Contrasting Behaviour of Octupole<br>Bands at High Spin in ${ }^{220}$ Ra and ${ }^{222} \mathrm{Th}$<br>and Lifetimes of States in the<br>Opposite-parity Bands of ${ }^{153} \mathrm{Eu}$



Thesis submitted in accordance with the requirements of the 15 University of Liverpool for the degree of Doctor in Philosophy, by


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This thesis is dedicated to my parents

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#### Abstract

The well-established octupole bands in ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ have been studied to high spins, using the Eurogam gamma-ray spectrometer. The nuclei were produced in the reaction ${ }^{208} \mathrm{~Pb}\left({ }^{18} \mathrm{O}, \alpha 2 \mathrm{n} / 4 \mathrm{n}\right)$. The yrast octupole bands have been observed up to $31 \hbar$ in ${ }^{220} \mathrm{Ra}$ and $25 \hbar$ in ${ }^{222} \mathrm{Th}$, but the gamma-ray multiplicities have been measured to be very nearly the same. This observation is interpreted as evidence for the crossing of the octupole band by a four quasi-particle aligned band in ${ }^{222} \mathrm{Th}$, which has been predicted by cranked HFB calculations. Values of intrinsic electric dipole moments have been inferred from $\mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2)$ ratios, and are found to be $0.34(4)$ efm for ${ }^{220} \mathrm{Ra}$ and $0.55(3)$ efm for ${ }^{222} \mathrm{Th}$. These observations are consistent with the interpretation of the structure of these nuclei in terms of octupole deformation.


The lifetimes of thirteen excited states in ${ }^{153} \mathrm{Eu}$ have been measured using the recoil-distance method. The states were populated using multiple Coulomb excitation of ${ }^{153} \mathrm{Eu}$ by a 220 $\mathrm{MeV}{ }^{58} \mathrm{Ni}$ beam. Electromagnetic transition moments have subsequently been extracted. The results are consistent with the conclusions of another recent experimental study; despite the apparent parity-doublet band structure, ${ }^{153} \mathrm{Eu}$ can adequately be described without introducing octupole deformation.

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## CHAPTER 1

## Introduction

This thesis describes two experiments in which state-of-the-art gamma-ray spectroscopy techniques were used to study nuclei with reflection-asymmetric deformations. Some of the theoretical concepts that are necessary to understand the relevance of the results may be unfamiliar, and the important points are covered in this introduction. This chapter introduces collective rotation, the Nilsson model and the cranked shell model, and shows how many of the nuclear properties, such as angular momenta and moments of inertia, are derived from simple considerations of classical and quantum mechanics. The models are presented with a qualitative slant; the specific details being referenced. A brief description of how these models relate to the subject matter of this thesis is given, and the chapter ends with an outline of the thesis, and a summary of its aims.

The shell model has had considerable success in describing nuclear properties, particularly around closed shells. Away from closed shells, coherence in the single-particle motion can result in collective effects and nuclear deformation. The effect of the deformed shape on single-particle motion can be investigated using the Nilsson model. The Nilsson model also enables the total energy of a static deformed nucleus to be calculated, and leads to the prediction of stable deformed nuclei. A consequence of deformation is that collective rotation is one of the allowed modes of excitation. The rotation also perturbs the singleparticle orbits, and these effects can be calculated using the cranked shell model.

### 1.1 Nuclear Structure: Deformation and Rotation

Three-hundred million-million $2^{+}$states can be constructed out of the 12 valence protons and 10 valence neutrons for the rare-earth nucleus ${ }_{62}^{154} \mathrm{Sm}_{92}$. Were it possible to calculate all these states, it would not be possible to understand and interpret them. This famous, albeit extreme, example of Talmi [Talmi 62] illustrates how the nuclear shell model [Heyde 89] becomes unfeasible at mid-shell where there are many valence nucleons. In such mid-shell nuclei, the excitation energies of the first excited states are well below that which would be predicted by the nuclear shell model. The reason for this is that the single-particle motion in these nuclei becomes correlated and the nucleons behave collectively in a manner analogous to molecules in a liquid drop [Ring 80] [Eisenberg 75].

## Nuclear Deformation

The forces between nucleons outside of closed shells can be broken down into residual interactions. The simplest residual interaction is the pairing interaction which acts between nucleons of the same orbital angular momentum, $j$. There is much empirical evidence for this interaction; the most obvious of which being the zero angular-momentum ground states of all even-even nuclei. Of the other residual interactions, the $2^{\lambda}$-pole interactions act most strongly between nucleons in $\Delta \mathrm{j}=\Delta \ell=\lambda$ orbitals. The interactions are short range and therefore only act between orbits which lie physically close together. Orbits within a major shell have $\Delta \mathrm{j}=\Delta \ell=2$, and so the quadrupole ( $\lambda=2$ ) interaction is usually the most important. Nucleons subjected to the quadrupole interaction will move in quadrupoleshaped orbits. With many nucleons in these orbits, and a strong quadrupole interaction, quadrupole vibrations or deformation of the nucleus can result.

