the standard deviations of such data series. The Fligner-Killeen Test is performed with the null H_0 and alternative hypothesis H_1 of:

$$\begin{aligned} H_0 &: \sigma_1 = \sigma_2 = ... = \sigma_n \text{ (reject at } p < 0.01) \\ H_1 &: \sigma_i \neq \sigma_j; \forall i, j. \end{aligned} \tag{6.2}$$

Therefore if the null hypothesis is accepted it is considered that the variance of the returns obtained by the different strategies for each test case are not significantly different. If the p–values are lower than the predefined significance level, the null hypothesis is rejected in favour of the alternative hypothesis and the variances of the returns may be considered significantly different among themselves.

If the variances of two data series are different, then it is safe to assume that the data series come from different distributions [40]. If this is the case, we will proceed with the assumption that the returns of the strategies are different.

6.1.2.1 α–Perfect Forecasting Experiments

Table 6.2 contains the resulting p-values and Fligner-Killeen statistics obtained from the test. The results suggest that the hypothesis H_0 can be rejected for all the test cases as the p–values for all are lower than the predefined significance level (that is $p < 0.01$). Therefore H_0 is rejected in favour of H_1 and the variances of the data series are considered different. This indicates that the data series can be treated as coming from different distributions.

6.1.2.2 SMA_n Forecasting Experiments

The data in Table 6.3 contains the resulting p–values and the statistic for the Fligner-Killeen Test of homogeneity of variances. The results show that for all the experiments, the p–values are less than the established significance level of 0.01. Therefore the hypothesis H_0 is rejected in favour of the H_1 hypothesis and the returns are considered significantly different among themselves.

α	Microsoft	Dell	IBM	Random A	Random B
0	$p = 8.1 \times 10^{-192}$	$p = 1.4 \times 10^{-312}$	$p = 9.4 \times 10^{-198}$	$p = 1.1 \times 10^{-215}$	$p = 6.9 \times 10^{-235}$
	$s = 897.77$	$s = 1455.40$	$s = 925.20$	$s = 1008.01$	$s = 1096.71$
0.2	$p = 1.4 \times 10^{-190}$	$p = 1.6 \times 10^{-306}$	$p = 3.1 \times 10^{-198}$	$p = 4.0 \times 10^{-220}$	$p = 1.4 \times 10^{-224}$
	$s = 892.03$	$s = 1427.44$	$s = 927.42$	$s = 1070.10$	$s = 1049.17$
0.4	$p = 2.6 \times 10^{-187}$	$p = 1.2 \times 10^{-290}$	$p = 4.6 \times 10^{-185}$	$p = 5.5 \times 10^{-221}$	$p = 5.8 \times 10^{-202}$
	$s = 876.91$	$s = 1354.17$	$s = 866.57$	$s = 1032.51$	$s = 944.63$
0.6	$p = 6.4 \times 10^{-173}$	$p = 5.2 \times 10^{-247}$	$p = 1.4 \times 10^{-153}$	$p = 1.5 \times 10^{-181}$	$p = 1.8 \times 10^{-98}$
	$s = 810.43$	$s = 1152.69$	$s = 720.99$	$s = 850.37$	$s = 465.92$
0.8	$p = 3.8 \times 10^{-40}$	$p = 5.8 \times 10^{-216}$	$p = 1.5 \times 10^{-19}$	$p = 1.6 \times 10^{-35}$	$p = 9.9 \times 10^{-22}$
	$s = 194.72$	$s = 1009.31$	$s = 97.84$	$s = 173.08$	$s = 108.19$
1	$p = 8.4 \times 10^{-37}$	$p = 8.5 \times 10^{-213}$	$p = 2.4 \times 10^{-09}$	$p = 8.0 \times 10^{-10}$	$p = 5.0 \times 10^{-21}$
	$s = 179.09$	$s = 994.69$	$s = 48.81$	$s = 51.17$	$s = 104.88$

Table 6.2: Resulting p-values and statistics for the Fligner-Killeen Test for homogeneity of variance among returns. Results obtained from the α–Perfect forecasting experiments.

SMA_n n	Microsoft	Dell	IBM	Random A	Random B
15	$p = 5.3 \times 10^{-35}$	$p = 3.5 \times 10^{-233}$	$p = 2.7 \times 10^{-10}$	$p = 1.8 \times 10^{-11}$	$p = 1.2 \times 10^{-24}$
	$s = 170.6$	$s = 1088.8$	$s = 53.4$	$s = 59.2$	$s = 121.9$
30	$p = 7.0 \times 10^{-36}$	$p = 2.4 \times 10^{-233}$	$p = 3.5 \times 10^{-12}$	$p = 1.3 \times 10^{-11}$	$p = 9.4 \times 10^{-23}$
	$s = 174.7$	$s = 1089.6$	$s = 62.5$	$s = 59.9$	$s = 113.0$
45	$p = 1.0 \times 10^{-36}$	$p = 1.9 \times 10^{-228}$	$p = 8.3 \times 10^{-13}$	$p = 9.6 \times 10^{-12}$	$p = 9.1 \times 10^{-23}$
	$s = 178.7$	$s = 1066.9$	$s = 65.6$	$s = 60.5$	$s = 113.0$
60	$p = 6.6 \times 10^{-38}$	$p = 4.3 \times 10^{-221}$	$p = 4.0 \times 10^{-11}$	$p = 5.8 \times 10^{-10}$	$p = 2.7 \times 10^{-23}$
	$s = 184.2$	$s = 1032.9$	$s = 57.4$	$s = 51.8$	$s = 115.5$
75	$p = 2.1 \times 10^{-41}$	$p = 4.0 \times 10^{-219}$	$p = 3.1 \times 10^{-12}$	$p = 3.7 \times 10^{-08}$	$p = 3.8 \times 10^{-23}$
	$s = 200.6$	$s = 1023.8$	$s = 62.8$	$s = 42.9$	$s = 114.8$
90	$p = 5.9 \times 10^{-42}$	$p = 2.8 \times 10^{-216}$	$p = 9.9 \times 10^{-14}$	$p = 1.4 \times 10^{-08}$	$p = 4.3 \times 10^{-24}$
	$s = 203.1$	$s = 1010.7$	$s = 70.0$	$s = 45.0$	$s = 119.3$

Table 6.3: Resulting p-values and statistics for the Fligner-Killeen Test for homogeneity of variance among returns. Results obtained from the SMA_n forecasting experiments.

6.2 Strategy Returns Performance Analysis

This section describes the results of performing the analysis of returns described in Section 5.5.1 to the data series obtained from the experiments.

6.2.1 α–Perfect Forecasting

Tables 6.4, 6.5 and 6.6 contain the relative returns of obtained by the strategies in the set of experiments A. Each row contains the relative returns of the strategies for one test case. The first column is the asset price series used in the corresponding test case. From the second column, each column contains the relative returns for the different strategies that participated in the market. For each row, the highest return is highlighted. The test cases in the tables are grouped by the value of the forecasting parameter α.

The results from the stock market price series (*Microsoft*, *Dell* and *IBM* price series) show that the *OTMinR* strategy obtains higher returns than the other strategies participating in the market. The *OTMinR* strategy also obtains higher returns than the other strategies in most of the test cases where the *Random A* price series is used as the asset price.

When the *Random B* price series is used as the asset price, the *OTMaxW* strategy outperforms the other strategies in most of the test cases. The test case when $\alpha = 0.6$ is the only case (using the *Random B* price series) where the *OTMix* strategy outperforms the *OT-MaxW*. The only test case where the *ATSpec* strategy outperforms the other strategies is when the *Random A* price series is used and $\alpha = 1$.

It is of special interest, the fact that for the test cases of the *Microsoft* asset price, the performance of the *OTMinR* is more than 100% higher than the average for the experiments with high and uncertainty (when $\alpha = 0, 0.2$) shown in Figure 6.4, and medium uncertainty (when $\alpha = 0.4, 0.6$) shown in Figure 6.5.

It should be recalled that, the *Microsoft* price series is characterized by a market crash where the price decreases abruptly in one time step. Considering this fact along with the high returns of the agents gives the intuition that the agents using the *OTMinR* strategy were able to prevent much losses due to the market crash. This phenomena is investigated furtherly in Section 6.3.

In the same way, the *IBM* price series is characterized by a similar market crash where the price decreases by a high in one time step. This may be the reason of why the returns of the *OTMinR* are more than 50% higher than the average.

From Table 6.5, it is possible to see that the advantage of the *OTMinR* strategy decreases when the forecasting accuracy is high (i.e., when $\alpha = 0.8, 1$). And in the case of the *Random*

A price series, it is outperformed by the *ATSpec* strategy.

Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
a) Relative returns when $\alpha = 0$						
Dell	**0.231**	-0.131	0.096	-0.046	-0.084	-0.066
Microsoft	**1.165**	-0.530	0.280	-0.054	-0.430	-0.430
IBM	**0.655**	-0.232	0.037	-0.090	-0.186	-0.184
Random A	**0.191**	-0.060	-0.037	-0.036	-0.027	-0.032
Random B	-0.324	**0.162**	-0.087	0.042	0.113	0.093
b) Returns when $\alpha = 0.2$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.225**	-0.126	0.086	-0.043	-0.083	-0.060
Microsoft	**1.166**	-0.522	0.265	-0.062	-0.431	-0.417
IBM	**0.665**	-0.230	0.047	-0.093	-0.195	-0.194
Random A	**0.230**	-0.066	-0.035	-0.042	-0.041	-0.045
Random B	-0.324	**0.158**	-0.026	0.028	0.095	0.069

Table 6.4: Relative Returns of the different strategies for the experiments with different price series using α–Perfect forecasting with $\alpha = 0$ and $\alpha = 0.2$. The highest values for each row are emphasised.

Further information is obtained by comparing the dispersion of the relative returns for one test case. This comparison is done obtaining the standard deviation of the relative returns for each test case (denoted by each row from Tables 6.4 to 6.6). The standard deviations are shown in Table 6.7. From these standard deviations it is possible to see that, in the test cases where the forecasting accuracy is high (when $\alpha = 0.8$ and $\alpha = 1$) the dispersion of the relative returns is lower than in the experiments where the forecasting accuracy is medium ($\alpha = 0.4$ and $\alpha = 0.6$) and when it is low ($\alpha = 0$ and $\alpha = 0.2$). This pattern indicates that as the uncertainty decreases in the market (i.e., as $\alpha \rightarrow 1$), the difference between the returns obtained by the use of the strategies is lower and consequently agents may have fewer opportunities to profit from the market. This may be due to the fact that as the forecast accuracy increases, all the agents will tend to make offers that lay on the same side of the market.

The differences in the returns of the Option trading strategies shown so far suggest that under the experimented conditions, trading Option contracts may provide an advantage to an agent by yielding higher returns than the non-Option trading strategies. Specifically, when the traders face more uncertainty in their forecasting (i.e., as $\alpha \rightarrow 0$), trading Options can give greater advantage over non trading Options. Figure 6.1 shows a chart with the standardised relative returns (scaling the values from Tables 6.4, 6.5 and 6.6 to make their mean 0 and standard deviation 1) for each strategy under the experimented scenarios. The results are grouped by the price series used as the price of the asset. For each group, the horizontal axis in the figures represents the standardised relative returns obtained by the strategies participating in the market. The vertical axis represents the different values of α. From this figure it is possible to see that the returns of the agents follow certain trends as the value

a) Relative returns when $\alpha = 0.4$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.214**	-0.123	0.066	-0.035	-0.070	-0.051
Microsoft	**1.174**	-0.498	0.200	-0.050	-0.418	-0.408
IBM	**0.637**	-0.219	0.029	-0.079	-0.185	-0.183
Random A	**0.235**	-0.068	-0.027	-0.041	-0.048	-0.051
Random B	-0.349	**0.154**	0.069	0.029	0.059	0.038
b) Returns when $\alpha = 0.6$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.187**	-0.124	0.022	-0.024	-0.034	-0.026
Microsoft	**1.177**	-0.461	0.027	-0.011	-0.375	-0.357
IBM	**0.570**	-0.191	-0.005	-0.053	-0.160	-0.161
Random A	**0.154**	-0.060	0.005	-0.025	-0.034	-0.040
Random B	-0.289	0.150	**0.156**	0.008	-0.014	-0.010

Table 6.5: Relative Returns of the different strategies for the experiments with different price series using α–Perfect forecasting with $\alpha = 0.4$ and $\alpha = 0.6$. The highest values for each row are emphasised.

a) Relative returns when $\alpha = 0.8$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.157**	-0.135	-0.020	-0.010	0.009	-0.001
Microsoft	**0.775**	-0.465	-0.436	-0.003	0.084	0.046
IBM	**0.268**	-0.138	-0.095	0.031	-0.030	-0.036
Random A	**0.043**	-0.028	0.035	0.003	-0.022	-0.031
Random B	-0.260	**0.152**	0.149	-0.007	-0.018	-0.016
b) Returns when $\alpha = 1$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.174**	-0.139	-0.043	-0.002	0.020	-0.009
Microsoft	**0.759**	-0.468	-0.447	-0.026	0.129	0.054
IBM	**0.205**	-0.142	-0.131	0.043	0.010	0.017
Random A	-0.001	-0.012	-0.008	0.015	**0.021**	-0.016
Random B	-0.264	**0.154**	0.153	-0.006	-0.013	-0.024

Table 6.6: Relative Returns of the different strategies for the experiments with different price series using α–Perfect forecasting with $\alpha = 0.8$ and $\alpha = 1$. The highest values for each row are emphasised.

α	Microsoft	Dell	IBM	Random A	Random B
0	0.59	0.12	0.30	0.08	0.16
0.2	0.58	0.12	0.31	0.10	0.15
0.4	0.57	0.11	0.29	0.10	0.16
0.6	0.55	0.09	0.26	0.07	0.14
0.8	0.41	0.08	0.13	0.03	0.13
1.0	0.41	0.09	0.11	0.01	0.14

Table 6.7: Standard deviations of the relative returns for each test case in the α–Perfect forecasting experiments.

of α is varied among experiments. Specifically, the *OTMinR* strategy is the top performing strategy in the experiments using the *Microsoft*, *IBM* and *Dell* price series. In the same experiments, the *OTMaxW* strategy is the worst performing strategy. Also, the performance of the *ATSpec* strategy (asset-only trading speculator) improves as the uncertainty decreases (i.e., as $\alpha \to 1$).

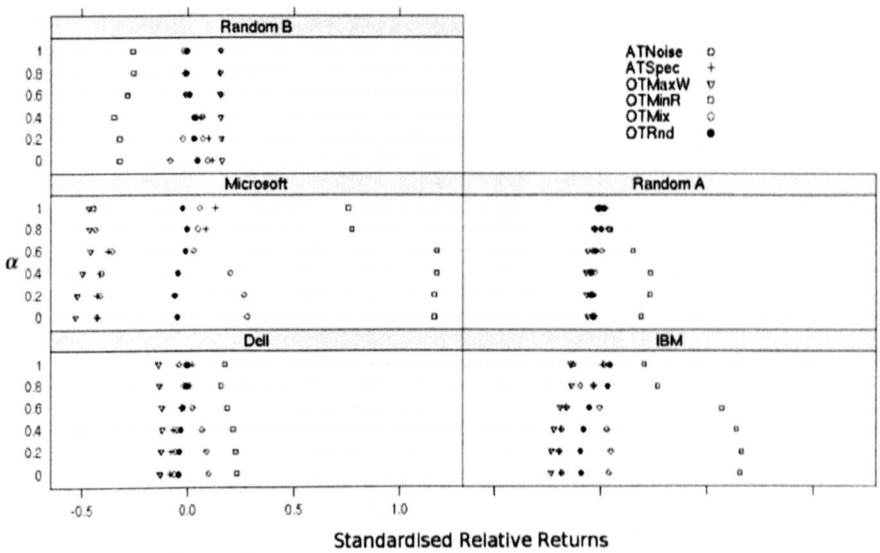

Figure 6.1: Standardised Relative returns of the different strategies for the α–Perfect forecasting experiments.

In the test cases where the *Random B* price series is used, the *OTMinR* performs worse than all the other strategies. However, in this same case, both the *OTMaxW* and the *OTMix* strategy have the best performance.In the cases where *Random A* price series is used, the *OTMinR* strategy performs better than the other strategies for the high and medium uncertainty test cases, however in the cases where the uncertainty is low, the difference between the returns of all the strategies is very low.

6.2.2 SMA_n Forecasting

The results for the analysis of relative returns in the experiments using the SMA_n forecasting function is described now. It must be remembered that the variation of the SMA_n forecasting parameter n does not directly imply a better or worse forecast; instead, the n parameter is used to define the number of values in the past that are considered to calculate the mean of the price for the forecasting. For this reason, the results obtained from the set of experiments B may not have well defined patterns due to the variation of this parameter.

The relative returns obtained by the strategies for the experiments SMA_n experiments are shown in Tables 6.8, 6.9 and 6.10. From the obtained data it is possible to see that the *OTMinR* strategy outperforms the other strategies in the test cases where the *Dell*, *Microsoft* and *IBM* stock prices are used as the price of the asset.

a) Relative returns when $n = 15$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.171**	-0.136	-0.042	0.007	-0.051	0.050
Microsoft	**0.741**	-0.468	-0.450	0.009	-0.021	0.188
IBM	**0.217**	-0.140	-0.133	0.049	-0.058	0.065
Random A	0.006	-0.012	-0.012	**0.019**	-0.019	**0.019**
Random B	-0.285	**0.153**	0.133	0.020	-0.040	0.019
a) Relative returns when $n = 30$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.165**	-0.133	-0.034	0.014	-0.103	0.092
Microsoft	**0.749**	-0.466	-0.445	0.026	-0.093	0.230
IBM	**0.217**	-0.140	-0.133	0.045	-0.041	0.052
RandomA	0.006	-0.012	-0.013	**0.017**	0.001	0.000
RandomB	-0.286	**0.150**	0.139	0.020	-0.023	0.000

Table 6.8: Relative Returns of the different strategies for the experiments with different price series using SMA_n forecasting with $n = 15$ and $n = 30$. The highest values for each row are emphasised.

a) Relative returns when $n = 45$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.164**	-0.134	-0.032	0.015	-0.116	0.103
Microsoft	**0.736**	-0.470	-0.446	0.014	0.073	0.094
IBM	**0.213**	-0.140	-0.131	0.042	-0.023	0.039
Random A	0.008	-0.013	-0.015	0.015	**0.022**	-0.017
Random B	-0.287	**0.144**	0.129	0.010	0.059	-0.054
a) Relative returns when $n = 60$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.162**	-0.133	-0.031	0.017	-0.121	0.105
Microsoft	**0.729**	-0.470	-0.443	0.021	0.065	0.099
IBM	**0.216**	-0.140	-0.131	0.042	-0.047	0.059
RandomA	0.002	-0.016	-0.018	0.014	**0.035**	-0.017
RandomB	-0.301	**0.140**	0.126	0.011	0.091	-0.067

Table 6.9: Relative Returns of the different strategies for the experiments with different price series using SMA_n forecasting with $n = 45$ and $n = 60$. The highest values for each row are emphasised.

