Labour Contract Theory : An Examination

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T.S. Worrall   Labour Contract Theory : An Examination

Abstract

The purpose of this research is to review and extent that model of the labour market which has become known as implicit contract or labour contract theory. The extensions examined in this thesis are persued at three levels. First a general equilibrium model of a labour contract economy is examined. Second the optimal labour contract when participants are asymmetrically informed is studied. Finally the introduction of imperfectly informed agents into the model is considered.

Chapter 1 provides an introduction to labour contract theory. The assumptions of the theory are delineated and contrasted with the assumptions of other more standard models. It is shown that the theory presented by Azariadis-Baily-Gordon is both eclectic and descriptive because it offers, essentially no new insights. However their presentation does provide a basis upon which to build and this is done in chapters 2 - 4. Chapter 1A generalizes the model examined in chapter I.

Chapter 2 presents a general equilibrium model of a labour contract economy. A labour contract equilibrium is defined and shown to exist. It is also shown that although agents will typically have an incentive to enter into labour contracts this will not necessarily be in the interests of society.

Chapter 3 is perhaps the key chapter of this thesis. It examines the optimal labour contract when the employer and employee have asymmetric information. It is commonly observed that the importance of labour contracts derives from the inability of agents to buy and sell labour forward. This, however is only justified if potential buyers and sellers are asymmetrically informed. It is shown that the optimal labour contract under asymmetric information is productively inefficient. In particular it is shown that when the employee is better informed than the employer there will be underemployment.

Chapter 4 examines the optimal labour contract when there is imperfect information. In the first part of chapter 4 the indexation of wages to prices is examined. It is shown how the degree of wage indexation depends upon the employers and the employees income elasciticity of demand, the employees coefficient of relative income risk aversion and the elasticity of the marginal product. The second part of chpater 4 tentatively suggest some routes whereby aggregate information can affect output and employment.

Preface

The purpose of this research is to review and extend that model of the labour market which has become known as implicit contract or labour contract theory. The extensions examined in this thesis are persued at three levels. First a general equilibrium model of a labour contract economy is examined. Second the optimal labour contract when participants are asymmetrically informed is studied. Finally the introduction of imperfectly informed agents into the model is considered.

The theory of labour contracts was developed independently by C. Azariadis, M.N. Baily and D.F. Gordon nearly ten years ago. The development of this theory was to a large extent due to a dissatisfaction with the treatment of the labour market by general equilibrium and auction market theories. It has been a widely held belief, both by microeconomists and macroeconomists that the labour market is somehow different from most other conventional commodity and financial markets. Both Edgeworth in the 1880's and Hicks in the 1930's devoted considerable attention to this problem. Hicks, in particular felt that the equation of the real wage and the marginal product of labour was not a sufficient condition for efficiency within the labour market. Hicks suggested that certain 'social' influences might be important. It is the purpose of labour contract theory to elucidate and examine these additional conditions of efficiency without however straying from the confines of the traditional neo-classical paradigm.

The theory of labour contracts is charmingly simple and can be easily summarized. Suppose there are just two time periods $t = 0,1$ and suppose that at time $t = 1$ there are S possible states of nature $s = 1 \ldots S$. Then consider an employer and employee who plan to trade labour at time $t = 1$. If either the employer or the employee are not risk netural, then it will be in their interest to negotiate a labour contract, at time $t=0$. The contract will specify how much labour the employee is to supply in each state of nature, $\ell(s)$ and the wage he is to receive in each state $w(s)$. Of course such a contract will be irrelevant if labour can be sold forward contingent upon the state of nature s. However the labour market is perhaps the most obvious case of a market where commodities cannot be sold forward. Anyone who has tried to borrow against his or her future labour income will vouch for this. To be more specific suppose the employee derives utility u, from a wage income w, according to a strictly concave function $u(w)$. Then, if the employer is risk neutral, a contract that offers a fixed wage income $\bar{w}$, will always dominate a contract with a variable wage income $w(s)$. This is essentially the Azariadis-Baily-Gordon result.

Chapter 1 provides an introduction to labour contract theory. The assumptions of the theory are delineated and contrasted with the assumption of other more standard models. It is shown that the theory presented by Azariadis-Baily-Gordon is both eclectic and descriptive because it offers essentially no new insights. However their presentation does provide a basis upon which to build and this is done in chapters 2 - 4. Chapter 1A generalizes the model examined in chapter 1.

Chapter 2 presents a general equilibrium model of a labour contract economy. A labour contract equilibrium is defined and shown to exist. It is also shown that although agents will typically have an incentive to enter into labour contracts this will not necessarily be in the interests of society.

Chapter 3 is perhaps the key chapter of this thesis. It examines the optimal labour contract when the employer and employee have asymmetric information. It has already been emphasized that the importance of labour contracts derives from the inability of agents to buy and sell labour forward. This, however, is only justified if potential buyers and sellers are asymmetrically informed. It is shown that the optimal labour contract under asymmetric information is productively inefficient. In particular it is shown that when the employee is better informed than the employer there will be under-employment.

Chapter 4 examines the optimal labour contract when there is imperfect information. In the first part of chapter 4 the indexation of wages to prices is examined. It is shown how the degree of wage indexation depends upon the employers and the employees income elasticity of demand, the employees coefficient of relative income risk aversion and the elasticity of the marginal product. The second part of chapter 4 tentatively suggest some routes whereby aggregate information can affect output and employment.

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CHAPTER 1

An Introduction to Labour Contract Theory

Section 0 : Introduction

It is almost ten years since the first articles developing the theory of labour contracts were presented and published. Azariadis (1975) was presented at Oslo in August 1973. Baily (1974) was initially accepted by 'The Review of Economic Studies' in February, 1973 and D.F. Gordon (1974) was presented at Rochester in April, 1973. The theory offered by these papers is charmingly simple and novel. Consider an employee who derives utility u, from a wage income w, according to a concave function $u = u(w)$. Suppose that the wage income w fluctuates for some reason so that it is a random variable. Then the employee would be better off if he received the certain mean or average wage income $\bar{w}$ instead of the random wage income w. The employer will be perfectly willing to pay this fixed wage income $\bar{w}$ if he is risk neutral. To put the point another way the employer is able to lower his labour costs and still offer the worker the same level of utility. From this analysis it appears that labour contract theory rationalizes a fixed wage divorced from the conventional theory of supply and demand. This would seem to open up a new vista for the theory of employment or unemployment.

In the subsequent years some fifty to sixty articles elaborating the theory have been published.[1] This discussion has provided a considerable and valuable contribution to the theory of the labour market. Nevertheless the literature has often been confused and repetitive, neither apparently has the theory been received into the corpus of labour economics.[2] Thus it is necessary to provide a coherent introduction to the theory of labour contracts to clear away some misunderstandings and irrelevances. This chapter does not present a survey or a review of the literature rather the theory of labour contracts is presented in its simplest and most abstract form.

The chapter is organized as follows. Section 1 traces briefly some historical detail. The dissatisfaction with previous theories of unemployment is emphasized and the aims or goals of labour contract theory discussed. Section 2 presents a conveniently simplified but hopefully insightful example. Various modes of analysis are examined in order to highlight the method and significance of labour contract theory. Finally in section 3 some concluding comments are offered.

Section 1 : Antecedence

This section will consider the historical antecedence of labour contract theory and examine some of the failure of competing theories of unemployment. First the tenets of labour contract theory are traced through a brief review of the relevant economic thought. Second alternative theories of enemployment are examined to illustrate the aims and ultimately judge the results of labour contract theory.

Labour economics began with Adam Smith.[3] Before Adam Smith the mercantilist believed wages 'ought' to be low but they had no theory of wages. Similarly the physiocrats had no theory of wages though they did make a clear distinction between labourers and capitalists and it is this distinction that is the starting point for the theory of labour contracts.

In the search for the historical foundations of labour contract theory many authors have quoted Adam Smith. Yet it is useful to remember Alexander Gray's dehortive that Smith has no

"tenable theory of wages: on the other hand, he suggests many trains of thought, and there are indeed few subsequent theories of wages which cannot appeal for support to some passage".

Despite this warning it does seem worthwile to give one quote from the "Wealth of Nations" in which Smith devoted a whole chapter to occupational wage differentials. He concluded that the main reasons for wage differentials were

"first. The agreeableness or disagreeableness of the employment themselves; secondly the easiness and cheapness or difficulty and expense of learning them; thirdly the constancy or inconstancy of employment in them; fourthly the small or great trust which must be reposed in those who exercise them; and fifthly the probability or improbability of success in them".

It is true to say that much of this is more relevant to human capital theorists, nevertheless the third and fourth points are directly applicable and will be the subject of study in subsequent analysis.

In the nineteenth century Thorton recognized the general fixity of wages in his essay 'Paper Credit' in 1802.

"A fall (in price) arising from temporary distress will be attended probably with no correspondent fall in the rate of wages; for the fall in price, and the distress will be understood to be temporary, and the rate of wages we know, is not so variable as the price of goods".

Of the nineteenth century economists Edgeworth[4] devoted most attention to the issue of labour contracts. As suited the time his main analysis focussed upon unionization, but he explicitly recognized the role of human capital and posed the problem well as a search for[5]

"a straightforward answer to an abstract question what is the effect of combinations on contract in an otherwise state of perfect competition as here supposed".

This is exactly the question to which labour contract theory tries to answer. How are wages and employment determined inside a firm which is subject to the outside forces of competition. It is in this context that the dichotomy between the long run and the short run is most apparent,in the short run labour is immobile.

It is in this vein that Hicks writes in his 'Theory of Wages' [6]

" .. the labour market is ... by nature ... a very special kind of market, a market that is likely to develop 'social' as well as purely economic aspects. The conditions for this to happen are: (1) that the worker should be free to change his employer ... (2) that employment should be regular, i.e. non-casual so that there is a presumption that the relation between the employer and .. his employees will be a continuing relation ... For the purely economic correspondence between the wage paid to a particular worker and his value to the employer is not a sufficient condition of efficiency; it is also necessary that there should not be strong feelings of injustice about the relative treatment of different employees and there should be some confidence about fair treatment over time ... Compromise is necessary with the result that wage rates are more uniform both between workers and over time than they would be if the labour market worked like a commodity market."

It is precisely the purpose of labour contract theory to encompass these 'social' aspects of the labour market into the economic sphere. Perhaps the clearest statement of the risk sharing role in the labour contract is to be found in Knight's "Risk, Uncertainty and Profit". In examining how the introduction of uncertainty affects the internal organization of the firm, Knight states

" ... the most fundamental change of all in the form of organization (is) the system under which the confident and venturesome (employers) 'assume the risk' or 'insure' the doubtful and timid (employees) by guaranteeing the latter a specified income in return for an assignment of the actual results".

To summarize it has been widely recognized that the labour market is dissimilar to a commodity market in many ways and that this induces wages to be more stable than they might otherwise have been.

Prior to 1973 there were basically two competing theories of unemployment. These can be conveniently classified as the new microeconomic theory associated with Phelps et al (1970) and the neo-Keynesian disequilibrium theory developed by Barro and Grossman (1971). There are a great variety and diversity of articles presented in the Phelps volume. The Lucas and Rapping paper however is somewhat different from the rest. They start from the presumption that participants in the labour market are sufficiently well informed so as to be able to substitute leisure for labour when the real wage is relatively low. This will tend to stabilize the real wage though as an explanation of unemployment and particularly youth unemployment it seems unsatisfactory. Nevertheless section 2 will show how closely this explanation is related to the explanation offered by labour contract theory.

However for the most part the new microeconomics emphasize the dislocated nature of the labour market. In particular if the employee is unable to distinguish real from relative price changes he may be "fooled" into taking more leisure even though the real wage is unchanged. Considerable sophistication can be introduced into these models by allowing employees to search among potential employers trading-off increased serach costs against possible increased wage offers.

Neo-Keynesian disequilibrium theory attempts to model quantity constrained equilibria and hence validate Keynesian unemployment theory. To this end the Walrasian price adjustment mechanism is replaced by the more Marshallian assumption that prices and wages are fixed in the short run. This rationalizes the use of the consumption function and investment accelerator in macroeconomics. If prices and wages are fixed at a level where there is excess supply in both the product and labour market then the usual Keynesian comparative statics results apply.

The usefulness of both theories is undoubted. Nevertheless both have been the subject of substantial reservations and criticism. It was the nature of these criticisms that stimulated the research and development of labour contract theory. The new-microeconomics for example adopted a partial-partial equilibrium approach, that is they considered only one side of the market, typically the sellers side of the labour market. Thus these theories tend to suggest that voluntary quits rise as real wages fall even though this is empirically unjustified. Equally search theories and more generally explanations that assume agents misperceive real and nominal variables ignore a wealth of economy-wide information that is readily available to employees. R.J. Gordon in particular has questioned this assumption, suggesting that agents can and do possess and process a considerable amount of economy wide information. At the same time the fix-price assumption of disequilibrium theory needs some rationale. It is not easy to see how prices can remain fixed if supply is not equated to demand, even in the short run if agents are assumed to behave rationally.

Given these criticisms labour contract theory seems to offer a convenient explanation. The theory lays equal stress on both sides of the labour market and the risk-sharing activities of employer and employee reconciles a fixed wage with perfect competition. It is the purpose of the next section to examine a simple model in which the success of labour contract theory in satisfying these criticisms can be assessed.

Section 2 : Model

Suppose that there is some country or economy, say Moravia, with just two agents and just two goods. Moravia is famed for its tea and the sweat or effort of its menfolk. These are the only two goods in the economy. Of the two agents, agent a, called Ada is an employer, she supplies no effort but drinks tea copiously. Her utility function u^a is linear in her consumption of tea or profits π, that is $u^a = \pi$. The other agent, agent b, called Bill is an employee, he drinks r units of tea and supplies ℓ units of labour or effort to Ada. Bills utility function is $u^b = \log(1 + r - \frac{1}{2}\ell^2)$. This utility function belongs to the class of functions $u = v(r - h(\ell))$ where v is concave and h is convex. This class has some nice properties : (i) the marginal disutility of labour is independent of consumption (ii) labour is neither a normal nor inferior good. The importance of this class of utility functions will become apparent.

Ada is able to use or transform Bill's effort into the production of output y. of tea. Neither Ada or Bill have any endowments of tea so the only way for Ada and Bill to survive is to combine as a firm. Output however depends not only on Bills effort but also upon a random variable θ. In particular output $y = \theta\ell$ so that labour exhibits constant but random returns to scale. More generally the production function may be written $y = f(\ell, \theta)$ where f is concave in ℓ . The amount of labour input Bill decides to supply is chosen after the result of the random process is known, that is once the state of nature is known. The random process θ may reflect any source of uncertainty in the relationship between the employer and the employee. For example θ might be the state of the weather. It is assumed that θ has

has a continuous probability density function $g(\theta)$ and where it is convenient this will be assumed to be normal with a unit mean and variance σ^2. Since both Ada and Bill have to evaluate risky prospects it is useful to assume they both obey the Von-Neumann-Morganstern axions so that their expected utility functions are linear in probabilities. The model is summarized in equation (1.1)

$$U^a = E\pi = E(y-r), \qquad U^b = Eu^b = E\log(1+r-\tfrac{1}{2}\ell^2)$$
$$y = \theta\ell \qquad \theta \sim N(1,\sigma^2) \tag{1.1}$$

where E is the expectations operator.

There are a number of possible ways of analysing a model of this type and the labour contract approach is only one of them. Four methods will be considered below in order to relate labour contract theory to some more conventional analyses. The four methods are a) the market equilibrium approach associated with Arrow and Debreu, b) the bargaining solution that has been suggested by Nash, c) a more sophisticated market equilibrium approach in the tradition of Radner or more generally auction market theorists such as Friedman and , d) the labour contract approach developed by Azariadis, Baily and Gordon.

Market Equilibrium

The market equilibrium approach has been fully specified by Arrow and Debreu. Perhaps more relevant to the issue at hand the Lucas and Rapping paper in Phelps (1970) is in the same basic tradition. There are two key assumptions upon which this approach is founded. These are a) that every agent acts competitively and, b) there is a complete set of contingent claims markets. If agent act competitively then each agent will take the set of prices determined by the market as a vector of parameters. In this context the price taking hypothesis may be justified by assuming that Ada and Bill are simplv representative of a large number of like-minded agents. The next question is how many prices are there? The market equilibrium economist will immediately point out that there are not just two goods in this Moravian economy. If there are a continuum of possible realizations of the random variable θ, that is there are a continuum of possible states of nature then there are a double continuum of goods. Effectively tea produced or effort supplied in one particular state of nature is a different commodity from tea produced or effort supplied in any other. The assumption that there is a complete set of contingent claims markets means there is one market for each and every commodity that is a double continuum of markets. Each agent will then decide how much tea or effort to buy or sell in each market that is contingent upon the state of nature. Thus each agent will be able to allocate his wealth to any state he wants constrained only by market prices.

Consider how each agent actually will allocate his or her wealth across states of nature. Ada chooses how much labour to demand from Bill and how much tea to drink in each state of nature in order to maximize expected utility. Formally Ada's problem is

P.1.1 $$\max_{y^a,\ \pi^a,\ \ell^a} \quad E\pi^a$$

s.t $$p(\pi^a - y^a) + \omega\ell^a = 0 \tag{1.2}$$

$$y^a(\theta) = \theta\ell^a(\theta) \quad \forall \quad \theta \tag{1.3}$$

where E is the expectations operator over the random variable

$\pi^a(\theta)$ is Ada's demand for tea in state θ

$y^a(\theta)$ is Ada's supply or production of tea in state θ

$p(\theta)$ is the price of tea in state θ

$\ell^a(\theta)$ is Ada's demand for labour in state θ

$\omega(\theta)$ is the price of labour in state θ

and π^a, y^a, p ℓ^a, are the associated infinite dimensional vectors of the demand for tea, supply of tea, price of tea, demand for labour, and price of labour in every state.

Thus Ada's problem, P.1.1 is to maximize her expected consumption of tea subject to two constraints. Equation (1.3), the second constraint, simply relates Ada's demand for labour to her supply or output of tea in each state, in other words, it is her production constraint. Equation (1.2) is more fundamental. In order to understand this constraint it is simplest to conceptually separate Ada's demand for and supply of tea. For example for a promise to produce $y^a(\theta)$ units of tea if state θ occurs Ada will receive p (θ) per unit. Thus for each possible state Ada's total revenue for her planned production of tea is py^a. This revenue is used in two ways, to finance her planned purchases of consumption $p\pi^a$ and to pay the wages ω of her planned labour input ℓ^a. It is important to notice that Ada's profit in any particular state $p(\theta)y^a(\theta) - \omega(\theta)\ell^a(\theta)$ need not equal the value of her consumption in that state $p(\theta)\pi^a(\theta)$, this is because Ada can trade both tea and labour contingent upon any state.

To solve P.1.1 let α be the Lagrangian multiplier for equation (1.2) and $\alpha(\theta)$ be costate variables for the set of constraints defined in equation (1.3). The first order conditions are:-

$$g(\theta) = \alpha p(\theta), \quad \alpha\omega(\theta) = \alpha(\theta)\theta, \quad \alpha(\theta) = \alpha p(\theta), \ \forall\, \theta \qquad (1.4)$$

From equation (1.4) it is readily apparent that the marginal product of labour, which is simply θ is equal to the real wage rate $\omega(\theta)/p(\theta)$ in every state. It is also apparent that the price of tea in any particular state is proportional to the probability of the state occurring.

Bill has a similar problem he must choose how much tea to demand and how much labour to supply in each state so as to maximize his expected utility. Formally Bill's problem is

$$\text{P.1.2} \qquad \max_{r^b \;\; \ell^b} \qquad E \log(1 + r^b(\theta) - \tfrac{1}{2}(\ell^b(\theta)^2)$$

$$\text{s.t} \qquad pr^b - \omega\ell^b = 0 \qquad (1.5)$$

where $r^b(\theta)$ is Bill's demand for tea in state θ

$\ell^b(\theta)$ is Bill's supply of labour in state θ

and r^b and P^b are the associated infinite dimensional vectors of Bill's demand for tea and supply of labour in every state.

Bill's budget constraint, equation (1.5) has the same interpretation of Ada's budget constraint equation (1.2). Bill promises to supply $\ell^b(\theta)$ units of labour if state θ occurs, thus his total income from labour supply is $\omega\ell^b$. This income Bill can allocate to planned consumption of tea in any particular or possible state. Again it is important to notice that Bill's wage income in any particular state $\omega(\theta)\ell^b(\theta)$ need not equal the value of his consumption in that state $p(\theta)r^b(\theta)$.

To solve P.1.2 let β be the Lagrangian mulitplier associated with the budget constraint equation (1.5), then the first order conditions are

$$g(\theta)u_1^b(\theta) = \beta p(\theta), \quad g(\theta)u_1^b(\theta)\ell^b(\theta) = y^a(\theta), \qquad \forall\ \theta \qquad (1.6)$$

where $u_1^b(\theta)$ is Bills marginal utility in state θ. From equation (1.6) it can be seen that the marginal disutility of labour $\ell^b(\theta)$ is equal to the real wage $\omega(\theta)/p(\theta)$ in every state.

In equilibrium the demand and supply of each good in each state must be equated. Thus there are a double continuum of market equilibrium clearing conditions, one of which is redundant by Walras' law. These equations are

$$\ell^a(\theta) = \ell^b(\theta) = \ell(\theta) \qquad \pi^a(\theta) + r^b(\theta) = y^a(\theta) \quad \forall\, \theta \tag{1.7}$$

In words Ada's demand for labour must equal Bill's supply and the sum of Ada's and Bill's demand for tea must equal Ada's supply in each and every state. Using equation (1.7) equations (1.4) and (1.6) can be combined to give equation (1.8)

$$\ell(\theta) = \theta \qquad u^b(\theta) = \beta/\alpha \quad \forall\ \theta\ . \tag{1.8}$$

Equation (1.8) contains two fundamental relationships. The first condition requires that the marginal disutility of labour $\ell(\theta)$ should equal the marginal product of labour θ in each state. If this condition holds it is impossible to reallocate labour so as to increase Ada's output of tea and Bill's utility simultaneously. Any solution that has this property is said to be productively efficient. The second relationship in equation (1.8) shows that Bill's marginal utility is a constant independent of the state of nature. This means there is no way to reallocate income across states of nature so as to increase either Ada's or Bill's utility. A solution exhibiting this property is said to share risk efficiently.

To summarize the market equilibrium that occurs when agents behave competitively and trade contingently is both productively efficient and shares risk efficiently. If θ is distributed normally the market equilibrium has the following properties

$$\left.\begin{array}{ll} \omega(\theta)/p(\theta) = \ell(\theta) = \theta\ , & r(\theta) = (\alpha/\beta) + \tfrac{1}{2}\theta^2 - 1 \\ \pi(\theta) = \tfrac{1}{2}\theta^2 - (\alpha/\beta) + 1 & u^b(\theta) = \log((\alpha/\beta) + \tfrac{1}{2}) \\ E(\omega(\theta)/p(\theta)) = E\ell(\theta) = 1 & Er(\theta) = (\alpha/\beta) + \tfrac{1}{2}(1 + \sigma^2) - 1 \\ E(\pi(\theta)) = \tfrac{1}{2}(1 + \sigma^2) - (\alpha/\beta) + 1 & \partial(\omega(\theta)/p(\theta))/\partial\theta = \partial\ell(\theta)/\partial\theta = 1 \\ \partial\, r(\theta)/\partial\theta = \partial\pi(\theta)/\partial\theta = \theta\ . & \end{array}\right\} \tag{1.9}$$

The Bargaining Game

The game theorist has a rather different approach from the equilibrium theorist. Instead of assuming that each agent acts competitively and trades contingently the game theorist assumes that a) each agent or player has full knowledge of the personal utility of the other and b) each agent or player can enter into a binding commitment with the other player. Which set of assumptions is most applicable depends upon the context, neither one has any prior claim. For the present purpose it is useful to consider both the approach of the market equilibrium theorist and the game theorist since it will be shown below the labour contract theorist is typically eclectic in his choice of assumptions.

The game theorist is likely to perceive the model as a two-person non-constant sum game or contest. The payoffs are in terms of utility, no interpersonal comparisons of utility being allowed. Nash (1950) developed a method of solving such bargaining problems. Consider the utility possibility set or characteristic function of the game

$$U = \{(U^a, U^b) \mid U^a \geq U^a(0), \quad U^b \geq U^b(0) \quad ;$$
$$r^b(\theta) + \pi^a(\theta) = \theta\,\ell(\theta) \ , \forall \ \theta\}$$

where $U^a = E\pi^a(\theta) = E(\theta\ell(\theta) - r^b(\theta)), \quad U^a(\theta) = E0 = 0$

$$U^b = E\log(1 + r^b(\theta) - \tfrac{1}{2}\ell(\theta)^2), \quad U^b(0) = E\log 1 = 0.$$

U is a convex, compact set the U^a and U^b axis bounding the set on two sides. This is drawn in diagram 1.1. $U^a(0)$ and $U^b(0)$ are the fixed threat points of the Nash bargaining game. To see this consider how Ada and Bill will play this game. Ada and Bill will enter into a contract that specifies how much labour Bill is to supply $\ell(\theta)$ and how much remuneration $r^b(\theta)$ Ada is to pay Bill for this in each state. Ada's consumption will then be the residual of output less remuneration. The only credible threat either Ada or Bill can make is not to enter into any contract but this will leave them both with an expected utility of zero.

Nash showed that if the contract satisfied the apparently reasonable axioms of feasibility, invariance, efficiency and independence then the contract chosen solved the problem

$$\text{P.1.3} \qquad \max_{r,\ell} \qquad (U^a - U^a(0))^{\lambda}(U^b - U^b(0))^{(1-\lambda)}$$

where λ represents the bargaining strengths of the two agents. The first order conditions for P.1.3 are

$$u_1^b(\theta) = \lambda U^b/(1-\lambda)U^a \qquad \ell(\theta) = \theta \qquad \forall\, \theta \qquad (1.10)$$

Thus the contract chosen by the two players in the bargaining game is productively efficient and shares risk efficiently. Indeed for some value of λ the bargaining solution and the market equilibrium will coincide. Both solutions are pareto efficient though neither may be distributionally desirable. For example if λ is close to unity Ada has considerably more bargaining strength than Bill. This means Bill's marginal utility will be very high and hence his total utility very low.

The spot market equilibrium

The spot market equilibrium approach extends the Arrow-Debreu model by dropping the assumption that agents can trade contingently upon any state of nature. It is a common observation that the real world does not have a complete set of contingent claims markets. The number of possible contingencies are innumerable, the number of markets is not. For example it is impossible to sell ones labour contingent upon ones disposition, that is being on or off form since this is not observable at all accurately by anyone but oneself.

This would seem to be a general problem, in an economy there will be a considerable amount of private information, and it will be difficult for markets to operate, contingent upon this information. Radner (1968) studied this problem. He suggested that trade between agents be restricted to contingencies that both parties can observe or verify. In this context it may be assumed that either Ada or Bill can observe the outcome of the random variable θ but not the other. Then it can be assumed that trade is restricted or confined to take place only after the state of nature has occurred. This type of market arrangement will be termed the spot or auction market.

The assumption that trade cannot be made contingent upon the state of nature is clearly justified if the cost to say, Bill of observing or verifying the true state of nature is infinite. But this assumption has the same status as the assumption that agents act competitively which is only justified when there are an infinite number of agents. The purpose of this subsection is not to assess the costs of trading contingently rather it is to find out what are the consequences if trade cannot be conducted contingently.

In the spot market Ada and Bill trade only after the value of θ has been realized. Thus there are only two prices in the spot market. It will be convenient to normalize the price of tea to unity so that the price of labour, w is the real wage.[7] If the state of nature realised is θ Ada will choose her labour demand ℓ^d to maximize profits or utility, that is

P.1.4 $$\max_{\ell^d} \quad \theta\ell^d - w\ell^d$$

Similarly Bills problem is to choose his labour supply, ℓ^s to maximize his utility,

P.1.5 $$\max_{\ell^s} \quad \log(1 + w\ell^s - \tfrac{1}{2}(\ell^s)^2).$$

It is important to realize that in the spot market Ada's profits and consumption in any particular state are synonymous as is Bill's consumption and wage income. Solving P.1.4 and P.1.5 and remembering that in equilibrium labour demand equals labour supply ($\ell^a = \ell^s = \ell$) the analogue of equation (1.9) can be written

$$\left.\begin{array}{llll} w(\theta) = \ell(\theta) = \theta & r(\theta) = w(\theta) & \ell(\theta) = \theta^2 & \pi(\theta) = 0 \\ u^b(\theta) = \log(1 + \tfrac{1}{2}\theta^2) & & E w(\theta) = E\ell(\theta) = 1. & \end{array}\right\}$$

$$\left.\begin{array}{lll} Er(\theta) = 1 + \sigma^2 & E\pi(\theta) = 0 & \partial w(\theta)/\partial\theta = \partial\ell(\theta)/\partial\theta = 1 \\ \partial r(\theta)/\partial\theta = 2\theta & & \partial u^b(\theta)/\partial\theta = \theta/(1 + \tfrac{1}{2}\theta^2) \end{array}\right\} \quad (1.11)$$

Equation (1.11) shows that the marginal product of labour equals the marginal disutility of labour in each state. The spot market equilibrium is productively efficient. However the spot market does not allocate risk efficiently since Bill's marginal utility of income is not a constant. In particular since $\partial u^b(\theta)/\partial\theta > 0$ and $\partial\pi(\theta)/\partial\theta = 0$ Bill is bearing all the risk and Ada none. Then it is possible to increase Bill's utility by transferring income from productive states, that is states with a high value of θ to less productive states, those with a low value of θ. Since Ada is risk netural her expected utility can be kept constant by such a transfer. Graphically the spot or auction market equilibrium is in the interior of the utility possibility set U. This is not to say the spot market equilibrium has no desirable welfare properties. Indeed Radner shows that the Spot market equilibrium is pareto optimal given the set of markets that are open.

The labour contract

It has already been noted that the labour contract theorist is typically eclectic in his choice of assumptions. In particular the labour contract theorist assumes, a) each agent has knowledge of the other agents personal utility, b) the employer and employee can enter into a binding commitment with each other, c) each agent acts competitively and d) goods and labour cannot be bought or sold contingent upon the state of nature. Each assumption will now be examined in turn.

It is most likely that each agent will know the others preferences if their trading with each other is continuous and durable. Hicks has emphasized that the labour market is more like a durable goods market than a commodity or fish market. Labour may be as perishable as cut flowers but there are a number of reasons for supposing that the relationship between an employer and an employee will be long lasting and durable. There are mobility costs in the labour market and specific job and occupational skills that inhibit divisibility and diversification. Even is these costs were zero however, it will be shown that agents have an incentive to contract rather than operate in the spot market so that the employment relationship is likely to be durable. Then the employment relationship may be stable because agents contract, or agents may contract because the employment relationship is stable. Needless to say these two factors tend to be reinforcing.

Assumption b is that the employer and employee can enter into a binding commitment, that is there is no possibility of default on the contract by either party. Clearly if mobility costs are very high then the issue of default does not arise since it is impossible. Suppose then that mobility costs are relatively low and suppose that the Ada-Bill economy is replicated n times, that is there are n identical Adas and n identical Bills. Default in this context essentially involves one of the agent reneging on the contract in some state and trying to sell and buy on the spot market. Suppose $\sigma^2 = 3$ then it is easy to show that Bill's income from the spot market is greater than or less than his income from the contract as θ is greater than or less than two. Conversely Ada's profits from the spot market are higher than or lower than her profits from the contract as θ is less than or greater than two. Thus Bill will only wish to default if θ is greater than two, but then there will be no employer who will wish to hire Bill on the spot market. The position is not really changed if each employer experiences a different value of θ for in that case the state of nature is described by a n vector of different θ's.

If employers or employees have different preference, that is not all Adas are identical and not all Bills are identical, then default may indeed be possible. However if time is introduced into the analysis reputation may become important in limiting default. An employee say, whose defaults will earn a 'bad' reputation and hence he will not be offered such a favourable contract in the future. In the limit if the time horizon is infinite there will be no default since default can be made infinitely costly.

Given that each agent knows the otherspreferences and both can enter into a binding commitment with each other, Ada and Bill will negotiate a contingent contract specifying the amount of effort to be supplied and the quantity of tea with which Bill will be paid in every state or contingency. This contract will be pareto optimal so that there are no unexpected gains to be made. This can be formalized by assuming the contract maximizes a weighted sum of each agents utility.

$$\text{P.1.6} \quad \max_{r,\ell} \quad \lambda E(\theta\ell(\theta) - r(\theta)) + (1-\lambda)E\log(1 + r(\theta) - \tfrac{1}{2}(\ell(\theta)^2)$$

The weights may be interpreted in the game theory tradition as representative of bargaining strengths. However there is another interpretation, more consonant with the assumption that agents behave competitively. Consider again the replicated Ada-Bill economy. Then

Ada will have to provide Bill with a certain level of expected utility say $\bar{u}$ in order to induce him to work for her. This is the supply price of employees, it will be determined by the number of employees and Ada's demand for each of them. If both Ada and Bill behave competitively they will take the supply price of employees $\bar{u}$, as a parameter. Then P.1.6 must be rewritten in another form, essentially the contract will be chosen to maximize Ada's expected profit subject to giving Bill an expected utility level of $\bar{u}$

$$\text{P.1.7} \qquad \max_{r,\ell} \quad E(\theta\ell(\theta) - r(\theta)) \text{ s.t. } E\log(1 + r(\theta) - \tfrac{1}{2}(\ell(\theta))^2) = \bar{u} .$$

if $\hat{\lambda}$ is the Lagrangian multiplier for P.1.7 then P.1.6 and P.1.7 are equivalent if $\hat{\lambda} = (1-\lambda)/\lambda$.

Labour contract theorists also recognize that it is unusual for labour to be bought or sold contingently because of moral hazard problems, hence assumption d. This seems a little strange because the labour contract has in fact been made state contingent. Actually there is no contradiction. For example suppose that contingent claims market cannot operate because Bill the employee is unable to observe the state of nature θ[8]. Then it might be expected that Ada has an incentive to conceal the true value of θ , implying that the contract is unenforceable and breaks down. To see why this is not the case examine the first order conditions for P.1.7

$$\left.\begin{array}{llll} & -1 + \hat{\lambda}u_1^b(\theta) = 0 & \theta + \hat{\lambda}u_1^b(\theta)(-\ell(\theta)) = 0 & \forall\, \theta \\ \text{or} & \ell(\theta) = \theta & \hat{\lambda} = 1 + r(\theta) - \tfrac{1}{2}(\ell(\theta))^2 & \forall\, \theta \end{array}\right\} \quad (1.12)$$

Since the marginal utility of Bills income is constant and utility has only a single argument, this argument $(1 + r(\theta) - \frac{1}{2}(\ell(\theta))^2)$ is constant. Then the marginal cost of hiring labour $\partial r(\theta)/\partial\ell(\theta)$ is equal to $\ell(\theta)$ which is the marginal disutility of labour. But $\ell(\theta)$ equals θ the marginal product of labour so the marginal cost of hiring labour equals the marginal product. Since only Ada can observe θ she can effectively choose that labour input that maximizes her profit by reporting or announcing the appropriate value of θ. She will clearly choose the level of labour input that equates the marginal cost of hiring labour to the marginal product. Since the contract derived from equation (1.12) automatically satisfies this condition it is incentive compatible, that is Ada has no incentive to misreport the true state of nature[9]. Suffice it to say incentive compatibility is not a general property and depends crucially upon a) the employers risk neutrality b) the employees utility function belonging to the class of functions satisfying $u = v(r-h(\ell))$.