In the work described in this thesis, it is the octupole ( $\lambda=3$ ) interactions, which occur between $\Delta \mathrm{j}=\Delta \ell=3$ orbits, that are of interest. They must occur between states from different major shells. They can, therefore, only come about when an intruder state is involved. An intruder state is one which is brought down from the ( $\mathrm{N}+1$ ) major shell into the N major shell by the spin-orbit interaction. (The spin-orbit interaction term was added to the nuclear potential [Haxel 49] in order to make it more realistic, and reproduce the experimentally observed magic numbers). Octupole interactions occur between these intruder states and the normal-parity states in a shell. As the shells fill, the Fermi surface approaches the region where the octupole interactions between the $\Delta \mathrm{j}=\Delta \ell=3$ states is largest, and the energy of the octupole-shaped orbits is lowered. Coherence in the octupole-shaped orbits of several valence nucleons results in an octupole-vibrational or octupole-deformed nucleus
depending on the number of valence nucleons involved and on the strength of the octupole interaction. The subject of octupole deformation is dealt with in chapter 2.

## Collective Rotation

Rotation of a spherical nucleus is not allowed, because rotations about a symmetry axis are quantum-mechanically indistinguishable. However, for a deformed nucleus, rotation becomes a possible mode of excitation. Collective rotation generates collective angular momentum, $\vec{R}$ which vectorially adds to the intrinsic angular momentum, $\vec{J}$, to give the total angular momentum, $\vec{I}$. This is illustrated schematically in figure 1.1 . This discussion is limited to axially-symmetric nuclei, with the $z$-axis being the axis of symmetry and the $x$-axis being the axis of rotation. The collective rotational angular momentum, $\vec{R}$ is perpendicular to the symmetry axis, therefore the projection of $\vec{I}$ on the symmetry axis is the same as the projection of $\vec{J}$, and is denoted by $\Omega$. The sum of the individual $\Omega$ values is known as K . The rotational properties of the nucleus can be understood by applying


Figure 1.1: Nuclear angular momentum: its various components and projections.
expressions for the rotation of a classical rigid body. For example the rotational frequency in classical mechanics is defined as,

$$
\begin{equation*}
\omega=\frac{d E}{d I} \tag{1.1}
\end{equation*}
$$

and the quantum-mechanical analogue of this expression is,

$$
\begin{equation*}
\hbar \omega=\frac{d E_{e x}}{d \sqrt{I(I+1)}}=\frac{E_{I}-E_{I-2}}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}} \tag{1.2}
\end{equation*}
$$

For a rotational band of stretched E 2 transitions $\hbar \omega \simeq \mathrm{E}_{\gamma} / 2$. By analogy to the expression for the kinetic energy of a classical rigid body, $\mathrm{E}=\frac{1}{2} \Im \omega^{2}$, where $\Im$ is the moment of inertia, the excitations of a rotating nucleus form sequences of states with energy,

$$
\begin{equation*}
E=\frac{\hbar^{2}}{2 \Im^{(0)}} I(I+1), \tag{1.3}
\end{equation*}
$$

where $\Im^{(0)}$ is the static moment of inertia. However the nucleus is not rigid, and deviations
from this simple expression occur. An expansion of the type,

$$
\begin{equation*}
E(I)=E_{0}+A[I(I+1)]+B[I(I+1)]^{2}+C[I(I+1)]^{3} \ldots \tag{1.4}
\end{equation*}
$$

can provide a more accurate description. Harris [Harris 65] introduced a parametrization in terms of rotational frequency,

$$
\begin{equation*}
E(I(\omega))=\alpha \omega^{2}+\beta \omega^{4}+\gamma \omega^{6} \ldots, \tag{1.5}
\end{equation*}
$$

which is often used. In that case,

$$
\begin{align*}
I(\omega) & =\frac{d E}{d \omega}=2 \alpha \omega+4 \beta \omega^{3}+6 \gamma \omega^{5} \ldots  \tag{1.6}\\
\Im^{(0)}(\omega) & =\frac{I}{\omega}=2 \alpha+4 \beta \omega^{2}+6 \gamma \omega^{4} \ldots \tag{1.7}
\end{align*}
$$

Taking this Harris parametrization to second order, a plot of $\Im^{(0)}$ against $\omega^{2}$ should be linear. This is only usually the case at low rotational frequencies, and deviations occur, the reasons for which are mentioned later in this introduction. The measured moments of inertia are also usually smaller than the rigid-body values, because of the effects of pairing which cause the nuclear matter to behave like a fluid. In addition to the static moment of inertia, two other moments of inertia are introduced which illustrate how the nucleus behaves under rotation. They are both related to the derivatives of the excitation energy with respect to aligned angular momentum. The kinematic moment of inertia, $\Im^{(1)}$, is defined as,

$$
\begin{equation*}
\Im^{(1)}=I_{x} \hbar^{2}\left[\frac{d E}{d I_{x}}\right]^{-1}=\hbar \frac{I_{x}}{\omega} \tag{1.8}
\end{equation*}
$$

and the dynamic moment of inertia, $\Im^{(2)}$, is defined as,

$$
\begin{equation*}
\Im^{(2)}=\hbar^{2}\left[\frac{d^{2} E}{d I_{x}^{2}}\right]^{-1}=\hbar \frac{d I_{x}}{d \omega} . \tag{1.9}
\end{equation*}
$$

In these definitions, the $I_{x}$ term is the aligned angular momentum on the axis of rotation and is given by, $I_{x}=\sqrt{I(I+1)-K^{2}}$.