In the cases where the *Random B* price series is used, the *OTMaxW* strategy outperforms the other strategies. In the same cases, the *OTMix* strategy also outperform the rest of the strategies and its relative returns are very close those of the *OTMaxW* strategy.

It is possible to see from the resulting data that the *OTRnd* strategy gets the highest returns in the cases where the *Random A* price series is used and $n = 15$ and 30 (shown in Figure 6.8). Finally, the *ATSpec* strategy obtains the highest returns in the cases where the asset price series is *Random A* and $n = 45, 60, 75$ and 90 (Figures 6.9 and 6.10).

Figure 6.2 contains charts with the standardised relative returns from the experiments. The horizontal axis represents the standard relative returns and the vertical axis represents the different value used for the SMA_n n parameter which represents the number of steps

a) Relative returns when $n = 75$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.159**	-0.132	-0.028	0.018	-0.129	0.112
Microsoft	**0.736**	-0.467	-0.441	0.013	0.133	0.027
IBM	**0.225**	-0.139	-0.129	0.044	-0.056	0.056
RandomA	-0.007	-0.019	-0.019	0.016	**0.046**	-0.017
RandomB	-0.316	**0.139**	0.132	0.017	0.095	-0.068

a) Relative returns when $n = 90$						
Price Series	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.155**	-0.133	-0.022	0.019	-0.139	0.119
Microsoft	**0.739**	-0.466	-0.442	0.019	0.089	0.061
IBM	**0.230**	-0.138	-0.124	0.043	-0.064	0.053
RandomA	-0.001	-0.021	-0.021	0.014	**0.051**	-0.023
RandomB	-0.321	**0.138**	0.126	0.023	0.103	-0.070

Table 6.10: Relative Returns of the different strategies for the experiments with different price series using SMA_n forecasting with $n = 75$ and $n = 90$. The highest values for each row are emphasised.

used for calculating the moving average. The results are grouped by the price series used as the asset price.

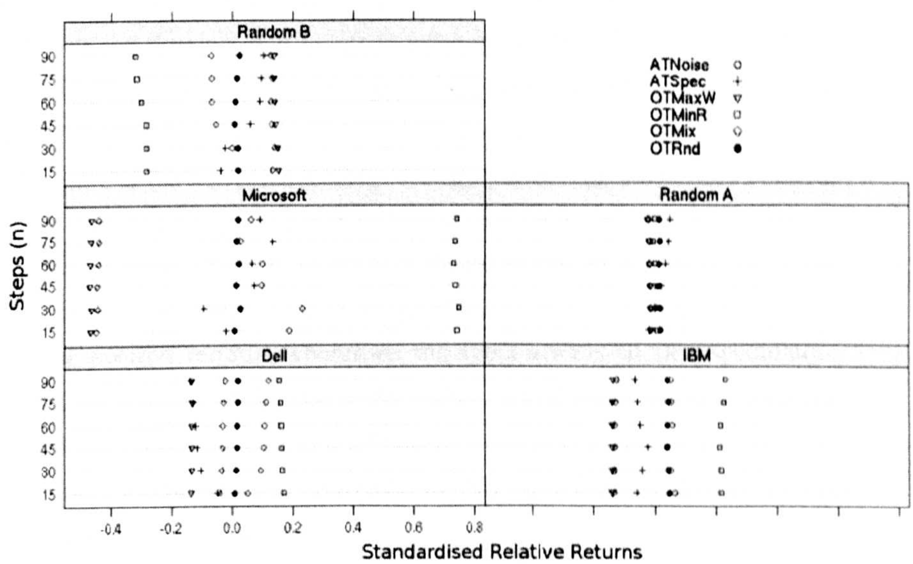

Figure 6.2: Standardised Relative returns of the different strategies for the SMA_n forecasting experiments.

The results in the figure show that for the experiments where the *Dell*, *Random A* and *IBM* series are used, the differences in the relative returns among the strategies are similar regardless of the value of n. However, in the cases where the *Random B* and *Microsoft* series are used, the performance of the *ATNoise* and *ATSpec* strategies varies depending on the value of n.

Another characteristic from the results when using the *Random B* price series (as shown in shown in Figure 6.2) is that the *OTMinR* strategy performs worse than the other strategies for all the tested values of n. Also in the same case and in the case where the *Random A* price series is used as the price of the asset, the *ATSpec* strategy performance increases with the number of steps for the SMA_n forecasting.

6.2.3 Discussion

The results obtained from the analysis of the returns of the different strategies strongly suggest that the the agents using an Option trading strategy obtained higher returns than those not using such strategies in the majority of the experimented scenarios. The specific scenarios where agents trading Option contracts did not obtain higher returns than those not trading Options are in few cases when using the *Random A* price series and when the *Random B* price series is used as the price of the underlying asset. For the experiments where the asset price was not the *Random B* series, the *OTMinR* strategy performed better than the other strategies. The *OTMaxW* strategy performed better than the other strategies in all the test cases where the *Random B* price series was used in the SMA_n experiments.

Relevant issues from the results of the experiments using α–Perfect forecasting are that not all the strategies using Options have a better performance than the strategies not using Options; notably, the *OTMaxW* Option trading strategy performed worse than both the *ATSpec* and the *ATNoise* asset trading strategies in almost every case, excepting the experiments using the *Random B* price series as the price of the asset. Given the definition of the *OTMaxW* strategy, an agent using that strategy will select the action for which there is more probability of obtaining positive returns. Therefore, the agent always adopt a speculator–like strategy according to its model of the risk in the market and its forecasting. From this result then, it is possible to conclude that using Options does not guarantee a better outcome than not using Options in all possible scenarios. Moreover, it shows that a bad Option trading strategy can cause a reduced performance.

The *OTMix* strategy, which is a strategy that combines the *OTMinR* and *OTMaxW* strategies has a good performance compared to the other strategies (excepting the *OTMinR* strategy). While the returns of the *OTMix* strategy are not as good as the returns of using only the *OTMinR* strategy, it still outperforms the asset only trading strategies in the experiments where uncertainty is high and medium ($\alpha = 0$, $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.6$). Moreover, the *OTMix* strategy has a better performance than *OTMinR* in the scenarios where the *Random B* price series is used as the asset price. This, due to the use of the *OTMaxW*-like strategy.

From the results of the experiments using SMA_n forecasting, it is possible to conclude that the *OTMinR* strategy also performs remarkably better than the other strategies in the experiments where the asset price is one of the stock market price series. However, for both of the randomly generated price series, the *OTMinR* strategy is outperformed by other strategies.

The fact that the *OTMinR* strategy performs bad in all the cases when *Random B* price series is used and in both set of experiments (when using α–Perfect forecasting or SMA_n forecasting) may indicate that this strategy is not good for scenarios where the variance of the price is as high as in the *Random B* price series. However, in these type of scenarios it is still possible for Option-trading agents to perform better than agents not trading Options as can be seen from the performance of the *OTMaxW* strategy, which outperforms the other strategies in the *Random B* cases.

Recapitulating the hypothesis established in Chapter 1, this analysis has shown that agents can indeed benefit from trading Options in the market; the analysis also shows that agents benefit most from trading Options in the cases where the asset prices are based on *real* price series. However, the performance of the Option-trading agents using the *OTMinR* strategy seems to degenerate in markets where the volatility is high (such as the markets represented in the two randomly generated price series). In this cases, the use of other Option trading strategies can still outperform asset-only traders.

Given that the forecasting model of the Option trading agents assumes Normally distributed price series (See section 3.2.2.1), it could be argued that the performance of the Option trading strategies should be expected to be higher than the non-Option trading strategies in the randomly generated price series. This because the Option-trading strategies base their valuation of the utility of the actions as a function of the Normal distribution. As the generated price series are more Normal (as shown in Section 5.3) than the stock market price series, it is possible to argue that the Option-trading agents' forecast should be more accurate. However, the difference in the performance in this cases is not as large as in the cases where the stock market prices are used.

The reason why the performance of the Option-trading agents is better in the stock market based price series may be caused by several factors. One of these factors is that in the cases of the *Microsoft* and *IBM* price series, there are some single steps where there is a high decrement in the price. This is demonstrated by the skewness of the price series (shown in Section 5.3) which is -6.18 for the Microsoft price series and -0.50 in the case of the *IBM* price series. Negative skewness indicates that there are more steps in time where the price is decreased compared to what is expected in a Normal distribution. In the case of the *Microsoft* price series, the Option trading agents may be protected from such high decrement in the price by holding Options.

The difference in the performance among the created Option-trading strategies is also well defined, given that the *OTMinR* strategy outperforms all the other Option Trading strategies in the majority of the experiments. However, the fact that the performance of the *OTMinR* strategy is very low in the two generated price series (*Random A* and *Random B*) indicates that it may be sensible to use the *OTMinR* strategy in combination with the other asset-only strategies depending on the price model.

The fact that the Option-trading strategies performed well in the experiments using the SMA_n price forecasting function indicates that agents can benefit from the trading of Options even if they have only a rough approximation of the price. This result is promising, because as it was explained in 2.2.4, the Simple Moving Average is a tool used in real markets to forecast prices which is not very accurate. Further tests with a better forecasting mechanism may improve the strategies performance.

The returns obtained by the *OTMix* strategy in both set of experiments are closer to those obtained by the *OTMaxW* than to the returns obtained by the *OTMinR*. This suggests that the *OTMaxW* is dominant for the *OTMix* strategy. Therefore, it may be possible to modify the heuristic of the *OTMix* strategy to make a better choice of strategies.

6.3 Correlation with Price Analysis

This section presents the results of the analysis of correlation between the agents strategies' returns and the asset price returns for the data obtained in the experiments. The background and description of this analysis is detailed in Section 5.5.2. First, the general results of the statistical correlation is developed and then the relation between the returns and the asset price returns is discussed in more detail.

While looking at the results of this analysis, it is important to keep in mind that because the wealth of the agents is defined as a linear function of the asset price (see Section 3.2), it is expected that the correlation between the returns of the agents and the corresponding price returns is going to be strong and direct. However, the aim of this analysis is to see whether the use of Option contracts has an impact on this correlation. The analysis of correlation is extended with the presentation and discussion of scatter plots for selected experiments results as a reinforcement of the obtained results.

a) Correlations when $\alpha = 0$

	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.49**	0.99	0.99	0.99	0.99	0.99
Microsoft	**0.50**	0.99	0.99	0.99	0.99	0.99
IBM	**0.63**	0.99	0.99	0.99	0.99	0.99
Random A	**0.60**	0.99	0.99	0.99	0.99	0.99
Random B	**0.52**	0.99	0.99	0.99	0.99	0.99

b)Correlations when $\alpha = 0.2$

	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.50**	0.99	0.94	0.99	0.99	0.99
Microsoft	**0.50**	0.99	0.99	0.99	0.99	0.99
IBM	**0.62**	0.99	0.99	0.99	0.99	0.99
Random A	**0.55**	0.99	0.99	0.99	0.99	0.99
Random B	**0.54**	0.99	0.99	0.99	0.99	0.99

Table 6.11: Correlations between the strategy returns and the price returns when $\alpha = 0$ and $\alpha = 0.2$. The lowest correlation for each row is emphasised.

6.3.1 α–Perfect Forecasting

The results from the analysis of correlation between the returns obtained by the strategies in the market and the returns of the asset price for the α–Perfect forecasting experiments are presented in Tables 6.11, 6.12 and 6.13. Each row in each table refers to a particular price series and each column refers to a specific trading strategy.

The value at each cell of the tables is the correlation coefficient obtained by applying the correlation function to the price returns data series and the strategies returns data series for the corresponding price and trading strategy.[3] The correlation coefficient has a range of $[-1, 1]$ where a value close to 1 indicates that there is a strong positive linear correlation and a value close to -1 would indicate a strong negative linear correlation. A value closer to 0 may indicate that the two data series are not linearly correlated [40].

Comparing the correlations among the different strategies for one row (i.e., one test case), it is apparent that the returns of the agents using the *OTMinR* strategy are the only ones that observe a significantly lower correlation with the price in every experiment. Also, contrasting the correlation of the *OTMinR* strategy between the experiments using the same price series but different values of α, it is possible to see that the correlation decreases as the uncertainty about the price increases (i.e. as $\alpha \rightarrow 0$). This suggests that in an uncertain market (where agents face more risk), the use of Options may provide the means necessary to stabilize their wealth when prices fluctuate in unwanted directions.

From the tables, it is possible to observe that neither use of the *OTMix*, *OTMaxW* and *OTRnd* strategies provokes a decrease in the correlation in any of the experimented scenarios. In the case of the *OTMaxW* strategy this performance is explained by the behaviour of the agents; as specified in Section 3.2.4, the *OTMaxW* strategy is similar to a speculator

[3]The calculated values are statistically significant at a level $p = 0.01$.

a) Correlations when $\alpha = 0.4$						
	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.52**	0.99	0.95	0.99	0.99	0.99
Microsoft	**0.51**	0.99	0.99	0.99	0.99	0.99
IBM	**0.67**	0.99	0.99	0.99	0.99	0.99
Random A	**0.56**	0.99	0.99	0.99	0.99	0.99
Random B	**0.64**	0.99	0.99	0.99	0.99	0.99
b) Correlations when $\alpha = 0.6$						
Dell	**0.60**	0.99	0.98	0.99	0.99	0.99
Microsoft	**0.54**	0.99	0.99	0.99	0.99	0.99
IBM	**0.77**	0.99	0.99	0.99	0.99	0.99
Random A	**0.68**	0.99	0.99	0.99	0.99	0.99
Random B	**0.92**	0.99	0.99	0.99	0.99	0.99

Table 6.12: Correlations between the strategy returns and the price returns when $\alpha = 0.4$ and $\alpha = 0.6$ The lowest correlation for each row is emphasised.

a) Correlations when $\alpha = 0.8$						
	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.75**	0.99	0.99	0.99	0.99	0.99
Microsoft	0.99	0.99	0.99	0.99	0.99	0.99
IBM	0.99	0.99	0.99	0.99	0.99	0.99
Random A	0.99	0.99	0.99	0.99	0.99	0.99
Random B	0.99	0.99	0.99	0.99	0.99	0.99
b) Correlations when $\alpha = 1$						
Dell	**0.81**	0.99	0.99	0.99	0.99	0.99
Microsoft	0.99	0.99	0.99	0.99	0.99	0.99
IBM	0.99	0.99	0.99	0.99	0.99	0.99
Random A	0.99	0.99	0.99	0.99	0.99	0.99
Random B	0.99	0.99	0.99	0.99	0.99	0.99

Table 6.13: Correlations between the strategy returns and the price returns when $\alpha = 0.8$ and $\alpha = 1$ The lowest correlation for each row is emphasised.

strategy choosing the action that yields most returns.

The use *OTMix* strategy (which itself uses the *OTMinR* strategy) may still yield a decrease in the correlation between returns and the price, as can be seen in Tables 6.11b and 6.12 on the cases when the Dell price series is used as the price of the asset. In this cases the correlation between the returns of the *OTMix* returns and the price returns is lower.

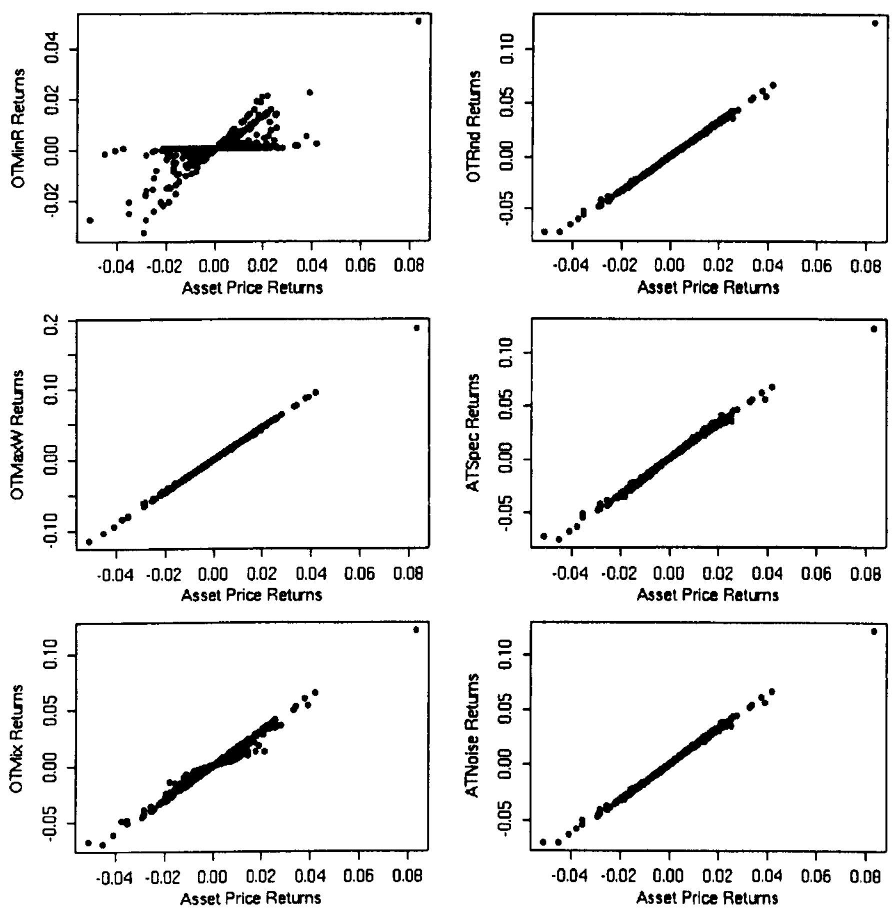

Figure 6.3: Scatter plot chart of the strategies' returns against the asset price returns for the experiment using *Dell* price series and $\alpha = 0.6$.

