ENTROPIC VECTOR OPTIMIZATION AND SIMULATED ENTROPY: THEORY AND APPLICATIONS

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In the Prince, which was written four centuries ago, Machiavelli observed:

> "There is nothing more difficult to take in hand, more perilous to conduct, or more uncertain in its success than to take the lead in the introduction of a new order of things".

A-Sultan, the author, replies:

"Entropy, as a measure of uncertainty, can help!!?"

ABSTRACT

This thesis explores the use of Shannon's (informational) entropy measure and Jaynes' maximum entropy criterion in the solution of multi-criteria optimization problems, for Pareto set generation, and for seeking the global minimum of single-criteria optimization problems. At present, traditional methods contain no information-theoretic basis. They all view optimization problems in terms of a topological domain defined deterministically by function hypersurfaces. Part of this thesis is a continuation of research aimed at developing a non-deterministic approach to optimization problems through the use of the Maximum Entropy Principle (MEP) and the Minimax Entropy Principle (MMEP). We treat the multi-criteria optimization problem as a statistical system which can be interpreted as transformations of the system to a sequence of equilibrium states which are characterized by certain entropy maxima depending upon a feasible entropy parameter.

This thesis also introduces, for the first time, simulated entropy for obtaining the global minima of single-criteria minimization problems. Several ways of using entropy in the optimization context are investigated. Two simulated entropy algorithms are developed. A class of structural optimization problems is chosen to provide example problems. Computational results demonstrate that the algorithms proposed in this work are efficient and reliable. The following parts of the present work are original:

1) An entropy-based method for generating Pareto set is presented. This new method yields additional insight into the nature of

entropy as an information-theoretic basis and clarifies some ambiguities in the literature about its use in optimization.

2) A new stochastic technique for reaching the global minimum of constrained single-criteria minimization problems is developed. The system may be interpreted as a statistical thermodynamic one which approaches spontaneously an equilibrium state which is characterized by lowering the temperature of the system to its limit along the entropy process transitions. However, in each transition, equilibrium is characterized by maximizing the entropy at that certain temperature. This is known as the Simulated Entropy.

3) Two simulated entropy techniques are developed which seek the global minimum of some deterministic objective function. The two techniques can be applied to constrained minimization problems only.

4) A practical structural design program of reinforced concrete frames is developed. It considers two criteria to be minimized, chooses the most optimal solution, applies an enumeration technique to round off some of the design variables, performs system reliability analysis to calculate the system reliability, and then applies the PNET method to choose an appropriate value for the system probability of failure. Computer programs are developed to apply these methods to multi-storey frames.

5) A new way for investigating the sensitivity of an optimized design of a multi-storey frame to parameter and criteria changes is developed.

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ACRONYMS

AC	Aggregated Constraint
AOF	Aggregated Objectives Function
BF	Bound Formulation
COF	Compartmentalized Objective Function
ECOF	Entropy-Based Constrained Objective Function
ECT	Entropy Constrained Transportation
ER	Entropy Research
ESC	Entropy-Based Surrogate Constraint
ESM	Elastic Safety Margin
EWOF	Entropy-Based Weighted Objectives Function
LHS	Left Hand Side
LP	Linear Programming
MED	Maximum Entropy Distribution
MEP	Maximum Entropy Principle
MMEP	Minimax Entropy Principle
MMFDCM	Modified Method of Feasible Directions for Constrained Minimization
MOLP	Multiple Objective Linear Program
NLP	Non-Linear Programming
OR	Operation Research
P	Primal
PNET	Probabilistic Network Evaluation Technique
PSM	Plastic Safety Margin
RHS	Right Hand Side
S	Surrogate
SA	Simulated Annealing
SAC	Sub-Aggregated Constraints
SCF	Surrogate Constraint Function
SE	Simulated Entropy
ST	Standard Transportation
WOF	Weighting Objectives Function

NOTATIONS

a	-	depth of equivalent rectangular stress block in.
A_c	-	area of rectangular core of column, sq. in.
A_g	-	gross area of cross section, sq. in.
A_s	-	area of tension reinforcement, sq. in.
$(A_s)_i$	-	area of tension reinforcement in ith column, sq. in.
$\acute{A}_s$	-	area of compression reinforcement, sq. in.
$(\acute{A}_s)_i$	-	area of compression reinforcement in ith column, sq. in.
A_{sh}	-	area of transverse hoop bar (one leg), sq. in.
A_{shb}	-	area of shear reinforcement, within one sixth of the height from bottom of a column, within a distance s; where s is taken to be d/4, sq. in.
$(A_{shb})_i$	-	area of shear reinforcement within one-sixth of the height from bottom of ith column, within a distance s, sq. in.
A_{shm}	-	area of shear reinforcement, within a height from one-sixth to two-thirds of the height of a column, within a distance s; where s is taken to be d/2, sq. in.
$(A_{shm})_i$	-	area of shear reinforcement within a height from one-sixth to two-thirds of the height of ith column, within a distance s, sq. in.

A_{sht} - area of shear reinforcement, within one-third of the height from top of a column, within a distance s; where s is taken to be d/4, sq. in.

$(A_{sht})_i$ - area of shear reinforcement, within one-third of the height from top of ith column, within a distance s, sq. in.

$(A_{s\ell})_i$ - area of main reinforcement at the top left of ith beam, sq. in.

$(A_{sm})_i$ - area of main reinforcement at the bottom near the midspan of ith beam, sq. in.

$(A_{sr})_i$ - are of main reinforcement at the top right of ith beam, sq. in.

$A_{v\ell}$ - area of shear reinforcement within a distance s which is in a quarter span from left end of a beam; where s is taken to be d/4, sq. in.

A_v - area of shear reinforcement within a distance sq. in.

$(A_{v\ell})_i$ - area of shear reinforcement within a distance s which is in a quarter span from left end of ith beam, sq. in.

A_{vm} - area of shear reinforcement within a distance s which is in a center half span of a beam; where s is taken to be d/2, sq. in.

$(A_{vm})_i$ - area of shear reinforcement within a distance s which is in a center half span of ith beam, sq. in.

A_{vr} - area of shear reinforcement within a distance s which is in a quarter span from right end of a beam; where s is taken to be d/4, sq. in.

$(A_{vr})_i$ - area of shear reinforcement within a distance s which is in a quarter span from right end of ith beam, sq. in.

b - width of beam, in.

b_i - width of ith beam, in.

c - total number of objective functions.

C_c - cost of concrete per sq. in. per linear ft.

C_s - cost of steel per sq. in. per linear ft.

C_1 - constant, in.

C_2 - constant, in.

C_3 - concrete cover; measured to the centroid of main reinforcement of beam, in.

C_4 - concrete cover; measured to the centroid of main reinforcement of column, in.

C - total cost of a structure.

d - effective depth of beam, in.

d' - distance from centroid of compression reinforcement to extreme compression fibre, in.

d'' - distance from the plastic centroid to the centroid of tension reinforcement, in.

d_b - specified nominal diameter of bars, ft.

d_b - nominal diameter of reinforcing bars, in.

$(d_b)_1$ - nominal diameter of longitudinal bar, in.

$(d_b)_2$ - nominal diameter of web steel bar, in.

D - overall depth of column, in.

D_i - overall depth of ith column, in.

E_c - concrete modules of elasticity.

E_s - steel modules of elasticity.

f'_c - cylinder strength of concrete, psi.

f'_s - compression stress of compression reinforcement of column, psi.

f_y - specified yield strength of reinforcement, psi.

$F(X)$ - objective function.

$g(X)$ - inequality constraint function.

$h(X)$ - equality constraint function.

h - overall depth of beam, in.

h_i - overall depth of ith beam, in.

H_i - height of ith column, ft.

k - total number of equality constraints; or Boltzmann's Constant.

K - stiffness ratio.

ℓ_c	-	height of storey, ft.
ℓ_d	-	development length, ft.
L	-	span length of beam, in.
$(L_b)_i$	-	length of ith beam, ft.
$L_{s\ell}$	-	length of main reinforcement, ft.
L_{sm}	-	length of main reinforcement, ft.
L_{sr}	-	length of main reinforcement, ft.
m	-	total number of inequality constraints.
M	-	moment at the base of each storey.
$\overline{M}_i$	-	mean value of elastic safety margin.
M_n	-	nominal flexural strength of a beam section, in-lbs.
$(M_n)_i$	-	nominal moment strength of a column section at location i on interaction diagram, ft-kips. (i=1,2,3,4.).
$M_{n\ell}$	-	nominal negative moment strength at left end of a beam, ft-kips.
M_{nm}	-	nominal moment strength at maximum positive moment section of a beam, ft-kips.
M_{nr}	-	nominal negative moment strength at right end of a beam, ft-kips.
M_u	-	factored moment of a column section, ft-kips.

M_{ua} = positive factored moment at section where parts of reinforcing bars from sections of maximum positive moment are theoretically to be cut, ft-kips.

M_{ub} = negative factored moment at section where parts of reinforcing bars from sections of maximum negative moment are theoretically to be cut, ft-kips.

$M_{u\ell}$ = factored moment at left end of a beam, ft-kips.

M_{um} = factored moment at maximum positive moment section of a beam, ft-kips.

M_{un} = negative factored moment at sections of maximum negative moment, ft-kips.

M_{up} = positive factored moment at sections of maximum positive moment, ft-kips.

M_{ur} = factored moment at right end of a beam, ft-kips.

n = total number of design variables or correlation matrix dimension.

N_b = number of beams in a structure.

N_c = number of columns in a structure.

$(N_{hb})_i$ = number of ties or hoops within one-sixth of the height from bottom of ith column.

$(N_{hm})_i$ = number of ties or hoops within a height from one-sixth to two-thirds of the height of ith column.

$(N_{ht})_i$ = number of ties or hoops within one-third of the height from top of ith column.

N_ℓ - number of layers of longitudinal reinforcement placed in beam.

N_s - total number of tension bars in beam, based on specifiedly based bars.

N_s - total number of tension or compression bars in column, based on specifiedly based bars.

N_v - total number of stirrups within a distance s, based on specifiedly based bars (beam).

N_v - total number of ties within a distance s, based on specifiedly based bars (column).

$(N_{v\ell})_i$ - number of stirrups within a quarter span from left end of ith beam.

$(N_s+N_\ell-1)/N_\ell$ - maximum number of bars placed in a layer in beam.

$(N_{vm})_i$ - number of stirrups within a center half span of ith beam.

$(N_{vr})_i$ - number of stirrups within a quarter span from right end of ith beam.

p_i - probability of an outcome i; or a parameter.

P - entropy parameter, temperature.

P_n - nominal axial laod strength of a column section, lbs.

$(P_n)_i$ - nominal axial load strength of a column at location i on interaction diagram, kips. (i=1,2,3,4.).

P_u - factored axial load of a column section, kips.

r	-	top storey number.
S	-	feasible direction vector, or search direction.
S	-	entropy.
S_i	-	spacing of stirrups; S_1 - d/4, S_2 - d/2, (beam) in.
S_i	-	spacing of ties or hoops; S_3 - d/4, S_4 - d/2, (column) in.
S_h	-	center to center spacing of hoops, in.
t	-	width of column, in.
t_i	-	width of ith column, in.
$(V_b)_i$	-	total volume of steel of ith beam, sq. in. per linear ft.
$(V_c)_i$	-	total volume of steel of ith column, sq. in per linear ft.
V_c	-	nominal shear strength provided by concrete, lbs.
V_n	-	nominal shear strength at a section, Kips.
V_{nb}	-	nominal shear strength at the bottom section of a column, lbs.
$V_{n\ell}$	-	nominal shear strength at left end of a beam, lbs.
V_{nm}	-	nominal shear strength at one-third height from the top or one-sixth height from the bottom section of a column, lbs.

V_{nq} — nominal shear strength at quarter span section of a beam, lbs.

$V_{nq\ell}$ — nominal shear strength at quarter span section from left end of a beam, kips.

V_{nqr} — nominal shear strength at quarter span section from right end of a beam, kips.

V_{nr} — nominal shear strength at right end of a beam, lbs.

V_{nt} — nominal shear strength at the top section of a column, lbs.

V_s — nominal shear strength provided by shear reinforcement, lbs.

V_u — factored shear force at a section, kips.

V_{ub} — factored shear force at the bottom section of a column, kips.

$V_{u\ell}$ — factored shear force at left end of a beam, kips.

V_{um} — factored shear force at one-third height from the top or one-sixth height from the bottom section of a column, kips.

V_{uq} — factored shear force at quarter span section of a beam, kips.

$V_{uq\ell}$ — factored shear force at quarter span section from left end of a beam, kips.

V_{uqr} — factored shear force at quarter span section from right end of a beam, kips.

V_{ur} — factored shear force at right end of a beam, kips.

V_{ut} - factored shear force at the top section of a column, kips.

W_i - preassigned weighting coefficient.

$\mathbf{X}$ - vector of design variables.

$\mathbf{X}^*$ - optimum point.

$\bar{Z}_i$ - mean value of plastic safety margin.

α - moment coefficient or a scalar multiplier or a given step size.

β - a scalar or reliability index.

γ_1 - factor which according to ACI Code is given as either 0.75 or 0.50.

γ_2 - factor which according to ACI Code is given as either 0.08 or 0.06.

γ_3 - factor which accordincg to ACI Code is given as 0.01.

γ_4 - constant which according to ACI Code is 2/5 for continuous beams and 4/5 for simple beams.

γ_5 - constant which according to ACI Code is 1/16 for simply supported beams; or 1/18.5 for one end continuous beams; or 1/21 for both ends continuous beams.

γ_6 - specified ratio of width to overall depth of beam.

γ_7 - factor to specify the relation of the width of beam and column.

γ_8 - factor to specify the relation of the overall depth of beam and column.

γ_9 - specified ratio of width to overall depth of column .

γ_a - ratio of the reinforcement to be continued at section of cut-off point of bars to the reinforcement required at sections of maximum positive moment.

γ_b - ratio of the reinforcement to be continued at section of cut-off point of bars to the reinforcement required at sections of maximum negative moment.

γ_c - coefficient to limit the design axial load strength of a section in pure compression to γ_c times its nominal strength, i.e. coefficient to achieve the purpose of the minimum eccentricity. According to ACI Code, it is given as 0.80 or 0.85.

λ_i - weighted coefficient.

μ_x - mean value of a discrete random variable X.

$[\bar{\rho}]$ - correlation matrix.

$\bar{\rho}$ - average correlation coefficient.

$\acute{\rho}$ - ratio of compression reinforcement.

ρ - ratio of tension reinforcement.

ρ_b - reinforcement ratio producing balanced strain conditions.

ρ_b - reinforcement ratio producing balanced strain condition for section with tension reinforcement only.

σ_X - standard deviation of X.

ϕ_b - strength reduction factor for flexure which according to ACI Code is given as 0.90.

ϕ_c - strength reduction factor for compression members which according to ACI Code is given as 0.70 or 0.75.

ϕ_{sh} - strength reduction factor for shear which according to ACI Code is given as 0.85.

φ_{ci} - column curvature at the bottom of the ith storey.

Ω - feasible design space.

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INTRODUCTION

1.1 INTRODUCTION

This thesis has two seemingly very different themes. The first is an essentially theoretical topic of developing new mathematical optimization methods, while the second theme is the very practical topic of optimal design of reinforced concrete frames. Since the most time-consuming and expensive operation in a design office is the preparation of specifications and schedules, comprising the final documents, more efficient optimization formulation is considered and coupled with a practical design program to meet these requirements. The link between the two themes is that the developed methods should be helpful in the design.

The theoretical part of the thesis deals with developing two sets of new methods of mathematical optimization: the first set includes a new method for generating Pareto sets for multi-criteria optimization problems using the Maximum Entropy Principle (MEP) and the second set includes two new methods for seeking the global minimum of a single-criteria minimization problem using the Minimax Entropy Principle (MMEP) by simulated entropy. This is considered in detail in chapter three.

Design is one of the primary functions of engineering. The objective of the designer is to proportion a structure in such a way that the requirements of safety and serviceability are met as economically as possible. In practice, ensuring safety and serviceability is controlled by various specifications and codes; however, the economy of the structural design has largely depended on the engineer's experience and intuition. The design problem is often handled by means of repeated analysis. The most important use of structural analysis is as a tool in structural design. As such, it will usually be a part of a trial-and-error procedure, in which an assumed configuration is analysed, and the members designed in accordance with the results of the analysis. This phase is called the preliminary design; since this design is still subject to change, usually a crude, fast analysis method is adequate. At this stage, the cost of the structure is estimated, loads and member properties are revised, and the design is checked for possible improvements. The changes are now incorporated in the structure, a more refined analysis is performed, and the member design is revised. This process is repeated until a final design, which satisfies the designer in all aspects, is achieved. This iterative process of analysis and modifications leading to a *'best'* design may be characterized as an informal optimization process. Since the number of possible designs satisfying the applicable criteria may be theoretically infinite, the iterative design-proportion process may be directed and controlled by mathematical means so as to minimize some important aspects, such as cost and drift. This is the motivation for the use of techniques of formal optimization.

Formal structural optimization, which is considered in this thesis, is a mathematical decision-making process aimed at producing a best possible design with the available resources.

Most of the available literature on the optimization of structural concrete frames has been concerned with highly idealised mathematical formulations of the problem in which many practical aspects of real concrete frame design are omitted. In considering a practical design program, especially in the optimization of reinforced concrete frames, both the member sizing and the practical details have to be taken into consideration. This can be done by considering all the applicable requirements of the given specifications or codes as design constraints. The formulation should be made in such a way as to easily accommodate any later modifications made in the specifications and the codes.

In practice, the design of a reinforced concrete structure is often a trial and adjustment procedure. It is evident that many trials, even with the availability of computers, are not always possible, and consequently the structure is generally conservatively designed. Recent developments in probabilistic analysis combined with simultaneous rapid growth of computing power provide the necessary conditions for the application of reliability concepts in the optimum design of complete structures. In the light of this design philosophy, which extends reliability and optimization concepts from structural elements to complete structures, part of the objective of this thesis is to develop a practical design program for large reinforced concrete frames incorporating optimization and reliability. Through the use of mathematical programming techniques and on the basis of modern codes such as ACI, (American Concrete Institute), Code (318-83), programs for the optimal design of reinforced concrete frames for use in professional practice are now possible, resulting in considerable cost savings and improved safety.

ACI Code (318-83) emphasises design based on both strength and serviceability. The strength design is the predominant approach for proportioning the structural elements, with elastic theory used to analyse the structure, and obtain the design forces. The elastic theory is also used to ensure serviceability. In order to design an optimal reinforced concrete structure as closely to the professional practice as possible, the design constraints should include the code requirements of flexural strength, shear strength, axial strength, ductility, serviceability, limits of web reinforcement, concrete cover, development length, cut-off points, spacing limits and requirements, i.e. everything of importance to a practising designer. Also, the discreteness of design variables must be taken into account. For example, in the optimization of reinforced concrete structure the dimensions of the member cross-sections are conventionally measured in multiples of ten millimeters; also, the reinforcement can only be selected from a range of discrete available sizes which vary non-uniformly. Any practical design programme must accommodate these practical conventions.

1.2 REVIEW OF PREVIOUS WORK IN DESIGN OPTIMIZATION

Optimal design problems have been of increasing interest to many engineers and researchers for the past two decades. Recent developments in mathematical programming techniques have made it possible to solve a wide spectrum of structural optimization problems.

A concrete structure can be designed by the working stress method or by the strength method. Some investigators have optimized concrete frames based on the working stress method for minimum cost (Munro, et.al., 1972). Shunmugavel (1974), used the method of feasible directions to obtain the solution for minimum cost in which the design variables are continuous variables. The primary objective of the work was to obtain the optimal relative stiffness of the members of the large frames studied. The cross-sectional dimensions and the amount of main reinforcement of the members were the design variables. The constraints were developed in accordance with the ACI Code (318-71) requirements. The arrangement of main reinforcement followed a specified pattern. No consideration was given to constraints due to shear, concrete cover, contribution of compression reinforcement to bending strength, clear spacing and possible unsymmetric arrangement of reinforcement in column sections.

Twisdale and Khachaturian (1975), used the dynamic programming method for the optimization of steel planar structural systems. Redundant forces of the structures were taken to be the state variables.

Balaguru (1980) used the Lagrange multiplier method for the cost optimum design of doubly reinforced concrete beams. Costs of concrete, steel and formwork were considered. Dimensions of the beam and the amount of reinforcement were formulated as continuous variables. Flexural strength constraints, based on ACI Code (318-77), were taken into consideration.

Gerlein and Beaufait (1980), used a storey-by-storey linear programming optimization approach for an optimum preliminary strength design of reinforced concrete frames. The total volume of reinforcing steel required by the members of the structure was minimised. The objective function was expressed in terms of moment capacities which were treated as functions of the amount of reinforcing steel at sections. The recommended bar details of the Concrete Reinforcing Steel Institute (CRSI) were used to obtain the moment capacity at sections. Collapse mechanisms were used to develop the constraints. A minimum amount of reinforcement required at any cross-section was based on ACI Code (318-77).

Liebman, Khachaturian and Chanaratna (1981), developed a discrete optimization method based on the integer gradient direction by using the interior penalty function for concrete structures. The method was used for optimal design of reinforced concrete beams. The amount of reinforcement and the dimensions of the concrete were formulated as discrete variables, and the cost of the structure was taken as the objective function. The constraints were based on the strength requirements only.

Yang (1981), developed a practical minimum cost design of reinforced concrete structures in which the design variables vary discretely. From a practical point of view, the variables can only assume certain discrete values and the optimization process can be based only on these discrete values. The design variables include cross-section dimensions, amount of reinforcements, amount of web reinforcement and the cut-off points of longitudinal reinforcement.

Zheng and Huanchun (1985), treated the highly nonlinear optimum design problem of reinforced concrete frames in two levels corresponding to global constraints and local constraints respectively, with iterations in each level. In the first level, the most flexible structure among those satisfying the global constraints, such as displacement constraints, size constraints, etc., is sought under the most unfavourable horizontal loads. The optimum solution from the first level is applied to construct bounds upon design variables. In the second level, using these values from the first level, the most economical structure satisfying all local constraints, such as those of strength, size and percentage of reinforcement is obtained.

Frangopol (1985), presented a reliability-based optimization technique to design reinforced concrete frames. The objective function to be minimized was the total cost of concrete and longitudinal steel subject to overall probability against plastic collapse specified as a reliability constraint. The optimum design vector represents the mean values of the positive and negative plastic moments associated with the frame critical sections.

From the above survey, it can be seen that relatively little research effort has been dedicated to the optimization of reinforced concrete frames in general, and to the multi-criteria reliability-based optimization of reinforced concrete frames in particular. Very little or nothing has been done in the area of developing the type of practical optimum design program which is required in professional practice.

1.3 REVIEW OF PREVIOUS WORK IN MATHEMATICAL OPTIMIZATION

There are many techniques available for multi-criteria optimization. Without loss of generality the multi-criteria problem can be stated in terms of minimization. Single-criteria optimization techniques, such as the method of feasible directions and sequential linear programming, have been combined with the Pareto set generating techniques described in chapter two to solve multi-criteria optimization problems. Because of the large volume of existing literature on mathematical optimization a more detailed survey of methods related to the work of this thesis is given in chapter 2.

1.4 PURPOSE AND SCOPE

The purpose of this thesis is to accomplish two things. The first is mathematical and can be divided into two main areas.

1) The development of new methods for generating Pareto solutions of multi-criteria optimization problems.
2) The development of new methods for seeking the global minimum of single-criteria minimization problems.

The second purpose is to consider the application of the above new methods in practical structural engineering and consists of a study of the multi-criteria optimization of discrete non-linearly constrained problems in the design of reinforced concrete frames. Specifically, it is intended to carry out the following:

1) To identify all the articles of the code that should be considered in the design of reinforced concrete frames, including the details.
2) To solve the above design problem by the new methods developed

in the first part. However, a practical formulation is going to be considered and it will be discussed later in chapter 7.

3) To develop a practical design program for one-storey and multi-storey frames incorporating optimization and reliability.

In this study, the geometric configuration and loading conditions of a frame are given. Design constraints are developed according to the ACI Code (318-83). The related Articles of the above chapters are listed in Table (1.1). The relevant chapters of the Code are chapters 7, 9, 10, 12 and Appendix A. The objective functions are the total cost of the frame and the drift (top storey lateral displacement). The cost of the formwork of the structure can be included, but, is omitted herein. The design variables are the cross-section dimensions of the members, the effective depth, the amount of longitudinal reinforcement, the cut-off points of longitudinal reinforcements, the area of web reinforcement and others. All the design variables are treated as discrete variables. The optimization of reinforced concrete frames for practical purposes involves a large number of design variables and highly non-linear constraint functions.

1.5 MOTIVATIONS AND SPECIFICATIONS OF THE PRESENT MATHEMATICAL WORK

In almost all previous work of mathematical optimization the geometric and deterministic viewpoint has played a large part in the development of methods. Methods for single- and multi-criteria optimization have been devised from considering the objective function as a *hypersurface* and using *geometric* arguments to devise a search strategy. In this geometric interpretation, the constraints are viewed as *deterministic boundaries* which must not be crossed. Terms such as

Table 1.1

Chapters and Articles of ACI Code (318-83) Concerned

Chapters	Articles
3	3.3.3.
7	7.1, 7.6.1, 7.6.2, 7.6.3, 7.7.1, 7.10.4.3, 7.10.5.1, 7.10.5.2, 7.10.5.3, 7.10.5.4.
9	9.1.1, 9.2.1, 9.2.2, 9.2.3, 9.3.1, 9.3.2, 9.5.1, 9.5.2,1.
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11	11.1.1, 11.1.3.1, 11.3.1.1, 11.5.1.1, 11.5.2, 11.5.3, 11.5.4.1, 11.5.4.3, 11.5.5.3, 11.5.6.2, 11.5.6.8.
12	12.2.1, 12.2.2, 12.2.3, 12.2.5, 12.3.2, 12.4, 12.11.2, 12.11.3, 12.11.4, 12.12.1, 12.12.3, 12.13.3, 12.14.1.
Appendix A	A.5.1, A.5.2, A.5.3, A.5.4, A.5.5, A.5.10, A.5.11, A.5.12, A.6.1, A.6.5.1, A.6.5.2, A.6.5.3.

"gradients", "steepest descent" and "barriers" all have topological associations. They use calculated "information" (function values, gradients, etc.) in a *geometrical way* to search a deterministic topological domain for an optimum point. Information theory appears to be incompatible with this as it is essentially concerned with probabilities. In contrast with conventional methods, a different viewpoint is adopted in the present work. One of its objectives is to set multi-criteria and single-criteria optimization in a *non-deterministic* context where calculated "information" are used also in a *gemoetrical way* to search a non-deterministic topological domain to generate Pareto solution sets for multi-criteria optimization problems and to locate the global minimum points of single-criteria minimization problems. The idea behind this approach is based upon the speculation that generating Pareto sets or locating global minima could be thought of as a *communication system* in which "messages" are received and transmitted alternatively. Design of the system requires that the messages be correctly translated and subsequently used effectively. Thus, methods in information theory can then be used in the generation of Pareto sets and in location of global minima. It is hoped that this approach will yield new insights into multi-criteria and single-criteria optimization processes and develop new and radically different algorithms.

From a statistical point of view, a communication system has mathematical similarities to a statistical thermodynamic system. A multi-criteria or single-criteria optimization problem is consequently simulated in the present work as a *statistical thermodynamic system* for convenience of the later presentations.

Several questions are then raised about how to do this detection and simulation. They are:

1) What are micro-states of this statistical thermodynamic system in an optimization context?

2) What are the probabilities of the micro-states?

3) What common characteristic is there in these two processes?

4) What common law governs them?

These questions will be answered in detail in chapter 8.

1.6 OBJECTIVE AND SCOPE

For multi-objective optimization problems, most, if not all, of the available Pareto set techniques have several difficulties. Firstly, they are computer-time consuming since in order to generate a representative or entire Pareto set the preassigned vector of weighting coefficients, bounds, etc. must be varied over a large number of combinations. Secondly, Pareto solutions are obtained randomly since the distribution characteristics of these solutions are unknown. Thirdly, where there are many criteria the amount of computation required may, itself, become a difficulty which dramatically affects the total number of Pareto solutions obtained and the selection of a better preferred solution.

For single-objective optimization problems, most, if not all, of the available deterministic techniques terminate in a local minimum which depends on an initial configuration given by the user and do not provide any information as to the amount by which this local minimum deviates from a global minimum.

The main purpose of this thesis is to study the possibility of eliminating or, greatly reducing, the above mentioned difficulties. Specifically, it is intended to carry out the following:

1) To view vector optimization problems in terms of a topological domain defined non-deterministically through the use of the Shannon's entropy and maximumum entropy principle.

2) To develop new methods based on the terminology described in 1 to eliminate the three difficulties involved with Pareto solution generation.

3) To view single-objective constrained minimization problems in terms of a topoligical domain defined non-deterministically through the use of the Shannon's entropy and minimax entropy principle. The whole problem will be treated as a thermodynamical system.

4) To develop new methods based on the terminology described in 3 which seek to reach the global minimum of such problems.

However, there is always a strong link between engineering and optimization in general. It is a further objective of this thesis to develop a practical design program which incorporates vector optimization, design variables standardization, and system reliability analysis together. In this work the program is applied to reinforced concrete frames.

CHAPTER TWO

MATHEMATICAL OPTIMIZATION

CHAPTER TWO

MATHEMATICAL OPTIMIZATION

SYNOPSIS

This chapter introduces the concepts of single-criteria optimization and multi-criteria optimization, their mathematical formulations and gives a brief survey of useable methods. For single-criteria constrained optimization, Sequential Linear Programming (SLP), the method of centres, and the method of feasible directions are reviewed. For single-criteria unconstrained optimization, the conjugate directions methods, the conjugate gradient method, and the variable metric methods are reviewed also. Finally, the traditional methods used for solving multi-criteria optimization are reviewed. These include the weighting method, the Non-Inferior Set Estimation (NISE), and the constraint method.

2.1 INTRODUCTION

The concept of optimization is basic to much of what we do in our daily lives. The desire to run a faster race, win a debate, or increase corporate profit implies a desire to do or be the best in some sense. In engineering, we wish to produce the *best possible design with the resources available*. Thus, in designing new products, we must use design tools which provide the desired results in a timely and economical fashion (Vanderplaats, 1984).

The numerical quantities for which values are to be chosen in producing a design will be called *design variables*. The design restrictions that must be satisfied in order to produce an acceptable design are collectively called *constraints*. These constraints are formulated in terms of the design variables and may be either equalities or inequalities.

A feasible design is defined as a set of design variables which satisfies all constraint restrictions. Of all feasible designs some are *better* than others. If this is true, then there must be some quality that the better designs have more of than the less desirable ones do. If this quality can be expressed as a computable function of the design variables, we can consider optimizing to obtain a *best* design. The function with respect to which the design is optimized is called the *objective function*. We conventionally assume that the objective function is to be minimized, which entails no loss of generality since the minimum of any function of [F] occurs where the maximum of [-F] occurs (Fox, 1971).

Numerous algorithms are available for single-criteria optimization and each major technique has its own desirable and undesirable characteristics. The most general approach to mathematical optimization is that of using *mathematical programming* techniques. Mathematical programming is a term coined by Robert Dorfman about 1950, and now is a generic term encompassing linear programming, integer programming, convex programming, non-linear programming, network flow theory, dynamic programming, and programming under uncertainty.

Most real-world problems have several solutions, and some may have an infinite number of solutions. The purpose of optimization is to find the best possible solution among the many potential solutions for a given problem in terms of some effectiveness or performance criteria. A problem which admits only one solution does not have to be optimized. For large, highly non-linear problems, numerical methods are used. Numerical methods use past information to generate better solutions to the optimization problem by means of iterative procedures.

The optimization process for a practical, real-world problem has two distinct phases. One consists of formulating a mathematical model which represents accurately the real-world problem; the other is concerned with the numerical methods which can be used to solve the mathematical model. This chapter is particularly concerned with the later phase.

The general single-criteria optimization problem with n design variables, m inequality constraints, k equality constraints can be written as follows:

$$\text{Minimize:} \quad F(X) \tag{2.1}$$

$$\text{S.t.:} \quad g_i(X) \geq 0 \qquad i = 1,2,\ldots,m \tag{2.2}$$

$$h_j(X) = 0 \qquad j = 1,2,\ldots,k \tag{2.3}$$

$$X^L{}_q \leq X_q \leq X^U{}_q \qquad q = 1,2,\ldots,n \tag{2.4}$$

Any vector X that satisfies all the constraints (2.2), (2.3) and (2.4) is called a *feasible point*. The set of all the feasible points constitutes the *feasible region*.

A point X^*, which satisfies all the constraints and at which F(X) attains its minimum, is called the *optimum point* and the pair X^* and $F(X^*)$ constitutes an *optimum solution* where $F(X^*)$ represents the optimum value of the objective F(X). If $F(X^*) \leq F(X)$ holds for all X in the feasible region, then X^* is called *global optimum* point, otherwise it is a *local optimum* point.

An inequality constraint $g_i(X)$ is said to be active if $g_i(X^*) = 0$ at the optimum point X^*. For a real-world problem, there almost always exist several constraints, so that the corresponding problem is constrained, linearly or non-linearly. Throughout this thesis mainly constrained and non-linear problems are considered.

An optimization problem is said to be a *linear programming* problem (LP) if all the problem functions are linear in the variables X_i, otherwise it is a *non-linear programming* problem (NLP).

The area of linear programming in the last three decades has achieved great success. The most significant development being the *simplex method* by which the solutions of most linear programming problems can be systematically obtained in a finite number of iterations. On the other hand, there has been no single algorithm devised that can handle all non-linear problems. The major difficulties presented in solving non-linear optimization problems arise from the presence of non-linear constraints. Keeping them satisfied involves considerable complexity. When inequality constraints are present, it is impossible for one to know in advance whether an inequality constraint will be active or not at the optimum.

However, solution methods depend upon the nature of the variables (discrete/continuous), the objective function F and constraints g_i (linear/non-linear), and the constraint type (equality/inequality).

In many engineering situations, the design variables are available only in discrete sizes. This fact can pose some formidable problems for the optimiser, because most of the well-known methods apply only to continuous valued variables. The fundamental difficulty of course, is that it is usually impractical, for reasons of computational cost, to check all possible designs for suitability (Fox, 1971).

Over the last three decades, there has been a massive body of published work on the solution of constrained optimization problems. A comprehensive review of all these methods is beyond the scope of this thesis and the reader is referred to Duffin (1967), Lasdon (1970), Gill (1976), Bazaraa (1971), Fletcher (1981), Bertsekas (1982) and Vanderplaats (1984) for surveys.

An extremum (maximum or minimum point) can be either global (truly the highest or lowest function value) or local (the highest or lowest in a finite neighbourhood and not on the boundary of that neighbourhood) as shown in Fig. (2.1). Virtually nothing is known about finding global extrema in general. There are two standard heuristics that everyone uses: (i) find local extrema starting from widely varying starting values of the independent variables and then pick the most extreme of these (if they are not all the same), or (ii) perturb a local extremum by taking a finite amplitude step away from it, and then see if your routine returns you to a better point, or *always* to the same one.

2.2 NON-LINEAR PROGRAMMING

Non-linear programming deals with the problem of optimizing an objective function in the presence of equality and inequality constraints. If all the functions are linear, we obviously have a linear program. Otherwise, the problem is called a non-linear program. The development of the simplex method for linear programming and the advent of high-speed computers have made linear programming an important tool for solving problems in diverse fields. However, many realistic problems cannot be adequately represented as a linear program owing to the non-linearity of the objective function and/or the non-linearity of any of the constraints. Efforts to solve non-linear problems efficiently have made rapid progress during the past three decades.

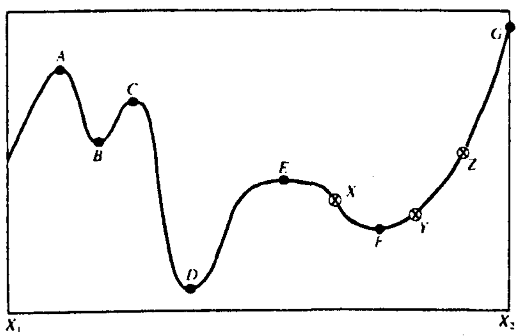


Fig. (2.1)

Extrema of a function in an interval. Points A, C, and E are local, but not global maxima. Points B and F are local, but not global minima. The global maximum occurs at G, which is on the boundary of the interval so that the derivative of the function need not vanish there. The global minimum is at D. At point E, derivatives higher than the first vanish, a situation which can cause difficulty for some algorithms. The points, X, Y, and Z are said to bracket the minimum F, since Y is less than both X and Z (Press et al., 1986).

Different algorithms are designed for solving various classes of non-linear programming problems, such as unconstrained optimization problems, problems with inequality constraints, problems with equality constraints, and problems with both types of constraints. Within each of these classes different algorithms make specific assumptions about the problem structure. For example, for unconstrained optimization problems, some procedures assume that the objective function is differentiable, whereas other algorithms do not make this assumption and rely primarily on functional evaluations only. For problems with equality constraints some algorithms can only handle linear constraints, while others can handle non-linear constraints as well. Thus generality of an algorithm refers to the variety of problems that the algorithm can handle and also to the restrictions of the assumptions required by the algorithm. Another important factor is the reliability or robustness of the algorithm which is very important in engineering design, since the program must produce a result. Given any algorithm, it is usually not difficult to construct a test problem that it cannot solve effectively. Reliability means the ability of the procedure to solve, with reasonable accuracy, most of the problems in the class for which it is designed. The relationship between the reliability of a certain procedure and the problem size and structure cannot be overlooked. Some algorithms are reliable if the number of variables is small, or if the constraints are not highly non-linear, but not otherwise reliable.

The convergence of non-linear programming algorithms usually occurs in a limiting sense, if at all. Thus, we are interested in measuring the quality of the points produced by the algorithm after a reasonable number of iterations. Convergence of an algorithm is important in a mathematical sense, but is of less importance to an engineer. In engineering design it is sometimes very valuable to be able to find a feasible design, and to then be able to generate an improved feasible design. The idea of exact convergence to an accurate optimum is largely irrelevant to an engineer.

2.3 NON-LINEAR PROGRAMMING METHODS FOR CONSTRAINED PROBLEMS

2.3.1 Sequential Linear Programming [Vanderplaats, 1984]

Even though most engineering analysis and design is non-linear in the design variables, it is often possible to linearize a particular problem and then obtain the solution to this linear approximation using linear programming methods like the simplex method. Having this approximate solution we can now linearize about this point and solve the new linear programming problem, repeating the process until a precise solution is achieved. This approach, whereby we repeatedly linearize and solve the resulting problem, is referred to as *Sequential Linear Programming* (SLP).

Consider the general non-linear programming problem of Eqs (2.1) to (2.4). Now linearize this problem via a first-order Taylor series expansion so we have:

Minimize: $$F(X) \approx F(X^o) + \nabla F(X^o).\delta X \quad (2.5)$$

S.t.: $$g_i(X) \approx g_i(X^o) + \nabla g_i(X^o).\delta X \geq 0; \quad i = 1,2,\ldots,m \quad (2.6)$$

$$h_j(X) \approx h_j(X^o) + \nabla h_j(X^o).\delta X = 0; \quad j = 1,2,\ldots,k \quad (2.7)$$

$$X^L{}_q \leq X^o{}_q + \delta X_q \leq X^U{}_q; \quad q = 1,2,\ldots,n \quad (2.8)$$

where

$$\delta X = X - X^o$$

$$\nabla F(X^o) = \text{the gradient of } F(X^o) = \begin{Bmatrix} \frac{\partial F(X^o)}{\partial X_1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial F(X^o)}{\partial X_m} \end{Bmatrix}$$

The zero superscript identifies the point about which this Taylor series expansion is performed.

It is clear that Eqs (2.5) to (2.8) represent a linear programming problem where the design variables are contained in the vector δX, and the functions and gradients at X^o are constants and coefficients, respectively.

Figure (2.2) provides a geometric interpretation of the SLP method. At the initial design X^o, the objective and constraints are linearized to give the straight-line representations of the functions. The optimum of this linear problem is found and is seen to be near the non-linear optimum, but it is infeasible. However, if we relinearize at this point and repeat the process, we would expect to approach the precise optimum in a few iterations.

In Fig. (2.2), the true optimum lies at a vertex in the design space. For fully constrained problems such as this, SLP often converges rapidly to the solution. However, for other problems in

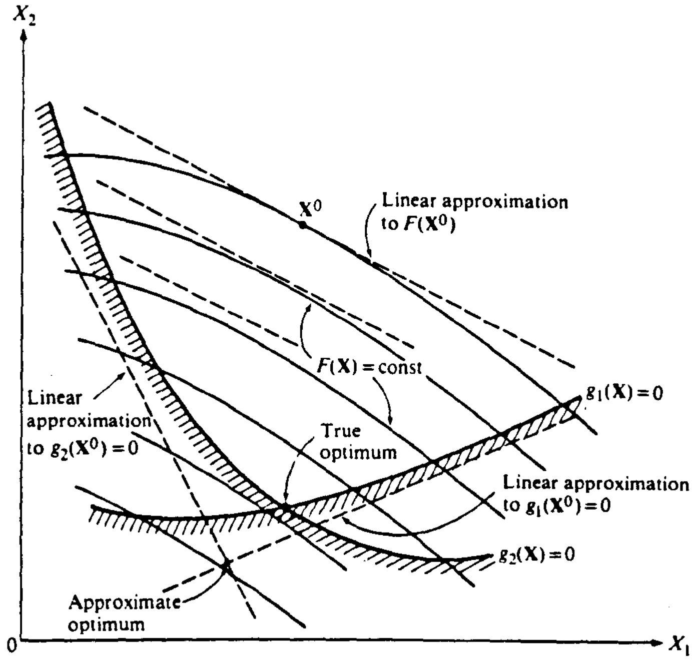


Fig. (2.2)

The Linearized Problem (Vanderplaats, 1984)

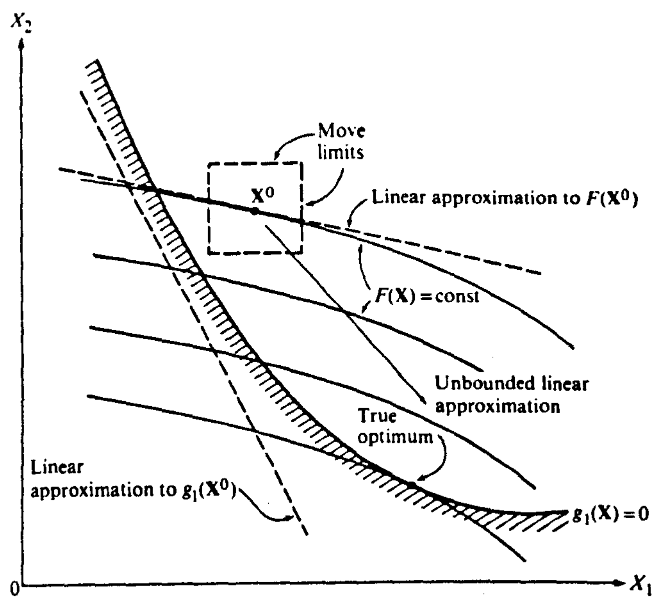


Fig. (2.3) Under Constrained Problem and Move Limits

(Vanderplaats, 1984)

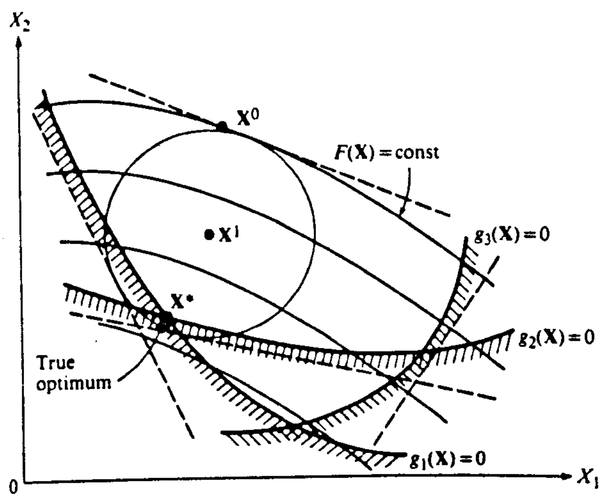
which there are fewer active constraints at the optimum than there are design variables, the method often performs poorly. The reason for this is seen from Fig. (2.3), which shows a two-variable problem with only one non-linear constraint. Here the linear approximation to the problem is unbounded. The difficulty of dealing with such problems is reduced by imposing more limits on the linear approximation as shown in the figure. This will ensure that the optimum will eventually be reached within a tolerance of the move limits. The critical difficulty in using the method is in choosing the move limits and the reduction factor on these limits so that the optimum is efficiently achieved.

2.3.2 The Method of Centers

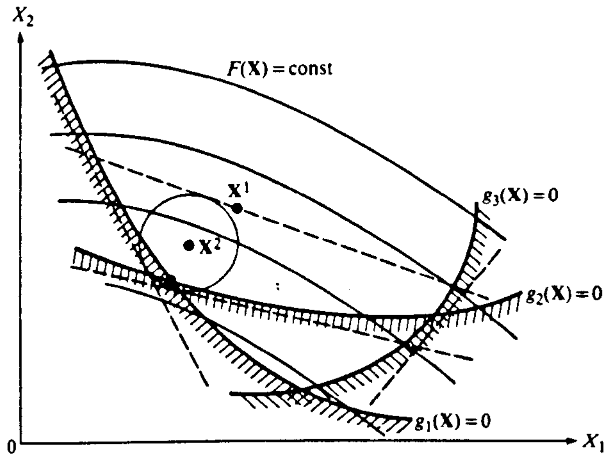
The SLP method discussed in the previous section produces a sequence of improving but infeasible design, in the usual case of a convex design space. From an engineering viewpoint, it is often desirable to approach the optimum from inside the feasible region. Also, it is desirable to produce a sequence of improving designs which follow a path down the *center* of the design space. In this way, if the optimization must be prematurely terminated, or if for some reason the final design is found to be unacceptable, several other *good* designs are available from which to choose. The method of centers, also known as the method of inscribed hyperspheres, is an SLP technique which has these features.

Consider Fig. (2.4a), which depicts a two-variable non-linear programming problem with three constraints. The straight lines are the linearized approximation to the problem at X^0. The basic concept here is to find the largest circle (hypersphere in n dimensions) which will fit in this linearized space and then to move to the center of that circle. This is shown in Fig. (2.4a), where the circle touches the line of constant objective and the constraint boundaries on $g_1(X)$ and $g_2(X)$. The circle does not touch the $g_3(X)$ boundary. We now move to design point X^1 at the center of this circle. The problem is linearized again and the linearized objective at X^1 is treated as a bound as shown in Fig. (2.4b). The center of the new circle inside the linearized design space is the new design X^2. The process is repeated until the solution has converged to a specific tolerance.

The method of centers is attractive for its feature of progressing down the middle of the design space. For more details about this method, the reader is referred to Vanderplaats (1984).



(a) First Linearization



(b) Second Linearization

Fig. (2.4) Geometric Interpretation of the Methods of Centers (Vanderplaats, 1984)

(a) First Iteration, (b) Second Iteration

2.3.3 The Method of Feasible Directions

We now turn from SLP methods to methods which attempt to deal directly with the non-linearity of the problem. This method consist of an iteration or step-by-step design evolution of the familiar form:

$$X_{q+1} = X_q + \alpha S_q \tag{2.9}$$

where the direction S_q and the *distance of travel* α are always chosen so that X_{q+1} is in the feasible domain. They are called feasible direction methods because of certain properties of S_q which will now be described. First, we consider two definitions:

1. A vector S is a *feasible direction* from the point **X** if at least a small step can be taken along it that does not immediately leave the feasible domain.
2. A vector **S** is a *usable feasible direction* from the point X if in addition to definition 1,

$$S^T.\nabla F < 0 \qquad (2.10)$$

For problems with sufficiently smooth constraint surfaces, definition 1 is satisfied if:

$$S^T.\nabla g_i < 0 \qquad (2.11)$$

or if a constraint is linear or *outward curving*:

$$S^T.\nabla g_i \leq 0 \qquad (2.12)$$

where ∇F and ∇g are the gradients of F and g respectively.

The significance of interpreting definition 1 in this way is that the vector **S** must make an obtuse angle with all constraint normals except that, for the linear or outward curving ones, the angle may approach 90°. Any vector satisfying the strict inequality lies at least partly in the feasible region of the space. In other words, there is some $\alpha > o$ for which $\mathbf{X}_{q+1}$ is in the feasible domain as shown in Fig. (2.5).

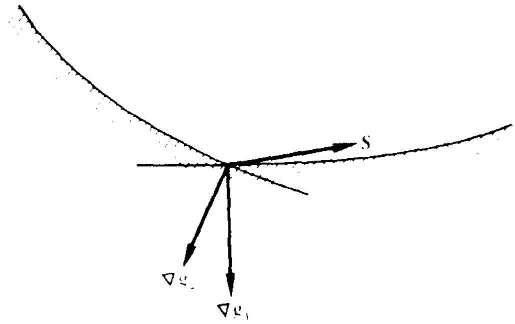


Fig. (2.5)

Feasible Direction Definition (Fox, 1971)

A vector satisfying the strict inequality of definition 2 is guaranteed to produce, for some $\alpha > 0$, an X_{q+1} that reduces the value of F as shown in Fig. (2.6).

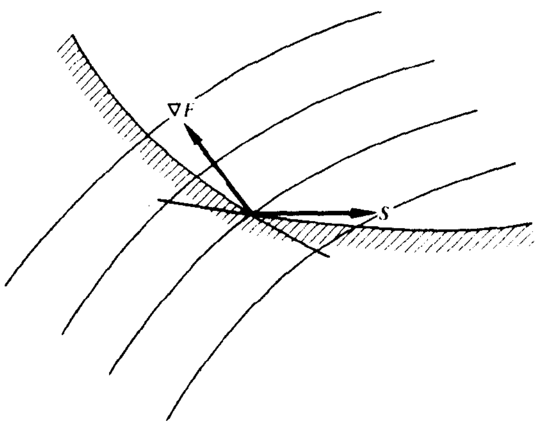


Fig. (2.6)

Usable Feasible Direction Definition (Fox, 1971)

Methods of feasible directions are those that produce an improving succession of feasible X_q vectors by moving in a succession of usable feasible directions. There are two general parts to those algorithms. First, a usable feasible vector must be determined for each step of the iteration, and second, the step size must be determined.

However, assuming the constraint is non-linear and convex, a small move in this direction would violate the constraint. This is not desirable, so we need a way to push away from the constraint boundary. We can do this by adding a positive push-off factor to Eq. (2.12) to give:

$$S^T.\nabla g_i + \theta \leq 0 \tag{2.13}$$

where θ is a non-negative constant. In practice, we would like the push-off factor to be affected by the direction of ∇F. Thus, remembering that $S^T.\nabla F$ is negative for usability, we can modify Eq. (2.13) to be:

$$S^T.\nabla g_i - [S^T.\nabla F]\theta \leq 0 \tag{2.14}$$

Now minimizing the quantity in Eq. (2.10) is equivalent to maximizing a scalar β in the following inequality:

$$S^T.\nabla F + \beta \leq 0 \tag{2.15}$$

and Eq. (2.15) becomes the usability condition. Clearly for β to be maximum, Eq. (2.15) will be satisfied with equality so $\beta = -S^T.\nabla F$. Therefore, the feasibility requirement of Eq. (2.14) becomes:

$$S^T.\nabla g_i + \theta\ \beta \leq 0 \qquad (2.16)$$

Figure (2.7) shows the effect that the push-off factor θ has on determining the possible S^T vectors.

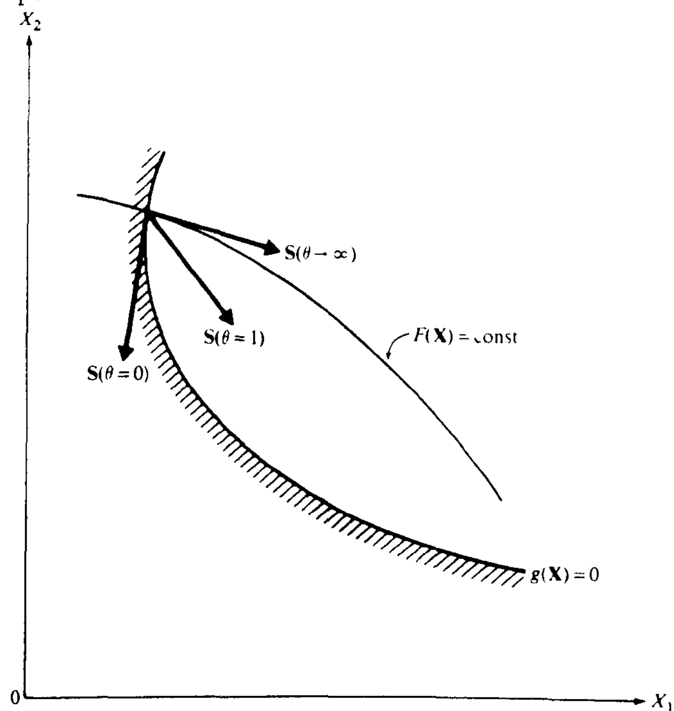


Fig. (2.7) Effect of θ on the Search Direction (Vanderplaats, 1984)

The general direction-finding problem can be written as:

Maximize: β (2.17)

S.t.: $S^T.\nabla F + \beta \leq 0$ (2.18)

$S^T.\nabla g_j + \theta_j\beta \leq 0$; $j \in J$ (2.19)

S^T bounded (2.20)

where J is the set of currently active constraints $g_j(\mathbf{X}) = 0$.

This method will be used later. However, for more details about this method and other methods, the reader is referred to Fox (1971) and Vanderplaats (1984).

2.4 NON-LINEAR PROGRAMMING METHODS FOR UNCONSTRAINED PROBLEMS

In this section we will examine methods of solving the following problem. Find **X** such that F(**X**) is minimum where there are no restrictions on the choice of **X**. Although there are relatively few practical applications in which a design problem is unconstrained many constrained problems employ unconstrained methods as part of their solution strategies. See Fox (1971) and Himmelblau (1972).

2.4.1 Powell's Method: Conjugate Directions

This method will minimize a quadratic function in a finite number of steps; hence it can be said to converge quadratically.

Powell's method can be understood intuitively as follows: Given that the function has been minimized once in each of the coordinate directions and then in the associated pattern direction, discard one of the coordinate directions in favour of the pattern direction for inclusion in the next m minimizations, since this is likely to be a better direction than the discarded coordinate direction. After the next cycle of minimizations, generate a new pattern direction and again replace one of the coordinate directions. The process is illustrated in Fig. (2.8).

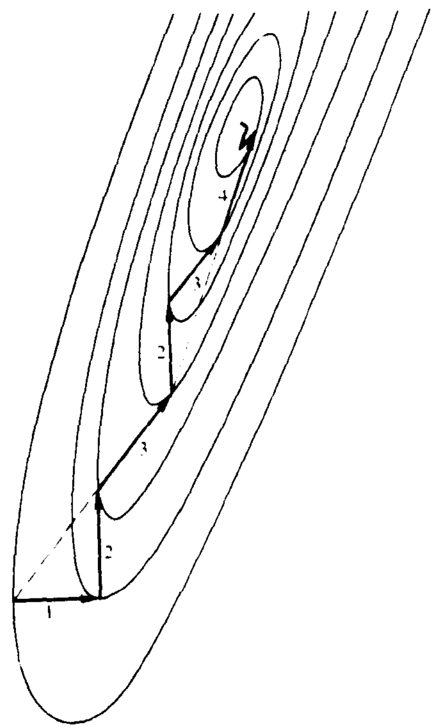


Fig. (2.8) Powell's Method (Fox, 1971)

In practice, a little careful programming is required to make the method truly efficient, but the idea is contained in the above description. The flow diagram shown in Fig. (2.9) is a codification of the simplest version of the method. Note that a pattern direction is constructed (block A), then used for a minimization step (blocks B and C), and then stored in S_n (block D) as all of the directions are up-numbered and S_1 is discarded. The direction S_n will then be used for a minimizing step just before the construction of the next pattern direction. Consequently, in the second cycle both **X** and **Y** in block A are points that are minima along S_n, the last pattern direction. This sequence will impart special properties $S_{n+1} = X - Y$ that are the source of the rapid convergence of the method.

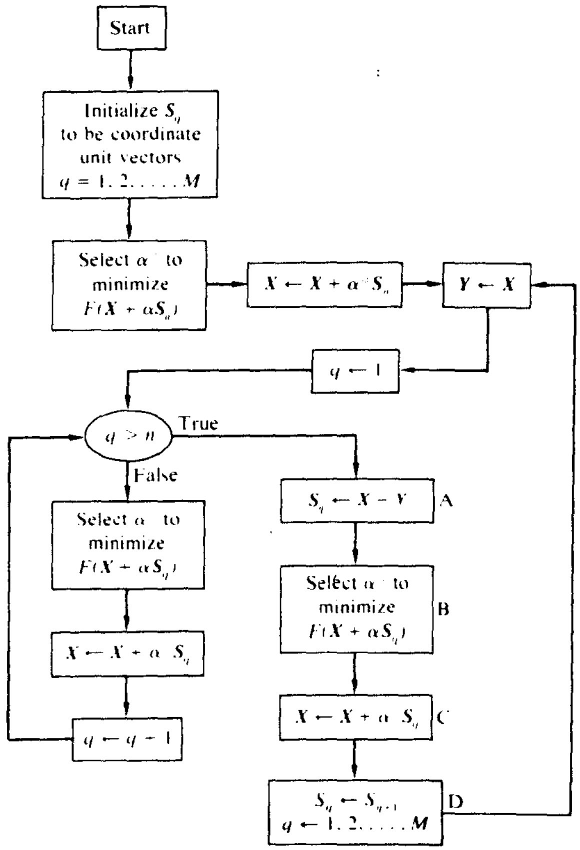


Fig. (2.9)

Computer Flow Diagram of Powell's Method (Fox, 1971)

2.4.2 Fletcher and Reeves Method: Conjugate Gradient

The Fletcher-Reeves conjugate gradient method generates a sequence of search directions S that are linear combinations of the current steepest descent direction, $-\nabla F(X^{(k)})$, and the previous search directions and $S^{(o)}, \ldots, S^{(k-1)}$.

$$- \nabla F(X^{(k)}) = - \left[\frac{\partial F(X^{(k)})}{\partial X_1}, \ \ldots, \ \frac{\partial F(X^{(k)})}{\partial X_n} \right]^T$$

The weighting factors are chosen to make the search directions conjugate. The weights turn out to be such that at $X^{(k)}$ only the current gradient and the most recent gradient are used to compute the new search direction.