Equation (1.12) shows that the optimal labour contract is productively efficient and also shares risk efficiently. Notice too that Bills wage income always equals his consumption in every state but his wage does not equal the marginal product. In the Arrow-Debreu approach Bill's real wage always equals the marginal product but the value of Bill's consumption need not equal his wage income in any particular state. Formally however the labour contract replicates the market equilibrium if $\hat{\lambda} = \alpha/\beta$. This is shown in euqation (1.13)

$$\left.\begin{array}{l} \ell(\theta) = \theta \quad r(\theta) = \hat{\lambda} + \tfrac{1}{2}\theta^2 - 1 \qquad \pi(\theta) = \tfrac{1}{2}\theta^2 + 1 - \hat{\lambda} \\ u^b(\theta) = \log\hat{\lambda} \quad E\ell(\theta) = 1 \quad Er(\theta) = \hat{\lambda} + \tfrac{1}{2}(\sigma^2-1) \\ E\pi(\theta) = \tfrac{1}{2}(\sigma^2+3) - \hat{\lambda} \quad Eu^b = \log\hat{\lambda} \quad \partial\ell(\theta)/\partial\theta = 1 \\ \partial r(\theta)/\partial\theta = \theta \quad \partial\pi(\theta)/\partial\theta = \theta \ . \end{array}\right\} \quad (1.13)$$

It has been traditional in the labour contract literature to compare the labour contract to the spot or auction market equilibrium. To do this it is convenient to set $\bar{u} = \log(\tfrac{1}{2}(\sigma^2+3))$, then $\lambda = \tfrac{1}{2}(\sigma^2+3)$ and equation (1.13) can be rewritten

$$\left.\begin{array}{l} \ell(\theta) = \theta \qquad r(\theta) = \tfrac{1}{2}\theta^2 + \tfrac{1}{2}(\sigma^2+1) \qquad \pi(\theta) = \tfrac{1}{2}\theta^2 - \tfrac{1}{2}(\sigma^2+1) \\ u^b(\theta) = \log(\tfrac{1}{2}(\sigma^2+3)) \qquad E\ell(\theta) = 1 \qquad Er(\theta) = \sigma^2+1 \\ E\pi(\theta) = 0 \qquad Eu^b(\theta) = \log(\tfrac{1}{2}(\sigma^2+3)) \quad \partial\ell(\theta)/\partial\theta = 1 \\ \partial r(\theta)/\partial\theta = \theta \qquad \partial\pi(\theta)/\partial\theta = \theta \, . \end{array}\right\} \quad (1.13')$$

The spot market equilibrium and the labour contract with $\bar{u} = \log(\frac{1}{2}(\sigma^2+3))$ can be compared by contrasting equations (1.11) and (1.13′). Expected profits are zero in both cases. However Bill is better off with the labour contract. This follows since the logarithmic transformation is a concave function. Therefore

$$\log(E(\tfrac{1}{2}\theta^2+1)) = \log(\tfrac{1}{2}(\sigma^2+3)) > E\log(\tfrac{1}{2}\theta^2+1) \qquad (1.14)$$

In this context the optimal labour contract (weakly) pareto dominates the spot market equilibrium. This reflects the fact that the labour contract is on the frontier of the utility possibility set U, whilst the spot market equilibirum is not. Thus there is always a potential welfare improvement in moving from the spot market equilibrium to the labour contract.[10] This is a descriptive and not prescriptive statement. For the spot market system to work it is not necessary that Ada should know Bills preferences or that Ada and Bill can enter into a binding commitment. Both assumptions are required if the labour contract is to operate. It is best to view the spot market equilibrium and labour contract not as competing hypotheses but as different tools to be brought out of the tool bag according to either tastes or needs.

Nevertheless there has been a great deal of emphasis placed in the literature in comparing the emmployment properties of the spot market equilibrium and the labour contract. In the Ada-Bill economy exactly the same amount of labour is used in all states in both the spot market equilibrium and the labour contract. This is of course not a general property but depends upon Bills marginal disutility of labour being independent of his consumption of tea. Since both the spot market equilibrium and the labour contract are productively efficient, the independence of the marginal disutility of labour and the employees consumption means equal amounts of labour will be allocated to each potential state in both systems.

It is interesting then to see why labour contract theory might be regarded as a theory of unemployment. Azariadis (1975) presents a model rather similar to the Ada-Bill economy. However Azariadis assumes Bills utility function is

$$u^b = U(w) \quad \text{if } \ell = 1 \qquad u' > 0 \quad u'' \leq 0 \quad (1.15)$$
$$u^b = U(w+R) \quad \text{if } \ell = 0$$

where w is income. Thus Azariadis assumes labour is supplied indivisibly in units of only one or zero. R is the reservation wage which represents the gain in utility from not working. In this case the reservation wage is simply the marginal disutility of labour and is independent of the employees consumption. From the above discussion it is clear that labour will be allocated in the same way in both the spot market equilibrium and the labour contract. Unfortunately Azariadis gives a different result, employment will be more variable if labour is allocated via the labour contract. His result obtains since Azariadis does not allow the employer to pay unemployment compensation to the employee. This effectively means that the marginal disutility of labour is not independent of income because income is dependent upon the employees employment status. It has been pointed out by several authors[11] that the restrictions on unemployment compensation is arbitrary. For more general utility functions the relationship between the employment patterns of the spot market equilibrium and labour contract is quite complicated and very little can be said in general.

Equally the fixity of wage rates noted in the opening paragraph to this chapter is only associated with the utility function described by equation (1.15). If the employees utility function is separable in income and leisure, $u = v(r) - h(\ell)$, then income is fixed and if the employees utility function has the form $u = v(r-h(\ell))$ then utility is constant. However in the general case $u = u(r,\ell)$ only the marginal utility of income $u_1(r,\ell)$ is fixed.[12]

This section has demonstrated in the context of a very simple model how labour contract theory relates to other more conventional methods of analysis. It has been shown that labour contract theory is both eclectic and descriptive. It is eclectic since it uses assumptions from both game theory and market equilibrium analysis. It is descriptive because it describes how agents might reach a point on the utility possibility frontier.

Section 3 : Conclusions

This chapter has illustrated how the theory of labour contracts relates to a number of other approachs adopted in economics, namely equilibrium theory and bargaining theory. It has been shown that at the present level of generality the differences between all three approaches is essentially descriptive rather than substantive.

It was stated in the introduction that the main aim of contract theory was to model unemployment and explain a fixed wage. However it has been shown that the theory does neither. The optimal labour contract is pareto efficient so that whatever allocation of labour is chosen is in this sense optimal. Equally the contract maintains a constant level of utility rather than a constant wage.[13] In retrospect Malinvaud's (1977) conjecture that contract theory provided a choicetheoretic foundation for fixed price models is incorrect. The crucial assumption of disequilibrium theory appears to be that quantities are rationed rather than that prices are fixed.

Nevertheless contract theory does represent an interesting departure from standard Walrasian analysis. It has been commonly observed that the labour market is dissimilar to commodity markets in many respects, most importantly because it is impossible to serve two masters simultaneously. Contract theory provides an attempt to analyse the implication of this to the labour market, rigorously and within the context of the neoclassical paradigm . There are therefore a number of extensions to the present model that deserve exploration.

In chapter 1A which is the appendix to this chapter, the basic model of the labour contract presented in this chapter is extended in a number of directions. In particular some of the more restrictive assumptions of this chapter are dropped. In chapter 2 a general equilibrium treatment of labour contract theory is presented. This builds upon the analysis of chapter 1A and it is shown how a conflict between private and social goals might arise. In chapter 3 the incentive compatible contracts discussed in section 2 are analysed within the more general framework of chapter 1A. It is shown that the optimal incentive compatible contracts are productively inefficient and share risk inefficiently. In chapter 4 the role of imperfect information in contract theory is examined. This has been extensively studied within the market equilibrium framework in recent years. It is shown under what conditions real wages are constant and how aggregate information can affect output and employment.

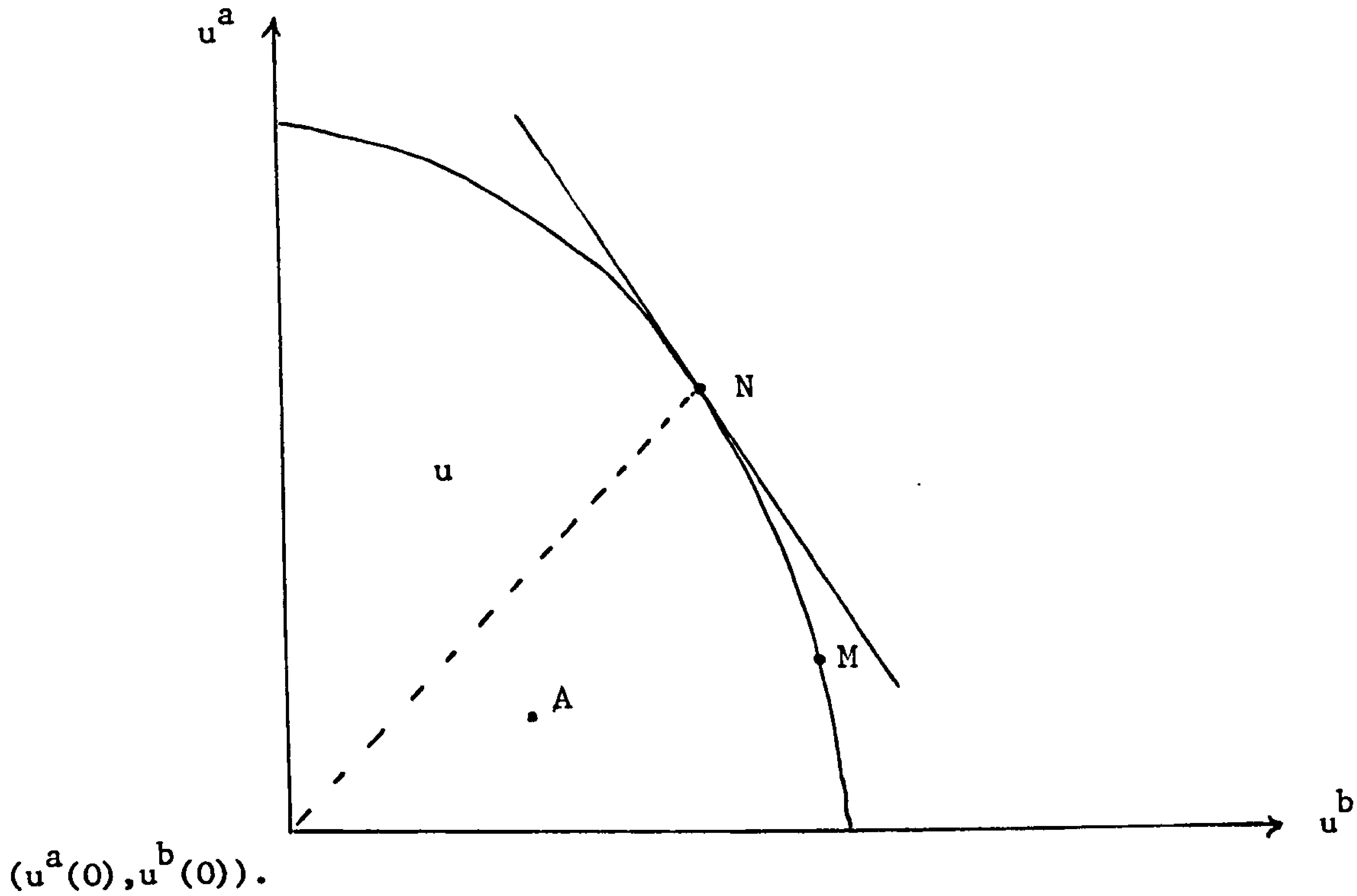

Diagram 1

N = Nash bargaining Solution

M = Market equilibrium solution

A = Auction market equilibrium solution

Notes

1. See bibliography page 164

2. See for example the book of readings, King (1980).

3. See McNulty (1980) for a concise history of thought of labour economics.

4. Edgeworth (1881) has an interesting and detailed discussion about the contract curve when a labourer can serve only one master. Hicks (1930) contrasts Egeworth's approach with that of Marshall.

5. Edgeworth (1881).

6. This quote is taken from the commentary to the second edition, published in 1963.

7. Notice that w is a scalar whereas ω is a vector. Similarly in this subsection ℓ^d and ℓ^s are also scalars.

8. The same analysis also applies if Bill can observe θ and Ada cannot.

9. The earliest examination of the incentive compatibility issue appears to have been made by Barro (1977) and Calvo and Phelps (1977).

10. This is not true if there are two goods. Suppose one firm produces wheat and the other barley and suppose that both wheat and barley are perfect substitutes in consumption, i.e. corn. In addition suppose there are just two states of nature, drought or monsoon. The wheat is assumed to survive the drought but not the monsoon and the barley to survive the drought but not the monsoon. Then the spot market for labour to harvest corn will always dominate the contract market simply because labour is mobile. See also chapter 2.

11. See for example Sargent (1980). Grossman and Hart (1981) show that Azariadis' model is incentive compatible when formulated in this way.

12. The general case is examined in chapter 1A.

13. See chapter 4.

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CHAPTER 1A : Appendix

The purpose of this appendix is to examine the optimal labour contract under more general assumptions than those presented in chapter one. It was noted in section two of chapter one that the optimal labour contract will not typically be incentive compatible. That is to say if the employer has more information about the state of nature than the employee it will not usually be in his interest to reveal this information. Indeed the simplicity of the model presented in chapter one reflected the desire to avert these problems which will be considered in greater depth in chapter three. However as a preliminary to that investigation it seems useful to analyse the optimal labour contract assuming that all the relevant information is public knowledge.

The model presented in this appendix extends that analysed in chapter one. A single firm will be considered, a general equilibrium treatment being deferred to chapter two. Suppose that the firm consists of a single risk neutral employer and N risk averse employees. The N employees are indexed by $n = 1, \ldots N$. Together the employer and N employees produce a single consumption good y, according to a random, concave production function f. The arguments of the production function are the labour input ℓ_n, $n = 1, \ldots N$ of each of the employees and a single random variable θ. θ has a strictly positive continuous probability density function g, over the interval (a,b) where $g(a) = g(b) = 0$. The production function f is assumed to have the following properties

1A.A.1 $$y = f(\ell_1, \ldots \ell_n, \ldots \ell_N, \theta) : [0,1]^N \times (a,b) \to R^+ \text{ ; is } C^2 \text{ and}$$
$$f_{N+1} > 0,\ f_n > 0,\ f_{nn} \leq 0,\ f_{n,N+1} \geq 0,\ \lim_{\ell_n \to 0} f_n = \infty, \quad n=1\ldots N.$$

where each employees labour input is defined over the unit interval.

Each employee has a positive marginal product but his labour input exhibits diminishing marginal returns. Since each employees marginal product is infinite at a zero labour input, every employee will supply a strictly positive quantity of labour. The random variable θ may represent many factors, it may represent weather conditions or perhaps some random management skill or effort or random market conditions. Equally it may reflect randomness in some of the employees ability to perform the requisite task. Whichever interpretation is accepted it will be assumed that an increase in θ always increases the marginal product of each employee so that θ is properly considered as a shock to efficiency or skill. It should be noticed that any labour input provided by the employer is assumed to be fixed and is notationally supressed. It is also assumed that every agent knows the relevant probability density function $g(\theta)$. This is a strong rational expectations assumption. It is justified if each agent has observed the drawing of the random variable on numerous occasions and hence knows the true objective probability that any particular state will occur.

The employer is assumed to be risk neutral so that he has a linear utility function v in consumption or profits π. This assumption is made principally for simplicity since the analysis of this appendix can be carried through without it. Risk neutrality is usually regarded as a surrogate for other factors that are not modelled. For example if employers are wealthier than employees and risk aversion declines with wealth the assumption of risk neutrality may be justified. Equally employers may have better access to capital or financial markets of diversifying risk. Another common justification for this assumption is the Knightian distinction between venturesome and and confident employees and inately timid employees. But on reflection this justification does not seem relevant in this context. Suppose for example there are a large number of identical agents who at least potentially may become employers or employees, that is each agent has an occupational choice[1] Then Knight might argue that entrepreneurship was the more risk occupation and therefore would be rewarded by a higher than average compensation. If this was the case then the more risk tolerant agents are likely to become employers. However in the labour contract model risk is allocated optimally among agents, so that identical agents will bear equal amounts of risk. This suggests that the Knightian justification for risk tolerant employers is not appropriate in the labour contract context. Nevertheless for pedagogic reason it will be assumed that the employers preferences are given by

1A.A.2 $v = v(\pi) : R^{+} \to R$; is c^2; $v(\pi) = \pi = f(\ell_1 \ldots \ell_n \ldots \ell_N, \theta) - r_1 \ldots - r_n \ldots - r_N$

where r_n is the remuneration the employers pay to the nth employee.

Each employee has a utility function that is concave in consumption or remuneration and labour input. It is assumed that both consumption and leisure are normal goods and that each employee is risk averse. The latter is equivalent to assuming utility is jointly concave in consumption and leisure. Formally the nth employees preferences are represented by

1A.A.3 $u^n = u^n(r_n, \ell_n) : R^{+} \times [0,1] \to R$; is c^2 and

$u_1^n > 0$, $u_2^n < 0$, $u_{11}^n \leq 0$, $u_{22}^n \leq 0$, $(u_{11}^n \; s^n + u_{12}^n) \leq 0$,

$$(u_{12}^n \; s + u_{22}^n) \leq 0 \quad (u_{11}^n \; u_{22}^n - (u_{12}^n)^2) \geq 0$$

where $s^n = - u_2^n / u_1^n$ is the marginal rate of substitution between consumption and leisure.

Notice that the nth employees utility does not depend on the labour input of any other employee. This simplifies the examination of the optimal contract. If there are any interactions between the employees these are incorporated in the production function.

A contract is an allocation within the firm that specifies the labour input to be supplied by each employee and the recompense offered in every contingency or state of nature. The contract is thus a 2xN-tuple of labour input and remuneration schedules

$\delta = \{r_1(\theta) \,....\, r_n(\theta) \,...\, r_N(\theta),\ \ell_1(\theta) \,....\, \ell_n(\theta) \,.....\, \ell_N(\theta)\}$.

A feasible contract satisfies the following conditions[2]

$$\int_a^b u^n(r_n(\theta),\ \ell_n(\theta))g(\theta)d\theta - \bar{u}^n = 0 \quad \forall\ n = 1 \,.....\, N \qquad (1A.1)$$

$$f(\ell_1(\theta)\,...\,\ell_n(\theta)\ ...\ \ell_N(\theta),\theta) - r_1(\theta)\ ...\ r_n(\theta)\ ...\ - r_N(\theta) \geq 0 \quad \forall\ \theta \qquad (1A.2)$$

$$r_n(\theta) \geq 0 \qquad \forall\ \theta \text{ and } \forall\ n = 1....N \qquad (1A.3)$$

$$1 \geq \ell_n(\theta) \geq 0 \qquad \forall\ \theta \text{ and } \forall\ n = 1....N \qquad (1A.4)$$

Condition (1A.1) states that each employees expected utility should equal some constant $\bar{u}^n$. The constant $\bar{u}^n$ is best thought of as being competitively determined. Suppose that the nth employee is representative of a large number of identical potential employees. Then $\bar{u}^n$ is determined by the supply and demand for employees of type n and both emplôyer and employee treat $\bar{u}^n$ parametrically. Alternatively $\bar{u}^n$ may represent the nth employees opportunity cost perhaps in terms of home production or it may represent the fixed threat point of a bargaining game. Equation (1A.2) - (1A.4) are simply non-negativity constraints.

Let Δ denote the set of feasible contracts, that is the set of δ's satisfying equations (1A.4) - (1A.7). Suppose

1A.A.4 There exist some $\delta \in \Delta$

Then the optimal labour contract δ^* is that feasible contract that maximizes the employers profits. Formally δ^* solves

$$\text{P.1A.1} \quad \max_{\delta} \int_a^b (f(\ell_1(\theta)\ldots\ell_n(\theta)\ldots\ell_N(\theta),\theta) - r_1(\theta) \ldots - r_n(\theta) \ldots - r_N(\theta)g(\theta)d\theta$$

$$\text{s.t} \quad \delta \varepsilon \Delta \qquad (1A.5)$$

Lemma 1A.1 There exists a unique optimal labour contract s.t for $\pi(\theta) > 0$ $r_n(\theta) > 0$ and $1 > \ell_n(\theta) > 0$ $n = 1 \ldots N$

$$f_n(\ell_1(\theta))\ldots\ell_n(\theta)\ldots\ell_N(\theta), \theta) = s^n(r_n(\theta),\ell_n(\theta)) \forall\theta, \forall n = 1 \ldots N \qquad (1A.6)$$

$$u_1^n (r_n(\theta) \quad \ell_n(\theta)) = \text{constant} \qquad \forall \theta \; \forall n = 1 \ldots N \qquad (1A.7)$$

Proof: The feasible set is convex and compact and the objective function is concave. Setting up the Lagrangian function for P.1A.1 and (1A.6) and (1A.7) to follow immediately.

Condition (1A.6) is productive efficiency, the marginal product of each employee equals his marginal rate of substitution between income and leisure. This is an intrastate efficiency condition it means it is impossible to change the employees labour input and increase output and each employees utility simultaneously. Equation (1A.7) shows that the optimal labour contract allocates risks efficiently. This is an interstate condition, it is impossible to reallocate any employees income between any two or more states withouth decreasing his utility. Thus the optimal labour contract is productively efficient and allocates risk efficiently.

It is interesting to know under what condition each employee will be offered the same contract, that is when each employee supplies the same input and remuneration. Lemma 2 provides a sufficiency condition.

Lemma 1A.2 A sufficient condition for each employee to be offered the same contract

$$r_n(\theta) = r(\theta),\ \ell_n(\theta) = \ell(\theta) \qquad \theta \text{ and } n = 1 \ldots\ldots N \qquad (1A.8)$$

$$\text{is } f(\ell_1(\theta),\ldots\ell_n(\theta),\ldots\ell_N(\theta),\ \theta) = f(\ell_1(\theta)+\ldots\ell_n(\theta),\theta) \qquad \theta \qquad (1A.9)$$

$$\text{and } u^n(r,\ell) = u(r,\ell),\ \bar{u}^n = \bar{u} \qquad n=1\ldots N \qquad (1A.10)$$

Proof: Define the feasible set of contracts Δ' satisfying (1A.1) - (1A.4) and (1A.8). If Δ' is non-empty, a unique equal treatment contract exists, the lagrangean for which is

$$\begin{aligned} L = \int_a^b & (f(\ell_1(\theta),\ldots\ell_n(\theta),\ldots\ell_N(\theta),\theta) - r_1(\theta)\ldots-r_n(\theta)\ldots-r_N(\theta))g(\theta)d\theta \\ & + \gamma^1(\theta)(f(\ell_1(\theta),\ldots\ell_n(\theta),\ldots\ell_N(\theta).\theta) - r_1(\theta)\ldots-r_n(\theta)\ldots-r_N(\theta)) \\ & + \sum_{n=1}^{N}\Big[\lambda^n\int_a^b u^n(r_n(\theta).\ \ell_n(\theta))g(\theta)d\theta + \gamma_n^2(\theta)r_n(\theta) \\ & + \gamma_n^3(\theta)\ell_n(\theta) - \gamma_n^4(\theta)(\ell_n(\theta)-1) - \gamma_n^5(\theta)(r_n(\theta)-r(\theta)) \\ & - \gamma_n^6(\theta)(\ell_n(\theta) - \ell(\theta))\Big] \end{aligned}$$

where $\lambda=(\lambda^1,\ldots\lambda^n\ldots\lambda^N)$ is the set of multipliers for equation (1A.1) and $\gamma=(\gamma^1\ldots\gamma^6)$ are the multipliers for (1A.2)-(1A.4) and (1A.8). Consider an interior optimum $\gamma^1=\gamma^2=\gamma^3=\gamma^4=0$. then if $\gamma^5=\gamma^6=0$ the first order conditions imply

$$f_n - f_m = \lambda^m u_2^m - \lambda^n u_2^n \qquad m\neq n \qquad (1A.11)$$

$$\lambda^n u_1^n = \lambda^m u_1^m \qquad m\neq n \qquad (1A.12)$$

The conditions (1A.11) and (1A.12) are satisfied when $f_n=f_m$, $u_1^n=u_1^m$, $u_2^n=u_2^m$ and $\lambda^n=\lambda^m$ which are implied by (1A.9) and (1A.10)

Lemma 1A.2 shows that when each employee is identical, both in utility terms and in the labour services, he provides and when the supply price of employees is determined competitively so that $\bar{u}^n=\bar{u}$, then each employee will be offered the same contract. In which case equation (1A.6) and (1A.7) can be written more succinctly as

$$f_1(N\ell(\theta),\theta) = s(r(\theta),\ell(\theta)) \quad \forall\,\theta \tag{1A.13}$$

$$U_1(r(\theta),\ell(\theta)) = \text{constant} = 1/\lambda \quad \forall\,\theta\ . \tag{1A.14}$$

The nature of the optimum contract can now be examined more closely. Lemma 1A.3 summarizes the labour input and remuneration schedules.

Lemma 1A.3 If all the employees are identical and each employees labour is a perfect substitute for any other then the following relationships hold.

$$\dot{\ell}(\theta) = -\lambda U_{11} f_1/\Delta \geq 0 \tag{1A.15}$$

$$\dot{r}(\theta) = \lambda U_{12} f_{12}/\Delta \sim U_{12} \tag{1A.16}$$

$$\dot{u}(\theta) = u_1\dot{r} + u_2\dot{\ell} = u_1(\dot{r} - s\dot{\ell}) = \lambda U_1 f_{12}(U_{11}s + U_{12})/\Delta \leq 0 \tag{1A.17}$$

$$\dot{\pi}(\theta) = f_1\dot{\ell} - \dot{r} + f_2 = s\dot{\ell} - \dot{r} + f_2 = -(\dot{u}/u_1) + f_2 \geq 0 \tag{1A.18}$$

where $\Delta = \lambda[f_{11}U_{11} + \lambda(U_{11}U_{22} - U_{12}^2)] \geq 0$

Proof: Rewriting equations (1A.13) and (1A.14) and normalizing so that N = 1

$$f_1(\ell(\theta),\theta) + \lambda U_2(r(\theta),\ell(\theta)) = 0 \quad \forall\,\theta \tag{1A.19}$$

$$-1 + \lambda U_1(r(\theta),\ell(\theta)) = 0 \quad \forall\,\theta \tag{1A.20}$$

Then partially differentiating (1A.21) and (1A.22) with respect to θ and omitting arguments gives

$$\begin{bmatrix} f_{11} + \lambda U_{22}, & \lambda U_{12} \\ \lambda U_{12} & \lambda_{\ 11} \end{bmatrix} \begin{bmatrix} \dot{\ell} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_{12} \\ 0 \end{bmatrix}$$

which gives (1A.15) - (1A.18).

Lemma LA.3 encapsulates the essence of labour contract theory at its most basic. It has been suggested by Akerlof and Migazaki (1980) that the employment schedule generated by the labour contract is never more variable than the employment schedule generated by the spot or auction market. A simple example will show this supposition is incorrect.

Example: Suppose $f(\ell,\theta) = \theta\ell$

$$u(r,\ell) = v(r) - h(\ell) = r^{\frac{1}{2}} - \theta^{3/2}$$

$$\theta = \theta_1 = \tfrac{1}{2} \text{ with probability } \tfrac{1}{2}$$

$$= \theta_2 = {}^{3}/_{2} \text{ with probability } \tfrac{1}{2}$$

Then in the spot market $\omega = \theta \quad \ell = \theta^{\frac{1}{2}}/3 \quad r = \theta^{3/2}/3$

$$y = \theta^{3/2}/3 \quad \pi = 0$$

and for the contract $\ell = (2\theta/3\lambda)^2 \quad r = (\lambda/2)^2$

$$y = \theta^3(2/3\lambda)^2\pi = \theta^3(2/3\lambda)^2 - (\lambda/2)^2$$

Letting $\lambda = 1.3$ the results can be tabulated as follows

	r_1	r_2	ℓ_1	ℓ_2	π_1	π_2
spot	0.1178	0.6173	0.2357	0.4802	0	0
contract	0.4225	0.4225	0.0657	0.5917	−0.3896	0.4651

	u_1	u_2	Er	$E\ell$	$E\pi$	Eu
spot	0.2289	0.5217	0.3650	0.3220	0	0.3573
contract	0.6331	0.1948	0.4225	0.3287	0.0378	0.4140 .

Employment is more variable if on average higher in the contract as opposed to the spot market. These results are drawn in figure 1A.1 and 1A.2. These diagrams show how the wage is divorced from the marginal product in the contract market.

In Lemma 1A.3 it was assumed that each employees labour is a perfect substitute for any other employees labour. Equivalently there is no distinction made between the number of men employed and the hours they work. Then total labour input is simply $L = N.\ell$.

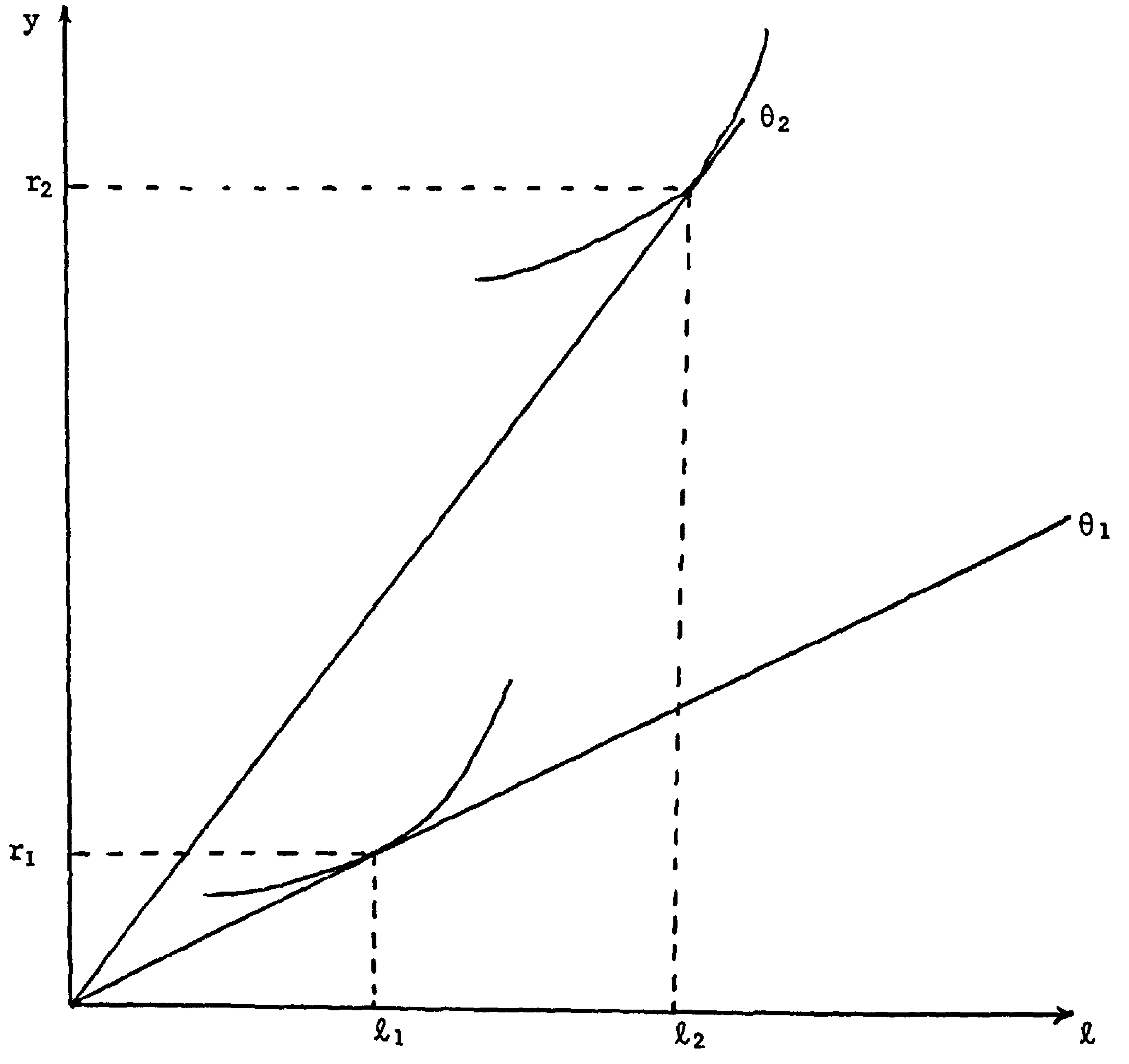

Diagram 1A.1 = Spot Market

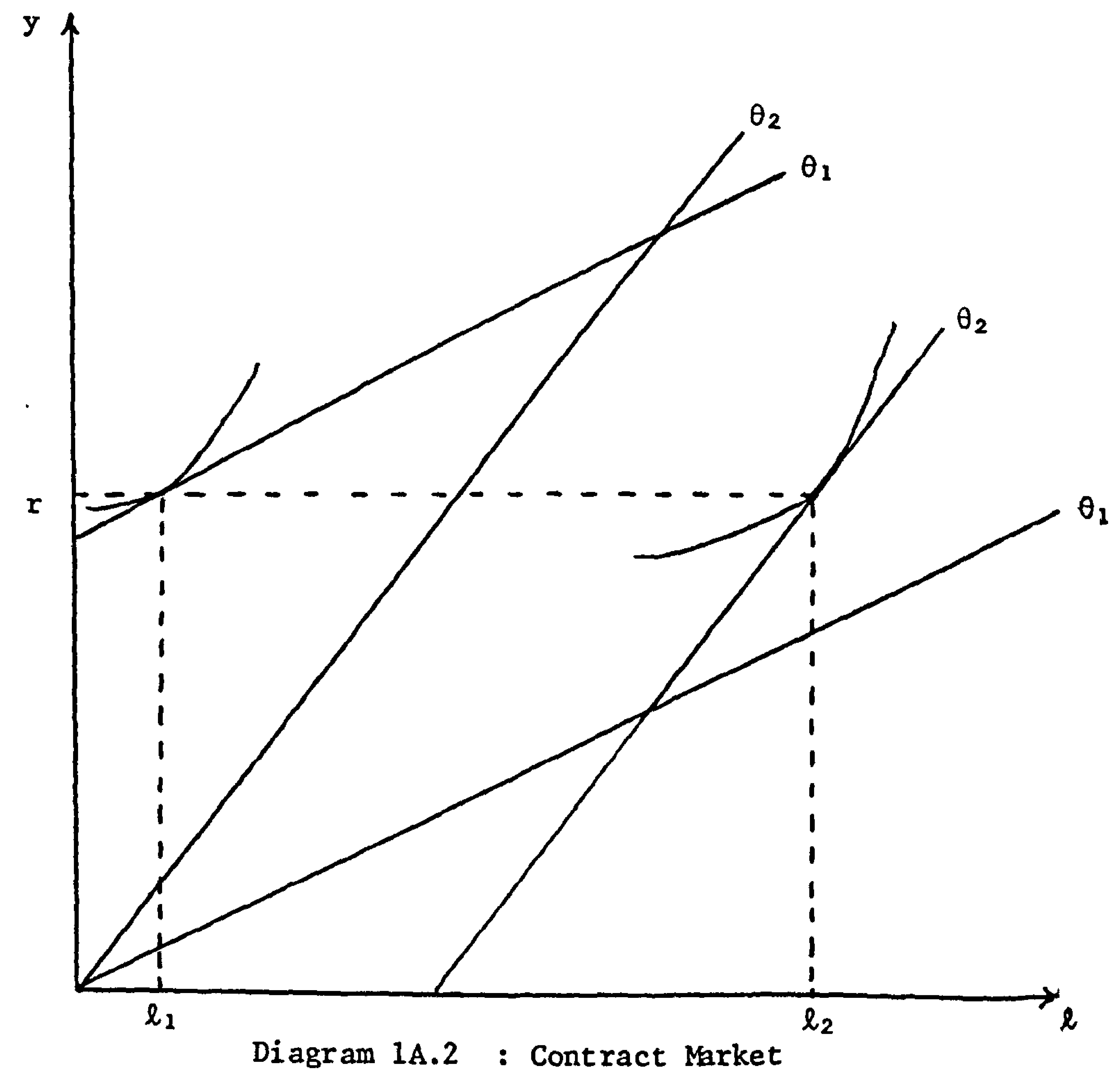

Diagram 1A.2 : Contract Market

Indeed up to now it has been assumed that N is exogeneous. Suppose however that N is endogeneous so that the employer determines the number of employees as well as the contract he offers to each of them. Then an augmented or employment contract can be defined as $\hat{\delta} = \{\delta . N\}$ where $\delta = \{r(\theta), \ell(\theta)\}$. The employers problem can then be redefined as

$$\text{P.1A.3} \qquad \max_{\hat{\delta}} \int_a^b (f(N.\ell(\theta),\theta) - N.r(\theta))g(\theta)d\theta \quad \text{s.t} \quad \delta \in \Delta \ .$$

To show that a solution exists an additional assumption is needed, namely

$$\text{1A.A5} \qquad \lim_{L\to\infty} f_1(L,\theta) = 0 \quad \theta.$$

Lemma 1A.4 Given 1A.A1 - 1A.A5 a solution for P.1A.3 exists.

Proof: The feasible set is compact and convex and by 1A.A5 the set of feasible N can be restricted to a compact set.

Unfortunately uniqueness cannot be proved because the objective function is not concave in the choice variables. Notice that the cost function is quasi-concave in N and r since the employer determines both the per unit cost and the number of units. Although $f(N\ell,\theta)$ is also quasi-concave in N and ℓ given equation 1A.A1 it is not unnatural to assume that the production function is concave in N and ℓ^3.
Fundamentally the non-concavity of the objective function is due to the formulation of the cost function.

