Revisiting Ancient Egyptian Mathematics
Implications for Science Studies and Egyptology

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Though their portals and mansions have crumbled and their ka servants are gone; their tombstones are covered with soil, their graves are forgotten. But their names are pronounced because of their writings, which were good. Their names will last for eternity.

The Song of the Harper
An Ancient Egyptian Text

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Abstract

This thesis will revisit the two major mathematical texts from ancient Egypt, the Rhind and Moscow mathematical papyri. This will be done in order to examine how historians of mathematics and Egyptologists commonly view these texts. Egyptian mathematics is commonly held to be limited to applied mathematics that does not have the abstract qualities that have led historians of mathematics to identify the first mathematicians as the Greeks. In order to examine these ideas, this thesis will initially examine some of the most important problems from the Rhind and Moscow mathematical papyri. The features of these problems that have been used to build up the picture of the character of Egyptian mathematics can then be discussed in detail. It will be seen that Egyptian mathematics has many features that are hard to reconcile with the traditional view. This is because most commentators on Egyptian mathematics have a naïve understanding of the philosophy of science. To add to this problem the aims of writing History of Science can be conflicting. Mathematicians have an interest in how theories they are familiar with arose and so write in chronological order. On the other hand, Egyptologists and other historians should be interested in the science and mathematics of the period they are studying for its own sake. This raises a philosophical problem; constructivism is one solution to this problem. This solution is unpopular with scientists and their objections will be explored. The questions which constructivism raises in relation to Egyptian mathematics will be identified and responded to in order that a more productive analysis of Egyptian mathematics can be achieved. This will lead to the conclusion that Egyptian mathematics does not deserve its current reputation and is in need of further investigation by specialist Egyptologists and archaeological theorists.

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Finally, thanks must go to my family: Mum, Dad, Chris and Becky. This is what happens when you teach a small child algebra and π!

List of Full Translations of Problems from the Rhind Mathematical Papyrus

Problem Number	Page
24	99
25	101
31	69 - 71
36	65 - 67
40	55 - 56
41	46 – 47, 120
42	121 - 122
72	51 - 52
79	60 - 62

List of Full Translations of Problems from the Moscow Mathematical Papyrus

Problem Number	Page
4	82 - 83
6	94 - 95
7	84 – 85
10	134
14	109 - 110
17	89 - 91
19	97
23	102
25	104

List of Commentaries on Problems from the Rhind Mathematical Papyrus.

Problem Number	Page
7	38
9	39
11	39
17	41
24	31, 50
25	35, 57
26	29
27	31
30	42
32	30
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49	48
51	30
52	30
53	38
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56	40
58	41
62	53
65	48 - 49
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67	42
69	34, 36
70	37
73	53
74	53
75	53
78	53
79	49
80	44
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Chapter One

Introduction

1.1: Introduction to the Study of the History of Science and Mathematics

The study of history is a complicated one full of potential traps and misconceptions. Even defining what is meant by the term 'history' is fraught with difficulty. It is too simplistic just to state that history is the study of documents from the past. Reading the texts is an active not a passive process. We all bring our own meanings to the text and this problem is compounded when the text is in a language that is now dead.

Nor is history just about storytelling, although narrative can play an important part. It is not sufficient merely to report the significant names and dates from the past; an element of analysis is required in order for the past to have meaning and for us to learn about why events occurred[1]. In the natural sciences, it is not sufficient to observe nature; the scientist attempts to define theories that explain why the universe behaves in certain ways. The same is true in historical research. The historian should not just observe the bare facts; an element of analysis is needed in order for the subject to have any meaning or interest. Without the layer of analysis, the sciences become nothing more than

[1] Green, A. and Troup K. (1999) *The Houses of History: A critical reader in twentieth-century history and theory*; University of Manchester Press; Manchester; pp 204-213.

dispassionate observation. It is only through analysis of the observations that scientists learn to understand the causes of their observations. In this way, scientists can then appreciate the next experiment that is needed in order to investigate the phenomenon further. The act of experimentation is not therefore the most important part of carrying out a scientific exploration; it is the analysis of the results that enables scientists to move on in their field. The same is true of history. It is not sufficient to be a dispassionate observer to the events of history, to give the study any meaning requires the historian to offer explanation.

If historical writing were confined only to the bare facts then it would be a dull subject devoid of anything to stimulate the reader. Explaining the motives behind the events and exploring the cultural, social, economic and political networks of a period is the most difficult part of writing history and is therefore the most interesting. It takes a skilled and knowledgeable historian to be able to turn a series of facts into a document that illuminates the workings of human culture and society.

The history of the sciences and mathematics is not a special case in this respect. It is not adequate simply to chart the discoveries of leading scientists and mathematicians: a chronicle of the important names, dates and places is not historical writing. A linear narrative of scientific discovery does not enable researchers to understand how scientific discoveries are made and the impact of those discoveries on other systems such as politics, economies and even artistic culture. If anything is to be learnt from the history of the sciences then analysis and contextualisation is not only desirable but also vital. Writings about the development of the sciences and mathematics have the ability to inspire modern scientists to make great discoveries. There is a place for books that chart discovery and praise the geniuses of the past. However, this cannot be described as history. Many anecdotal tales are told about the past, including anecdotes about scientists who are long since dead. These anecdotes have become part of our culture but it would be foolish to describe them as historical. History of the sciences has the power to help in the identification of the conditions necessary for the sciences to thrive. This is an important avenue of research yet in order to do this we have to be able to draw parallels across historical periods. This cannot be accomplished if researchers are engaged in only narrating the course of developments. A deeper understanding is required.

In this way, it is similar to other areas of historical study. Yet, there are problems that are peculiar to the study of the history of the sciences and mathematics. Science and mathematics have developed along with progress in epistemology and the philosophical appreciation of science and mathematics.

In a modern society, we have become familiar with disciplinary boundaries. However, many of the boundaries are only used for convenience: it is hard to define where many of them are placed and it is not always possible to remain within them. For example, the subject of physical chemistry is both physics and chemistry, while the subject of genetics needs a good understanding of both biological and chemical systems. There is a growing awareness of the need for inter-disciplinary research. Therefore, even within a modern academic system that is familiar, it is hard to classify some individual pieces of research. This problem can only be made more complicated when studying historical texts. The further back in time travelled, the more dissimilar the mode of investigation and the setting of the investigation. This can lead to problems when discussing the nature of the work in the text. It is too easy to allow our modern judgments about the nature of science and the desirable qualities of that science to interfere with an objective analysis of it. Not only is it difficult to identify accurately the boundaries between the elements of science, it is also difficult to draw boundaries between science and technology or science and engineering. One area which is common to all is mathematics. This investigation of ancient Egyptian mathematics will show how modern conceptual problems of definition have influenced our attitude to the science and mathematics of the past. It will also show how a slightly different approach to the study of scientific and mathematical texts of the past can radically alter our opinion of those texts.

This thesis will examine the problems of the identification of texts for study. It will discuss how our definitions of science have been allowed to interfere with the production of an inclusive holistic picture of the development of the sciences and mathematics. It will examine the approach taken by many historians of science and mathematics and the linear narrative style adopted by large numbers of them. This linear narrative style only charts the development of mathematical ideas, rather than analysing the nature of science and mathematics and the meaning of scientific and mathematical texts from the past within their own context.

In modern academic culture, we have come to prize science and mathematics because of their abstract nature and the apparent objectivity of the method of enquiry. One of the strengths of science is that statements can be made with a level of certainty that cannot be achieved in the arts and humanities subjects.

Lewis Wolpert has even gone as far as calling science 'the defining feature of our age'. He proposes that science has the power to change our lives in a way that no other subject can[2]. This power is a worthy subject of study and there are many ways in which this can be done, some more popular with scientists than others are.

Philosophy of science, sociology and history can all contribute to our insight into the methods of science and how it interacts with society and culture in the wider sense. However, the results of these studies are hard to integrate as each has its own point of view from which to study the problem. The way in which ancient Egyptian mathematics has been studied will be investigated in order to show how philosophy of science, sociology and history affect our understanding of the Egyptian mathematical achievement. This thesis aims to investigate these areas and try to combine the ideas obtained from each. Ancient Egyptian mathematics is a good example to use to identify the most salient points of the nature of the sciences and mathematics because there are so many factors that affect the modern reader's interpretation of the texts. The extreme age of the texts is one of their most important factors. The texts come from a time that is much removed from our own in space, religion and philosophy. The way in which they are written about has the ability to highlight many of the problems that we have when trying to write a history of science or mathematics.

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[2] Wolpert, L. (1992) *The Unnatural Nature of Science*; Faber and Faber; London; p. ix.

1.2: Ancient Egyptian Mathematics

This thesis aims to study the problems of writing history of mathematics with special reference to the problems it causes to our understanding of the mathematics of ancient Egypt. This thesis will contain some of the more interesting problems of ancient Egyptian mathematics in order to give the reader a flavour of the nature and language of the extant texts. This thesis is not intended to be a complete survey of the contents of the papyri as this is unfeasible in a work of this size and type. The texts have been published in full in other volumes and the reader is advised to consult these for a complete edition of the texts[3]. The Egyptian mathematical material will be considered in order to investigate how our current method of studying and writing about the history of mathematics can colour our understanding of the mathematical product of another culture. The comments on the character of ancient Egyptian mathematics will be studied in order to identify the reasons why Egyptian mathematics has largely been ignored. This thesis will consider the comments of the earlier Egyptologists, such as Peet, who produced translations, as well as more modern editions such as Couchoud, Clagett and Imhausen. A comparison between the different editions will highlight how the ideas about Egyptian mathematics have changed, and also the changes in the histographical approach taken by the different researchers.

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[3] For the Moscow Mathematical Papyrus see: Struve W.W. (1930) *Mathematisher Papyrus des Staatlichen Museums der Schönen Künste in Moskau, QSGM,* Abt. A: Quellen; Berlin.
For the Rhind Mathematical Papyrus see: Peet, T.E. (1923) *The Rhind Mathematical Papyrus: British Museum 10057 and 10058;* University of Liverpool Press; London.
Facsimiles of the Struve edition of the Moscow Papyrus and an edition of the Rhind Papyrus by Chace, Manning and Archibald (Chace, Manning and Archibald (1927) *The Rhind Mathematical Papyrus*; Vol.1; Oberlin; Ohio.) can be found along with facsimiles of other Egyptian mathematical texts and their translations in: Clagett, M. (1999) *Ancient Egyptian Science: A Source Book Vol. 3; Ancient Egyptian Mathematics*; Memoirs of the American Philosophical Society vol. 232; Philadelphia.

The comparisons made between Greek mathematics and earlier mathematical traditions will also be considered. These comparisons are extremely illuminating as they draw out the reasoning behind the generally high opinion of Greek mathematics when it is compared to the earlier mathematical traditions. This contrast between the Greek and the Egyptian mathematical traditions allows the study of the opinions formed about both traditions simultaneously. This will make clear the reasoning behind the negative comments made about ancient Egyptian mathematics.

Chapter Eight will look at how our understanding of Egyptian mathematics can be improved when we recognise how philosophy can distort our appreciation of the achievements of the ancient Egyptians. The final section in Chapter Eight will offer some suggestions for the furtherance of the study of ancient Egyptian mathematics. This section will offer observations on the problems of the reluctance or unwillingness of many in the Egyptological community to study of Egyptian mathematics because it is considered to be hard. The problems of science communication and the popular image of mathematics and the sciences will be considered in order to offer proposals for the integration of the study of Egyptian mathematics and science into a wider Egyptological context.

1.3: Philosophy of Science and Its Relationship To Historiography

This work will examine how modern philosophy of science has produced definitions of what science is that are now used retrospectively to appraise the science of the past. Through studying the works of the philosophers Thomas Kuhn, Karl Popper and Imre Lakatos and more recent commentaries, this thesis will examine the problems of definition and demarcation.

The demarcation question was of special interest to Popper and Lakatos. They believed that it was possible to define a demarcation criterion or criteria that would be able to distinguish a scientific subject from a non-scientific or pseudo-scientific subject. Popper and Lakatos were both very influential thinkers and their ideas are still important. Their ideas are still taught to science research students so the students can learn to identify good practice in scientific research and make the students think about how they design their research and experiments to gain maximum benefit. The ideas of Popper and Lakatos are also very influential on how scientists perceive their work and the position that work has in the context of academia and the wider context of society and culture. As many researchers in the history of the sciences are trained as scientists, the work of such philosophers is therefore very influential on the approach and methodologies utilised in the history of the sciences.

Kuhn is most famous for what may be the last macro-history of science - *The Structure of Scientific Revolutions*[4]. His ideas about the way in which science advances through periods of normal science followed by paradigm shift has also found its way into the consciousness of the scientific community. Kuhn appears in the same company as Popper and Lakatos in the philosophy of science but his work, especially *The Structure of Scientific Revolutions*, deals with a different set of issues. Kuhn is more interested in studying the process by which science advances. This is therefore an extremely important text for anyone wishing to study the history of the sciences. Kuhn's work has also been identified as the

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[4] Kuhn, T. (1962) *The Structure of Scientific Revolutions*; University of Chicago Press.

'harbinger of the constructivist movement'[5]. Constructivism is an exceptionally important idea in the current historiography of science and constructivism will be looked at in detail in a later chapter. However, before constructivism can be fully appreciated it will be necessary to study the ideas of Kuhn and how his macro-history of science has come to be one of the most frequently quoted idea in scientific epistemology.

1.3.1: Constructivism

Recently there has been a lot of interest in the idea of constructivism in the history of science. Constructivism proposes that science is influenced by culture. The extent of this influence is a matter of opinion and various types of constructivism can be identified. Different authors also have different attitudes towards whether constructivism necessitates an anti-realist stance about science. This idea will be discussed and the importance of this approach not only to the understanding of history of science and mathematics itself, but also to the integration of the history of science into intellectual history and the much broader historical picture.

The idea that the people engaged in doing science are affected by their culture also opens up the question of objectivity in science. It is the apparent objectivity of the subject that gives it a claim to be able to find the Truth. Objectivity is one of the most highly prized virtues of any piece of academic research and it is because science seems to have the claim to be the most objective of all the academic disciplines that it maintains the highly regarded position that it currently enjoys. Anything that brings this idea into question is seen as an attack

[5] Golinski, J. (1998) *Making Natural Knowledge: Constructivism and the history of science*; Cambridge University Press; Cambridge; p. 13.

upon science. This thesis will examine whether constructivism does mean that objectivity is unobtainable and whether this implies an attack on science.

1.3.2: Science Wars

The study of the sciences from a humanities point of view can be informative. Unfortunately, some of the members of the science community have taken the idea of a cultural aspect of science as a criticism of it. Some researchers engaged in this aspect of research have been branded as anti-science. They have also been accused of being ignorant of science, the implication being that they do not understand the nature of the issues that they are studying. There is also a charge that the appreciation of science as a cultural phenomenon will undermine science teaching and science communication. The main proponents of this attack are Paul Gross and Norman Levitt. The final section of this thesis will explore the charges they make as they are outlined in their book, *Higher Superstition*[6].

This thesis will examine whether supposing that there is a cultural aspect to science does in fact undermine science, science teaching and science communication, or whether in fact accepting and understanding the cultural aspect has the power to assist in a greater appreciation of science and mathematics.

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[6] Gross, P and Levitt, N. (1998) *Higher superstition: The academic left and its quarrels with science.* John Hopkins University Press, Baltimore.

1.4: Thesis Structure and Content

This thesis will begin by examining material from ancient Egyptian mathematical papyri. This material will constitute the core of the thesis. In later chapters it is this material that will serve as a type example of the difficulties that occur when studying the history of mathematics and science. However, this thesis is primarily concerned with the way in which this material has been studied and what this can tell us about the practice of the history of science and mathematics, rather than the mathematical achievements of the ancient Egyptians.

Chapters Two, Three and Four will not, therefore, be a complete investigation of Egyptian mathematical texts. Instead, they will highlight the important features of ancient Egyptian mathematics needed to gain a good general understanding of the nature of ancient Egyptian mathematics. They will particularly look at those features that appear most often in the work of historians of mathematics of both specialist books on ancient Egyptian mathematics and general histories of mathematics.

Chapter Two will look at the basic procedures of addition, subtraction, multiplication and division. These arithmetical operations will serve as an introduction to the way in which ancient Egyptian mathematical texts are composed and laid out. An understanding of these operations will also assist the reader in understanding some of the more complicated problems from Egyptian mathematical texts that are explored in later chapters. This chapter will also survey the employment of unit fractions, as this is one of the most unfamiliar techniques used in Egyptian mathematics to a modern reader and one that appears in many problems from the ancient Egyptian texts.

Chapter Three will begin to look at some of the more advanced problems contained in the Egyptian mathematical texts. These will include geometrical problems and also problems known as aha problems, that work out the value of an unknown heap (*ʿḥʿ* in hieroglyphs) that are similar to algebraic problems.

Chapter Four is devoted to the two most interesting problems to have survived from ancient Egypt, Problems 10 and 14 of the Moscow Mathematical Papyrus. These problems are dealt with in a separate chapter to allow for a more thorough investigation. Problem 10 is especially interesting and it is also the problem that has attracted the most discussion about its meaning. It is to be hoped that this thesis will present the most comprehensive investigation of this problem and that it will offer a reasonably conclusive argument for the adoption of the translation that it presents. Problem 14 attracts a fuller discussion because of what the method of the solution reveals about the nature of ancient Egyptian mathematics.

Once the mathematical achievements of the Egyptians have been explored, the way in which these achievements have been studied will be reviewed; this will make up the content of Chapter Five. This review will be achieved through a study of each of the major authors of specialist books on Egyptian mathematics from the early works by Peet to the most up to date works by authors such as Imhausen and Couchoud. Authors of major chronological histories of mathematics will also be considered to see how Egyptian mathematics is perceived to fit into a general history of mathematics. There is also an increasing number of popular books on the history of mathematics that adopt a thematic, rather than a chronological approach to the history of mathematics. These will be analysed as they often have interesting observations to make and they illuminate unique aspects of the nature of history of mathematics and science.

This survey of the way in which Egyptian mathematics has been written about will lead into a discussion about the need for a re-examination of the way in which history of science and mathematics is studied and its place within modern academic culture. This discussion will examine how the picture we have about the achievements of ancient Egyptian mathematics falls short of a complete understanding of the material within the context of the study of Egyptology.

Chapter Six will begin an investigation into the philosophical context of work in the history of mathematics and science. Whilst investigating the work of some of the major philosophers of science and mathematics it will be made clear how this work affects presumptions made about science and mathematics in the study of its history and in turn how this might distort the picture gained about the mathematical and scientific achievements of cultures of the past.

This examination of the philosophical context of work in the history of the sciences will continue into Chapter Seven. Chapter Seven will be particularly concerned about the ideas of Constructivism and considerations that emerge from a Constructivist approach to the history of science. The main aim of this chapter is to consider how an understanding of the mathematics and science produced by an ancient civilisation can be incorporated into the wider understanding. This chapter will then also explore the relationship between science and technology as the boundary between science and technology is not necessarily as clear in an ancient civilisation as it is in a modern academic culture. Therefore, assumptions that are made about the difference may obscure the true nature of the science or mathematics of that culture.

Once the problems of writing history of mathematics and science have been investigated and solutions to these problems suggested, this thesis will return to the specific problems of ancient Egyptian mathematics. Chapter Eight will consist of a commentary on the character and nature of ancient Egyptian mathematics based on the discussions of the previous chapter. This chapter will refer back, when necessary, to the problems from the Egyptian texts contained in Chapters Two, Three and Four. Chapter Eight will look at how our understanding of Egyptian mathematics can be improved when we recognise how philosophy can distort our appreciation of the achievements of the ancient Egyptians and will offer some suggestions for the furtherance of the study of ancient Egyptian mathematics. This chapter will offer suggestions for dealing with the unwillingness of Egyptologists to study mathematics and science.

Chapter Nine will consider some of the implications of this thesis for science studies in general. This thesis will raise issues about the nature of science that are relevant in current debates about science, science communication and public engagement with science. Whilst these issues do not relate directly to the ancient Egyptian material, they are important because the process of studying the ancient Egyptian material and examining the philosophical context of books written on the subject will necessarily highlight the prejudices for and against science that are causing many difficulties currently. It was felt that these aspects should therefore be included so that the reader can consider the wider implications of this thesis.

1.5: Notes on the Presentation of Problems from the Egyptian Mathematical Texts

The chapters covering the Egyptian mathematical texts will largely consist of translations prepared by the author of selected problems from the Rhind and Moscow Mathematical Papyri. These two texts are the most complete texts to have survived, both in terms of their coverage of the range of mathematical material and their physical condition.

Each problem will be translated on a separate page. Because some of the techniques are unfamiliar to a modern reader, it is helpful for each of the problems to be presented separately so that the problems can be easily seen and followed. Whilst this may make the material appear fragmentary, the benefits for ease of comprehension outweigh this difficulty. It is intended that the reader should be made familiar with the techniques employed so that he or she will be able to follow the analysis of the material that will appear in later chapters. The translations will present a transliteration of the hieroglyphic text maintaining the original lines. The transliteration is the first step in producing a translation of the hieroglyphics, this transliteration provides a phonetic equivalent of the hieroglyphs and will also separate out each of the words.

Any diagrams that are part of the original texts will be presented after the translation, on a separate page if necessary. Where the accompanying diagrams contain hieroglyphic numbers, these will be presented at the top of the page, with a translation of the numbers presented on a copy of the drawing at the bottom of the page.

Arithmetical workings will be presented in a format as close to the original format as possible. Where substantial changes have had to be made either to elucidate the problem to a modern reader, or for reasons of physically fitting the problem on an A4 page, these will be made clear in a footnote.

A commentary will follow each translated problem in a separately headed section. These commentaries will describe the methods employed by the Egyptian scribe and provide explanations employing modern mathematical notation where appropriate. These explanations have been kept separate from the translated problems in order that the reader should gain skill in deciphering the problems. The methods employed need to be understood for their own worth. If the reader is too reliant on the modern notation then this will act as a barrier between the reader and the material. The reader may find it useful to have a pen and paper on hand so that their understanding of the problem can be checked before reading the commentary provided.

Some problems are included because they employ vocabulary that appears in other problems of a more enigmatic nature. A correlation between the instances of one particular word can assist in deciphering its exact meaning by comparing the contexts in which they appear. Where problems have been included for this reason this will be highlighted in the commentary and a cross-reference to other problems will be included in a footnote appended next to the word in question.

In some cases, there are rival interpretations of the problems; where these are significant they will follow the translation prepared by the author and any judgments made by the author will be explained. In the case of a few of the geometrical problems from the Moscow Mathematical Papyrus there are arguments over the hieroglyphic transcription from the original hieratic text, either because of gaps in the papyrus, or because of the notoriously bad handwriting of the scribe who prepared it. Where these arguments arise, the translation will be produced from the original full publication of the Moscow Mathematical Papyrus prepared by Struve, as this volume contains a complete facsimile of the Moscow Mathematical Papyrus and a complete transcription into hieroglyphs. The differences in the transcription will be discussed in a separate section after the translation and commentary of the appropriate problems. Any changes that the differences may make to the meaning of the text will be explained in this section.

The Egyptians had a system of expressing fractions that is largely unfamiliar. Except for two-thirds, fractions were expressed as unit fractions, these are fractions with a numerator of 1. In this thesis, fractions are shown as the numerator with a line over the top. A half is thus shown as $\bar{2}$, a quarter as $\bar{4}$ etc. Two-thirds will be shown as a three with two lines over the top: $\bar{\bar{3}}$. A further explanation of unit fractions will be given in Section 2.4.

Parts of some of the mathematical problems were written in red ink. Where this occurs any transliterations will be underlined and the translation will be given in bold. This follows standard procedure for the publication of mathematical texts from ancient Egypt.

Chapter 2

Arithmetical Procedures

This chapter will consider arithmetical procedures from ancient Egyptian mathematics. In addition to exploring the ways in which the Egyptians performed arithmetic procedures, it is also intended to familiarise the reader with the methods employed, as an understanding of these methods is useful when reading later chapters. The texts indicate that the Egyptians had a considerable proficiency in mental arithmetic. The features of Egyptian arithmetic that facilitate mental arithmetic will be explored in detail.

2.1: Introduction

Section 2.2 looks at addition in the mathematical texts and the suggestion that the Egyptians needed an addition table to complete their arithmetic. The aim of this chapter is to show the flexibility of the arithmetical procedures and their use in mental arithmetic. It is hoped that by the end of this chapter the reader will have no doubt whether the scribes that prepared the mathematical texts needed this table.

Section 2.3 explores the different methods of multiplication contained in the Rhind Mathematical Papyrus. This is intended to be a complete and definitive list of the techniques used. Each example of multiplication contained in the

Rhind Mathematical Papyrus has been identified and categorised. This is to demonstrate how the method of multiplication was manipulated to suit the context of the problem. In some problems multiplications were carried out with great attention to detail, in other problems the scribe who prepared the text is more interested in demonstrating a geometrical method so the arithmetic is kept simple.

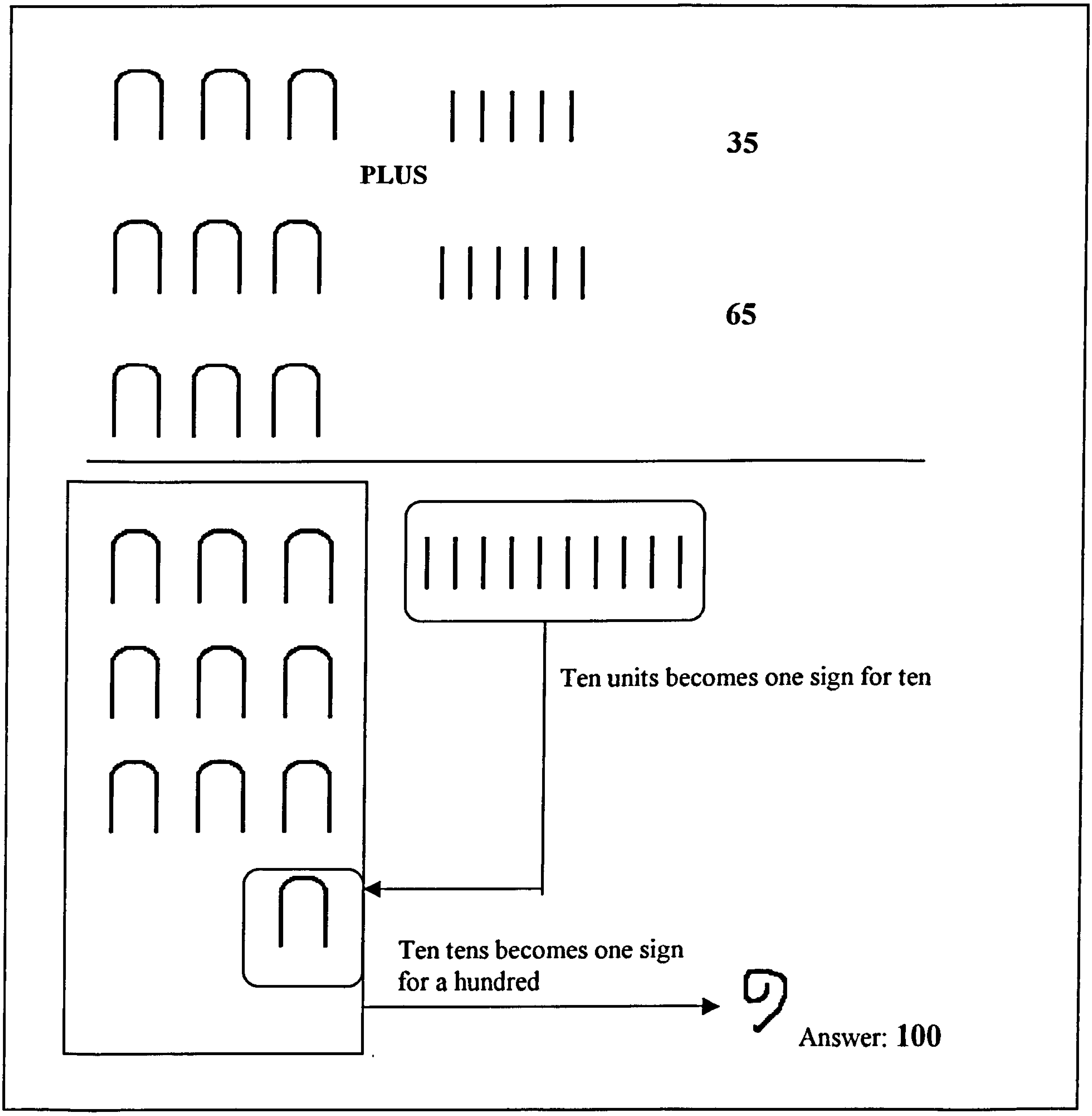
Section 2.4 investigates the techniques employed in division. While the procedures are similar to those employed in multiplication, as can be expected, there are several features that show how the ideas are adapted to suit the needs of the scribe.

The final section of this chapter will discuss the use of unit fractions in ancient Egypt. Most commentators on the use of fractions in Egypt find their methods clumsy and inferior to the method of writing fractions that we use today. However, through the detailed investigation of a group of three problems from the Rhind Mathematical Papyrus, it will be shown that the scribe, through considerable practice, was skilful in their use and this will highlight some of the advantages of unit fractions. Far from being a hindrance, unit fractions can be an asset in practical mathematics.

2.2: Addition and Subtraction

The method for working out the result of an addition or subtraction is not explained in any of the extant mathematical problems; the answer is simply given below or to the side of the numbers.

Some writers of history of mathematics books[1] dismiss the process of addition as a particularly simple process in Egyptian mathematics because of their number system. In hieroglyphs, addition is a very simple process, as the hieroglyphs only need to be rearranged and then groups of ten converted into one symbol of the next power of ten. For example:



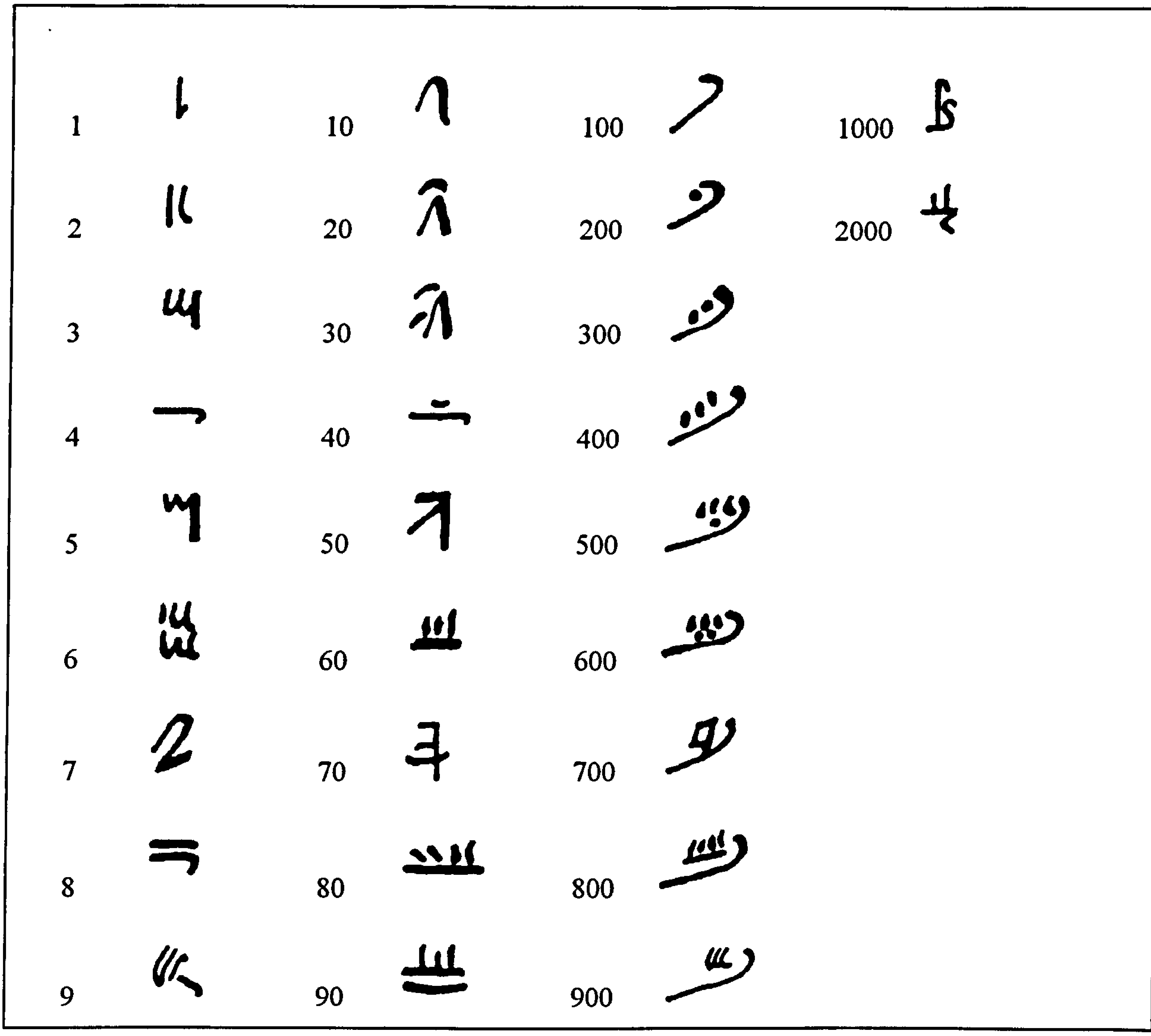
[1] See, for example, Teresi, D (2002) *Lost Discoveries. The Ancient Roots of Modern Science - from the Babylonians to the Maya;* Simon & Schuster, New York; pp.39-40. See also Peet, T.E. (1923) *The Rhind Mathematical Papyrus British Museum 10057 and 10058*; University of Liverpool Press; p.12. and Neugebauer, O. (1952) *The Exact Sciences in Antiquity*; Princeton University Press; Princeton, New Jersey; p. 73.

However, this process only works if the addition sum is written in hieroglyphic notation, whereas the mathematical texts were written in hieratic. In hieratic the process does not work, as there are different signs for each of the units, multiples of ten, multiples of a hundred etc.

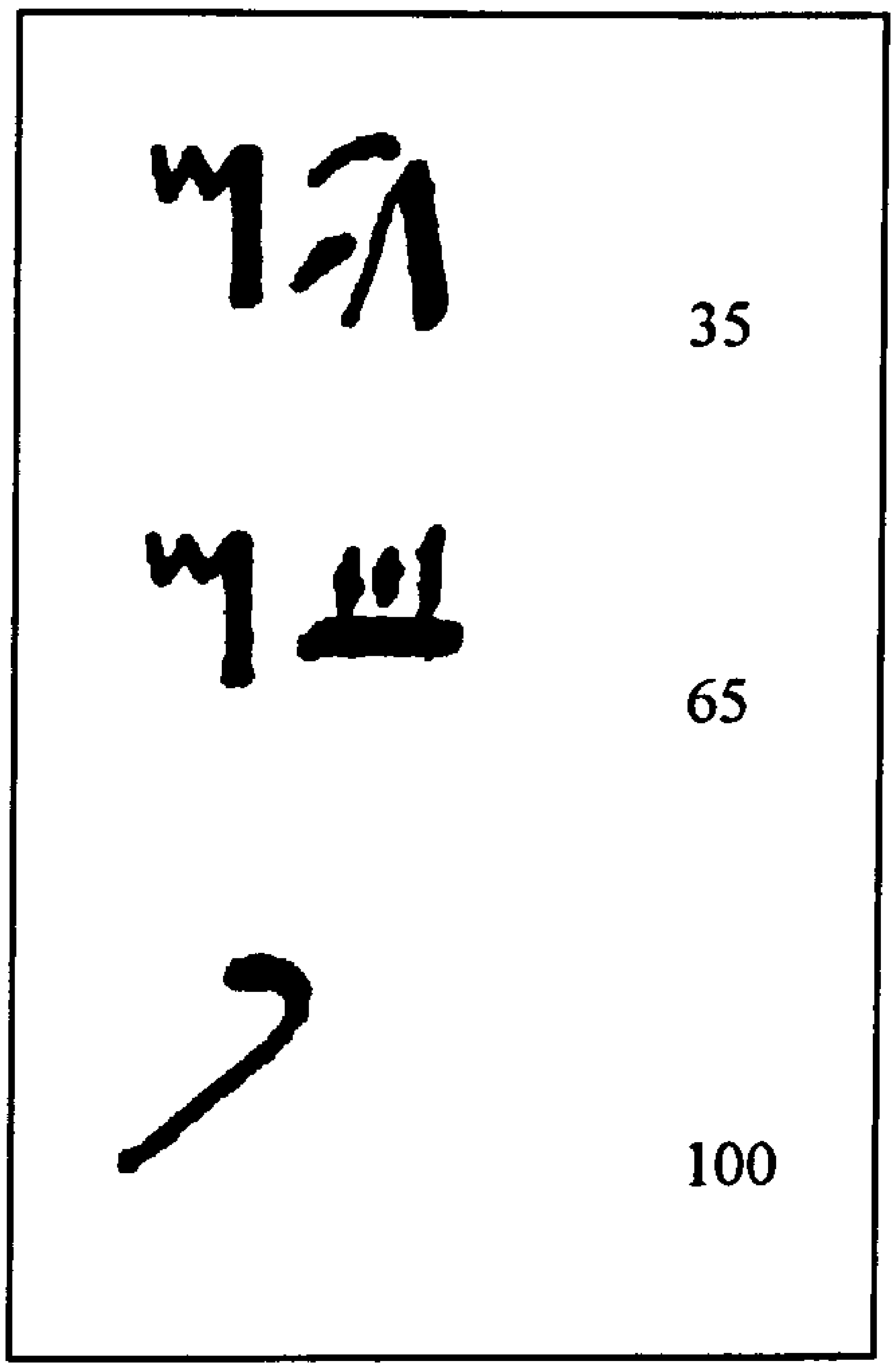
In this case, addition requires the mathematician to be aware of number combinations and to be able to work out the sum in another way.

The hieratic number system works as a cipher. Instead of using the same sign several times to show a multiple of units or tens, the scribe would use a separate sign for different multiples of units, tens, hundreds, thousands etc. The numerals looked like this[2]:

[2] Numerals copied from Gillings, R. (1972); *Mathematics in The Time Of The Pharoahs*; MIT Press, Cambridge, Mass.; p. 257-8



Whilst this system is not a place-value system, addition could not be performed by collecting together symbols and then converting them. The scribe would need to know number combinations. The addition completed above in hieroglyphic numerals would appear thus in hieratic:



Additions are usually performed with no explanation in the text. This means that they were either performed mentally or on another piece of papyrus or on an ostracon. Given that Egyptians tend to write out in full the method for completing a multiplication sum, it seems that it is not unreasonable to presume that additions were carried out mentally. It is a very pessimistic interpretation of the texts to consider the idea that the mathematician might have had to write out an addition sum.

2.2.1: Gillings' Addition Table.

In his book, *Mathematics in the Time of the Pharaohs* Gillings[3] asks whether the Egyptian mathematicians worked out an additions table like they had worked out a 2/n table in the Rhind papyrus and a list of fractional equalities in the Egyptian

[3] Gillings, R. (1972) *Mathematics in the Time of the Pharaohs;* Cambridge Massachusetts; MIT Press.

Mathematical Leather Roll. He gives a possible table on page 13 of his book. Its translation into Arabic numbers looks like this:

2 9 11	2 8 10	2 7 9	2 6 8	2 5 7	2 4 6	2 3 5
3 9 12	3 8 11	3 7 10	3 6 9	3 5 8	3 4 7	
4 9 13	4 8 12	4 7 11	4 6 10	4 5 9		
5 9 14	5 8 13	5 7 12	5 6 11			
6 9 15	6 8 14	6 7 13				
7 9 16	7 8 15					
8 9 17						

He does not go as far as to suggest the Egyptians did not know what one plus one equals, but he comes close. Gillings suggests that modern children learn addition of simple numbers by a "look and say" method, so that number combinations are learned by heart, that children do not perform simple additions by counting. In much the same way we add words to our vocabulary by looking them up so many times that we eventually remember, this is how we learn simple number combinations. Is Gillings suggesting that the ancient Egyptians did not have the intellect of a modern child? He writes that hieratic numbers do not lend themselves to counting. This is a very curious statement. In what way are the hieratic symbols any different from our own? How are they different from hieroglyphic numbers? It is true that modern mathematics uses a place value system that mean only ten symbols need to be learnt. However, hieratic is a cursive form of hieroglyphic numbers. The importance of learning number combinations that add to ten is the same in both numeral systems. We must assume that the Egyptians had words for three and four, and could say them. It does not matter anyway; if you can hold up three fingers on one hand and four on the other you do not need to be able to count in hieratic to see that you are holding up seven fingers. Even when we get beyond sums with an answer of ten or less we can still work them out in our heads. It is only when we are dealing with very big numbers that we need to write them down, but even then we do not require addition tables, we can just work out the sum of the units, the tens, the hundreds and so on until we have completed the sum. Is there any reason to believe that the Egyptians did not have this mental capacity?

2.3: Multiplication

The Rhind Mathematical Papyrus is our best source for examples of Egyptian multiplication technique. In the Moscow Mathematical Papyrus, there are no examples of multiplication sums with the working shown. In the Moscow Mathematical Papyrus, the workings of a problem are not shown beneath a narrative version of the problem, as is the case in the Rhind Mathematical Papyrus. Conversely, the narrative versions of the problems are all that appear and contain in them multiplications where the result is given without any apparent recourse to working. In MMP 11, for example, it is necessary to square 5. The result, 25, is given without any further explanation. It is possible that the scribe used another piece of papyrus or a broken piece of pottery to work out the correct answer before inserting this in the papyrus. This is pure speculation though and does not assist us in an understanding of how the Egyptians performed arithmetical procedures. This section will therefore concentrate on the Rhind Mathematical papyrus and look in detail at how multiplications are performed, and how they are written in the text.

There are surprisingly few examples of multiplication sums whose working is shown in the extant mathematical texts. It is therefore possible to look in detail at all the examples that come from the main source of the evidence, the Rhind Mathematical Papyrus. What follows is an attempt to classify these examples. Whilst it is understood that all the examples of multiplication that are included here are all superficially the same, there are distinct groups that have some correlation to the type of problem that they are taken from. These differences show that Egyptian multiplication technique can adapt to the problem at hand and is not a rigid way of performing the arithmetic.

Multiplication Type	Number of Examples in Rhind Mathematical Papyrus	RMP Problem Numbers
A Multiplication by Repeat Doubling, with No Fractions	6	26, 32, 41, 48, 50, 52,
B Multiplication by Repeat Doubling of Fractions	7	24, 25, 27, 43, 54, 55, 69
C Multiplication by Repeat Doubling and Fractions	5	35, 42, 53, 69, 70
D Multiplication of Fractions. No Doubling	19	7-20, 30, 35, 56, 58, 67
E Multiplication of Fractions by Repeat Halving	2	80, 81
F Other Methods	8	41- 46, 49, 79
G Trivial Multiplication in ʿḥʿ Problems	7	24, 25, 27, 35, 36, 37, 38
H Multiplication with No Working Shown	9	40, 61, 62, 72, 73, 74, 75, 78, 82

Table 2.1 Multiplication Types in the Rhind Mathematical Papyrus

Type A - Multiplication by Repeat Doubling, with No Fractions

Multiplications were sometimes worked out by repeated doubling. They could be written in two columns, one column showing the multiplicator, the other the multiplicand. For example, in RMP 41 it is necessary to work out 8 multiplied by 8[4]. The working could appear as follows[5]:

1	8
2	16
4	32
/8	64

The first row of the right-hand column shows one of the numbers to be multiplied. The left-hand column[6] shows the multiplier of that number. Row one for example shows that 1 lot of 8 is 8, the second shows that 2 lots of 8 equals 16 and so on. The numbers in both columns are doubled from one row to the next. This process is continued until the answer required is reached, that of 8 multiplied by 8. The tick to the left of the 8 in the left-hand column shows that this is the correct multiplier reached.

These ticks become important when it is necessary to perform a multiplication where neither of the numbers to be multiplied is a power of 2. In these cases it becomes necessary to add up the numbers in some of the columns. For example,

[4] 8 multiplied by 8 also appears in RMP 48 and RMP 50. The method is the same so these problems are not shown here.
[5] In the Rhind Mathematical Papyrus itself, this sum appears over two columns to save on space. They have been placed in one column here to assist the reader.
[6] The Rhind Mathematical Papyrus was written from right to left so in fact the numbers were reversed. However, for the assistance of the reader multiplications will always be shown with the multiplier in the left-hand column and the multiplicand in the right-hand column. This convention follows all the major publications of the Rhind Mathematical Papyrus.

RMP 48 requires the scribe to calculate 9 multiplied by 9. The working could appear as follows[7]:

\1	9
2	18
4	36
\8	72
Total	81

The total is reached by adding the numbers in the right-hand columns of ticked rows. The two ticked numbers in the left-hand columns add up to the required 9.

There is also an of example in RMP 26, showing the multiplication of 3 by 4:

1	3
2	6
\4	12

The last example of repeat doubling that does not involve fractions is found in part of RMP 32. In one part of the problem it is necessary to find the square of 12. The working appears in the problems thus:

[7] In this case, the numbers do appear in a single column.

1	12
2	24
\4	48
\8	96
Total	144

Apart from these six examples, there are very few other examples of repeated doubling in the extant mathematical texts that do not involve fractions. One of these examples is RMP 79, which deals with a geometric progression. A full translation is given in Section 2.4.

RMP 52 is a problem dealing with reckoning a truncated triangular area of land, or a trapezium. Although the text of the problem and the subsequent working is muddled, the scribe works out 2000 times 5. The working appears as[8]:

\1	2000		
2	4000		
\4	8000	Total	10000

Multiplication by two has been omitted from this type, as they cannot be considered to show a repeat doubling as only one has occurred. An example of this can be found in RMP 51. In this case, 1000 is doubled to give 2000.

[8] This is the same layout as appears in the Rhind Mathematical Papyrus.

Type B - Multiplication by Repeat Doubling of Fractions[9]

There are several examples of multiplication of fractions by the repeat doubling method. The method for this multiplication is unchanged to those above. The only difference is that the numbers in the right-hand column contain fractions. These examples appear in RMP 24, 27 and 54. RMP 24 is one of the ꜥḥꜥ problems[10]. In the final step of the problem, it is necessary to multiply $2\ \bar{4}\ \bar{8}$ by 7. This is achieved as follows[11]:

\1	$2\ \bar{4}\ \bar{8}$
\2	$4\ \bar{2}\ \bar{4}$
\4	$9\ \bar{2}$

RMP 27 is also an ꜥḥꜥ problem. Again, it is the final step in the problem that requires multiplication. In this case, it is 5 multiplied by $3\ \bar{2}$. It is carried out as follows:

\1	$3\ \bar{2}$
2	7
\4	14

The quantity is 17 $\bar{2}$

[9] For an explanation of the notation used for Egyptian Fractions, see Chapter 1.
[10] For an explanation of ꜥḥꜥ problems see Chapter 3.
[11] The total of this multiplication, $16\ \bar{2}\ \bar{4}$, is not explicitly stated. Instead, the answer is contained in the next step of the problem.

In RMP 43, it is necessary to work out 113 $\bar{\bar{3}}$ $\bar{9}$ by 4. The working appears as follows:

1	113 $\bar{\bar{3}}$ $\bar{9}$
2	227 $\bar{2}$ $\overline{18}$
\4	455 $\bar{9}$

It should be noted that in performing the doubling from row one to row two, the scribe has had to work out a unit fraction identity. In this example each term in the first row doubles as follows:

Term:	113	$\bar{\bar{3}}$	$\bar{9}$
Doubled:	226	$1 + \bar{3}$	$\bar{6} + \overline{18}$
Combined[12]:	227 $\bar{2}$ $\overline{18}$		

This technique will be explored later in this chapter in a section on unit fractions.

Another example of this type of multiplication is in RMP 54. This problem uses setats, which were a unit of area equal to a square khet. Since khet was a unit of length equal to 100 cubits, a setat was therefore equal to 10,000 square cubits. Cubit-strips were a long rectangle 1 khet by 1 cubit, so 100 cubit strips made one setat[13]. The problem involves dividing 7 setat into ten fields of equal area. The final stage is a demonstration that the correct figure has been reached. The working appears as[14]:

[12] $\bar{3} + \bar{6} = \bar{2}$

[13] Gillings, R. (1972) *Mathematics in the Time of the Pharaohs*; MIT Press; Cambridge, Mass.; p. 209-210.

[14] The total of 7 setat is not stated.

1	$\bar{2}\ \bar{8}$	setat	$7\ \bar{2}$	cubit-strips
\2	$1\ \bar{4}\ \bar{8}$	setat	$2\ \bar{2}$	cubit-strips
4	$2\ \bar{2}\ \bar{4}$	setat	5	cubit-strips
\8	$5\ \bar{2}$	setat	10	cubit-strips

It should be noted that the scribe performed the change in units without any further explanation. The doubling from row one to row two requires a change from cubit-strips to setat. Double a half of a setat and an eighth of a setat gives one and a quarter setat, the first two terms of the second row. A half setat is equal to fifty cubit strips, so a quarter is equal to 25 cubit strips and an eighth of a setat is equal to twelve and a half cubit strips. Doubling the $7\ \bar{2}$ cubit-strips from row one gives 15 cubit strips. This is equal to an eighth of a setat, with two and a half cubit-strips left over. Although all of these quantities could be expressed as a fraction of a setat, the scribe has chosen to only use unit fractions with a small, even denominator. The reasons why this makes practical sense will be explored later in this chapter. It can be noted though, that the scribe's choice of fractions makes the doubling from one row to another as simple as possible. This choice of fractions also makes the final step of adding the ticked rows together simple.

RMP 55 is another example of multiplication that mixes setats with cubit-strips. Again, it employs only repeat doubling to work out 5 multiplied by $\bar{2}$ setat and 10 cubit-strips. The working appears as follows:

\1	$\overline{2}$	setat	10	cubit-strips
2	1 $\overline{8}$	setat	7 $\overline{2}$	cubit-strips
\4	2 $\overline{4}$ $\overline{8}$	setat	2 $\overline{2}$	cubit-strips

As in RMP 54, the scribe converts the number of cubit-strips into a fraction of a setat only when this assists with the computations for the reasons explained above.

In RMP 69, there is another example of repeat doubling which also involves a mixture of units. In this case, the units are hekat and ro, which are units of dry measure of grain. There were 320 ro in a hekat so a sixty-fourth of a hekat is equal to 5 ro[15]. The working appears thus[16, 17]:

1	$\overline{\mathit{32}}$	[hekat]	4 ro
2	$\overline{\mathit{16}}$ $\overline{\mathit{64}}$	[hekat]	3 ro
4	$\overline{\mathit{8}}$ $\overline{\mathit{32}}$ $\overline{\mathit{64}}$	[hekat]	1 ro
8	$\overline{\mathit{4}}$ $\overline{\mathit{16}}$ $\overline{\mathit{32}}$	[hekat]	2 ro
\16	$\overline{\mathit{2}}$ $\overline{\mathit{8}}$ $\overline{\mathit{16}}$	[hekat]	4 ro
32	1 $\overline{\mathit{4}}$ $\overline{\mathit{8}}$ $\overline{\mathit{64}}$	[hekat]	3 ro
\64	2 $\overline{\mathit{2}}$ $\overline{\mathit{4}}$ $\overline{\mathit{32}}$ $\overline{\mathit{64}}$	[hekat]	1 ro

The result is 3 $\overline{\mathit{2}}$ hekat

[15] Gillings, R. (1972) *Mathematics in the Time of the Pharaohs*; MIT Press; Cambridge, Mass.; p. 210.
[16] The working actually appears as four columns as the rows for 32 times and 64 times were written beside the original two columns. For the assistance of the reader, here they have been shown in two columns only.
[17] Horus-eye fractions are shown in italic. For a further explanation of Horus-eye fractions see Section 2.6.1.

Horus-eye fractions can be used easily in this example because it is easy to convert ro into Horus-eye fractions of a hekat because there are 320 ro to a hekat. Horus-eye fractions have the advantage that it is simple to keep doubling them.

There is one further example, that of RMP 25, another ʿḥʿ problem. In this example is the multiplication of $5\ \bar{3}$ by 2 to give $10\ \bar{\bar{3}}$. It has been included in this section but it is debateable whether it should be counted as an example of repeat doubling because there is only one doubling. However, as the other ʿḥʿ problems use repeat doubling in this stage of the calculation it was felt that this was the best classification, even though it must be seen as a trivial example.

Type C - Multiplication by Repeat Doubling of Fractions

The repeat doubling method does not take into account multipliers that contain fractions. There are several examples of multiplications in the Rhind Mathematical Papyrus that are based on the repeat doubling method but which also contain multipliers that are fractional. For example in RMP 35 it is necessary to multiply $\bar{5}\ \overline{10}$ by $3\ \bar{3}$. This multiplication cannot be completed by repeat doubling alone. Instead, the first part of the working shows the repeat doubling and the third row shows the fractional part of the multiplication. The working appears in the papyrus thus:

\1	$\overline{5}\ \overline{10}$	
\2	$\overline{2}\ \overline{10}$	
\\$\overline{3}$	$\overline{10}$	**Total 1**

RMP 42 is concerned with finding the volume of a cylindrical granary. In the first step in a problem of this type, once a ninth of the diameter has been subtracted it is squared. In this example, it is necessary to square $8\ \overline{\overline{3}}\ \overline{6}\ \overline{18}$. Because this is a complicated multiplication, the working takes several rows to complete[18]:

1	$8\ \overline{\overline{3}}\ \overline{6}\ \overline{18}$
2	$17\ \overline{\overline{3}}\ \overline{9}$
4	$35\ \overline{2}\ \overline{18}$
\8	$71\ \overline{9}$
\\$\overline{\overline{3}}$	$5\ \overline{\overline{3}}\ \overline{6}\ \overline{18}\ \overline{27}$
$\overline{3}$	$2\ \overline{6}\ \overline{12}\ \overline{18}$
\\$\overline{6}$	$1\ \overline{3}\ \overline{12}\ \overline{24}\ \overline{72}\ \overline{108}$
\\$\overline{18}$	$\overline{3}\ \overline{9}\ \overline{27}\ \overline{108}\ \overline{324}$
Total	$79\ \overline{108}\ \overline{324}$

RMP 69 is the first in a series of problems that deals with a unit known as the pesu. The pesu was a measurement of quality of bread and beer. It is given as the number of loaves or jugs of beer that were made out of one hekat of grain, the

[18] Again, this multiplication has been split so that in the text it appears in four columns. It is shown here in two columns to assist the reader.

lower the pesu, the greater the quality of the bread or beer[19]. In this problem it is necessary to multiply 22 $\bar{\bar{3}}$ $\bar{7}$ $\overline{21}$ by 3 $\bar{2}$ in order to show that the preceding division of 80 by 3 $\bar{2}$ is correct. The total is not explicitly given as this multiplication adds up to 80 and is performed only as a check. The working appears thus:

\1	22 $\bar{\bar{3}}$ $\bar{7}$ $\overline{21}$
\2	45 $\bar{3}$ $\bar{4}$ $\overline{14}$ $\overline{28}$ $\overline{42}$
\\$\bar{2}$	11 $\bar{3}$ $\overline{14}$ $\overline{42}$

RMP 70 is another example dealing with pesu. This example of Type C multiplication also comes from the second stage of the problem as the method of solution follows the same pattern as the preceding problem. As an example of multiplication involving a larger multiplier, it serves as a better example of Type C multiplication than that of RMP 69. Again, this multiplication is performed to demonstrate that the preceding multiplication is correct. The multiplication is 12 $\bar{\bar{3}}$ $\overline{42}$ $\overline{126}$ multiplied by 7 $\bar{2}$ $\bar{4}$ $\bar{8}$:

\1	12 $\bar{\bar{3}}$ $\overline{42}$ $\overline{126}$
\2	25 $\bar{3}$ $\overline{21}$ $\overline{63}$
\4	50 $\bar{\bar{3}}$ $\overline{14}$ $\overline{21}$ $\overline{126}$
\\$\bar{2}$	6 $\bar{3}$ $\overline{84}$ $\overline{252}$
\\$\bar{4}$	3 $\bar{6}$ $\overline{168}$ $\overline{504}$
\\$\bar{8}$	1 $\bar{2}$ $\overline{12}$ $\overline{336}$ $\overline{1008}$

[19] See Gillings, R. (1972) *Mathematics in The Time of The Pharaohs*; MIT Press, Cambridge, Mass; p. 212.

RMP 53 is an unclear problem. The normal statement of the problem to be solved is missing in this case and there are also mistakes in the arithmetic. The accompanying diagram shows that this is a geometric problem concerned with the areas of a triangle and trapezoids contained in that triangle[20]. The second multiplication in this problem involves finding the area of a triangle with a height of 7 and a base of $2\ \bar{4}$, The scribe therefore has to work out 7 multiplied by $2\ \bar{4}$ and then multiply the total by $\bar{2}$. This example has been classified in this type because it does contain a doubling even though there is only one doubling.

Type D - Multiplication of Fractions. No Doubling

This type of multiplication is the most common to be found in the Rhind Mathematical Papyrus. This reflects the fact that much of the Rhind Mathematical Papyrus is concerned with the manipulation of fractions.

Problems 7 to 20 of the Rhind Mathematical Papyrus[21] all deal with multiplications of two unit fraction series: $1\ \bar{2}\ \bar{4}$ and $1\ \bar{\bar{3}}\ \bar{3}$. These problems ,like most of the Rhind Mathematical Papyrus, do not state a problem to be solved at the beginning. At the beginning of RMP 7 it states: *tp n škm* which translates as: *an example of making complete* as *škm* is the causative of *km* which

[20] See Clagett, M. (1999) *Ancient Egyptian Science A Source Book: Volume Three Egyptian Mathematics*; American Philosophical Society, Philadelphia; Fig. IV.5a-c; pp 382-4.
[21] See Gillings, R. (1972) *Mathematics in the Time of the Pharaohs*; MIT Press; Cambridge, Mass; p109-109-10.

means to complete[22]. This seems to serve as a title for this section. Instead, they seem to be examples of multiplying unit fractions in preparation for later examples. A couple of examples will suffice to show the method employed in these problems. RMP 9 appears thus[23]:

1	$\bar{2}\ \overline{10}$
$\bar{2}$	$\bar{4}\ \overline{20}$
$\bar{4}$	$\bar{8}\ \overline{50}$
Total 1	

The scribe here makes a mistake, as the third row is not double the second row. Either the total is incorrect, or as Peet suggests[24] the fraction intended in the first row was $\bar{2}\ \overline{14}$, as this would make the second row $\bar{4}\ \overline{28}$ and the third row. This would give the correct total of one.

This is not the only problem in this section of the papyrus where mistakes appear. In RMP 11 the problem appears thus:

1	$\bar{7}$
$\bar{2}$	$\bar{9}\ \overline{14}$
$\bar{4}$	$\overline{18}$
Total $\bar{4}$	

This is clearly incorrect. It is possible that the scribe made the first error by giving twice $\bar{7}$ as $\bar{9}$, and then later realising a mistake and writing $\overline{14}$ in later.