To illustrate the idea, let the initial direction of search be $S^{(o)} = -\nabla F(X^{(o)})$, then let $X^{(1)} - X^{(o)} = \lambda^{*(o)}. S^{(o)}$ and make:

$$S^{(1)} = -\nabla F(X^{(1)}) + \omega_1 \ S^{(o)}$$

where ω_1 is a scalar weight that will be chosen so as to make $S^{(1)}$ and $S^{(o)}$ conjugate with respect to H:

$$(S^{(o)})^T \ H \ S^{(1)} = 0,$$

so that

$$\omega_1 = \frac{\nabla^T F(X^{(1)}).\nabla F(X^{(1)})}{\nabla^T F(X^{(o)}).\nabla F(X^{(o)})}$$

and so on for ω_k:

$$\omega_k = \frac{\nabla^T F(X^{(k)}).\nabla F(X^{(k)})}{\nabla^T F(X^{(k-1)}).\nabla F(X^{(k-1)})}$$

All the weighting factors prior to ω_k, namely ω_{k-1}, $\omega_{k-2}, \ldots,$ prove to be zero - a very neat arrangement. The major steps in the algorithm are:

1. At $X^{(o)}$ compute $S^{(o)} = -\nabla F(X^{(o)})$.

2. In the k^{th} stage determine the minimum of $F(\mathbf{X})$ by a unidimensional search in the $S^{(k)}$ direction. This locates $\mathbf{X}^{(k+1)}$.

3. Evaluate $F(\mathbf{X}^{(k+1)})$ and $\nabla F(\mathbf{X}^{(k+1)})$.

4. The direction $S^{(k+1)}$ is determined frome:

$$S^{(k+1)} = -\nabla F(\mathbf{X}^{(k+1)}) + S^{(k)} \frac{\nabla^T F(\mathbf{X}^{(k+1)}) . \nabla F(\mathbf{X}^{(k+1)})}{\nabla^T F(\mathbf{X}^{(k)}) . \nabla F(\mathbf{X}^{(k)})}$$

After (n+1) iterations (k=n), the procedure cycles again, $\mathbf{X}^{(n+1)}$ becomes $\mathbf{X}^{(o)}$.

5. Terminate the algorithm when $\|S^{(k)}\|<\epsilon$, where ϵ is an arbitrary constant. Note that the superscript k, denotes the point in E^n as shown in Fig. (2.10).

2.4.3 Variable Metric Methods

Variable metric methods gather information about previous iterations. This information is stored in an n dimensional array. Because more information can be stored in this way, one would expect these methods to be somewhat more efficient and reliable.

The basic concept here is to create an array which approximates the inverse of the Hessian matrix as the optimization progresses. In these methods, the search direction at iteration q is defined as:

$$S^k = -H \; . \; \nabla F(\mathbf{X}^k) \tag{2.21}$$

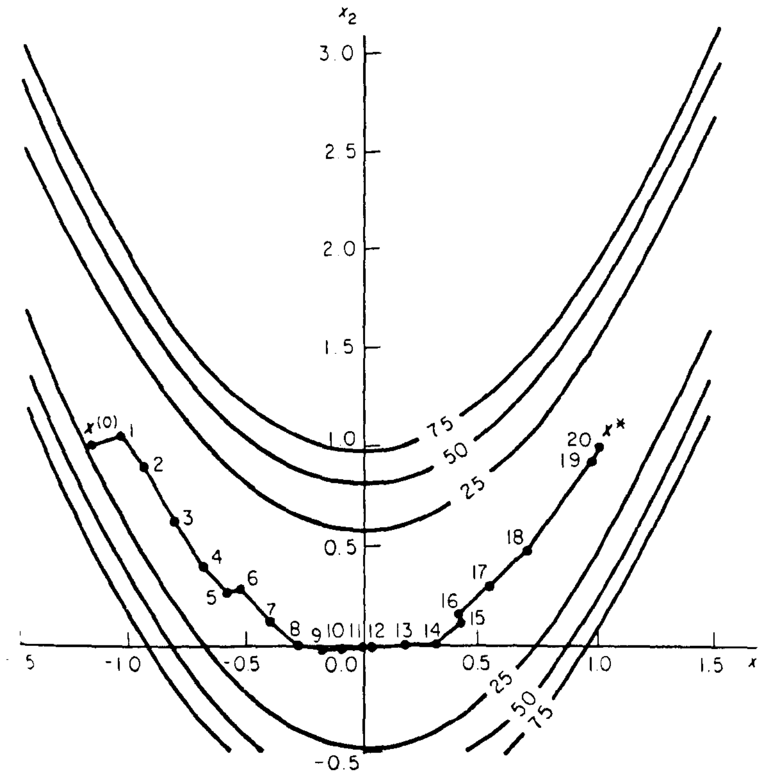


Fig. (2.10)

Trajectory of the search to minimize Rosenbrock function by the Fletcher-Reeves method (numbers indicate stages, i.e., different search directions) (Himmelblau, 1972).

Here H approaches the inverse of the Hessian matrix during the optimization process for quadratic functions. Because we are creating an approximation to H^{-1}, these methods have convergence characteristics similar to second-order methods such as Newton's Method,. Thus, variable metric methods are often called quasi-Newton methods.

Given the search direction S^k, a one-dimensional search is performed according to the following equation:

$$X^k = X^{k-1} + \alpha^*_q . S^k \qquad (2.22)$$

Here q is the iteration number, **X** is the vector of design variables, α^*_q is a scalar multiplier determining the amount of change in **X** for this iteration. At the initial design point, the **H** matrix is taken as the identity matrix H = I, so that the initial search direction is simply the direction of the steepest descent. At the end of iteration q, a new **H** matrix is defined as:

$$H^{k+1} = H^k + Dk^k \qquad (2.23)$$

where D is a symmetric update matrix

$$D^k = \frac{\sigma + \theta . \tau}{\sigma^2} pp^T + \frac{\theta - 1}{\tau} H^k y(H^k y)^T - \frac{\theta}{\sigma} \left[H^k yp^T + p(H^k y)^T \right] \qquad (2.24)$$

The change vectors **p** and **y** are defined as:

$$p = X^k - X^{k-1} \qquad (2.25a)$$

$$y = \nabla F(X^k) - \nabla F(X^{k-1}) \qquad (2.25b)$$

and the scalars σ and τ are defined as:

$$\sigma = p\, y \qquad (2.26a)$$

$$\tau = y^T H^k y \qquad (2.26b)$$

Equation (2.24) is actually part of a family of variable metric methods. Probably the most popular of the variable metric methods are the Davidson-Fletcher-Powell (DFP) and the Broydon-Fletcher-Goldfarb-Shanno (BFGS) methods. The choice of θ determines the update formula to be used:

1) $\theta = 0$, DFP method

2) $\theta = 1$, BFGS method

2.5 MULTI-CRITERIA MATHEMATICAL OPTIMIZATION

2.5.1 Introduction

Multi-criteria optimization problems arise in different engineering fields and scientists are devoting considerable attention to developing methods for solving them. The aim of these methods is to help the engineer to make the right decision in conflicting situations, i.e., in situations in which several objectives must be satisfied.

As in many fields of operational research, the theory of multi-criteria optimization is more developed and better represented in the mathematical literature than in practical use for engineering applications.

In complex engineering optimization problems, there often exist several non-commensurable criteria which must be considered. This situation is formulated as a multi-criteria optimization problem (also called multi-performance, multiple objective or vector optimization) in which the engineer goal is to optimize not a single objective function but several functions simultaneously.

Formulation of an optimization problem consists in constructing a mathematical model that describes the behaviour of a physical system encompassing the problem area. This model must closely approximate the actual behaviour of the system for the solution obtained to be adequate and useful.

The general multi-criteria optimization problem with n design variables, m inequality constraints, k equality constraints and c criteria can be written as:

$$\text{Minimize:} \quad \mathbf{F}(\mathbf{X}) = \left\{F_1(\mathbf{X}), F_2(\mathbf{X}), \ldots, F_c(\mathbf{X})\right\} \tag{2.27}$$

$$\text{S.t.:} \quad g_i(\mathbf{X}) \geq 0 \qquad i = 1,2,\ldots,m \tag{2.28}$$

$$h_j(\mathbf{X}) = 0 \qquad j = 1,2,\ldots,k \tag{2.29}$$

$$X^L{}_q \leq X_q \leq X^U{}_q \qquad q = 1,2,\ldots,n \tag{2.30}$$

In other words, we wish to determine from among the set of all values which satisfy Eqs (2.28) to (2.30) that particular set $X^*{}_1$, $X^*{}_2, \ldots, X^*{}_n$ which yields the optimum values of all the objective functions.

Usually the solution point $\mathbf{X}^*$ will not minimize all objective functions simultaneously. As **X** alters, some objectives will increase while others decrease. Consequently it is necessary to establish some criteria for determining what constitutes a *solution* to the problem stated above. Clearly, such solutions must involve compromises or trade-offs among the separate objective functions.

The concept of Pareto optimization was introduced by V. Pareto in 1896 and it still plays a most important role in multi-criteria optimization. Pareto set optimization can produce both qualitative and quantitative information for designs where there are multiple design goals. It explores the relationships between the design decisions and solution performances which are fundamental to design.

A point $\mathbf{X}^* \in \Omega$, the set of feasible solutions, is Pareto optimal if for every $\mathbf{X} \in \Omega$ either,

$$\forall i \in c, \; [F_i(\mathbf{X}) = F_i(\mathbf{X}^*)]$$

or, there is at least one $i \in I$ such that:

$$F_i(\mathbf{X}) > F_i(\mathbf{X}^*)$$

This definition is based upon the intuitive conviction that the point $\mathbf{X}^*$ is chosen as the optimal if no criterion can be improved without worsening at least one other criterion.

There are two basic problems in multi-criteria optimization, namely:

1. The generation of the Pareto set; and
2. The selection of a preferred solution.

There are many techniques for multi-criteria optimization. Many of these techniques are confined to the generation of the complete or approximate Pareto optimal solution set for a problem, and do not require an articulation of preferences among objectives by the searcher. The remaining techniques require a prior, or a sequential, articulation of preferences, with the result being a single Pareto optimal solution rather than a set of Pareto solutions.

Commonly used generative techniques include weighting approaches, the NISE methodology, and the constraint-based methods. Some of these are now examined and discussed.

2.5.2 The Weighting Method

Perhaps one of the simplest approaches to problems with multiple-criteria is to combine them into one scalar objective function as the weighted sum of the criteria. The above multi-criteria problem is replaced with the following:

$$\underset{\mathbf{X}\in\Omega}{\text{Maximize:}} \quad F(\mathbf{X}) = \sum_{i=1}^{c} W_i\, F_i(\mathbf{X}) \tag{2.31}$$

$$\text{S.t.:} \quad g_i(\mathbf{X}) \geq 0 \quad i = 1,2,\ldots,m \tag{2.32}$$

$$h_j(\mathbf{X}) = 0 \quad j = 1,2,\ldots,k \tag{2.33}$$

$$X^L_q \leq X_q \leq X^U_q \quad q = 1,2,\ldots,n \tag{2.34}$$

with $$\sum_{i=1}^{c} W_i = 1;\ W_i \geq 0$$

In order to generate the entire Pareto set, the vector W must be varied over a large number of combinations. However, since the shape and the distribution characteristics of the Pareto set are unknown, it is impossible to determine beforehand the nature of the variations required in W so as to produce a new solution at each pass. The second important disadvantage of the method is that it will not identify

Pareto solutions in a non-convex part of the set. This can be seen by a geometrical interpretation of the weighting method in the criterion space.

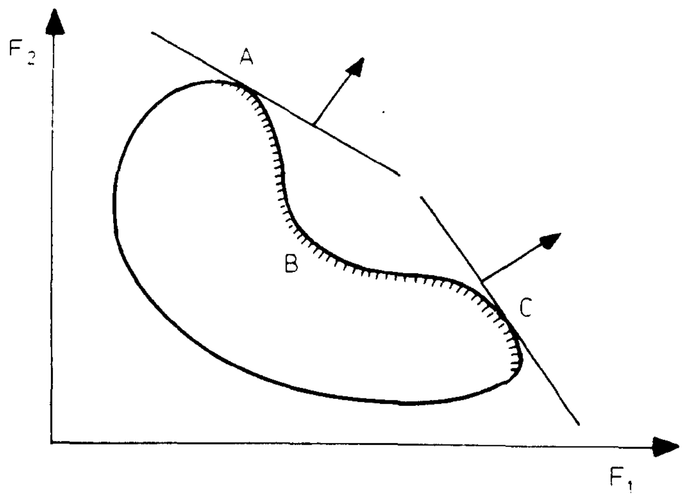


Fig. (2.11) (Balachandran, 1989)

Geometrical Interpretation of the Weighting Method in Two-Criteria Case

Figure (2.11) shows the geometrical interpretation of the weighting method in the case of two objectives. The Pareto optimal point B is missed no matter what weights are used, see Balachandran (1989).

2.5.3 The Non-Inferior Set Estimation (NISE) Method

The NISE method is a technique for systematically calculating the weighting vector W so as to guarantee the identification of a new Pareto solution, if one exists, on the convex hull of the feasible performance space. In addition, the accuracy of any approximation to the Pareto set can be controlled by the use of an error criterion. The

NISE method is thus an extension of the weighting method with a mechanism for quickly converging onto the Pareto set.

Cohon (1978) provides a detailed account of the NISE method for bicriterion problems. This technique operates by finding a number of Pareto optimal points in the criteria space and evaluating the properties of the line segments between them. The method begins by optimizing each objective individually, yielding the points A and B in Fig. (2.12). The next solution should be that feasible solution furthest out along the indicated direction. This point is found by solving the weighted problem with weights W_1 and W_2 that satisfy:

$$-W_1/W_2 = \text{slope of AB}.$$

The new solution obtained by solving the weighted problem is then located in the criteria space. If the new solution lies above AB, two new line segments are generated as shown in Fig. (2.12). The case in which the new solution lies on AB indicates that there is no other solution above the line segment AB. The above procedure is repeated with each line segment until a satisfactory approximation of the Pareto optimal set has been found, Balachandran (1989).

2.5.4 <u>The Constraint Method</u>

The constraint method is another technique which transforms a multi-criteria objective function into a single criterion one by retaining one selected criterion as the primary criterion to be optimized and treating the remaining p-1 criteria as constraints. The multi-criteria problem is thus replaced with the following:

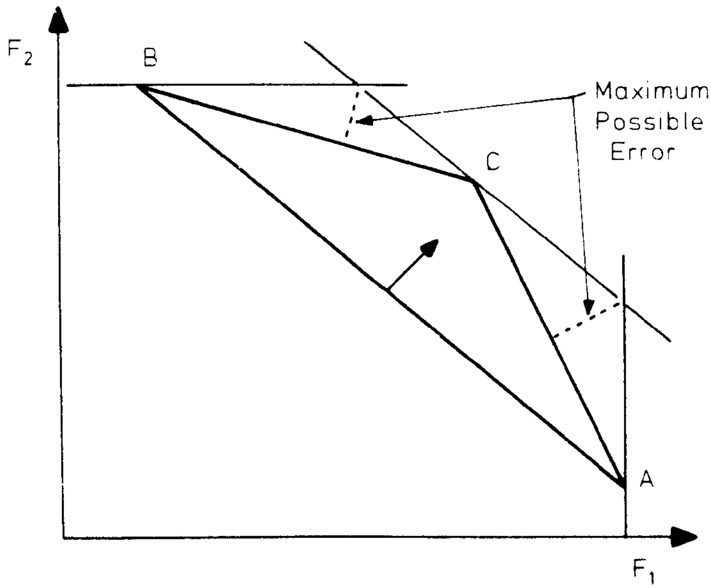


Fig. (2.12)

Concept of the NISE Method (Balachandran, 1989)

$$\underset{X\epsilon\Omega}{\text{Maximize:}} \quad F_1(\mathbf{X}) \tag{2.35}$$

S.t.:

$$F_r(\mathbf{X}) - b_r \geq 0 \qquad r = 2,3,\ldots,c \tag{2.36}$$

$$g_i(\mathbf{X}) \geq 0 \qquad i = 1,2,\ldots,m \tag{2.37}$$

$$h_j(\mathbf{X}) = 0 \qquad j = 1,2,\ldots,k \tag{2.38}$$

$$X^L_q \leq X_q \leq X^U_q \qquad q = 1,2,\ldots,n \tag{2.39}$$

where b_r are the lower bounds on the remaining c-1 objectives. The

Pareto optimal set is then generated by solving the above single objective problem with a parametric variation of b_r.

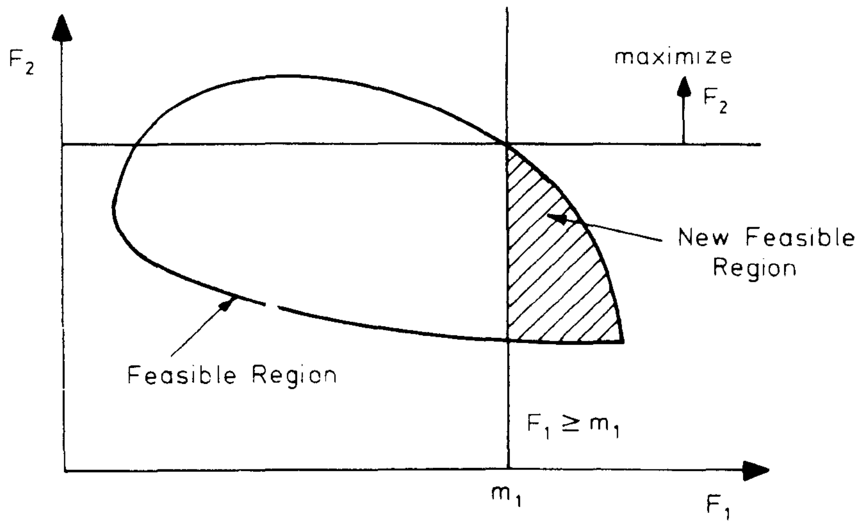


Fig. (2.13) Concept of the Constraint Method (Balachandran, 1989)

Figure (2.13) shows the concept of the constraint method. A bound is specified on F_1 and F_2 is maximized to find a Pareto optimal solution. The two objective functions were chosen for maximization.

As in the weighting method, many different combinations of values for each b_c must be examined in turn to generate the entire Pareto set. The constraint method will generate non-convex Pareto sets and can thus be used to investigate *gaps* in the Pareto set generated by the NISE method. The constraint method thus provides direct control of the generation of members of the Pareto set and generally provide an efficient method for defining the shape of the Pareto set. However, each constraint set, (b_r; r = 2,3,...,c),does not guarantee the feasibility of the resulting solution so that for problems of three or more dimensions many iterations may not be useful, Gero (1983).

CHAPTER THREE
ENTROPIC VECTOR OPTIMIZATION AND SIMULATED ENTROPY

CHAPTER THREE

ENTROPIC VECTOR OPTIMIZATION AND SIMULATED ENTROPY

SYNOPSIS

This chapter introduces the concept of entropy and the relationships between informational entropy and the much better known classical thermodynamic entropy. Entropy is used as a measure of uncertainty. The development of Shannon's informational entropy is described and its further development into entropy-based minimax methods for solving multi-criteria minimization problems is presented. The relationships among entropy, simulated annealing and free energy in optimization are investigated theoretically and are used for development of a new subject for solving global minimization problems.

In summary, two families of entropy-based methods are described in detail. The first set of methods is used for generating Pareto solutions of multi-criteria optimization problem while the second is used for seeking the global minimum of single criteria optimization problems.

3.1 ENTROPY AND MAXIMUM ENTROPY THEORY

3.1.1 Introduction

The entropy concept has played a central role in a number of areas such as *thermodynamics, statistical mechanics and information theory*. It springs from two roots. On the one hand, in classical thermodynamics, entropy is defined as a macroscopic thermodynamic variable of the system under consideration; on the other hand, in statistical mechanics and information theory, it is defined as a measure of the number of ways in which components of a system may be arranged under given circumstances.

In classical thermodynamics, this concept has significance no less fundamental than that of *energy*. According to the *second law of thermodynamics*, the entropy is an isolated system tends to a maximum so that this provides a *criterion* for the direction in which processes can take place. On the relationship between energy and entropy, Emden wrote (Fast, 1968):

> *In the huge manufactory of natural processes, the principle of entropy occupies the position of manager, for it dictates the manner and method of the whole business, whilst the principle of energy merely does the book-keeping, balancing credits and debits.*

Classical thermodynamics in which the entropy concept originated is concerned only with the *macroscopic* states of matter, i.e., with the experimentally *observable* properties. Thus, it does not enquire into the mechanisms of phenomena and is, therefore, unconcerned with what happens on a microscopic scale. The microscopic picture, however, can help to give deeper meanings to the thermodynamic laws and concepts. The branch of science concerned with this aspect is *statistical mechanics*.

When a small number of macroscopic variables such as, pressure, temperature, volume, chemical composition, etc. of a system are known, the thermodynamic state of this system is known. It is clear that a description of this kind still leaves open many possibilities as regards the detailed state on a microscopic scale. One and the same state, in a thermodynamic sense, thus comprises very many states on the microscopic scale; that is, a thermodynamic state can be realised in many ways or *micro-states*. If the number of these micro-states is denoted by m, then entropy of the system is defined as:

$$S = k.\ell n\ m \qquad (3.1)$$

where k is Boltzmann's constant. The quantity S in Eq. (3.1) may be considered as the statistical interpretation of entropy.

That entropy tends to a maximum means, according to Eq. (3.1), a tendency towards the macro-state with a maximum number of possibilities of realization, i.e. a tendency towards the most probable state. In order to be able to apply Eq. (3.1) directly, all the micro-states must have the same probability of occurring. A definition of the entropy with more general validity than Eq. (3.1) will be introduced in the next section.

Entropy of a system was first defined by Clausius as a function of some macroscopic variables that can be directly measured. The Clausius' entropy is a *non-probabilistic* concept, and is usually referred to as the *classical entropy*. Boltzmann was the first to emphasize the *probabilistic* meaning of the entropy. He noticed that the entropy of a physical system can be considered as a measure of *disorder* in the system and that in a system having many degrees of freedom, the number measuring the disorder of the system also measures the *uncertainty* about individual micro-states. However, he made no *explicit* reference to *information* and his entropy is, therefore, referred to as *statistical entropy*. It was Shannon (1948) who first introduced the entropy concept as a measure of uncertainty or *information* in an information theory context. Thus Shannon's entropy is referred to as *informational entropy* which has wider applicability than statistical entropy (Li, 1987).

3.1.2 Definitions and Properties of Informational Entropy

One of the fundamental building blocks of modern information theory is the paper by Shannon (1948) in which a new mathematical model of communication systems was proposed and investigated. The most important innovation of this model was that it considered the components of a communication system as probabilistic entities. In his paper, Shannon proposed a *quantitative* measure of the amount of uncertainty about the possible outcome of a probabilistic experiment.

Consider a probabilistic experiment having n discrete possible outcomes $a_1, \ldots, a_n$ with associated discrete probabilities $p_1, \ldots, p_n$, satisfying the following axiomatic conditions.

$$p_i \geq 0 \ (i = 1,2,\ldots,n) \quad \text{and} \quad \sum_{i=1}^{n} p_i = 1$$

Such an experiment, of course, contains an amount of *uncertainty* about the particular outcome which will occur if we perform the experiment. It can be seen that this amount of uncertainty, contained *a priori* by the probabilistic experiment, essentially depends on the *probabilities* of the possible outcomes of the experiment. For instance, if we have a probabilistic experiment having only two possible outcomes a_1, a_2 with two different sets of probability distributions ($p_1 = 0.5$, $p_2 = 0.5$) and ($p_1 = 0.96$, $p_2 = 0.04$), it is obvious that the first case contains more uncertainty than the second. In the second case, the result of the corresponding experiment is *almost surely* a_1, while in the first case we cannot make any prediction on the particular outcome which will occur. This shows that uniform probability distribution has a larger amount of uncertainty associated with it than a non-uniform distribution.

Shannon was able to postulate a measure of such uncertainty on a quantitative basis in the following way. He proposed that a measure for uncertainty should satisfy the following requirements:

1. It should be continuous in the $p_i (i = 1,2,\ldots,n)$.

2. If all the p_i are equal, it should be a monotonically increasing function of the number of outcomes n.

3. The uncertainty about two independent events A and B should be the sum of the uncertainties about A and B taken separately.

Shannon demonstrated that these criteria were sufficient to define uniquely the function:

$$S = -k \sum_{i=1}^{n} p_i . \ell n \ p_i \tag{3.2}$$

where k is merely a positive constant depending on a suitable choice for the units of measure, and it is defined that $0 \ell n 0 = 0$. The function S in Eq. (3.2) is referred to as the *informational entropy*.

3.1.3 <u>The Maximum Entropy Principle (MEP)</u>

Shannon's entropy measure was an important step forward in that it allowed the amount of uncertainty in a probabilistic experiment to be quantified provided that the probabilities of all outcomes are known. The next important advance was made by Jaynes (1957, 1968, and 1983), who realized that in many probabilistic experiments the probabilities of discrete outcomes are often not known *(unknown prior probabilities)*. Jaynes extended the use of the Shannon entropy measure to calculate the unknown prior probabilities from observable data on the probabilistic experiment, and hence extended the role of Shannon's function from a simple measure to a crucial role in an *inference* process; i.e. given a probabilistic process and observed aggregated data from that process, what does the Shannon measure of uncertainty allow us to logically infer about the p.d's underlying the process?

Suppose there exists an observable probabilistic process in which a discrete random variable can take on any one of n outcomes of value

$X_1, \ldots, X_n$. Also, suppose that as a result of observations on the process it can be deduced that the outcomes satisfy certain aggregated functional relationships $y_1(\mathbf{X})$, $y_2(\mathbf{X}), \ldots$, such as *mean values*, *variance*, etc. Let there be m such functions where $m \leq n$. What can be deduced about the probabilities $p_1, \ldots, p_n$ of the random variable attaining values $X_1, \ldots, X_n$? Clearly an infinite number of p.d's can satisfy the m observed functions $y_1, \ldots, y_m$. Which one should be chosen? Some selection criterion is then needed.

Jaynes, in a brilliant paper (1957), wrote:

> *In making inference on the basis of partial information we must use that probability distribution which has maximum entropy subject to whatever is known. This is the only unbiased assignment we can make; to use any other would amount to arbitrary assumption of information which by hypothesis we do not have.*

Jaynes, therefore, recognized that the infinite number of p.d's which satisfy the m observable behaviour functions all contain different amounts of uncertainty. The one with the highest entropy value should be chosen as this introduces minimum *artificial bias* into the choice.

Jaynes referred to the above statements as the *principle of maximum entropy*, implying that all else is consequent upon its fundamental nature. Mathematically, to maximize the entropy S of Eq. (3.2) subject to the given information, leads to a mathematical optimization problem:

(MEP) Maximize: $$S = -k \sum_{i=1}^{n} p_i \cdot \ell n\ p_i \qquad (3.2)$$

S.t.:

$$\sum_{i=1}^{n} p_i = 1 \tag{3.3}$$

$$\sum_{i=1}^{n} p_i . y_j(X_i) = E\ [y_j];\ j = 1, \ldots, m \tag{3.4}$$

where E[.] is the expectation operator and it is axiomatic that $p_i \geq 0$. The constant k is arbitrarily set to unity below, which has no effect on the solution but simplifies the algebra.

3.1.4 Some Applications of the Maximum Entropy Principle

Shannon's entropy measure and Jaynes' maximum entropy criterion have found applications in a variety of areas of science and engineering such as *thermodynamics*, *statistical mechanics*, *civil engineering*, *queuing theory*, *transportation planning*, etc.

Recently, civil engineering has also become an active area in which there have appeared many applications of informational entropy, Siddall and Diab (1974), and Basu and Templeman (1984) have used the **(MEP)** in structural reliability analysis and probabilistic design. In traditional approaches to reliability analysis an analytical p.d. must be chosen by the analyst/designer to represent random loads or strengths. Any bias or lack of fit between the chosen distribution and the available data will be magnified in the extrapolation of the distribution to the tail regions which are important in reliability analysis, as the probability of failure is heavily related to the overlapping tails of load and strength distributions. The Maximum Entropy Distribution **(MED)**, as the least biased p.d., is a more appropriate substitute for the analytical distributions. Basu and Templeman (1984) argue that by fitting a **MED** to available data itself by means of the **MEP**, a more logical and rigorous approach to structural reliability analysis results.

Munro and Jowitt (1978) used the **MEP** in decision analysis in the ready-mixed concrete industry. The principal problem is that of making optimal decisions in the face of the uncertainty associated with the unknown state of future orders. They argued that the evaluation of prior probabilities for the order states should be made objectively and should not be affected by any personal bias. Thus in their approach the **MEP** was used to produce the least biased p.d. associated with orders for each mix.

Guiasu (1977) has also used the **MEP** in queuing theory. For some single-server queuing systems, when the expected number of customers is given, the **MEP** gives the same p.d. of the possible states of the systems as the birth-and-death process applied to an M/M/1 system in a steady-state condition. For other queueing systems, such as M/G/1 for instance, the **MEP** gives a simple p.d. of possible states, while no closed-form expression for such a p.d. is known in the general framework of a birth-and-death process. In this work, Guiasu argues that use of the **MEP** strengthens belief that fundamental assumptions in the birth-and-death process which is axiomatic to such queuing systems are correct and justifiable. Furthermore, it strengthens some of the initial axioms and improves insight into the problems

Many researchers have used Shannon's entropy in the transportation problem, which is generally formulated as an LP problem [Wilson, (1974), Dinkel, (1977 and 1979) and Erlander, (1981)]. A typical transportation problem is to predict values of a set of variables X_{ij} which represent volumetric traffic flow between zones i and j (trip distribution). Suppose we are given the totals of flows leaving each zone i, O_i, and entering each zone j, D_j, and the cost of travel between each zone, c_{ij}, then an LP model to minimize the total travelling cost takes the form:

(ST) Minimize:
$$C = \sum_i \sum_j c_{ij} X_{ij} \tag{3.5}$$

S.t.:
$$\sum_j X_{ij} = O_i \tag{3.6}$$

$$\sum_i X_{ij} = D_j \tag{3.7}$$

and
$$X_{ij} \geq 0 \tag{3.8}$$

which is referred to below as the Standard Transportation (ST) problem. The solution of this linear programming problem (ST) will have several non-zero variables, but also several variables with values of zero. This means that some possible links between zones will have been eliminated. Whilst this corresponds to a least cost solution it is also a solution which has low flexibility. Erlander (1977 and 1981), among others, adds an entropy constraint:

$$- \sum_i \sum_j X_{ij} \cdot \ell n\ (X_{ij}) \geq H \tag{3.9}$$

to the problem **ST** in order to prevent variables becoming zero and to ensure a more uniform distribution of variable values. This modified problem with an entropy constraint is referred to as the Entropy Constrained Transportation (**ECT**) problem.

The entropy constraint may be looked upon as a measure of the spread of the distribution of journeys over the cells of the trip matrix. The higher value of the entropy H the more even the distribution. Thus, it seems natural to investigate the possibilities of using the entropy as a broad measure of accessibility.

The procedure for solving problems **ST** and **ECT** are not repeated here but can be found in the aforementioned papers. It is interesting that by adding the entropy constraint Eq. (3.9), an analytical solution to problem **ECT**, expressed in terms of the Lagrange multipliers associated with the constraints Eqs. (3.6-3.9), is obtained. It turns out that the analytical solution is of the well-known gravity model type. Evans (1973) has formally proved that the solution to problem **ST** is the limit of the gravity model solution as the Lagrange multiplier associated with the entropy constraint Eq. (3.9) approaches infinity.

Erlander (1981) has stressed some advantages of this modified model over problem **ST**. First of all, the solution of problem **ST** is replaced by the *smoother* solution of problem **ECT**. This character may be advantageous in some planning problem where the *uncertainty* of the real world has been replaced in the models by *deterministic* relationships. The entropy constraint may, in such cases, be viewed as a substitute for lost complexity. Also, the solution obtained has high *accessibility* since all variables in the solution of problem **ST** that may be zero, become strictly positive in problem **ECT**. Thus adding the entropy constraint has made the mathematical modelling more sensible.

Erlander (1981) has extended the entropy constraint approach to solve the following **LP** problem.

(LP) Minimize: $C^T X$ (3.10)

$$\text{S.t.:} \quad [A]\,\mathbf{X} = \mathbf{b} \tag{3.11}$$

$$\sum_{i=1}^{n} X_i = 1 \quad \text{and} \quad \mathbf{X} \geq 0 \tag{3.12}$$

where **C** is a given n-Vector, **X** is an n-Vector of variables, [A] is an m × n coefficient matrix and has full rank (= m), and **b** is an m-Vector of resources. By adding an entropy constraint:

$$\sum_{i=1}^{n} X_i \,.\, \ell n\, X_i \geq H \tag{3.13}$$

to problem **LP**, he examines the solution to this modified problem. It is obvious that the solution depends upon values of H. When H is less than a certain value H_{min}, the entropy constraint is slack and therefore, has no effect on the solution. For $H > H_{min}$, the entropy constraint becomes active and the objective function increases as H increases until some maximum value H_{max} is reached. A further increase of H makes the problem infeasible. Hence, the values of H which are of interest lie in a certain interval $[H_{min}, H_{max}]$, where the entropy constraint is active and there are feasible solutions.

Ben-Tal (1985) has used entropy in **NLP** problems with stochastic constraints. The stochastic program is replaced by a deterministic program by penalizing solutions which are not feasible in the mean. The penalty term is given in terms of a relative entropy functional and is accordingly called an *entropic penalty*.

Templeman and Li (1987) have shown that there are links between entropy and optimization by investigating several ways of using them and studying their relations to some well-established results of optimization theory.

The present chapter aims to explore the use of Shannon entropy and Jaynes' maximum entropy criterion in a multi-criteria optimization context and single-criteria optimization context.

3.2 ENTROPY-BASED MINIMAX METHODS AND VECTOR OPTIMIZATION

3.2.1 Definitions

Research into optimum engineering design now has a thirty-year history and continues to flourish. Over the past twelve years or so it has been recognized that real-world design is rarely adequately represented by a single criterion scalar optimization problem. Most design optimization problems require a multi-criteria formulation in order to represent the many conflicting goals which the designer is attempting to optimize simultaneously in the design process. Consequently, over the last decade considerable attention has been given to multi-criteria optimization methods.

A single criterion (or scalar) optimization problem may be stated as:

$$\underset{\mathbf{X} \in \Omega}{\text{Minimize:}} \qquad F(\mathbf{X}) \tag{3.14}$$

in which F is a single objective function of independent variables X_i, $i = 1,2,\ldots,n$, forming a variable vector $\mathbf{X}$. Ω represents a constrained feasbible region of $\mathbf{X}$ so problem (3.14) represents a standard constrained scalar optimization problem. A multi-criteria (or vector) optimization problem may be stated as:

$$\underset{\mathbf{X} \in \Omega}{\text{Minimize:}} \qquad \mathbf{F} = \{ F_1(\mathbf{X}),\ F_2(\mathbf{X}), \ldots,\ F_c(\mathbf{X}) \} \tag{3.15}$$

in which $\mathbf{F}$ is now a vector of p different and independent objective functions, each a function of variable vector **X** and for each of which a reduction in value represents improvement. The constrained feasible region of **X** is again represented by Ω.

The single greatest difference between scalar and vector optimization problems lies in the fact that scalar problem (3.14) usually has a unique solution, $\mathbf{X}^*$, at least locally, wheras vector problem (3.15) usually has very many solutions; i.e. there is no unique vector $\mathbf{X}^*$ which simultaneously minimizes all the functions F_1, $F_2, \ldots$, F_c which compr ise **F**. It is, therefore, necessary to define very carefully what is meant by an *optimum* solution of vector problem (3.15). One such definition is that of a Pareto solution.

A Pareto solution of problem (3.15) consists of any vector **X** such that when **X** is locally perturbed by a small amount $\Delta\mathbf{X}$ none of the individual objectives F_j, $j = 1,2,\ldots,c$ in **F** decreases in value without at least one of the other objectives F_j increasing in value. Clearly, from this definition, if each of the individual objectives F_j in **F** is minimized independently of the others as a scalar optimization problem (3.14) then the resulting optimum variable vector $\mathbf{X}^*$ will be a Pareto solution of problem (3.15). Also, the scalar minimization of any linear combination of any of the F_j in **F** will also yield a Pareto solution. Figure (3.1) explains the idea of Pareto solutions graphically for the case in which problem (3.15) has two objective functions F_1 and F_2 which form the axes of the graph. Any feasible vector of variables **X** will yield particular values of the two objectives and will correspond to a unique point on the graph. The shaded region represents all possible feasible points. The curved boundary AB represents all Pareto solutions of the problem.

Mathematically, Pareto optimality is identical to noninferiority. Kuhn and Tucker (1951) stated conditions for noninferiority in vector optimization problems. If a solution $\mathbf{X}^*$ is noninferior, then there exist multipliers $u_i \geq 0$, $i = 1,2,\ldots, c$ and:

$$\mathbf{X}^* \in \Omega \tag{3.16}$$

$$\sum_{i=1}^{c} u_i . \nabla F_i(\mathbf{X}^*) = 0 \tag{3.17}$$

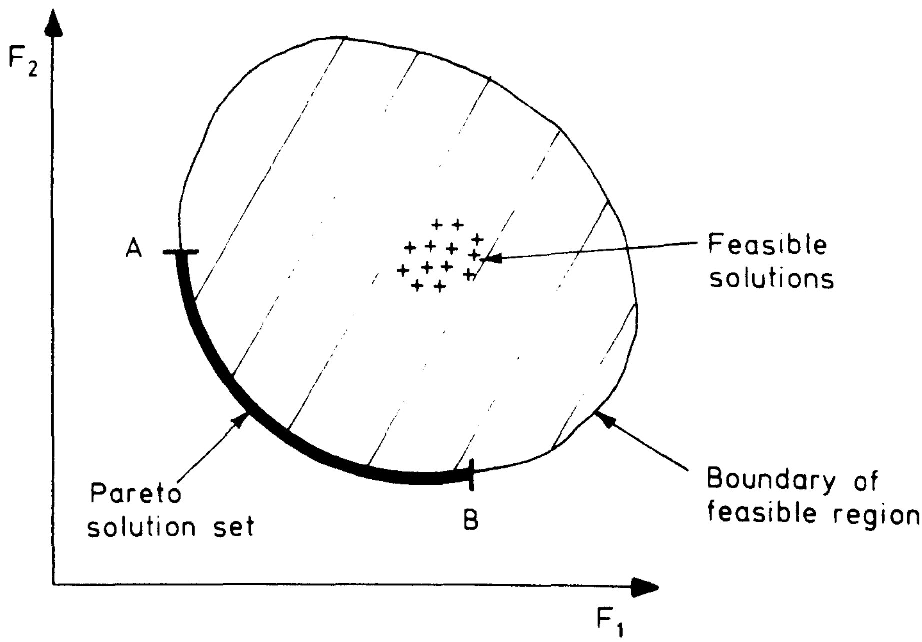


Fig. (3.1)

Pareto Solutions

The first condition, (3.16), requires feasibility. The conditions in (3.16-3.17) are necessary for noninferiority. They are also sufficient if the $F_i(\mathbf{X})$ are concave for $i = 1,2,\ldots,c$, Ω is convex, and $u_i > 0$ for all i (Cohon, 1978).

Engineering design may often be formulated as a vector optimization problem (3.15) in instances where the designer wishes to find some design in which several conflicting criteria must all be made as small as possible in value. In general, minimum cost and minimum failure probability are examples of conflicting criteria for structures; minimum cost and minimum surface roughness are similarly in conflict in machining operations. The set of Pareto solutions for such problems represents many possible solutions each of which is *best* for a particular combination of weights or preferences which may be assigned to the individual objectives. Problem (3.15) must, therefore, be solved to generate the complete set of Pareto solutions. This is a difficult task and one which has been the subject of much research over the last decade. Having generated the Pareto set the designer may then select from among them that particular solution which best represents his own desired balance of preferences.

An alternative strategy to that of the last paragraph is for the designer first to articulate his own set of preferences of weights for each of the individual objectives in **F**. A sum of all the objectives, each weighted according to the desired preferences, may then be formed as a single objective function and the design problem solved as a scalar optimization problem (3.14). This will result in a unique solution which corresponds to one particular point in the Pareto set. This approach is much simpler to solve than the first, but it requires the prior articulation of preferences which may not be known in the absence of any information about alternative good solutions such as is provided by the Pareto set.

This section is concerned with the first approach above and presents new methods for the generation of Pareto solution sets. Several methods already exist for generating Pareto solutions and a good survey is provided in chapter 2. The methods described here are particularly useful in instances where c is large, i.e. the vector objective function has very many different conflicting objectives.

3.2.2 Preliminaries

In order to develop the Pareto set generation methods it is first necessary to introduce several established ideas, without further proof. The first of these is a definition of *minimax optimization*. Minimax optimization is closely related to vector optimization as will be demonstrated later and can be stated as follows:

$$\underset{\mathbf{X} \in \Omega}{\text{Minimize}} \quad \underset{i=1,..,c}{\text{Maximum:}} \quad < F_i(\mathbf{X}) > \tag{3.18}$$

Minimax optimization problem (3.18) requires that of the c individual objectives in vector optimization problem (3.15) the one with the currently largest value should be minimized over variables **X**. In any solution algorithm for problem (3.18) it is apparent that as **X** changes, the values of all the individual objectives will change. Consequently, the one with the largest current value will also change during the course of the solution process and this presents difficulties to any solution algorithm. A good example of a minimax optimization problem is that of the shape optimization of an engineering component which has been analyzed by the finite element method. This will have calculated values of some stress resultant at c points in the component. The designer may then wish to alter the values of some design variables **X** of the component (such as thickness and boundary shape parameters in such a way that the value of the maximum stress occurring at any of the c designated points in the component is made as small as possible. Problem (3.18) represents this design problem precisely.

The second idea is the Shannon entropy function Eq. (3.2). It is a measure of the uncertainty in a discrete random process in which p_i is the probability associated with discrete event i. Jaynes used the Shannon entropy function in his Maximum Entropy Principle which has been examined in section (3.1.3). Eqs. (3.2-3.4) represent the mathematical form of Jaynes' Maximum Entropy Principle. The maximum entropy problem has an explicit solution:

$$p_j = \exp\,[\beta . y_j(\mathbf{X})\,/k] / \left\{ \sum_{i=1}^{c} \exp\,[\beta . y_i(\mathbf{X})\,/k] \right\} ; \; j \in c \qquad (3.19)$$

in which β is the Lagrange multiplier associated with the expected value constraint.

3.2.3 The Entropy-Based Weighted Method And Its Aggregated Form

This section examines the developments of the above mentioned method for the solution of vector optimization problems. It is specifically concerned with problems of minimax optimization and with the close relationships which exist between minimax optimization and Pareto solution set generation for problems of general vector optimization.

The two vector optimization problems investigated here are first defined. The most general problem of multicriteria optimization may be stated as:

$$\min_{\mathbf{X} \in \Omega} \quad F_i(\mathbf{X}) \qquad i = 1,2,\ldots,c \qquad (3.20)$$

in which **X** is a vector of variables X_q; $q = 1,2,\ldots,n$ and Ω represents some feasible region within which Ω must lie. **F(X)** is a vector of goals or objectives $F_i(\mathbf{X})$; $i = 1,2,\ldots,c$ which are assumed to be formulated such that a reduction in value of each objective function is desirable. It is also assumed that all objective functions **F** are expressed in dimensionless form. Problem (3.20) is the standard multi objective optimization problem which has many Pareto solutions. $\mathbf{X}^*$ is defined as a Pareto solution of problem (3.20) if, within a local perturbation region of $\mathbf{X}^*$ no other point $\mathbf{X} \in \Omega$ can be found such that:

$$F_i(\mathbf{X}) \leq F_i(\mathbf{X}^*) \qquad \forall i = 1,2,\ldots,c$$

A Pareto solution of problem (3.20) may be found by solving the scalar optimization problem:

$$\min_{\mathbf{X}\in\Omega} \sum_{i=1}^{c} W_i \, F_i(\mathbf{X}) \tag{2.31}$$

in which **W** is a vector of specified multipliers W_i ; $i = 1,2,\ldots,c$ satisfying normality and non-negativity conditions. Problem (3.20) generally has many Pareto solutions, each corresponding to a particular choice of values of **W**.

The second problem which is closely related to problem (3.20) is the minimax optimization problem:

$$\min_{\mathbf{X}\in\Omega} \; \max_{i\in c} \; \langle F_i(\mathbf{X}) \rangle \tag{3.21}$$

The solution of the minimax problem (3.21) consists of one particular Pareto solution of problem (3.20) and is that solution in which the value of the largest objective or objectives among **F** is made as small as possible.

The first three theorems to be proved in this section are listed below.

<u>Theorem 1</u>:

The vector X^* which solves the vector minimax problem (3.21), where **F** is a vector of dimensionless objective functions, is generated by solving the scalar optimization problem, aggregated objective functions problem: (**AOF**)

$$\min_{\mathbf{X}\in\Omega} \quad \frac{1}{p} \cdot \ell n \sum_{i=1}^{c} \exp\,[p.F_i(\mathbf{X})] \qquad (3.22)$$

where p is a positive parameter of increasing value towards infinity.

<u>Proof</u>:

This requires the use of Jensen's inequality (p-th norm inequality) (Hardy, 1934), which states that for any set of positive numbers U_i, $i=1,2,\ldots,c$ and $p \geq q \geq 1$,

$$\left\{ \sum_{i=1}^{c} U^p{}_i \right\}^{1/p} \leq \left\{ \sum_{i=1}^{c} U^q{}_i \right\}^{1/q} \qquad (3.23)$$

Inequality (3.23) shows that the p-th norm of the set U decreases monotonically as its order, p, increases. An important property of the p-th norm is its limit as p tends towards infinity:

$$\lim_{p\to\infty} \left\{ \sum_{i=1}^{c} U^p{}_i \right\}^{1/p} = \max_{i\epsilon c} \langle U_i \rangle \qquad (3.24)$$

Let $\qquad U_i = \exp\,[F_i(\mathbf{X})\,]\,; \qquad i=1\ldots,c \qquad (3.25)$

Since the objectives **F** are dimensionless, the **U** defined by equation (3.25) are all positive numbers and their substitution into result (3.24), gives:

$$\lim_{p\to\infty} \left(\sum_{i=1}^{c} \exp\ [p.F_i(\mathbf{X})] \right)^{1/p} = \underset{i\epsilon c}{\text{Max}} \quad <\exp\ [F_i(\mathbf{X})]> \tag{3.26}$$

Taking natural logarithms of both sides and noting that:

$$\ell n \lim\ (\bullet) \equiv \lim\ \ell n\ (\bullet)$$

$$\ell n\ \text{Max}\ (\bullet) \equiv \text{Max}\ \ell n\ (\bullet)$$

equation (3.26) becomes:

$$\lim_{p\to\infty} (1/p).\ell n \sum_{i=1}^{c} \exp\ [p.F_i(\mathbf{X})] = \underset{i\epsilon c}{\text{Max}} <F_i(\mathbf{X})> \tag{3.27}$$

Equation (3.27) holds for any set of objectives **F(X)** including that set which results from minimizing both sides of equation (3.27) over $\mathbf{X} \epsilon \Omega$. Thus, equation (3.27) may be extended to:

$$\underset{\mathbf{X}\epsilon\Omega}{\text{Min}} \quad \underset{i\epsilon c}{\text{Max}} \quad <F_i(\mathbf{X})> = \underset{\mathbf{X}\epsilon\Omega}{\text{Min}} \quad (1/p)\ .\ \ell n \sum_{i=1}^{c} \exp\ \left[p.F_i(\mathbf{X})\right] \tag{3.28}$$

as p in the range $1 \leq p \leq \infty$ increases towards ∞. This completes the proof. Theorem 1 does not directly involve entropy in any way but has proved the first main result of this section. The next theorem shows how the scalar function (3.22), and hence, by virtue of theorem 1, the minimax optimization problem (3.21), is related to the general vector optimization problem (3.20). It is at this point that entropy is seen to be the link between the two problems (3.20) and (3.21).

<u>**Theorem 2**</u>:

For any value of P, a parameter with any non-zero positive or negative value:

$$(1/P)\ .\ \ell n \sum_{i=1}^{c} \exp\ \left[P.F_i(\mathbf{X})\right] \geq \sum_{i=1}^{c} \lambda_i\ F_i(\mathbf{X}) - \frac{1}{P} \sum_{i=1}^{c} \lambda_i\ .\ \ell n\ \lambda_i \tag{3.29}$$

with equality when the RHS is maximized over variables $\boldsymbol{\lambda}$ such maximizing values of $\boldsymbol{\lambda}$ are:

$$\lambda_i = \exp\left[P.F_i(\mathbf{X})\right] / \sum_{i=1}^{c} \exp\left[P.F_i(\mathbf{X})\right] \; ; \; i = 1,2,\ldots,c \qquad (3.30)$$

<u>Proof</u>:

Let $$U_i = \exp\left[P.F_i(\mathbf{X})\right] \; ; \; i = 1,2,\ldots,c \qquad (3.31)$$

with dimensionless objectives $\mathbf{F}$, as before. Thus the $\mathbf{U}$ defined by equation (3.31) will be positive numbers for any value of P. First, Cauchy's inequality (the arithmetic-geometric mean inequality) (Hardy, 1934) states that for U_i and λ_i ; $i=1,2,\ldots,c$ satisfying,

$$\sum_{i=1}^{c} \lambda_i = 1 \text{ and } \lambda_i \geq 0$$

then:

$$\sum_{i=1}^{c} U_i \geq \prod_{i=1}^{c} (U_i/\lambda_i)^{\lambda_i} \qquad (3.32)$$

Taking natural logorithms of inequality (3.32) gives:

$$\ell n \left\{ \sum_{i=1}^{c} U_i \right\} \geq \sum_{i=1}^{c} \lambda_i \, \ell n \, U_i - \sum_{i=1}^{c} \lambda_i \, \ell n \, \lambda_i \qquad (3.33)$$

substituting (3.31) into (3.33) yields:

$$\frac{1}{P} \ell n \sum_{i=1}^{c} \exp\left[P.F_i(\mathbf{X})\right] \geq \sum_{i=1}^{c} \lambda_i F_i(\mathbf{X}) - \frac{1}{P} \sum_{i=1}^{c} \lambda_i \, \ell n \, \lambda_i \qquad (3.29)$$

Inequality (3.33) becomes an equality for any value of P when the RHS is maximized over variables $\boldsymbol{\lambda}$ subject to non-negativity and normality of the weights. The Lagrangean of (3.33) is:

$$L(\boldsymbol{\lambda}, \alpha) = \sum_{i=1}^{c} \lambda_i \, \ell n \, U_i - \sum_{i=1}^{c} \lambda_i \, \ell n \, \lambda_i + \alpha \left(\sum_{i=1}^{c} \lambda_i - 1 \right) \tag{3.34}$$

There is no need to include the non-negativity conditions explicitly as the middle term of equation (3.34) imposes this indirectly. Stationarity of equation (3.34) with respect to λ_i ; $i = 1,2,\ldots,c$ and α gives:

$$\lambda_i = U_i \Big/ \sum_{i=1}^{c} U_i \; ; \qquad i = 1,2,\ldots,c \tag{3.35}$$

Result (3.30) can be shown to correspond to a maximizing point of the RHS of inequality (3.33) by examining the second derivative matrix of the Lagrangean (3.34) which is negative definite.

Substituting result (3.35) into the RHS of inequality (3.33) gives, after some algebraic simplification:

$$\underset{\boldsymbol{\lambda}}{\text{Max}} \left[\sum_{i=1}^{c} \lambda_i \, . \, \ell n \, U_i - \sum_{i=1}^{c} \lambda_i \, \ell n . \lambda_i \right] = \ell n \left(\sum_{i=1}^{c} U_i \right)$$

Inequality (3.29), also, becomes an equality for any value of P when the RHS is maximized over variables $\boldsymbol{\lambda}$, i.e. when the variables $\boldsymbol{\lambda}$ take values given by equation (3.35) on substitution of equation (3.31):

$$\lambda_i = \exp\left[P.F_i(\mathbf{X})\right] \Big/ \sum_{i=1}^{c} \exp\left[P.F_i(\mathbf{X})\right] \; ; \; i = 1,2,\ldots,c \tag{3.30}$$

and Theorem 2 is proved.

The RHS of inequality (3.29) consists of the vector optimization problem (3.20) in its scalar weighting form (2.31) but with an additional entropy term which is a function only of the mulipliers (weights) λ and the parameter P. In its equality form with multipliers given by equation (3.30), relationship (3.29) shows that the entropy of the multipliers measures the difference between the scalar function (3.22) and the weighting objective function in problem (2.31).

It is clear that if both sides of relationship (3.29) in its equality form with multipliers given by equation (3.30) are minimized over variables $X \in \Omega$ and P becoming increasingly large and positive, the entropy term in (3.29) will tend towards zero. The LHS will generate a Pareto solution of the general vector optimization problem. This is expected, but it shows that the values of the multipliers **W** in problem (2.31) which correspond to a minimax solution rather than just to any Pareto solution must be given by equation (3.30). Thus far we have shown that the scalar function (3.22) can be used to generate a vector minimax solution of problem (3.21) and that this solution is related through entropy to all Pareto solutions of the general vector optimization problem (3.20). The second main result of this section still remains to be proved and Theorem 3 formally states this.

<u>Theorem 3</u>:

The vector X^* which solves the scalar optimization problem (3.29) either in its aggregated form, the LHS, or its entropy-based weighted form, the RHS, for any non-zero value of P is a Pareto solution of the general vector optimization problem (3.20).

<u>Proof</u>:

From Theorem 2, relationship (3.29) is an equality for any P when λ is given by equation (3.30). This result holds for any set of objectives **F** including those evaluated at $\mathbf{X}^*$ which minimizes the RHS of equality (3.29).

Thus:

$$\frac{1}{P}\ \ell n \sum_{i=1}^{c} \exp\left[P.F_i(\mathbf{X}^*)\right] = \sum_{i=1}^{c} \lambda^*_i\ F_i(\mathbf{X}^*) - \frac{1}{P} \sum_{i=1}^{c} \lambda^*_i\ \ell n\ \lambda^*_i \qquad (3.36)$$

when λ^* is given by:

$$\lambda^*_i = \exp\left[P.F_i(\mathbf{X}^*)\right] / \sum_{i=1}^{c} \exp\left[P.F_i(\mathbf{X}^*)\right] \ ; \ i = 1,2,\ldots,c \qquad (3.30)$$

$\mathbf{X}^*$ will be a Pareto solution of problem (3.20) if it is a solution of problem (2.31) with **W** defined by (3.30), i.e. $\mathbf{X}^*$ must satisfy the necessary stationarity condition for problem (2.31) which is of the form (Kuhn - Tucker conditions for non-inferiority):

$$\sum_{i=1}^{c} u_i \left[\partial F_i(\mathbf{X}^*)/\partial X_j\right] = 0 \ ; \qquad \forall j \in n \qquad (3.37)$$

Name the LHS function and the RHS function in the equation (3.36) V_1 and V_2 respectively. The theorem will be proved if it can be shown that $(\partial V_1/\partial X^*_j)$ and $(\partial V_2/\partial X^*_j)$ are equal to (3.37).

Examining the first derivative of V_1 yields:

$$(\partial V_1/\partial X^*_j) = (1/P)\ (\partial/\partial X_j)\ \left[\ell n \sum_{i=1}^{c} \exp\ (P.F_i(\mathbf{X}^*))\right]$$

$$= \sum_{i=1}^{c} \lambda_i\ \left[\partial F_i(\mathbf{X}^*)/\partial X_j\right] = 0 \qquad (3.38)$$

Examining the first derivative of V_2 which yields the same result in equation (3.37) is summarized in Appendix (A).

<u>Discussion</u>:

An important feature of Theorems 2 and 3 is that they impose no restrictions upon permissable values of P. Pareto solutions of the general vector optimization problem (3.20) may be generated by solving the scalar optimization problem (3.36) either in its aggregated form, the LHS, or its entropy-based weighted form, the RHS, for a range of different positive or negative values of P. This means that the scalar optimization problem, either side of (3.36), forms a very useful and efficient means of generating Pareto solutions. However, it is clear from Theorem 3 that values of P close to zero should not be used; both sides of (3.36) tend towards infinity as P tends towards zero.

In Theorem 1 a restriction is imposed upon P by the use of Jensen's inequality which is valid only in the range of $1 \geq P \geq \infty$. However, this theorem is used only to prove the vector minimax property of the solutions of either side of the problem (3.36) as P tends towards infinity. Consequently the results of this section may be summarised in the following statement:

> Pareto solution of the vector optimization problem (3.20) may be generated by solving the scalar optimization problem (3.36), on either side, for any values of P in the range $-\infty \leq P \leq \infty$ except those close to zero. For increasingly Positive values of P, the Pareto solutions generated will approach the solution of the vector minimax optimization problem (3.21).

It is clear that P plays an important role in generating individual solutions within the Pareto set. Any chosen value of P will generate a Pareto solution. Varying P will generate different Pareto solutions.

The Pareto solution generation properties of the scalar optimization problem (3.36), on either side, for any P have only recently become apparent. One big advantage that the method appears to have is that Pareto solution sets may be generated by specifying values for only one parameter, P. The currently popular weighting method (2.31) also involves scalar optimization but requires the specification of values for each of the objective weights W_i; $i = 1,2,\ldots,c$. Investigating the many different combinations of these weight values can be time-consuming, particularly for problems with many objectives.

However, the scalar optimization problem (3.36) can be interpreted geometrically. Consider the two criteria optimization problem presented in Fig. (3.2). In the space of objectives we can draw a line L with slope $(-\ \lambda_1/\lambda_2)$ or $[-\ \exp\ (P(F_1 - F_2))]$. The set L which represents this line is such that:

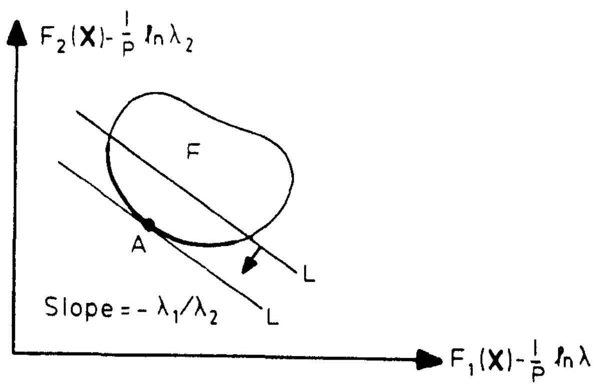


Fig. (3.2)

Geometrical Interpretation of the EWOF Method

Based on (Osyczka, 1984)

$$\frac{1}{P} \ln \sum_{i=1}^{2} \exp\left[P.F_i(X)\right] = \lambda_1 \left[F_1 - \frac{1}{P} \ln \lambda_1 \right] + \lambda_2 \left[F_2 - \frac{1}{P} \ln \lambda_2\right] = C$$

where C is a constant. The minimization of (3.36), using either side, can be interpreted as moving the line L with variable (λ_1 and λ_2) or [exp (PF_1) and exp (PF_2)] in a positive direction as close as possible to the origin, but keeping the intersection of the sets L and F. The point A for which L is tangent to F will be the minimum of (3.36). Note that for a non-convex problem, a great part of the set of non-inferior solutions may not be available, i.e., no values of λ_i or exp (PF_i) can locate the points in a certain region of the set F^{Pareto}. Consider the problem presented in Fig. (3.3). The line L_1 which is tangent at A with slope ($-\lambda_1/\lambda_2$) or $\left[-\exp\left(P(F_1-F_2)\right)\right]$ can be moved further in a positive direction until it is tangent at point B. Thus, the scalar optimization problem (3.37) with the values of $\lambda_i(\mathbf{X})$ or $\exp[PF_i(\mathbf{X})]$ will find point B but not A. Other values of $\lambda_i(\mathbf{X})$ or exp $[PF_i(\mathbf{X})]$ will find point C. It is easy to see for this problem that the set of non-inferior solutions between D and E is not available.