## The Nilsson Model

The effect of the deformed nuclear potential on the single-particle orbits can be calculated using the Nilsson model [Nilsson 69]. In this model the states are characterized by the quantum numbers,

$$
\Omega^{\pi}\left[\mathrm{N}_{\mathrm{z}} \Lambda\right]
$$

where $\Omega$ is the projection of the total single-particle angular momentum on the symmetry axis and $\Lambda$ is the projection of the orbital angular momentum on the symmetry axis. Hence,
$\Omega=\Lambda+\Sigma$, where $\Sigma\left( \pm \frac{1}{2}\right)$ is the projection of intrinsic spin of the orbit on the symmetry axis. N is the principal quantum number, $\pi$ is the parity, $(-1)^{N}$, and $\mathrm{n}_{z}$ is the number of nodes in the wavefunction in the $z$-direction, (where $z$ is the symmetry axis). In labelling the states it is often useful to know that if N is even then $\mathrm{n}_{z}+\Lambda$ must be even, and if N is odd then $\mathrm{n}_{z}+\Lambda$ must be odd, and that $\mathrm{n}_{z}$ can take values up to a maximum of N .

Many of the principles of the Nilsson model can be understood qualitatively. The nuclear force is attractive so an orbit will have lower energy if it lies close to the rest of the nuclear matter, than if it lies at a large distance from it. Considering, for example, a nucleus with a positive quadrupole prolate deformation, as shown in figure 1.2 the orbit A will, therefore, have lower energy than orbit B. This is equivalent to saying that orbits with low $-\Omega$ values will have lower energy than those with high $\Omega$ values. Also the orbit $A$ is more extended in the $z$ direction than orbit B . This means that orbit A will have more nodes in its wavefunction in the $z$ direction. So the lowest energy orbits also have the largest $n_{z}$ values. The rate of change of the energy of the orbit as a function of $\Omega$, can be estimated by considering the angle which the plane of the orbit makes with the $z$ axis. The orbits A and $B$ in figure 1.2 illustrate that the energy of the orbit depends on the angle $\theta$ [Casten 90]. The classical angle of an orbit is given by $\theta=\sin ^{-1}(\Omega / \mathrm{j})$. It is easy to show that $\theta$ changes slowly for low $-\Omega$ and rapidly for high $-\Omega$, and so it follows that the energy difference between low $-\Omega$ values is small, and that between high $-\Omega$ values is large.


Figure 1.2: Two schematic orbits around a nucleus with a prolate deformation. The nuclear force is attractive so the orbit A will have lower energy that the orbit $B$. The angle $\theta$ represents the angle which the plane of the orbit B makes with the symmetry axis.

The $\Omega$ and $n_{z}$ dependence of the energy means that as the deformation is changed some of the Nilsson states will approach each other. The Pauli exclusion principle says that no two particles with the same quantum-numbers can occupy the same state. This has the implication that, if the energy of the state is plotted against deformation, then the trajectories of no two states can cross, As they get closer together they will repel. Each trajectory will start out as a straight line but will curve as it approaches another state


Figure 1.3: A Nilsson diagram for $\mathrm{N}=82-126$. The solid lines represent positive-parity states and the dashed lines represent negative-parity states. The deformation parameter $\epsilon$ is approximately $0.95 \beta$. Taken from [Casten 90].
of the same $\Omega$ and $\pi$. Starting with the single-particle shell model energies of a given potential, and considering the $\Omega$ splitting and level repulsion, an entire Nilsson diagram can be constructed. A Nilsson diagram for the $\mathrm{N}=82-126$ shell is given in figure 1.3.

Each ( $2 \mathrm{j}+1$ )-fold degenerate j -state is split into ( $\mathrm{j}+\frac{1}{2}$ ) two-fold degenerate levels. The sequence of states can be understood by recalling that ( $n_{z}+\Lambda$ ) is even (odd) if $N$ is even (odd) and that the states with the highest $\mathrm{n}_{z}$ lie the lowest in energy. A feature of the Nilsson diagram that is worth pointing out is that the highest- $\Omega$ orbitals are very straight due the lack of nearby $j$-shells, with the same high- $\Omega$ component with which they can mix. An example from the figure is the $\Omega=\frac{9}{2}$ orbit from the $h_{\frac{\rho}{2}}$ state. Also, the orbits emanating from an intruder orbital such as the $i_{\frac{13}{2}}$ in figure 1.3 have opposite parity to all the nearby orbitals and therefore, do not mix and are very pure states.


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Although different potentials have been used within the Nilsson model, such as a WoodsSaxon potential in [Dudek 78], Nilsson originally used a modified-oscillator potential, and in that case, the Hamiltonian is written,

$$
\begin{equation*}
\mathrm{H}=\frac{\mathrm{p}^{2}}{2 \mathrm{~m}}+\frac{1}{2}\left[\omega_{\mathrm{x}}^{2}\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right)+\omega_{\mathrm{z}}^{2} \mathrm{z}^{2}\right]-2 \kappa \hbar \omega_{0}\left[\ell . \mathrm{s}-\mu\left(\ell^{2}-\left\langle\ell^{2}>_{\mathrm{N}}\right)\right] .\right. \tag{1.10}
\end{equation*}
$$