Figures 6.3 and 6.4 show scatter plots of the returns obtained by the different strategies against the returns of the asset price used in the corresponding experiments. The figures reinforce the fact that the strategies' returns increase or decrease in direct correlation with the price as indicated by the statistical analysis in the Tables 6.11 to 6.13. That is, when the price has a positive return (i.e. when it increments from one time step to the next), the wealth of the agents will increase and similarly, when the price observes a negative return (i.e. when it decrements from one step to the next), the agents' wealth will decrease (negative returns).

Figure 6.3 shows the scatter plots for the different strategies for a single test case (using the Dell price series and $\alpha = 0.6$). While the *ATNoise*, *ATSpec*, *OTMaxW* and *OTRnd* strategies observe a very strong linear relationship as all the points in the respective plots are aligned forming a straight line, the *OTMinR* strategy does not follow the same pattern. Instead, the *OTMinR* strategy observes a reduction in the magnitude of the positive and positive returns under the same price returns. Lastly, the *OTMix* strategy follows a linear relationship similar to the other strategies, however it can be seen that some of the points lay outside the linear pattern. This is caused by the choice of actions similar to the use of the *OTMinR* strategy.

Figure 6.4 shows the scatter plots for the *OTMinR* strategy in the experiments where the market price the *Random A* price series under different values of α. The data in the figure shows how as the value of α increases, the points in the plots become more aligned to a linear correlation. The data in this figure demonstrates the effect of the differences in the knowledge of the future price over the outcome of the strategy. In the scenarios where there is high uncertainty (when $\alpha = 0$ and $\alpha = 0.2$) the agents using the *OTMinR* strategy can hedge the risk, thus minimizing the losses which is reflected by the spread of the prices in the charts. However as the accuracy of the information about the price increases, the number of cleared Options will decrease (such pattern is shown afterwards in Section 6.4). A decrease in Option trading causes the agents' wealth to be more dependent on the price variations.

6.3.2 SMA_n Forecasting

The results from the application of the analysis of correlation to the data series obtained from the experiments using the SMA_n forecasting function are detailed in Figures 6.14 to 6.16.

The data in the results show that a significant difference in correlation is observed when using the *Dell* price series as the price of the asset. In this case, the *OTMinR* strategy is the only strategy on which the correlation between the returns of the strategies and the price returns is decreased. This correlation increases as as the value of the window parameter n is increased.

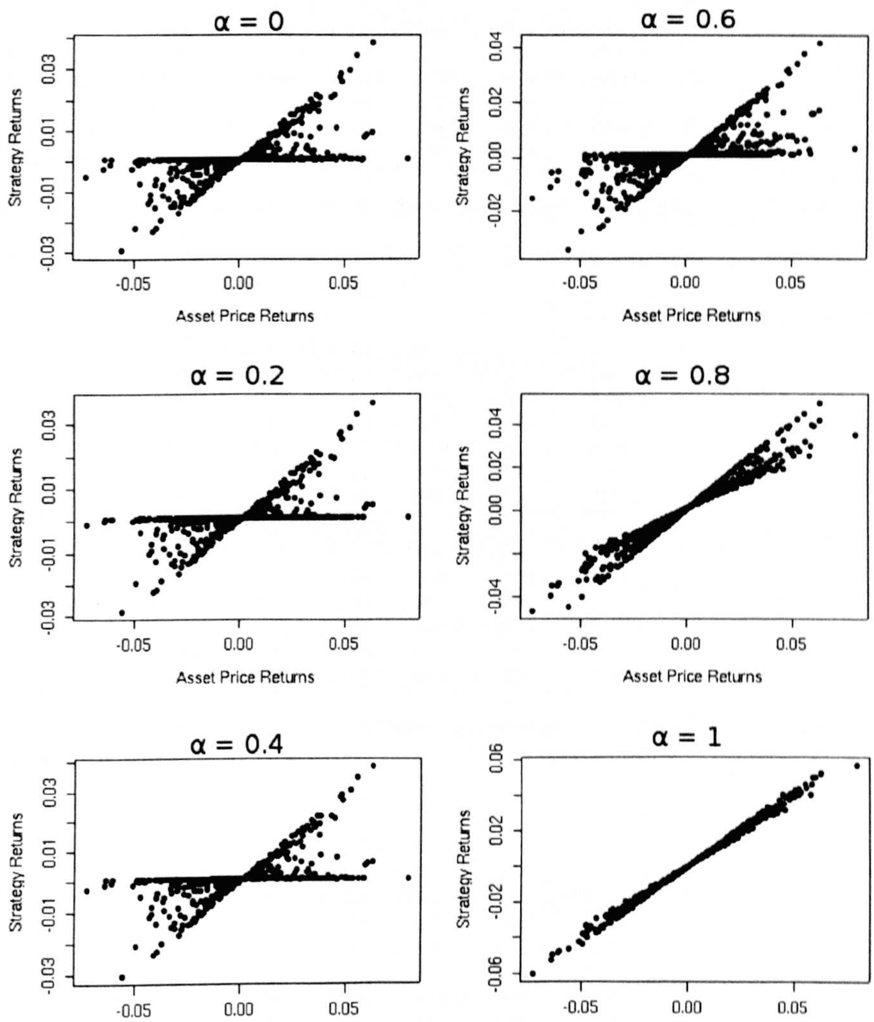

Figure 6.4: Scatter plot chart of the returns for the *OTMinR* strategy with different values of α against the asset price returns for the experiments using the *Random A* price series.

a) Correlations when $n = 15$						
	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.73**	0.99	0.99	0.99	0.99	0.99
Microsoft	0.99	0.99	0.99	0.99	0.99	0.99
IBM	0.99	0.99	0.99	0.99	0.99	0.99
Random A	0.99	0.99	0.99	0.99	0.99	0.99
Random B	0.99	0.99	0.99	0.99	0.99	0.99
b) Correlations when $n = 30$						
	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.77**	0.99	0.94	0.99	0.99	0.99
Microsoft	0.99	0.99	0.99	0.99	0.99	0.99
IBM	0.99	0.99	0.99	0.99	0.99	0.99
Random A	0.99	0.99	0.99	0.99	0.99	0.99
Random B	0.99	0.99	0.99	0.99	0.99	0.99

Table 6.14: Correlations between the strategy returns and the price returns when $n = 15$ and $n = 30$. The lowest correlation for each row is emphasised.

a) Correlations when $n = 45$						
	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.80**	0.99	0.99	0.99	0.99	0.99
Microsoft	0.99	0.99	0.99	0.99	0.99	0.99
IBM	0.99	0.99	0.99	0.99	0.99	0.99
Random A	0.99	0.99	0.99	0.99	0.99	0.99
Random B	0.99	0.99	0.99	0.99	0.99	0.99
b) Correlations when $n = 60$						
	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.82**	0.99	0.94	0.99	0.99	0.99
Microsoft	0.99	0.99	0.99	0.99	0.99	0.99
IBM	0.99	0.99	0.99	0.99	0.99	0.99
Random A	0.99	0.99	0.99	0.99	0.99	0.99
Random B	0.99	0.99	0.99	0.99	0.99	0.99

Table 6.15: Correlations between the strategy returns and the price returns when $n = 15$ and $n = 30$. The lowest correlation for each row is emphasised.

a) Correlations when $n = 75$						
	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.83**	0.99	0.99	0.99	0.99	0.99
Microsoft	0.99	0.99	0.99	0.99	0.99	0.99
IBM	0.99	0.99	0.99	0.99	0.99	0.99
Random A	0.99	0.99	0.99	0.99	0.99	0.99
Random B	0.99	0.99	0.99	0.99	0.99	0.99
b) Correlations when $n = 90$						
	OTMinR	OTMaxW	OTMix	OTRnd	ATSpec	ATNoise
Dell	**0.85**	0.99	0.94	0.99	0.99	0.99
Microsoft	0.99	0.99	0.99	0.99	0.99	0.99
IBM	0.99	0.99	0.99	0.99	0.99	0.99
Random A	0.99	0.99	0.99	0.99	0.99	0.99
Random B	0.99	0.99	0.99	0.99	0.99	0.99

Table 6.16: Correlations between the strategy returns and the price returns when $n = 75$ and $n = 90$. The lowest correlation for each row is emphasised.

6.3.3 Discussion

A notable pattern from the returns of the *OTMinR* agents can be observed in the cases where the correlation of the strategies' returns with the price is lower than 0.99. In such cases there are steps in the market where the price decreases but the agents using the *OTMinR* strategy still obtain positive returns. This demonstrates that that the use of the *OTMinR* strategy has prevented the agent from losing wealth in the case of a price fall, and in consequence indicates that the strategy can be used to hedge the risk of losing wealth in such cases.

However, it must also be noted that in the same cases where the *OTMinR* returns are lowly correlated with the price returns, there is also a decrease in the profits obtained from the positive returns in the price. This behaviour is presumably caused by the nature of the *OTMinR* strategy that always chooses the action which presents the lowest risk of losing wealth according to its beliefs. This behaviour makes the agents choose less risky actions which at the same time may provide lower returns.

6.4 Trading Volume Analysis

6.4.1 α–Perfect Forecasting

The results from the trading volume analysis (which is described in Section 5.5.3) for the α–Perfect forecasting experiments are presented next. The analysis is divided in three sections: First, the resulting R_H^C ratios (the proportion of hold offers made by the agents against those cleared in the market) for the different experiments using α–Perfect forecasting is described; next, the resulting R_W^C ratios (the proportion of write offers made by the agents against those offers cleared in the market) for the same experiments are presented; finally, the R_E^H ratios (the proportion of offers to hold owned by the agents against the number of exercised offers) are analysed.

6.4.1.1 Cleared Hold Offers Ratio

As defined in Section 5.5.3, R_H^C represents the proportion of the volume of offers to hold Options made by the different strategies ($vol_{OH}(\cdot)$) compared to the number of those offers that are cleared by the market and therefore traded by the agents ($vol_{CH}(\cdot)$). Charts depicting the results of this analysis for the α–Perfect forecasting experiments are shown in Figure 6.5.[4] The data in this figure is grouped by the price series used as the price of the asset. For each group, the different test cases with the corresponding α values experimented

[4]The tables containing the data used to generate these charts can be found in Appendix C.

are listed. Figure 6.5a shows the number of offers made by the strategies and Figure 6.5b shows the R_H^C ratios corresponding to the same experiments.

The data in the figures shows that volume of offers submitted by the *OTMaxW* and the *OTMix* strategies observe a similar pattern indicating that the *OTMix* strategy adopts the *OTMaxW* strategy with more frequency in the experiments. Also, as expected, the *OTRnd* strategy submits a similar number of offers for all the test cases. For the experiments where the asset prices are the *Random B*, *Microsoft* and *Dell* price series, the *OTMinR* submits less offers to hold Options than any other agent.

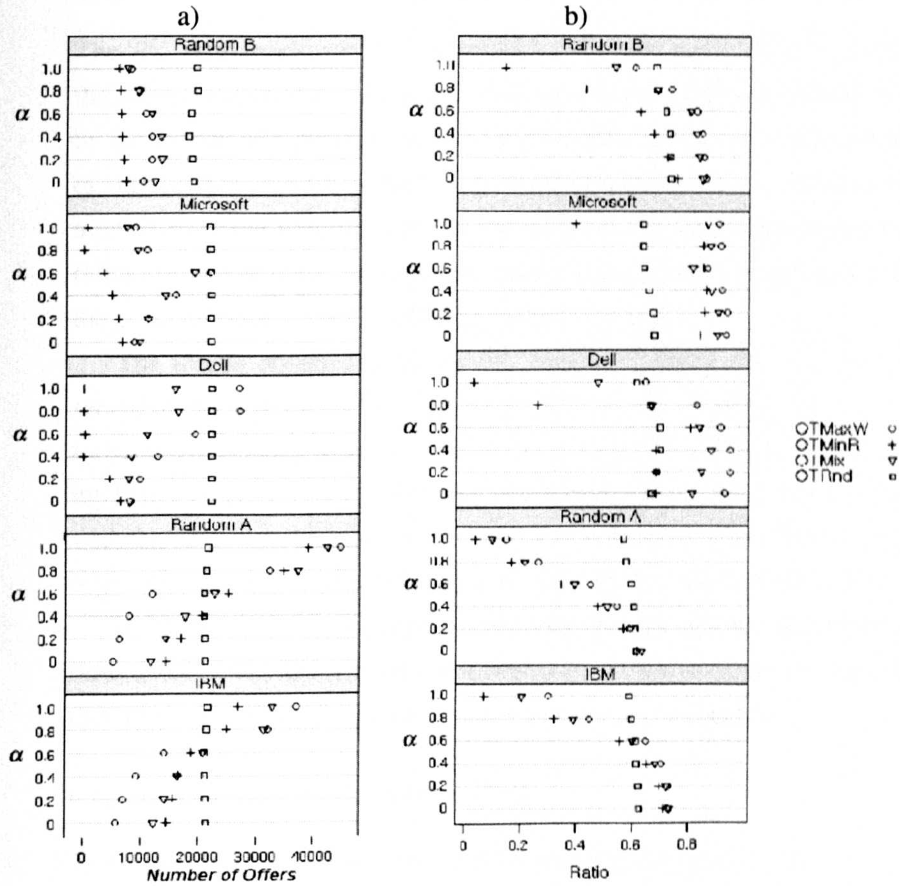

Figure 6.5: For the different strategies: a) Total of hold offers submitted to the market ($vol_{OH}(\cdot)$). b) R_H^C ratios. Results from the α–Perfect forecasting experiments.

For the same experiments which use the *Random B*, *Microsoft* and *Dell* price series, the volume of offers submitted by the *OTMinR* strategy ($vol_{OH}(\cdot)$) to the market increases as the forecast uncertainty increases ($\alpha \rightarrow 0$), and the the ratio of cleared offers (R_H^C) decreases as the forecast accuracy increases. ($\alpha \rightarrow 1$). Considering this behaviour along with the returns analysed in Section 6.2.1, it is possible to see that the *OTMinR* strategy effectively used Option contracts to increase its profits. Moreover, as it can be concluded from the data in Figure 6.5b, the number of hold offers cleared in the market decreases as the uncertainty about the future price decreases. This pattern arises because as the forecasting of the prices becomes more accurate for all the agents, the agents will make similar offers to the market (offers which provide the best performance according to their forecasts), and therefore the market will not be able to find matching offers to trade.

6.4.1.2 Cleared Write Offers Ratio

The R_W^C ratio represents the proportion of the volume of offers to write Options made by the different strategies ($vol_{OW}(\cdot)$) compared to the number of those offers that are cleared by the market and therefore traded by the agents ($vol_{CW}(\cdot)$). Figure 6.6 shows the results of this analysis for the α–Perfect forecasting experiments.[5] Figure 6.6a contains the the number of offers to write made by the different strategies and Figure 6.6b shows the R_W^C ratios corresponding to the same test cases. The test cases are grouped by the series used as the price of the asset under different values of α tested.

The results obtained from the R_W^C analysis indicate that, contrary to the pattern observed in the hold offers, the volume of offers to write Options ($vol_{OW}(\cdot)$) increases as the forecasting accuracy increases (that is, as $\alpha \longrightarrow 1$). This pattern may be explained considering that, as Option trading agents have a more accurate forecast, the possibility of profiting with the premium obtained from writing an Option which will not be exercised in the future offers the best benefits for the agents in terms of the combination of risk and their returns.

In a similar way as the offer to hold Options, the number of cleared offers to write Options decreases as the certainty about the price increases (shown in Figure 6.6b), suggesting that agents are submitting similar types of offers to the market.

6.4.1.3 Exercised Hold Options Ratio

The results from Figure 6.7 suggest that, as the price forecasting certainty increases (i.e., as $\alpha \longrightarrow 1$) the ratio of exercised Options increases (Figure 6.7b). This pattern is also observed with the average of the R_E^H ratios among the strategies. The average of these ratios across the test cases is shown in Table 6.17. Each column in the table contains the average (across the four Option trading strategies) of R_E^H ratios for the experiments using the price series specified in the first column. Results for the different experimented α values are shown on each row of Table 6.17.

[5] The tables containing the data used to generate this charts can be found in Appendix C.