[22] Peet, T. (1923) *The Rhind Mathematical Papyrus British Museum 10057 and 10058*; University of Liverpool Press, London; p. 54. Faulkner, R. (1999) *A Concise Dictionary of Middle Egyptian*; Griffith Institute, Oxford; p.251.
[23] These problems do not use the customary ticks.
[24] Peet, T.E. (1923) *Op. Cit.* p. 55.

The mistake persists to the third row. Despite these errors, the total is given correctly, perhaps indicating that this problem has been copied badly from another papyrus.

RMP 17 uses a different set of fractional multipliers and appears thus:

1	$\bar{3}$
$\bar{\bar{3}}$	$\bar{6}$ $\overline{18}$
$\bar{3}$	$\bar{9}$

Total $\bar{\bar{3}}$

From these examples, it can be seen that these problems are not genuine mathematical problems, but practice in the manipulation and properties of fractions. It is clear that the scribe was in need of some practice. The second of these examples uses the multipliers 1, $\bar{\bar{3}}$ and $\bar{3}$. It cannot have escaped the attention of the scribe that these add to 2. Therefore, if these problems were only performed to solve the arithmetical problem the scribe would surely have multiplied by 2.

RMP 56 requires the multiplication of $\bar{2}$ $\bar{5}$ $\overline{50}$ by 7 in the process of working out the seked, a measure of the slope, of a pyramid. The scribe proceeds thus:

1	7
$\bar{2}$	3 $\bar{2}$
$\bar{5}$	1 $\bar{3}$ $\overline{15}$
$\overline{50}$	$\overline{10}$ $\overline{25}$

Its seked is 5 $\overline{25}$

The scribe could have carried out this multiplication using the repeated doubling technique. The working would have looked like this[25]:

1	$\overline{2}\ \overline{5}\ \overline{50}$
2	$1\ \overline{3}\ \overline{15}\ \overline{25}$
4	$2\ \overline{\overline{3}}\ \overline{10}\ \overline{30}\ \overline{15}\ \overline{75}$
Total $5\ \overline{25}$	

Although the method the scribe used takes more lines of working, using repeated doubling leads to more complicated fractional expressions. The final addition is also harder to complete. It should be noted, though, that the actual method requires the scribe to work out one fiftieth of 7, which he does without further explanation. This illustrates some of the problems using unit fractions and this example will be discussed in more detail in section 2.4.

RMP 58 is a very similar problem as the problem is also concerned with finding the seked of a pyramid. In this example it is necessary to work out $93\ \overline{3}$ multiplied by $\overline{2}\ \overline{4}$. The working appears thus:

1	$93\ \overline{3}$
\\$\overline{2}$	$46\ \overline{\overline{3}}$
\\$\overline{4}$	$23\ \overline{3}$

[25] All doubles of unit fractions have been worked out using the 2/n table from the recto of the Rhind Mathematical Papyrus. See Appendix 1.

No total is given, but the tick marks are used to show that the first line should not be included in the total.

RMP 30 uses a long string of unit fractions. The problem is to find a quantity that when multiplied by $\bar{\bar{3}}$ $\overline{10}$ becomes 10. The multiplication part of this problem is demonstrating that the correct answer was reached. It is very similar to RMP 58. The multiplication appears thus:

1	13 $\overline{23}$
\\$\bar{\bar{3}}$	8 $\bar{\bar{3}}$ $\overline{46}$ $\overline{138}$
\\$\overline{10}$	1 $\bar{5}$ $\overline{10}$ $\overline{230}$

As in RMP 58, the tick marks are used to show that only the bottom two rows need to be summed, but the total is not given.

RMP 67 is a unique problem in the Rhind Mathematical Papyrus as it deals with the number of cattle one herdsman needs to give in tribute. The herdsman needs to give two-thirds of one third of the cattle entrusted to him. It is interesting not only for the uniqueness of the problem type, but also for its treatment of fractions. The multiplication in this example is also a check that the correct answer has been reached. The herdsman has bought 70 cattle out of the herd to give as tribute. The previous working of the problem has arrived at the answer that the herd is 315 strong. The check is performed thus[26]:

[26] Here it is written in two columns although the end of this problem is a little confused and the working appears over 4 columns.

1	315
$\bar{\bar{3}}$	210
$\bar{3}$ of it	105

$\bar{\bar{3}}$ of $\bar{3}$ of it is 70. These are what he bought.

It is standard in Egyptian mathematical texts for one third to be worked out by halving two thirds of the quantity, as the scribe has done here. It is interesting because although an earlier step in the problem showed that $\bar{\bar{3}}$ of $\bar{3}$ is equal to $\bar{6}$ $\overline{18}$, the scribe performs the check with the numbers stated in the original problem, even though this requires more working.

RMP 35 is also a check on the preceding work. In this case, it is necessary to check that 320 multiplied by $\overline{10}$ $\bar{5}$ equals 96:

1	320
$\overline{10}$	32
$\bar{5}$	64

Total 96.

It should be noted that the scribe selects the easier task of multiplying by a tenth before doubling that result to get 320 multiplied by a fifth.

Type E - Multiplication of Fractions by Repeat Halving

This method of multiplication is the reverse of Type A multiplication. Instead of each row being double the preceding row, it is half. There are only two examples of this method of multiplication in the Rhind Mathematical Papyrus in two successive problems, RMP 80 and 81. Both of these problems are concerned with fractions of the hekat measure for grain in henu and ro, where there are 10 henu in a hekat and 320 ro in a hekat. It is possible that RMP 80 is a scribal error because the first part of RMP 81 is identical. Only the titles given to the problems are different. The title of RMP 80 reads "*As for vessels used in measuring by the functionaries of the granary*", the title for RMP 81 reads simply "*Another reckoning of the henu*"

The fractions of a hekat shown in these problems are all Horus-eye fractions, which makes the use of repeat halving appropriate. The table covers 34 rows, although some of them are repeated. The first six rows are the repeated section and appear thus:

$\overline{2}$	(hekat is)	5	(henu)
$\overline{4}$	"	$2\ \overline{2}$	"
$\overline{8}$	"	$1\ \overline{4}$	"
$\overline{16}$	"	$\overline{2}\ \overline{8}$	"
$\overline{32}$	"	$\overline{4}\ \overline{16}$	"
$\overline{64}$	"	$\overline{8}\ \overline{32}$	"

The rest of the problem deals with other Horus-eye parts of the hekat, mostly sums of several Horus-eye parts that can be derived from the results above.

Type F - Other Methods

The repeat doubling method for multiplication is a laborious way to carry out a multiplication if the required multiplier is a large number. To account for this problem the Egyptians could use other techniques when multiplying.

Multiplying by ten is a fairly easy process and so in multiplications above ten, ten appears in the left-hand column as a multiplier. For example, in RMP 45 it is necessary to multiply 75 by 20[27]. The working appears as follows:

1	75
10	750
20	1500

This procedure is for more economical than repeat doubling. If the sum was to be performed by repeat doubling it would have appeared thus:

1	75
2	150
\4	300
8	600
\16	1200
Total	1500

This is obviously more laborious than the way it actually appears in the Rhind Mathematical Papyrus as it takes more rows to reach the required multiplier and it also involves an addition of two rows. This method is also used to multiply 75 by 20 in RMP 44.

There are several more examples of this technique in the Rhind Mathematical Papyrus, using 10 as a multiplier, rather than the longer repeat-doubling method. Several of these examples come from problems concerned with working out how much grain a granary of a given size can hold. In these problems, it is necessary

[27] RMP 46 contains a very similar calculation, 25 multiplied by 20. It is worked out using the same method, multiplying by 10 and then by 2.

to convert the volume into 100 quadruple-hekat, or 400 hekat. Once the volume in cubic cubits had been worked out, then the volume was converted into khar, there being one and a half khar in a cubic cubit. This was then converted to 100 quadruple hekat by multiplying by $\overline{20}$. The entire method can be seen in RMP 41:

<u>Rhind Mathematical Papyrus Problem 41</u>

1) *tp n ỉrt šsꜥ dbn n 9 10 ḫb.ḫr=k* $\overline{9}$ *n 9 m 1 ḏꜣ.t 8*

Example of working out a granary, round 9, 10 (in height). You are to subtract a ninth of nine namely 1; remainder 8

2) *wꜣḥ-tp m 8 r sp.w 8 ḫpr.ḫr 64 ỉr.ḫr=k wꜣḥ-tp m 64*

You are to make the function of 8 times 8, becomes 64. You are to make the function of 64

3) *r sp 10 ḫpr.ḫr=f m 640 di* $\overline{2}$*=f ḥr=f ḫpr.ḫr=f m 960 rḫt =f m ḫꜣr.w*

times 10, it becomes 640. Put half of it on it, becomes 960. **Its content in khar.**

4) *ỉr.ḫr=k* $\overline{20}$ *n 960 m 48 hꜣꜣ.t pw r=f m 4-ḥḳꜣt šs 4800 ḥḳꜣt*

Make one twentieth of 960, namely 48. This is the amount that will go into it in quadruple-hekat. **4800 quadruple hekat of grain.**

5) *ḳi n sšm.t =f*

Form of its reckoning[28]

1	8
2	16
4	32
\8	64

1	64
\10	640
\\$\overline{2}$	320

Total 960

$\overline{10}$	96
\\$\overline{20}$	48

The last two stages of working are the most interesting in this kind of problem. In the last stage, a twentieth is worked out by using the intermediate step of working out a tenth. In the preceding step, the scribe mixes the number that the multiplier in the left hand column refers to, which occurs only in problems working out the volume of a granary in the Rhind Mathematical Papyrus. The second row shows ten times 64, the third row does not show a half of 64, but a half of 640. It is more common that the scribe should explain if the row follows

[28] In the Rhind Mathematical Papyrus, each double column of working is set alongside the previous one. Due to limitations of space, here they will be placed underneath.

on from the previous multiplicand, as he does in RMP 45 where he notes that a row is a $\overline{10}$ of a $\overline{10}$ of it.

In other problems concerned with the volume of a granary the multiplication by $\overline{20}$ appears frequently as this is a conversion factor. It appears in RMP 42, 43, 44 and 46.

In RMP 42, there is a repeat of the multiplication by ten and then by half of the ten. Again, the scribe does not distinguish the two rows even though they use different multiplicands. In RMP 43, it is necessary to square 10 $\overline{\overline{3}}$. This is performed with the second row showing the multiplication of 10 with 10 $\overline{\overline{3}}$.

RMP 49 shows a multiplication of 1000 with 100. In this case multiplying each row by 10 makes most sense[29]:

1	1000
10	10,000
100	100,000.

RMP 65 should also be included in this type, although at first it does not appear to be an example of multiplication. However, as addition of columns is a central part to all Egyptian multiplication, it has been included here. The problem is to work out the distribution of 100 loaves among 10 men including a sailor, a foreman and a watchman who are to receive double portions. The number of loaves is divided by the number of portions needed to give 100 divided by 13

[29] This is the multiplication that is worked out in the text, even though the text of this problem suggests the scribe should multiply 10 khet by 2 khet.

which is $7\ \bar{\bar{3}}\ \overline{39}$. The scribe then demonstrates that this is the correct answer, as is customary. However, instead of multiplying 7 by $7\ \bar{\bar{3}}\ \overline{39}$ and then adding six lots of $7\ \bar{\bar{3}}\ \overline{39}$ the scribe writes out the portions in full and adds them together[30]:

	$7\ \bar{\bar{3}}\ \overline{39}$
	$7\ \bar{\bar{3}}\ \overline{39}$
	$7\ \bar{\bar{3}}\ \overline{39}$
	$7\ \bar{\bar{3}}\ \overline{39}$
	$7\ \bar{\bar{3}}\ \overline{39}$
	$7\ \bar{\bar{3}}\ \overline{39}$
	$7\ \bar{\bar{3}}\ \overline{39}$
Sailor	$15\ \bar{3}\ \overline{26}\ \overline{78}$
Foreman	$15\ \bar{3}\ \overline{26}\ \overline{78}$
Watchman	$15\ \bar{3}\ \overline{26}\ \overline{78}$
Total	100

The final example of this type of multiplication is the second section of RMP 79. This problem is concerned with finding the sum of a geometric progression. The first term of the progression is 7 and the common ratio is 7. Therefore, the scribe does not carry out a repeat doubling, but a repeat multiplication by 7. The translation of this problem is given in full in Section 2.5.

[30] The actual working is carried over three columns with the total given in a fourth.

Type G - Trivial Multiplication in ꜥḥꜥ Problems

RMP 24, 25 and 27 are all ꜥḥꜥ problems. In these problems a quantity has a fraction of itself added on to arrive to a given total. Because of the Egyptian method of solving this type of problem, in all these problems it is necessary to multiply an integer by a fraction, with the fraction being the multiplier to be reached as a sum of terms of the left-hand column. These examples are, however, trivial examples of multiplication as the integer is selected in order that the multiplication should take the fewest number of rows to complete, often in only two rows. One example shall be given here to illustrate this type of multiplication. In RMP 24, the problem is to find an unknown quantity such that when a seventh of this quantity is added to itself it becomes 19. The Egyptians used a false assumption method to work out this quantity. To make the first step easy in this case the scribe chooses 7, so the first step, to multiply 7 by 7 $\bar{7}$ appears thus:

\1	7
\\$\bar{7}$	1

The total is not explicitly given in this case, although in other examples it is. For a fuller explanation of this type of problem, see Chapter 3.

RMP 35 to 38 are a similar group of problems. In this group of problems the total required is always 1, but the method remains unchanged. The first step of these problems is to take a false position, which is then scaled up or down to give the correct figure. In these examples, the false position selected is always 1.

Type H - Multiplications with No Working Shown

There are many problems in the Rhind Mathematical Papyrus where the scribe has performed calculations without explicitly stating the method. Every type of multiplication looked at so far, with perhaps the exception of Type F, has necessitated some arithmetical working as part of the whole procedure. This is particularly true of the examples that used fractions. Many of these examples require manipulation of fractions and this is carried out with no further explanation. However, the examples of multiplication that are included in this type are included in the narrative of the problem, in a similar way to that found in the Moscow Mathematical Papyrus. Only a few of the examples of this type will be investigated in detail to show the contexts in which this type of multiplication occurs.

Rhind Mathematical Papyrus Problem 72

This problem deals with the exchange of loaves of different quality. The quality is the number of that strength loaf that can be made from one measure of grain, the hekat. The method of solution is convoluted. It should be simple to scale up the number of loaves according to the strength, as the measure is inversely proportional to the amount of grain used. The procedure used, however, is to work out the excess of the quality, 35, and to work out the corresponding number of loaves, adding this to the original 100 loaves.

1) *tp n ḏbꜣ tꜣ.w m tꜣ.w mi ḏd.n=k tꜣ.w 10 r 100 ḏbꜣ m ꜥḥꜥ*[31] *tꜣ.w 45*

Example of exchange, loaves for loaves. If it is said to you 100 loaves of strength 10 exchanged for a heap of loaves of strength 45

2) *ir.ḫr=k ꜥꜣw n 45 r 10 ḫpr.ḫr 35 ir.ḫr=k 10 r gm.t 35 ḫpr.ḫr 3* $\bar{2}$

You are to make the excess of the 45 over the 10, it becomes 35. You are to work with 10 in order to find 35, it becomes 3 and a half.

3) *ir.ḫr=k 100 r spw 3* $\bar{2}$ *ḫpr.ḫr 350 wꜣḥ.ḫr=k 100 ḥr=s ḫpr.ḫr 450*

You are to make 100 times 3 and a half, it becomes 350. You add 100 to it, it becomes 450.

4) *ḏd.ḫr=k ḏbꜣ pw ꜣt tꜣ.w 10 100*

You will then say, this means 100 loaves of strength 10

5) *m tꜣ.w 45 r 450*

are exchanged for 450 loaves of strength 45,

6) *ir m wḏy.t 10*

making in wedyat flour 10 hekat.

Although the example of multiplication in this problem is a reasonably simple one, 100 multiplied by 3 $\bar{2}$, yet in other problems of the Rhind Mathematical Papyrus simple multiplications have been worked out in full. It may be the case that the scribe in this example wanted to draw out the method of solving the

[31] For a discussion of the meaning of *ꜥḥꜥ* in Egyptian mathematical texts see Chapter 3 and the commentary on MMP 19.

problem as a whole and did not wish to break it up by demonstrating 100 multiplied by 3 $\overline{2}$.

RMP 73 to 75 and 78 are also problems concerned with the exchange of bread and beer of different strengths. They use a very similar method. The multiplications contained within them are mostly relatively simple. It seems to be the scribe's intention to explore the method of working out the exchanges over several examples. As noted with RMP 72, to work out the multiplications would break up the continuity of the solution. The multiplications given are as follows:

RMP 73	10 multiplied by 15
RMP 74	100 multiplied by 10
	100 multiplied by 20
RMP 75	30 multiplied by 7 $\overline{2}$ $\overline{4}$
RMP 78	10 multiplied by 20

RMP 62 is also concerned with a kind of exchange. In this case, there is a bag containing equal weights of gold, silver and lead. Different amounts are paid for each metal, which are given. The multiplications worked out in this problem are 4 multiplied by 12, 6, 3 and 21.

RMP 82 is concerned with feeding geese. The amount need to feed the geese for one day is given. The multiplications performed are 2 $\overline{2}$ multiplied by 10 and by 40.

The two other examples of multiplications in this type form rudimentary tables. An example of this kind that can be found in RMP 61 is a table of multiplications of fractions. Part of this problem is lost due to the condition of the papyrus. What remains is a list of multiplications of $\bar{\bar{3}}$, $\bar{3}$ and $\bar{2}$ by other unit fractions. Peet suggests that this problem was not originally intended as part of the papyrus as it is carelessly written in the margin. He also suggests that it is written here to provide reference for other problems written close to it on the papyrus roll[32].

[32] Peet, T. (1923) *The Rhind Mathematical Papyrus British Museum 10057 and 10058*; University of LIverpool Press; London; p. 103.

Rhind Mathematical Papyrus Problem 40

RMP 40 gives a list of multiplications of 1 $\bar{\bar{3}}$. The problem is to share out 100 loaves among 5 men so that the number of loaves is an arithmetical progression. The problem reads:

1) *t3.w 100 n s 5 $\bar{7}$ n 3 ḥry.w*

One hundred loaves to 5 men. One seventh of the first three men

2) *n s 2 ẖry.w*

to the two last.

3) *pty twnw*

What is the excess?

4) *ir.t mi ḫpr twnw 5 $\bar{2}$*

The doing as it occurs, the excess 5 and a half.

\1	23
\1	17 $\bar{2}$
\1	12
\1	6 $\bar{2}$
\1	1
Total 60	

\1	60
$\backslash\bar{\bar{3}}$	40
Total	100

ir.ẖ[r] =k w3ḥ

You are to count

tp m 1 $\bar{3}$

with one and a third

r sp	23	*ḫpr.ḫr.f*	38 $\bar{3}$	23	times, it becomes		38 $\bar{3}$
"	17 $\bar{2}$	"	29 $\bar{6}$	17 $\bar{2}$	"	"	29 $\bar{6}$
"	12	"	20	12	"	"	20
"	6 $\bar{2}$	"	10 $\bar{\bar{3}}$ $\bar{6}$	6 $\bar{2}$	"	"	10 $\bar{\bar{3}}$ $\bar{6}$
"	1	"	1 $\bar{\bar{3}}$	1	"	"	1 $\bar{\bar{3}}$

In the second part of this problem, it is necessary to scale up the assumed shares of the first half to obtain the correct shares. This requires each of the shares in the first part to be multiplied by 3 $\bar{2}$, which the scribe performs without showing the working in the normal way.

2.4: Division

As division is the reverse of multiplication, the process looks the same in the Egyptian texts, as there are still two columns. An Egyptian would not ask 'what is 8 divided by 2?'; instead he would ask 'by what must I multiply 2 to get 8?'. In the process of multiplication, it is necessary to add up the figures in the right-hand column until the correct answer has been reached. It is also known what factors are needed in the left hand column. In the process of division, it is the numbers in the left-hand column that have to be summed to reach the correct answer. This means that it is not known what figures need to be in this column and this is the column that is consciously manipulated. This requires a certain amount of educated guesswork on the part of the scribe. As a consequence of this guesswork, the repeated doubling method is far more prevalent in divisions. The strength of the repeat doubling method is that any number can be made up of a combination of the numbers in the doubling sequence. A modern mathematician would recognise these numbers as binary.

Only a few examples are needed to show this method, as there is very little variation. A simple example of the method can be found in RMP 25, where it is necessary to divide 16 by 3. The working would appear as follows[33]:

\1	3
2	6
\4	12
$\bar{\bar{3}}$	2
\\$\bar{3}$	1

[33] In this example, the working is divided into four columns, instead of the two shown here.

The scribe starts with 3 and uses the repeat doubling method until the target figure is approached. Another line of doubling would be unnecessary, as this would make the number in the right-hand column larger than the target. Fractions are therefore necessary. The only unusual feature of this problem for a modern reader is the use of two-thirds in row 4. It seems strange that the Egyptians would first find two-thirds and then halve it to find one-third, but this is standard procedure in the Egyptian mathematical texts.

RMP 66 involves the daily portion of ten hekat of fat that has been issued for one year. The hekat are converted into ro, giving 3200 ro. The division required is then to divide 3200 by 365, or to multiply 365 to find 3200. The working is shown as:

1	365
2	730
4	1460
$\bar{\bar{3}}$	243 $\bar{3}$
$\overline{10}$	36 $\bar{2}$
$\overline{2190}$	$\bar{6}$
Total	8 $\bar{\bar{3}}$ $\overline{10}$ $\overline{2190}$

This example follows the general pattern of the previous example. Repeat doubling is used until that process can go no further, then fractions are used to find the remainder of the target number. This example is interesting, however, because of the final row of working, which is presented with no further explanation. Up to this point the total of 355 $\frac{5}{6}$ has been reached. The scribe must now work out the number in the left-hand column that would give $\bar{6}$ in the right-hand column. This in itself can be considered an example of division, but not one that the scribe feels it necessary to explain. The required denominator can be found by multiplying 365 by 6; it is however speculative how the scribe actually achieved this answer.

This problem also brings up questions of the utilitarian nature of the mathematical papyri. This problem appears to be a purely practical problem involving the rations of a commodity and therefore can be considered practically. Indeed, the reader is told at the end of this problem that "You may do similarly for any problem put to you resembling this example.". However, the accuracy to which this problem is solved is impractical to administer. The suggestion that these problems are *purely* practical in nature is therefore inaccurate. The impetus for this problem may be practical, but the way in which it is carried out is not.

An interesting example of division is RMP 69. It is necessary to multiply 80 to get 1120. The working appears as follows:

1	80
\10	800
2	160
\4	320
Total	1120

The scribe first selects to multiply by ten; this is because the target of 1120 is greater than ten times the starting number of 80. Once the scribe has multiplied by ten the remainder to be found is 1120 minus 800, which equals 320. The scribe then returns to a repeat doubling method to find the remaining 320. This is achieved in two further rows. This is an interesting example because it reinforces that the Egyptians were flexible in their approach to arithmetic and could modify their procedures to fit the example at hand.

2.5: Rhind Mathematical Papyrus Problem 79[34]

1) *wʿ.t imy.t-pr*

An inventory of a household.

1	2801
2	5602
4	11204
Total	19607
7	*pr.w* Houses
49	*myw.w* Cats
343	*pnw.w* Mice
2301[35]	*bd.t* Emmer
16807	*ḥḳꜣ.t* Hekat

[34] The translation of this problem is discussed in Peet, T.E (1929) *Op. Cit.* p.121-2.
[35] Sic. This should read 2401. Perhaps another copying error.

The problem that is posed here is similar to the nursery Rhyme "As I was going to St. Ives". A geometric progression is to be summed. Its first term is 7, the common ratio is 7 and it contains 5 terms. The sum is worked out in two ways. The second way is simply to work out the terms of the progression and then add them together. This is only remarkable because the repeat-doubling method is not used, See Section 2.3. Type F.

The first method shown is more remarkable. The formula that we would use to work out the sum is:

$$a\frac{r^n - 1}{r - 1}$$

where a is the first term,
r is the common ratio
and n is the number of terms.

Replacing the terms of this progression:

$$7 \times \frac{16807 - 1}{7 - 1} = 7 \times \frac{16806}{6} = 7 \times 2801$$

This is exactly the calculation that is performed in the first part of this problem. How the scribe achieved this answer is not explained and as this is the only example of its type it cannot be compared to any other problem from the Rhind Mathematical Papyrus. It may be that the Egyptians only knew how to calculate the sum of a geometrical progression of this type, one in which the first term is also the common ratio. This makes the formula simpler:

$$a\frac{l-1}{r-1}$$ where l is the last term.

Any further comments on this section of the problem are purely speculative as this is the only example of a geometric progression in the extant papyri.

2.6: The Use of Unit Fractions in Ancient Egyptian Mathematics

Unit fractions are one of the most distinctive features of Egyptian mathematical texts. As Section 2.3. has shown, the Rhind Mathematical Papyrus devotes a lot of attention to their use and manipulation. In this section, the main features of unit fractions will be explored. This will not be an exhaustive examination. However, it will aim to show the most important points and also to draw out the features that have attracted the most criticism. Section 2.6.3 will give an in-depth analysis of RMP 31, 32 and 33. These problems are some of the most difficult problems concerning the manipulation of unit fractions

2.6.1: General Description

The ancient Egyptians used only unit fractions, with the exception of 2/3 in their mathematical papyri. Other fractions were expressed as a sum of unit fractions, with the largest fraction (smallest denominator) first and then decreasing in size. No fraction could be repeated.

The use of unit fraction has intrigued many writers on Egyptian mathematics. It has been described as: "... at once the glory and the straitjacket of Egyptian methodology"[36]. The "glory" refers to the technical skill shown by the texts in the manipulation of the unit fractions. The scribes show great talent in their use.

The use of unit fractions in ancient Egyptian mathematics is one of the most debated topics in its study. There is a feeling in the literature on the subject that the Egyptians used unit fractions only because they could not think of anything better. Unit fractions have also been described as a negative influence on the development of mathematics[37].

Gillings, one of the most optimistic commentators on ancient Egyptian mathematics wrote:

> " Today, if a new concept arises, mathematicians devise at once a new notation for it, but the Egyptians, never thinking to improve or alter their notation for fractions developed instead special techniques for dealing with the notation that they already had." [38]

Gillings links the use of unit fractions to the need for fairness in the division of commodities. Not only would loaves be distributed fairly it would be obvious that it had been done as well as each worker would get the same number of pieces of the same size. This argument is one of the most persuasive for the use and the continued use of unit fractions into the Graeco-Roman era. Unit fractions were the preferred method of expression of factional quantities in the Almagest.

Horus-eye fractions were a special type of unit fractions. They are the fractions that are obtained through the repeat halving of one.

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[36] Robins G. and Shute. (1987) *The Rhind Mathematical Papyrus*; British Museum Publications; London; pp. 58-9.
[37] Neugebauer, O. (1952) *Op. Cit.* p. 72.
[38] Gillings, R. (1972) *Op. Cit.* p. 105.

2.6.2: Addition using Auxiliaries

It is often necessary in Egyptian mathematical problems to add together a large number of unit fractions. This is one of the main weaknesses of the method. In RMP 31, for instance, in the first division sum it is necessary to add 11 unit fractions with 7 separate denominators.

To make this process easier, auxiliaries were used. Auxiliaries are numbers placed below or to the side of a fraction, often but not exclusively in red ink, that show the numerator of that fraction as if it had been laced over a common denominator. For example in RMP 37 it is necessary to add up 8 unit fractions each with a different denominator. The scribe wrote out each of the fractions to be summed and beside or beneath each of the fractions with a large denominator an auxiliary was written. The actual working of the scribe is a little confusing as the fractions to be summed are written out over several columns of the text. Placed on two lines the working appears thus[39]:

$\overline{2}$	$\overline{4}$	$\overline{8}$	$\overline{72}$	$\overline{16}$	$\overline{32}$	$\overline{64}$	$\overline{576}$	Total	$\overline{8}$
			8	*36*	*18*	*9*	*1*		*72*

The first three fractions in this group have a large, even denominator and so they are easy to add together. These fractions do not have auxiliaries placed underneath. However, the other fractions are difficult to handle. The auxiliaries underneath show the numerator of an equivalent fraction if the common denominator was 576. The total is only the total of the fractions with auxiliaries, $\frac{72}{576}$ is equal to $\frac{1}{8}$. This method of using auxiliaries to add fractions with large

[39] Auxiliaries are shown under the fractions in italics.

denominators shows the skill of the Egyptian scribe. It also shows that the Egyptians had a deeper understanding of fractions than simply using unit fractions. The practice of using auxiliaries to add fractions is directly comparable to our use of common denominators.

2.6.3: Problems Demonstrating the Use of Fractions from The Rhind Mathematical Papyrus

To illustrate the techniques that have already been explored in this chapter, two problems from the Rhind Mathematical Papyrus are presented here in full. These two problems exemplify the mathematical skill of the scribe who prepared these texts. A translation will be given in full, followed by a full explanation of the method.

Rhind Mathematical Papyrus Problem 36[40]

1) *ꞽw=ꞽ hꜣꜣ.k-wꞽ sp.w 3* $\overline{3}$*=ꞽ* $\overline{5}$*=ꞽ ḥr=ꞽ ꞽw=ꞽ mḥ.k-wꞽ*

I go down three times, a third of me, a fifth of me, on me. I return satisfied.

2) *pty pꜣ ꜥḥꜥ ḏd sw*

What is the heap that says this?

[40] The parts of this problem have been placed below each other to make the process of the solution clear. In the Rhind Mathematical Papyrus, they are placed side by side. In this example, the auxiliaries are all written in red ink. They are shown here with italics.

1	1
1	1
1	1
$\overline{3}$	$\overline{3}$
$\overline{5}$	$\overline{5}$

1	106
$\overline{2}$	53
$\backslash\overline{4}$	26 $\overline{2}$
$\backslash\overline{106}$	1
$\backslash\overline{53}$	2
$\backslash\overline{212}$	$\overline{2}$
Total	1

1	$\overline{4}$ $\overline{53}$ $\overline{106}$ $\overline{212}$
2	$\overline{2}$ $\overline{30}$ $\overline{318}$ $\overline{795}$ $\overline{53}$ $\overline{106}$
$\overline{3}$	$\overline{12}$ $\overline{159}$ $\overline{318}$ $\overline{636}$
$\overline{5}$	$\overline{20}$ $\overline{265}$ $\overline{530}$ $\overline{1060}$

$\overline{53}$	$\overline{106}$	$\overline{212}$			*35*
20	*10*	*5*			
$\overline{30}$	$\overline{318}$	$\overline{795}$	$\overline{53}$	$\overline{106}$	*70*
35 $\overline{3}$	*3* $\overline{3}$	*1* $\overline{3}$	*20*	*10*	
$\overline{12}$	$\overline{159}$	$\overline{318}$	$\overline{636}$		*100*
88 $\overline{3}$	*6* $\overline{\overline{3}}$	*3* $\overline{3}$	*1* $\overline{\overline{3}}$		
$\overline{20}$	$\overline{265}$	$\overline{530}$	$\overline{1060}$		*80* [sic. read *60*]
53	*4*	*2*	*1*		
					265 $\overline{4}$

$\overline{2}$	530
$\overline{4}$	265
$\overline{4}$	265
Total	1060

The problem is to find a value which solves the following equation:

$$x(3+\overline{3}+\overline{5})=1$$

The first step in the solution is the curious need for the Egyptians to show the multiplication of 3 $\overline{3}$ $\overline{5}$ by 1. The next step of working is not shown in the papyrus. The terms have to be added, which may have been done with auxiliaries. The common denominator chosen is 30. This gives the answer to the sum as $\frac{106}{30}$. The answer can now be obtained by dividing 1 by $\frac{106}{30}$. In the Egyptian fashion, this requires multiplying 106 to find 30. The scribe is able to achieve this answer in the normal way. Two halvings are performed to reach 26 $\bar{2}$. The extra needed to reach 30 are found after seeing that $\overline{106}$ of 106 is 1. It is interesting to note that the total given is not 30, but 1. This shows that the Egyptian scribe never lost sight of the fact that his target was 1, even though he has had to change the fractions using the denominator of 30.

The next part of this problem in the Rhind Mathematical Papyrus is a demonstration that the correct answer has been reached. This is the more complicated part of the problem because of the nature of unit fractions in Egyptian arithmetic. Each fraction has to be summed, but the fractions have large denominators that make this process difficult. To achieve the addition, the fractions are all written out with the auxiliary written in red ink underneath. The denominator chosen is 1060. In this example, we can see the strength of the Egyptian use of auxiliaries as several of the auxiliaries are themselves fractions. The fractions with small denominators: a half and two quarters are left out of this stage, and are added in later. The auxiliaries are first summed by row, and then the total of all of the rows is worked out. The total of all of the auxiliaries is 265, which the scribe shows is a quarter. This added together with the omitted fractions gives the desired total of 1.

Rhind Mathematical Papyrus Problem 31

1) *ꜥḥꜥ* $\overline{\overline{3}}$ *=f* $\overline{2}$ *=f* $\overline{7}$ *=f ḥr =f ḫpr =f m 33*

A heap, two-thirds of it, a half of it, a seventh of it, on it, becomes as 33

1	1 $\overline{\overline{3}}$ $\overline{2}$ $\overline{7}$	
/2	4 $\overline{3}$ $\overline{4}$ $\overline{28}$	
/4	9 $\overline{6}$ $\overline{18}$ (read $\overline{14}$)	
/8	18 $\overline{3}$ $\overline{7}$	
$\overline{2}$	$\overline{2}$ $\overline{3}$ $\overline{4}$ $\overline{14}$	
/$\overline{4}$	$\overline{4}$ $\overline{6}$ $\overline{8}$ $\overline{28}$	Total 32 $\overline{2}$ Remainder $\overline{2}$

/ $\overline{97}$	$\overline{42}$	1	1	42	
/ $\overline{56}$ $\overline{679}$ $\overline{776}$	$\overline{21}$	2	$\overline{\overline{3}}$	28	
/ $\overline{194}$	$\overline{84}$	$\overline{2}$	$\overline{2}$	21	
/ $\overline{388}$	$\overline{168}$	$\overline{4}$	$\overline{7}$	6	Total 99 (should read 97)

Total 33

$\overline{7}$ $\overline{8}$ $\overline{14}$ $\overline{28}$ $\overline{28}$

6 *5*$\overline{4}$ *3* *1*$\overline{2}$ *1*$\overline{2}$

17$\overline{4}$

3 $\overline{2}$ $\overline{4}$ (Remainder) $\overline{2}$ (is) 21

To make this problem easier to follow, the parts of it need to be rearranged from the order of the Rhind papyrus so they are in the order of calculation. The line placed through the centre of the first part has been added in accordance with the edition of Peet[41]. All changes have been made in an attempt to facilitate the reader's understanding. The two steps to the solution of the problem are also identified. These steps correspond to the commentary below.

<u>Step 1</u>

1	1 $\bar{\bar{3}}$ $\bar{2}$ $\bar{7}$	
/2	4 $\bar{3}$ $\bar{4}$	$\overline{28}$
/4	9 $\bar{6}$	$\overline{18}$ (read $\overline{14}$)
/8	18 $\bar{3}$	$\bar{7}$
$\bar{2}$	$\bar{2}$ $\bar{3}$ $\bar{4}$ $\overline{14}$	
/$\bar{4}$	$\bar{4}$ $\bar{6}$	$\bar{8}$ $\overline{28}$

Total 32 $\bar{2}$ Remainder $\bar{2}$

$\bar{7}$	$\bar{8}$	$\overline{14}$	$\overline{28}$	$\overline{28}$
6	*5* $\bar{4}$	*3*	*1* $\bar{2}$	*1* $\bar{2}$

17 $\bar{4}$

3 $\bar{2}$ $\bar{4}$ (Remainder) $\bar{2}$ (is) 21

41 Peet, T.E. (1923) *The Rhind Mathematical Papyrus British Museum 10057 and 10058*; University of Liverpool Press, p 66.

Step 2

1	42	
$\overline{\overline{3}}$	28	
$\overline{2}$	21	
$\overline{7}$	6	Total 99 (should read 97)

/ $\overline{97}$			$\overline{42}$	1
/ $\overline{56}$	$\overline{679}$	$\overline{776}$	$\overline{21}$	2
/ $\overline{194}$			$\overline{84}$	$\overline{2}$
/ $\overline{388}$			$\overline{168}$	$\overline{4}$

Total 33

In RMP 31, the working of the problem shows that the scribe had some understanding of fractions beyond just using unit fractions. The scribe was able to add fractions by the use of a method similar to the modern technique of finding the common denominator. On the first reading of this problem, this is not obvious and it is for this reason that RMP 31 has been overlooked by other authors. The detailed description below is given in an attempt to show the importance of this problem for an understanding of Egyptian arithmetical procedure and to challenge notions about Egyptian fractions.

The problem stated is to find a quantity that when two-thirds and a half and a seventh are added to it, equals 33. Algebraically this can be expressed as:

$$x\left(1+\frac{2}{3}+\frac{1}{2}+\frac{1}{7}\right)=33$$

To solve this problem using modern fractions, a common denominator would be found so that the series of fractions can be expressed as one fraction and then both sides would be divided by this fraction:

$$x\left(\frac{42+28+21+6}{42}\right)=33$$

$$x\left(\frac{97}{42}\right)=33$$

$$x=\frac{33\times 42}{97}=\frac{1386}{97}$$

$$x=14\frac{28}{97}$$

Step one of the Egyptian method is to try to obtain the result using the doubling and halving technique. This technique yields an answer of 32 and a half when all the ticked rows on the left hand side of our imaginary line are added together, giving the stated remainder of a half. It can be seen that the fractions on the left hand side of this line are all fractions with a small, even denominator. This makes the process of adding them reasonably simple. There are fractions, though, that are not included in this sum.

The next stage then is to include these fractions to find out how much of a remainder from 33 has been achieved. This stage is carried out using auxiliaries. The fractions that have not been included have been written out in a horizontal row. Below each of these fractions, a number is written, the auxiliary (here shown in italics to highlight and separate them from the rest of the problem). This number shows how many times $\frac{1}{42}$ has to be multiplied to get the fraction above. This is a process extremely like finding a common denominator; the strength of the Egyptian method is that they use auxiliaries that are themselves fractions. This is not general practice. In modern arithmetic a common denominator would be used that would render all the numerators as whole numbers. Using these auxiliaries allows the fractions that were not included in the first addition to be summed. The auxiliaries sum to 17 and a quarter. The remainder is 3 and a half and a quarter, the difference between 21 and 17 and a quarter. This is because, as the scribe reminds us, we are looking for the half that was a remainder after the first sum and in the auxiliaries this is equal to 21.

Step two of the Egyptian method is to calculate the necessary quantity to find the remainder left from part one. This can be considered as a new problem, expressed algebraically it would appear thus:

$$y\left(1+\frac{2}{3}+\frac{1}{2}+\frac{1}{7}\right)=\frac{\left(3\frac{1}{2}+\frac{1}{4}\right)}{42}$$

To obtain the answer both sides must be divided by $1+\frac{2}{3}+\frac{1}{2}+\frac{1}{7}$. To do this it is necessary to express this sum in one common denominator. The obvious choice is to use 42, as the right hand side is already expressed using this denominator. The scribe does this and calculates that it is equal to $\frac{97}{42}$.

Therefore, we can now express our problem as:

$$y\left(\frac{97}{42}\right)=\frac{\left(3\frac{1}{2}+\frac{1}{4}\right)}{42}$$

Following established procedures from other problems in the Rhind Mathematical Papyrus, it would seem that the scribe should now ask 'By what must I multiply 97 in order to get $3\frac{1}{2}+\frac{1}{4}$?'. This is not how the scribe proceeds in this case. The first stage of this problem is the scribe's attempt to solve the problem as a straight division. However, the usual method for working out a division requires some guess work. In stage one the scribe has gone as far as possible with this method and found that this example is too complicated to be solved in this manner, so another method has to be found. This method is now used here. The scribe multiplies $3\frac{1}{2}+\frac{1}{4}$ by the reciprocal of 97. Because auxiliaries have been used this is shown in a three-column sum, although it should be seen as an extension of the division in the first step of the problem. In the Rhind Mathematical Papyrus itself, it is shown as a continuation. The left-hand column and the central column serve the same function as those columns they are a continuation of. The left-hand column shows the multiplicators and the central column shows the multiplicands of the original terms. However, the central column also shows the multiples of $\frac{1}{42}$. This stage is achieved by multiplying $\frac{1}{42}$ by $3\frac{1}{2}+\frac{1}{4}$. The left-hand column shows the number of multiples of $\frac{1}{42}$. The technique of multiplication is a simple one in this case because $3\frac{1}{2}+\frac{1}{4}$ splits easily into 1, 2, $\frac{1}{2}$ and $\frac{1}{4}$, which can be worked out using the doubling and halving technique.

The answer to the problem is not stated explicitly. To obtain the answer parts one and two have to be combined. Thus the final answer is:

$$14\ \overline{4}\ \overline{56}\ \overline{97}\ \overline{194}\ \overline{388}\ \overline{679}\ \overline{776}$$

This problem shows that the Egyptians could conceive of other ways of using fractions other than straight unit fractions. It this problem, not only are auxiliaries used, which is roughly equivalent to our use of common denominators, but the final step requires the use of reciprocals. The consequences to our understanding of the Egyptian use of fractions cannot be emphasised enough. Rather than presuming a deficiency in the mathematical skill and imagination of the Egyptian scribes, instead we find the positive properties of unit fractions that encouraged their use.

2.7: Conclusions

This exploration of the arithmetical procedures of the ancient Egyptians shows that even the most basic functions of Egyptian mathematics are more sophisticated that it appears at first. Once the context of the arithmetic is factored into the assessment of it, it can be seen that the Egyptian scribes were able to modify their procedures to reflect the circumstances. This suggests that the Egyptians were not simply following rules but had a sophisticated mathematical imagination and an affinity for numerical manipulation.

This understanding also raises the possibility that the Egyptians were skilled in mental arithmetic. The places where the working out of a multiplication or division is absent are just as illuminating as those places where it is present. It is often the case that the places where they are absent are in problems of a geometrical nature. The absence of any arithmetical working in the Moscow Mathematical Papyrus, a papyrus mostly concerned with geometry and which contains the two most sophisticated problems in the extent corpus, is highly suggestive. It seems likely that the reason that the working is absent in these cases is that the arithmetic is not the focus of these problems; the scribe who composed the text was more interested in showing the method for solving the problem. In those cases where the arithmetic is displayed clearly, it is the arithmetical procedures that the problem is concerned with practising.

The assumed utilitarian nature of Egyptian mathematics also has to be questioned. Whilst many of the problems that this chapter has examined take their inspiration from everyday situations, the accuracy and care taken over their solution is not everyday. This is evident not only in the use of fractions with very high denominators but also in RMP 79, which was discussed in Section 2.5. The problems of abstract features in Egyptian mathematics will be explored further in Chapter 5, once geometrical and algebraic problems have been investigated in more detail.

Chapter 3

Geometrical and Algebraic Problems

Chapter 2 began this inspection of the processes of Egyptian mathematics by a thorough study of arithmetical procedures. This chapter will examine geometrical and algebraic problems from the Moscow Mathematical Papyrus, except for Problems 10 and 14, which due to their importance and uniqueness will be examined in Chapter 4. A few examples of geometrical problems from the Rhind Mathematical Papyrus will be included where they illuminate either the mathematical procedure or the vocabulary of problems from the Moscow Mathematical Papyrus.

3.1: Introduction

The Moscow Mathematical Papyrus is one of the most important mathematical texts from ancient Egypt. Although it does not have as many problems as the better-known Rhind Mathematical Papyrus, it contains some unique geometrical problems and so should be afforded equal status to the Rhind Mathematical Papyrus by historians of mathematics. An understanding of the full scope of ancient Egyptian mathematics cannot be acquired without a comprehensive study of the perplexing problems from the Moscow Mathematical Papyrus.

The Moscow Mathematical Papyrus is in the Museum of Fine Arts in Moscow. It is numbered 4676 in their collection, and it was discovered in Egypt in a tomb not far from the Ramesseum. It was in a small building very close to the Ramesseum that the Rhind Mathematical Papyrus was found. It is part of the collection of W.S. Golenischeff, who donated his collection to the Museum of Fine Arts in 1912. For this he was supposed to receive a life annuity from the Russian Government. Unfortunately for Golenischeff, the change of government after the Russian Revolution resulted in the annuity being stopped. He died in 1947[1].

Unlike the Rhind Mathematical Papyrus in which the problems are arranged in broad categories such as fractional identities, division of loaves among workers, *pesu* problems, geometrical problems etc, the Moscow Mathematical Papyrus appears to be arranged in no logical order. It has been suggested that this can provide clues about the nature of the text and the method of its preparation. Clagett, remarking on the original edition of the text by Struve, suggests that it shows the papyrus is the work of a student. He says:

> " This leads to the conclusion that the author of the Moscow Papyrus was a student whose training has progressed enough for the teacher to present various problems to be solved in order to test the skill of the student"[2]

The idea of a student trying to pass his final exam in a scribal school is an attractive idea. However, we can only theorise on how this document could find its way into a tomb on the West Bank at Thebes. Also, given the very small sample size of mathematical texts that have survived, we should be careful before indulging in flights of fancy. The Rhind Papyrus is a document that has been copied from another. It may be that the original was not ordered in the way that Ahmose, the scribe named as the copyist at the beginning of the text, copies

[1] Gillings, R. (1972), *Mathematics In The Time of The Pharaohs*, MIT Press, Cambridge MA. p. 246.

[2] Clagett, M. (1999) *Ancient Egyptian Science: A Source Book. Vol.3 Egyptian Mathematics*, American Philosophical Society, Philadelphia. p209.

it. The ordering may be editorial judgement on his part. This means that care should be taken when discussing the importance of the ordering.

3.2: Contents of the Moscow Mathematical Papyrus[3]

Problem Number	Description of problem
1	Damaged and unreadable
2	Damaged and unreadable
3	Working out a cedar mast
4	Area of a triangle.
5	Pesu of bread
6	Dimensions of a rectangle with known area
7	Dimensions of a triangle with known area
8	Pesu of bread
9	Pesu of bread.
10	Area of the curved surface of a hemisphere
11	Loaves and basket
12	Pesu of beer.
13	Pesu of loaves and beer
14	Volume of a truncated pyramid.
15	Pesu of beer.
16	Pesu of beer.
17	Dimensions of a triangle with known area.
18	Measuring cloth.
19	Working out a heap.
20	Pesu of 1000 loaves.

[3] Adapted from Gillings, R. (1972) *Op. Cit.* Appendix7 pp 246-7.

21	Mixing bread.
22	Pesu of loaves and beer.
23	Unclear problem concerning the work of a cobbler.
24	Exchange of loaves and beer
25	Elementary equation.

Each problem is translated below, with the transliteration. Each translated problem is followed by my commentary on its mathematical content, trying to be as true to the original text as possible. It is a mistake to try and render the text in modern mathematical language, as these modern terms have very strict definitions and connotations leading from their use in the precise texts that are produced today. These problems are presented under a general heading for the convenience of the modern reader, used to dealing with different types of mathematics. As discussed in the introduction to this chapter, there is little and tenuous evidence that the Egyptians generally treated their mathematics in this way. To order by type without a clear statement of the order that they appear in the original text is to add a layer of interpretation by stealth. Any diagrams that appear are copied as precisely as possible from the drawings that appear in the Moscow Mathematical Papyrus.

The hieroglyphic transcriptions that were used to produce the transliteration and translation are those produced by Struve[4] as this is the only complete transcription and the one that Clagett chose to reproduce in his sourcebook[5]. Where the transcription from the original hieratic to hieroglyphs produced by

[4] Struve W.W. (1930) *Mathematischer Papyrus des Staatlichen Museums der Schönen Künste in Moskau*, *QSGM*, Abt. A: Quellen, Berlin.
[5] Clagget M. (1999) *op. cit.* Figs 4.6a-t

Gunn and Peet[6] is different, these differences are highlighted according to line number after the full translation. The translations accompanying Struve's transliteration are this author's; those accompanying Gunn and Peet's transliterations are those of Gunn and Peet.

3.3: Rectangles and Triangles

Moscow Mathematical Papyrus Problem 4

1) *tp n irt spdt*

Example of working out a triangle.

2) *mi ḏd n=k spdt nt 10 m mryt*

If it is said to you a triangle of ten in height

3) *4 ḥr tp-r ḥ3 di=k rḫ*

four on the shortest side, may you give knowledge

4) [ ] *-pn*

[ ] this

5) [ ] *sp 2*

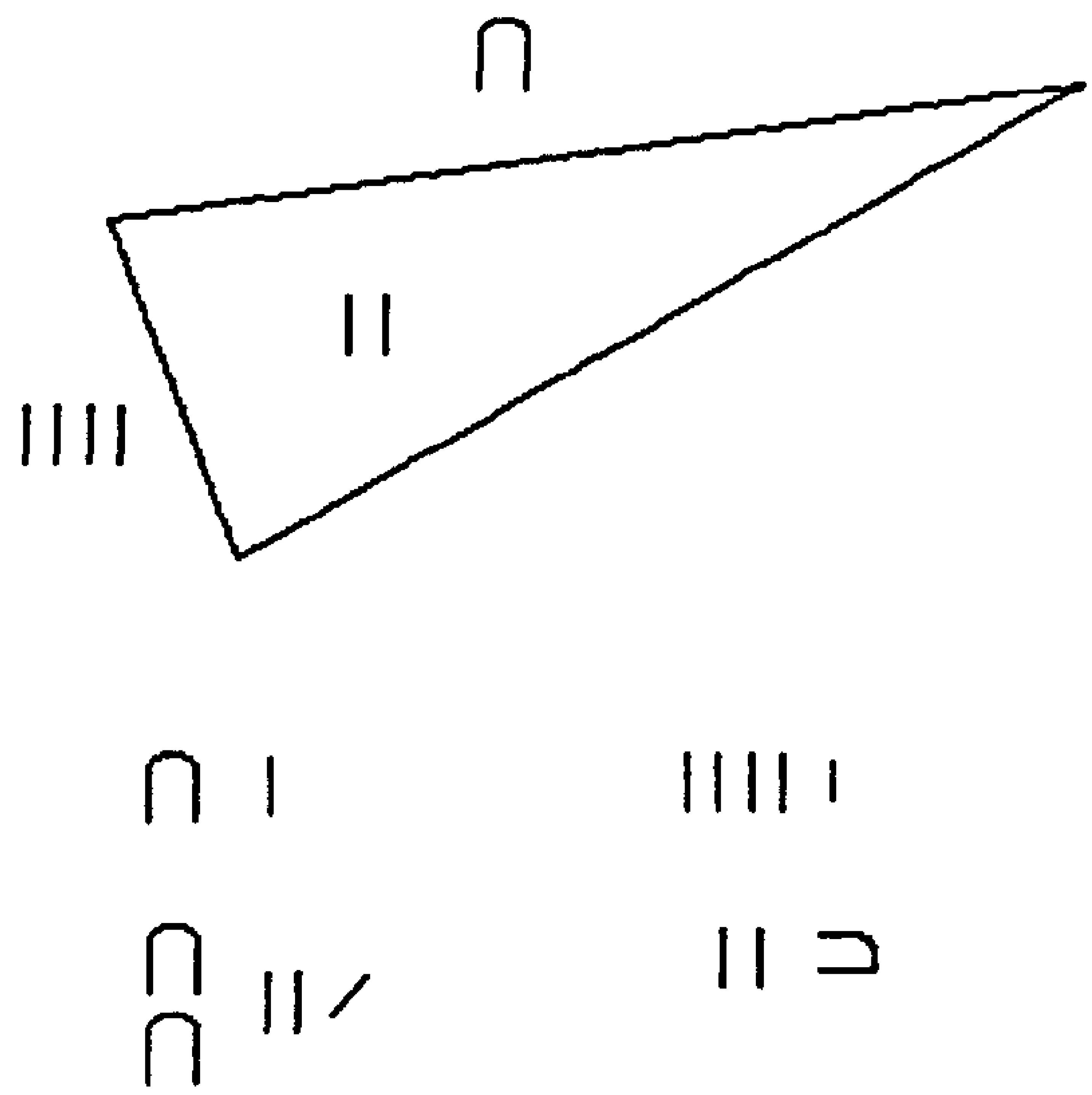
[ ] times two

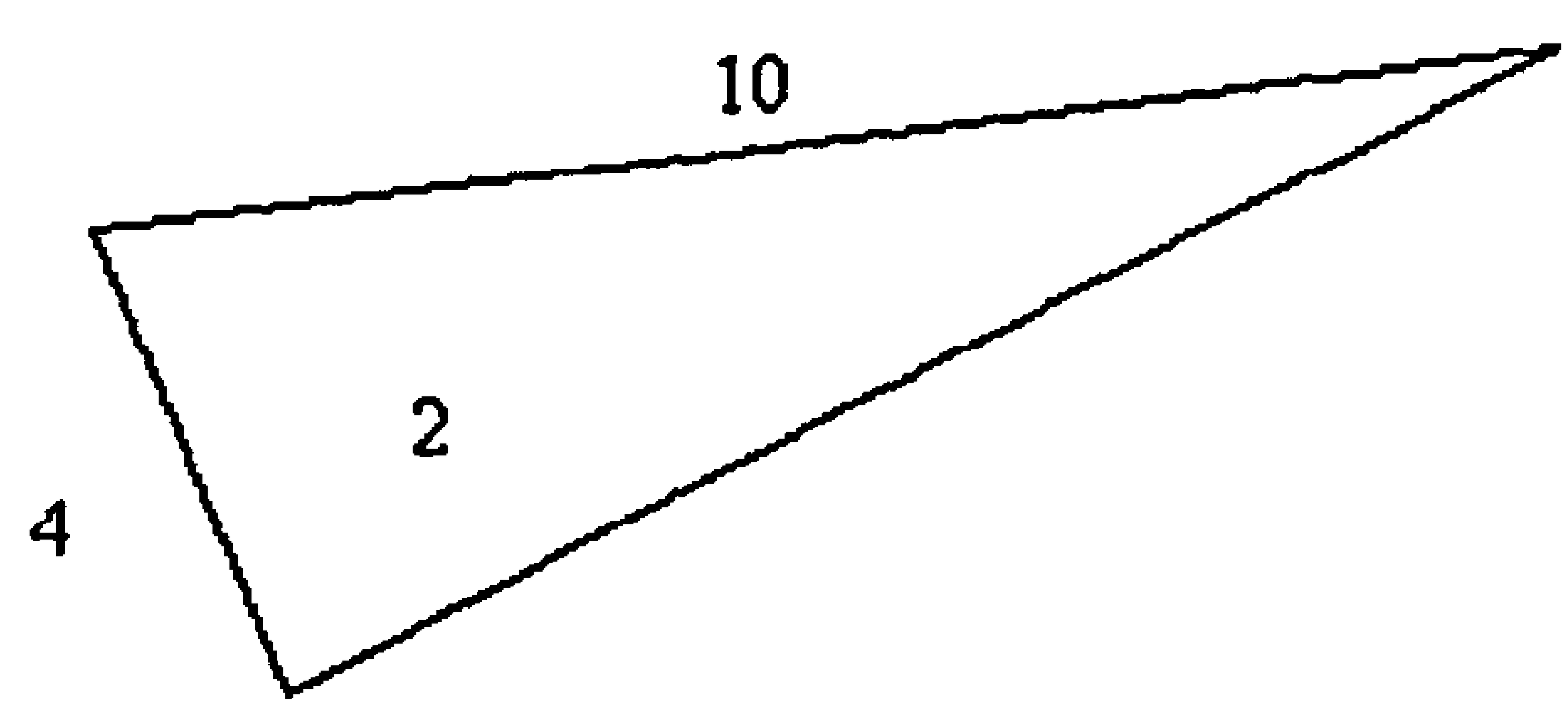
6) [ ] *pw*

[ ] this

[6] Gunn, B. and Peet, T. (1929) "Four Geometrical Problems from the Moscow Mathematical Papyrus" *The Journal of Egyptian Archaeology* Vol. 15 pp 167-85.

A line may be lost here.





10	1	4	1
20	2/	2	1/2

Commentary on Moscow Mathematical Papyrus Problem 4

This is an incomplete problem that deals with the dimensions of a triangle. Even though the last half of this problem is missing the diagram at the end of the problem gives us the full working. The mathematical content of this problem is straightforward. We are given a triangle where the length of one side is ten, and that of another is four. The arithmetic at the bottom shows the scribe found a half of one side and multiplied it by the other to give the area. Problem six proves that the scribe knew the relationship between the area of a triangle and a rectangle, so this problem is elementary. The workings in this problem are shown in the direction that they appear in the papyrus, so they should be read from right to left.

Moscow Mathematical Papyrus Problem 7

1)*tp n ỉrt spdt*

Example of working out a triangle.

2)*mỉ ḏd n=k spdt nt ꜣḥt 2 ỉdb n 2 ½*

If it is said to you a triangle, area two (thousands of land), bank of two and a half

3) *ỉr.ḫr=k kꜣ[b=k ꜣ]ḥt ḫpr.ḫr 40 ỉr sp 2 ½*

You are to make it double area becomes forty[7] make (it) times two and a half.

[7] This appears to be a mistake, but the scribe has changed units. See commentary.

4)ḫpr.[ḫr 100 ir ḳnbt ḫpr.ḫr] 10 nis wʿ ḫnt 2 ½

it becomes [one hundred, make the square root becomes ten]. Summon one from two and a half,

5)ḫpr [.n im p]w 1/3 1/15 ir n 10 ḫpr.ḫr 4

this becomes two-fifths (1/3 + 1/15 = 2/5) do this to ten, becomes four.

6)10 pw m ꜣw r 4 m sḫw

It is 10 in length by 4 in breadth.

Differences In Gunn and Peet's Transcription

Line 4

ḫpr.[ḫr100 ir ḳnbt=f m] 10 nis wʿ ḫnt 2 ½

result [one hundred. Take its square root, namely ten.] Evoke 1 from 2 ½

Here the only difference lies in the reconstructed section. Peet inserts the second person singular suffix pronoun. The meaning of the line is not significantly altered.

Line 5

ḫpr [t im p] w 1/3 1/15 ir n 10 ḫpr.ḫr 4

what results is two-fifths . Apply this to 10 result four.

Again the only difference is in the reconstructed section. Peet reconstructs a *t* at the end of the verb *ḫpr* where Struve places an *n*. The mathematical sense of this sentence is not changed.