This can be expressed in terms of the deformation parameter $\delta(\simeq 0.95 \beta)$ if the oscillator frequencies are expressed as,

$$
\begin{align*}
\omega_{\mathrm{x}}^{2}=\omega_{\mathrm{y}}^{2} & =\omega_{0}^{2}\left[1+\frac{2}{3} \delta\right]  \tag{1.11}\\
\omega_{\mathrm{z}}^{2} & =\omega_{0}^{2}\left[1-\frac{4}{3} \delta\right] \tag{1.12}
\end{align*}
$$

where $\omega_{0}$ is the oscillator frequency, equal to $41 \mathrm{~A}^{-\frac{1}{3}}$ in the spherical potential with $\delta=0$. (Volume conservation requires that the product $\omega_{x} \omega_{y} \omega_{z}$ is constant). The Nilsson Hamiltonian then becomes,

$$
\begin{equation*}
\mathrm{H}=\frac{\mathrm{p}^{2}}{2 \mathrm{~m}}+\frac{1}{2} \mathrm{~m} \omega_{0}^{2} \mathrm{r}^{2}-\mathrm{m} \omega_{0}^{2} \mathrm{r}^{2} \delta \frac{4}{3} \sqrt{\frac{\pi}{5}} \mathrm{Y}_{20}(\theta, \phi)+2 \kappa \hbar \omega_{0}\left[\vec{\ell} . \overrightarrow{\mathrm{S}}-\mu\left(\overrightarrow{\ell^{2}}-\langle\vec{\ell}\rangle_{\mathrm{N}}\right)\right] \tag{1.13}
\end{equation*}
$$

For small deformation, the nucleus is approximately spherical and j is almost a good quantum number. If the $\mathrm{Y}_{20}$ term in equation 1.13 is treated as a perturbation, then the shift in energy relative to the $\delta=0$ case can be calculated [Casten 90] to be,

$$
\begin{equation*}
\Delta \mathrm{E}(\mathrm{~N} \ell \mathrm{j} \Omega)=-\frac{2}{3} \hbar \omega_{0}\left[\mathrm{~N}+\frac{3}{2}\right] \delta \frac{\left[3 \Omega^{2}-\mathrm{j}(\mathrm{j}+1)\right]\left[\frac{3}{4}-\mathrm{j}(\mathrm{j}+1)\right]}{[2 \mathrm{j}-1] \mathrm{j}(\mathrm{j}+1)[2 \mathrm{j}+3]}, \tag{1.14}
\end{equation*}
$$

This result illustrates that the energy shift is proportional to $\delta$, depends on $\Omega^{2}$ and is linearly dependent on N . All these features can be seen in the Nilsson diagram. The dependence on N indicates that the Nilsson states slope down more for heavy nuclei, so they are more easily deformed than light nuclei.

For large deformation, the $\vec{\ell} . \vec{s}$ and $\overrightarrow{\ell^{2}}$ terms are negligible and the Hamiltonian reduces to that of an anisotropic harmonic oscillator. The energy of the states is given by,

$$
\begin{equation*}
\mathrm{E}=\hbar \omega_{\mathrm{x}}\left(\mathrm{~N}-\mathrm{n}_{\mathrm{z}}+1\right)+\hbar \omega_{\mathrm{z}}\left(\mathrm{n}_{\mathrm{z}}+\frac{1}{2}\right) \tag{1.15}
\end{equation*}
$$

The $\delta$ dependence of the oscillator frequencies means that energies are still proportional to the deformation. The slope depends on $n_{z}$, which is obvious in the Nilsson diagram as all the states with the same $n_{z}$ are parallel. The lowest-lying orbits at the largest deformations have $\mathrm{n}_{z}=\mathrm{N}$ or $\mathrm{n}_{z}=\mathrm{N}-1$, while the highest-lying orbits have $\mathrm{n}_{z}=0$ or $\mathrm{n}_{z}=1$. The energies are independent of $\Omega$ in the large-deformation limit.

## The Nilsson Model and the Strutinsky Shell-correction Method

With the Nilsson model, it is possible to calculate the total energy of the nucleus as a function of deformation. This leads to the prediction of stable deformed nuclear shapes. The total many-body Hamiltonian can be expressed by,

$$
\begin{equation*}
\hat{H}=\sum_{i=1}^{A} t_{i}+\frac{1}{2} \sum_{i, j} V(i, j), \tag{1.16}
\end{equation*}
$$

where $t_{i}$ is the kinetic energy of the $i^{\text {th }}$ nucleon and $V(i, j)$ represents a two body interaction between the $\mathrm{i}^{\text {th }}$ and $\mathrm{j}^{\text {th }}$ nucleons. To calculate which shapes are stable, this Hamiltonian is minimized and it can be shown [Heyde 94] that the total nuclear energy $\mathrm{E}_{0}$ is,

$$
\begin{equation*}
E_{0}(\delta) \equiv<\hat{H}>=\frac{3}{4} \sum_{i=1}^{A} \varepsilon_{i}(\delta), \tag{1.17}
\end{equation*}
$$

where $\varepsilon_{i}(\delta)$ denotes the eigenvalues in the Nilsson potential. A typical example of the behaviour of $\mathrm{E}_{0}$ with $\delta$ is shown in figure 1.4 , where the total energy of a nucleus with a second minimum and that predicted by the liquid-drop model are presented. The calculation