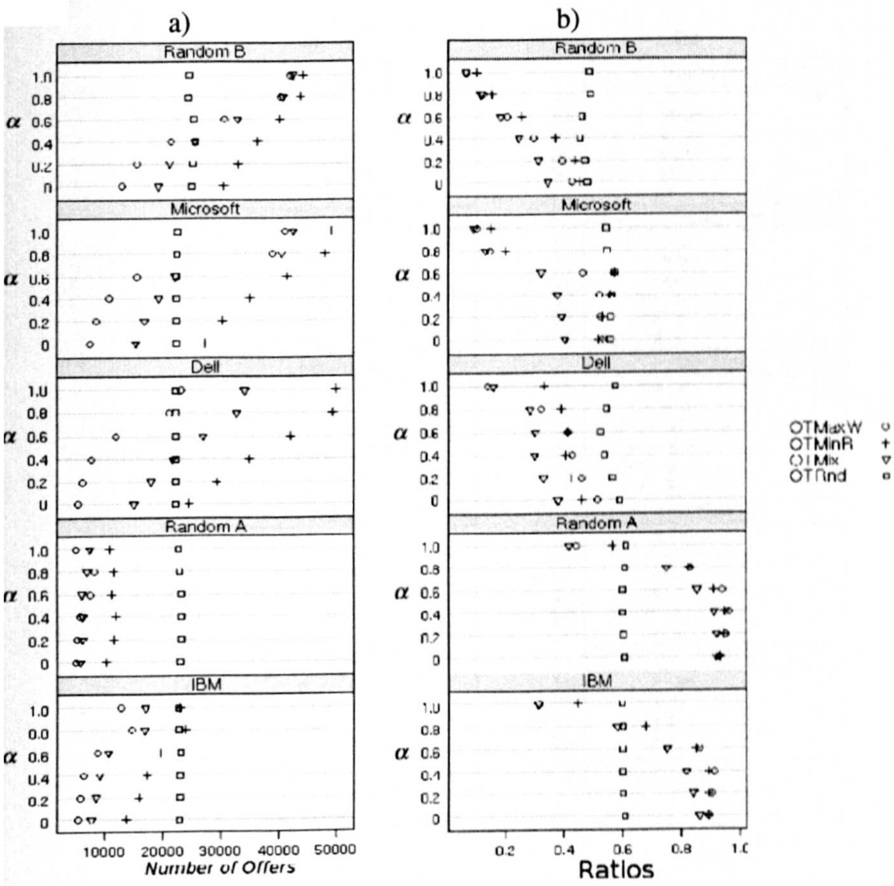

Figure 6.6: For the different strategies a) Total of write offers submitted to the market ($vol_{OW}(\cdot)$). b) R_W^C ratios. Results from the α–Perfect forecasting experiments.

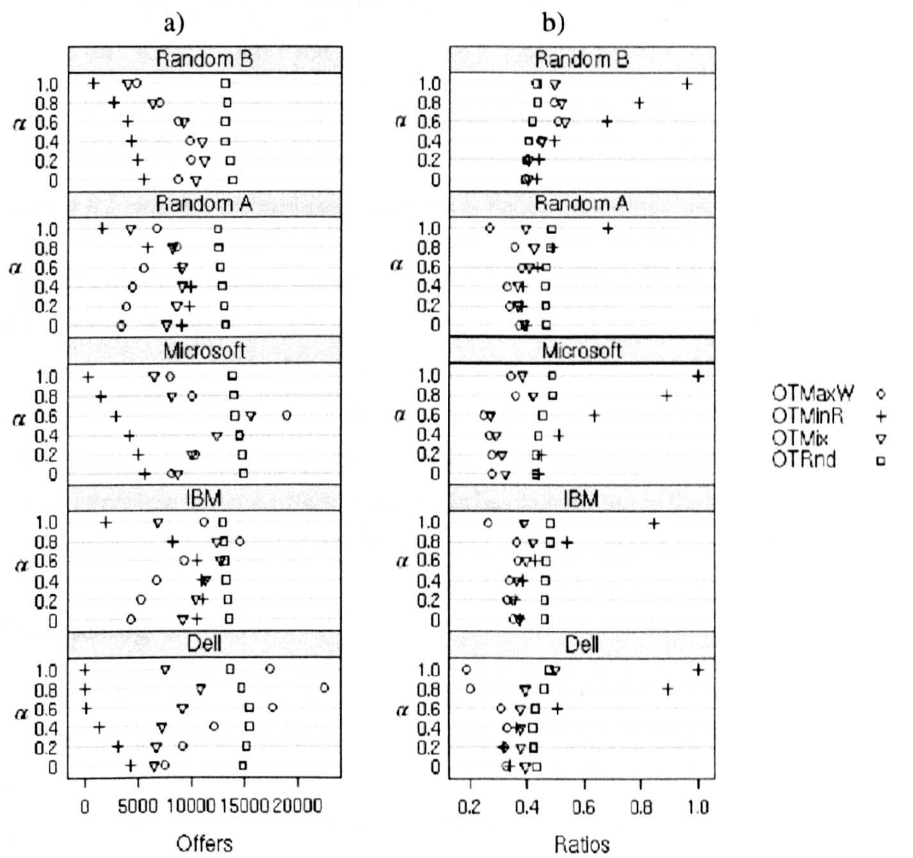

Figure 6.7: For the different strategies a) Total of hold offers cleared by the market ($vol_{CH}(\cdot)$). b)R_E^H ratios. Results from the α–Perfect forecasting experiments.

The results in Table 6.17 indicate that in a scenario with low forecasting uncertainty, the few Options contracts held by the agents were offered as a result of a more accurately informed choice by the agents. That is, the agents which traded such Options are *more certain* that they will exercise them. In these scenarios, any time the strategy leads to an action to hold an Option, it is more probable (due to the fact that the model of the future price is accurate) that the agent will exercise the Option at a future step.

α	Dell	Microsoft	IBM	Random A	Random B
0	0.37	0.37	0.39	0.41	0.41
0.2	0.37	0.36	0.38	0.39	0.41
0.4	0.38	0.37	0.39	0.39	0.45
0.6	0.40	0.41	0.42	0.42	0.53
0.8	0.54	0.49	0.45	0.44	0.56
1.0	0.55	0.54	0.50	0.46	0.58

Table 6.17: Average R_E^H ratios for the experiments using the α–Perfect forecasting function.

A decrease in the volume of market cleared offers to hold Options ($vol_{CH}(\cdot)$) as the agents' forecasting certainty increases (i.e., as $\alpha \rightarrow 1$) can be seen throughout Figure 6.7a. This suggest that as the uncertainty in the future price decreases, the forecasts of the agents will become increasingly similar, provoking them to submit the same offers to the market. Therefore, the market would not be able to clear such offers due to the lack of matching offers. However, in this experiment some offers will always be cleared due to the presence of agents using a random offering strategy implemented as the *OTRnd* and *ATNoise* strategies.

6.4.2 SMA_n Forecasting

The results obtained from the trading volume analysis for the experiments which used the SMA_n forecasting function are described now. As with the α–Perfect forecasting experiments results, the analysis is divided in three sections: The first section contains the analysis of the R_H^C ratios; in the second section, the analysis of the R_W^C ratios is presented; finally, the last section contains the analysis of the R_E^H ratios,

6.4.2.1 Cleared Hold Offers Ratio

Figure 6.8 presents graphical results of the analysis of hold offers for the SMA_n experiments.[6] Figure 6.8a depicts the number of offers to hold Options submitted by the different strategies to the market and Figure 6.8b shows the R_H^C ratios, which is the proportion of those offers submitted against the number of those Options which were cleared by the market.

[6]The data used to generate these figures can be found in Appendix C

In Figure 6.8a, the horizontal axis represents the volume of offers submitted to the market ($vol_{OH}(\cdot)$)and the vertical axis denotes the number of steps used in the SMA_n forecasting function. The test cases are grouped by the series used as the price of the asset. Similarly in Figure 6.8b, the horizontal axis denotes the R_H^C ratios while the vertical axis represents the number of steps used for the SMA_n forecasting.

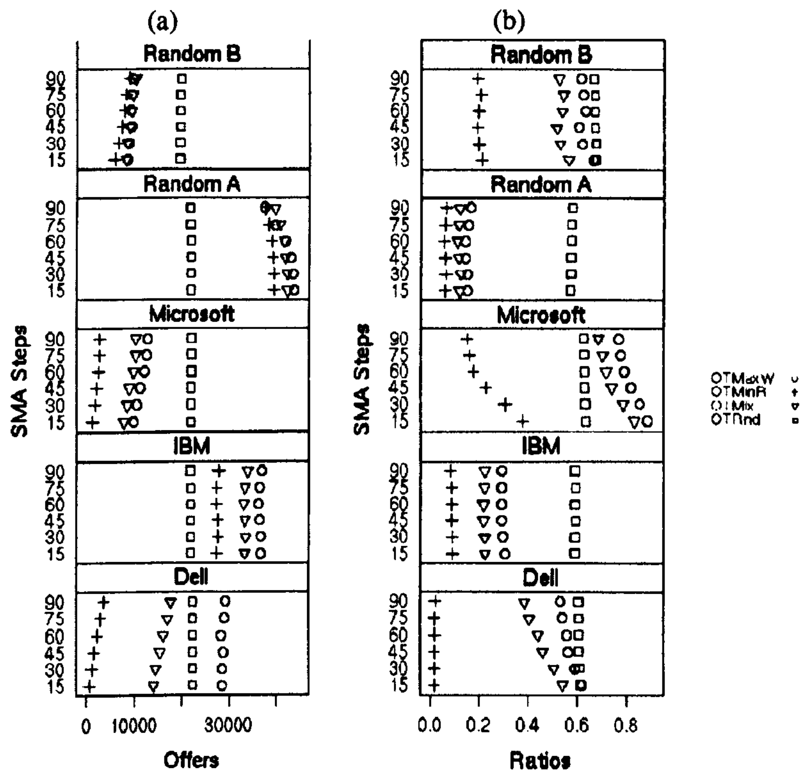

Figure 6.8: For the different strategies a) Total of hold offers submitted to the market ($vol_{OH}(\cdot)$). b)R_H^C ratios. Results from the SMA_n forecasting experiments.

From the graphs presented in Figure 6.8, it is possible to see that the *OTRnd* strategy has similar volume of offers (Figure 6.8a) and a similar R_H^C ratio in all test cases. This pattern is explained if it is considered that the random choice of the agents using such strategy is not affected by any of the parameters varied for the agents.

For the three non-random trading strategies, the results show that for all the test cases the *OTMinR* strategy has a lower volume of offers to hold Options than the *OTMaxW* and *OTMix*. Specifically:

$$vol_{OH}(OTMinR, p, n) < vol_{OH}(OTMix, p, n) < vol_{OH}(OTMaxW, p, n) \quad (6.3)$$

for any price series p and any number of steps n used for the SMA_n forecasting strategy. Similarly, the resulting R_H^C ratios for these strategies are characterised by the formula:

$$R_H^C(OTMinR, p, n) < R_H^C(OTMix, p, n) < R_H^C(OTMaxW, p, n) \qquad (6.4)$$

also for any price series p and any number of steps n used for the SMA_n forecasting strategy.

6.4.2.2 Cleared Write Offers Ratio

Similarly to the hold offers, the results of the analysis of hold offers for the SMA_n experiments is shown in Figure 6.9.[7] Figure 6.9a depicts the number of offers to hold Options submitted by the different strategies to the market and Figure 6.9b shows the R_W^C ratios which is the proportion of those offers submitted against the number of those Options which were cleared by the market.

In Figure 6.6a, the horizontal axis represents the number of offers submitted to the market and the vertical axis denotes the number of steps used in the SMA_n forecasting function. The test cases are grouped by the series used as the price of the asset. Similarly in Figure 6.9b, the horizontal axis denotes the R_W^C ratios while the vertical axis represents the number of steps used for the SMA_n forecasting.

From the data presented in the figure, it is possible to see that the *OTRnd* strategy has similar number of offers (Figure 6.9a) and a similar R_W^C ratio in all test cases. This pattern is explained considering that the random choice of the agents using the *OTRnd* strategy is not affected by any of the parameters varied for the agents. This is the same behaviour as observed for the volume of offers to hold Options described previously.

In contrast with the volumes obtained from the offers to hold Options for the three non-random trading strategies, the results show that for all the test cases the *OTMinR* strategy has a higher volume of offers to write Options than the *OTMaxW* and *OTMix*. The observed pattern can be formalised as:

$$vol_{OW}(OTMinR, p, n) > vol_{OW}(OTMix, p, n) > vol_{OW}(OTMaxW, p, n) \qquad (6.5)$$

for any price series p and any number of steps n used for the SMA_n forecasting strategy. Similarly, the resulting R_W^C ratios for these strategies is characterised by the formula:

$$R_W^C(OTMinR, p, n) > R_W^C(OTMix, p, n) > R_W^C(OTMaxW, p, n) \qquad (6.6)$$

[7]The data used to generate this figures can be found in Appendix C

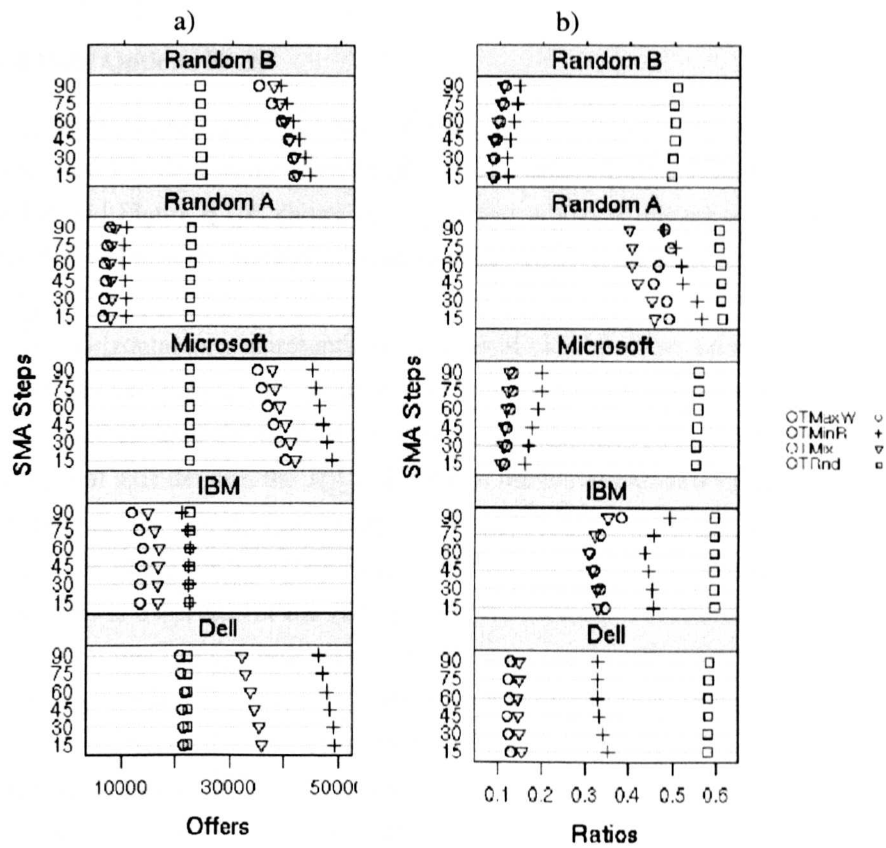

Figure 6.9: For the different strategies a) Total of write offers submitted to the market. b)R_W^C ratios. Results from the SMA_n forecasting experiments.

also for any price series p and any number of steps n used for the SMA_n forecasting strategy.

Finally, comparing the data in Figures 6.8a and 6.9a it is possible to see a relation between between volume of offers to hold Options ($vol_{OH}(\cdot)$) and the volume of offers to write Options ($vol_{OW}(\cdot)$. The volume of offers to hold Options inversely proportional to the volume of offers to write Options. That is, in the cases where $vol_{OH}(\cdot)$ is high, $vol_{OW}(\cdot)$ is lower and *vice versa*.

6.4.2.3 Exercised Hold Options Ratio

The results of the analysis of exercised hold offers for the SMA_n experiments is shown in Figure 6.10.[8] Figure 6.10a depicts the number of offers to hold Options which where cleared by the market and Figure 6.10b shows the R_E^H ratios which is the proportion of those offers cleared offers against the number of those Options which were exercised by the agents.

In Figure 6.7a, the horizontal axis represents the number of cleared offers submitted to the market and the vertical axis denotes the number of steps used in the SMA_n forecasting function. The test cases are grouped by the series used as the price of the asset. Similarly in Figure 6.7b, the horizontal axis denotes the R_E^H ratios while the horizontal axis represents the number of steps used for the SMA_n forecasting.

The number of cleared offers for the *OTRnd* strategy is similar across all the test cases (6.10a). The same pattern is observed for the R_E^H ratios of the *OTRnd*. The reason of this pattern is that agents using this strategy will make a similar total number of offers in every test case (as the agents choose the offers from a Uniform random distribution). It is possible to see that the R_E^H ratios for the *OTRnd* strategies are close to 0.5 which means that half of the cleared Options offers submitted by the agents are exercised. This is explained if it is considered that from the possible Option offers available to the agents, one half are offers to write Options while the other half are offers to hold Options. Therefore, when the offers are cleared in the market, there is the a 50% probability that the offer chosen is an offer to write an Option, and the same probability that the offer chosen is to hold an Option.

Another observation from the exercise hold analysis is the fact that the *OTMinR* strategy yields a low volume of cleared offers to hold (Figure 6.10a). This result is consistent with the data in Figure 6.8 which shows low R_H^C ratios. However, the ratio of exercised holds (R_E^H) for this strategy is higher than any of the other strategies as shown in Figure 6.10b, which may indicate that the majority of offers to hold Options cleared by the market were

[8] The data used to generate these figures can be found in Appendix C

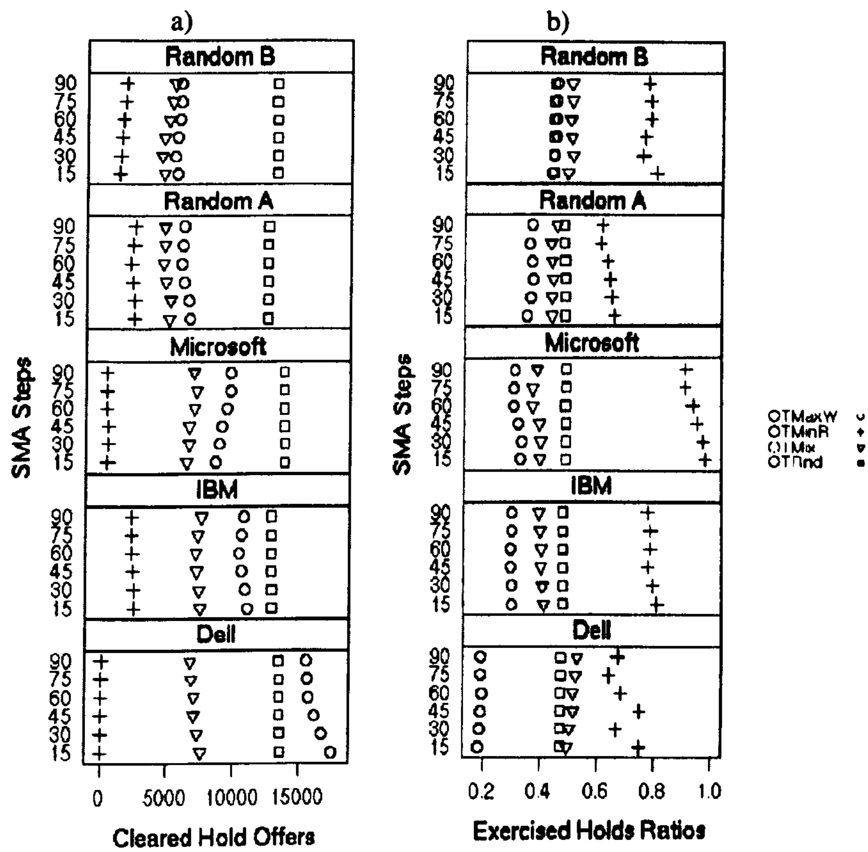

Figure 6.10: For the different strategies a) Total of hold offers cleared by the market; b) R_E^H ratios. Results from the SMA_n forecasting experiments.

exercised by the agents using this strategy.