Both forms of the verb *ḫpr* are unusual in a mathematical context. The most usual form of the verb is in the *sḏm.ḫr=f* form. This verb form is common in injunctions and as statements of result[8]. In mathematical texts it lends a feeling of a continuing process as it has a very strong narrative sense. In fact, the verb *ḫpr* appears in the *sḏm.ḫr=f* form in every case in the Moscow Mathematical Papyrus (apart from a special case noted below), except for once in Problem 9. At the beginning of line 26 it appear as just *ḫr.*

The verb is also used in the expression *irt mi ḫpr*: *The doing as it occurs*. This is an expression that is used to introduce a section of arithmetical working. However, in this case the verb is not serving as part of the continuing, narrative process, instead it is a phrase that commonly comes at the end of the narrative section.

Unlike the Rhind Mathematical Papyrus, the Moscow Mathematical Papyrus has very few arithmetical sections. They occur only in Problems 4, 6, 14 and 17. In only one case is the phrase *irt mi ḫpr* used, in Problem 6, however, the end of Problem 4 has been lost, and it is possible that it occurred here. Problem 9 is again an exception to this rule as there is no arithmetical section at the end of the problem but this phrase is still used. The last line of the problem reads: *irt mi ḫpr gm=k nfr* : *The doing as it occurs, you will find it correct.* This is in stark

[8] Gardiner, A. (1957) *Egyptian Grammar*; 3rd ed.; Griffith Institute; Oxford; p. 346.

contrast to the Rhind Mathematical Papyrus where the vast majority of problems finish with the arithmetical working. The problems that deal with the doubling of unit fractions contain very few words at all. The majority of the other problems in the Rhind Mathematical Papyrus each give a brief title to the problem, which then consists almost entirely of arithmetical procedure.

Commentary on Moscow Mathematical Papyrus Problem 7

This problem is similar in nature to MMP 6[9] because it deploys the same methodology, but the starting point for this problem is a triangle. The triangle has a known area and the length of one side is known and the ratio of the length of one side to the other is also known. This problem gives us a triangle of a known area and we are asked to find the lengths of the sides so that one side is two-fifths the length of the other side. The only difference to the method used in MMP 6 is that you first have to multiply the area by two, because a triangle is half the area of the rectangle that contains it. From this point it is easy to multiply the area by two and a half and find the square root. Note that the area is not scaled up by the reciprocal of the ratio of the two sides in this case, because the ratio given is greater than one. Therefore, after we have found the square root, we do find the reciprocal and multiply it by the length that we have found, giving us the length of the shorter side.

It is interesting to note that in this problem the scribe moves from one unit of measurement to another without explaining this. In line two the area is given as two thousands of land. This is a unit equal to the area of a thousand strips one

[9] See later section.

cubit wide by 100 cubits long (100 cubits = 1 *khet*.). To make the mathematics easier the scribe coverts this measurement into *arurae* which are equal to a square *khet*. At the end of this problem we are not supplied with a demonstration that we have found the correct answer.

Moscow Mathematical Papyrus Problem 17

1)*tp n irt sbdt*[10]

Example of working out a triangle.

2)*mi ḏd n=k sbdt nt 20(?) m 3ḥt=s*

If it is said to you a triangle of twenty(?) in area;

3) *ir di.t=k ḥr 3w di.n=k 1/3 1/15 iw=f iry ḥr sḫw*

that which you put on the length, you put two fifths of it on the breadth .

4) *ir.ḫr=k k3b=k 20 ḫpr.ḫr 40*

You are to double the twenty, it becomes forty.

5) *ir.ḫr=k ir=k 1/3 1/5 r gmt 1 ḫpr.ḫr r sp 2 ½*

You are to work on a third and a fifteenth in order to find one, it becomes two and a half times

6) *ir.hr=k ir=k 40 sp 2 ½ ḫpr.ḫr 100 ir.ḫr=k ir=k ḳnbt=s*

You are to make forty times a third and a fifteenth; it becomes one hundred. You are to take its square [root];

7) *ḫpr.ḫr 10 mk 10-pw m 3w ir.ḫr=k ir=k 1/3 1/15*

it becomes ten. Look, it is ten in length. You are to take a third and a fifteenth

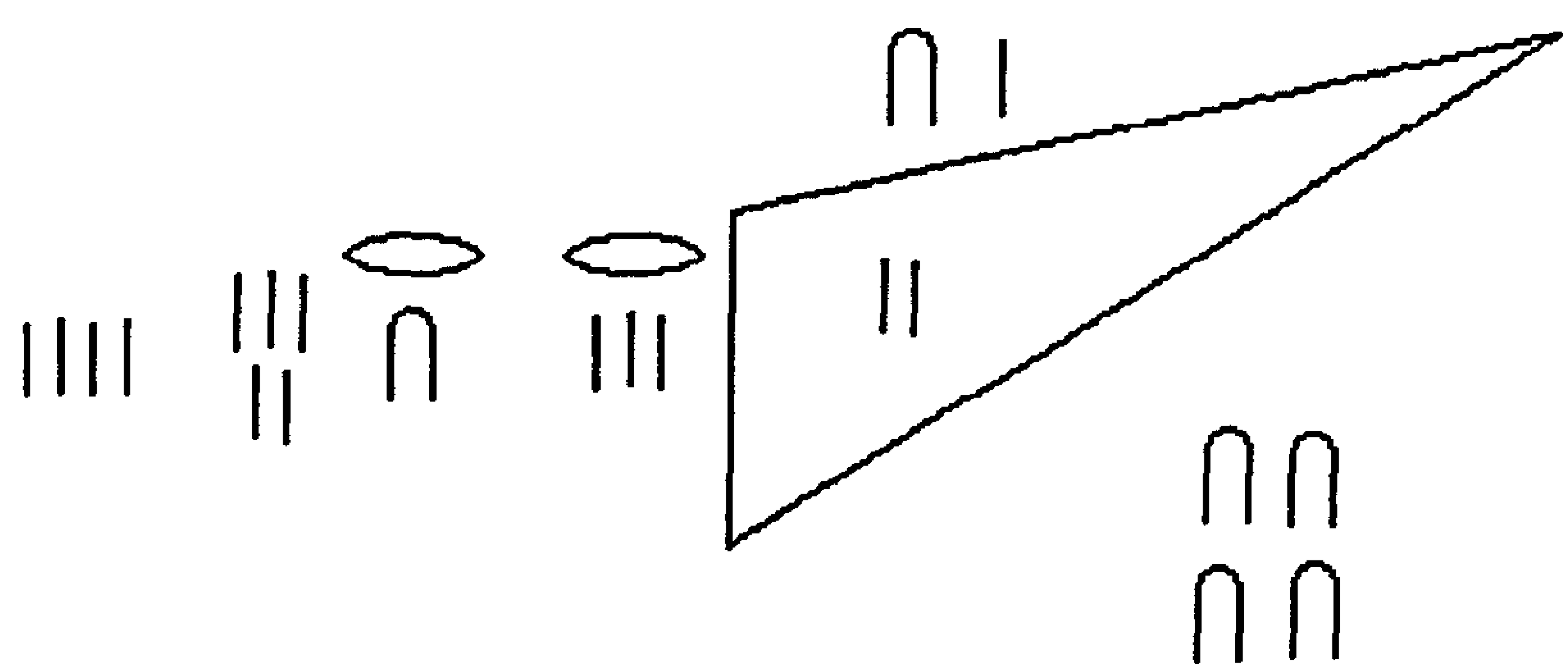
[10] Transliterated from Struve (1930) *Op. Cit. sbdt* is given instead of the usual noun for a triangle, *spdt.*

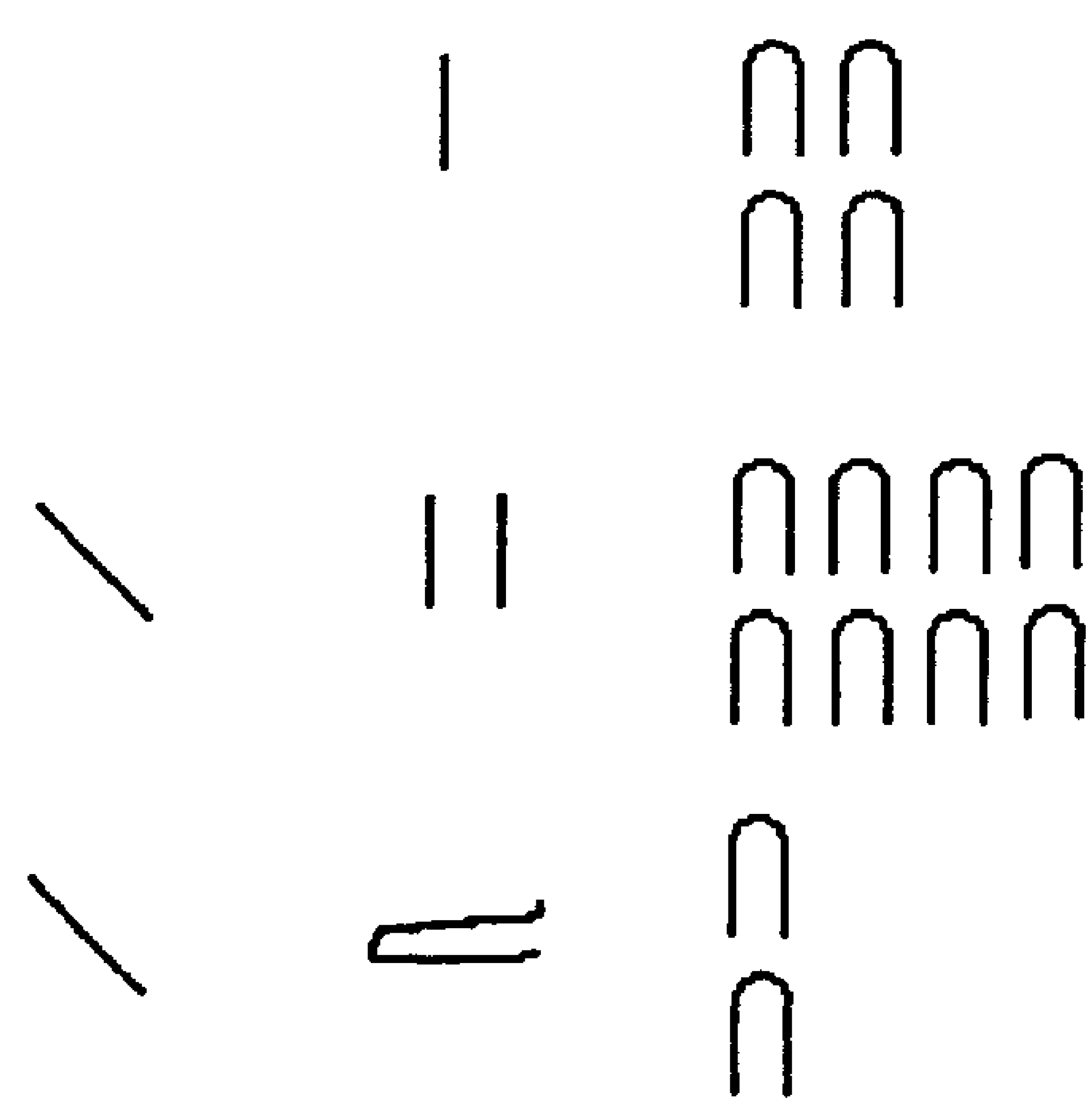
8) *n 10 ḫpr.ḫr 4 mk 4 -pw ḥr sḫw*

of ten, it becomes 4. Look, it is 4 on the width.

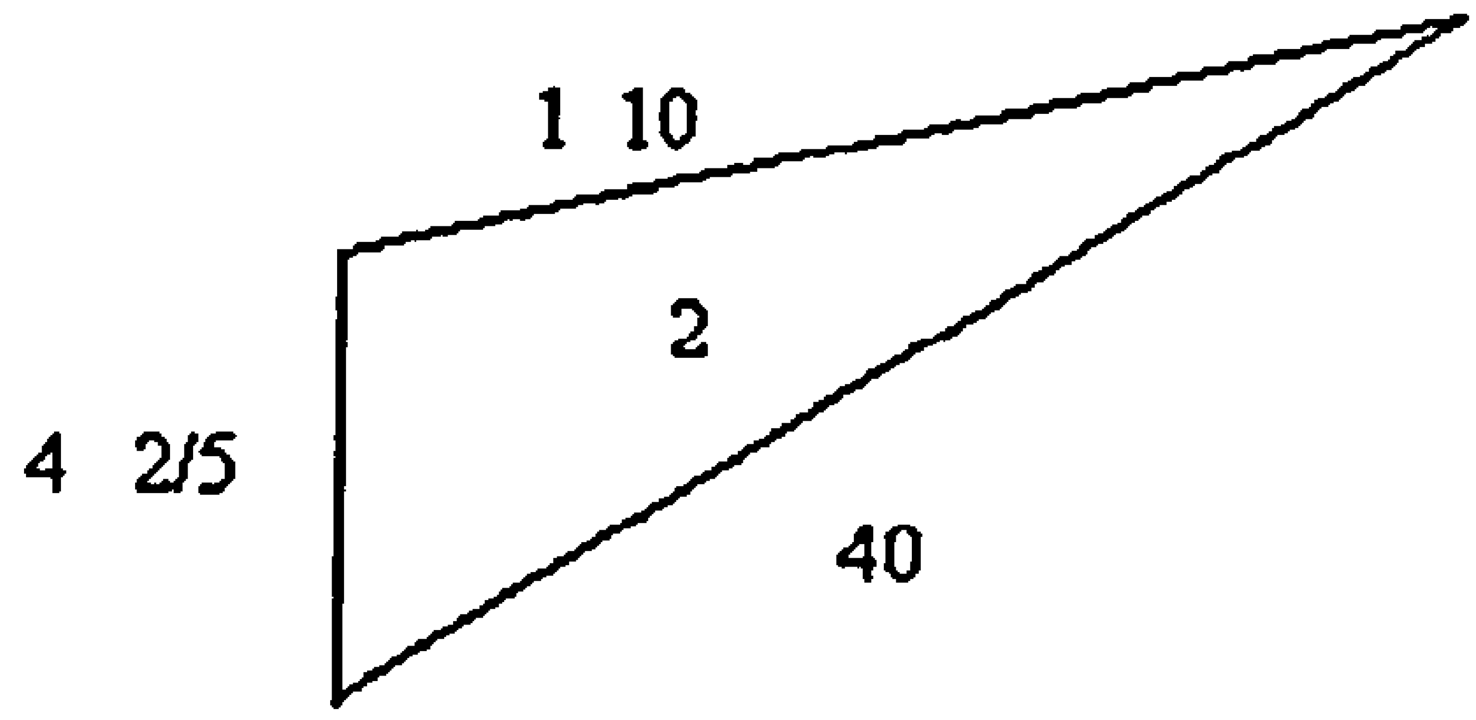
9) *gm=k nfr*

You will find it correct.





dmḏ 100 ḳnbt 10



1	40
\2	80
\ ½	20

Total 100 Square root 10.

Differences in Gunn and Peet's Transcription

Line 2

mi ḏd.n=k sbdt nt 2000 m ꜣḥt=s

If you are told: A triangle of 2 thousand(s-of-land) in its area,

Struve places a question mark over his transcription of 20 in the transcription accompanying the plates. In the text of the translation he translates the number as 20 quadruple *khet.* Gunn and Peet note that the hieratic sign for two thousand is abnormally formed but that, in their opinion, the reading is not in doubt (Gunn and Peet, 1929, p. 174) For a discussion of units in Egyptian mathematics see below. The accompanying diagram to MMP17 shows a triangle with a figure 2 clearly written in the centre. This probably suggests that Gunn and Peet are correct in their reading of two thousands-of-land.

Line 3

ir dit=k ḥr ꜣw dd=k 1/3 1/15 ir ḥr sḫw

and what you put on the length, you must put 2/5 thereof on the breadth.

The transliteration of the accompanying diagrams also shows differences. In Gunn and Peet's paper a second 4 is inserted on the left hand-side where Struve conjectures a second line of a multiplication sum.

Commentary on Moscow Mathematical Papyrus Problem 17

This is a very similar problem to MMP 7, where the area of the triangle and the ratio of the lengths are again known. It is interesting, though, because it highlights a problem with Egyptian fractions. Again we are dealing with a triangle with a known area and we are told that the breadth of the triangle is two-fifths of the length. This is in reality exactly the same problem as Problem 7. We can see using modern fractions that two–fifths is simply the reciprocal of two and a half. This problem is included in the papyrus because of the difference this makes to the method of calculation. In Problem 6 (see below) we use the factor given in the problem to scale up the rectangle into the imaginary square. In this problem, because the factor stated is less than one we must use the reciprocal to turn the rectangle into a square. After we have found the square root, we can use the stated factor to find the breadth. This is the same method of operation as Problem 6, when the factor stated at the beginning of the problem was also less than one.

At the end of this problem, unlike Problem 7, we are shown a diagram to demonstrate that the answer is correct. The diagram that we are shown is more complicated than the diagram accompanying Problem 6. We are given a picture of the triangle inside the triangle the area 2 is shown. Surrounding the triangle we are given details of the arithmetic necessary to complete the problem. At the bottom the calculation of 2 and a half times forty is shown we are told this equals one hundred and the square root is quoted as ten. We are also told on the left hand side that 1/3 and 1/15 times ten is equal to four, although we are not shown how this calculation is carried out.

Moscow Mathematical Problem 6

1) *tp n irt p[t]*

Example of working out a rectangle[11]

2) *mi ḏd n=k [p]t nt stty ½ ¼ n ȝw n sḫw*

If it is said to you a rectangle of <12 in> area a half and a quarter of the length is the breadth:

3) *ir.ḫr=k ir=k ½ ¼ r gmt wꜥ ḫ*pr.*ḫ*r *m 1 1/3*

You are to take three-quarters in order to find one. It becomes one and a third.

4) *ir=k* [12] *pn nt[t] m stty 1 1/3 ḫpr.ḫr 16*

You take this twelve, which is the area {and multiply by} one and a third, becomes 16.

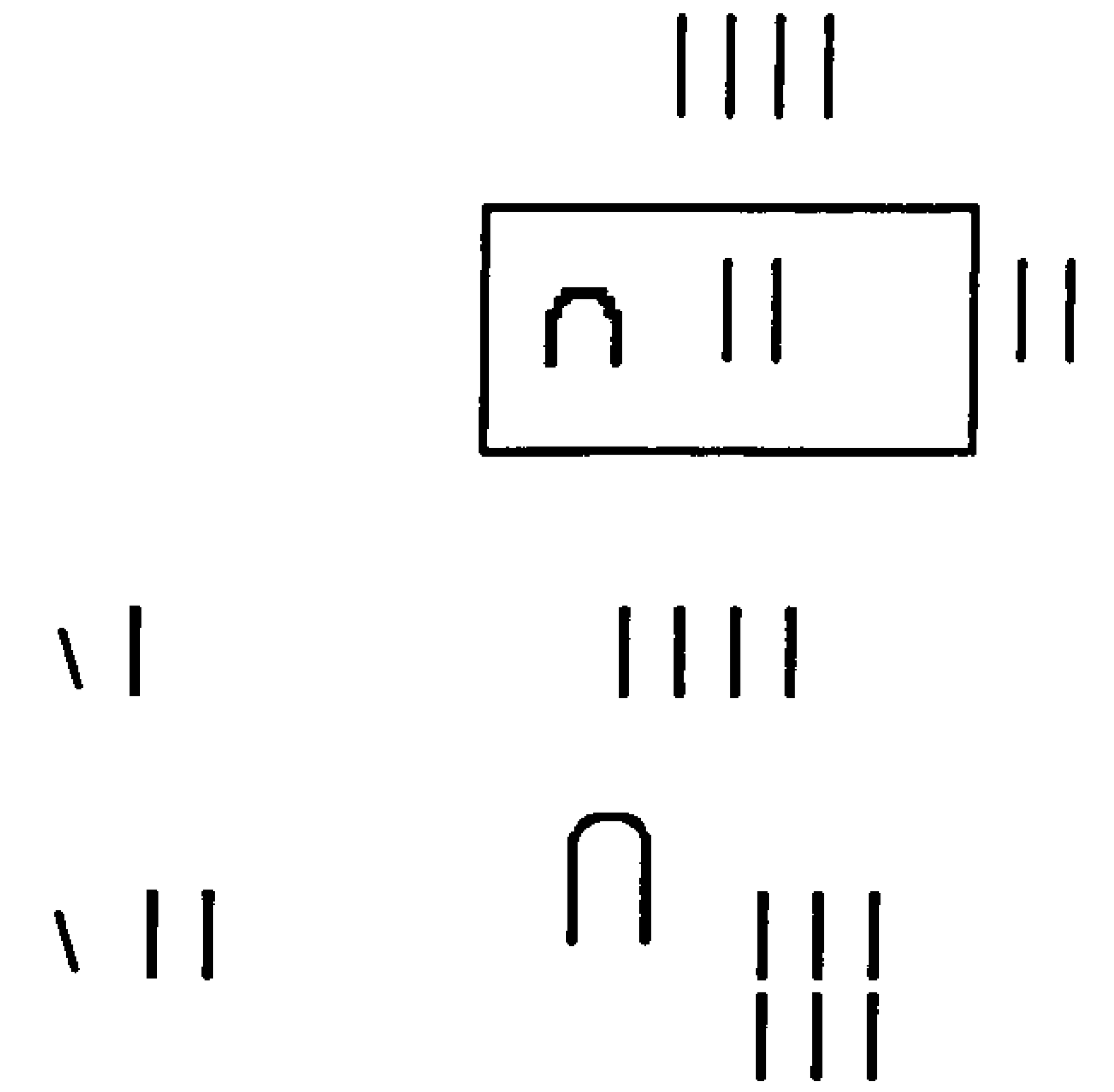
5) *ir.ḫr=k ir=k ḳnb.t ḫpr.ḫr 4 n ȝw ¾=f m 3 n*

You are to make a square root; result 4 for the length three quarters of it namely 3 for

6) *sḫw irt mi ḫpr*

the breadth. The doing as it occurs:

[11] *pt* is translated as rectangle because the accompanying diagram shows the shape that we have to work with. There can be no doubt that *pt* refers in this problem to a geometrical figure. The title of the problem contains the name of the figure that we have to work on. It seems that here *pt* is related to the word for base, *p*. However, this is the only problem in the Moscow Mathematical Papyrus to deal with a rectangle. Compare to RMP49 where a different word for rectangle is used: *ifd* a word derived from *fdw* the numeral 4 (Peet. T.E. (1923) *The Rhind Mathematical Papyrus: British Museum 10057 and 10058,* University of Liverpool Press, London. p.85)



\ 1 4

\ 2 16

Commentary on Moscow Mathematical Papyrus Problem 6

In this problem we are faced with a rectangle with an area of 12 square *khet* and we need to find its dimensions. We are told that the breadth is three-quarters of the length. The first step is to take the reciprocal of three quarters, which is one and a third. Note that because we are dealing with the unit fractions of a half and a quarter finding a reciprocal in Egyptian mathematics is much harder than with modern fractions that have both the denominator and a numerator. With a modern fraction the numerator and denominator are swapped, so that the fraction is turned upside down. With the Egyptian fractions the scribe has to operate on the fraction to find one, i.e. he has to work out what number is needed to multiply three-quarters to become one. This calculation is done so that it is possible to scale up the rectangle so that it becomes an imaginary square. It is then easy to work out the longest side because this will be the square root of the area of the imaginary square that has been made. The ratio of the longest to the shortest sides is given to us when the problem was stated at the beginning, so this factor, three-quarters, is used to calculate the breadth.

The modern algebraic notation for the mathematics contained in this problem would run as follows:

$$x \times \frac{3}{4}x = 12$$

$$\frac{3}{4}x^2 = 12$$

$$x^2 = 12 \times \frac{4}{3}$$

$$x^2 = 16$$

$$x = 4$$

At the end of the problem we are shown that we have found the correct answer. We are given a drawing of the enclosure with one side marked with a four, and the other side marked with a three. The arithmetic following from the drawing seems incorrect. Struve hypothesised[12] that a line is missing from the calculation and that it should read:

1	4
2	8
\4	16

[12] Struve. (1930) *Op. Cit.* p.127

3.4: Algebraic Problems from the Moscow Mathematical Papyrus

Moscow Mathematical Papyrus Problem 19

1) *tp n irt ꜥḥꜥ iry sp 1 ½ ḥnꜥ*

Example of working out a heap. Make one and a half times together with

2) *4 ii.n=f r 10 m ꜥḥꜥ dd–sw*

four, it has come as ten. What heap says this?

3) *ir.ḫr=k ir=k ꜥꜣ n pꜣ 10 r pꜣ 4 ḫpr.ḫr 6*

You shall work out the excess of the ten over the four; it becomes six.

4) *ir.ḫr=k ir=k 1 ½ r gmt 1 ḫpr.ḫr 2/3 ir.ḫr=k*

You shall work out one and a half in order to find one, it becomes two thirds.

5) *ir=k 2/3 n pꜣ 6 ḫpr.ḫr r 4 mk m 4*

You shall work out two-thirds of the six, it becomes four. See, it is four,

6) *dḏ – sw gm=k nfr*

says it. You will find it good.

Commentary on Moscow Mathematical Papyrus Problem 19

This problem deals with the solution of a simple equation, but it is interesting because of the way the problem is solved. This problem shows the use of mathematical techniques that are used when solving algebraic problems, even though there is no evidence that the Egyptians used any form of algebra. It is solved in exactly the same way a modern mathematician would; there is just different notation. Expressed with modern algebraic notation the problem would read:

$$\frac{3}{2}x+4=10$$

$$\frac{3}{2}x=10-4$$

$$\frac{3}{2}x=6$$

$$x=6\times\frac{2}{3}$$

$$x=4$$

This method is interesting because it shows that the author of the Moscow Mathematical Papyrus did not rely on the method of false assumption to solve equations, as the author of the Rhind Mathematical Papyrus seems to have done. The method of false assumption is illustrated by the following problems from the Rhind Mathematical Papyrus. To simplify the presentation of the problem I have omitted the hieroglyphic numbers, which form the arithmetical calculation.

Rhind Mathematical Papyrus Problem 24

1) *ꜥḥꜥ $\overline{7}$ =f ḥr=f ẖpr=f m 19*

A heap, a seventh of it on it becomes nineteen,

2) *ꜥḥꜥ irt mi ẖpr*

[what is the] heap? The doing as it occurs:

\1	7	1	8	\\$\overline{4}$	2
\\$\overline{7}$	1	\2	16	\\$\overline{8}$	1
		$\overline{2}$	4		

\1	2 $\overline{4}$ $\overline{8}$
\2	4 $\overline{2}$ $\overline{4}$
\4	9 $\overline{2}$

1	16 $\overline{2}$ $\overline{8}$
$\overline{7}$	2 $\overline{4}$ $\overline{8}$
Total	19

The method of solving the problem can be explained thus: suppose the heap (i.e. the unknown quantity) is equal to one. The total of one times one and a seventh is eight. This sum is shown in the first column. The scribe then takes eight and works out what factor is needed to multiply eight by to give nineteen. This is worked out in the second and third columns. The answer is two and a quarter and an eighth. The same factor that is needed to convert eight into nineteen is needed to convert seven into the correct quantity of the heap. The next column shows two and a quarter and an eighth times seven. This gives the total of sixteen, a half and an eighth. The last column shows that one and a seventh times sixteen a half and an eighth is equal to nineteen.

A very similar problem using the same method is given below to further illustrate the method:

Rhind Mathematical Papyrus Problem 25

1) *ꜥḥꜥ* $\bar{2}$*=f ḥr=f ḫpr m 16*

A heap, half of it on it, becomes sixteen.

1	2	\1	3	$\bar{\bar{3}}$	2
$\bar{2}$	1	2	6	\\$\bar{3}$	1
		\4	12		

1	5 $\bar{3}$
\2	10 $\bar{\bar{3}}$

irt mi ḫpr

The doing as it occurs:

The heap:	1	10 $\bar{\bar{3}}$
	$\bar{2}$	5 $\bar{3}$
	Total	16

Here the numbers are different but the process is exactly the same. We take the false position that the amount in the heap is two. One and a half times by two is three. We then ask what factor is needed to turn three into sixteen, i.e., what do we have to do to three to get sixteen. The answer is given by the sum of the ticked numbers in the left-hand side of the columns two and three. The answer is the sum of one, four and a third which gives five and a third. We are finally shown that one and a half times by ten and two thirds is indeed sixteen. MMP 19 uses the method that is taught today for the solution of algebraic problems. Instead of using a trial example, as in the problems from the Rhind Mathematical papyrus, MMP 19 manipulates the equation until the required variable is found.

Moscow Mathematical Papyrus Problem 23

1) *tp n irt b3kw ṯbw*

Example of working out the work of a sandal maker.

2) *mi dd n=k b3kw ṯbw ir wd*ʿ*=f*

If it is said to you, the work of a sandal-maker, if he cuts

3) *n r' 10 ir db3=f n r*ʿ *5*

for 10 days if he replaces for five days.

4) *in iw wd'=f db3=f n r' 1*

What does he cut and replace in one day?

5) *iw=f r wr r ir.ẖr=k ir=k rmny n p3 10 ḥn' p3 5*

the most is done, you shall work out the arms of the ten and the five.

6) *ẖpr.ẖr dmd r 3 ir.ẖr=k ir=k -sw r gmt 10 ẖpr.ẖr sp 3 1/3*

it becomes a total of three. You shall work on it in order to find ten, it becomes three and a third.

5) *mk 3* $\bar{3}$ *-pw n r*ʿ *1 gm=k nfr*

Look it is three and a third in one day. You will find it good.

Commentary on Moscow Mathematical Papyrus Problem 23

This problem is the most enigmatic in the whole extant corpus of Egyptian mathematics. We cannot tell for sure what this problem means, and unlike other problems where the translation is made easier by the mathematical context, here that mathematics serves to make it more obscure. Like the *pesu* problems it possibly has a basis in the calculations of economics that a scribe could be expected to perform in his duties. The function of 'finding the arms' is particularly curious. We start with ten and five, and finish with three. It is possible that it is referring to some kind of balance. If we were to balance ten and five we would require a balance with the fulcrum a third of the way along. However, how this relates to the activities of the sandal maker is not made clear.

The problem appears to be trying to find the work that a sandal maker does in one day if in a fifteen-day period he cuts for ten days and replaces for five of those days. To make this problem make sense we require more information about what it is trying to achieve. We may also require more information about the work of a sandal maker.

Moscow Mathematical Papyrus Problem 25

1) *tp n irt ꜥḥꜥ iry sp 2 hnꜥ ii r 9*

Example of working out a heap, two times it on it, comes as nine.

2) *inꜣ ꜥḥꜥ dd-sw ir.ḫr=k ir=k dmd n pꜣ ꜥḥꜥ ḥnꜥ pꜣ 2*

What heap says it? You shall work out the total of the heap together with the two

3) *ḫpr.ḫr r 3 ir.hr=k ir=k pꜣ 3 r gmt 9 ḫpr.ḫr sp 3*

it becomes three. You will work on the three in order to find nine, it becomes three times.

4) *mk 3 dd-sw gm=k nfr*

Look it is three says it. You will find it good.

Commentary on Moscow Mathematical Papyrus Problem 25

This is another problem involving an unknown quantity. The same word *'h'* for the unknown quantity. This problem may be expressed algebraically as:

$$x + 2x = 9$$
$$3x = 9$$
$$x = 9 \div 3$$
$$x = 3$$

Given the skill that the scribe showed in problem 19 this is an elementary problem.

3.5: Conclusions

The Moscow Mathematical Papyrus contains a wide range of geometrical and algebraic problems, although these problems are not ordered in any way. The Rhind Mathematical Papyrus, in contrast, ordered the problems in broad groups. It is also interesting that very few of the problems from the Moscow Mathematical Papyrus show any arithmetic. Section 2.7. proposed the idea that the texts only contain arithmetic where this is the object of completing the problem, in geometrical problems of the Rhind Mathematical Papyrus the arithmetic was sometimes absent. The Moscow Mathematical Papyrus follows this trend. Most of the problems in the Moscow Mathematical Papyrus are geometrical or algebraic in nature as can be seen from the contents list in Section 3.2. This shows that the object of writing, or copying, the Moscow Mathematical Papyrus was to explore geometry and algebra.

The arithmetic of the Rhind Mathematical Papyrus showed an adaptability and skill on the part of the scribe who prepared it. The same is true of the geometry of the Moscow Mathematical Papyrus. MMP 19 shows that the Egyptians could conceive of algebraic problems in an abstract way. They were not limited by an inability to conceive of quantities in a purely physical and definite way. The geometrical problems investigated in this chapter demonstrate that the Egyptians had a curiosity about different shapes and their properties. These problems are, however, elementary when compared with MMP 14 and MMP 10, the subject of the next chapter.

Chapter 4

Moscow Mathematical Papyrus Problems Fourteen and Ten

Following the investigation of some of the simpler problems in Egyptian geometry, this chapter will turn to two of the most complicated and enigmatic. Both of these problems are contained in the Moscow Mathematical Papyrus and both question the notion that Egyptian geometry was straightforward and incapable of being abstract.

4.1: Introduction

This chapter will look at the two most intriguing problems not just from the Moscow Mathematical Papyrus but also from the whole of Egyptian mathematics. The first edition of the Moscow Mathematical Papyrus[1] appeared after many of the editions of the Rhind Mathematical Papyrus. When Struve's edition was published, Peet reviewed it for the *Journal of Egyptian Archaeology*. Peet, although admiring the work that Struve put into preparing the edition, repeated his opinion that the Moscow Mathematical Papyrus had nothing, apart from problem 14, dealing with the truncated pyramid, that would alter the opinion he had of Egyptian mathematics gained from other texts, most notably

[1] Struve W.W. (1930) *Mathematisher Papyrus des Staatlichen Museums der Schönen Künste in Moskau, QSGM,* Abt. A: Quellen; Berlin.

the Rhind Mathematical Papyrus[2]. He includes MMP 10 in this assessment. There had already been plenty of interest in the MMP 14[3], presumably because of Peet's opinion of the problem. Before the full publication of the Moscow Mathematical papyrus, Peet and Gunn published an article in the *Journal of Egyptian Archaeology*, which discussed the problem at length and offers several possible dissections of the truncated pyramid. This article discussed three other problems: numbers six, seven and seventeen. It makes no mention of number ten. Peet gives his most detailed opinion of Moscow Mathematical Papyrus Problem 10 in another paper published in the *Journal of Egyptian Archaeology*[4], in this article Peet presents his arguments against the translation of the problem given in Struve's edition of the Moscow Mathematical Papyrus. Peet uses linguistic arguments for his interpretation of the problem. Since the publication of the article, there have been few commentators on the problem with the necessary knowledge of hieroglyphs to examine the differences and give a definitive translation of the problem. It is usual to find that when this problem is discussed, both Peet's and Struve's interpretations are given with no further comment because the author does not have the expertise to judge the rival claims[5]. Section 4.2 will explore Peet's arguments in detail and provide examples from other mathematical problems to elucidate his points. It will also

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[2] Peet, T.E. (1931) Review of Struve's edition of the Moscow Mathematical Papyrus; *Journal of Egyptian Archaeology*; vol. 17; pp. 154-60.

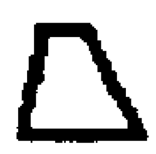
[3] W. R. (1931) "Moscow Mathematical Papyrus, No. 14" *Journal of Egyptian Archaeology*; vol. 17; pp 50-2. Vogel, K. (1930) "The truncated pyramid in Egyptian Mathematics"; *Journal of Egyptian Archaeology*; vol. 16; pp 242-49. Gunn, B. and Peet, T.E, (1929) "Four Geometrical Problems from the Moscow Mathematical Papyrus" *Journal of Egyptian Archaeology*; vol. 15; pp 167-85. Turaiev, B. (1917) "The Volume of the truncated pyramid in Egyptian mathematics," *Ancient Egypt*; London; pp 100-02;

[4] Peet, T.E. (1931) "A Problem in Egyptian Geometry"; *Journal of Egyptian Archaeology*; vol. 17; pp 100-06.

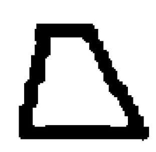
[5] See for example: Joseph, G. G. (2000) *The Crest of the Peacock: Non-European Roots of Mathematics*; 2nd ed.; Penguin, Harmondsworth; pp 87-9. Also Van Der Waerden, B.L. (1954) *Science Awakening*; P. Noordhoff ltd; Groningen; pp 33-34.

offer an alternative translation of the problem, which satisfies both the arguments of Peet and Struve.

4.2: Moscow Mathematical Papyrus Problem 14

1) *tp n irt* 

Example of working out a truncated pyramid[6]

2) *mi ḏd n=k*  *n 6 n štwti*

If it is said to you a truncated pyramid of six in vertical height[7]

3) *r 4 ḥr ẖr r 2 ḥr ḥry*

by four on the base by two on top.

4) *ir.ḫr=k ir=k 4 -pn m iw ḫpr.ḫr 16*

You are to square this four, it becomes sixteen.

5) *ir.ḫr=k ḳꜣb=k 4 ḫpr.ḫr 8*

You are to make double four, it becomes eight.

6) *ir.ḫr=k ir=k* 2 *pn m iw ḫpr* 4

You are to square this 2 ,it becomes four.

7) *ir.ḫr=k dmḏ=k pꜣ* 16

You are to make the total of the sixteen

8) *ḥnʿ pꜣ* 8 *ḥnʿ pꜣ 4*

together with the eight together with the four.

[6] This problem is unusual because the name of the figure it is concerned with is not spelt out. The only derivation for this symbol, suggested by Gunn and Peet is the determinative of *is*, the word for the plinths of old solar obelisks that had roughly the shape of a truncated pyramid. However, these were regular figures, where the figure in Moscow Mathematical Papyrus 14 is clearly not.

[7] The derivation and meaning of this word is unknown. It does not appear in texts outside of the Moscow Mathematical Papyrus. However, it clearly refers to the height because of the accompanying diagram. For further discussion, see Gunn, B. and Peet, T.E. (1929) *Op Cit.* p.178.

9)*ḫpr.ḫr* 28 *ir.ḫr=k ir=k*

it becomes twenty-eight. You are to make

10) $\bar{3}$ *n 6 ḫpr.ḫr 2 ir.ḫr=k*

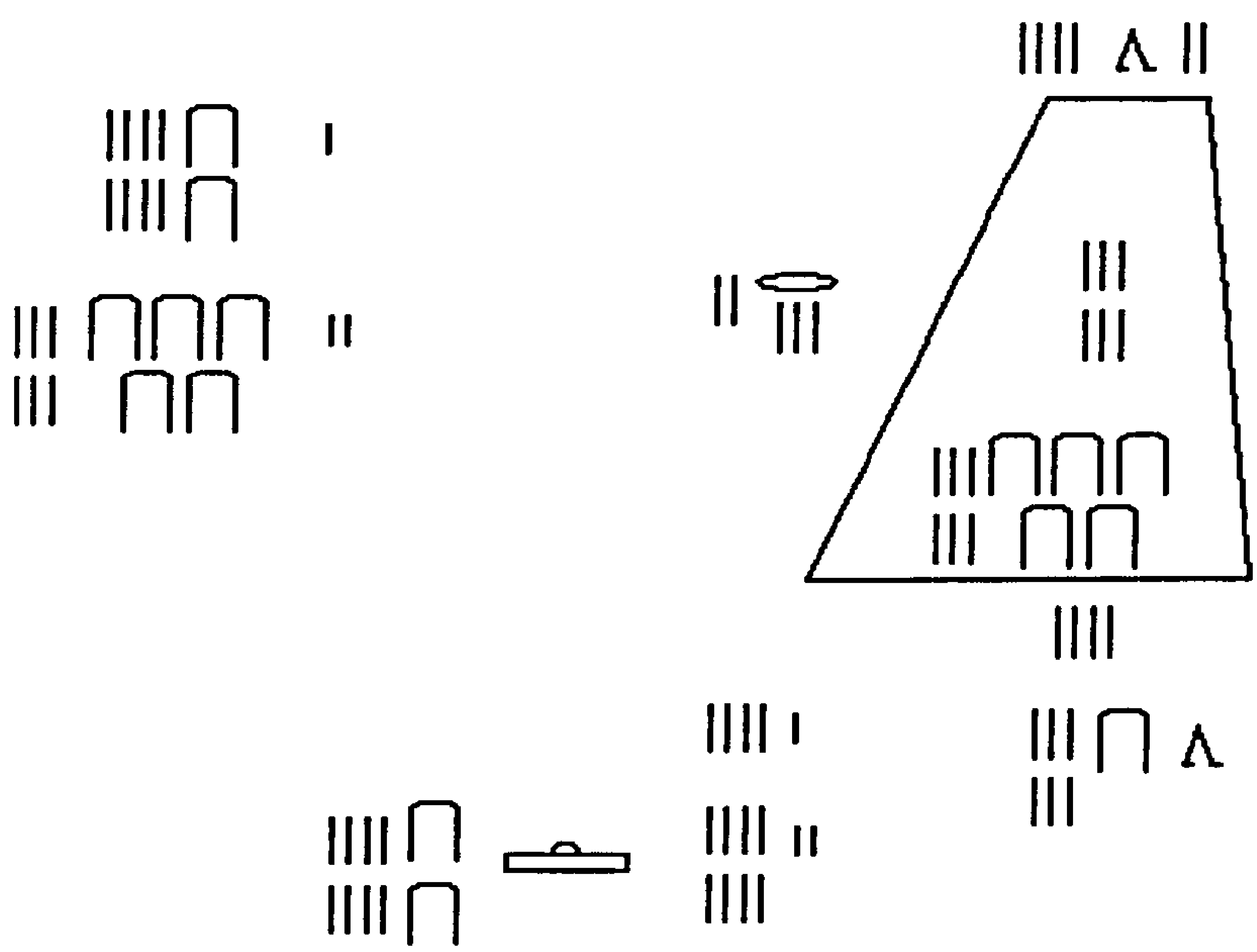
a third of six, it becomes two. You are to make

11) *ir=k 28 sp 2 ḫpr.ḫr 56*

twenty-eight times two, it becomes fifty-six.

12) *mk n=f-sw 56 gm=k nfr*

See, it is of fifty-six. You will find it good.



4.2.1: Commentary on Moscow Mathematical Papyrus Problem 14

Together with MMP 10, this problem is one of the greatest Egyptian mathematical achievements. In this problem, the correct formula for working out the volume of a truncated pyramid is used. The formula can be represented algebraically[8] as:

$$V = h/3\ (a^2 + ab + b^2)$$

Where *h* is the height, *a* the length of the base and *b* the length of the truncated side.

It is not known how the Egyptians achieved this remarkable result[9]. The simplest solution is to suggest that the Egyptians worked out the solution using a model made out of Nile mud that was cut into small pieces and weighed to give the proportions. However, I feel that this ignores the evidence of the skill of the mathematicians as indicated by other problems in this papyrus. Other problems in the Moscow Mathematical Papyrus suggest that the scribe had at least some skills that we would associate with algebra. A plausible suggestion is that the truncated pyramid was split into sections and reformed to make solids of known volume. This did not have to be achieved by cutting up Nile mud, but through the imagination and reasoning of the scribe. We have seen from several of the problem in the Moscow Mathematical Papyrus that the mathematicians were able to work out the area of a figure of known dimensions and then scale it to the

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[8] See Fig. 3.1. for a diagram.

[9] For a detailed summary of the earliest theories on the derivation of this formula see Clagett, M. (1999) *Ancient Egyptian Science: A Source Book. Vol. Three: Egyptian Mathematics*; American Philosophical Society; Volume 232; Philadelphia.

figure in hand, their method of solving *ꜥḥꜥ* [10] problems by the method of false assumption is a similar process. This can also be achieved in three dimensions, albeit with increased difficulty, although there is no explicit evidence in the mathematical texts that the ancient Egyptian did this. There are several ways that a dissection of the truncated pyramid can be achieved to form a sum of shapes which is easier to manipulate. The exact procedure used by the Egyptian scribe that first came up with the correct formula for working out the volume of a truncated pyramid has been lost.

A recent paper by Flora Vafea[11] has suggested a possible geometrical transformation. The premise of the transformation is the division of the height by three and then considering each of the parallelepipeds. This has the virtue of immediately introducing the h/3 term in the eventual equation. However, the geometrical reasoning involved is extremely complicated and there is no evidence that the Egyptians could reason to that level. There are many other interpretations of how this result was achieved[12]

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[10] See Section 3.4.

[11] Vafea, F (2002) "The Mathematics of Pyramid Construction in Ancient Egypt" *Mediterranean Archaeology and Archaeometry* Vol 2:1; pp 111-124. Reproduced with kind permission from the author and publisher.

[12] Gillings, R.J. (Dec. 1964) "The volume of a truncated pyramid"; *The Mathematics Teacher*; Part I: Vol. 59, No. 4, pp 552- 55. Vetter Q. (1933) "Problem 14 of the Moscow Mathematical Papyrus" *Journal of Egyptian Archaeology*; vol. 19; pp 16-18. Thomas, W. R. (1931) "Moscow Mathematical Papyrus, No. 14" *Journal of Egyptian Archaeology*; vol. 17; pp 50-2. Vogel, K. (1930) "The truncated pyramid in Egyptian Mathematics"; *Journal of Egyptian Archaeology*; vol. 16; pp 242-49. Gunn, B. and Peet, T.E, (1929) "Problems for the Moscow mathematical Papyrus" *Journal of Egyptian Archaeology*; vol. 15; pp 167-85. Turaiev, B. (1917) "The Volume of the truncated pyramid in Egyptian mathematics," *Ancient Egypt*; London; pp 100-02;

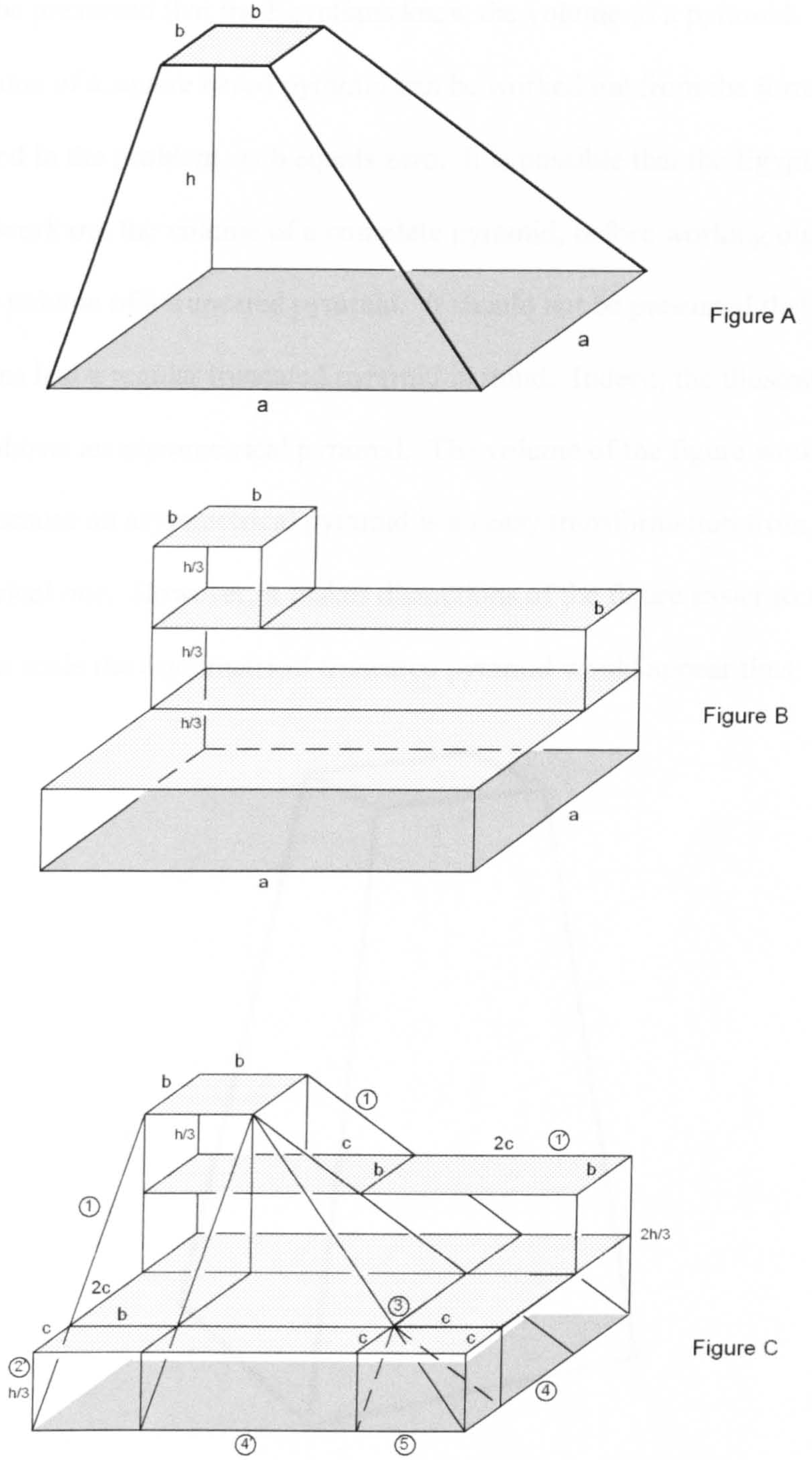

Fig. 3.1 Transformation of a truncated pyramid into a set of three parallelepipeds

It is to be presumed that the Egyptians knew the volume of a pyramid. Indeed, the volume of a square based pyramid can be worked out from the formula contained in the problem, as b equals zero. It is possible that the Egyptians knew how to work out the volume of a complete pyramid, before working out how to find the volume of a truncated pyramid. It should not be presumed that the Egyptians had a regular truncated pyramid in mind. Indeed, the illustration clearly shows an asymmetrical pyramid. The volume of the figure would be the same, because an asymmetrical pyramid is an easy transformation from a symmetrical one. However, it makes dissections of the figure easier to see. Drawn to scale the asymmetrical truncated pyramid would appear thus:

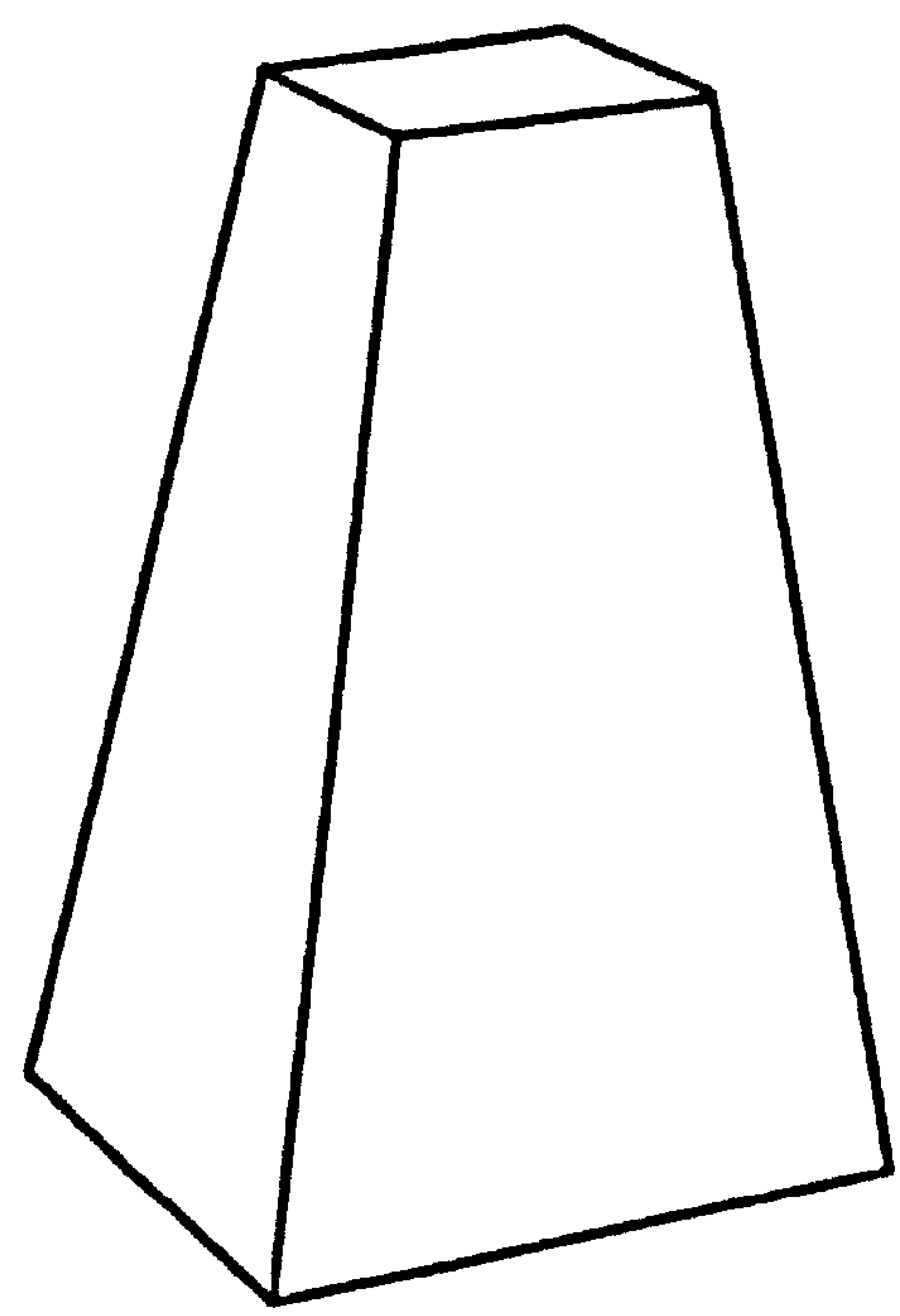

Fig. 3.2: The Truncated Pyramid from MMP 14 Drawn to Scale

This figure has a height of six and a base of four. The section cut off at the top has a base measurement of two. The truncated pyramid can be dissected into a square prism, a square-based pyramid and two triangular pyramids, a common idea for the derivation of the method seen in the Moscow Mathematical Papyrus:

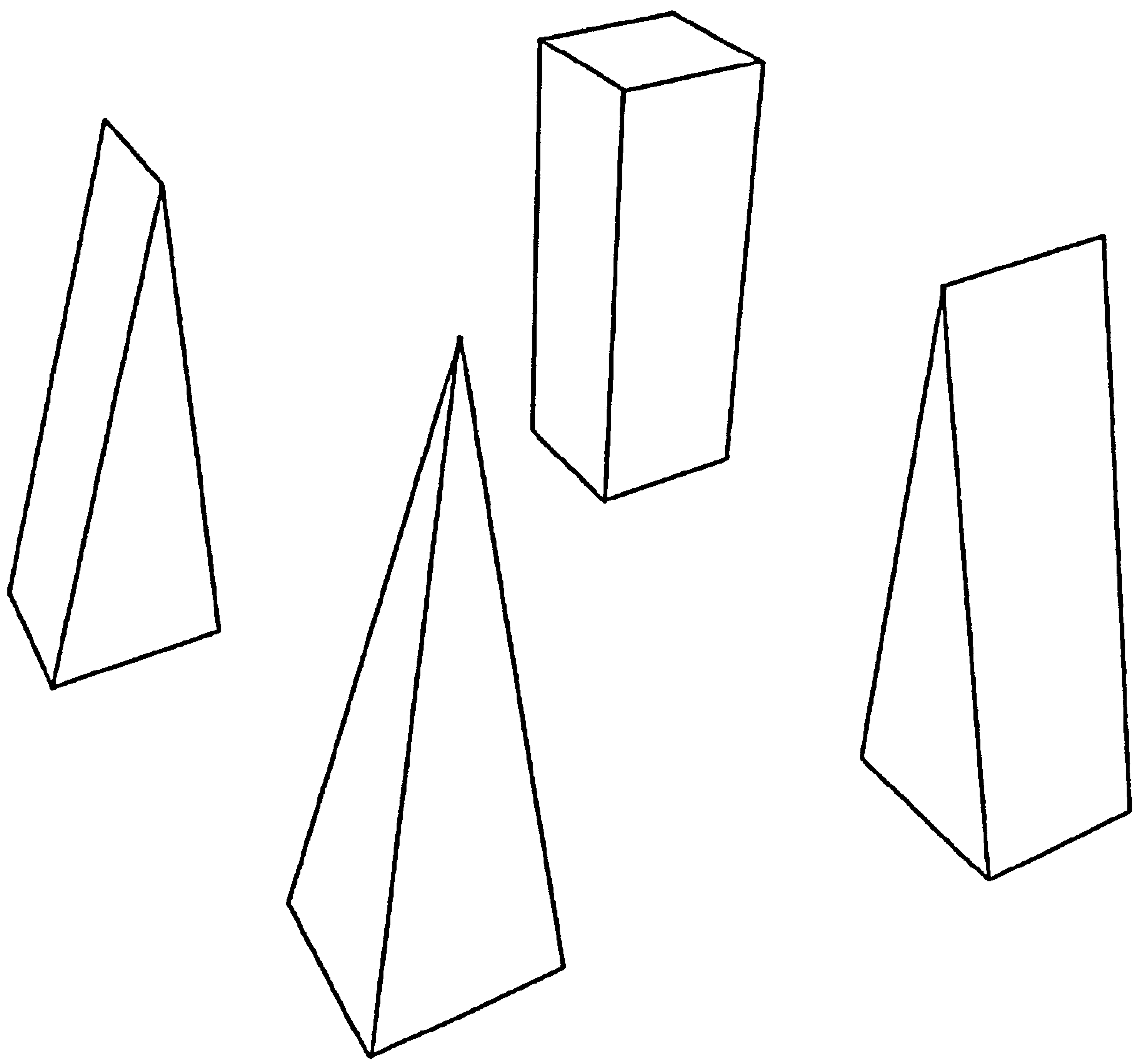

Fig. 3.3: A Possible Dissection of the Truncated Pyramid

The square prism will have a volume of: $h a^2$

The pyramid will have a volume of: $\frac{h}{3}a^2$

The two triangular prisms will each have a volume of: $\frac{h}{2}a^2$

This truncated pyramid is a special case, as the base is twice the length of the cut-off. This enables the simple dissection and also means that the volumes of the four pieces can be expressed as in terms of a. This dissection does not however offer an explanation of the method described in the Moscow Mathematical Papyrus. The first calculation is carried out in line 4. This calculation clearly deals with the length of the bottom side. If the truncated pyramid is cut up in any way then this dimension is cut. Explaining the method seen in MMP 14 will require more algebra than any previous writer on Egyptian mathematics has cared to admit.

The practical applications of this problem are not hard to see considering that this problem comes from the land that built the pyramids. However, the existence of a practical application should not distract us from the abstract nature of the reasoning involved. This is not mere arithmetic brought to bear on an everyday problem. The pyramids were enormous projects undertaken with vast workforces. As such they would have attracted the keenest minds of the kingdom. It should not surprise us then that the building of the pyramids sparked a piece of brilliant deduction and reasoning. This problem, although grounded in application, should be considered as an example of pure mathematics. The precise nature of the reasoning is lost, but it must have existed at some time. Papyrus is a very friable material and does not survive well. The scribe engaged in the process of manipulating the mathematics may not have used papyrus anyway. Ostraca were used for writing on in ancient times as well and discarded readily. These do not survive to the present day unless they were preserved in extraordinary circumstances. When dealing with such a small sample of material, we cannot be too rigid in our criteria for investigation and categorisation. The benefit of the doubt in cases like this should be afforded to the Egyptians. The most pessimistic perception of the achievements of Egyptian mathematics is an extreme position. It is easy to become dogmatic in the assumption that the Egyptians had no mathematical thought beyond the use of arithmetic for the solution of practical problems.