Figure 1.4: Schematic variation of the energy with deformation for a nucleus with a second minimum. The dashed line corresponds to the liquid-drop model barrier. After [Heyde 94].
with the liquid-drop model overlooks fluctuations in the single-particle energies and therefore fails to predict the second minimum. Fluctuations in the single particle-energies give rise to large errors in the binding energies. A method was developed to obtain both the bulk (liquid-drop model) and the local (shell model) effects as a function of deformation, by Strutinsky [Strutinsky 66] [Strutinsky 67] which results in adding a shell-correction term to the liquid-drop energy.

$$
\begin{equation*}
\mathrm{E}=\mathrm{E}_{l d m}+\mathrm{E}_{\mathrm{sc}} \tag{1.18}
\end{equation*}
$$

with,

$$
\begin{equation*}
\mathrm{E}_{s c}=\sum_{\mathrm{i}=1}^{\mathrm{A}} \varepsilon_{\mathrm{i}}(\delta)-\tilde{\mathrm{E}}_{s c} \tag{1.19}
\end{equation*}
$$

The term $\tilde{E}_{s c}$ subtracts the part of the total energy that has already been accounted for by the liquid-drop model, and leaves the shell-model fluctuations. The shell correction term is related to the level density at the Fermi surface [Heyde 94]. The nucleus will be more strongly bound if the level density is small. Strutinsky replaced sharply-defined (delta function) level densities with smooth (Gaussian) descriptions. Areas of low level density away from closed shells have lead to predictions of nuclear deformation, which have been experimentally observed.

## The Cranked Shell Model

So far rotational motion has been described as a purely macroscopic, collective effect but collective motion is built from an underlying microscopic structure. The cranked shell model is a microscopic description of nuclear rotation and can be applied up to the highestspin states. The first discussion of cranking was that by Inglis [Inglis 54] [Inglis 56] and it has since been thoroughly described by Bengtsson and Garrett [Bengtsson 85]. The model provides a means of investigating the effects of Coriolis and centrifugal forces on singleparticle orbits.

If the wavefunction of the nucleus in the intrinsic (non-rotating) frame is $\Psi^{i n t}$, then that in the laboratory (rotating) frame, $\Psi^{l a b}$, is given by,

$$
\begin{equation*}
\Psi^{l a b}=\mathrm{R}(\omega \mathrm{t}) \Psi^{i n t}=\exp \left[\frac{-\mathrm{i} \omega t \mathrm{j}_{\mathrm{x}}}{\hbar}\right] \Psi^{i n t}, \tag{1.20}
\end{equation*}
$$

when the rotation is about the $x$ axis. Substitution of this expression into the timedependent Schrödinger equation and summing over all nucleons leads to the cranking Hamiltonian,

$$
\begin{equation*}
\mathrm{H}^{i n t}=\mathrm{H}^{l a b}-\omega \mathrm{J}_{x}, \tag{1.21}
\end{equation*}
$$

where the superscripts $l a b$ and int represent the laboratory (rotating) and intrinsic (nonrotating) frames, respectively. The eigenvalues, $\mathrm{E}^{\text {lab }}$ of this cranking Hamiltonian are known as Routhians, a term adopted from classical mechanics.

$$
\begin{equation*}
\mathrm{E}^{i n t}=\mathrm{E}^{l a b}-\omega \mathrm{J}_{\boldsymbol{x}} . \tag{1.22}
\end{equation*}
$$

The Routhians are usually plotted as a function of the rotational frequency, $\hbar \omega$, and in such a plot the negative-slope of the Routhian gives the alignment $\mathrm{I}_{\mathrm{x}}$,

$$
\begin{equation*}
\mathrm{I}_{x}=-\frac{\mathrm{dE}}{\mathrm{int}} . \tag{1.23}
\end{equation*}
$$

The $\omega \mathrm{J}_{\mathrm{x}}$ terms describe the Coriolis and centrifugal forces which act to align the singleparticle orbits thus increasing the total nuclear angular momentum on the rotation axis.

If the experimental Routhians are to be compared with the theory, the values must first be separated into collective and single-particle components, and a reference Routhian must be subtracted [Bengtsson 85]. Under rotation the Nilsson quantum numbers become invalid, so the single-particle Routhians for a reflection-symmetric nucleus are labelled with parity $\pi$ and signature $\alpha$, the only remaining good quantum numbers. (For a reflection asymmetric nucleus the Routhians are labelled with simplex, introduced in the next chapter).

## Pairing and Quasi-particles

Paired nucleons in time-reversed orbits will collide twice per orbit, and may scatter into different pairs of time-reversed orbits. In the deformed nucleus the components of a state with angular momentum j are no longer degenerate and the scattering results in the occupation of levels of different energies. This means that the Fermi surface is no longer sharply-defined but becomes smeared. The random scattering into different-energy states means that a state near the Fermi surface may be occupied or it may not. The concept of quasi-particles is introduced, which describe a state in terms of its probability of being occupied by a particle, $V_{j}$, and the probability that it is not $U_{j}$ (equivalent to being occupied by a hole). Far below the Fermi level, $\mathrm{V}_{\mathrm{j}}=1$ and $\mathrm{U}_{\mathrm{j}}=0$. Far above the converse is true and around the Fermi level, the probabilities are mixed.