The volume of cleared offers for the *OTMaxW* strategy is higher than the other two non-random strategies. However, the R_E^H ratio is lower in every test case. This may indicate that the agents using this strategy do not need to exercise all the Option contracts which they hold.

Similarly to the results from the R_H^C and R_W^C analysis, the volume of cleared hold offers and the R_E^H ratios of the *OTMix* strategy have values that lie between the values obtained by the *OTMinR* and the *OTMaxW* strategies. Such a pattern confirms expected behaviour according to the design of the *OTMix* strategy which switches between the use of both the *OTMinR* and *OTMaxW* strategies.

6.4.3 Discussion

The results obtained from the volume analysis show that the *OTMinR* strategy (which obtained the best returns in the majority of experiments according to the analysis in Section 6.2) achieved its performance by submitting more offers to write Options than offers to hold Options. However, the ratios of cleared offers to write Options (R_W^C) were low compared to the ratios of cleared offers to hold Options (R_H^C). Generally, when all the non-random Option trading strategies submitted a high number of similar offers to market (in the α–Perfect forecasting experiments when $\alpha = 0.8$ and 1), the clearing ratios were low. This is due to the fact that the only matching offers were provided by the random Option trading strategy *OTRnd*.

The results from the experiments using the α–Perfect forecasting function showed that there are differences in the submitted offers among the three non-random Option trading strategies. These results also demonstrate the differences in the effects that the degree of forecast accuracy has in the offer volumes of the strategies.

The analysis of volume for the SMA_n forecasting demonstrates that there are differences between the number of offers made by the different strategies when using this type of forecasting function. Similarly as in the α–Perfect forecasting experiments, the number of cleared offers to write form the *OTMinR* strategy are higher than the number of cleared offers to hold. This may indicate that the action of writing Options is used by agents to obtain higher returns even when the price of the asset is decreasing.

6.5 Discussion

The results of the experiments show that there are differences in performance between the agents which trade Options to those not trading Options in the three performed analyses. Specifically the results of the analysis of returns presented in Section 6.2 shows that the agents which used the *OTMinR* Option trading strategy (allowing them to trade Options) performed better than those not trading Options in 41 of the 60 experimented scenarios (that is, in 68.3% of the test cases). Also, the *OTMaxW* trading strategy performed better than the asset–only trading strategies in 11 of the experimented scenarios in which the *OTMinR* strategy did not. This makes a total of 52 of 60 experimented scenarios (or 86% of the test cases) where Option trading strategies outperformed asset–only trading strategies. Therefore, it is possible to conclude that in these cases Option trading offered a benefit over simple asset trading in the market.

The second important result obtained from this section is the comparison of the different Option trading strategies. The *OTMinR* strategy outperforms all of the other strategies in the majority of the cases, whereas the *OTMaxW* strategy performs very poorly. It is only in the experiments where the asset price observed very high volatility (using the *Random B* price series) that the performance of the *OTMinR* strategy decreased dramatically and the *OTMaxW* strategy performed better.

It could be expected that because the *OTMix* strategy uses both the *OTMinR* and *OT-MaxW* strategies to participate in the market, this strategy should perform better than each of the other two strategies. However the obtained results from the analysis in this chapter indicate that the heuristic used to mix the strategies is not optimum and in some cases it reduces the performance obtained by the use of those strategies as standalone strategies.

The results of the correlation analysis demonstrated that the price-returns correlations obtained by the *OTMinR* strategy are lower than for the other strategies in 22 of 30 of the test cases using the α–Perfect forecasting function and in 6 of the 30 test cases where the SMA_n forecasting function was used. The price–returns correlation of the *OTMinR* strategy did not decrease in the test cases where agents uncertainty low (i.e., when $\alpha = 0.8$ and $\alpha = 1$) when using the α–Perfect forecasting function.

Further examination of the data also showed that as the traders get more accurate information, the effect of Option trading is decreased. This is due to the decrease in offered and cleared Options. However, even in those cases where the agents had perfect knowledge, the Option-trading agents can outperform non Option-trading agents in some of the experiments. The correlation analysis also shown that Options may also be used to reduce the agents' loss of wealth due to the negative fluctuations of the asset price.

The results from the volume analysis gave further insight into the trading patterns of the different strategies. The high volume of Option-trading indicates that the Option trading strategies make use of such possible actions. It is of special interest that some of the profits obtained by the agents were caused due to Option writing (selling Option contracts). This result indicates that both Option trading positions can be used to obtain a better performance.

As the market constrained the number of assets possible to trade to one asset per transaction, the magnitude of the benefits and disadvantages of trading the different strategies was reduced. This constraint also avoided the use of more complex Option–trading strategies which are commonly used in real markets to gain more control over the risk of trading or to take advantage of the beliefs about the future price.

In summary, these experiments showed that agents using Option trading can perform better in the cases where there is more uncertainty about the price in the market. However, a very high volatility in the market may also cause very high Option prices, making them unattractive to the agents. The results obtained also shown that agents trading Options can be less susceptible to price variations; however, this does not equate to a better performance as the use of Options can decrease the returns obtained from positive price fluctuations. This result agrees with the theory of risk and returns [62] which states that more risky investments tend to yield higher returns. The use of Options would therefore be suggested in scenarios where there is uncertainty about the asset price in the market and that the volatility of the asset price makes the prices of Options appealing in comparison to the prices of the asset.

Trading Options is not a good strategy when there is perfect forecasting in the market as a simple asset speculating strategy will perform better. However, if other conditions of the market are changed, such as a market with heterogeneous forecasting accuracy or allowing the trading of more than one good and Option contracts at each step, the use of Options could still be beneficial in the aforementioned cases.

Finally, the proper design of the strategy to trade Options is one of great importance. The results from the experiments showed that trading Options using an unsuitable strategy will result in higher losses than for agents not trading Options. Further research should be done focusing on the design of strategies which make use of trading a combination of Options and assets and the implementation of search heuristics and genetic algorithms to optimise the choice of the actions.

Future price forecasting is another issue which spawns a separate research area. Although the presented model only dealt with the forecasting of the asset price, it is unlikely that such forecasting accuracy is possible in real markets. However, a combination of asset price forecasting and the forecasting of the future volatility of the market might improve the utility of Option trading.

It is possible to argue that given an agent with perfect forecasting function (when α = 1), the best strategy would be to buy or sell assets at the present time because holding an Option will always incur a price overhead in the premium. However, it should be noted that in the model developed, the asset's future price is not the only source of information in the agents' model of the future state of the market; the agents also considers the volatility of the asset price in order to make an informed judgement of how risky each action will be according to the forecasted price.

In view of this fact, it would be possible for an agent to behave as in the speculator-like strategy (selling assets when their price is decreasing and buying assets when their price is increasing) if given a perfect forecast (when $\alpha = 1$), the agent *knew* that such forecasting was exact and in consequence ignored or modified the value of its perceived volatility of the market. This can be achieved modifying the action valuation function of the Option trading strategies by reducing the agent's perceived variance of the price when evaluating the outcome of each possible action.

In markets where real resources are being traded, it is difficult for an agent to obtain such information. Such cases were experimented with the SMA_n forecasting function. In those cases, the agent could improve its action valuation function by detecting decrements in the volatility and therefore decreasing its perceived variance of the price. With the current strategies, the agents perceived variance is calculated using the complete price returns beginning from the first time step. With the proposed modification, the agents would drop some of the past prices and only consider the variance of the price returns from the $t - k$ steps for some predefined value of k.

Moreover, as was demonstrated in the experiments, Option trading agents can also use their knowledge about the future price to write Options which (according to their price forecast) will not be exercised at expiration time. They write these Options in order to profit from the price of the Option (the premium).

In summary the outcome from the analysis can be summarised by the following claims:

- Trading Options can be beneficial to an agent by preventing it from losing wealth by negative trends in the market.
- It is possible for an agent to decrease the dependency of its wealth to the asset price by using Options.

- Using the wrong Option trading strategy can lead to a performance which is worse than not trading Options.
- Trading Options are more beneficial to the agents in scenarios where the forecasting accuracy is low.
- Agents can hedge the risk caused by price fluctuations inherent in a market by trading Options.

6.6 Summary

In the present Chapter the results of the simulation experiments performed to test the created Option trading model were presented. The analysis of the obtained data using the three performance metrics given in Section 5.5 was detailed and the results were discussed with reference to the objectives of this research. The outcomes of the strategies were compared for the different test cases and several conclusions about the performance of the strategies were drawn from the performed analysis.

From the results, it is possible to conclude that the framework created provides enough tools for agents to trade Options and obtain a benefit from the trading of these Options by decreasing the risk of losing wealth. The results also showed that from all the designed Option trading strategies the *OTMinR* strategy performed best.

The experimental results demonstrated that using some of the novel developed strategies it is possible for agents to benefit by the use of Option contracts. Moreover, the interactions in the system indicate that the designed Option market is liquid enough to allow agents to trade Options and benefit by trading them. A novel result is the fact that even though the market is closed and agents have to compete for the assets and the Option contracts, there was enough trading in the market to provide significant advantage for a particular strategy.

It is also clear that one of the drawbacks of the model is the constraint in the volume of offers to trade Options or assets that can be submitted at each step and the number of goods that can be traded at each time step. These constraints reduced the complexity of the strategies available for the agents to single actions. The ability to make a combination of offers would make possible different Option trading strategies such as *covered call* (when the trader buys a call Option and at the same time buys assets) [23]. These kind of strategies may provide the agents with a better management of the risk of the market.

Chapter 7

Future Work and Conclusion

Option Derivatives have been used as a mechanisms for risk management for some time in real markets. In my thesis I have created an Option trading model and a software framework that allows autonomous software agents to trade in online software Option markets. This framework has been developed with two objectives in mind: first, to demonstrate that it is possible for software agents to trade in a market which is similar to real world Exchanges and secondly, to create an Option trading framework which can be used for risk management in distributed software systems.

Throughout the thesis I have shown the properties of my proposed Option trading model. I have also described the development of a software implementation of the proposed model. This implementation was used to perform several experiments to test the hypotheses established at the beginning of the thesis.

My research has shown that it is possible to develop a software market with a structure similar to the one used in real Exchanges where Options are traded. This result is an advancement from the current Option trading models reviewed in Section 2.4.1 where the markets clear all the offers submitted by the agents.

7.1 Addressing the Research Questions

The results obtained from the simulation experiments shall now be used to answer the research questions previously established. From Chapter 1:

> *1. Can software agents benefit from the exchange of Options in the software market?*

The experimental results shown that agents which used the *OTMinR* strategy obtained higher returns than agents using all of the other strategies on 23 of the 30 test cases when using the α–Perfect forecasting function. Similarly, agents using the *OTMinR* strategy obtained higher returns on 18 of the 30 test cases when using the SMA_n forecasting function. This result shows that the agents using such strategy gained a significant advantage from using Options against agents using other strategies.

> 2. *Is it possible to characterise specific cases where software agents trading Options have a better performance than those not using them?*

From the experimental results, it is possible to see that the Option trading strategies consistently outperformed the asset–only trading strategies in the cases where the prices based on real stock prices (i.e., the *Microsoft, Dell* and *IBM* price series) were used. The Option trading strategies were outperformed in some of the cases where the generated price series were used (i.e., the *Random A* and *Random B* price series). One of the differences between the generated and the stock–based price series is the sudden high increments and decrements in the price (i.e. a high skewness of the distributions) present in the stock–based price series. Agents using the *OTMinR* strategy reduced the loss caused by such decrements; this reduction resulted in higher returns. Hence, in the test cases when the *Microsoft* and *IBM* price series were used (which have a skewness higher than the other price series), the *OTMinR* strategy had higher relative returns than in the other test cases.

> 3. *Are agents trading Options less susceptible to price variations than those not using Options?*

Although not all the strategies allowing agents to trade Options showed a decrease in the correlation of their returns with the price, agents that used the *OTMinR* strategy observed a significant decrease in correlation. During the analysis of correlation it was observed that the reduction of the correlation sometimes affected the positive returns of the agents; although this behaviour is not necessarily desired, it was observed due to the design of the *OTMinR* strategy. A better strategy may aim to minimize only the correlation which causes negative profits. The analysis of correlation shown some patterns indicating that Options can be used to prevent losses due to negative price variations. Thus, and according to the used definition of risk, some agents used Option contracts to manage their risk effectively.

> 4. *What is the difference in the performance among the developed Option trading strategies?*

From the four designed Option trading strategies (i.e., the *OTMinR*, *OTMaxW*, *OTMix* and *OTRnd*), the *OTMinR* strategy performed better than the others. The *OTMaxW* strategy performed very poorly in the majority of the test cases. However, an interesting result is that in the test cases where it performed well, the *OTMinR* strategy performance was poor. Of the two strategies used in the *OTMix* (which used the *OTMinR* and *OTMaxW* strategies), the *OTMix* strategy performance was always better than the worst performing strategy but not as good as the better performing strategy. This demonstrates that the algorithm for the selection of the strategy is not optimal. However, with the current selection algorithm, the agents still have some benefit from trading Options. Finally, the *OTRnd* strategy was mostly within the average of the other strategies' performance (that is, the relative returns were close to 0).

7.2 Model Architecture

My research has shown that the developed market framework can be used as a market for Option trading. Although the implemented market and strategies allow agents to trade the underlying resource, it is possible to add such mechanism without much effort, in order to use the Option market in parallel with an exfisting Market-Based system.

Similarly, the developed trading strategies can be used as a foundation for more complex strategies which allow agents to make more better choices. The use of the risk loss factor can be extended to consider combinations of actions to perform more complex strategies. The results show that agents using the *OTMinR* strategy were successful in outperforming agents using the asset only trading strategies by using Options. However, the performance of such Option trading agents could be increased by developing strategies that make a better use of the positive changes in the underlying asset.

One of the main assumptions made for the used model of risk is that the underlying asset follows a Normally distributed random-walk stochastic process. Under this assumptions, the agents create a probabilistic model of the price series with the objective of quantifying the risk exposed by the available actions. The accurate valuation of the actions' risk is then dependent on the accuracy of the agent model. In the developed strategies, the complete history of prices was used to estimate the variance of the Normal distribution model. This approach may be improved by allowing the calculation of such variance considering a smaller number steps in the past. This would allow an agent to detect when a market price has stabilized, providing it a more accurate model of the present state of the market.

7.3 Lessons learnt

As in every research project, several issues were raised during the development of the Option trading framework and the simulation experiments. First, an intended logic-based approach to model the agents' reasoning mechanism was unsuccessful. This was due to the nature of the probability modelling. The main issue against modelling the reasoning engine of the agents using production rules is that, to represent all the possible reasoning paths available to the agent would take a huge number of rules. Secondly, the decision mechanism would have to be *hard-coded* in the agents, with the decisions of the different actions being selected by the agent creator at design time.

The second issue raised was related to the implementation of the framework. The current implementation provides logging mechanisms that output information to plain data files. Although convenient at the time of programming, this is not beneficial when trying to analyse the obtained data. A better approach would have been to implement logging mechanisms to output data to a data base engine. With such an approach, the data would be structured and readily available for its processing.

Finally, given the structure of the RepastJ ABM framework, the code of the graphical interface for the Option Market had to be mixed with the code containing the logic of the market. This kind of software practice is not desired and can be prevented by using a Model–View–Controller [41] approach to the design of a software system.

7.4 Framework Extensions

Two main lines of further research could be performed. First, the framework can be extended to allow agents to trade more than one asset at each time step. To achieve this the strategies will have to be modified to consider a portfolio of assets. In this case, the maximisation of the agents' performance could be based on portfolio theory considering risk and Option based portfolios. This might be solved using tools such as Value at Risk [54]. The strategies can also be extended to provide the means for the agents to consider external constraints. This, in order to allow agents to impose specific objectives of the number of assets and cash that must be available at future times.

Although the developed framework is focused on the trading of European Option contracts, it should be possible to use a similar approach to provide other kind of Derivatives to Market–Based systems. This can be achieved by using a parallel market approach, where the software market where the Derivatives are traded is treated as a separate market from the primary market where the resources are traded.

Using this kind of parallel market mechanism it should be possible to develop markets where contracts such as Swaps and Futures are traded using similar auctioning mechanisms as in typical Market–Based systems which are based on auctions. An interesting addition would also be the creation of agent's coalitions which join efforts for maximising a portfolio of resources by trading on different markets. With this coalition approach, each agent would be able to concentrate its efforts in only one market.

Finally, although this thesis did not addressed the issue of the measuring of efficieny in the allocation of the resources among the market system, it contributed several strategies and the model of the mechanism for a parallel market where Derivatives are traded. Such strategies may be considered by a designer when modelling mechanisms for the automatic allocation of resources. For this, the utility and preferences of the available actions will have to be established depending on the concrete mechanism to model. However, the work presented in this thesis provides the basic Option trading mechanism to use.