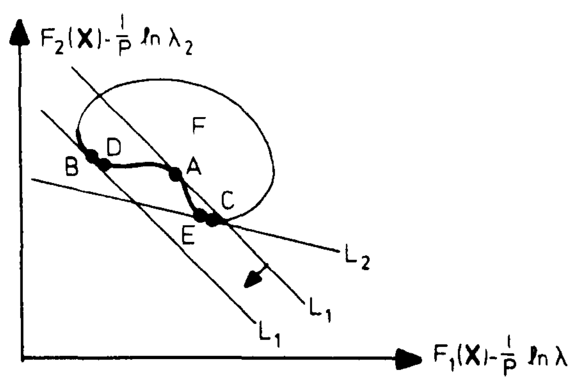


Fig. (3.3)

The EWOF Method for a Non-Convex Problem

Based on (Osyczka, 1984)

In conclusion, the scalar optimization problem (3.36), which its LHS represents the aggregated objectives form and its RHS represents the entropy-based weighted objectives form, alone are inadequate since non-convex Pareto solutions can't be generated. Other entropy-based techniques which can generate non-convex Pareto solutions are needed so the two sets of methods can be used simultaneously to investigate gaps in the Pareto solution obtained by any of the sets alone and the next section deals with this.

3.2.4 The Entropy-Based Constrained Method And Its Compartmentalized Form

This section examines the development of the above mentioned method for the solution of vector optimization problems. Optimizing one objective while all of the others are constrained to some value is perhaps the most intuitively appealing generating technique. Marglin (1967) appears to be the first to have suggested such an approach to vector optimization problems. The method follows directly, like the weighting method (2.31), from the necessary conditions of non-inferiority, (3.16-3.17). If **X** satisifies these conditions, then it is also an optimal solution to:

$$\min_{\mathbf{X}\in\Omega} \quad \mathbf{F(X,L)} = \left\{ \begin{array}{l} F_h(\mathbf{X}) \\ \text{S.t.: } F_i(\mathbf{X}) \le L_i \;;\quad \forall i \ne h \end{array} \right\} \tag{3.39}$$

since

$$\nabla \mathbf{F(X,L)} = u_h \;.\; \nabla F_h(\mathbf{X}) + \sum_{\substack{i=1 \\ i\ne h}}^{c} u_i . \nabla F_i(\mathbf{X}) \tag{3.40}$$

where the hth objective was arbitrarily chosen for minimization and L_i is preassigned upper bound on objective i. The RHS's in (3.39) seem to have appeared magically which, of course, they did. They are not at issue in the Kuhn - Tucker condition (3.40), for which any RHS would do. They are, of course, important for feasibility.

<u>Theorem 4</u>:

The vector X^{*}_{j} which solves the vector minimax problem (3.21) is generated by solving the compartmentalized objective function problem (COF)

$$\left.\begin{array}{lll} \underset{X\epsilon\Omega}{\text{Min}} & \frac{1}{p}\ \lambda_h(X).\ell n \sum_{i=1}^{c} \exp\ [p.F_i(X)] & \\ \text{S.t.} & \frac{1}{p}\ \lambda_i(X).\ell n \sum_{i=1}^{c} \exp\ [p.F_i(X)] \le 0\ ; & \forall i \ne h \epsilon c \end{array}\right\} \tag{3.41}$$

with positive values of the parameter p increasing towards infinity.

<u>Proof</u>:

This requires recalling the entropy-based weighted problem:

$$\underset{X\epsilon\Omega}{\text{Min}} \quad \sum_{I=1}^{c} \lambda_i\ F_i(X)\ -\ \frac{1}{p} \sum_{i=1}^{c} \lambda_i\ \ell n\ \lambda_i \tag{3.42a}$$

with

$$\lambda_i(X) = \exp\ \left[pF_i(X)\right] / \sum_{i=1}^{c} \exp\ \left[pF_i(X)\right];\ \sum_{i=1}^{c} \lambda_i(X) = 1$$

which after substitution leads to the aggregated problem (3.22):

$$\underset{X\epsilon\Omega}{\text{Min}}\ \frac{1}{p}\ .\ \sum_{i=1}^{c} \lambda_i(X)\ .\ \ell n \sum_{i=1}^{c} \exp\ \left[pF_i(X)\right] \tag{3.42b}$$

Note that (3.42b) is nothing else but the LHS of (3.29) since:

$$\sum_{i=1}^{c} \lambda_i(X)\ .\ \ell n \left(\sum_{i=1}^{c} (\cdot) \right) = \ell n \left(\sum_{i=1}^{c} (\cdot) \right) \sum_{i=1}^{c} \lambda_i(X)$$

$$= \ell n \left(\sum_{i=1}^{c} (\cdot) \right)$$

Assuming that:

$$\sum_{i=1}^{c} \exp\left[p.F_i(\mathbf{X})\right] = y(\mathbf{X},p)$$

Then, (3.41) can be rewritten as:

$$\underset{\mathbf{X}\epsilon\Omega}{\text{Min}} \quad \frac{1}{p} \exp\left[p.F_h(\mathbf{X})\right] . \left[[\ell n\ y(\mathbf{X},p)]/y(\mathbf{X},p)\right]$$

$$\text{S.t.:} \quad \frac{1}{p} \exp\left[p.F_i(\mathbf{X})\right] . \left[\ell n\ y(\mathbf{X}),p)/y(\mathbf{X},p)\right] \le 0 \ ; \ \forall i \neq h \epsilon c$$

Increasing p towards ∞ yields:

$$\left.\begin{array}{lll} \underset{\mathbf{X}\epsilon\Omega}{\text{Min}} & \underset{h\epsilon c}{\text{Max}} & \langle F_h(\mathbf{X})\rangle \\ \text{S.t.:} & \underset{i\neq h\epsilon c}{\text{Max}} & \langle F_i(\mathbf{X})\rangle \le 0 \end{array}\right\} \qquad (3.43)$$

Equation (3.43) holds for any set of objectives **F(X)**. Thus equation (3.43) may be extended to:

$$\left.\begin{array}{l} \underset{X\epsilon\Omega}{\text{Min}} \quad \underset{h\epsilon c}{\text{Max}} \ \langle F_h(\mathbf{X})\rangle = \underset{X\epsilon\Omega}{\text{Min}} \quad \frac{1}{p} \lambda_h(\mathbf{X}) \ . \ \ell n \sum_{i=1}^{c} \exp\ [pF_i(\mathbf{X})] \\ \text{S.t.:} \quad \underset{i\neq h\epsilon c}{\text{Max}} \ \langle F_i(\mathbf{X})\rangle = \frac{1}{p} \lambda_i(\mathbf{X}) \ . \ \ell n \sum_{i=1}^{c} \exp\ [p.F_i(\mathbf{X})] \le 0 \end{array}\right\} \qquad (3.44)$$

as p in the range $1 \le p \le \infty$ increases towards ∞. This completes the proof. Theorem 4 does not directly involve entropy in any way. The next theorem shows how the compartmentalized problem (3.41), and hence, by virtue of theorem 4, the minimax optimization problem (3.43), is related to the general vector optimization problem (3.20). It is at this point that entropy is seen to be the link between the two problems (3.20) and (3.43).

Theorem 5:

For any value of P, a parameter with any non-zero positive or negative value:

$$(1/P).\ \lambda_h(\mathbf{X})\ .\ \ell n \sum_{i=1}^{c} \exp\left[PF_i(\mathbf{X})\right] \geq \lambda_h(\mathbf{X}).F_h(\mathbf{X}) - \frac{1}{P}\lambda_h(\mathbf{X})\ .\ \ell n\ \lambda_h(\mathbf{X})\ ;\ h \epsilon c \tag{3.45}$$

with equality when the RHS is maximized over variables $\boldsymbol{\lambda}$. Such maximizing values of $\boldsymbol{\lambda}$ are:

$$\lambda_h = \exp\left[P.F_h(\mathbf{X})\right] / \sum_{i=1}^{c} \exp\left[P.F_i(\mathbf{X})\right]\ ;\ h \epsilon c \tag{3.46}$$

Proof:

Inequality (3.45) becomes an equality for any value of P when the RHS is maximized over variables $\boldsymbol{\lambda}$ subject to non-negativity and normality of the weights. The Lagrangean of (3.44) is:

$$L\ (\boldsymbol{\lambda},\alpha) = \lambda_h(\mathbf{X})\ .\ F_h(\mathbf{X}) - \frac{1}{P}\lambda_h(\mathbf{X})\ .\ \ell n\ \lambda_h(\mathbf{X}) +$$

$$\sum_{\substack{i=1 \\ i\neq h}}^{c} \beta_i \left[\lambda_i\ F_i - \frac{1}{P}\lambda_i\ \ell n\, \lambda_i\right] + \alpha\left[\sum_{i=1}^{c}\lambda_i - 1\right] \tag{3.47}$$

There is no need to include the non-negativity conditions explicitly as the middle term of equation (3.47) imposes this indirectly. Stationarity of equation (3.47) with respect to λ_i ; $i = 1,\ldots,c$ and α gives:

$$\lambda_h = \exp\left[P.F_h(\mathbf{X})\right] / \sum_{i=1}^{c} \exp\left[P.F_i(\mathbf{X})\right]\ ;\ h \epsilon c \tag{3.46}$$

substituting result (3.46) into the RHS of inequality (3.45) gives, after some algebraic simplification:

$$\underset{\lambda_h}{\text{Max}} \quad \left[\lambda_h\, F_h(X) - (1/P)\, \lambda_h . \ell n\, \lambda_h\right] = (1/P)\, \lambda_h . \ell n \sum_{i=1}^{c} \exp\left[P . F_i(X)\right]$$

and Theorem 5 is proved.

<u>Theorem 6</u>:

The vector $\mathbf{X}^*$ which solves the compartmentalized problem (3.41) or its entropy-based constrained form for any value of P is a Pareto solution of the general vector optimization problem (3.20).

<u>Proof</u>:

From Theorem 5, relationship (3.45) is an equality for any P when $\boldsymbol{\lambda}$ is given by equation (3.30). This result holds for any set of objectives **F** including those evaluated at $\mathbf{X}^*$ which minimizes (3.45), both sides. Thus:

$$\left.\begin{matrix} \underset{\mathbf{X}\in\Omega}{\text{Min}} & \frac{1}{P}\lambda^*_h . \ell n \sum_{i=1}^{c} \exp\,[PF_i] \\ \\ \text{S.t.:} & \frac{1}{P}\lambda^*_i . \ell n \sum_{i=1}^{c} \exp\,[PF_i] \le 0 \end{matrix}\right\} = \left\{\begin{matrix} \underset{\mathbf{X}\in\Omega}{\text{Min}}\ \lambda^*_h\, F_h & - & \frac{1}{P}\lambda^*_h . \ell n\, \lambda^*_h & \\ \\ \text{S.t.:}\ \lambda^*_i\, F_i & - & \frac{1}{P}\lambda^*_i . \ell n\, \lambda^*_i & \le 0 \end{matrix}\right\} \forall i \ne h \epsilon c \tag{3.48}$$

when $\boldsymbol{\lambda}$ is given by (3.30).

$\mathbf{X}^*$ will be a Pareto solution of problem (3.20) if it is a solution of problem (2.35-2.36) with $\boldsymbol{\lambda}$ defined by (3.30), i.e., $\mathbf{X}^*$ must satisfy the necessary stationarity condition for problem (3.38).

Examining the first derivative of Eq. (3.48) which yields the same result in equation (3.38) is summarized in Appendix (B).

3.2.5 <u>A Method for Dealing with Alternative Optima</u>

Alternative Optima for both the aggregated and compartmentalized problems and their entropy-based forms can cause difficulties in that some of the optimal solutions may be inferior. All alternative optima are non-inferior in the aggregated problem or its entropy-based form if all weights are strictly positive, and in the compartmentalized problem or its entropy-based form if all constraints on objectives are binding. It is only in the case of some zero weights or some non-binding objective constraints that some alternate optima may be inferior.

A method for checking the non-inferiority of alternative optima in the appropriate situations is presented below which can be used when the number of objectives is more than two.

When some alternative optima may be inferior we want to search among all of them for the non-inferior solutions and discard the inferior ones. In the case of the entropy-based weighted method (3.42a) or its aggregated form (3.42b), suppose that $\lambda_i > 0$ for $i = 1,2,\ldots,b$ and $\lambda_i = 0$ for $i = b+1, b+2,\ldots,c$ and that in the solution to problem (3.42a) or (3.42b) we found alternative optima which gave $F_i(\mathbf{X}) = F_i(\mathbf{X}^*)$ for $i = 1,2,\ldots,b$. Then we can solve a new problem to find the alternative optima:

$$\min_{\mathbf{X}\in\Omega} \sum_{i=b+1}^{c} \lambda_i F_i(\mathbf{X}) - \sum_{i=b+1}^{c} \frac{1}{P} \lambda_i \ell n \lambda_i \qquad (3.49)$$

$$\text{S.t.:} \quad F_i(\mathbf{X}) = F_i(\mathbf{X}^*); \quad i = 1,2,\ldots,b \qquad (3.50)$$

or:

$$\min_{\mathbf{X}\in\Omega} \frac{1}{P} \sum_{i=b+1}^{c} \lambda_i \, . \, \ell n \sum_{i=b+1}^{c} \left[PF_i(\mathbf{X})\right] \qquad (3.51)$$

$$\text{S.t.:} \quad F_i(\mathbf{X}) = F_i(\mathbf{X}^*); \quad i = 1,2,\ldots,b \qquad (3.50)$$

where λ_i; i = b+1, b+2, ..., c is given by equation (3.30). In (3.49) or (3.50), $\lambda_i > 0$ for i = b+1, b+2, ..., c. The values of the entropy parameter P varies automatically to find an approximation of that portion of the non-inferior set which lies among the alternative optima.

To understand the procedure, a problem with three objectives will be considered. Suppose we began by minimizing $F_1(\mathbf{X})$ individually, i.e., $\lambda_1 = 1$ and $\lambda_2 = \lambda_3 = 0$ (single-objective optimization problem). Alternative optima were found to exist since the feasible region in objective space is as shown in Fig. (3.4a). All of the solutions on the crosshatched face Ω_0 yield the minimum of objective $F_1(\mathbf{X})$, called $F_1(\mathbf{X}^*)$. We proceed then by solving:

$$\min_{\mathbf{X}\in\acute{\Omega}_0} \sum_{i=2}^{3} \left[\lambda_i F_i(\mathbf{X}) - (1/P)\, \lambda_i \ln \lambda_i\right] \tag{3.52}$$

$$\text{S.t.:} \quad F_1(\mathbf{X}) = F_1(\mathbf{X}^*) \tag{3.53}$$

or:

$$\min_{\mathbf{X}\in\acute{\Omega}_0} \frac{1}{P} \sum_{i=2}^{3} \lambda_i \,.\, \ln \sum_{i=2}^{3} \left[P\, F_i(\mathbf{X})\right] \tag{3.54}$$

$$\text{S.t.:} \quad F_1(\mathbf{X}) = F_1(\mathbf{X}^*) \qquad (3.53)$$

By adding the constraint in (3.53) to the original feasible region Ω_0, we have created a new feasible region $\acute{\Omega}_0$, which is just the face of Ω_0 that gives $F_1(\mathbf{X}^*)$. This face is redrawn in Fig. (3.4b). All of the feasible solutions of $\acute{\Omega}_0$ yield $F_1(\mathbf{X}^*)$; those that are also non-inferior are shown as the crosshatched portion of $\acute{\Omega}_0$ in Fig. (3.4b). These are found by generating different values of P which give combinations of strictly positive values of λ_2 and λ_3 in (3.52) or (3.54). The reader will note that the procedure is equivalent to the entropy-based weighted method or its aggregated form as applied to a subset of the original feasible region.

For alternative optima in the entropy-based constrained method or its compartmentalized form, we proceed in a similar fashion, but instead of defining the set of objectives F_1, $F_2, \ldots, F_h$ as these with strictly positive weights, we define them as those objectives that had binding, active, constraints and that objective selected for optimization. Furthermore, the set of objectives F_{h+1}, $F_{h+2}, \ldots, F_c$ are those that had non-binding constraints. The procedure is then pursued in an identical manner.

3.2.6 Discussion

Section (3.2) has surveyed the new developments and use of informational entropy in the context of mathematical optimization processes. There are currently three areas in which entropy and the **MEP** appear to have considerable potential for methodological development. The first of these is as a means of measuring uncertainty, the second is as a means of making inferences in the presence of uncertain or incomplete information, and the third is as a means of solving a variety of optimization problems. The third of these has been studied here in some detail but the first two are certainly of equal importance and worthy of much more research.

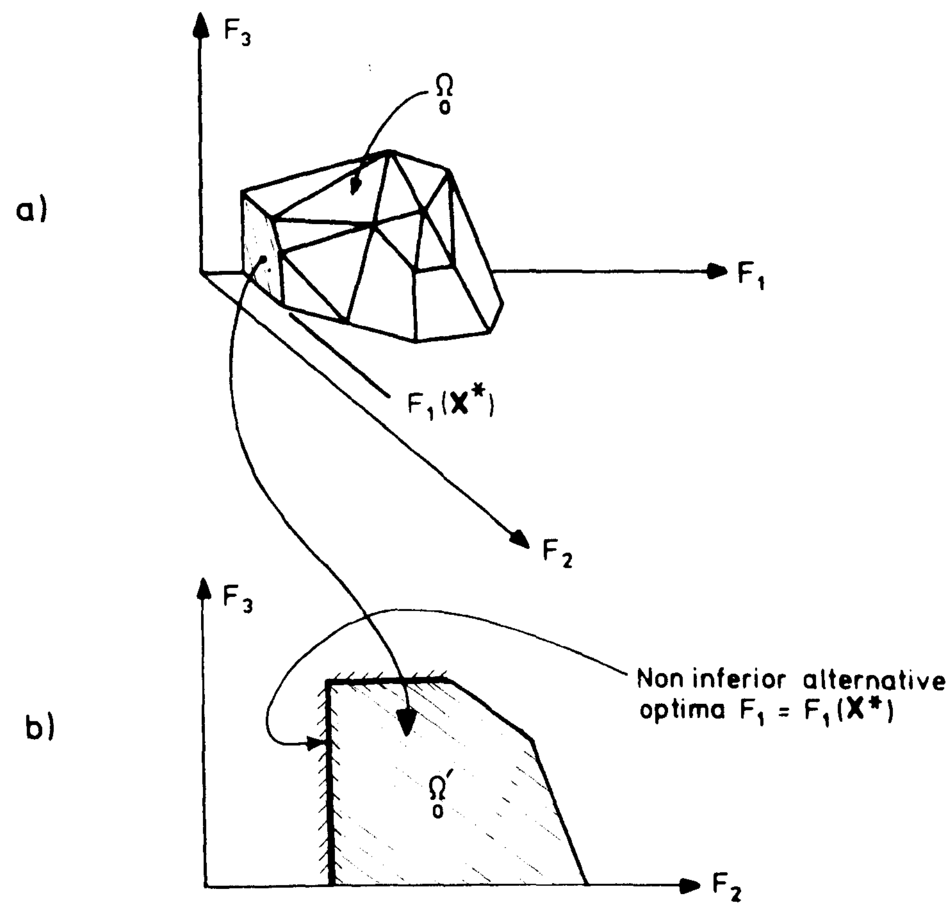


Fig. (3.4): Dealing with alternative optima when some of them may be inferior: (a) 3-D non-inferior set. (b) face that minimize F_1. Based on (Cohon, 1978)

The use of entropic inference in mathematical optimization processes is relatively new. The entropy-based weighted method or its aggregated form developed by Templeman (1988) and the entropy-based constrained method or its compartmentalized form developed by the author for multi-criteria optimization have so far been the most successful. Computational work is concerned with assessing the efficiency of generating Pareto solution sets using the scalar minimization problem, either side of (3.36), and the constrained minimization problem, either side of (3.48), for varying P values in comparison with other conventional methods such as the weighting objective method (2.31), the constraint method (3.39), or the bound formulation. The objectives of developing new techniques to eliminate the difficulties described in section (1.6) are successfully achieved here. Because of that, the new methods are expected to replace the traditional methods for generating Pareto set since considerable amounts of computational work and computer time can be saved.

3.3 BOUND FORMULATION AND VECTOR OPTIMIZATION

In mathematical programming, a vector optimization problem can be formulated as follows:

$$\text{Find } \mathbf{X}^* \text{ such that: } \qquad \mathbf{F}(\mathbf{X}^*) = \text{Min } \mathbf{F}(\mathbf{X}) \tag{3.55}$$

where $\mathbf{X} = [X_1, X_2, \ldots X_q]^T$ is a vector of design variables defined in q-dimensional Euclidean space of variables E^q. $\mathbf{F}(\mathbf{X}) = \left[F_1(\mathbf{X}), F_2(\mathbf{X}), \ldots, F_c(\mathbf{X})\right]^T$ is a vector function defined in c-dimensional Euclidean space of objectives E^c. It is important to point out that all goals or objectives are nondimensionalized.

The objective of bound formulation is to determine **X** such that:

$$F_i(\mathbf{X}) \leq \beta; \; i \epsilon c \qquad (3.56)$$

where β is an unrestricted scalar variable. For any fixed value of **X**, we have:

$$\beta \geq F_i(\mathbf{X}); \qquad i = 1,2,...,c \qquad (3.57)$$

Therefore, the smallest possible β satisfying Eq. (3.57) is given as a function of the design variables **X** by:

$$\beta(\mathbf{X}) = \underset{i \epsilon c}{\text{Max}} \; F_i(\mathbf{X}) \qquad (3.58)$$

Hence, the determination of the minimum value of β leads to the unconstrained minimization problem:

$$\underset{\mathbf{X}\epsilon\Omega}{\text{Min}} \; \beta(\mathbf{X}) = \underset{\mathbf{X}\epsilon\Omega}{\text{Min}} \; \underset{i \epsilon c}{\text{Max}} \; F_i(\mathbf{X}) \qquad (3.59)$$

which is equivalent to the constrained minimization problem, bound formulation:

$$\textbf{(BF)} \qquad \text{Min } \beta \qquad (3.60)$$

$$\text{S.t:} \; \mathbf{F(X)} \leq \beta \qquad (3.61)$$

It has been proved in this thesis, Eq. (3.27), that minimization of the RHS of Eq. (3.58) is equivalent to:

$$\underset{\mathbf{X}\epsilon\Omega}{\text{Min}} \; \underset{i \epsilon c}{\text{Max}} < F_i(\mathbf{X}) > = \underset{\mathbf{X}\epsilon\Omega}{\text{Min}} \; \frac{1}{P} \, .\ell n \left\{ \sum_{i=1}^{c} \exp P.F_i(\mathbf{X})] \right\} \qquad (3.62)$$

with varying P in the range $\infty \geq P \geq -\infty$ and $P \neq 0$.

Now, the LHS of Eq. (3.62) is the minimum of the RHS of Eq. (3.58), thus:

$$\underset{\mathbf{X}\epsilon\Omega}{\text{Min}}\ \beta(\mathbf{X}) = \underset{\mathbf{X}\epsilon\Omega}{\text{Min}}\ \frac{1}{P}\ .\ell n \left\{ \sum_{i=1}^{c} \exp\ [P.\mathbf{F}_i(\mathbf{X})] \right\} \tag{3.63}$$

That is, mathematically, the solution of the constrained minimization problem, **BF**, is equivalent to the solution of the unconstrained minimization problem (3.62). However, the minimization problem **BF** is not practical because $\beta(\mathbf{X})$ is not continuously differentiable everywhere with respect to **F**, which may cause numerical problems when minimizing β.

3.4 MINIMAX ENTROPY PRINCIPLE AND GLOBAL MINIMIZATION

3.4.1 Definitions

Given some function F(X) and some constraint functions, the process of locating a minimum value of F(X) commences with no numerical information whatsoever. An initial point is then chosen and information is calculated about the objective and constraint functions, typically their numerical values and gradients at the design point. This numerical information is then used in some deterministic mathematical programming algorithm to infer where the next trial point should be placed, so as to get closer to the constrained optimum of the problem. The new trial generates more information from which another point is inferred and eventually the solution is reached by this process of gathering better and better information and using it in an inference-based algorithm. Almost all such optimization algorithms use some form of geometrical inference to generate a sequence of improving trial points. The functions in the problem are interpreted as geometrical hypersurfaces with contours, slopes and gradients. Actually, we never have sufficient information to be able to plot the geometry, except for the simplest of problems, but it is convenient to imagine that these hypergeometrical shapes exist because it helps us to visualize what a numerical search algorithm is doing. Our general knowledge of geometry can be used to develop solution strategies which can use in a geometrical way any numerical information which is generated about the problem. We pretend, for convenience, that everything in the problem before us has a totally deterministic geometric representation. However, it is equally valid to argue that the problem should be interpreted non-deterministically: if we have made evaluations of the objective and contraint functions at some point this represents concrete information, but the introduction of some geometrical prediction strategy to estimate where to place the next trial represents, according to Jaynes', "an arbitrary assumption of information which by hypothesis we do not have". Actually, we have no grounds for assuming that the particular problem we are solving will conform precisely to the geometrical strategy we have introduced. The use of any particular geometrical strategy in this situation is analogous to the use of a particular probability distribution to represent data from some random process. According to the **MEP** both assumptions are wrong and introduce bias. The only correct strategy is to employ the **MEP** to infer where the next trial should be placed, using only the concrete data available.

It is important to realize that the geometrical interpretations placed upon optimization processes are only interpretations. There is nothing sacrosanct about them. It is perfectly reasonable to discard this deterministic, geometric interpretation of optimization and to attempt to develop new optimization algorithms based upon totally non-geometrical, non-deterministic concepts or geometrical, non-deterministic concepts. To do so, one must understand the basics of statistical mechanics.

Statistical mechanics is the study of the behaviour of very large systems of interacting components, such as atoms in a fluid, in thermal equilibrium at a finite temperature. Suppose that the configuration of the system is identified with the set of spatial positions of the components. If the system is in thermal equilibrium at a given temperature T, then the probability $\pi_T(s)$ that the system is in a given configuration s depends upon the energy E(s) of the configuration and follows the Boltzmann distribution:

$$\pi_T(s) = \frac{e^{\frac{-E(s)}{kT}}}{\sum_{w \in S} e^{\frac{-E(w)}{kT}}}$$

where k is Boltzmann's constant and S is the set of all possible configurations.

In studying systems of particles, one often seeks to determine the nature of the low energy states, for example, whether freezing produces crystalline or glassy solids. Very low energy configuration is not common, when considering the set of all configurations.

However, at low temperatures they predominate, because of the nature of the Boltzmann distribution. To achieve low-energy configurations, *it is not sufficient* to simply *lower the temperature*. One can use an annealing process, where the temperature of the system is *elevated*, and then *gradually lowered*, spending enough time at each temperature to reach thermal equilibrium. *If insufficient time* is spent at each temperature, especially near the freezing point, then the probability of attaining a very low energy configuration is greatly reduced. This is known as the annealing process.

Simulated Annealing is a *stochastic* computational technique derived from statistical mechanics for finding near globally-minimum-cost solutions to *large* optimization problems. In general, finding the global minimum value of an objective function with many degrees of freedom subject to conflicting constraints is an NP-Complete (non-deterministic polynomial) problem, since the objective function will tend to have many local minima. A procedure for solving hard optimization problems should sample values of the objective function in such a way as to have a high probability of finding a near-optimal solution and should also lend itself to efficient implementation. Consider a discrete minimization problem with N feasible solutions numbered in ascending order of their objective function values (F_i; $i = 1,2,...,N$). Iterative search methods specify a neighbourhood $N_i \in \{1,2,...,N\}$ for each solution, and proceed to its lowest valued neighbour, provided this switch results in a strict improvement. The methods repeat this step until no improving neighbour can be found, i.e., until they reach a local optimum. The classical deterministic methods encounter two major problems:

(i) the solution obtained is heavily dependent on the starting solution; and

(ii) the methods often converge to inferior local optima.

Simulated annealing methods randomize the procedure to overcome these problems, allowing for occasional switches that worsen the solution. This is equivalent to annealing in condensed matter physics where a solid in a heat bath is heated up by increasing the temperature of the heat bath to a maximum value at which all particles of the solid randomly arrange themselves in the liquid phase, followed by cooling through slowly lowering the temperature of the heat bath. In this way, all particles arrange themselves in the low energy ground state of a corresponding lattice, provided the maximum temperature is sufficiently high and the cooling is carried out sufficiently slowly. At each temperature, the solid is allowed to reach thermal equilibrium, characterized by a probability of being in a state with energy E given by the Boltzmann distribution. To simulate the evolution to thermal equilibrium of a solid for a fixed value of the temperature, Metropolis et al. (1953) proposed a Monte Carlo Method, which generates sequences of states of the solid (Laarhoven, 1987). The Metropolis algorithm can also be used to generate sequences of configurations of a combinatorial optimization problem. In that case, the configurations assume the role of the states of a solid while the objective function and the control parameter take the roles of energy and temperature, respectively. The simulated annealing algorithm can now be viewed as a sequence of Metropolis algorithms evaluated at a sequence of decreasing values of the control parameter. It is based on randomization techniques. It also incorporates a number of aspects related to iterative improvement algorithms. Since these aspects play a major role in the understanding of the simulated annealing algorithm (Laarhoven, 1987), and - later on in this thesis - the development of the simulated entropy algorithms, we first elaborate on the iterative improvement technnique.

3.4.2 Iterative Improvement Algorithms

The application of an iterative improvement algorithm presupposes the definition of configurations, a cost function and a generation mechanism, i.e. a simple prescription to generate a transition from a configuration to another one by a small perturbation. The generation mechanism defines a neighbourhood R_i for each configuration i. Iterative improvement is, therefore, also known as neighbourhood search or local search. The algorithm can now be formulated as follows. Starting off at a given configuration, a sequence of iterations is generated, each iteration consisting of a possible transition from the current configuration to a configuration selected from the neighbourhood of the current configuration. If this neighbouring configuration has a lower cost, the current configuration is replaced by this neighbour, otherwise another neighbour is selected and compared for its cost value. The algorithm terminates when a configuration is obtained whose cost is no worse than any of its neighbours. The disadvantages of iterative improvement algorithms can be formulated as follows:

a) By definition, iterative improvement algorithms terminate in a local minimum and there is generally no information as to the amount by which this local minimum deviates from a global minimum;

b) The obtained local minimum depends on the initial configuration, for the choice of which generally no guidelines are available.

To avoid some of the aforementioned disadvantages, one might think of a number of alternative approaches:

1) Execution of the algorithm for a large number of initial configurations, say N (e.g., uniformly distributed over the set of configurations R) at the cost of an increase in computation time; for $N \to \infty$, such an algorithm finds a global minimum with probability 1, if only for the fact that a global minimum is encountered as an initial configuration with probability 1 as $N \to \infty$;

2) (Refinement of 1) use of information gained from previous runs of the algorithm to improve the choice of an initial configuration for the next run (this information relates to the structure of the set of configurations);

3) Introduction of a more complex generation mechanism (or, equivalently, enlargement of the neighbourhoods), in order to be able to **jump out** of the local minima corresponding to the simple generation mechanism;

4) Acceptance of transitions which correspond to an **increase** in the cost function in a limited way (in an iterative improvement algorithm only transitions corresponding to a **decrease** in cost are accepted).

The first approach is a traditional way to solve combinatorial optimization problems approximately. The second and the third approaches being usually strongly problem-dependent. An algorithm that follows the fourth approach was independently introduced by Kirkpatrick et al. (1982) and Cerny (1985). It is generally known as simulated annealing, Monte Carlo annealing, statistical cooling, probabilistic hill climbing, stochastic relaxation, or probabilistic exchange algorithm.

In this section, informational entropy is used to develop simulated entropy techniques for solving combinatorial constrained minimization problems. Since the first technique is based on using surrogate constraint approach, we first elaborate on it.

3.4.3 Surrogate Approach to Constrained Optimization

A surrogate problem is one in which the original constraints are replaced by only one constraint, termed the surrogate constraint. For the inequality constrained problem, primal problem:

$$(P) \qquad \min_{X \in \Omega} \quad F(X)$$

$$\text{S.t.:} \qquad g_i(X) \le 0 \qquad j = 1,\ldots,m$$

the corresponding surrogate problem has the form:

$$(S) \qquad \min_{X \in \Omega} \quad F(X)$$

$$\text{S.t.:} \qquad \sum_{i=1}^{m} W_i \cdot g_i(X) \le 0$$

where W_i $(i = 1,\ldots,m)$ are non-negative weights, termed surrogate multipliers, which may be normalized without loss of generality by requiring:

$$\sum_{i=1}^{m} W_i = 1; \; W_i \geq 0$$

Some of the relationships between the primal problem (P) and the surrogate problem (S) are (Glover, 1965):

1) The feasible region of problem (P) is always included by the feasible region of problem (S).

2) If $\mathbf{X}^s$ solves problem (S) and $\mathbf{X}^*$ solves problem (P), then $F(\mathbf{X}^s) \leq F(\mathbf{X}^*)$ for all $W \geq 0$.

3) If $\mathbf{X}^s$ solves problems (S) and is also feasible in problem (P), then $\mathbf{X}^s$ also solves problem (P).

4) If at least one of original constraints is active at the optimum, then the surrogate constraint must be active at the optimum.

A comparison of the primal feasible region with the surrogate one is illustrated in Fig. (3.5) which shows that the primal feasible region is always a part of the surrogate feasible regions for any value λ. Problem (S) can, therefore, be viewed as a relaxation of problem (P).

3.4.4 <u>Simulated Entropy</u>

While annealing define a process of heating a solid and cooling it slowly, entropy in an isolated system tends to a maximum so that this variable is a criterion for the direction in which processes can take place. In simulated entropy (SE), a process which tends to a maximum entropy must be defined first. The two main results to be proved in this section are as follows:

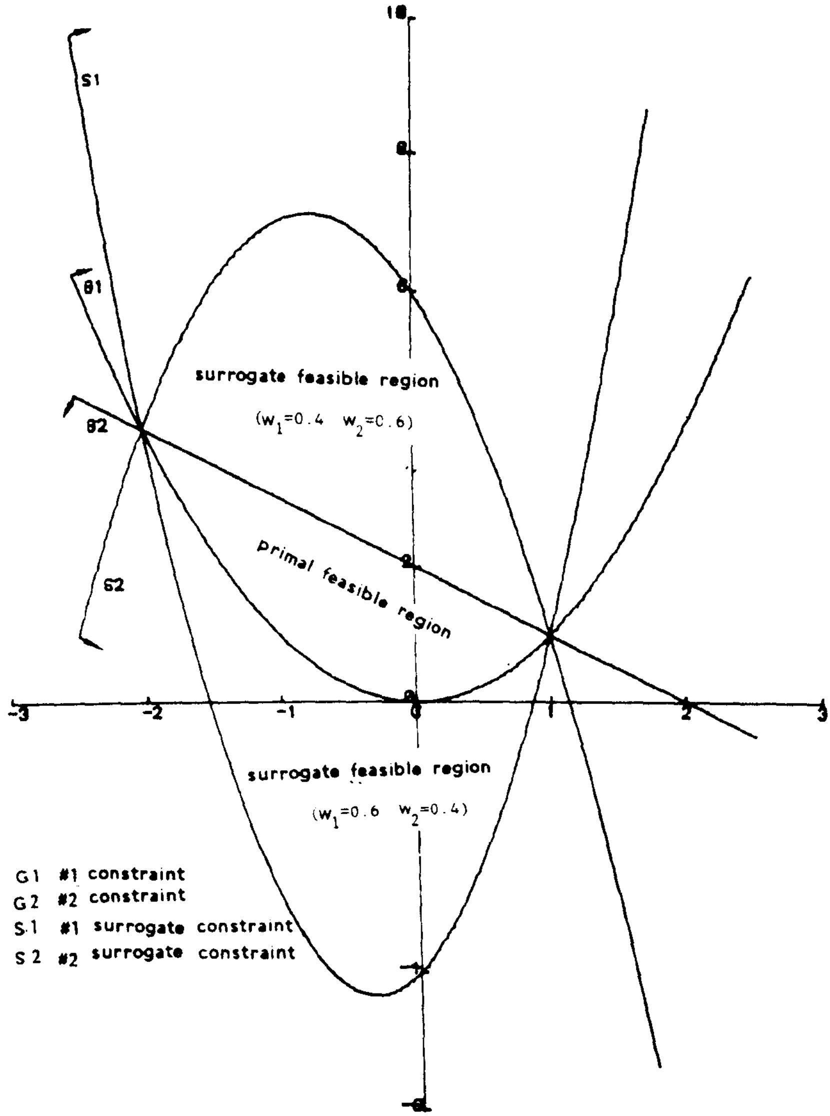


Fig. (3.5)

Primal and Surrogate Feasible Regions (Li, 1987)

a) For positive and decreasing values of P, minimizing the objective function F(X) over variables $X \epsilon \Omega$ subject to the entropy-based surrogate constraint function:

$$\sum_{i=1}^{m} \lambda_i . g_i(X) - P \sum_{i=1}^{m} \lambda_i . \ell n \lambda_i = 0$$

converges, hopefully, to the global minimum of the primal problem (P).

b) For positive and decreasing values of P, minimizing the objective function F(X) over variables $X \epsilon \Omega$ subject to the entropy-based constraints functions:

$$\lambda_i \, g_i(X) - P \lambda_i . \ell n \lambda_i \leq 0; \qquad i = 1,2,\ldots,m$$

converges, hopefully, to the global minimum of the problem (P). Theorem 7 formally states the first of these results.

<u>Theorem 7</u>:

The vector X^* which seeks the global minimum of the single-criteria optimization problem:

$$\text{(P)} \qquad \underset{X \epsilon \Omega}{\text{Min}} \quad F(X)$$

$$\text{S.t.: } g_i(X) \leq 0; \qquad i = 1,\ldots,m$$

is generated by solving the entropy-based surrogate constraint problem, simulated entropy problem:

$$(SE) \quad \min_{X \in \Omega} \quad F(X)$$

$$\text{S.t.:} \quad \sum_{i=1}^{m} \lambda_i \, g_i(X) - P \sum_{i=1}^{m} \lambda_i \ln \lambda_i = 0 \tag{3.64}$$

where **g** is a vector of dimensionless constraint functions, with positive values of the parameter P decreasing towards zero and:

$$\lambda_i = \exp\left[\frac{1}{P} g_i(X)\right] \Big/ \sum_{i=1}^{m} \exp\left[\frac{1}{P} g_i(X)\right] \tag{3.65}$$

Proof:

This requires the use of Cauchy's inequality (the arithmetic-geometric mean inequality), equation (3.32):

$$\sum_{i=1}^{m} U_i \geq \prod_{i=1}^{m} (U_i/\lambda_i)^{\lambda_i} \tag{3.32}$$

or

$$\left\{\sum_{i=1}^{m} U_i^{1/P}\right\}^{P} \geq \left\{\prod_{i=1}^{m} (U_i^{1/P}/\lambda_i)\right\}^{P} \; ; \; P > 0 \tag{3.66}$$

Inequality (3.66) shows that the (1/P-th) norm of the set U increases monotonically as its order, (1/P), increases. It tends to its limit as P tends towards zero:

$$\lim_{p \to 0} \left\{\sum_{i=1}^{m} U^{p}_{i}\right\}^{1/p} = \max_{i \epsilon m} \langle U_i \rangle \tag{3.67}$$

Let $$U_i = \exp\,[g_i(\mathbf{X})\,] \qquad (3.68)$$

Since the constraints g are dimensionless, the U defined by equation (3.68) are all positive numbers and their substitution into result (3.67), gives:

$$\lim_{p \to 0} \left\{ \sum_{i=1}^{m} \exp\,[(1/p).g_i(\mathbf{X})] \right\}^{p} = \underset{i \epsilon m}{\text{Max}} \quad \langle \exp\,[g_i(\mathbf{X})]\,\rangle \qquad (3.69)$$

Taking natural logarithms of both sides and noting that:

$$\ell n \lim\,(\bullet) \equiv \lim \ell n\,(\bullet)$$

$$\ell n\, \text{Max}\,(\bullet) \equiv \text{Max}\, \ell n\,(\bullet)$$

equation (3.69) becomes:

$$\underset{p \to 0}{\text{lin}} \; P\,.\,\ell n \left\{ \sum_{i=1}^{m} \exp\,[(1/P).g_i(\mathbf{X})] \right\} = \underset{i \epsilon m}{\text{Max}} \; \langle g_i(\mathbf{X}) \rangle \qquad (3.70)$$

Result (3.70) shows that the constrained minimization problem, **(P)**, can be solved by minimizing F(**X**) subject to the aggregated constraints function; **AC**:

$$\left.\begin{array}{lll} \textbf{(SE)} & \underset{\mathbf{X} \epsilon \Omega}{\text{Min}} & F(\mathbf{X}) \\ & \text{S.t.:} & P\,.\,\ell n \left\{ \sum_{i=1}^{m} \exp\,[(1/P)\; g_i(\mathbf{X})] \right\} = 0 \end{array}\right\} \qquad (3.71)$$

as P in the range $0 < P \leq \infty$ decreasing towards zero. Taking natural logarithms of both sides of inequality (3.66) with U defined by:

$$U_i = \exp\ [(1/P).g_i(\mathbf{X})] \qquad (3.72)$$

gives:

$$P\ .\ \ell n \left\{ \sum_{i=1}^{m} \exp\ [(1/P).g_i(\mathbf{X})] \right\} \geq \sum_{i=1}^{m} \lambda_i\ g_i(\mathbf{X}) - P \sum_{i=1}^{m} \lambda_i\ \ell n\ \lambda_i \qquad (3.73)$$

which becomes an equality when the variables $\boldsymbol{\lambda}$ takes the values given by equation (3.35) on substitution of equation (3.72):

$$\lambda_i = \exp \left(\frac{1}{P} g_i(\mathbf{X}) \right) \Big/ \sum_{i=1}^{m} \exp \left(\frac{1}{P} g_i(\mathbf{X}) \right) \ ;\ i = 1,\ldots,m \qquad (3.65)$$

and theorem (7) is proved.

The RHS consists of the vector constraint function in its surrogate form **(S)** but with an additional entropy term which is a function only of the multipliers $\boldsymbol{\lambda}$ and the parameter P. In its equality form with multipliers given by equation (3.65), relationship (3.73) shows that the entropy of the multipliers measures the difference between the aggregated constraint function (3.62) and the surrogate constraint function **(S)**.

It is clear that if we minimize $F(\mathbf{X})$ over variables $\mathbf{X} \in \Omega$ subject to either side of inequality (3.73) with multipliers given by equation (3.65) and P becoming decreasingly small and positive, the entropy term in (3.73) will tend towards zero.

The second main result of this section still remains to be proved and Theorem 8 formally states this.

<u>Theorem 8</u>:

The vector X^* which seeks the global minimum of the single-optimization problem, (P), is generated by solving the entropy-based constrained problem:

$$\text{(SE)} \quad \left.\begin{array}{ll} \min\limits_{X \in \Omega} & F(X) \\ \text{S.t.:} & \lambda_i \, g_i(X) - P \, \lambda_i \, \ell n \, \lambda_i \leq 0; \; i = 1, \ldots, m \end{array}\right\} \tag{3.74}$$

with positive values of the parameter P decreasing towards zero and:

$$\lambda_i = \exp\left[\frac{1}{P} g_i(X)\right] \Big/ \sum_{i=1}^{m} \exp\left[\frac{1}{P} g_i(X)\right] \tag{3.65}$$

<u>Proof</u>:

This requires recalling the entropy-based surrogate constrained problem:

$$\left.\begin{array}{ll} \min\limits_{X \in \Omega} & F(X) \\ \text{S.t.:} & \sum\limits_{i=1}^{m} \lambda_i \, g_i \, (X) - P \sum\limits_{i=1}^{m} \lambda_i \ln \lambda_i = 0 \end{array}\right\} \tag{3.64}$$

with $\lambda_i(X)$ is given by (3.65). Without loss of generality, (3.64) may be rewritten as follows:

$$\left.\begin{array}{ll} \underset{X\in\Omega}{\text{Min}} & F(\mathbf{X}) \\ \\ \text{S.t.:} & \lambda_i\, g_i(\mathbf{X}) - P\, \lambda_i\, \ell n\, \lambda_i \le 0;\quad \forall i\epsilon m \end{array}\right\} \qquad (3.75)$$

substituting (3.65) into (3.75) and assuming that:

$$\sum_{i=1}^{m} \exp\left[\frac{1}{P}\, g_i(\mathbf{X})\right] = y(\mathbf{X},\ P)$$

yields to:

$$\left.\begin{array}{ll} \underset{X\in\Omega}{\text{Min}} & F(\mathbf{X}) \\ \\ \text{S.t.:} & P.\exp[(1/P).g_i(\mathbf{X})].[\ell n\ y(\mathbf{X},P)/y(\mathbf{X},P)] \le 0;\quad \forall i\epsilon m \end{array}\right\}$$

decreasing P towards zero yields to:

$$\left.\begin{array}{ll} \underset{X\in\Omega}{\text{Min}} & F(\mathbf{X}) \\ \\ \text{S.t.:} & \text{Max} <g_i(\mathbf{X})> \le 0;\quad \forall i\epsilon m \end{array}\right\} \qquad (3.76)$$

Result (3.76) shows that the primal constrained minimization problem, **(P)**, can be solved by minimizing F(**X**) subject to the sub-aggregated constraints; **SAC**:

$$\textbf{(SE)}\quad \left.\begin{array}{ll} \underset{X\in\Omega}{\text{Min}} & F(\mathbf{X}) \\ \\ \text{S.t.:} & P.\lambda_i(\mathbf{X})\ .\ \ell n \sum_{i=1}^{m} \exp\left[\frac{1}{P}\, g_i(\mathbf{X}))\right] \le 0\ ;\ \forall i\epsilon m \end{array}\right\} \qquad (3.77)$$

as P in the rane $0 \leq P \leq \infty$ decreasing towards zero. This is also true when (3.75) is used with $\lambda_i(\mathbf{X})$ given by (3.65). This completes the proof.

3.4.5 Discusion:

There are several problems in statistics which can be formulated so that the desired solution is the global minimum of some explicitly defined objective function. In many cases the number of candidate solutions increases exponentially with the size of the problem making exhaustive search impossible, but descent procedures, devised to reduce the number of solutions examined, can terminate with local minima. In section (3.5.4) we have described two simulated entropy algorithms which can escape local minima. The simulated entropy algorithms (SE) are stochastic search procedures which seek the minimum of some deterministic objective function. It applies small perturbations to the current solution. A deterministic descent method will always go to the minimum in whose basin it finds itself, whereas the simulated entropy techniques can climb out of one basin into the next. Because of its ability to make **small uphill steps**, the simulated entropy techniques avoid being trapped in local minima. However, it is far from apparent at first, that one could not always do at least as well by running a deterministic algorithm many times, with randomly chosen starting points.

Although simulated entropy algorithm details vary when applied to different problems, we always use the same basic simulated entropy idea:

At each temperature, the process must tends to a maximum entropy.

We define a descent algorithm to generate a solution like the method of feasible directions; a perturbation scheme to perturb the current solution using the informational entropy defined before; and, analogous to the decreasing temperature regime of the physical cooling process, a sequence of control parameters P which starts at some initially high value P_0, decreases by dP to a final near zero value P_f. Indeed, it is not apparent so far, why we should vary P at all: why not just choose $P = P_f$ in the first place? Two temperature (control parameter, or entropy parameter) scheduling mechanisms can be used, either one, in the prototype entroper. Temperature is exponentially reduced using the formula (Kirkpatrick, 1983):

$$P_i = (P_1/P_0)^i \; P_0 \; ; \; i = 1,2,\ldots,I$$

Temperature scheduling is completely defined by specifying the temperature at which simulated entropy to begin, P_0, and the ratio of subsequent to initial temperature, P_1/P_0. The second scheduling mechanism, where the temperature is reduced at intervals it uses the formula:

$$P_i = P_0 - \Delta P; \; i = 1,2,\ldots,I$$

where ΔP is the step size to be defined in addition to the temperature at which simulated entropy to begin, P_0. The simulated entropy techniques developed in this thesis repeat the following steps until at least one of the original constraints is active, assuming we are given value P_0, ΔP or P_1/P_0 of the central parameter and a starting point:

1) Using an iterative technique, generate a neighbour $F_i(\mathbf{X})$ of the current solution $F_{i-1}(\mathbf{X})$. This is a potential candidate for $F_{i+1}(\mathbf{X})$.

2) Set P_i to $(P_0 - \Delta P)$ or $[P_1/P_0]^i P_0$ and replace i with i + 1.

Finally, the simulated entropy techniques developed before can be interpreted geometrically as discussed before in the previous section. Theorem 7 which implies two forms of the first simulated entropy technique seeks the global minimum if it is in a convex region, while Theorem 8 which implies two forms of the second simulated entropy technique seeks the global minimum if it is in a concave or convex region.

CHAPTER FOUR
ENTROPIC OPTIMIZATION APPLICATIONS

CHAPTER FOUR

ENTROPIC OPTIMIZATION APPLICATIONS

SYNOPSIS

In this chapter, the set of entropy-based methods developed in the previous chapter is tested, discussed and compared.

4.1 INTRODUCTION

Several points emerge from the present work. First, dealing with the entropy-based methods requires that all objective functions in the goal vector **F** or all constraints in the surrogate constraint **g** must be dimensionally homogenous. It is therefore, essential in such applications that they should all be non-dimensionalized before use in the developed methods. In multi-criteria optimization applications this suggests the specification for each goal of cost C(**X**) or displacement $\delta(\mathbf{X})$ of a desirable value $\acute{C}$ or $\acute{\delta}$ in which case the goals $F_i(\mathbf{X})$ then have the form:

$$F_i(\mathbf{X}) \equiv \frac{C(\mathbf{X})}{\acute{C}} \quad \text{or} \tag{4.1}$$

$$F_i(\mathbf{X}) \equiv \frac{\delta(\mathbf{X})}{\acute{\delta}} \tag{4.2}$$

Similarly, in single-criteria minimization applications this suggests the specification for each constraint in the surrogate constraint vector of a desirable value $\acute{d}$ in which case each constraint has the form:

$$g_i(\mathbf{X}) \equiv \frac{G_i(\mathbf{X})}{\acute{d}} - 1 \tag{4.3}$$

More important property of this nondimensionalizing may be added. Exponentiation within the entropy-based methods may cause computational underflow if P becomes too small or computational overflow if P becomes too large. Nondimensionalizing reduces such problems considerably for different applications.

The maximum entropy problem has an explicit solution:

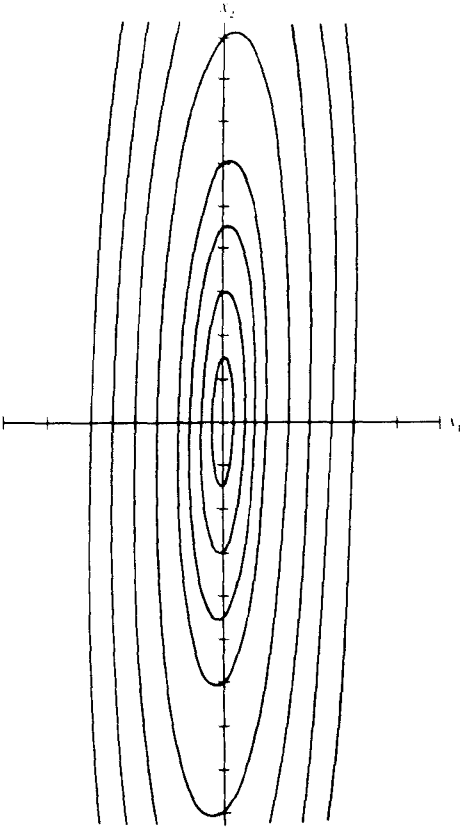
$$\lambda_i = \exp (P\, Y_i) \;/\; \sum_{i=1}^{n} \left\{ \exp (P\, Y_i) \right\}$$

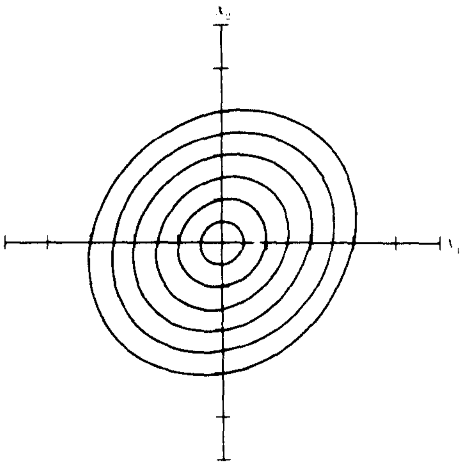
A technique which materially improves the convergence of nearly all minimization problems is the scaling of Y_i. It is ordinarily considered good practice in engineering to choose balanced units of measure or intelligently nondimensionalize the Y_i of a problem in order to improve the numerical behavior of the solution. In minimization the eccentricity, and hence the difficulty, can be greatly influenced by the scales chosen. For example, consider the functions:

$$Y_1(\mathbf{X}) = 144\, X^2{}_1 + 4X^2{}_2 - 8X_1X_2$$

$$Y_2(\mathbf{X}) = X^2{}_1 + X^2{}_2 - (1/3)\, X_1\, X_2 = Y_1\, [(X_1/12),\ (X_2/2)]$$

Function contours of both Y_1 and Y_2 are shown in Fig. (4.1). Clearly, Y_1 is much more eccentric than Y_2 and would be harder to minimize.





$Y_1(\mathbf{X}) = 144X^2_1 + 4X^2_2 - 8X_1X_2$ $\qquad$ $Y_2(\mathbf{X}) = X^2_1 + X^2_2 - (1/3)X_1X_2$

Fig. (4.1) Eccentricity (Fox, 1971)

The objective of scaling is to accomplish a coordinate expansion or contraction which will minimize the eccentricity. As the latter example illustrates, the coordinate transformation of simple scaling may still leave considerable eccentricity. Furthermore, the techniques of making the coefficients of the squared terms equal is not, in general, optimal scaling even in quadratic problems, beyond the two-variable case. The theory of optimal scaling is difficult and beyond the scope of this thesis (Fox, 1971). The value of the objective function is the same in both the scaled and the unscaled space.

As we have seen that one explicit solution of Jaynes **MEP** is:

$$\lambda_i = \frac{e^{PY_i}}{\sum_{i=1}^{n} e^{PY_i}} = \frac{1}{1 + \sum_{j=1}^{n} e^{P(Y_j - Y_i)}} \tag{4.4}$$

Scaling the term $(Y_j - Y_i)$ to different values with fixed P, gives different distribution to the values of λ_i which always satisfy the normality condition:

$$\sum_{i=1}^{n} \lambda_i = 1; \ \lambda_i \geq 0$$

This property is very helpful in discovering more solutions.

Three philosophies are available for rescaling the objective functions (Steuer, 1986):

(a) normalization,

(b) use of 10 raised to an appropriate power, and

(c) the application of range equalization factors

(a) Normalization

Normalization may be applied as described before or by using norms. Norms measure the lengths of vectors. The L_p-norms of a vector when p equals 1,2, and ∞ are:

$$\|V\|_1 = \sum_{i=1}^{n} |V_i|$$

$$\|V\|_2 = \left(\sum_{i=1}^{n} |V_i|^2 \right)^{0.5}$$

$$\|V\|_\infty = \max_{i \in n} (|V_i|)$$

Dividing each component of a vector by the norm of the vector normalizes a vector. The resultant length of the normalized vector is one.

(b) Use of 10 Raised to an Appropriate Power

Assume that our purpose is numerical (i.e., to bring all objective function coefficients into the same order of magnitude). Besides normalization, another option is to rescale each objective function by 10 raised to an appropriate power. Whereas normalization is likely to change the coefficients to unrecognizable numbers, the recognition of each coefficient is retained with 10 raised to an appropriate power because only the decimal point moves.

Example 1:

Consider the MOLP problem:

$$\underset{X \in \Omega}{\text{Max}} \quad F_1(\mathbf{X}) = 63400.0\ X_1 + 189600.0\ X_2 - 50400.0\ X_3$$

$$\underset{X \in \Omega}{\text{Max}} \quad F_2(\mathbf{X}) = 9.61\ X_1 - 2.35\ X_2 + 6.78\ X_3$$

If we normalize each objective function using the L_1-norm, we have:

$$\underset{X \in \Omega}{\text{Max}} \quad F_1(\mathbf{X}) = 0.209\ X_1 + 0.625\ X_2 - 0.166\ X_3$$

$$\underset{X \in \Omega}{\text{Max}} \quad F_2(\mathbf{X}) = 0.513\ X_1 - 0.125\ X_2 + 0.362\ X_3$$

However, if we rescale the first objective by 10^{-5} and the second objective by 10^{-1}, we have:

$$\underset{X \in \Omega}{\text{Max}} \quad F_1(\mathbf{X}) = 0.634\ X_1 + 1.896\ X_2 - 0.504\ X_3$$

$$\underset{X \in \Omega}{\text{Max}} \quad F_2(\mathbf{X}) = 0.961\ X_1 - 0.235\ X_2 + 0.678\ X_3$$

In this way, we have brought all objective function coefficients into the same order of magnitude, yet we have retained the recognizability of the objective function coefficients.

(c) Use of Range Equalization Factors

Suppose our purpose is to equalize the ranges of the criterion values over the efficient set. One way to do this is to multiply each objective by its representative range equalization factor:

$$\pi_i = \frac{1}{R_i} \left(\sum_{j=1}^{K} \frac{1}{R_j} \right)^{-1}$$

where R_i is the range width of the ith criterion value over the efficient set.

Example 2:

Suppose the range of $F_1(\mathbf{X})$ over the efficient set E (set of all efficient points) is [600,800] and the range of $F_2(\mathbf{X})$ is [-24,12]. Thus, $R_1 = 200$ and $R_2 = 36$. Hence, $\pi_1 = 0.1525$ and $\pi_2 = 0.8475$. After we scale the objectives by the π_i in Example 2, the range of $F_1(\mathbf{X})$ over E is [91.5,122.0] and the range of $F_2(\mathbf{X})$ is [-20.3,10.2]. Suppose it is also desired to equalize the midpoints of the scaled range at a given value. This can be accomplished by adding K_i constant terms to the scaled objective functions. If the given value is 50, the range midpoints would be equalized with $K_1 = -56.75$ and $K_2 = 55.05$. Therefore, if:

$$\max_{\mathbf{X}\in\Omega} \quad F_1(\mathbf{X})$$

$$\max_{\mathbf{X}\in\Omega} \quad F_2(\mathbf{X})$$

were the starting **MOLP** problem of example 2, the ending **MOLP** would be:

$$\max_{\mathbf{X}\in\Omega} \ \pi_1 \ [F_1(\mathbf{X})] + K_1$$

$$\max_{\mathbf{X}\in\Omega} \ \pi_2 \ [F_2(\mathbf{X})] + K_2$$

where π_1, π_2, K_1, and K_2 are as given above.