The introduction of quasi-particles into cranking scheme is known as the Hartree-FockBogolyubov (HFB) formalism, and the cranked HFB Hamiltonian is given by,

$$
\begin{equation*}
\mathrm{H}^{\mathrm{HFB}}=\mathrm{H}^{\mathrm{int}}-\omega \mathrm{J}_{x}-\Delta\left(\mathrm{P}^{+}-\mathrm{P}\right)-\lambda \mathrm{N}, \tag{1.24}
\end{equation*}
$$

where $\Delta$ is known as the pair-gap parameter and is half of the energy needed to break a pair of nucleons in the even-even nucleus, or core. $\mathrm{P}^{+}$and P are the quasi-particle creation and annihilation operators, and the $\lambda \mathrm{N}$ term is included to keep the number of nucleons constant. Calculations made with this Hamiltonian produce quasi-particle Routhians, like those shown in figure 4.23.

## Alignments and Backbends

The pairing of nucleons causes the nuclear matter to behave like a fluid [Bardeen 57] which is the reason why the measured moments of inertia are lower than the rigid-body values.


Figure 1.5: States generated by variou: nuclear potentials. The degeneracies and quantum numbers are given. Taken from [Ejiri 89].

As the nucleus rotates, the paired nucleons in their time-reversed orbits are subjected to the Coriolis force which acts in different directions on each nucleon, acting to split the pair [Mottelson 60]. As the pairs break, the nucleus becomes less fluid, causing changes in the moment of inertia. This effect causes a discontinuity in a plot of moment of inertia against rotational frequency, because $\omega$ remains constant while the moment of inertia increases rapidly. The moment of inertia bends back or bends up, hence the name backbend or upbend [Johnson 72]. In the paired state, the nucleons' angular momenta are of equal magnitude and in opposite directions and therefore cancel each other out. When the pair is broken the two individual nucleon angular momenta add together constructively and their contribution to the total angular momentum is known as aligned angular momentum, $\mathrm{I}_{\mathrm{x}}$. The process of pair-breaking can be called a quasi-particle alignment, and rotational bands can be built on the aligned configuration. Backbending was first interpreted as the crossing, or interaction, of the ground-state band with a band built on an aligned quasi-particle state [Stephens 72]. Quasi-particle alignments often result in structural changes within the nucleus and in extreme cases can cause shape transitions.

A summary of what has been described in this section is given in figure 1.5 , where the s - d states generated by various nuclear potentials are shown.

### 1.2 Aims of this Work

The purpose of the work in this thesis is to study the behaviour of nuclei in which octupole correlations occur. Two experiments have been performed: a comparison of the high-spin behaviour of two nuclei that are known to be octupole deformed; and a lifetime measurement for a nucleus in which the octupole deformation interpretation is disputed. Both experiments used standard experimental techniques.

The first study concerns the isotones ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ which have well-established octupole bands (chapter 4). Cranked HFB calculations have predicted [Nazarewicz 87] that a four quasi-particle aligned band will cross the octupole band at about $\sim 26 \hbar$ in ${ }^{222} \mathrm{Th}$. Observation of this band-crossing would provide further validation of the octupole model.

The ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ nuclei were simultaneously populated using a heavy-ion fusionevaporation reaction in order to reach states with high angular momentum. De-excitation gamma rays were collected with the Eurogam gamma-ray spectrometer which was able to detect high-fold gamma-ray coincidences. The success of such experiments relies on the fact that with high-fold coincidence events, specified gamma-ray coincidences can be demanded before a spectrum is incremented. In demanding coincidences, gamma rays from the background, from other populated nuclei and from other structures within the same nucleus can be reduced. The remaining 'gated' spectrum then contains only gamma rays from a certain structure of interest. This technique is important when studying heavy nuclei like ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ because they have very small cross-sections for production in compound nucleus reactions. Most of the cross-section goes into fission. Gating allows weak structures to be separated from the large fission background.

The second experimental study is a lifetime measurement of some of the excited states in the rare-earth nucleus ${ }^{153} \mathrm{Eu}$. The motivation for the measurement of lifetimes, is to extract electromagnetic transition probabilities and moments (chapter 3), which give information on the collectivity of the nucleus. The level structure forms an apparent parity-doublet band with strong E1 transitions, characteristic of octupole deformation (chapter 2). However a recent high-spin gamma-gamma coincidence study [Pearson 94] has shown the states in the opposite parity components of the parity-doublet bands to have different $g_{\mathrm{K}}$ values (defined in chapter 3) which contradicts the parity-doublet interpretation. The E1 strengths were inferred from $B(E 1) / B(E 2)$ values assuming the quadrupole moment to be constant and
equal to that of the ground state. A measurement of the excited-state quadrupole moments is necessary to check that this assumption is correct. Electric dipole moments and $g_{\mathrm{K}}$ values can also be inferred from the lifetimes and branching ratios. The results of a lifetime study would be complementary to the gamma-gamma coincidence study of [Pearson 94] and would help in determining whether the bands in ${ }^{153} \mathrm{Eu}$ are indeed parity-doublet bands built on an intrinsic mixed state, or if, as suggested in [Pearson 94], the bands are built on accidentally degenerate reflection-symmetric Nilsson states.

Rough approximations of the lifetimes of excited states, based on the previously-measured $\mathrm{B}(\mathrm{E} 2)$ values for the lowest states, predicted that the lifetimes would lie in the range of picoseconds to hundreds of picoseconds. Lifetimes of this order can be measured using the recoil-distance method. Use of Coulomb excitation allows the exact population of all the states to be calculated, and hence the effect of feeding from higher-lying states can be determined. Gamma rays were collected in coincidence with backscattered beam ions because this selects the head on collisions which populate the highest angular-momentum states. The recoil-distance method used in this experiment was slightly different than the standard method, and is described in chapter 5.