7.5 Application Domains

The second possible extension to my work is the implementation of my Option trading model into a Multi–Agent System application. This may be done to test the benefits and drawbacks of using Option contracts in specific domains. There are several domains where the created framework can be used. However, one of the main limitations of my model is that the underlying system must use a Market–Based mechanism and as such, the system must use the concept of cash, goods (or resources) and wealth. Some of the possible application domains will be briefly described here and the use of the Option trading framework in such domains will be discussed.

As the main objective of the Option trading framework is the management of risk in Multi–Agent systems, it may be possible to use it in any conceivable domain where a group of agents share resources. From a computational point of view, it is interesting to apply this framework to agent based applications where uncertainty is a factor in the decisions of the agents.

7.5.1 Supply Chain Management

One of the possible domains for the application of my Option framework is in Agent–Based supply chain management systems. Supply Chain Management (SCM) has an evident need to control well defined risks. Also, there have been several research studies undertaken about risk management of SCM and modelling of supply chains as Multi–Agent systems.

The management of risk in supply chains has been previously studied in [1], where a model which introduces *intermediaries* was proposed; these intermediaries provide Option like contracts to the retailers in order to minimise their risk. However, when using intermediaries there is the need to modify the supply chain by introducing another player in the chain, moving the risk management problem to this special type of agent.

The work presented in [2] discusses the management of risk in a chemical manufacturing supply chain. In this work, the authors propose a model for the risk management of supply chains based on the Economic theory of risk of investment and rate of returns. The authors propose the use of a *risk premium* or *the increase in expected return in exchange for a given amount of variance* as a measure used for risk management. This approach can be used as part of a risk management strategy for agents trading in an Agent-Based supply chain.

From the Market-Based Control point of view, a supply chain has been modelled as a series of markets where different agents trade resources [69]. This model of a supply chain can be used as an underlying market where my developed Option trading framework could be attached. If a supply chain is modelled as a series of markets, then it is possible to enhance each market by attaching the developed Option trading model to allow the agents to trade Option contracts on the underlying assets. This would provide the agents with an alternative market to trade Options to hedge the risks exposed by the supply chain.

7.5.2 Grid Resource Allocation

Grid resource allocation is another domain that would benefit from the use of Option trading. The modelling of distributed resource allocation with Market-Based mechanisms has been widely researched (as for example in [21, 25, 26, 85]). One of the approaches used when modelling Grid systems as Multi-Agent systems is the use of auctioning mechanisms for the distribution of resources. In using this approach, the systems are modelled as markets where agents gather to trade resources. This approach makes the addition of my proposed Option trading framework very straightforward. In fact, the use of Derivatives in Grid systems has been previously proposed in [55] where the use of Futures and Options to commercialise Grid resources is proposed.

7.5.3 Sensor Networks

Sensor networks present a special type of distributed resource allocation scenario. A sensor network consists of a set of autonomous sensors distributed among some physical environment [74]. The sensor network aims to monitor specific properties of the environment, making each sensor cooperate in a coordinated manner. In order to improve the efficacy of sensor networks, the allocation of resources such as bandwidth and time must be considered. However, sensor networks usually present the difficulty of having to consider energy constraints. Market–Based models of sensor networks have been recently proposed in works like [60] and [96]. In these works, each sensor in a sensor network is modelled as an agent which trades some of the required resources according to its objectives. As traders participating in a market, the agents acting in such sensor networks are also subject to market risks. Thus my developed Option trading framework could be used to provide a risk management mechanism for such sensor agents.

7.6 High Order Beliefs

In a market where different agents trade Options for different objectives, it may be useful to provide agents with the ability to form beliefs about the beliefs of other traders. Including such analysis in the agents' reasoning mechanism may give advantages to the agents; first in knowing the expectations of other participants in the market and thus being able to form a prediction of the demand at future times (which may affect the price of the assets); second, when creating a model about the other traders in the market it may be possible to make advantage of the use of Options. A game theoretic analysis of Option contracts has been addressed previously (see [17, 104]) by separating the decision process when trading Options into the valuation of the payoffs and the analysis of strategic interactions. This approach may be useful for trading agents which have interest in forming coalitions when participating in the market (such as for example, in a Supply Chain).

7.7 Final Remarks

In general, it may be possible to adapt my Option trading framework to any system that can be modelled as a market where some resource is traded. Although the constraints in the presented model did not allow more complex agent behaviour, the development of more complex strategies and better reasoning mechanisms can increase the utility of my proposed model. However, the developed Option market allowed me to address the stated research questions.

Although the developed framework is focused on the trading of European Option contracts, it should be possible to use a similar approach to provide other kind of Derivatives to Market–Based systems. This can be achieved by using a parallel market approach, where the software market where the Derivatives are traded is treated as a separate market from the primary market where the resources are traded.

Using this kind of parallel market mechanism it should be possible to develop markets where contracts such as Swaps and Futures are traded using similar auctioning mechanisms as in typical Market–Based systems which are based on auctions. An interesting addition would also be the creation of agent's coalitions which join efforts for maximizing a portfolio of resources by trading on different markets. With this coalition approach, each agent would be able to concentrate its efforts in only one market. y Chain Management and the different problems which will try to be corrected by using the Option Trading framework. A review of the current research into the solution of such problem will be developed.

Appendix A

Statistics Background

This appendix contains a revision of some of the concepts of Statistics that are used throughout this thesis. The aim of this appendix is to provide a brief introduction of these concepts for the reader who is not familiar with the statistic concepts used throughout this thesis. The majority of the information presented in this appendix was obtained from [40, 46, 73, 89].

A.1 Central Moments

When the values of a random variable X have a strong central tendency (i.e., they tend to cluster around some particular value), the variable can be described by a set of numbers called the *central moments*.

The most common central moments are the mean and the variance of a distribution. These central moments will be described now, along with the skewness.

A.1.1 Mean

The arithmetic mean of a distribution X is its first central moment. It is also called the expected value $E[X]$ of the distribution. The arithmetic mean of a discrete distribution X for $x_i \in X$ and N is the total number of elements in X can be obtained with the formula:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{A.1}$$

The arithmetic mean is a measure of central tendency. The unit of the arithmetic mean is the same as the units of the elements in the distribution. The arithmetic mean is greatly influenced by *outlier values* (values that lie very far away from the central tendency). Those values can be characterized by higher moments shown next.

A.1.2 Variance

The variance of a distribution X is its second central moment. The variance indicates the *width* or variability of the values around the mean. The variance σ^2 of a discrete finite distribution can be obtained with the formula:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2 \tag{A.2}$$

for a distribution X with N total elements, a mean of $\overline{x}$.

The variance is used to describe how spread are the values of a distribution around its mean. However, the unit of measurement of the variance is the square of the original units of the distribution. Therefore, usually the standard deviation σ is used as the most common measure of statistical dispersion. The standard deviation σ is calculated as:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2} \tag{A.3}$$

for a distribution X with N total elements, a mean of $\overline{x}$.

A.1.3 Skewness

The skewness of a distribution is its third central moment. The skewness measures the asymmetry of the distribution around its mean. The skewness is a *non–dimensional* value which is used to characterise the shape of the distribution. That is, it is not measured in any specific types of units as the mean, variance or the standard deviation.

The skewness γ of a distribution X with N total elements, a mean of $\overline{x}$ and a standard deviation of σ is calculated with the formula:

$$\gamma = \frac{1}{N}\sum_{j=1}^{N}\left[\frac{x_j - \overline{x}}{\sigma}\right]^3 \tag{A.4}$$

A positive value of skewness indicates that the tail of a distribution extends to the right of the x axis and a negative skewness indicates that the tail of a distribution extends to the left of the x axis (see Figure A.1).

A.2 Uniform Distribution

A random variable X is said to follow a Uniform distribution when all its values are equally probable. That is, when picking randomly one of the values of the distribution, all the

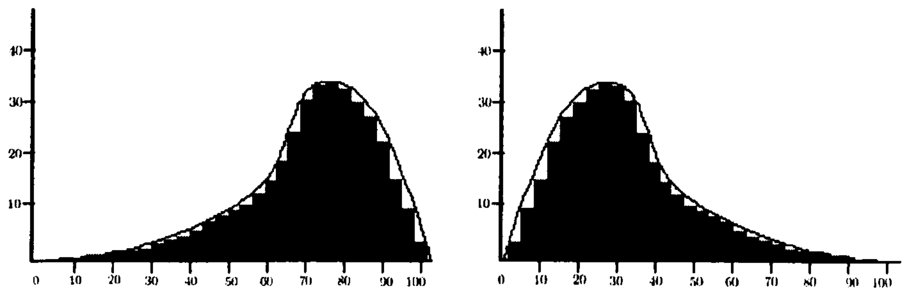

Figure A.1: Graphical representation of a histogram with (a) Negative skewness and (b) Positive skewness

values that conform the distribution have the same probability of being chosen. A uniform distribution is characterised by a location parameter a and a scale parameter b. The location parameter indicates the minimum value of the distribution and the scale parameters indicates the maximum value. Thus, the probability density function (p.d.f.) of a Uniform distribution X with a range of values $[a, b]$ is:

$$P[x] = \frac{1}{b-a} \tag{A.5}$$

for any value of $x \in X$.

A.3 Normal Distribution

The Normal distribution or Gaussian Distribution is a family of continuous probability distributions. Each member of the family is defined by two parameters, the *mean* μ and the variance σ^2 (the square of the standard deviation σ). The Normal distribution which has a mean of $\mu = 0$ and a variance of $\sigma^2 = 1$ is called the *standard Normal distribution*. Under these assumptions, a real-valued random variable $X \in \mathbb{R}$ is said to be Normally distributed with mean μ (also called the expected value) and variance σ^2 using the formula:

$$X \sim N(\mu, \sigma^2) \tag{A.6}$$

A.3.1 Probability Density Function

The *probability density function* (p.d.f.) of a Normal distribution is called the Gaussian function.[1] The function is defined as:

$$\varphi_{\mu,\sigma^2}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R} \tag{A.7}$$

[1] After the mathematician Carl F. Gauss.

The graphical representation of this function is the bell curve which is shown in Figure A.2. The p.d.f of the standard Normal distribution is defined as:

$$\varphi(x) = \varphi_{0,1}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad x \in \mathbb{R} \tag{A.8}$$

The integral of the Normal p.d.f. $\varphi_{\mu,\sigma^2}(x)$ over the Real line is equal to 1, specifically:

$$\int_{-\infty}^{\infty} \varphi_{\mu,\sigma^2}(u)\, du = 1 \tag{A.9}$$

This is, the area under the curve of the Normal distribution function (depicted in Figure A.2) will be equal to 1.

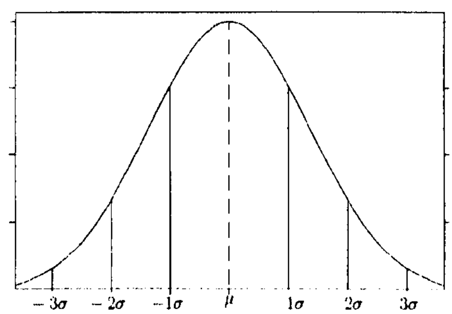

Figure A.2: Graphical representation of the Normal Probability Density Function.

A.3.2 Cumulative Distribution Function

The *cumulative distribution function* (c.d.f) of a Normal probability distribution denoted by $\Phi_{\mu,\sigma^2}(x)$ and evaluated at x, indicates the probability that a random variable X with such distribution has a value of less or equal to x, that is:

$$P[X \leq x] = \Phi_{\mu,\sigma^2}(x) \tag{A.10}$$

when X is a random variable which follows a Normal distribution with mean μ and variance σ^2. The value of $\Phi_{\mu,\sigma^2}(x)$ is obtained with the formula:

$$\int_{-\infty}^{x} \varphi_{\mu,\sigma^2}(u)\, du = 1 \tag{A.11}$$

that calculates the area under the curve within the range of $[-\infty, x]$ of the Normal distribution function (the grey area in Figure A.3).

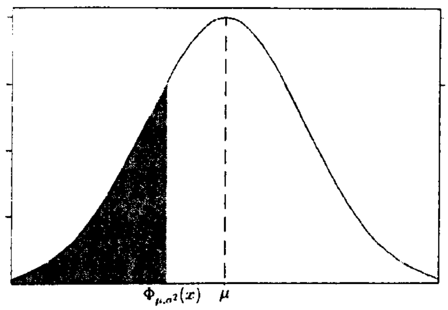

Figure A.3: Graphical representation of the Normal cumulative distribution function.

A.3.2.1 Standard Normal Cumulative Distribution

The c.d.f of a variable X that follows the standard Normal distribution (with $\mu = 0$ and $\sigma^2 = 1$) is called the standard Normal c.d.f. and is defined as:

$$\Phi(x) = \Phi_{0,1}(x) \tag{A.12}$$

The complement of the standard Normal c.d.f. is defined as $1 - \Phi(x)$ and represents the complement of the cumulative probability obtained by $\Phi(x)$. That is

$$\begin{aligned} \text{if} \quad \Phi(x) &= P[X \leq x] \\ \text{then} \quad 1 - \Phi(x) &= P[X > x] \end{aligned} \tag{A.13}$$

Thus if $\Phi(x)$ is the grey area under the curve in Figure A.3 then $1 - \Phi(x)$ will be the white are under the curve of the same figure.

A.3.2.2 Distribution Standardization

When looking for the probability value of $P[X \leq y]$ in the cumulative density function of the variable X that is different to the standard Normal c.d.f., the value can be standardised to obtain the probability value using the standard Normal c.d.f. The standardised value of y is obtained with the formula:

$$z = \frac{y - \mu}{\sigma} \tag{A.14}$$

Where μ is the mean of the non-standard Normal distribution function and σ is is the square root of its variance (i.e., its standard deviation). After doing this, then it is possible to obtain the value of the probability $P[X \leq y]$ using the standard Normal c.d.f. $\Phi(z)$.

A.3.3 Confidence Intervals

When working with random data samples it is useful to adopt a level of confidence. This confidence interval indicates that when measuring the random variable several times and estimates are made from these samples, the resulting values will lie within the true population in the established confidence level.

In a random variable X that follows a Normal distribution, 68% of all the values will like within one standard deviation 1σ, similarly 95% of the values will lie within 2σ and 99.7% of the values will lie within 3σ.

A.3.4 Log-Normal Distribution

A log-Normal distribution is a distribution whose natural logarithm is Normally distributed. That is, a real-valued random variable X is said to be log–Normally distributed if $ln(X)$ is Normally distributed.

Appendix B

Normality Statistical Analysis Data Tables

This appendix presents the detailed results of the Shapiro-Wilk statistical test used to analyse the returns of the strategies obtained from the simulation experiments.

Each table details the Shapiro-Wilk analysis results of the experiments for a different price series. For each Table, each row in the tables groups the statistics for an specific test case with the forecasting parameter specified in the first column; the rest of the columns show the p–value for test and the and the W Shapiro-Wilk statistic, a higher p–value indicates that there is more probability that the tested data series is Normally distributed.

Shapiro-Wilk test results for the test cases using *Microsoft* asset price.							
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$\alpha = 0$	p-value	2.38×10^{-42}	3.39×10^{-45}	3.27×10^{-40}	4.64×10^{-41}	5.50×10^{-42}	3.40×10^{-42}
	W	0.568	0.477	0.627	0.605	0.579	0.572
$\alpha = 0.2$	p-value	2.38×10^{-42}	4.05×10^{-45}	3.11×10^{-40}	3.64×10^{-41}	5.71×10^{-42}	3.35×10^{-42}
	W	0.568	0.479	0.627	0.602	0.579	0.572
$\alpha = 0.4$	p-value	2.54×10^{-42}	7.14×10^{-45}	1.70×10^{-40}	2.77×10^{-41}	4.18×10^{-42}	2.69×10^{-42}
	W	0.569	0.488	0.62	0.598	0.575	0.569
$\alpha = 0.6$	p-value	2.82×10^{-42}	1.48×10^{-43}	1.66×10^{-41}	2.04×10^{-41}	2.25×10^{-42}	1.63×10^{-42}
	W	0.57	0.531	0.592	0.595	0.567	0.563
$\alpha = 0.8$	p-value	1.71×10^{-42}	4.53×10^{-37}	1.27×10^{-42}	1.43×10^{-41}	4.89×10^{-41}	2.58×10^{-41}
	W	0.564	0.703	0.56	0.59	0.605	0.598
$\alpha = 1.0$	p-value	1.72×10^{-42}	1.58×10^{-37}	1.44×10^{-42}	1.13×10^{-41}	1.44×10^{-40}	1.83×10^{-41}
	W	0.564	0.692	0.561	0.588	0.618	0.593

Shapiro-Wilk test results for the test cases using *Dell* asset price.							
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$\alpha = 0$	p-value	1.08×10^{-17}	4.39×10^{-47}	6.58×10^{-29}	1.42×10^{-16}	9.73×10^{-16}	1.95×10^{-15}
	W	0.945	0.408	0.843	0.952	0.956	0.958
$\alpha = 0.2$	p-value	1.37×10^{-17}	7.30×10^{-47}	1.86×10^{-28}	1.97×10^{-16}	2.06×10^{-15}	1.52×10^{-15}
	W	0.946	0.416	0.849	0.953	0.958	0.957
$\alpha = 0.4$	p-value	1.08×10^{-17}	3.25×10^{-46}	6.40×10^{-27}	1.84×10^{-16}	4.81×10^{-15}	2.12×10^{-15}
	W	0.945	0.441	0.867	0.952	0.96	0.958
$\alpha = 0.6$	p-value	2.35×10^{-18}	1.09×10^{-43}	3.33×10^{-22}	1.88×10^{-16}	1.38×10^{-14}	1.25×10^{-15}
	W	0.941	0.527	0.914	0.952	0.962	0.957
$\alpha = 0.8$	p-value	2.17×10^{-18}	2.72×10^{-38}	2.56×10^{-14}	2.16×10^{-16}	8.90×10^{-18}	5.64×10^{-18}
	W	0.941	0.675	0.963	0.953	0.945	0.944
$\alpha = 1.0$	p-value	2.21×10^{-18}	7.42×10^{-36}	1.07×10^{-13}	1.55×10^{-17}	8.47×10^{-18}	2.62×10^{-18}
	W	0.941	0.728	0.966	0.946	0.945	0.942

Shapiro-Wilk test results for the test cases using *IBM* asset price.							
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$\alpha = 0$	p-value	2.86×10^{-21}	2.40×10^{-41}	4.10×10^{-24}	2.01×10^{-21}	3.21×10^{-21}	2.85×10^{-21}
	W	0.921	0.597	0.897	0.92	0.921	0.921
$\alpha = 0.2$	p-value	2.78×10^{-21}	1.48×10^{-41}	2.07×10^{-24}	2.43×10^{-21}	2.94×10^{-21}	2.69×10^{-21}
	W	0.921	0.591	0.894	0.921	0.921	0.921
$\alpha = 0.4$	p-value	2.54×10^{-21}	3.33×10^{-40}	8.73×10^{-24}	2.77×10^{-21}	2.30×10^{-21}	2.07×10^{-21}
	W	0.921	0.628	0.9	0.921	0.92	0.92
$\alpha = 0.6$	p-value	2.09×10^{-21}	8.16×10^{-37}	3.40×10^{-22}	4.23×10^{-21}	2.10×10^{-21}	2.02×10^{-21}
	W	0.92	0.708	0.914	0.922	0.92	0.92
$\alpha = 0.8$	p-value	2.24×10^{-21}	5.68×10^{-22}	1.74×10^{-21}	4.34×10^{-21}	5.72×10^{-21}	3.96×10^{-21}
	W	0.92	0.916	0.919	0.922	0.923	0.922
$\alpha = 1.0$	p-value	2.00×10^{-21}	7.49×10^{-21}	1.85×10^{-21}	3.52×10^{-21}	9.76×10^{-21}	1.93×10^{-21}
	W	0.92	0.924	0.92	0.922	0.925	0.92

Table B.1: Results of applying the Shapiro-Wilk normality test to the returns obtained from the α-Perfect forecasting set of experiments using the *Microsoft*, *Dell* and *IBM* price series as the price of the asset.