4.3: Moscow Mathematical Papyrus Problem 10: What Shape is a Basket?

There have been several interpretations of this problem, most notably those of Struve and Peet[13]. This section will show that the most likely interpretation is that of a hemisphere, in preference to a semi-circle or a half cylinder. This is therefore a singular piece of mathematics. It is not intuitive to see a curved surface in the same terms that you see a flat surface. The curved surface of half a cylinder can be transformed into a flat surface very easily but the rounded surface of a sphere cannot.

Explaining how this result was achieved is extremely difficult and will rely on a thorough understanding of how the Egyptians thought about the area of a circle, it will be this knowledge that may eventually yield an answer. It is not possible to tell whether this result was achieved through experimentation into how much material was needed to weave a basket of a known size, or whether it came about through logical reasoning. It may be possible that the Egyptians noticed that it takes twice as much material to weave a hemi-spherical basket than it takes to

[13] See Section 4.1.

weave the lid. What can be said, however, is that this problem represents the pinnacle of mathematical achievement in all the surviving papyri.

4.3.1: Overview and examples

This problem is probably the most interesting problem in the mathematical texts that have survived from ancient Egypt. It is certainly the most debated problem. Several rival interpretations have been proposed. The first interpretation put forward by Struve[14] suggested that this problem deals with a hemisphere. This would put this problem apart from the other achievements of Egyptian mathematics because it would be the only problem to deal with a surface that cannot be flattened. To be able to conceive a curved surface in terms of a flat area is no mean feat and strongly suggests that the Egyptians were capable of abstract mathematical thought. Peet, however, was not convinced of this translation of the text and proposed opposing translations that involved either a semicircle or a semi-cylinder. These interpretations of the problem are not as abstract, because they only need the conceptualisation of a two-dimensional space and are less exciting to a historian of mathematics than Struve's interpretation. The arguments surrounding Peet's translations are dependent on our understanding of other mathematical problems, particularly in the way that they are formulated and communicated. Although the mathematical content of this particular problem is unique, the format is not. This problem should therefore only be considered after the character of all the other problems and their unique use of Egyptian words has been understood.

[14] Struve, (1930) *Op. Cit*.p. 157

To this end, I have produced my own translations of some of the more enlightening problems from the Rhind Mathematical Papyrus, which are presented here[15]. These problems deal with points that are raised in the translation of Moscow Mathematical Papyrus Problem 10. I have tried to present the translations in the lines that they appear in the Rhind Papyrus. I have presented them in this way because breaks in line are sometimes, but not always, helpful in following the working of the problem. The arithmetical working that appears at the end of the problems preserves the columns used in the Rhind Mathematical Papyrus as far as possible. These columns are sometimes extremely confusing to a modern reader but they have been preserved because of this difficulty. Mathematics should always be clearly set out if the reader is to learn anything from the papyrus. Some arguments surrounding the mathematical ability of the Egyptians have been based on the untidy nature of their mathematical papyri. I have tried to be true to the layout of the original text, where possible within the confines of the word processor, to retain the confusion to try and be as true as possible to the original text, with apologies to the reader. For further notes on presentation style, see Section 1.5.

[15] These use my own terms "function" and "guidance" that will be justified later.

Rhind Mathematical Papyrus Problem 41

1) *tp n irt š3ᶜ dbn n 9 10 hb.ḫr = k* $\bar{9}$ *n 9 m 1 ḏ3t 8*

Example of working out a container, round of 9 (by) 10; you deduct a ninth of 9, namely 1, remainder 8.

2) *w3ḥ -tp m 8 r spw 8 ḫpr.ḫr 64 ir.ḫr = k w3ḥ -tp m 64*

The function of 8 by times 8 becomes 64. You make the function of 64

3) *r sp 10 ḫpr.ḫr = f 640 di* $\bar{2}$ *= f ḥr = f ḫpr.ḫr = f m 960 rḫt = f m ḫ3rw*

by times 10, it becomes 640; put half of it on it, it becomes 960. The amount of it in khar.

4) *ir.ḫr = k* $\overline{20}$ *n 960 m 48 h33t -pw r = f m 4-ḥk3t šs 4800 ḥk3t*

You make a twentieth of 960 namely 48, this is what goes into it in quadruple hekat. Grain 4800 hekat

5) *ki n sšmt = f*

Form of its guidance

				dmd	
1	8	\8	64	total	960
2	16	1	64	$\overline{10}$	96
4	32	\10	640	$\overline{20}$	48
		\2	320		

Rhind Mathematical Papyrus Problem 42

1) *š3ʿ dbn n 10 10 ḫb.ḫr= k* $\overline{9}$ *n 10 m 1* $\overline{9}$ *d3t m 8* $\overline{\overline{3}}\ \overline{6}\ \overline{18}$

Round container of 10 (by) 10; you deduct a ninth of ten, namely one and a ninth, remainder is 8, two thirds, a sixth and an eighteenth

2) *ir.ḫr= k w3h -tp m 8* $\overline{\overline{3}}\ \overline{6}\ \overline{18}$ *r spw 8* $\overline{\overline{3}}\ \overline{6}\ \overline{18}$ *ḫpr.ḫr 79* $\overline{108}\ \overline{324}$

Carry out the function of (8, two thirds, a sixth and an eighteenth) by times (8, two thirds, a sixth and an eighteenth) becomes 79, a hundred and eighth and a three-hundred and –twenty-fourth

3) *ir.ḫr =k w3 h- tp m 79* $\overline{108}\ \overline{324}$ *sp 10 ḫpr.ḫr = f m 790* $\overline{8}\ \overline{27}\ \overline{54}$

Carry out the function of 79, a hundred and eighth and a three-hundred-and –twenty -fourth times 10, it becomes 790 an eighth, a twenty-seventh and a fifty-fourth

4) *di* $\overline{2}$ *=f ḥr=f ḫpr.ḫr =f m 1185 w3h -tp m 1185* $\overline{20}$ *m 59* $\overline{4}$ *h33t - pw r =f m 4-ḥk3t*

put half of it on it, it becomes 1,185; the function of 1,185 (times) a twentieth is 59 and a quarter, this is what goes into it in quadruple hekat

5) *sš 5900 ḥk3t 4*

grain, 5900 hekat 25 ro

6) *ki n sšmt = f*

Form of its guidance

1	$8\ \bar{\bar{3}}\ \bar{6}\ \overline{18}$	$\bar{3}$	$2\ \bar{\bar{3}}\ \bar{6}\ \overline{12}\ \overline{36}\ \overline{54}$	1	$79\ \overline{108}\ \overline{324}$
2	$17\ \bar{\bar{3}}\ \bar{9}$	$\backslash\bar{6}$	$1\ \bar{3}\ \overline{12}\ \overline{24}\ \overline{72}\ \overline{108}$	10	$790\ \overline{18}\ \overline{27}\ \overline{54}$
4	$35\ \bar{2}\ \overline{18}$	$\backslash\overline{18}$	$\bar{3}\ \bar{9}\ \overline{27}\ \overline{108}\ \overline{324}$	$\bar{2}$	$395\ \overline{36}\ \overline{54}\ \overline{108}$
8	$71\ \bar{9}$	*dmḏ*	$79\ \overline{108}\ \overline{324}$	*dmḏ*	1185
$\bar{\bar{3}}$	$5\ \bar{\bar{3}}\ \bar{6}\ \overline{18}\ \overline{27}$	total		total	

10	$118\ \bar{2}$
\20	$59\ \bar{4}$

The above problems demonstrate some of the difficulties of translating mathematical papyri into English. The main trouble is trying to translate into a language that has very strict definitions for mathematical terms. I have deliberately avoided using terms such as multiply, diameter and cylinder wherever possible because of the connotations they imply. The mathematical problems do deal with these concepts but the exact meaning of the Egyptian words is not known to us. The meaning of words such as *wȝh –tp* can be understood from the context and the arithmetic that follows, but they do not warrant translation into technical English because of the imprecision in our understanding of the exact meaning of the words. In some cases in the problems translated above, *wȝh –tp* is used to introduce a multiplication, in other cases it is used as a general word for an arithmetic operation. I have therefore translated it to mean 'function' as this is a word that is used to describe arithmetical or algebraic operations. It is difficult to find an English equivalent, because our mathematical terminology has such specific meanings, that although they can have a range of meaning in colloquial English, their use in mathematical contexts renders the meaning precise. In previous translations of the Rhind Mathematical Papyrus, it has been translated as 'operate' 'multiplication' and 'multiply', depending on the context.[16] The translation of 'function' always allows us to translate *w3h –tp* in the same way. I believe that this eliminates some of the difficulties of the translator although I recognise the inexactness of the comparison with modern mathematical language.

The other word that I have tried not to use in my translation is 'proof'. The nature of proof is a thorny problem. Mathematicians would hold that proof is entirely abstract and that the Egyptian way of showing that their working is correct is not proof in any way. This is a narrow way to view a civilisation that had no use for algebra, but so as not to become involved in that argument at this stage I have used the word 'guidance'. I have chosen this word because the arithmetic is given at the bottom of each problem for exactly that purpose; to guide us through what has gone before. The use of technical words is also a problem when trying to translate the words used to give the values of dimensions of a geometrical figure. I have compiled a list of the words used in the Rhind Mathematical Papyrus to give dimensions, shown in Table 3.1.

It becomes apparent when we examine this selection of words that some of them have a doubtful derivation from other hieroglyphic words. The only reason that we understand some of them at all is because of the context and how the dimensions are used in the arithmetical working. A good example of this

[16] Chace, Bull and Manning, (1929) *Op. Cit.* Peet, (1923) *Op. Cit.*

problem is that *wḫꜣ -ṯbt* has been translated literally as 'what the sole requires'[17] however, it is difficult to see why this particular word should be used. The Egyptian names for the dimensions do not inform us what the dimension is; it is for the context to determine. An understanding of the Moscow Mathematical Papyrus problem 10 should have grounding in the understanding of the Rhind Mathematical Papyrus. The above data is therefore vital in trying to select an interpretation of the Moscow Mathematical Papyrus.

[17] Peet, T. E. (1923) *Op. Cit.* p. 98

Word	Translation	RMP Problem Numbers
ꜣw	length	44
wsẖ	width	43, 44
wr r wr	base (of a pyramid)	56, 57, 58
wẖꜣ ṯbt	height (of a pyramid)	56, 57, 58
pr-m -wš	height	43, 44
kꜣ	height	43, 44
kꜣ =f-n-ḥrw	height (of a cone?)	60
rẖt	amount, dimension	46
ḥr mryt	on the long side (of a triangle)	51, 52
hꜣk	the truncation (of a triangle)	52
sntt	base width (of a cone)	60
sḳd	gradient (of a pyramid)	56, 57, 58
tp -r	the short side (of a triangle)	51, 52
dbn	round (diameter of a circle)	41, 42, 50

Table 4.1: Translations of Dimensions from the Rhind Mathematical Papyrus

4.3.2: Struve's Translation

The first translation of Moscow Mathematical Papyrus Problem 10 was published by Struve.[18] The following English translation of his German translation was produced by Peet[19]:

1 Form of working out a basket.
2 If they mention to you a basket with a mouth
3 of four and a half in preservation.
4 Let me know its surface.
5 Take a ninth of nine, since the basket
6 is half an egg; result 1.
7 Take the remainder, namely eight.
8 Take a ninth of eight;
9 result three and a sixth and an eighteenth
10 Take the remainder of these eight
11 after (the subtraction of) this three and a sixth and an eighteenth; result seven and a ninth.
12 Reckon with seven and a ninth, four and a half times;
13 result thirty two. Behold, that is its surface.
14 You have found rightly.

Peet has several objections to Struve's translation. He suggests that a dimension has been left out by scribal error from line 2. His argument is based on the use of

[18] Struve, W.W. (1930) *Op. Cit.* p.157
[19] Peet, T. E, (1931) *Op. Cit.* p. 101

m to introduce a dimension of a geometrical figure and *r* to separate two dimensions. The basket or *nbt* is described as:

nbt m tp-r r 4 $\bar{2}$ *m ꜥḏ*

Struve translates this as "einen Korb mit einer Mündung zu 4 ½ in Erhaltung"[20] "a basket with a mouth of four and a half in preservation". The problem with this translation becomes apparent when we examine how the other geometrical problems are posed. The main problem is the use of *r* to introduce a dimension of a geometrical shape. There are plenty of examples of its use meaning 'by' i.e. 6 cubits by 2 cubits. For example, Line 2 of RMP 49 reads:

ꜣḥt n ḥt 10 r ḥt 2
A field of 10 khet by 2 khet

In the majority of examples, the *r* is left out, RMP 41 and RMP 42 for example, but in no example is it used to mean 'of' in the way Struve proposes. The use of the preposition *m* in this case is also a problem. *m* is used to give the name of a dimension. If this rule is to be followed both *tp-r* and *ꜥḏ* should be the names of two dimensions of the figure. A good example of this can be seen in the first line of problem 43:

sꜣꜥ dbn n mḥ 9 m kꜣ =f 6 m wsḫ =f
A round container of 9 cubits in its height, 6 in its width.

In this example, the two dimensions are *kꜣ* and *wsḫ*: height and width. Each of these dimensions is introduced with the preposition *m*, in this case the preposition *r* is not used to separate the two dimensions.

---

[20] Struve. (1930) *Op. Cit.* p. 157

If Moscow Mathematical Papyrus Problem 10 is to fit with the rules of using these prepositions, it would necessitate the restoration of the value of *tp-r*. The final point in the favour of the restoration of a value is the use of *n*. In all the examples of geometrical problems *n* is always used in the first line of the problem in its exposition. This seems to be the most concrete rule of all. In this case it is missing. The problem as posed in the Moscow Mathematical Papyrus is nonsensical. Therefore Peet's argument for the restoration of the value of *tp-r* in line two of this problem must be followed, making the reading of this line as follows:

nbt [*nt* x] *m tp-r r 4* $\bar{2}$ *m* *ʿd̲*

4.3.3: The Possible Restorations of the Value of *tp-r*

The omission of this value leaves us with a problem. Which value has been left out? There are three mathematically possible solutions to this problem, i.e. they would all have a surface with an approximate area of 32.

The first, implied by Struve, is a hemi-sphere with a diameter of 4.5. Although Struve did not recognise that there was a missing dimension, the missing dimension can be deduced from his figure:

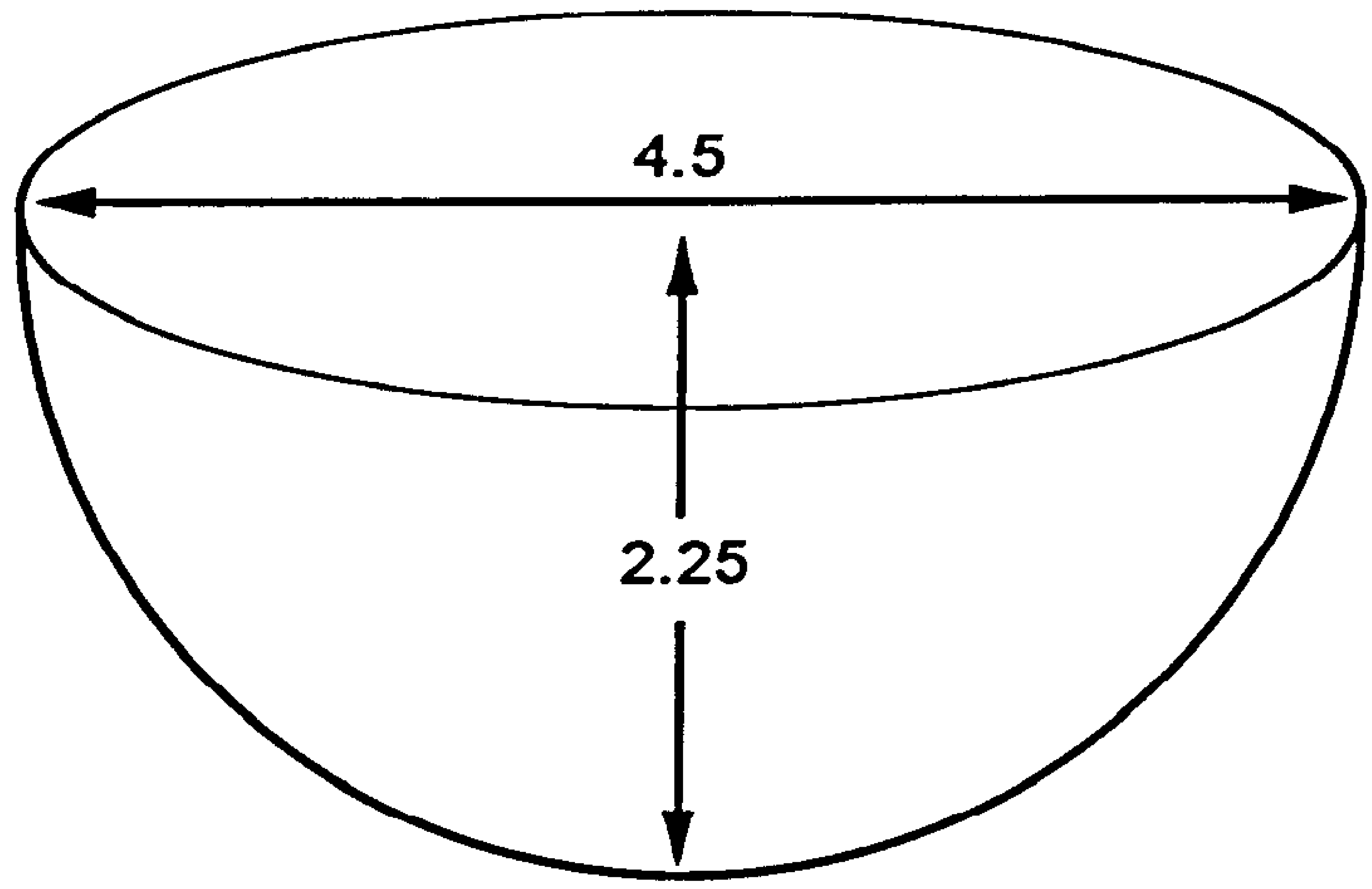

Fig. 4.4: MMP10 as a Hemisphere

The first of Peet's interpretations is a two-dimensional semicircle with diameter of 9:

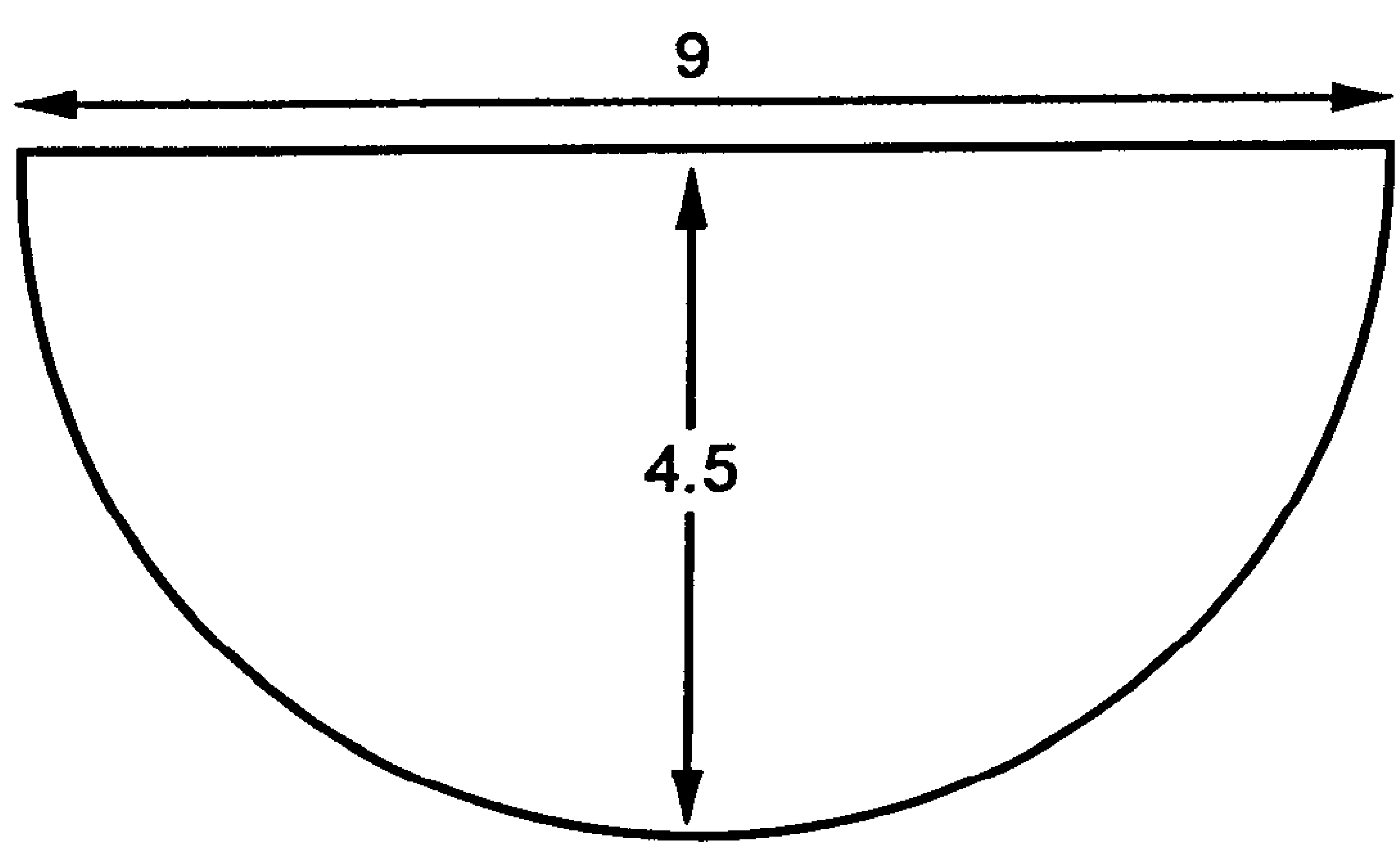

Fig. 4.5: MMP10 as a Semicircle

The second of Peet's interpretations involves the curved surface (not the two semi-circular ends) of half a cylinder with a radius of 4.5 and a width of 4.5:

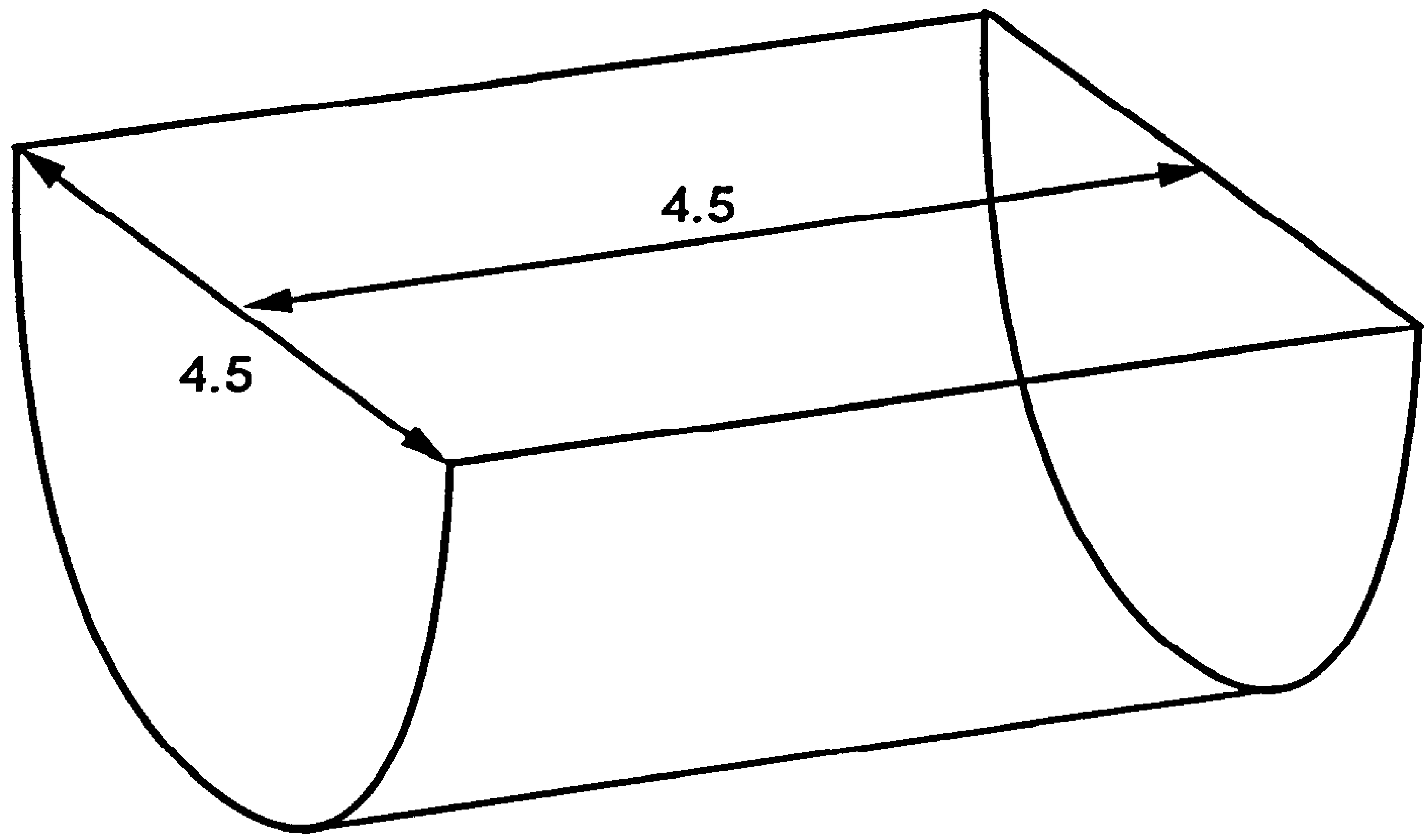

Fig. 4.6: MMP10 as a Half-cylinder

Peet's translations both use technical mathematical language. For the reasons discussed above I feel that using this language is a bad idea. He tries to translate *nbt* as either semicircle or semi-cylinder, depending on which interpretation of the problem he is discussing. I would prefer to leave the translation as basket. Problems from the Rhind papyrus that deal with finding the volume of a cylinder all give the problem as finding the volume of a container, presumably a granary[21], even though a granary would only be an approximation of the shape of a cylinder. The author(s) of these papyri may be using everyday objects to make it easier for the reader to visualise the shape that he is referring to. Peet also gives technical meanings to *tp-r* and *ꜥḏ.*, depending on the context. The translation of these words is extremely difficult for the reasons discussed above. *tp-r* was used in the Rhind Mathematical Papyrus to mean the shorter side of a triangle. It

[21] The word used for this figure *sꜣꜥ*, is unknown in other Egyptian texts. Peet, 1923, p.80. It can be presumed that it is a granary however, because it's volume is shown by the amount of grain that it contains.

clearly does not mean that here, the use of *tp-r* in the Rhind papyrus was limited to problems that deal with the area of a triangle. Our translation of this word should have a link to the use of *tp-r* in the Rhind Mathematical papyrus, but be applicable to a curved shape. In his translation of the Rhind Papyrus, Peet is happy with the translation of 'mouth' for *tp-r*, the *tp* adding little to the meaning of the compound word[22]. I put forward the idea that the translation should be the shorter of two dimensions. This would fit in with the ideas of the hemisphere and the semi-circle, as *tp-r* is the shorter dimension in both these cases. If this is true then *ʿḏ* could be translated with the sense of 'whole'; meaning that it is the larger of the two dimensions.

Peet dismisses the idea that this problem could deal with a hemisphere. He offers the interpretations of the semicircle and the cylinder for consideration. I do not understand why he dismisses the hemisphere solution so easily. He objects to Struve's restoration from the damaged hieratic of the word *inr* meaning egg in line 6. The reading of this word may be doubtful, but as Peet himself admits it is difficult to restore any word with certainty. In Peet's translations of the problem the word that he restores in translation is the word for half the object that he is interpreting the problem to be dealing with. This means that neither of Peet's translations addresses this difficulty. At least Struve offers a hieroglyphic word for our consideration. The second objection that Peet has is the fact that two dimensions have been given. He claims that this is not necessary, because a hemisphere can be described with only one dimension. There are several counter arguments to this. The first is the word used to describe this figure is *nbt* and this should be translated as basket so as to avoid the problems of using technical

[22] Peet, T. E. (1923) *Op. Cit.* p.91

English. What shape is a basket? There are several different shapes of basket and to make sure that the reader knew which particular form was meant two dimensions would have to be given, to ensure that the reader knew a hemisphere was meant. The second counter argument is that there is no evidence for the Egyptians using a radius in their mathematics. The relationship between the diameter and the radius may be obvious to modern mathematicians, but this does not mean that it would have been apparent to an Egyptian who is used to always using the diameter only. It may not have entered into the mind of the scribe that drew up this papyrus that there was a relationship between the two dimensions that he was quoting. The first of Peet's interpretations also gives the two dimensions, the diameter and the radius. If he thinks that it is possible that the Egyptian would quote the diameter and the radius in the heading of the same problem, why does he dismiss the hemisphere? The third objection is the introduction of the number 9 in line five of the problem. This is quoted without explanation. This is unusual in Egyptian mathematical texts, but by no means impossible. In line 3 of RMP 43, it is not obvious where the four and two thirds quoted come from, other than their relation to the breadth of six cubits:

> *w3ḥ-tp m 113* $\bar{\bar{3}}\,\bar{9}$ *r spw 4* $\bar{\bar{3}}$ *–pw n mḥ 6 nty m wsḫ =f ḫpr.ḫr =f m 455*$\bar{9}$ *rḫt =f –pw m ḫ3rw*
>
> the function of 113, two thirds and a ninth by times 4 and two thirds, this is of the 6 cubits which is its breadth; it becomes 445 and a ninth. This is the amount in khar

The translations that Peet suggests are not free of objections. It seems strange that the Egyptians would use such a complicated method to work out the area of a semi-circle. The rule to work out the area of a circle was well known by the

Egyptians. It would have been easier to use that rule and then divide the result by two. The other objection to this rendering of the problem is that other problems that deal with flat areas always pose the problems in the terms of a field. Whatever the shape of the two-dimensional plane the word used to describe it is always *ȝḫt*. The use of the word *nbt* is doubly confusing; why a basket should be an approximation to a semi-circle is not apparent.

There are fewer, less conclusive, arguments against the translation of the curved surface of a semi-cylinder. The approximation of the figure, a basket, does not fit well with the translation, as it would have to be a basket with no sides for the arithmetical component of the problem to fit. The completion of the figure to give the surface area of the whole semi-cylinder would be very easy. It would only require the working out of the area of a circle with a diameter of 4.5, which we know the Egyptians could do. This rendering also suffers because it does not fit in with the translation of *tp-r* that is proposed here. In this case, the two dimensions are equal. However the main objection to this solution is that the Egyptian term for a cylinder is well known, in all the problems in the Rhind Mathematical Papyrus that deal with a cylinder, it is always referred to as *sȝʿ dbn*.

The interpretation of a hemisphere does not suffer from these problems. It is a shape like no other dealt with in mathematical papyri, so it is logical that the problem should be posed using terms that no other mathematical problems use. If the reader were unfamiliar with working out the area of this figure then the explanation in lines 5 and 6 would also be necessary. The objections that Peet has with this interpretation seem superficial and unless new evidence is brought forward the hemisphere is the best interpretation of the problem.

Moscow Mathematical Papyrus Problem 10

1) *tp n ỉrt nbt*
Example of working out a basket

2) *mỉ ḏd n=k nbt* [*nt* 2 $\bar{4}$] *m tp-r*
If it is said to you a basket [of 2 and a quarter] in the short dimension

3) *r* 4 $\bar{2}$ *m ʿḏ hꜣ*
by four and a half in the long dimension, Ha!

4) *dỉ=k rḫ=ỉ ꜣḥt=s ỉr.ḫr=k*
let me know its area. You shall make

5) *ỉr=k* $\bar{9}$ *n 9 ḥr ntt ỉr nbt*
a ninth of nine since the basket

6) $\bar{2}$ *-pw n ỉ*[?] *ḫpr.ḫr* 1
is half of a [?] becomes 1

7) *ỉr.ḫr=k ḏꜣt m* 8
you take the remainder, namely 8

8) *ỉr.ḫr=k ỉr=k* $\bar{9}$ *n 8*
you shall make a ninth of 8

9) *ḫpr.ḫr* $\bar{\bar{2}}\,\bar{6}\,\overline{18}$ *ỉr.ḫr=k*
becomes two thirds, a sixth and an eighteenth. You shall make

10) *ỉr=k ḏꜣt nt pꜣ* 8 *r sꜣ*
the remainder of the 8 after subtracting

11) *pꜣ* $\bar{\bar{2}}\,\bar{6}\,\overline{18}$ *ḫpr.ḫr* 7$\bar{9}$
the two thirds a sixth and an eighteenth, becomes seven and a ninth

12) *ỉr.ḫr=k ỉr=k* 7$\bar{9}$ *sp* 4$\bar{2}$
you shall make seven and a ninth times four and a half

13) *ḫpr.ḫr* 32 *mk ꜣḥt=s -pw*
becomes 32. Behold! This is its area

14) *gm=k nfr*
you have found it correctly.

The mathematical workings of this problem are very interesting. The modern formula for working out the surface area of a hemi-sphere is:

$$\text{Area} = 2\pi r^2$$

It is doubtful that this formula would have been known to the Egyptians, as it is based on the length of the radius. In none of the extant problems in Egyptian geometry does this dimension appear. The Egyptian method of finding the area of a circle was to find eight ninths of the diameter and square it. The formula used in MMP10 expressed in modern notation is[23]:

$$d\left[\left(2d - \frac{2d}{9}\right) - \frac{1}{9}\left(2d - \frac{2d}{9}\right)\right]$$

The derivation of the modern formula from the ancient Egyptian one is a case of applying simple algebraic manipulation.

Factorising the brackets by $2d$:

$$d\left[2d\left\{\left(1 - \frac{1}{9}\right) - \frac{1}{9}\left(1 - \frac{1}{9}\right)\right\}\right]$$

Simplifying:

$$d\left[2d\left\{\left(\frac{8}{9}\right) - \frac{1}{9}\left(\frac{8}{9}\right)\right\}\right]$$

$$2d^2\left(\frac{8}{9}\right)^2$$

[23] where d equals the diameter.

The Egyptian approximation to π is $\frac{256}{81}$, so $\left(\frac{8}{9}\right)^2$ is approximately equal to $\frac{\pi}{4}$

Replacing $\left(\frac{8}{9}\right)^2$ with $\frac{\pi}{4}$:

$$2d^2\left(\frac{\pi}{4}\right)$$

Replacing *d* with 2r, because the diameter is twice the radius and simplifying:

$$\textbf{Area} = 2\pi\, r^2$$

Algebraic transformations are only easy, however, when one is working towards a known goal. Working from the modern formula to the ancient Egyptian formula would involve non-intuitive factorisation. Although I am convinced that this problem deals with a hemisphere, it is clear that much additional work will be required before a fuller understanding of how the ancient Egyptians were able to prepare this remarkable problem in abstract geometry can be achieved.

4.4: Conclusions

These two problems from the Moscow Mathematical Papyrus are the two most advanced problems in the extant corpus of Egyptian mathematics. They are also two of the most contested problems; the reasoning behind their production is unclear. They question the notion that Egyptian mathematics was incapable of being abstract. The method utilised in these problems could not have been arrived at by chance. A now anonymous mathematician used some form of reasoning to work from simple observations of the figures to the sophisticated method employed. It is immaterial whether this reasoning was performed on papyrus, or with a more practical method using models made out of mud or observing the work of basket weavers, the reckoning is still an abstraction from a particular case to a general solution. There are several ways in which a truncated pyramid can be cut up to form simpler geometrical shapes, which can then be combined. However, all of these methods still require further spatial reasoning to reach the formula contained in the method of MMP 14. There is an apparent link between the Egyptian method for finding the area of a circle and the surface area of a hemisphere, though this link is far from clear. Both methods utilise the fraction of eight-ninths of the diameter which can give a good approximation to π. The method for finding the surface area of a hemisphere is complicated and how it is possible to arrive at this method from a circle is unclear. Together these problems show that the ancient Egyptians were capable of abstract reasoning, but that how it was achieved is uncertain.

Chapter 5

Review of the Study of Ancient Egyptian Mathematics

The study of Egyptian mathematics has produced some startling comments. For an Egyptologist, interested in how mathematics affected the lives of the ancient Egyptians, many of these comments are less than helpful. Yet texts on the history of mathematics present ideas that have permeated the thinking of Egyptologists about mathematics in Egypt. This chapter will explore how Egyptian mathematics has been viewed and written about. It will include a survey of some of the common ideas about Egyptian mathematics and what these comments accentuate about how the history of science and mathematics has been studied. This chapter will not deal will specific comments made about the Egyptian mathematical problems, rather it will focus on general comments made about the character of Egyptian mathematics. It will also begin to explore how the study of Egyptian mathematics has been coloured by philosophical ideas.

5.1 Introduction

The preceding chapters have provided translations of parts of the extant Egyptian mathematical texts and given a commentary on the mathematical content of these texts. It should be clear from these translations and commentaries that the content is more mathematically sophisticated than it appears on the first reading of the problems. For example, the process of multiplication appears to be simple: repeated doubling until the answer can be obtained with addition. However, on a closer reading of the texts, it is found that the actual processes used offer a wider variety of technique, which is influenced by the context. It is this apparent simplicity that has struck many of the authors of books and paper on Egyptian mathematics: such books are often slow at noticing the depths and subtleties of the Egyptian mathematical texts.

This chapter will explore the ways in which the subject of Egyptian mathematics has been approached. It will not deal with the specific comments of writers on Egyptian mathematics. Rather, it will look at the general comments about the character of Egyptian mathematics. The major writers and the major works on the subjects will be analysed to identify the common ideas about Egyptian mathematics and how these ideas arise.

5.2: History of Mathematics Textbooks

Reference to ancient Egyptian mathematics can be found in general textbooks on the history of mathematics. It is often found in a shared chapter with Babylonian mathematics before the author moves on to chapters on Greek mathematics. The treatment that Egyptian mathematics receives falls far short of a detailed examination. These chapters often concentrate on numeration and arithmetic, rather than on algebraic and geometrical elements. Even those textbooks that take a more sympathetic approach to Egyptian mathematics still do not deal adequately with the range of material present in the Egyptian texts. In addition, because they are written by non-Egyptologists, mistakes and misunderstandings are bound to occur. These accounts of Egyptian mathematics will also necessarily rely on the work of others so will have little to say that is new.

For example, Teresi's book on the ancient roots of modern science[1] dedicates most of his section on Egyptian mathematics to addition in hieroglyphic notation, something that does not appear in any of the Egyptian texts as they were written in hieratic script. In his explanation of multiplication, he is only aware of the repeat doubling method of multiplication. He writes:

> "Today, we can do the above problem in our heads: 180 × 20 = 3,600. Ancient Egyptians and medieval Europeans couldn't."[2]

This shows that as well as being unaware of the subtleties of Egyptian multiplication techniques, he is also unaware that the Moscow Mathematical Papyrus contains many examples of multiplication that have been completed without showing the working out[3]. This quote also seems to presume that the Egyptians were less intelligent than modern Europeans are. It seems that it is as much of a sweeping statement to say that all modern Europeans are capable of doing mental arithmetic, as it is to say that no ancient Egyptians could. Even his explanation of the placing of hieroglyphic numbers is flawed. He explains at length how the numbers are written and says that numbers were written in the opposite order to the way they are written now; with the units on the left hand side increasing powers of ten to the right. What he fails to realise is that the mathematical texts were written from right to left, so when reading across a line, the higher powers of ten would be read first moving across to the units. Most of these problems can be accepted as Teresi is not an Egyptologist and so he is not

[1] Teresi, D. (2002) *Lost Discoveries : The Ancient Roots of Science - from the Babylonians to the Maya*; Simon and Schuster, New York; pp 38 - 47.
[2] Teresi, D; *Op. Cit.* p.43.
[3] See Section 2.3.

able to deal with the primary sources for himself. However, although Teresi quotes Herodotus' account of the birth of geometry being in Egypt as surveying was necessary after the annual inundation, he does not mention any of the geometrical achievements of the Egyptians. He does not even mention that they could work out the area of a quadrilateral, let alone the curved surface of a hemisphere.

Unfortunately, Teresi's book is not an unusual example of the type of material that is written about Egyptian mathematics.

George Gheverghese Joseph's book, *The Crest of the Peacock*[4], has become a standard text for those interested in the roots of mathematics, and non-European mathematics in particular, since its first publication in 1991. Joseph is a specialist in mathematics of India, Tibet and the Indus valley. He raises many of the problems that dog the study of Egyptian mathematics, showing that the problems faced are not peculiar to Egyptian mathematics but a deeper problem with the way that the history of mathematics is researched. He justifies his book by appealing to the necessity of understanding non-European cultures so they do not remain a footnote. The problems of chasing the origins of mathematics are compounded by the way in which Europeans and their cultural dependencies write history only from their own viewpoint. Thus, Africa is only included in history after its peoples' encounter with Europe. He sees that the history of the sciences and mathematics has difficulties in this regard because of the prestigious nature of science and mathematics.

[4] **Joseph, G. G (2000) *The Crest of the Peacock: Non-European Roots of Mathematics*; 2nd ed.; Penguin, Harmondsworth.**

Joseph describes the ancient Egyptian's advances in geometry and algebra as well as arithmetic; although most of the space he affords Egyptian mathematics deals with multiplication, division and unit fractions. The only geometrical problems he deals with directly are RMP 48 and 50 and MMP 10 and 14. The two problems from the Rhind Mathematical Papyrus are included because they deal with the areas of circles. Joseph notes the implicit value for π contained in these problems, $\left(\frac{16}{9}\right)^2$ or approximately 3.1605, and gives a few explanations for how this value was achieved.

The two problems from the Moscow Mathematical Papyrus are those dealt with in Chapter 4 of this work, as they are the pinnacle of Egyptian achievement in geometry. In his presentation of MMP 14, Joseph shows the translation of the Egyptian text on the left-hand side of the page and a modern algebraic equivalent on the right-hand side of the page, showing that the Egyptians were using the correct formula. Again, he gives several explanations for the Egyptian's derivation of the formula.

Joseph's attitude to the geometrical problems that he discusses and the proposed explanations of how the Egyptians achieved them is that the Egyptians took a very practical approach to their mathematics. He favours those explanations that have some link to everyday life. His suggestions for finding the area of a circle and of MMP 10 involve everyday objects such as basket lids and matting. He refers to "the 'concrete' approach to geometry that the Egyptians favoured"[5]. In his assessment of the character of Egyptian mathematics, Joseph argues against the ideas of Morris Kline, whose views he labels as Eurocentric. In particular,

[5] Joseph, G.G; *Op. Cit*; p 87.

Joseph highlights Kline's views on the contrast between Egyptian, Babylonian and Greek mathematics[6].

Most of Joseph's complaints about scholarship of Egyptian mathematics are related to his views that historians of mathematics tend to ignore new ideas that go against the idea of a Greek miracle, thus leading to views such as Kline's.

He answers five main criticisms of pre-Greek mathematics:

1) They had no general rules

2) The texts contain no proofs

3) They lacked abstraction

4) They failed to distinguish clearly between exact and approximate results

5) There was no discernable activity which we may label 'mathematics' and which was studied for its own sake.[7]

Each of these points are answered in detail. However, although Joseph uses mathematical ideas from both Egyptian and Babylonian texts, he does not give examples of problems from the texts that can be used to argue against these ideas. This may be because of the limited space in a book of this nature, or because Joseph is not a specialist in these areas and is reliant on the experience of other scholars. Either way, his ideas are in need of expansion and solid examples. He answers Point 5 with general observations on the need for a leisured class before mathematics can be undertaken for purely aesthetic reasons. He also notes that the idea of mathematics as an end in itself is derived from the Greek ideals and is therefore a debatable statement. These arguments are persuasive, but the Egyptian material contains problems that after a little analysis

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[6] See Section 5.4

[7] Joseph, G.G; *Op. Cit*; p 126.

show only a tenuous link to practical need. RMP 79[8], for example, although couched in everyday terms has scant basis in everyday problems. It is exactly analogous to puzzle rhymes. Points 4 and 5 are answered from only the Babylonian texts. Joseph makes no mention of Egyptian geometry, presumably because he perceives the Egyptian material having a concrete nature. This is, however, a gut feeling on Joseph's part. His arguments in this matter are self-affirming. He believes that the Egyptian material is concrete, so he prefers concrete explanations of the derivations, which affirm his assessment that the mathematics is concrete. Many of the ideas in Joseph's assessment question definitions of mathematics and mathematical terms and their appropriateness in the study of ancient mathematics.

Joseph finishes his assessment of Egyptian and Babylonian mathematics by noting that any perception of the achievements of these civilisations will rest on the definition of algebra. If 'true' algebra requires the symbolism with which it has become identified, then the perception of the achievements of the Egyptians and Babylonians will suffer. However, it may be possible to argue that the Egyptian and Babylonian material represents an early phase in the development of algebra. This is a matter of the philosophy of mathematics.

Joseph begins his appraisal of Egyptian mathematics by pointing out evidence that the Egyptian civilisation had African roots. He quotes Diodorus in support of his argument, even though this source was written millennia after the events he posits. He feels that it is important to emphasise the supposed African roots of the Egyptian civilisation and refers to the 'black' origins of Egypt. It is

[8] See Section 2.5.

troubling to find mentions such as these of race in Joseph's work. We may have sympathy with his ideas about the subjugation of non-European peoples and the denial of their influence in European culture, but this should not be allowed to cloud our judgement of the Egyptian mathematical texts. Racial grouping is a modern political concept and as such, it should be left to modern politics and not influence our opinion, either way, of the Egyptian civilisation and its mathematical achievements.

5.3: Specialist Writers

5.3.1: Professor T. E. Peet

Peet, late Brunner Professor of Egyptology at the University of Liverpool, deserves a section of his own. This is not only because of his immense output on the subject of Egyptian mathematics, but also because - as one of the first translators and editors of the Egyptian mathematical texts - his opinions have greatly influenced other authors' opinions. I also owe a personal gratitude to Peet; many of the books that made me interested in the subject were from Peet's personal library.

Peet's 1923 edition of the Rhind Mathematical Papyrus[9] was not the first edition of the text, but with its commentaries and full hieroglyphic transcription, it remains one of the best editions. Its size is also an advantage as the translations to the problems are laid out clearly. Gunn's review of 1926 says :

[9] Peet, T.E (1923) *The Rhind Mathematical Papyrus: British Museum 10057 and 10058;* University of Liverpool Press; London.

> "The author is to be congratulated on a very able piece of work; it may be added that the volume is handsome small folio, of unusually tasteful appearance inside and out, and does credit to its printers and publishers"[10]

The first edition of the text was published by Eisenlohr[11] some fifty years previously but advances in understanding of the ancient Egyptian language warranted a fresh treatment of the text. This edition of the papyrus was followed a few years later by an edition of the Rhind Mathematical Papyrus[12] that gave a reproduction of the hieratic text with a hieroglyphic transcription and a transliteration. Each problem was given a separate page and the clarity of this edition has reserved it an important place in the study of the Rhind Mathematical Papyrus, recently being produced in facsimile in Clagett's source book on ancient Egyptian mathematics[13]. Gunn's edition is clearer and so is reproduced in Clagett's sourcebook, but the assessment of the text owes a lot to that of Peet.

In Peet's 1923 edition of the Rhind Mathematical Papyrus, written before the first full publication of the Moscow Mathematical Papyrus, Peet devotes a section to his impressions of the character of ancient Egyptian mathematics. He only specifically mentions MMP 14[14] as this had been published by Turaiev in 1917[15]. Peet does not mention MMP 10[16], for example, it was not until 1931 that Peet publishes his article on the problem[17] in the same volume of the *Journal of Egyptian Archaeology* as his review of Struve's edition of the Moscow

[10] Gunn. B (1926) review of Peet T.E; 1923; *The Rhind Mathematical Papyrus: British Museum 10057 and 10058*; in *Journal of Egyptian Archaeology*; vol. 12; pp 123-37.
[11] Eisenlohr (1877) *Ein mathematsches Handbuch der alten Ägypter, übersetzt und erklärt*; Leipzig.
[12] Chace, A.B, Bull, L.S, Manning, H.P. and Archibald R.C (1927/9) *The Rhind Mathematical Papyrus*; 2 volumes; Buffalo; NY; Mathematical Association of America.
[13] Clagett, M. (1999) *Ancient Egyptian Science, A Source Book, Vol. 3 Ancient Egyptian Mathematics;* Memoirs Of The American Philosophical Society; Vol. 232; Figs. IV. 2a-aaa.
[14] See Chapter 4.
[15] Turaiev, B.A (1917) "The Volume of the Truncated Pyramid in Egyptian Mathematics"; *Ancient Egypt*; pp.100-02.
[16] See Chapter 4.
[17] Peet, T.E. (1931) "A Problem in Egyptian Geometry"; *Journal of Egyptian Archaeology*; vol. 17; pp 100-06.

Mathematical Papyrus[18]. In this review, he states that in his studies of photographs of the Moscow Mathematical Papyrus he had found nothing to modify our opinion of Egyptian mathematics apart from MMP 14, a statement he stands by. His treatment of MMP 10 is studied in detail in Chapter 4. Nothing much needs to be added here, except to note that Peet takes a pessimistic view of the ability of the Egyptians and looks for a simpler translation of the problem than that of the surface area of a hemisphere. The rest of the review concerns Peet's arguments for alternative transliterations and translations. Also, there is no argument that the Egyptians took the inspiration for their mathematics from everyday life, the word for the object in MMP 10 is *nbt*, a word that does not appear in any other of the surviving mathematical texts, but is usually translated as basket[19]. Peet, in his insistence that the figure must relate to a figure that has two different dimensions, is ascribing a mathematical meaning to the word *nbt*. Yet, Peet does not believe that Egyptian mathematics was 'scientific'. Baskets in ancient Egypt come in many shapes and sizes; a lot of them are roughly hemispherical. If the scribe who prepared MMP 10 was thinking of an actual basket then two dimensions would be necessary to distinguish between a hemispherical basket and any other shape basket. This fits with Peet's own arguments about the practical nature of Egyptian mathematics.

Peet has a lot to say on the character of Egyptian mathematics. In 1931, he gave a lecture on mathematics in ancient Egypt at the John Rylands library in

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[18] Peet, T.E (1931) Review of Struve W.W; 1930; *Mathematisher Papyrus des Staatlichen Museums der Schönen Künste in Moskau, QSGMI;* in. *Journal of Egyptian Archaeology*; vol.17; pp 154-60.

[19] See Faulkner, R.O. (1962) A *Concise Dictionary of Middle Egyptian*; Griffith Institute, Oxford; p. 128.

Manchester, which was published in the bulletin the same year[20]. This lecture came after Struve's publication of the Moscow Mathematical Papyrus. In the conclusion of the lecture, Peet returned to his statements about the character of mathematics that he made in his edition of the Rhind Mathematical Papyrus, and found that the publication of the Moscow Mathematical Papyrus did not alter his main opinion that "The outstanding feature of Egyptian mathematics is its intensely practical character"[21]. He does concede, though, that there are problems in the Rhind Mathematical Papyrus that deal with abstract numbers. These problems are the *ꜥḥꜥ* problems. Peet's change of mind over these problems stems from the determinative given to the word in the mathematical texts. In the mathematical texts the word is given the papyrus roll determinative, which is used for abstract concepts such as truth, new, know and great. This changes the nuance of the word. Instead of being a literal heap, a physical entity, it is this argument that renders the translation of this word from 'heap' to quantity'. Despite this concession, Peet is adamant that Egyptian mathematics is only practical. The examples of geometrical progressions are dismissed, putting him at odds with the opinion of Chace who believed that these problems are theoretical problems in practical form[22]. Peet simply states:

> "Examples of this kind do suggest that, while mainly occupied with practical problems, the Egyptians occasionally allowed themselves to observe and even to record a result or method which had no direct application to the concrete facts of life. But there is no sign that such things were regarded as more than idle curiosities"[23]

The absence of a philosophical work written by the Egyptians on the intentions of their mathematicians makes any comment on how these problems were

[20] Peet. T.E. (1931) "Mathematics in Ancient Egypt"; *Bulletin of the John Rylands Library Manchester*; vol. 15; pp 409-441.
[21] Peet T. E. (1931) *Op. Cit.* p. 437
[22] Chace, A.B (tr. and ed.) (1927) *The Rhind Mathematical Papyrus;* Mathematical Association of America; p. 43.
[23] Peet T. E. (1931) *Op. Cit.* p. 438

regarded as speculative. The very fact of the inclusion of these problems in the ancient texts shows that they were afforded some importance. The level of this importance is not known. However, it would be as debatable to say that they were regarded as the crowning achievement of mathematical activity as it is to say they were regarded as idle curiosities.

Peet also speculates on whether Egyptian mathematics can be regarded as "scientific" as modern mathematics can. He cites several authors, notably Vogel, who consider Egyptian mathematics to be abstract because they are able to deal with numbers in themselves rather than a number of objects. Vogel also points to the ordering of the Rhind Mathematical Papyrus and to the use of verifications at the end of the problems as evidence of a scientific element to Egyptian mathematics. Peet rejects each of these points in turn. To him, the presence of abstract numbers does not make science, only a form of proto-science. The ordering of the Rhind papyrus is - he says - nothing more than evidence of the orderly mind of the Egyptians and the use of verifications is an *a posteriori* method, it is not *a priori* as a modern proof, so these verifications cannot be used as evidence for a scientific system. Peet writes:

> "That they did not reach the conception of scientific mathematics and its dependence on cogent *a priori* demonstration is merely another instance of the vast debt which the world owes to the Greeks"[24]

[24] Peet T. E. (1931) *Op. Cit.* p. 441. For commentary on the idea of a Greek miracle, see Section 8.4.3.

5.3.2: Neugebauer

Neugebauer achieved fame in the history of mathematics for his work deciphering Babylonian clay tablets. It is in Babylonian mathematics that he has the most to say. In his most important work, *The Exact Sciences in Antiquity*[25], the vast majority of the book is given to Babylonian mathematics and science. Egyptian mathematics and astronomy is given only one chapter of twenty pages in a book of 191 pages. However, because of his authority on Babylonian mathematics, his writings on Egyptian mathematics were taken seriously and Neugebauer's work is widely referenced[26].

Neugebauer seems to have been particularly influenced by Peet. He calls Peet's lecture at the John Rylands library "an excellent brief summary"[27]. The parallels between the views of Peet and Neugebauer are clear. Peet commonly characterises the mathematics of the Egyptians as practical and concrete. Neugebauer does not overtly speculate on whether Egyptian mathematics can be seen as scientific, although the tone of his work and the assumptions he makes about the ability of the Egyptians shows that he does not consider it to be scientific. Perhaps he felt this was a question that had been satisfactorily dealt with by Peet in his John Rylands lecture. This assumption runs through all the writings of Neugebauer and so does the influence of Peet.

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[25] Neugebauer, O (1952) *The Exact Sciences in Antiquity*; Princeton University Press; Princeton, New Jersey.
[26] See for example: Bell, E.T. (1945) (2nd ed.); *The Development of Mathematics*; McGraw-Hill; New York. Also: Boyer, C.B. rev. Merzbach; V.C. (1989) *A History of Mathematics*; John Wiley and sons inc, New York.
[27] Neugebauer, O. (1952) *Op. Cit.* p. 87.

Neugebauer is one of the most pessimistic commentators on Egyptian mathematics. He compares Egyptian mathematics unfavourably with not only Greek mathematics, but with Babylonian mathematics as well, writing:

> "Ancient science was the product of a very few men; and these men happened not to be Egyptian"[28]

His appraisal of Egyptian mathematics is nothing short of damning. The first section of the chapter explains how mathematics and astronomy had no effect on the lives of the Egyptians and that:

> "The mathematical requirements for even the most developed economic structures of antiquity can be satisfied with elementary household arithmetic that no mathematician would call mathematics."[29]

His specific appraisal of the individual achievements of the Egyptians works from this assumption; that the mathematics they produce is nothing more than elementary arithmetic that arises from practical, everyday problems. Working from this assumption leads Neugebauer to fail to grasp the significance of some of the contents of the Egyptian texts. The procedure for multiplication, for example, is sophisticated and developed for ease of computation[30]. The repeat-doubling method enables the Egyptian scribe to complete multiplication sums with the minimum amount of fuss. Yet, Neugebauer misses the subtleties of the Egyptian method and describes only the repeat-doubling method. He then writes:

> "In general, multiplication is performed by breaking up one factor into a series of duplications. It certainly never entered the minds of the Egyptians to ask whether this process will always work. Fortunately it does; and it is amusing to see that modern computing machines have made use of this principle to exactly the same end, namely, to reduce multiplication to a simple process of counting."[31]

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[28] Neugebauer, O. (1952) *Op. Cit.* p.86
[29] Neugebauer, O (1952) *Op. Cit.* pp. 71-2
[30] See Section 2.3.
[31] Neugebauer, O (1952) *Op. Cit.* p. 73.

This statement is misguided for several reasons. Pronouncements on the intentions, motives and thoughts of the Egyptian scribes who produced the mathematical texts are speculative and should be approached with trepidation. There is no evidence in the Egyptian material to support the idea that the Egyptians were not aware of the effectiveness of their multiplication system. It is far from certain that the Egyptians had not noticed the ease in which they were able to perform multiplications. Indeed, the Moscow Mathematical Papyrus shows that they did not find it necessary to always show how they achieved their results if the problems involved geometry. Moreover, it should serve as a positive indication that modern computing machines use the same technique, as it shows the efficiency of this system. It should not be amusing to see this parallel. Rather, while it may at first seem surprising that the Egyptians had found such a marvellous system, this surprise should be tempered with the realisation that the scribes involved in accounting and field measurements would have had to perform many of these calculations, maybe under pressure.

His appraisal of unit fractions is similar in tone. Because he assumes that the Egyptians had not adopted their mathematical procedures through practice and convenience, rather through a lack of inquisition, he misunderstands the significance of the 2/n table contained in the Rhind Mathematical Papyrus. His analysis is complicated and relies on the idea of 'natural' and 'algorithmic' fractions, a distinction that arises not from the mathematical texts, but from Neugebauer's own feelings. This analysis falls far short of that of Gillings[32] whose treatment, stemming from the idea that the Egyptians selected fractions that made arithmetic simpler to perform, is elegant and simple.

[32] See Section 5.3.3.