### 1.3 Organization of this Thesis

The common theme between the experimental studies described in this thesis, is that of a reflection-asymmetric nuclear shape, and a description of the spectroscopy of 'octupoledeformed' reflection-asymmetric nuclei is given in chapter 2. Chapter 3 is devoted to gamma-ray spectroscopy, covering transition probabilities, measurable properties and detection and data-analysis methods. Also described are the large germanium-detector arrays that have been developed specifically to study rotating nuclei, and the types of reaction that were used in this thesis. Chapters 4 and 5 are self-contained descriptions of the experiments in the form: introduction; experimental details; results; discussion; and conclusion. Chapter 4 describes the high-spin study of ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ and chapter 5 describes the measurement of lifetimes of excited states in ${ }^{153} \mathrm{Eu}$.

## CHAPTER 2

## Pear-shaped Nuclei

The distance from the centre of mass of an axially-symmetric nucleus to its surface can be parametrized in terms of spherical harmonics as,

$$
\begin{equation*}
\mathrm{R}(\theta, \phi)=\mathrm{R}_{0}\left[1+\sum_{\lambda=2}^{\infty} \beta_{\lambda} \mathrm{Y}_{\lambda 0}(\theta, \phi)\right] . \tag{2.1}
\end{equation*}
$$

The $\beta_{\lambda}$ coefficients are used to quantify nuclear deformation. Quadrupole deformation, given by the $\beta_{2}$ parameter, is usually dominant and describes a reflection-symmetric shape, but not all deformed nuclei are spheroidal. A nucleus which has a non-zero $\beta_{3}$ term is octupole deformed and will, to an extent, be reflection asymmetric or pear shaped. This chapter begins with an introduction to the theory and origins of octupole deformation, together with a discussion of the spectroscopic features which are characteristic of an octupoledeformed nucleus, such as simplex bands and parity doublets. This type of deformation only occurs in localized regions of the nuclear chart, and a survey of the most susceptible nuclei is presented together with reasons why this is so. The effects of high-frequency rotation have been shown to reinforce and even induce octupole shapes in some nuclei, and this is discussed. A depiction of which nuclei are octupole deformed ends the chapter.

### 2.1 Introduction

The shape and symmetries of an octupole-deformed nucleus lead to the characteristic spectroscopic features which are listed below:

- Interleaved states of positive and negative parity [Ward 83].
- Near-degenerate states with the same spin and opposite parity in odd-mass nuclei, known as parity doublets [Dahlinger 88].
- Intrinsic electric dipole moments and enhanced electric dipole transitions between the positive- and negative-parity states [Butler 91].
- Intrinsic electric octupole moments, and large associated $\mathrm{B}(\mathrm{E} 3)$ transition probabilities [Ibbotson 93] [Spear 90].
- Enhanced alpha-decay probabilities [Leander 84].
- Inverted or attenuated staggering of the isotope shift [Sheline 88].

The first three of these points were used in this work, while the last three are included for completeness. These features are discussed in section 2.3.

The association of negative-parity states with a reflection-asymmetric nuclear shape comes from an analogy with the spectroscopy of reflection-asymmetric molecules such as $\mathrm{NH}_{3}$ and HCl . (These molecules have rotational bands of alternating positive- and negativeparity states, connected by transitions of the order of 10 meV [Kroto 75] [Dixon 65]). Soon after the collective model of the nucleus was established, low-lying negative-parity states were observed in some light-actinide nuclei, using high-resolution alpha-spectroscopy experiments [Asaro 53] [Stephens 54] [Stephens 55]. The observation of these states, together with the that of strong E1 transitions, led to these states being interpreted as octupole-vibrational excitations [Alder 56]. Early calculations, mainly based on the Nilsson model, were unable to show that any nucleus could possess a stable octupole deformation in its ground state [Lee 57]. The octupole-vibrational model was able to successfully calculate the energies of all the octupole-vibrational states in nuclei with $152 \leq \mathrm{A} \leq 190$ [Neergård 70], but the model proved less successful when it was applied to the actinide region [Neergård 70a] [Neergård 70b]. For example, the model predicted a $0^{+}$two-octupole phonon state at about twice the energy of the $1^{-}$state, which was not observed in any case. (Discounting the model on this premise may have been a mistake, as
the two-octupole phonon states have not been observed in nuclei which are yet thought to be octupole vibrational, such as ${ }^{232} \mathrm{Th}$ ).

Calculations using the Strutinsky shell-correction method [Strutinsky 66] [Strutinsky 67] with a harmonic-oscillator potential, predicted minima in the potential-energy surface at non-zero values of $\beta_{3}$. However these calculations still failed to predict stable octupole deformation in the ground state of any nucleus. The first indication of stable octupole deformation came when parity doublets were predicted to exist for the ground states of some actinium, thorium and protactinium nuclei [Chasman 79] [Chasman 80]. Later in 1981, Möller and Nix [Möller 81] used the Strutinsky shell-correction method, but with a more realistic folded-Yukawa potential [Nix 69] [Nix 72], and they calculated that ${ }^{222} \mathrm{Ra}$ should have an octupole-deformed ground state. They found that including octupole deformation into their potential-energy calculations removed the discrepancies between the calculated and measured nuclear masses in the light-actinide region. Subsequent work by Leander [Leander 82] showed that several nuclei near ${ }^{222} \mathrm{Ra}$ have octupole-deformed groundstate shapes.