4.2 A GENERAL-PURPOSE ENTROPY-BASED OPTIMIZATION COMPUTER PROGRAM

A new general-purpose entropy-based optimization computer program is described. All the entropy-based methods developed in this thesis are coupled with ADS (Automated Design Synthesis) computer program to perform the optimization task. The optimization process is segmented into three levels, Strategy ISTRAT, Optimizer IOPT, and One-dimensional search IONED. At each level, several options are available so that a total of nearly 100 possible combinations can be created. The program has been developed by the author and coupled with ADS which has been developed by Vanderplaats (1986).

The ADS computer program solves the general optimization problem in the standard form: find the set of n design variables contained in the vector **X** that will minimize:

$$F(\mathbf{X}) \qquad (4.5)$$

$$\text{S.t.: } g_i(\mathbf{X}) \le 0 \;; \qquad i = 1,\ldots,m \qquad (4.6)$$

$$h_j(\mathbf{X}) = 0 \;; \qquad j = 1,\ldots,k \qquad (4.7)$$

$$X^L_q \le X_q \le X^U_q \;; \qquad q = 1,\ldots,n \qquad (4.8)$$

where $F(\mathbf{X})$ is the objective function, $g_i(\mathbf{X})$ are the inequality constraints, and $h_j(\mathbf{X})$ are the equality constraints. The bounds X^L_q and X^U_q on the design variables are referred to as side constraints, and could be included in the general inequality constraint set of Eq. (4.6). It is usually preferable to deal with side constraints separately because they directly limit the region of search for the optimum. For more information about ADS capabilities see the above mentioned reference.

The *Entropy-ADS* computer program solves the general vector and single optimization problems in different standard forms as follows:

(1) <u>Multi-Criterion Minimization</u>

Find Pareto set solutions for which each one has n design variables contained in the vector **X** that will minimize any of the functions shown in Table (4.1). The below objective functions, for constrained problems, are subject to Eqs. (4.6-4.7).

(2) <u>Single-Criterion Minimization</u>

Find the set of n design variables constrained in the vector **X** that will globablly seek the minimum of F(**X**) subject to one of the constraints sets listed in Table (4.2).

Finally, it is important to point out that two aspects have to be considered in computer programming:

1) In each of the minimax vector optimization methods, each feasible value of the entropy parameter P generates a Pareto point. A standard way of generating different values of P is by using the following formula:

$$P_i = P_{i-1} + \alpha$$

Minimax Vector Optimization Methods		
NO.	Name	Form
1.	The Entropy-Based Weighted Objective Function Methods, EWOF	$\sum_{i=1}^{c} \lambda_i . F_i(\mathbf{X}) - (1/P) . \sum_{i=1}^{c} \lambda . \ln \lambda_i$
2.	The Aggregated Objective Functions Method, AOF	$(1/P) . \sum_{i=1}^{c} \exp [P.F_i(\mathbf{X})]$
3.	The Entropy-Based Constrained Objective Function Method, ECOF	$\lambda_h . F_h . (\mathbf{X}) - (1/P) . \lambda_i . \ln \lambda_i$ S.t.: $\lambda_i . F_i(\mathbf{X}) - (1/P) \lambda_i . \ln \lambda_i \le 0;\ i \ne h \epsilon c$
4.	The Compartmentalized Objective Function Method, COF	$(1/P) . \lambda_h \sum_{i=1}^{c} \exp [P.F_i(\mathbf{X})]$ S.t.: $(1/P) . \lambda_i . \sum_{i=1}^{c} \exp[P.F_i(\mathbf{X})] \le 0;\ i \ne h \epsilon c$

$$\sum_{i=1}^{c} \lambda_i = 1; \quad \lambda_i = e^{P.F_i(\mathbf{X})} \Big/ \left\{ \sum_{i=1}^{c} e^{P.F_i(\mathbf{X})} \right\}; \quad +\infty \ge P \ge -\infty$$

Table (4.1)

Simulated Entropy Methods		
NO.	Name	Form
1.	The Entropy-Based Surrogate Constraint Method, ESC	$\sum_{i=1}^{m} \lambda_i . g_i(\mathbf{X}) - P \sum_{i=1}^{m} \lambda . \ell n\ \lambda_i = 0$
2.	The Aggregated Constraint Method, AC	$P. \sum_{i=1}^{m} \exp\ [(1/P).g_i(\mathbf{X})] = 0$
3.	The Entropy-Based Constrained Method, EC	$\lambda_i . g_i(\mathbf{X}) - P.\lambda_i.\ell n\ \lambda_i \le 0;\ i = 1,2,\ldots,m$
4.	The Sub-Aggregated Constrained Method, SAC	$P.\lambda_i \sum_{i=1}^{m} \exp\ [(1/P).g_i(\mathbf{X})] \le 0;\ i=1,2,\ldots,m$

$$\sum_{i=1}^{m} \lambda_i = 1; \quad \lambda_i = e^{(1/P).g_i(\mathbf{X})} \Big/ \left\{ \sum_{i=1}^{m} e^{(1/P).g_i(\mathbf{X})} \right\}; \quad \infty \ge P > 0$$

Table (4.2)

where α is a given step size between two consecutive values of P_i. Remember that the smaller step size we choose, the better the chance of having more Pareto solutions if there are any, $\alpha = 0.025$ has been used very often in this thesis.

2) The entropy multipliers λ_i can be written in FORTRAN as follows:

```
      DO  10  I=1,N
      A = 0.0
      DO  20  J=N,1,-1
      AMBDA(I) = A+EXP(C*(Y(J) - Y(I)))
      A = AMBDA(I)
20    CONTINUE
```

```
      AMBDA(I) = 1.0/AMBDA(I)
10    CONTINUE
```

where:

N,Y = Number of objective functions and the objective function value if the minimax methods are used, C = P.

Number of constraints and the constraints values if the simulated entropy techniques are used, C = 1/P.

4.3 PARETO SET GENERATION OF MULTI-CRITERIA OPTIMIZATION

Six numerical examples are presented. The Pareto set for each example is generated by different methods. The first two examples are purely mathematical with no physical or engineering interpretation is given. The third example represents an optimal-shape design problem where the Pareto set performances and the associated designs, are represented as diagrams of the two parts of the beam. The form of these solutions goes through several distinct stages in progressing along the Pareto boundary of the feasible performance space. The form of these solutions and the trends they follow provide valuable design information for the designer or manufacturer. Even if none of them are chosen as the final design, the prescriptive advice that they offer on the likely form of good solutions in relation to different strategies performance can be used in the synthesis of that final design and is very difficult to obtain in any other way. The fifth example, which is a linear programming one, is a production planning problem to determine how many units to produce of each of two products, denoted as A and B. The last example is a three criterion optimization problem.

4.3.A. Numerical Examples

Example 4.3.A.1 (Osyczka, 1984)

This first example is purely numerical without any physical or engineering interpretation.

Optimize:

$$F_1(\mathbf{X}) = X^2{}_1 + X^2{}_2 + 12\ (X_1 + X_2) \rightarrow \quad \text{Min}$$

$$F_2(\mathbf{X}) = X_1\ X_2 \qquad\qquad \rightarrow \quad \text{Max}$$

$$\text{S.t.:}\ g_1(\mathbf{X}) \equiv -\ 0.5\ X^2{}_1 + 5\ X_1 - X_2 - 6 \ \geq 0$$

$$g_2(\mathbf{X}) \equiv -\ X^2{}_1 + 6\ X_1 - X_2 + 14\ X_2 - 42 \ \geq 0$$

$$g_3(\mathbf{X}) \equiv -\ X^2{}_1 + 16\ X_1 - X^2{}_2 + 6\ X_2 - 48 \geq 0$$

$$X_1,\ X_2 \geq 0$$

We transform the second objective function to a minimization form, i.e., we multiply it by (-1). Hence, we have:

$$F_2\ (\mathbf{X}) = -\ X_1\ X_2 \qquad \longrightarrow \ \text{Min}$$

The graphical illustration of the above problem is shown in Fig. (4.2) and the results are summarised in Table (4.3) and Fig. (4.3). Note that only two solutions can be obtained by the first method since the rest of Pareto solutions are concave

The Entropy-Based Weighted Objective Functions Method

No.	$X = [X_1, X_2]$	$F(X) = [F_1(X), F_2(X)]$
1	[5.9874,6.0134]	[216.019,-36.0045]
2	[3.0,3.0]	[89.9983,-8.9997]

The Entropy-Based Constrained Objective Function Method

1	[3.0,3.0]	[90.,-9.0]
2	[3.3703,3.4035]	[104.2291,-11.47095]
3	[3.3162,36426]	[107.7719,-12.07969]
4	[4.0083,4.0205]	[128.5761,-16.1153]
5	[4.3031, 4.5412]	[145.2714,-19.54137]
6	[4.7752,4.9711]	[164.4717,-23.73837]
7	[5.0072,5.0205]	[170.6086,-25.13837]
8	[5.6178,5.6180]	[197.9528,-31.56117]
9	[5.6103,5.7799]	[201.5647,-32.42685]
10	[5.9791,6.0216]	[216.0159,-36.0033]

Table (4.3)

Results of Calculations for Example (4.3.A.1)

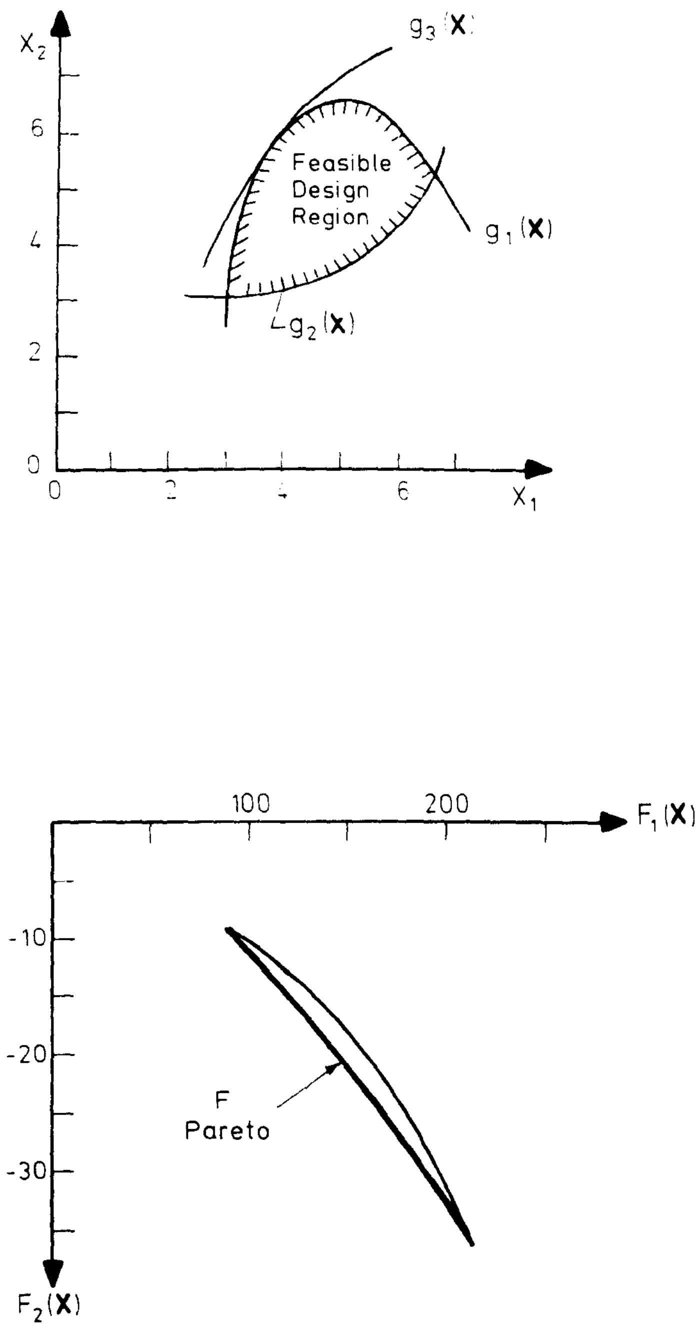
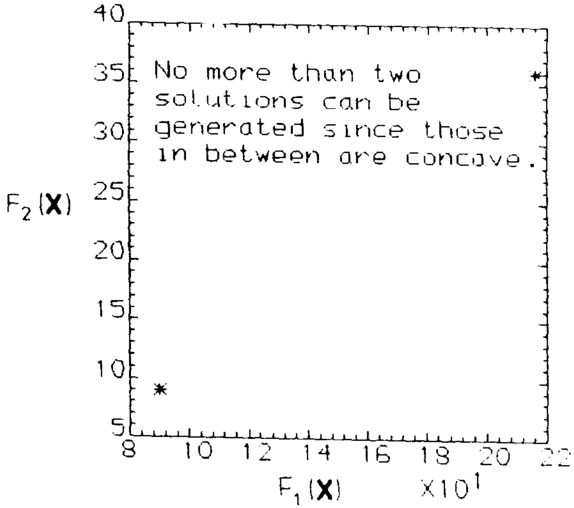
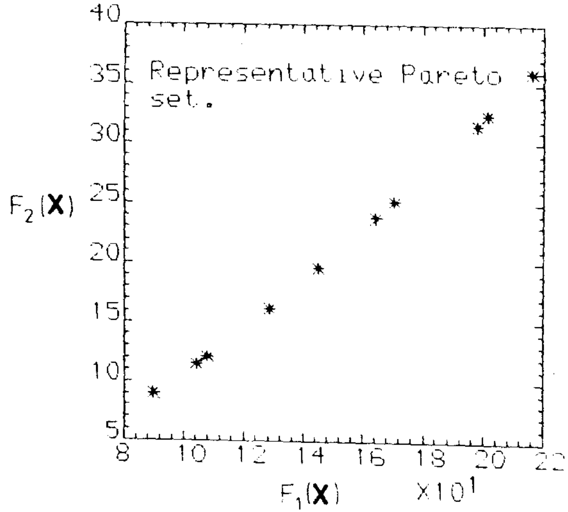


Fig. (4.2)
Graphical Illustration of Example (4.3.A.1) Based on (Osyczka, 1984)



The Weighted Objectives Method



The Constrained Objectives Method

Fig. (4.3)

Pareto Optimal Sets by the Entropy-Based Minimax Methods of Example (4.3.A.1)

Example 4.3.A.2 (Osyczka, 1984)

This is another well known numerical example.

Optimize:

$$F_1\ (\mathbf{X}) = X_1 + X^2{}_2 \rightarrow \text{Min}$$

$$F_2\ (\mathbf{X}) = X^2{}_1 + X_2 \rightarrow \text{Min}$$

S.t.:

$$g_1\ (\mathbf{X}) \equiv -\ 12 - X_1 - X_2 \geq 0$$

$$g_2\ (\mathbf{X}) \equiv -\ X^2{}_1 + 10\ X_1 - X^2{}_2 + 16\ X_2 - 80 \geq 0$$

$$X_1,\ X_2 \geq 0$$

The graphical illustration of the above problem is shown in Fig. (4.4) and the results are summarized in Table (4.4) and Fig. (4.5). To make the right decision about the satisfactory solution, we would like to have some representative subset of $\mathbf{X}^{Pareto}$ and let us assume that it would be convenient for us if this subset were such that the values of the objective functions would more or less uniformly cover the set $\mathbf{F}^{Pareto}$. Note that such a subset is obtained by the second method rather than the first method.

Example 4.3.A.3 (Osyczka, 1984)

Consider the third example in which the exterior diameters of the beam, Fig. (4.6), are assumed to be 100 mm and 80 mm. If these diameters are not predetermined, i.e., can be chosen in the process of design, then they can be treated as the third and fourth design variables. A similar question arises when we consider the beam material. If the designer is free to choose the material of the beam

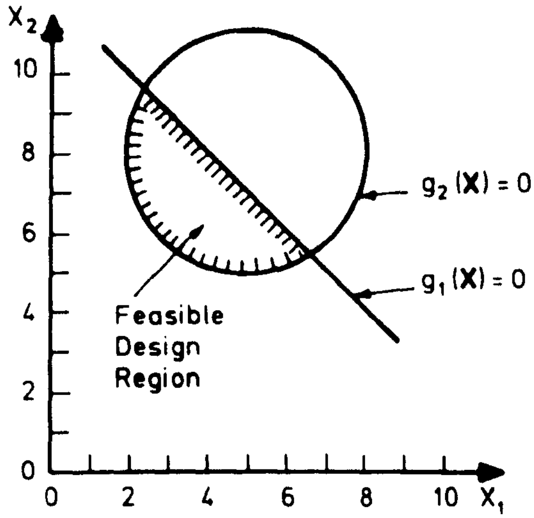


Fig. (4.4)

Graphical Illustration of Example (4.3.A.2).

Based on (Osyczka, 1984)

The Entropy-Based Weighted Objective Functions Method

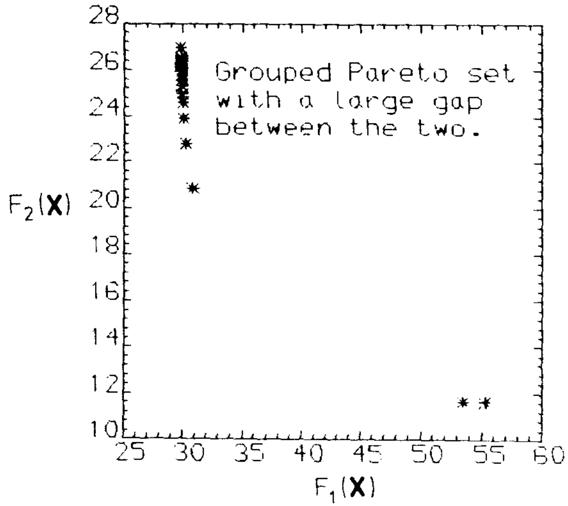
No.	$X = [X_1, X_2]$	$F(X) = [F_1(X), F_2(X)]$
1	[4.6840,5.0167]	[29.8512,26.9561]
2	[4.6389,5.0219]	[29.8578,26.5409]
3	[4.6315,5.0227]	[29.8592,26.4735]
4	[4.6232,5.0239]	[29.8627,26.3978]
5	[4.6140,5.0250]	[29.8649,26.3139]
6	[4.6042,5.0263]	[29.8676,26.2251]
7	[4.5948,5.0275]	[29.8704,26.1395]
8	[4.5722,5.0307]	[29.8807,25.9362]
9	[4.5594,5.0326]	[29.8862,25.8204]
10	[4.5338,5.0366]	[29.9008,25.5918]
11	[4.5013,5.0418]	[29.9206,25.3035]
12	[4.4605,5.0488]	[29.9513,24.9448]
13	[4.4184,5.0568]	[29.9895,24.5789]
14	[4.3342,5.0747]	[30.0869,23.8603]
15	[4.2043,5.1075]	[30.2913,22.7834]
16	[3.9583,5.1866]	[30.8587,20.8547]
17	[2.1179,7.1673]	[53.4886,11.6528]
18	[2.0849,7.2913]	[55.2486,11.6382]

The Entropy-Based Constrained Objective Function Method

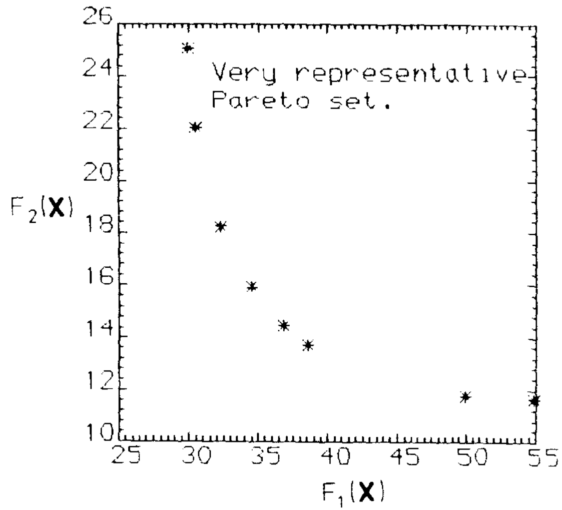
1	[4.4757,5.0463]	[29.94075,25.07806]
2	[4.1126,5.1344]	[30.47424,22.04742]
3	[3.5874,5.3533]	[32.24579,18.22313]
4	[3.2135,5.5898]	[34.45959,15.91654]
5	[2.9425,5.8167]	[36.77594,14.47511]
6	[2.7789,5.9833]	[38.57903,13.70582]
7	[2.2052,6.9096]	[49.94713,11.77235]
8	[2.0917,7.2641]	[54.85886,11.63919]

Table (4.4)

Results of Calculations for Example (4.3.A.2)



The Weighted Objectives Method



The Constrained Objective Method

Fig. (4.5)

Pareto Optimal Sets by the Entropy-Based Minimax Methods of Example (4.3.A.2) from the given set of materials then we shall have a new design variable. The beam should satisfy two objectives:

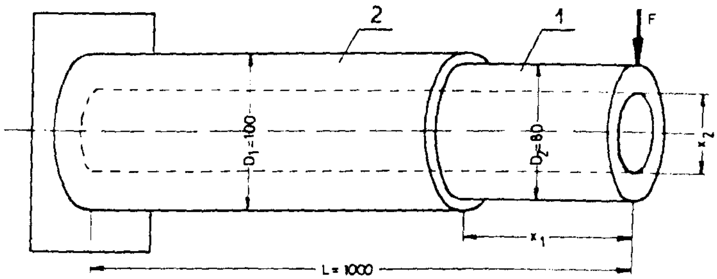


Fig. (4.6)

Drawing of the Beam (Osyczka, 1984)

(1) the minimization of the volume of the beam, and

(2) the minimization of the static compliance of the beam.

The function describing the volume of the beam is:

$$F_1(X) = (\pi/4)\ [X_1\ (D^2_2 - X^2_2) + (\ell - X_1)\ (D^2_1 - X^2_2)]$$

The static compliance of the beam for the displacement under the force F is:

$$F_2\ (X) = \frac{64}{3\pi E}\left\{\left(\frac{1}{D^4_2 - X^4_2} - \frac{1}{D^4_1 - X^4_2}\right) X^3_1 + \frac{\ell^3}{D^4_1 - X^4_2}\right\},$$

where E is Young's modulus. We assumed that ℓ = 1000 mm, D_1 = 100 mm, D_2 = 80 mm and E = 2.06×10^5 N/mm^2.

Also, it is assumed that the beam should resist the maximum force $F_{max} = 12000$ N and the permissible bending stress of the beam material is $\sigma_g = 180$ N/mm. Thus, the bending strength constraints are:

for part 1:

$$\frac{X_1 \; F_{max}}{[(D^4{}_2 - X^4{}_2)/(32D_2)]} \leq \sigma_g$$

and for part 2:

$$\frac{\ell \; F_{max}}{[(D^4{}_1 - X^4{}_2)/(32D_1)]} \leq \sigma_g$$

Assuming also that the interior diameter of the beam is to be no less than 40 mm. After substitution, the optimal-shape design problem may be formulated as follows:

Optimize:

$$F_1(\mathbf{X}) = .785 \left[X_1 \ (6400. - X^2{}_2) + (1000. - X_1) \ (10^4 - X^2{}_2)\right] \text{ mm}^3 \rightarrow \text{Min}$$

$$F_2(\mathbf{X}) = 3.298 \times 10^{-5} \left\{\left(\frac{1}{4.096 \times 10^7 - X^4{}_2} - \frac{1}{10^8 - X^4{}_2} \right) X^3{}_1 + \frac{10^9}{10^8 - X^4{}_2}\right\} \text{ mm/N} \rightarrow \text{Min}$$

S.t.:

$$g_1(\mathbf{X}) = 180 - 9.78 \times 10^6 X_1/ (4.096 \times 10^7 - X^4{}_2) \geq 0$$

$$g_2(\mathbf{X}) = 75.2 - X_2 \geq 0$$

$$g_3(\mathbf{X}) = X_2 - 40 \geq 0$$

$$X_1, \ X_2 \geq 0$$

The results are summarized in Table (4.5) and Figs (4.7-4.9). Referring to Table (4.5), the designer chooses one solution. This choice is based on his intuition and experience.

The Entropy-Based Weighted Objectivs Method			
No.	$X = [X_1, X_2]$		$F(X) = [F_1(X), F_2(X)]$
1	[165.29,75.2]		[0.2943695e+07,0.49924662e-03]
2	[183.58,74.609]		[0.2961524e+07,0.495371176e-03]
3	[192.75,74.307]		[0.2970895e+07,0.493597705e-03]
4	[202.39,73.986]		[0.2981043e+07,0.491856365e-03]
5	[212.51,73.644]		[0.2992045e+07,0.490157399e-03]
6.	[219.88,73.392]		[0.3000292e+07,0.489001861e-03]
7	[223.67,73.262]		[0.3004618e+07,0.488433288e-03]
8	[152.71,40.0]		[0.6162431e+07,0.340317842e-03]
9	[145.21,40.0]		[0.6183632e+07,0.340058003e-03]
10	[137.95,40.0]		[0.6204248e+07,0.33983076e-03]
11	[132.72,40.0]		[0.6218924e+07,0.33968105e-03]
12	[131.05,40.0]		[0.6223628e+07,0.33963611e-03]
13	[124.50,40.0]		[0.624214e+07,0.339468941e-03]
14	[110.72,40.0]		[0.6281057e+07,0.339171384e-03]
15	[101.02,40.0]		[0.6308531e+07,0.339000951e-03]
16	[96.227,40.0]		[0.6321995e+07,0.338929007e-03]
17	[74.569,40.0]		[0.6385737e+07,0.33867266e-03]
18	[1.0,40.0]		[0.6591174e+07,0.33846451e-03]
The Entropy Based Constraint Objectives Method			
1	[165.30,75.199]	A	[0.2943734e+07,0.4992401e-03]
2	[182.30,74.651]		[0.2960245e+07,0.4956268e-03]
3	[194.01,74.265]		[0.2972194e+07,0.4933646e-03]
4	[206.16,73.859]		[0.2985099e+07,0.4912079e-03]
5	[217.11,73.487]		[0.2997181e+07,0.4894268e-03]
6	[221.51,73.336]		[0.3002174e+07,0.4887516e-03]
7	[221.43,71.549]		[0.3205612e+07,0.4663430e-03]
8	[232.23,71.191]		[0.3215188e+07,0.4652811e-03]
9	[225.57,67.174]		[0.3670394e+07,0.4277397e-03]
10	[236.85,66.252]		[0.3735082e+07,0.4232714e-03]
11	[219.06,66.058]		[0.3805428e+07,0.4189061e-03]
12	[225.08,65.402]		[0.3856105e+07,0.4156467e-03]
13	[208.21,64.240]	C	[0.4022139e+07,0.4063430e-03]

14	[231.21,62.351]		[0.4144831e+07,0.3994876e-03]
15	[234.36,62.070]		[0.4163319e+07,0.3985558e-03]
16	[242.77,61.416]		[0.4203013e+07,0.3966531e-03]
17	[234.36,58.967]		[0.4458198e+07,0.3850318e-03]
18	[231.06,58.955]		[0.4468607e+07,0.3845755e-03]
19	[228.60,58.361]		[0.4530231e+07,0.3820444e-03]
20	[213.70,57.052]		[0.4691012e+07,0.3758790e-03]
21	[232.50,55.351]		[0.4787910e+07,0.3725172e-03]
22	[226.02,54.971]		[0.4839180e+07,0.3707132e-03]
23	[216.04,54.552]		[0.4903343e+07,0.3685560e-03]
24	[228.51,53.746]		[0.4936624e+07,0.3675970e-03]
25	[228.51,52.481]		[0.5042167e+07,0.3644039e-03]
26	[228.51,51.345]		[0.5134723e+07,0.3617750e-03]
27	[228.51,50.906]		[0.5169965e+07,0.3608146e-03]
28	[228.51,50.241]		[0.5222748e+07,0.3594169e-03]
29	[228.51,49.383]		[0.5289871e+07,0.3577087e-03]
30	[193.97,48.748]		[0.5436395e+07,0.3538036e-03]
31	[202.94,47.265]		[0.5522785e+07,0.3518865e-03]
32	[192.28,46.483]		[0.5610471e+07,0.3499517e-03]
33	[175.45,46.399]		[0.5664181e+07,0.3488639e-03]
34	[175.45,44.620]		[0.5791270e+07,0.3463724e-03]
35	[175.45,42.870]		[0.5911458e+07,0.3442250e-03]
36	[192.55,40.0]		[0.6049859e+07,0.3421793e-03]
37	[175.45,40.0]		[0.6098175e+07,0.3412750e-03]
38	[154.20,40.0]		[0.6158251e+07,0.3403721e-03]
39	[147.19,40.0]		[0.6178043e+07,0.3401239e-03]
40	[134.01,40.0]		[0.6215292e+07,0.3397167e-03]
41	[132.52,40.0]		[0.6219488e+07,0.3396757e-03]
42	[126.21,40.0]		[0.6237336e+07,0.3395106e-03]
43	[115.18,40.0]		[0.6268511e+07,0.3392596e-03]
44	[109.69,40.0]		[0.6284024e+07,0.3391511e-03]
45	[103.31,40.0]		[0.6302049e+07,0.3390382e-03]
46	[92.308,40.0]		[0.6333136e+07,0.3388738e-03]
47	[85.273,40.0]		[0.6353017e+07,0.3387872e-03]
48	[77.439,40.0]		[0.6375157e+07,0.3387062e-03]
49	[51.448,40.0]		[0.6448607e+07,0.3385353e-03]
50	[48.834,40.0]		[0.6455994e+07,0.3385250e-03]
51	[44.642,40.0]		[0.6467846e+07,0.3385106e-03]
52	[42.974,40.0]		[0.6472553e+07,0.3385057e-03]
53	[0.0,40.0]	B	[0.6593964e+07,0.3384645e-03]

Table (4.5)

Results of Calculations for Example (4.3.A.3)

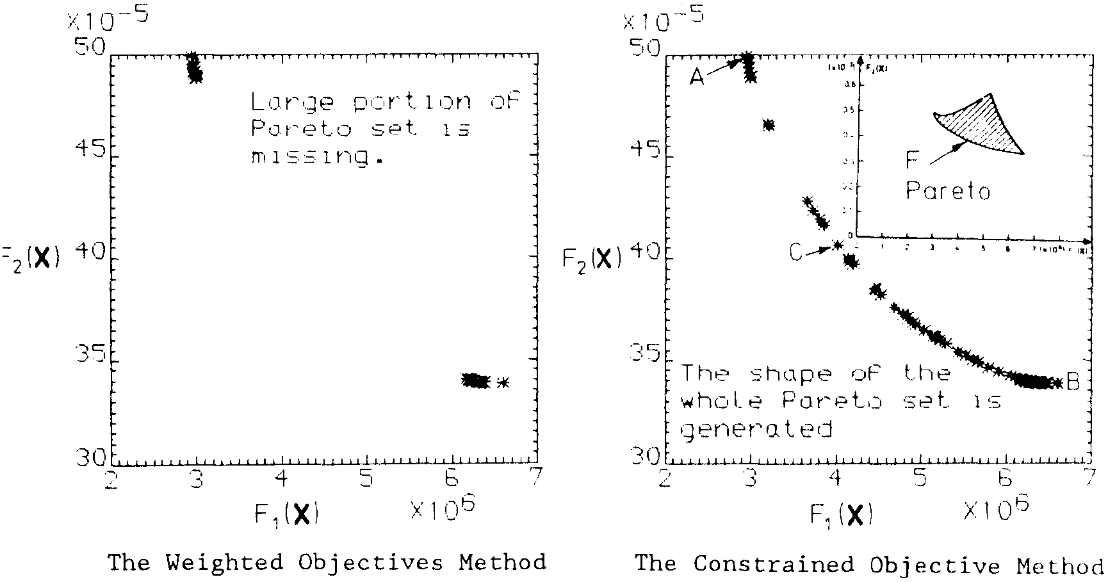


Fig. (4.7)
Pareto Sets Generated by the Entropy-Based Minimax Methods

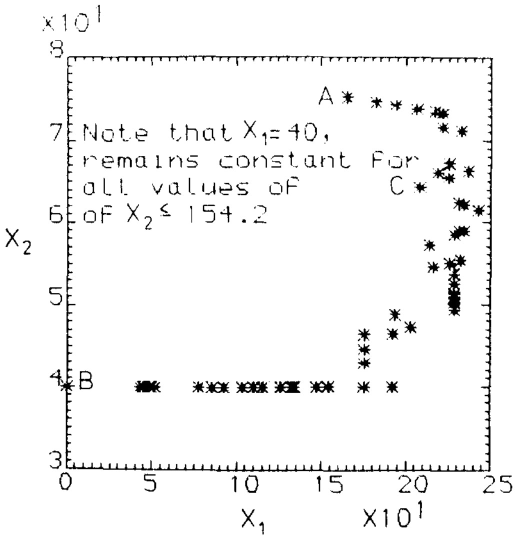


Fig. (4.8)
The Design - Design Pareto Set Generated by the ECOF Method

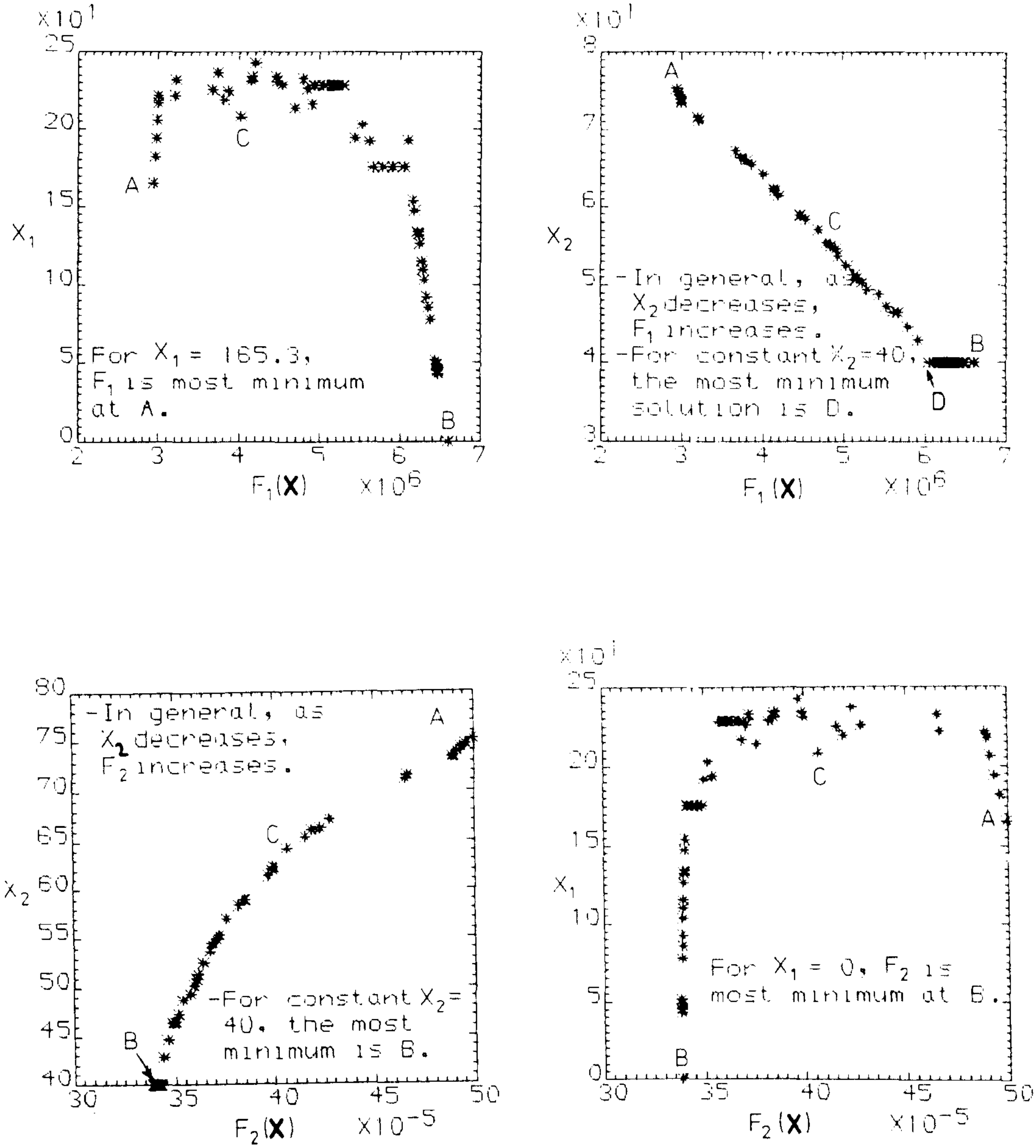


Fig. (4.9)

The Design-Performance Pareto Sets Generatd by the ECOF Method

Example 4.3.A.4 (Singh, 1987)

The dressing depth of cut and the dressing lead are the two important dressing variables which significantly affect the surface roughness and specific grinding energy. Normally, to ensure good grinding performance, it is desirable to have, among other things, low surface roughness and, at the same time, low specific grinding energy. A finely dressed wheel gives low surface roughness but at the same time, a higher specific griding energy, and vice versa. Therefore, both criteria of good grinding performance are in conflict and cannot be achieved simultaneously. Experiments were conducted to establish functional relationships between the dressing variables and the surface roughness as well as specific griding energy under reciprocating plunge surface grinding conditions which may be cast as follows:

Optimize:

$$F_1(X) = -2.4798 + 0.55\,X_1 + 0.22\,X_2 \rightarrow \text{Min}$$

$$F_2(X) = 3.7366 - 0.56\,X_1 - 0.27\,X_2 \rightarrow \text{Min}$$

S.t.: $g_1(X) = 3.18 - X_1 \geq 0$

$g_2(X) = X_1 - 2.40 \geq 0$

$g_3(X) = 2.70 - X_2 \geq 0$

$g_4(X) = X_2 - 2.17 \geq 0$

$X_1,\ X_2 \geq 0$

The results are summarized in Table (4.6).

The Entropy-Based Weighted Objective Functions Method

No.	$\mathbf{X} = [X_1, X_2]$	$F(\mathbf{X}) = [F_1(\mathbf{X}), F_2(\mathbf{X})]$
1	[3.18,2.7]	[-0.1368,1.2268]
2	[2.4,2.7]	[-0.5658,1.6636]
3	[2.4,2.5987]	[-0.5881,1.691]
4	[2.40,2.5159]	[-0.6063,1.7133]
5	[2.40,2.4594]	[-0.61873,1.7286]
6	[2.40,2.426]	[-0.6261,1.7376]
7	[2.40,2.373]	[-0.6377,1.7519]

The Entropy-Based Constrained Objective Function Method

No.	$\mathbf{X} = [X_1, X_2]$	$F(\mathbf{X}) = [F_1(\mathbf{X}), F_2(\mathbf{X})]$
1	[3.18,2.70]	[-0.1368,1.2268]
2	[3.0560,2.70]	[-0.2050,1.2962]
3	[2.9648,2.70]	[-0.2552,1.3473]
4	[2.8292,2.70]	[-0.3297,1.4233]
5	[2.7395,2.70]	[-0.3791,1.4735]
6	[2.6024,2.70]	[-0.4545,1.5503]
7	[2.4794,2.70]	[-0.5221,1.6191]
8	[2.40,2.2886]	[-0.6563,1.7747]
9	[2.4,2.17]	[-0.6824,1.8067]

Table (4.6)

Results of Calculations for Example (4.3.A.4)

Example 4.3.A.5 (Osyczka, 1984)

This is a production planning problem to determine how many units to produce of each of two products, denoted as A and B. We could then define:

X_1 = number of units of product A to produce,

X_2 = number of units of product B to produce.

Both products A and B require time in two departments. Product A requires 1 hour in the first department and 1.25 hours in the second department. Product B requires 1 hour in the first department and 0.75 hours in the second department. The available hours in each department are 200 monthly. Furthermore, there is a maximum market potential of 150 units for product B. Assume that product A is of high quality and product B is of lower quality. The profits are \$4 and \$5 per product respectively. The best customer of the company wishes to have as many as possible of type A product. We realise that two objectives:

(1) the maximization of profit,

(2) the maximum production of product A,

should be considered in this problem. The graphical illustration of the above problems is shown in Fig. (4.10). Hence, the objective functions and constraints can be written as follows:

Optimize:

$$F_1(\mathbf{X}) = 4\,X_1 + 5X_2 \qquad \rightarrow \text{Max}$$

$$F_2(\mathbf{X}) = X_1 \qquad \rightarrow \text{Max}$$

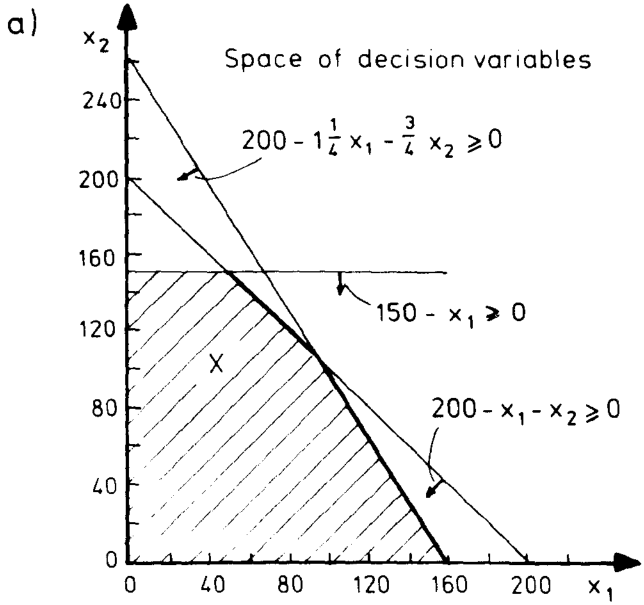
S.t.: $g_1(\mathbf{X}) \equiv X_1 + X_2 - 200 \leq 0$

$$g_2(\mathbf{X}) \equiv 1.25\,X_1 - 0.75\,X_2 - 200 \leq 0$$

$$g_3(\mathbf{X}) \equiv X_2 - 150 \leq 0$$

$$X_i \geq 0;\ i = 1,2$$

The results are summarized in Table (4.7) and Fig. (4.11). This example is a linear programming problem. The decision maker decides a priority that in this problem the profit is more important than the number of units of product A which reflects the wishes of the best customer. He must now express his preferences in a formalized way. He assumes that he can afford to satisfy the wishes of the customer providing that it will cost him less than 5.26% of the profit he would make if he disregarded these wishes. The solution which gives the



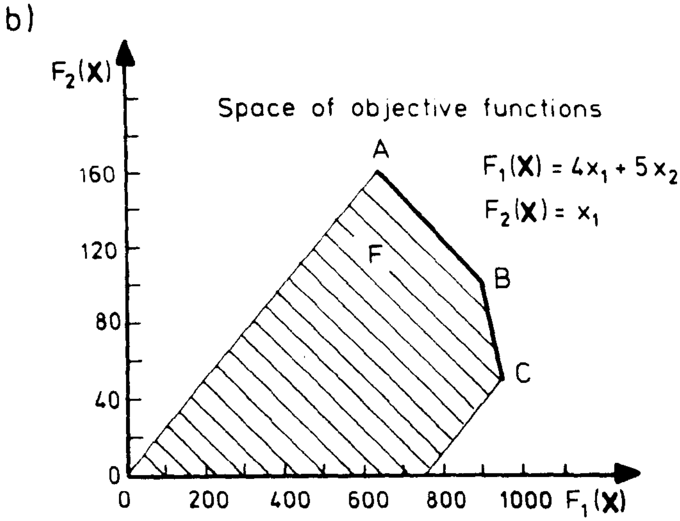
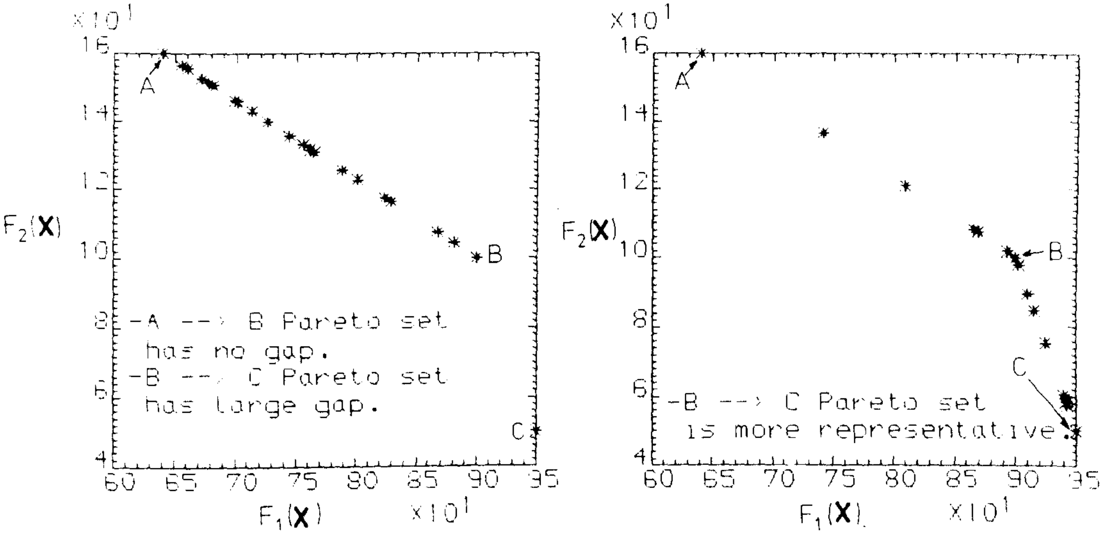


Fig. (4.10) (Osyczka, 1984)

Graphical Illustration of the Production Planning Problem maximum profit is F(X) = [950.0,50.0]. The compromise solution which gives 5.26% of the profit decrement is F(X) = [900.0,100.0]. However, the manager may be unable to determine his preferences by giving the strict values of the function decrement and then he may want to compare solutions for which the values of the profit decrements change in some range, for example between 4% and 12%. In this case, the resuls from Table (4.7) can help him to make the right decision.

No.	$X = [X_1, X_2]$	$F(X) = [F_1(X), F_2(X)]$
The Entropy-Based Weighted Objective Functions Method		
1	[50.0,150.0]	[950.0,50.0]
2	[100.0,100.0]	[900.0,100.0]
3	[104.43,92.619]	[880.8125,104.428864]
4	[107.755,87.418]	[867.286377,107.549561]
5	[116.45,72.584]	[828.718994,116.450287]
6	[117.72,70.475]	[823.234131,117.715149]
7	[122.90,61.842]	[800.793213,122.895233]
8	[125.90,56.831]	[787.761719,125.901764]
9	[131.35,47.742]	[764.128906,131.354828]
10	[132.00,46.659]	[761.312012,132.004852]
11	[133.40,44.338]	[755.281494,133.397659]
12	[135.93,40.117]	[744.303467,135.929932]
13	[140.15,33.077]	[725.999023,140.153976]
14	[143.07,28.217]	[713.364258,143.069763]
15	[145.90,23.506]	[701.118408,145.896698]
16	[146.44,22.593]	[698.740479,146.444427]
17	[150.75,15.418]	[680.087891,150.750015]
18	[151.47,14.224]	[676.981934,151.465668]
19	[152.62,12.308]	[672.0,152.615234]
20	[155.45,7.5848]	[659.720703,155.449219]
21	[156.53,5.7849]	[655.041748,156.529419]
22	[159.99,0.0]	[640.0,160.0]
The Entropy Based Constrained Objective Function Method		
1	[50.0,150.0]	[950.0,50.0]
2	[57.404,142.60]	[942.5955,57.40358]
3	[58.193,141.81]	[941.8069,58.19331]
4	[60.399,139.60]	[939.6001,60.39946]
5	[75.527,124.47]	[924.4707,75.52701]
6	[84.416,115.58]	[915.5845,84.41553]
7	[89.620,110.38]	[910.3801,89.61966]
8	[97.794,102.21]	[902.2051,97.79425]
9	[100.0,100.0]	[900.0,100.0]
10	[101.71,97.148]	[892.585,101.7117]
11	[107.15,88.091]	[869.0393,107.1455]
12	[107.82,86.962]	[866.1006,107.8228]
13	[120.96,65.076]	[809.1987,120.9553]
14	[136.69,38.845]	[740.9973,136.6929]
15	[160.0,0.0]	[640.0281,160.0]

Table (4.7)
Results of Calculations for Example (4.3.A.5)



The Weighted Objectives Method The Constrained Objective Method

Fig. (4.11)

Pareto Sets Generated by the Entropy-Based Minimax Methods

Example 4.3.A.6

This example is also purely numerical and has no physical or engineering interpretation except that it is three-criteria optimization problem.

Optimize:

$$F_1(\mathbf{X}) = X_1 + X^2_2 \rightarrow \text{Min}$$

$$F_2(\mathbf{X}) = X^2_1 + X_2 \rightarrow \text{Min}$$

$$F_3(\mathbf{X}) = (X_1 - 5)^2 + (X_2 - 8)^2 + 25 \rightarrow \text{Min}$$

S.t.: $g_1(\mathbf{X}) = - X^2_1 + 10\, X_1 - X^2_2 + 16\, X_2 - 80 \geq 0$

$X_1,\ X_2 \geq 0$

The results are summarized in Table (4.8) and Fig. (4.12).

4.3.B Discussion

Quadratic programming, dynamic programming or convex programming were developed originally for the solution of single criteria problems. However, they have been combined with the Pareto set generating techniques to solve multi-criteria optimization problems. In this thesis, mathematical programming has been combined with the developed entropy-based techniques to solve multi-criteria optimization problems by generating its Pareto sets.

In the preceding section several examples were solved and their solutions were presented. Although it is difficult to select a preferred solution when the problem is purely numerical and has no physical or engineering interpretation, which is the case with the first two examples, the only remaining purpose is showing the Pareto solutions. In such cases the shape of the Pareto optimal curve is the main significance to the decision maker and this can be reached by generating as many solutions as possible. As shown in Figs. (4.3 and 4.5), the Pareto optimal set defines a straight line or a very flat curve which means the tradeoff between $F_1(X)$ and $F_2(X)$ will remain constant over the span of the Pareto optimal set.

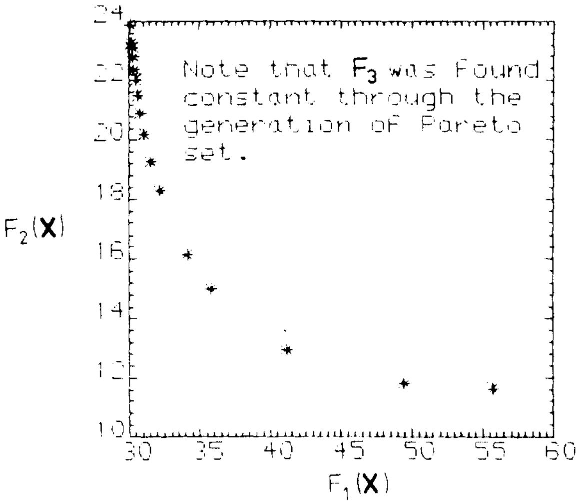
The Entropy-Based Weighted Objective Functions Method

No.	$X = [X_1, X_2]$	$F(X) = [F_1(X), F_2(X), F_3(X)]$
1	[4.3393,5.0737]	[30.0813,23.9031,34.0]
2	[4.2636,5.0919]	[30.1909,23.2699,34.0]
3	[4.2434,5.0971]	[30.2233,23.1033,34.0]
4	[4.2063,5.1069]	[30.287,22.7996,34.0]
5	[4.1555,5.1212]	[30.3819,22.3892,34.0]
6	[4.1090,5.1354]	[30.4811,22.0189,34.0]
7	[4.0439,5.1564]	[30.6328,21.5096,34.0]
8	[3.9656,5.1840]	[30.8392,20.9102,34.0]
9	[3.8685,5.2216]	[31.1336,20.1869,34.0]
10	[3.7422,5.2755]	[31.5754,19.2947,34.0]
11	[3.5971,5.3482]	[32.2006,18.2874,34.0]
12	[3.2552,5.5597]	[34.1654,16.1561,34.0]
13	[3.0471,5.7226]	[35.7958,15.0077,34.0]
14	[2.5939,6.2082]	[41.1352,12.9363,34.0]
15	[2.2189,6.875]	[49.4843,11.7984,34.0]
16	[2.0771,7.3243]	[55.7229,11.6387,34.0]

The Entropy-Based Constrained Objective Function Method

1	[3.0341,8.3716]	[73.11850,17.57733,29.00291]
2	[3.3923,7.9887]	[67.21158,19.49637,27.58484]
3	[2.3873,7.8146]	[63.45506,13.51391,31.86044]
4	[2.0814,7.3168]	[55,61745,11.64924,33.98468]
5	[2.3800,7.232]	[54.68159,12.89636,32.45427]
6	[3.13,6.8473]	[50.01587,16.64418,29.82558]
7	[2.7875,6.7001]	[47.67912,14.47050,31.58466]
8	[3.0143,6.5957]	[46.51712,15.68193,30.91498]
9	[3.5143,6.3821]	[44.24493,18.73265,29.82492]
10	[2.5181,6.3147]	[42.39381,12.65540,34.0]
11	[3.5876,6.0952]	[40.73869,18.96632,30.62314]
12	[3.6693,6.066]	[40.46576,19.52956,30.51114]
13	[3.8961,5.9873]	[39.74387,21.16669,30.26959]
14	[4.3961,5.8283]	[38.36531,25.15375,30.08095]
15	[3.4574,5.4269]	[32.90865,17.38052,34.0]
16	[3.5035,5.3999]	[32.66278,17.67416,34.0]
17	[3.852,5.2283]	[31.18718,20.06621,34.0]
18	[3.9074,5.2060]	[31.01013,20.47374,34.0]
19	[4.0132,5.1670]	[30.71066,21.27275,34.0]
20	[4.1499,5.1231]	[30.3960,22.34462,34.0]
21	[4.4244,5.0557]	[29.98438,24.63060,34.0]

Table (4.8)
Results of Calculations for Example (4.3.A.6)



$[F_3 = 34]$

Fig. (4.12)

Pareto Optimal Set by the EWOF Method of Example (4.3.A.6)

The third example is about design. The exploration of the relationships between the design variables (X_1 and X_2) and solution performances (F_1 and F_2) is possible only through multi-criteria optimization by generating the Pareto solutions. Three types of relationships can be recognised:

1) The performance-performance relationships, Fig. (4.7).

2) The design-design relationships, Fig. (4.8).

3) The design-performance relationships, Fig. (4.9).

The performance space and the design space relationships can directly be investigated by generating the Pareto set efficiently. Pareto solution A gives us the least volume while Pareto solution B gives us the least static compliance. These two solutions are extremes and the rest of the Pareto set solutions are in between, such as solution C.

By the performance-performance relationships we mean the implications of choosing a certain level of performance in one criterion on the performances that are then attainable in other criteria. In terms of our beam design problem, the designer has to decide how much worsening of the beam volume, F_1, we are prepared to accept in order to lower static compliance. In this case, the Pareto set of performances follows the classic convex shape where improvement in volume has a steadily increasing rate of effect on static compliance. In the central portion, where C exists, the balance is even and even performances here likely to represent good compromise solutions.

By the design-design relationships we mean the implications of choosing or restricting values for one deign variable on the values that need to be given to other design variables if acceptable performances are to be maintained. By studying these three types of relationships, the designer can easily select his preferred solution. The information we can get from the design-design space is that the interior diameter of the inner beam, X_2, stays constant for values of $X_1 \leq 154.2$. However, the relationship between X_1 and X_2 is not stable since design variables do not need to follow a stable relationship necessarily, because they are not objectives, but decisions vary in one way or another to satisfy optimality.

By the design-performance relationships, we mean the implications of choosing a certain level of one design on the performances that are then attainable in other criteria. The design-performance spaces, (F_1, X_2) and (F_2, X_2), follow stable shape while the design-performance spaces, (F_1, X_1) and (F_2, X_1), follow an unstable one for the same reasons mentioned before.

In the fourth example, we were able to obtain more Pareto solutions using the entropy-based methods than the ones obtained by non-entropy based methods. The shape of the Pareto optimal set defines a straight line which means the tradeoff between $F_1(\mathbf{X})$ and $F_2(\mathbf{X})$ will remain constant over the span of the Pareto optimal set. In addition, the same thing about the exploration of the relationships between the design variables (X_1 and X_2) and solution performances (F_1 and F_2) can be mentioned here.

The last example is also purely mathematically numerical which has no physical or engineering interpretation. By investigating the relative merits of the entropy-based weighted method and the entropy-based constrained method through solving several numerical examples, we have shown clearly that the entropy-based weighted method alone is inadequate to produce representative Pareto set because of a large gap between two groups of solutions. There is no indication of whether these gaps are by reason of the set being concave in that region or there being no solutions in that region. The entropy-based constrained method provides more evenly distributed information and should be used in addition to the first method to investigate large gaps.

The numerical results obtained by the developed minimax methods make these methods very efficient computer-aided design tools. They are not only computationally efficient and easy to use, but they also generate very good representations of a complete Pareto set.

It is very important to note that, no matter how many criteria we consider, we always need to play with one parameter to generate the Pareto set. This property of the developed methods is very unique. In addition, it is notable that the size of the Pareto set generated by the developed methods is, in general, larger than the one generated by one of the known methods since generating Pareto set can be done quicker than before and more solutions are obtained.

In the preceding section, s x examples were presented concisely on two-dimensionally plots of the approximated Pareto optimal set in the performance space as shown before.

The results of the new entropic vector optimization techniques developed in this thesis provide for the first time a quantity of information (objective values, design values, trade-offs) upon which designers can base their decisions at **relatively low computational costs**. This is a unique and important property, particularly when applying these techniques for problems with more than two objectives. The accuracy of the obtained results may be noted by comparing them with the corresponding graphical illustration figure or the reference taken from it. In all cases, previoulsy published results obtained by other researchers were generated by the present methods to very high accuracy.

4.4 SIMULATED ENTROPY, MINIMAX ENTROPY AND GLOBAL MINIMIZATION

Five numerical examples are solved using the new entropy-based simulated entropy techniques developed in chapter three. General discussion showing the characteristics of these techniques is included also.