Shapiro-Wilk test results for the test cases using *Random A* asset price.							
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$\alpha = 0$	p-value	0.544	1.57×10^{-37}	0.029	0.464	0.521	0.544
	W	0.998	0.692	0.996	0.998	0.998	0.998
$\alpha = 0.2$	p-value	0.551	1.04×10^{-39}	0.001	0.511	0.556	0.566
	W	0.998	0.64	0.994	0.998	0.998	0.998
$\alpha = 0.4$	p-value	0.551	4.74×10^{-39}	0.002	0.548	0.567	0.577
	W	0.998	0.657	0.994	0.998	0.998	0.998
$\alpha = 0.6$	p-value	0.520	6.07×10^{-33}	0.163	0.595	0.503	0.514
	W	0.998	0.782	0.997	0.998	0.998	0.998
$\alpha = 0.8$	p-value	0.506	1.82×10^{-04}	0.628	0.554	0.649	0.650
	W	0.998	0.993	0.998	0.998	0.999	0.999
$\alpha = 1.0$	p-value	0.523	0.699	0.534	0.327	0.777	0.434
	W	0.998	0.999	0.998	0.998	0.999	0.998
Shapiro-Wilk test results for the test cases using *Random B* asset price.							
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$\alpha = 0$	p-value	1.19×10^{-33}	6.04×10^{-43}	6.47×10^{-21}	1.83×10^{-26}	6.30×10^{-29}	1.21×10^{-29}
	W	0.77	0.55	0.924	0.872	0.842	0.833
$\alpha = 0.2$	p-value	4.43×10^{-34}	2.28×10^{-42}	1.55×10^{-26}	1.65×10^{-27}	1.23×10^{-29}	1.81×10^{-30}
	W	0.762	0.567	0.872	0.86	0.833	0.821
$\alpha = 0.4$	p-value	5.41×10^{-35}	2.42×10^{-39}	3.94×10^{-35}	2.70×10^{-29}	1.47×10^{-29}	1.37×10^{-30}
	W	0.745	0.65	0.742	0.838	0.834	0.819
$\alpha = 0.6$	p-value	5.50×10^{-35}	4.58×10^{-23}	8.61×10^{-35}	4.82×10^{-27}	5.85×10^{-26}	3.32×10^{-28}
	W	0.745	0.906	0.749	0.866	0.878	0.852
$\alpha = 0.8$	p-value	4.05×10^{-35}	1.11×10^{-09}	4.60×10^{-35}	1.69×10^{-24}	8.16×10^{-18}	8.35×10^{-22}
	W	0.743	0.98	0.744	0.893	0.945	0.917
$\alpha = 1.0$	p-value	2.21×10^{-35}	1.38×10^{-09}	2.43×10^{-35}	6.60×10^{-25}	2.40×10^{-16}	3.38×10^{-23}
	W	0.738	0.981	0.738	0.889	0.953	0.905

Table B.2: Results of applying the Shapiro-Wilk normality test to the returns obtained from the α-Perfect forecasting set of experiments using the *Random A* and *Random B* price series as the price of the asset.

		Shapiro-Wilk test results for the test cases using *Microsoft* asset price					
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$n = 15$	p-value	1.73×10^{-42}	1.28×10^{-37}	1.51×10^{-42}	1.12×10^{-41}	2.34×10^{-40}	1.35×10^{-41}
	W	0.564	0.69	0.562	0.587	0.624	0.59
$n = 30$	p-value	1.70×10^{-42}	1.10×10^{-37}	1.44×10^{-42}	1.34×10^{-41}	1.45×10^{-40}	1.84×10^{-41}
	W	0.564	0.689	0.561	0.59	0.618	0.593
$n = 45$	p-value	1.67×10^{-42}	9.66×10^{-38}	1.36×10^{-42}	1.11×10^{-41}	5.37×10^{-39}	2.20×10^{-42}
	W	0.563	0.688	0.561	0.587	0.658	0.567
$n = 60$	p-value	1.65×10^{-42}	9.11×10^{-38}	1.27×10^{-42}	1.38×10^{-41}	7.17×10^{-40}	9.71×10^{-42}
	W	0.563	0.687	0.56	0.59	0.636	0.586
$n = 75$	p-value	1.61×10^{-42}	8.71×10^{-38}	1.21×10^{-42}	1.23×10^{-41}	1.49×10^{-39}	7.88×10^{-42}
	W	0.563	0.687	0.559	0.589	0.644	0.583
$n = 90$	p-value	1.59×10^{-42}	7.12×10^{-38}	1.20×10^{-42}	1.32×10^{-41}	7.64×10^{-40}	1.29×10^{-41}
	W	0.563	0.685	0.559	0.589	0.637	0.589
		Shapiro-Wilk test results for the test cases using *Dell* asset price					
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$n = 15$	p-value	2.23×10^{-18}	4.54×10^{-39}	1.34×10^{-13}	4.90×10^{-18}	3.87×10^{-15}	1.49×10^{-20}
	W	0.941	0.656	0.966	0.943	0.959	0.927
$n = 30$	p-value	2.40×10^{-18}	2.70×10^{-37}	1.94×10^{-13}	2.95×10^{-18}	4.38×10^{-13}	3.25×10^{-23}
	W	0.941	0.698	0.967	0.942	0.968	0.905
$n = 45$	p-value	2.50×10^{-18}	5.46×10^{-36}	1.41×10^{-13}	3.08×10^{-18}	2.07×10^{-13}	3.58×10^{-23}
	W	0.942	0.725	0.966	0.942	0.967	0.905
$n = 60$	p-value	2.63×10^{-18}	4.62×10^{-35}	9.79×10^{-14}	3.22×10^{-18}	5.44×10^{-14}	2.58×10^{-22}
	W	0.942	0.744	0.966	0.942	0.964	0.913
$n = 75$	p-value	2.54×10^{-18}	2.00×10^{-34}	6.24×10^{-14}	3.42×10^{-18}	3.85×10^{-14}	2.69×10^{-22}
	W	0.942	0.756	0.965	0.942	0.964	0.913
$n = 90$	p-value	2.33×10^{-18}	1.13×10^{-33}	5.81×10^{-14}	3.43×10^{-18}	1.35×10^{-14}	6.11×10^{-22}
	W	0.941	0.77	0.965	0.942	0.962	0.916
		Shapiro-Wilk test results for the test cases using *IBM* asset price					
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$n = 15$	p-value	2.04×10^{-21}	7.35×10^{-21}	1.94×10^{-21}	3.12×10^{-21}	2.63×10^{-20}	5.68×10^{-22}
	W	0.92	0.924	0.92	0.921	0.928	0.916
$n = 30$	p-value	2.04×10^{-21}	8.21×10^{-21}	1.91×10^{-21}	3.13×10^{-21}	5.70×10^{-21}	9.77×10^{-22}
	W	0.92	0.925	0.92	0.921	0.923	0.917
$n = 45$	p-value	2.03×10^{-21}	9.34×10^{-21}	1.82×10^{-21}	2.55×10^{-21}	3.65×10^{-20}	1.69×10^{-22}
	W	0.92	0.925	0.92	0.921	0.929	0.911
$n = 60$	p-value	2.01×10^{-21}	9.56×10^{-21}	1.82×10^{-21}	1.58×10^{-21}	1.77×10^{-18}	8.94×10^{-24}
	W	0.92	0.925	0.92	0.919	0.941	0.9
$n = 75$	p-value	2.02×10^{-21}	6.73×10^{-21}	1.80×10^{-21}	1.63×10^{-21}	2.13×10^{-18}	6.28×10^{-24}
	W	0.92	0.924	0.92	0.919	0.941	0.898
$n = 90$	p-value	2.03×10^{-21}	4.01×10^{-21}	1.73×10^{-21}	1.38×10^{-21}	4.61×10^{-18}	3.18×10^{-24}
	W	0.92	0.922	0.919	0.919	0.943	0.896

Table B.3: Results of applying the Shapiro-Wilk normality test to the returns obtained from the SMA_n forecasting set of experiments using the *Microsoft*, *Dell* and *IBM* price series as the price of the asset.

Shapiro-Wilk test results for the test cases using *Random A* asset price							
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$n = 15$	p-value	0.513	0.789	0.517	0.329	0.789	0.086
	W	0.998	0.999	0.998	0.998	0.999	0.997
$n = 30$	p-value	0.512	0.799	0.515	0.330	0.694	0.011
	W	0.998	0.999	0.998	0.998	0.999	0.996
$n = 45$	p-value	0.511	0.747	0.510	0.351	0.692	0.006
	W	0.998	0.999	0.998	0.998	0.999	0.995
$n = 60$	p-value	0.515	0.735	0.491	0.341	0.701	0.002
	W	0.998	0.999	0.998	0.998	0.999	0.995
$n = 75$	p-value	0.512	0.769	0.486	0.352	0.732	3.20×10^{-04}
	W	0.998	0.999	0.998	0.998	0.999	0.993
$n = 90$	p-value	0.509	0.745	0.473	0.354	0.771	1.55×10^{-04}
	W	0.998	0.999	0.998	0.998	0.999	0.992
Shapiro-Wilk test results for the test cases using *Random B* asset price							
		OTMaxW	*OTMinR*	*OTMix*	*OTRnd*	*ATSpec*	*ATNoise*
$n = 15$	p-value	2.83×10^{-35}	1.30×10^{-09}	4.59×10^{-35}	1.56×10^{-25}	8.80×10^{-15}	7.20×10^{-25}
	W	0.74	0.98	0.744	0.882	0.961	0.889
$n = 30$	p-value	3.51×10^{-35}	1.88×10^{-09}	3.87×10^{-35}	1.85×10^{-26}	3.49×10^{-11}	8.40×10^{-30}
	W	0.742	0.981	0.742	0.872	0.975	0.831
$n = 45$	p-value	3.61×10^{-35}	1.97×10^{-09}	3.78×10^{-35}	1.57×10^{-26}	1.46×10^{-11}	4.50×10^{-30}
	W	0.742	0.981	0.742	0.872	0.974	0.827
$n = 60$	p-value	3.87×10^{-35}	2.56×10^{-09}	3.99×10^{-35}	8.80×10^{-27}	3.74×10^{-11}	1.08×10^{-30}
	W	0.742	0.981	0.743	0.869	0.976	0.818
$n = 75$	p-value	3.79×10^{-35}	3.53×10^{-09}	3.93×10^{-35}	2.05×10^{-27}	2.20×10^{-10}	2.77×10^{-32}
	W	0.742	0.982	0.742	0.861	0.978	0.793
$n = 90$	p-value	3.63×10^{-35}	5.16×10^{-09}	3.84×10^{-35}	1.01×10^{-26}	2.65×10^{-10}	2.12×10^{-33}
	W	0.742	0.982	0.742	0.869	0.978	0.774

Table B.4: Results of applying the Shapiro-Wilk normality test to the returns obtained from the SMA_n forecasting set of experiments using the *Random A* and *Random B* price series as the price of the asset.

Appendix C

Trading Volume Analysis Data Tables

This appendix contains the tables with the data used to generate the figures used in Section 6.4 for the trading volume analysis. The tables show the values of the R_H^C, R_W^C and R_E^H ratios which are defined in Section 5.5.3. Table C.3 shows the ratios obtained from the set of experiments A, which used the α-Perfect forecasting function. Table C.6 shows the ratios obtained from the set of experiments B, which used the SMA_n forecasting function. The data is included as an appendix because it forms the basis of the presented results.

Offers volumes for the experiments using the *Microsoft* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$\alpha = 0$	7558	8669	27300	6664	15387	9666	22433	21978
$\alpha = 0.2$	8579	11085	30269	5937	16935	11043	22426	21968
$\alpha = 0.4$	10884	15856	35082	4829	19389	14080	22411	21983
$\alpha = 0.6$	15700	21877	41560	3370	22248	19123	22423	21974
$\alpha = 0.8$	38895	10910	48087	1800	40575	9302	22507	21852
$\alpha = 1$	41154	8796	49248	702	42514	7436	22602	21776
Offers volumes for the experiments using the *Dell* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$\alpha = 0$	5458	8091	24508	6388	15051	8014	22251	22176
$\alpha = 0.2$	6226	9717	29293	4534	18024	7893	22253	22181
$\alpha = 0.4$	7800	12863	34887	2009	21729	8238	22236	22198
$\alpha = 0.6$	12029	19265	41984	244	26925	10975	22255	22192
$\alpha = 0.8$	21129	27078	49346	27	32748	16351	22212	22232
$\alpha = 1$	23063	26937	49950	50	34124	15876	22205	22240
Offers volumes for the experiments using the *IBM* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$\alpha = 0$	5439	5881	13616	14676	7623	12520	22717	21571
$\alpha = 0.2$	5688	7182	15763	15806	8412	14376	22844	21468
$\alpha = 0.4$	6332	9454	17217	16757	8990	16617	22862	21410
$\alpha = 0.6$	8635	14332	19528	18903	10446	21087	22905	21378
$\alpha = 0.8$	14502	32337	23707	25139	16729	31577	22626	21725
$\alpha = 1$	12814	37186	22850	27150	16872	33128	22499	21876
Offers volumes for the experiments using the *Random A* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$\alpha = 0$	4931	5500	10096	14639	5797	12037	22838	21440
$\alpha = 0.2$	5160	6493	11421	17185	6123	14485	22948	21333
$\alpha = 0.4$	5845	8147	11849	20786	6196	17814	23026	21210
$\alpha = 0.6$	7473	12150	11133	25373	6030	22969	23017	21227
$\alpha = 0.8$	8086	32430	11431	35024	6883	37482	22748	21583
$\alpha = 1$	5081	44919	10660	39340	7350	42650	22508	21876
Offers volumes for the experiments using the *Random B* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$\alpha = 0$	13094	10150	30407	7243	19349	12238	24943	18793
$\alpha = 0.2$	15690	11626	33048	6797	21233	13415	25170	18559
$\alpha = 0.4$	21492	11628	36290	6465	25536	13286	25590	17998
$\alpha = 0.6$	30705	10501	40270	6320	32976	11508	25312	18339
$\alpha = 0.8$	40439	9439	43793	6164	40726	9227	24430	19456
$\alpha = 1$	42019	7981	44248	5752	42528	7472	24587	19256

Table C.1: Values of the volumes of offers submitted by the Option-trading strategies in set of experiments using the the α-Perfect forecasting function.