To understand the nature of this nonconcavity suppose that P.1A.3 is ammended to

$$\text{P.1A.4} \qquad \max_{N} \ \{ \max_{\delta} \int_a^b (f(N\ell(\theta),\theta) - Nr(\theta))g(\theta)d\theta \mid \delta \in \Delta \}$$

Lemma 1A.5 Given (1A.A1) - (1A.A5) a unique solution exists for P.1A.4

Proof: Let $\pi(N,\bar{u}) = \{\max \int_a^b (f(N.\ell(\theta),\theta) - Nr(\theta))g(\theta)d\theta \mid \delta \in \Delta \}$

Then $\partial\pi(N.\bar{u})/\partial N = \int_a^b (f_1 \ell - r) + f_1 N \partial\ell/\partial N - N\partial r/\partial N) g(\theta) d\theta$

$$= \int_a^b (f_1 \ell - r) g(\theta) d\theta$$

since $\int_a^b u(r(\theta), \ell(\theta)) g(\theta) d\theta = \bar{u}$

therefore $\partial\pi(N,\bar{u})/\partial N = \int_a^b f_{11} \ell^2 \; g(\theta) d\theta \leq 0$

so that P.1A.4 is a concave programming problem.

Lemma 1A.5 explains how the non-concavity of P.1A.3 may be viewed as a type of market imperfection. Suppose that there are a large number of identical potential employees and suppose that there are a large number of identical potential employers. Then it is appropriate to view the contract δ, as determined by the forces of competition. Each employer and employee will take the contract δ as given. In this case $\bar{u}$ is a consequence of equilibria and not a fundamental parameter. Notice that if there were several types of employers that is distinguished by their production function or the industry to which they belong then though each industry may offer a different contract each industry would have to provide each employee with the same expected utility level $\bar{u}$.

Thus in the labour market it is possible to distinguish three types of competitive behaviour. These are a) the wage-taking hypothesis, b) the utility taking hypothesis and c) the contract taking hypothesis. The wage taking hypothesis assumes that there are a large number of perhaps non-identical employers and a large number of employees. For the utility taking assumption to be validated it is necessary only that there be a large number of employees since the employee is unable to influence the supply of employees in general. The contract-taking hypothesis is only justified if there are a large number of identical employers and a large number of employees. It is the utility-taking hypothesis that is maintained throughout the remainder of this thesis. Despite the non-concavity associated with P.1A.3 it would seem to be a natural assumption to make and it will be used in the general equilibrium model of chapter 2. Indeed the implementation of the contract taking hypothesis seems much more problematical.

Notes

1. Kanbur (1979) analyses a model of occupational choice. The following discussion is based loosely upon his work.

2. Notice that each labour input ℓ_n will always be strictly positive because of assumption 1A.A.1.

3. This assumption is used by Chan and Ioannides (1982). They show that layoffs can occur even when hours can be varied as well as the number of men. However they assume each employee is offered the same contract although Lemma 1A.2 shows that this is not optimal. Nevertheless strong welfaristic grounds can be made for treating similar employees in a similar manner and this may justify their assumption. That is to say there is a constraint that any labour contract be horizontally equitable.

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CHAPTER 2

An Equilibrium Model of a Labour Contract Economy

Section 0 : Introduction

In this chapter a general equilibrium model of a labour contract economy is presented and examined. As a starting point the general equilibrium model of an economy with incomplete markets developed by S. Grossman (1977) is considered. It is then suggested that within the context of this market structure agents will have an incentive to combine to arrange some form of mutual insurance. The arrangement of mutual insurance will be restricted by the same set of factors that cause markets to be incomplete. However it is suggested below that one plausible mechanism by which mutual insurance arrangements can be made is the labour contract in which employer and employee agree to allocate income and profits to the benefit of both parties.

It is important to examine the labour contract economy within the context of a general equilibrium model for four reasons:

(a) It shows that the partial equilibrium models of, for example, Azariadis (1975) are amenable to a general equilibrium treatment.

(b) It illustrates the nature and context of a labour contract in a general and rigorous manner.

(c) It allows the efficiency properties of the labour contract economy to be examined in greater detail.

(d) It serves a basis from which subsequent examples can be considered. Most important it motivates the discussion of asymmetric and imperfect information models of chapters 3 and 4.

It is well known that the Arrow-Debreu theory of value was originally formulated for an economy without uncertainty. However an appropriate reinterpretation allows uncertainty to be introduced without fundamentally altering the analysis. With this reinterpretation a commodity is distinguished not only by its physical characteristics and location but also by the date and eventin which the commodity is consumed. Therefore the two fundamental theorems of welfare economics continue to apply even in an economy with uncertainty. That is every competitive equilibrium is Pareto efficient and every Pareto efficient allocation can be sustained as a competitive equilibrium.

However the reinterpretation of the Arrow-Debreu theory of value in terms of a complete set of contingent claims markets is empirically unjustified. This can be seen clearly from the fact that if there were a complete set of contingent claims markets each agent would face a single budget constraint. This means that each agents decisions are essentially atemporal and that the entire course of the economy is determined at the first date. There are a number of reasons why complete markets do not exist. These can be generally summarized under the heading of set up costs. One example may suffice. Consider the possibility of selling labour services contingent upon ones disposition. Since only the supplier can know his truedisposition this is clearly impossible.

Suppose however that there are no set up costs in the explicit sense, contingent claims markets might still be inoperative. It is clear that trading over time or across different states of nature requires some degree of trust.[1] For example it is no good buying "jam tomorrow", if there is no guarantee that the potential supplier will deliver. Of courseeven in a completely static or certain economy there are implicitly some legal or ethical codes that make the market system tenable. Nevertheless when time and uncertainty are introduced the problems become much more acute. Indeed there are some commodities and labour is one of them for which the moral and legal concensus actually prohibit the establishment of contingent claims markets.

If the set of contingent commodity markets is incomplete it cannot be expected that agents will react passively to this situation. Competition is likely to induce the development of some 'non-Walrasian' institutions. The labour contract is one of these institutions. It provides a degree of mutual insurance between the employer and the employee given the structure of other markets. Clearly the labour contract is not the only possible response to a situation of incomplete markets. However delineating the optimal response is an impossible task and therefore the purpose of this chapter is to examine an economy where the only form of mutual insurance arrangement is the labour contract.

The first point to be considered is why is it that contracts occur in the labour market rather than the product market. Casual empiricism suggests that relationships in the labour market tend to be more stable than relationships in the product market. It is an old maxim that a man can serve only one master, and this tends to be borne out upon closer examination. Human capital is inherently indivisible and non-diversifiable. For example, an economist cannot suddenly become a plasterer or a plumber, no matter how desirable this may be. Human capital may also be firm specific as well as job specific, for instance, familiarity with working practices or colleagues etc. may be important. All such considerations can generally be subsumed under the heading of mobility costs which impart a stability to the labour market, that does not appear to be so important in the product market. The point was made in chapter 1 that the observed stability of the labour market is really a chicken and egg type problem. Is the labour market stable because participants contract or do participants contract because the labour market is stable. Whichever is the truth the continuity of employment in the labour market fosters an element of trustworthiness between the employer and the employee that is not apparent in other markets . Therefore the employer and the employee are able to trade risk in the form of a labour contract when perhaps it would be impossible to trade risk on other markets because this type of trade would not be credible.

There are a number of reasons why the employer and employee will wish to combine to form a labour contract. First, of course, the firm needs the labour that the employee provides in order to produce goods. Second, the employer and the employee will trade or share risk if there is a local difference in their risk aversion parameters. Knight suggested that employers are innately less risk averse than their employees and therefore will offer them insurance. Similarly it is often suggested that employers tend to be wealthier than their employees and have better access to capital markets for diversifying risks. Both of these factors indicate a role for employers by insuring their employees. Third the employer and employee may have different beliefs as well as differences in tastes. Knight also suggested that employers are confident and employees timid. Possible differences in beliefs make for a good horse race, for example, if the employer believes a downturn in demand is unlikely but the employees believe that a downturn is more than likely the optimal arrangement or bet will indicate that wages should be relatively high if a downturn actually occurs.[2]

Equilibrium models of an economy with labour contracts have been studied by Polemarchakis (1979) and Holmstrom (1982). Both Polemarchakis and Holmstrom assume that labour is supplied indivisibly, that is either one unit of labour is supplied or none at all. Polemarhakis examines a single period multi-sector model and shows that a continuum of Nash equilibria exists, one of which is the full employment equilibrium. Holmstrom studies a multi-period model with a single consumption good. He assumes that employees are mobile and can default on any contract in the final period. He assumes employers cannot default. He proves that a competitive equilibrium exists and shows that an economy with contracts pareto dominates an economy without contracts.

In this chapter, in order to capture the idea that the employment relation is relatively stable it will be assumed that an employee can work for only one employer, not just at one particular instant but throughout time. This is equivalent to assuming that the mobility costs of labour are infinitely high. Since a labour market of this type cannot be wage mediated it is important to understand how employees are allocated to employers. Each employer must offer each employee a labour contract which specifies how much labour is to be provided and the amount of remuneration offered in each state of nature or contingency and time period. The employee has a single simple choice, either accept or reject the offer. The offer or contract will be accepted or rejected on the basis of the lifetime expected utility it provides. The forces of competition will bid up or bid down this expected level of utility until the number of employees demanded equals the supply. The level of expected utility is effectively the price of an employee. If agents behave competitively this level of expected utility will be treated parametrically. The similarity between the price of an employee and the familiar notion of the price of a commodity will be exploited in the equilibrium analysis.

This chapter examines an economy with a sequence of markets for goods and securities and a set of labour contracts. The equilibrium concept adopted is the equilibrium of plans, prices and price expectations or rational expectations equilibrium developed by Radner (1972) and Hart (1975). In order to keep the analysis relatively simple Grossman's (1977) adaptation of this model will be used. The model has two periods but there are S possible states of nature in the second period. An equilibrium for a given set of labour contracts is a set of prices for the first period and a set of price expectations for the second period such that each agents plans given these prices and price expectations are mutually consistent. This means that in equilibrium each agent will have a common set of price expectations. That is each agents price expectations are quasi-rational in the sense that they each expect the same price to rule in some state s, but they may widely differ in the probability they assign to the occurrence of this state. Then if each agent is intemporally consistent the prices they expect will be the equilibrium prices.

It will also be assumed that each agent is atomistic or that there is a continuum of economic agents. Despite its unrealism this assumption justifies utility and price taking behaviour by agents. There are however two more technical reasons why this assumption is made. First employers hire employees rather than labour services, and although the latter is assumed to be perfectly divisible, the number of employees will not be perfectly divisible unless there is a continuum of employees, Second it was emphasized in chapter 1A that the employers hiring decision was fundamentally non-convex because of the form of the cost function. If there are a large number of different employers this will tend to convexify the economy, or alternatively each individual non-convexity becomes insignificant.

With suitable modifications a rational expectations equilibrium with production can be defined. In particular it is possible to allow for inputs or investments that contribute to output in some future date-event pair. However in this chapter it will be assumed that only contemporaneous transformations of inputs and labour services into outputs are possible. The importance of this assumption is that it allows the employers consumption and production decisions to be treated separately. It is then possible to model the economy as if it were a pure exchange economy.

It has already been noted that there are assumed to be S possible states of nature in the second period. This uncertainty is completely exogeneous and is not characterized or specified in any further way. Uncertainty may be due to randomness in either preferences or technology, but for convenience it will be assumed that only production is state dependent. The most important point to realize is that it is assumed that not all of the uncertainty or risk is individual. This is important because it has been shown by Malinvaud (1973) that if all risks are individual then a Pareto optimal allocation can be supported by a set of non-contingent prices.

This chapter is organized in two parts. Section 1 develops a model of an economy with labour contracts. assumed that both employers and employees are risk averse and that employers have access to an asset market so that they can transfer wealth from one period to another. In contrast to Polemarchakis and Holmstrom it is assumed that labour services are perfectly divisible so that an optimal contract allocates both men and hours. The supply and demand functions for goods, assets and employees are identified and it is shown that a labour contract equilibrium exists.

In section 2 some of the efficiency aspects of a labour contract equilibrium are examined. It is shown that an optimal contract between an employer and an employee is both productively efficient and shares risk efficiently. It is shown that given this contract the employee will be no better off even if he has access to the asset markets in which the employer trades. To highlight the nature of the labour market equilibrium two further definitions of equilibrium are given. The first is the standard auction market equilibrium studied by Radner and Hart. The second is the auction market equilibrium with specialized skills, (where labour is immobile) suggested by Holmstrom. Theorem 3 contrasts these three definitions. An example is then given where at the auction market equilibrium every agent has an incentive to enter into a labour contract, but if every agent does do this every agent is made worse off. Finally a constrained pareto optimum is defined and it is shown that the labour contract equilibrium is not a constrained pareto optimum. Concluding comments are contained in section 3.

Section 1 : The model

In this section a model of a labour contract economy is developed. This is done in a number of phases. First the structure of the economy is described, this includes the temporal structure, the number and type of agents, goods and assets. Second the preferences and technology available to each agent is described. Third the price vector for this economy is specified and hence each agents budget and demand sets are delineated. Finally the labour contract equilibrium is defined and shown to exist.

Temporal Structure

There are two time periods $t = 0, 1$. In the first period there is no uncertainty. In the second period there are S possible states of nature $s = 1 \ldots\ldots S$. It will often be convenient to denote the first time period as $s = 0$. It is not known at the first date which of the S possible states of nature will prevail in the second period. Therefore the time path of the economy is fundamentally uncertain. However it is assumed that once the second period arrives each agent knows which state of nature has occurred. Thus each agent has complete and symmetric information or each agent has the same information partition. This is to be contrasted with the models of asymmetric and imperfect information models developed in chapters 3 and 4. The restriction to two periods is inessential, more generality can be easily bought, but at the cost of some considerable complication in notation.

Agents

There is a measure space of agents (A,B,v)[3] A denotes the set of agents and B the Borel set of subsets of A. The probability measure v, is defined on (A,B) where $v(A) = 1$.

There are two classes of agents, employers and workers. These two classes will be represented by two subsets of A, W the set of workers and E the set of employers. It will be assumed that all workers are identical. Again this is inessential, W could be sub-divided into sets of types of workers. However it will be assumed that all employers are non-identical. A representative employer will be indexed by a superscript, e and where necessary a worker by the superscript w. Since it is assumed that all workers are identical in every respect except the employer for whom they work, it will often be convenient to distinguish workers by the same superscript e. Whether e refers to an employer or a worker should always be obvious from the context.

The following assumptions about the measure space of agents are made

A.2.1 (i) $W = A \setminus E = \{a \in A | a \notin W\}$

(ii) $v(W) = \tau \qquad v(E) = 1 - \tau$

(iii) $v(\{w\}) = v(\{e\}) = 0.$

Assumption A.2.1 (i) shows that the class of employers and the class of workers are exhaustive and mutually exclusive. In particular no agent is both an employer and a worker. The number τ represents the relative number of workers in the economy. On average each employer will employ $(\tau/(1-\tau))$ workers. However the actual number of employees in each firm will be determined in equilibrium by preferences and technology. Each agent a is atomless, $v(\{a\}) = 0$, this means that each agent acting individually makes no contribution to the economy. This justifies the assumption that agents take the economy's price vector as given. It is also technically convenient for the reasons outlined in section 0.

Goods

There are G physically distinct commodities indexed $g = 1 \ldots G$, and labour. It will be convenient to denote labour as good 0. Since there are (S+1) states of nature, including the initial time periods the commodity space has (G+1)(S+1) dimensions. The assumption that there is only one type of labour can be relaxed without altering the analysis.

Assets

There are K assets indexed k = 1 .... K, where K < S. That is to say the economy is not 'spanned' by the set of assets. Associated with the kth asset is the vector $(b_k \ldots b_{ks})$ where b_{ks} is a vector of commodities. The gth component of this vector is denoted b_{ksg}. If an agent holds a_k units of the kth asset in the initial period then he has a claim to $a_k b_{ksg}$ units of good g if state s occurs in the second period. If $a_k < 0$ then the agent promises to deliver $a_k b_{ksg}$ units of good g if state s occurs. It will be assumed that $b_{sk} \geq 0$ for each asset and for each state. Defining $B_s = (b_{1s} \ldots b_{ks})$ an agent holding an asset portfolio a, has a claim to a vector of commodities $B_s a$ in state s = 1 ... S.

In the same way that it is assumed that the set of markets is incomplete it will be assumed that only employers and not workers can trade in asset markets. This rather strong assumption is meant to reflect the common observation that employers have greater access to capital markets for diversifying risks.

Preferences

All workers are indentical, bar the employer for whom they work. Therefore it will be convenient to let each workers allocation decisions be identified by their employer's superscript e. An employee who works for an employer e consumes goods x^e and supplies labour ℓ^e. The consumption vector x^e has G(S+1) dimensions and x^e_{gs} is the consumption of good g in state s of a worker employed by firm e. The vector x^e_s

denotes the bundle of goods consumed in states = 0,1 ... S. The labour supply vector ℓ^e has (S+1) dimensions, ℓ^e_s is the labour supply of a worker employed by firm e in state s = 0,1 ... S. The following assumptions are made about the workers preferences

A.2.2 (i) All workers have the same consumption possibility set $X = R_+^{(G+1)(S+1)}$

(ii) All workers preferences can be represented by a twice continuously differentiable strictly concave, bounded and strictly monotone utility function, i.e.,

$U = U(x^e, \ell^e) : R_+^{(G+1)(S+1)} \to R$; is c^2 and strictly concave

where $\partial u / \partial x^e_{gs} > 0 \quad g = 1 \dots G, \quad s = 0,1 \dots S$ and $\forall\, e \in E$.

$\partial u / \partial \ell^e_s < 0 \quad s = 0,1 \dots S$ and $\forall\, e \in E$

(iii) $\lim\limits_{x^e_{gs} \to 0} \partial u / \partial x^e_{gs} = +\infty$ for some $g = 1 \dots G \quad s = 0,1 \dots S$ and $\forall\, e \in E$.

The strict concavity of the utility function implies that every worker is income risk averse, that is they will never accept a fair gamble on their income. Notice that since labour is measured as positive and because all goods are 'good' the consumption possibility set can be placed in the positive orthant of the commodity space. Notice also that it is not assumed that the workers possess a Von-Neumann-Morgenstern utility function or that the workers know the objective probability distribution over the set of states in the second period. In this context the monontonicity of preferences implies that every worker believes that every state s = 1 ... S, may occur. Assumption A.2.2(iii) guarantees that every workers wage income is strictly positive. This in turn implies that every workers budget set has a non-empty interior.[4]

An employer e consumes goods z^e. The consumption vector z^e has

G(S+1) dimensions where z^e_{gs} is the consumption of good g in state s by employer e and z^e_s is the bundle of commodities consumed by the employer e in state s. The set of assumptions A.2.3 made about each employers preferences are analogous to the assumptions A.2.2 made about workers preferences and therefore the same comments apply

A.2.3 (i) Each employer $e \in E$ has the same consumption possibility set $Z = R_+^{G(S+1)}$

(ii) Each employer $e \in E$ has preferences that can be represented by a twice continuously differentiable concave and strictly monotone, utility function that is

$$v^e = v^e \quad (z^e) : R_+^{G(S+1)} \rightarrow R \text{ ; is } c^2 \text{ and concave}$$

where $\partial v^e / \partial z^e_{gs} > 0 \quad g = 1 \ldots G, \quad s = 0 \ldots S \text{ and } \quad \forall e \in E$

(iii) $\lim\limits_{z^e_{ys} \rightarrow 0} \partial v^e / \partial z^e_{gs} = \infty$ for some $g = 1 \ldots G$, $s = 0,1 \ldots S$ and $\forall e \in E$.

Technology

Each employer $e \in E$ has access to a production technology that transforms current inputs into current outputs. It was noted in the introduction that this technology does not admit investment. This allows a conceptually simple separation between the employers production and consumption decisions. In general, however, the technology will be both firm specific and state dependent. The total amount of labour used by a firm e in state s is denoted L^e_s. This labour input consists of the labour supplied by each employee of firm e in state s, ℓ^e_s multiplied by the number of men hired by firm e, n^e. Hence, it is being assumed that the number of men employed and the number of hours they work are perfectly substitutable in production. It was shown in chapter 1A that this assumption is sufficient for each employee to be offered the same contract. The current output of firm e in state s is denoted y_s^e. The gth component of y_s^e is y_{gs}^e which is positive if y_{gs}^e is an output and negative if it is an input. It is assumed that the technology can be defined by a production set $F^{e,s}(y_s^e, L_s^e) \geq 0$ where the production frontier $F^{e,s}(y_s^e, L_s^e)$ satisfies

A.2.4 (i) For each employer $e \in E$ and each state $s = 0,1 \ldots S$ the production frontier $F^{e,s}(y_s^e, L_s^e)$ is twice continuously differentiable and strictly concave.

(ii) For each employer $e \in E$ and each state $s = 0, 1 \ldots S$

$$\partial F^{e,s}(y_s^e, L_s^e)/\partial L_s^e > 0$$

$$\lim_{L_s^e \to 0} \partial^2 F^{e,s}(y_s^e, L_s^e)/\partial (y_{gs}^e)^2 = 0 \text{ for some } g = 1 \ldots G$$

$$\lim_{L_s^e \to \infty} \partial^2 F^{e,s}(y_s^e, L_s^e)/\partial (y_{gs}^e)^2 = -\infty \qquad g = 1 \ldots G$$

(iii) For each employer $e \in E$ and for each state $s = 0, 1 .. S$

If $L_s^e = 0$, $y_s^e \geqq 0$ and $F^{e,s}(y_s^e, L_s^e) \geq 0$, then $y_s^e = 0$

If some y_s^e and $L_s^e > 0$ s.t. $F^{e,s}(y_s^e, L_s^e) > 0$

For $L_s^e \geq 0$ $F^{e,s}(0, L_s^e) \geq 0$.

If $L_s^e \geq 0$, $F^{e,s}(y_s^e, L_s^e) \geq 0$ then for $y_s^{e\prime} \leq y_s^e$

$$F^{e,s}(y_s^{e\prime}, L_s^e) \geq 0.$$

Assumption A.2.4(ii) guarantees that labour will always be used as an input into the production process. That is each employee will supply a strictly positive labour input. Assumptions A.2.4(iii) are respectively, not free production, the production set has an interior point, zero production is always possible and free disposal.

Prices

Consider the G physically distinct commodities. The price of good g in state s is denoted p_{gs}. p_s is the vector of commodity prices in state s and p is the collection of all such vectors, one for each state. The vector p has G(S+1) dimensions. Notice that period two prices, p_s, s = 1, ... S are essentially expected spot prices. This will be made explicit when the appropriate budget constraints are specified. For the moment it is important to realize that future prices are expected with certainty by every agent and if every agent is intertemporally consistent in his decision making these prices will equilibriate the market.

In addition to the G physically distinct goods there are the K assets or securities which the employers trade amongst themselves in the initial period. The vector of asset prices is denoted q a typical element of which is q_k.

The only price left to specify is the price of labour or more specifically the price of employees. In a labour contract economy where labour is immobile ex post, employees or workers will be allocated to employers on the basis of the lifetime expected utility the employer offers them. To make this more precise suppose there is some initial position, say t = -1, in which a contract market is operative. A contract offered by an employer e, $\hat{\delta}^e$ will specify the amount of labour to be supplied by the worker ℓ_s^e and the recompence offered r_s^e in each state and time period s = 0, 1 ... S. Therefore the contract is specified by the pair $\hat{\delta}^e = \{r^e, \ell^e\}$. Employees will be attracted to the employer who offers the best contract. This will be judged upon the expected lifetime utility the contract yields the worker. Competition between employers and between employees will force every employer to offer a contract yielding the same level of expected utility, $\bar{u}$. Employers must also and simultaneously choose how many workers to employ, that is they must choose an employment plan $\delta^e = \{\delta^e, n^e\}$. The price of employees $\bar{u}$ will adjust so as to equate the demand for workers to the supply. Since every agent is insignificant no agent will be able to alter $\bar{u}$ or any other price by his own actions. Hence $\bar{u}$, p and q can each be treated parametrically by agents. Therefore the price system for the labour contract economy is the triple $(p,q,\bar{u})$.

Budget Sets

In this subsection the budget set of each agent will be defined for a given set of employment plans that is $\{\delta^e\}_{e \in E}$. Therefore for each employer $e \in E$ the vector ℓ^e and r^e and the number n^e are taken as given. However this is not enough to define the employers wealth which will be specified first.

Since the employer can only transform contemporaneous inputs into contemporaneous outputs the best the employer can do is to maximize profits in each state. This is done by choosing the input-output vectors that solves

$$\text{P.2.1} \qquad \max \qquad \pi^e_s = p_s y^e_s - R^e_s \quad \text{s.t. } F^{e,s}(y^e_s, L^e_s) \geq 0.$$

where R^e_s is the total wage bill of employer e in state s and L^e_s is the total labour input of employer e in state s. Letting $\gamma^{e,s}$ be the Lagrangian multiplier for P.2.1 the first order conditions are:

$$p_{gs} + \gamma^{e,s} \; F^{e,s}(y^e_s, L^e_s)/\partial y^e_{sg} = 0 \qquad g = 1 \ldots G \qquad (2.1)$$

The solution to P.2.1 is $y^e_s = \chi^e_{y_s}(p_s, L^e_s)$, where $\chi^e_y(p, L^e)$ is the (S+1) dimensional vector of supply functions. The supply function $\chi^e_{y_s}(p_s, L^e_s)$ is unique and continuous. Hence the restricted or variable profit function $\tilde{\pi}^{e,s}(p_s, L^e_s, R^e_s)$ can be defined as

$$\tilde{\pi}^{e,s}(p_s, L^e_s, R^e_s) = p_s \chi^e_{y_s}(p_s, L^e_s) - R^e_s \qquad (2.2)$$

Lemma 2.1 The restricted profit function $\tilde{\pi}^{e,s}(p_s, L_s^e, R_s^e)$ has the following properties:

(i) Continuous

(ii) Non-negative variable costs

(iii) Sheppards Lemma, $\partial\tilde{\pi}^{e,s}(p_s, L_s^e, R_s^e)/\partial p_s = \chi_{y_s}^e$

(iv) Homogeneous of degree 1 in p_s and R_s^e

(v) Convex in p_s

(vi) Concave in L_s^e .

Proofs: (i) Since $\chi(p, L)$ is continuous $\tilde{\pi}(p,L,R)$ is continuous

(ii) Since $y = 0$ is feasible $\tilde{\pi}(p,L,R) \geq -R$.

(iii) $\partial\tilde{\pi}(p,L,R)/\partial p = \chi + p\partial\chi(p,L)/\partial p = \chi$ since $F(y,L) = 0$ at the production frontier.

(iv) Multiplying p and R by $\alpha > 0$, $\alpha py - \alpha R = \alpha\pi$, therefore the solution to P.2.1 is unchanged and $\tilde{\pi}(p,L,\alpha R) = \alpha\tilde{\pi}(p,L,R)$

(v) For p and p′ , $p(p',L) - R \leq \tilde{\pi}(p,L,R)$ and $p'y(p',L) - R = \tilde{\pi}(p',L,R)$. Therefore $\tilde{\pi}(p,L,R) \geq \tilde{\pi}(p',L,R) + (p-p')\chi(p',L)$. Consequently $\chi(p',L)$ is a subgradient of $\tilde{\pi}(p,L,R)$ at p′ and hence $\tilde{\pi}(p,L,R)$ is convex in p.

(vi) Let $\tilde{\pi}(p,L',R) = py' - R$ and $L = \alpha L + (1-\alpha)L'$ and $\hat{y} = \alpha y + (1-\alpha)y'$ for some $\alpha \in (0,1)$. Then because $F(y,L)$ is concave $F(\hat{y},\hat{L}) \geq 0$ and therefore $\tilde{\pi}(p,\hat{L},R) \geq p\hat{y} - R = \alpha py - \alpha R + (1-\alpha)py' - (1-\alpha)R = \alpha\tilde{\pi}(p,L,R) + (1-\alpha)\tilde{\pi}(p,L',R)$. Hence $\tilde{\pi}(p,L,R)$ is concave in L.

Lemma 2.1 specifies the employers wealth in each state of nature and time period. Given this information it is now possible to delineate the employers budget set. It is fundamental that in an economy with an incomplete set of markets, that is in a sequential economy, each agent faces a series of budget constraints rather than just a single budget constraint. In the economy outlined above each agent faces S+1 budget constraints, one for each time period and state.

Consider first the employers budget constraint at the first date t = 0. The employers wealth in the first period is simply the profits of the firm that he owns, π_o^e . He can allocate this wealth in one of two ways. First he can purchase commodities, z_o^e for current consumption. Second he can choose to hold a portfolio of assets a^e that promise him delivery of, or commits him to deliver a given vector of goods in the second period. Therefore the first period budget constraint of an employer e is

$$p_o z_o^e + qa^e \leq \pi_o^e \tag{2.3}$$

In the second period in some state s = 1... S the employers wealth consists both of the profits of the firm he owns and the implicit interest payments on his portfolio holdings. Therefore an employer e's budget constraint in state s = 1 ... S is

$$p_s z_s^e \leq \pi_s^e + p_s B_s a^e \tag{2.4}$$

Together the budget constraints of equations (2.3) and (2.4) constitute the budget set of employer e

$$\hat{B}^e(p,q,\pi^e) = \{(z^e,a^e) \mid p_o z_o^e + qa^e \leq \pi_o^e ,$$
$$p_s z_z^e \leq \pi_s^e + p_s B_s a^e \quad s = 1\ldots S . \}$$

Although no employer plans to be insolvent in any state of nature the budget set $\hat{B}^e(p,q,\pi^e)$ may still be unbounded. This is possible because one group of employers may go very long on one group of assets and very short on another group of assets whilst another group of employers does the exact opposite. Therefore in order to make each employers budget set bounded it is necessary to place an arbitrary set of bounds, c on the employers asset trading possibilities. Then the bounded budget set $B^e(p,q,\pi^e c)$ can be defined

$$B^e(p,q,\pi^e c) = \{(z^e,a^e) \mid (z^e,a^e) \in \hat{B}^e(p,q,\pi^e),$$

$$|a_k^e| \leq c_k \qquad k = 1 \dots K \} . \tag{2.5}$$

The worker employed by firm e also faces (S+1) budget constraints. However the worker is unable to transfer wealth over time because he cannot trade in assets. Therefore the budget set for a worker employed by firm e is simply[6]

$$D^e(p,r^e) = \{x^e \mid p_s x_s^e \leq r_s^e \qquad s = 0,1 \; \dots . \; S \} \tag{2.6}$$

Demand Sets

In this subsection each agents demand sets are examined. First for a given set of employment plans the demand of the employer for goods and assets and the demand of the worker for goods is considered. Then the employers demand for workers is examined.

Consider an employer e, given his wealth π^e the employer has a two stage decision process. In period zero he will chosse a consumption bundle and an asset portfolio. In period one the employer allocates both profit and portfolio wealth to consumption. However the employers decision can be treated as a one period problem that is maximizing utility subject to the budget set given in equation (2.5). In doing this it must be remembered that the choices of second period consumption bundles are entirely hypothetical except for the state of nature that actually occurs. Then an employer e solves

P.2.2 $\max_{z^e a^e} \quad V^e \; (z^e) \quad \text{s.t} \quad (z^e,a^e) \in B^e(p,q,\pi^e,c)$

Letting α^e denote the set of S+1 multipliers for the budget constraints (2.3) and (2.4) and ξ be the set of K multipliers for the asset bounds the first order conditions for P.2.2. are

$$\partial v^e (z^e)/\partial z^e_{gs} = \alpha^e_s p_{sg} \qquad s = 0, 1 \ldots S \text{ and } g = 1 \ldots G \quad (2.7)$$

$$\alpha_o q_k - \sum_{s=1}^{S} \alpha_s p_s b_{sk} - \xi_k = 0 \quad k = 1 \ldots. K. \qquad (2.8)$$

A solution or demand set for z^e and a^e can be written as $\chi^e_z(p,q,\pi^e,c)$ and $\chi^e_a(p,q,\pi^e,c)$ respectively.

Lemma 2.2 For every strictly positive price vector $(p,q) \gg 0$ the demand correspondences $\chi^e_z(p,q,\pi^e,c)$ and $\chi^e_a(p,q,\pi^e,c)$ are non-empty, compact, convex and upper semi-continuous.

Proof: (1) Consider the budget constraint (2.4). It will be shown below that $\pi_s > 0 \quad s = 0,1 \ldots S$. Given A.2.3(iii) it can be assumed that $\pi_s + p_s B_s a > 0$. Therefore for $p \gg 0$ the budget constraint will be non-empty and compact and continuous in p_s, π_s and a. In the second period the employer maximizes utility by choosing z_s subject to (2.4) for a given z_o and a. From the maximum theorem the demand correspondence for z_s, $\chi_{z_s}(z_o, zp_s, \pi_s)$ will be non-empty, compact, convex and upper semi-continuous and the maximum value function $\tilde{v}(z_o, a, p^1{}_s\pi')$ will be continuous, where $p^1 = (p_1 \ldots p_s)$ and $\pi^1 = (\pi_1 \ldots \pi_s)$.

(2) Consider the constraint (2.3). Since $\pi_0 > 0$ and $(p_o,q) \gg 0$ the budget constraint (2.3) is non-empty, compact and continuous in p_0, q and π_0. The employer chose z_o and a in the first period to maximize $\tilde{v}(z_o,a,p',\pi')$ subject to equation (2.3). By the maximum theorem $\chi_{z_o}(p,q,\pi,c)$ and $\chi_a(p,q,\pi,c)$ are non-empty compact convex and upper semi-continuous. Then substituting into $\chi_{z_s}(z_o,a_s,p_s,\pi_s)$ proves the Lemma.

Lemma 2.3 The indirect utility function for P.2.2 $\hat{v}^e(p,q,\pi^e,c)$ has the following properties:

(i) Continuous

(ii) Homogeneous of degree 0 in p,q and π^e

(iii) Non-decreasing and concave in π^e

(iv) Quasi-convex in p and q

(v) Roys identity, $\partial\hat{v}^e/\partial p_s = \partial\hat{v}/\partial\pi^e_s(-z^e_s) = -\alpha^e_s z^e_s \quad s = 0,1 \ldots S$

$$\partial\hat{v}^e/\partial q_k = \partial\hat{v}/\partial\pi^e_s(-a^e_k) = -\alpha^e_s a^e_k \quad s = 0,1 \ldots S$$

Proof:(i) From Lemma 2.2 and the maximum theorem $\hat{v}(p,q,\pi,c)$ is continuous.

(ii) Multiplying p,q, and π by $\alpha > 0$ leaves the budget set $B(p,q,\pi,c)$ unchanged.

(iii) (1) Clearly $B(p,q,\pi,c) \subseteq B(p,q,\pi',c)$ for $\pi' \geq \pi$. Therefore $\hat{v}(p,q,\pi,c)$ is non-decreasing in π.

(2) Let $\alpha\varepsilon(0,1)$, and $\hat{\pi} = \alpha\pi + (1-\alpha)\pi'$. Then $\hat{v}(p,q,\hat{\pi},v) = \max\{ v(z) | (z,a) \in B(p,q,\hat{\pi},c)\} \geq \max\{ v(\hat{z}) | \hat{z} = \alpha z + (1-\alpha)z',$ $\hat{a} = \alpha a + (1-\alpha)a', (z,a)\varepsilon B(p,q,\pi,c)\ (z',a\ ) \in B(p,q,\pi',c)\} \geq$ $\alpha\max\{v(z) | (z,a) \in B(p,q,\pi,c)\} + (1-\alpha)\max\{v(z\ ) | \ (z',a') \in B(p,q,\pi',c)\} =$ $\alpha\hat{v}(p,q,\pi,c) + (1-\alpha)\ \hat{v}(p,q,\pi',c)$.