4.4.A Numerical Examples

Example (4.4.A.1) (Li, 1987)

Minimize:

$$F(\mathbf{X}) = (X_1 - 2)^2 + (X_2 - 1)^2$$

$$S.t.: \; g_1(X) \equiv X^2_1 - X_2 \leq 0$$

$$g_2(X) \equiv X_1 + X_2 - 2 \leq 0$$

$$X_1, X_2 \geq 0$$

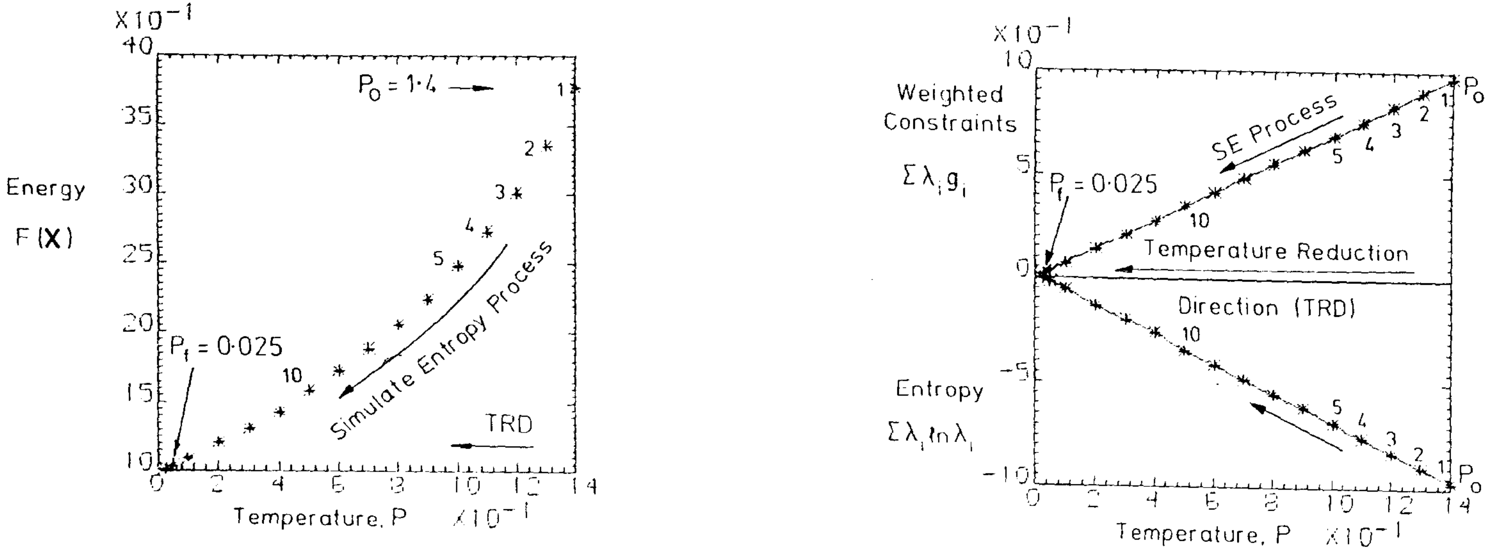
The results are summarized in Table (4.9) and Fig. (4.13).

P	F	X_1	X_2	g_1	g_2
1.4	3.7791	.0563	.9656	-.96245	-.97808
1.3	3.3557	.1685	.9654	-.937	-.8662
1.2	3.0168	.2634	.9698	-.9004	-.7669
1.1	2.7322	.3472	.9762	-.8556	-.6766
1.0	2.4862	.4233	.9838	-.8046	-.5928
0.9	2.2387	.5084	.8824	-.6239	-.6092
0.8	2.0592	.5697	.8836	-.5590	-.5467
C.7	1.8872	.6311	.8848	-.4866	-.4841
0.6	1.7286	.6902	.8857	-.4093	-.4241
0.5	1.5835	.7468	.8859	-.3282	-.3673
0.4	1.4256	.8062	.9798	-.3299	-.2140
0.3	1.3076	.8580	.9418	-.2057	-.2003
0.2	1.2018	.9052	.9432	-.1237	-.1516
0.1	1.0826	.9596	.9848	-.064	-.0555
0.05	1.0345	.9829	1.0009	-.0349	-.0162
0.025	1.0104	.9948	.9962	-.0066	-.0089

Table (4.9)

The Global Minimum by the ESC Method of Example (4.4.A.1)

As noted, we started the simulated entropy process at a maximum temperature of 1.4 which is correspondent to a maximum energy of 3.7791 and minimum values of constraints $g_1(X)$ and $g_2(X)$. Lowering the temperature continuously towards zero along the process, the energy of the system F(X) decreases continuously along the process to its minimum and the corresponding constraints values increase continuously towards zero. The simulated entropy process stops when at least one original constraint approaches zero.



(A)

(B)

Fig. (4.13)

Example (4.4.A.1)
(A) Temperature, P Vs. Energy, F
(B) Temperature Vs. Entropy ($\sum \lambda_i \ell n\ \lambda_i$) & the SCF ($\sum \lambda_i F_i$)

Example (4.4.A.2) (Li, 1987)

$$\text{Minimize:} \quad F(\mathbf{X}) = (X_1 X_2 X_3)^{-1}$$

$$\text{S.t.:} \quad g_1(\mathbf{X}) \equiv 2X_1 + X_2 + 3X_3 - 1 \le 0$$

$$g_2(\mathbf{X}) \equiv X_1 + X_2 + X_3 - 1 \le 0$$

$$g_3(\mathbf{X}) \equiv X_1 + 3x_2 + 2X_3 - 1 \le 0$$

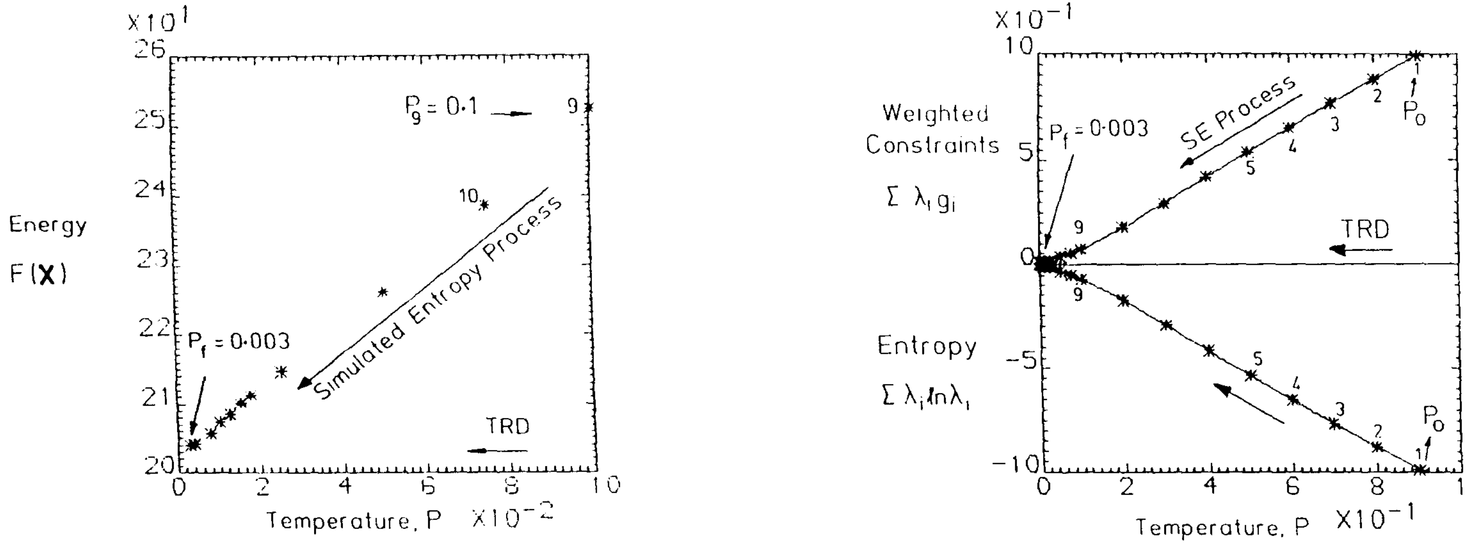
$$X_1,\ X_2,\ X_3 \ge 0$$

The results are summarized in Table (4.10) and Fig. (4.14).

P	F	X_1	X_2	X_3	g_1	g_2	g_3
.9	84749000.0	.002998	.002233	.00176	-.9865	-.993	-.9868
.8	78000.0	.02002	.02351	.02724	-.8547	-.9292	-.855
.7	11564.0	.03744	.04452	.05189	-.7249	-.8662	-.7252
.6	3708.8	.05444	.06508	.07611	-.5977	-.8044	-.5981
.5	1434.5	.1150	.08881	.06825	-.4764	-.7279	-.4821
.4	778.05	.1299	.1079	.09172	-.3572	-.6705	-.3631
.3	487.20	.1438	.1256	.1136	-.2459	-.617	-.2521
.2	331.83	.1844	.1484	.1101	-.1524	-.5571	-.1502
.1	252.37	.1942	.1614	.1264	-.07096	-.518	-.0687
.075	238.63	.1963	.1643	.1299	-.0533	-.5095	-.051
.05	226.14	.1984	.1671	.1334	-.0359	-.5011	-.0336
.025	214.62	.2005	.1699	.1368	-.0188	-.4928	-.0162
.0175	211.27	.2011	.1708	.1378	-.0136	-.4903	-.0109
.015	210.17	.2013	.1711	.1382	-.01185	-.4895	-.00912
.0125	208.49	.2065	.1689	.1376	-.00547	-.4871	-.0118
.01	207.41	.2063	.1742	.1341	-.0108	-.4853	-.0027
.0075	205.72	.2097	.1709	.1356	-.00279	-.4838	-.0063
.004	204.13	.2098	.1731	.1349	-.0026	-.4822	-.00113
.003	203.95	.2112	.1713	.1355	-.00031	-.482	-.0000388

Table (4.10)

The Global Minimum by the ESCM of Example (4.4.A.2)



(A) (B)

Fig. (4.14)

Example (4.4.A.2)
(A) Temperature Vs. Energy
(B) Temperature Vs. Entropy & the SCF

Example (4.4.A.3) (Hock, et. al. 1981)

Minimize:

$$F(\mathbf{X}) = 9X^2_1 + X^2_2 + 9X^2_3$$

S.t.:

$$X_1 X_2 - 1 \geq 0$$

$$-10 \leq X_1 \leq 10$$

$$1.0 \leq X_2 \leq 10$$

$$-10 \leq X_3 \leq 1.0$$

$$X_1, X_2, X_3 \geq 0$$

The results are summarized in Table (4.11) and Fig. (4.15).

Note that during the simulated entropy process, the third design variable and the last two constraints were constant. The lowest free energy, $F(\mathbf{X}) = 6.0448$, was obtained at temperature $P = 0.2$ in which the first constraint was active. So far, each of the minimum energy obtained by simulated entropy is very close to the one given in its corresponding reference.

Example (4.4.A.4) (Hock, et. al. 1981)

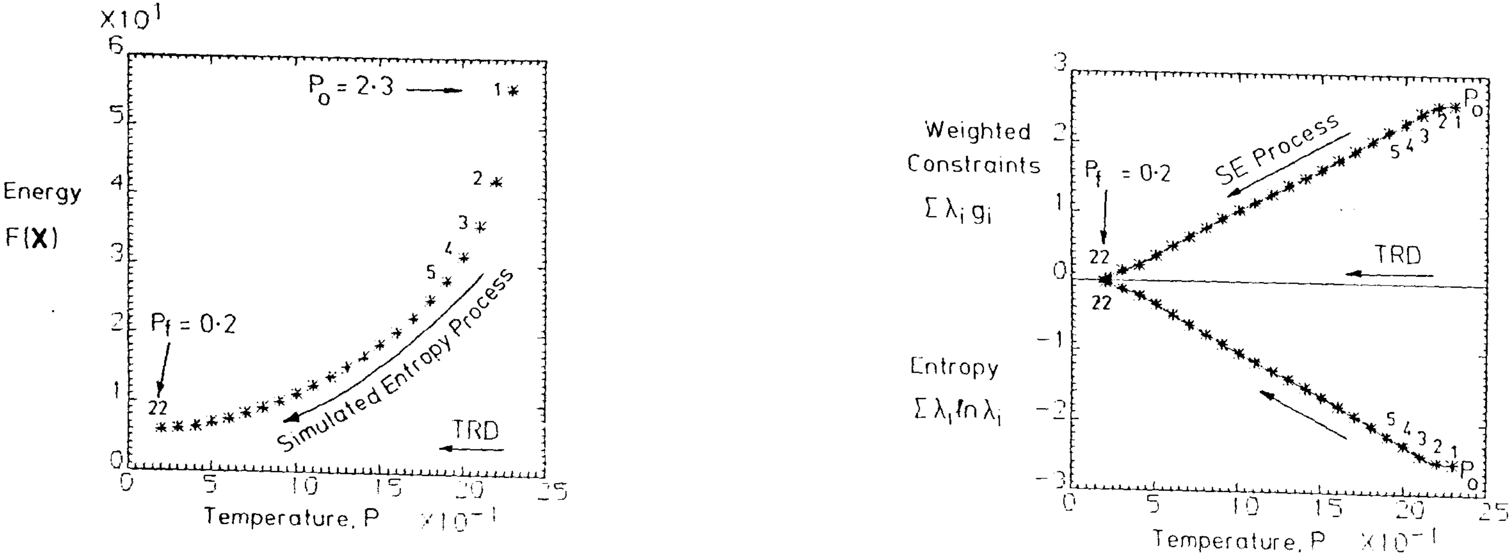
Minimize:

$$F(\mathbf{X}) = X^2_1 + X^2_2 + X_1 X_2 - 14X_1 - 16X_2 + (X_3 - 10)^2$$
$$+ 4(X_4 - 5)^2 + (X_5 - 3)^2 + 2(X_6 - 1)^2 + 5X^2_7$$
$$+ 7(X_8 - 11)^2 + 2(X_9 - 10)^2 + (X_{10} - 7)^2 + 45$$

P	F(X)	X_1	X_2	X_3	g_1	g_2	g_3	g_4	g_5	g_6	g_7
2.3	55.553	1.7058	5.4189	.0001	-8.2438	-8.2942	-11.7058	-4.5811	-4.4189	-.9999	-10.0
2.2	42.368	1.2837	5.2476	.0001	-5.7363	-8.7163	-11.2837	-4.7524	-4.2476	-.9999	-10.0
2.1	35.739	1.1883	4.7991	.0001	-4.7026	-8.8117	-11.1883	-5.2009	-3.7991	-.9999	-10.0
2.0	31.245	1.1604	4.3733	.0001	-4.075	-8.8396	-11.1604	-5.6267	-3.3733	-.9999	-10.0
1.9	27.814	1.0146	4.3069	.0001	-3.3698	-8.9854	-11.0146	-5.6931	-3.3069	-.9999	-10.0
1.8	24.967	0.9246	4.1561	.0001	-2.8427	-9.0754	-10.9246	-5.8439	-3.1561	-.9999	-10.0
1.7	22.45	.8775	3.9396	.0001	-2.4568	-9.1225	-10.8775	-6.0604	-2.9396	-.9999	-10.0
1.6	20.3	.8392	3.7366	.0001	-2.1358	-9.1608	-10.8392	-6.2634	-2.7366	-.9999	-10.0
1.5	18.437	.7975	3.5656	.0001	-1.8434	-9.2025	-10.7974	-6.434	-2.5656	-.9999	-10.0
1.4	16.669	.7647	3.3774	.0001	-1.5825	-9.2354	-10.7646	-6.6226	-2.3774	-.9999	-10.0
1.3	15.073	.7487	3.1667	.0001	-1.3709	-9.2513	-10.7487	-6.8333	-2.1667	-.9999	-10.0
1.2	13.684	.7193	3.0045	.0001	-1.1612	-9.2807	-10.7193	-6.9955	-2.0045	-.9999	-10.0
1.1	12.433	.6684	2.9005	.0001	-.9386	-9.3316	-10.6684	-7.0995	-1.9005	-.9999	-10.0
1.0	11.249	.6419	2.7462	.0001	-.7627	-9.3581	-10.6419	-7.2538	-1.7462	-.9999	-10.0
.9	10.147	.6262	2.5724	.0001	-.6109	-9.3738	-10.6262	-7.4276	-1.5724	-.9999	-10.0
.8	9.243	.588	2.4724	.0001	-.4537	-9.412	-10.588	-7.5276	-1.4724	-.9999	-10.0
.7	8.3427	.574	2.3157	.0001	-.3326	-9.4246	-10.5754	-7.6843	-1.3157	-.9999	-10.0
.6	7.6045	.5573	2.1931	.0001	-.2221	-9.4427	-10.5573	-7.8069	-1.1931	-.9999	-10.0
.5	7.0363	.5349	2.1121	.0001	-.1298	-9.4651	-10.5349	-7.8879	-1.1121	-.9999	-10.0
.4	6.5037	.5402	1.9691	.0001	-.0367	-9.4598	-10.5402	-8.0309	-.9691	-.9999	-10.0
.3	6.2028	.6070	1.699	.0001	-.0313	-9.393	-10.607	-8.301	-.699	-.9999	-10.0
.2	6.0448	.939	1.6941	.0001	-.00618	-9.4061	-10.5939	-8.3059	-.6941	-.9999	-10.0

Table (4.11)

The Global Minimum by the ESC Method of Example (4.4.A.3)



(A)

(B)

Fig. (4.15)

Example (4.4.A.3)
(A) Temperature Vs. Energy
(B) Temperature Vs. Entropy & the SCF

S.t.: $g_1(\mathbf{X}) \equiv 105 - 4X_1 - 5X_2 + 3X_7 - 9X_8 \geq 0$

$g_2(\mathbf{X}) \equiv - 10 X_1 + 8X_2 + 17X_7 - 2X_8 \geq$

$g_3(\mathbf{X}) \equiv 8X_1 - 2X_2 - 5X_9 + 2X_{10} + 12 \geq 0$

$g_4(\mathbf{X}) \equiv - 3(X_1 - 2)^2 - 4(X_2 - 3)^2 - 2X^2_3 + 7X_4 + 120 \geq 0$

$g_5(\mathbf{X}) \equiv - 5X^2_1 - 8X_2 - (X_3 - 6)^2 + 2X_4 + 40 \geq 0$

$g_6(\mathbf{X}) \equiv - .5 (X_1 - 8)^2 - 2(X_2 - 4)^2 - 3X^2_5 + X_6 + 30 \geq 0$

$g_7(\mathbf{X}) \equiv X^2_1 - 2(X_2 - 2)^2 + 2 X_1 X_2 - 14 X_5 + 6X_6 \geq 0$

$g_8(\mathbf{X}) \equiv 3X_1 - 6X_2 - 12(X_9 - 8)^2 + 7X_{10} \geq 0$

$X_i \geq 0 \; ; \; i = 1,...,10$

The results are summarized in Table (4.12) and Fig. (4.16). The minimum energy obtained by the simulated entropy, $F(\mathbf{X}) = 0.8834$, is much less than the value given by Hock et al (1981), $F(\mathbf{X}) = 24.306$. That means by using a descent algorithm alone we might escape the global minimum of the system.

<u>Example (4.4.A.5)</u> (Hock, et. al. 1981)

Minimize:

$$F(\mathbf{X}) = X^2_1 + 0.5X^2_2 + X^2_3 + 0.5X^2_4 - X_1 X_3 + X_3 X_4 - X_1 - 3X_2 + X_3 - X_4$$

S.t.: $5 - X_1 - 2X_2 - X_3 - X_4 \geq 0$

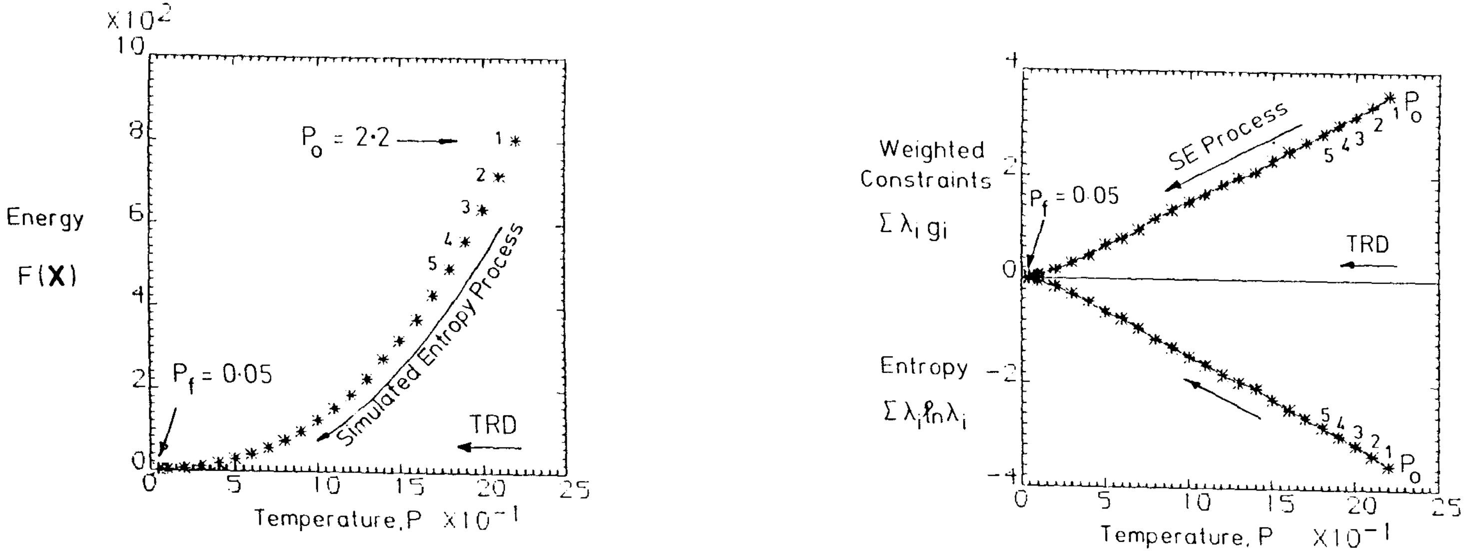
$4 - 3X_1 - X_2 - 2X_3 + X_4 \geq 0$

$X_2 + 4X_3 - 1.5 \geq 0$

$X_i \geq 0; \quad i=1,...,4$

P	F	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
2.2	804.51	5.7983	3.1609	2.5926	6.3209	.0001	11.406	4.033	3.9635	6.7673	18.2670	-2.83	-9.59	-3.65	-7.16	-12.26	-2.50	-4.58	-7.20
2.1	716.97	5.6702	3.1021	2.7512	6.2058	.0001	11.025	3.7643	4.528	6.7206	18.061	-2.49	-9.10	-3.58	-7.19	-11.85	-2.45	-4.45	-7.01
2.0	636.1	5.5323	3.0365	3.0473	6.0157	.00011	10.38	3.5219	4.9062	6.6732	17.696	-2.27	-8.65	-3.48	-7.07	-11.47	-2.37	-9.21	-6.74
1.9	559.72	5.397	3.0204	3.4602	5.7937	.00012	9.581	3.3005	5.1629	6.6548	17.144	-2.12	-8.26	-3.34	-6.8	-11.11	-2.28	-3.93	-6.42
1.8	491.08	5.257	2.980	3.8037	5.6463	.00011	8.9196	3.0889	5.4704	6.6312	16.612	-1.94	-7.87	-3.21	-6.58	-10.72	-2.21	-3.69	-6.11
1.7	427.96	5.124	2.9464	4.1661	5.5211	.00011	8.2503	2.889	5.7487	6.6174	16.018	-1.78	-7.5	-3.07	-6.31	-10.36	-2.13	-3.44	-5.79
1.6	369.53	4.9899	2.9177	4.5546	5.4207	.00011	7.5636	2.7045	5.9961	6.6152	15.323	-1.64	-7.15	-2.91	-5.97	-9.99	-2.05	-3.19	-5.45
1.5	317.38	4.8636	2.8639	4.8761	5.3472	.0001	7.0167	2.5133	6.3059	6.5970	14.756	-1.47	-6.78	-2.78	-5.68	-9.65	-1.97	-2.99	-5.14
1.4	273.26	4.7317	2.8108	5.0365	5.3089	.0001	6.6978	2.3153	6.7232	6.5476	14.477	-1.23	-6.38	-2.7	-5.59	-9.28	-1.90	-2.87	-4.89
1.3	226.25	4.5209	2.7678	5.5788	5.2341	.000125	5.8276	2.1558	6.2373	6.5600	13.486	-1.20	-6.02	-2.45	-5.01	-8.69	-1.78	-2.56	-4.43
1.2	187.59	4.2899	2.6808	5.9453	5.1956	.00011	5.2836	1.978	7.0805	6.5349	12.737	-1.11	-5.59	-2.25	-4.64	-8.06	-1.66	-2.36	-4.01
1.1	153.11	4.1783	2.6174	6.2268	5.1716	.00012	4.789	1.8184	7.3752	6.527	12.154	-.95	-5.26	-2.12	-4.26	-7.78	-1.58	-2.16	-3.72
1.0	122.7	3.9431	2.5607	6.5706	5.142	.0001	4.2743	1.6484	7.6168	6.5058	11.433	-.85	-4.85	-1.92	-3.84	-7.15	-1.96	-1.98	-3.31
.9	96.185	3.7858	2.4868	6.8592	5.127	.0001	3.789	1.4956	7.878	6.4927	10.865	-.73	-4.49	-1.76	-3.41	-6.75	-1.36	-1.78	-2.97
.8	74.556	3.595	2.3804	7.0814	5.1016	.0001	3.4435	1.3403	8.1862	6.4337	10.359	-.60	-4.09	-1.64	-3.08	-6.31	-1.23	-1.63	-2.64
.7	55.195	3.4637	2.3823	7.3272	5.0886	.00011	2.9955	1.1958	8.4593	6.5071	9.8941	-.45	-3.81	-1.48	-2.69	-5.96	-1.16	-1.48	-2.57
.6	39.926	3.2985	2.2822	7.5236	5.0665	.0001	2.6872	1.0602	8.747	6.338	9.5801	-.32	-3.45	-1.42	-2.34	-5.6	-1.05	-1.34	-2.01
.5	28.851	3.0879	2.121	7.7585	5.0358	.13811	2.3754	.9444	8.964	6.1715	9.2954	-.28	-3.06	-1.35	-1.88	-5.18	-.88	-1.06	-1.43
.4	17.447	2.79	2.2357	7.9968	5.0538	.08846	1.9241	.08846	9.1851	6.2537	8.6542	-.16	-2.74	-1.06	-1.55	-4.48	-.81	-.99	-1.26
.3	10.166	2.6574	2.158	8.1931	5.0304	.3267	1.7179	.7281	9.3699	6.1984	8.3974	-.096	-2.5	-.98	-1.12	-4.22	-.69	-.67	-.99
.2	5.262	2.5846	2.1306	8.3349	5.0148	.509	1.6071	.6775	9.4936	6.0828	8.2579	-.04	-2.36	-.97	-.81	-4.06	-.61	-.45	-.58
.1	1.8131	2.5278	2.0773	8.4856	5.0008	.7123	1.5044	.6373	9.579	5.9912	8.1477	-.014	-2.24	-.96	-.45	-3.94	-.51	-.21	-.25
.05	.8834	2.5038	2.053	8.4652	4.9941	.7471	1.4788	.6015	9.6114	5.946	8.0866	-.014	-2.16	-.96	-.49	-3.92	-.47	-.16	-.08

Table (4.12)
The Global Minimum by the ESCM of Example (4.4.A.4)



(A)

(B)

Fig. (4.16)

Example (4.4.A.4)
(A) Temperature Vs. Energy
(B) Temperature Vs. Entropy & the SCF

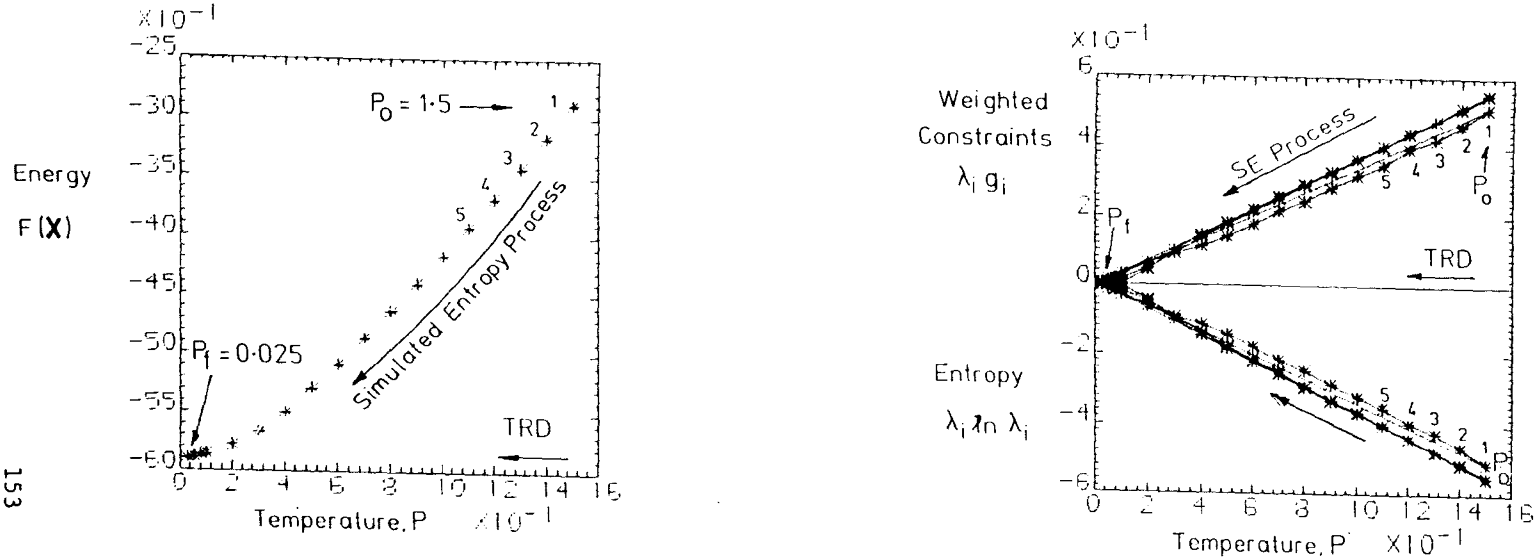
The results are summarized in Table (4.13) and Fig. (4.17). Also here, the minimum energy obtained by the SE methods, F(**X**) = - 5.8971, is less than the value given by Hock et al, (1981), F(**X**) = -4.682. This assures the necessity of using the simulated entropy techniques to secure our search for the global minimum.

P	F	x_1	x_2	x_3	x_4	g_1	g_2	g_3
1.5	-2.8795	0.00011	1.4091	0.3661	0.29904	-1.5165	-2.1574	-1.3735
1.4	-3.162	0.0001	1.5283	0.31264	0.28463	-1.3461	-2.1308	-1.2788
1.3	-3.4176	0.041	1.5785	0.273	0.29587	-1.2331	-2.0483	-1.1705
1.2	-3.6759	0.10919	1.6194	0.2575	0.30198	-1.0924	-1.84	-1.1495
1.1	-3.9249	0.14549	1.6635	0.20588	0.31367	-1.0079	-1.8019	-0.9871
1.0	-4.1706	0.2857	1.7041	0.18542	0.32017	-0.8776	-1.6195	-0.9458
0.9	-4.4028	0.26179	1.74	0.15398	0.3313	-0.7729	-1.4979	-0.8559
0.8	-4.6366	0.31235	1.7787	0.11222	0.32702	-0.691	-1.3868	-0.7276
0.7	-4.8679	0.37211	1.8238	0.091488	0.33322	-0.5556	-1.2102	-0.6897
0.6	-5.0853	0.4255	1.8601	0.05629	0.33269	-0.4654	-1.036	-0.5852
0.5	-5.2948	0.48263	1.8925	0.024625	0.33426	-0.3734	-0.9446	-0.491
0.4	-5.5005	0.5483	1.9251	0.0001	0.3293	-0.272	-0.7591	-0.4255
0.3	-5.6671	0.61991	1.9468	0.0001	0.33583	-0.1506	-0.5291	-0.4472
0.2	-5.7836	0.6751	1.9639	0.000863	0.33104	-0.0651	-0.3401	-0.4674
0.1	-5.8623	0.7306	1.9620	0.0001	0.3229	-0.0223	-0.1688	-0.4624
0.075	-5.8662	0.72921	1.9635	0.0001	0.3279	-0.0157	-0.1765	-0.4639
0.05	-5.8867	0.73933	1.9814	0.0001	0.29016	-0.007702	-0.0906	-0.4818
0.025	-5.8971	0.74389	1.9911	0.0001	0.27119	-0.00258	-0.0482	-0.4915

Table (4.13)
The Global Minimum by the ESCM of Example (4.4.A.5)

4.4.B Discussion

We have described a stochastic minimization procedure in chapter 3 and tested its applicability in this section by solving different minimization problems. The methods, which the author chose to call simulated entropy incorporates a scheme for simulating the equilibrium behaviour of atoms at constant temperature, in a procedure analogous to chemical entropy. It has the attraction that it is general yet extremely easy to apply. The two simulated entropy methods use a descent method but it can accept small objective function increases. The procedure can be made to behave as simulated annealing by defining randomly a topology via a perturbation scheme, so that neighbours may be generated from any given solution.



(A)

(B)

Fig. (4.17)

Example (4.4.A.5)

(A) Temperature Vs. Energy

(B) Temperature Vs. Entropy Components ($\lambda_i \ell n \lambda_i$) & its Corresponding Constraint ($\lambda_i g_i$)

Two of the most important factors which influence the convergence are the descent method and the way the sequence of control paramets P_i is chosen. The justification for using the simulated entropy algorithm with P > 0 is that we may not be able to tell a priority whether a given descent method will produce potential local optima or not. In other words, using the simulated entropy algorithm with P > 0 provides extra security against bad or uncertain descent method.

We have given examples which show that it is not possible to ensure convergence in less than exponential time for a general problem. However, we have not developed any rigorous theory concerning the class of problems for which the typical behaviour is good. Nevertheless, many applications of the simulated entropy algorithm including the first three examined in this chapter, have displayed good performance of the method in terms of a comparison with the results obtained by using a descent method alone, when applied to the same problem.

We have applied the method to different mathematical examples and demonstrated how the algorithm may be applied. While these applications do indicate that the simulated entropy idea is likely to be a useful one, a good deal of further work, possibly in the form of a large scale simulation, is required to study the effect of alternative perturbation schemes and a satisfactory method of using feedback to determine the control.

For each example, the temperature versus energy relationship is plotted. It shows that simulated entropy takes place at initial maximum temperature P_0 at which energy is maximum too. Temperature reduction continues slowly towards P_f at which energy is minimum. Also, the temperature versus the weighted constraints, the upper vertical axis, and entropy, the lower vertical axis, is plotted. It shows that the two are symmetric with respect to the temperature axis alongside the simulated entropy process. At the minimum and final temperature P_f, very close to zero, the weighted constraints and entropy coincide at a value very close to zero also, i.e. the equilibrium configuration is reached and the simulated entropy process is terminated.

In summary, we see that, for most applications, given a particular starting solution and a sufficient number of transitions, the simulated entropy algorithm, with $P > 0$, is more likely to obtain a good solution as opposed to a local optimum, than the descent method when it is used by itself ($P = 0$). At first glance these results suggest that the simulated entropy algorithm with $P > 0$ is to be preferred to the descent method by itself ($P = 0$). However, one question which must be answered is whether the number of iterations, $I_{P=0}$, required by the descent method by itself, to first observe the corresponding global or new local optima is significantly less than those, say $I_{P>0}$, required by the simulated entropy algorithm, with $P > 0$. For, if this is true, it may well be possible to carry out several applications of the descent method ($P = 0$), each from a different starting point and for $I_{P=0}$ iterations, so that the total number of iterations required is less than the $I_{P>0}$ iterations required in one application of the algorithm with $P > 0$.

We have found that for some applications $I_{P=0}$ is comparable to $I_{P>0}$, $I_{P=0}$ is considerably less than $I_{P>0}$. This means that it may be preferable to make several applications of the descent method ($P = 0$), each for a small number of iterations and from a different random start. However, note that, in this second case, there is still a chance that the global optimum may not be among the set of local optima.

Finally, the two simulated entropy techniques developed and employed here provide a secure way so that designers and mathematicians can detect their minima. However, as with all global optimization-seeking methods, computational costs and time requirements are high.

CHAPTER FIVE
RELIABILITY CONCEPTS

CHAPTER FIVE

RELIABILITY CONCEPTS

SYNOPSIS

This chapter introduces the concepts of uncertainties in reinforced concrete structures, and structural reliability. The research of the late 1960's to the early 1980's, which led to the development of structural optimization in the probabilistic sense, is surveyed.

Also, the concept of limit states is introduced and its relation with failure is discussed. A brief discussion of human errors is included since many structural failures are related to human errors.

5.1 INTRODUCTION

Many problems in engineering involve natural processes and phenomena that are inherently random; the states of such phenomena are naturally indeterminate and thus cannot be described with definiteness. For these reasons, decisions required in the process of engineering planning and design must usually be made, and are made, under conditions of uncertainty. The effects of such uncertainty on design and planning are important, to be sure; however, the quantification of such uncertainty, and the evaluation of its effects on the performance and design of an engineering system, properly, should include concepts and methods of probability. Furthermore, under conditions of uncertainty, the design and planning of engineering systems involve risks, and the formulation of related decisions requires risk-benefit trade-offs, all of which are properly within the province of applied probability (Ang, 1975).

Given a random experiment with a sample space S, a function X that assigns each element in S one and only one real number $X(s) = x$ is called a *random variable* (rv). The space of X is the real numbers $\{x : x = X(s), s \in S\}$ where $s \in S$ means element s belongs to the set S. If the sample space contains a finite countable number of elements, then the rv is known as *discrete rv*, while *continuous rv can take any value within its domain.*

There are three conceptual interpretations of probability:

i) *<u>The Equally Likely Interpretation (ELI)</u>: If an event can occur in N equally likely and different ways, and if n of these ways has an attribute A, then the probability of occurrence of A is defined as: $P_r(A) = n/N$.*

ii) *<u>The Frequency Interpretation (FI)</u>: If an experiment is conducted N times and a particular attribute A occurs n times, then the probability of occurrence of A is defined as $P_r(A) = (n/N)$ as $N \rightarrow \infty$.*

iii) *<u>The Subjective Interpretation (SI)</u>: The probability is a measure of degree of belief one holds in a specified proposition.*

A probability density function (pdf) is a special type of mathematical expression which governs the assigned probability that a random variable, x, takes: i) a particular value of X in the case of a continuous rv or, ii) less than and equal to a particular value X_1, or, iii) greater than and equal to a particular value X_2, or, iv) values within a range from X_1 to X_2. The functional value of the pdf should always be positive where X, X_1 and X_2 ϵS and real numbers. The *cumulative distribution function* (cdf) gives the probability that the value of a rv will be less than or equal to some real number X (Basu, 1981).

The most important probabilistic characteristics of a random variable may be classified as follows:

1) Central Values:

(a) Mean: Mean value or average value or expected value of a discrete random variable with a Probability Mass Function PMF, $p_X(x_i)$, is:

$$\mu_x = E(X) = \sum_{\text{all } x_i} x_i \cdot p_X(x_i)$$

(b) Median: For a discrete random variable, it is the value at which values above and below it are equally probable, that is, if x_m is the median of X, then:

$$F_X(x_m) = 0.50$$

(c) Mode: is the value of rv which gives the maximum probability density value of pdf.

Mean, median and mode of a symmetrical distribution are the same.

2) Measures of dispersion: or variabilities; i.e., the quantities that give a measure of how closely the values of the variate are clustered around the central value.

(a) Variance: If the deviations are taken with respect to the mean value, then a suitable average measure of dispersion is the variance. For a discrete random variable X with PMF, $p_X(x_i)$, the variance of X is:

$$Var(X) = \sum_{\text{all } x_i} (x_i - \mu_X)^2 \cdot p_X(x_i)$$

(b) Standard Deviation: a more convenient measure of dispersion

$$\sigma_X = [\ Var(X)\ ]^{0.5}$$

(c) Coefficient of Variation: A nondimensional measure of dispersion, small or large,

$$COV(X) = \delta_X = \frac{\sigma_X}{\mu_X}$$

3) Measures of Skewness: or the symmetry or lack of symmetry of a rv probability distribution:

(a) Third central moment: For a discrete random variable X with PMF, $p_X(x_i)$, the third central moment of X is:

$$E(X - \mu_X)^3 = \sum_{\text{all } x_i} (x_i - \mu_X)^3 \cdot p_X(x_i)$$

(b) Skewness Coefficient: A nondimensional measure of skewness

$$\Theta = \frac{E(X - \mu_X)^3}{\sigma^3_X}$$

5.2 UNCERTAINTIES IN REINFORCED CONCRETE STRUCTURES

An accurate assessment of the probable strength of an element of a reinforced concrete structure is complex, as a result of the many factors which can contribute to the variability. The major sources of variability are:

1) Yield strength of reinforcement.
2) Compressive strength of concrete in the cross-section
3) Area of reinforcement
4) Gross area of section
5) Location of reinforcement
6) Stress distribution in section at failure

Deviations of any of the above from their design values have differing effects on the strength of reinforced concrete members, depending on the magnitude of the deviation and the structural form of the element, column or beam. However, for factors such as the position of the reinforcement, there is never likely to be any really reliable data, and in these circumstances the only solution is to adopt an idealized distribution for the variable (Baker, 1970).

5.2.1 Yield Strength of Reinforcement Uncertainties

The strength of a reinforced concrete section is frequently governed by the strength of the reinforcement, and it is fortunate that information is more readily available for this property than for the other sources of variability mentioned above (Baker, 1970).

Uncertainties in the yield strength, f_y, of the reinforcement may be evaluated from mill or laboratory test data. Julian (1957) reported data from 171 tests on No. 3 to No. 10 bars with a nominal yield strength of 40 ksi (280 MN/m^2); values ranged from 38.95 ksi (268 MN/m^2) to 64.9 ksi (447 MN/m^2), with a mean of 47.7 ksi (329 MN/m^2), and a COV of .12. The COV is high because the test data for different bar sizes are lumped together. Test by Chow and Gardner (1971) of 20 No. 5 bars of intermediate grade 40 ksi (280 MN/m^2) steel show a mean 49.9 ksi (344 MN/m^2) and COV of 0.073. Baker (1970) gives data for nominal 40 ksi (280 MN/m^2) reinforcement which indicates a decrease in mean value from 50.4 ksi (347 MN/m^2) for No. 3 bars to 44.1 ksi (304 MN/m^2) for No. 8 bars; the associated COV range is approximately 0.07-0.11. The measured mean value is clearly affected by the bar size. Other factors also contribute to the uncertainty in $\bar{f}_y$. Commercial testing procedures, on which $\bar{f}_y$ is based, tend to increase the apparent $\bar{f}_y$ due to high applied strain rates (Ellingwood, 1974).

5.2.2 <u>Compressive Strength of Concrete Uncertainties</u>

The variability in f'_c is evaluated from tests on standard cylinders (Ellingwood, 1974). For concrete mixes designed for a nominal compressive strength f'_c = 3000 Psi (21 MN/m^2), $\bar{f}_c$ = 3456 Psi (23.8 MN/m^2), with $\delta_{f'c}$ = 0.12. Additional data, defining the mean f'_c delivered by a ready-mix company during separate monthly periods, indicates an error for the measured f'_c = 0.07. This estimate must be augumented by other factors affecting the mean strength in the actual structure; such factors would include the strain rate and duration of loading, casting direction, curing of the member, and the effects of creep, shrinkage, and confinement. A comparison of the strength of in-situ concrete with that determined from standard cylinders indicates that under field conditions, the strength is 10% - 21% lower than the strength observed under laboratory conditions. Assuming these factors contribute a combined uncertainty of 0.16, we obtain:

$$\Delta_{f'c} = \sqrt{(0.07)^2 + (0.16)^2} = 0.18$$

The total uncertainty in f'_c is then (Ellingwood, 1974):

$$\Omega_{f'c} = \sqrt{(0.18)^2 + (0.12)^2} = 0.21$$

5.2.3 Gross Area of Section Uncertainties

Uncertainties in member geometry are functions of the care and quality control exercised in construction. Uncertainties in the beam width, b, are assumed to be the same as those in the total member thickness, h; analysis of data indicate $\delta_b = \delta_h = 0.04$; and $\Delta_b = \Delta_h = 0.02$ (Ellingwood, 1974). For other uncertainties of other sources, the reader is referred to Refs. (Baker, 1970) and (Ellingwood, 1974).

5.2.4 Area of Reinforcement Uncertainties

Variability in steel area may be found from the variabilities in the individual bar areas, assuming that the individual areas are perfectly correlated within a member. Although current American Society for Testing and Materials (ASTM) specifications for bars smaller than No. 10 require tests on full size bars and strength calculations based on nominal areas, thus effectively yielding $A_s f_y$ rather than f_y, the variability of A_s is considered separately herein because the variability of f_y might be considered using test data based on actual rather nominal areas. Even with the present specifications, this separation would still be required for bars larger than No. 10 if the tension tests are made on reduced-section specimens rather than full-size bars. In addition, if A_s or f_y appear separately in certain design equations, the individual uncertainties must be known. Baker's results (1970), indicate the COV of the bar diameter is approximately 0.015 for small bars, with a tendency to decrease for large bars; thus $\delta_{As} = 0.03$. Other data (Ellingwood, 1972), suggested that $\delta_{As} = 0.02$ and $\Delta_{As} = 0.03$. The latter uncertainty arises from fabrication errors, carelessness in placement at the site, and the great unpredictability of mean areas for smaller diameter bars.

5.2.5 Limiting Concrete Strain Uncertainties

Statistics of the limiting concrete strain, ϵ_{cu}, are necessary for evaluating the uncertainties in ρ_b, reinforcement ratio producing balanced strain conditions. Test results in the form of plots of $\bar{\epsilon}_{cu}$ versus $\bar{f}'_c$ indicate that $\bar{\epsilon}_{cu}$ is approximately 0.004. From these plots, the basic variability in ϵ_{cu} is found to be approximately 0.12. The mean concrete strain will be affected by the degree of confinement of the concrete and the gauge length used to determine the strain; the prediction error is assumed to be 0.10.

5.2.6 Stress Distribution in Section at Failure Uncertainties

The concrete stress block parameter, η, and the equivalent stress factor, β_1, are related to the concrete stress distribution in the compression zone at ultimate load. β_1 decreases with f'_c. The ACI Code specifies the average concrete stress to be 0.72 for $f'_c \leq 4000$ psi (28 MN/m^2). Analysis of data suggests that $\delta_{\beta 1} = 0.12$; the prediction error is assumed to be approximately 0.05. A study of beams failing in tension indicates that $\bar{\eta} = 0.59$ and $\delta_{\bar{\eta}} = 0.05$; the prediction error is assumed to be negligible. Some of the uncertainty in η may be attributed to uncertainty in the steel stress at failure in the beams tested.

A summary of the basic variability and prediction error of each variable, along with the resulting total uncertainty, is presented in Table (5.1), see (Ellingwood, 1974).

Parameter (1)	Predicted mean (2)	Basic variability, δ (3)	Prediction error, Δ (4)	Total uncertainty, Ω_t (5)
f_y (nominal 40 ksi)	47.7 ksi	0.09	0.12	0.15
f_c (nominal 3,000 psi)	3.5 ksi	0.12	0.18	0.21
A_s		0.02	0.03	0.036
b		0.04	0.02	0.045
d		0.07	0.05	0.086
h		0.04	0.02	0.045
$\beta_1\beta_3$	0.72	0.12	0.05	0.13
η	0.59	0.05	0.0	0.05
ϵ_{cu}	0.004	0.12	0.10	0.156

Note: 1 ksi = 6.89 MN/m².

Table (5.1)

Uncertainties in Design Parameters (Ellingwood, 1974)

5.3 STRUCTURAL RELIABILITY

In view of numerous uncertainties underlying the design and construction of a structure, or other engineering systems, there is some risk (of structural inadequacies including failure) that is unavoidable. This risk should be recognized, and structures ought to be designed on the basis of an acceptably small measure of risk. Within this premise, the consideration of safety in the design of structures, therefore, requires probabilistic analyses. In practice, such analyses, however, must recognise the following:

1) Available data and information are generally insufficient to determine the correct probability distributions; indeed, quite

often, only the means and variances may be obtainable.

2) There are, inevitably, various factors (tangible as well as intangible) in design for which information and knowledge are lacking or imperfect; the uncertainties arising there from must be included in the design process.

The calculated risk, or probability of failure, is meaningful only as a comparative measure of safety; an absolute meaning, therefore, cannot be attributed to a calculated risk, especially when this involves extremely small probabilities. Design on the basis of an acceptable risk, therefore, should be interpreted in this light; for this purpose, the calculated measure of risk would be a proper measure of safety and a viable basis for design if it is self-consistent under the conditions prevailing in practice.

In the classical formulation, failure of an engineering system (e.g., a structure or one of its components) is realized when the calculated system capacity R is exceeded by a load (or load effect) S; i.e.,

$$\text{Failure} = R < S \tag{5.1}$$

The risk of failure then is the probability of failure:

$$P(\text{Failure}) = P\ (R < S) \tag{5.2}$$

Presumably, in practice, an acceptable risk P_f can be specified, and a design would be obtained on the basis of:

$$P\ (R < S) = P_f \qquad (5.3)$$

Theoretically, the above basis for design is sound; however, in actual design situations, wherein the prevailing conditions are those described above, serious shortcomings have been recognised (Freudenthal, 1968), including the following:

1) Within the range of acceptable risks, the probability $P(R < S)$ and the designs derived therefrom, are extremely sensitive to the distribution functions of R and S. On the other hand, the scarcity of data makes the precise determination of these distributions impossible.

2) All uncertainties are necessarily described through the distributions of R and S and thus are tacitly assumed to be of the objective type, objective in the sense that the uncertainties are associated with measured statistical or probabilistic information.

In spite of these shortcomings, however, the reliability concept described above constitutes the basic foundation of structural safety analysis (Ang, 1972).

The safety of a structure, in the probabilistic sense is described by the chance of withstanding the effect of loads or, in another way, the inverse risk of failure subject to load. Mathematically, this can be measured as the probability of failure (P_f) or probability of survival (P_s). Probability of survival is also known as *reliability*. The relationship between P_f and P_s is:

$$P_s = 1 - P_f \qquad (5.4)$$

The method of safety analysis using the concept of probability theory is known as reliability analysis. The points given below are very important and should be clarified in the context of reliability analysis:

1) All the recognized methods for structural analysis remain valid when carrying out a probabilistic design of a structure.
2) Reliability analysis is performed with derived or assumed statistical properties like moments or distribution functions of the design variables.
3) The safety analysis used should be consistent with the state of the art of structural mechanics used to analyze a specified failure mode.
4) A probabilistic approach to design can only take care of the random behavior of those design parameters which can be quantified, e.g. load and strength. Characteristics such as construction, fabrication and computational inadequacy may only be considered indirectly. Strict quality control during construction and fabrication may reduce the magnitude of the danger of failure. Another possible way to consider these factors is to develop statistical models of load and strength considering operational limitations (Basu, 1981).

Structures can be represented as series, parallel, or combined systems. A series system, Fig. (5.1), is one in which any component failure is system failure. In structural engineering, in general, statically determinate trusses are series systems. With reinforced concrete structures, the failure of a floor beam in shear or bending can be equivalent to the failure of a series owing to the fact that the entire design becomes suspect and the loss from failure involves the entire system. In any event, statically determinate systems tend to be series systems insofar as the major design elements are concerned.

A parallel system is one in which two or more paths of resistance must fail for the system to fail, Fig. (5.2). A combined system, Fig. (5.3), includes both series and parallel elements.

5.4 MODELLING OF STRUCTURAL SYSTEMS

A real structural system is so complex that exact calculation of the probability of failure is completely impossible. The number of possible different failure modes is so large that they cannot all be taken into account, and even if they could all be included in the analysis exact probabilities of failure cannot be calculated. It is, therefore, necessary to idealize the structure so that the estimate of the reliability becomes manageable. The main objective of a structural reliability analysis is to be able to design a structure so that the probability of failure is minimized in some sense. Therefore, the model must be chosen carefully so that the most important failure modes for real structures are reflected in the model. The system reliability is based on the assumption that the total reliability of the structural system can be sufficiently accurately estimated by considering a finite number of failure modes as dominating and then combining them in complex reliability systems.

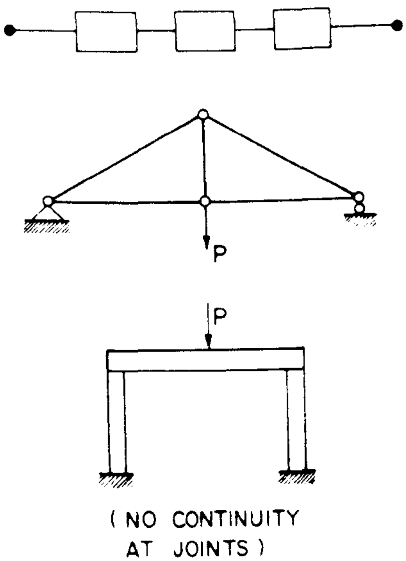


Fig (5.1)

Series Systems

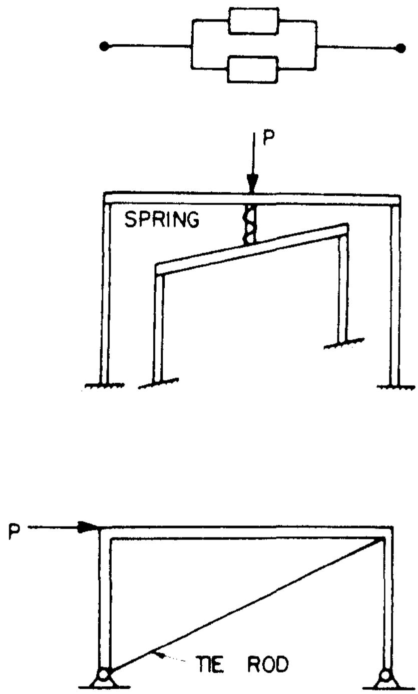


Fig. (5.2)

Parallel Systems

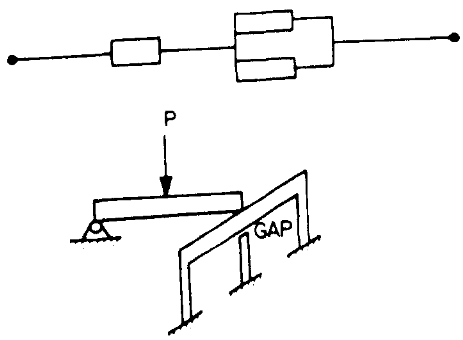


Fig. (5.3)

Combined Systems

Failure in a single structural element will for a statically indeterminate (redundant) structure as shown in Fig. (5.4) not always results in failure of the total system. The reason for this is that remaining structural elements may, by redistribution of the load effects, be able to sustain the external loading. For statically indeterminate structures, total failure will usually require that failure takes place in more than one structural element. Total failure of a structural system means formation of a mechanism which requires simultaneous failure in a number, say n, of failure elements. This is symbolized by the parallel system shown in Fig. (5.5).

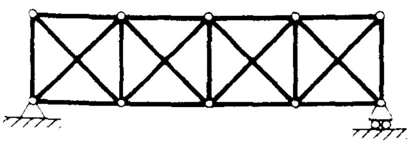


Fig. (5.4)

Indeterminate Structure

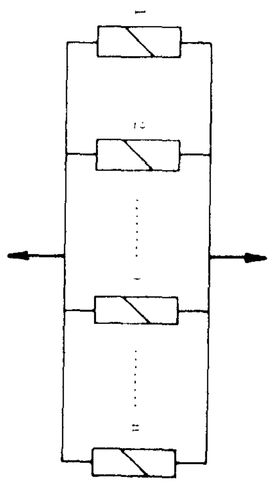


Fig. (5.5)

Parallel System

Clearly for a real redundant structure, the number of potential failure modes will usually be very high. Each failure mode will then be modelled as an element of a parallel system. Failure of such a structure will take place when the weakest failure mode (parallel system) fails. Therefore, the parallel systems (failure modes) are combined in a series system as shown in Fig. (5.6).

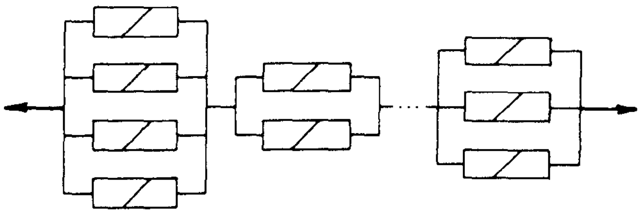


Fig. (5.6) Combined (Parallel and Series) System

5.5 REVIEW OF PROBABILITY BASED DESIGN

The probability based design philosophy differs from the deterministic philosophy in evaluating safety. Unlike the deterministic approach, the probability based formulation assumes that the preassigned design values of load and strength are not constants; rather they are random variables. Safety is measured in terms of probability of failure (P_f) and evaluated by using the theory of probability from the random variations of load and strength.

The fundamental case as illustrated in Fig. (5.7) is the basic conceptual problem in the field of structural reliability. It is an idealized component, (a single member subject to a single load) of a complex structure with many members and many load conditions. However, any structure can be approximated by a combination of several fundamental cases. Any mathematical modelling of such a combination depends on the type of structure, the nature of the loadings, material properties etc. By virtue of this combination, the safety of an entire structure is expressed in terms of P_f, i.e., $P_{f,\ allowable}$.

There are several approaches in estimating P_f for the fundamental case and for entire structure. Most of them assume that the probability density functions (pdf) of load (S) and strength (R) are predefined. In general, a suitable analytical model of infinite range is selected from the statistical properties of available limited data of R and S, using standard statistical procedures. The reason for picking up an analytical distribution of infinite range is that it makes provisions to take account of the values of R and S in the extremum regions of their respective spectra which from the data may not be available. However, once the P_f's for fundamental cases are determined, the next problem is to find the P_f of the entire structure from these discretized fundamental cases by suitable combination. Two models of such combination are commonly used and they are *Weakest-Link Structures* and *Fail-Safe Structures*. Fig. (5.8) illustrates the reliability modellings of a three bar truss with two statistically independent loadings.