Neugebauer's attitude to the mathematics and the logical scientific thinking of the Egyptians is beginning to be replaced. In 1980, he wrote a short article on the orientation of the pyramids[33]. In this article, he proposes that the Egyptians achieved the accuracy of the alignment of the Great pyramid using a solar method. Neugebauer begins from the assumption that astronomical theories of alignment - those that depend on the stars - are beset with practical difficulties. His theory needs only the "...primitive experience of symmetry of shadows..."[34]. Whilst it should be recognised that Neugebauer is only presenting a theory as to how the accuracy could have been achieved, it is interesting to note that the theory he supplies is one of the most basic theories that fits the facts. It requires only observation and experimentation and very few scientific evaluations. In a recent paper, Kate Spence[35] has shown that the pyramids were probably aligned using the stars. She shows that the processional movement of two stars, Kochab and Mizar match the drift in the alignment of the pyramids. This seems to suggest that the Egyptians were using these two stars to prove the alignments. This evidence suggests that the Egyptians were more acute observers than Neugebauer gives them credit for. In *The Exact Sciences in Antiquity*, Neugebauer writes that he does not believe that the Egyptian astronomical texts lend themselves to accurate computation. Yet, from the work of Spence, we know that the accuracy of the pyramid alignments is due to the Egyptians' careful observations of astronomy.

[33] Neugebauer, O (1980) "On the Orientation of Pyramids"; *Centaurus*; vol. 24; pp 1-3.
[34] Neugebauer, O. (1980) *Op. Cit.*; p. 1.
[35] Spence, K. (Nov. 200) "Ancient Egyptian Chronology and Astronomical Orientation of Pyramids"; *Nature* 408; pp 320 – 324.

He is also dismissive of the Egyptian calendar, drawing our attention to its agricultural significance[36]. The Egyptian calendar was divided into three seasons, which reflected the rising and falling of the Nile. The heliacal rising of Sirius, or Sothis to give it its Egyptian name, was used as a marker because this event coincided with the start of the inundation. The utility of the calendar is used as a negative indicator of the scientific abilities of the Egyptians, in a similar way to the utility of their mathematics being used as a counter-argument to claims of abstraction. Yet calendars are practical concepts, the measurement of the year must necessarily fit with the yearly cycle of the seasons. The internal divisions of that cycle are completely arbitrary and so no divisions will have basis in science. The Egyptians also used lunar calendars, in fact their calendrical system is very complicated as it involves the observation of several astronomical phenomena. To dismiss this, as Neugebauer does, is short-sighted.

Neugebauer's work suffers because he works from pessimistic estimates of the abilities of the Egyptians and so fails to perceive the significance of many of the more subtle features of Egyptian arithmetic. He rarely comments on geometry and so misses some of the most important achievements of the Egyptians. If Neugebauer is being replaced, then it is a development to be welcomed.

5.3.3: Gillings

One of the most prolific writers on the subject of ancient Egyptian mathematics was Richard Gillings. His works include the most complete book on the subject of Egyptian mathematics, *Mathematics in the Time of the Pharaohs*. He is also the author of many journal articles written in the 1950's and 60's[37].

[36] Neugebauer, O. (1952) *Op. Cit.* p.82
[37] For a list of these journal articles see the bibliography.

Mathematics in the Time of the Pharaohs covers the contents of all the mathematical papyri in themed chapters. Gillings also includes appendices where he tries to answer some of the critics of Egyptian mathematics. The only downfall to Gillings' work is that in all the time he spent studying the texts, he never learnt to read hieroglyphs, so his interpretations of the texts are distant and he lacks the authority to present a definitive edition of the texts. In fact, he relies on the help of T.G.H. James, then Assistant Keeper of the Department of Egyptian Antiquities of the British Museum, to explain the difficulties translating the mathematical problems. In some cases, such as MMP 10 he does not have the knowledge to analyse the competing translations of Struve and Peet. He seems to prefer the idea that the problem deals with a hemisphere but his exploration of the problem relies on reconstructing the arithmetic that may have accompanied this problem. Of course, the scribe of the Moscow Mathematical Papyrus did not include arithmetic in the papyrus so Gilling's reconstruction is guesswork. Gillings also shows the formula worked out in the problem in modern algebraic notation. Here, though, he is a little disingenuous, as he does not make it clear that the formula he shows has to be derived from the Egyptian method with several lines of algebra. Also, the formulae for the surface area of a semi-cylinder and the semi-circle can equally be extracted from the Egyptian method. Whilst the translation as a hemisphere does seem after careful examination of the text to be the most probable one, Gillings arguments have little weight.

Apart from these problems, Gillings remains one of the best and most complete commentators on Egyptian mathematics. His treatment of unit fractions is one of his best achievements. He analysed the unit fraction identities of the 2/n table of the recto of the Rhind Mathematical Papyrus and came up with five precepts which explain why one identity is chosen over another. These five precepts are:

Precept 1

Of the possible equalities, those with the smaller numbers are preferred, but *none* as large as 1,000.

Precept 2

An equality of only 2 terms is preferred to one of 3 terms, and one of 3 terms to one of 4 terms, but an equality of more than 4 terms is *never* used.

Precept 3

The unit fractions are always set down in descending order of magnitude, that is, the smaller numbers come first, but *never* the same fraction twice.

Precept 4

The smallness of the first number is the main consideration, but the scribe will accept a *slightly* larger first number, if it will *greatly* reduce the last number.

Precept 5

Even numbers are preferred to *odd* numbers, even though they might be larger, and even though the numbers of terms might thereby be increased.[38]

Gillings then demonstrates that these precepts explain virtually every entry in the table. These precepts show that the Egyptians produced a table that had

[38] Gillings, R. (1972) *Op. Cit*; p. 49. Emphasis of Gillings.

maximum value for ameliorating the processes of arithmetic. Although Gillings does not state it directly, his admiration for the work of the 2/n table is clear. He points to the many variations and extensions of this table that have been found in other mathematical fragments, and to the fact that they can be found as late as the sixth century C.E. His opinion is therefore in conflict with that of Peet who wrote:

> "The Recto is a monument to the lack of scientific attitude of mind"[39]

The appendices at the back of the book show Gillings' attitude to Egyptian mathematics. Appendix 1 deals with the nature of proof, and Appendix 4 offers a response to the views of Morris Kline. In these appendices, Gillings attempts to answer the most common criticisms of Egyptian mathematics. Of the nature of proof[40], Gillings points to the numerical solutions present in many of the problems. He feels that although the Egyptians did not use symbolic proofs, their method could be rigorous without it. He reasons that if the value chosen is typical and any further generalizations are immediate then a nonsymbolic argument can be rigorous. Yet, Gillings agrees that the Egyptians did not reason in the same way as the Greeks. Unfortunately, this discussion misses a discussion of the purpose of the Egyptian texts. If the texts that have survived are training manuals, then it is not possible to draw direct comparisons between these texts and the theoretical treatises of the Greeks. Of course, this argument is to ascribe motive to the Egyptian scribes, and that should be done with caution. However, it is a possibility that deserves exploration. The paucity of available evidence will always be a problem when trying to evaluate ancient Egyptian

[39] Peet, T.E. (1931) *Op. Cit.;* p.413.
[40] Gillings, R (1972) *Op. Cit;* Appendix 1; pp 232-233.

mathematics: different ideas should be explored to give the most complete picture possible.

Appendix 4 is an argument against the views of Morris Kline, a historian of mathematics and the author of *Mathematics a Cultural Approach*[41], a volume of seven hundred pages. In particular, Gillings disputes the statement that Egyptian mathematics is like the scrawling of children. Gillings does not try to conceal his contempt for the poor scholarship of Kline. The idea that the Egyptians did not recognise mathematics as a separate subject comes under the most scorn. Gillings feels that the presence of the Rhind Mathematical papyrus, a text devoted entirely to mathematics, is a counter to this. Unfortunately, he does not elaborate on this point. The idea that the Egyptians did not do mathematics as a separate subject has found its way into modern Egyptology[42]. This argument hinges on the idea that their mathematics was not abstract. It appears that it is not enough to have produced texts such as the Rhind and Moscow mathematical papyri, Kemp's argument is based on the idea that these texts deal with individual cases, rather than with mathematical principles. This is a restatement of Peet's 'concrete mathematics'.

The importance of Gillings' work is that it showed that another interpretation of the texts was possible. He questions many of the assumptions made by Peet, Kline, Van der Waerden and other historians of mathematics. His treatment of the texts is sympathetic and he realises the importance of considering the contents within the context they were written. His book brings together all the

[41] Kline, M. (1962) *Mathematics, A Cultural Approach*; Addison-Wesley; Reading, Mass.
[42] For example see Kemp; B. J. (1989) *Ancient Egypt: Anatomy of a Civilization;* Routledge; London; p. 117.

Egyptian texts and gives a clear list of their contents. It is an invaluable reference for anyone interested in Egyptian mathematics.

5.4: Current commentators on Egyptian Mathematics

Recently there has been a renewed interest in Egyptian mathematics, partly owing to the interest in non-European mathematics. This section will survey the most important of these commentators, particularly those who are studying the Egyptian mathematical texts for an Egyptological audience.

Generally, the approach taken by more recent commentators has been a more favourable towards the achievements of the Egyptians. It has also been recognised that Egyptian mathematics is far from being exhaustively studied and that more work and a wider appreciation of the material within the Egyptological community is warranted. There is even some appreciation that the context of mathematics in ancient Egypt is important[43]. However, these commentators still have opinions about the nature of science and mathematics that are ill considered and naïve. The exception to this is the set of sourcebooks written by Clagett[44], which through their completeness have made a substantial contribution to the subject and which present material that has relevancy to the context of Egyptian mathematical texts so they can be considered in their proper setting. A fourth

[43] Imhausen, I. (2003) "Egyptian Mathematical Texts and Their Contexts" *Science in Context*; vol. 16(3) pp 367-89.
[44] Clagett, M. (1992) *Ancient Egyptian Science Volume I: Knowledge and Order*; Memoirs of the American Philosophical Society vol. 184; Philadelphia.
Clagett, M. (1995) *Ancient Egyptian Science Volume 2: Calendars, Clocks and Astronomy*; Memoirs of the American Philosophical Society vol. 214; Philadelphia.
Clagett, M. (1999) *Ancient Egyptian Science Volume 3: Ancient Egyptian Mathematics*; Memoirs of the American Philosophical Society vol. 232; Philadelphia.

volume is intended that will discuss medicine, anatomy and the way in which nature was rendered in art.

Clagett's first volume is perhaps the most important as it presents texts which may not be considered scientific, but do illuminate the way in which the Egyptians thought about knowledge and its place in Egyptian culture. This selection includes texts that are more traditionally considered historical such as the Westcar papyrus and the Palermo stone. These texts are incorporated because of what they show about the nature of the Egyptian court and the importance that was placed on knowledge and accurate records. Other texts include tomb inscriptions that discuss the idea of scribal immortality through the production of knowledge:

> "Blessed nobles too are buried in their tombs.
> (Yet) those who built tombs,
> Their places are gone.
> What has become of them?
> I have heard the words of Imhotep and Hardedef,
> Whose sayings are recited whole."[45]

Onomastica are included that list objects and ideas by general categories. These were produced by the ancient Egyptians in order that they would be able to bring forth what is named. For an Egyptologist they are instructional because of the way they demonstrate that religious ideas intersect with what would now be considered scientific ideas. They show that to make the distinction with regard to ancient Egypt is nonsensical.

Volume three deals solely with Egyptian mathematical texts and is the most complete volume on the subject as it contains not only the Rhind and Moscow

[45] Clagett, M. (1992) *Op. Cit.* p. 220.

Mathematical Papyri, but the Kahun fragments, the Berlin papyrus, the Reisner Papyrus and the Mathematical Leather Roll. Part one of the volume is a commentary on all the contents of these texts arranged into themes, part two presents full translations of them.

Clagett's work is an important text for those wishing to know more abut ancient Egyptian science as it provides the most complete edition of the relevant texts. However, his conclusions are not particularly original and so do not give a student of Egyptian science much more than the bare essentials. The three published volumes of these source books are impressively detailed, but lack the overall view that authors such as Gillings bring to the study.

For his conclusion, Clagett discusses the predominance of arithmetical workings in the extant texts. In his opinion, these texts were composed to provide scribes with a manual to which they could refer in the course of their work. Although the texts use concrete numbers, in many cases the procedures can be generalised. Clagett draws attention to the final line of RMP 66:

> "You shall proceed in this way [given above] in any example like this."[46]

Through this evidence, he seems to imply that he believes that the Egyptians were engaged in science. He draws attention to this debate, but does not give any opinions of his own. Indeed, he seems reluctant to do so as his intention when compiling the source books was to document the actual mathematical procedures.

[46] Clagett, M. (1999) *Op. Cit.* p. 94.

The most important commentator on Egyptian mathematics at present is Annette Imhausen[47]. She has been greatly influenced by the work of Sabetai Unguru[48] and so she recognises the great importance of the context of the production of the Egyptian mathematical texts. The definition Imhausen advances for "mathematics" does not include texts that show evidence of mathematical knowledge, such as accounting texts. Mathematical texts are only those that have been written to teach or learn mathematics. Thus, the texts Imhausen identifies from ancient Egypt as being mathematical are problem texts, such as the Rhind and Moscow Mathematical Papyri and table texts, such as the Mathematical Leather Roll. She proposes a new way of studying the mathematical texts by examining their algorithmic structure. Egyptian texts have been described as numeric, rhetorical and algorithmic[49]. This means that the texts are written with concrete numbers, that they are written in a narrative style and the solution is given as a sequence of instructions. It is through studying the algorithmic structure of the mathematical texts that Imhausen believes a new, better understanding of the texts can be achieved. She warns against using modern algebraic notation in translations of the problems, as this will lead to a loss of the three features of the Egyptian mathematical texts. Instead, she has devised a notational system that can summarise the main features of the algorithms used in the separate mathematical problems. This will then allow for a comparison between different problems. This approach to the problems is new and as yet untested. Imhausen's work deserves time to develop and a chance to prove that her new approach has applications. The only caveat is that the

[47] Imhausen, I. (2003) *Op. Cit.* and Imhausen, I. (2002) "The Algorithmic Structure of the Egyptian Mathematical Problem Texts" in Steele, J. and Imhausen, I. (eds.) *Under One Sky: Mathematics and Astronomy in the Ancient Near East; Alter Orient und Altes Testament*; Band 297; Ugarit-Verlag; Minster; pp149-66.
[48] See Chapter 7.
[49] Ritter, J. (1989) "Chacun sa vérité: les mathématiques en Égypte et en Mésopotamie." in Serres, M. (ed.) *Éléments d'histoire des sciences*; Bordas; Paris; pp. 39-61.

algorithmic structures she identifies can lose a lot of detail. For example the algorithmic structure for RMP 31[50] appears thus[51]:

Translation	*Numerical Algorithm*	*Symbolical Algorithm*
1 a quantity	1	D_1
its $\bar{\bar{3}}$	$\bar{\bar{3}}$	D_2
its $\bar{2}$	$\bar{2}$	D_3
its $\bar{7}$ are added to it	$\bar{7}$	D_4
so that 33 results.	33	D_5
	[$1+\bar{\bar{3}}+\bar{2}+\bar{7}=$ $1+\bar{\bar{3}}+\bar{2}+\bar{7}$]	(1) $D_1 + D_2 + D_3 + D_4$
2-21 [calculation]	$33 : 1+\bar{\bar{3}}+\bar{2}+\bar{7}$	(2) D_5 : (1)

The first column gives a translation of the problem as it appears in the text, the second column shows the numbers that are in each section of the problem and the third column is Imhausen's symbolic interpretation of it. Each *D* is one particular dimension or numerical value; the subscript numbers show which one. Where these dimensions are operated on, the operation is shown and it is given a number in brackets. Subsequent operations will use these bracketed numbers to show where the result of this operation has been used. The small numbers on the left hand side of the first column shows the line numbers from the original text. It can be seen that in this example, most of the table is concerned with only the first line of the problem. The majority of the problem is concerned with carrying out the calculation, and the means by which it is calculated is the most important part of this problem.

[50] See Section 2.6.3 for a full translation.
[51] Imhausen, A. (2002) *Op. Cit.* p.164.

These concerns aside, finding new ways of evaluating the Egyptian mathematical texts is a worthwhile pursuit. The work of Imhausen has just started and it will be interesting to see what her analysis uncovers.

Recent commentaries on ancient Egyptian mathematics have taken a far more optimistic view about the achievements of the Egyptians and the value in studying the texts in more detail. It is beginning to be recognised that context is important and that comparisons to other mathematical traditions are largely redundant. However, this is yet to have gained any credence in general mathematical history, nor are Egyptologists aware of these developments. Also, the changes in attitude are tentative and need a solid theoretical basis if these developments are to be sustained.

5.5: Conclusions

The definition of mathematics that is used to examine and to evaluate the competence of the ancient Egyptians is one that has been arrived at through millennia of use, development and practice. Yet, this is a definition that is projected back on a time before it was developed. It is this definition that is used to provide comparisons with Greek mathematics. It should not be surprising that this definition finds in favour of Greek mathematics. The mathematics of the Greeks has been held in high opinion for centuries and it has been used as an ideal, a standard to be followed. The opinion of Kant was written long before the Egyptian mathematical texts were discovered and translated. Yet the discovery of the Egyptian texts does not appear to have altered the opinion that the

Egyptians were no more than a prologue, they were 'groping around' before the revolution of the Greeks. This should be troubling and it is time that our assumptions about the nature of mathematics and its use and place in society should be re-evaluated. If we are to do justice to Egyptian mathematics then new methods need to be found to appraise it and to describe its place within the culture that it was created.

In no other area of History of Archaeology are comparisons made as they are in the History of Mathematics. It would seem absurd to try to draw comparisons between the Pharaonic system of kingship and Athenian democracy and then criticise the Egyptians: it is accepted that the two civilisations had different needs. However, this is analogous to what is taking place in the History of Mathematics. Instead of trying to understand why the Egyptians produced the mathematical texts that have been found, as Egyptologists do when studying any other aspect of the civilisation, strict definitions have been applied to show how right the Egyptians were. They are being judged on our standards, instead of their own.

The two following chapters will attempt to address this problem. Chapter 6 will investigate some of the most important philosophers of science so that the definitions by which the Egyptians are being judged can be examined in more detail. The assumptions of these philosophers can also be examined

Identifying science is problematic. We need to work from the assumption that every text that deals with looking at the natural world is scientific and produce criteria for non-inclusion in the scientific corpus, rather than assuming that they are non-scientific and producing criteria for inclusion. Our own assumptions about what science is and how to go about doing it should not influence our decision, hard as this may be.

The relationship between science and technology is also important. Distinctions between science and technology often rely on technology being the realisation of scientific ideas. Technology is practical, where science is abstract. This is a clear parallel with distinctions drawn between pure and applied mathematics. Technology is a phenomenon that has been explored in detail in archaeology as technology leaves more material remains than scientific ideas. Not only does science need to be written down, a medium that survives poorly in the archaeological record, but only a fraction of people were engaged in science. Chapter 7 will examine this relationship in more detail so that lessons learnt in this area can be applied to the study of Egyptian mathematics.

After this evaluation of philosophy and its effects on the way that ancient Egyptian mathematics is studied, a new approach to the topic - useful for Egyptologists - will be attempted. This approach will try to maximise our understanding of the contents of the texts without resorting to crude comparisons. The evaluation will take into account the criticisms of mathematics that have been revealed from a study of the literature. These criticisms are:

1) Egyptian mathematics is not abstract.
2) The Egyptians did not recognise mathematics as a distinct subject.
3) The Egyptians were a retarding force on the development of mathematics.
4) The texts only provide concrete examples, rather than general principles.

5) The geometry of the Egyptians was merely applied arithmetic.

In addition to discussing these criticisms, an approach that is valuable to an Egyptologist also needs to provide answers that enable the study of mathematics to be integrated into a wider understanding of the Egyptian civilisation. For example, if it is assumed that these texts provide training materials for Egyptian scribes, how effective are they? What does that say about the duties of scribes? The evaluation of the Egyptian sources can also be approached from the other direction; what were the needs of the Egyptians? What would constitute good mathematics to an ancient Egyptian?

Chapter 6

Philosophy of Science and Ancient Egyptian Mathematics

In Chapter 5, it could be seen that the ancient Egyptian mathematics was being evaluated according to definitions of science and mathematics that have been arrived at after millennia of use. The source of these definitions was largely philosophical. These definitions were then projected back onto the ancient Egyptian material, which unsurprisingly fell short. In this chapter, philosophical definitions of science will be examined, particularly those that use the history of science as a guide. From this examination, it will be possible to study to what extent Egyptologists interested in Egyptian science should pay heed to the philosophy of science.

6.1: Introduction

The works of some of the major philosophers of science will be investigated so that the thinking behind their definitions of science can be identified. This will then lead into a discussion of whether these definitions are appropriate in the History of Science[1]. In particular, this chapter will investigate the demarcation question set by Popper and investigated by others such as Lakatos and

[1] I will make the distinction between "the history of science" and "History of Science" in line with Ivor Grattan-Guinness. The first denotes history in the past and its development, whereas the second is the academic discipline and study of the former. cf. Grattan Guiness I. (1990) "Does History of Science Treat of the History of Science? The Case of Mathematics" (sic) *History of Science*; vol. 28, pp 149-73.

Feyerabend. This question and the proposed answers are very illuminating because they show some of the assumptions that philosophers make whilst exploring the nature of science. Again, the affect this has on the History of Science will be examined. This chapter will also begin to identify the reasons why the philosophy of science has such an important place in the History of Science. An assessment of the goals of research in the History of Science and the relationship between History of Science and Philosophy of Science will be explored in detail in Chapter 7.

When research is carried out in the History of Science and Mathematics, it may be done for a variety of reasons, dependent on the experience and interests of the researcher. The History of Science must necessarily be concerned with the past and with science; this may seem to be an obvious and frivolous statement to make but it is one that should be borne in mind when we try to identify the nature and goals of research. The relative importance of each of these major aspects is a question that should be considered before the research is attempted. Are we studying the past to see what science was produced by each period of the human past or looking at the past of a particular idea, theory or paradigm? The emphasis of the study will dictate the research goals and it should also dictate the research methodology.

The study of history is the study of the human past. Scientific and mathematical ideas have been very important in shaping civilisations and have affected the way in which we live in many different ways. This influence can be quite subtle, science explores our place in the universe and has to refer to observable truths about the universe, but it affects the way humans see themselves through their place in the universe. This means that in the History of Science it is possible to judge how correct a particular person or idea was, in a way it is not possible to do in other areas of history. However, a scientific study cannot survive without the practitioners of that study and this creates problems when trying to assess these ideas. For the purposes of an approach which is more interested in the ideas, judgements of 'correctness' are important. However, if the object of study is those people who created the ideas, then this becomes less important: in this approach a wrong answer can be just as revealing as a correct one. This people-centred methodology should encompass the entirety of the scientific and mathematical output of a period or culture, it should also try to describe the achievements of the culture in a less passionate manner and reserve judgement. Any comments made about that culture or period should be rooted in the ideas of that culture.

This is not to say that the 'correctness' of the mathematics should be ignored. Yet, using modern philosophical definitions is problematic. The criticisms of ancient Egyptian mathematics rely on definitions of mathematics and also its place in academic enquiry. The idea that mathematics should be abstract and that applied mathematics, such as the problems in the Egyptian texts, is less important than pure mathematics is an assumption that is largely unquestioned. Yet, definitions of science and mathematics are not rigid. This chapter will examine some of the more important ideas about science and mathematics in order that the origin of these criticisms can be more fully appreciated.

6.2: Philosophical Definitions of Science

The nature of science and mathematics is something that the History of Science attempts to explore, but it is also central to defining which texts are studied and how they are perceived. The philosophy of science and mathematics, therefore, has a strong impact on the way that the subject is studied. What constitutes a scientific text worthy of study will depend upon the philosophical criteria that are used to define science. These philosophies also determine the boundaries between mathematics, science and technology.

Giving a precise definition of the nature of science is very difficult. Definitions tend to be based on how science operates. The features of experimentation or close observation, an ordered area of study, an ability to build on answers and a logical approach are the main features of science that have made it successful. However, these features are not the goal of scientific research, only the process by which it is most efficiently achieved. The purpose of science is an attempt to gain an insight into the universe, to understand its inner workings and perhaps use that knowledge for our own advantage. This is one of the reasons that science has gained such a respected place within the confines of academia. As well as being able to produce theoretical, abstract ideas describing fundamental properties of nature, it is also able to develop these ideas into concrete realities that affect and improve the standard of living. It is this endeavour to gain an insight into the natural world that makes science so attractive. Because it deals with small incremental observations that can be developed into complex theories and models, it has enabled the human race to develop into one of the most successful species on the planet. It is this property of science - the ability to form complex theories - that should be the focus of investigative research into the origins of scientific thought. By only using a narrow definition of science, we are closing interesting lines of research. Study into the importance of magical and theological texts in ancient societies does not have the same prestige as examinations of texts that are recognised as scientific, because magic does not have the same modern cultural implications. However, such texts form an important body of material that should be reviewed with the same vigour and with the view to understanding the thought processes that went into their production. The modern world has come to trust scientific answers over superstitious ones in many areas of our lives, but when the scientific explanations fall short, we still turn to superstitious or alternative methods. This is particularly true in medicine, where we still have an incomplete understanding of the workings of the human body. When magical methods are used in this situation then they may secure a result, if only though accident or coincidence or a placebo effect. This does not equate with people consulting their horoscope in the national papers. Here the objectives of reading the horoscope and the advice it contains are generic and applicable to any situation. Magical texts that are aiming for a specific result are comparable with scientific texts.

The definition of science that has been built up by philosophers and writers such as Lakatos, Kuhn and Popper is written to explore the best practice within the scientific disciplines and how scientific knowledge is created. The writings of Popper, for example, investigate the nature of evidence and how scientific knowledge is obtained from this evidence. His two main concerns were induction and demarcation. He was interested in inductive reasoning because of the conflicting arguments associated with it. On the one hand it can be argued that observation and experiment cannot be used to justify laws since they transcend human experience. One the other hand, science uses laws all the time and in science empirical evidence is held as the only way to pronounce on the validity of scientific laws. The apparent disagreement of these ideas is the foundation of the problem of induction[2]. The problem of demarcation is to find a normative criterion for distinguishing between science and pseudo-science. These are closely linked in the minds of the philosophers who examined it, and have obvious implications for the assessment of ancient Egyptian mathematics. The Demarcation question will be discussed in detail in Section 6.4.

These two problems are key to understanding the epistemology of modern science and to appreciating how scientific advance can be made. Understanding scientific advance is an admirable aim, but where does it leave researchers in the History of Science? How much attention do we need to pay to modern ideas about the place of science in modern society and the way in which it operates now? It is tempting to see the modern view of science provided by the Philosophy of Science as the best model for the operation of science. This model has been built up after millennia of scientific research. It reflects all the examples of advancement in science. If science is progressing then surely its Philosophy must progress too to reflect changing paradigms and in order for scientists to make new advances within those paradigms.

[2] Popper, K. (1974) *The Problem of Induction*; http://dieoff.org/page126.htm; retrieved 17/05/04.

6.2.1: Realism and Instrumentalism[3]

The argument between realist and antirealist philosophies rests on the question of whether scientific knowledge exists independently of the minds of scientists. Realism asserts that we can know what exists and what laws govern the behaviour of the universe. Antirealist philosophies challenge the notion of objectivity in several ways and link science to the actions of humans and their creativity in varying degrees. Realism is popular with practising scientists. Science and its technological applications have come to hold an important place in Western culture. Many scientists instinctively feel that science must have some relationship to the Universe: that we are discovering truths, not inventing them[4]. One of the main arguments for realism is the 'cosmic coincidence' argument[5]. This argument rests on the success of science; if the entities employed by science did not exist and scientific theories were not at least approximately true then the massive convergence of evidence would be a coincidence of implausibly cosmic proportions. Realism is therefore attractive to scientists who deal with these entities and who see the success and the wide applications for their work. Antirealist theories are sometimes seen as an attack on science because they threaten to undermine the privileged position of science and its claims to objectivity[6]. However, in History of Science, ideas, which have been proved false, have to be evaluated. For this reason, antirealist philosophies of science deserve further investigation. Instrumentalism holds that concepts are merely useful instruments for human purpose. Truth is therefore not central to

[3] Fine, A. (1998) "Scientific realism and antirealism" In Craig, E. (ed.); *Routledge Encyclopaedia of Philosophy*; London: Routledge. Retrieved May 17, 2004, from http://www.rep.Routledge.com/article/Q094
[4] Rescher, N. (1987) *Scientific Realism*; Reidel; Dordrecht; pp31-54.
[5] Klee, R (1997) *Introduction to the Philosophy of Science: Cutting Nature at its Seams*; Oxford University Press; Oxford.
[6] See Chapter 9

scientific enquiry, rather reliability and usefulness are. It is allied to pragmatism. Two prevalent anti-realist theories are instrumentalism and Constructionalism. Instrumentalism is the belief that scientific theories are not necessarily true, but they are useful as instruments for prediction. Lakatos was critical of instrumentalism. He sees instrumentalists as conventionalists[7] who do not have the education to see that some hypotheses can be true whilst being unproven and others false whilst having true consequences[8]. Constructionalism is - in essence –an idea that science is constructed by human beings and is affected by cultural beliefs, it will be explored in detail in Chapter 7.

For the study of ancient Egyptian science, the arguments for realism that depend on the predictive power of science and the reaction of scientists are less important that the idea of the human element in scientific activity. Because the scientific ideas being studied have been superseded and comparisons to later scientific cultures have no interest for an Egyptologist, the arguments for realism hold little interest.

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[7] Conventionalism sees science as a series of pigeon holes which organise facts. These pigeon holes are not held to be true, they are only true by convention.

[8] Lakatos, I. (1981) "History of Science and its Rational Reconstructions" in Hacking, I (ed.) *Scientific Revolutions*; Oxford Readings in Philosophy; Oxford University Press; Oxford; p110-13

6.2.2: Approximation to the Truth

This discussion is based on the fact that the ideas of Newton and Galileo have been superseded by the ideas of Einstein and quantum mechanics. Each new theory is likely to be investigated further and found to be if not untrue, then unable to explain new observations of the fundamental nature of the universe. This leaves the historian of science with a problem. If science does operate in this way then how can we sit in judgement on earlier ideas knowing them to be false, or less true than current ideas? It is all too easy to criticise previous ideas because they do not match up to the rigour of modern scientific ideals and yet it is just as hard to abandon our own cultural prejudices in favour of a more primitive model. This idea is fundamental to chronological Histories of Science. The notion that each idea is replaced eventually by one that is a better approximation is very pervasive and persuasive. This model of the progress of scientific thought takes no account, however, of the impact that these ideas have on society. There are many examples in the history of science where one particular theory gains favour over another because of influences from culture and religion that predispose the reviewers and general scientific community towards that idea. The resistance to Darwin's theories on evolution can be seen as an example of this. Personalities are also very important to consider. Lord Kelvin's views on the age of the Earth, for example, remained important for a long time because of the personality and status of Lord Kelvin, and also greatly impeded the acceptance of Darwin's idea because he proposed such a young age for the Earth.

Other definitions of science depend on the logic of scientific discovery. They attempt to define science by the way in which scientific knowledge is created. These definitions are discussed in detail by philosophers interested in the demarcation question. Each of these definitions is discussed in the following section.

6.3: The Demarcation Question

The demarcation question was set by Popper. It asks whether there is one criterion that can be used to distinguish between science and non-science, which he identifies as including, amongst others, logic, metaphysics, psychoanalysis. This is an extremely important question for anyone engaged in the history of science. The criticisms of Egyptian mathematics show that the question of whether or not something is scientific can radically alter our opinion of it. However, this judgement is often performed on a gut feeling of what science is. There is a feeling that we all know what science is when we come across it. There may be some truth in this when we are dealing with modern science. The people engaged in producing the science have all had a standard training and they will fit into standard roles. This cannot be assumed when dealing with texts from the past. There is no reason to suppose that the practitioners of the past were dealing within these standard roles. In this case, to ask for clarification on what is meant by the term science is not as facetious as it may appear. By examining Popper's demarcation question and some of the reactions to it, it will be possible to see that the question "what do you mean by science?" is complicated and deserves thought before making pronouncements on the scientific achievements of the past. Popper is concerned with contemporary science, by showing that working out what is science is a complicated process; it will be shown that asking this question of the past is even more complicated.

6.3.1: Description of Demarcation Problem

Popper believed that all observation in science is selective and bound by current theories. This means that there can be no freestanding observations and thus that the primacy of 'pure' observation is mistaken. He therefore questions the traditional view that there is one methodology that can distinguish science from non-science or pseudo-science. Instead, Popper believes that science is like any other human activity and consists largely of problem solving.

Popper's solution to this question was falsification, having rejected inductive reasoning. Falsification asserts that it is not proving a theory that is important in science, since it is easy to obtain evidence to support any hypothesis. Confirmations should count only if they are the result of risky predictions, which might have conceivably been false. Thus, any theory which is to be considered scientific must have a test that has the power to falsify it: a crucial experiment. Theories should also be able to make new predictions of phenomena as yet unseen.

The demarcation problem is at the heart of many philosophical uses for the history of science. Philosophical approaches want to use the history of science as a sourcebook of experimental data in order empirically to test hypotheses about the workings of science. Philosophers involved in this methodological approach claim to be interested in the whole of science: they certainly do not attempt to define the object of their enquiry more closely, nor do they explicitly state whether they are excluding any part of science from this enterprise. However, from the assumptions and examples given, it is clear that they are only interested in defining the distinguishing feature of modern, pure, academic science. The examples used are from the recognised science of academic study. Science that may have happened outside the structure of academia is not important in this study. They are also only looking at science that has informed modern academic science; non-Western approaches and results are ignored. One can only assume that these are not considered part of the scientific venture and that their logical structure is of no interest to those philosophers and scientists who are concerned with defining what science is.

There is also a much more fundamental problem with this approach, both in its application to history of mathematics and in answering the question posed. In trying to define what science is, examples are used from history of science to test whether or not they fit a particular definition of science. Whether inductivist, constructionalist or falsificationalist, these ideas are checked against the history of science. This approach therefore claims to be testing the foundations of a subject, but uses the writer's assumptions of what science is in order to answer the question. The assumptions underlying what science is are therefore applied retrospectively to select episodes from history that can tell us what science is. The writer's own assumptions must then become part of the definition that is reached. If we were to assume that science includes non-academic science and selected our examples accordingly, then we would reach a very different definition. The conclusion that Lakatos reaches, that science is defined by Research Programmes, shows that he is only interested with how science develops within the confines of academia. He does not feel that it is necessary to explain how these ideas might find their way into the collective consciousness of society as a whole, as this is not where he sees science operating. Nor is he interested in non-Western approaches, as none of the examples come from non-Western societies. He therefore excludes from his study the idea that there may be another way to do science.

However, Popper's response to the demarcation question holds interesting ideas for a historian of science. Popper rejects inductive reasoning and the primacy of observation from his answer because he recognises that these observations are selective and bound by current theories. Therefore, he recognises that the context of scientific work can affect its outcome. Falsification does not imply that there is any correct way for finding knowledge, nor is there any correct way of presenting it. It does not even suggest what a scientist should be interested in. It merely states that any theories should have the ability to be shown as incorrect. Recognising that the context is all-important is necessary if an objective analysis of ancient science and Egyptian mathematics is to be achieved.

In addition, the idea of falsification is an important one in studying Egyptian mathematics because it is closely allied with the idea of proof. There is much discussion over whether the Egyptians had the concept of proof and this will be discussed further in Chapter 8. However, it should be noted that falsification and the idea of proof are closely aligned. There are important differences, but to evaluate the Egyptian texts it is necessary to question these terms because they have all been defined with modern scientific practice in mind.

6.3.2: Pseudo-science

The concept of pseudo-science is prevalent in attempts to answer the demarcation problem. If an idea does not live up to the high standards exacted by the various philosophical schools then it is relegated to the realms of pseudo-science. Here such ideas become uninteresting and subject to no further study. Instead, I believe these ideas should be investigated further. Even if they are not science, then identifying the reasons behind their creation becomes just as important as classifying those ideas and theories that are considered scientific. History is a complicated subject and all the variables have to be studied in order to come up with a complete picture. This approach is championed by Feyerabend[9]. It does not usually find favour amongst scientists, as they are keen to maintain the supremacy of science. It is true that science has been a great success in modern society and that we owe a lot to it. However, in academic study there should be no room for hierarchies of knowledge. If science and its place in society are to be understood then a more inclusive attitude to different forms of knowledge is needed. This fact was recognised by Lakatos[10] who argued that if the threshold of rationality is set too high then too much of the history of science appears irrational. This compels historians of science who treat the growth of scientific knowledge as the epitome of rationality either to give a curtailed account of the development of scientific thought, or to twist the facts to conform to this idea. This suggests that the most useful theories for the use of historians of science are the most inclusive because these would stimulate the least distorted account of the growth of science. It should also suggest that

[9] See Section 2.5.
[10] Lakatos, I. (1978) "On Popperian historiography" *Mathematics, Science and Epistemology* Worrall and Currie, G (eds.) *Philosophical Papers volume 2*; Cambridge University Press; Cambridge; pp 201-10.

historians of science should not be too quick to assume that the growth of science is the epitome of rationality and that it is possible for scientists in the past not to live up to the standards of rationality that we now prize in our scientists. Thus the idea of pseudo-science is problematic for historians of science and caution should be taken when applying it to the history of science.

6.3.3: Imre Lakatos and the Demarcation Question

Lakatos was also interested in the demarcation question, as he had been a pupil of Popper. It is in his writings about the demarcations question that the link between history of science and philosophy of science is made plain. He wrote as the opening line to a paper:

> "Philosophy of science without history is empty. History of science without philosophy of science is blind" [11]

This is evocative of Kant who wrote:

> "Concepts without percepts[12] are empty, percepts without concepts are blind.[13]"

By using this quote from Kant, Lakatos is saying that philosophy of science gives concepts, where history of science gives percepts, or that history of science is there to give evidence for the concepts in the philosophy of science.

In this paper, Lakatos explores the demarcation question. He states his aims as arguing:

[11] Lakatos, I (1978) "History of Science and its Rational Reconstructions" in Worral J. and Currie, G. (eds.) *The Methodology of Scientific Research Programmes. Philosophical Papers vol. 1*; Cambridge University Press; Cambridge; pp102- 138.
[12] A percept: The mental product or result of perceiving as distinguished from the action.
[13] Kant, I. (1787) Critique of Pure Reason, A 51/B 75 Translation Goyer, P. & Wood, A. Cambridge Edition of the Works of Immanuel Kant (1998) Cambridge University Press, Cambridge

> "a) Philosophy of science provides normative methodologies in terms of which the historians construct 'internal history' and thereby provides a rational explanation of the growth of objective knowledge.
>
> b) Two competing methodologies can be evaluated with the help of (normatively interpreted) history
>
> c) Any rational reconstruction of history needs to be supplemented by an empirical (socio-psychological) 'external' history."[14]

Internal history is usually defined as intellectual history, whereas external history is identified as social history. Lakatos here tries to redefine the distinction between them. The difference between the two sets of definitions is slight and the attempt to split the history of science into two distinct sections remains unresolved. Lakatos argues that understanding external history is unimportant for the understanding of the history of science. He sees internal history as being normative; internal history should search for the rules by which science advances. On the other hand, external history is empirical; it judges the observation of events. Thus Lakatos argues that external history is guided only by the methodology used and the choice of problem. Because he sees internal history as normative, in his opinion internal history does not suffer from the biases of problem selection. This argument presumes, of course, that there is a normative logic about the advance of scientific knowledge, that there is a single way by which knowledge can be found. It also presumes that internal history and external history can be separated and that scientific advance is not altered by external religious, social and cultural factors. This idea is the antithesis of constructivism[15].

[14] Lakatos, I (1978) *Op. Cit.* p.107.
[15] For more on constructivism see Chapter 7.

Lakatos compares the rival methodologies of science that are proposed by the philosophy of science. Each of these methodologies represents rational reconstructions as guides to history, and inspired by history. By methodology, Lakatos means codes or logics of discovery, a set of rules by which existing theories can be appraised. He says that normative no longer means rules for arriving at solutions but only for appraising what is there. These rules have a double function and are often what are cited when we talk about what science is. Firstly, they are a kind of ethical code for scientists. Secondly, they are the cores of historiographical research programmes. It is worth looking in detail at the arguments Lakatos advances for the acceptance or rejection of each of the methodologies. There are features in each of them that are reflected in previous approaches to ancient Egyptian mathematics. By examining why these methodologies are accepted or not it will illuminate the assumptions made by philosophy of science about the nature of the History of Science. The internal/external divide is prevalent in the arguments of Lakatos. Understanding this divide is of extreme importance, as it will be argued that Egyptian mathematical texts can only be fully understood if they are incorporated into our wider understanding of Egyptian culture and society.

6.3.3.1: Inductivism

> "According to inductivism, only those propositions can be accepted into the body of science, which either describe hard facts or are infallible inductive generalizations from them."[16]

Inductivism has its problems because of its preoccupation with establishing the truth: if a proposition cannot be inductively argued from observed facts then it is ignored. If actual history does not fit the inductivist standards, inductivists may suggest starting the whole business of science anew. Criticism is sceptical, it consists of showing that a proposition is unproven, or pseudo-scientific rather than showing it is false.

There are two sorts of discovery in this method: fact propositions and inductive generalisations. A historian of science following the inductivist approach will scan the history of science looking for these two types as these are the backbone of internal history. A revolution in inductivism terms happens when errors are unmasked and what were scientific theories are shown to be pseudo-science. Radical inductionism invalidates propositions when external influence can be shown, this is a kind of radical internalism. It is noted, though, that this is a utopian position. Scientists cannot select observations with an empty mind. An inductivist approach, although vigorous in its scientific standards, cannot explain why things happen. This defeats Lakatos' attempts to find a normative internal history and so he does not favour this approach.

[16] Lakatos, I. (1978) *Op. Cit.* p. 103

6.3.3.2: Conventionalism

This allows for the pigeon-holing of ideas to build a coherent whole. This system is not proven true, it is only 'true by convention'. Genuine progress in conventualist science is cumulative and takes place at the ground level of 'proven facts'. The changes to the pigeon-hole system are on the theoretical level and are merely instrumental, thus progress in theory is only in convenience and not at the truth level.

Conventionalism recognises that false theories may have true consequences; they may also have great predictive ability, which is, of course, a means of experimental verification. Instrumentalism is a form of conventionalism[17]. In the opinion of Lakatos, instrumentalism ignores the problems of some propositions being true while unproven and others being false while having true consequences; theories are merely instruments for prediction. Lakatos considers this to be a lower form of conventionalism, caused by a lack of proper logical training.

Conventionalism does not brand discarded systems as unscientific. The conventionalist sees much more of the actual history of science as rational (internal) than does the inductivist. Major discoveries are primarily inventions of new and better pigeon-hole systems, where better systems are simpler ones, as these require fewer assumptions. This constitutes the backbone of history. Unfortunately, this method of history cannot give a rational explanation of why different ideas are selected and different systems are tried at different times when

[17] See Section 6.2.1.

the benefits of the new system are unproven. Thus, like inductivism, external programmes have to be appealed to.

6.3.3.3: Methodological Falsification

Methodological falsification was Popper's solution to the demarcation question. Lakatos respects Popper's methodology because of its simplicity and force. He feels, however, that there are problems with it because it cannot explain how some crucial experiments are taken to be confirming rather than falsifying. Thus external theories are needed to supplement internal history. Also, in Lakatos' view, each scientific theory is born falsified because of the anomalies that surround every scientific experiment.

6.3.3.4: Methodology of Scientific Research Programmes[18]

Lakatos felt that each of these three demarcation criteria fails when the history of science is appealed to. This is because Lakatos was examining history for rational episodes. Because Lakatos felt that internal history should be able to explain the growth of knowledge, any methodology that had to appeal to external history could not constitute the demarcation criterion. Lakatos is dismissive of any scientist's claim to inductive reasoning, including Newton, who Lakatos says did not derive his laws from Kepler, because Newton knew Kepler's laws, based on prefect ellipses, to be false[19]. Conventionalism is dismissed through the idea

[18] Larvor, B. (1998) *Lakatos: An Introduction*; Routledge; London; pp 50 – 58.
[19] Lakatos, I (1978) "Newton's Effect of Scientific Standards" in Worral J. and Currie, G. (eds.) *The Methodology of Scientific Research Programmes. Philosophical Papers vol. 1*; Cambridge

that Copernicus' theories did not constitute a simpler system than those that they replaced[20].

To solve the demarcation question Lakatos believes that instead of focussing on separate theories, research programmes are the appropriate way to examine the history of science. Research programmes are the historically linked theories. Theories are static because their content does not change, whereas the research programme may have central ideas, but the details can change over time.

The idea of the research programme, rather than the theory, being the correct level of detail for study, allows Lakatos to modify Popper's ideas on falsification. The anomalies that Lakatos sees in scientific experiments that would falsify a theory are not a problem in the context of a research programme. Crucial experiments disappear, because it is only long after the event that an anomaly is recognised as falsifying a scientific hypothesis, when one programme has been defeated by another one.

The idea, which is central to Lakatos' solution to the demarcation problem, is that there have been extended wars during the history of science between research programmes, some of which are progressing and some of which are degenerating. A discipline can be considered scientific as long as progressive programmes triumph over degenerating ones. The consequences of this line of reasoning for Egyptian mathematics are clear. The enterprise of Egyptian

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University Press; Cambridge; pp193- 222. See p. 210 for a detailed discussion on Lakatos' views of the advance of Newton's laws.

[20] Lakatos discussed Copernicus in detail in: Lakatos, I (1978) "Why Copernicus's Programme Superceeded Ptolemy's"in Worral J. and Currie, G. (eds.) *The Methodology of Scientific Research Programmes. Philosophical Papers vol. 1*; Cambridge University Press; Cambridge; pp168 – 92..

science should be taken as a whole rather than isolating particular mathematical problems, or even separating mathematics from other scientific disciplines in ancient Egypt. The entire corpus of Egyptian scientific literature needs to be examined and if possible a chronological account built up. Lakatos has his critics and his theories on scientific research programmes may not fully explain the growth of scientific knowledge, but it is clear that understanding programmes rather than separate theories opens up a new line of enquiry for ancient Egyptian science that should be followed.

6.4: Imre Lakatos and Mathematical Philosophy

Lakatos was not only a philosopher of science, but also of mathematics. His most important work on the philosophy of mathematics[21] was a dialectic exploration of concepts in mathematics, such as proof and its rôle within mathematical theorems. Lakatos was interested in the development of concepts in mathematics and in order to explore the relations between these concepts, Lakatos invents pupils who represent various viewpoints, drawing on the tradition of Socratic dialogue. These pupils discuss the topics in the presence of a teacher who prompts them into discussion. Much of the book is concerned with proof, and it is the concept of proof that is of most interest to the Egyptologist.

Lakatos' views on proof are neatly summed up in a paper that he wrote for a seminar at Cambridge, although he himself had no intentions of publishing it. It

[21] Lakatos, I. (1976) *Proof and Refutations*; Worrall J. and Zahar E. (eds); Cambridge University Press; Cambridge.

does however include the relevant material from *Proofs and Refutations.* For example, Euler's conjecture and the problems of proving it take up 102 pages out of 170 in *Proofs and Refutations*. Much of the interesting part of this argument is summarised in Lakatos' description of pre-formal proofs.

Lakatos opens the paper with the observation that:

> "Pure mathematicians disown the proofs of applied mathematicians, while logicians in turn disavow those of pure mathematics."[22]

In this statement Lakatos is recognising that there are several forms of proof and the needs of different groups will require a different form of proof. Lakatos defines three types of proof: pre-formal, formal and post formal. Pre-formal proof is illustrated with a proof of Euler's theorem of polyhedra which states that:

$$V - E + F = 2.$$

Where V is the number of vertices, E is the number of edges and F is the number of faces.

The premise for this proof lies not in the formal manipulation of formulae, but in an argument that relies on a thought experiment. The proof requires the reader to imagine the figures and to manipulate them mentally.

Lakatos asserts that there is more to proof than the instinctive formalist[23] definition of mathematics and proof. He argues that proof is a "finite sequence of formulae of some given system, where each formula of the sequence is either

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[22] Lakatos, I.. (1978) "What does a mathematical proof prove?" *Mathematics, Science and Epistemology* Worrall and Currie, G (eds.) *Philosophical Papers volume 2*; Cambridge University Press; Cambridge; p.61.

[23] Lakatos defines formalism as "the school of mathematical philosophy which tends to identify mathematics with its formal axiomatic abstraction" Lakatos, I. (1976) *Op. Cit.* p.1.

an axiom or a formula derived by a rule of the system from some proceeding formulae"[24]. He also believes that there is not a definition of proof that would allow us to decide what is a proof and what is not. Instead, Lakatos takes a quasi-Popperian approach and appeals to falsification. Euler's theorem can be shown to have omitted possibilities that do not agree with his conjecture; the conjecture assumes a simple polygon, whereas a polygon with a hole in the centre would not fit the conjecture. Thus the conjecture has to be modified.

Formal proofs are those that are most often associated with proof. They are the formal statements of formulae that follow on from axioms. Formalisation of a pre-formal proof is possible, and it can lend authority to a proof, as it will show that there is no counter-example. For many mathematicians, this is the only permissible definition of a proof, and it is the definition that is now used to judge the Egyptian mathematical papyri.

Lakatos goes on to define a type of proof that need not concern the student of Egyptian mathematics; it is included here only for completeness. Post-formal proofs are those which define more than the mathematician wished to prove. Lakatos gives examples such as Peano's axioms[25] which are satisfied by more than natural numbers.

Lakatos concludes his paper by stating that pre and post-formal proofs prove things that are sometimes clear and empirical, and sometimes vague. Formal

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[24] Lakatos, I (1978) *Op. Cit*; p. 62
[25] There is a natural number 0.
Every natural number a has a successor, denoted by $S(a)$.
There is no natural number whose successor is 0.
Distinct natural numbers have distinct successors: if $a \neq b$, then $S(a) \neq S(b)$.
If a property is possessed by 0 and also by the successor of every natural number which possesses it, then it is possessed by all natural numbers.

proofs are reliable, but given the elusive nature of mathematics, it is sometimes not clear what they are reliable about[26].

Where does this leave the idea of proof in Egyptian mathematics? It is clear that in Lakatos' opinion the Egyptian texts should not be dismissed as quickly as they often are. The arguments that are used to argue that the Egyptians did not use proofs are based solely on the formal definition of proof. Yet, the ideas about proof articulated by Lakatos show that there is more scope than this idea will admit. From his opening remarks it can be seen that different groups of people have different needs for mathematics, and so require different standards in their proofs. So we should not be too critical of the Egyptian texts, instead we should identify the needs of their mathematicians and then judge the standard of their proofs within that context.

Proofs and Refutations is anti-positivist in its historiography[27]. Positivist historiography takes the view that to explain a historical fact is to subsume it into a historical law. It will attempt to offer the same logical analysis for history as positivism does for the sciences. Lakatos makes no attempt to discover general laws of mathematical development; he rejects the idea that it is possible. He admits that there are logical patterns in mathematics, but that those patterns do not hold any explanation for why the episodes of mathematical history occurred they way they did. His loathing for positivist historiography is shown in other papers. For example, in his paper "What does mathematical proof prove?" he

[26] Lakatos, I (1978) *Op. Cit*; p. 69
[27] Larvor, B. (1998) *Op. Cit*; p. 22

writes that he does not wish to let "a disastrous historicism into sound mathematical philosophy" [28].

6.5: Kuhn and The Structure of Scientific Revolutions

While writing *The Structure of Scientific Revolutions*[29] Kuhn realised that as he found out about science in the past, his preconceptions were shattered. His background was in physics and he received a PhD. in physics in 1949. However, when he examined the history of science he found that neither inductivist nor falsificationist theories explained the way in which one scientific idea replaces another. His theories of revolutions in science reflect the way in which one set of theories and dogma are replaced by another incompatible with the original. His idea of the progress of the sciences can be summarized by the following scheme:

> *pre-science - normal science - crisis - revolution*
>
> *– new normal science - new crisis.*

The idea behind this scheme is that there is plenty of disorganised activity that predates the conception of a scientific theory. Once a theory is established, a period of normal science is entered in which scientists work within the paradigm of the theory. This experimentation will create anomalous results which will build up until a crisis in the theory ensues and eventually the original paradigm will be abandoned in favour of a new one and a paradigm shift will occur. There

[28] Lakatos, I. ed. Worrall and Currie, G. (1978) *Mathematics, Science and Epistemology. Philosophical Papers volume 2*; Cambridge University Press; Cambridge; p.61.
[29] Kuhn, T. S. (1970) *The Structure of Scientific Revolutions*; 2nd ed.; University of Chicago Press; Chicago.

will then be another period of normal science as the new paradigm is developed and explored and new anomalies arise, until a new crisis is encountered. [30]

The main problem with Kuhn's theories is the idea of paradigm. Kuhn never explicitly explains what is meant by a paradigm and consequently what is meant by a paradigm shift. It is possible to describe some of the typical components of a paradigm, however. Loosely, a paradigm is a system of beliefs shared by scientists. It can, however, be a singular scientific achievement such as Newton's *Principia* that came to serve as the basis for the work of many scientists that followed him. Paradigms will also include the ways in which fundamental laws are applied to various situations. Kuhn felt that there is more to a paradigm than can be laid down in the form of rules or directions. If one tries to characterise a specific paradigm then it will always turn out that some work within the paradigm violates the characterisation[31]. This should not make the idea of a paradigm untenable, however. Many of the features of a paradigm are similar to the research programmes advanced by Lakatos.

The criticisms that Kuhn's work has attracted come mostly from scientists who thought that Kuhn was attacking scientific objectivity. There is an inherent conservatism about Kuhn's ideas as scientists become not independent thinkers, but people guided by what they have been taught and the paradigm in which they are working. This brought attention to the need for a historical approach to the

[30] Chalmers, A.F. (1978) *What is this thing called Science?*; Open University Press; Milton Keynes; p. 86.
[31] Chlamers A.F. (1978) *Op. Cit.*; pp. 87-9.

philosophy of science. Lakatos in particular was influenced by the work of Kuhn[32].

6.5.1: Pre-Science

It is the idea of *Pre-Science* that is troubling, particularly for the study of Egyptian mathematics. This model evades the question of the origin of science. Scientific thought is one area of human cognition and behaviour that separates us from the rest of the animal kingdom. However, identifying the point in our evolution when scientific ideas happened is as difficult as identifying the first time our ancestors used language. Whilst this is partly a problem of the non-survival of evidence, it is also partly due to definitions. When considering the origin of language, non-verbal communication has to be considered. This leads to problems over the definition of language. Just as non-verbal communication may not fulfil all the criteria for identification as a language, pre-science may not fulfil all the criteria for science. Archaeologies of pre-history concern themselves with technology. It is argued that advances in flint-knapping techniques and the production of stone tools of increasing complexity and specialisation of use do not constitute scientific thought. It is presumed that they were produced in a trial and error fashion; likened to an evolutionary process, where those tools that were better gave the people who made them an advantage. Our ancestors are not often credited with the intellectual capability to think through the processes used to make flint tools in a way that is considered scientific. Yet, this is to place too much importance on the way in which science operates now, rather than to understand the process of early man.

[32] Rorty, R. (2000) "Kuhn" in Newton-Smith, W. H. (ed.) *A Companion to the Philosophy of Science*; Blackwell Publishers Oxford; pp 203 – 6.

6.6: Feyerabend and Epistemological Anarchy

Paul Feyerabend was a fierce critic of philosophers who claim that there is only one methodology for successful science; he was critical of the idea that there is only one way to increase our knowledge of the universe. For Feyerabend, there should be no restrictions on activities that create knowledge: in his view, anarchy, while not being a good political philosophy, is excellent in the philosophy of science[33]. In Feyerabend's view, the idea that there should be rules to scientific epistemology is not just misguided but also detrimental to science because it neglects the social and cultural factors that can lead to scientific change[34].

Against Method is dedicated to Lakatos "Friend and fellow-anarchist". For some time Feyerabend and Lakatos taught at the LSE together. He sees Lakatos as an anarchist because Lakatos' methodology of research programmes does not give rules for scientific advance, rather it provides standards[35]. Lakatos was originally to write a reply to *Against Method* but sadly died before he could do so. Lakatos and Feyerabend had limited differences in their outlook. The main difference in their approach was whether scientists need philosophers to explore the dialectics of their work by producing the type of rational reconstructions of the advance of knowledge that Lakatos was enthusiastic about.[36]

Anarchy should not be understood to mean that there is no distinction between reasonable scientists and cranks. However, the difference Feyerabend sees

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[33] Feyerabend, P. (1975) *Against Method*; New left Books; London.
[34] Feyerabend, P. (1975) *Op. Cit.* p.295- 309.
[35] Chalmers, A.F. (1982) *What is This Thing Called Science?*; 2^{nd}. Ed.; Open University Press, Milton Keynes; p 135.
[36] Larvor, B (1998) *Op. Cit; pp 84-5.*

between these groups is not that one group can suggest what is plausible and bound to work whereas the other is bound to fail. Within epistemological anarchy there is no way to predict which theories will be successful; the difference arises because the reasonable scientist will develop an argument and adapt it in response to contrary evidence, whereas the crank will defend his point of view in its original form regardless of such evidence[37]. Feyerabend's view of epistemological anarchy is, therefore, one where he recognises that the human beings who engage in scientific thinking do not always follow rules of discovery, rather that they must at times break them in order to advance science.

Feyerabend complains that the defenders of science do not adequately investigate other forms of knowledge and that science cannot be seen as the only form of knowledge that should be taken seriously. He argues that witchcraft can and should be studied with the same seriousness as science by immersing oneself in witchcraft with the view to become a witch[38]. Feyerabend also recognises the importance of having a historical component to the study of the philosophy of science, but he argues for the usefulness of studying other forms of knowledge.

[37] Feyerabend, P. (1964) "Realism and Instrumentalism : Comments on the Logic of Factual Support"; Bunge E. (ed.) *The Critical Approach to Science and Philosophy*; Free Press; New York; p. 305.
[38] Williams, M. (1998). "Feyerabend, Paul Karl". In E. Craig (Ed.), *Routledge Encyclopedia of Philosophy*. London: Routledge. Retrieved May 20, 2004, from http://www.rep.routledge.com/article/Q114

He writes:

> "The history of science, after all, does not just consist of facts and conclusions drawn from facts. It also contains ideas, interpretations of facts, problems created by conflicting interpretations, mistakes, and so on.... the history of science will be as complex, chaotic, full of mistakes and entertaining as the ideas it contains, and these ideas in turn will be as complex, chaotic, full of mistakes and entertaining as the minds of those who invented them. Conversely, a little brainwashing will go a long way in making the history of science duller, simpler, more uniform, more 'objective' and more easily accessible to treatment by strict and unchangeable rules."[39]

This approach has been accused of being relativistic, of thinking that scientific theories are not true absolutely, that they only have truth-value from a particular standpoint. He is also charged with wishing for a world where science is not special[40]; some go so far as to call Feyerabend an irrationalist[41]. These comments are largely unfair and miss the point. Feyerabend is reacting against the philosophies of Popper and Lakatos and suggesting that to understand science, other ideas have to be explored with equal emphasis.