(iv) Let (p,q) and (p',q') be chosen so that $\hat{v}(p,q,\pi,c) \leq k$ and $\hat{v}(p',q',\pi,c) \leq k$ for some number k. Let $\hat{p} = \alpha p + (1-\alpha)p'$ and $q = \alpha q + (1-\alpha)q'$ for $\alpha\ \varepsilon(0,1)$. Then $B(\hat{p},\hat{q},\pi,c) \subset (B(p,q,\pi,c) \cup B(p',q',\pi,c))$. To see this suppose $(\tilde{z},\tilde{a}) \in B(\hat{p},\hat{q},\pi,c)$ and $(\tilde{z},\tilde{a}) \notin B(p,q,\pi c)$ or $B(p',q',\pi,c)$. Then $\hat{p}_o\tilde{z} + \hat{q}\tilde{a} \leq \pi_o$, $\hat{p}_s\tilde{z}_s \leq \pi_s + \hat{p}_s B_s\tilde{a}$, $p_o\tilde{z} + q\tilde{a} > \pi_0$ $p_s\tilde{z} > \pi_s + p_s B_s\tilde{a}$, $p'_o\tilde{z} + q'\tilde{a} > \pi_0$and $p'_s\ \tilde{z}_s > \ \pi_s + p'_s B_s\tilde{a}$. Then multiplying the third and fourth inequalities by α and the fifth and sixth by $(1-\alpha)$ gives a contradiction. Therefore $\hat{v}(\hat{p},\hat{q},\pi,c) \leq \max\{\hat{v}(p,q,\pi,c)\ ,\ \hat{v}(p',q',\pi,c)\} \leq k$ and hence $\hat{v}(p,q,\pi,c)$ is quasi-convex in p and q.

(v) To prove Roy's identity it is necessary to assume the demand correspondences χ_z and χ_a are differentiable functions. Then $\partial\hat{v}/\partial\pi_{s'} = \sum_s (\partial\hat{v}/\partial z_s)(\partial z_s/\partial\pi_{s'}) = \alpha_{s'}$ using equations (2.7) and (2.8)

Similarly $\partial\hat{v}/\partial p_{s'} = \sum_s (\partial v/\partial z_s)(\partial z_s/\partial p_{s'}) = -\alpha_{s'} z_{s'}$

and $\partial\hat{v}/\partial q_k = \sum_s (\partial\hat{v}/\partial z_s)(\partial z_s/\partial q_k) = -\alpha_s a_k$ using equations (2.7) and (2.8).

Corollary 2.1 Writing $\pi^e_s = \tilde{\pi}^e_s (p_s, L^e_s, R^e_s)$ and substituting $\hat{v}^e(p,q,\pi^e,c) = \hat{v}^e(p,q,L^e,R^e,c)$, then $\hat{v}^e(p,q,L^e,R^e,c)$ is concave in L^e and R^e.

Proof: From Lemma 2.1 $\tilde{\pi}^e_s$ is concave in L^e_s and R^e_s and from Lemma 2.3 $\hat{v}(p,q,\pi,c)$ is concave in the vector π. Therefore $\hat{v}(p,q,L,R,c)$ is concave in L and R.

The worker employed by firm e has an essentially similar decision problem to his employer except that workers are unable to trade in assets. Therefore for a given set of employment plans the worker solves

$$\text{P.2.3} \quad \max_{x^e} \quad U(x^e, \ell^e) \qquad \text{s.t. } x^e \in D^e(p, r^e)$$

Letting β^e_s be the appropriate set of Lagrangian multipliers the first order conditions for P.2.3. are

$$\partial u\,(x^e,\ell^e)/\partial x^e_{gs} = \beta^e_s p_{sg} \qquad s = 0,1 \ldots S \text{ and } \quad g = 1 \ldots G \qquad (2.9)$$

It will be shown below that $r^e_s > 0$ for all $e \in E$ and for all $s = 0, 1 \ldots S$. Therefore for $p \gg 0$ the budget set $D^e(p,r^e)$ is non-empty and continuous in p and r^e. This means that the workers demand function $\chi^e_x(p,r^e)$ is differentiable. In addition χ^e_x can be decomposed into its component functions $\chi^e_{x_s}(p_s, r^e_s)$ because the workers cannot directly trade across time or states.

Lemma 2.4 The indirect utility function for P.2.3 $\hat{u}^e(p,r^e,\ell^e) = u^s(\chi^e_x(p,r^e), \ell^e)$ has the following properties

(i) Continuous and bounded above

(ii) Non-decreasing and strictly concave in r^e

(iii) Strictly quasi-convex in p

(iv) Homogeneous of degree 0 in p and r^e

(v) Roy's identity, $\partial\hat{u}^e/\partial p_s = \partial\hat{u}^e/\partial r_s^e(-x_s^e) = -\beta_s^e x_s^e$

$$s = 0,1 \ldots S.$$

Proof: As Lemma 2.3 mutatis mutandis.

It is only left to specify how the optimal employment plan or contract $\delta^e = \{n^e, r^e, \ell^e\}$ is chosen. The employer will choose that employment plan δ^e, that maximizes his expected indirect utility. Competition between employers will ensure that each worker is offered the same level of expected indirect utility $\bar{u}$. Then the employers choice of employment plan is restricted to the budget set

$$\Delta(p,\bar{u}) = \{(r^e,\ell^e) \,|\hat{u}^e(p,r^e,\ell^e) \geq \bar{u}\} \tag{2.10}$$

The set $\Delta(p,\bar{u})$ is convex and continuous in p and $\bar{u}$ for $p \gg 0$.

At this stage it is necessary to introduce one further assumption

A.2.5 $\exists\, \delta^e \varepsilon\, \Delta(p,\bar{u}) \quad$ s.t. $\hat{v}^e(p,q,\pi^e(p,\delta^e),c) > v^e(p,q,o,r)$

$$\forall\, e \in E.$$

Assumption A.2.5. guarantees that every employer will choose to offer a labour contract, that is $n^e > 0$. Notice that the employers objective function $\hat{v}^e(p,q,\delta^e,c) = \hat{v}^e(p,q,\pi^e(p,\delta^e),c)$ is not concave in the choice variables δ^e. It was noted in chapter 1A that the revenue function is quasi-concave in n^e and ℓ_s^e and that the cost function is quasi-concave in n^e and r_s^e for each state of nature $s = 0,1 \ldots S$. Thus, in general the choice of the optimal employment plan will not be unique.

The employer chooses δ^e by maximizing $\hat{v}^e(p,q,\delta^e,c)$ subject to equation (2.10), that is he solves

P.2.4 $\max_{\delta} \quad v^e(p,q,\delta^e,c) \quad s.t \ \delta^e \varepsilon \ \Delta\ (p,\bar{u})$

Because of assumption A.2.5 the constraint $\hat{v}^e(p,q,\pi^e,c) \geq \hat{v}^e(p,q,o,c)$ need not be taken into account in P.2.4. Similarly it will be shown that $\bar{u} > \hat{u}^e(r, 0, 0)$ so that workers will always be willing to participate in the optimal contract. The demand set for P.2.4, is defined as

$$\chi_\delta^e\ (p,q,\bar{u},c) = \{\ \delta^e \varepsilon\ \ \Delta(p,\bar{u}) \mid \hat{v}^e(p,q,\delta^e,c)$$

$$\geq v^e(p,q,\delta^{e'},c) \quad \forall \delta^{e'} \varepsilon\ \ \Delta(p,\bar{u})\ \}.$$

Lemma 2.5 The demand set $\chi_\delta^e(p,q,\bar{u},c)$ has the following properties

(i) Non-empty and compact for $p \gg 0$

(ii) Upper semi-continuous in p, q and $\bar{u}$ for $p \gg 0$.

Proof: (i) Since $u(p,r,\ell)$ is bounded $\Delta(p,\bar{u})$ is non-empty and compact for $p \gg 0$. By assumption A.2.4.(ii) n can be restricted to a compact set. Therefore since $v(p,q,\pi,c)$ and $\tilde{\pi}_s(p_s,L_s,R_s)$ are continuous for $p \gg 0$. $\chi_s(p,q,\bar{u},c)$ is non-empty and compact.

(ii) This follows from the maximum theorem since $\hat{v}(p,q,\pi,c)$ $\tilde{\pi}_s(p_s,L_s,R_s)$ and $\Delta(p,u)$ are continuous for $p \gg 0$.

Lemma 2.6 The optimal employment plan arranged between an employer and a worker has the following properties

(i) $(r^e,\ell^e,n^e) \gg 0$ and $\tilde{u}^e(p,r^e,\ell^e) = \bar{u}$

(ii) Risk is shared efficiently between each employer and his workers, that is, the ratio of the employers marginal utility of income and the workers marginal utility of income is a constant independent of the state of nature, or

$$\alpha_s^e/\beta_s^e = \lambda_o^e/n^e \quad = \text{constant} \tag{2.11}$$

(iii) Within each firm the employment plan is productively efficient, that is, the marginal rate of transformation between labour and any good is equal to the marginal rate of substitution between labour and that good or

$$\partial y^e_{gs}/\partial L^e_s = -\ \partial u^e/\partial \ell^e_s/\partial u^e/\partial x^e_{gs} \quad g = 1 \ldots G \text{ and } \quad s = 0,1 \ldots S \tag{2.12}$$

(iv) Labour is allocated so that the expected marginal product of labour equals the expected real wage when weighted by the employers marginal utility of income, that is

$$\sum_{s=0}^{S} \alpha^e_s(p_s(\partial y^e_s/\partial L^e_s)\ell^e_s - r^e_s) = 0 \tag{2.13}$$

Proof (i) From A.2.5 $n > 0$. To show $(r,\ell) \gg 0$ let λ_0 be the Lagrangian multiplier for the constraint (2.10) and let λ^1_s and λ^2_s be the multipliers for the non-negativity constraints $r_s \geq 0$, $\ell_s \geq 0$ respectively. Then the first order conditions for P.2.4 are

$$(\partial\hat{v}/\partial\pi_s)(\partial\pi_s/\partial r_s) + \lambda_0\partial\tilde{u}/\partial r_s + \lambda^1_s = 0 \tag{2.14}$$

$$(\partial\hat{v}/\partial\pi_s)(\partial\pi_s/\partial\ell_s) + \lambda_0\partial\hat{u}/\partial\ell_s + \lambda^2{}_s = 0 \tag{2.15}$$

$$\sum_{s=0}^{S}(\partial\hat{v}/\partial\pi_s)(\partial\pi_s/\partial n) = 0. \tag{2.16}$$

To show $r_s > 0$ suppose $r_s = 0$. Then $\lambda^1_s \geq 0$ but this contradicts A.2.2(iii) that $\partial\hat{u}/\partial r_s = \beta_s = \infty$ at $r_s = 0$. Therefore $r_s > 0$, $\lambda^1_s = 0$. Similalry if $\ell_s = 0$, $\lambda^2_s \geq 0$ but then $\partial\pi_s/\partial\ell_s = nz_g p_{sg}(\partial^2F/\partial y_{sg}\partial L_s)/(\partial^2F/\partial(y_{sg})^2) = \infty$ at $\ell_s = 0$ from A.2.4(ii) which is a contradiction. Therefore $\ell_s > 0$ and $\lambda^2_s = 0$. Since $\hat{v}(p,q,\pi,c)$ is increasing in $\pi, \hat{u}(p,r,\ell) = \bar{u}$.

(ii) - (iv) These follow directly upon substituting equations (2.1), (2.7) and (2.9) into equations (2.14) - (2.16).

Lemma 2.5 summarizes the nature of the optimal employment contract. The most important feature for the present purposes is how the optimal employment contract depends upon the prices $(p,q,\bar{u})$. Using this information each agents demand for goods or assets will satisfy

$$z^e \in \chi_z^e(p,q,\bar{u}) = \chi_z^e(p,q,\pi^e(p,\chi_L^e(p,q,\bar{u}), \chi_R^e(p,q,\bar{u}))$$

$$a^e \in \chi_a^e(p,q,\bar{u}) = \chi_a^e(p,q,\pi^e(p,\chi_L^e(p,q,\bar{u}), \chi_R^e(p,q,\bar{u}))$$

$$y^e \in \chi_y^e(p,q,\bar{u}) = \chi_y^e(p,\chi_L^e(p,q,\bar{u}))$$

$$x^e \in \chi_x^e(p,q,\bar{u}) = \chi_x^e(p,\chi_r^e(p,q,\bar{u}))$$
$$n^e \in \chi_n^e(p,q,\bar{u})$$

for every $e \in E$.

Equilibrium

This subsection shows that a labour contract equilibrium exists. First the labour contract equilibrium is defined as

Definition 2.1 An allocation (z^e,z^e,y^e,a,n^e) for all $e \in E$ and a price system $(p,q,\bar{u})$ is called a labour contract equilibrium if

(i) $z^e \in \chi_z^e(p,q,\bar{u})$, $x^e \in \chi_x^e(p,q,\bar{u})$ $y^e \in \chi_y^e(p,q,\bar{u})$

$a^e \in \chi_a^e(p,q,\bar{u})$ and $n^e \in \chi_n^e(p,q,\bar{u})$ $\forall e \in E$

(iii) $\int_E (z^e + n^e x^e - y^e)\, dv = 0,$

$\int_E a^e\, dv = 0$ and $\int_E n^e\, dv - \tau = 0.$

Proposition 2.1. A labour contract equilibrium exists.

Proof (1) Notice it has been implicitly assumed that $p_s = (1,p_{2s} \ldots p_{gs})$ for all $s = 0,1 \ldots S$ because profits and wage income have been defined in terms of the numeraire commodity good 1. Therefore it is possible to redefined p and q in the space $\Delta = \{(p,q) | \sum_g p_o g + \sum_k q_k = 1, \sum_g p_{sg} = 1.$ $s = 0,1 \ldots S\}$. Since it has already been shown that $\bar{u}$ can be restricted to a convex compact set $(p,q,\bar{u}) \in T$ where T is a convex compact set.

(2) Each demand correspondence $\chi_i^e(p,q,\bar{u})$, $i = z,x,y,a,n$ is upper semi-continuous, therefore the vector valued excess demand function

$$\xi(p,q,\bar{u}) = \left(\int_E (z^e + n^e x^e - y^e)\, dv, \int_E a^e dv, \int_E n^e dv - \tau \right)$$

is also upper semi-continuous and convex-valued because each agent is atomistic.

(3) Because of assumptions A.2.2 the only possible values of $\int_E n^e dv - \tau$ are non-negative.

(4) It is possible to show that the set of attainable allocations for this economy is bounded, therefore it can be shown a la Radner that the fixed point of the correspondence $\xi(p,q,\bar{u})$ is an equilibrium.

Section 2 : Efficiency

This section examines some of the normative implications of the analysis of section 1. This will be done in three stages. First, it was assumed in section 1 that only employers and not employees can trade in assets. This assumption was made to reflect the common empirical observation that employers have better or easier access to capital markets than do employees. Proposition 2.2 shows that even if employees do have access to capital markets the equilibrium allocation will remain essentially unchanged.

Second the labour contract equilibrium is compared with two further definitions of equilibrium, the auction market equilibrium and the auction market equilibrium with specialized skills. It was noted in chapter 1 that the labour contract equilibrium has been traditionally compared with the auction market or spot market equilibrium. In the partial equilibrium analysis of chapter 1 it was shown that the labour contract equilbrium pareto dominates the auction market equilibrium. This is not longer the case in a general equilibrium model. To show this an example is given in which agents taking prices as given, have an incentive to contract but ex post each agent ends up in a worse off position. Proposition 2.3 contrasts the efficiency properties of the three types of equilibria defined.

Finally the efficiency of the labour contract equilibrium is contrasted with the constrained pareto efficient allocation. The constrained pareto efficient alllocation is that allocation that could be made by a central planner with complete information but constrained to operate through the same set of markets that are operative in the labour contract equilibrium. Proposition 2.4 shows that the labour contract equilibrium is not in fact a constrained pareto efficient.

Consider the first question. Suppose that the employee can trade in $k' \leq K$ assets. It is immediately apparent that this does not change the efficiency properties of the labour contract equilibrium.

Proposition 2.2 The efficiency of the labour contract equilibrium is unaltered even if the employee can trade in k′ assets.

Proof: Suppose the employee can trade in k′ assets. Then the employees choice of consumption must satisfy $\beta\ {}_{o}q_{k} = \sum_{s=1}^{S} \beta_s p_s\ b_{sk} \quad k = 1 \ldots k'$ in addition to (2.10). But this is automatically satisfied from equations (2.8) and (2.11).

Proposition 2.2 suggests that the differences in allocation when employees can trade in assets and when they cannot are cosmetic. That is to say that any observed difference between an employers and a workers access to capital markets in a labour contract economy is unimportant. Effectively the employer will trade in assets on behalf of his workers or actually offset any asset trading by his workers, by amending their remuneration.

To examine the efficiency of the labour contract equilibrium consider the following definitions of alternative equilibria.

Definition 2.2 An allocation (z^e, y^e, a^e, L^e) for all $e \in E$ and an allocation (x,ℓ) for all $w \in W$ and a price system (p,q,ω) is called an auction market equilibrium if

(i) (y_s^e, L_s^e) max $\pi_s^e = p_s y_s^e - \omega_s L_s^e$ s.t. $F^{e,s}(y_s^e, L_s^e)$ $s = 0,1 \ldots S$.

(z^e, a^e) max $V^{e,s}(z^e)$ s.t $(z^e, a^e) \in B^e(p,q,\pi^e)$

(x,ℓ) max $u(x,\ell)$ s.t $(x,\ell) \in \{(x,\ell) \mid px_s = \omega\ell$, $s = 0, 1 \ldots S\}$.

(ii) $\int_E (z^e - y^e)dv - \tau x = 0$

$\int_E a^e dv = 0$

$\int_E L^e dv - \tau\ell = 0$

Definition 2.3 An allocation $(z^e, y^e, a^e, x^e, \ell^e, n^e)$ for all $e \in E$, and a price system $(p,q,\omega^e,\bar{u})$ is called an auction market equilibrium with specialized skills if

(i) $(y_s^e, \hat{\ell}_s^e)$ max $\pi_s^e = p_s y_s^e - \omega_s^e n^e \hat{\ell}_s^e$ s.t $F^{e,s}(y_s^e, n^e \hat{\ell}_s^e)$

$s = 0, 1 \ldots S$.

(z^e, a^e) max $v^{e,s}(z^e)$ s.t. $(z^e, a^e) \in B^e(p,q,\pi^e)$

$(x^e, \tilde{\ell}^e)$ max $u^s(x^e, \tilde{\ell}^e)$ s.t $(x^e, \tilde{\ell}^e) \in \{(x^e, \tilde{\ell}^e) \mid$

$p_s x_s^e = \omega_s^e \tilde{\ell}_s^e$ $s = 0, 1 \ldots S\}$

n^e max $\tilde{v}^e(p,q,\pi^e)$ s.t $\tilde{u}(p,\omega^e) = \bar{u}$.

(ii) $\int_E (z^e - y^e + n^e x^e)\, dv = 0$

$\int_E a^e dv = 0$

$\int_E n^e dv - \tau = 0$

$\ell^e = \tilde{\ell}^e \; \forall e \in E$.

The notation used in definition 2.2 and 2.3 should be fairly clear from the analysis of section 1. Definition 2.2 is the standard definition of a Walrasian allocation in an economy with incomplete markets. Notice that the labour market is wage mediated at every date event pair so that there is no distinction in this definition between the number of hours worked and the number of men.

definition 2.3 has been introduced by Homlmstron (1982). The idea behind definition 2.3 is that labour is immobile and therefore workers must be assigned to employers at the first time period. This is similar to the labour contract equilibrium, however unlike the labour contract equilibrium the labour market is wage mediated at each date event pair. The equilibrium wage is of course firm specific since there is no labour mobility. Wage mediation of this type can only really be justified if there are a large number of identical firms, say in one country, between which labour is mobile. Thus labour in the auction market equilibrium with specialized skills is properly considered industry specific.

These definitions are considered in order to contrast them with definition 2.1 of a labour contract equilibrium. The following example shows that for reasonable parameter values the auction market equilibrium may pareto dominate the labour contract equilibrium. The example also illustrates, how the equilibrium of definitions 2.1 and 2.2 are determined.

Example 1. There are two goods, no assets, two employers and a continuum of employees. It is assumed that there is no production and hence no trade at t=0. At t=1 there are two equiprobable states of nature that directly affect employees preferences. Each employee has a utility function

$$u=\tfrac{1}{2}\left[a\log x_1(1) + (1-a)\log x_2(1) + (1-a)\log x_1(2) + a\log x_2(2) - (1-b)\ell(1) - (1+b)\ell(2)\right] + 2.$$

where $x_g(s)$ denote, the consumption of good g=1,2 in state s=1,2. The first employer is risk neutral and consumes only the first good so that his utility function is

$$V^1= \tfrac{1}{2}(Z_1(1) + Z_1(2).).$$

and similarly the second producer consumes only the second good

$$V^2=\tfrac{1}{2}(Z_2(1) + Z_2(2)).$$

The first employer produces only the first good and the second employer produces only the second so that

$$y_1^1(L) = (L^1(1))^{\frac{1}{2}},\ y_1^1(2) = (L^1(2))^{\frac{1}{2}},\ y_2^2(1)=(L^2(1))^{\frac{1}{2}},\ y_2^2(2)=(L^2(2))^{\frac{1}{2}}$$

where $y_g^e(s)$ is the output of good g by employer e in state s and $L^e(s)$ is the total labour employed by firm e in state s.

Labour Contract Equilibrium: In the labour contract equilibrium each employer chooses n^e the number of men to hire before the state of nature is known. Hence $L^e(s)=n^e\ell^e(s)$ where $\ell^e(s)$ is the labour service provided by an employee of firm e in state s. Since there are no assets the price of good one can be normalized to unity in each state. Then the price of good two in state s can be represented by p(s). An employee of firm e faces two budget constraints one for each state

$$x_1^e(s) + p(s)x_2^e(s) = r^e(s) \qquad s= 1,2$$

Therefore the employees demand functions are

$$x_1^e(1) = ar^e(1),\ x_2^e(1)=(1-a)r^e(1)/p(1),\ x_1^e(2)=(1-a)r^e(2),\ x_2^e(2)=ar^e(2)/p(2).$$

and the indirect utility function is

$$\hat{u}^e= a \log a + (1-a) \log (1-a) + 2 + \tfrac{1}{2}\big[\log r^e(1) + \log r^e(2) - (1-a) \log p(1) - a \log p(2) - (1-b)\ell^e(1) - (1+b)\ell^e(2)\big].$$

Each employers demand is determined directly from the budget constraint

$$Z_1(1) = (n^1\ell^1(1))^{\frac{1}{2}} - n^1r^1(1) \quad , Z_1(2) = (n^1\ell^1(2))^{\frac{1}{2}} - n^1r^1(2).$$

$$Z_2(1) = (n^2\ell^2(1))^{\frac{1}{2}} - n^2r^2(1)/p(1).\ Z_2(2) = (n^2\ell^2(2))^{\frac{1}{2}} - n^2r^2(2)/p(2).$$

Each employer chooses a contract $\delta^e=\{r^e(1),\ r^e(2),\ \ell^e(1),\ \ell^e(2)\}$ and a labour pool n^e to maximize his utility subject to offering each employee a given level of utility $\bar{u}$. By symmetry $n^e=\frac{1}{2}$ and therefore it is easy to show that the optimal contract satisfies

$$\ell^e(1) = (1+b)/(1-b),\ \ell^e(2) = (1-b)/(1+b) \qquad e=1,2.$$

$$r^1(s) = \sqrt{2}/2\sqrt{(1-b^2)} \quad r^2(s) = p(s) = \sqrt{2}/2\sqrt{(1-b^2)} \qquad s = 1,2$$

The two equilibrium conditions that determine p(1) and p(2) are

$$\tfrac{1}{2}(x_1^1(1) + x_1^2(1)) + Z_1(1) = y_1^1(1)$$

$$\tfrac{1}{2}(x_1^1(2) + x_1^2(2)) + Z_1(2) = y_1^1(2)$$

hence $p(1) = (1-a)/a \quad p(2) = a/(1-a)$

and $\hat{u} = \tfrac{1}{2}[a \log a + (1-a) \log (1-a) + a \log (1-a) + (1-a) \log a - \log (1-b) - \log (1+b)] - {}^3{}_2 \log 2.$

$$\hat{V}^1 = \hat{V}^2 = \sqrt{2}/4\sqrt{(1-b^2)}.$$

Auction Market Equilibrium: In the auction market equilibrium there is no way of trading across states of nature so that each state may be considered separately. In state one the employee solves

$$\max_{x_1(1),\, x_2(1),\, \ell(1)} \quad a \log x_1(1) + (1-a) \log x_2(1) - (1-b)\ell(1)$$

$$\text{s.t.} \quad x_1(1) + p(1)\, x_2(1) = w(1)\ell(1).$$

Then the employees demand functions are

$$x_1(1) = aw(1)/(1-b),\ x_2(1) = (1-a)w(1)/(1-b)p(1),\ \ell(1) = 1/(1-b)$$

In state one each employer chooses total labour input to maximize profits and therefore the employers labour demand functions are

$$L^1(1) = 1/4\ (w(1))^2,\ L^2(1) = (p(1))^2/4(w(1))^2.$$

The two equilibirum conditions that determine w(1) and p(1) are

$$x_1(1) = y_1^1(1) - Z_1(1) \qquad x_2(1) = y_2^2(1) - Z_2(1)$$

hence $w(1) = \sqrt{(1-b)}/2\sqrt{a} \qquad p(1) = \sqrt{(1-a)}/\sqrt{a}$

and by symmetry $w(2) = \sqrt{(1+b)}/2\ (1-a) \quad p(2) = \sqrt{a}/\sqrt{(1-a)}.$

$$\hat{u} = \tfrac{1}{2}[a \log a + (1-a) \log (1-a) - \tfrac{1}{2} \log (1-b) - \tfrac{1}{2} \log (1+b)] - \log 2.$$

$$\hat{V} = \hat{V}^2 = \tfrac{1}{4}\left[\frac{\sqrt{(1+b)}\sqrt{a} + \sqrt{(1-b)}\sqrt{(1-a)}}{(1-b^2)}\right]$$

To contrast the auction market equilibrium with the labour contract equilibrium suppose a=0.6 and b=0.2. Then it can be seen that each employer is equally well off in the contract and auction market equilibrium because

$$\sqrt{(1.2)}\sqrt{(0.6)} + \sqrt{(0.8)}\sqrt{(0.4)} = (0.6)\sqrt{2} + (0.4)\sqrt{2} = \sqrt{2}.$$

The difference between the employees utility in the labour market equilibrium and the auction market equilibrium is

$$\Delta = \tfrac{1}{2}\left[a \log (1-a) + (1-a) \log a - \tfrac{1}{2} \log(1-b) - \tfrac{1}{2} \log (1+b) - \log 2\right]$$
$$= - 0.46.$$

Therefore each employee prefers the spot market allocation, so that the auction market equilibrium (weakly) pareto dominates the labour contract equilibrium for these parameter values.

This is not, however the end of the story. Consider an employer e at the auction market equilibrium. Then it can be shown that the employer who takes the spot market prices of goods as given will be able to increase profits by offering a suitable contract to over half of the workforce. This suggests that if contracting is allowed the auction market equilibrium will not necessarily constitute a Nash equilibria. This is a similar result to that given by Polemarchakis and Weiss (1978) who give an example, with costly labour mobilitiy, where both the spot market equilibrium and the labour contract equilibrium constitute a Nash equilibria.

Since labour is mobile in the auction market equilibrium and immobile in the labour contract equilibrium a better comparison is between the auction market equilibrium with specialized skills and the labour contract equilibrium, where labour is immobile in both. However such a comparison does not negate the validity of the discussion following example 1. Indeed it is a general proposition in second best theory that multiple equilibria may be pareto ranked (see Hart (1975)), and in particular opening up new markets may lead to pareto inferior allocation in some instances. Therefore it may be conjectured that there are some economy's for which the auction market equilibrium with specialized skills pareto dominates the labour contract equilibrium.

Proposition 2.3 seeks to compare the efficiency properties of the three types of equilibria defined in definitions 2.1 - 2.3. To do this let the three equilibria be defined as X^1, X^2, and X^3 efficient and define the partial ordering > as follows; X^i is said to be at least as efficient as X^j, $X^i > X^j$ if definition i meets all the marginal conditions met by definition j and perhaps some more besides. Then

Proposition 2.3. Both the labour contract equilibrium (definition 2.1) and the auction market equilibrium (definition 2.2) are at least as efficient as the auction market equilibrium with specialised skills (definition 2.3) i.e. $X^1 > X^3$, $X^2 > X^3$.

Proof: Compare all the relevant first order conditions.

Notice that the auction market equilibrium and the labour contract equilibrium cannot be ranked according to this criteria. This is because the auction market equilibrium allocates labour efficiently across firms whereas the labour contract equilibrium allocates risk optimally within firms. The auction market equilibrium with specialised skills does neither. This suggests that the auction market equilibrium is preferable when tastes and technology are variable but that the labour contract equilibrium is preferable when workers are very risk averse.

It is not easy, in general, to characterize the efficiency of a labour contract equilibrium. It is well known that the auction market equilibrium is a social Nash Optimum (Grossman, 1977) and it is readily apparent that the auction market equilibrium with specialized skills is a Social Nash Optimum when labour is immobile. However the labour contract equilibrium depends on specifically intra-firm arrangements. Therefore a characterization of the labour contract equilibrium would probably involve a multitude of planners one for each firm, or perhaps industry. However the purpose of Proposition 2.4 is rather more limited, that is to show that a labour contract equilibrium is not a constrained pareto efficient allocation.

A constrained pareto efficient allocation is that allocation that could be achieved by a hypothetical central planner who is restricted only by the same set of factors that restricts the sets of markets to be incomplete. Unfortunately, as Hart (1975) has pointed out it is not easy to define the set of feasible allocations from which the planner may choose. Of course it is reasonable to suppose that the set of competitive equilibria belong to this set of feasible allocations. Therefore if competitive equilibria can be pareto ranked it follows that there are some competitive equilibria which are not constrained pareto efficient. This is the procedure adopted by Hart and the one to be followed below.

Consider again example 1. The marginal rates of substitution of good one and two between states one and two, in the labour contract equilibrium are

$$\partial u^e / x_1^e(1) / \partial u^e / \partial x_1^e(2) = r^e(2)/r^e(1)$$

$$\partial u^e / \partial x_2^e(1) / \partial u^e / \partial x_2^e(2) = p(1)r^e(2)/p(2)r^e(1)$$

which are not independent of the employer for whom the employee works. This suggests that an improvement can be made by introducing assets for transferring wealth. Consider the following example based on Hart's example 1.

Example 2. This is the same as example 1 except two assets are introduced at time t=0. The first asset promises to deliver one unit of the first good if state one arises and (a/(1-a)) units of good one if state two occurs. Similarly the second asset will deliver (a/(1-a)) units of good two is state one occurs and one unit of good two if state two arises. Then the labour contract equilibrium of example 1 illustrates the equilibrium when the vector of monetary returns on assets are linearly dependent across states and hence there is no trading in assets. Suppose however the vector of monetary returns is linearly independent, then since there are two goods and two assets, the economy is spanned by the set of assets and it is as if there was a complete set of markets with appropriate contingent market prices. Let these contingent market prices be $(q_1(1), q_1(2), q_2(1), q_2(2))$. Since both the employers are risk netural, in equilibrium $q_1(1)=q_1(2)$ and $q_2(1) = q_2(2)$. Letting $q_1(1) = 1$ and $q_2(1) = q$ it is possible to define the spot market prices of commodities as $p_1(1) = 1$ $p_1(2) = (1-a)/a$. $p_2(1) = q(1-a)/a$. $p_2(2) = q$. Then using the methodology of example 1 it can be shown that

$$q = \lambda^1(1-a)/\lambda^2 a$$

where $\lambda^1 = \frac{1}{4}\left[\frac{1}{1-b} + \frac{(1-a)}{a(1+b)}\right]^{\frac{1}{2}}$

$$\lambda^2 = \frac{1}{4}\left[\frac{(1-a)}{(1-b)a} + \frac{1}{1+b}\right]^{\frac{1}{2}}$$

and $x_1^1(s) = 2a\lambda^1$, $x^2(s) = 2a\lambda^2 q$ $\qquad$ s=1,2.

$$x_2^1(s) = 2a\lambda^1/q \text{ , } x_2^2(s) = 2a\lambda^2 \qquad s=1,2.$$

$$\pi^1(1) + \pi^1(2) = 2\lambda^1$$

$$(\pi^2(1) + \pi^2(2))/q = 2\lambda^2$$

$$\ell^1(2) = (1-a)^2/8(\lambda^1)^2 a^2 (1+b)^2 \qquad \ell^1(1) = 1/8(\lambda^1)^2 (1-b)^2$$

$$\ell^2(2) = 1/8(\lambda^2)^2(1+b)^2 \quad , \ell^2(1) = (1-a)^2/8(\lambda^2)^2 a^2(1-b)^2$$

Notice that provided $a \neq \frac{1}{2}$, $b \neq 0$ the vector of monetary returns in equilibrium is linearly independent. Then it is simply tedious to show that for $a = 0.6$, $b = 0.2$ the allocation of example 2 pareto dominates the allocation given by example 1. Since both allocations are potential competitive equilibria the following proposition can be derived.

Proposition 2.4 There are labour contract equilibria which are not constrained pareto optimal.

Proof: By counter example given above.

Section 3: Conclusion.

This chapter has presented a model of a labour contract economy when the set of markets is incomplete. It was suggested that labour contracts are one of the instruments available to agents in choosing how to react to the incompleteness of markets. An example was given where starting from the auction market equilibrium agents would have an incentive to contract in the labour market. The example also showed that such behaviour could make everybody worse off. It was also shown by way of a counter example that the labour contract equilibrium need not be a constrained pareto optimum.

The model presented adopted a number of unrealistic assumptions. First it was assumed that every agent had complete information, that is each agent knew which state of nature had actually occured. Formally this involved each agent knowing the production technology of every employer. An example with incomplete information will be examined in chapter four. Secondly it was assumed that agents were symmetrically informed. Therefore it was assumed that employers knew the workers preferences and each worker knew his employers technology and preferences. Examples of asymmetrically informed agents will be examined in the next chapter.

Notes

1. For a discussion of the role of trust in general equilibrium see Gale (1982).

2. This situation may be 'quasi-rational', that is agents beliefs are not contradicted by prices. This terminology is due to Gale (1982).

3. See Hildenbrand (1974) for a treatment of general equilibrium when there is a measure space of agents.

4. It also implies that in all feasible allocations every worker will be amployed.

5. Superscripts and subscripts will be dropped in proofs where this will not cause confusion.

6. Notice that both π and r are defined in terms of a numeraire commodity the price of which is unity, see section 3.

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CHAPTER 3

Implicit Contracts and Asymmetric Information

Section 0: Introduction

One of the main purposes in studying implicit contracts is to examine any informational asymmetries between the contracting parties. In particular it has been a widely held belief that moral hazard problems are prelevant in the labour market. Azariadis for example, writes in the introduction to his classic 1975 article

"...the obvious bears repeating that no market exists for direct exchange of claims on future labour services: the costs of monitoring and enforcement and 'moral hazard' are some of the reasons why such markets have not arisen".

Moral hazard can be defined as any situation in which the very fact that an agent is 'insured' against some outcome changes his best course of 'action'. Of course if the insurer can observe the agents action then he can stipulate what the insurees action should be. If the insuree deviates then all bets are off. However if the insurer cannot observe the insurees action, there is an information asymmetry, then the insurance coverage will have to be reduced, possibly to zero.[1]

Moral hazard can arise on both sides of the labour market. For example it is difficult for an employee to sell his labour services contingent upon his own disposition because this is not easily observable by any other agent. This is not to say that private institutions will not provide some measure of sickness or unemployment insurance if these were not provided by the Government. Rather sickness or unemployment insurance will never be complete, that is enough to maintain a constant marginal utility of income. Equally Government provision of sickness insurance will meet the same problems, but Government intervention in this area is usually justified on other welfaristic grounds. On the opposite side of the labour market an employer will usually be unable to purchase labour services contingent upon say profits unless these are independently audited.

It is interesting to note that Demsetz (1969) who is rather critical of Arrow's treatment of moral hazard perceives the labour contract as the appropriate response of the market to these information asymmetries. He writes

"The real economic system does in fact allow exchange of commodity options. A labour contract with adjustment for changes in the consumer price index is such a commodity option. Such a contract specifies one wage rate if nature reveals one price level and another wage conditional upon the appearance of a different price level".

Thus at least for Demsetz implicit contracts and asymmetric information are inseparable.

The contract analysed in chapter one, and indeed the contract analysed by Azariadis (1975) was pareto efficient. That is to say that the contract was productively efficient in each state of nature and risk was allocated optimally. However this result was achieved either by assuming that the employees utility function had a particularly simple form or by assuming that both parties to the contract could observe which state of nature had actually occurred. Neither assumption is very satisfactory. In particular if both agents can observe the true state of nature, then contingent claims markets may operate unhindered. In this situation a contract would only replicate the allocation achieved by a complete set of contingent claims markets and therefore is of little independent interest.[2] However if the true state of nature is not readily observable by some agent, there is an information asymmetry and the optimal labour contract must take this into account.