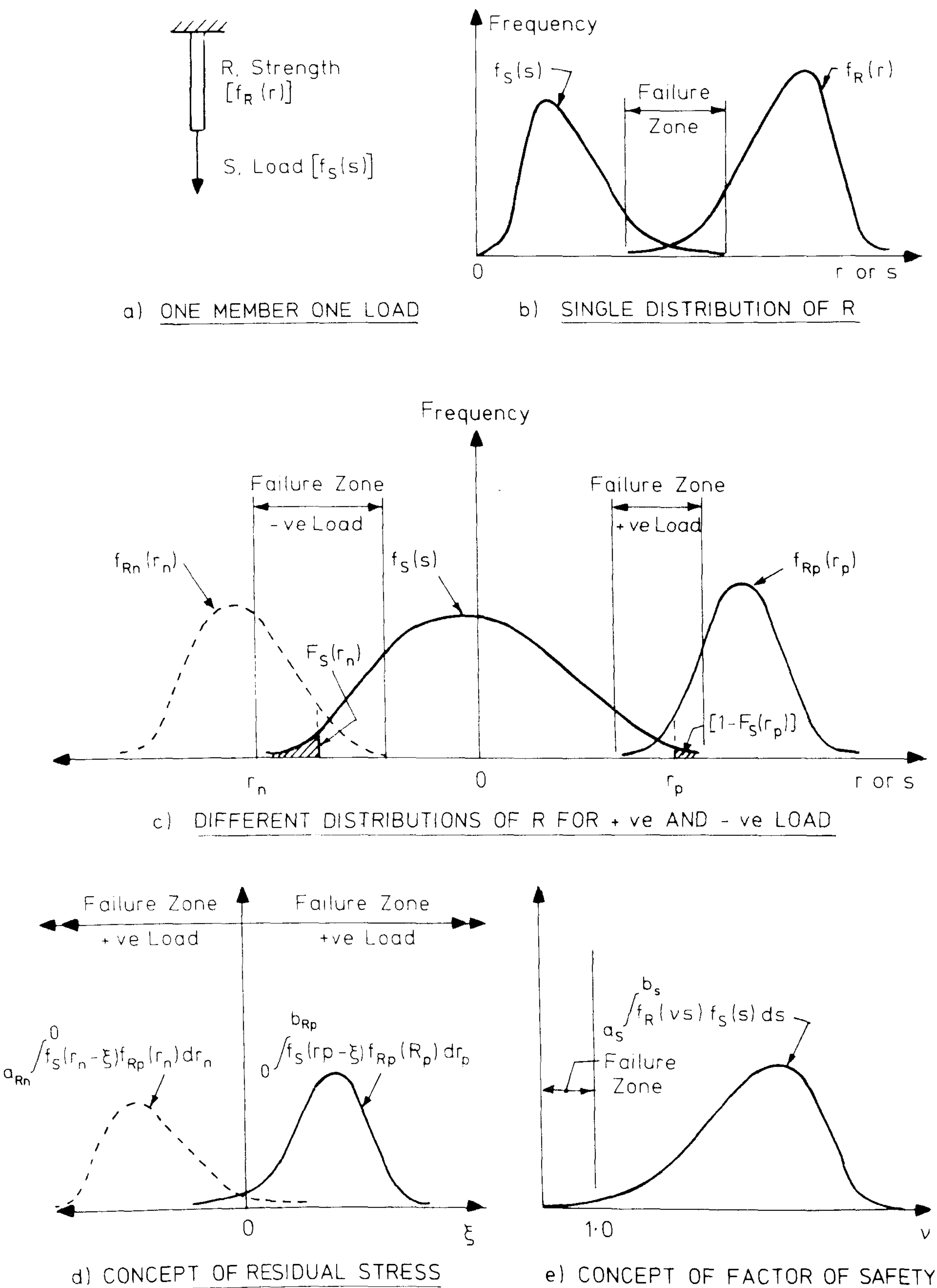


Fig. (5.7)

Fundamental Case (Basu, 1981)

Weakest-Link Structures: This model assumes that a structure will fail if any single critical member fails. Freudenthal (1966), introduced this model. It is useful for prismatic structures in which many elements or members are subjected to a loading of single or multi-origins. The assumption behind the Weakest-Link model suggests that this is true for a statically determinate structure. However, it can still be applied if this criterion of first member failing is taken as overall failure. This determines a conservative estimation of P_f of the entire system. The true estimation can be done by the Fail-Safe criterion.

Fail Safe Structures: Unlike Weakest-Link modelling, the Fail-Safe criterion assumes a structure would fail when several components exceed their capacity simultaneously. The number of such components depends on the type of structures. Computationally, this approach is complicated because of the numerous alternate load paths and yielding of combinations of members to produce failure. This model is suitable for structures when limit or ultimate analysis of structures are carried out.

One of the first publications in the field of probability based optimization was made by Hilton and Feigen (1960). They used a Lagrange multiplier formulation to minimize weight subject to the constraints on P_f for one load condition assuming that failures of individual components of a structural system are statistically independent. Based on Hilton and Feigen's approach, Switsky (1964) derived a simple scheme to proportion the member sizes using the relationship:

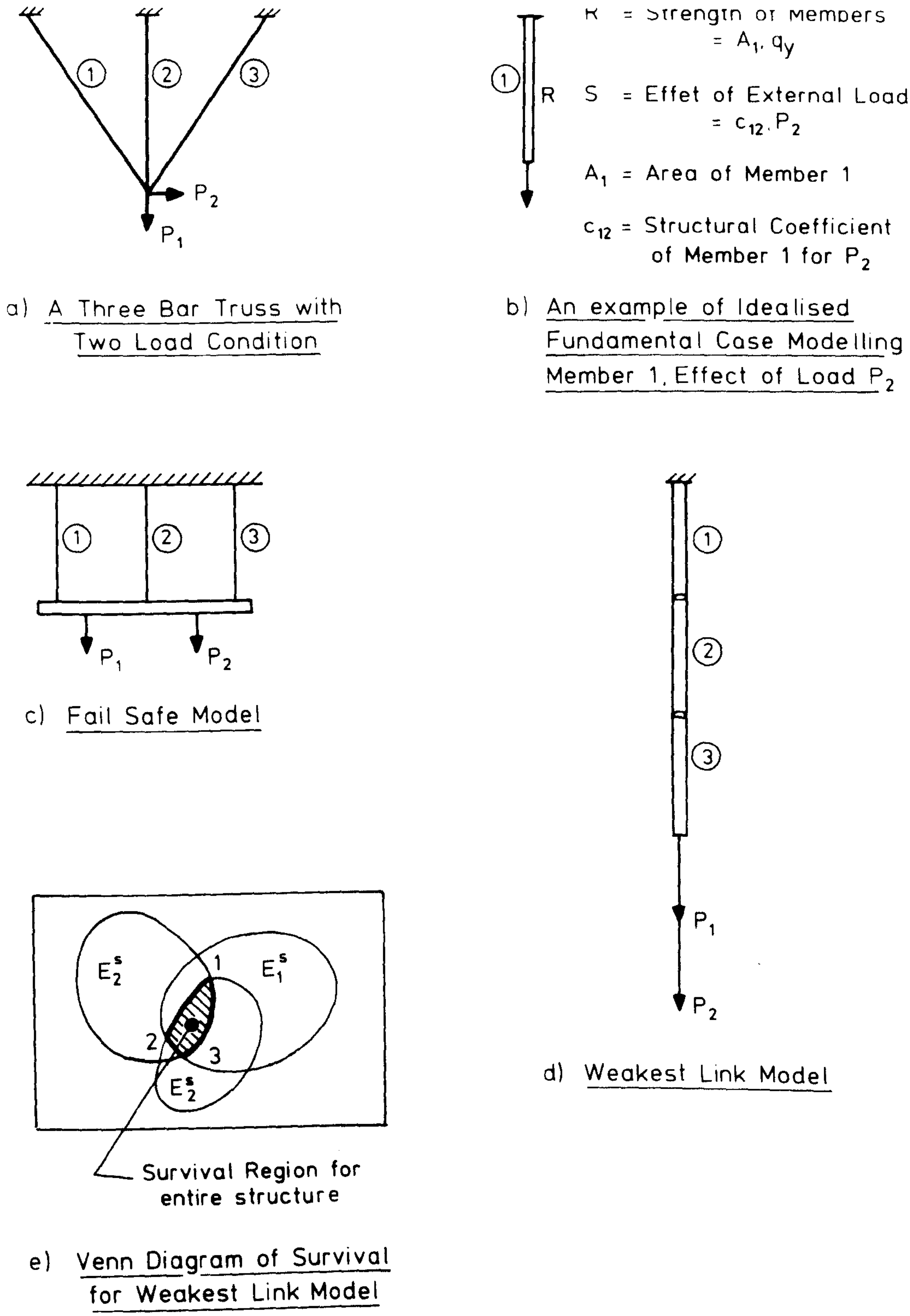


Fig. (5.8) (Basu, 1981)

Reliability Modelling of a 3-Bar Truss for Multi-Load Condition

$$P_{fi} = \frac{W_i}{\sum W_i} P_{f,allowable} \qquad (5.5)$$

where W_i and P_{fi} are the weight and probability of failure of the ith member. Moses and Kinser (1967), developed an ordering technique for statically intermediate structures to take account of correlations between the probability of failure of individual members. Moses and Stevenson (1970) report the reliability based optimization of a framed structure using the collapse mechanism and the Fail-Safe criterion.

Nevertheless, by 1970 it had become apparent that the concepts and methods of probability are the proper bases for the development of optimum criteria for structural design. As a result, during the past 15 years many investigators focuses their efforts on developing:

1) Strategies for identifying failure modes for large structural systems which incorporate brittle or ductile behavior, or both (Moses, 1977).
2) More sophisticated methods, taking failure mode correlation into account, for determining overall system reliability of both brittle and ductile structures (Vanmarcke, 1971), (Ditlevsen, 1979), (Ang, 1981) and (Chau, 1983).
3) Automated reliability-based optimization procedures for minimum material cost with failure probability constraint, for maximum safety based on fixed weight or total expected cost, and for a minimum total expected cost (Mau, 1971), (Vanmarcke, 1971, 1972), (Moses, 1973) and (Basu, 1981).

4) Multi-criteria reliability-based optimization procedures for structural systems subject to probability constraints imposed at both serviceability and ultimate limit states (Parimi, 1978).

5.6 PRINCIPAL STEPS IN THE PROBABILISTIC APPROACH

There are three principal steps in the probabilistic approach to structural deign:

1) Selection of Failure-Modes:

This depends on the purpose of the type of structures to be built, the loading, and the service life. A rigid frame member, under the effect of external load may be susceptible to several modes of failure, e.g., axial, bending and shear modes. The complexity of failure modes increases with the involvement of phenomena like creep, fatigue, expansion, vibration, etc.

2) Structural Analysis:

Once the failure modes are selected, the next step is the analysis of structures, or in other words the estimation of the effect of external load. There are both linear and non-linear methods of analysis. A rigid frame may be analyzed by a linear method such as the stiffness method or the finite element method, or a non-linear analysis can be carried out using some collapse mechanism technique. This structural analysis is carried out at the mean values of load and strength.

3) Reliability Analysis:

Reliability analysis is carried out from the quantified effect of loads and material properties on the structures, determined by analysis. This measures the level of safety. There are several methods available to evaluate the probability of failure. Each one has its own merits and demerits and their suitability depends on the type of problem. Load and strength are considered to be the only random variables in reliability analysis.

5.7 FUNDAMENTAL CASE

The situation of one member - one load, Fig. (5.7a), is the basic conceptual problem in the field of structural reliability and is called the fundamental case. There are several existing methods to evaluate probability of failure of the fundamental case. Some of them are discussed here:

1) Integral Equation Technique:

This technique assumes that the distributions of load and strengths are defined and there is no statistical correlation between them. The structural capacity of different failure modes may vary over two distinct ranges of strength. For example, tension and compression of a member in an axial failure mode. These two distinct ranges are denoted as positive and negative strength. Distribution functions of strength may be different from these two ranges. There are two approaches to evaluate P_f by the integral equation technique. The first approach assumes the strength has a single distribution, Fig. (5.7b). Freudenthal and others (1966) developed an integral equation to evaluate P_f using this approach. Different distributions of strength are considered in the second approach. Ang, et. al. (1968) developed a model to evaluate P_f in this approach.

Integral equations for P_f can be derived in three ways; A) directly from load and strength, B) from the concept of residual stress, and C) from the concept of a factor of safety. Expressions of P_f for these three cases are given below;

(A) Directly from Load (S) and Strength (R):

1) Single Distribution of R for a given S: Failure occurs whenever the magnitude of the load is greater than that of the strength. This is illustrated in Fig. (5.8b). P_f is expressed as:

$$P_f = \int_{a_S}^{b_S} F_R(s).f_S(s)ds \qquad a_R \leq R < s; \quad a_S \leq s \leq b_S \tag{5.6a}$$

or

$$P_f = \int_{a_R}^{b_R} [1 - F_S(r)]\ f_R(r)dr \qquad R < s \leq b_S; \quad a_R \leq R \leq b_R \tag{5.6b}$$

2) Different Distribution of R for Positive and Negative Ranges: Failure occurs whenever the positive load is greater than the positive strength or the negative load is less than the negative strength.

$$P_f = \int_{0}^{b_S} F_{Rp}(s).f_S(s)ds + \int_{a_S}^{0} [1 - F_{Rn}\ (s)].f_S(s)\ ds \qquad \begin{matrix} 0 \leq Rp < S;\ 0 \leq s \leq b_S \\ a_S \leq s < Rn;\ a_S \leq s \leq 0 \end{matrix} \tag{5.7a}$$

$$P_f = \int_{0}^{b_{Rp}} [1.\ F_{Rp}\ (s)].f_{Rp}(rp).drp + \int_{a_{Rn}}^{0} F_s(rn).drn \qquad \begin{matrix} Rp < s \leq 0;\ 0 \leq Rp \leq b_{Rp} \\ a_S \leq s < Rn;\ a_{Rn} \leq Rn \leq 0 \end{matrix} \tag{5.7b}$$

These equations are illustrated in Fig. (5.7c)

(B) From Residual Strength

Residual Strength is given by: $M = R - S$ (5.8a)

Failure occurs whenever residual strength is less than zero for a positive load and greater than zero for negative loads. Fig. (5.7d) illustrates this concept. P_f is expressed as:

$$P_f = \int_{aM}^{0} \left[\int_{0}^{b_{RP}} f_S\,(rp-M).f_{Rp}\,(rp).drp\right].dM + \int_{0}^{bM} \left[\int_{a_{Rn}}^{0} f_S(rn-M).f_{Rn}(rn) .drn\right] dM \qquad (5.8b)$$

(C) From Factor of Safety:

Factor of Safety is given by: $\nu = (R/S)$ (5.9a)

Failure occurs whenever F attains the value in the range 0 to 1. Fig. (5.7e) shows this concept.

$$P_f = \int_{0}^{1} \left[\int_{0}^{b_s} f_{Rp}\,(\nu s).f_S\,(s).ds\right] s\, d\nu + \int_{0}^{1} \left[\int_{a_S}^{0} f_{Rn}(\nu s).f_S(s).ds\right]. s.d\nu \qquad (5.9b)$$

Detailed derivations of these equations are given in Basu (1981). P_f can be calculated for some distributions of R and S directly from the Error function. For example, if both R and S have normal distributions, then:

$$P_f = \Phi \left[- \frac{\mu_R - \mu_S}{\sqrt{\sigma^2_R + \sigma^2_S}} \right] \tag{5.10}$$

where, the Error Function, $\Phi[u]$, is given by:

$$\Phi[u] = \int_{-\infty}^{u} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)\, dt \tag{5.11}$$

The expression of P_f for log normal distributed R and S, is:

$$P_f = \Phi \left[- \frac{\ell n \left[\nu_c \sqrt{(1 + V^2_S)/(1 + V^2_R)} \right]}{\sqrt{\ell n[(1 + V^2_S)\ (1 + V^2_R)]}} \right] \tag{5.12}$$

where V_R and V_S are coefficients of variation of R and S, ν_c is known as central factor of safety. Equations (5.10) and (5.12) are derived assuming strength has a single distribution and the derivations are given in Basu (1981).

5.8 FAILURE AND LIMIT STATES

The design of any structure by any method entails the selection of one or several limit states. A limit state value strictly is a quantity, generally numerical, which divides *acceptable* and *unacceptable* performance as governed by some possible failure mode. Thus a limit state, for reinforced concrete frame, could be a strength requirement in a load and resistance design format. Some limit states are quite obvious because the associated failure modes, such as the fracture of a tension member, are quite obvious and well defined. Others, including serviceability limit states, are often difficult to quantify since the relevant failure is less dramatic and its definition is of a more subjective nature.

The true acceptability of the performance of a designed structure of given nominal materials is influenced by three interrelated items:

1) The load intensity and pattern assumed.
2) The refinement of the resistance evaluation or analysis, and
3) The value selected for the relevant limit state.

If either the load or the resistance analysis is changed, the numerical value of the limit state should also be reexamined (Criswell, 1979).

As pointed out by Ang and Cornell (1974),

Failure should be interpreted with respect to some predefined limit state; it may be an excessive deflection, major cracking in a concrete beam, or the total collapse of a structure. Therefore, depending on the limit state under consideration, the concept of a failure probability is applicable to both the safety and performance of structures.

In this thesis, several limit states are considered which might be classified to two groups as follows:

1) Element limit states.

2) Structure limit states.

For optimal design of reinforced concrete frames, the element limit states are represented as constraints which are as follows:

For a beam element:

(1) Flexural and shear strength limit states

(2) Ductility and serviceability limit states

(3) Concrete cover for reinforcement limit states

(4) Web Reinforcement limit states

(5) Development length for Longitudinal Reinforcement limit states

(6) Cut-off points of Longitudinal Reinforcement limit states

(7) Spacing limits

For a column element:

(1) Axial, shear and flexural strength limit states

(2) Ductility and serviceability limit states

(3) Concrete cover for Reinforcement limit states

(4) Limits of Lateral Reinforcement

(5) Spacing limits.

while the structure limit states correspond to the collapse of that structure at which the structure becomes unfit for use.

In practical situations, the structure is composed of several components. The main concern in such a system is not the failure of a component, but the overall system failure. In such practical cases the description of the structural model by an explicit expression is extremely difficult. However, an overall system reliability analysis is considered explicitly in chapter 7.

Finally, it should be stated that the probabilities of failure calculated by different methods are based on failures caused by random fluctuations in the basic variables such as extremely low strength capacities or extremely high loads.

CHAPTER SIX

REINFORCED CONCRETE FRAME STRUCTURES

CHAPTER SIX

REINFORCED CONCRETE FRAME STRUCTURES

SYNOPSIS

This chapter is a general discussion of the multi-criteria optimization problems as applied to structural design. Specifically, the presentation includes formulation of the objective functions and the design constraints. The design constraints, which are based on the ACI (318-83), are developed separately for a beam and a column. Finally, the bounds of discrete design space which conform with practice for each of the design variables are discussed.

6.1 INTRODUCTION

Design is one of the primary functions of engineering. The objective of the designer is to proportion the structures in such a way that the requirements of safety and serviceability are met as economically as possible. In practice, the provisions of safety and serviceability are provided by various specifications and codes. In practice, the design of a reinforced concrete structure is a trial and adjustment procedure. It is evident that many trials, even with availability of computers, are not always possible, and consequently the structure is generally over-designed. However, the optimal design of reinforced concrete structures for professional practice can be achieved, resulting in considerable savings.

However, structural optimization within a deterministic design philosophy is characterized essentially by a well defined (deterministic) structural code for checking constraints, and usually an element optimization technique for finding the best values of design variables, such as element dimensions and reinforcement ratios, which result in the minimization of material weight. However, methods of optimization based on deterministic safety concepts, while minimizing the weight or cost of an element may be changing its level of safety. This is a major limitation of deterministic optimization formulations, in which the inherent random nature of both structural loading and strength is not included and, consequently, the safety criteria are not specified in terms of a risk value.

It is now generally recognized that structural problems are non-deterministic and, consequently, that engineering optimum design must cope with uncertainties. Clearly, the proper tool for the assessment and analysis of such uncertainties requires methods and concepts of reliability. Therefore, it is not an over-statement to affirm that the combination of reliability-based design procedures and optimization techniques is the only means of providing a powerful tool to obtain a practical optimum design solution (Frangopol, 1985).

In order to design an optimal reinforced concrete frame as closely to professional practice as possible, the discreteness of the design variables has to be taken into consideration. Furthermore, the design constraints should cover the code requirements of flexural strength, axial load strength, shear strength, ductility (plasticity), serviceability, limits of web reinforcement, concrete cover, development length, cut-off points, spacing limits and other requirements. In this thesis, the American Code ACI (318-83) will be considered as a code of design.

6.2 REVIEW OF PREVIOUS WORK

Optimal design problems have been of increasing interest to many engineers and researchers. Shunmugavel (1974) used the method of feasible directions to obtain the minimum cost design solution for reinforced concrete structures in which the design variables are continuous variables. The primary objective of the work was to obtain the optimal relative stiffness of the members of the large frames studied. The cross-sectional dimensions and the amount of main reinforcement of the members were the design variables. The constraints were developed in accordance with the ACI Code (318-71) requirements. The arrangement of main reinforcement followed a specified pattern. No consideration was given to constraints due to shear, concrete cover, contribution of compression reinforcement to bending strength, clear spacing and possible unsymmetric arrangement of reinforcement in column sections.

Gerlein and Beaufait (1980), used a story-by-story linear programming optimization approach for an optimum preliminary strength design of reinforced concrete frames. The total volume of reinforcing steel required by the members of the structure was minimized. The objective function was expressed in terms of moment capacities which were treated as functions of the amount of reinforcing steel at sections. The recommended bar details of the Concrete Reinforcing Steel Institute (CRSI) was used to obtain the moment capacity at sections. Collapse mechanisms were used to develop the constraints. The minimum amount of reinforcement required at any cross-section was based on the ACI Code (318-77).

Yang (1981), used discrete mathematical programming methods for obtaining an optimum design of a reinforced concrete frame for a specified geometry. The design variables were the overall depth, effective depth, width of members and area of longitudinal reinforcement. In addition, the details such as the amount of web reinforcement and cut-off points of longitudinal reinforcement were also considered as variables. Total cost has been used as the objective function, and has been optimized taking into account the constraints developed according to the ACI Building Code (318-77) requirements. The objective function was the cost of the structure which is based on the costs per unit volume of the steel and concrete. The constraints included requirements such as flexural strength, shear strength, ductility (plasticity), serviceability, concrete cover, spacing, web reinforcement, development length and cut-off points of longitudinal reinforcement. The Generalized Reduced Gradient, Rounding and with Neighbourhood Search was used.

Zheng and Huanchun (1985), studied the optimum design of engineering reinforced concrete frames by a two-level optimization technique. They divided their constraints into local and global ones and their optimum design problem into two sub-problems. The global constraints were the displacement constraints and the local constraints were the flexural, shear and axial constraints and constructional requirements constraints. In the first level, the top horizontal displacement (drift) of the frame was taken as the objective function, and was maximized to satisfy all global constraints. Because the top displacement of the frame in the summation of the relative displacements between two adjacent storeys, so that, if the top displacement of the frame is maximum and none of the displacement constraints is violated, this frame will correspond to the most flexible one. This is why the top horizontal displacement is maximized in the first level. The first sub-problem was converted to a Sequential Linear Programming (SLP) problem. The optimum solution of the first level was used to obtain the bounds of the design variables. In the second level, the cost of total frame material was taken as the objective function, and was minimized to satisfy all members constraints. This problem was solved by a two-dimensional search method.

From the above discussion, it can be seen that only a small part of the research effort has been dedicated to multi-criteria optimization, in general, and multi-criteria reliability-based optimization of reinforced concrete frames in particular.

6.3 DEVELOPMENT OF OBJECTIVE FUNCTIONS

6.3.1 The Material Cost Objective Functions

Objective functions based on cost are generally the most often used in structural engineering. Reinforced concrete structures are made of two different materials, namely steel and concrete, having different relative strengths and costs. Cost is taken as one criterion for optimization in this study.

For a reinforced concrete frame with prismatic members and a specified geometric configuration, the total cost of the frame is a non-linear function of the design variables and can be expressed as follows:

$$C = C_s \sum_{i=1}^{N_b} (V_b)_i + C_s \sum_{i=1}^{N_c} (V_c)_i + C_c \sum_{i=1}^{N_b} (bhL_b)_i + C_c \sum_{i=1}^{N_c} (tDH)_i \tag{6.1}$$

where:

C	=	total material cost of a frame.
C_c	=	cost of concrete per square inch per linear foot.
C_s	=	cost of steel per square inch per linear foot.
N_b	=	number of beams in a frame.
N_c	=	number of columns in a frame.
$(V_b)_i$	=	total volume of steel of ith beam in square inches per linear foot.
$(V_c)_i$	=	total volume of steel of ith column in square inches per linear foot.
b_i	=	width of ith beam in inches.
h_i	=	overall depth of ith beam in inches.
$(L_b)_i$	=	length of ith beam in feet.
t_i	=	width of ith column in inches.
D_i	=	overall depth of ith column in inches.
H_i	=	height of ith column in feet.

The volume of the total reinforcement in beam i, $(V_b)_i$, can be expressed as follows:

$$(V_b)_i = (A_{s\ell}\, L_{s\ell} + A_{sm}\, L_{sm} + A_{sr}\, L_{sr})_i + (N_{v\ell}\, A_{v\ell} + N_{vm}\, A_{vm} + N_{vr}\, A_{vr})_i \;.[2\,(b + h - C_1)_i] \tag{6.2a}$$

where:

$(A_{s\ell})_i$	=	area of main reinforcement at the top left of ith beam in square inches.
$(A_{sr})_i$	=	area of main reinforcement at the top right of ith beam in square inches.

$(A_{sm})_i$ = area of main reinforcement at the bottom near the mid-span of ith beam in square inches.

$L_{s\ell}$ = length of main reinforcement in feet.

L_{sm} = length of main reinforcement in feet.

L_{sr} = length of main reinforcement in feet.

C_1 = constant in inches.

$(A_{v\ell})_i$ = area of share reinforcement within a distance s which is in a quarter span from left end of ith beam in square inches.

$(A_{vr})_i$ = area of shear reinforcement within a distance s which is in a quarter span from right end of ith beam in square inches.

$(A_{vm})_i$ = area of shear reinforcement within a distance s which is in a center half span of ith beam in square inches.

$(N_{v\ell})_i$ = number of stirrups within a quarter span from left end of ith beam.

$(N_{vr})_i$ = number of stirrups within a quarter span from right end of ith beam.

$(N_{vm})_i$ = number of stirrups within a center half span of ith beam.

Similarly for column i,

$$(V_c)_i = (A_s H + \acute{A}_s H)_i + (N_{ht} A_{sht} + N_{hm} A_{shm} + N_{hb} A_{shb})_i . [2 (t + D - C_2)_i] \quad (6.2b)$$

where:

$(A_s)_i$ = area of tension reinforcement in ith column.

$(\acute{A}_s)_i$ = area of compression reinforcement in ith column.

C_2 = constant in inches.

$(A_{sht})_i$ = area of shear reinforcement within one-third of the height from top of ith column, within a distance s in square inches.

$(A_{shb})_i$ = area of shear reinforcement with one-sixth of the height from bottom of ith column, within a distance s in square inches.

$(A_{shm})_i$ = area of shear reinforcement within a height from one-sixth to two-thirds of the height of ith column, within a distance s in square inches.

$(N_{ht})_i$ = number of ties or hoops within one-third of the height from top of ith column.

$(N_{hb})_i$ = number of ties or hoops within one-sixth of the height from bottom of ith column.

$(N_{hm})_i$ = number of ties or hoops within a height from one-sixth to two-thirds of the height of ith column.

f'_c = cylinder strength of concrete in psi.

f_y = specified yield strength of reinforcement in psi.

Figures (6.1), (6.2) and (6.3) show the above variables in typical beams and columns.

6.3.2 The Drift (Top Lateral Deflection) Objective Function

In a frame-type structure, the drift may be thought of as consisting of two parts: one due to bending in the columns and beams, and the other due to axial deformation of the columns. As the height-to-width ratio of the structure increases, the effect of column axial deformation assumes greater significance (Fintel, 1974).

Figure (6.4) shows the curvature distribution in a typical column, when the lateral loading has increased to the extent of just causing yield in the frame. The lateral deflection at the top of the rth story at first yield relative to the bottom of the structure, assuming that the points of contraflexure occur at 0.6 of the column height from the bottom of the columns of the bottom story, and at mid-height in the columns of all other storeys, is:

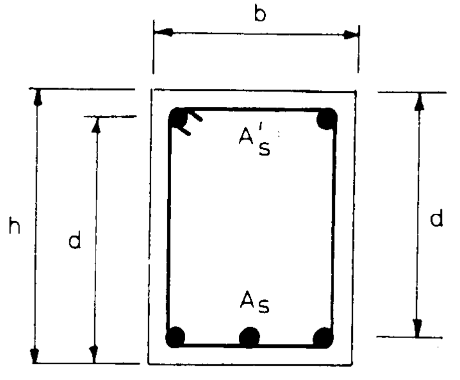


Fig. (6.1a)

Cross Section of a Beam

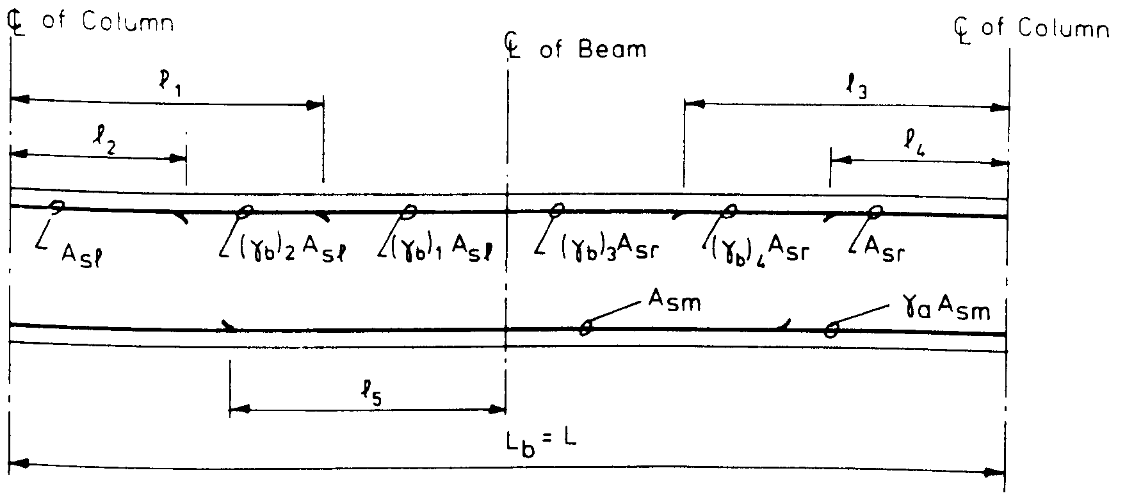


Fig. (6.1b)

Arrangement of Main Reinforcement in a Beam

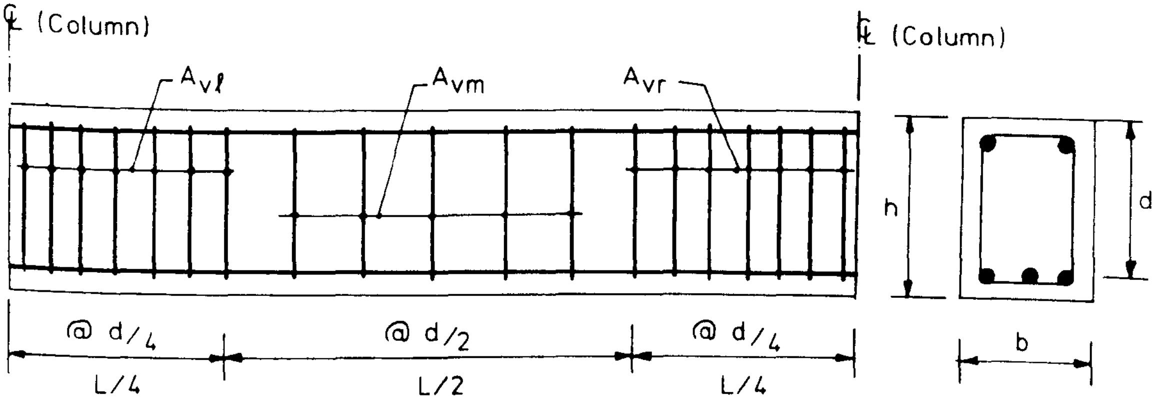


Fig. (6.2)

Arrangement of Web Reinforcement in a Beam

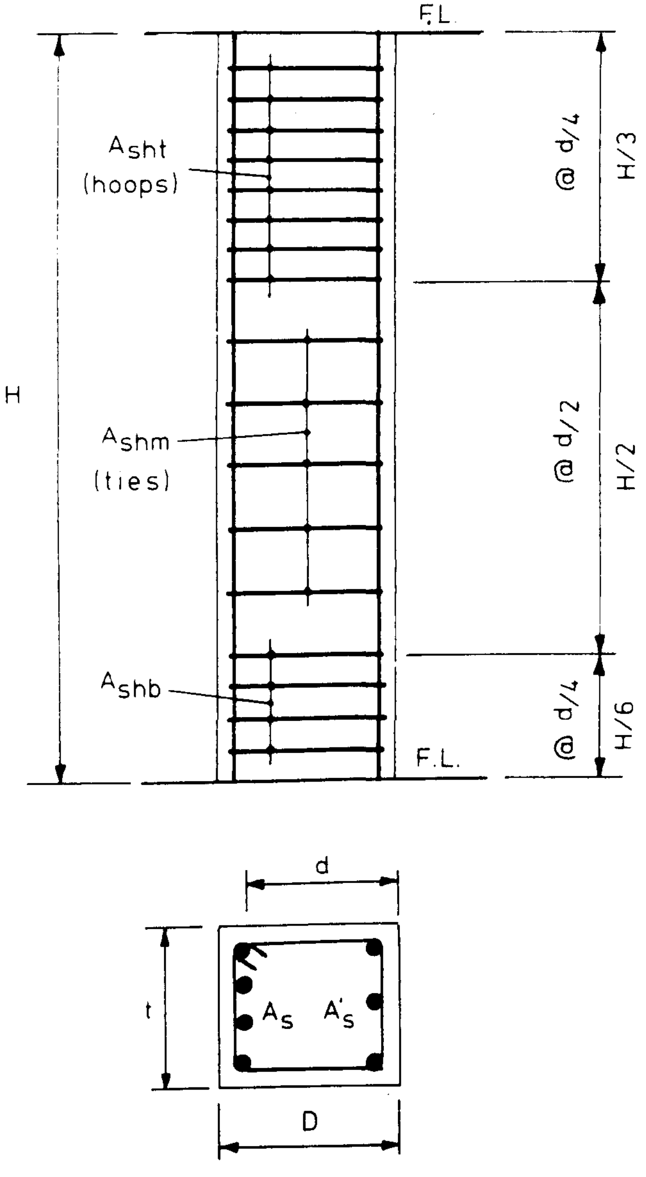


Fig. (6.3a)

Arrangement of Reinforcement in a Column

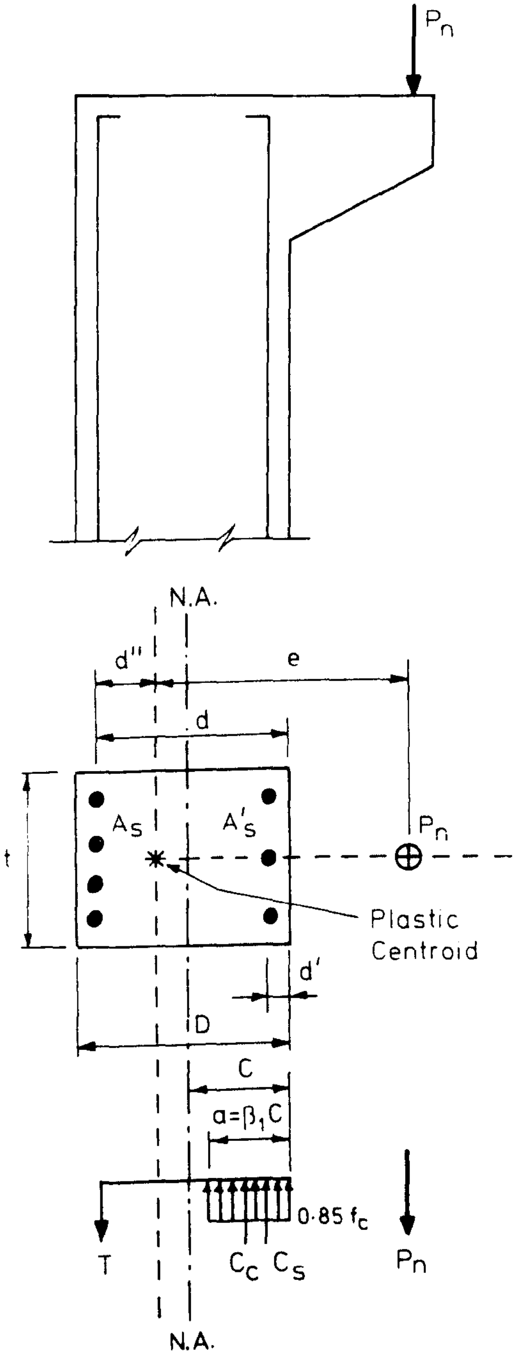


Fig. (6.3b)

Eccentrically Loaded Column

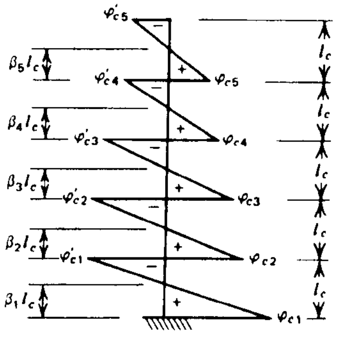


Fig. (6.4)

Curvature Distribution (Park et al, 1975)

$$\Delta_y = (\ell_c^2/6)\left[\ell_{c1}\ (r + 1/3) + \ell_{c2} + \ell_{c3} + \ldots + \ell_{cr}\right] \quad (6.3)$$

where $\beta_1 = 0.6$ and $\beta_2 = \beta_3 = \ldots. = \beta_r = 0.5$.

The curvature distribution in the column follows the shape of the bending moment diagram because the moments are still in the initial linear region of the moment-curvature relationships:

$$\varphi_{ci} = \frac{M_i}{E_c\ I_i}, \quad i = 1,2,\ldots r \quad (6.4)$$

where:

E_c = modulus of elasticity of the concrete.

I_i = moment of inertia of the cracked section transformed to concrete at storey i.

M_i = bending moment at storey i.

The curvature in the columns will differ from story to story because of different section properties and axial load levels. (Park, et.al. 1975).

6.4 DEVELOPMENT OF CONSTRAINTS

A constraint is a restriction imposed by the code in order to satisfy safety and serviceability requirements. In a reinforced concrete structural design problem, the constraints are used to describe the provisions of the code. Based on the ACI Code (318-83), the provisions to be considered in this study are strength, serviceability, ductility (plasticity), and other provisions such as spacing limits, concrete cover, cut-off points and development lengths of the reinforcement. The design constraints corresponding to the provisions of ductility, serviceability and architectural considerations can be indirectly considered by limiting the section dimensions and reinforcement of each member (Yang, 1981).

The design constraints considered in this study are discussed in the following paragraphs.

6.4.1 Constraints for a Beam

6.4.1.1 Flexural Strength

The nominal flexural strength of a beam section with constant width is given by:

$$M_n = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c b}\right) \qquad (6.5a)$$

The nominal flexural strength of a beam section with compression reinforcement and with a constant width can be written as:

$$M_n = 0.85 f'_c a b \left(d - a/2 \right) + A'_s f_y (d - d') \qquad (6.5b)$$

For equilibrium, $0.85\ f'_c\ ab = (A_s - A'_s)\ f_y$ hence, the above equation can be expressed as:

$$M_n = (A_s - A'_s)\ f_y\ (d - a/2) + A'_s\ f_y\ (d - d') \tag{6.5c}$$

Therefore, in terms of the design variables, the nominal flexural strength of a beam section with constant width is given by:

$$M_n = A_s\ f_y\ d + A'_s\ f_y\ d - A'_s\ f_y\ h - \frac{f^2_y}{1.7f'_c}\ \frac{(A_s - A'_s)^2}{b} \tag{6.5d}$$

where h = overall depth of beam in inches.

In order to represent the practical situation as closely as possible, part of the reinforcement is carried through, and the remainder is cut off based on moment resistance or on development length requirements. Each part may consist of several bars of the same size. In Eq. (6.5d), the contribution of compression reinforcement to flexural strength is taken into account by assuming the compression reinforcement yields at ultimate capacity. In practice, this assumption is reasonable, since the compression reinforcement in beams at ultimate capacity generally reaches the yield stress except when the beam is shallow or when high strength steel is used or when the beam is very ductile. For example, if ϵ_c is .003 and d' is 2.5 in., the compression reinforcement of grade 60 will reach yield strength under balanced conditions when the depth d is greater than 14 in. The distance from the centroid of the compression reinforcement to the extreme tension fibre of a beam can be considered as a design variable together with the effective depth d. However, in this case, one effective depth is used for both tension and compression reinforcement.

The compression reinforcement can be placed in the beam either at a distance d from the extreme tension fibre or as closely as the extreme compression fibre as possible.

In accordance with Articles (9.1.1), (9.2.1), (9.3.1) and (9.3.2) of the Code, design strength provided by a member or cross-section in terms of load, moment, shear, or stress shall be taken as the nominal strength multiplied by a reduction factor ϕ, and the design strength shall be at least equal to the required strength. Hence, the flexural strength constraints at the critical or controlling sections in any beam can be expressed as follows:

$$\phi_b \, M_{n\ell} - M_{u\ell} \geq 0 \qquad (6.6a)$$

$$\phi_b \, M_{nm} - M_{um} \geq 0 \qquad (6.6b)$$

$$\phi_b \, M_{nr} - M_{ur} \geq 0 \qquad (6.6c)$$

where:

ϕ_b = strength reduction factor for flexure which according to ACI Code is given as 0.90.

$M_{n\ell}$ = nominal negative moment strength at left end of a beam in feet - kips.

M_{nm} = nominal moment strength at maximum positive moment section of a beam in feet - l.ips.

M_{nr} = nominal negative moment strength at right end of a beam in feet - kips.

$M_{u\ell}$ = factored moment at left end of a beam in feet - kips.

M_{um} = factored moment at maximum positive moment section of a beam in feet -kips.

M_{ur} = factored moment at right end of a beam in feet - kips.

6.4.1.2 <u>Shear Strength</u>

The nominal shear strength of a beam section is given by:

$$V_n = V_c + V_s \quad (6.7)$$

where:

V_c = nominal shear strength provided by concrete in pounds.

V_s = nominal shear strength provided by shear reinforcement in pounds.

In accordance with Articles (11.3.1.1) and (11.5.6.2) of the Code, unless a more detailed computation is made based on Article (11.3.2), the shear strength V_c for members subject to shear and flexure only is computed by:

$$V_c = 2\sqrt{f_c'}\ b\ d \quad (6.8a)$$

The shear strength V_s of shear reinforcement, perpendicular to the axis of a member, is computed by:

$$V_s = A_v\ f_y\ d/s \quad (6.8b)$$

where A_v is the area of shear reinforcement within a distance s.

Articles (11.5.4.1) and (11.5.4.3) of the Code state that the spacing of shear reinforcement placed perpendicular to the axis of the member shall not exceed d/2 in non-prestressed members and when V_s exceeds $4\sqrt{f_c'}$ bd, maximum spacing is reduced to d/4. It is good practice to assume that the spacing, s, of shear reinforcement within a quarter span from the ends of a beam is equal to d/4 and that of the rest of the span is equal to d/2. Therefore, the nominal shear strength of sections at ends and at quarter span of a beam, as shown in Fig. (6.2), can be expressed as follows:

$$V_{n\ell} = 2\sqrt{f'_c}\, bd + 4f_y A_{v\ell} \quad (6.9a)$$

$$V_{nr} = 2\sqrt{f'_c}\, bd + 4f_y A_{vr} \quad (6.9b)$$

$$V_{nq} = 2\sqrt{f'_c}\, bd + 2f_y A_{vm} \quad (6.9c)$$

where:

$A_{v\ell}$ = area of shear reinforcement within a distance s which is in a quarter span from left end of a beam; where s is taken to be d/4, in square inches.

A_{vr} = area of shear reinforcement within a distance s which is in a quarter span from right end of a beam; where s is taken to be d/4, in square inches.

A_{vm} = area of shear reinforcement within a distance s which is in a center half span of a beam; where s is taken to be d/2, in squre inches.

$V_{n\ell}$ = nominal shear strength at left end of a beam in pounds.

V_{nr} = nominal shear strength at right end of a beam in pounds.

V_{nq} = nominal shear strength at quarter span section of a beam in pounds.

Since the design shear strength shall be at least equal to the required shear strength, according to Articles (9.1.1), (9.1.2), (9.3.1) and (9.3.2) of the Code, the shear strength constraints at the critical or controlling sections in any beam can be expressed as follows:

$$\phi_{sh} V_{n\ell} - V_{u\ell} \geq 0 \quad (6.10a)$$

$$\phi_{sh} V_{nr} - V_{ur} \geq 0 \quad (6.10b)$$

$$\phi_{sh} V_{nq\ell} - V_{uq\ell} \geq 0 \quad (6.10c)$$

$$\phi_{sh} V_{nqr} - V_{uqr} \geq 0 \quad (6.10d)$$

where:

ϕ_{sh} = strength reduction factor for shear which according to ACI Code is given as 0.85.

$V_{nq\ell}$ = nominal shear strength at quarter span section from left end of a beam in kips.

V_{nqr} = nominal shear strength at quarter span section from right end of a beam in kips.

$V_{u\ell}$ = factored shear force at left end of a beam in kips.

V_{ur} = factored shear force at right end of a beam in kips.

$V_{uq\ell}$ = factored shear force at quarter span section from left end of a beam in kips.

V_{uqr} = factored shear force at quarter span section from right end of a beam in kips.

In the above expressions the units of $V_{n\ell}$ and V_{nr} are in kips.

6.4.1.3 <u>Ductility (Plasticity)</u>

In designing a beam, the amount of tension reinforcement in flexural members must be limited to ensure a level of ductile behavior. Articles (10.3.3) and (10.5.1) of the Code state that for flexural members, the ratio of reinforcements ρ provided shall not exceed 0.75 of the ratio ρ_b that would produce balanced strain conditions for the section under flexure without axial load. For member with compression reinforcement need not be reduced by the 0.75 factor. At any section of a flexural member, where positive reinforcement is required by analysis, the ratio ρ provided shall not be less than that given by $200/f_y$. Therefore, the ductility constraints can be expressed as follows:

$$\gamma_1 \rho_b + \frac{A'_s}{bd} - \frac{A_s}{bd} \geq 0 \qquad (6.11a)$$

$$\frac{A_s}{bd} - \frac{200}{f_y} \geq 0 \qquad (6.11b)$$

where:

γ_1 = factor which according to ACI Code is given as either 0.75 or 0.5 (in seismic zone).

ρ_b = reinforcement ratio producing balanced strain condition for section with tension reinforcement only.

6.4.1.4 Serviceability

The assessment of the performance of the structure at the service load is an important consideration when members are proportioned on the basis of the strength method. Article (9.5.1) of the Code indicates that reinforced concrete members subject to flexure shall be designed to have adequate stiffness to limit deflections that may adversely affect serviceability of a structure at service loads. Article (9.5.2.1) of the Code states that minimum depth stipulated in Table (9.5a) of the Code shall apply for one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections. Article (10.7.1) of the Code states that for flexural members the ratio of overall depth to clear span shall be less than 2/5 for the continuous spans or 4/5 for simple spans. For larger ratios, the beam will become a deep beam. In addition, some other provisions due to architectural requirements may also be necessary. The serviceability constraints can be expressed as follows:

$$\bar{h} \quad \text{or} \quad \gamma_4 L \geq h \geq \gamma_5 L \qquad (6.12a)$$

$$\bar{b} \quad \text{or} \quad h \geq b \geq \gamma_6 h \qquad (6.12b)$$

where:

$\bar{h}$ = specified overall depth of beam in inches.

$\bar{b}$ = specified width of beam in inches.

γ_4 = constant which according to ACI Code is 2/5 for continuous beams and 4/5 for simple beams.

γ_5 = constant which according to ACI Code is 1/16 for simply supported beams; or 1/18.5 for one end continuous beams; or

1/21 for both ends continuous beams.

γ_6 = specified ratio of width to overall depth of beam.

L = span length of beam in inches.

h = overall depth of beam in inches.

6.4.1.5 <u>Concrete Cover for Reinforcement</u>

In the design of reinforced concrete structures, minimum concrete cover is always provided for the protection of the reinforcement. This cover is measured to the outer edge of stirrups, ties, or spiral if transverse reinforcement encloses main bars; to the outermost layer of bars if more than one layer is used without stirrups or ties. Article (7.7.1) of the Code states that for cast-in-place concrete not exposed to weather or in contact with the ground, the minimum concrete cover of 1.5 inches shall be provided for primary reinforcement, ties, stirrups, spirals of members. Since the concrete cover and effective depth are components of overall depth of a member, the concrete cover constraint of a beam can be expressed as follows:

$$h - d \geq (1.5 + N_\ell \cdot d_b) \quad \text{or} \quad C_3 \qquad (6.13)$$

where:

N_ℓ = number of layers of longitudinal reinforcement placed in beam.

d_b = nominal diameter of reinforcing bars in inches.

C_3 = concrete cover measured to the centroid of main reinforcement of beam in inches.

6.4.1.6 <u>Limits of Web Reinforcement</u>

From consideration of diagonal compression struts in beams, higher shear reinforcement content implies larger concrete compression stresses. This indicates that shear reinforcement content can not be increased indefinitely. Also, irrespective of the shear intensity, a minimum shear reinforcement should be provided in every beam even though analysis shows that shear reinforcement is not required. As is pointed out in Articles (11.5.5.3) and (11.5.6.8) of the Code, minimum area of shear reinforcement for members shall be computed by $A_v = 50 bs/f_y$ and shear strength V_s shall be taken less than $8\sqrt{f'_c}bd$. Therefore, the shear reinforcement content constraints can be expressed as follows:

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{A_{v\ell}}{bd} \geq 0 \qquad (6.14a)$$

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{A_{vr}}{bd} \geq 0 \qquad (6.14b)$$

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{A_{vm}}{2bd} \geq 0 \qquad (6.14c)$$

$$\frac{A_{v\ell}}{bd} - \frac{50}{4f_y} \geq 0 \qquad (6.14d)$$

$$\frac{A_{vr}}{bd} - \frac{50}{4f_y} \geq 0 \qquad (6.14e)$$

$$\frac{A_{vm}}{bd} - \frac{50}{4f_y} \geq 0 \qquad (6.14f)$$

6.4.1.7 <u>Development Length for Longitudinal Reinforcements</u>

Development length is a consideration at sections of maximum moment along a beam and where a neighbouring bar is cut. Articles

(12.11.3) and (12.11.4) of the Code state that: reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member or 12 d_b, whichever is greater, and, continuing reinforcement is no longer required to resist flexure. Hence, for a beam, as shown in Fig. (6.1b), the development length constraints can be expressed as follows:

$$\ell_i \geq 1.0 \qquad i = 2, 4, 5 \tag{6.15a}$$

$$\ell_i \geq \ell_d \qquad i = 2, 4, 5 \tag{6.15b}$$

$$\ell_{i-1} - (\ell_i - d) \geq \ell_d \qquad i = 2, 4 \tag{6.15c}$$

$$\ell_i - (\ell_i - 12\ d_b) \geq \ell_d \qquad i = 2, 4 \tag{6.15d}$$

where:

ℓ_d	=	development length in feet.
ℓ_j	=	length of reinforcement from sections of maximum moment in feet, j=1,2,...,5.
d	=	effective depth of beam in feet.
d_b	=	specified nominal diameter of bars in feet.

6.4.1.8 Cut-Off Points of Longitudinal Reinforcement

It is evident that after the formation of diagonal cracks in beams, the tension force in the flexural reinforcement becomes larger than that required to resist the external moment at that section. This and other causes may shift the location of maximum moments of the moment diagrams which are customarily used in design. To provide for shifts in the location of maximum moments, Article (12.11.3) of the

Code requires that flexural reinforcement be extended beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member or $12d_b$ whichever is greater. Therefore, the constraints of cut-off points of main reinforcement of a beam can be expressed as follows:

$$\gamma_a M_{up} - M_{ua} \geq 0 \tag{6.16a}$$

$$-\gamma_b M_{un} + M_{ub} \geq 0 \tag{6.16b}$$

where:

γ_a	=	ratio of the reinforcement to be continued at section of cut off point of bars to the reinforcement required at sections of maximum positive moment.
γ_b	=	ratio of the reinforcement to be continued at section of cut off point of bars to the reinforcement required at sections of maximum negative moment.
M_{ua}	=	positive factored moment at section where parts of reinforcing bars from sections of maximum positive moment are theoretically to be cut, in feet - kips.
M_{ub}	=	negative factored moment at section where parts of reinforcing bars from sections of maximum negative moment are theoretically to be cut, in feet - kips.
M_{up}	=	positive factored moment at sections of maximum positive moment, in feet - kips.
M_{un}	=	negative factored moment at sections of maximum negative moment in feet - kips.

6.4.1.9 <u>Spacing Limits</u>

In order to ensure no honeycomb between bars and between bars and forms, and no shear or shrinkage cracking due to concentration of bars on a line, the spacing limits for reinforcement are required. According to Articles (7.6.1) and (7.6.2) of the Code which state that clear distance between parallel bars in a layer shall not be less than d_b or 1.0 inch; where parallel reinforcement is placed in two or more layers, bars in the upper layer shall be placed directly above bars in the bottom layer with clear distance between layers not less than 1.0 inch. Therefore, in a beam, the spacing limits constraints can be expressed as follows:

$$b \geq 3.0 + 2\,(d_b)_2 + \left[\frac{N_s + N_\ell - 1}{N_\ell} - 1\right](d_b)_1 + \frac{N_s + N_\ell - 1}{N_\ell}(d_b)_1 \qquad (6.17a)$$

$$b \geq 2.0 + 2\,(d_b)_2 + \frac{N_s + N_\ell - 1}{N_\ell}\,(1.0 + (d_b)_1) \qquad (6.17b)$$

$$\frac{S_1 - N_v\,(d_b)_2}{N_v} \geq 1.0 \qquad (6.18a)$$

$$\frac{S_2 - N_v\,(d_b)_2}{N_v} \geq 1.0 \qquad (6.18b)$$

where:

$(d_b)_1$	=	nominal diameter of longitudinal bar in inches.
$(d_b)_2$	=	nominal diameter of web steel in inches.
N_s	=	total number of tension bars in beam, based on specifically based bars.
$(N_s+N_\ell-1)/N_\ell$	=	maximum number of bars placed in a layer in beam.
N_v	=	total number of stirrups within a distance s, based on specifically based bars.
S_i	=	spacing of stirrups; $S_1 = d/4$, $S_2 = d/2$ in inches.

6.4.2 Constraints for a Column

6.4.2.1 Axial and Flexural Strength

The column is a structural member used primarily to carry compressive loads. For a short column, the ultimate capacity at a given eccentricity is governed only by the strength of materials and the dimensions of the cross section. It is evident that the ultimate load of an axially loaded, reinforced concrete column is the sum of the yield strength of the steel plus the strength of the concrete. Therefore, the axial strength of either a tied or a spiral column can be expressed as:

$$P_n = 0.85\, f'_c \, (A_g - A_s - A'_s) + f_y \, (A_s + A'_s) \tag{6.19}$$

where:

- A_g = gross area of the cross section in square inches.
- A_s = area of longitudinal reinforcement at the left face of the column in square inches.
- A'_s = area of longitudinal reinforcement at the right face of the column in square inches.

Figures (6.3a) and (6.3b) show the arrangement of the reinforcement. In accordance with Figure (6.3b), A_s corresponds to the tensile steel and A'_s corresponds to the compressive steel. For small eccentricities both A_s and A'_s will be in compression.

In practical situations columns are usually subjected to a certain amount of bending, which is conveniently represented by assuming that the axial load is applied eccentrically. The eccentricity is usually measured from the plastic centroid of the column section.

In columns, since the axial loads may be larger than that at balanced conditions, a compression failure cannot be avoided by limiting the area of reinforcement. The compression reinforcement in an eccentrically loaded column at ultimate capacity generally reaches the yield strength except when the load level is low, when high strength steel is used or when the column is small so that the cover is relatively large. For an eccentrically loaded column with bars at two faces, when tension failure occurs the axial load strength of the section as in Fig. (6.3b) is given by:

$$P_n = 0.85\ f'_c\ a\ t + A'_s\ f'_s - A_s\ f_y \qquad (6.20a)$$

if the stress in the compression reinforcement reaches the yield point $f'_s = f_y$, otherwise, f'_s should be determined in such a way that the strain in steel corresponding to f'_s is equal to the strain in concrete at the level of the steel. By assuming that the compression reinforcement yields at ultimate capacity, the axial strength of the section can be expressed as:

$$P_n = 0.85\ f'_c\ a\ t + A'_s\ f_y - A_s\ f_y \qquad (6.20b)$$

and the flexural strength measured from the plastic centroid of the section as in Fig. (6.3b) is given by:

$$M_n = 0.85\ f'_c\ a\ t\ (d - d'' - 0.5\ a) + A'_s\ f_y\ (d - d' - d'') + A_s\ f_y\ d'' \qquad (6.20c)$$

where:

P_n	=	nominal axial load strength of a column section in pounds.
M_n	=	nominal moment strength of a column section in pounds inches.
a	=	depth of equivalent rectangular stress block in inches.
d	=	effective depth of column in inches.
d'	=	distance from centroid of compression reinforcement to extreme compression fibre in inches.
d''	=	distance from the plastic centroid to the centroid of tension reinforcement in inches.
t	=	width of column in inches.

The distance d'' can be calculated from the following expression:

$$d'' = \frac{0.85\, f'_c\, t\, D\, (d - 0.5\, D) + A'_s\, f_y\, (d - d')}{0.85\, f'_c\, t\, D + (A_s + A'_s)\, f_y} \quad (6.21)$$

where:

D = overall depth of column in inches.

On the basis of the above equations, it is possible to obtain points on the interaction diagram so long as the column fails in tension. Table (6.2) shows the expression for P_n and M_n for four different locations.

In accordance with Articles (9.3.2) and (10.3.5) of the Code, the strength reduction factor ϕ, for reinforced concrete members, may be increased from 0.7 or 0.75 for compression members to 0.90 for pure flexural members as the design axial load strength ϕP_n decreases from the specified value of 0.10 $f'_c A_g$ to zero. The ϕP_n value shall not be taken greater than that given in Eqs. (10.1) and (10.2) of the Code. It is evident that, in practice, for most compression members, the axial loads are normally greater than that value specified in the Code at which an increase in the ϕ factor is permitted. However, there may be cases in which the axial loads in columns are relatively small. In this study, for convenience, the strength reduction factor for columns with large axial loads will be assumed to be $\phi = 0.70$ and that for columns of small axial loads in the tension failure range to be $\phi = 0.80$.