Feyerabend's view of epistemological anarchy seems very attractive, considering that Egyptian mathematics does not fit into the traditional ideas of science and that it is necessary to contextualise the texts into the wider Egyptian culture. It can certainly be seen that mathematics and science should not be given a privileged position in Egyptologists' attempts to understand Egyptian culture. However, within Egyptology the opposite has been true. Mathematics and science have been pushed to the fringes partly because of Egyptologists' unwillingness to deal with mathematical and scientific texts, but also because of the generally negative opinion that can be found in textbooks. Despite this, to

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[39] Feyerabend, P. (1975) *Op. Cit.*; p. 19.
[40] Couvalis , G (1997) *The Philosophy of Science; Science and Objectivity*; SAGE Publications; London; pp 111-2.
[41] Stove, D. (1982) *Popper and After: Four Modern Irrationalists*; Pergamon Press; Oxford; p. 3.

follow Feyerabend and consider Egyptian mathematics as only one way in which the Egyptians discovered knowledge would be to go too far. Egyptian mathematics deserves serious, considered thought. To accept anarchy and the view that science and mathematics is no different from other forms of knowledge would not help facilitate this. Nor would this approach be helpful for Egyptologists who are reluctant to look at mathematical texts.

Feyerabend can tend towards the idea that science has no special place. This should be particularly avoided in the case of Egyptian mathematics. Egyptian mathematics has a strong practical component. It is not entirely practical, there are abstract elements and problems that do not have an obvious application, but much of the impetus for Egyptian mathematics comes from everyday problems. Thus Egyptian mathematics has a purpose in Egyptian society. To adopt a relativistic approach when the Egyptians were clearly using their mathematics to solve practical problems would be to ignore the importance that the Egyptians themselves place on their scientific knowledge.

6.7: Conclusions

This chapter has only been able to chart some of the more important ideas in the philosophy of science and its relationship to the history of science. Many philosophers of science have been attracted to the subject because of their admiration for the seemingly rigorous nature of science and its apparent logic[42]. The idealistic nature of the philosophy of science has led to some accounts of scientific advances that do not take into account the often disorderly nature of

[42] Larvor, B. (2000) "History, Role in the Philosophy of Science" in Newton-Smith, W. H. (ed.) *A Companion to the Philosophy of Science*; Blackwell Publishers Oxford; pp 154-161.

human beings. This has led to the distinction between internal and external history. This distinction allows science to be seen as purely logical, with anything that does not fit within this logical pattern of the growth of scientific knowledge, being explained as external influence. Some say that philosophers of science should not be allowed to engage in the History of Science at all:

> "Philosophers tend to be interested in ideas, their logical connections and their logical consequences. They do not seem to find it very interesting to ask where ideas came from, how they developed and how they were interpreted by others who claim to have been influenced by them. They are, therefore, at their best when analysing a system; as we have seen, they are at their worst when trying to account for the evolution of one."[43]

It seems clear that a definition of science, used for the purposes of the history of science should not be based solely on modern concepts of the philosophy of science. Because the philosophy of science changes with shifts in scièntific thought, we should not use a philosophy that has evolved to cope with post-Newtonian physics in order to judge and evaluate the achievements of ancient science. The definition that is used should reflect the essence of the purpose of science, but not include areas of philosophy that deal with the way in which science operates. This would allow a more inclusive and less judgemental approach. The demarcation question attempts to distinguish between science and pseudo-science. The criteria selected for this demarcation are ideas which are concerned with the operation of science and the way in which it is explored and is built up. Thus, ideas such as Conventionalism do not try to engage with the purpose of science, but rather with systems of pigeon-holes which allow for the organisation of scientific 'facts'. Historians are often concerned with the reasons why historical texts were produced. Discovering motivation is a key aim in History. For example, understanding the motivation of a particular account of a

[43] Pearce Williams, L. (1975) "Should philosophers be allowed to write history?" *British Journal of the Philosophy of Science*; 26; 241-53; p. 252.

battle can also reveal bias and help build up an objective analysis of the events and outcome of the battle. History of Science should not be an exception to this rule. The purpose of producing a text should still be of paramount importance.

When an historian tries to write history, it is important to try to relieve oneself of any cultural or societal assumptions in order to have an objective approach. This may be impossible to do but it is important the attempt is made and it is vital for historians to be vigilant in their search for assumptions that affect our view of what happened. We should be interested in the past for its own merits and ideas. In contrast, Lakatos and others using history of science to inform the question of demarcation are applying cultural assumptions to history. By only selecting those ideas that they would recognise as being scientific, they invalidate the entire line of enquiry. Lakatos states that he is interested in writing normative history. The attempt to place strict rules on human behaviour in the past, particularly rules that have a notion of 'correctness' attached to them, is not only diverse from the best practice in History but it is in opposition to it. The more a researcher probes into an area of history the more complicated it becomes. The more factors behind a particular event are recognised and the more links between these different events are discovered. This builds an ever more complex picture. Just because an idea does not fit neatly into a model does not mean that it can be discarded or relegated into a lesser category.

To answer the question 'What is Science?', what is not science should be considered as a matter of equal urgency. We also have to consider the claims of people from the past that their ideas are scientific. They cannot be relegated to pseudo-science because that would be placing our own assumptions at the forefront of our research. They must be considered as having equal merit in the examination of what makes science. It may be that to the modern observer the theories appear ludicrous and they may be wrong but that does not make them unimportant in our approach to History of Science.

If the distinction between external and internal history appears strange to an Egyptologist or a historian, then they should learn from the problems with this distinction. In Egyptology this division is alive and healthy. There are many who consider mathematics and science to be too difficult and so do not engage with the texts on even the most superficial level. This is to promote the cause of those who would like to see science and mathematics as separate from the rest of human achievement. It will serve to separate the study of science and mathematics and allow it to be the sole province of historians of science, who do not have the necessary education to see the links between the content of texts such as the Rhind Mathematical Papyrus and the rest of Egyptian culture. Egyptologists will have to take more note of the science of the ancient Egyptians if the artificial internal/external divide is to be replaced with a more astute analysis.

Egyptologists interested in ancient Egyptian science and mathematics certainly need to be aware of philosophical issues. Philosophy of science has an important rôle in making sense of scientific thought. An approach to ancient science that does not take account of philosophical issues will always be naïve and will not properly address the issues of scientific thought in its proper context. However, that is not to say that Egyptologists need to follow the philosophical arguments in great detail, nor should philosophical analyses of science be adhered to too strictly. The uses that philosophy of science has for the history of science are at times in conflict with the interests of Egyptologists. For example, the distinction between internal and external history. To understand the ancient Egyptian mathematical texts, knowledge of the system of education, scribes and the state is important when evaluating the types of problems that were solved. Therefore Egyptologists need to temper their appreciation of the philosophy of science with an understanding of the best way to evaluate the texts with the correct perspective and emphasis. It is of primary importance for Egyptologists to decide what information they wish to glean from the texts and how an understanding of ancient Egyptian science might fit with more traditional aspects of Egyptology. Once this has been achieved, it will be possible to approach the philosophy of science and epistemology within an Egyptological framework.

Chapter 7

Science, Egyptology and Culture

In the previous chapter, it was argued that definitions of science that rely on the presumption of a rational method of discovery do not do justice to science and mathematics in ancient Egypt. It was proposed that the definition of science that is used takes into account the purposes for which science is produced. It should also be a matter of priority to research ancient Egyptian epistemology. The production of a contextualised History of Mathematics and Science requires that the context is researched as much as the content. This chapter will explore whether it is possible to define science as the process by which scientific theories are discovered. The definitions of science explored in the previous chapter rely on ideas such as observation, experimentation and falsification. These ideas define science as the means by which we arrive at the understanding of observed data and how this understanding is turned into an understanding of the workings of the universe.

7.1: Introduction

Attempts to define science by what is produced in scientific journals are difficult because there are no set definitions of what is produced. There are conventions regarding how a scientific paper is written, but these change. There are also gaps in our understanding and present models of the fundamental workings of the universe will probably be superseded by ones that are more complete. What, therefore, is the product of the process of science? How can these products be identified in a text that is known to be incomplete or simply wrong? It is possible to look back into the past, read the treatises of scientists and thinkers of the past, and then judge them for their understanding of the truth compared to the understanding of the modern scientist, to compare the understanding in the past, to modern understanding. We cannot possibly mark them against an absolute concept of reality and truth (whatever these concepts may mean), because there is no guarantee that the most efficient method for doing science has been discovered. This means that the only absolute standard that we can use to assess the achievements of previous cultures is the methodological approach that they take. Therefore, for the purposes of the History of Science we should define science in terms of process.

There is always a need in academic studies to categorise things. The use of language dictates that we assign definite terms to abstract concepts. These concepts will often overlap and the difference between two ideas will be unclear. It is obstructive to productive discourse to be pedantic about the use of terms and deconstruct the ideas until there is no word that can be used without reams of explanation. However, this is not to say that we should take classificational terms for granted and use them without consideration. This leads to an over-simplification of the situation and will lose the rich variety present in scientific and mathematical texts. Modern mathematics has to make use of a precise language and each of its terms has been precisely defined, but it should not be presumed that this was the case in the past.

This discussion shows that to research usefully and understand scientific texts from the past a more historically sensitive approach needs to be taken which takes into account the purposes for which science was produced and the way in which people organised themselves to produce it. Modern terms and their definitions should not be taken too seriously, but a healthy appreciation of them is helpful and should form the basis of the discussion.

7.2: Approaches to the History of Mathematics

Two types of historian can be identified in the History of Mathematics: Cultural Historians (those who approach mathematics as historians of science, ideas and institutions) and Mathematical Historians (those who study the history of mathematics primarily from the standpoint of modern mathematics). The argument between the relative advantages and disadvantages of each approach is characterised in the debate between André Weil and Sabetai Unguru about ancient Greek mathematics and the existence of geometrical algebra. Unguru took the cultural approach, Weil the mathematical one.

Weil went as far as saying that only mathematicians are qualified to write History of Mathematics and that the better the mathematician, the better the History was likely to be[1]. This is because Weil sees the major audience for History of Mathematics as mathematicians. To illustrate his point he quotes Leibniz:

[1] Dauben, J. W. (1993) "Mathematics: An Historians Perspective" *Philosophy and the History of Science: A Taiwanese Journal*; Vol. 2, No. 1, April 1993; pp 1-21.

> "Its use is not just that History may give everyone his due and that others may look forward to similar praise, but also that the art of discovery be promoted and its method known through illustrious examples."[2]

He also sees History of Mathematics as auxiliary to general History; its use for general Historians being to supply them with catalogues of facts in chronological order, ordered by themes, countries and author. These would then be used by the general Historian to understand the social developments in the time period of study. Weil does not see that general Historians would be interested in the specifics of mathematics or science. An Historian of the 19th Centaury, while recognising the influence of the steam engine is not - in Weil's view - interested in how it works, nor the effort that went into the discovery of the laws of thermodynamics[3]. This view reflects the arguments of philosophers such as Lakatos who have tried to use History of Science to provide rational reconstructions of the advance of knowledge. It exactly reflects the division between internal and external history. However, Weil does not see this link himself. Indeed, he does not see the relationship between history and philosophy of science whatsoever. On the contrary, he believes that there is no link at all. Because of this unawareness of the relationship, he has little to offer on one of the most difficult questions, and the question most pertinent to an Egyptologist: what is a mathematical idea? His only answer is to paraphrase Housman on the definition of poetry: a mathematician may not be able to define a mathematical idea, but he knows one when he smells one[4]. This naïve understanding of the nature of mathematics forms the basis of many opinions on the character of Egyptian mathematics.

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[2] Weil, A. (1978) "History of Mathematics: Why and How" *Proceedings of the International Congress of Mathematicians.* Helsinki, 1978; pp 227-236.
[3] Weil, A. (1978) *Op. Cit.* p. 228.
[4] Weil, A. (1978) *Op. Cit.* p. 230.

Neither does Weil engage with the problem of identifying suitable texts for study. Unless some thought is put into ascertaining which texts should be considered scientific then there is a danger of a self-confirming hypothesis. Historians are active agents in this process. It is reasoned that what looks like science to a modern reader constitutes what looked like science in the past[5]. This is often not a problem as some distinctions do have to be drawn and for historians of science who are tracing back a current scientific idea to explore its origins, it is not a problem at all. It does become a major problem when studying science in ancient Egypt. Chapter 5 explored some of the most common ideas about Egyptian mathematics and it was shown that many of these problems come from the current definitions that are projected back on the past. The nasal receptors of commentators have come across something they are unaccustomed to and this leads to problems as they are unsure what to make of it.

It has not escaped the attention of historians of mathematics that the views of Weil and others like him severely limits the number of people likely to have an interest in the History of Mathematics. There are few enough Historians of Science and within this, there are still fewer Historians of Mathematics. History of Mathematics written by mathematicians focuses on the technical detail and specific papers, rather than constructing broad histories written by themes. This loses some of the richness and interest of the past as it divorces mathematics from mathematicians and the human stories that provide so much interest to the story. It has been pointed out even Weil's quotation of Leibniz is not as straightforward as he leads us to believe. Leibniz was not only a philosopher, he is one of the contenders for the discovery of differential calculus. Both Leibniz

[5] Cunningham, A. (1988) "Getting the Game Right: Some Plain Words on the Identity and Invention of Science" *Studies in History and Philosophy of Science*; Vol. 19, no. 3; pp. 365-89.

and Newton, the other contender, wrote histories of their discovery of calculus to promote their respective claims. The debate between Newton and Leibniz became acrimonious and Leibniz eventually asked the Royal Society to investigate their claims. A year later, the Royal Society produced its own history, a collection of documents on which Newton had been allowed to comment. Unsurprisingly, the Royal Society found in favour of Newton. This episode in the history of mathematics sheds new light on the motivation for Leibniz's comments. In this case, it was not possible to write a purely objective History of Mathematics. There were too many reputations at stake and it was impossible to be purely objective[6].

This example is extreme, but it does show some of the dangers of writing History and that the History of Mathematics is not immune from these problems.

Weil's argument that only mathematicians should be allowed to write History of Mathematics can lead to other problems in the emphasis of their work. Naturally, a mathematician will be most interested in the branch of mathematics that they have worked in. Their lack of historiographical training also lends them to the idea that History should be chronological and narrative. The resulting work is anecdotal and lacks detailed research. As Ivor Grattan-Guiness describes it:

[6] Dauben. J, (1993) "Mathematics: An Historian's Perspective" *Philosophy and the History of Science: A Taiwanese Journal*; Vol 2, no. 1; April 1993; pp. 1-21.

> "Even those mathematicians who become somewhat interested in history usually assert its importance only for trivial reasons of anecdote and general heuristic without consideration of basic questions of historiography. Further and more importantly, they usually view history as the record of a "royal road to me" – that is, an account of how a particular modern theory arose out of older theories instead of an account of those older theories in their own right. In other words, they confound the question, "How did we get here?", with the different question, 'What happened in the past?'"[7]

A good example of this problem can be seen in relationship to ancient mathematics.

B.L. Van der Waerden is particularly overt in his linear developmental model for the advancement of mathematics in the ancient world[8]. In reviewing the evidence for the use of what modern mathematicians would call 'Pythagorean triples' in ancient civilisations from many different cultures including India, England, China and the Near East, he concludes that there must have been a single discovery of the triples which then spread across the world. The origin of this discovery, he postulates, is a Neolithic culture of Central Europe dating to around 3000 and 2500 B.C.[9].

This is a diffusionist hypothesis. Diffusionism is defined as a:

> 'Tendency to explain cultural change and cultural similarities in terms of the adoption of technologies and stylistic traits from neighbouring or trading-partner cultures.'[10]

Diffusionism has lost credence in archaeology since the radiocarbon revolution, when it was shown that the megalithic tradition may have been invented in several different places and could have not followed the diffusionist path suggested by earlier archaeologists. Even when introduction of a technology can

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[7] Grattan Guiness I. (1990) "Does History of Science Treat of the History of Science? The Case of Mathematics" (sic) *History of Science*; vol. 28, pp 149-73; p. 157.
[8] Van der Waerden (1983) *Geometry and Algebra in Ancient Civilisations* Springer Verlag, Berlin
[9] Van der Waerden (1983) *op cit.* p. xi
[10] Jameison, R. (1999) "Diffusionism" in Shaw and Jameison (eds) *A Dictionary of Archaeology* Blackwell Publishers, Oxford. p.200

be shown to have happened, the focus on the archaeological research is why the innovation was accepted and the effect that it had on the culture as a whole. The use of diffusionist principles by Van der Waerden shows his lack of appreciation of historical and archaeological theory. He has taken the linear narrative to its extreme and reaches conclusions which the evidence does not support.

The work of Sabetai Unguru is used as a counterargument to Weil. Unguru is critical of the plundering of history for anecdotes and argues for the need for a cultural approach. His work has been highly influential on, for example, the work by Imhausen on ancient Egyptian mathematics[11]. Unguru draws attention to the assumption of many mathematicians that mathematical entities reside in the realm of Platonic ideas and consequently are eternal and unchanging and there to be discovered rather than invented[12]. This will also predispose mathematicians to ask "Who was first?" and so follow linear modes of thought. Unguru is direct in his opinion of this approach, it is "ahistorical and should be recognised as such by the community of mathematicians"[13]. Instead, historians of mathematics should strive for a faithful reconstruction of the past, although Unguru recognises that this is unachievable as it is impossible to think like an ancient mathematician. His main concern is the discussion of geometrical algebra in ancient Greece. It is posited by some that Euclid's geometrically expressed statements are actually an algebraic reasoning; only he did not have

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[11] Imhausen, I. (2003) "Egyptian Mathematical Texts and Their Contexts" *Science in Context*; vol. 16(3) pp 367-89.

[12] Unguru, S. (1979) "History of Ancient Mathematics. Some reflections on the state of the art." *Isis*; vol. 70, no. 254; pp. 556-65. For the work of Unguru see also: Unguru, S. (1975) "On the need to Rewrite the History of Greek Mathematics"; *Archive For The History of The Exact Sciences*; Vol. 15; pp. 67-114. Unguru, S. and Rowe, D. (1981) "Does the Quadratic Equation have Greek Roots? A Study of 'Geometric Algebra', 'Application of Areas', and related problems (Part 1)"; *Libertas Mathematic*; Vol. 1; pp. 1-49. Unguru, S. and Rowe, D. (1982) "Does the Quadratic Equation have Greek Roots? A Study of 'Geometric Algebra', 'Application of Areas', and related problems (Part 2)"; *Libertas Mathematic*; Vol. 2; pp. 1-62

[13] Unguru, S. (1979) *Op. Cit.* p. 556.

the symbolism to articulate it properly. To Unguru, this is equivalent to saying that Euclid would have written in Sanskrit, but only if he was aware of the Sanskrit alphabet. The point Unguru is making is that mathematical texts from the past should only be discussed in terms that are applicable to the time in which they were produced. He says:

> "The only acceptable meta-language for a historically sympathetic investigation and comprehension of Greek mathematics seems to be ordinary language, not algebra"[14]

The consequences of these ideas for the study of Egyptian mathematics are clear. Wherever possible they must be written about in terms that the Egyptians might recognise. The dissociation of the texts from their analysis is to be avoided; the texts should be allowed to speak for themselves. Linear narratives and the "royal road to me" are unconstructive and will lead to an impoverished analysis of the texts. There is a definite danger that if an approach like that advocated by Weil is taken, then the achievement of the ancient Egyptians will be overlooked.

7.3: Constructivism

One way of incorporating the ideas of Unguru into research into the History of Science is to adopt a constructivist approach. Constructivism describes a range of philosophical ideas about the nature of science, within which there are several viewpoints that are different from each other in terms of how rigorously the central ideas of constructivism are believed.

[14] Unguru, S. (1979) *Op. Cit.* p. 556.

Constructivism is a philosophical idea that defines science in terms of how scientific ideas were achieved. Broadly speaking, constructivism sees scientific ideas as a construct of humans. It, therefore, emphasises the relationship between science and society and its cultural notions. However, there are many different degrees of significance within constructivist thinking and the degree to which it is thought that culture influences scientific methodology and research varies quite considerably from author to author. Whilst some use constructivism as a way of investigating how scientists organise themselves in the process of research, others suggest that society and culture affect the product of science.

Constructivism, according to the Routledge Encyclopaedia of Philosophy, is the view that scientific knowledge is made by scientists and not determined by the world, thus this links constructivism to anti-realism. Constructivism is also linked to relativism, because constructivist philosophers do not suppose that scientists are guided by one particular method, rather they try to observe the workings of science without bias[15]. However, this approach is necessary because if we are to understand the processes by which scientific ideas are achieved, there needs to be unbiased examination of the processes involved.

Constructivism within science studies can be more of a methodological rather than a philosophical way of thinking. Those wishing to study the way in which scientific knowledge is gained, and how this knowledge is accepted or rejected by the wider scientific community, usually have some form of constructivist opinion. However, because it is not a strictly defined phenomenon, there are as many ideas about the details of constructivism as there are researchers who use

[15] Downes, S (1998) "Constructivism". In Craig, E. (ed.) *Routledge Encyclopaedia of Philosophy*. London: Routledge. Retrieved July 29, 2003, from http://rep.routledge.com/article/Q017

it. The different approaches vary most in the degree in which they see scientific ideas as a product of society.

Barnes, Bloor and Henry tried to research the potential of "the sociology of knowledge" and their approach became known as the Strong Programme, as it used an extreme form of constructivism[16]. The question they wished to answer is whether sociology could investigate the content and the nature of scientific knowledge[17]. To answer this question the Strong Programme took a social constructivist argument. This is one of the most extreme versions of constructivism as it is prone to relativism. This version of constructivism argues that scientific ideas are only considered true because a body of scientists for their own reasons have decided that they are true. Social constructivism questions the link between observations and objective analysis. This version of constructivism at first seems to be one of the best for a historian interested in science. If historians are interested in the ways in which society manifests itself in its scientific ideas then it seems that the assumption that society directly affects scientific ideas is inherent.

However, the problem with this approach is that it is one of the aims of history to study the way in which society works. To use the version of constructivism that is proposed by the Strong Programme, we have already made too many assumptions about the society that the historian is trying to understand. For a method to have any strength then it needs to work from the known into the unknown. Society is an unknown; therefore trying to understand science through its society cannot ever be a secure methodology.

[16] Barnes, B, Bloor, D. and Henry, J. (1996) *Scientific Knowledge: A Sociological Analysis*, Chicago University Press, Chicago.
[17] Bloor, D. (1991) *Knowledge and Social Imagery*. 2nd ed. University of Chicago Press. p.3

Golinski is another strong advocate for the use of constructivism in the History of Science [18]. His version of constructivism is weaker than that of Bloor. He states in the preface:

> "I argue implicitly that the best justification of an approach is to show that it can be used productively to generate new knowledge and to deepen understanding."[19]

He defends his approach from relativism by stating that he sees his version of constructivism as methodological relativism. He makes no claims that all forms of knowledge are the same, only that they should be studied in the same manner. This distinction takes its cue from the philosophy of Feyerabend. A constructivist approach to the History of Science does not imply that the researcher necessarily believes that scientific knowledge does not bear any relationship to the ideas or truth or reality. It merely implies that there are better ways to study the way in which human beings approach the subject and better ways to think about history.

There are several problems in the History of Science that Golinski feels are an impediment to productive research. The most fundamental of these problems is the disciplinary structure of the education system. The strict disciplinary boundaries have led to confusion over whether History of Science is a form of science or a form of history. This may seem to be a trivial question but, in fact, there are serious methodological and cultural differences between the ways these two disciplines work. Science tends to be associated with technical language, objectivity and the non-human world. Its outlook is also towards the future, to the next big discoveries and how these will change the world. History is the

[18] Golinski, J. (1998) *Making Natural Knowledge*; Cambridge University Press; Cambridge.
[19] Golinski, J. (1998) *Op. Cit.* p. xi.

opposite, it uses common language, it is subjective, it is concerned with human culture and it looks backwards. The challenge for historians of science is to find a way that these objectives can be combined to produce new ideas about what science is and its role in human culture. Golinski warns that to achieve this goal requires scientists to handle the ideas of social influence in science and historians to look at the analytical issues involved in looking for evidence. Historians are as prone to generalisations and assumptions in their thinking about the History of Science as scientists are. To solve these problems will require a level of frankness and cooperation that has not been seen before.

Constructivism likes to see science as practice rather than as a body of ideas. This is an important consideration because it opens up a field of research that is more interested in the peripheries of science and the intellectual context in which science operates. However, this still suffers from the problems that Lakatos' approach to History of Science does. Lakatos was interested in looking at the logical structure underpinning science and providing a rational reconstruction[20]. He presumed that science could be identified in the past by looking for a particular mode of thought. The attempt to define science in the past whether by method or by product still requires that the modern writer can recognise in documents and treatises from the past the mode of thought or the desired outcome of a writer. Historians are active agents and this leads to problems of subjectivity. It is impossible to be philosophically neutral towards science. The best an historian can do is to attempt to understand the philosophy of the time that the texts being studied were written. This is the most productive reading of constructivist thought. The weaker versions of constructivism attempt to explore

[20] See Chapter 5.

the link between society and the production of knowledge. Any attempts to write History of Science should be critically aware of this philosophy, even if it is not subscribed to, as it has important ramifications for the selection of texts for study and so to the minimisation of subjective thinking.

7.4: The Relationship between Science and Technology

The relationship between science and technology is pertinent to the study of Egyptian mathematics because of the distinction that is made between applied and pure mathematics. It is generally taken by general historians of mathematics that Egyptian mathematics deals with only concrete examples and so cannot be considered as abstract mathematics. This is similar to the distinction that is made between science, the abstract, and technology, the applied.

I agree with Lewis Wolpert when he says that technology is not science. However, the fact that he had to make the distinction between science and technology the subject of an entire chapter of his book *The Unnatural Nature of Science*[21] shows that the nature of the association is complicated and may run contrary to public, and academic, perceptions of the two fields. He goes to great lengths to explain how advancement in technology goes along evolutionary lines. He is particularly keen to separate technology from science as he wishes to promote the cause of science and its position within the hierarchy of knowledge. Technology is defined as a practical or industrial art. It is often used to refer to machinery and tools that are used for a practical end. As such, one of the main differences Wolpert sees between science and technology is the rewards to be

[21] Wolpert, L (1992) *The Unnatural Nature of Science*, Faber and Faber, London.

gained from them. Advance in technology is driven by the market place, so rewards in technology are monetary. In opposition, science is, in Wolpert's view, driven by the search of scientists for esteem and personal satisfaction. He also believes that in the past craftsmen organised themselves into guilds in order to keep their ideas secret, whereas scientists made their work freely available. He believes that it is impossible to patent scientific advance[22]. This distinction is not valid for ancient Egypt. Knowledge was kept within the scribal class, so it was not possible to become a mathematician if one was not a scribe. So, by Wolpert's criteria, technology and science in ancient Egypt become much closer than is true in more modern times. Again, the question of the intent of the people who produced these texts becomes extremely pertinent. This question will be explored in Chapter 8.

Technology is often considered to progress in a way akin to biological evolution. When considering ancient and prehistoric technology this is particularly the case. There are no names in the early history of technology. Technological advancement seems to happen without the conscious influence of people. It is therefore presumed that technological advance happens on a trial and error basis. George Basalla[23] is the most recent author to probe this question in detail and to explore the limits of an evolutionary metaphor. He feels that evolution is a good metaphor as machines display many of the features of evolving organisms. They change slowly over time, they can have vestigial features, and machines engage in a struggle for survival, albeit with the aid of humans. Pitt-Rivers was one of the first to attempt to fit the theories of evolution to archaeology. He had seen the difference it had made to the biological sciences and was keen to replicate

[22] Wolpert, L. (1992) *Op. Cit.* p. 31-2
[23] Basalla, G. (1988) *The Evolution of Technology*; Cambridge University Press; Cambridge.

this success. The idea is that humans select the best artefacts for the job and thus gradually modify the surviving artefacts so they perform their tasks better. As a result, a progressive path can be reconstructed even though artisans are not aware of the long-term implications of their changes. Basalla is critical of the Pitt-Rivers model of technological change because there is no room in it for titular inventors, gifted individuals who make a significant advance. Basalla's model is based on four concepts: diversity, continuity, novelty and selection. He feels it is better than a cumulative theory of invention because in this model there is little room for innovation of gifted individuals. His model takes account of big advances while recognising periods of smaller changes.

> "This *diversity* can be explained as the result of technological evolution because artefactual *continuity* exists; *novelty* is an integral part of the made world; and a *selection* process operates to choose novel artefacts for replication and addition to the stock of made things."[24]

The problem with the comparison to evolutionary biology is that there are no conscious beings making selections in biological evolution, nor is there any end product. Biological organisms evolve to fit whatever niche is available; technology must change to suit the needs of humans. The diversity of technological tools cannot always be explained by niches of human need for different tools. The concept of human need is also problematic as there are several different levels of need. The need for food and shelter is the most important, but in a civilisation that has a hierarchy and a surplus there is demand for technology that surpasses these basic needs. It is presumed that mathematics in Egypt started because of the demands of building the pyramids. The existence of the pyramids can hardly be described as a basic need, yet their construction prompted advances in science, mathematics, engineering and technology.

[24] Basalla, G. (1988) *Op. Cit.* p. 25.

Science is dependent on advances in technology. Astronomy for example is dependent on the ability to make observations. Our ability to make better, more precise observations leads us to a better, more precise understanding of stars, nebulae, galaxies etc. Modern particle physics would be impossible unless we had the technology, such as a particle accelerator, to make it possible.

There is a complicated interaction between science and technology. Technological advancement relies on science and scientific advancements are sometimes dependent on having the technology available to make the experimental observations that science relies on. They are different parts of the same human endeavour: to understand and control the natural world. Classifications of human intellectual activity will always have arbitrary boundaries. Where these are placed is not normally a problem and we can resort to a common-sense approach to the question. Most of the time, we do not have to consider the reasoning behind placing the boundary at a particular point. However, when challenged, we may find it hard to justify the position that we take. The common-sense ideas that we have are not always based on any rational thought. Indeed, there are very few people who even consider what the difference might be. A common-sense idea is that technology is the application of scientific ideas, technology is the physical embodiment of abstract ideas. This difference is satisfactory for most modern uses of the terms "science" and "technology" but problematic when studying the origins of science. It becomes a circular argument. If technology is the embodiment and application of scientific ideas then science must come before technology. Yet, stone tools are classified as technological and there are very few if any historians of science that are happy with the idea that early hominids thought scientifically. For science to occur historians of science need hard evidence to show that abstract thought and reasoning has taken place. Further to this, attempts at solving problems even if those problems would be considered science in a modern context, are dismissed from the scientific canon as not being abstract enough. The intellectual achievements of the Egyptians is criticised in this way. Because their mathematical texts are expressed in terms of solutions to a definite problem, rather than as general solutions to general conceptual problems they are rejected because they are not abstract enough.

There are many similarities between the relationship of science and technology and the argument about abstractness in ancient Egyptian mathematics. The presumption that technological advance is made in a manner akin to biological evolution, rather than through the intellectual work of some now anonymous inventor is particularly reminiscent of some of the arguments about advances in ancient Egyptian mathematics. The perceived concrete nature of ancient Egyptian mathematics has led many to argue for explanations of problems, such as MMP 14, that have the least abstract thought possible. It seems that the use of mathematics for practical needs has placed it on the level of technology, which is considered to advance along lines divergent from scientific advance.

7.5: Historical Archaeology and the History of Science

Broadly speaking, we identify science with texts and technology with more tangible things. This distinction can also be seen in the different disciplines of history and archaeology. Here the same broad definitions can be used with few reservations. Archaeology is about material culture and environmental remains, things to be dug up and investigated. History, on the other hand, is about the analysis of the textual remains of the past. If Egyptian science is to be understood properly then these two disciplines must be able to interact productively.

The problems of combining archaeological investigation of artefacts with textual history are well known and have become the subject of recent research. Some of the problems are due to the differences in standing perceived by researchers of their own discipline and that of the discipline of others. Dirt archaeologists deal with very different things from historians interested in the words. However, the same people who engaged in written historical activity also made and used material culture. Traditionally, it was the duty of history to reconstruct events that the objects of archaeology could illustrate. Archaeology took a subservient role to that of history. However, recently there is a growing realisation that the two disciplines complement each other. Evidence from archaeology can even overturn the received historical view. An amazing study has been carried out by Fox[25] on the battle of Little Big Horn. Fox traced the progress of the battle of the Little Big Horn by using ballistic analysis on cartridge shells found on the battle site. Through this analysis, he was able to reconstruct the paths of individual guns around the battle site. The weight of the archaeological evidence shows that Custer's battalion disintegrated and the weapons were seized by the attacking forces and turned against Custer's battalion. This picture is far from the popular idea of a heroic last stand. This study prompted Fox to reflect on the character of history, archaeology and their interaction or lack of it. He found

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[25] Fox, R. A. (1993) *Archaeology, History and Custer's Last Battle: The Little Big Horn Re-examined*, University of Oklahoma Press, Norman.

while presenting work at academic conferences that there was a feeling that he should not have attempted the analysis. Instead, he should have just made his findings available to people with more expertise on the events of the battle, i.e. historians of the battle. He defines the two disciplines. He says:

> "Paraphrasing Deetz, then, we can define the practice of archaeology as the value-laden construction of past actuality based on fragmentary remains...But historians will now agree that histories represent constructions of the past and are the product of contemporary values imposed on excerpts of past actuality" [26]

The paper by Deetz[27] introduces a new term to the mix, that of 'archaeography' which he defines as "... the writing of contexts from the material culture of past actuality"[28]. In current British archaeology, we are more likely to identify this with theoretical archaeology. However, the idea that archaeological data can be used to write overarching commentaries on the nature of past societies and culture is an idea familiar to archaeologists on both sides of the Atlantic.

The comparison between these two conflicts between research fields only works to a certain point. Archaeology and history are both attempting to understand 'the past', whatever that may be. Science and technology, however, are two distinct entities. Yet the two conflicts have at their core the same notion: the idea of the primacy of text over objects. Both science and history deal with textual evidence and traditionally the word has been seen to be more important. John Moreland[29] shows this relationship diagrammatically on what he calls a 'Hawkesian ladder', named after the archaeologist who first articulated the idea of the hierarchical steps in archaeological inference.

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[26] Fox, R. A. (1993) *Op. Cit.* p. 327.
[27] Deetz, J. (1988) 'History and Archaeological Theory: Walter Taylor Revisited' *American Antiquity* Vol 53:1; pp 13-22.
[28] Deetz, J. (1988) *Op. Cit.* p 18
[29] Moreland, J. (2001) *Archaeology and Text*, Duckworth, London.

Religion/ Ideology [Science]	Elites	History
Social Relations/ Politics	Elites	History
Subsistence	'Peasants'	Archaeology
Technology	'Peasants'	Archaeology

Fig 7.1. Archaeology and History on a Hawkesian ladder[30]

7.6: Conclusions

If history is written by the victors, then History of Science is written by the most successful scientists. Even an historian with little scientific training will be aware of the power of science and its technological applications. It is impossible to forget knowledge of π when reading ancient Egyptian mathematical texts. The notion of letting the past speak for itself is therefore challenging and requires consideration of not only the aims of the study but also its methodology and assumptions. Writing the History of Science and particularly the History of Mathematics requires the combination of many disciplines, some of which have competing methodologies and aims. The further back in the past travelled, and

[30] Taken from Moreland, J. (2001) p.14. This is a diagrammatic representation of arguments published in: Hawkes, C. (1954) 'Archaeological theory and method: some suggestions from the Old World', *American Anthropologist* 56: 155-68

the scarcer the evidence, the harder this problem becomes. Any analysis of ancient Egyptian mathematics must consider these problems. For an Egyptologist, some of these problems are made simpler because the goals of Egyptology have been explored and so a clearer picture is possible. However, abstractness in Egyptian mathematics is important and to make a judgement it is essential that the competing claims of different disciplines are considered.

The relationship between science and technology is the most significant problem to consider. Egyptian mathematics has been placed on the level of technology, yet this distinction is based on features of science and technology that have little comparison with ancient Egypt. Consequently, this identification has no basis in the general analysis of Egyptian culture. If science and technology are comparable then it makes sense to recognise Egyptian mathematics as science. The problems of integrating history and archaeology are not unfamiliar to Egyptologists. Attempts to find archaeological evidence for scientific thought are, however, going to meet with difficulties. The accuracy of the alignment of the pyramids at Giza has shown astronomical knowledge, but the application of mathematics leaves less evidence. The debate between Weil and Unguru does not present any difficulties to an Egyptologist. The plundering of history for episodes to inspire current mathematicians is not a concern. A more sensitive approach is necessary if the Egyptian mathematical texts are to be properly understood.

Chapter 8

A New Approach to Ancient Egyptian Mathematics

This chapter will explore how a better understanding of the context of research into the history of mathematics may alter our appreciation of ancient Egyptian mathematics. It will draw on the conclusions reached in Chapters 6 and 7 on the nature of science and the best way in which to study the history of science and apply these findings to the study of ancient Egyptian mathematics. It will also return to the observations made in Chapters 2 to 4 about the achievements of ancient Egyptian mathematics.

8.1: Introduction

A contextualised approach to ancient Egyptian mathematics is important, but understanding what that context is is very difficult. To understand this context a consideration of the aims behind the production of the texts is necessary. The Rhind and Moscow Mathematical Papyri each contain a series of problems with concrete, numerical terms. They have been copied and stored. It is largely agreed that they were produced as training manuals so that Egyptian scribes could learn mathematics to carry out their tasks. Each of the factors by which the texts are to be judged need to have significance in relationship to these observations. These factors can be very simple ideas such as ease of reading but once these simple factors have been appraised, more complicated factors can be brought into consideration, such as the idea of an ancient Egyptian epistemology. Building up an epistemology requires the combination of all the branches of ancient Egyptian science, such as mathematics, medicine, anatomy, veterinary science and astronomy. This will then need to be integrated with what is known of the ancient Egyptian's religious beliefs. This is a possibility for further research but it will require the combined efforts of many experts on other areas of Egyptian science. This thesis can, however, start to investigate some of the simpler factors. The questions that this chapter will attempt to answer are:

Are the texts easy to understand?

Is the mathematics easy to use?

Does the mathematics provide an insight?

Was the mathematics useful?

Can the mathematics solve complex problems?

In addition to these questions, this chapter will examine some further considerations that have been considered important by other historians of mathematics:

Is the mathematics abstract?

Do the Egyptians have a place in the development of mathematics?

Did the Egyptians have the concept of proof?

8.2: To What Extent Can we Talk about the 'Character' of Ancient Egyptian Mathematics?

Arguments about whether or not the Egyptians were able to calculate and use various mathematical formulae or concepts generally lead to comments about the 'character of ancient Egyptian mathematics'[1]. This trend appears to have been started by Peet in the original translation and publication of the Rhind Mathematical Papyrus[2]. This implies that ancient Egyptian mathematics, or at least Egyptian mathematical texts show particular attributes that separate them from other mathematics or mathematical texts. As Egyptian mathematics is seen to lie outside the main corpus of mathematical works, these comments are usually directed towards discounting an achievement. Yet, the ideas we have about the character of Egyptian mathematics are based on what can be presumed to be only a sample of texts produced. Not only that, the extant texts hint at further mathematical work that does not survive, perhaps because it was never written down. MMP 10 is perhaps the type example. The method used to calculate the surface area of the hemisphere is not an obvious method[3]. Indeed, the conception of a three dimensional curved surface in terms of a flat area is an idea that takes more thought than is evident in the surviving text. The existence of a link between the method of calculation of the hemisphere and a circle[4] is evident. The use of the same fraction [8/9] and the squaring of the diameter are devices used in both methods to achieve the answer. This problem and others like it show that there was more thought and rationalising in ancient Egypt than

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[1] See for example Rossi, C and Tout, C. (2002) "Were the Fibonacci Series and the Golden Section Known in Ancient Egypt.?" *Historia Mathematica*; 29; pp. 101-113.
[2] See Chapter 5.
[3] See Chapter 4.
[4] See RMP 41, Chapter 4.

survive in their texts. This should not surprise us, as papyrus is a fragile material and prone to many forms of decay. The similarity between the methods employed to find the surface of a hemisphere and a circle, suggests that there may have been a particular way in which the Egyptians thought about mathematical problems and therefore a particular way they went about solving them, which in turn suggests a 'character'. Unfortunately, there are very few surviving texts so any commentators on Egyptian mathematics need to be mindful that anything they say will be a generalisation.

We cannot suppose that the texts that survive from ancient Egypt are anything other that the work of skilled mathematicians. It must be assumed that the mathematics they contain is representative of the level of achievement for the time they were written. The method achieved in MMP 14 could not have been reached by chance. It represents an end product of a long line of mathematical reasoning. Some now anonymous mathematician(s) sat down and reasoned out the arguments behind the mathematics. This knowledge should also inform any discussion on the achievements of the Egyptians.

8.3: An Appraisal of Egyptian Mathematics

8.3.1: Is the Mathematics Easy to Understand?

This is perhaps one of the most difficult questions to answer because the language that it is expressed in is idiomatic, and thus hard to decipher. There are also technical terms used in the problems that do not appear in any other type of text. We must assume their meaning from the mathematical context they are given in. To make it even harder, some of the technical mathematical terms are also idiomatic. Some of these idiomatic phrases are reasonably clear, others are more opaque, as is the nature of idiomatic phrases. As they are embedded in the culture that produced them, and this culture is not fully understood, we will not be able to retrieve the meaning from all the phrases. For example, the term for a square root is reasonably straight forward as *knbt* is also the word for a corner[5], but others are more obscure. For example, in MMP 7 we are given the ratio of the height to the breadth of a triangle as *idb*: the 'bank', referring to a river bank[6]. We can only understand what this term means from the preceding and following arithmetic. The precise meaning of the term and any nuance implied to an Egyptian reader is lost on us. We must presume that the Egyptian scribe that prepared the text knew what was meant. We must also presume that he was using idiom that was apparent to his intended readership.

As to the mathematical processes involved, it does take a knowledgeable eye to ascertain exactly what is meant in the problems and the methods concerned. For example, the problems dealing with false supposition do not explain the reasoning behind the argument. It is up to the readers to work it out for themselves. This is certainly a defect in any mathematical text, it means that the texts could not be used other than in a lesson with a master. This is not a self-study manual.

[5] Faulkner, R. (1999) *Dictionary of Middle Egyptian*; Griffith Institute; Oxford; p. 280.
[6] Faulkner, R. (1999) *Op. Cit.* p. 35.

8.3.2: Is the Mathematics Easy to Use?

The foundation of a mathematical system that is easy to use is clear arithmetical procedures and good notation. If the procedures are too complex then it is easy to make mistakes. The arithmetic in the papyri is unfamiliar to a modern reader, but there is no reason to see this as a fault. There is also no reason to suppose that because the Egyptians spelt out every arithmetical step in the problems that they were intellectually inferior.

The system for multiplication is extremely clear, once it has been explained what the two columns of numbers represent. The system of repeat doubling is the best system for working out a multiplication sum, and the Egyptians could modify this system to fit the specific problem. It is superior to our system because it does not necessitate the learning of multiplication tables. Neugebauer writes that:

> "... the whole procedure of Egyptian mathematics is essentially additive. ... It certainly never entered the minds of the Egyptians to ask whether this process will always work. Fortunately it does; and it is amusing to see that modern computing machines have made use of this principle to exactly the same end, namely to reduce multiplication to a simple process of counting" [7]

What Neugebauer fails to recognise is that multiplication is a process of repeat addition; it is essentially counting. The modern way of working out a multiplication of numbers with more than two digits splits the sum into several smaller parts. Thus, the sum twenty six times fourteen is split into the sums six times four, twenty times four, six times ten and twenty times ten all added together. The answers to these multiplications we only know because we learnt

[7] Neugebauer, O: (1952) *The Exact Sciences in Antiquity:* Princeton: Princeton University Press p. 73.

them parrot fashion at school. It is only by an accident of nature that human beings are so obsessed with base ten. A binary system is the simplest system to use.

There is a problem with unit fractions. They are very awkward for modern mathematicians to use as they are so used to fractions with a denominator and a numerator,. However, with practice and experience they do become easier to understand. The 2/n table is an invaluable reference. The use of other tables such as the Egyptian Mathematical Leather Roll must have made calculations easier, but the use of unit fractions does give rise to complications in the arithmetic. The difference between MMP 7 and 17 is only in the way the mathematics is calculated. This problem would not arise if top-heavy fractions could be used, as there would be no difference in the method.

There are several reasons why the Egyptians may have used unit fractions. The first is the problem of notation. Apart from Horus-eye fractions, fractions are written with an *r* over the number. A new system of notation would have to be invented to allow anything other than unit fractions. Two-thirds is a special case, but this has its own symbol. It is impossible to create a unique symbol for every fraction. The second reason is that splitting loaves into unit fractions appeals to our sense of fairness. If loaves that are to be shared equally among a group of men are split into pieces the same size than each man can see that he has the same amount as his companion, discounting the crumbs that would fall off and the crusts. Whatever the reasons for the use of unit fractions, Section 2.6.3. shows that the Egyptians could conceive of non-unit fractions. This means that there were positive reasons for the use of unit fractions, which must have something to do with the way mathematics was used in ancient Egypt. It is possible that the use of mathematics in accounting was the incentive.

8.3.3. Does the Mathematics Provide an Insight?

It is interesting to see that the geometrical problems in the Moscow Mathematical Papyrus use numbers that make the arithmetic easier. For example, Problem 10 uses a hemisphere with a diameter of nine because this is the smallest value that gives a whole number when divided by nine. Similarly, problem 6 gives a case where the values given make the arithmetic simple. This was no doubt done deliberately and improves the instructional value of the text. The student of mathematics does not have to worry about the sharpness of his arithmetical skill to be able to follow the text. He is therefore free to follow the mathematical reasoning of the problem with as little hindrance as possible. However, Gunn and Peet argue:

> "Now from our standpoint this simplicity is a defect; we feel that some difficult divisions and square roots would have been more instructive to the student as showing that the problem does not depend for being solved on containing only fours and sixes and hundreds, and that in the case of square roots he would see what degree of accuracy was expected when the root would not "come out" exactly." [8]

I do not agree with this point of view. There are plenty of mathematical problems that would allow the student to practise his arithmetical skill. When dealing with problems that contain geometrical ideas, the arithmetic should be kept as simple as possible. Gunn and Peet suggest that the geometrical problems are kept simple so that they can be learned by heart. It is suggested that a student faced with a truncated pyramid of any size could then recall MMP 14 and so

[8] Gunn, B and Peet, T.E (1929) 'Problems from the Moscow Mathematical Papyrus,' *Journal of Egyptian Archaeology* Vol.15 pp.166-185; p. 185.

solve the problem. I do not think this is the case. The most obvious application of the mathematics in MMP 14 is to the volume of a mud brick ramp. The scribe may very well have had to find the volume of a figure that approximated a truncated pyramid, or had elements of it. To apply the knowledge an insight into the mathematics is needed, this is what the Egyptians were trying to achieve when they gave values that made the sums easy.

8.3.4: Was the Mathematics Useful?

Egyptian mathematics is often dismissed as 'purely practical'. The problems certainly lend themselves to the measurement of land, the reason given by Greek writers for the invention of geometry in Egypt. However, the practical nature of Egyptian mathematics does not mean that it had no time for intellectual curiosity. The most famous example of this is problem 79 from the Rhind Mathematical Papyrus which seems to be the forerunner of the rhyme "As I was going to St. Ives ... ". However, the vast majority has a practical use and I think this is to be praised, as long as the other ideas of good mathematics are not sacrificed.

8.3.5: Can the Mathematics Solve Complex Problems?

The Egyptian mathematician had many tools at his disposal. He could find the volume of rectangular and circular prisms and pyramids, he knew about the relationships that the areas of different shapes have, he had a wide range of arithmetic skills, and he could work out ratios and proportion. The problems in the papyri are not prescriptions for solving particular problems, they are a recipe book of skills. It is difficult to say if these skills were complex enough to solve all of the problems that an Egyptian mathematician would face. Some work may have been solved by trial and error. However, we can hardly criticise the Egyptians for not having a sound appreciation of calculus.

8.4: Further Considerations

Further to the ideas of good mathematics in an ancient Egyptian context, there are other important considerations that historians of mathematics have used in order to evaluate the importance of Egyptian mathematics.

8.4.1: Is the Mathematics Abstract?

Egyptian mathematics is commonly criticised for not being abstract. The main consequence to this fact is the perception that the Egyptians had no algebraic system, either in concept or notation. Gunn and Peet wrote in their commentary on MMP 6:

> "This is hardly the place in which to discuss the psychological meaning of the use of the unknown x in mathematics, but it is at least clear that this, whether used explicitly in an algebraical solution or implicitly in some purely "arithmetical" one, gives to the modern method an abstract character entirely foreign to Egyptian mathematics."[9]

To Gunn and Peet algebra is the most abstract form of mathematics. Algebra uses letters to show the unknown quantities and rearranges equalities until the unknown is found. However, algebra is a modern convention to express the problem in a simple, visual way. Modern mathematicians have got so used to using algebra that they will 'think' in algebra. Egyptian mathematics does suffer

[9] Gunn and Peet, (1929) "Problems from the Moscow Mathematical Papyrus" *Journal of Egyptian Archaeology*; Vol.15; pp. 166-85; p. 169.

because it does not use algebraic notation, but this does not mean that Egyptian mathematicians could not use the abstract ideas that underlie algebra to solve their mathematical problems. As we have already seen in MMP 19, the scribe who wrote the papyrus was able to manipulate the equality in exactly the same way that a modern mathematician using algebra would. In MMP 19, the formula used in the solution could only have been achieved by knowledge of abstract mathematical arguments. There are also Egyptian mathematical problems that do use a word that I believe exactly corresponds to the unknown x used in modern mathematics. The word *'h'* is translated as heap. What is a heap if it is not a pile of unknown quantity? The fact that the Egyptians are using a word where we would use a single letter does not alter the mathematical concepts behind their use. The word 'algebra' is a European corruption of an Arabic phrase which means "restoration and reduction". Restoration refers to the idea that the same quantity can be added or subtracted to either side of an equation, but leave the same equality. This idea is used in many problems including problem 19 of the Moscow Papyrus. The idea of reduction refers to the way algebra is used to simplify an equation until it can be solved. This idea is also present in Egyptian mathematics. The person who prepared the formula used in MMP 14 was certainly using this principle, whatever his starting point was.

It is not only from algebra that mathematics gets its abstract quality. The fact that the Egyptians had a number system that is independent of objects is the basis of an abstract philosophy of mathematics. In many histories of mathematics the authors say that the Egyptians were not able to think of numbers independently of the counted object. They say that the mathematics is all about practical problems, so that in every part of the text the numbers are always counting something. Contemporary wisdom states that the Egyptians were not able to think of 'three' but would have to think of 'three bulls', 'three *hekat* of grain' or 'three loaves of bread'. There are two rejoinders to this argument. Firstly, most people in our society do not consider that number is an abstract concept; the idea of number is so fixed in the cultural zeitgeist that the count and the object are inseparable. But number is, ipso facto, abstract; one cannot hold three in the palm of one's hand; only three things, not three alone. The sign for the concept remains the same; three is applicable to atoms as much as it is to elephants. This was well understood by the Egyptians and is the second opposition to the argument.

The way that the Egyptians represented 'three' did not change according to the object. It did not change whether it was representing a number of objects, a linear measure, a measure of area or a measure of volume. In MMP 6 the numbers refer to different properties of the rectangle: we are given a ratio and the area and have to work out the two lengths. The numbers are used together; there is no difference to how they are treated. This means that the Egyptians did know that 'three' has properties that transcend the object. This is pure and abstract mathematics.

The Egyptians' abstract conceptualisation of mathematics goes further. In MMP 10, among others, there are no units given. Therefore, the reader may assume what units are used, but none are stated. This shows that the Egyptians understood that units are arbitrary and that any system of measurement will be a ratio scale. Ratio is a concept not purely of measurement but of the inherent properties of analogy. A ratio scale is a scale where the units of measurement are arbitrary, so several numerical values could be given for the same concepts; whether they be length, area, mass or volume; but the ratios between two measurements will be identical as long as they are both stated in the same units. For example, the ratio between the distances from London to Edinburgh and from London to Paris, will always be the same, but the numerical values given for the distances will change depending on whether the measurement is given in miles, metres or yards. There is nothing more abstract in mathematics than the idea of analogy or of the irrelevance of number and the supremacy of concept.

Another reason for supposing that Egyptian mathematics is not abstract is because the problems are expressed in ways that lead us to believe that the mathematics is for purely practical purposes. We have already dealt with the need of practicality in the mathematics, but we can also ask: "Are all the problems practical?". There are problems in the mathematics that expound with implausible, and in some instances, impossible numbers. For example, RMP 50 deals with a circular piece of land with a diameter of 9 *khet*. One *khet* is equal to one hundred cubits. The area of the piece of land is given as 64 square *khet* so the piece of land has an area of 640, 000 square cubits or over 600 km^2. It is hard to see how being able to measure a piece of land this size would be practical for everyday agrarian problems, particularly as it is a circular piece of land and fields are normally quadrilateral in dimension.

The numbers used in the problem are chosen because the Egyptian method requires that eight ninths of the diameter be found: this is easy to do when the diameter is nine. The methodology used is to find eight ninths of the diameter and square it; the concept of 'field' is purely arbitrary, it has no intrinsic significance to the problem. It could have easily been a building or a well, but the idiom used is agrarian because the society was so. RMP 50 just shows the method. It is not an example of a 'real' mathematical problem. This shows that the Egyptians could see that their methods would work for cases of which they had no practical experience. Thus, the concept of 'household arithmetic' is invalid in such a proposition.

Philosophers of science also write about the beauty of mathematics. Kline writes:

> "To describe mathematics as only a method of inquiry is to describe da Vinci's 'Last Supper' as an organisation of paint on canvas"[10]

Considering the fact that Leonardo da Vinci's 1498 depiction of the 'Last Supper' in the Convent of Santa Maria delle Grazie (Refectory) in Milan is a fresco and is therefore not on canvas how far can one rely on the research of Kline? In fact any pictorial representation is an organisation of pigment on medium; any meaning is not inherent in the pigment itself but on the ability of the viewer to make sense of the pattern of pigmentation. Thus, mathematics may be viewed as both a methodology and an abstraction, without detracting from either viewpoint. Egyptian art could also be functional, to suggest that the Egyptians should see aesthetic value in their mathematics is another example for the need to have a contextualised approach to the history of mathematics.

Kline asserts that the reason that the great mathematicians have striven to produce new algorithms and proofs is to satisfy their higher intellectual need and their want for beauty. Mathematics obtains its beauty because of the logical

[10] Kline, M. (1964) *Mathematics in Western culture*, Oxford: Oxford University Press, p147.

reasoning and the ability to express complicated ideas in a simple way. Symmetry is perceived as the ultimate expression of beauty in nature and much of mathematics and modern physics lends itself to this ideal. Mathematics can be very intellectually satisfying because it appeals to our logical temperament. If beauty is a necessary component for mathematics then it is a component that the Egyptians were well aware of. At the end of the problems we are told 'Look, you will find it good." The word used for correct is *nfr* which is also the word for beauty. For an ancient Egyptian truth is beauty, literally.

8.4.2: Did the Egyptians Have the Concept of Proof?

A question that follows on from the ideas surrounding the abstract nature of the mathematics is whether Egyptian mathematics is scientific. David Oldroyd in his *The Arch of Knowledge* defines science as

> "The scientific movement is made up of a community of people who seek to gain knowledge of the world by the use of various kinds of observational and experimental procedures, not merely by thinking or talking about problems as do philosophers – though obviously a deal of though and talk is involved along the way in scientific work. The scientist seeks to discover the regularities of nature and the laws describing these regularities; and theoretical explanations are proposed to account for such laws."[11]

This regularity is important to the concept of proof; the necessity that an instance is reproducible and applicable to extended circumstances not necessarily reliant upon the initial exempla. So, one of the important features of science is proof as a demonstration of logical reasoning. The Egyptians obviously saw the need to demonstrate the results that they obtained. We see phrases such as "the doing as

[11] Oldroyd, D. (1989) *The Arch of Knowledge: an Introductory Study of the History of the Philosophy and Methodology of Science* New South Wales: New South Wales University Press p.2.

it occurs" appearing at the end of the problem followed with the argument behind the solution of the problem.

In MMP 6, we are shown a diagram of the completed answer. Next to the two sides of the diagram we are given the two measurements, and inside the rectangle we are given the area. By the side we are shown that 3, the breadth, times 4, the height, do in fact equal the area, 12. After MMP 17 and 14, we are given more of the arithmetic needed to solve the problem. These are not proofs in the modern sense. They are demonstrations, but they serve some of function that a modern proof does. Science needs to have an objective demonstration that the answer it has reached was not arrived at by chance, but because the method is sound. Modern mathematicians will not use any definite values when constructing a proof. In the Egyptian problems we can see the beginnings of scientific thought because they saw the need for demonstration of the result. They were not able to express this in the objective way that is necessary in modern proof, because algebraic notation is the contemporary means of transmitting such proof.

In Chapter 6, the idea of mathematical proof was examined through the work of Lakatos. Lakatos was keen to impress on his readers the idea that proofs do not necessarily have to be formal to be sound. In fact, he sees mathematicians who only give the epithet 'proof' to abstract, formulaic proofs as naïve. His description of pre-formal proofs holds much that is relevant to an Egyptologist trying to explore the nature of proof in ancient Egyptian mathematics. In pre-formal proofs, the premise lies not in the formal manipulation of formulae, but in an argument that relies on a thought experiment. The proof requires the reader to imagine the figures and to manipulate them mentally. This has similarities to the Egyptian approach to writing mathematical texts. The extant texts do not show a formal understanding of proof, but to deny that the ancient Egyptians did not have any concept of proof is clearly false. The use of modern terminology to describe a civilization that has been dead for thousands of years is not productive, and this is a clear example of how modern terminology can cause problems. It may be that the idea of clarification, rather than proof, is more appropriate to the processes of the ancient Egyptians. They used their numerical workings at the end of the problems to demonstrate that, in the specific case at least, the method is valid. The numerical arguments are therefore showing the procedure in a different way. There are no examples of the Egyptians using a method that only works by chance. The procedures that went into the production of these texts must have been rigorous, even if they were not as rigorous as modern standards.

We may not be able to recognise Egyptian mathematics as scientific, but is it fair to ask whether Egyptian mathematics conforms to our modern idea of science? In no other areas of Egyptology is this question asked. We do not discuss the failings of the political system because it lacked democracy. We do not criticise the Egyptians for transporting material by raft and sledge, when if only they had thought of it they could have used a speedboat and a lorry. We learn nothing new about either the nature of mathematics or the Egyptians. We can hardly expect one of the first civilisations to have achieved the mathematical maturity that is present in a modern society. We have inherited thousands of years of thought; it is no wonder that we are more scientific in our representation. In art and religion the Egyptians had an entirely different outlook. They required different things of their art than we do. It is therefore not appropriate to use our ideas of using perspective in art when looking at Egyptian art. We can see that the Egyptians could think in a logical manner. The fact that this is not always the case should not shock us. When we cannot see the logic behind Problem 23 of the Moscow Mathematical Papyrus we should not condemn the entirety of Egyptian mathematics for being illogical. There may have been a sound reason that is now lost to us. One must resist importing contemporary constructs and ideas into an ancient world.