It is the purpose of this chapter to examine the optimal labour contract when there is asymmetric information, that is a situation in which some agent has information to which others are not privy. Although Azariadis (1975) was clearly motivated by moral hazard issues his analysis of the labour contract did not extend to its examination. It is only recently that Grossman and Hart (1981) Green and Kahn (1982) and Hart (1983) have developed a rigorous analysis of the moral hazard problem within the labour contract framework. Their work builds on previous work by Spence and Zeckhauser (1971), Harris and Raviv (1979) and Laffont and Maskin (1980) on incentives. The problem of describing the optimal labour contract turns out to be essentially similar to the problem of finding the optimal income tax. Hence the analysis of optimal income taxation studied by Mirrlees (1971, 1976) is especially relevant. Green and Kahn make this analogy explicit.

This chapter will build upon the model developed in chapter 1A. As such the analysis will be very similar to Green and Kahn. Within this context there are two obvious polar cases of information asymmetry In the first case the employee may observe the true state of nature whilst the employer cannot. Such a case is consistent with the state of nature being identified with the employees disposition, or indeed any factor that affects the employees productivity which the employer cannot observe. In the second case the roles are reversed so that it is the employer who has the superior information. For example the employer may have better information about randomness in the production process.[3]

Section 1 presents a somewhat homely and discursive example of the first type of information asymmetry. The example examines the role of information asymmetry in the relationship between a Ph.D. student and his supervisor. This is meant to provide an introduction to section

2 and emphasis is placed on intuition rather than rigour. Section 2 is rather more technical, it examines the case where the employer has superior information. It is shown how this information asymmetry produces too much employment in a sense to be defined below. Section 3 concludes this chapter.

Before proceeding it is perhaps worthwhile to make a few caveats at this point. These should be remembered when reading sections 1 and 2. Firstly, Arrow has emphasized that in markets affected by moral hazard some forms of moral behaviour are likely to evolve to compensate. For example the relationship between the employer and the employee might be governed by established codes of conduct or ethical standards different from those predicted by economic or optimizing behaviour. Secondly and not unrelatedly is the issue of reputation. Once time is explicitly introduced into the analysis an agents concern for his reputation becomes important, and the temporal nature of information flows must be taken into account. These are thorny problems and no resolution is readily available,[4] though they are obviously important to an understanding of labour contracts since there are predicated upon a stability in the employer/employee relationship.

Section 1 : How to pay for a doctoral thesis

This section asks the question: What is the optimal grant award scheme for a doctoral student? It is assumed that the students research is financed out of his supervisors salary. If the student is better informed about how he conducts his research than his supervisor then choosing the optimal payment scheme is equivalent to finding the optimal contract between the student and his supervisor under asymmetric information. The analysis can be easily reinterpreted in the labour market context by reading employee for student and employer for supervisor. It will be shown that the optimal grant to the student will be increasing and convex function of his output.

Consider a student who is enroled at the Moravian state university for a course leading to the degree of doctor of philosophy in the department of economics. The course lasts some fixed period of time, say three years. For tuition the student is assigned to some eminent and august professor who will act as a supervisor. The requirements of the course are to produce a short thesis on a particular topic, or perhaps some articles of inclusion in the learned journals of the profession.

In order to drastically simplify the problem it will be assumed that the students output y is related to his input ℓ in the following way

A.3.1 $\quad y = f(\ell,\theta) : [0,1] \times (a,b) \to R_+$; is C^2 and bounded above

$$\text{where} \quad f_1 > 0 \quad f_2 > 0,\ f_{11} \geq 0,\ f_{12} \geq 0 \quad \lim_{\ell \to 0} f_1 = \infty$$

where θ is a random variable defined on (a,b) which has a continuous probability density function $g(\theta)$. Some comments are obviously called for about the assumption A.3.1. First notice that y is a undimensional variable so that the quality and quantity of the students output are commensurable, and that y can be objectively assessed and is readily apparent to both the student and the supervisor. Second the students input or effort or work load again is undimensional. This is not however the only input. The professor will also provide an input through his supervision and encouragement of the student. For simplicity it is assumed that the professors input is constant and it is therefore supressed in the notation. Third the random variable θ is to be interpreted as a shock to the efficiency of the student since a high value of θ is associated with a high value of the students marginal product. Of course in general θ will be a vector of random variables the components of which might be luck, the skill or lack of skill of the student, the degree of difficulty or simplicity of the subject matter, the students personal circumstances etc.

However again for simplicity it will be assumed that θ is a single parameter and not a vector of parameters. What is more it is assumed that the student chooses his labour input after the random variable θ has been drawn from $g(\theta)$ and that the professor cannot observe this drawing. To motivate these assumptions suppose that θ represents the easiness of the subject area chosen by the student. That is to say a high value of θ indicates that the subject area is very easy and a low value of θ indicates that the subject is particularly difficult. Of course to actually ascertain the true value of θ requires a considerable amount of effort and investment of resources. For the first year of the degree course the student is devoted almost solely to this task. He will then decide on how much work or effort to supply in the following two years dependent upon the actual degree of difficulty of the subject. The professor however is far too busy with his own work or with other students to find out the true value of θ. Of course if the professor can observe ℓ and knows f then he can deduce θ by inversion so it will be assumed quite reasonably that the professor does not observe the students input, ℓ .

The professor has to choose how much of his salary, X he is to give to the student as a grant to fund his research. The grant will be denoted r. Since the professor cannot observe either θ or ℓ , he will make the grant r conditional upon the students output y. This is shown rigorously later on. For the moment notice that the grant r may be properly interpreted as the amount of patronage. This of course was a common form of funding scientific work in previous centuries.

The professor will however not give anything for nothing. He will not patronize the student unless this increases his utility in some appropriately defined sense. There are at least two ways in which the students output may affect the professors utility. First the students efforts may stimulate or encourage the professor in his own work. Second the professor may derive a certain kudos from his patronage or his students work. It will be assumed that the professors utility v has the following simple functional form.

A.3.2 $$v = v(I) : R_+ \to R : v(I) = I = X + y - r$$

where I is the professors gross income. Thus it is assumed that the students output y is commensurable with the professors net income X-r, and that the professor is risk neutral. There is really no justification for the former assumption. However if the professor is thought of as an employer then I represents profits. The professor's risk neturality might be justified if, for example, he supervised a large number of students for each of whom the drawing of θ is independent of any other students drawing.

The students utility u, depends both on the grant r and effort ℓ. The utility function u is assumed to satisfy

A.3.3 $$u = u(r,\ell) : R_+ \times [0,1] \to R \; ; \; C^2 \text{ and}$$
$$u_1 > 0,\ u_2 > 0,\ u_{11} \le 0,\ u_{22} \le 0,\ s = -u_2/u_1 \ge 0$$
$$(u_{11}s + u_{12}) \le 0 (u_{12}s + u_{22}) \le 0\ (u_{11}u_{22} - u_{12}^2) \ge 0.$$

Before proceeding it is convenient to facilitate the graphical and algebraic analysis by rewriting the production and utility functions. The production $y = f(\ell,\theta)$ can be inverted to give

$$\ell = h(y,\theta) h_1 = 1/f_1 > 0,\ h_2 = -f_2/f_1 < 0,\ h_{11} = -f_{11}/f_1^3 \ge 0,$$
$$h_{12} = (-1/f_1^3)(f_1 f_{12} - f_{11} f_2) \le 0. \tag{3.1}$$

Using equation (3.1) the students utility function can be rewritten as

$$u = u(r,y,\theta) = u(r,h(y,\theta)) \tag{3.2}$$

which can also be inverted to give

$$r = r(u,h(y,\theta)) \ ; \quad r_1 = 1/u_1 > 0 \ r_2 = s > 0 \qquad (3.3)$$

It is now possible to examine exactly what is meant by a contract with asymmetric information. In chapter 1A it was assumed that the components r and ℓ of a contract could be made contingent upon the state of nature θ . However it is less clear how such a contract can be impmemented if there is asymmetric information. In particular it might be assumed that the student will have an incentive to lie about the true state of nature. Nevertheless Dasgupta, Hammond and Maskin (1979) have shown that any contract that can be implemented is implementable by a contract in which there is no incentive to lie. This has been demonstrated graphically by Harris and Townsend (1981).

Harris and Townsend's analysis can be adapted to the present case. First consider the students and the professors indifference maps in (r,y) space. These are drawn in diagrams 3.1 and 3.2. The professor's indifference curves are 45° lines and his utility increases towards the south-east. The students indifference curves are convex to the south-east and his utility increases towards the north-west. However the student has a different indifference map for every state of nature. Suppose for the moment that there are just two states of nature θ_1 and θ_2 . That is to say that the students chosen subject is hard or harder still. Then at any given point in (r,y) space the students indifference curve associated with $\theta = \theta_2$ has a lower slope than the indifference curve associated with $\theta = \theta_1$.

Consider the fixed income contract (c_1, c_2) depicted in diagram 3.3 where $c_1 = (r(\theta_1), y(\theta_1))$ and $c_2 = (r(\theta_2), y(\theta_2))$. Clearly the student prefers the allocation c_1 to c_2 which ever state occurs. If the state of nature is $\theta = \theta_2$ then the student gets higher utility at the point c_1. Similalry if $\theta = \theta_1$ the students utility is lower utility at the allocation c_2

Thus if the supervisor cannot observe θ the student will announce that the state of nature is $\theta = \theta_1$ regardless of the true state of nature and therefore the contract (c_1, c_2) is not implementable. This is quite natural, since the student receives the same remuneration independent of his output he will always choose to produce less output because this requires less effort. Notice that the fixed income contract (c_1, c_2) is typical of the sort of contract offered to doctoral students in the U.K. Therefore it seems quite likely that U.K. students will always claim that their subject is especially difficult and produce low quality output.[5]

On the other hand consider the alternative contract (c_1, c_2') where $c_1 = (r(\theta_1), y(\theta_1))$ and $c_2' = (r'(\theta_2), y'(\theta_2))$. This contract is incentive compatible. That is the student will never have any incentive to lie about the true state of nature. For instance if θ_2 is the true state of nature the student cannot gain by announcing the true state of nature is θ_1 and if θ_1 is the true state of nature he will always benefit by announcing the true state to be θ_2. Thus it can be seen that the only contracts that are implementable are incentive compatible and any incentive compatible contract can be implemented by the student telling the truth.

Incentive comptaible contracts have some very interesting properties. Examination of all of these properties must await a complete algebraic treatment but diagram 3.4 indicates that not all first best contracts are incentive compatible. In chapter 1A it was shown that the optimal contract equated the marginal disutility of labour to the marginal product of labour and left the employees marginal utility of income independent of the state of nature. Such a contract is called a first best contract because it is pareto efficient. An example of a first best contract is (c_1, c_2'') drawn in Diagram 3.4 The dotted lines represent lines of equal marginal utility of income. Since labour is a normal good they have a lower slope than indifference curves. The two dotted lines through c_1 and c_2'' represent the same level of marginal utility but in different states. However the contract (c_1, c_2'') is not incentive compatible because the student will always announce the state to be $\theta = \theta_1$ independent of the true state of nature. In fact it can be shown that the first best contract is only incentive compatible if labour is neither a normal nor inferior good. This was the basis for most of the analysis in chapter 1.

The algebraic analysis is most easily understood if it is assumed that there is a continuum of possible states of nature so that the density function $g(\theta)$ is continuous. Now it has been shown that an incentive compatible contract can be written $\delta = \{r(\theta'), y(\theta')\}$ in precisely the same way that the first best contract could except is that value of θ the student chooses to announce. Given the contract δ the student is in a position to announce any state of nature he perceives to be to his advantage. That is the student will choose to announce that state of nature θ' that maximizes his utility when the true state of nature is θ . Therefore for each possible value of θ the student sovles :

$$\text{P.3.1} \qquad \max_{\theta'} \quad u(r(\theta'), \; h(y(\theta'),\theta)) \quad \text{s.t } \theta' \in [a,b]$$

A contract δ is incentive compatible if the student always announces the true state of nature, or equivalent if the solution to P.3.1 is $\theta'(\theta) = \theta$. The first and second order conditions for P.3.1 are

$$\dot{r}(\theta) = s(r(\theta), h(y(\theta),\theta)) \, h_1(y(\theta) \, ,\theta) \, \dot{y}(\theta) \tag{3.4}$$

$$\ddot{r} - sh_1\ddot{y} - \dot{y}^2 \, (sh_{11} + h_1^2(s_1 s \; + s_2)) \le 0 \tag{3.5}$$

Totally differentiating equation (3.4) equation (3.5) can more conveniently be written as

$$\dot{y}(sh_{12} + s_2h_1h_2) \geq 0 \tag{3.6}$$

Given assumptions A.1 and A.3 equation (3.6) implies that $\dot{y}(\theta)$ is positive. It will be convenient to take $y(\theta)$ to be strictly positive in what follows though it will be shown in section 2 that the results are unaffected. Then inverting $y = y(\theta)$

$$\theta = y^{-1}(y) \qquad (3.7$$

and

$$r(\theta) = r(y^{-1}(y)) = r(y) \tag{3.8}$$

Equation (3.8) shows that remuneration is dependent upon output. It also shows that the contract δ can be viewed alternatively as choosing the reward function $r(y)$. This is essentially a piece-rate system of operation. The student chooses how much output to produce given the reward scheme $r(y)$. That is in each state of nature the student solves

P.3.1′ $\max_{y} \quad u(r(y),\ h(y, .)) \quad \text{s.t} \quad h(y, .) \ \varepsilon [0,1]$

The first and second order conditions for P.3.1 are

$$r'(y) = s(r(y), h(y,.) \ h \ (y,.) \tag{3.4'}$$

$$r''(y) \leq sh_{11} + h_1^2(s_1s + s_2) \tag{3.5'}$$

Differentiating equation (3.8) shows that equations (3.4′) and (3.5′) are the same as equations (3.4) and (3.5) and therefore P.3.1 and P.3.1′ are equivalent. Equation (3.4′) has a natural interpretation. It states that the student will equate the marginal rate of substitution between income and leisure to the marginal benefit of supplying labour which is

$$\partial r/\partial \ell = r'(y)/h_1(y,.) \tag{3.9}$$

Notice that the student can only do this because it is being assumed that the professor cannot observe the students labour input. Consequently in order to induce the student to produce good quality work the reward offered must increase as output increases.

Perhaps the most natural way to write the incentive compatible contract is $\delta = \{u(\theta), y(\theta)\}$. The output schedule $y(\theta)$ is the solution to P.3.1 and is chosen directly by the student. The schedule $u(\theta)$ is the maximum value function for P.3.1 . Once $u(\theta)$ and $y(\theta)$ are determined $\ell(\theta)$, $r(\theta)$ and $v(\theta)$ are determined directly by equation (3.1) and (3.3) and assumption A.3.2.

The contract δ must offer the student a given level of expected utility $\bar{u}$ or else the student will enrole at another university or perhaps take up a non-academic post, therefore

$$\int_a^b u(\theta)g(\theta)d\theta = \bar{u} \qquad (3.10)$$

It must also satisfy the incentive compatibility constraint (3.4′). This can be rewritten in terms of $y(\theta)$ and $u(\theta)$ as

$$\dot{u}(\theta) = u_2(r(u(\theta), h(y(\theta),\theta)), h(y(\theta),\theta)).h_2(y(\theta),\theta) \qquad (3.4'')$$

It will also be assumed that the set of contracts satisfying equations (3.10) and (3.4″) Δ is non-empty.[6] The professor's expected utility is

$$\int_a^b (X + y(\theta) - r(u(\theta), h(y(\theta),\theta)))\ g(\theta)d\theta \qquad (3.11)$$

The optimal contract δ^* maximizes the professors utility (3.11) subject to the constraints (3.10) and (3.4″). This is an optimal control problem, where $y(\theta)$ is the control variable, controlled by the student and $u(\theta)$ is the state variable. Letting λ be the multiplier for equation (3.10) and $p(\theta)$ be the costate variable for equation (3.4″) for the Hamiltonain equation is

$H(y(\theta), u(\theta), \lambda, p(\theta)) = (y(\theta) - r(u(\theta), h(y(\theta),\theta)) + \lambda u(\theta))g(\theta)$

$$+ p(\theta) u_2(r(u(\theta), h(y(\theta),\theta)), h(y(\theta),\theta)).h_2(y(\theta),\theta). \qquad (3.12)$$

The optimal contract δ^* is the solution to

P.3.2 $\max_{\delta} \quad H(\delta, \lambda\, p(\theta))$.

The important point to grasp is the sign of the costate variable. From equation (3.4″) it is clear that $\dot{u}(\theta)$ is positive. However if the professor could observe θ then the optimal contract is the first best contract in which case $\dot{u}(\theta)$ is negative. Therefore the incentive compatibility constraint contrains $\dot{u}(\theta)$ from below and therefore $p(\theta)$ is negative.

Differentiating the Hamiltonian with respect to the control variable $y(\theta)$ gives directly

$$(1 - r_2h_1) = pu_1(sh_{12} + s_2h_1h_2)/g > 0. \qquad (3.13)$$

Equation (3.13) states that the marginal rate of substitution between income and leisure $r_2 = s$ is less than the marginal product of labour $f_1 = 1/h_1$. That is to say in any particular state of nature, or ex post both the professor and student could be made better of by some increase in remuneration or effort. To put it more strikingly there is underemployment. This cannot however be described as involuntary unemployment since the contract is ex ante optimal, that is there is no way in which to increase both the professors and the students expected utility. In this sense the contract is constrained pareto efficient. It is also clear that the student will not wish to supply any more labour at the going wage because the wage or the marginal benefit to supplying labour is equated to the marginal rate of substitution.

On the other hand the professor will always prefer that labour supply is greater at the going wage but it is less clear how important this is because the wage is not taken parametrically but varies as the students effort changes.

Some insight into this result can be obtain by comparing the first best contract with the optimal incentive compatible contract. The first best contract equates the marginal rate of substitution between income and leisure to the marginal product but both exceed the marginal benefit to supplying labour. That is

$$\partial r/\partial \ell \leq s(r,\ell) = f_1(\ell,\theta) \qquad \theta \tag{3.14}$$

where $\partial r/\partial \ell = \partial r/\partial \theta / \partial \ell/\partial \theta$ is defined to be the marginal benefit to supplying labour. Whereas in the optimal incentive compatible contract

$$\partial r/\partial \ell = s(r,\ell) \leq f_1(\ell,\theta) \qquad \theta \tag{3.15}$$

so that at least the divergence between the marginal cost of labour and the marginal product is preserved.

To conclude, since the professor cannot observe the students input he is quite likely to under supply and produce low quality output. These effects are amelioated by making the students grant an increasing function of output. It is tempting to suggest that if welfare is assessed purely on an ex post basis then an increase in grant and effort would be beneficial to all parties.

Section 2: More on Incentive Compatible Contracts

In this section the employer/employee model is rehabilitated but is now assumed that the employer can verify which state of nature has occurred but that the employee cannot. It is not difficult to think of examples where this might be the case. For example the firm may have more information about the demand conditions affecting the firm or more information about the operation and effectiveness of other inputs into the production process. This section examines the optimal incentive compatible contract in this situation.

Assumption A.3.1 - A.3.3 of section 1 are maintained throughout this section but the information sets of the employer and employee now satisfy

A.3.4 $$I^v = \{\theta, y, r, \pi, \ell, u, v, f, g\}$$

$$I^u \quad \{r, \ell, u, v, f, g\}$$

where I^v is the employers information set and I^u is the employees information set. Notice that there is really no contradiction in the employee knowing the objective probability density function $g(\theta)$ and not the realized value of θ. For example the employee may be able to learn the true value of θ with a one period lag, then after some time he will be able to deduce $g(\theta)$ but still as far as the present contract is concerned be unable to observe the current θ. Alternatively with the important exception of lemma 3 the analysis goes through if the employee has a subjective probability density function $k(\theta)$. The coincidence of $g(\theta)$ and $k(\theta)$ can be seen as a strong rational expectations assumption. Equally the fact that f is included in I^u is not restrictive since any uncertainty about f might be included with the state of nature θ. But this explains why y and π are not included in I^u since observing either output or profits is equivalent to knowledge of θ through the knowledge of f and y. This being said the examination of the optimal incentive can proceed along very similar lines to the analysis of section 1 except that more attention will be paid to specific details.

In section 1 it was shown that any contract under asymmetric information could be written as a state contingent contract. In this section the employer is assumed to be able to observe θ but the employee cannot. Therefore the employer will choose to announce that state of nature θ' that is in his own best interest. In general the employers profits will depend both upon the true state of nature θ and the state of nature θ' the employer chooses to announce has occurred. The employer will choose to announce θ' so as to maximize his ex post utility or profits, that is to say he solves

$$\text{P.3.3} \qquad \max_{\theta'} \quad \pi(\theta',\theta) \quad \text{s.t} \quad \theta' \varepsilon [a,b]$$

The solution set for P.3.3 is $\theta'(\theta)$ where

$$\theta'(\theta) = \{\theta' \varepsilon [a\ b] \quad \pi(\theta',\theta) \geq \pi(\theta'',\theta) \ \forall \ \theta'' \ \varepsilon [a,b]\} \tag{3.16}$$

A contract δ is said to be incentive compatible if and only if the employer never has any incentive to lie, that is

$$\theta \ \varepsilon \ \theta'(\theta) \tag{3.17}$$

There are two points to be made about equation (3.17). First it is being assumed that the employer will report the state of nature honestly unless he can actually gain by lying. This is equivalent to saying that if the employer is indifferent between any two allocations he will always choose the one that the employee most prefers. Second incentive compatibility implies that the solution set to P.3.3 $\theta'(\theta)$ is non-empty. This is guaranteed if $\pi(\theta',\theta)$ is upper semi-continuous in the choice of variable θ'.

The maximum value function for P.3.3. is

$$\pi(\theta) = \pi(\theta,\theta) \tag{3.18}$$

which is continuous if $\pi(\theta',\theta)$ is continuous in θ' and θ. In fact the absolute continuity of $\pi(\theta)$ will be assumed below but for the moment suppose that $\pi(\theta',\theta)$ is continuous and twice differentiable. Then the first and second order condition for P.3.3 can be written as

$$\pi_{\theta'}(\theta',\theta) = 0 \quad (3.19)$$

$$\pi_{\theta'\theta'}(\theta',\theta) \leq 0 \quad (3.20)$$

If equation (3.20) is satisfied as an inequality the solutions of $\theta'(\theta)$ will be a continuous and differentiable function. Then the following four equations are all equivalent definitions of incentive compatibility

$$\theta'(\theta) = \theta \quad (3.21)$$

$$\partial\theta'(\theta)/\partial\theta = 1 \quad (3.21)$$

$$\pi_{\theta'\theta'}(\theta'(\theta),\theta) + \pi_{\theta'\theta}(\theta'(\theta),\theta) = 0 \quad (3.23)$$

$$\dot{\pi}(\theta) = \pi_{\theta'}(\theta'(\theta),\theta) + \pi_{\theta}(\theta'(\theta),\theta) = \pi_{\theta}(\theta,\theta) \quad (3.24)$$

Equation (3.22) is obtained by differentiating equation (3.21). Equation (3.23) is obtained by totally differentiating equation (3.19) and using equation (3.22). Equation (3.24) is obtained by differentiating equation (3.18) and using equation (3.19). Given equation (3.23) the second order condition equation (3.20) can be conveniently rewritten

$$\pi_{\theta'\theta}(\theta'(\theta),\theta) \geq 0 \quad . \quad (3.25)$$

It is perhaps more helpful to write the profit function $\pi(\theta',\theta)$ explicitly as revenue minus costs. That is

$$\pi(\theta',\theta) = f(\ell(\theta'),\theta) - r(\theta') \quad (3.26)$$

where r is remuneration and ℓ is labour input. Therefore rewriting equation (3.24) incentive compatibility implies

$$\dot{\pi}(\theta) = f_2(\ell(\theta),\theta) \quad . \quad (3.27)$$

This will be referred to as the incentive compatibility constraint. It constrains profits to increase with θ only to the extent that output increases with θ. Again rewriting equation (3.25)

$$\pi_{\theta'\theta}\,(\theta'(\theta),\theta)\big|_{\theta'=\theta} = f_{12}(\ell(\theta),\theta)\;\dot{\ell}(\theta) \geq 0 \qquad (3.28)$$

This suggests yet another interpretation of incentive compatibility. Suppose that $\dot{\ell}(\theta)$ is strictly positive then the function $\ell = \ell(\theta)$ can be inverted so that $\theta = \ell^{-1}(\theta)$ and $r(\theta) = r(\ell^{-1}(\ell)) = r(\ell)$. Thus remuneration is some function of labour input, there is essentially a time-rate system of payments. Choosing a contract is equivalent to choosing the optimal time-rate system. This does not seem inconsistent with modern industrial experience. In particular wages usually rise with hours worked, and the latter is often chosen unilaterally by the employer. In this sense incentive compatibility is reduced to the employer choosing the labour input to maximize ex post profits given the function $r = r(\ell)$. That is to say the employer solves

P.3.3′ $\quad \max\limits_{\ell} \quad f(\ell,\theta) - r(\ell) \quad$ s.t $\ell \in [0,1]$

The first and second order conditions for P.3.3 are

$$\partial r(\ell)/\partial\ell = f_1(\ell,\theta) \qquad (3.29)$$

$$\partial^2 r(\ell)/\partial\ell^2 \geq f_{11}(\ell,\theta) \quad . \qquad (3.30)$$

Equation (3.29) shows that the employer will equate the marginal cost of hiring labour to the marginal product of labour. Equations (3.29) and (3.30) can be rewritten as

$$\dot{r}(\theta) = f_1(\ell(\theta),\theta)\dot{\ell}(\theta) \qquad (3.29')$$

$$\ddot{r}(\theta) = f_{11}(\ell(\theta),\theta)\ddot{\ell}(\theta) + f_1(\ell(\theta),\theta)(\dot{\ell}(\theta))^2 \qquad (3.30')$$

since
$$\dot{r}(\theta) = (\partial r(\ell)/\partial\ell)\dot{\ell}(\theta) \qquad (3.31)$$

and
$$\ddot{r}(\theta) = (\partial^2 r(\ell)/\partial\ell^2)\ddot{\ell}(\theta) + (\partial r(\ell)/\partial\ell)(\dot{\ell}(\theta))^2 \qquad (3.32)$$

Equations (3.28′) and (3.30′) are simply equations (3.19) and (3.20) written explicitly in terms of the remuneration and labour input schedules. This shows that P.3.3 and P.3.3′ are equivalent. Thus the incentive compatible contract can be thought of in two possible ways. First the contract may be thought of as negotiating a pair of state contingent schedules such as $r(\theta)$ and $\ell(\theta)$ such that the employer always has an incentive to announce the true state of nature. Second the contract may be thought of as the time rate payment scheme $r(\ell)$ where control of the labour input is relinquished solely to the employer. The second interpretation is perhaps more natural but analytically it is simpler to deal with the first. In fact P.3.3′ suggests an obvious way to do this. The solution to P.3.3′ is $\ell = \ell(\theta)$ and the resulting profit function or maximum value function is

$$\pi(\theta) = f(\ell(\theta),\theta) - r(\ell(\theta)) \quad . \tag{3.33}$$

Given the incentive compatibility constraint equation (3.27) $\ell(\theta)$ can be treated as the control variable and $\pi(\theta)$ as the state variable in an optimal control problem. That is to say that a contract δ is defined as the set of pairs $\{\pi(\theta), \ell(\theta)\}$.

Before proceeding to the analysis of the optimal incentive compatible contract, consider the following utility possibility set

$$U^* = \{(E_\pi, E_u) \mid E_\pi \geq 0 \; Eu \geq Eu(0,0),\; r(\theta) + \pi(\theta) = f(\ell(\theta),\theta),$$
$$\dot{\pi}(\theta) = f_2(\ell(\theta),\theta) \;\forall\; \theta\}.$$

The set U^* is the set of possible levels of expected utility that can be achieved by the employer and the employee when the employee cannot observe the true state of nature. This is to be contrasted with the utility possibility set when the employee can verify θ

$$U = \{(E_\pi, E_u) \mid E_\pi \geq 0 \; E_u \geq Eu(0,0), r(\theta) + \pi(\theta) = f(\ell(\theta),\theta), \forall \theta\} .$$

Lemma 1 The set U^* is a subset of U and is convex.

Proof: 1. $U^* \subset U$. To show this consider points on the boundary of U. In chapter 1A it was shown that:

(i) $\dot{u}(\theta) \le 0$ with strict inequality if labour is a normal good.

(ii) $\dot{\pi}(\theta) = (-\dot{u}(\theta)/u(\theta)) + f_2(\theta) \ge f_2(\theta)$ with strict inequality if labour is a normal good. Therefore U* will include the boundary points of U only if labour is neither normal nor inferior.

2. U* is convex. To show that U* is convex it suffices to show that randomnized contracts are never optimal. Consider the randomnized contract $\tilde{\delta} = \{\tilde{\pi}(\theta'), \tilde{\ell}(\theta')\}$. This contract determines profit and labour input by some appropriate lottery if the employer announces that the state of nature is θ' . Then the non-random contract $\delta = \{\pi(\theta), \ell(\theta)\}$ can be defined by

$$\pi(\theta') = E_{\tilde{\pi}}\, \tilde{\pi}(\theta') \quad f(\ell(\theta'),\theta) = E_{\tilde{\ell}} f(\tilde{\ell}(\theta'),\theta) \;\; \forall \theta \;.$$

Then the contract δ is incentive compatible if and only if $\tilde{\delta}$ is incentive compatible because they yield the same level of (expected) profits in each state. If $\tilde{\delta}$ is incentive compatible then $\theta' = \theta$ and therefore

$$E_{\tilde{\ell}} \tilde{\ell}(\theta) \ge \ell(\theta)$$
$$f(\ell(\theta),\theta) = E_{\tilde{\ell}} f(\tilde{\ell}(\theta),\theta) \le f(E_{\tilde{\ell}} \ell(\theta),\theta)$$

by concavity and therefore using the concavity of the utility function

$$E_{\tilde{\delta}}\, u(f(\tilde{\ell}(\theta),\theta) - \tilde{\pi}(\theta), \ell(\theta)) \le u(E_{\tilde{\ell}} f(\tilde{\ell}(\theta),\theta) - E_{\tilde{\pi}} \tilde{\pi}(\theta) \;,$$
$$E_{\tilde{\ell}} \tilde{\ell}(\theta)) \le u(f(\ell(\theta),\theta) - \pi(\theta), \ell(\theta)) \;.$$

That is to say the non-random contract δ offers the same level of (expected) profits in each state as the random contract $\tilde{\delta}$ and also no less (expected) utility. Therefore the random contract weakly (strongly) dominates the random contract and hence U* is weakly (strictly) convex .

Thus the optimal incentive compatible contract is non-random. The optimal incentive compatible contract $\delta^* = \{\pi^*(\theta).\ell^*(\theta)\}$ maximizes the employers expected profits subject to three constraints.[7] The first of these is quite familiar, namely that the employer must offer the employee his reservation price, $\bar{u}$. The second constraint is the incentive compatibility constraint equation (3.27) and the third constraint is the second order condition for P.3.3, that is $\dot{\ell}(\theta) \geq 0$. Thus the optimal contract is the solution to

$$\text{P.3.4} \qquad \max_{\delta} \int_a^b \pi(\theta)g(\theta)d\theta$$

$$\text{s.t} \int_a^b u(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta))g(\theta)d\theta = \bar{u} \qquad (3.34)$$

$$\dot{\pi}(\theta) = f_2(\ell(\theta),\theta) \qquad \text{almost everywhere} \qquad (3.27)$$

$$\dot{\ell}(\theta) \geq 0 \qquad \text{almost everywhere} \qquad (3.35)$$

It will be convenient to ignore the constraint $\dot{\ell}(\theta) \geq 0$ initially and examine

$$\text{P.3.4}' \qquad \max_{\delta} \int_a^b \pi(\theta)g(\theta)d\theta$$

$$\text{s.t} \int_a^b u(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta))g(\theta)d\theta = \bar{u} \qquad (3.34)$$

$$\dot{\pi}(\theta) = f_2(\ell(\theta),\theta) \quad \text{almost everywhere.} \qquad (3.27)$$

This can be done without any loss of generality because the solution to P.3.4′ will apply to those states where $\dot{\ell}(\theta) > 0$ in the solution to P.3.4. These issues are taken up again below.

There is a problem with P.3.4′ as it stands since a solution may not exist. This is true even if $\bar{u}$ is chosen so that some incentive compatible contract δ' is feasible, in addition to being incentive compatible. However if the following additional assumptions are made then it can be shown that a solution to P.3.4′ exists.

A.3.5 i) $\pi(\theta)$ and $\ell(\theta)$ are absolutely continuous on $[a,b]$

ii) There exists a number k s.t $\dot{\ell}(\theta) \leq k$ almost everywhere

iii) The reservation price $\bar{u}$ satisfies $\bar{u} \leq \{\sup Eu | Eu \in U^*\}$.

Assumptions A.3.5(i) and (ii) are technical conditions, any control system that satisfies these conditions is called an inertial control system because it rules out a large number of discrete jumps. This would not appear to be unduly restrictive in the present context. Assumptiom A.3.4 (iii) guarantees that at least one feasible contract exists.

Lemma 2: Given A.3.1 - A.3.5 an optimal contract $\delta^* = \{\pi^*(\theta), \ell^*(\theta)\}$ for P.3.4 exists.

Proof: The functions, u, f and f_2 are continuous and $(\theta, \pi(\theta))$ can be restricted to a compact set. Therefore the conditions of theorem V 2.1 in Berkovitz (1974) are satisfied.

To show that the optimal contract is unique requires a further assumption

A.3.6 i) $u(r,\ell)$ is strictly concave in r and ℓ

ii) $f(\ell,\theta)$ is strictly concave in ℓ

iii) $f_2(\ell,\theta)$ is strictly cancave in ℓ

Assumption A.3.6(i) and (ii) simply strengthen A.3.1 and A.3.3. Assumption A.3.6 (iii) will then be automatically satisfied if uncertainty is multiplicative that is

$$y = \phi(\theta)\ f(\ell) \qquad \phi'(\theta) > 0. \tag{3.36}$$

If λ is the mulitplier for equation (3.34) and $p(\theta)$ is the costate variable for equation (3.27) then the Hamiltonian function for P.3.4′ is

$$H(\pi(\theta),\ell(\theta),p(\theta),\lambda) = (\pi(\theta) + \lambda u(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta)))g(\theta) + p(\theta)f_2(\ell(\theta),\theta)$$

The first order or necessary conditions for P.3.4′ are therefore

$$-\dot{p}(\theta) = (1 - \lambda u_1(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta)))g(\theta) \tag{3.37}$$

$$\lambda(u_1(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta))\ f_1(\ell(\theta),\theta) + u_2(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta)))g(\theta)$$

$$= - p(\theta)f_{12}(\ell(\theta),\theta) \tag{3.38}$$

$$p(a) = p(b) = 0 \quad . \tag{3.39}$$

Lemma 3. Given the assumptions A.3.1 - A.3.6 the necessary conditions equations (3.37) - (3.39) are sufficient and the optimal contract $\delta^* = \{\pi^*(\theta), \ell^*(\theta)\}$ is unique.

Proof. Given A.3.1 - A.3.6 all the conditions of theorem 8.c.s. of Takayama (1974) are satisfied except $p(\theta) \geq 0$. Thus it is only necessary to show that $p(\theta) \geq 0$.