Based on the Code and a linearised interaction diagram, as shown in Fig. (6.4), which is convenient and on the safe side, the axial and flexural strength constraints of a column can be expressed as follows:

For compression failure when $\phi(P_n)_1 \geq P_u \geq \phi(P_n)_2$:

$$\gamma_c \phi_c (P_n)_1 - P_u \geq 0 \tag{6.22a}$$

$$\phi_c - \frac{P_u}{(P_n)_1} - \frac{[(P_n)_1 - (P_n)_2] M_u}{(P_n)_1 (M_n)_2} \geq 0 \tag{6.22b}$$

For tension failure when $\phi(P_n)_2 \geq P_u \geq \phi(P_n)_3$:

$$\phi_c + \frac{P_u [(M_n)_2 - M_n)_3]}{(P_n)_2 (M_n)_3 - (P_n)_3 (M_n)_2} - \frac{M_u [(P_n)_2 - (P_n)_3]}{(P_n)_2 (M_n)_3 - (P_n)_3 (M_n)_2} \geq 0 \tag{6.22c}$$

For tension failure when $\phi (P_n)_3 \geq P_u \geq 0$:

$$\phi_c + \frac{P_u [(M_n)_3 - (M_n)_4]}{(P_n)_3 (M_n)_4} - \frac{M_u}{(M_n)_4} \geq 0 \tag{6.22d}$$

where:

ϕ_c = strength reduction factor for compression members which according to ACI Code is given as 0.7 or 0.75

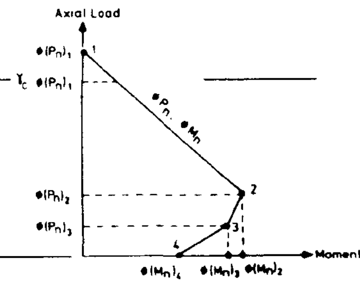
γ_c = coefficient to limit the design axial load strength of a section in pure compression to γ_c times its nominal strength, i.e. coefficient to achieve the purpose of the minimum eccentricity. According to ACI Code, it is given as 0.8 or 0.85.

In the above expressions the moments and forces have units of feet - kips and kips, respectively.

6.4.2.2 Shear Strength

The nominal shear strength of sections at top, middle height and bottom of a column, as shown in Fig. (6.3a), respectively, will have the same forms as that described in a beam, that is,

Table (6.2)
Nominal Axial and Flexural Strength of Columns



Location on the Interaction Diagram	Axial Strength P_n	Flexural Strength (Measured from the Plastic Centroid) M_n
1	$a_2tD+a_1A_s+a_1\acute{A}s$	0
2	$b_1td-a_3\acute{A}_s a_3\acute{A}_s$	$\frac{1}{a_2tD+a_3A_s+a_3\acute{A}_s}(\tfrac{1}{2}a_2b_1t^2D^2d-\tfrac{1}{2}a_2a_3A_st^2D-\tfrac{1}{2}a_2a_3\acute{A}_stD^2+a_2a_3A_stDd+(a_2a_3+b_1a_3)\acute{A}_stDd-2a^2{}_3A_s\acute{A}_sd+4a^2{}_3A_s\acute{A}_sd+a_3b_1A_std^2-a_3b_1\acute{A}_std^2)$ $-\frac{b^2{}_1\ td^2}{2a_2}$
3	$b_2td-a_3A_s+a_3\acute{A}_s$	$\frac{1}{a_2tD+a_3A_s+a_3\acute{A}_s}(\tfrac{1}{2}a_2b_2t^2D^2d-\tfrac{1}{2}a_3a_3A_stD^2-0.5a_2a_3\acute{A}_stD^2+a_2a_3A_stDd+(a_2a_3+b_2a_3)\acute{A}_stDd-2a^2{}_3A_s\acute{A}_sD+4a^2{}_3A_s\acute{A}_sd+a_3b_2A_std^2-a_3b_2\acute{A}_std^2)$ $-\frac{b^2{}_2\ td^2}{2a_2}$
4	0	$-a_3\acute{A}_sD+a_3A_sd+a_3\acute{A}_sd+a_3\frac{A_s\acute{A}_s}{t}-a_5\frac{A_s{}^2}{t}-a_5\frac{\acute{A}_s{}^2}{t}$

The expressions of the constants introduced above are as follows:

$a_1 = f_y-0.85\acute{f}_c,\ a_2 = 0.85\acute{f}_c,\ a_3 = f_y,\ a_4 = 1.18f_y{}^2/\acute{f}_c,\ a_5 = 0.59f_y{}^2/\acute{f}_c$

$b_1 = 0.85\acute{f}_c\ \beta_1\ C_b/d,\ b_2 = 0.85\ \acute{f}_c\ \beta_1\ C_b/d$, where C_b is neutral axis depth for balanced condition.

$$V_{nt} = 2\sqrt{f'_c}\ t\ d + 4\ f_y\ A_{sht} \qquad (6.23a)$$

$$V_{nb} = 2\sqrt{f'_c}\ t\ d + 4\ f_y\ A_{shb} \qquad (6.23b)$$

$$V_{nm} = 2\sqrt{f'_c}\ t\ d + 2\ f_y\ A_{shm} \qquad (6.23c)$$

where:

A_{sht} = area of shear reinforcement, within one third of the height from top of a column, within a distance s; where s is taken to be d/4; in square inches.

A_{shb} = area of shear reinforcement, within one sixth of the height from the bottom of a column, within a distance s; where s is taken to be d/4; in square inches.

A_{shm} = area of shear reinforcement, within a height from one sixth to two thirds of the height of a column, within a distance s; where s is taken to be d/2; in square inches.

V_{nt} = nominal shear strength at the top section of a column in pounds.

V_{nm} = nominal shear strength at one-third height from the top or one-sixth height from the bottom section of a column in pounds.

V_{nb} = nominal shear strength at the bottom section of a column in pounds.

As in beams, the shear strength constraints at the critical or controlling sections in any column can be expressed as follows:

$$\phi_{sh}\ V_{nt} - V_{ut} \geq 0 \qquad (6.24a)$$

$$\phi_{sh}\ V_{nb} - V_{ub} \geq 0 \qquad (6.24b)$$

$$\phi_{sh}\ V_{nm} - V_{um} \geq 0 \qquad (6.24c)$$

where:

V_{nt} = factored shear force at the top section of a column in kips.

V_{um} = factored shear force at one-third height from the top or one-sixth height from the bottom section of a column in kips.

V_{ub} = factored shear force at the bottom section of a column in kips.

6.4.2.3 Ductility (Plasticity)

As described in Articles (10.9.1), (10.3.3) and (A.6.1) of the Code, area of longitudinal reinforcement for non-composite compression members shall not be less than 0.01 nor more than 0.08 or 0.06 times the gross area of the section. For members subject to combined flexure and compressive axial load when the design axial load strength $\phi \rho_n$ is less than the smaller of $0.10 f_c' A_g$ or $\phi \rho_b$, the ratio of reinforcement ρ provided shall not exceed 0.75 of the ration ρ_b that would produce balanced strain conditions for the section under flexure without axial load. For members with compression reinforcement the portion of ρ_b equalised by compression reinforcement and not be reduced by the 0.75 factor. Therefore, the ductility constraints can be expressed as follows:

$$\gamma_2 - \frac{A_s + \acute{A}_s}{tD} \geq 0 \qquad (6.25a)$$

$$\frac{A_s + \acute{A}_s}{tD} - \gamma_3 \geq 0 \qquad (6.25b)$$

$$\gamma_1 \bar{\rho}_b - \frac{A_s \acute{f}_s}{td \; f_y} - \frac{A_s}{td} \geq 0 \qquad (6.25c)$$

where:

γ_2 = factor which according to ACI Code is given as either 0.08 or 0.06.

γ_3 = factor which according to ACI Code is given as 0.01.

$\acute{f}_s$ = compression stress of compression reinforcement of column, in this study $\acute{f}_s$ is taken to be equal to f_y.

6.4.2.4 Serviceability

Based on architectural and other requirements, the serviceability constraints can be expressed as follows:

$$\bar{t} \geq \gamma_7 \; b \qquad (6.26a)$$

$$\bar{D} \geq \gamma_8 \; h \qquad (6.26b)$$

$$\bar{D} \geq t \geq \gamma_9 \; \bar{D} \qquad (6.26c)$$

where:

$\bar{t}$ = specified width of column in inches.

$\bar{D}$ = specified overall depth of column in inches.

γ_7 = factor to specify the relation of the width of beam and column.

γ_8 = factor to specify the relation of the overall depth of beam and column.

γ_9 = specified ratio of width to overall depth of column.

6.4.2.5 Concrete Cover for Reinforcement

As mentioned before, the concrete cover and effective depth are components of overall depth of a member. Unlike beams, the longitudinal reinforcement in columns is placed in one layer in most cases. Therefore, in accordance with Article (7.7.1) of the Code, the concrete cover constraint of a column can be expressed as follows:

$$D - d \geq C_4 \qquad (6.27)$$

where:

C_4 = concrete cover measured to the centroid of main reinforcement of column in inches.

6.4.2.6 Limits of Lateral Reinforcement

The purpose of lateral reinforcement in columns in threefold. First, column bars carrying compression loads are liable to buckle.

Lateral ties must provide adequate lateral support to each column bar, to prevent instability due to outward buckling. Second, for columns of buildings subjected to lateral loading which often carry large flexural shear loads, shear reinforcement must be required. Third, lateral ties are to provide confinement to the concrete core. Based on Articles (11.5.5.3) and (11.5.6.8) of the Code, the shear reinforcement content constraints can be expressed as the same forms as that of Eqs. (6.19), that is:

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{A_{sht}}{td} \geq 0 \qquad (6.28a)$$

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{A_{shb}}{td} \geq 0 \qquad (6.28b)$$

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{A_{shm}}{2td} \geq 0 \qquad (6.28c)$$

$$\frac{A_{sht}}{td} - \frac{50}{4\ f_y} \geq 0 \qquad (6.28d)$$

$$\frac{A_{shb}}{td} - \frac{50}{4\ f_y} \geq 0 \qquad (6.28e)$$

$$\frac{A_{shm}}{2td} - \frac{50}{4\ f_y} \geq 0 \qquad (6.28f)$$

For special ductile frame columns subject to flexure and axial loads, confinement reinforcement shall be provided above and below beam -column connections for a certain distance. As is prescribed in Articles (10.9.3), (A.6.5.2) and (A.6.5.3) of the Code, for a column, where rectangular loop reinforcement is used, the required area of transverse loop bar (one leg) shall be computed by:

$$A_{sh} = 0.225\ \ell_h\ S_h\ \left(\frac{A_g}{A_c} - 1\right) \frac{f'_c}{f_y} \qquad (6.29)$$

or by:

$$A_{sh} = 0.06\ \ell_h\ S_h\ \frac{f'_c}{f_y} \qquad (6.30)$$

whichever is greater.

where:

A_c = area of rectangular core of column in square inches.

A_{sh} = area of transverse bar (one leg) in square inches.

ℓ_h = maximum unsupported length of rectangular hoop in inches.

s_h = center to center spacing of hoops in inches.

Therefore, for a column, as shown in Fig. (6.3a), the lateral reinforcement content constraints to take account of providing confinement to the concrete core can be expressed as follows:

$$A_{sht} - 0.3375\ \frac{(t + D - 3.0)\ d\ f'_c}{(D - 3.0)\ f_y} \geq 0 \qquad (6.31a)$$

$$A_{shb} - 0.3375\ \frac{(t + D - 3.0)\ d\ f'_c}{(D - 3.0)\ f_y} \geq 0 \qquad (6.31b)$$

$$A_{sht} - 0.03\ (t - 3.0)\ d\ \frac{f'_c}{f_y} \geq 0 \qquad (6.32a)$$

$$A_{shb} - 0.03\ (t - 3.0)\ d\ \frac{f'_c}{f_y} \geq 0 \qquad (6.32b)$$

where:

A_{sht} and A_{shb} are the area of transverse hoop bar (two legs) in square inches.

Article (7.10.5.2) of the Code states that vertical spacing of ties shall not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or least dimension of the compression member. This requirement is normally met when the dimension of the compression member and the size of the reinforcing bars are specified. It is not the strength but rather the stiffness of the ties that matters. Therefore, no constraints are needed to be given for this case.

6.4.2.7 Spacing Limits

Articles (7.6.3) and (7.10.4.3) of the Code state that in spirally reinforced or tie reinforced compression members, clear distance between longitudinal bars shall not be less than 1.5 d_b nor 1.5 inch; clear spacing of lateral reinforcement shall not be less than 1.0 inch. Therefore, the spacing limits constraints can be expressed as follows:

$$t \geq 3.0 + 2\ (d_b)_2 + N_s\ (d_b)_1 + 1.5\ (N_s - 1)\ (d_b)_1 \qquad (6.33a)$$

$$t \geq 1.5 + 2\ (d_b)_2 + (1.5 + (d_b)_1)\ N_s \qquad (6.33b)$$

$$\frac{S_3 - N_v\ (d_b)_2}{N_v} \geq 1.0 \qquad (6.34a)$$

$$\frac{S_4 - N_v\ (d_b)_2}{N_v} \geq 1.0 \qquad (6.34b)$$

where:

N_s = total number of tension or compression bars in column, based on specifically based bars.

N_v = total number of ties within a distance s, based on specifically based bars.

S_i = spacing of ties or hoops; S_3 = d/4, S_4 = d/2 in inches.

Other than flexure, shear and axial forces, torsion may arise as a result of primary or secondary actions. Even though design for torsion can be included in the multi-criteria optimisation procedure, it is not considered at present in order not to increase the complexity of the problem.

6.5 BOUNDS OF DESIGN SPACE

6.5.1 Bound Values of Design Space of Overall Depth of Members

The bound values of overall depth of members depend on loading conditions, on span lengths of members and on types of structures. According to Articles (9.5.2) and (10.7.1) of the Code which state that the deflection of beams and one-way slabs need not be computed if the minimum overall thickness of members satisfy these values specified in Table (9.5a) of the Code and provided that the members are not supporting or not attached to partitions or other construction likely to be damaged by large deflection; in order not to be a deep flexural member, the overall depth to clear span ratio of members should be less than 2/5 for continuous span, or 4/5 for simple span. Therefore, by using limiting span-thickness ratios, the upper and lower bound values of the design space of overall depth of members can be defined. However, it should be pointed out that, in practice, for members with a span length less than 40 ft, it is not necessary to set the upper bound value of the design space of the overall depth of members to be greater than say 50 inches.

6.5.2 Bound Values of Design Space of Effective Depth of Members

As mentioned before, the effective depth and the concrete protection are components of overall depth of a member. Therefore, the bound values of both overall thickness and effective depth of members can be defined at the same time. Also, reinforcing bars of suitable sizes are used as typical bars to check the spacing requirements and the number of layers of main reinforcement in a member. Thus, with a concrete clear cover of 1.5 in., and assuming that, in practice, the maximum number of four layers of reinforcing is allowed in a member, the upper and lower bound values of the design space of the effective depth could be set to be 2.5 inches and 6 inches less than that of the overall depth of members, respectively. In this study, it is assumed that both overall depth and effective depth of members have the same design space.

6.5.3 Bound Values of Design Space of Width of Members

Considering the cross section of a member, it is not common to design it as very shallow or very narrow. The range of the width-depth ratio of the cross-sections of members can be very large, theoretically. However, in practice, the above range could lie between 0.4 and 1.0. Having defined the bound values for the overall depth of the member, the width to depth ratio may be used to obtain the upper and lower bound values for the width of the section. Practically, in most cases, it is not desirable to have the lower bound value of the width of a member to be less than 8 inches.

6.5.4 Bound Values of Design Space of Cut-Off Points of Main Reinforcement

It is evident that the curtailment of the flexural reinforcement in beams can be determined from the bending moment diagram. We assume that it is practical to divide the moment reinforcement, required at sections of maximum moment under given loads, into equal or unequal parts and curtail each divided part at a suitable position. In order to meet the Code requirement, it is also assumed that at least one divided part of the flexural reinforcement goes through the whole span length of a member. Hence, the cut-off points of flexural reinforcement, based on moment diagram, from the sections of maximum moment will be somewhere between the midspan and the end sections of the member. That is, the upper and lower bound values of the design space of the cut-off points could be set to be the half span length of the member and the development length ℓ_d, respectively.

6.5.5 Bound Values of Design Space of Web Reinforcement

In design, the spacing of web reinforcement could usually be assumed to be 0.25d within a quarter span from member end and 0.50d within the middle half span. Since, in practice, reinforcing bars from number 2 through number 4 are used as web reinforcement, it is reasonable to assume that only bars of the same size are allowed in each design and to use number 4 bars as a typical one to check the clear spacing of the web reinforcement in members. Under certain conditions, more than one number 4 closed stirrup may be needed within 0.25d or 0.50d in order that the shear reinforcement requirements are met. Those stirrups should be uniformly distributed such that the clear spacing limitations are also satisfied. As mentioned before, the area of web reinforcement is between $50bd/(4f_y)$ and $2\sqrt{f'_c}\ bd/f_y$. Therefore, through the use of selected bars and the bound values of the dimensions of members, the upper and lower values of the design space of the web reinforcement could be well defined.

6.5.6 Bound Values of Design Space of Main Reinforcement

Considering the ASTM standard reinforcing bars, it is evident that the available bar sizes are from number 3 through number 18. However, in practice, it is not common to use all the allowable bar sizes for the main reinforcement of a given structure. This indicates that only certain suitable bar sizes can be selected to be the main reinforcement in most structural design. It is good practice to assume that the longitudinal reinforcement of beams in a structure is made up of number 4 through number 9 bars and that of compression members is number 5 through number 9 or even larger. The bound values of longitudinal reinforcement may vary from one structure to another. However, it is evident from the ACI Code (318-83), that the allowable limits of longitudinal reinforcement is between 0.75 $\bar{\rho}_b + \acute{\rho}\ \acute{f}_s/f_y$ and $200/f_y$ for flexural members and is between 0.01 A_g and 0.06 A_g - 0.08 A_g for compression members. Therefore, once the bound values of the dimensions of the member are defined, the upper and lower bound values of the design space of longitudinal reinforcement can be set accordingly.

In general, the upper and lower bound values of a discrete design space can be established by the engineer's experience and juugement.

6.6 DESIGN VARIABLES

For each one-bay one storey frame, a total of 18 design variables are considered in this study as shown in Table (6.3). In terms of the considered design variables, derivations of the total cost, and drift, objective functions are summarised in Table (6.4).

Notes	Design Variables	
	Beam	Column
Cross Section Dimensions	$X_1=h$ $X_2=b$ $X_3=d$	$X_4=D$ $X_5=t$ $X_6=d_c$
Reinforcement Lengths	$X_7=\ell_1$ $X_8=\ell_2$	
Main Reinforcement Amounts	$X_9=A_{sm}$ $X_{10}=A_{s\ell}$	$X_{11}=A_s$ $X_{12}=\acute{A}_s$
Shear Reinforcement Amounts	$X_{13}=A_{vm}$ $X_{14}=A_{vr}$ $X_{18}=A_{v\ell}$	$X_{15}=A_{sht}$ $X_{16}=A_{shm}$ $X_{17}=A_{shb}$

Table (6.3)
One-Bay One-Storey Design Variables

Name	Objective Functions
Total Cost	$C = C_s \left[X_9 \left(L + 2\,X_7 - \frac{X_4}{12} + 2\,\ell_{d2}\right) + X_{10} \left(L + X_8 + 5.6\,\ell_{d1}\right) + 2\,L \left(\frac{X_1 + X_2 + \ell_{st}}{X_3}\right) \left(X_{13} + X_{14} + X_{18}\right)\right] + C_s \left[2(X_{11} + X_{12}) \left(H + \frac{X_1}{24}\right) + H \left(\frac{X_4 + X_5 + \ell_{st}}{X_6}\right) \left(\frac{16}{3} X_{15} + \frac{8}{3} X_{17} + 4X_{16}\right)\right] + C_c \left[2\,X_1\,X_2 \left(L - \frac{X_4}{12}\right)\right] + C_c \left[2\,X_4\,X_5 \left(H + \frac{X_1}{24}\right)\right]$; \$ where $\ell_{st} = -6 + 4\,d_b$
Drift	$\Delta = \frac{(12H)^2}{6} \left[\frac{12000\,(.25)\,P_h\,H}{E_c\,I_c}\,1.333\right]$; inch

Table (6.4)
One-Bay One-Storey Objective Functions

CHAPTER SEVEN

PRACTICAL ENGINEERING DESIGN PROGRAM FOR REINFORCED CONCRETE FRAMES

CHAPTER SEVEN

PRACTICAL ENGINEERING DESIGN PROGRAM FOR REINFORCED CONCRETE FRAMES

SYNOPSIS

In this chapter, a practical design program (PDP) of reinforced concrete frames is introduced. The PDP is reliability based and the structures considered have the following features:

1) The frames are made up of prismatic members.
2) Loads are fixed, static and include the weight of the members.
3) The geometric configuration of the frames is given and is independent of the design variables.

Several examples are included in detail. The Pareto set for each example is generated using the entropy-based minimax method derived in chapter three.

7.1 INTRODUCTION

The design of safe and functionally reliable structural framed systems within the constraints of economy is a subject of continuing concern to the profession of structural engineering. Because of the unpredictability of loads, strengths, and response of actual structures there generally exists a small probability of structural failure, locally, or structural collapse, globally. It has been established by Ang and Cornell (1974) that failure should be interpreted with respect to some predefined limit states; it may be an excessive deflection or the total collapse of a structure. In general, depending on the limit state under consideration, the concept of a failure probability is applicable to both the in-service conditions and the ultimate performance of structural systems. In this sense, the failure probability serves as a common and logical basis for the evaluation of performance of non-deterministic structural components and systems subject to random loads.

The optimization of building structures involves usually a number of requirements that should be met at the same time to obtain the fully useful design. In the case of single criterion optimization, one of the requirements is selected accordingly to be the criterion, while the remaining ones are met by including them into the constraint set. But with such an approach, it is necessary to determine in advance the level of meeting these constraints. Multi-criteria optimization helps to take into account numerous criteria that are often mutually conflicting. It is then possible to find a compromise solution which, although none of the criteria involved attains its extreme, guarantees meeting all the requirements in the best possible way following on from the global criterion. It is a minimum cost criterion which is most frequently assumed in structural optimization problems. Another criterion most frequently appearing in the theory of structures is the minimum displacements at selected points of a structure (Gero, 1983). The total material cost and top lateral displacement of a frame are the criteria considered in this thesis.

The concern for safety of structures must ultimately include the consideration of the reliability of a complete structural system. In probabilistic reliability analysis, the different potential ways in which a structure might become less than fully serviceable are referred to as failure modes. For a structural system of multiple components and/or multiple failure modes, there are several levels of damage states. The most serious level of damage, of course, is the ultimate collapse. Prior to its ultimate collapse, however, a structural system may undergo one or more intermediate stages of damage. Statistical correlation among failure modes arises through common loading variables or correlated strengths.

In this practical design program, we are optimizing reinforced concrete frame under vertical and horizontal loads for two different performance criteria, the total material cost and drift subject to flexural strength, shear strength, axial strength, ductility, serviceability, concrete cover, spacing limits, beam cut-off points of longitudinal reinforcement and development length of longitudinal reinforcement in column constraints. The considered design variables represent cross-section dimensions and amounts of reinforcements. The two criteria and all constraints must be expressed as functions of these design variables.

Unlike beams, there are no design constraints corresponding to the development length and the cutoff points of longitudinal reinforcement in columns since, in practice, no part of the longitudinal reinforcement in columns is assumed to be cut. In this case, for a typical frame, the longitudinal tensile reinforcement of the beam is divided into two equal parts and it is assumed that one part carries throughout the span. The span length of the beam is taken to be the center span of the columns. Although the height of the columns can be measured from the surface of the beam, in this study, for convenience, the height of the columns is measured from the mid-depth of the beam. Both the span lengths of the beams and the heights of the columns of the frame are considered as parameters in this study.

For the design of the web reinforcement of the beam, the governing sections are assumed to be at the end span and the quarter span measured from the center of the columns, and that for the design of ties or hoops in the columns are at the top, the one-third and the five-sixth height of the column measured from the mid-depth of the beam.

The development length of the tension and the compression reinforcement, within an exterior beam-column joint, of the beam are computed from the center and the face of the column, respectively.

Design spaces for the dimensions and the reinforcement of the columns can be different from that of the beam. However, for convenience, it is assumed that both beam and column have the same design space in this case.

This practical design program goes through three phases:

1) Optimization Phase: where Pareto set is generated using the new developed methods in chapter 3.
2) Rounding-off and Standardization Phase: where cross-section dimensions are rounded off and amounts of reinforcement are standardized.
3) Reliability Phase: where the multi-optimal design is analyzed probabilistically to determine its reliability.

To demonstrate how this practical design program is working, we shall turn first to a classical frame design problem in the literature of research in optimal frame design, one-storey one-bay frame. The next section clearly demonstrates that.

7.2 THE PDP PHASES

7.2.1 The Optimization Phase

The first example considered is a one-bay one-storey reinforced concrete frame as shown in Fig. (7.1), (Yang, 1981). The frame is subjected to uniformly distributed line load of 1.8 K/ft. The wind force is assumed to be 5.5 Kips and the vertical concentrated load is 15.0 Kips.

The frame is 40 ft wide and 15 ft high. The yield strength of reinforcement is 60 Ksi. The compressive strength of concrete is 4 Ksi. The overall depth of the column is assumed to be not less than one half of the overall depth of the beam, and both the column and the beam have the same width. The strength reduction factor is assumed to be 0.8 for columns with tension failures. The tension reinforcement at point 3 of the interaction diagram is assumed to have a strain of 0.007 which is equivalent to assuming that the axial load at point 3 of the interaction diagram is one half of that at point 2 when the longitudinal reinforcement of the columns is assumed to be symmetrical. The cost of concrete is $0.08 per sq. in. per linear ft. The cost of steel is $2.00 per sq. in. per linear ft. The width - overall depth ratio of the beam is assumed to be greater than or equal to 0.40. The problem has 18 design variables and 53 constraints. The computer program inputs are summarized in Table (7.1). The optimal design

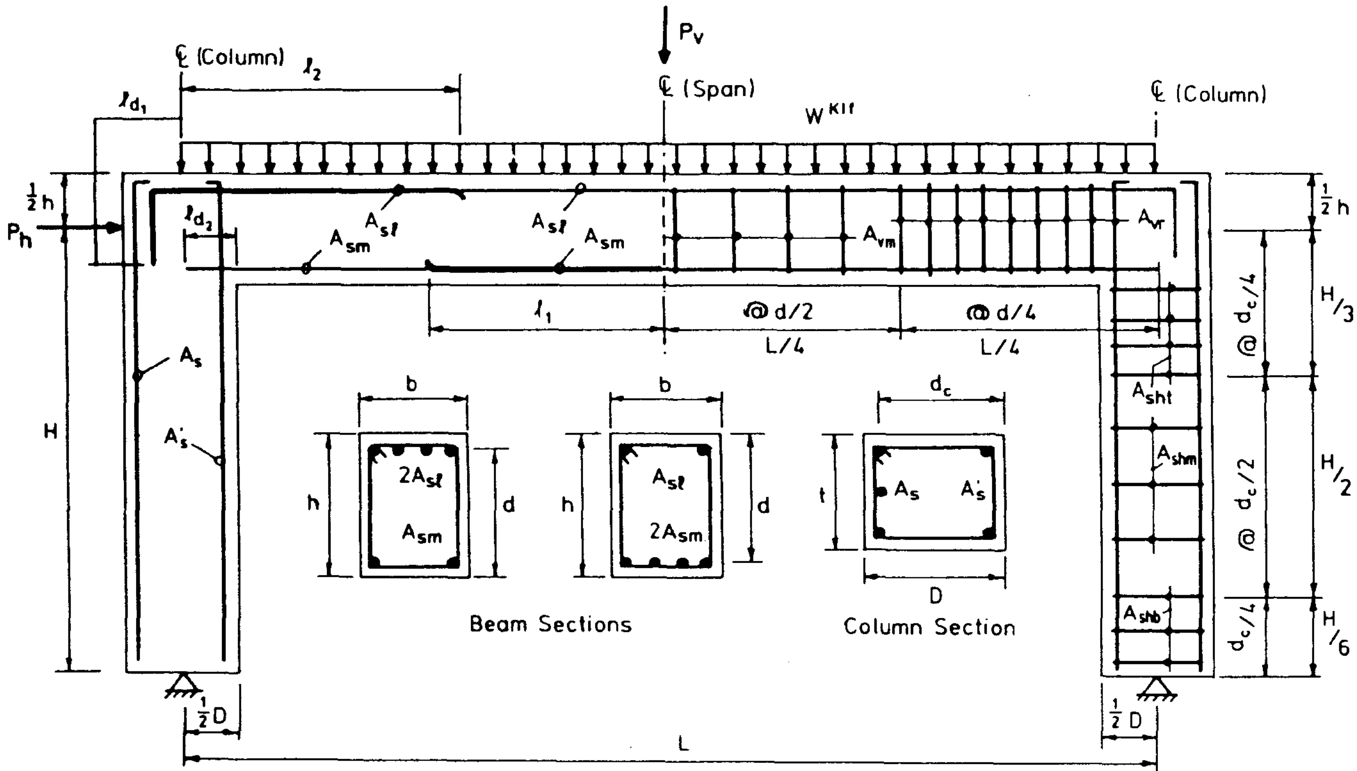


Fig. (7.1)

One-Bay One-Storey Reinforced Concrete Frame

Name	Variable	Unit
Compressive Strength of Concrete	f'_c	psi
Tensile Strength of Steel	f_y	psi
External Vertical Applied Load	P_v	kips
External Horizontal Applied Load	P_h	kips
Beam Span	L_b	ft
Column Height	H	ft
Cost of Steel	C_s	$\$/in^3$
Cost of Concrete	C_c	$\$/in^3$
Modules of Elasticity of Steel	E_s	Ksi
Uniform Live Load	LL	K/ft
No. of Tension Steel Layers in a Beam	($n\ell b$)	
No. of Tension Steel Layers in a Column	($n\ell c$)	
Total No. of Longitudinal Bars at Midspan of a Beam	($ns\ell$)	
Total No. of Bars at the Ends of a Beam	(ns2)	
Total No. of Bars at the Outer Side of a Column Cross Section	(ns3)	
Total No. of Bars at the Inner Side of a Column Cross Section	(ns4)	
Total No. of Shear Steel Within the Spacing's Based on #4 Bars of Both Beam and Columns	(nv2, nv4)	
Shear Steel Bar No. of Both Beam and Columns (#4 is OK)	(nt)	
The "Bar No." of Longitudinal Steel (Tension) in a Beam and Columns	(nb, nc)	
Ratio of Width/Overall Depth of a Beam	γ_6	
Ratio of Beam Cross Section/Column Cross Section Width	γ_7	
Ratio of Steel to be Continued at Section of Cut-off Point to the Steel Required at Sections of Max Positive Moment.	γ_8	
Ratio of Steel to be Continued at Section of Cut-off Point to the Steel Required at Sections of Max Negative Moment.	γ_9	

Table (7.1)
ADS Input Data of R.C. Frames

#	Cost ($)	Drift (in)	Notes
1	7711.977	0.001363676	Optimizer: MMFDCM
2	6172.785	0.003921729	
3	5049.387	0.01477914	No. of criteria: 2 (cost, drift)
4	4342.375	0.0300479	
5	4208.5	0.05196794	Dominant criteria: cost
			No. of Constraints: 48
			Case: One Bay - One Storey R.C. Frame

Table (7.2) Pareto Set

solutions considering two objective functions by the new developed entropy-based method are summarized in Table (7.2).

If we consider our preferred criterion is the total material cost, then we choose solution #5 where C = $4208.5. Next, using the enumeration technique described below, the cross-section dimensions should be rounded off to its neighbourhood in the corresponding design space, Tables (7.3-7.4). Similarly the amounts of reinforcements should be standardized to its neighbourhood in the corresonding design space, Tables (7.5-7.7).

7.2.2 The Rounding Off and Standardization Phase

It is often required to round off some of the design variables and/or standardize others by assigning an upper and lower bound on each design variable for different combinations. If the number of design

Table (7.3)

Design Space for Overall Depths and Effective Depths of Members (Increments of 1/2 in.)

Design Number X_1, X_3 X_4, X_6	Member Depth (in.)	Design Number X_1, X_3 X_4, X_6	Member Depth (in.)
1	16.50	25	28.50
2	17.00	26	29.00
3	17.50	27	29.50
4	18.00	28	30.00
5	18.50	29	30.50
6	19.00	30	31.00
7	19.50	31	31.50
8	20.00	32	32.00
9	20.50	33	32.50
10	21.00	34	33.00
11	21.50	35	33.50
12	22.00	36	34.00
13	22.50	37	34.50
14	23.00	38	35.00
15	23.50	39	35.50
16	24.00	40	36.00
17	24.50	41	36.50
18	25.00	42	37.00
19	25.50	43	37.50
20	26.00	44	38.00
21	26.50	45	38.50
22	27.00	46	39.00
23	27.50	47	39.50
24	28.00	48	40.00

Table (7.4)

Design Space for Widths of Members
Increments of 1/2 in.)

Design Number X_2, X_5	Member Width (in.)	Design Number X_2, X_5	Member Width (in.)
1	9.50	12	15.00
2	10.00	13	15.50
3	10.50	14	16.00
4	11.00	15	16.50
5	11.50	16	17.00
6	12.00	17	17.50
7	12.50	18	18.00
8	13.00	19	18.50
9	13.50	20	19.00
10	14.00	21	19.50
11	14.50	22	20.00

Table (7.5)

Design Space for Web Reinforced Bars

Design Number X_{13}, X_{14}, X_{15}, X_{16} X_{17}, X_{18}	Bar Type	Design Area (sq. in.)	Design Number X_{13}, X_{14} X_{15}, X_{16} X_{17}, X_{18}	Bar Type	Design Area (sq. in.)
1	1#3	0.11	15	10#3	1.10
2	1#4	0.20	16	6#4	1.20
3	2#3	0.22	17	11#3	1.21
4	3#3	0.33	18	12#3	1.32
5	2#4	0.40	19	7#4	1.40
6	4#3	0.44	20	13#3	1.43
7	5#3	0.55	21	14#3	1.43
8	3#4	0.60	22	8#4	1.60
9	6#3	0.66	23	15#3	1.65
10	7#3	0.77	24	16#3	1.76
11	4#4	0.80	25	9#4	1.80
12	8#3	0.88	26	17#3	1.87
13	9#3	0.99	27	18#3	1.98
14	5#4	1.00	28	10#4	2.00

Table (7.6)

Design Space for Cutoff Points of Longitudinal Reinforcing Bars (Increments of 1/4 ft.)

Design Number X_7, X_8	Bar Length (ft.)	Design Number X_7, X_8	Bar Length (ft.)	Design Number X_7, X_8	Bar Length (ft.)
1	1.00	27	7.50	53	14.00
2	1.25	28	7.75	54	14.25
3	1.50	29	8.00	55	14.50
4	1.75	30	8.25	56	14.75
5	2.00	31	8.50	57	15.00
6	2.25	32	8.75	58	15.25
7	2.50	33	9.00	59	15.50
8	2.75	34	9.25	60	15.75
9	3.00	35	9.50	61	16.00
10	3.25	36	9.75	62	16.25
11	3.50	37	10.00	63	16.50
12	3.75	38	10.25	64	16.75
13	4.00	39	10.50	65	17.00
14	4.25	40	10.75	66	17.25
15	4.50	41	11.00	67	17.50
16	4.75	42	11.25	68	17.75
17	5.00	43	11.50	69	18.00
18	5.25	44	11.75	70	18.25
19	5.50	45	12.00	71	18.50
20	5.75	46	12.25	72	18.75
21	6.00	47	12.50	73	19.00
22	6.25	48	12.75	74	19.25
23	6.50	49	13.00	75	19.50
24	6.75	50	13.25	76	19.75
25	7.00	51	13.50	77	20.00
26	7.25	52	13.75		

Table (7.7)

Design Space for Longitudinal Reinforcing Bars

Design Number X_i	Bar Type	Design Area (sq. in.)	Design Number X_i	Bar Type	Design Area (sq. in.)
1	1#4	0.20	40	5#8	3.95
2	1#5	0.31	41	9#6	3.96
3	2#4	0.40	42	4#9	4.00
4	1#6	0.44	43	13#5	4.03
5	1#7	0.60	44	7#7	4.20
6	2#5	0.62	45	14#5	4.34
7	1#8	0.79	46	10#6	4.40
8	4#4	0.80	47	15#5	4.65
9	2#6	0.88	48	6#8	4.74
10	3#5	0.93	49	8#7	4.80
11	1#9	1.00	50	11#6	4.84
12	2#7	1.20	51	5#9	5.00
13	4#5	1.24	52	12#6	5.28
14	3#6	1.32	53	9#7	5.40
15	7#4	1.40	54	7#8	5.53
16	5#5	1.55	55	13#6	5.72
17	2#8	1.58	56	6#9	6.00
18	8#4	1.60	57	14#6	6.16
19	4#6	1.76	58	8#8	6.32
20	3#7	1.80	59	11#7	6.60
21	6#5	1.86	60	7#9	7.00
22	2#9	2.00	61	9#8	7.11
23	7#5	2.17	62	12#7	7.20
24	5#6	2.20	63	13#8	7.80
25	3#8	2.37	64	10#8	7.90
26	4#7	2.40	65	8#9	8.00
27	8#5	2.48	66	14#7	8.40
28	13#4	2.60	67	11#8	8.69
29	6#6	2.64	68	9#9	9.00
30	9#5	2.79	69	12#8	9.48
31	14#4	2.80	70	13#8	10.27
32	3#9	3.00	71	11#9	11.00
33	7#6	3.08	72	14#8	11.06
34	10#5	3.10	73	15#8	11.85
35	4#8	3.16	74	11#9	12.00
36	11#5	3.41	75	13#9	13.00
37	8#6	3.52	76	14#9	14.00
38	6#7	3.60	77	15#9	15.00
39	12#5	3.72			

where i = 9, 10, 11 and 12

variables which has to be rounded off or standardized is n, then the total number of combinations will be $N = 2^n$. For example, if $n = 3$ then $N = 8$ as shown in Fig. (7.2).

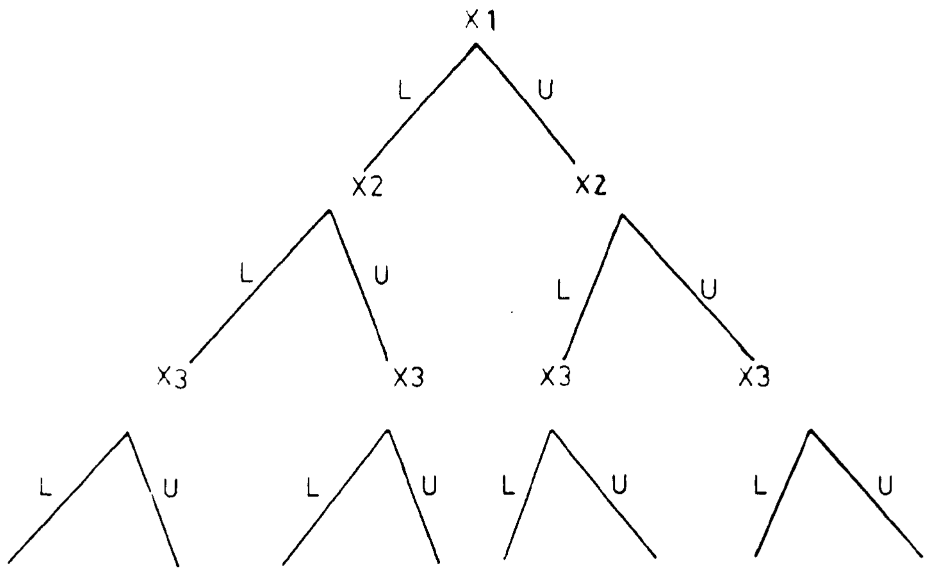


Fig. (7.2)

An Enumeration Technique

The basic idea of this technique is as follows:

1) Obtain the real-valued most optimal solution and determine the number of design variables to be rounded off or standardized.

2) Assign upper and lower neighboring integer or standardized bounds to each of the determined design variables.

3) Consider all possible combinations. For each combination, violation of any constraint must be checked.

4) Pick up all feasible combinations.

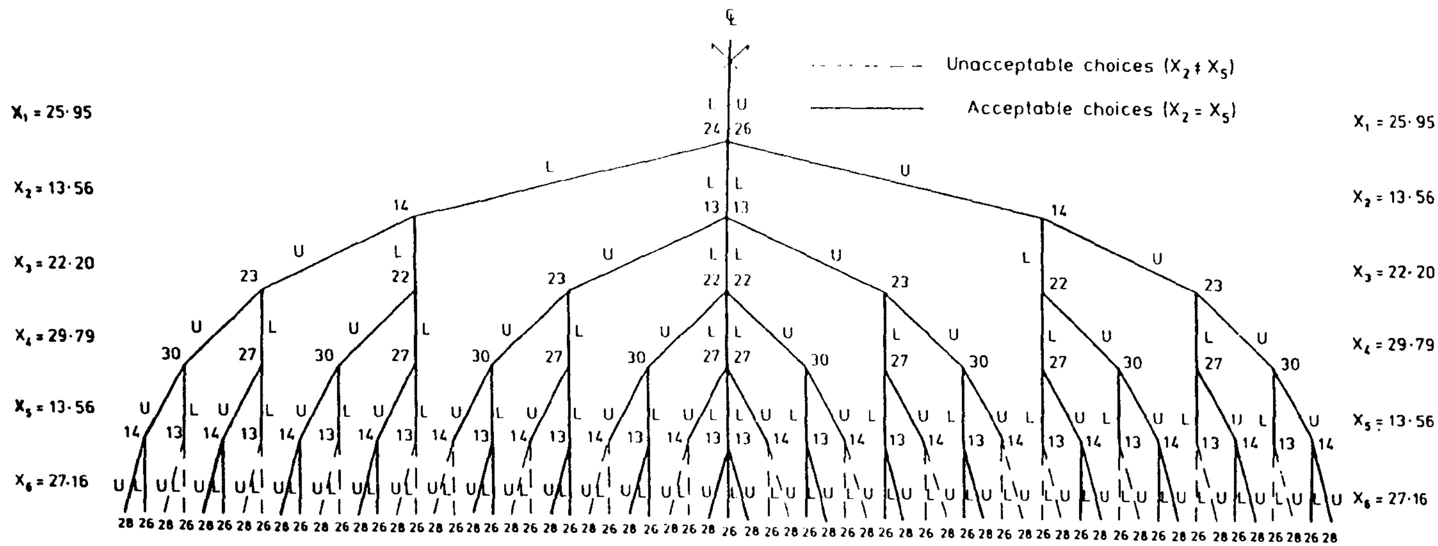


Fig. (7.3)

Design Variables Rounding Off

5) Choose the most optimal solution.

The results are clearly shown in Fig. (7.3) and Table (7.8).

Design Variables of Sol.#5	Optimal Value	
Design Variable	Real	Mixed
$X_1 = h$	25.95	26.0
$X_2 = b$	13.56	14.0
$X_3 = d$	22.20	22.0
$X_4 = D$	29.79	30.0
$X_5 = t$	13.56	14.0
$X_6 = d_c$	27.17	26.0
$X_7 = \ell_1$	5.30	5.25
$X_8 = \ell_2$	7.41	7.50
$X_9 = A_{sm}$	4.83	4.84
$X_{10} = A_{s\ell}$	4.90	4.84
$X_{11} = A_s$	4.63	4.65
$X_{12} = \acute{A}_s$	2.11	2.17
$X_{13} = A_{vm}$	0.33	0.33
$X_{14} = A_{vr}$	0.33	0.33
$X_{15} = A_{sht}$	0.41	0.40
$X_{16} = A_{shm}$	0.41	0.40
$X_{17} = A_{shb}$	0.41	0.40
$X_{18} = A_{v\ell}$	0.33	0.33
Cost (\$)	4208.5	4304.4
Drift (in)	0.05196794	0.056958

Table (7.8)

Pareto Set by the entropy-based constraint method

Comparing the total material cost obtained here, \$4304.4, with the one obtained by Yang (1981), \$3606.78, shows that there is an increase of around 16%. However, Yang's formulation was that of minimizing a single objective function, cost, and did not include a drift constraint. In optimal design of high rise frames the two criteria become important and a compromise solution becomes most desirable. The results of the same example in the above mentioned reference are as follows:

$X_1 = 31.0$	$X_2 = 15.0$	$X_3 = 26.5$
$X_4 = 15.5$	$X_5 = 15.0$	$X_6 = 13.0$
$X_7 = 17.00$	$X_8 = 5.75$	$X_9 = 3.96$
$X_{10} = 1.86$	$X_{11} = 4.00$	$X_{12} = 2.17$
$X_{13} = 0.20$	$X_{14} = 0.33$	$X_{15} = 0.11$
$X_{16} = 0.11$	$X_{17} = 0.11$	$X_{18} = 0.33$

Comparing the two sets of results shows that the two-criteria formulation is consistent and compatable with other formulations and a reasonable level of accuracy is achieved.

The final step is applying system reliability analysis to determine the system failure probability of our optimal design.

7.2.3 The Reliability Phase

The β-unzipping method is a method by which the reliability of structures can be estimated at a number of different levels. The aim has been to develop a method which is at the same time simple to use and reasonably accurate. The method was first suggested by Thoft-Christenson (1982) and is further developed by Thoft-Christensen and Sorensen (1982) and (1983). The β-unzipping method is quite general in the sense that it can be used for two-dimensional and three-dimensional framed and trussed structures, for structures with ductile or brittle elements and also in relation to a number of different failure mode definitions.

A structural system as well as a single structural component may have several parallel ways of carrying its load. Each of these defines a partial failure mode that may be defined by an equation, $M(\mathbf{X}) = 0$, in the space physical variables. The failure event may be described in terms of linear safety margins $[M] = (M_1, \ldots, M_n)$ such that the failure event is [M] < [0]. The number of potential yield hinges m is usually chosen such that m exceeds the degree of redundancy n. The condition m > n is sufficient for the existence of at least one geometrically possible collapse mechanism.

The reliability of a framed structure is estimated on the basis of a modelling by a series of systems where the elements in the series system are parallel systems. The single parallel systems are called failure modes and a failure mode is defined as a mechanism. For one-bay one-storey frame, this can be seen clearly in Fig. (7.4).

The procedure summarized here explains, in detail, how to estimate structural system reliability with reasonable accuracy.

Step (1):

By applying Linear Elasto-Static Finite Element Analysis **(LESFEA)**, determine the coefficients for all failure elements in terms of P_v and P_h, Fig. (7.5). These coefficients of influence might be determined for axial forces or bending moments. For example:

$$N_i = \xi_i \; P_v + \eta_i \; P_h; \qquad i = 1, 2, \ldots, 7 \tag{7.1}$$

$$B_i = \alpha_i \; P_v + \gamma_i \; P_h; \qquad i = 1, 2, \ldots, 7 \tag{7.2}$$

Margin of safety for failure element (i) can be determined as follows:

$$M_i = \min (M_{i1}, M_{i2}) \begin{cases} M_{i1} = R^+_i - S_i & \text{(+ means tension failure)} \quad (7.3) \\ M_{i2} = R^-_i + S_i & \text{(- means compression failure)} \quad (7.4) \end{cases}$$

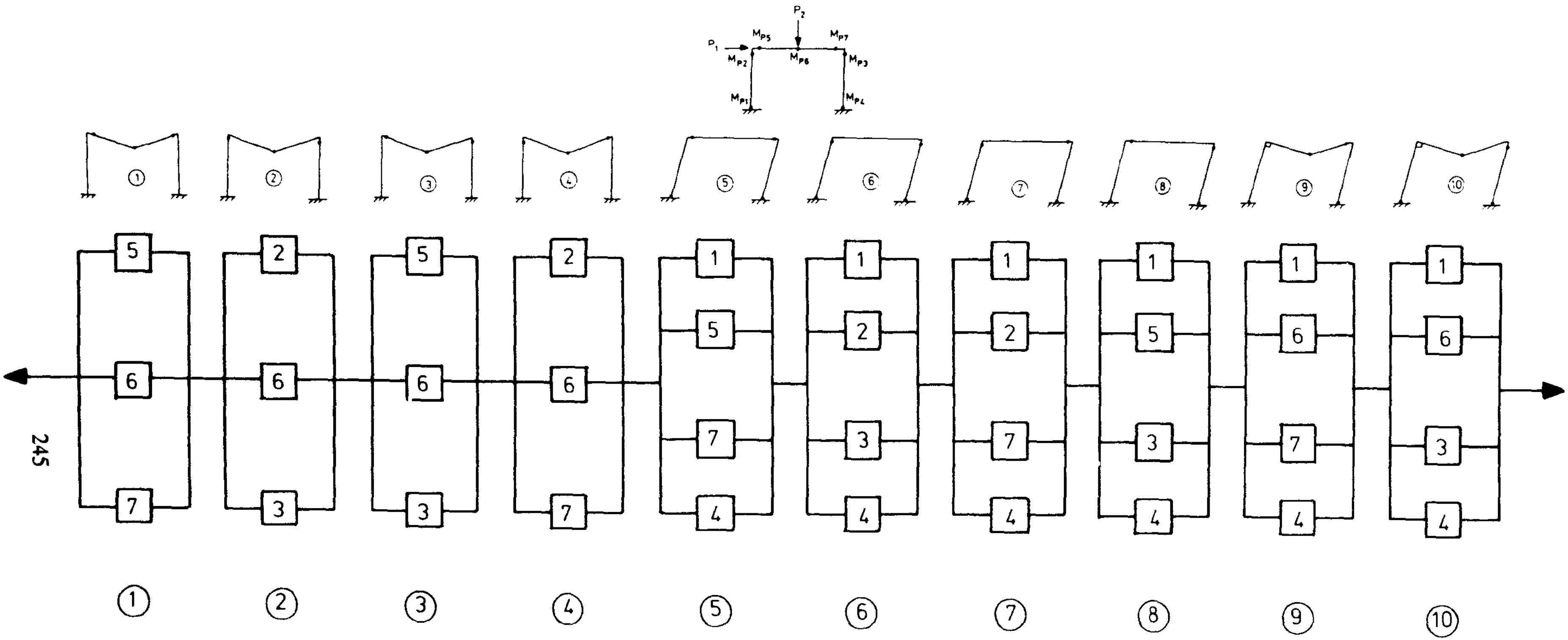


Fig. (7.4)

One-Bay One-Storey Frame Systems Modelling at Mechanism Level

☐ Critical Section Number, ⊙ Mechanism Number

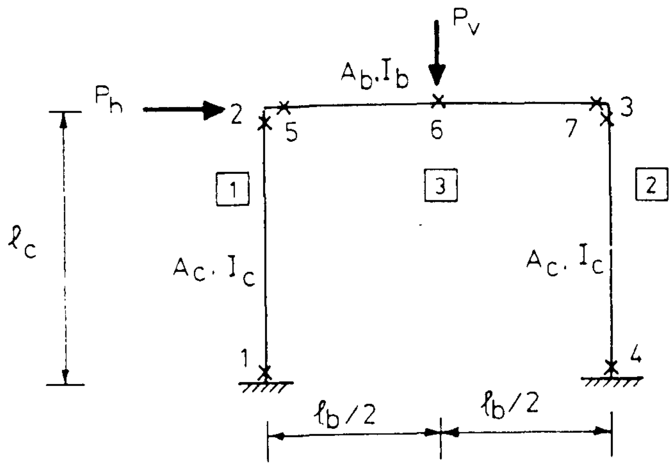


Fig. (7.5)

One-Bay One-Storey Frame (LESFEA)

where:

R_i = The (yield) strength capacity

S_i = The load-effect (force or moment) of the failure element i. ($S_i = B_i$ or $S_i = N_i$).

Note that in the presentation above, a safety margin is linear in P_v and P_h. If the safety margin M is non-linear in P_v and P_h, then approximate values can be obtained using a linearized safety margin M.

The moment influence coefficients of applied loads are shown in Table (7.9). In this design program, the random variables are the compressive strength of concrete, tensile strength of steel, cross-section dimensions, amounts of tension and compression reinforcements and external applied loads.

No. of Critical Section	MIC of P_h	MIC of P_v
1	-5.2496	2.1429
2	2.3324	-4.3921
3	-2.3036	-4.3921
4	5.1145	2.1429
5	2.3324	-4.3921
6	0.0144	5.6079
7	-2.3036	4.3921

Table (7.9)

Moment Influence Coefficients of Applied Loads

Step (2):

In general, for a plastic framework, several plastic hinge mechanisms are conceivably possible; occurrence of any one of them would lead to the collapse of the system. Following Stevenson and Moses (1970), the *perform function* for a mechanism, say mechanism i, may be defined as:

$$Z_i = \sum a_{ij} M_j + \sum b_{ik} S_k \qquad (7.5)$$

where: a_{ij} and b_{ik} are resistance and load coefficients, respectively.

M_j = the fully plastic moment capacity at the plastic hinge j.

S_k = the loads that are active in the producing mechanism i.

such that (Z_i<0) means the occurrence of mechanism i. Therefore, if there are n possible mechanisms of the system, the probability of collapse of the system is:

$$P(\text{collapse}) = P(Z_1<0 \cup Z_2<0 \cup .... Z_n<0)$$

By applying Plastic Analysis **(PA)**, Fig. (7.6), we can determine the margin of safety for system failure mode (j) where j = 1, 2,...,10 failure modes, Table (7.10).

Mode No.	Plastic Margin of Safety	Symbol	Notes
(1)	$Z_{(1)} = M_{P5} + 2M_{P6} + M_{P7} - S_v$	$Z_{(1)/5,6,7}$	$S_v = P_v \cdot \ell_h/2$
(2)	$Z_{(2)} = M_{P2} + 2M_{P6} + M_{P3} - S_v$	$Z_{(2)/2,6,3}$	$S_h = P_h \cdot \ell_c$
(3)	$Z_{(3)} = M_{P5} + 2M_{P6} + M_{P3} - S_v$	$Z_{(3)/5,6,3}$	
(4)	$Z_{(4)} = M_{P2} + 2M_{P6} + M_{P7} - S_v$	$Z_{(4)/2,6,7}$	
(5)	$Z_{(5)} = M_{P1} + M_{P5} + M_{P7} + M_{P4} - S_h$	$Z_{(5)/1,5,7,4}$	
(6)	$Z_{(6)} = M_{P1} + M_{P2} + M_{P3} + M_{P4} - S_h$	$Z_{(6)/1,2,3,4}$	
(7)	$Z_{(7)} = M_{P1} + M_{P2} + M_{P7} + M_{P4} - S_h$	$Z_{(7)/1,2,7,4}$	
(8)	$Z_{(8)} = M_{P1} + M_{P5} + M_{P3} + M_{P4} - S_h$	$Z_{(8)/1,5,3,4}$	
(9)	$Z_{(9)} = M_{P1} + 2M_{P6} + 2M_{P7} + M_{P4} - S_v - S_h$	$Z_{(9)/1,6,7,4}$	
(10)	$Z_{(10)} = M_{P1} + 2M_{P6} + 2M_{P3} + M_{P4} - S_v - S_h$	$Z_{(10)/1,6,3,4}$	

Table (7.10)

where $Z_{(1)/5,6,7}$, for example, can be read as "system failure mode (1), given failure in failure element 5, 6 and 7".