Cleared offers volumes for the experiments using the *Microsoft* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$\alpha = 0$	4051	8100	2255	14318	5599	2465	6284	8732	2833	12644	14867	6397
$\alpha = 0.2$	4564	10381	2886	16249	5068	2290	6744	10007	3132	12651	14754	6372
$\alpha = 0.4$	5732	14571	3928	19707	4158	2123	7422	12395	3615	12724	14461	6370
$\alpha = 0.6$	7351	18970	4685	23932	2864	1820	7252	15556	4234	12936	14081	6415
$\alpha = 0.8$	5829	9978	3607	9823	1529	1362	5498	8163	3443	12443	13921	6812
$\alpha = 1$	4479	7986	2747	7574	279	279	4125	6448	2479	12381	13845	6751

Cleared offers volumes for the experiments using the *Dell* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$\alpha = 0$	2837	7509	2444	11373	4374	1475	5779	6507	2568	13213	14811	6411
$\alpha = 0.2$	2889	9200	2975	12569	3091	974	6017	6681	2533	12674	15177	6413
$\alpha = 0.4$	3370	12202	4005	14322	1374	501	6594	7260	2748	12071	15522	6509
$\alpha = 0.6$	5003	17627	5441	17484	197	100	8239	9198	3467	11807	15511	6661
$\alpha = 0.8$	6879	22454	4522	19574	7	6	9444	10917	4297	12210	14729	6752
$\alpha = 1$	3314	17381	3233	16809	2	2	5599	7566	3779	12876	13649	6516

Cleared offers volumes for the experiments using the *IBM* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$\alpha = 0$	4875	4288	1516	12181	10488	3970	6575	9194	3429	13848	13508	6244
$\alpha = 0.2$	5152	5243	1726	14126	11059	3988	7078	10418	3668	13761	13396	6184
$\alpha = 0.4$	5776	6667	2256	15381	10975	4229	7365	11390	4167	13730	13221	6142
$\alpha = 0.6$	7439	9308	3452	16621	10501	4509	7877	12727	5084	13725	13127	6135
$\alpha = 0.8$	8713	14511	5282	16086	8222	4441	9738	12396	5226	13635	13043	6267
$\alpha = 1$	4047	11225	2999	10221	1997	1692	5271	6879	2697	13477	12914	6206

Cleared offers volumes for the experiments using the *Random A* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$\alpha = 0$	4593	3383	1260	9447	9035	3598	5330	7633	2993	13868	13187	6157
$\alpha = 0.2$	4923	3829	1298	10822	9749	3724	5650	8599	3177	13836	13053	6066
$\alpha = 0.4$	5635	4436	1472	11255	9924	3850	5664	9119	3344	13798	12873	5974
$\alpha = 0.6$	7041	5521	2102	10169	8831	3881	5157	9108	3730	13798	12704	5942
$\alpha = 0.8$	6734	8604	3056	9458	5888	2892	5185	8202	3491	13844	12527	6011
$\alpha = 1$	2258	6753	1828	6029	1654	1131	3078	4266	1688	13795	12488	6042

Cleared offers volumes for the experiments using the *Random B* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$\alpha = 0$	5677	8714	3442	13898	5494	2389	6754	10362	4192	12055	13814	5438
$\alpha = 0.2$	6241	9909	3991	14629	4891	2158	6728	11201	4547	11987	13585	5379
$\alpha = 0.4$	6441	9823	4363	13663	4353	2148	6422	10969	4956	11755	13136	5320
$\alpha = 0.6$	6404	8682	4406	10562	3946	2683	6190	9249	4925	11828	13107	5487
$\alpha = 0.8$	5222	6932	3407	6903	2659	2106	4981	6316	3281	12080	13279	5782
$\alpha = 1$	2994	4839	2081	4708	808	774	2992	3993	1979	12034	13088	5672

Table C.2: Values of the volumes of cleared offers and exercised hold Options by the Option-trading strategies in set of experiments using the the α-Perfect forecasting function.

Ratios for the experiments using the *Microsoft* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H
$\alpha = 0$	0.54	0.93	0.28	0.52	0.84	0.44	0.41	0.90	0.32	0.56	0.68	0.43
$\alpha = 0.2$	0.53	0.94	0.28	0.54	0.85	0.45	0.40	0.91	0.31	0.56	0.67	0.43
$\alpha = 0.4$	0.53	0.92	0.27	0.56	0.86	0.51	0.38	0.88	0.29	0.57	0.66	0.44
$\alpha = 0.6$	0.47	0.87	0.25	0.58	0.85	0.64	0.33	0.81	0.27	0.58	0.64	0.46
$\alpha = 0.8$	0.15	0.91	0.36	0.20	0.85	0.89	0.14	0.88	0.42	0.55	0.64	0.49
$\alpha = 1$	0.11	0.91	0.34	0.15	0.40	1.00	0.10	0.87	0.38	0.55	0.64	0.49
Ratios for the experiments using the *Dell* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H
$\alpha = 0$	0.52	0.93	0.33	0.46	0.68	0.34	0.38	0.81	0.39	0.59	0.67	0.43
$\alpha = 0.2$	0.46	0.95	0.32	0.43	0.68	0.32	0.33	0.85	0.38	0.57	0.68	0.42
$\alpha = 0.4$	0.43	0.95	0.33	0.41	0.68	0.36	0.30	0.88	0.38	0.54	0.70	0.42
$\alpha = 0.6$	0.42	0.91	0.31	0.42	0.81	0.51	0.31	0.84	0.38	0.53	0.70	0.43
$\alpha = 0.8$	0.33	0.83	0.20	0.40	0.26	0.90	0.29	0.67	0.39	0.55	0.66	0.46
$\alpha = 1$	0.14	0.65	0.19	0.34	0.03	1.00	0.16	0.48	0.50	0.58	0.61	0.48
Ratios for the experiments using the *IBM* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H
$\alpha = 0$	0.90	0.73	0.35	0.89	0.71	0.38	0.86	0.73	0.37	0.61	0.63	0.46
$\alpha = 0.2$	0.91	0.73	0.33	0.90	0.70	0.36	0.84	0.72	0.35	0.60	0.62	0.46
$\alpha = 0.4$	0.91	0.71	0.34	0.89	0.65	0.39	0.82	0.69	0.37	0.60	0.62	0.46
$\alpha = 0.6$	0.86	0.65	0.37	0.85	0.56	0.43	0.75	0.60	0.40	0.60	0.61	0.47
$\alpha = 0.8$	0.60	0.45	0.36	0.68	0.33	0.54	0.58	0.39	0.42	0.60	0.60	0.48
$\alpha = 1$	0.32	0.30	0.27	0.45	0.07	0.85	0.31	0.21	0.39	0.60	0.59	0.48
Ratios for the experiments using the *Random A* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H
$\alpha = 0$	0.93	0.62	0.37	0.94	0.62	0.40	0.92	0.63	0.39	0.61	0.62	0.47
$\alpha = 0.2$	0.95	0.59	0.34	0.95	0.57	0.38	0.92	0.59	0.37	0.60	0.61	0.46
$\alpha = 0.4$	0.96	0.54	0.33	0.95	0.48	0.39	0.91	0.51	0.37	0.60	0.61	0.46
$\alpha = 0.6$	0.94	0.45	0.38	0.91	0.35	0.44	0.86	0.40	0.41	0.60	0.60	0.47
$\alpha = 0.8$	0.83	0.27	0.36	0.83	0.17	0.49	0.75	0.22	0.43	0.61	0.58	0.48
$\alpha = 1$	0.44	0.15	0.27	0.57	0.04	0.68	0.42	0.10	0.40	0.61	0.57	0.48
Ratios for the experiments using the *Random B* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H	R_W^C	R_H^C	R_E^H
$\alpha = 0$	0.43	0.86	0.39	0.46	0.76	0.43	0.35	0.85	0.40	0.48	0.74	0.39
$\alpha = 0.2$	0.40	0.85	0.40	0.44	0.72	0.44	0.32	0.83	0.41	0.48	0.73	0.40
$\alpha = 0.4$	0.30	0.84	0.44	0.38	0.67	0.49	0.25	0.83	0.45	0.46	0.73	0.40
$\alpha = 0.6$	0.21	0.83	0.51	0.26	0.62	0.68	0.19	0.80	0.53	0.47	0.71	0.42
$\alpha = 0.8$	0.13	0.73	0.49	0.16	0.43	0.79	0.12	0.68	0.52	0.49	0.68	0.44
$\alpha = 1$	0.07	0.61	0.43	0.11	0.14	0.96	0.07	0.53	0.50	0.49	0.68	0.43

Table C.3: Values of the R_W^C, R_H^C and R_E^H ratios obtained by the Option trading strategies in set of experiments using the the α-Perfect forecasting function.

Offers volumes for the experiments using the *Microsoft* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$n = 15$	40062.56	9687	48639	1111	42041	7709	22512	21873
$n = 30$	38956.22	10444	47813	1587	41029	8371	22509	21878
$n = 45$	37919.34	11131	47035	2015	40156	8894	22517	21870
$n = 60$	36858.92	11941	46476	2324	39068	9732	22486	21908
$n = 75$	35771.48	12479	45701	2549	38066	10184	22443	21937
$n = 90$	35086.56	12663	45050	2700	37586	10164	22452	21934
Offers volumes for the experiments using the *Dell* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$n = 15$	21607.66	28392	49455	545	35867	14133	22236	22209
$n = 30$	21603.68	28395	49079	921	35452	14548	22236	22219
$n = 45$	21302.14	28696	48510	1490	34654	15346	22218	22234
$n = 60$	21730.28	28266	47925	2075	33853	16147	22205	22238
$n = 75$	21045.96	28948	47187	2813	33039	16961	22212	22238
$n = 90$	20842.34	29149	46504	3496	32289	17711	22222	22236
Offers volumes for the experiments using the *IBM* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$n = 15$	13394.88	36605	22679	27321	16794	33206	22543	21814
$n = 30$	13415.68	36584	22489	27511	16690	33310	22554	21795
$n = 45$	13620.56	36379	22462	27538	16743	33257	22614	21734
$n = 60$	13836.92	36113	22623	27327	16916	33034	22586	21753
$n = 75$	13159.28	36291	22131	27319	16177	33273	22605	21754
$n = 90$	11784.28	36866	20987	27663	14802	33848	22639	21708
Offers volumes for the experiments using the *Random A* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$n = 15$	6422.26	43578	10681	39319	7772	42228	22500	21866
$n = 30$	6758.94	43241	10792	39208	7944	42056	22568	21798
$n = 45$	6842.18	42758	10582	39068	7904	41746	22554	21795
$n = 60$	6635.40	41715	10192	38758	7525	41425	22607	21744
$n = 75$	7113.44	39437	10326	38074	7879	40521	22659	21689
$n = 90$	7498.80	37101	10606	37394	8485	39415	22693	21635
Offers volumes for the experiments using the *Random B* price series								
	OTMaxW		OTMinR		OTMix		OTRnd	
	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}	vol_{OW}	vol_{OH}
$n = 15$	41614.92	8385	44346	5654	41869	8131	24444	19461
$n = 30$	41236.22	8764	43529	6471	41621	8379	24424	19474
$n = 45$	40371.70	9228	42369	7281	40685	8965	24327	19611
$n = 60$	39135.54	9214	41263	7687	39708	9242	24336	19588
$n = 75$	37162.56	9387	40145	8255	38741	9659	24393	19521
$n = 90$	35086.00	9514	39068	8932	37583	10317	24220	19744

Table C.4: Values of the volumes of offers submitted by the Option-trading strategies in set of experiments using the the SMA_n forecasting function.

Cleared offers volumes for the experiments using the *Microsoft* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$n = 15$	4541	8608	2855	7874	418	410	4447	6404	2551	12431	13863	6783
$n = 30$	4658	8930	2964	8140	480	465	4597	6596	2631	12454	13842	6770
$n = 45$	4536	9091	2935	8341	459	435	4582	6594	2623	12495	13811	6763
$n = 60$	4666	9452	2897	8847	416	389	4629	7007	2611	12533	13799	6734
$n = 75$	4739	9707	2957	9083	403	365	4698	7188	2663	12555	13777	6738
$n = 90$	4651	9724	2994	8990	400	364	4663	6987	2697	12556	13749	6724
Cleared offers volumes for the experiments using the *Dell* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$n = 15$	2798	17507	3223	17454	11	8	5578	7641	3780	12923	13595	6423
$n = 30$	2717	16789	3166	16740	18	12	5333	7358	3700	12934	13559	6419
$n = 45$	2631	16202	3099	16111	27	20	5141	7071	3663	12931	13514	6399
$n = 60$	2787	15778	3099	15746	42	29	4946	7096	3651	12942	13505	6402
$n = 75$	2655	15653	3031	15535	56	36	4923	6894	3611	12981	13491	6412
$n = 90$	2720	15522	3020	15294	73	49	4866	6816	3618	12994	13464	6401
Cleared offers volumes for the experiments using the *IBM* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$n = 15$	4623	11121	3343	10347	2480	2006	5542	7481	3078	13479	12910	6216
$n = 30$	4492	10880	3258	10214	2453	1952	5452	7405	3032	13476	12895	6187
$n = 45$	4392	10606	3146	10022	2414	1879	5279	7270	2928	13471	12875	6180
$n = 60$	4305	10461	3113	9907	2339	1844	5194	7186	2899	13453	12872	6176
$n = 75$	4420	10639	3203	10110	2329	1836	5194	7355	2947	13481	12881	6183
$n = 90$	4489	10809	3258	10339	2302	1787	5194	7580	3003	13505	12837	6166
Cleared offers volumes for the experiments using the *Random A* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$n = 15$	3156	6587	2323	6024	2382	1564	3561	5071	2228	13785	12487	6065
$n = 30$	3270	6521	2369	5980	2429	1573	3589	5170	2263	13779	12498	6084
$n = 45$	3115	6129	2279	5512	2261	1447	3298	4799	2125	13751	12487	6053
$n = 60$	3095	5937	2172	5273	2161	1367	3049	4590	2001	13761	12490	6036
$n = 75$	3511	6091	2179	5207	2346	1428	3181	4692	2027	13726	12495	6024
$n = 90$	3592	6101	2245	5046	2447	1496	3374	4650	2105	13717	12530	6046
Cleared offers volumes for the experiments using the *Random B* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}	vol_{CW}	vol_{CH}	vol_{HE}
$n = 15$	3620	5581	2419	5308	1180	946	3490	4595	2265	12102	13165	5813
$n = 30$	3560	5428	2405	5075	1263	954	3478	4422	2243	12131	13132	5772
$n = 45$	3729	5588	2513	5256	1387	1058	3555	4620	2321	12183	13128	5792
$n = 60$	3919	5818	2613	5540	1511	1183	3752	4959	2480	12214	13138	5789
$n = 75$	3991	5869	2649	5692	1691	1319	3973	5226	2644	12214	13085	5751
$n = 90$	3974	5859	2669	5733	1721	1333	4099	5407	2718	12315	13135	5838

Table C.5: Values of the volumes of cleared offers and exercised hold Options by the Option-trading strategies in set of experiments using the the SMA_n forecasting function.

Ratios for the experiments using the *Microsoft* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E
$n = 15$	0.11	0.89	0.33	0.16	0.38	0.98	0.11	0.83	0.40	0.55	0.63	0.49
$n = 30$	0.12	0.86	0.33	0.17	0.30	0.97	0.11	0.79	0.40	0.55	0.63	0.49
$n = 45$	0.12	0.82	0.32	0.18	0.23	0.95	0.11	0.74	0.40	0.55	0.63	0.49
$n = 60$	0.13	0.79	0.31	0.19	0.18	0.93	0.12	0.72	0.37	0.56	0.63	0.49
$n = 75$	0.13	0.78	0.30	0.20	0.16	0.91	0.12	0.71	0.37	0.56	0.63	0.49
$n = 90$	0.13	0.77	0.31	0.20	0.15	0.91	0.12	0.69	0.39	0.56	0.63	0.49

Ratios for the experiments using the *Dell* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E
$n = 15$	0.13	0.62	0.18	0.35	0.02	0.75	0.16	0.54	0.49	0.58	0.61	0.47
$n = 30$	0.13	0.59	0.19	0.34	0.02	0.67	0.15	0.51	0.50	0.58	0.61	0.47
$n = 45$	0.12	0.56	0.19	0.33	0.02	0.75	0.15	0.46	0.52	0.58	0.61	0.47
$n = 60$	0.13	0.56	0.20	0.33	0.02	0.68	0.15	0.44	0.51	0.58	0.61	0.47
$n = 75$	0.13	0.54	0.19	0.33	0.02	0.64	0.15	0.41	0.52	0.58	0.61	0.48
$n = 90$	0.13	0.53	0.19	0.33	0.02	0.67	0.15	0.38	0.53	0.58	0.61	0.48

Ratios for the experiments using the *IBM* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E
$n = 15$	0.35	0.30	0.30	0.46	0.09	0.81	0.33	0.23	0.41	0.60	0.59	0.48
$n = 30$	0.33	0.30	0.30	0.45	0.09	0.80	0.33	0.22	0.41	0.60	0.59	0.48
$n = 45$	0.32	0.29	0.30	0.45	0.09	0.78	0.32	0.22	0.40	0.60	0.59	0.48
$n = 60$	0.31	0.29	0.30	0.44	0.09	0.79	0.31	0.22	0.40	0.60	0.59	0.48
$n = 75$	0.34	0.29	0.30	0.46	0.09	0.79	0.32	0.22	0.40	0.60	0.59	0.48
$n = 90$	0.38	0.29	0.30	0.49	0.08	0.78	0.35	0.22	0.40	0.60	0.59	0.48

Ratios for the experiments using the *Random A* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E
$n = 15$	0.49	0.15	0.35	0.56	0.06	0.66	0.46	0.12	0.44	0.61	0.57	0.49
$n = 30$	0.48	0.15	0.36	0.55	0.06	0.65	0.45	0.12	0.44	0.61	0.57	0.49
$n = 45$	0.46	0.14	0.37	0.52	0.06	0.64	0.42	0.11	0.44	0.61	0.57	0.48
$n = 60$	0.47	0.14	0.37	0.52	0.06	0.63	0.41	0.11	0.44	0.61	0.57	0.48
$n = 75$	0.49	0.15	0.36	0.50	0.06	0.61	0.40	0.12	0.43	0.61	0.58	0.48
$n = 90$	0.48	0.16	0.37	0.48	0.07	0.61	0.40	0.12	0.45	0.60	0.58	0.48

Ratios for the experiments using the *Random B* price series												
	OTMaxW			OTMinR			OTMix			OTRnd		
	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E	R^C_W	R^C_H	R^H_E
$n = 15$	0.09	0.67	0.43	0.12	0.21	0.80	0.08	0.57	0.49	0.50	0.68	0.44
$n = 30$	0.09	0.62	0.44	0.12	0.20	0.76	0.08	0.53	0.51	0.50	0.67	0.44
$n = 45$	0.09	0.61	0.45	0.12	0.19	0.76	0.09	0.52	0.50	0.50	0.67	0.44
$n = 60$	0.10	0.63	0.45	0.13	0.20	0.78	0.09	0.54	0.50	0.50	0.67	0.44
$n = 75$	0.11	0.63	0.45	0.14	0.20	0.78	0.10	0.54	0.51	0.50	0.67	0.44
$n = 90$	0.11	0.62	0.46	0.15	0.19	0.77	0.11	0.52	0.50	0.51	0.67	0.44

Table C.6: Values of the R^C_W, R^C_H and R^H_E ratios obtained by the Option trading strategies in set of experiments using the the SMA_n forecasting function.

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