To do this suppose $p(\theta) \leq 0$ over some interval (θ', θ''). Then by the continuity of the costate variable

$$p(\theta') = p(\theta'') = 0 \tag{3.40}$$

and substituting into equation (3.38)

$$s(\theta) - f_1(\theta) < 0 \qquad \theta \in (\theta', \theta'') \tag{3.41}$$

where

$$s(\theta) = - u_2(f(\ell(\theta),\theta) - \pi(\theta), \ell(\theta))/u_1(f(\ell(\theta),\theta) - \pi(\theta), \ell(\theta)) = - u_2(\theta)/u_1(\theta)$$

and $f_1(\theta) = f_1(\ell(\theta),\theta)$. Therefore

$$u_{11}(\theta)f_1(\theta) + u_{12}(\theta) < u_{11}(\theta)s(\theta) + u_{12}(\theta) \leq 0 \quad \theta \in (\theta', \theta'') \tag{3.42}$$

because labour is a normal good. Hence

$$\partial(1 - \lambda u_1(\theta))/\partial\theta = -\lambda(u_{11}(\theta)f_1(\theta) + u_{12}(\theta))\ell(\theta) > 0 \quad \theta \in (\theta', \theta'') \tag{3.43}$$

This is important because integrating equation (3.37)

$$\begin{aligned} - p(\theta'') &= \int_a^{\theta'} (1 - \lambda u_1(\theta))g(\theta)d\theta + \int_{\theta'}^{\theta''} (1 - \lambda u_1(\theta))g(\theta)d\theta \\ &= p(\theta') + \int_{\theta'}^{\theta''} (1 - \lambda u_1(\theta))g(\theta)d\theta \end{aligned} \tag{3.44}$$

and substituting equation (3.40) into this gives

$$\int_{\theta'}^{\theta''} (1 - \lambda u_1(\theta))g(\theta)d\theta = 0 \quad . \tag{3.45}$$

From equation (3.45) and equation (3.43) it can be seen that there exists a unique $\theta'''\varepsilon(\theta',\theta'')$ such that

$$1 - \lambda u\ (\theta''') = 0 \quad . \tag{3.46}$$

Then consider some $\theta\ \varepsilon(\theta',\theta''')$

$$- p\ (\theta) = - p(\theta') + \int_{\theta'}^{\theta} (1 - \lambda u_1(\hat{\theta}))g(\theta)d\hat{\theta}$$

$$= \int_{\theta'}^{\theta} (1 - \lambda u_1(\hat{\theta}))g(\hat{\theta})d\hat{\theta} < 0 \quad \forall\ \theta\ \ \varepsilon(\theta',\theta''') \tag{3.47}$$

where equation (3.47) follows via equations (3.40) and (3.43). However equation (3.47) shows that $p(\theta) > 0$ for all $\theta\ \varepsilon\ (\theta',\theta''')$ which contradicts the initial assertion that $p(\theta) < 0$. Similarly consider some $\theta\ \varepsilon\ (\theta''',\theta'')$ then

$$- p(\theta) = - p(\theta'') - \int_{\theta}^{\theta''} (1 - \lambda u_1(\hat{\theta}))g(\hat{\theta})d\hat{\theta}$$

$$= - \int_{\theta}^{\theta''} (1 - \lambda u_1(\hat{\theta}))g(\hat{\theta})d\hat{\theta} < 0 \quad \theta\ \varepsilon(\theta''',\theta'') \tag{3.48}$$

again using equations (3.40) and (3.43). Therefore taken together equations (3.47) and (3.48) imply that $p(\theta) > 0$ for all $\theta\ \varepsilon(\theta',\theta'')$ which contradicts the initial assertion that $p(\theta) < 0$ for all $\theta\ \varepsilon(\theta',\theta'')$. Since the choice of the interval (θ',θ'') is arbitrary this completes the proof.

Lemma 3 is of some independent interest because of its similarity to the proof given in Seade (1982) that the optimal marginal rate of income taxation is positive. The following series of remarks or corollaries are meant to illustrate the sensitivity of the result.

Corollary 1 : If labour is a strictly normal good, that is $u_{11}s + u_{12} < 0$ then $p(\theta) > 0$ for all $\theta\ \varepsilon(a,b)$.

Proof: Suppose $p(\theta) \leq 0$ for all $\theta\ \varepsilon(\theta',\theta'')$. Then working through the arguments of Lemma 3, equation (3.41) becomes

$$s(\theta) - f_1(\theta) \leq 0 \qquad \theta \ \varepsilon \ (\theta',\theta'') \qquad (3.41')$$

and equation (3.42) becomes

$$u_{11}(\theta)f_1(\theta) + u_{12}(\theta) \leq u_{11}(\theta)s(\theta) + u_{12}(\theta) < 0 \qquad \theta \ \varepsilon(\theta',\theta'') \qquad (3.42')$$

but the remainder of the analysis is unaltered.

Corollary 1 is most important result. Since $p(\theta) > 0$ for all $\theta \ \varepsilon(a,b)$ it follows directly from equation (3.38) that the marginal product of labour is less than the marginal rate of substitution between income and labour. Thus the optimal incentive compatible labour contract is productively inefficient. In particular it is possible to reduce employment in any one state and increase both profits and utility in that state. Incentive compatibility however must mean that utility or profits are decreased in some other state.

This result needs some careful interpretation. Since the marginal rate of substitution exceeds the marginal rate of transformation there is overemployment. This does not however mean that the employment level for any particular state in the incentive compatible contract exceeds what it would be if the first best contract was operative. Equally it is difficult to say that this overemployment is involuntary even in an ex post sense. The employer is hiring labour exactly up to the point where the marginal cost of hiring labour services is just equal to the marginal product of these services, i.e. $f_1(\ell,.) = \partial r/\partial \ell$. For the employee however the marginal benefit to supplying labour is less than the marginal disutility of labour, $s(r,\ell) \geq \partial r/\partial \ell$. Thus the employee wishes to reduce his labour input at the going wage rate. This all sounds rather Keynsian and indeed the optimal incentive compatible contract has done away with the second classical postulate namely that $s(r,\ell) = \partial r/\partial \ell$, but maintained the first which is what Keynes suggested should be done. However this interpretation is inappropriate in this context. The employee takes, $\bar{u}$ as given ex ante and $r(\ell)$ as given ex post but not the wage rate.

The first best contract also dissassociates the marginal rate of substitution from the marginal rate of remuneration. In particular in the first best contract $f_1(\ell,.) = s(r,\ell) \geq \partial r/\partial \ell$ but this in no way reflects suboptimal levels of employment. Notice that in the first best contract the marginal product of labour exceeds the marginal cost of hiring labour. Therefore if it is attempted to implement the first best contract when the employee cannot observe the true value of θ the employer will wish to hire labour up to the maximum level and hence overemployment will result. The optimal incentive compatible contract may be seen as ameliorating but not illiminating this effect.

Consider again equation (3.35). It is difficult to know how the divergence between the marginal product and the marginal rate of substitution varies as θ varies because of the complicated relationship between $p(\theta)$ and $g(\theta)$. However at the endpoints of the distribution there is no productive inefficiency. This is obvious if $g(a) = g(b) > 0$. However, if $g(a) = g(b) = 0$ l'hopitals rule can be applied[8] to show that $f_1(a) - s(a) = f_1(b) - s(b) = 0$ providing that $f_{12}(\ell(\theta),\theta)$ is bounded.

Corollary 2 : If and only if labour is neither a normal nor inferior good, i.e. $u_{11}s + u_{12} = 0$ then the optimal incentive compatible contract δ^* for P.3.4 is a first best contract.

Proof: Necessity: Given $(u_{11}s + u_{12}) = 0$, proceed along the lines of Lemma 3 to show that both $p(\theta) > 0$ or $p(\theta) < 0$ gives a contradiction.

Sufficiency: Given $(u_{11}s + u_{12}) \neq 0$, proceed along the lines of Lemma 3 to show that $p(\theta) = 0$ gives a contradiction.

Therefore $p(\theta) = 0$ for all $\theta \in (a,b)$ so that the incentive compatibility constraint does not bind. Hence δ^* is a first best contract

Notice that if labour is neither a normal nor inferior good the employees utility function can be written as

$$u(r,\ell) = \hat{u}(r - h(\ell)) \tag{3.49}$$

where $\hat{u}$ is concave and h is convex. Corollary 2 provides the justification for the assumption made in chapter 1 that the employee utility function belonged to this class of functions. Hence the incentive compatibility problem could be conveniently ignored in that chapter.

Corollary 3: Given the assumptions A.3.1 and A.3.6 then the optimal incentive compatible contract δ^{**} that solves P.3.4 is unique. In addition if $\dot{\ell}(\theta) = 0$ over the interval (θ',θ'') then

$$\int_{\theta'}^{\theta''} (f_1(\theta) - s(\theta))g(\theta)d\theta + p(\theta'')(f_1(\theta'')-s(\theta''))-p(\theta')(f_1(\theta')-s(\theta')) = 0 \tag{3.50}$$

Proof 1 : To show that δ^{**} is unique it is necessary to show that $p(\theta) \geq 0$ for all $\theta \in (a,b)$. Let $q(\theta)$ be the multiplier for equation (3.35) in P.3.4 then the first order conditions for P.3.4 are

$$-\dot{p}(\theta) = (1 - \lambda u_1(f(\ell(\theta),\theta) - \pi(\theta), \ell(\theta)))g(\theta) \tag{3.37'}$$

$$-\dot{q}(\theta) = \lambda(u_1(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta))f_1(\ell(\theta),\theta)$$
$$+ u_2(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta)))g(\theta) + p(\theta)f_{12}(\ell(\theta),\theta) \tag{3.38'}$$

$$p(a) = p(b) = q(a) = q(b) = 0. \tag{3.39'}$$

By the complementary slackness condition, for those intervals of (a,b) for which $\dot{\ell}(\theta) > 0$, $q(\theta) = 0$ so that the analysis of Lemma 3 applies unammended to these regions. Suppose then there is some typical region (θ',θ'') for which $\dot{\ell}(\theta) = 0$. It has already been shown that $p(\theta') \geq 0$ and $p(\theta'') \geq 0$. In addition

$$(1 - \lambda u_1(\theta))/\partial\theta = 0 \qquad \forall\, \theta \quad \in(\theta',\theta'') \tag{3.51}$$

Therefore let $(1-\lambda u_1(\theta)) = c$ for all $\theta \varepsilon(\theta', \theta''')$. Suppose that $p(\theta''') < 0$ for some $\theta''' \varepsilon(\theta',\theta'')$ then integrating equation (3.37)

$$p(\theta''') = p(\theta') - c(\varepsilon(\theta''') - G(\theta'')) < 0 \quad (3.52)$$

and

$$p(\theta''') = p(\theta'') + c(G(\theta'') - G(\theta''')) < 0 \quad (5.53)$$

where G is the distribution function. Since G is monotone increasing equation (3.52) and (3.53) imply $c > 0$ and $c < 0$ respectively, which is a contradiction. So $p(\theta) \geq 0$ for all $\theta \varepsilon(a,b)$ and δ^{**} is unique.

2. To show that equation (3.50) holds integrate equation (3.38) for $\theta \varepsilon(\theta',\theta'')$

$$-q(\theta) = \int_{\theta'}^{\theta} (\lambda(u_1(\hat{\theta})f_1(\hat{\theta}) + u_2(\hat{\theta}))g(\hat{\theta}) + p(\hat{\theta})f_{12}(\hat{\theta}))d\hat{\theta} \quad (3.54)$$

$$= \int_{\theta'}^{\theta} ((f_1(\hat{\theta}) - s(\hat{\theta}))g(\hat{\theta}) + \dot{p}(\hat{\theta})(f_1(\hat{\theta}) - s(\hat{\theta})) + p(\hat{\theta})f_{12}(\hat{\theta}))d\hat{\theta}$$

(3.55)

where equation (3.55) is obtained from equation (3.54) by using equation (3.37'). The term $\dot{p}(\theta)(f_1(\theta) - s(\theta))$ can be integrated by parts remembering that $\dot{\ell}(\theta) = 0$. Therefore

$$-q(\theta) = \int_{\theta'}^{\theta} (f_1(\hat{\theta}) - s(\hat{\theta}))g(\hat{\theta})d\hat{\theta} + p(\hat{\theta})(f_1(\hat{\theta}) - s(\hat{\theta}))\Big|_{\theta'}^{\theta} . \quad (3.56)$$

Since $q(\theta'') = 0$ by the complementary slackness condition, equation (3.50) follows directly from equation (3.56) .

Corollary 3 shows that the only difference between P.3.4 and P.3.4′ occurs when $\dot{\ell}(\theta) = 0$. Suppose $\dot{\ell}(\theta)=0$ for $\theta\varepsilon(\theta',\theta'')$. Then if $\theta'' < b$ a simple argument establishs that the overemployment result continues to hold for $\theta\varepsilon(\theta',\theta'')$. That is

$$s(r(\theta),\ell(\theta))=s(r(\theta''),\ell(\theta'')) \geq f_1(\ell(\theta''),\theta'')> f_1(\ell(\theta),\theta) \quad (3.57)$$

This chain follows from (3.27), Lemma 3 and A.3.1 respectively. However if $\theta'' = b$ the middle inequality need not hold. Indeed since $\dot{q}(b)$ must be non-positive using equation (3.38') and l' hospitals rule where necessary shows

$$s(r(b), \ell(b)) < f_1(\ell(b), b) \quad (3.58)$$

Therefore if $\dot{\ell}(\theta)=0$ for $\theta\varepsilon(\theta',b)$ there may be underemployment for some set of states (θ^*,b), $\theta^*\varepsilon(\theta', b)$.

Until now it has been assumed that the employer is risk neutral. Suppose however that A.3.2 is modified so that

A.3.2′ $\quad v = v(\pi) : R \to R$, is C^2 and $v'(\pi) > 0 \; v''(\pi) < 0$

Assumption A.3.2′ states that the employer is strictly risk averse. Then the optimal contract δ^{***} solves the following programming problem.

P.3.4″
$$\max_{\delta} \int_a^b v(\pi(\theta))g(\theta)d\theta$$

$$\text{s.t} \int_a^b u(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta))g(\theta)d\theta = \bar{u} \qquad (3.34)$$

$$\dot{\pi}(\theta) = f_2(\ell(\theta),\theta). \qquad (3.27)$$

Notice that the incentive compatibility constraint is unchanged because any monotonic transformation leaves the maximum the same in P.3.3. Then the following interesting result has been proved by Grossman and Hart.

Corollary 4. Given assumptions, A.3.1, A.3.2′, A.3.3-A.3.6 and if labour is neither a normal nor inferior good then δ^{***} is unique and $f_1(\theta) > s(\theta)$ for all $\theta\ \varepsilon(a,b)$.

Proof: The first order conditions for P.3.4″ are

$$-\dot{p}(\theta) = (v'(\pi(\theta)) - \lambda u_1(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta)))g(\theta) \qquad (3.37'')$$

$$\lambda(u_1(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta))f_1(\ell(\theta),\theta) + u_2(f(\ell(\theta),\theta) - \pi(\theta),\ell(\theta))g(\theta)$$

$$= - p(\theta)f_{12}(\ell(\theta),\theta) \qquad (3.38'')$$

$$p(a) = p(b) = 0 \quad . \qquad (3.39'')$$

Then let $p(\theta) \geq 0$ over some interval (θ', θ''). This implies $f_1(\theta) \leq s(\theta)$ via equation (38″). Therefore $u_{11}(\theta)f_1(\theta) + u_{12}(\theta) \geq u_{11}(\theta)s(\theta) + u_{12}(\theta) = 0$ since labour is neither normal nor inferior. This then implies

$$\partial(v'(\theta) - \lambda u_1(\theta))/\partial\theta = v''(\theta)f_2(\theta) - \lambda(u_{11}(\theta)f_1(\theta) + u_{12}(\theta))\ \ell(\theta) < 0 \quad \forall\ \theta\ \varepsilon(\theta', \theta'') \qquad (3.59)$$

Then upon integration equation (3.37″) provides a contradiction. Therefore $p(\theta) < 0$ for all $\theta\ \varepsilon(a,b)$ and hence $f_1(\theta) > s(\theta)$ for all $\theta\ \varepsilon(a,b)$. The uniqueness of δ^{***} follows because $p(\theta)$ is uniformly signed which is all that is required in Takayma's theorem .

Corollary 4 shows that if labour is neither a normal nor inferior good and the employer is risk averse then the marginal product of labour will exceed the marginal rate of substitution between income and labour in each state. In other words there is underemployment. It is interesting to compare and contrast Corollary 1 with Corollary 4. Equation (3.59) explains the difference between the two results. In Corollary 4 equation (3.59) is assumed to be positive. In fact, in general the sign of equation (3.59) is very complicated and it may also change sign. Equally equation (3.59) does not directly determine the sign of the costate variable $p(\theta)$ which also depends on the normality of labour. It is clear for example that Corollary 4 still obtains if labour is an inferior good. Hence it is best to think of Corollary 1 and Corollary 4 as delineating circumstances when an unambiguous result can be obtained. One's choice can be made upon taste and situation.

The risk neutrality of the employer is usually justified by an appeal to a Knightian distinction between entrepreneurs and workers or by appeal to casual empirism that suggest emplyers are more wealthy or have better access to financial markets than workers or that firms are widely held by diverse individuals. However firms do take part in risk reducing activities and their purchases of insurance do not seem less than for individuals or workers. Equally firms cannot diversify against collective risks such as a 'world recession' or perhaps even against sectoral risks. This seems to suggest that employers should be modelled as risk averse. On the other hand the contract market is still a competitive market, and will tend to drive out risk averse employers. By the same token the normality of leisure as a good is widely acceptable. If for example there is no income effect in the supply of labour, the labour supply curve will always be upward sloping. Conclusions based on such an assumption can clearly be erroneous. Therefore it is perhaps true that Corollary 1 has wide applicability.

A simple example of an incentive compatible contract is given below. It illustrates the results of Lemma 3. In particular it is shown how the optimal incentive compatible contract responds to changes in the distribution of the set of states.

Example : The production, utility and density function satisfy

$$u(r,\ell) = 2\sqrt{r} - \ell : f(\ell,\theta) = \theta\ell : g(\theta) = \begin{cases} a & \text{if} \quad \theta = \theta_1 \\ 1-a & \quad \theta = \theta_2 \end{cases} \tag{3.60}$$

Thus the density function has a Bernoulli distribution where

$$E\theta = a\theta_1 + (1-a)\theta_2 = \mu. \tag{3.61}$$

$$\text{Var}\theta = a(1-a)(\theta_1-\theta_2)^2 = (a/(1-a)(\mu-\theta_1)^2 \tag{3.62}$$

Then if μ and θ_1 are taken as given an increase in a represents a mean preserving increase in variance, that is

$$\partial(\text{Var}\theta)\partial a = (\mu-\theta_1)|/(1-a)^2 \geq 0 . \tag{3.63}$$

This will be used to examine how the contract changes as the variance of the distribution changes but with the mean held constant. Such a procedure is only meant to be illustrative since the Bernoulli distribution is not really suited to a mean-variance treatment.

There are two incentive compatibility constraints for this distribution namely, that the employer cannot increase his profits by misreporting $\theta = \theta_1$ when in fact $\theta = \theta_2$ and vice-versa

$$\theta_1 \ell_1 - r_2 \geq \theta_1 \ell_2 - r_2 \qquad (3.64)$$

$$\theta_2 \ell_2 - r_2 \geq \theta_2 \ell_1 - r_1 \qquad (3.65)$$

where r_j and ℓ_j are the remuneration and labour input levels in state $j = 1, 2$. Notice that if equation (3.64) holds as an equality then equation (3.65) holds as an inequality and vice-versa. That is only one of the incentive compatibility constraints binds at any one time. It will be shown that only equation (3.64) binds. Intuitively this is plausible because the first best contract will automatically satisfy equation (3.63) but not equation (3.64).

The optimal incentive compatible contract δ^* solves the following problem.

P.3.5

$$\max_{\delta = \{r_1, r_2, \ell_1, \ell_2\}} \quad a(\theta_1 \ell_1 - r_1) + (1-a)(\theta_2 \ell_2 - r_2)$$

$$\text{s.t} \quad a(2\sqrt{r_1} - \ell_1) + (1-a)(2\sqrt{r_2} - \ell_2) = \bar{u}$$

$$\theta_1(\ell_1 - \ell_2) - (r_1 - r_2) \geq 0$$

$$\theta_2(\ell_2 - \ell_1) - (r_2 - r_1) \geq 0 .$$

Let the Lagrangian multipliers for the three constraints be λ, p_1 and p_2 respectively. Then the first order conditions for P.3.5 are

$$a\theta_1 - \lambda a + p_1\theta_1 - p_2\theta_2 = 0 . \qquad (3.66)$$

$$(1-a)\theta_2 - \lambda(1-a) - p_1\theta_1 + p_2\theta_2 = 0 . \qquad (3.67)$$

$$-a + \lambda a r_1^{-\frac{1}{2}} - p_1 + p_2 = 0. \qquad (3.68)$$

$$-(1-a) + \lambda(1-a)r^{-\frac{1}{2}} + p_1 - p_2 = 0. \qquad (3.69)$$

$$p_1(\theta_1(\ell_1 - \ell_2) - (r_1 - r_2)) \geq 0 \text{ with c.s.} \qquad (3.70)$$

$$p_2(\theta_2(\ell_2 - \ell_1) - (r_2 - r_1)) \geq 0 \text{ with c.s.} \qquad (3.71)$$

Notice from equations (3.66) and (3.67) that $\lambda = \mu$ which is set equal to unity for convenience. Since equation (3.70) and (3.71) imply $p_1 \cdot p_2 \geq 0$ with complementary slackness it is evident from equation (3.66) or (3.67) that $p_2 = 0$ and

$$p_1 = a(1-\theta_1)/\theta_1 = (1-a)(\text{var}\theta)/(1-\theta_1)\theta_1$$
$$= (\text{var}\theta)/\theta_1(\theta_2-\theta_1) \qquad (3.72)$$

Therefore using equations (3.66) and (3.68) and (3.67) and (3.69)

$$r_1 = \theta_1^2$$
$$(\theta_2 r_2^{-\frac{1}{2}}-1) = -\, p(\theta_2-\theta_1)/(1-a)$$
$$= -\,(\text{var}\theta)/(1-a)\theta_1 \qquad (3.74)$$

The L.H.S. of equation (3.74) represents the divergence between the marginal product of labour θ_2, and the marginal rate of substitution between income and labour $r^{\frac{1}{2}}$, in state two. It shows that the marginal product is less than the marginal rate of susbstitution or that there is overemployment in state two. Equation (3.73) on the other hand shows that the contract is productively efficient in state one. Differentiating the L.H.S. of equation (3.74) for a fixed value of μ and θ_1 with respect to a gives

$$\partial(\theta_2 r_2^{-\frac{1}{2}} - 1)/\partial a\big|_{\mu,\theta} = -[(1-a)(\partial \text{var}(\theta)/\partial a) + \text{var}\theta]/(1-a)^2\theta_1 \leq 0. \quad (3.75)$$

This shows that the deviation between the marginal product and the marginal rate of substitution decreases as the probability that state one occurs increases. At first sight this result seems rather counter-intuitive. It might be expected that as the probability that state two occurs decreases the employees income in that state is raised to maintain a constant level of expected utility, hence raising the marginal rate of substitution still further above the marginal product.

In fact it is known that in the general case, the transversality conditions, equation (3.39) ensure that the optimal incentive compatible contract is productively efficient at the end points of the distribution. The reason for this appears to be that by definition, the contract at the end points is constrained by incentive comptaibility on only one side. But the only reason the contract is productively inefficient is to meet the incentive comptaibility constraints. Therefore any inefficiency at the endpoints is simply a dead-weight loss. The same principles may apply to equation (3.75). For example as the probability that state two occurs falls the incentive compatibility constraint becomes less important so that productive inefficiency can be reduced. This is all very tentative and the really important point to notice is that the optimal incentive compatible contract actually depends on the probability density function $g(\theta)$. This is important because the first best contract does not depend on g though it may depend upon the parameters of the distribution through the expected utility constraint. The dependence of the optimal incentive compatible contract upon the density function will be important in any macroeconomic or general equilibrium context.

The next section presents some concluding remarks.

Section 3 : Conclusions

This chapter has shown how the optimal labour contract is affected if the contracting parties have asymmetric information. It has been shown that the optimal incentive compatible contract will be both productively inefficient and also share risk inefficiently. In general if the employee has better information there will be underemployment and if the employee has better information there will be overemployment.

It was shown that the optimal incentive compatible contract depends upon the distribution function of the state of nature. This is important because the first best contract does not depend upon the density function. This feature of the optimal incentive compatible contract will be used in chapter 4 to examine optimal contracts when there is imperfect information.

The two cases of asymmetric information studied in this chapter are not the only instance of information asymmetry in labour contracts. For example when considering a potential employee, the employer might not know the distribution function $g(\theta)$ that indicates whether the employee is on average a good or bad worker. To be more specific the density function may be parameterized $g(\theta,\alpha)$ where α is known to the potential employee but not to the employer. A situation of this type has been studied by Rothschild and Stiglitz (1976) in the context of an insurance market. Alternatively the situation might be reversed so that the employer knows α but the employee does not. This would appear to be a fairly typical and interesting information asymmetry which would make an interesting extension to the present model.

Notes

1. A result of this kind is given by Akerlof (1970).

2. See chapter 1.

3. Green and Kahn only examine the second case. The first case has also been studied by Cooper (1981).

4. See Homlmstrom (1981) for a preliminary investigation.

5. It is at this point that a concern for reputation becomes important.

6. It is assumed that the non-negativity constraint on the supervisors income is not binding, or alternatively X is sufficiently large.

7. Again profits are always assumed to be non-negative. An examination of this constraint is given by H. Grossman (1977).

8. See Stiglitz (1977).

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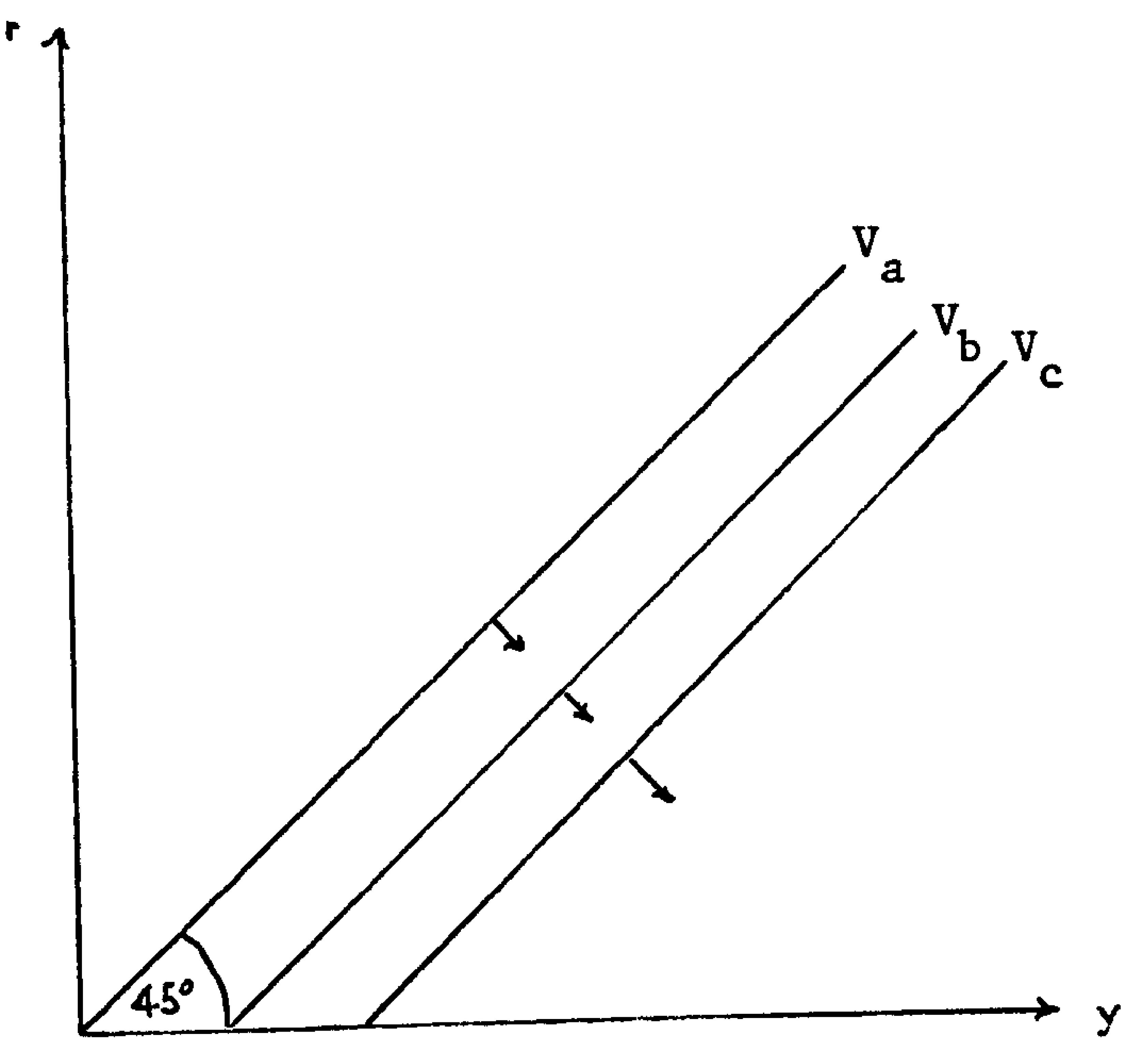

Diagram 3.1

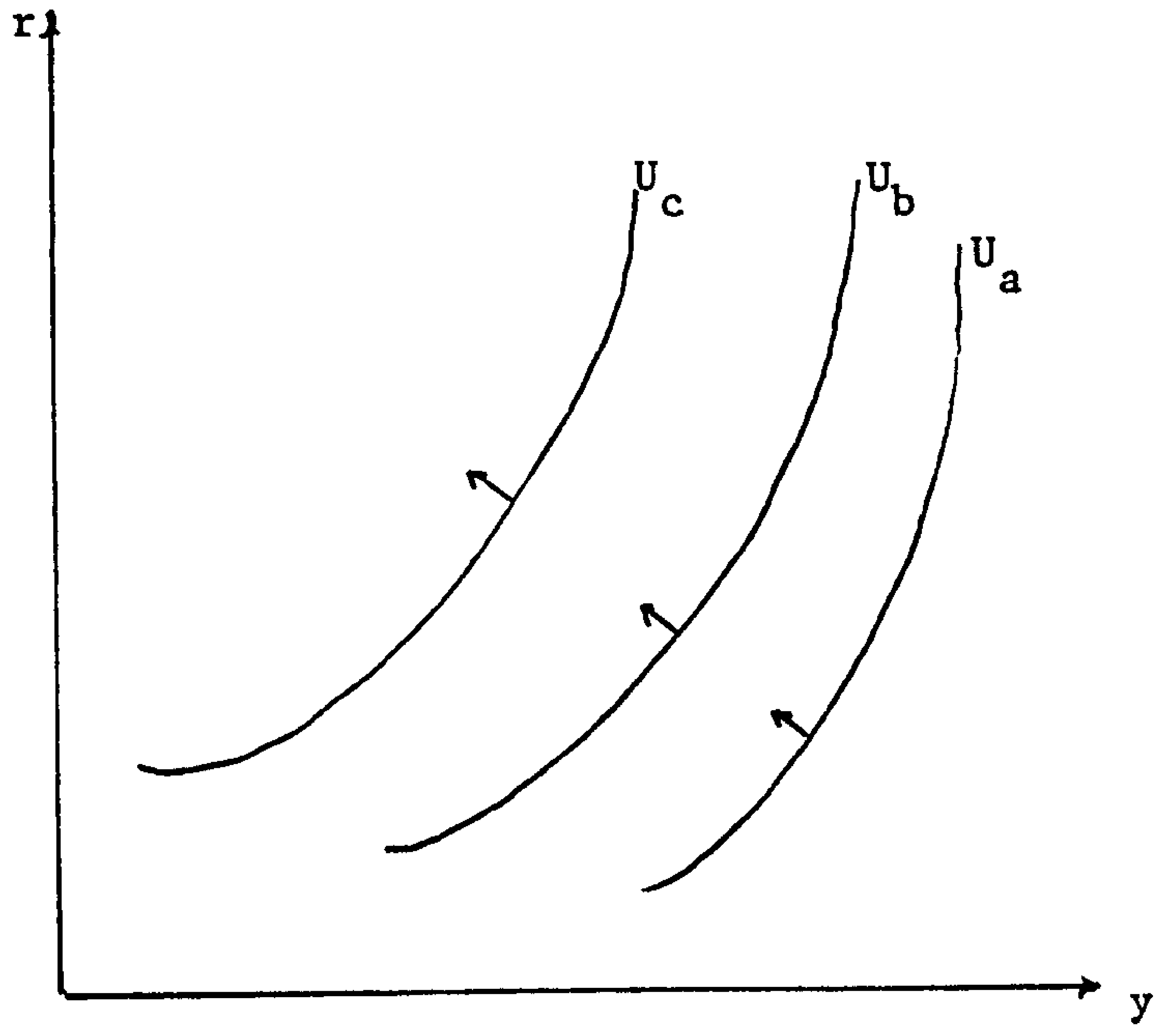

Diagram 3.2

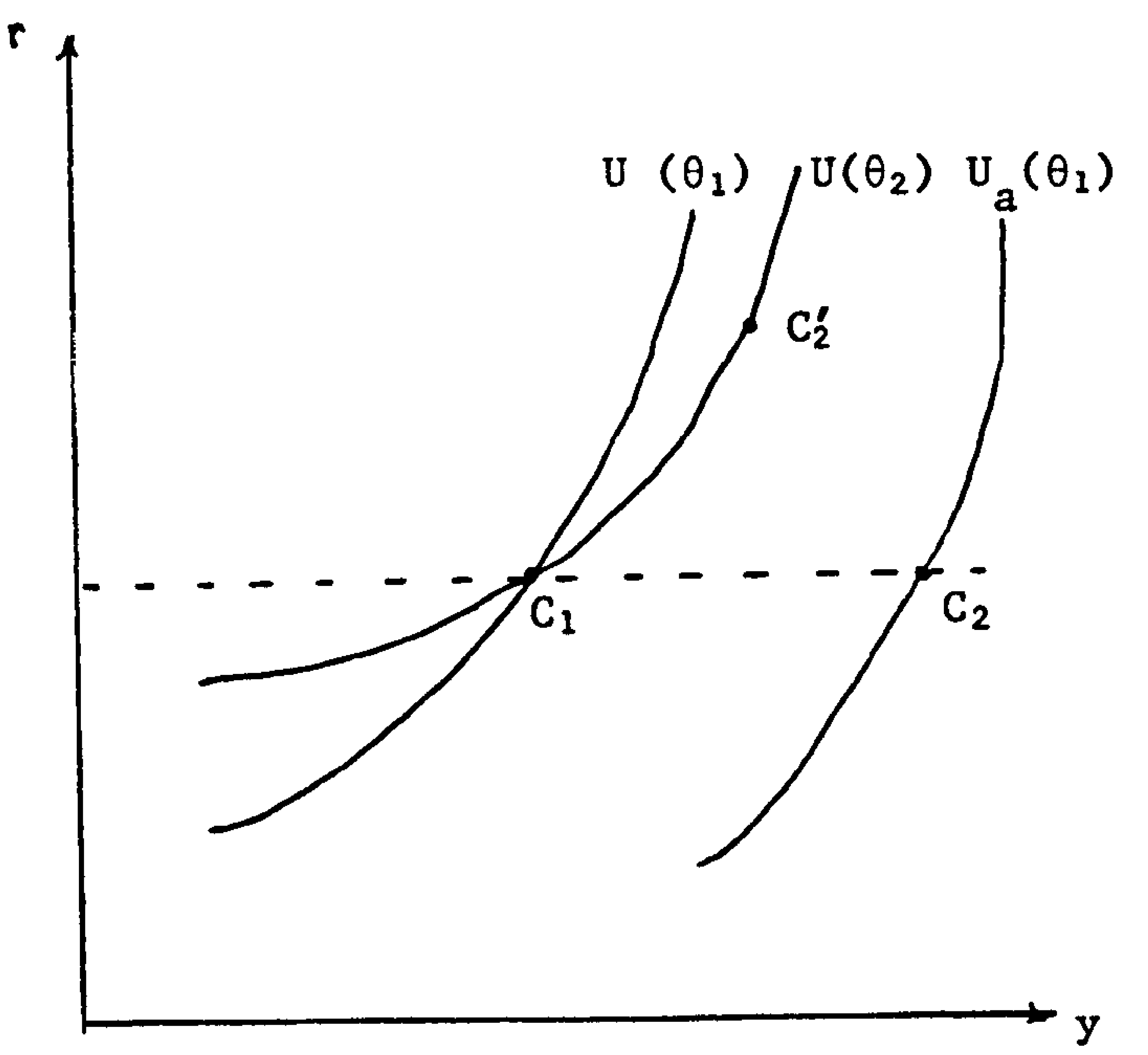

Diagram 3.3

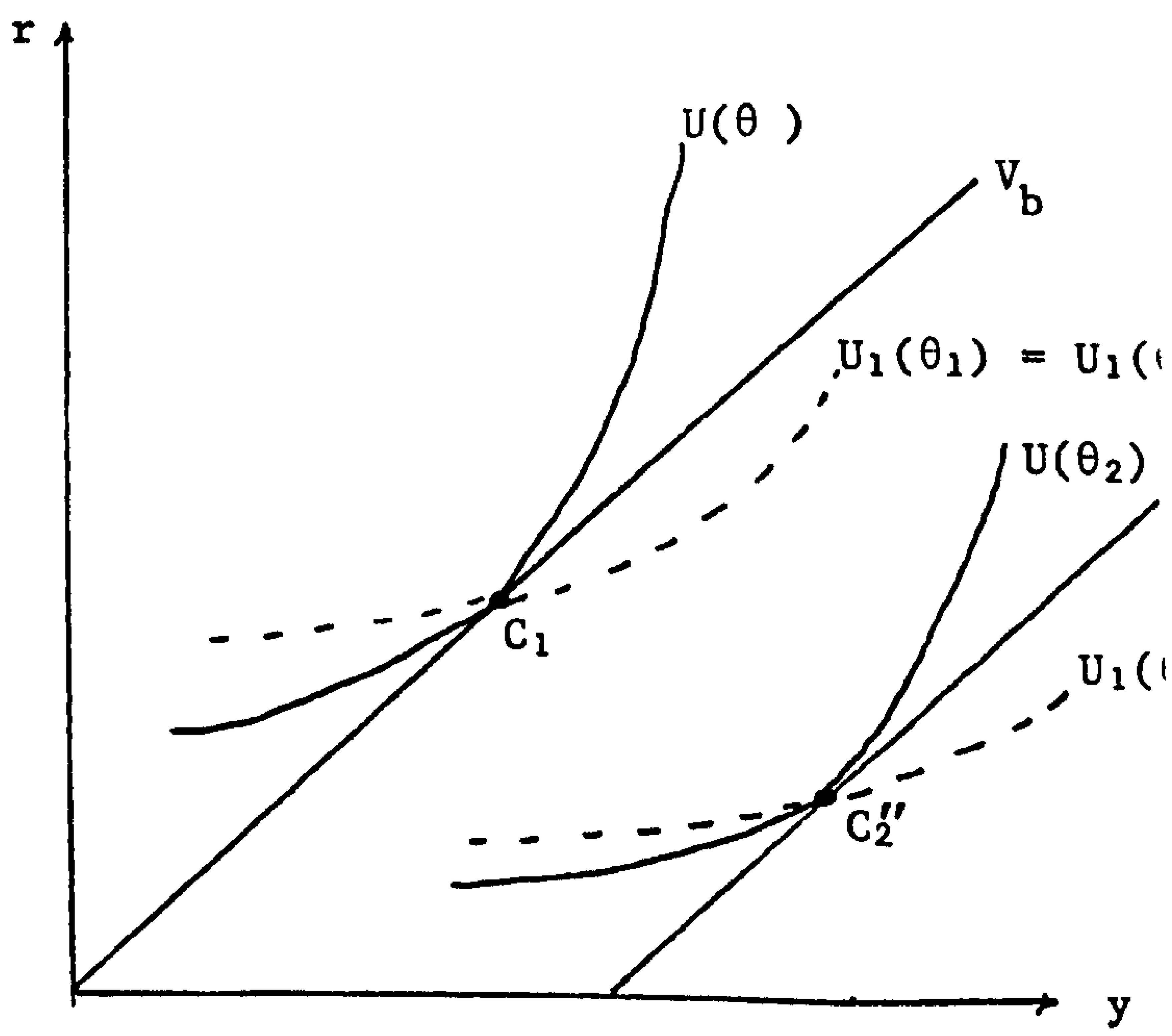

Diagram 3.4

Seade, J., (1982), "On the Sign of the Optimum Marginal Income Tax", Review of Economic Studies, 49, 637-644.