8.4.3: Do the Egyptians Have a Place the in History of Mathematics?

> "There is hardly a culture, however primitive, which does not exhibit some rudimentary kind of mathematics. The mainstream of western mathematics as a systematic pursuit has its origin in Egypt and Mesopotamia. It spread to Greece and to the Graeco-Roman world. For some 500 years following the fall of Rome, the fire of mathematical creativeness was all but extinguished in Europe; it is thought to have been preserved in Persia. After some centuries of inactivity, the flame appeared again in the Islamic world and from there mathematical knowledge and enthusiasm spread through Sicily and Italy to the whole of Europe" [12]

The received view of the history of mathematics is a linear model where ideas are discovered and then passed on to the next generation of mathematicians. The arrow of time is the arrow of progress. The writers of these comments are usually fixed into a linear narrative model where discoveries are made only once and the next mathematician in the chain will improve on the idea or use it to make a new discovery. Models are constructed where one person makes a mathematical discovery, and then either through a written or a spoken medium,

[12] David, P and Hersh, R (1981) *The Mathematical Experience*; Birkhauser, Boston.

this idea is passed on wholesale to the next generation of mathematicians. Through this mechanism, the corpus of mathematical literature is built up in a linear fashion with ideas being built upon in each successive stage. Working backwards, it follows that the mathematics of an antecedent culture is only valuable if it had a contribution to make to the culture under study and therefore an impact on the whole of the process of development of mathematical thinking. This model necessitates the idea of a progenitor of mathematics, who started the chain, usually identified as Thales of Miletos. It is a commonly held view that mathematics started with the ancient Greeks. Indeed, the idea that mathematics started with the ancient Greeks and that everything before it was no more than a confused prologue goes back at least as far as Kant:

> "Yet it must not be thought that it was as easy for it [mathematics] as for logic - in which reason has to do only with itself - to find the royal path, or rather itself to open up; rather, I believe that mathematics was left groping about for a long time (chiefly among the Egyptians), and that its transformation is to be ascribed to a revolution, brought about by the happy inspiration of a single man [Thales]."[13]

Thales of Miletos has been called the Father of Mathematics[14]. If Thales was the progenitor of mathematics then this precludes there being anything mathematical before he made his discoveries. Therefore, the numerate reckonings of the Egyptians are in some way short of the benchmark that is set by the working of Thales. One of the reasons for this is the perceived utility of Egyptian mathematics. Egyptian mathematics is seen to be merely applied mathematics

[13] Kant, I (1786) *Critique of Pure Reason*, preface to second edition. Bxi.
Translation: Goyer, P. & Wood, A. Cambridge Edition of the Works of Immanuel Kant (1998) Cambridge University Press, Cambridge.

[14] See, for example, Boyer, C, rev. Merzbach, U (1991) *A History of Mathematics*, 2nd ed; John Wiley and Sons; p. 46. Also: Lloyd, G.E.R. (1970); *Early Greek Science: Thales to Aristotle*; Norton; London; p.8.

and that it lacks abstract elements. The way that the Egyptians phrased their mathematical problems is often cited as evidence for this[15].

Most Western historians of mathematics will hold that mathematics as a pure subject was born in ancient Greece, an invention of Europeans. In histories of mathematics, Egypt is confined to a short opening chapter, if given any space at all. The book will then move on to the important business of discussing Greek mathematics, as this quote from Morris Kline's 'Mathematics in Western Culture illustrates:

> "We shall see that the last few hundred years during which the Greeks flourished and the last few hundred years of our modern era produced infinitely more knowledge and progress than the millenniums of the two ancient civilisations."

It is surprising to see a man who would call himself a mathematician fall prey to mathematical error. We are not progressing with mathematics at an infinite rate, therefore either he is distorting the picture by vastly exaggerating the situation, or he believes that there was no progress made in mathematics and the sciences in Egypt or Mesopotamia. Kline also writes that Egyptian mathematics compared to Greek mathematics is like comparing "the scrawlings of a child just learning how to write" to great literature[16]. In his history of mathematics Egyptian and Babylonian mathematics is only given three pages in a volume of seven hundred. This indicates the proportion of our mathematical ability he thinks we owe the Egyptians.

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[15] Peet, T.E. (1923) *The Rhind Mathematical Papyrus British Museum 10057 and 10058*; University of Liverpool Press, Liverpool; p. 10-11
Also Boyer, C, rev. Merzbach, U (1991). *Op. cit.* p. 21.
[16] Kline, M. (1962) *Mathematics a Cultural Approach,* Reading Mass.: Addison-Wesley, p. 258.

There are some writers who believe that Egyptian mathematics was a retarding force on progress. The main charge against them is their use of unit fractions, the use of which persisted well into Greek and Roman times. The use of unit fractions was taught to administrators and so the use of unit fractions spread to other areas of the Roman Empire. Fractions were sometimes expressed as unit fractions, although the problem had been posed in sexagesimal fractions, which can give a more accurate result. However, the Egyptians cannot be blamed for the continuing use of their practices. Rather, the use of unit fractions was presumably due to the fact that the system that was easy to understand, this is particularly evidenced as results were changed into unit fractions. There is no evidence that the continued use of unit fractions in any way impeded the mathematical output of the Greeks and Romans.

Even those sympathetic to the achievements of Egyptian mathematics are still in the practice of denigrating the achievements that the Egyptians did make. Newman remarks that the Egyptians were remarkably pertinacious in solving everyday problems, yet:

> "As to the question of how Egyptian mathematics compares with Babylonian or Mesopotamian or Greek mathematics, the answer is comparatively easy and comparatively unimportant" [17]

Neugebauer goes even further saying:

> "To some extent Egyptian mathematics has had some, though rather negative, influence on later periods."[18]

> "The role of Egyptian mathematics is probably best described as a retarding force upon numerical procedures"[19]

[17] Newman, J. (1956) *The World of Mathematics*, London: Allen and Unwin p. 84
[18] Neugebauer, O. (1952) *The Exact Sciences in Antiquity*; Princeton University Press; Princeton, New Jersey; p. 72
[19] Neugebauer, O; *Op. Cit.* p. 80.

Do the Egyptians have a place in the history of mathematics or are we to believe that the contribution that they made is worth only three pages out of seven hundred? The indebtedness of the Greeks to Egyptian and Babylonian mathematics has to be explained in order for the linear model to work. Many Greek writers wrote about how much they owed to the Egyptians. These comments also have to be explained.

The Greeks themselves were not shy at acknowledging the debt they felt they owed to the Egyptians. They certainly thought they geometry was born in Egypt. Herodotus[20] wrote:

> "The king moreover (so they say) divided the country among all the Egyptians by giving each an equal square parcel of land, and made this his source of revenue, appointing the payment of a yearly tax. And any man who was robbed by the river of a part of his land would come to Sesostris and declare what had befallen him; then the king would send men to look into it and measure the space by which the land was diminished, so that thereafter it should pay in proportion to the tax originally imposed. From this, to my thinking, the Greeks learned the art of geometry."

Proclus[21] wrote:

> "According to most accounts geometry was first discovered among the Egyptians, taking its origin from the measurement of areas. For they found it necessary by reason of the rising of the Nile, which wiped out everybody's proper boundaries. Nor is there anything surprising in that the discovery both of this and of the sciences should have had its origin in practical need, since everything which is in process of becoming progresses from the imperfect to the perfect. Thus the transition from perception to reasoning and from reasoning to understanding is natural. Just as exact knowledge of numbers received its origin among the Phoenicians by reason of trade and contracts, even so geometry was discovered among the Egyptians for the aforesaid reason."

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[20] Herodotus, *History*, II, 109; tr. A.D. Godley, (1920) Heinemann, Portsmouth.

[21] Proclus, *On Euclid*, I ;tr. I. Thomas,(1939) *Greek Mathematical works I*, Heinemann, Portsmouth

The Greeks acknowledged the debt they had to the Egyptians. It is often claimed that the Greeks were exaggerating when they recognised geometry as an invention of the Egyptians. Some claim that there is no evidence that the Greek mathematicians ever visited Egypt to learn about its mathematics[22]. Yet it does not seem to be a coincidence that the early Greek mathematicians studied in Alexandria. Many Greek mathematicians such as Eudoxus, Thales, Democritus and Pythagoras were supposed to have learnt from the Egyptians. We have no direct source that this is the case, but there is no reason not to believe it. It seems absurd that the Greeks should not be believed in this case. The writings of Proclus contain ideas similar to those that are seen Egyptian mathematics as a prologue.

Peet does not argue against these accounts of the Greeks. Indeed, he writes:

> "The statement with regard to the equal division of land between the people has probably little historical value, but we need have no doubts as to the soundness of the derivation of Greek geometry from Egypt. Geometry is an intensely practical science and would naturally first appear in a country where land was of very great value, in other words, in a highly agricultural country"[23]

Yet, Peet in his John Rylands lecture speaks of "the vast debt which the world owes to the Greeks"[24]. There is obviously a discrepancy in these opinions. The key to this discrepancy lies in philosophical definitions of mathematics and science. These definitions have been built up from the beginnings of philosophy in Greece. There are works written by the Greeks explaining what they saw as desirable in mathematics. Our philosophy stems from these works. Therefore, the definitions that are used to evaluate Greek and Egyptian mathematics come

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[22] See for example, Mary Leftkowitz (1996) *Not out of Africa, How Afrocentrism became an excuse to teach myth as history*; Basic books; New York.
[23] Peet. T. E. (1923) *Op. Cit.*; p. 31.
[24] Peet T. E. (1931) *Op. Cit.* p. 441.

from the ideas of the Greeks. Since Egyptian mathematics precedes Greek mathematics, effectively ideas are being retrospectively applied. So while the Greeks are happy to credit the Egyptians for their contributions, because philosophy was in its infancy, modern writers are coming after millennia of philosophical enquiry which is affecting their treatment of the mathematical texts.

With the publication of *Black Athena*[25], the level to which the Greeks borrowed mathematical ideas from the Egyptians has become a political one. The question is now loaded with modern assumptions about race. This is most unfortunate as it is impossible to have an unimpassioned academic debate about this issue. It is my feeling that *Black Athena* makes too many claims for the Egyptians, however, this does not mean that the idea that the Greeks should have learnt from the Egyptians is flawed. It is extremely difficult to trace cultural influence, particularly of such an intellectual kind, and extreme care should be taken over speculations of this kind. However, racial considerations and the modern political climate should not interfere with our efforts.

It seems clear, therefore, that we should not dismiss the mathematics of the Egyptians as unscientific, rather we should see it as proto-science that had an important role in the beginnings of mathematics as a separate scientific subject. We should not underestimate the importance of the first attempts to write an instructional text on mathematics.

[25] Bernal, M. (1987) *Black Athena : The Afroasiatic Roots of Classical Civilization Vol. 1: The Fabrication of Ancient Greece 1785-1985*; Free Association Books; London.

8.5: Conclusions

Egyptian mathematics does not fulfil some of the complex principles that are sought in modern mathematics. The texts do not show the intricate proofs that are so highly valued now, nor do all the problems show detachedness from the everyday. However, this is not to say that the Egyptian texts show no evidence for the prizing of mathematics for its own sake: there was certainly recognition of the separate subject of mathematics. The appraisal of the texts in the appropriate context shows that they do exceed the immediate need for numerical manipulation. Once this is understood, then it is impossible to deny that the Egyptians have an important place in the development of mathematics.

If it is assumed that these texts represent a teaching aid for new scribes, then questions about how easy it is to learn mathematics from them and how easy they are to read are fundamental to a balanced appraisal. The Rhind Mathematical Papyrus gives the reader plenty of examples to practise the manipulation of unit fractions. Unit fractions had significance in ancient Egyptian mathematics, as the study of RMP 31 shows they were productive in their mathematical system and fulfilled some need. The appearance of tables to facilitate their use and the number of examples should be seen as a positive characteristic of the texts. The fact that the texts have nominal practical value is also important for the training of new scribes, this is as true today as it was in the past. It is easier to learn mathematics once the lessons are placed in a recognisable situation. It would have made it easier for the scribes taking lessons to imagine the shapes and so to work out their volumes and areas.

The nature of proof is a philosophical problem and the Egyptians did not live up to the more vigorous definitions of proof, but this does not explain what the Egyptians were capable of. It is easy to say what they did not do, it is far harder to understand the strengths and weaknesses of the procedures found in the texts. There is also the far more profound issue of why the Egyptians were satisfied with their demonstrations and where the impetus for a more formal understanding of proof came from. It is not satisfactory to simply explain away this problem as a Greek Miracle, to do so implies that the Egyptians were less intellectually and culturally sophisticated. The discrepancies between the modern reconstruction of the Greek Miracle and the Greek's own accounts of the debt that they owe to the Egyptians should be a matter of concern. Of course, there are problems with the survival of evidence, MMP 10 and 14 suggest that reasoning beyond what has survived took place. It is possible that this work was not recorded, or it was not deemed sufficiently important for the Egyptians to put them in a place that would promote their survival. If the Moscow Mathematical Papyrus had not survived, then our opinion of Egyptian mathematics would be severely altered. The possibility is that other texts that would have had the power to do the same have not survived. If this is the case, then more attention needs to be paid to people who had the opportunity to meet ancient Egyptian mathematicians.

It is clear that answering these few seemingly easy questions will require a much more considered approach than has formerly been taken. These questions raise important issues about the nature of evidence in the History of Mathematics, and how this evidence should be used. In the context of research into ancient Egyptian mathematics, these questions need to be explored within a wider framework and include what is known about other branches of Egyptian science and religion. Only then can the philosophical assumptions and naïve opinions about science and mathematics be eradicated from our understanding of some of the most enigmatic texts to survive from ancient Egypt.

Chapter 9

Conclusions and Consequences for Science Studies, History of Mathematics and Egyptology

The previous three chapters in this thesis have been dedicated to challenging our ideas about mathematics and science. They have shown that by understanding that science has a place within culture, a new approach to Egyptian mathematics is possible. Unfortunately, this definition of science has many critics. In this chapter, the comments of these critics will be examined and answered.

9.1: Introduction

This chapter will explore the reaction that many scientists have had to the notions of constructivism, and attempts to place science within a social context. There has been at times a vociferous and ill-tempered reaction to science studies, perhaps best exemplified by the Sokal affair[1]. The prevailing idea amongst the critics of science studies and constructivist thought is that it is in some way anti-scientific. Why Science Studies are anti-scientific is often poorly defined and elucidated, there is almost a conspiratorial feeling in much of the writing. Often the authors of this scientific backlash against Science Studies accuse their

[1] See section 9.6

opposites of being ignorant of science. They seem to think that only qualified research scientists should have the authority to write about science. Some of the most vociferous and active in this backlash will be discussed in sections 9.1 to 9.3. This is by no means a complete review of the field nor is it meant to be a complete reply. However, an understanding of the approach taken by these authors is instructive when considering how mathematics and science are portrayed in the history of science and mathematics. This is important to Egyptologists and the study of Egyptian mathematics because much of what has been proposed as the new approach to Egyptian mathematics is prone to criticism from this quarter. The belief in their superiority that these authors display and their love of the purest forms of science, untainted and uncontaminated by society, culture or any external factor is evident.

This position is often accompanied by a reference to the "Two Cultures". This phrase was first used by C.P. Snow in his lecture of the same name. In academic circles the phrase "Two Cultures" is used to encapsulate the idea of a schism between science and the arts. The phrase is used almost to justify this schism and argue that this status quo is natural. This is a complete misappropriation of C.P. Snow's original view. Snow's original idea and the effect that the misappropriation of his phrase "Two Cultures" is having on the relationship between the different disciplines is discussed in section 9.4.

This chapter will also explore the wider implications of this study. Initially, it will look at the consequences for science communication and the public understanding of science by considering the impact that the social setting of science and mathematics might make. The position of scientists over this issue will also be examined in order to show how this attitude can be detrimental to the popular image of science. Finally, this chapter will show the general implications for Egyptology and offer suggestions for the improvement of the study of Egyptian science and its contextualisation.

9.2: Constructivism and The Position of Scientists

Constructivism does not try to place doubt on the products of science. Constructivism as it is used in the history of science tries to understand the processes by which scientists come to formulate hypotheses and laws. Constructivism, in this sense, is not necessarily anti-realist. The main idea in this theory is that science is a human creation. It is the nature of the formulation that is constructivist, not the product of this process. Therefore, it should not be taken that a researcher taking a constructivist approach believes that science can be reduced to a purely linguistic or a cultural activity, nor that the product of science is a kind of collective delusion or hallucination. Neither is Constructivism a firm set of philosophical ideas, rather it should be seen as a methodological approach to the problems of writing the History of Science.

Treating scientists as social actors is in no way derogatory, nor should constructivist ideas be used to detract from the wonder of the product of the sciences. However, some scientists have taken the constructivist argument as an attack. Gross and Levitt[2] felt it necessary to launch a counter-attack on constructivist thinkers in order to defend what they saw as their reputations under

[2] Gross, P. and Levitt, N. (1998) *Higher Superstition: The academic left and its quarrels with science.* (2nd ed.) John Hopkins University Press, Baltimore.

condemnation. They allow the point that scientists are human beings and as such should not be immune from study by the social sciences[3]. They even acknowledge that the questions and investigations that scientists are interested in reflect the interests and prejudices of culture. In this they have comprehended the usefulness of the constructivist argument to those engaged in trying to understand how human beings can produce knowledge about the natural world.

It is impossible to prove a realist or an anti-realist philosophy of science. If it is universally accepted that human beings perform scientific research and that they will be, as all humans are, influenced by their own cultural preconceptions, then it is impossible for anyone to have an opinion about science that goes beyond human nature. There can never be any knowledge about the Universe that is not prey to the problems of human sensual experience and is not bounded by human language and all of the difficulties inherent in that. The argument about realism and anti-realism in science is therefore an argument, which while intriguing, is unsolvable. It requires us to examine ourselves and our own understanding of the Universe, but with no other tools than we have used to gain experience of the Universe. We are trapped by our own nature and we cannot leave behind our nature. This was recognised by Kant in the Critique of Pure Reason:

> "Human reason, in one sphere of its cognition, is called upon to consider questions, which it cannot decline, as they are presented by its own nature, but which it cannot answer, as they transcend every faculty of the mind."[4]

[3] Gross, P. and Levitt, N. (1998) *op. cit.* p.42

[4] Kant, I. (1786) *The Critique of Pure Reason.* Preface to the 2nd ed. Translation: Goyer, P. & Wood, A. Cambridge Edition of the Works of Immanuel Kant. (1998) Cambridge University Press, Cambridge.

However, accepting a constructivist argument, even in its strong form, does not necessitate a complete breakdown of scientific thinking.

This situation is not a problem for historians. Historians are used to dealing with the idea that their representation of the past may not be how it 'actually was'. This idea is important in historical research and it is vital that students learn to critically appreciate the nature and pitfalls of historical research. It is through understanding these problems that researchers can become better.

Gross and Levitt maintain that to work with the strong form of constructivism is to believe that scientists deludes themselves when they assert that they can know reality. They develop this idea to conclude that to follow that argument means that there is no way to distinguish between science and superstition[5]. This is clearly a mistaken argument and reveals the prejudices of the authors. Using the parallel with historical research, there may be no way to be certain that historians have discovered any truth about the past. Nevertheless, not every statement about the past is given equal credibility. Writers such as Eric von Daniken[6] propose that extra-terrestrial beings constructed the pyramids of Giza and other ancient monuments. This clearly is not afforded the same integrity as I.E.S Edwards'[7] account of the construction of the same pyramids. There are groups of people that believe the account of von Daniken and other similar writers but these are generally confined to non-academic circles. Since there is no way for academia to control the beliefs of the populace, nor, it can be argued, is control a

[5] Gross, P and Levitt, N; (1998) *Higher superstition: The academic left and its quarrels with science.* John Hopkins University Press, Baltimore. p. 45.
[6] Von Daniken, E (1969) *The Chariots of the Gods* London : Souvenir Press.
[7] Edwards, I.E.S. (1993) 9th ed. *The Pyramids of Egypt* Penguin Books, Harmondsworth, Chapters 4 and 8. pp 98-151; 245-294.

desirable ideal, von Daniken and his associates need not concern Egyptologists greatly. Clearly the acceptance by Egyptologists that their hypotheses about ancient Egypt may not be how it actually was and their acknowledgement of the incomplete understanding of ancient Egypt is not detrimental to the subject. In fact it is a truth that must be understood and recognised in order to study the subject successfully. Similarly, the ideas of constructivism in scientific thinking do not reduce science to the same level as superstition. Constructivism, even in its strong form, does not deny that scientific ideas about the workings of the universe are more likely to be correct than superstitious beliefs. Constructivism does not deny the ideal of objectivity; it merely suggests that the objectivity of scientific experiments happens within a framework. Scientists work within a framework of economic, social, political and ideological factors; yet, this does not preclude the fact that scientists will try to suspend those ideas and be vigorous in their research. Far from seeing scientists as deluded individuals caught up in a type of mass hysteria, scientists are seen as cultural agents.

As the distinctions that we make between different academic disciplines is also necessarily cultural, it is important when reviewing scientific material from the past that we do not allow these hierarchies to cloud our judgement of the nature of the texts. Arguments of a constructivist nature do not try to place one system of ideas above another. Instead, neutrality is made important in research in the history of sciences and mathematics. For this reason constructivism is a necessary methodological approach to take.

9.3: Is Science Studies Anti-Science?

In this thesis I have supported the view that constructivism is a useful idea when we are studying the history of science. My reason for this is that it allows us to study the people that produced the science or mathematics as well as the ideas themselves; it allows us to consider both the process and the product of science.

There have been many attacks on the discipline of Science Studies. These have often centred on the idea that in some way Science Studies and those researchers who partake in it are anti-science or ignorant of the nature of the subject they are studying. It is presumed that through studying the links between science and culture both in the past and in the present, science itself comes under attack. There are many reasons for this idea and yet who those engage in Science Studies do so because they realise the power of science and recognise that science is in itself a subject worthy of study.

History of Science is an area of research that I would place under the greater umbrella of science studies. As discussed in Chapter 2 there are several approaches to the history of science and mathematics. One is defined as the 'mathematical' approach where the mathematical ideas are the data used for analysis. The other is the cultural approach where the relationships between mathematics, science, technology, culture and society in the past are studied. This is an aspect of science studies. Yet, scientists feel that they are under attack from this area of research and are highly critical of it. One of the most often repeated criticisms that scientists level at those engaged in science studies is that they don't understand science.

Surely, scientists do not want nor expect academic culture or the general public to take what they say on faith? Therefore, they should encourage debate on what science is and how it fits into society. This idea is reflected in the move from Public Understanding of Science to Public Engagement with Science. Through this understanding we have a better chance of making science more accessible and through this increase the level of understanding of science. Promotion of a healthy respect for science is of paramount importance if there is to be constructive debate about the power that science has in modern society. A healthy respect does not entail gazing in wonderment from afar. A healthy respect means recognising the true nature of the subject with all its flaws whilst still recognising the potential and power that it has. Science undoubtedly has the power to change our lives. Surely then we need to study what science is in as many ways as possible in order to gain a better understanding of what it is and how it works?

The idea of the two cultures, first introduced by C. P. Snow[8] has so pervaded our ideas about the relationship between different academic disciplines that it is now extremely difficult to have a discussion between disciplines without one side being suspicious of the other's motives.

We should not shun the idea that science is a part of human culture. It does not denigrate the enterprise of science to recognise that it is done by fallible human beings. On the contrary, it makes science even more special as we have had to struggle against our human instincts and our 'common sense' view of the world

[8] See Section 9.1

in order to practise science. Indeed, most major advances in science are counter-intuitive.

9.4: Objectivity and Scientism

Objectivity is often argued as the defining factor in distinguishing between an arts and a science discipline. It is this ideal that some writers[9] have argued places science in a superior position and allows it to pursue the truths of the universe. It is also why the idea of using constructivist philosophy is so widely condemned. If there is any social factor in the production of scientific knowledge then this brings the question of objectivity in scientific research under scrutiny. Objectivity also conjures up notions of fairness, impartiality and truth. It also suggests that there are no distorting factors. If society and culture are affecting scientific research, then surely there is a distorting factor and the claim science has to objectivity is suspect. Another problem is whether it is possible to remain objective whilst working within a community of scientists. Because objective has a natural association with ideas such as fairness, it is also supposed that the scientific method is better equipped to discover truths about the Universe.

These ideas can lead to the extreme of scientism. Scientism is the philosophical belief that science is the best kind of knowledge there is. This is believed because science is seen as the most authoritative, objective and so the most serious branch of learning. The idea of objectivity is one of the central values of the academic system. To be objective is to not let personal ideas and feelings

[9] See for example, Porter, T. M. (1996) *Trust in Numbers: The Pursuit of Objectivity in Science and Public Life*, Princeton University Press, Princeton New Jersey. Wolpert, L. (1992) *The Unnatural Nature of Science*; Faber and Faber; London.

interfere with the search for the truth. The reliance of science on experimentation and the quantitative nature of data are seen as the pinnacle of this ideal. The desire of subjects such as linguistics, and to a certain extent, archaeology and history, to be considered a science is one of the symptoms of scientism. These subjects desire the label that will enable them to enter the inner sanctum of academic culture. The desire is not to change the aims of the subjects or to significantly change the methodology, but to gain status within the academic community.

There have been claims that scientific objectivity can be equated with moral superiority:

> "In defending the scientific community's just claims to knowledge I am also defending the moral superiority of that community relative to any other human association"[10]

The association with moral values is linked to the idea of objectivity in science. Objectivity not only means truth to nature but also to ideas of impersonality, universality and to fairness[11].

These ideas are important when considering the way in which the study of the history of science is considered. Scientists who write history of science and inform what is seen to be desirable in science are used to the idea that objectivity in research is everything. Yet when writing history the aims are different from those of a natural science. It takes a different philosophy to write history than to do science. Historians have to be able to weigh up evidence that may conflict and may have been written with ulterior motives.

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[10] Harrè R. (1986) *Varieties of Realism: a rationale for the natural sciences, Blackwell*, Oxford.
[11] Porter, T. M. (1996) *Trust in Numbers: The Pursuit of Objectivity in Science and Public Life*, Princeton University Press, Princeton New Jersey.

9.5: The Misappropriation of C.P. Snow's Two Cultures

In 1959, C.P. Snow gave the Rede lecture at the University of Cambridge. It was called "The Two Cultures and the Scientific Revolution" and was later published[12]. 'The Two Cultures' is a phrase that has found its way into the academic consciousness and yet few have read Snow's original lecture. The fact that it was a lecture on the scientific revolution as well is also a fact that has been lost in the majority of cases. The phrase has been misappropriated to such an extent that Snow himself would be appalled. It has come to justify attitudes that are, in sentiment, the precise opposite of what Snow was arguing. Many use the phrase to sum up what they see as a great divide between the arts on one side and the sciences on the other and to maintain that divide. It is also used to excuse an ignorance of another academic's field of research. This is as true of scientists misunderstanding the arts as it is of arts researchers misunderstanding the sciences.

It should be remembered when considering the Two Cultures that the two disciplines Snow had in mind are the sciences and literary culture. The literary culture, as discussed by Snow, does not extend to cover the humanities. His examples of the literary culture are all part of the literary culture in the narrowest sense. His examples are novelists such as Dickens, playwrights, such as Shakespeare and poets like Yeats. Only once is history mentioned in the original

[12] Snow, C.P. (1962) *The Two Cultures and the Scientific Revolution*, Cambridge University Press, Cambridge.

lecture and that in negative comments about the attitude of scientists when describing the culture of scientists. Snow writes:

> "Their culture is in many ways an exacting and admirable one. It doesn't contain much art, with the exception, an important exception, of music... Of books, though, very little. And of the books which to most literary persons are bread and butter, novels, history, poetry, plays, almost nothing at all."[13].

Despite this crucial point, some writers would include historians and classicists in the literary culture[14].

The definition of culture is one point at which Snow attracts critics. In his original lecture it is not clear what he means by the term, so he is forced to expand the idea in a second extended version. In this second version, Snow defines culture in two ways. The first he calls the dictionary definition "intellectual development, development of the mind" which he backs up with a quote from Coleridge "Qualities and faculties which characterise our humanity"[15]. However, by this definition, as Snow admits, no one academic disciple can count as culture, only sub-cultures. The second of Snow's definitions he borrows from anthropology: "A group of persons living in the same environment, linked by common habits, common assumptions, a common way of life"[16]. This is the stronger definition of the two when trying to understand Snow's idea of the two cultures. However, Snow's examples seem to us simplistic. The notion of culture is problematic and one that different disciplines describe in different ways and have different uses for.

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[13] Snow, C.P. (1962) pp. 12-13
[14] Gross,P. & Levitt, L. (1998) p.7
[15] Snow, C.P. (1964) *The Two Cultures: A Second Look*, Cambridge University Press, Cambridge. p.62 Original italicisation .
[16] Snow, C.P. (1964) p.64

Robin Dunbar misrepresents Snow's ideas when he tries to use the lecture to prove that people are concerned by the apparent destruction of traditional values. He portrays the content of Snow's lecture as being "science versus the arts"[17] thus giving the impression that Snow's purpose was to highlight and maintain, even promote a pugilistic relationship between the two cultures. The quote that he uses from the lecture is a highly perverted and one-sided account of the true content and meaning of Snow's original lecture. He plucks from Snow's lecture a quote that "literary intellectuals are natural Luddites"[18]. He does not include the parts of the lecture where Snow is critical of the attitude of scientists toward literary culture. Snow was very critical of a scientist's ignorance of traditional (literary) culture. He says of scientists:

> "The whole literature of the traditional culture doesn't seem to them relevant ... They are, of course, dead wrong. As a result their imaginative understanding is less than it could be. They are self impoverished"[19].

Dunbar does note that it would be unfair to presume that all intellectuals and members of the humanities (a group not included in Snow's definition of the literary culture) are anti-science. However, Dunbar also believes that there is growing evidence that the humanities are feeling the pressure from scientists and so are turning against the sciences.

Snow is horrified by the lack of communication between his Two Cultures. He says: "The degree of incomprehension on both sides is the kind of joke which has gone sour"[20]. He does not advocate a position where there is a divide between the sciences and the arts, on the contrary he feels that not only academia and the

[17] Dunbar, R (1995) *The Trouble With Science*; Faber and Faber; London. p.2
[18] Dunbar, R (1995) *Op. Cit.* p.2
[19] Snow, C.P. (1962) Op. Cit.p.13
[20] Snow, C.P. (1962) *Op. Cit.*p.11

academics it contains are worse off for it, but society as a whole. The difference between academia and science is characterised by Snow as a distinction between intellectual life and practical life. These two ideas he sees as linked at a deep level.

In his second version he identifies what he considers a hope for better communication between his Two Cultures: a Third Culture. Snow identifies this Third Culture as an American phenomenon. It consists of various fields of the humanities: social history, sociology, demography, political science, economics, government studies and social arts and also psychology and medicine. Snow groups these disciplines together because they are all interested in researching how human beings live and as Snow puts it they are concerned "not in terms of legend, but of fact"[21]. He feels that this Third Culture has the power to unite the other two as it is aligned with literary culture but "for such a culture to do its job, it has to be on speaking terms with the scientific one".

Clearly, history of science and mathematics should be seen as part of this third culture. In addition, the works of people such as Latour, Golinski and other sociologists of science should also been included in this culture. However, there needs to be cooperation between the different cultures so that there can be a free flow of ideas. The scientific community has reacted with horror at some of the findings of this Third Culture[22]. They have been keen to point out that because the researchers are not part of the scientific culture, they cannot understand the science they are writing about. Instead of a value-free discussion of the issues,

[21] Snow, C.P. (1964) p.70.
[22] See Gross & Levitt (1998) *Op. Cit.*, Porter (1996) *Op. Cit* and Dunbar (1995)

scientists have severely criticised and even ridiculed the Third Culture. They are guilty of precisely what Snow was complaining about in his lecture on the Two Cultures: "the politeness has gone, and they just make faces"[23].

C.P. Snow also talked about how scientific culture and traditional culture have become separated. He saw traditional culture being embodied by the great authors of the past. He therefore associates traditional culture with literary culture, using the terms almost synonymously.

9.6: Science Communication

Science communication is a growing field of research that covers a diverse range of topics. This discipline covers science journalism, the public understanding of science and how science is represented in texts. Central to science communication is the desire to raise the profile and preconceptions about science. There are many contentious issues surrounding science and the applications of science.

Dunbar points out a crisis in our education system[24]. Fewer students are opting to study the sciences at university. A lot of money is therefore being spent on public understanding of science initiatives. As Dunbar notes, one of the big problems facing these initiatives is the problem of the image of the sciences as 'hard'. Students perceive that the sciences are harder than humanities and arts subjects. The rivalry between students from the two sides is legendary. As

[23] Snow, C.P. (1962) p.17
[24] Dunbar, R (1995)

Dunbar points out there is an idea that in the sciences there is a right or a wrong answer, "Whereas in the humanities you can get away with waffle even if you don't know what the answer really should be"[25]. This is a double failure on the part of the education system and society that propagates these ideas. Not only do we need to encourage the idea that the sciences are not too difficult and thus encourage more students to take them up; we also need to put an end to the silly notion that the arts and humanities are a soft option.

The roots of this argument are promoted by many scientists. The foundation of the idea that the sciences are hard as opposed to the undemanding option of the arts and humanities comes from the notion that the sciences are purely objective whilst the humanities are simply subjective. Thus, the argument goes that in the humanities any opinion is valid whereas in the sciences there is only one right answer, which must be learnt. By denigrating the humanities and the arts, the sciences look harder by comparison. Consequently, in order for the sciences to improve their image, there must be a greater understanding and cooperation between the disciplines. Scientists must refrain from out of hand rebuffs of the arts and humanities. For example, Dunbar argues that:

> "Not everyone intuitively understands the principles of photosynthesis, but everyone knows what a good story is or how to sing a song. Anyone can paint (even if it is not Michelangelo's class), but not everyone can write out the geometric proof of Pythagoras' theorem"[26].

This argument does great harm to both the sciences and the arts. Simultaneously it reduces the achievement of those in the arts and at the same time perpetuates the idea that you have to be a special kind of person to study science. Whilst we

[25] Dunbar, R. (1995) p.181
[26] Dunbar, R. (1995) p.181

may all be able to sing in the shower, performing Handel's "Messiah" to a large audience requires training and musical expertise.

There has also been confusion over the polysemious meaning of the word 'hard'. When used to compare the sciences to the arts, 'hard' is being used as the antonym of 'soft'. However, it has now been adopted as a description of the sciences. Physics is 'hard' where sociology is 'soft'. On its own, rather than in company of the word 'soft' it appears that the sciences are more difficult as 'hard' can also be the antonym of 'easy'.

The history of science is full of examples where it is the process of overcoming difficulties and finding resourceful and original ways to study the world that has the power to inspire. The pyramids of Giza are an example of this point. Even though the Egyptians were working with primitive tools, they were able to find inventive solutions and therefore build the only surviving wonder of the ancient world. We can now only hypothesise about which stars were used to align the pyramids to the compass points to within a fraction of a degree. Through the scant remains of their mathematical texts we can have a guess at some of the mathematical procedures that they used to lay out the ground, yet the details of the process remain elusive. Newton also has the ability to inspire, not just because his understanding of optics and gravity enabled us to develop scientifically and technologically, but also because he approached old problems in an ingenious way.

Cooperation between the arts and the sciences is necessary, not only in the History of Science, but also for gaining a better understanding of the nature of science. When this cooperation does not work, then it can cause mistrust and difficulties for both sides, as evidenced by the Sokal affair. This was[27] a notorious event that instigated the science wars. This affair is still written about even though the article that began the trouble was published in 1996. In a recent column in The Guardian[28], Sokal was proclaimed as a hero, "The man who pulled the greatest academic scam of our times". Sokal had sent a parody article for publication in a Science Studies journal. Sokal's method was unscientific, yet it is one that scientists have defended because they believe themselves under attack[29]. The affair had caused many problems and has not advanced Sokal's premise to anyone who was not already in agreement. It is not possible to carry out research in an atmosphere of suspicion, Historians of Science should learn from Sokal's mistake and ensure that dialogue is kept productive at all times.

9.7: Lessons for the study of Egyptian Mathematics

For any useful understanding of ancient Egyptian mathematics to be produced there needs to be a consideration of different methodologies and philosophies. On the one hand, the purely mathematical content of the texts needs to be established and the place of this mathematics in the development of Western mathematics needs to be understood. This requires mathematical training and an appreciation of the philosophy of mathematics and how this philosophy relates to other episodes in the development of Western mathematics. Platonic philosophy is often used in this case; it is assumed that numbers and mathematical constructs are above and beyond human invention.

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[27] Relevant material is available through Sokal's website at http://www.physics.nyu.edu/faculty/sokal/

[28] B. Goldacre, (5th June 2003) Bad Science Column, *The Guardian,*

[29] Hilgartner,S. (1997) 'The Sokal Affair in context' in *Science, Technology & Human Values,* Vol.22, No. 4, pp 506-522.

On the other hand, an appreciation of the context in which the texts were produced and how the mathematics fits into the broader understanding of the culture that produced it is also necessary. I have argued in this thesis that a constructivist philosophy is the most useful in this case. Mathematics may be beyond human invention but the ways in which it is explored and the methods used to record findings will vary according to cultural and societal needs. This potentially brings different areas of academic research into conflict. To date, this conflict has been dealt with very badly, bringing bitterness and acrimony.

Whatever our opinions of the mathematics of ancient Egypt, the Egyptian mathematical texts should be of intense interest to an Egyptologist. It is unpardonable that mathematics is such a marginal subject in the field of Egyptology. Even if the Egyptians were simply interested in using mathematics as a tool to solve everyday problems, that still is a valuable insight into the minds of the ancient Egyptians. In science, a null experiment is often more telling than a positive one.

It should be the work of Egyptologists to pronounce judgement on Egyptian texts, whatever their content. An understanding of the wider context of the production of the texts is vital in trying to approach the topic with any credibility. In addition, the texts should be studied in their original language: too much is lost if the authors have to rely on translations, there is too much room for mistake. Science and mathematics are not special cases in this respect. Egyptologists have traditionally had a background in the humanities and arts, which is reflected in the way the subject is approached and the topics that can be found in the undergraduate curriculum. Yet, mathematics has been allowed to take a special place in the academic hierarchy as it is that it believed that it is a more difficult subject. It is common to hear arts and humanities students state: "I can't do maths". This attitude has been allowed to prosper so that it is an acceptable thing to shut off from mathematics and not even attempt to understand it. This is an attitude that I have encountered whilst giving lectures and conference papers. I am yet to find a mathematician who says: "I can't do history". For this reason mathematics in ancient Egypt has become such a specialised topic that it is not even mentioned in most Egyptology lecture courses and general textbooks. This attitude is so engrained that Kemp finds it necessary to apologise to his readers when introducing the most basic mathematics[30]. The standard view seems to be:

> "These are exact sciences which fall outside the usual purview of the humanities, and they can be treated properly only by specialists, while Egyptologists must accept their judgement with respect and gratitude"[31]

However, being an Egyptologist should entail studying the whole culture, even if it is not possible to become an expert on every topic.

The mathematical papyri are unfamiliar; it takes patience to master the arithmetic and particularly the use of unit fractions. With patience, however, and a lecturer who understands the material and can guide the student through that initial unfamiliarity, it is possible to gain insight into the material. Just because the student has not achieved good results in traditional mathematics exams, it does

[30] Kemp, B. (1989) *Ancient Egypt: Anatomy of a Civilization*; Routledge; London; p. 116.
[31] Sauneron, S (Trans. Lorton, D); 2000; *The Priests of Ancient Egypt*; Cornell University Press; New edition. First published 1957.

not mean that with a little patience they should not master the Egyptian material. Indeed, much of the mathematics, particularly arithmetic, that is taught in schools is prescriptive. Egyptian arithmetic requires a different approach so it is necessary to start from the beginning and work towards more complicated problems. In essence, the Egyptian texts should be used to teach mathematics to Egyptologists. The Rhind Mathematical Papyrus in particular has many problems that given the right guidance have relatively simple mathematical content, but show interesting features of Egyptian mathematics.

It is to be hoped that mathematics becomes a standard part of an Egyptology curriculum, even as only part of a module. The place of mathematics in art, accounting, religion and every other aspect of the Egyptian civilisation is poorly understood because mathematics has always been left to the mathematicians. To understand these relationships, it is necessary that Egyptologists who specialise in these areas become aware of the Egyptian mathematical procedures. It is only then that Egyptology will begin to recognise where these links exist and then allow specialists to investigate. Egyptologists are prone to misunderstand the idea of the Two Cultures and use it as a defence for scientific ignorance and even science phobia. In Section 9.5, it was shown that Snow did not intend his phrase to be used as an excuse. On the contrary, he wanted to encourage cooperation and mutual understanding. History of Science is an obvious area where this could become a reality. Egyptologists need to be more open minded about science and show more willingness to engage with the science of ancient Egypt.

It has been my experience that Egyptologists in general are disinclined to investigate the nature of Egyptian mathematics. There are, I feel, a number of reasons for this. First, it is a hard subject to break into. For example, Mathematical texts are not particularly suitable for classes in Egyptian hieroglyphs. As the translations in Chapter 2 to 4 show, there are either very few words amongst the numerical workings, or the meaning of the words is too uncertain for the translations of the texts to be a useful linguistic exercise. Also the use of the *sḏm.ḫr=f* form is extremely unusual and so mathematical texts are a specialist area, not particularly useful for the general student of Egyptian hieroglyphs. The study of Egyptian mathematics has no tradition of being taught on an undergraduate degree and so it has become a specialised topic. This means that only a few who already have an interest in mathematics seek out books on the subject. It is then hard to find like-minded researchers to discuss problems with or find conferences at which papers can be presented alongside papers on a similar topic. Also, the many negative comments made about Egyptian mathematics give the impression that the subject is unworthy of attention.

The main problem, though, is the image that science and mathematics has. Science is perceived as being a subject that only a specialised few can understand. It has become acceptable for students in arts subjects to confess to a lack of understanding of the sciences or mathematics. Indeed, it has become common that arts students not only confess to not understanding mathematics but that they also express fear or loathing of the subject. It is therefore hard to engage with these students, as they are unwilling to consider the texts because of their perception that they are no good at mathematics. The use of unit fractions and other functions and methods that are unfamiliar to students are also a factor that confuses and so causes another barrier to be raised between the student and the texts. These barriers are largely a matter of preconceptions, once enough problems have been explained and worked through in front of the student, the student can grasp the essentials of the methods involved. This holds true not only for Egyptology students but also for school pupils of all ages.

9.8: Overall Conclusions

Egyptian mathematics is worth revisiting as the frameworks and philosophies that have been used previously to evaluate the material are naïve and give an incomplete picture. There is much work to be done if the Egyptian mathematical Papyri are to be afforded the recognition they deserve: as some of the most important texts to survive from ancient Egypt. Egyptian mathematics deserves a more prominent place in chronological histories of mathematics. The Greeks acknowledged the debt they owed to the Egyptians. The idea of a Greek miracle can no longer be sustained. It is time that mathematicians examine the dogma that is present in their understanding of the development of mathematics. The ancient Egyptian mathematical texts reveal a rich mathematical culture, albeit in infancy. Egyptologists should certainly pay more attention to these fascinating texts. It is not possible to fully appreciate the intellectual culture of ancient Egypt without reference to their mathematical and other scientific achievements. To achieve this modern preconception of the nature of science and mathematics should be set aside. Egyptologists have become accustomed to being cautious when evaluating Egyptian religious and ritual practices as many of the features of these practices do not fit with modern ideas of religion. The same should be true for mathematics and science.

It is suggested that a constructivist philosophy can open up interesting avenues of research. By understanding mathematics and science as part of human culture, it can be seen that even apparently trivial questions can open up interesting possibilities for research. Whether or not Platonism is followed, the production of mathematics and science is cultural, the uses to which it is put and the questions that are deemed worthy of study are determined by society. If another culture has different needs for science and mathematics this does not make their texts less worthy of study. In contrast, they are possibly more interesting as they have the ability to question our presumptions and stereotyped ideas.

The boundary between internal and external influences is illogical. This distinction presupposes that the advance of science is purely rational. It assumes that there is an arrow of progress that is always followed. Archaeologists have come to be suspicious of arrows drawn on maps to show the advance of a culture, it is recognised that the mechanisms for advance are important and that reality is more complicated than these diagrams can show. Diffusionism has been shown to be simplistic, independent invention is possible. The mechanisms behind scientific advance are not clear. It should not be assumed that they are rational, nor should it be thought that scientific advance is mapped out. The 'royal road to me' can be of interest to practicing mathematicians, but it should not concern a serious researcher in the History of Science and Mathematics.

Egyptian mathematics should not be derided because it has its origins in practical need, nor should this fact be shocking. There are strong abstract elements in the mathematical texts, even if these elements are not immediately obvious. The vocabulary and linguistic structure of the mathematical texts merits further research. The translations offered in this thesis are not definitive, merely the best possible given the present level of understanding. It would also be interesting to carry out a comparative study between the mathematical texts and other scientific texts such as medical papyri. The method of communication of science is an important factor in understanding its status and function in society. There is scope for building a better framework for understanding ancient mathematics in the relatively new field of Ethnomathematics. Ethnomathematics investigates mathematical knowledge in small-scale indigenous cultures. An anthropology of mathematics may be able to provide ways of discussing mathematics that do not rely on received understandings of the nature of mathematics. Whatever the approach taken, it is imperative that ancient Egyptian mathematics becomes more widely understood within Egyptology so a debate about its importance in ancient Egypt can take place.

Bibliography

Barnes, B, Bloor, D. and Henry, J. (1996) *Scientific Knowledge: A Sociological Analysis*; Chicago University Press, Chicago.

Barnes, B. and Edge, D. (eds.) (1982) *Science in Context, Readings in the Sociology of Science*; Open University Press; Milton Keynes.

Barrow, J.D. (1968) *The Word Within the World* Oxford University Press; Oxford.

Basalla, G. (1988) *The Evolution of Technology*; Cambridge University Press; Cambridge.

Bell, E.T. (1945) (2nd ed.); *The Development of Mathematics*; McGraw-Hill; New York.

Bernal, M. (1987) *Black Athena : The Afroasiatic Roots of Classical Civilization Vol. 1: The Fabrication of Ancient Greece 1785-1985*; Free Association Books; London Bloor, D. (1991) *Knowledge and Social Imagery*. 2nd ed. University of Chicago Press, Chicago.

Bird, A. (1998) *Philosophy of Science*; UCL Press, London.

Bottazzini, U. and Fraser, C. (2000) "Editorial. At the Turn of the Millennium: New Challenges for the History of Mathematics and for *Historia Mathematica*" *Historia Mathematica*; Vol. 27; pp. 1 – 3.

Boyer, C, rev. Merzbach, U (1991) *A History of Mathematics*; 2nd ed; John Wiley and Sons.

Brett, A. (2002) "What is Intellectual History Now?" in Cannadine, D. (ed.) *What Is History Now?*; Palgrave Macmillan; London; pp. 113 – 131.

Bruins, E. M. (1981) "Egyptian Arithmetic: 2/n" *Kon Nederland Akad Wetanschappen*; Vol. 55, Ser. A; pp. 81 – 91.

Bruins, E.M (1975) "The Part in Ancient Egyptian Mathematics" *Centaurus* Vol 19, no 4, pp 133-171.

Bruins, E. M. (1981) "Egyptian Arithmetic" *Janus*; Vol. 68; pp. 33 – 52.

Callinger, R. (ed.) (1996) *Vita Mathematica. Historical research and integration with teaching*; The Mathematical Association of America; Buffalo; NY.

Campbell, P. J. (1977 - 1978) "A Practical Approach to Egyptian Mathematics" *Journal of Recreational Mathematics*; Vol. 10; pp. 81 – 86.

Carr, E. H. (1990) *What is History?* (2nd ed.) Penguin; Harmonsworth. First Published by Macmillan 1961.

Chace, A. B. (1931) "The Egyptian Fraction Reckoning" *Archeion*; Vol. 13; pp. 40 – 41.

Chace, A.B, Bull, L.S, Manning, H.P. and Archibald R.C (1927/9) *The Rhind Mathematical Papyrus*; 2 volumes; Mathematical Association of America; Buffalo; NY.

Chalmers, A.F. (1978) *What is this thing called Science?*; Open University Press; Milton Keynes.

Clagett, M. (1992) *Ancient Egyptian Science Volume I: Knowledge and Order*; Memoirs of the American Philosophical Society vol. 184; Philadelphia.

Clagett, M. (1995) *Ancient Egyptian Science Volume 2: Calendars, Clocks and Astronomy*; Memoirs of the American Philosophical Society vol. 214; Philadelphia.

Clagett, M (1999) *Ancient Egyptian Science: A Source Book Vol. 3. Ancient Egyptian Mathematics*, Memoirs of the American Philosophical Society vol. 232, Philadelphia.

Couchoud, S. (1993) *Mathématiques égyptiennes*; Le Léopard d'Or; Paris.

Couvalis , G (1997) The Philosophy of Science; Science and Objectivity; SAGE Publications; London.

Cunningham, A. (1988) "Getting the Game Right: Some Plain Words on the Identity and Invention of Science" Studies in History and Philosophy of Science; Vol. 19, no. 3; pp. 365-89.

D'Ambrosio, U, (1991) "Ethnomathematics and its place in the history and pedagogy of Mathematics" in Harris, M, (ed.) *School Mathematics and Work*; Falmer; London.

Dauben, J. W. (1993) "Mathematics: An Historians Perspective" *Philosophy and the History of Science: A Taiwanese Journal*; Vol. 2, No. 1, April 1993; pp 1-21.

David, P and Hersh, R (1981) *The Mathematical Experience*; Birkhauser, Boston.

Deetz, J. (1988) 'History and Archaeological Theory: Walter Taylor Revisited' *American Antiquity* Vol 53:1 pp13-22.

Downes, S (1998) "Constructivism". In Craig, E. (ed.) *Routledge Encyclopaedia of Philosophy*. London: Routledge. Retrieved July 29, 2003, from http://rep.routledge.com/article/Q017

Dunbar, R (1995) *The Trouble With Science*; Faber and Faber ; London.

Edwards, I.E.S. (1993). *The Pyramids of Egypt*; (9th ed) Penguin Books, Harmondsworth.

Eglash, R. (1997) "When Math Worlds Collide: Intention and Invention in Ethnomathematics" *Science, Technology & Human Values*; Vol. 22, Issue 1; pp 79 – 97.

Eisenlohr (1877) *Ein mathematsches Handbuch der alten Ägypter, übersetzt und erklärt*; Leipzig.

Engels, H. (1977) "Quadrature of The Circle In Ancient Egypt" *Historia Mathematica*; Vol. 4; pp 137 – 140.

Faulkner, R. (1972) *Dictionary of Middle Egyptian*; Griffith Institute; Oxford.

Fauvel, J and Gray, J. (1987) *The History of Mathematics: a Reader*; Macmillan education ltd, Basingstoke.

Fine, A. (1998) "Scientific realism and antirealism" In Craig, E. (ed.); *Routledge Encyclopaedia of Philosophy*; London: Routledge. Retrieved May 17, 2004, from http://www.rep.Routledge.com/article/Q094

Feyerabend, P. (1964) "Realism and Instrumentalism : Comments on the Logic of Factual Support"; Bunge E. (ed.) *The Critical Approach to Science and Philosophy*; Free Press; New York; p. 305.

Feyerabend, P. (1975) *Against Method*; New left Books; London.

Feyerabend, P. (1981) *Realism, Rationalism and Scientific Method. Philosophical Papers vol. 1*; Cambridge University Press; Cambridge.

Fox, R. A. (1993) *Archaeology, History and Custer's Last Battle: The Little Big Horn Re-examined*; University of Oklahoma Press, Norman.

Gardiner, A. (1957) Egyptian Grammar; 3rd ed.; Griffith Institute; Oxford; p. 346.

Gazalè, M. (2000) *Number: from Ahmes to Cantor*; Princeton University Press; Princeton.

Gennaro, A. (1998) "Practical Aspects of Egyptian Mathematics Shown by Mathematical Papyri" in Eyre, C. J. (ed.) *Proceedings of the 7th International Congress of Egyptologists*; Peeters; Leuven.

Gillings, R.J. (1955) "The Division of 2 by the Odd Numbers 3 to 101 from the Recto of the Rhind Mathematical Papyrus (BM. 10058)" *The Australian Journal of Science*; Vol. 18; pp. 43 – 49.

Gillings, R.J. (1959) "The Egyptian 2/3 Table for Fractions" *The Australian Journal of Science*; Vol. 22; pp. 247 – 250.

Gillings, R.J. (1962) "Problems 1 – 6 of the Rhind Mathematical Papyrus" *The Mathematics Teacher*; Vol. 56; pp 61 – 69.

Gillings, R.J. (Dec. 1964) "The Volume of a Truncated Pyramid"; *The Mathematics Teacher*; Part I: Vol. 57, No. 4, pp 552 – 55.

Gillings, R. J. (1965) "The Addition of Egyptian Unit Fractions" *Journal of Egyptian Archaeology*; Vol. 51; pp. 95 – 106.

Gillings, R. J. (Nov, 1966) "Mathematical Fragment from the Kahun Papyrus" *The Australian Journal of Science*; Vol. 29, no. 5; pp 126 – 130.

Gillings, R J. (1968) "Sum of n Terms of an Arithmetical Progression in Ancient Egypt" *The Australian Journal of Science*; Vol. 31, No. 1; pp. 47 – 50.

Gillings, R. J. (1972), *Mathematics In The Time of The Pharaohs*, MIT Press, Cambridge MA.

Gillings, R. J. (1980) "The Recto of the Rhind Mathematical Papyrus and the Egyptian Mathematical Leather Roll" *Historia Mathematica*; Vol 6; pp. 442 – 447.

Gillings, R. J. and Rigg, W. J. A. (1969) "The Area of a Circle in Ancient Egypt" *The Australian Journal of Science*; Vol. 32, No. 5; pp 197 – 200.

Gjertsan, D. (1989) *Science and Philosophy, Past and Present*; Pelican Books; London.

Glanville, S. (1927) "The Mathematical Leather Roll in the British Museum" *The Journal of Egyptian Archaeology*; Vol. 13; pp 232 –239.

Glanville, S. (1930) "A Working Plan for a Shrine" *Journal of Egyptian Archaeology*; Vol. 16; pp 237 – 239.

Goldacre, B. (5th June 2003) Bad Science Column, *The Guardian*, London.

Golinski, J. (1998) *Making Natural Knowledge: Constructivism and the history of science*; Cambridge University Press, Cambridge.

Griffith, F. (1898) *The Petrie Papyri: Hieratic Papyri from Kuhun and Gurob*; (2 vols.) B. Quaritch; London.

Grattan Guiness I. (1990) "Does History of Science Treat of the History of Science? The Case of Mathematics" *History of Science*; vol. 28, pp 149-73.

Green, A. and Troup K. (1999) *The Houses of History: A critical reader in twentieth-century history and theory*; University of Manchester Press; Manchester.

Gross, P and Levitt, N; (1998) *Higher superstition: The academic left and its quarrels with science*. John Hopkins University Press, Baltimore.

Gunn. B (1926) "Review of Peet T.E (1923) The Rhind Mathematical Papyrus: British Museum 10057 and 10058" in *Journal of Egyptian Archaeology*; vol. 12; pp 123-37.

Gunn, B. and Peet, T. (1929) "Four Geometrical Problems from the Moscow Mathematical Papyrus" The *Journal of Egyptian Archaeology* Vol. 15 pp 167-85.

Harrè R. (1986) *Varieties of Realism: a rationale for the natural sciences*; Blackwell, Oxford.

Harris, J. R. (1971) "Mathematics and Astronomy" in Harris, J. (ed.) *The Legacy of Egypt*; Clarendon Press; Oxford; pp. 27 – 54.

Hawkes, C. (1954) 'Archaeological theory and method: some suggestions from the Old World', *American Anthropologist*; 56: pp. 155-68.

Herodotus, trans. De Selincourt, A. (1954) *The Histories*; Penguin; Harmondsworth.

Hilgartner,S. (1997) 'The Sokal Affair in context' *Science, Technology & Human Values*, Vol.22, No. 4, pp 506-522.

Hornung, E. trans. Baines , J. (1982) *Concept of God in Ancient Egypt.* Cornell University Press; Ithaca.

Hurry, J. B. (1926) *Imhotep*; Oxford University Press; Oxford.

Imhausen, I. (2002) "The Algorithmic Structure of the Egyptian Mathematical Problem Texts" in Steele, J. and Imhausen, I. (eds.) Under One Sky: Mathematics and Astronomy in the Ancient Near East; *Alter Orient und Altes Testament*; Band 297; Ugarit-Verlag; Minster; pp149-66.

Imhausen, I. (2003) "Egyptian Mathematical Texts and Their Contexts" *Science in Context*; vol. 16(3) pp 367-89.

Isler, M. (2001) *Sticks, Stones and Shadows. Building the Egyptian Pyramids*; University of Oklahoma Press; Norman.

Jameison, R. (1999) "Diffusionism" in Shaw and Jameison (eds) *A Dictionary of Archaeology* Blackwell Publishers, Oxford.

Joseph, G.G. (1991) "Foundations of Eurocenricism" in Harris, M (ed.) *School Mathematics and Work*, Falmer, London.

Joseph, G. G. (2000) *The Crest of the Peacock: Non-European Roots of Mathematics*; 2nd ed.; Penguin, Harmondsworth.

Kant, I (1786) *Critique of Pure Reason,* Translation: Goyer, P. & Wood, A. (1998) *Cambridge Edition of the Works of Immanuel Kant*; Cambridge University Press, Cambridge.

Kemp, B. (1989) *Ancient Egypt: Anatomy of a Civilization*; Routledge; London.

Kline, M. (1962) *Mathematics, A Cultural Approach*; Addison-Wesley; Reading, Mass.

Kline, M. (1964) *Mathematics in Western culture*; Oxford: Oxford University Press.

Klee, R (1997) *Introduction to the Philosophy of Science: Cutting Nature at its Seams*; Oxford University Press; Oxford.

Knorr, W. (1982) "Techniques of Fractions in Ancient Egypt and Greece" *Historia Mathematica;* Vol. 9; pp 133-171.

Kuhn, T. S. (1970) The Structure of Scientific Revolutions; 2nd ed.; University of Chicago Press; Chicago.

Lakatos, I. (1976) *Proof and Refutations*; Worrall J. and Zahar E. (eds); Cambridge University Press; Cambridge.