For practical purposes Fig. (7.5), we consider:

$$M_{P1} = M_{P2} = M_{P3} = M_{P4} = R_{[1]} = R_{[2]}$$

$$M_{P5} = M_{P6} = M_{P7} = R_{[3]}$$

Step (3):

Determine mean and standard deviation of the elastic and plastic safety margins as follows:

Elastic Safety Margin (ESM)

$$\bar{M}_i = \bar{R}_i \pm (\alpha_i \bar{P}_v + \gamma_i \bar{P}_h) \quad (7.6)$$

$$\sigma^2_{Mi} = \sigma^2_{Ri} + \alpha^2_i \sigma^2_{Pv} + \gamma^2_i \sigma^2_{Ph} \quad (7.7)$$

and:

10th Plastic Safety Margin (PSM)

$$\bar{Z}_{(10)} = 4\bar{R}_{[1]} \pm 2\bar{R}_{[3]} - \bar{S}_v - \bar{S}_h) \quad (7.8)$$

$$\sigma^2_{Z(10)} = 16\sigma^2_{R[1]} + 4\sigma^2_{R[3]} + \sigma^2_{Sv} + \sigma^2_{Sh} \quad (7.9)$$

Step (4):

Determine the reliability index of the elastic and plastic safety margins as follows:

$$\beta_i = \frac{\bar{M}_i}{\sigma_{Mi}} \; ; \quad i = 1, 2, \ldots, 7 \quad (7.10)$$

$$\beta_{(j)} = \frac{\bar{Z}_{(j)}}{\sigma_{Z(j)}} \; ; \quad j = 1, 2, \ldots, 10 \quad (7.11)$$

Step (5):

For each failure mode, determine the correlation matrix $[\rho]$ which corresponds to the correlation between each series system and each of its parallel systems and vice versa. For example, the 10th failure mode correlation matrix is given as follows:

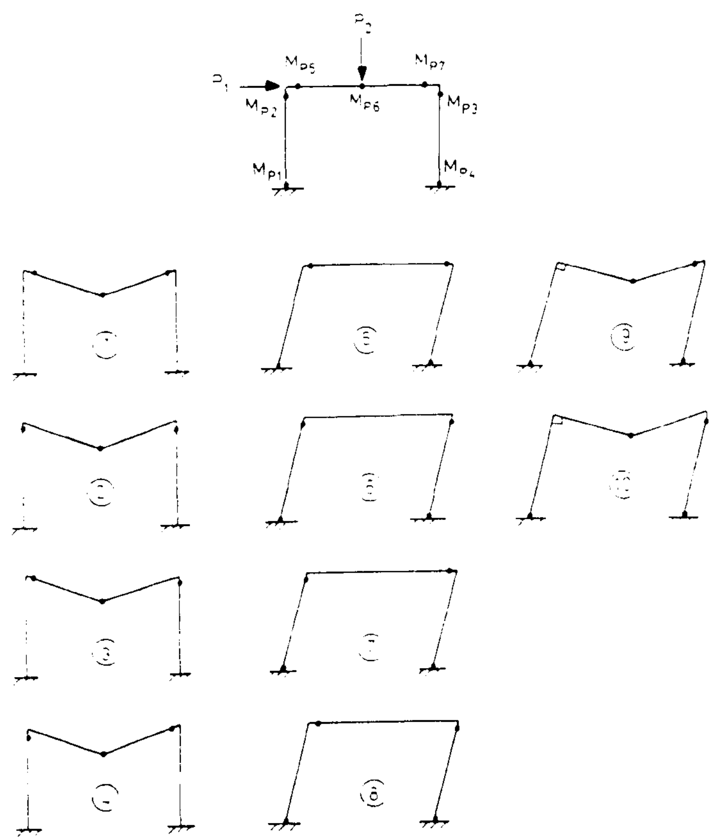


Fig. (7.6)

One-Bay One-Storey Frame (PA)

$$[\bar{\rho}]_{(10)/1,6,3,4} = \begin{bmatrix} 1 & \bar{\rho}_{1,6} & \bar{\rho}_{1,3} & \bar{\rho}_{1,4} \\ & 1 & \bar{\rho}_{6,3} & \bar{\rho}_{6,4} \\ & & 1 & \bar{\rho}_{3,4} \\ & & & 1 \end{bmatrix} \tag{7.12}$$

and its corresponding average correlation coefficient:

$$\bar{\rho}_{(10)} = \frac{1}{n(n-1)} \sum_{i,j=1}^{n} \bar{\rho}_{ij} \tag{7.13}$$

where:

n = the correlation matrix dimension

$$\bar{\rho}_{ij} = \frac{Cov[M_i,\ Z_j]}{\sigma_{Mi}\ \sigma_{Zj}} \tag{7.14}$$

$Cov[M_i,\ Z_j]$ = the covariance of M_i and Z_j

σ_{Mi} = standard deviation of elastic safety margin M_i

σ_{Zj} = standard deviation of plastic safety margin Z_j

Step (6):

Determine the probability of failure for each failure mode using Ditlevsen bounds, Table (7.11), where $\phi_1, \ldots, \phi_{10}$ are multi-dimensional functions:

$$\left. \begin{aligned} P_i &= \phi(-\beta_i).\phi\left[-\frac{\beta_{(i)} - \bar{\rho}_{(i)}\,\beta_i}{[1 - \bar{\rho}^2_{(i)}]^{0.5}} \right]; \quad & i = 1,\ 2, \ldots, 7 \\ P_{(i)} &= \phi(-\beta_{(i)}).\phi\left[-\frac{\Sigma\beta_{ij} - \bar{\rho}_{(i)}\,\beta_{(i)}}{[1 - \bar{\rho}^2_{(i)}]^{0.5}} \right]; \quad & (i) = 1,\ 2, \ldots, 10 \end{aligned} \right\}$$

Mode No.	Failure Mode Probability	Lower Bond	Upper Bond
(1)	$P(F1) = \phi_1 [-\beta_5, -\beta_6, -\beta_7, -\beta_{(1)}; \bar{\rho}_{(1)}]$	$Max [p_5, p_6, p_7 \; p_{(1)}]$	$p_5 + p_6 + p_7 + p_{(1)}$
(2)	$P(F2) = \phi_2 [-\beta_2, -\beta_6, -\beta_3, -\beta_{(2)}; \bar{\rho}_{(2)}]$	$Max [p_2, p_6, p_3 \; p_{(2)}]$	$p_2 + p_6 + p_3 + p_{(2)}$
(3)	$P(F3) = \phi_3 [-\beta_5, -\beta_6, -\beta_3, -\beta_{(3)}; \bar{\rho}_{(3)}]$	$Max [p_5, p_6, p_3 \; p_{(3)}]$	$p_5 + p_6 + p_3 + p_{(3)}$
(4)	$P(F4) = \phi_4 [-\beta_2, -\beta_6, -\beta_7, -\beta_{(4)}; \bar{\rho}_{(4)}]$	$Max [p_2, p_6, p_7 \; p_{(4)}]$	$p_2 + p_6 + p_7 + p_{(4)}$
(5)	$P(F5) = \phi_5 [-\beta_1, -\beta_5, -\beta_7, -\beta_4, -\beta_{(5)}; \bar{\rho}_{(5)}]$	$Max [p_1, p_5, p_7, p_4, p_{(5)}]$	$p_1 + p_5 + p_7 + p_4 + p_{(5)}$
(6)	$P(F6) = \phi_6 [-\beta_1, -\beta_2, -\beta_3, -\beta_4, -\beta_{(6)}; \bar{\rho}_{(6)}]$	$Max [p_1, p_2, p_3, p_4, p_{(6)}]$	$p_1 + p_2 + p_3 + p_4 + p_{(6)}$
(7)	$P(F7) = \phi_7 [-\beta_1, -\beta_2, -\beta_7, -\beta_4, -\beta_{(7)}; \bar{\rho}_{(7)}]$	$Max [p_1, p_2, p_7, p_4, p_{(8)}]$	$p_1 + p_2 + p_7 + p_4 + p_{(7)}$
(8)	$P(F8) = \phi_8 [-\beta_1, -\beta_5, -\beta_3, -\beta_4, -\beta_{(8)}; \bar{\rho}_{(8)}]$	$Max [p_1, p_5, p_3, p_4, p_{(8)}]$	$p_1 + p_5 + p_3 + p_4 + p_{(8)}$
(9)	$P(F9) = \phi_9 [-\beta_1, -\beta_6, -\beta_7, -\beta_4, -\beta_{(9)}; \bar{\rho}_{(9)}]$	$Max [p_1, p_6, p_7, p_4, p_{(9)}]$	$p_1 + p_6 + p_7 + p_4 + p_{(9)}$
(10)	$P(F10) = \phi_{10} [-\beta_1, -\beta_6, -\beta_3, -\beta_4, -\beta_{(10)}; \bar{\rho}_{(10)}]$	$Max [p_1, p_6, p_3, p_4, p_{(10)}]$	$p_1 + p_6 + p_3 + p_4 + p_{(10)}$

Table (7.11)

Ditlevsen Bounds For Each Failure Mode

where:

$$\Sigma \beta_{ij} = \begin{Bmatrix} \beta_5 + \beta_6 + \beta_7; \ i = 1 \\ \beta_2 + \beta_6 + \beta_3; \ i = 2 \\ . \\ . \\ . \\ \beta_1 + \beta_6 + \beta_3 + \beta_4; \ i = 10 \end{Bmatrix}$$

The lower bound, $(\underset{i=1}{\overset{n}{\mathrm{Max}}} \; P(Z_i < 0)$, represents the probability of failure of the system if all mechanisms are perfectly correlated (i.e., $\rho_{ij} = 1.0$ for all i and j). The upper bound, $1 - \prod_i [1 - P(Z_i < 0)]$, $i=1,\ldots,n$, represents the probability of failure of the system if all mechanisms are statistically independent (i.e. $\rho_{ij} = 0$ for all i and j).

Now, if there is a dominant failure mechanism, the lower bound of the failure probability may be a good approximation; if there is more than one significant mechanism, this approximation may be seriously on the unsafe side, and thus more precise determination of the collapse probability would be necessary. In this regard, the correlation between mechanism i and mechanism j should be taken into account in the evaluation.

A Probabilistic Network Evaluation Technique (PNET) has previously been developed for the analysis of activity networks (Ang, et al, 1975). This technique is applicable also for the approximation analysis of framed structures. Applied to the collapse probability of redundant plastic frameworks, the PNET method is based on the premise that those plastic mechanisms that are highly correlated (e.g. with ρ_{ij}

$> \rho_o$) may be assumed to be perfectly correlated whereas those with low correlations (i.e. with $\rho_{ij} \leq \rho_o$) may be assumed to be statistically independent.

On this basis, the system failure mechanisms can be divided into several groups in accordance with their mutual correlations such that within each group the mechanisms are mutually highly correlated. Then, according to the lower bound the mechanism within each group can be represented by the single mechanism having the highest probability of failure in the group, i.e. Max_i $P(Z_i<0)$; whereas, the representative mechanisms between the different groups may be assumed to be statistically independent. Central to the PNET approach is the value of ρ_0. Although a value of $\rho_0 = 0.5$ is appropriate for activity networks, the same value may not be suitable for the collapse probability analysis of structural frameworks. The system reliability analysis results are shown in Table (7.12).

Table (7.12) illustrates the validity of the developed system reliability analysis in section (7.2.3). The first part of this table shows separately values of the elastic reliability indices at the 7 critical sections and values of the plastic reliability indices correspond to the possible failure modes. Combining the two systems as described in steps (4, 5 and 6), the second part of table (7.12) shows the upper and lower bounds of failure probability for each mechanism. Finally, by assigning a value for the demarcating correlation ρ_0, the mechanisms are classified as perfectly correlated or statistically independent with respect to the critical mechanism #1 as shown in the third part. Using the PNET as described before, the system failure

No. of Critical Section	Elastic Reliability Index	No. of Failure Mode	Plastic Reliability Index
1	3.592	1	3.007
2	3.526	2	3.5
3	3.491	3	3.335
4	3.514	4	3.335
5	1.629	5	3.623
6	3.346	6	3.325
7	1.610	7	3.235
		8	3.524
		9	3.481
		10	3.755

Upper Bound	Probability of	Lower Bound
1.89×10^{-5}	1st Failure Mode	9.71×10^{-6}
5.33×10^{-9}	2nd Failure Mode	2.76×10^{-9}
2.81×10^{-6}	3rd Failure Mode	2.80×10^{-6}
2.99×10^{-6}	4th Failure Mode	2.98×10^{-6}
3.97×10^{-6}	5th Failure Mode	2.03×10^{-6}
1.31×10^{-7}	6th Failure Mode	3.83×10^{-8}
4.26×10^{-6}	7th Failure Mode	4.24×10^{-6}
4.04×10^{-6}	8th Failure Mode	4.03×10^{-6}
3.92×10^{-6}	9th Failure Mode	3.9×10^{-5}
9.93×10^{-9}	10th Failure Mode	4.2×10^{-9}

So, the critical failure mode is # 1 and lets consider the value of $\rho_0 = 0.6$

Perfectly Correlated Failure Mode with the Critical One	Statistically Independent Failure Modes from the Critical One
$\rho(1, 1) = 1.0$	$\rho(1, 5) = 0.444$
$\rho(1, 2) = 0.721$	$\rho(1, 6) = 0.000$
$\rho(1, 3) = 0.889$	$\rho(1, 7) = 0.206$
$\rho(1, 4) = 0.889$	$\rho(1, 8) = 0.206$
$\rho(1, 9) = 0.807$	$\rho(1, 10) = 0.553$

So, the System Failure Probability using the PNET Method is: 2.86×10^{-5}

Table (7.12)

System Reliability Analysis of the One-Bay One-Storey Frame Example probability is found to be 2.86 x 10^{-5}. This value is very close to the value obtained by Frangopol (1985) who considered 10 failure modes and used Ditlevsen's bounds approach, and consequently confirms that the present approach gives sensible and practical results. It is important to point out that this algorithm is computer programmable and appears to be powerful in handling the uncertainties included in the formulation, the statistical correlation structure between different collapse mechanisms and evaluating global failure probabilities of structural framed systems.

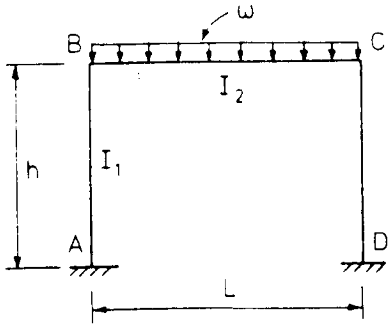
7.3 OPTIMIZATION FORM OF MULTI-STOREY RIGID FRAME END MOMENTS

When multi-storey frames and loads are given, it is very important for designers to find design moments formulas with speed and reasonable accuracy.

In practice, as the rigid frames are statically indeterminate in nature, it is impossible to determine directly stresses when the member sections are unknown. After having assumed, therefore, the section to be designed, we are compelled to start the calculation of end moments and check the safety of results before completing the final designs.

The external end moments for a rigid one bay - one storey frame subjected to different loads with different stiffness ratios, I/ℓ, can easily be derived or taken from many references. For example, the external end moments for a rigid frame under uniform load are shown in Fig. (7.7).

Takabeya, (1965) in his book, "Multi-Storey Frames", put Moment-Tables for different multi-storey frames under different loads with constant stiffness ratios. He claimed that his moment values for a constant stiffness frame gave in most cases, a good coincidence with the exact calculation for the reinforced concrete frame of variable stiffness.



$$\left.\begin{aligned} M_A &= M_D = \frac{WL^2}{12N_1} \; ; \; K = \frac{I_2}{I_1}\,\frac{h}{L} \\ M_B &= M_C = -\frac{WL^2}{6N_1} = -2M_A \\ V_A &= V_D = \frac{WL}{2} \; ; \; N_1 = K + 2 \end{aligned}\right\} \qquad (7.15)$$

Fig. (7.7) Rigid Frame Under Uniform Load

For the above example, the following values are given:

$$\left.\begin{aligned} M_A &= M_D = 0.333\,\frac{WL^2}{12} \\ M_B &= M_C = -0.667\,\frac{WL^2}{12} = -\;2M_A \end{aligned}\right\} \qquad (7.16)$$

Considering constant stiffness ratio; $k = 1 \Rightarrow N_1 = 3$, and substituting into (7.15), we get, Eq. (7.16):

Consider the formulas given in (7.15) and the values given in (7.16) as a reference, that should allow us to write the end moments as follows:

$$M_A = M_D = \frac{\alpha}{0.333} \frac{WL^2}{12N_1}$$

$$M_B = M_C = \frac{\beta}{0.667} \frac{WL^2}{6N_1} = \frac{\beta}{0.333} \frac{WL^2}{12N_1}$$

where α, β are the moment coefficients at these sections, $\alpha = 0.333$ and $\beta = 0.667$.

Example

For a two storey - one bay rigid frame Fig. (7.8), subjected to uniform vertical loads, the following moment coefficients are given by Takabeya:

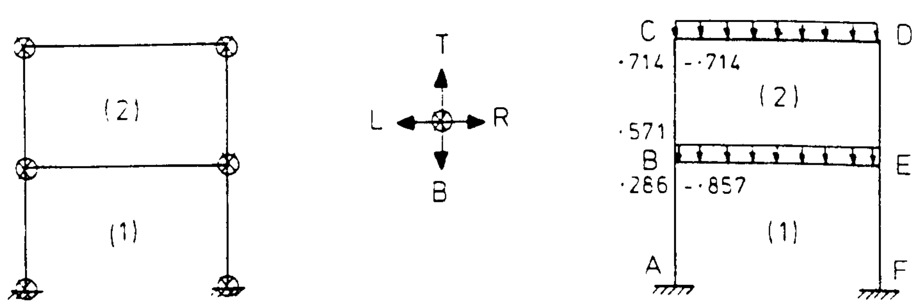


Fig. (7.8)

One-Bay Two-Storey Frame

For (2):

$$M_{CR} = -\frac{0.714}{0.667}\,\frac{WL^2}{6N_1} = -M_{BB}; \qquad M_{CR} = -\frac{0.714}{12}WL^2 = -0.0595WL^2$$

$$M_{BT} = \frac{0.571}{0.333}\,\frac{WL^2}{12N_1} = M_{DT}; \qquad M_{BT} = \frac{0.571}{12}WL^2 = 0.0476WL^2$$

For (1):

$$M_{BR} = -\frac{0.857}{0.667}\,\frac{WL^2}{6N_1} = -M_{BB}; \qquad M_{BR} = -\frac{0.857}{12}WL^2 = -0.0714WL^2$$

$$M_{AT} = \frac{0.743}{0.333}\,\frac{WL^2}{12N_1} = M_{DT}; \qquad M_{AT} = \frac{0.743}{12}WL^2 = 0.0619WL^2$$

Optimization Form *Takabeya Form*

The two forms coincide for frames with constant stiffness ratios. However, the optimization form is still a good approximation for design purposes, even for frames with variable stiffness. This conclusion was reached by the author after having made a thorough investigation on different types of multi-storey frames:

(1) For Constant Stiffness Ratios

(a) for the top storey

$$M_{CR} = \frac{-0.714}{0.667}\,\frac{WL^2}{6(3)} = -0.0595WL^2 = -M_{CB}$$

$$M_{BT} = \frac{0.571}{0.333}\,\frac{WL^2}{12(3)} = 0.0476WL^2 = M_{ET}$$

(b) for the bottom storey:

$$M_{BR} = \frac{-0.857}{0.667} \quad \frac{WL^2}{6(3)} = -\ 0.0714WL^2$$

$$M_{AT} = \frac{0.743}{0.333} \quad \frac{WL^2}{12(3)} = 0.0620WL^2$$

(2) For Variable Stiffness Ratios:

The stiffness ratio is given by:

$$K = \frac{I_2\, h}{I_1\, L}$$

where I_1, I_2, h and L are as shown in Fig. (7.7).

For constant stiffness ratios, K = 1, the end moments given by Takabeya or the optimization form must be the same. For variable stiffness ratios, $K \neq 1$, the end moments given by Takabeya or the optimization form gave in all cases a good coincidence with each other within a small range of error (ϵ = 0.0 - 0.20) when (K = 0.5 - 1.6). Lack of coincidence rate is the ratio of difference between two values in relation to one of them. In optimization, to control this, additional constraints may be added as follows:

$$K^L \leq K\ (I_1,\ I_2) \leq K^U$$

where K^U and K^L are the lower and upper bounds given by the designer. The relationship between K and ϵ is constant and summarized in Table (7.13). This procedure is considered in the next section where a four-storey frame example is presented.

K	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
ϵ_1	0.167	0.133	0.1	0.067	0.033	0.0	0.033	0.067	0.1	0.133	0.167	0.2	0.233	0.267	0.3	0.333
ϵ_2	0.2	0.154	0.111	0.071	0.034	0.0	0.032	0.063	0.091	0.118	0.143	0.167	0.189	0.211	0.231	0.25

$$\epsilon_1 = \frac{M_{OF} - M_{TF}}{M_{OF}} \qquad \epsilon_2 = \frac{M_{OF} - M_{TF}}{M_{TF}}$$

OF: Optimization Form Value

TF: Takabeya Form Value

M: End Moments Coefficient

Table (7.13)

Stiffness Ratio Variation Versus Lack of Coincidence Rate

7.3.1 One-Bay Multi-Storey Frame

The structure considered here is a one-bay four-storey frame as shown in Fig. (7.9). Each storey is subjected to a uniformly distributed line load of 1.8 (K/ft) and concentrated live load of 15 (Kips) The wind force is assumed to be 2.75 at the top and 5.5 at the other 3 storeys. Each storey is 40 ft wide and 15 ft high. The yield strength of reinforcement is 60 (Ksi). The compressive strength of concrete is 4 (Ksi). At each storey, the beam cross section width to overall depth is assumed to be not less than 0.4 and both the column and the beam have the same width. The cost of concrete is 0.08 ($/in^2/ft). The cost of steel is 2.0 ($/in^2/ft). The overall depth of the column is assumed to be not less than one half of the overall depth of the beam. Each storey has 18 design variables and 48 constraints. The results are summarized in Fig. (7.10) and Table (7.14).

7.3.2 Sensitivity Analysis

As a final topic of this thesis, the concept of a trade-off, or sensitivity study, is common to engineering design, so, we now consider three different types of sensitivity analysis:

1) *Design Parameter Sensitivity* **(DPS)**: In engineering design, if materials or design requirements are changed after we have already found an optimum solution to the original problem, we wish to estimate the effect that this will have on the design. This type of sensitivity analysis is not included in this thesis since it has already been published (Frangopol, 1985).

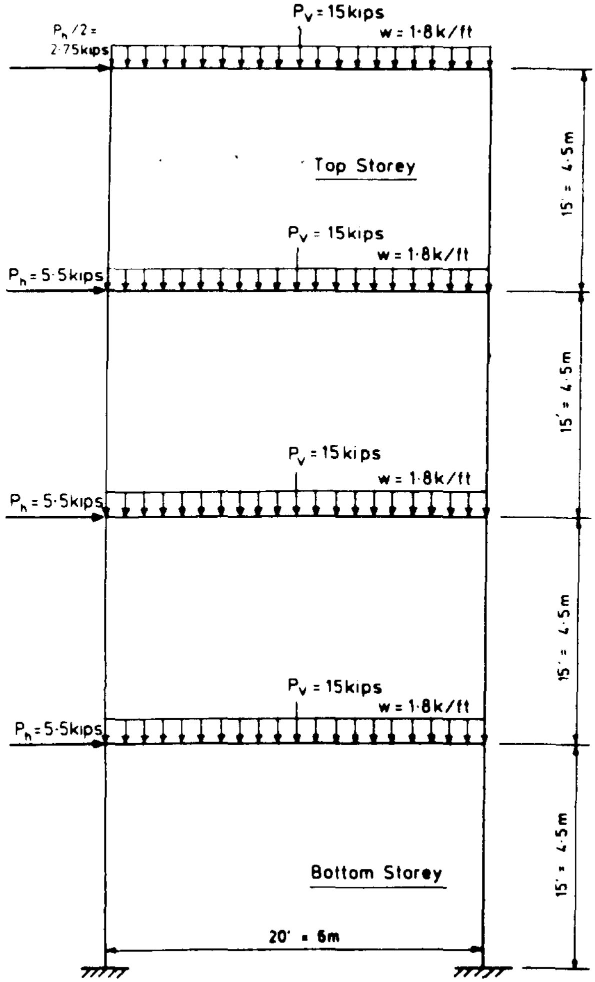
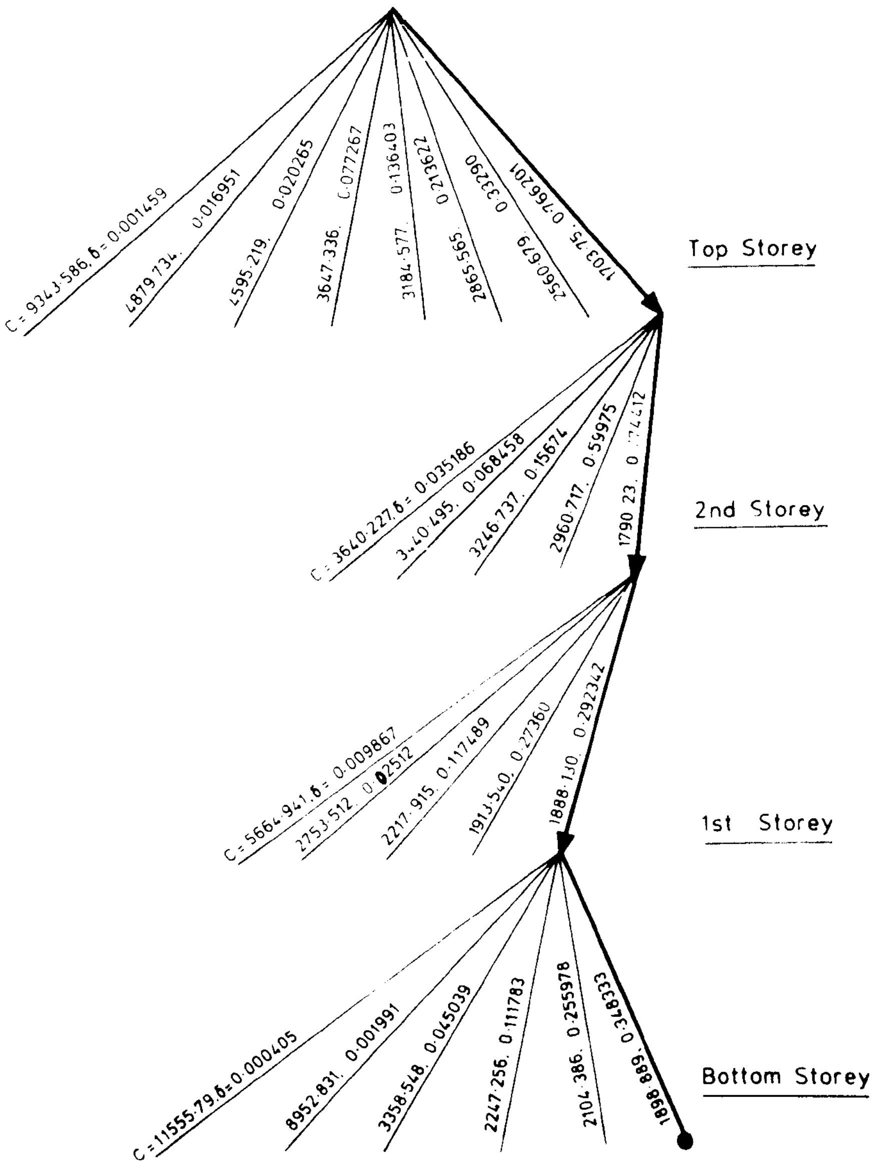


Fig. (7.9)

One-Bay Four-Storey R.C. Frame

Fig. (7.10)

The Multi-Optimal Path Tree



Design Variable	3rd Storey (Top)	2nd Storey	1st Storey	Ground (Bottom)
$X_1 = h$ (in.)	21.51	22.66	23.15	24.07
$X_2 = b$	13.56	13.56	13.56	13.56
$X_3 = d$	17.76	18.91	19.40	20.32
$X_4 = D$	14.19	13.56	17.56	17.58
$X_5 = t$	13.56	13.56	13.56	13.56
$X_6 = d_c$	11.56	10.94	14.93	14.95
$X_7 = \ell_1$ (ft)	3.14	3.14	3.32	7.07
$X_8 = \ell_2$	4.99	4.97	5.28	9.16
$X_9 = A_{sm}$ (in.^{2})	1.88	1.9	5.18	7.07
$X_{10} = A_s\ell$	1.76	1.87	5.2	5.74
$X_{11} = A_s$	1.71	2.01	2.90	2.32
$X_{12} = A'_s$	0.00	0.796	1.29	1.98
$X_{13} = A_{vm}$	0.154	0.289	0.299	0.957
$X_{14} = A_{vr}$	0.262	0.334	0.334	0.957
$X_{15} = A_{sht}$	0.224	0.228	0.380	0.572
$X_{16} = A_{shm}$	0.208	0.288	0.380	0.572
$X_{17} = A_{shb}$	0.196	0.288	0.381	0.519
$X_{18} = A_v\ell$	0.154	0.262	0.334	0.957
Total Material Cost ($)	1703.75	1790.23	1888.13	1898.889
Relative Horizontal Displacement (in.)	0.7662	0.7741	0.2923	0.3483
Drift (in.) = 2.1809		Total Cost ($) = 7281		

Table (7.14)

One-Bay Four-Storey Multi-Optimal Design

2) *Criterion Preference Sensitivity* **(CPS)**: If a criterion preference is changed after we have already found an optimum solution to the original problem, we wish to estimate the effect that this will have on the design. In building design, this can be simplified by creating a *multi-optimal path tree* **(MOPT)** along the whole building. In Fig. (7.10), for each storey, the Pareto set is represented by several branches arranged in descending or ascending order according to a preferred criterion. On each branch, values of all criteria are listed so different multi-optimal paths could be seen clearly. In multi-criteria multi-storey frame design optimization, the preference of one criteria or another might change from time to time or case to case or all of these together, as follow:

a) <u>Time to Time</u>

At the time of construction, suppose that the unit price per volume of concrete, or the unit price per volume per length of steel, or both of the prices are much higher than those used at the time of design. This will magnify the importance of the cost criteria and its preference and vice versa. The solution which minimizes cost to its lower value has to be the dominant solution.

b) <u>Case to Case</u>

In the case of the skyscraper building design, the drift criteria becomes very significant, which is not the case with a low-rise building design. The solution which minimizes drift to its lowest value has to be the dominant one. However, if the considered price of concrete, steel, or both is very high, then for high-rise buildings, both cost and drift criterion should be considered on the same level of importance and, in this case, a different solution has to be adopted.

A flowchart which shows the Pareto set of each storey and the interrelationships among all Pareto sets of all storeys in the building mapped will be called the *Multi-Optimal* Path Tree, Fig. (7.10).

In Fig. (7.10), the Pareto set for each storey is generated using the methods developed in chapter 3. For multi-storey rigid frames with loads are given, we recognise two cases:

1) Constant stiffness ratio, $K = 1$: The optimization process can be run for each storey with no need for feedback since the end moments given by Takabeya are independent of structural member cross sections.

2) Variable stiffness ratio, $K \neq 1$: By constraining the stiffness ratio to lie between upper and lower bounds assigned by the designer as explained before in section (7.3), the optimization process can still be run for each storey with no need for feedback and reasonable level of accuracy can be achieved (Takabeya, 1965).

Table (7.14) shows the design variables of the preferred Pareto solution at each storey. Since values of the design variables of the top two storeys are close to each other, one of the two designs or a compromise of one can be considered if no constraint is violated. Similar circumstances apply also to the bottom two storeys.

7.4 DISCUSSION

The concern for safety of structures must ultimately include the consideration of the reliability of a complete structural system. A Practical Design Program (PDP) requires a more complete knowledge of the design or performance space. Multi-criteria optimization is not just a tool for finding the best answer but is, in addition, one for learning about the design - performance space. PDP alone has to deal with mixed design variables problems. It has been demonstrated that it is possible to determine the complete detailed description of the structure by multi-criteria optimization. From a practical standpoint, the end moments optimization form developed can be a valuable tool for the optimal design of multi-storey frames. For the design of reinforced concrete frames, the design variables such as the member cross sections and the amount of reinforcement in each member can only be obtained from a set of discrete elements. From a practical point of view the variables can only assume certain discrete values and the optimization process can be based only on these discrete values. In addition, the design variables should include the details such as the amount of web reinforcement and the cut-off points of longitudinal steel. The design constraints which are developed based on the ACI Code (318-83) should cover the requirements mentioned in chapter six. Discrete design space, varied either uniformly or non-uniformly, for each of the design variables can be generated for practical purposes.

The PDP has proved to be useful since it incorporates vector optimization and reliability together in a very systematic way. At various stages during execution of the examples it was possible to compare results obtained with those of other researchers using different formulations. These comparisons showed that the present results are realistic and sensible in a practical engineering context.

CHAPTER EIGHT

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

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8.1 CONCLUSIONS

This thesis has examined the use of Shannon's (informational) entropy measure and Jaynes' maximum entropy principle in connection with the solution of single-criteria and multi-criteria optimization problems. At first glance, the two concepts, *entropy* and *optimization*, seem to have no direct link as the Shannon entropy is essentially related to probabilities while optimization is usually viewed in terms of a deterministic topological domain. To explore possible links between them, an optimization problem has been simulated as a *statistical thermodynamic system* that spontaneously approaches its equilibrium state under a specified *temperature*, which is then characterized by the maximum entropy. An attacking line:

Entropy → Thermodynamic Equilibrium → Optimization

was then postulated. Several questions are then raised about how to do this simulation. They are:

1) What are micro-states of this statistical thermodynamic system in an optimization context?
2) What are the probabilities of the micro-states?
3) What common characteristic is there in these two processes?
4) What common law governs them?

In multi-criteria optimization, these questions are briefly answered as follows:

1) Each micro-state corresponds to a criteria in which it is optimized by randomly taking or adding a finite amplitude step from or to it respectively so that a Pareto set can be generated.

2) The multiplier associated with each criteria is interpreted as the probability of the system being in the corresponding micro-state.

3) The Pareto set generation process can be thought of as a sequence of feasible transitions of the system to its equilibrium states such that the *equilibra* become the common characteristics in the two processes.

4) That the entropy of the system attains a maximum value in equilibrium states represent the common law to govern the two processes.

In single-criteria constrained minimization, these questions are briefly answered as follows:

1) Each micro-state corresponds to a constraint in which the objective function is minimized, subject to all constraints randomly taking a finite amplitude step from it.

2) The multiplier associated with each constraint is interpreted as the probability of the system being in the corresponding micro-state.

3) A minimizing process can be thought of as a sequence of transitions of the system to its equilibrium states such that the *equilibrium* becomes the common characteristic in the two

processes.

4) That the entropy of the system attains a maximum value at an equilibrium state but has a monotonically decreasing value during the minimization process represents the common law to govern the two processes.

In the course of this study, the concept of entropy was further examined by presenting a new entropy-based minimax method for generating Pareto solutions sets for multi-criteria optimization problems. The subject of simulated entropy for seeking the global minimum of single-criteria constrained minimization problems was explored and proved by developing two simulated entropy techniques. Finally, a practical optimal structural design programs for reinforced concrete frames was developed.

The main developments made in the present study are summarized as follows:

1) An entropy-based method for generating Pareto solution sets was developed in terms of Jaynes' maximum entropy formalism. This new method has provided additional insight into entropic optimization as well as affording a simple means of calculating the least biased probabilities, see section (3.2.4).
2) Two new entropy-based stochastic techniques were developed to reach the global minimum of constrained single-criteria minimization problems and belong to a new class of algorithms called simulated entropy, see section (3.4.4).

3) An optimal practical structural design program for reinforced concrete frames was developed. The design program is reliability-based, see chapter 7.

4) Numerical examples were presented, tested and compared using all the new entropy-based methods described in Chapter 3.

5) A new way for investigating the sensitivity of an optimal design of a multi-storey frame due to changes in criterion preference was developed, see section (7.7.3).

The present work has shown that there are links between entropy and optimization. There is no doubt that good entropy-based optimization algorithms can be devised based upon the present research. Several conclusions, drawn from the present research, are summarized as follows:

1) The development of the new entropy-based methods has provided not only an alternative convenient solution strategy but also additional insights into entropic processes.

2) Uncertainty contained in the solution of multi-criteria optimization problems is similar to that contained in thermodynamic systems: thus it is reasonable to employ a statistical thermodynamic approach, i.e., the entropy maximization approach, to estimate multi-criteria multipliers.

3) Uncertainty contained in the solution of constrained minimization problems is also similar to that contained in thermodynamic systems, so it is reasonable to employ a statistical thermodynamic approach, i.e., the entropy minimaximization approach, to estimate the entropy multipliers.

4) The exploration of a new class of methods called simulated entropy methods has a significance far beyond that subject itself. A fact which must be emphasized is, that it is the informational entropy approach which has made this possible. During the simulated entropy process simulation of the entropy was a close parallel to minimization of the maximum entropy at each configuration.

5) Entropic optimization methods developed in this thesis have a very unique property which make it very easy to be programmed . this unique property may be formulated as follows:

No matter how many goals or constraints we have, only one parameter controls the process: the entropy parameter, P.

6) The development of the entropic optimization methods have enabled the mathematical optimization examples considered in Chapter 4 to be solved easily. The computer results have shown that the developed methods have fast and stable convergence. Through the developments made here, it can be seen that the entropic optimization methods deserve to be more widely recognized than hitherto.

7) Exponentiation and the use of logarithms within the entropy function have very little influence on computer execution time if time is optimized and full efficiency of the computer system is used.

8) Also, the development of the entropic optimization methods have enabled the structural optimization problems considered in Chapter 7 to be solved very easily and successfully. They are practical enough to be recognized as efficient tools for solving multi-optimal design problems in particular and Computer Aided Design (CAD) problems in general.

8.2 RECOMMENDATIONS FOR FUTURE WORK

The present work is exploratory. However, it has opened up new avenues in the study of some classes of minimization problems, such as general stochastic optimization problems. Some potential research topics, which become possible due to the present work, are summarized as follows:

1) The present work is mainly oriented towards providing a practical basis for using Shannon entropy through the Maximum Entropy Principle (**MEP**) and the Minimax Entropy Principle (**MMEP**) in an optimization context. It has left many aspects of theoretical algorithmic development to be explored. For example, the Maximin Entropy Principle.
2) Further practical refinements are also required for more deeply understanding the minimum entropy principle and the maximin entropy principal and extending its applications to more optimization areas.
3) It is now clear that the explored relationship between simulated entropy and simulated annealing has a close relationship to global minimization. Thus, the Minimax Entropy Principle (**MMEP**) can be readily adapted to solving large simulation problems of a

stochastic nature. This has still to be explored.

4) Because of the explored relationship between simulation, simulated entropy, entropy and the Minimax Entropy Principle, and since simulation is an experimental arm of operations research (Hillier, et.al., 1980), the author strongly believes that Entropy Research (**ER**), which involves research on entropy may be described as a scientific approach to decision-making that involves uncertainty and as a result its concept is so general that it is equally applicable to many other fields as well.

5) The roots of Operations Research (OR) can be traced back many decades to the Second World War. Because of the war effort, there was an urgent need to allocate scarce resources to the various military operations and to the activities within each operation in an effective manner. However, because of the strong waves of technological development, the world is shaken. America, itself, is facing a very difficult challenge for the rest of the century (Kennedy, 1987). A new strategic system of management is urgently needed. Entropy may be used efficiently for such development. In words, operations research deals with what is available today while entropy research deals with what is unpredictable tomorrow which is the case we are facing.

In conclusion, the main contribution to knowledge contained in this thesis centres around the various demonstrations that informational entropy, stochastic simulations and optimization processes are closely linked. Through this work it is now possible to view the traditional deterministic, topological interpretation of optimization processes as not the only interpretation, but as just one possible interpretation. It is equally valid to treat optimization processes in a probabilistic, information-theoretic way and to develop solution methods from this interpretation. This new insight opens up new avenues for research into optimization methods.

APPENDICES

APPENDIX A

Given:

$$V_2 = \sum_{i=1}^{c} \lambda_i F_i - \frac{1}{P} \sum_{i=1}^{c} \lambda_i \ln \lambda_i$$

$$\text{with } \lambda_i = \exp (PF_i)/(\sum_{i=1}^{c} \exp (PF_i))$$

show that:

$$\partial V_2/(\partial X^*_j) = \sum_{i=I}^{c} \lambda_i \, \partial F_i/(\partial X^*_j) = 0; \ \forall j \epsilon J$$

where X^*_j is an optimal solution to V_2

Proof:

$$\frac{\partial V_2}{\partial X^*_j} = \sum_{i=1}^{c} \left(\lambda_i \frac{\partial F_i}{\partial X^*_j} \right) + \sum_{i=1}^{c} \left(F_i \frac{\partial \lambda_i}{\partial X^*_j} \right) - \frac{1}{P} \sum_{i=1}^{c} \left(\frac{\partial \lambda_i}{\partial X^*_j} (\ln \lambda_i + 1) \right) = 0 \tag{A.1}$$

$$\frac{\partial \lambda_i}{\partial X^*_j} = \frac{P(\partial F_i/\partial X^*_j) \, . \, \exp(PF_i)}{\sum_{i=1}^{c} \exp(PF_i)} - \frac{\exp(PF_i) \sum_{i=1}^{c} P(\partial F_i/\partial X^*_j) \, . \exp(PF_i)}{\left\{ \sum_{i=1}^{c} \exp (PF_i) \right\}^2}$$

or:

$$\frac{\partial \lambda_i}{\partial X^*_j} = P \lambda_i \frac{\partial F_i}{\partial X^*_j} - \frac{\sum_{i=1}^{c} P(\partial F_i/\partial X^*_j) \exp(PF_i)}{\sum_{i=1}^{c} \exp (PF_i)}$$

which after substitution into (A.1) yields:

$$\sum_{i=1}^{c} \lambda_i \, (\partial F_i/\partial X^*_j) + \sum_{i=1}^{c} PF_i \, (\partial F_i/\partial X^*_j) \, \lambda_i - \sum_{i=1}^{c} F_i \, \lambda_i \frac{\sum_{i=1}^{c} P(\partial F_i/\partial X^*_j) \, \exp(PF_i)}{\sum_{i=1}^{c} \exp\,(PF_i)}$$

$$- \sum_{i=1}^{c} \lambda_i \, (\partial F_i/\partial X^*_j) \, (1 + \ell n \, \lambda_i) + (1/P) \sum_{i=1}^{c} \lambda_i \, (1 + \ell n \, \lambda_i) \frac{\sum_{i=1}^{c} P(\partial F_i/\partial X^*_j) \, \exp\,(PF_i)}{\sum_{i=1}^{c} \exp\,(PF_i)}$$

or:

$$\sum_{i=1}^{c} \lambda \, (\partial F_i/\partial X^*_j) + \sum_{i=1}^{c} PF_i \, (\partial F_i/\partial X^*_j) \, \lambda_i - \sum_{i=1}^{c} F_i \, \lambda_i \left(\sum_{i=1}^{c} P(\partial F_i/\partial X^*_j) \, \lambda_i \right)$$

$$- \sum_{i=1}^{c} (1 + \ell n \, \lambda_i) \, \lambda_i \, (\partial F_i/\partial X^*_j) + (1/P) \sum_{i=1}^{c} (1 + \ell n \, \lambda_i) \, \lambda_i \left(\sum_{i=1}^{c} P(\partial F_i/\partial X^*_j) \, \lambda_i \right) = 0$$

or:

$$\sum_{i=1}^{c} \lambda_i \, (\partial F_i/\partial X^*_j) \left(1 + PF_i - P \sum_{i=1}^{c} F_i \, \lambda_i - (1 + \ell n \, \lambda_i) + \sum_{i=1}^{c} \lambda_i \, (1 + \ell n \lambda_i) \right) = 0$$

or:

$$\sum_{i=1}^{c} \lambda_i \, (\partial F_i/\partial X^*_j) \left(1 + P(F_i - \sum_{i=1}^{c} \lambda_i \, F_i) - 1 - \ell n \, \lambda_i + 1 + \sum_{i=1}^{c} \lambda_i \, \ell n \, \lambda_i) \right) = 0$$

or:

$$\sum_{i=1}^{c} \lambda_i \, (\partial F_i/\partial X^*_j) \left(1 - \ell n \, \lambda_i + \sum_{i=1}^{c} \lambda_i \, \ell n \, \lambda_i + P \, (F_i - \sum_{i=1}^{c} \lambda_i \, F_i) \right) = 0$$

But, $$\ell n\lambda_i = \ell n \left\{ \frac{\exp(PF_i)}{\sum_{i=1}^{c} \exp(PF_i)} \right\} = PF_i - \ell n \sum_{i=1}^{c} \exp(PF_i)$$

Substituting for $\ell n\ \lambda_i$ in above we get:

$$\sum_{i=1}^{c} \lambda_i\ (\partial F_i/\partial X^*_j) \left[1 - PF_i + \ell n \sum_{i=1}^{c} \exp(PF_i\ ) + \sum_{i=1}^{c} \lambda_i PF_i \right.$$

$$\left. - \sum_{i=1}^{c} \lambda_i\ \ell n \sum_{i=1}^{c} \exp(PF_i) + PF_i - P \sum_{i=1}^{c} \lambda_i\ F_i \right] = 0$$

or:

$$\sum_{i=1}^{c} \lambda_i\ (\partial F_i/\partial X^*_j) \left[1 + \ell n \sum_{i=1}^{c} \exp(PF_i)\ [1 - \sum_{i=1}^{c} \lambda_i] \right] = 0$$

but $\sum_{i=1}^{c} \lambda_i = 1$

$\therefore$ Finally: $$(\partial V_2/\partial X^*_j) = \sum_{i=1}^{c} \lambda_i\ (\partial F_i/\partial X^*_j) = 0\ ;\ \forall j \epsilon J$$

APPENDIX B

Given:

$$V_3 = \begin{cases} \underset{\mathbf{X}\epsilon\Omega}{\text{Min}} \quad \gamma_h \left[\lambda_h \; F_h \; (\mathbf{X}) \; - \; (1/P) \; \lambda_h \; \ell n \; \lambda_h \right]; \; \gamma_h > 0 \\ \\ \text{S.t.: } \lambda_i \; F_i \; (\mathbf{X}) \; - \; (1/P) \; \lambda_i \; \ell n \; \lambda_i \; ; i \neq h \epsilon c \end{cases}$$

with:

$$\lambda_i = \exp \; [P \; F_i \; (\mathbf{X})] / \sum_{i=1}^{c} \exp \; [P \; F_i \; (\mathbf{X})]; \; i = 1, \ldots, c$$

show that:

$$(\partial V_3/\partial X^*_j) = \sum_{i=1}^{c} \alpha_i \; (\partial F_i/\partial X^*_j) = 0 \; ; \; \alpha_i \geq 0 \quad \forall \; i = 1, \ldots, c$$

where X^*_j is an optimal solution to V_3

Proof:

$$(\partial V_3/\partial X^*_j) = \gamma_h \left[(\partial \lambda_h/\partial X^*_j) \; F_h + \lambda_h \; (\partial F_h/\partial X^*_j) \; - \; (1/P) \; [(\partial \lambda_h/\partial X^*_j)(1 + \ell n \; \lambda_h] \right]$$

$$+ \sum_{\substack{i=1 \\ i \neq h}}^{c} \gamma_i \left[(\partial \lambda_i/\partial X^*_j) \; F_i + \lambda_i \; (\partial F_i/\partial X^*_j) \; - \; (1/P) \; [(\partial \lambda_i/\partial X^*_j)(1 + \ell n \; \lambda_i] \right]$$

or:

$$(\partial V_3/\partial X^*_j) = \sum_{i=1}^{c} \gamma_i \left[(\partial \lambda_i/\partial X^*_j) \; F_i + \lambda_i \; (\partial F_i/\partial X^*_j) \; - \; (1/P) \; [(\partial \lambda_i/\partial X^*_j)(1 + \ell n \; \lambda_h] \right]$$

which we have already proved in Appendix (A) that is equal to:

$$(\partial V_3/\partial X^*_j) = \sum_{i=1}^{c} \gamma_i \lambda_i (\partial F_i/\partial X^*_j) = 0$$

$$= \sum_{i=1}^{c} \alpha_i (\partial F_i/\partial X^*_j) = 0$$

since $\alpha_i = \gamma_i \lambda_i \geq 0$.

APPENDIX C

PLASTIC MOMENTS

This appendix presents the derivations of the plastic moments, ultimate strength, of reinforced concrete cross-sections as function of the design variables.

C.1 THE PLASTIC MOMENTS FOR COLUMNS

$$R_1 = (1/12) \left[(X_{11} - X_{12}) (X_6 - a_c/2) f_y + X_{12} (X_6 - (X_4 - X_6)) \right] \quad (C.1)$$

with

$$a_c = \frac{(X_{11} - X_{12}) f_y}{0.85 \acute{f}_c X_5}$$

In practice, $R_1 = R_2 = R_6 = R_7$

C.2 THE PLASTIC MOMENTS FOR A BEAM

$$R_3 = (1/12) \left[(2X_{10} - X_9) (X_3 - a_{brl}/2) f_y + X_9 (X_3 - (X_1 - X_3)) \right] = R_5 \quad (C.2)$$

with

$$a_{brl} = \frac{(2X_{10} - X_9)}{0.85 \acute{f}_c X_2} f_y$$

$$R_4 = (1/12) \left[(2X_9 - X_{10}) (X_3 - a_{bm}/2) f_y + X_{10} (X_3 - (X_1 - X_3)\right]] \quad (C.3)$$

with

$$a_{bm} = \frac{(2X_9 - X_{10})}{0.85 \acute{f}_c X_2} f_y$$

APPENDIX D

THE CONSTRAINTS FUNCTIONS

This appendix presents derivations of the Constraint Functions for one bay-one storey reinforced concrete frame in terms of the considered design variables.

D.1 CONSTRAINTS FOR A BEAM

D.1.1 Flexural Strength

$$0.9\, f_y\, (X_9 X_3 + 2 X_{10} X_3 - X_9 X_1) - \frac{0.9}{1.7} \frac{f^2_y}{f'_c} \frac{(2X_{10} - X_9)^2}{X_2} - M_{PR} \geq 0$$

$$0.9\, f_y\, (2 X_9 X_3 + X_{10} X_3 - X_9 X_1) - \frac{0.9}{1.7} \frac{f^2_y}{f'_c} \frac{(2X_9 - X_{10})^2}{X_2} - M_{PM} \geq 0$$

where M_{PR} and M_{PM} are the moments at the right and the middle side of the beam.

D.1.2 Shear Strength

$$1.7 \sqrt{f'_c}\, X_2 X_3 + 3.4\, f_y X_{18} - 1000\, (WL/2 + P_v/2) - 2M_B/(12L) \geq 0$$

$$1.7 \sqrt{f'_c}\, X_2 X_3 + 3.4\, f_y X_{14} - 1000\, (WL/2 + P_v/2) - 2M_B/(12L) \geq 0$$

$$1.7 \sqrt{f'_c}\, X_2 X_3 + 1.7\, f_y X_{13} - 1000\, (WL/2 + P_v/2) - 2M_B/(12L) \geq 0$$

where M_B is the moment at the left side of the beam when the frame is under horizontal applied load at the left side of the beam.

D.1.3 Ductility (Plasticity)

$$\gamma_1 \rho_{b1} - \frac{2X_9 - X_{10}}{X_2 \quad X_3} \geq 0$$

$$\gamma_1 \rho_{b2} - \frac{2X_{10} - X_9}{X_2 \quad X_3} \geq 0$$

$$\frac{2X_9}{X_2 \ X_3} - \frac{200}{f_y} \geq 0$$

$$\frac{2X_{10}}{X_2 \ X_3} - \frac{200}{f_y} \geq 0$$

where

$$\rho_{b1} = \frac{0.003(0.85)\beta_1 \ E_s \ \acute{f}_c}{f_y \ (0.003E_s + f_y)} + \frac{X_{10}}{X_2 \ X_3}$$

$$\rho_{b2} = \frac{0.003(0.85)\beta_1 \ E_s \ \acute{f}_c}{f_y \ (0.003)E_s + f_y} + \frac{X_9}{X_2 \ X_3}$$

D.1.4 Serviceability

$$X_1 - \frac{12L}{18.5} \geq 0$$

$$X_1 - X_2 \geq 0$$

$$- X_1 + \frac{2(12L)}{5} \geq 0$$

$$X_2 - \gamma_6 X_1 \geq 0$$

D.1.5 Concrete Cover for Reinforcement

$$X_1 - X_3 - (1.5 + d_b . N_\ell) \geq 0$$

where d_b: specified nominal diameter of bars, inch.

D.1.6 Limits of Web Reinforcement

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{X_{18}}{X_2 X_3} \geq 0 \qquad \frac{X_{18}}{X_2 X_3} - \frac{50}{4 f_y} \geq 0$$

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{X_{14}}{X_2 X_3} \geq 0 \qquad \frac{X_{14}}{X_2 X_3} - \frac{50}{4 f_y} \geq 0$$

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{X_{13}}{2X_2 X_3} \geq 0 \qquad \frac{X_{13}}{2X_2 X_3} - \frac{50}{4 f_y} \geq 0$$

D.1.7 Development Length for Longitudinal Reinforcement

$$X_7 - \frac{0.04}{12} \frac{\pi d^2_b}{4} \frac{f_y}{\sqrt{f'_c}} \geq 0$$

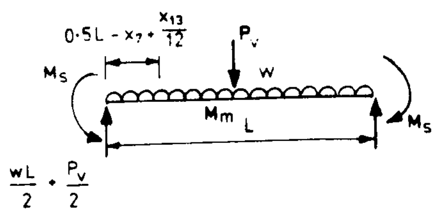
$$X_8 - \frac{X_4}{24} - 1.4 \frac{0.04}{12} \frac{\pi d^2_b}{4} \frac{f_y}{\sqrt{f'_c}} \geq 0$$

where d_b: specified nominal diameter of bars, inch.

D.1.8 <u>Cut-off Points of Longitudinal Reinforcement</u>

$$\gamma_a M_m + 0.5(12)(1000)\left[-(P_v + WL)(0.5L - X_7 + X_{13}/12) + W(0.5L - X_7 + X_3/12)^2 + M_s\right] \geq 0$$

where M_m and M_s are shown as follows:



Also:

$$X_2 - (4 - d_b) - 2\, d_b \frac{(N_s)_1 + N_\ell - 1}{N_\ell} \geq 0$$

$$\left(\frac{X_3}{4} - \frac{N_s}{8} (N_v)_2 \right) / (N_v)_2 - 1 \geq 0$$

where:

d_b = specified nominal diameter

$(N_s)_1$ = total no. of longitudinal bars placed at the midspan of beam

N_ℓ = no. of tension steel layers placed in beam

N_s = no. of shear steel bar (ties)

$(N_v)_2$ = total no. of stirrups within a distance, based on specifiedly based bars.

D.1.9 Compression Steel Yield Condition

$$\frac{2X_9 - X_{10}}{X_2\ X_3}\left[1 - \frac{0.85 f'_c}{f_y}\right] - 0.85\ \beta_1 \frac{f'_c}{f_y}\ \frac{1.5 + d_b\ N_\ell \quad 0.003\ E_s}{X_3 \quad 0.003\ E_s - f_y} \geq 0$$

$$\frac{2X_{10} - X_9}{X_2\ X_3}\left[1 - \frac{0.85 f'_c}{f_y}\right] - 0.85\ \beta_1 \frac{f'_c}{f_y}\ \frac{1.5 + d_b\ N_\ell \quad 0.003\ E_s}{X_3 \quad 0.003\ E_s - f_y} \geq 0$$

D.1.10 Tension Steel Yield Condition

$$\frac{0.003\ (0.85)\ \beta_1\ E_s\ f'_c}{f_y\ (0.003\ E_s + f_y)} + \frac{X_{10}}{X_2\ X_3} - \frac{2X_9}{X_2\ X_3} \geq 0$$

$$\frac{0.003\ (0.85)\ \beta_1\ E_s\ f'_c}{f_y\ (0.003\ E_s + f_y)} + \frac{X_9}{X_2\ X_3} - \frac{2X_{10}}{X_2\ X_3} \geq 0$$

D.2 CONSTRAINTS FOR A COLUMN

D.2.1 Axial and Flexural Strength

$$(P_n)_1 = 0.85\ f'_c\ \ X_5\ X_4 + (f_y - 0.85\ f'_c)\ (X_{11} + X_{12})$$

$$(P_n)_2 = (0.85)^2\ \frac{0.003E_s}{f_y + 0.003E_s} + f'_c\ X_5\ X_6 + f_y\ (X_{12} - X_{11})$$

$$(P_n)_3 = (0.85)^2\ \frac{0.003E_s}{2(f_y + 0.003E_s)} + f'_c\ X_5\ X_6 + f_y\ (X_{12} - X_{11})$$

$$(M_n)_2 = \frac{1}{0.85\ f'_c\ X_4\ X_5 + f_y\ (X_{11} + X_{12})}\Bigg(0.307\ f'^2_c\ \frac{0.003E_s}{f_y + 0.003E_s}\ (X_4\ X_5)^2\ X_6\ -$$

$$0.425\ f'_c\ f_y\ X_5\ X_{11}\ X^2_4\ -\ 0.425\ f'_c\ f_y\ X_{12}\ X^2_4\ X_5 + 0.85\ f'_c\ f_y\ X_{11}\ X_4\ X_5\ X_6$$

$$+ \left(0.85\ f'_c\ f_y + 0.7225\ f'_c\ f_y\ \frac{0.003E_s}{f_y + 0.003E_s}\right)\ X_{12}\ X_4\ X_5\ X_6\ -$$

$$2\ f^2_y\ X_{11}\ X_{12}\ X_4 + 4\ f^2_y\ X_{11}\ X_{12}\ X_6 + (0.85)^2\ f'_c\ f_y \frac{0.003E_s}{fy + 0.003E_s}\ X_{11}\ X_5\ X^2_6\ -$$

$$(0.85)^2\ f'_c\ f_y\ \frac{0\ .003E_s}{f_y + 0.003E_s}\ X_{12}\ X_5\ X^2_6\ \Bigg)\ -\ 0.3071\ f'_c\ \left(\frac{0.003E_s)^2}{f_y + 0.003E_s}\right) X_5\ X^2_6$$

$$(M_n)_3 = \frac{1}{0.85\ f'c\ X_4\ X_5 + f_y\ (X_{11} + X_{12})}\Bigg(0.307\ f'^2_c \frac{0.003E_s}{2(fy + 0.003E_s)}\ X_4^2\ X_5^2\ X_6\ -$$

$$0.425\ f'_c\ f_y\ X_{11}\ X_5\ X^2_4\ -\ 0.425\ f'_c\ f_y\ X_{12}\ X_5\ X^2_4 + 0.85\ f'_c\ f_y\ X_{11}\ X_4\ X_5\ X_6$$

$$+ \left(0.85\ f'_c\ f_y + 0.7225\ f'_c\ f_y\ \frac{0.003E_s}{2(f_y + 0.003E_s)}\right)\ X_{12}\ X_4\ X_5\ X_6\ -$$

$$2\ f^2_y\ X_{11}\ X_{12}\ X_4 + 4\ f^2_y\ X_{11}\ X_{12}\ X_6 + (0.85)^2\ f'_c\ f_y \frac{0.003E_s}{2(fy + 0.003E_s)}\ X_{11}\ X_5\ X^2_6\ -$$

$$(0.85)^2\ f'_c\ f_y\ \frac{0.003E_s}{2(f_y + 0.003E_s)}\ X_{12}\ X_5\ X^2_6\ \Bigg) -\ 0.3071\ f'_c \left(\frac{0\ .003E_s}{2(f_y + 0.003E_s}\right)^2 X_5\ X^2_6$$

$$(M_n)_4 = f_y\ (X_6\ X_{11} + X_6\ X_{12} - X_4\ X_{12}) + f_y\ \frac{X_{11}\ X_{12}}{X_5} - \frac{0.59 f^2_y}{f'_c\ X_5}\ (X^2_{11} + X^2_{12})$$

So, the constraints are:

$$0.56\ (P_n)_1 - P_u \geq 0$$

$$0.7 - \frac{P_u}{(P_n)_1}\ \frac{M_u}{(M_n)_2}\ \frac{(P_n)_1 - (P_n)_2}{(P_n)_1} \geq 0$$

$$0.8 + \frac{P_u}{(P_n)_3}\ \frac{(M_n)_3 - (M_n)_4}{(M_n)_4} - \frac{M_u}{(M_n)_4} \geq 0$$

$$0.8 + \frac{P_u\ [(M_n)_2 - (M_n)_3] - M_u\ [(P_n)_2 - (P_n)_3]}{(P_n)_2\ (M_n)_3 - (P_n)_3\ (M_n)_2} \geq 0$$

D.2.2 <u>Shear Strength</u>

$$1.7\ \sqrt{f'_c}\ X_5\ X_6 + 3.4\ f_y\ X_{15} - M_c/12H \geq 0$$

$$1.7\ \sqrt{f'_c}\ X_5\ X_6 + 1.7\ f_y\ X_{16} - M_c/12H \geq 0$$

$$1.7\ \sqrt{f'_c}\ X_5\ X_6 + 3.4\ f_y\ X_{17} - M_D/12H \geq 0$$

where M_c and M_D are the moments at C and D.

D.2.3 <u>Ductility (Plasticity)</u>

$$0.08 - \frac{X_{11} + X_{12}}{X_4\ X_5} \geq 0 \qquad -0.01 + \frac{X_{11} + X_{12}}{X_4\ X_5} \geq 0$$

$$- \gamma_1 \rho_{b3} - \frac{X_{11} + X_{12}}{X_5 X_6} \geq 0$$

$$\text{where } \rho_{b3} = \frac{0.003(0.85)\beta_1 E_s f'_c}{f_y (0.003E_s + f_y)} + \frac{X_{12}}{X_5 X_6}$$

D.2.4 Serviceability

$$X_4 - X_5 \geq 0$$

$$X_4 - 0.5 X_1 \geq 0$$

D.2.5 Concrete Cover for Reinforcement

$$X_4 - X_6 - (1.5 + (N_c/8) N_{\ell c}) \geq 0$$

where $N_{\ell c}$ = Number of tension steel layers placed in a colum.

N_c = Bar No. of longitudinal steel placed in a column.

D.2.6 Limits of Lateral Reinforcement

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{X_{15}}{X_5 X_6} \geq 0 \qquad \frac{X_{15}}{X_5 X_6} - \frac{50}{4f_Y} \geq 0$$

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{X_{17}}{X_5 X_6} \geq 0 \qquad \frac{X_{17}}{X_5 X_6} - \frac{50}{4f_Y} \geq 0$$

$$\frac{2\sqrt{f'_c}}{f_y} - \frac{X_{16}}{2X_5 X_6} \geq 0 \qquad \frac{X_{16}}{2X_5 X_6} - \frac{50}{4f_Y} \geq 0$$

D.2.7 Spacing Limits

$$X_5 - (3 + 2\ (N_s/8) - 1.5\ d_b) - 2.5\ d_b \cdot (N_s)_3 \geq 0$$

$$\frac{\left[X_6/4 - (N_s/8)\ (N_v)_4\right]}{(N_v)_4} - 1 \geq 0$$

where:

N_s = No. of shear steel bar (ties)

d_b = Specified nominal diameter

$(N_v)_4$ = Total no. of ties within a distance s, based on specifically based bars.

$(N_s)_3$ = Total no. of bars placed in the outer-side column cross section.

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