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CHAPTER 4

Labour Contracts and Imperfect Information

Section 0 : Introduction

This chapter examines the optimal contract when the participants to the contract have only imperfect information about the state of nature. In chapters 1-3 the optimal labour contract was studied within the confines of a closed system. In chapters 1-2 it was assumed that each participant to the contract had complete information about the state of nature. In chapter 3 it was assumed that there was asymmetric information, that is either the employer or employee was able to observe the true state of nature, but not both. This being said imperfect information does not exclude asymmetric information because it may still be possible for the employer (employee) to be better informed about the true state of nature than the employee (employer). There are two important issues that can be addressed under the heading of imperfect information. These are wage indexation and the implications of labour contracts for monetary policy.

The most important assumption of Keynesian and Neo-Keynesian analysis is the assumption that nominal wages and prices are sticky or fixed and therefore do not adjust to clear markets. Because markets do not clear agents are quantity constrained and it is the spillover effect of these constraints from one market to another that generates the Keynesian multiplier process. What has to be explained of course is why wages and prices do not adjust to clear markets.

One of the earliest claims for labour contract theory was that it could explain fixed or sticky wages, though it was not always made clear whether this fixity applied to nominal or real wages. In fact if the employer is allowed to pay the employee unemployment compensation the contract studied by Azariadis (1975) exhibits a constant real wage. A fixed real wage,however circumvents the usual impact of Keynesian monetary policy on output and employment which reduces the real wage by increasing prices. This suggests that what is important about Keynesian models is the non-clearance of markets rather than the fixity of prices.

Recently however Mordecai Kurz (1982) has suggested that a equilibrium with certain Keynesian features, namely the non-clearance of markets arises when some set of prices are linked to some other set of prices. The essential idea is that if some prices are linked or indexed then there will be too few independent prices to clear all markets simultaneously. Kurz does not examine why or how prices are linked but this is precisely what the labour contract does do. Hence section one will examine how wages are indexed to prices by a labour contract and in particular whether wages are fully indexed.

It has also been suggested, notably by Gray (1976) and Fischer (1977) that contracts with sticky wages imply that monetary disturbances can affect employment and output even when these disturbances are recognized as contemporaneous and monetary in nature. This is far from clear however because if monetary disturbances are perceived as contemporaneous the optimal labour contract will index directly to these disturbances so as to be completely offsetting. Fischer's viewpoint on this appears to be that transactions costs will rule out this type of sophisticated indexation, but this seems to be equivalent to assuming that the monetary disturbances are not recognized as such or not sufficiently important to be recognized as such. Thus any effect of monetary shocks on output and employment would be relatively minor.

In section 2 it is shown how, if there is asymmetric information, additional information inside or outside the labour market can influence the efficiency and output and employment levels of the labour contract. An example is given where for some parameter values a change in this additional information will uniformly reduce the productive inefficiency of the labour contract. However examination of the implications of this for business cylcle theory must await the development of a more general model.

Section 1 : Wage Indexation

This section examines how wages are indexed to prices. Azariadis (1978), Fischer (1977), Jones (1980) and Shavell (1976)[1] have each found contract theory a natural and amenable framework in which to discuss wage indexation. For example consider a contract between one employer and one employee. A contract δ negotiated at say, time zero specifies the wages to be paid to the employee, w in each possible state of nature, s at time one, and the labour to be supplied by the employee ℓ in each state of nature at time one. That is to say the contract is completely specified by the tuple of functions $\delta = \{w(s), \ell(s)\}$. Suppose however that neither the employer nor the employee observe the true state of nature and that the only signal they have of s is the price or price vector p. Then the contract will be written as $\delta = \{w(p), \ell(p)\}$. The function w(p) relates wages to prices and therefore summarizes or represents the indexation of wages to prices.

Parkin (1977) has suggested, however, that the function w(p) does not capture what is usually thought of as wage indexation. He maintains that wage indexation is designed to ensure that the relative prices agreed upon by the contract are sustained throughout the life of the contract independent of the rate of inflation. This seems a rather narrow view of indexation since by definition every agent will wish to maintain agreed upon relative prices. Therefore any deviation from complete indexation will be sub-optimal and can only be explained by appeal to some sort of transactions costs not specified by the model.

Since it cannot be argued that indexation is intrinsically expensive[2] w(p) will be taken to represent the dergee of wage indexation or escalation for the purposes of this section.

In addition to representing the degree of wage indexation w(p) can also be interpeted as defining a price index. This is quite an unusual definition of a price index and obviously requires some elaboration. It is often quite difficult to understand what is meant by a price index. Afrait (1976) somewhat uncharitably suggests that "when an unambiguous meaning for a number is not known it is an index". However recent developments in index number theory, e.g. Diewart (1981) suggests that the cost function c(p,u) be interpreted as an index. The cost function measure how income responds to price changes in order to maintain a constant level of utility. For any given level of u the cost function c(p,u) represents constant real income. In other words it shows how income should be escalated as prices change to maintain its real value. Analogous to the cost function c(p,u) is a wages function $\hat{w}(p,u)$ which measures how wages should be escalated as prices change in order to maintain its real value. The wage function w(p,u) represents the minimum wages necessary to achieve a utility level u by choice of consumption and leisure[3] Notice that the contract wage w(p) also depends upon the level of expected utility $\bar{u}$ the employee is guaranteed by the contract, that is $w = w(p,\bar{u})$. This function $w(p,\bar{u})$ defines a implicit price index in the same way that $\hat{w}(p,u)$ does, however w(p,u) does not in general maintain a constant real wage.

Price indices have traditionally been used in two quite distinct and different ways. First they have been used descriptively to convert nominal into real values. This is the purpose of the wages function $\hat{w}(p,u)$[4]. Second price indices have also been used operationally to escalate deferred monetary payments. But this is the function of

$w(p,u)$[4], it indicates how wages are to respond to prices at time one in a way specified by the contract negotiated at time zero. In this sense $w(p,u)$ implicitly defines the unit of account. It is interesting to note that before national income accounting became so important, in the forties and fifties, it was the operational use of index numbers rather than their descriptive content which was believed to be the most important. For example Irving Fisher (1922) thought that "perhaps the most important purpose of index numbers is to serve as a basis for loan contracts", that is for escalating debts.

The principal objective of this section is to examine the relationship between the two types of wage indices $w(p,u)$ and $\hat{w}(p,u)$. The wages function $\hat{w}(p,u)$ reflects only preferences, it is concerned with desirability rather than feasibility. On the other hand the contractual index $w(p,u)$ is chosen optimally as a hedge against uncertain future prices and therefore reflects both objectives and constraints. An important special case arises if the two indices coincide. In this case the contract is said to fully index wages because utility is kept constant. In general $w(p,u)$ will maintain a constant marginal utility of income and there $w(p,u)$ and $\hat{w}(p,u)$ will not coincide so that wage indexation will be incomplete.

There are two further conceptual issues that need to be addressed at this point. Both relate to the role of money or more generally assets within the model. First it will be assumed that wages are paid in terms of money, that is so many pound notes. This is doubly problematical. It is not enough to say that wages are paid in terms of the numeraire commodity because the numeraire is not unique. It is necessary to justify money as a medium of exchange, this is usually done by appealing to some sort of transactions costs. However the problem is still more complicated in the labour contract model because it is assumed that the employer knows the employees preferences.

It is thus possible that the employer might conveniently 'pay' the employee with an appropriate bundle of commodities in much the same way as he might provide maintainance and lubrication for machinary. There are legal constraints to such arrangements in the U.K. in the form of the truck acts but many professions do provide some form of non-momentary remuneration or fringe benefits. One story that can be told and the one that will be used here to justify the employer paying only money wages is that workers are better able or simply have more time to buy the goods they want. For example the employee might constitute one half of a worker-housewife partnership or marriage. The worker is to be viewed as dedicated to earning a wage by supplying labour and the housewife as dedicated to spending it.

Another conceptual problem of wage indexation has been raised by Alan Blinder (1977). Suppose for example that the employee has some portfolio choice problem. If this choice involves indexed bonds then this will affect the degree to which wages are indexed in the labour contract. The employee only desires that his wealth is indexed to prices so as to hedge against uncertainty. He is indifferent as to whether his wages are indexed or his assets. If the contract is negotiated after it is known how bonds are indexed then there is no problem. However if bonds are optimally indexed and indexed simultaneously with wages it is not clear what will result. The best that can be said is that the more are bonds indexed the less will wages be indexed and vice-versa.

The section proceeds as follows. First Azariadis' model is examined in order to highlight some conceptual issues. Then Azariadis' model is generalized in a number of ways. First it is assumed that agents are unable to disentangle relative from absolute price movements. Second it is assumed that employees live for two periods and are able to transfer wealth from one period to another. Third it is assumed that there is more than one good. Finally some concluding comments are offered to draw the analysis together.

Initially it will be helpful to reconsider the simple model developed in Azariadis (1975). This will be useful in putting subsequent more general models into perspective. In fact some liberties will be taken with Azariadis' model in order to simplify and reshape it into a form or framework more conducive to the examination of wage indexation.

To make the model as simple as possible suppose there is one employer and one employee. The assumption that there is only one employee is not restrictive providing that if there is more than one employee they are each identical, that is each employee has the same utility function and every employee's labour supply is a perfect substitute for any others. Assume also that there is a single consumption good y, that is produced by the employer according to a twice continuously differentiable concave production function $f(\ell)$ where ℓ is the employees labour input.

The price of the consumption good p is a random variable. The simple space of p is $P = (\underline{p},\bar{p})$ it is assumed that p has a continuous density function g(p). Because p is a random variable the employer and employee have the opportunity to make a contract δ before p is known, but conditional upon p. Therefore a contract δ will specify the remuneration r the employer will pay the employee, and the labour supply ℓ the employee will provide the employer for each possible value of the price of output. More succinctly $\delta = \{r(p),\ell(p)\}$.

It is important to notice that the interpretation of the contract δ is slightly different from that given in chapter 1 or chapter 1A. The level of remuneration r is to be interpreted as an amount of money, so many pound notes, transferred from the employer to the employee rather than a certain quantity of the consumption good. With this interpretation of δ the price of output p is to be thought of as a relative price, it is the price of the consumption good relative to the price of money, say

p_m. For the moment the price of money is conveniently normalized to unity. Therefore the relative price of consumption is a random variable, but its variability may be attributable to either real, that is changes in the price of output, or nominal, that is changes in the price of money disturbances.

Given some contract δ the employers ex post nominal profits π are given by

$$\pi = pf(\ell) - r \quad . \tag{4.1}$$

However the employer is interested in 'real' profits π/p, rather than nominal profits since real profits determine the level of his consumption. Real profits are

$$\pi/p = f(\ell) - r/p \; . \tag{4.1'}$$

It is assumed that the employees preferences are representable by a twice continuously differentiable utility function which is additively separable in consumption x, and labour supply ℓ. That is

$$u = u(x) - h(\ell) \tag{4.2}$$

where u is increasing and concave and h is increasing and convex. Since the employee receives a remuneration level r and there is only one consumption good his budget constraint is

$$x = r/p \quad . \tag{4.3}$$

Therefore the employees consumption is determined directly by his wage income or remuneration r. The wage or wage rate w is defined implicitly by the contract via equation (4.4)

$$w = r/\ell \quad . \tag{4.4}$$

The optimal contract δ^* between the employer and the employee is the solution to P.4.1

P.4.1 $$\max_{\delta\varepsilon\Lambda} \int_P (f(\ell(p)) - r(p)/p)g(p)dp$$

$$\text{s.t} \quad \int_P (u(r(p)/p) - h(\ell(p)))g(p)dp = \bar{u} \qquad (4.5)$$

where Λ, the set of contracts satisfying equation (4.5)[5], is assumed to be non-empty. In words the optimal contract δ^* maximizes the employers expected real profits subject to providing the employee with a given level of expected utility $\bar{u}$. As usual $\bar{u}$ may be interpreted as determined by competition in the market for employees or as determined by bargaining strengths.

The first order conditions for P.4.1, equations (4.6) and (4.7) are both necessary and sufficient

$$h'(\ell(p))/f'(\ell(p)) = 1/\lambda \qquad p \;\varepsilon\; P \qquad (4.6)$$

$$u'(r(p))/p) = 1/\lambda \qquad p \;\varepsilon\; P \qquad (4.7)$$

where λ is the Lagrangian multiplier associated with equation (4.5). It is readily apparent from equation (4.6) and (4.7) that both the real rate of remuneration r/p and the labour input ℓ are constant and independent of the relative price of output. Therefore the real wage w/p is also constant so that the terms wage and remuneration can be used synonymously. For the moment consider the remuneration level and rewrite equation (4.5) as

$$u(r(p)/p) - h(\ell) = \bar{u} \qquad (4.5\ )$$

Then solving for r as a function of p and $\bar{u}$

$$r(p,\bar{u}) = p.u^{-1}(\bar{u}+h(\ell(\bar{u}))) \qquad (4.8)$$

where $\ell(\bar{u})$ is determined from equation (4.6) because λ and $\bar{u}$ are directly related. Equation (4.8) shows that the nominal remuneration is proportional to the price level. Taking $\ell(\bar{u})$ as given the right hand side of equation (4.8) can be given a different interpretation. It is the level of income necessary to maintain a constant level of utility $\bar{u}$. In other words it is the cost function $c(p,\bar{u})$ defined as the solution to

$$\text{P.4.2} \qquad \min_x \quad px \qquad \text{s.t } u(x) - h(\ell(\bar{u})) = \bar{u} \quad .$$

That is solving P.4.2 directly by inverting the constraint

$$c(p,\bar{u}) = p.u^{-1}(\bar{u} + h(\ell(\bar{u}))) \ . \qquad (4.9)$$

Combining equations (4.8) and (4.9) gives

$$r(p,\bar{u}) = c(p,\bar{u}) \ . \qquad (4.10)$$

Equation (4.10) shows that given the optimal choice of labour input determined by equation (4.6), the remuneration schedule negotiated by the employer and the employee is precisely that cost function necessary to maintain the employees utilitv at a constant level. Therefore the remuneration schedule is linearly homogeneous in prices or alternatively nominal remuneration is fully escalated to keep utility constant. Now equation (4.10) can be rewritten in terms of the wage schedule $w(p,\bar{u})$ and the wage function $\hat{w}(p,u)$ since $\ell(\bar{u})$ is given. That is

$$w(p,\bar{u}) = r(p,\bar{u})/\ell(\bar{u}) = c(p,\bar{u})/\ell(\bar{u}) = \hat{w}(p,\bar{u}) \qquad (10')$$

so that wages are, equally, fully escalated. All this is of course fairly trivial since it follows directly from equation (4.7). That is to say a constant marginal utility of income must imply a constant level of total utility because the utility function has only a single argument. Nevetheless this result provides a convenient benchmark for subsequent analysis and the same type of reasoning will be used extensively below.

It is important to notice that constant real wages are being defined by the coincidence of the optimal wage schedule w(p,u) and the wage function $\hat{w}(p,u)$. The wage schedule w(p,u) defines an implicit price index in the same way that $\hat{w}(p,u)$ defines the appropriate deflator for converting nominal into real wages. Now what is usually understood by the term wage indexation is the extent to which wages change as prices change, that is the price derivative of w(p,u), $w_p(p,u)$. Therefore the degree of wage indexation (1-d) can be defined by equation (4.11)

$$w_p(p,u) \doteq \hat{w}_p(p,u)(1-d) \tag{4.11}$$

If d = 0 then wages are said to be fully indexed, if d = 1 then wages are unindexed or nominal wages are fixed and if $d \ \varepsilon (0,1)$ then wages are only partially indexed to price. The remainder of this section will be devoted to examining the optimal degree of wage indexation in some slightly more general models.

Therefore having established a method of analysing wage indexation it is possible to extend Azariadis' model in a number of different directions. The first extension to be considered is the possibility that the employer and the employee are unable to distinguish relative from absolute price movements.[6] In Azariadis' model it was assumed that the price of money p_m was observed by both employer and employee and therefore it was normalized to unity. However the price of money may not be so readily observable in which case the contract will have to be executed before p_m is known. That is to say the contract δ will be made contingent on the abolute price of output $p, \delta = \{ r(p), \ell(p) \}$. Therefore the employees actual consumption or equivalently the employers actual wage payments will not become known until the true value of p_m is known. Thus there are three distinct stages to any contract. At stage one the employee and employer negotiate a contract conditional upon the output price p. At stage two the value of p becomes known, and the agreed upon labour is supplied and remuneration received by the employee. At the third stage the value of p_m is revealed and this determined both the employers and the employees consumption.

The misperception of nominal for real variables has been used by Friedman (1968) as an explanation of the short-run trade-off between output and inflation, the Phillips curve. Similar misperceptions of transitory for permanent and local for general price changes have been used by Lucas and Rapping (1970) and Mortensen (1970) as alternative explanations of the Phillips curve. If these type of misperceptions are widespread it is important to understand how wages and prices are related. This is especially important in the Phillips curve context. It will be shown that the optimal labour contract will fully index wages to prices.

There is no reason to suppose that p_m and p are correlated. Therefore it will be assumed that p is drawn from a density function $g_p(p)$ with a sample space P and p_m is drawn from a density function $g_{p_m}(p_m)$ with a sample space P_m. Both density functions are assumed to be known to both the employer and the employee.

The optimal contract δ^* maximizes the firms expected profits, subject to offering the employer a level of expected utility $\bar{u}$. That is $\delta^* = \{r^*(p), \ell^*(p)\}$ solves

$$\text{P.4.3} \quad \max_{\delta \in \Lambda} \int_P \int_{P_m} (f(\ell(p)) - p_m r(p)/p) g_p(p) g_{p_m}(p_m) dp dp_m$$

$$\text{s.t.} \quad \int_P \int_{P_m} (u(p_m r(p)/p) - h(\ell(p))) g_p(p) g_{p_m}(p_m) dp dp_m = \bar{u} \tag{4.12}$$

where Λ , the set of contracts satisfying equation (4.12) is assumed to be non-empty.[7] The first order conditions for P.4.3 are

$$\int_{p_m} (f\ (\ell(p)) - \lambda\ h'(\ell(p)))g_{p_m}(p_m)dp_m = 0 \qquad p \in P \qquad (4.13)$$

$$\int_{p_m} (-p_m/p + \lambda u'(p_m(p)/p)\cdot p_m/p)g_{p_m}(p_m)dp_m = 0 \qquad p \in P \qquad (4.14)$$

where λ is the Lagrangian multiplier for equation (4.12). Equations (4.13) and (4.14) can be simplified as follows

$$h'(\ell(p))/f'(\ell(p)) = 1/\lambda \qquad (4.15)$$

$$\int_{p_m} u'(p_m\cdot r(p)/p)\cdot p_m g_{p_m}(p_m)dp_m = 1/\lambda\left(\int_{p_m} p_m g_{p_m}(p_m)dp_m\right). \qquad (4.16)$$

Equation (4.15) is exactly the same as equation (4.6). It shows that labour is independent of prices and depends only on $\bar{u}$ through its dependence on λ. Equation (4.16) is rather more difficult to interpret though in the same way as equation (4.7) it shows that the remuneration schedule also depends upon $\bar{u}$. Differentiating equation (4.16) with respect to p gives

$$\partial r(p,\bar{u})/\partial p = r(p,\bar{u})/p \qquad (4.17)$$

or integrating

$$r(p,\bar{u}) = \alpha p \qquad (4.18)$$

where α is a factor of proportionality. However the cost function c(p,u) depending on the absolute price p before price of money is observed is defined by

$$\int_{p_m} u(p_m\cdot c(p,\bar{u})/p)g_{p_m}(p_m)dp_m = \bar{u} + h(\ell(\bar{u})) \qquad (4.19)$$

Therefore differentiating equation (4.19) with respect to p

$$\partial c(p,\bar{u})/\ p = c(p,\bar{u})/p \qquad (4.20)$$

or integrating

$$c(p,\bar{u}) = \alpha p \quad . \tag{4.21}$$

Therefore the degree of wage indexation is unity, d = 0, so that wages are fully escalated even if the price of money is not observed at the time the contract is honoured. Of course ex post once the value of p_m is revealed wages are not fully indexed. Indeed ex post the degree of wage indexation is simply p_m. However this is not really relevant because wages have to be paid out before p_m is known. Therefore Azariadis' result stands, wages will be fully indexed even when there is misperceptions of real and nominal variables by the employer and the employee.

One obvious drawback of Azariadis' model is that although money is introduced explicitly it is not clear what money is or does. However an important role for money can be introduced if the employee is allowed to use money to transfer wealth from one period to another.

Suppose that the employee lives for two periods. In the first period he supplies labour to the employer and saves for his old age out of his wage income. In the second period the employee retires and consumes out of his savings. It will be assumed that the employer lives for only one period, the first, in which he transforms the employee's labour input into output which he then sells. This assumption will avoid a number of thorny issues and keep the analysis relatively simple.[8] It also rules out a number of interesting problems such as the optimal provision of pension schemes.

If it is preferred the model can be thought of in the overlapping generations context. In each period a fixed number of employers and employees are born. At any one time there is only ever one generation of employers because employers live for only one period. On the other hand there will be both old (retired) and young (employed) employees at any one time both competing for consumption. Alternatively it might be supposed that employers do live for two or more periods, but that they discount the future so heavily that the future becomes unimportant relative to their current optimizing decisions.

The model can then be conveniently characterized as having three dates and four goods. The four goods are present and future consumption, present leisure and a single asset called money by which wealth is transferred from the present to the future. The time structure of the model is drawn in diagram 1

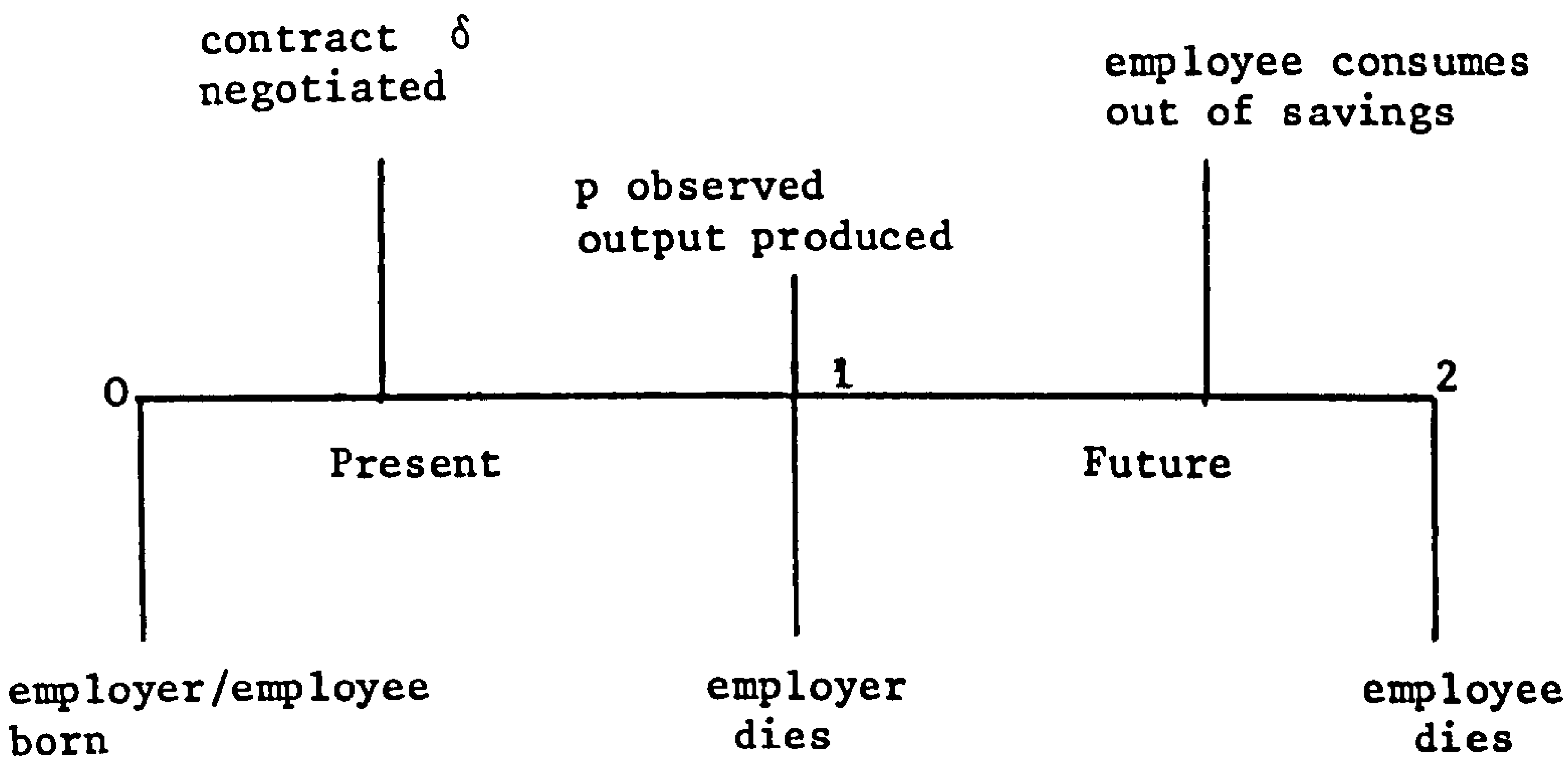

Diagram 1

At date zero, the start of the first period the employer and employee negotiate a contract δ which depends upon the price of present consumption p at date one. Viewed from date zero p is a random variable. It will be assumed that the density function of p $g(p)$ is known objectively to the employer and employee. As usual the sample space of p is P. Therefore a contract δ will specify the remuneration to be paid to the employee $r(p)$ and the labour he is to supply $\ell(p)$ if the price of present consumption at date one is p.

At date one, once p is known the contract is executed. The employer then consumes out of his profit which are

$$\pi = pf(\ell(p)) - r(p)) \ . \tag{4.22}$$

Therefore the employers consumption or real profits at date one is

$$\pi/p = f(\ell(p)) - r(p)/p \ . \tag{4.22 }$$

This is precisely the same as equations (4.1) and (4.1′). The employee however has to decide how much of his remuneration he should spend on present consumption and how much to save for future consumption. His decision will depend on what he believes will be the future price of consumption, $\hat{p}$. In general $\hat{p}$ will be a random variable the density function for which depends upon the price of present consumption p However, and in order to keep the analysis simple, it will be assumed that $\hat{p}$ is known with certainty once p is known according to the formula

$$\hat{p} = \psi(p) \tag{4.23}$$

The employees utility depends upon his present consumption c, his future consumption $\hat{c}$ and his labour supply ℓ . It will be assumed that the employees preferences are representable by a utility function that satisfies

A.4.1 $u = u(c,\hat{c}) - h(\ell) : R_+ \times R_+ \times [0,1] \to R$; is c^2 and

$h' > 0,\ h'' \geq 0,\ u_1 > 0,\ u_2 > 0,\ u_{11} \leq 0,\ u_{22} \leq 0.$

$(u_{11}u_{22} - u_{12}^2) \geq 0,$

$(u_{11}s + u_{12}) \leq 0,\ (u_{12}s + u_{12}) \leq 0,\ s = -u_2/u_1$

The Assumption A.4.1 requires that utility is concave in all its arguments and that both present and future consumption are normal goods.

Therefore at date one, once p is known and given the contract δ

the employee faces a budget set

$$B(p,\hat{p},r) = \{(c,\hat{c},m) \in R^3_+ | \ pc+m = r, \ \hat{p}\hat{c} = m\} \quad . \tag{4.24}$$

This budget set can be rewritten remembering that future consumption

$$\hat{c} = m/\psi(\ p) \tag{4.25}$$

therefore $$B(p,r) =\{ (c,m) \in R^2_+ | \ pc+m = r \} \quad . \tag{4.26}$$

The employee will choose his present consumption c and his savings m to maximize his utility subject to the budget constraint defined in equation (4.26). That is the employee solves

P.4.4 $$\max_{c,m} \quad u(c,m/\psi) \quad \text{s.t. } (c,m) \in B(p,r) \quad .$$

Letting β denote the Lagrangian multiplier for equation (4.26) the first order conditions are

$$u_1(c,m/\psi) = \beta p \quad u_2(c,m/\psi) = \beta\psi \ . \tag{4.27}$$

Therefore the employees present consumption $c = c(p,r)$ and his savings $m = m(p,r)$ are defined by equation (4.27). The employees indirect utility function is

$$u^*(p,r,\ell) = u(c(p,r),\ m(p,r)/\psi(p)) - h(\ell) \tag{4.28}$$

where $$u^*_p(p,r,\ell) = -\ \beta(c(p,r) + m(p,r)\varepsilon(p)/p) \tag{4.29}$$

$$u^*_r(p,r,\ell) = +\ \beta \tag{4.30}$$

$$u^*_\ell(p,r,\ell) = -\ h'(\ell) \tag{4.31}$$

and $$\varepsilon(p) = p\psi'(p)/\psi(p) \quad . \tag{4.32}$$

Equation (4.32) has a ready interpretation, it is the elasticity of price expectations. If $\varepsilon(p)$ is unity then the price of future consumption is believed to be proportional to the price of present consumption. If price expectations are very inelastic the $\varepsilon(p)$ is zero and the price of present consumption does not alter the expected price of future consumption.[9] In general the elasticity of price expectations will lie somewhere between these two extremes, that is $\varepsilon(p) \in (0,1)$.

Recapitulating at date one the employee decides upon his present consumption and savings by solving P.4.4. At date two the employees future consumption is determined directly by his savings via equation (4.25).

The optimal contract δ^* must take into account the employees opportunity to save at date one. Therefore δ^* will be chosen to maximize the employers expected profits subject to providing the employee with a given level of expected indirect utility $\bar{u}$. Formally the optimal contract is the solution to

$$\text{P.4.5} \qquad \max_{\delta \in \Lambda} \int_P (f(\ell(p)) - r(p)/p)g(p)dp$$

$$\text{s.t.} \quad \int_P u^*(p,r(p),\ell(p))g(p)dp = \bar{u} \tag{4.33}$$

where Λ is the set of contracts satisfying equation (4.33). It is assumed to be non-empty.[10] Letting λ be the Lagrangian multiplier for equation (4.33) the first order conditions for P.4.5 are

$$f'(\ell(p)) + \lambda u^*_r(p,r(p),\ell(p)) = 0 \qquad p \in P \tag{4.34}$$

$$-(1/p) + \lambda u^*_r(p,r(p),\ell(p)) = 0 \qquad p \in P\ . \tag{4.35}$$

Using equations (4.27) , (4.30) and (4.31) these two conditions can be rewritten

$$h'(\ell(p))/f'(\ell(p)) = 1/\lambda \tag{4.36}$$

$$u_1(c(p,r),\ m(p,r)/\psi(p)) = 1/\lambda\ . \tag{4.37}$$

Equation (4.36) is by now familiar, it shows that the labour input is independent of the price of current consumption. Equation (4.37) is similar to equation (4.7). It shows that the optimal contract maintains the marginal utility of current consumption of the employee at a constant level. Equation (4.37) or equation (4.35) also determines the optimal degree of wage indexation. Differentiating equation (4.35) with respect to p gives

$$r_p(p,\bar{u}) = -(u^*_r + p\ u^*_{rp})/pu^*_{rr} \tag{4.38}$$

remembering that r also depends upon $\bar{u}$. To interpret this equation consider the indirect utility function equation (4.28) and set $u^* = \bar{u}$, then

$$u^*(p,r,\ell) = \bar{u} \quad . \tag{4.39}$$

Inverting equation (4.39) gives the cost function $\hat{c}(p,\bar{u})$ where

$$\hat{c}_p(p,\bar{u}) = u^*_p(p,r,\ell)/u^*_r(p,r,\ell) \quad . \tag{4.40}$$

But using equations (4.29) and (4.30) gives

$$- u^*_p(p,r,\ell) = u^*_r(p,r,\ell)(c(p,r) + m(p,r)\varepsilon\ (p)/p) \quad . \tag{4.41}$$

Equation (4.41) is the analogue of Roy's identity. Differentiating it with respect to r gives

$$-u^*_{pr}/u^*_{rr} = c + (m/p)\varepsilon \ + u^*_r(c_r/p)\varepsilon)/u^*_{rr} \quad . \tag{4.42}$$

Substituting equation (4.41) into equation (4.38) and using equations (4.40) and (4.41)

$$r_p(p,\bar{u}) = \hat{c}_p(p,\bar{u}) \ +(u^*_r/u^*_{rr})(c_r + (m_r/p)\cdot\varepsilon-1/p) \quad . \tag{4.43}$$

Equation (4.43) relates the slope of the optimal wage index to the slope of the cost of living index $c_p(p,u)$. It will be useful to find out the degree of wage indexation (1-d) defined in equation (4.11). Recalling the budget constraint it can be shown that

$$c + (m/p)\varepsilon = (r/p)(1 - s(1-\varepsilon)) \tag{4.44}$$

$$c_r + (m_r/p)\varepsilon = 1/p = -1/p\,((1-(1-s)n)(1-\varepsilon)) \tag{4.45}$$

where $s = m/r$ (4.46)

and $n = c_r.r/c$ (4.47)

Notice that s is the savings ratio and n is the income elasticity of the employees demand for present consumption. Then substituting equations (4.45) and (4.46) into equation (4.43)

$$r_p(p,\bar{u}) = \hat{c}_p(p,\bar{u})\left[1 + \frac{1}{\rho}\;\frac{(1 - (1-s)n)(1-\varepsilon))}{(1 - s(1-\varepsilon))}\right] \tag{4.48}$$

where $\rho = - u^*_{rr}.r/u^*_r$.