Lakatos, I. (1978) Worrall and Currie, G. (eds.) *Mathematics, Science and Epistemology. Philosophical Papers volume 2*; Cambridge University Press; Cambridge.

Lakatos, I. (1978) "On Popperian historiography" *Mathematics, Science and Epistemology* Worrall and Currie, G (eds.) Philosophical Papers volume 2; Cambridge University Press; Cambridge; pp 201-10.

Lakatos, I (1978) "Newton's Effect of Scientific Standards" in Worral J. and Currie, G. (eds.) *The Methodology of Scientific Research Programmes.* Philosophical Papers vol. 1; Cambridge University Press, Cambridge.

Lakatos, I. (1978) "What does a mathematical proof prove?" Mathematics, Science and Epistemology Worrall and Currie, G (eds.) *Philosophical Papers volume 2*; Cambridge University Press; Cambridge.

Lakatos, I (1978) "Why Copernicus's Programme Superceeded Ptolemy's"in Worral J. and Currie, G. (eds.) *The Methodology of Scientific Research Programmes. Philosophical Papers vol. 1*; Cambridge University Press; Cambridge; pp168 – 92.

Lakatos, I. (1981) "History of Science and its Rational Reconstructions" in Hacking, I (ed.) *Scientific Revolutions*; Oxford Readings in Philosophy; Oxford University Press; Oxford; p110-13.

Larvor, B. (1998) *Lakatos: An Introduction*; Routledge; London.

Larvor, B. (2000) "History, Role in the Philosophy of Science" in Newton-Smith, W. H. (ed.) *A Companion to the Philosophy of Science*; Blackwell Publishers Oxford; pp 154-161.

Latour, B. (1987) *Science in Action: How to Follow Scientists and Engineers through Society*; Harvard University Press; Cambridge, MA.

Leftkowitz, M. (1996) *Not out of Africa, How Afrocentrism became an excuse to teach myth as history*; Basic books; New York.

Lloyd, G.E.R. (1970); *Early Greek Science: Thales to Aristotle*; Norton; London.

Long, R. L. (1986) "Remarks on the History and Philosophy of Mathematics" *American Mathematical Monthly*; Vol. 93, Issue 8; pp 609 – 619.

Maier, E. (1991) "Folk Mathematics" in Harris, M, (ed.) *School Mathematics and Work*; Falmer, London.

Moreland, J. (2001) *Archaeology and Text*; Duckworth, London.

Morley, N. (1999) *Writing Ancient History*; Duckworth; London.

Neugebauer, O. and Dunnington, G. W. (1936) "The History of Mathematics" *National Mathematics Magazine*; Vil. 11, Issue 1; pp 17 – 23.

Neugebauer, O. (Jun., 1951) "The Study of Wretched Subjects" *Isis*; Vol. 42, Issue 2; p. 111.

Neugebauer, O. (1952) *The Exact Sciences in Antiquity*; Princeton University Press; Princeton, New Jersey.

Neugebauer, O. (1980) "On the Orientation of Pyramids"; *Centaurus*; vol. 24; pp 1-3.

Newman, J. (ed.) (1956) The *World of Mathematics*; Allen and Unwin; London.

Numbers, R. (1985) "Science and Religion" *Osiris*; 2nd Series; Vol. 1; pp. 59 – 80.

Oldroyd, D. (1989) *The Arch of Knowledge: an Introductory Study of the History of the Philosophy and Methodology of Science*; New South Wales University Press; New South Wales.

Pearce Williams, L. (1975) "Should philosophers be allowed to write history?" *British Journal of the Philosophy of Science*; 26; 241-53.

Peet, T.E. (1923) *The Rhind Mathematical Papyrus: British Museum 10057 and 10058*, University of Liverpool Press, London.

Peet, T. E. (1923b) "Arithmetic in the Middle Kingdom" *Journal of Egyptian Archaeology*; Vol. 9; pp. 91 – 95.

Peet, T.E. (1931a) Review of Struve's edition of the Moscow Mathematical Papyrus; *Journal of Egyptian Archaeology*; vol. 17; pp. 154-60.

Peet, T.E. (1931b) "A Problem in Egyptian Geometry"; *Journal of Egyptian Archaeology*; vol. 17; pp 100-06.

Peet. T.E. (1931c) “Mathematics in Ancient Egypt”; *Bulletin of the John Rylands Library*; Manchester; vol. 15; pp 409-441.

Poincarè, H. (1914) *Science and Method*; Nelson; London.

Popper, K. (1974) “The Problem of Induction” http://dieoff.org/page126.htm; retrieved 17/05/04

Porter, T. M. (1996) *Trust in Numbers: The Pursuit of Objectivity in Science and Public Life*, Princeton University Press, Princeton New Jersey.

Teresi, D. (2002) *Lost Discoveries : The Ancient Roots of Science - from the Babylonians to the Maya*; Simon and Schuster, New York.

Turaiev, B. (1917) “The Volume of the truncated pyramid in Egyptian mathematics,” Ancient Egypt; London; pp 100-02.

Renfrew, A. C. (1973) *Before Civilisation: The Radiocarbon Revolution and Prehistoric Europe*; Pelican; Harmondsworth.

Renfrew, A. C. (1994a) “The Archaeology of Religion” in Renfrew, A. C. and Zubrow, E. B. *The Ancient Mind: Elements of Cognitive Archaeology*; Cambridge University Press; Cambridge.

Renfrew, A. C. (1994b) “Towards a Cognitive Archaeology” in Renfrew, A. C. and Zubrow, E. B. *The Ancient Mind: Elements of Cognitive Archaeology*; Cambridge University Press; Cambridge.

Rescher, N. (1987) *Scientific Realism*; Reidel; Dordrecht.

Richter, M. (1972) *Science as a Cultural Practic*; Schenkman Publishing; Guildford.

Ritter, J. (1989) “Chacun sa vérité: les mathématiques en Égypte et en Mésopotamie.” in Serres, M. (ed.) *Éléments d’histoire des sciences*; Bordas; Paris; pp. 39-61.

Ritter, J. (1995) "Measure for Measure: Mathematics in Egypt and Mesopotamia" in Serres, M. (ed.) *A History of Scientific Thought*; Blakwell; Oxford; pp 44 – 72.

Ritter, J. (2000) "Egyptian Mathematics" in Selin, H. *Mathematics Across Cultures : The History of Non-Western Mathematics*; Kluwer Academic Publishers; London; pp. 115 – 136.

Robins, G. and Shute, C. (1985) "Mathematical Bases of Ancient Egyptian Architecture and Graphis Art" *Historica Mathematica*; Vol. 12; pp 107-122.

Robins G. and Shute. (1987) *The Rhind Mathematical Papyrus*; British Museum Publications; London.

Rorty, R. (2000) "Kuhn" in Newton-Smith, W. H. (ed.) *A Companion to the Philosophy of Science*; Blackwell Publishers Oxford.

Rossi, C and Tout, C. (2002) "Were the Fibonacci Series and the Golden Section Known in Ancient Egypt?" *Historia Mathematica*; Vol. 29; pp. 101-113.

Sarton, G. (Sept., 1936) "On a Curious Subdivision of the Egyptian Cubit" *Isis*; Vol. 25, Issue 2; pp. 399 – 402.

Silver, B. L. (1998) *The Ascent of Science*; Oxford University Press; Oxford.

Silverman, D. (1975) "Fractions in the Abu Sir Papyri" *Journal of Egyptian Archaeology*; Vol. 61; pp. 248 – 249.

Sauneron, S (2000) *The Priests of Ancient Egypt*; Trans. Lorton, D; Cornell University Press; New edition. First published 1957.

Snow, C.P. (1962) *The Two Cultures and the Scientific Revolution*; Cambridge University Press, Cambridge.

Snow, C.P. (1964) *The Two Cultures: A Second Look*; Cambridge University Press, Cambridge.

Sorrel, T. (1994) *Scientism: Philosophy and the Infatuation with Science*; Routledge; London.

Spence, K. (Nov. 2000) "Ancient Egyptian Chronology and Astronomical Orientation of Pyramids"; *Nature* 408; pp 320 – 324.

Stove, D. (1982) *Popper and After: Four Modern Irrationalists*; Pergamon Press; Oxford.

Struve W.W. (1930) Mathematisher Papyrus des Staatlichen Museums der Schönen Künste in Moskau, QSGM, Abt. A: Quellen, Berlin.

Teresi, D (2002) *Lost Discoveries. The Ancient Roots of Modern Science - from the Babylonians to the Maya*; Simon & Schuster, New York.

Thomas, W. R. (1931) "Moscow Mathematical Papyrus, No. 14" Journal of Egyptian Archaeology; vol. 17; pp 50-2.

Turaiev, B. (1917) "The Volume of the truncated pyramid in Egyptian mathematics," *Ancient Egypt*; London; pp 100-02;

Unguru, S. (1975) "On the need to Rewrite the History of Greek Mathematics"; *Archive For The History of The Exact Sciences*; Vol. 15; pp. 67-114.

Unguru, S. (1979) "History of Ancient Mathematics. Some reflections on the state of the art." *Isis*; vol. 70, no. 254; pp. 556-65.

Unguru, S. and Rowe, D. (1981) "Does the Quadratic Equation have Greek Roots? A Study of 'Geometric Algebra', 'Application of Areas', and related problems (Part 1)"; *Libertas Mathematic*; Vol. 1; pp. 1-49.

Unguru, S. and Rowe, D. (1982) "Does the Quadratic Equation have Greek Roots? A Study of 'Geometric Algebra', 'Application of Areas', and related problems (Part 2)"; *Libertas Mathematic*; Vol. 2; pp. 1-62.

Vafea, F (2002) "The Mathematics of Pyramid Construction in Ancient Egypt" *Mediterranean Archaeology and Archaeometry* Vol 2:1; pp 111-124.

Van Der Waerden, B.L. (1954) *Science Awakening*; P. Noordhoff ltd; Groningen.

Van Der Waerden, B.L. (1980) "The (2:n) Table in the Rhind Papyrus" *Centarus*; Vol. 23; pp. 259 – 274.

Van der Waerden, B.L. (1983) *Geometry and Algebra in Ancient Civilisations*; Springer Verlag, Berlin.

Van Siclen, C. C. (1986) "Ostracon BM 41228: A Sketch Plan of a Shrine Reconsidered" *Göttinger Miszellen Beiträge zur ägyptologischen Diskussion*; Heft 90; pp 71 – 77.

Vetter Q. (1933) "Problem 14 of the Moscow Mathematical Papyrus" *Journal of Egyptian Archaeology*; vol. 19; pp 16 -18.

Vogel, K. (1930) "The truncated pyramid in Egyptian Mathematics"; *Journal of Egyptian Archaeology*; vol. 16; pp 242 - 49.

Von Daniken, E (1969) *The Chariots of the Gods*; London; Souvenir Press.

Weil, A. (1978) "History of Mathematics: Why and How" *Proceedings of the International Congress of Mathematicians.Helsinki, 1978*; pp 227 - 236.

Williams, M. (1998). "Feyerabend, Paul Karl". In E. Craig (Ed.), *Routledge Encyclopedia of Philosophy*. London: Routledge. Retrieved May 20, 2004, from http://www.rep.routledge.com/article/Q114

Wilson, A. M. (1995) *The Infinate in the Finite*; Oxford University Press; Oxford.

Wolpert, L. (1992) *The Unnatural Nature of Science*; Faber and Faber; London.

Appendix 1

The 2/n from the Recto of The Rhind Mathematical Papyrus

Divisor	Unit Fractions			
3	$\bar{\bar{3}}$			
5	3	15		
7	4	28		
9	6	18		
11	6	66		
13	8	52	104	
15	10	30		
17	12	51	68	
19	12	76	114	
21	14	42		
23	12	276		
25	15	75		
27	18	54		
29	24	58	174	232
31	20	124	155	
33	22	66		
35	30	42		
37	24	111	296	
39	26	78		
41	24	246	328	
43	42	86	129	301
45	30	90		
47	30	141	470	
49	28	196		
51	34	102		
53	30	318	795	
55	30	330		
57	38	114		
59	36	236	531	
61	40	244	488	610
63	42	126		
65	39	195		
67	40	335	536	
69	46	138		
71	40	568	710	
73	60	237	316	790
75	50	150		
77	44	308		
79	60	237	316	790
81	54	162		
83	60	332	415	498
85	51	255		
87	58	174		
89	60	332	415	498
91	70	130		
93	62	186		
95	60	380	570	
97	56	679	776	
99	66	198		
101	101	202	303	606

Taken from Gillings, R. (1972), *Mathematics In The Time of The Pharaohs*, MIT Press, Cambridge MA; p. 50. All unit fractions are written without overbars for simplicity.

Appendix 2

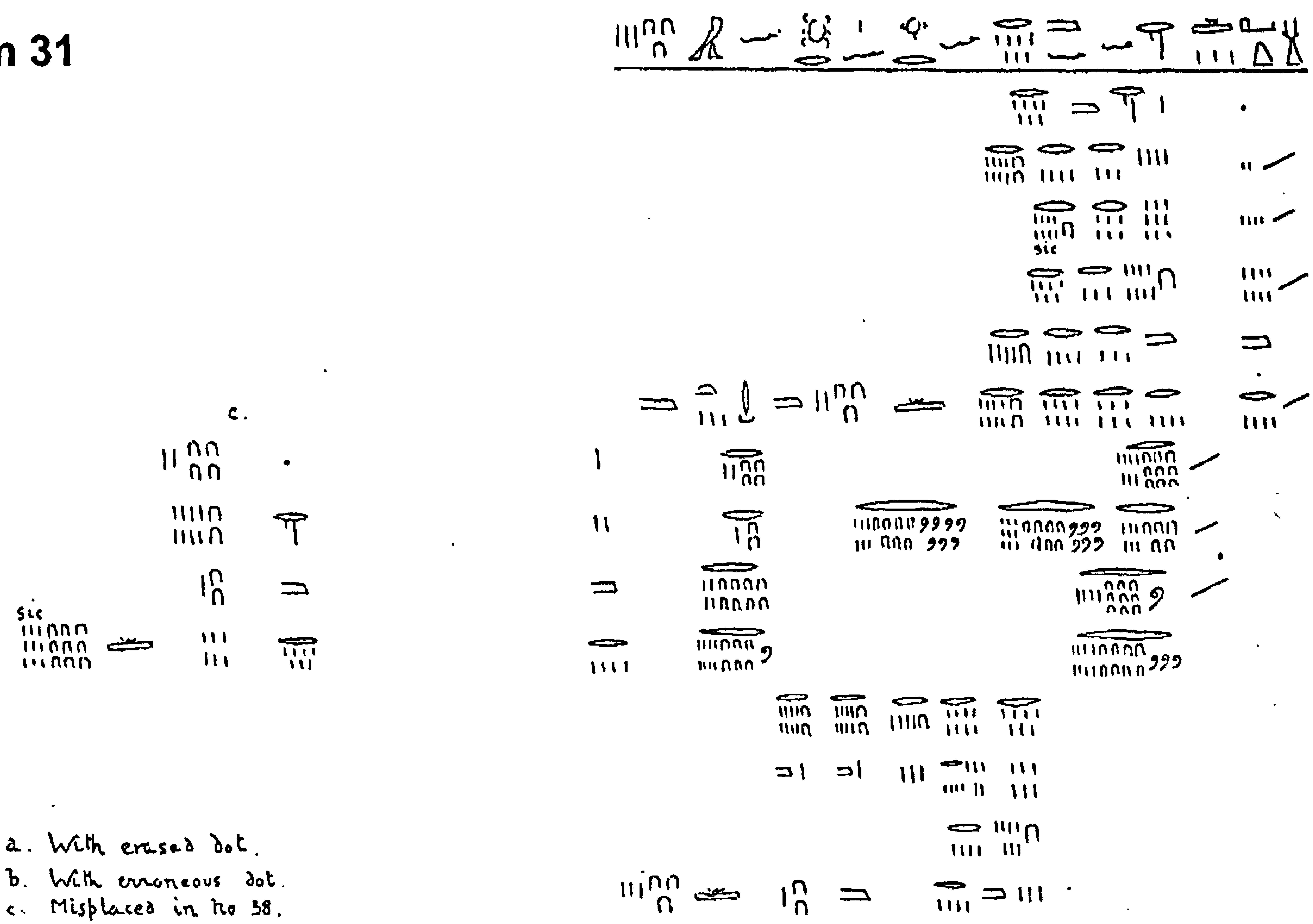
Hieroglyphic Texts of Problems from the Rhind Mathematical Papyrus Translated in this Thesis

Reproduced from Peet, T.E. (1923) *The Rhind Mathematical Papyrus: British Museum 10057 and 10058*, University of Liverpool Press, London

Problem 24

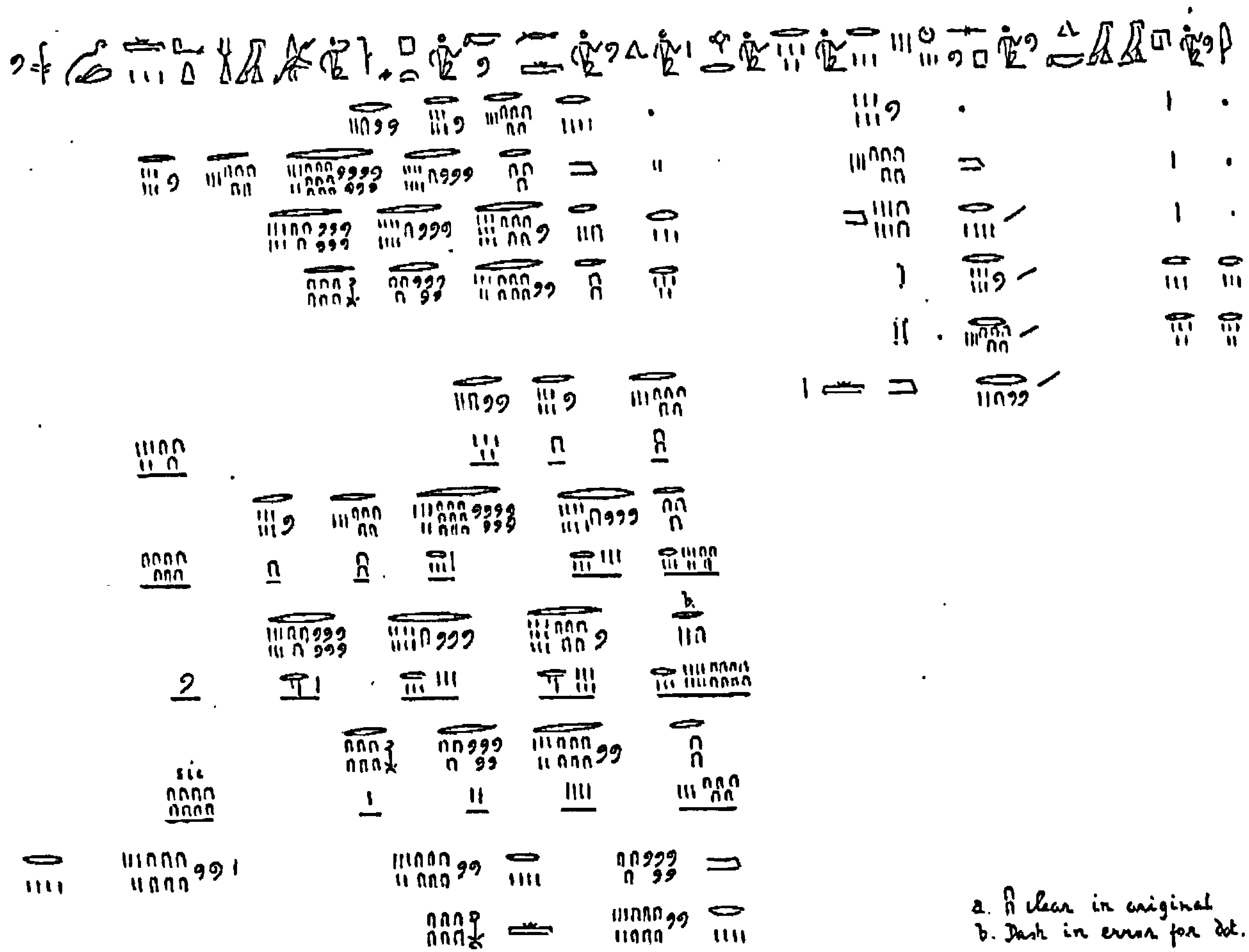
Problem 25

Problem 31



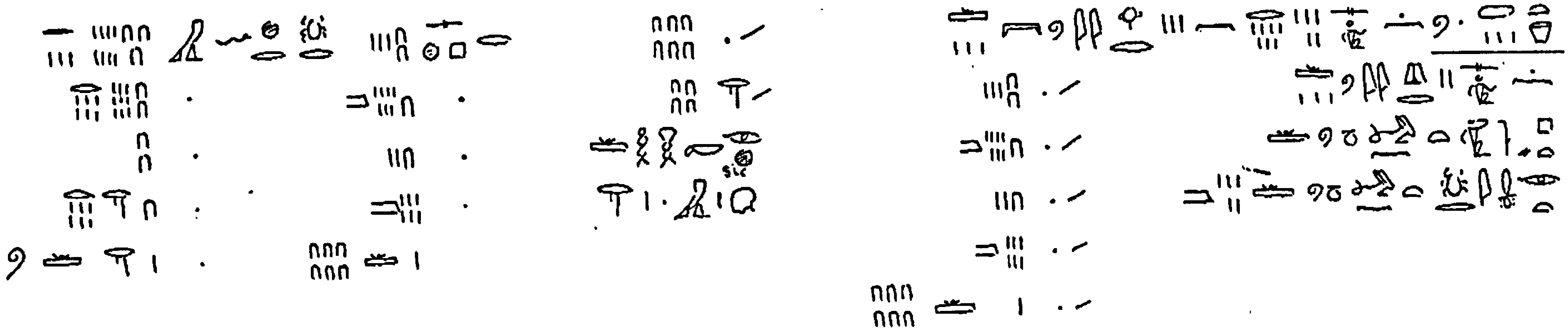
a. With erased dot.
b. With erroneous dot.
c. Misplaced in no 38.

Problem 36

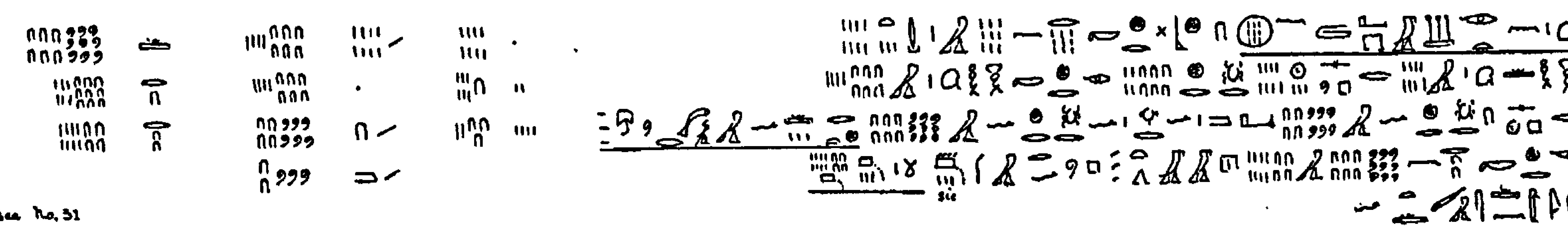


a. [sign] clear in original
b. Dash in error for dot.

Problem 40



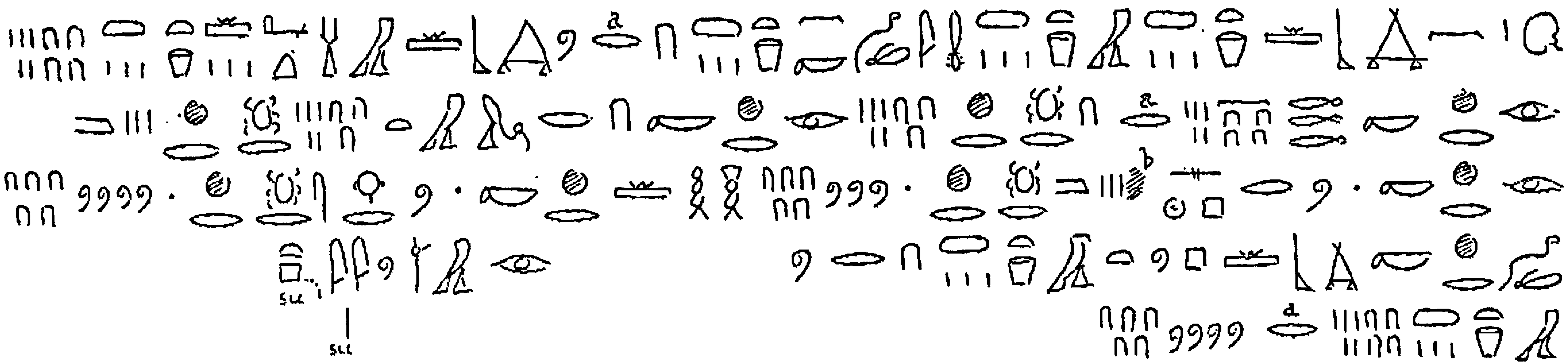
Problem 41



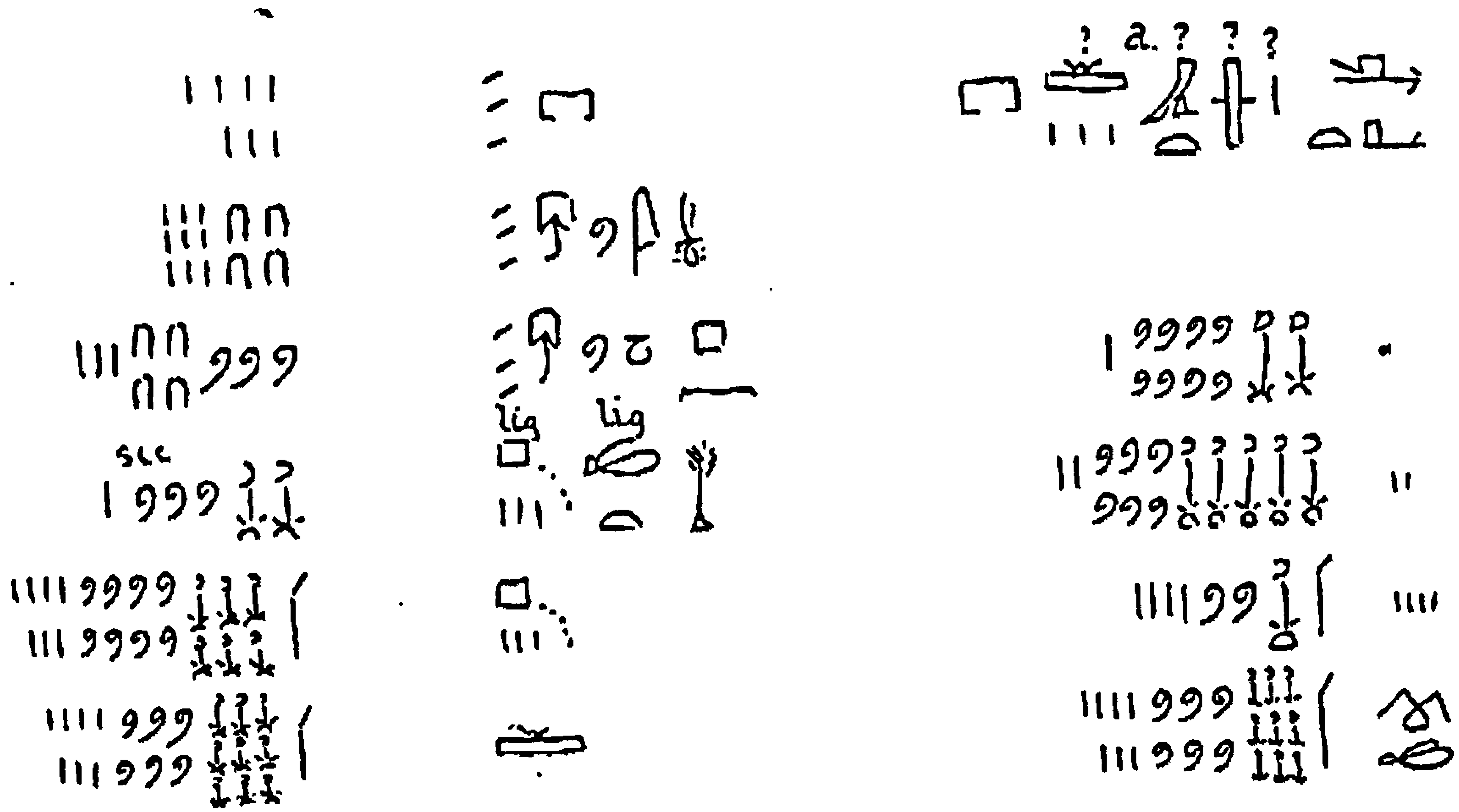
a. For group misplaced here see No. 31
b. Added in black.

Problem 42

Problem 72



Problem 79

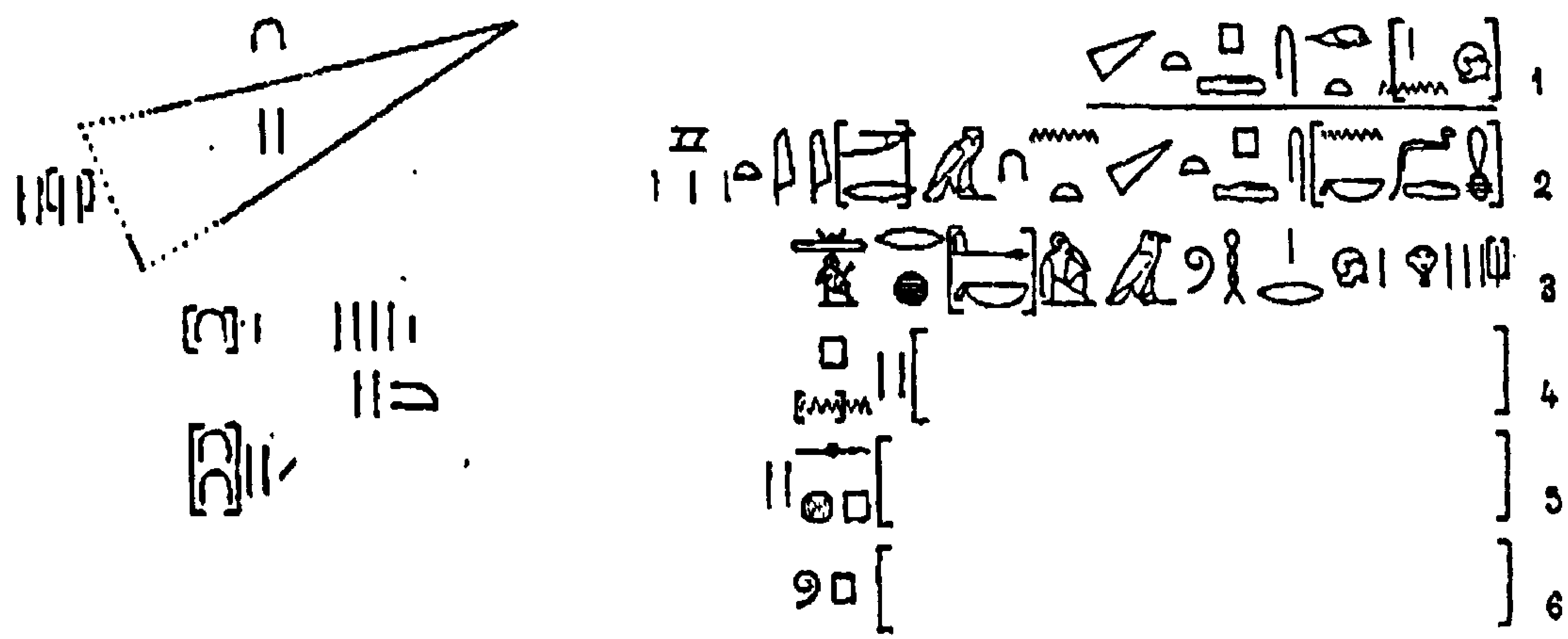


Appendix 3

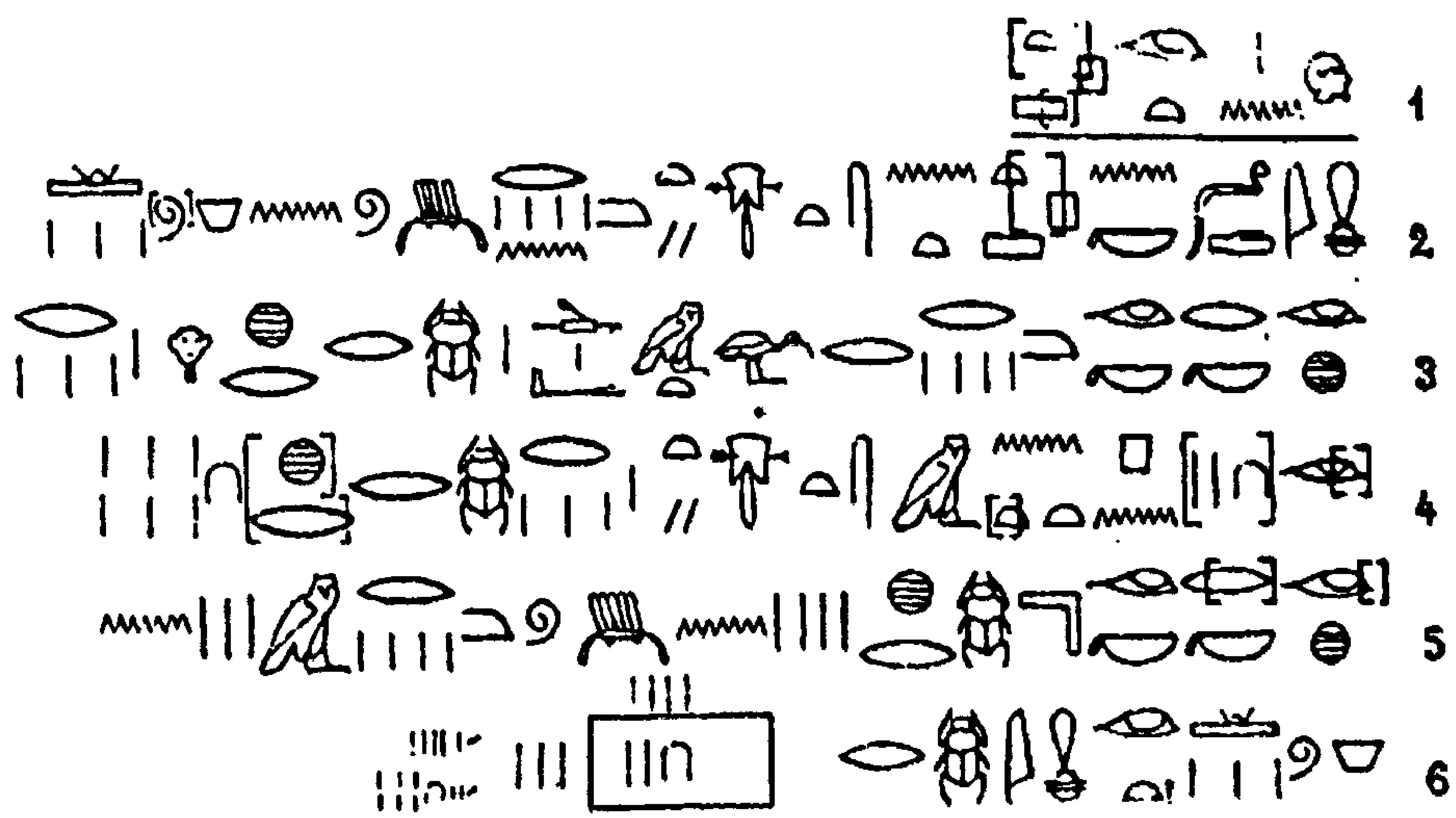
Hieroglyphic Texts of Problems from the Moscow Mathematical Papyrus Translated in this Thesis

Struve W.W. (1930) Mathematisher Papyrus des Staatlichen Museums der Schönen Künste in Moskau, QSGM, Abt. A: Quellen, Berlin.

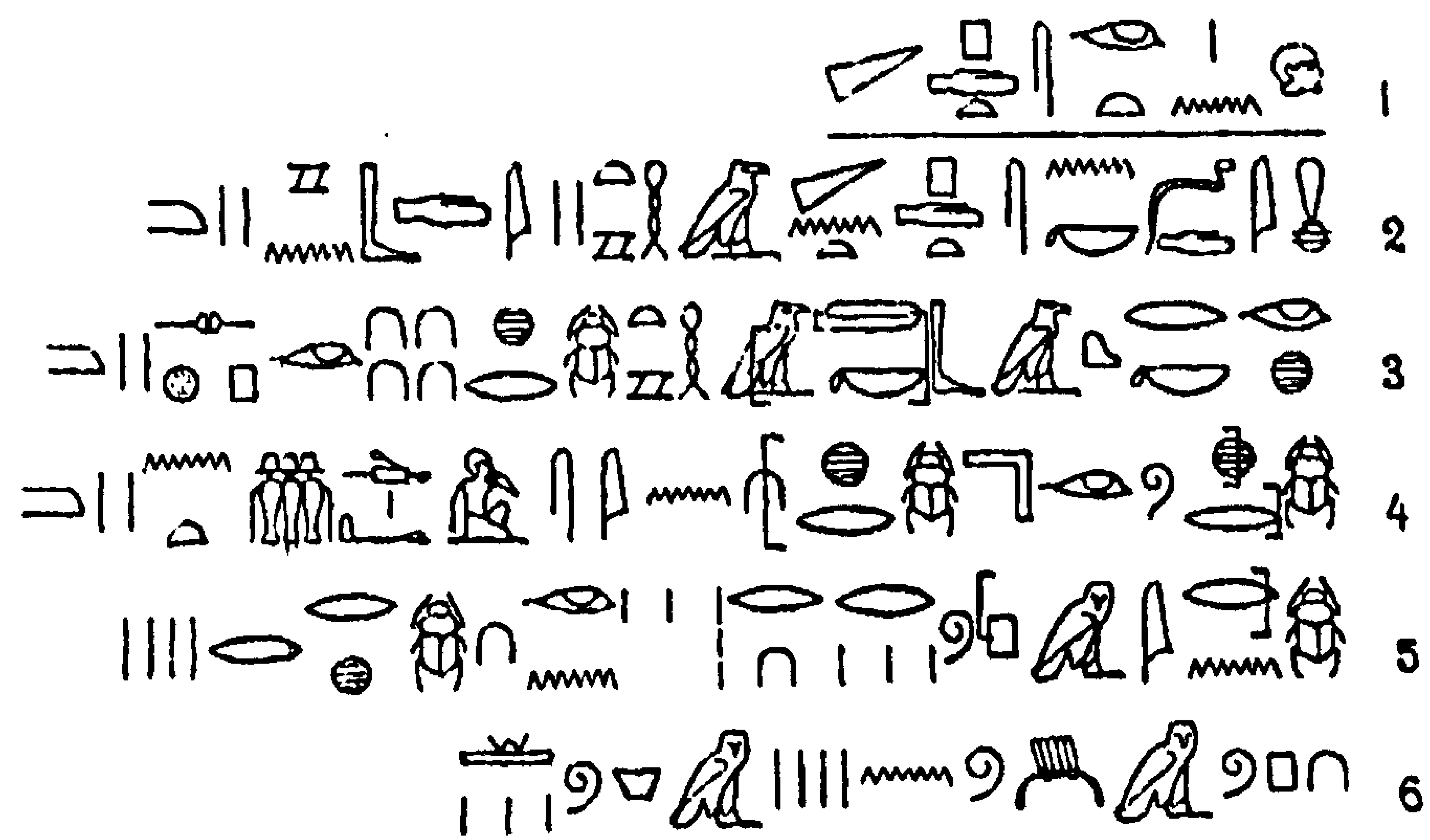
Problem 4



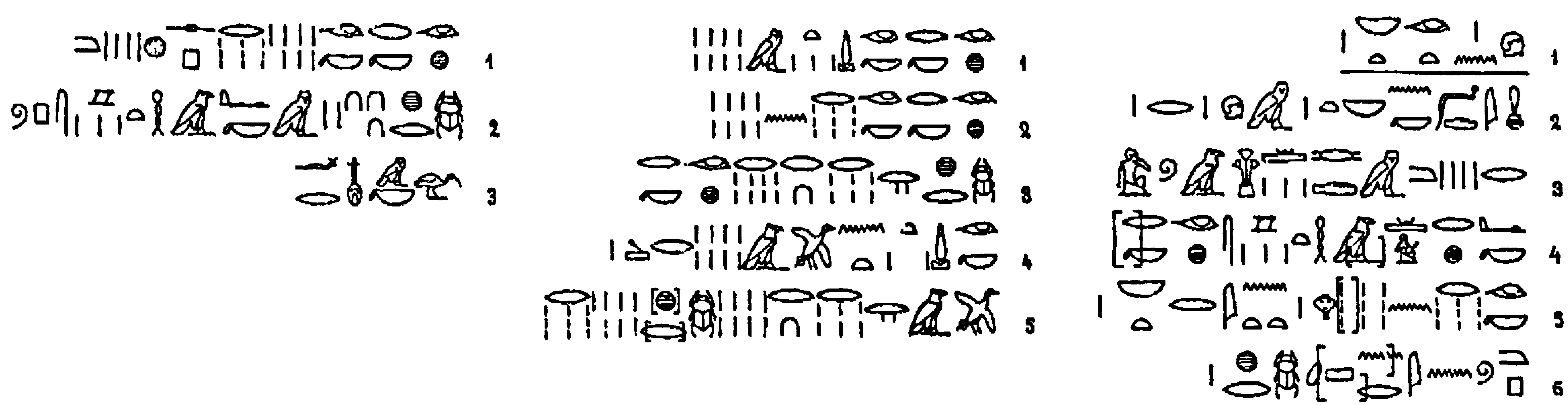
Problem 6



Problem 7

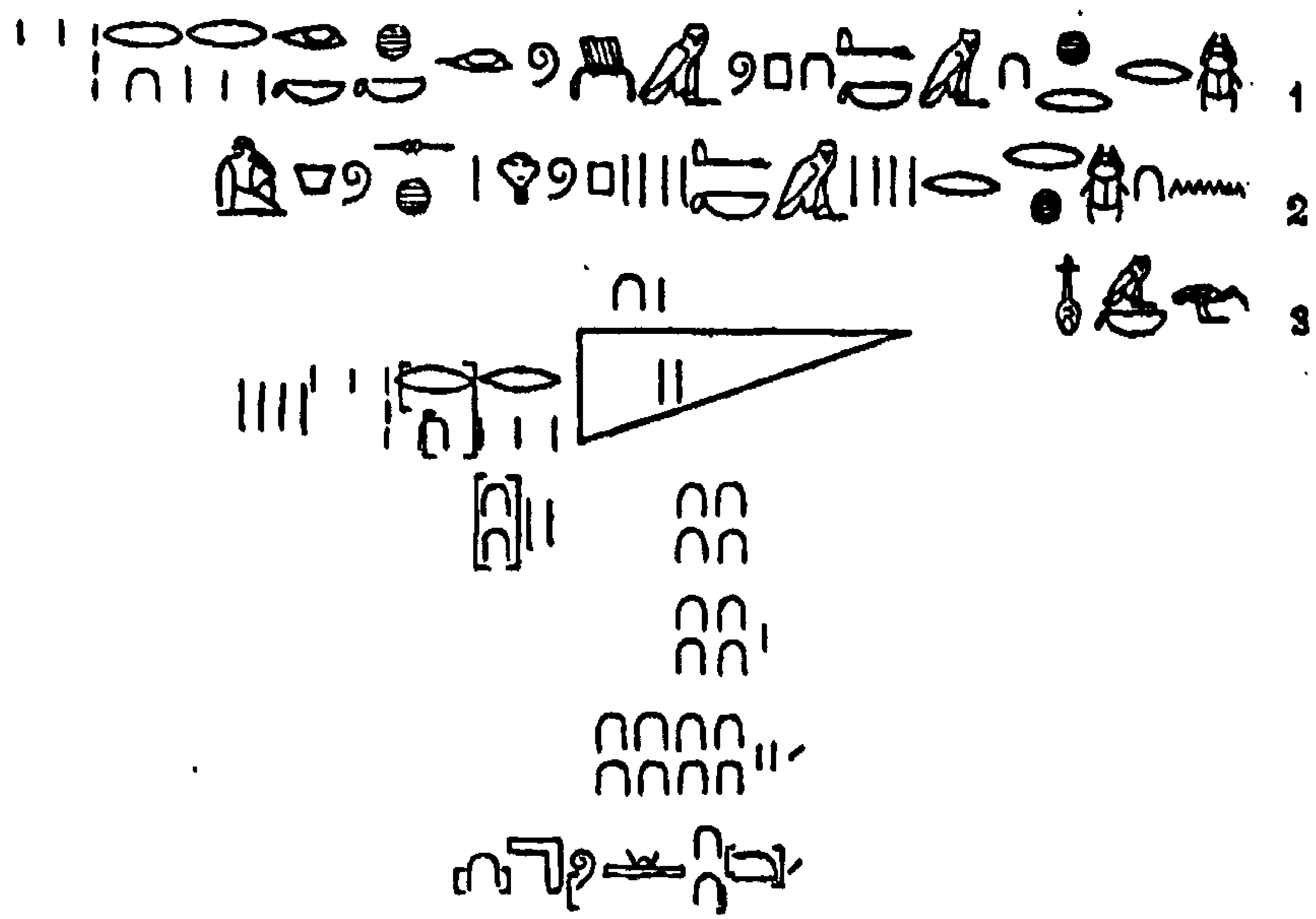


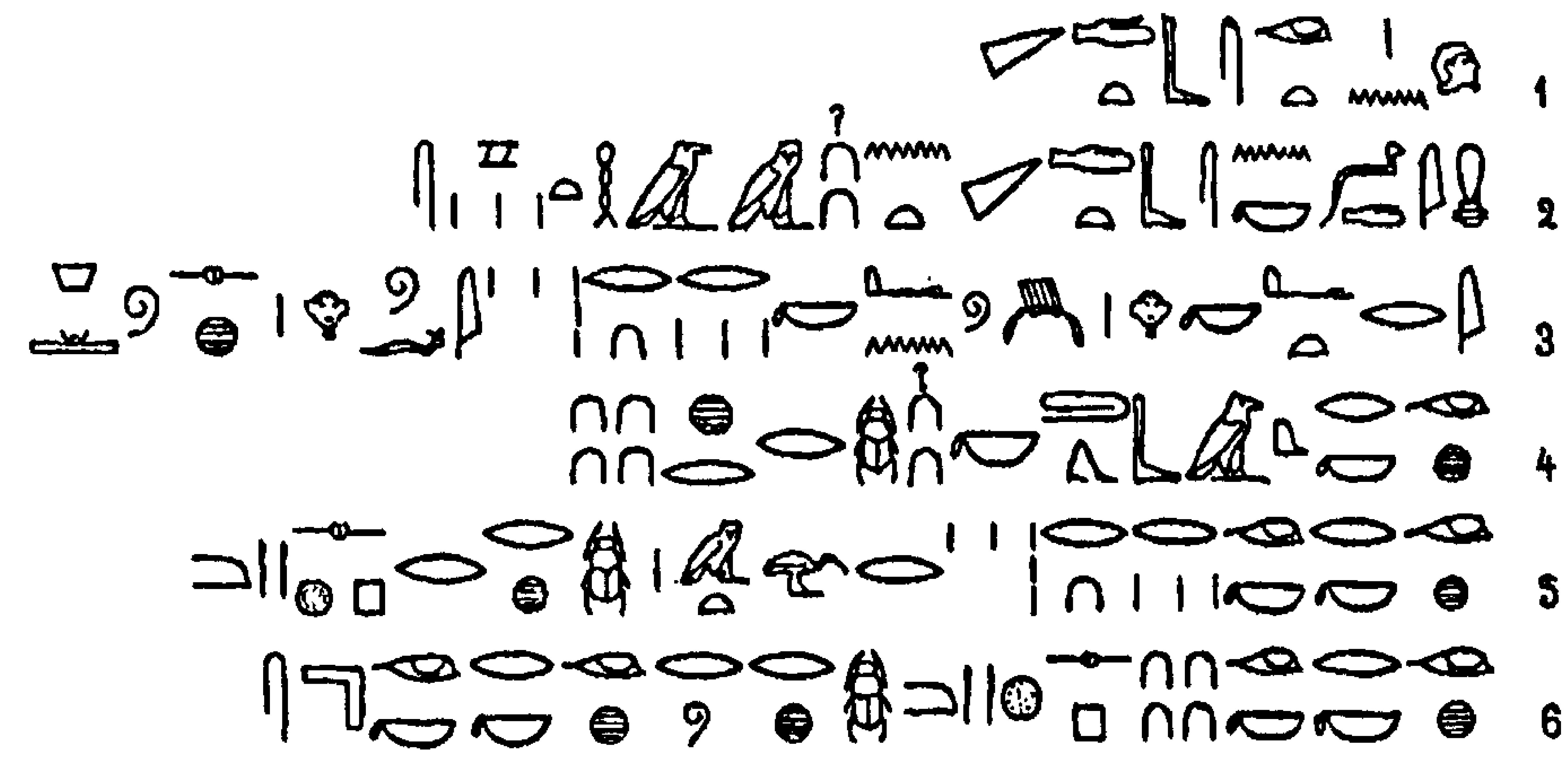
Problem 10



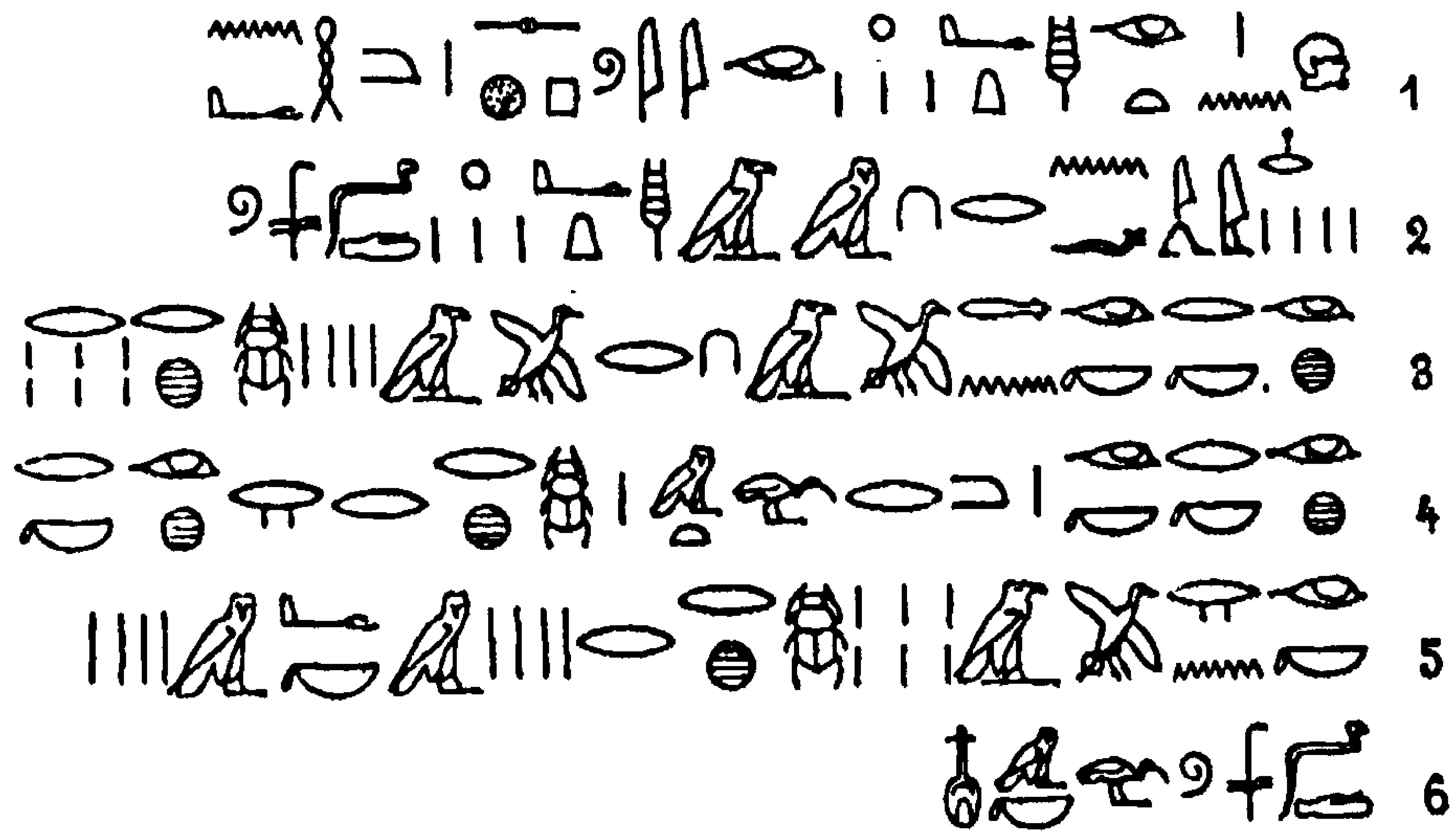
Problem 14

Problem 17

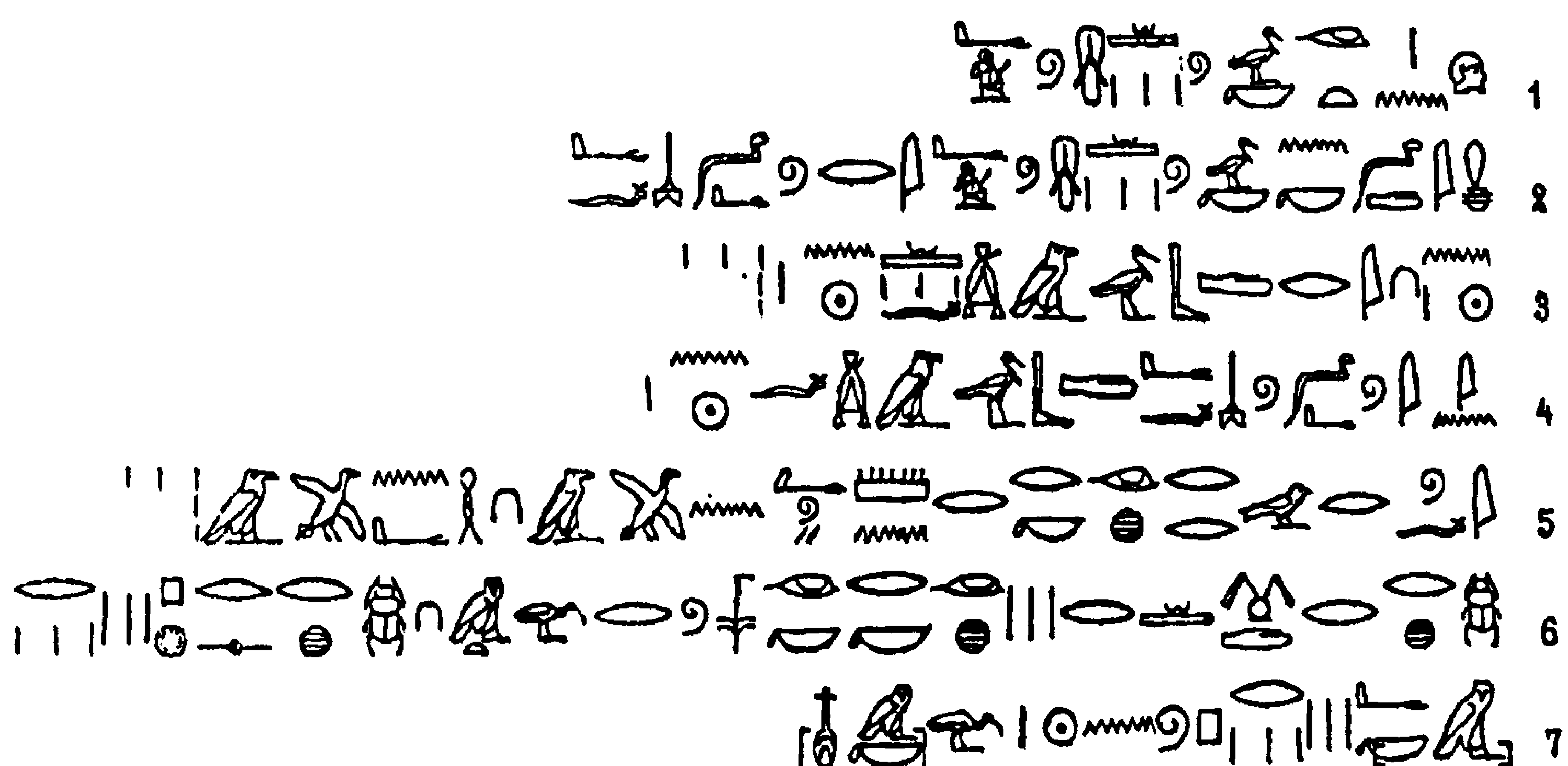




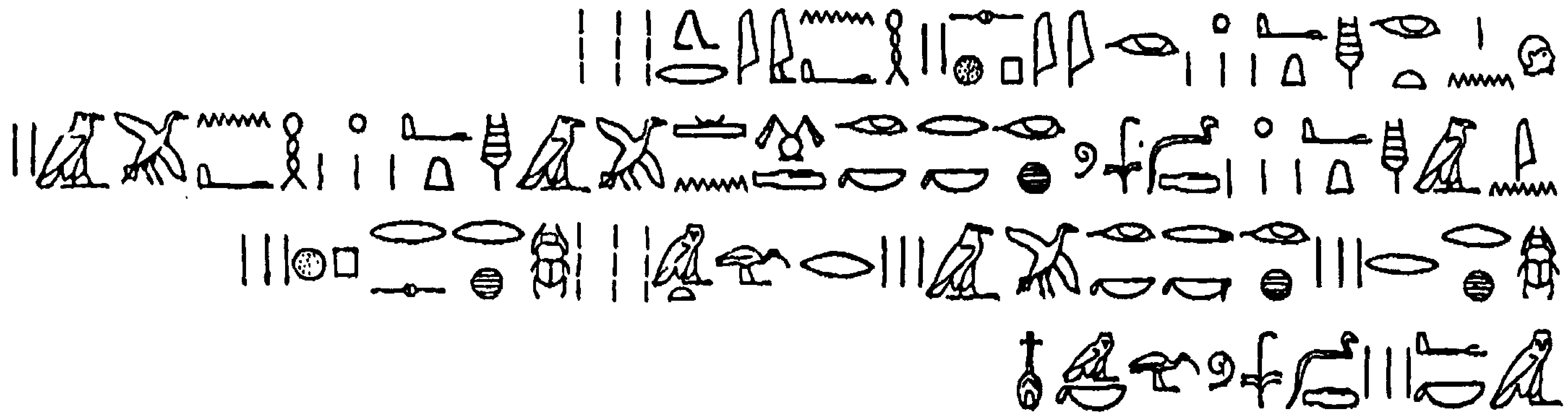
Problem 19



Problem 23



Problem 25



1
2
3
4