The term ρ is the employees coefficient of relative risk aversion. Therefore the degree of wage indexation (1-d) is

$$(1-d) = \left[\;1 + \frac{1}{\rho}\;\frac{(1 - (1-s)n)(1-\varepsilon)}{((1-s) + s\varepsilon)}\;\right] \tag{4.49}$$

Examination of equation (4.49) reveals three special cases when wages are fully indexed to prices, that is $d = 0$.

First wages will be fully indexed to prices if the elasticity of future price expectations is unity. Patinkin has called expectations satisfying this property static expectations because expected future prices are proportional to or the same as current prices. Unitary or static expectations will be justified if the shift in current price is perceived as a permanent or nominal shock. In a somewhat different context Jo Anna Gray (1976) has shown that wages will be fully indexed to prices if the only shocks to the system are purely nominal.

Second wages will be fully index to prices if the employees coefficient of relative risk aversion is infinite. If the employee is infinitely risk averse he will wish to maximize his welfare in the worst possible situation. This implies that utility will be kept constant independent of the price of present consumption, and hence wages are fully indexed.

Third the two schedules $r_p(p,\bar{u})$ and $c_p(p,\bar{u})$ will coincide if the term (1-(1-s)n) is zero. This term can be easily interpreted. It is the income effect in the demand for money, $m_r(p,r)$. It will therefore be zero if future consumption is neither a normal nor an inferior good. This will be true if present and future consumption enter into the utility function in an additively separable way, that is

$$u = u(c + \phi(\hat{c})) - h(\ell) \quad u' > 0,\ u'' > 0,\ \phi' > 0, \phi'' \leq 0. \tag{4.50}$$

If this is true the present and the future are essentially collapsed back into a single period, so that Azariadis' analysis can be reapplied which implies that wages should be fully indexed.

Notice that in general $(1-(1-s)n) > 0$ and it can be assumed that $\varepsilon(p) \ \varepsilon(0,1)$, which implies that $(1-d) > 1$. This means that wages and utility move directly with prices or that there is overindexation, or rewriting equation (4.48)

$$r_p(p,u) > \hat{c}_p(p,u) \tag{4.51}$$

This suggests that real wages tend to move procyclically with prices. This is in contrast to Gray's result that wages are only partially indexed to prices when there are both real and nominal shocks.

The degree of indexation (1-d) can be differentiated with respect to ρ, m,s and ε if these are taken to be constant and independent. Then it can be shown that

$$\partial(1-d)/\partial\rho = -\frac{1}{\rho^2}\left[\frac{(1-(1-s)n)(1-\varepsilon)}{((1-s) + s\ \varepsilon)}\right] \leq 0 \tag{4.52}$$

$$\partial(1-d)/\partial n = -\frac{(1-s)}{\rho} \times \frac{(1-\varepsilon)}{(1-s) + s\varepsilon)} \leq 0 \tag{4.53}$$

$$\partial(1-d)/\partial s = +\frac{1}{\rho} \times \frac{(1-\varepsilon)(1+\varepsilon(n-1))}{(1-s) + s\varepsilon)^2} \tag{4.54}$$

$$\partial(1-d)/\partial\varepsilon = -\frac{1}{\rho} \times \frac{(1-(1-s)n)}{(1-s) + s\varepsilon\ )^2} \leq 0 \tag{4.55}$$

Only equation (4.54) cannot be signed. Equation (4.52) shows that as the degree of relative risk aversion rises the degree of indexation falls to unity. Similarly equation (4.55) shows that as expectations become more elastic the degree of wage indexation falls to unity. Equation (4.53) shows that as the income elasticity of demand rises, proportionally less is saved, therefore the wage income is more nearly completely indexed.

Another disadvantage of Azariadis's model is that it assumes there is only one consumption good. This rules out interesting income and substitution effects from the model. This subsection will examine an optimal contract $\delta^*(r^*(p).\ \ell^*(p))$ between an employer and an employee when there is more than one consumption good. It will be shown that the optimal contract will not in general keep real wages constant. In particular it will be shown how the degree of wage indexation depends upon the income elasticities of demand of both the employer and the employee and upon the employees coefficient of relative risk aversion.

In this subsection it will be assumed that there are G consumption goods g = 1, ... G. The price of the gth consumption good will be denoted p_g and the G-dimensional vector of all consumption goods prices will be denoted p. It will be assumed that p is a joint random variable defined on the G-dimensional sample space P and possessing a continuous density function g(p). Therefore a contract

$\delta = \{r(p), \ell(p)\}$ specifies the remuneration level r and the labour supply ℓ conditional upon the price vector p.

Because there is more than one consumption good it is necessary to respecify the employers preferences and technology and the employees preferences in an appropriate manner. However the essential simplicity of Azariadis' model will be maintained by assuming that the employer is risk netural and that the employee's preferences are additively separable in consumption and leisure.

It is convenient to separate out the employer's production and consumption decisions. It will be assumed that the employer has to use the employee's labour to be able to produce. However he may use several other inputs and produce several outputs. The number y_g will represent the employers output (if positive) or input (if negative) of good g, and y will represent the employers input/output vector. It will be assumed that the employers technology can be represented by a production frontier $F(y, \ell) = 0$ where $F(y,\ell)$ satisfies

A.4.2 (i) $F(y,\ell)$ is twice continuously differentiable and strictly concave

(ii) $\partial F(y,\ell)/\partial\ell > 0$ $\quad \lim_{\ell\to 0} \partial^2 f(y,\ell)/\partial y_g^2 = 0$ for, some $g = 1, \ldots G$.

Assumption A.4.2(ii) guarantees that the employers labour is always used as an input into the production process.[11]

Given the contract $\delta = \{r(p), \ell(p)\}$ the employer as a producer will choose his input/output vector to maximize expost profits that is he solves

P.4.6 $$\max_y \quad py - r \qquad \text{s.t. } F(y,\ell) \geq 0.$$

The maximum value function for P.4.6 is the restricted profit function

$$\pi = \pi(p,\ell,r) = py(p,\ell) - r \tag{4.56}$$

where $\pi(p,\ell,r)$ is convex in p and concave in ℓ.

As a consumer the employer must decide how to allocate his profits among consumption goods. The employers consumption of good g will be denoted z_g and his consumption bundle will be denoted by a G-dimensional vector z. It is assumed that the employer's preferences are representable by a utility function v(z) which satisfies

A.4.3 (i) $v = v(z) : R_+^G \to R$; is c^2 and affine with $\partial v/\partial z_g > 0$

$y = g = 1 \ldots G$

It is assumed that the employer maximizes his utility by the choice of the consumption bundle z, subject to his budget constraint $pz = \pi$. Therefore given the contract δ and profits π , the employer solves

P.4.7 $\max_{z} \quad v(z) \qquad \text{s.t.} \quad pz = \pi$

Letting α be the Lagrangian multiplier for the budget constraint in P.4.7 the first order conditions are

$$v_{z_y}(z) = \alpha p_g \quad \forall g \quad g = 1 \ldots\ldots G$$

Therefore $z = z(p,\pi)$ and $\alpha = \alpha(p)$ solves P.4.7. Notice that α does not depend on π because v is affine. The employers indirect utility function is

$$v^*(p,\pi) = v(z(p,\pi)) \tag{4.57}$$

where

$$v^*_{p_g}(p,\pi) = -\alpha(p)z_g(p,\pi) \tag{4.58}$$

$$v^*_{\pi}(p,\pi) = \alpha(p) \tag{4.59}$$

$$v^*_{\pi\pi}(p,\pi) = 0 \tag{4.60}$$

Equation (4.60) shows that the employer is income or profit risk neutral because his preferences are affine. This is a natural generalization of Azariadis' assumption that the employer is risk neutral.

The employees consumption of good g will be denoted by x_g and x will denote his G-dimensional consumption bundle. It will be assumed that the employees preferences satisfy

A.4.4 $u = u(x) - h(\ell) : R_+^G \times [0,1] \mapsto R$; is c^2 with u strictly concave and h convex

$$\text{and } \partial u(x)/\partial x_g > 0 \qquad g \; y = 1 \ldots G.$$

In A.4.4 it is assumed that the employees preferences are additive separable in consumption and labour supply. Notice that ℓ is restricted to a closed interval. The employee's budget constraint is

$$px = r + m \tag{4.61}$$

where r is the income he receives from the contract and m is exogenous money income. It is possible that m may vary with p. It has been suggested by Blinder (1977) that if bonds are indexed then wages need not be indexed. If indexed bonds might be allowed for in this model by letting m = m(p). However for simplicity it will be assumed that m is independent of p.

The employees choice problem is ex post given the contract δ to maximize utility subject to the budget constraint equation (4.61)

P.4.8 $\max_x \quad u(x) - h(\ell) \qquad \text{s.t. } px = r + m$

Letting β denote the appropriate Lagrangian multiplier utility maximization implies

$$U_{x_g}(x) = \beta_{p_g} \qquad \forall g \quad g = 1 \ldots . G.$$

Therefore the solution to P.4.8 can be written $x = x(p, r+m)$ $\beta = \beta(p, r+m)$ and the employees indirect utility function is

$$u^* = u^*(p,\ r+m,\ \ell) = u(x(p,\ r+m)) - h(\ell) \tag{4.62}$$

where
$$u^*_p(p,\ r+m, \ell\ ) = -\ \beta(p,\ r+m)x(p,\ r+m) \tag{4.63}$$

$$u^*_m(p,\ r+m,\ \ell) = \beta(p,\ r+m) \tag{4.64}$$

$$u^*_\ell(p,\ r+m,\ \ell) = -h'(\ell) \tag{4.65}$$

Notice that the marignal utility of income $\beta(p,\ r+m)$ is the derivative of indirect utility with respect to gross income, r+m.

Having described the ex post choice problems of the employer and the employee it is now possible to examine the optimal contract δ^*, which is the solution to

P.4.9
$$\max_{\delta\varepsilon\Lambda} \int_P v^*(p,\ \pi(p,\ \ell(p),\ r(p))g(p)dp.$$
$$\text{s.t.} \quad \int_P u^*(p,\ r(p) + m,\ \ell(p))g(p)dp = \bar{u} \tag{4.66}$$

where Λ the set of contract, satisfying equation (4.66) is assumed to be non-empty.[12] In words the optimal contract maximizes the employers expected indirect utility subject to providing the employee with a given level of expected indirect utility.

Assumptions A.4.2 - A.4.4 guarantees that both v* and u* are strictly concave in ℓ and r. Therefore the optimal contract δ^* is unique. Letting λ be the appropriate Lagrangian multiplier for P.4.9 the first order conditions are

$$v^*_\pi(p,\pi)\pi_\ell(p,\ell,\ r) + \lambda u^*_\ell(p,\ r+m,\ \ell) = 0 \qquad p\ \varepsilon\ P \tag{4.67}$$
$$v^*_\pi(p,\pi)\pi_r(p,\ell,r) + \lambda u^*_r(p,\ r+m,\ell) = 0 \qquad p\ \varepsilon\ P \tag{4.68}$$

which can be rewritten as

$$\alpha(p)\ py_\ell(p,\ell) - \lambda h'(\ell) = 0 \qquad p\ \varepsilon\ P \tag{4.69}$$
$$-\ \alpha(p) + \lambda\beta(p,\ r+m) = 0 \qquad p\ \varepsilon\ P \quad . \tag{4.70}$$

Equation (4.69) determines the labour input schedule and equation (4.70) the remuneration schedule.

Consider first the remuneration schedule. Since P.4.9 is a cancave programming problem λ is directly related to $\bar{u}$ say by

$$\lambda = k(\bar{u}), k'(\bar{u}) > 0 \tag{4.71}$$

Then equation (4.70) determines r as a function of p, m and $\bar{u}$

$$r = r(p,m,\bar{u})$$

$$r_p(p,m,\bar{u}) = (\alpha_p(p)\beta(p,r+m)/\alpha(p)\beta_m(p, r+m)) - \beta_p(p, r+m)/\beta_m(p, r+m) \tag{4.72}$$

$$r_m(p,m,\bar{u}) = -1 \tag{4.73}$$

$$r_{\bar{u}}(p,m,\bar{u}) = -\beta(p, r+m)k'(\bar{u})/\beta_m(p, r+m)k(\bar{u}) > 0 \tag{4.74}$$

Equation (4.72) can be simplified by differentiating Roy's identity with respect to income for both the employer and the employee. Therefore

$$-\alpha_{p_g}(p)/\alpha(p) = \partial z_g(p,\pi)/\partial\pi \tag{4.75}$$

$$-\beta_{p_g}(p, r+m)/\beta_m(p, r+m) = x_g(p, r+m)$$
$$+ (\beta(p, r+m)\partial x_g(p, r+m)/\partial m)/\beta_m(p, r+m) \tag{4.76}$$

Then substituting equations (4.75) and (4.76) into equation (4.72), suprressing arguments

$$r_{p_g} = x_g(1 + \frac{1}{s_g\rho}(t_g n_{z_g} - s_g n_{x_g})) \tag{4.77}$$

where

$$\rho = -(r+m)\beta_m/\beta \qquad s_g = p_g x_g/(r+m) \qquad t_g = p_g x_y/\pi$$

$$n_{z_g} = (\partial z_g/\partial\pi)(\pi/z_g) \qquad n_{x_g} = (\partial x_g/\partial m)(r+m/x_g) .$$

The parameter ρ is the elasticity of the employees marginal utility of income, or the coefficient of relative income risk aversion; s_g is the share of good g in the employees budget; t_g is the share of good g in the employers budget; n_{z_g} is the income elasticity of the employers demand for good g and n_{x_g} is the income elasticity of the employees demand for good g.

To examine the labour input schedule differentiate equation (4.69). Then

$$\ell = \ell(p,\bar{u})$$

$$\ell_p(p,\bar{u}) = [-\alpha(p)\pi_\ell(p,\ell,r) - \alpha(p)\ \pi_{\ell p}(p,\ell,r)]/$$

$$[\alpha(p)\pi_\ell(p,\ell,r) - \alpha(p)\pi_\ell(p,\ell,r)h\ \ (\ell)/h''(\ell)] \qquad (4.78)$$

$$\ell_{\bar{u}}(p,\bar{u}) = h'(\ell)k'(u)/[\alpha(p)\pi_\ell(p,\ell,r) - k(\bar{u})h''(\ell)].$$

The wage schedule $\omega(p,m,\bar{u})$ is defined implicity by

$$\omega(p,m,\bar{u}) = r(p,m,\bar{u})/\ell(p,\bar{u})$$

$$\omega_{p_g}(p,m,\bar{u}) = (x_g/\ell)((r_{p_g}/x_g) - (r\ell_{p_g}/\ell x_g)) \qquad (4.80)$$

To simplify equation (4.80) it is convenient to make two further assumptions. First it will be assumed that the employee has no exogeneous income, that is $m = 0$. Second it will be supposed that the employer is a specialist, that is he produces only one good and uses only labour as an input. Therefore it the employer produces good j according to a concave production function $f(\ell)$ his ex post profits are

$$\pi(p_j,\ell,r) = p_j f(\ell) - r \qquad (4.81)$$

Given these two assumptions equation (4.80) can be rewritten as

$$\omega_{p_g}(p,m,\bar{u}) = \frac{x_g}{\ell}\left[1 + \frac{1}{s_g\rho}(t_g n_{z_g} - s_g n_{x_g}) - \frac{t_g n_{z_g}}{s_g(\varepsilon-\rho)}\right] \quad g \neq j \quad g = 1 \ldots G \qquad (4.82)$$

$$\omega_{p_j}(p,m,\bar{u}) = \frac{x_j}{\ell}\left[1 + \frac{1}{s_j\ell}(t_j n_{z_j} - s_j n_{x_j}) - \frac{t_j n_{z_j} - 1)}{s_j(\varepsilon-\rho)}\right] \qquad (4.83)$$

where $\varepsilon = f'(\ell).\ell/f(\ell) < 0$, $\hat{\rho} = h'(\ell).\ell/h(\ell) > 0$.

The parameter ε is the elasticity of the marginal product of labour and $\hat{\rho}$ is the elasticity of the marginal disutility of labour.

Next consider the constant real wage function $\hat{\omega}(p,m,\bar{u})$. This function is the minimum wage necessary to attain a given level of utility $\bar{u}$, it is the minimum value function for:

P.4.10 $\quad \min\limits_{x,\ell} \quad (px-m)/\ell \qquad \text{s.t.} \quad u(x) - h(\ell) = \bar{u}$.

If γ is the Lagrangian multiplier for P.4.10 the first order conditions are

$$(px-m)/\ell^2 = \gamma h \ , \ p_g/\ell = \gamma u_{x_g} \qquad g = 1 \ldots G \qquad (4.84)$$

Therefore the constant real wage function is

$$\hat{\omega}(p,m,\bar{u}) = (px(p,m,\bar{u})-m)/\ell(p,m,\bar{u})$$

where $\quad \hat{\omega}_{p_g}(p,m,\bar{u}) = x_g(p,m,\bar{u})/\ell(p,m,\bar{u}) \qquad (4.85)$

using equations (4.84).

It is now possible to relate the wage schedule and the constant real wage function. Using equation (4.85) the term in brackets in equations (4.82) and (4.83) becomes the optimal degree of wage indexation.[13] Consider first equation (4.82). This equation shows how wages are indexed to the price of a commodity not produced by the employer. This is perhaps the most likely case. If it is assumed that all goods are normal goods for both the employer and the employee then the greater is n_{z_g} and t_g the more likely it is that d is positive.

Similarly the higher is n_{x_g} and s_g the more likely it is that d is negative. Broadly speaking wages will be under indexed to the prices of those goods which are relatively important to the employee or for which he has a large income elasticity of demand. This must be counterbalanced by the fact that if the employee is sufficiently risk averse then wages will be over indexed because in that case the third term in the brackets dominates the second.

Examination of equation (4.83) reveals Azariadis' result as a special case. In Azariadis' model there is only one good, therefore each income elasticity and share of budget term is unity, wages are fully indexed. Similarly equation (4.48) which shows how wages are indexed when the employee lives for two periods can be interpreted within the present context. If the elasticity of price expectation $\varepsilon(p)$ is zero then equation (4.48) can be rewritten

$$r_p(p,\bar{u}) = c_p(p,\bar{u})(1 + \frac{1}{\rho} \quad \frac{(1 - (1-s)n))}{(1-s)} \qquad (4.48')$$

Equation (4.48′) is a special case of equation (4.82) where the employers share of the consumption good in his budget and the income elasticity of his demand are both unity. The consumption ratio (1-s) is simply the proportion of income the employee spends on present consumption. The over indexation of equation (4.48′) is then to be explained in the following terms. Present consumption is more important to the employer than the employee because he lives for only one period and therefore is uninterested in future consumption. The reason for this result is the familiar utilitarian principle. If the good in question is relatively unimportant to the employee it is necessary, as the price of this good changes, to give him a larger wage rise to maintain a constant level of marginal utility of income.

Equations (4.82) and (4.83) are interesting not least because they provide some basis for testing the theory empirically. In particular at the end of the nineteenth and beginning of the twentieth century miners wages were indexed to the price of coal according to quite complicated formulae.

Section 2 : Imperfect and Asymmetric Information

It has been suggested by Fischer (1977) and Phelps and Taylor (1977) that contracts with sticky wages imply that monetary disturbances can affect output and employment even when these disturbances are recognized as contemporaneous and monetary in nature. Barro (1977) has challenged this assertion. He shows that the rule for determining employment in Fischer's model, essentially employment is determined along the labour demand curve, is sub-optimal. Barro argues that employment is properly determined by the intersection of the supply and demand curves for labour. It therefore follows that any monetary disturbance which is perceived as contemporaneous will be completely offset by the terms of the labour contract. That is to say the optimal labour contract will index directly to the money stock or to nominal national income so as to be completely independent of nominal changes.

Barro goes on to suggest a number of extensions to Fischer's model which he then examines. The first extension Barro proposes is to assume that agents, or participants to the contract, have incomplete current information, that is to say they are unable to fully disentangle real from monetary shocks. The usual justification for this type of incomplete current information is that agents operate in locally separated markets and are therefore incompletelyinformed about global variables such as the price level or the money stock. Lucas (1972) for example uses this type of model to generate a Phillips curve. Lucas also shows that this Phillips curve does not provide an exploitable basis for monetary policy. As stated in chapter 1 all models of this type are open to R.J. Gordons criticism that agents do possess and process a considerable amount of aggregate information. Gordon points out that wage levels in non-local markets can be easily found out by a phone call. Equally the authorities publish regular figures for the unemployment level, price level, money stock, etc., which provide reasonably reliable aggregate information, with a typically short lag. The analysis presented in section 1 goes some way to examining the optimal labour contract when there is imperfect or incomplete information. In particular it has been shown how wages respond to prices when real and monetary shocks are confused. However it is apparent that in these models, the perceived part of monetary shocks have no impact on real variables.

Barro then goes on to briefly examine the role of moral hazard in the labour contract model. Barro gives the example of an employer who can observe an aggregate real shock but his employees cannot. Barro therefore argues that the employer will have an incentive to misrepresent the true value of the aggregate shock to his employees. The optimal labour contract when there is asymmetric information of this type has been studied in chapter 3, section 2.

In chapter 3, section 2 it was shown that the optimal incentive compatible contract was productively inefficient and also shared risk inefficiently. It was also shown that the optimal incentive compatible contract equated the marginal rate of remuneration to the marginal product. Interestingly the rule used for determining employment in Fischer's model is the equation of the wage rate to the marginal product, because employment is determined along the labour demand curve. Therefore the purpose of this section is to examine the optimal incentive compatible contract when there is imperfect information.

In chapter 3, section 2 the optimal incentive compatible contract $\delta = \{\pi(\theta), \ell(\theta)\}$ was made contingent upon the productivity shock θ which the employer could observe but which the employee could not. For the purposes of this section suppose that at the time that θ becomes known to the employer, the employee observes some additional information I which is correlated with θ[14]. This information may for example be the price of output or profits of the firm, or more generally the unemployment rate or the price level. In general the information I will effect both the employers profits and the employees utility. Of course if I and θ are perfectly correlated then observing I is equivalent to observing θ, so that the first best contract can be implemented, If, on the other hand, I and θ are only partially correlated then the optimal contract $\delta = \{\pi(I,\theta), \ (I,\theta)\}$ will be made contingent upon both I and θ.

Suppose for concreteness that the conditional distribution of I, can be specified for each possible value of θ. That is to say let $g_{I/\theta}(I/\theta)$ represent the continuous conditional probability density function of I given θ. There is a whole family of such conditional probability density functions one for each possible value of θ. In addition assume that there is a continuous probability density function $g(\theta)$ for the parameter θ. It will now be shown that the optimal incentive compatible contract, conditional upon the additional information I, satisfies Lemma 3.3 and Corollaries 3.1-3.4 when appropriately reinterpreted. It will then be possible to discuss the role of imperfect information in the optimal incentive compatible contract.

Consider the optimal incentive compatible contract. Because the employee cannot observe the true value of θ but only the information I correlated with it, the employer will announce that value of θ, θ' that maximizes his ex post profits. That is to say θ' solves

$$\text{P.4.11} \qquad \max_{\theta'} \quad (I,\theta,\theta')$$

so that the first order conditions and the solution function is

$$\pi_{\theta'}(I,\theta,\theta') = 0 \tag{4.86}$$

$$\theta' = \theta'(I,\theta) \tag{4.87}$$

Notice that the announced value of θ ,θ' must be consistent with the information I observed by the employee. However this is not a restriction because if the contract is incentive compatible

$$\theta'(I,\theta) = \theta \tag{4.88}$$

The maximum value function or profit function for P.4.11 is

$$\pi(I,\theta) = \pi(I,\theta,\theta'(I,\theta))$$

where $\pi_\theta(I,\theta) = \pi_\theta(I,\theta,\theta'(I,\theta)) + \pi_{\theta'}(I,\theta,\theta'(I,\theta))\partial\theta'(I,\theta)/\partial\theta$

$$= \pi_\theta(I,\theta,\theta') \tag{4.89}$$

using equations (4.86) and (4.88). Equation (4.89) is the natural extension of the incentive compatibility constraint discussed in chapter 3. If the employers production function is written $f(\ell,I,\theta)$ then equation (4.89) can be rewritten

$$\pi_\theta(I,\theta) = f_3(\ell(I,\theta), I,\theta) \tag{4.90}$$

Equation (4.90) holds for each value of θ and each value of I. In terms of the labour input schedule $\ell(I,\theta)$ and the remuneration schedule $r(I,\theta)$ equation (4.90) can be interpreted as equating the marginal product of labour to the marginal rate of remuneration for each value of I.

For notational simplicity assume that I is unidimensional and takes on values in the range (c,d). The optimal incentive compatible contract maximizes the employers expected profits subject to providing the employee with a given level of expected utility $\bar{u}$ and to the incentive compatibility constraint, equation (4.90), that is it solves

$$\text{P.4.12} \qquad \max_{\delta} \qquad \int_a^b\int_c^d \pi(I,\theta)g_{I/\theta}(I/\theta)g_\theta(\theta)dId\theta$$

$$\text{s.t.} \qquad \int_a^b\int_c^d u(f(\ell(I,\theta),I,\theta) -\pi(I,\theta),\ell(I,\theta),I)g_{I/\theta}(I/\theta)g_\theta(\theta)dId\theta = \bar{u} \tag{4.91}$$

$$\text{and} \qquad \pi_\theta\,(I,\theta) = f_3(\,(I,\theta),I,\theta) \quad \forall\,\theta \text{ and } \forall I \tag{4.90}$$

If λ is the multiplier for equation (4.91) and $p(I,\theta)$ is the set of costate variables for equation (4.90) then the Lagrangian function for P.4.12 is

$$L = \int_a^b\int_c^a ((\pi(I,\theta) + u(f(\ell(I,\theta),I,\theta)-\pi(I,\theta),\ell(I,\theta),I))g_{I/\theta}(I/\theta)g_\theta(\theta)$$

$$+ p(I,\theta)(f_3(\ell(I,\theta),I,\theta) -\pi_\theta(I,\theta)))dId\theta$$

$$= \int_a^b \int_c^d((\pi(I,\theta) + \lambda u(f(\ell(I,\theta),I,\theta) = \pi(I,\theta),\ell(I,\theta),I))g_{I/\theta}(I/\theta)g_\theta(\theta)$$

$$+ \ p(I,\theta)f_3(\ell(I,\theta),I,\theta) + p_\theta(I,\theta)\pi(I,\theta))dId\theta$$

$$- \int_c^d (p(I,b)\pi(I,b) - p(I,a)\pi(I,a))dI. \tag{4.92}$$

If profits are constrained to be non-negative and the utility function is assumed to be bounded then the order of integration in equations (4.92) can be reversed. This allows the Lagrangian L to be written as

$$L = \int_c^d\{\int_a^b ((\pi(I,\theta) + \lambda u(f(\ell(I,\theta),I,\theta) -\pi(I,\theta),\ell(I,\theta),I))g_{I/\theta}(I/\theta)g_\theta(\theta)$$

$$+ p(I,\theta)f_3(\ell(I,\theta),I,\theta) + p_\theta(I,\theta)\pi(I,\theta))d\theta$$

$$- (p(I,b)\pi(I,b) - p(I,a)\pi(I,a))\}dI. \tag{4.93}$$

The term inside the curly brackets is familiar from chapter 3. It is essentially the maximand studied in chapter 3, for a given value of I. In chapter 3 it was shown under what circumstances the optimal contract for this problem exists and is unique. Equation (4.93) poses no new conceptual problems, the optimal contract $\delta^* = \{\pi^*(I,\theta)$ $\ell^*(I,\theta)\}$ is found by maximizing the Hamiltonian

$$H = (\pi(I,\theta) + \lambda u(f(\ell(I,\theta),I.\theta) - \pi(I,\theta),\ell(I,\theta),I))g_{I/\theta}(I/\theta)g_\theta(\theta)$$

$$+ p(I,\theta)f_3(\ell(I,\theta),I,\theta),I,\theta). \qquad (4.94)$$

and the first order conditions are

$$- p_\theta(I,\theta) = (1-\lambda u_1(f(\ell(I,\theta),I,\theta) -\pi(I,\theta),\ell(I,\theta),I))g_{I/\theta}(I/\theta)g_\theta(\theta)$$

$$\forall\ I \text{ and } \forall\ \theta \qquad (4.95)$$

$$\lambda(u_1(f(\ell(I,\theta),I,\theta)- \pi(I,\theta),\ell(I,\theta),I)f_1(\ell(I,\theta),I,\theta)$$

$$+ u_2(f(\ell(I,\theta),I,\theta) - \pi(I,\theta),\ell(I,\theta),I))g_{I/\theta}(I/\theta)g_\theta(\theta)$$

$$= - p(I,\theta)f_{13}(\ell(I,\theta),I,\theta) \quad \forall\ I \text{ and } \forall\ \theta$$

$$p(I,a) = p(I,b) = 0 \qquad \forall\ I \qquad (4.97)$$

It can easily be seen from equations (4.95) - (4.97) that P.4.12 satisfies Lemma 3.3 and Corollaries 3.1 - 3.4 mutatis muntandis, for every given value of I.

To interpret equation (4.93) notice that dividing through by $g_I(I)$ will make no difference to the maximand. But then the term

$$g_{I/\theta}(I/\theta)g_\theta(\theta)/g_I(I) = g_{\theta/I}(\theta/I) \qquad (4.98)$$

where $g_I(I)$ is the marginal probability density function of I and $g_{\theta/I}(\theta/I)$ is the conditional probability distribution of θ given that the additional information is I. Thus $g_{\theta/I}(\theta/I)$ is the a posteriori distribution of θ, it is the distribution of θ once I is known. Thus the only difference between a contract chosen once the additional information I is observed is the use of the a posteriori rather than the a priori distribution of θ .

The following example illustrates how the efficiency of the optimal contract $\delta^* = \{\pi(I,\theta),\ \ell(I,\theta)\}$ may depend upon the additional information I.

Example: Let $f(\ell,I,\theta) = \ell/\theta$, $u(x,\ell,\ I) = -(1/A)(\exp(-Ax)) - \ell$

$g_{I/\theta}(I/\theta) = \theta\exp(-I\theta)$, $g_\theta(\theta) = \alpha\exp(-\alpha\theta), \theta \ \varepsilon (0,\infty)$

Therefore equations (4.95) and (4.96) can be rewritten

$$- p_\theta = (1 - \lambda\exp(-Ax))\alpha\ \theta\ \exp(-\theta(\alpha+I)) \qquad (4.95\ )$$

$$\lambda(\exp)\text{-}Ax)/\theta)-1))\alpha\ \theta\exp(-\theta(\alpha+I)) = p/\theta^2 \qquad (4.96\ )$$

Substituting equation (4.96) into (4.95) gives a differential equation in p

$$p_\theta + (p/\theta) - \alpha\theta(1-\lambda\theta)\exp(-\theta(\alpha+I)) = 0 \qquad (4.99)$$

which can be solved to give

$$p.\theta = \int \alpha(1-\lambda\theta)\ \exp(-\theta(\alpha+I))d\theta \qquad (4.100)$$

therefore

$$p = \frac{\alpha}{(\alpha+I)^2\theta}\ (\lambda-(\alpha+I))\exp(-\theta(\alpha+I)) \qquad (4.101)$$

Substituting into equation (4.96)

$$((\exp(-Ax)/\theta)-1) = (\lambda-(\alpha+I))/\lambda\theta^4(\alpha+I)^2 \qquad (4.102)$$

Differentiating equation (4.102) with respect to I gives

$$\partial((\exp(-Ax)/\theta)-1)/\partial I \gtrless 0 \text{ as } (\alpha+I) \gtrless 2\lambda\ . \qquad (4.103)$$

Equation (4.103) shows that productive efficiency is increased or reduced as α+I is greater than or less than 2λ . Since λ is determined by the parameter $\bar{u}$, there is no reason to expect either one sign or the other. However equation (4.103) illustrates the fact that the productive efficiency of the optimal contract will depend upon any additional information the employee observes that is correlated with the unknown value of θ.

The most obvious example of the additional information the employee migh observe is the output price of the firms production. Suppose for example that the employer is an intermediate goods producer who sells his output on a perfectly competitive market at a price q. Assume that the industry demand curve is

$$D = D(q,\varepsilon) . \; D_q(q,\varepsilon) < 0 \tag{4.104}$$

where ε is a random variable or shock to the demand curve. Suppose too that the industry consists of J employees indexed j 1 ... $\bar{J}$. The industry supply is simply the sum of each individual employers output.

$$Y = \sum_{j=1}^{\bar{j}} y^j \qquad y^j = f^j(\ell^j,\theta^j) \quad j = 1 \;.....\bar{J} \tag{4.105}$$

where f^j is the jth employers production function, ℓ^j is the jth employers labour input and θ^j is the value of the random shock to the jth employers production function. The values of the various θj's may be correlated reflecting some industry wide shock, such as a change in technology or purely firm specific and individualistic. Equating equations (4.104) and (4.105)

$$q = q(Y,\varepsilon) \quad q_Y(Y,\varepsilon) < 0 \tag{4.106}$$

Equation (4.106) shows how the output price reflects the impact of the various supply and demand shocks. It is assumed that apart from θ^j the employer and employee of firm j cannot observe the supply and demand shocks, therefore the employee uses the price q as an imperfect predictor of the supply shock θ^j. There are two special cases. First if all the supply shocks are uncorrelated, that is the shocks are completely individualistic, then because there are a large number of firms in the industry the output price will provide no information about θ^j. On the other hand if there is no demand shock and the supply shock is industry wide that is $\theta^j = \theta$ for all $j = 1 \ldots \bar{J}$ and all firms have the same production function, then observing the output price q is equivalent to observing θ. That is the optimal incentive compatible contract is a first best contract. In general q will provide only imperfect information about the supply and demand shocks ε and θ^j.

Grossman, Hart and Maskin (1982) have examined a model where an increase in the variability of final goods prices increase the employees uncertainty about his actual productivity. They show how this affects both output and employment comparative to the first best contract. Grossman,Hart and Maskin go on to suggest a number of causes of relative price variability such as changes in tastes or technology or wealth. The latter factor is particularly interesting because if monetary policy changes the wealth distribution this will change output and employment.

Section 3 : Conclusion

This chapter has examined wage indexation (section 1) and indexation to additional information (section 2). These are not mutually exclusive, the additional information I, may in fact be a price vector and hence wages will be indexed to prices. Section 1 shows how the degree of wage indexation depends upon various elasticities of the employer and the employee. This provides an implicit test for the theory and it would be interesting to examine equations such as (4.82) and (4.83) against the data. One possibility is to test the theory against data from labour contracts of the late nineteenth and early twentieth century. It might be expected that the theory will fit this data better than data for current contracts. This is because in the early twentieth and late nineteenth centuries there were fewer means of transferring wealth across time or states and therefore the labour contract is perhaps a more important mechanism for doing this.

In section 2 it was shown how, if there is asymmetric information, additional information inside or outside the workplace can influence the efficiency andoutput and employment levels of the labour contract. This provides a mechanism by which aggregate information and aggregate shocks can influence output and employment. Although this is rather tentative it does appear to provide an interesting and realistic model of the labour market, one perhaps better suited to macroeconomic analysis than its auction market counterpart.

Notes

1. See also Dreze (1979) and Eden (1979).

2. Given the current state of computer technology it is difficult to believe that indexing wages to any well defined variable is anything but costless.

3. The cost function is derived from the solution to $\min_{x,\ell} m = px - w$ s.t $u(x) = u$, whereas the wages function is derived from $\min_{x,\ell} w = \frac{px-m}{\ell}$ s.t $u(x) = u$. See below.

4. Cleeton (1982) uses the wages function for this purpose.

5. In addition to satisfying equation (4.5) it is assumed that Δ also satisfies the non-negativity constraints, $r \geq 0$, $0 \leq \ell \leq 1$, $f(\ell) - r/p \geq 0$.

6. H. Grossman (1981) analyses a model of this type.

7. See footnote 5.

8. If the employer lives for two periods but is risk neutral his savings are indeterminate.

9. This formula allows for both static expectations, $\hat{p} = p$ and inelastic expectations, p = constant. In terms of the economy's underlying disturbances static expectations hold if all shocks are nominal and inelastic expectations hold if all shocks have real causes. Therefore ψ reflects the degree to which shocks are believed to be due to nominal and real causes.

10. See footnote 5.

11. See chapter 2.

12. See footnote 5 again.

13. Notice that this is essentially a local comparison.

14. Hart (1983) also addresses this problem but in a slightly different way.

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For the sake of brevity, the following abbreviations have been used for those journals that are frequently referenced.

A.E.R.	American Economic Review
J.E.T.	Journal of Economic Theory
J.M.E.	Journal of Monetary Economics
J.P.E.	Journal of Political Economy
R.E.S.	Review of Economic Studies
Q.J.E.	Quarterly Journal of Economics