Although the existence of octupole deformation has yet to be proved conclusively, there is now much evidence to support it. About 50 nuclei have been shown to display the characteristic alternating positive- and negative-parity bands. The study of these nuclei has been the subject of several review articles and monographs. A complete coverage of all things concerned with reflection-asymmetric shapes including a list of over 700 related references is presented by Butler and Nazarewicz in [Butler 95]. Other review articles include: [Ahmad 93] (octupole shapes); [Åberg 90] (nuclear shapes); [Rohosiński 88] (octupole-vibrational states); and [Leander 88] (odd-mass octupole deformation). The reflection-asymmetric mean-field approach, or the 'octupole model' has proved very successful in describing the properties of these nuclei. However, other models such as the alpha-cluster model [Iachello 82] may equally well describe these nuclei, and may be independent of the octupole model. Any alternative models have yet to make predictions that can be tested, and are therefore not discussed in this thesis.

### 2.2 The Origins of Octupole Deformation

The single-particle level sequence for a harmonic-oscillator potential is shown in figure 2.2. Octupole correlations occur when states with $\Delta \ell=\Delta j=3$ come close enough together to


Figure 2.1: The sequence of levels for a modified harmonic-oscillator potential. The states labelled on the right are the octupole driving states, and the numbers are the nucleon numbers which give rise to the strongest octupole correlations [Nazarewicz 84].
interact. This occurs when the specific single-particle levels are lowered by the $\overrightarrow{\ell^{2}}$ and $\vec{\ell} . \vec{s}$ [Haxel 49] terms, into the shell below. Figure 2.2 illustrates that pairs of states with $\Delta \ell=\Delta \mathrm{j}=3$ come close together just above all the major shell gaps, in four distinct regions. These pairs of octupole-driving states, $\left(j_{\frac{15}{2}}\right.$ and $\left.g_{\frac{8}{2}}\right)$, $\mathrm{i}_{\frac{13}{2}}$ and $\mathrm{f}_{\frac{7}{2}}$ ), ( $\mathrm{h}_{\frac{11}{2}}$ and $\mathrm{d}_{\frac{5}{2}}$ ) and ( $\mathrm{g}_{\frac{9}{2}}$ and $\mathrm{p}_{\frac{3}{2}}$ ), are labelled on the right-hand side of figure 2.2. The strongest octupole correlations arise when the octupole-driving states lie close to the Fermi surface, and so the nuclei most likely to be octupole deformed will have N or Z close to $34,56,90$ or 134 . The best candidates will have both N and Z equal to one of these values. Furthermore, because the interacting levels get closer together as the particle-number increases [Leander 82], the heaviest nuclei have the strongest octupole correlations. For these reasons ${ }_{90}^{224} \mathrm{Th}_{134}$ is predicted to be the 'most octupole-deformed' nucleus, with its close neighbours also exhibiting strong octupole correlations [Sheline 87]. The next-strongest region of octupole correlations is that around ${ }_{56}^{146} \mathrm{Ba}_{90}$ [Nazarewicz 84]. Some other nuclei are also predicted to display weaker octupole effects, such as ${ }_{54}^{112} \mathrm{Xe}_{58}$ and ${ }_{40}^{96} \mathrm{Zr}_{56}$ [Nazarewicz 84].


Figure 2.1: The sequence of levels for a modified harmonic-oscillator potential. The states labelled on the right are the octupole driving states, and the numbers are the nucleon numbers which give rise to the strongest octupole correlations [Nazarewicz 84].
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Figure 2.2: Potential-energy minima calculated with Modified-Oscillator (MO), Woods-Saxon (WS) and folded-Yukawa (FY) potentials, relative to a reflection-symmetric $\beta_{3}=0$ minimum. Taken from reference [Nazarewicz 84].

In the calculations, the strength of the octupole correlations depends on the proximity of the interacting single-particle states and therefore on the choice of potential. While minima tend to stay at the same value of $\beta_{3}$, they are found to be deepest for the folded-Yukawa potential, as shown in figure 2.1 [Nazarewicz 84].

### 2.3 Spectroscopy of Octupole-deformed Nuclei

### 2.3.1 Alternating-parity Bands and Simplex

Pear-shaped nuclei break two symmetries which reflection-symmetric nuclei would possess [Bohr 75]. Both Signature, R, which describes a rotation of $180^{\circ}$ about an axis perpendicular to the symmetry axis and parity, P , which describes space inversion, are broken by the pear shape. The only symmetry remaining is that of simplex
$\hat{S}$ [Goodman 74] [Nazarewicz 85], which is a combination of the two broken symmetries,

$$
\begin{equation*}
\hat{S}=\hat{P} \hat{R}^{-1}, \tag{2.2}
\end{equation*}
$$

and is equivalent to a reflection in a plane containing the symmetry axis. The eigenvalue, s, of the simplex operator, can be used to label the states in an octupole-deformed nucleus. The relationship between simplex, s, parity, p, and spin, I, is,

$$
\begin{equation*}
\mathrm{p}=\mathrm{se} \mathrm{e}^{i \pi \mathrm{I}} \tag{2.3}
\end{equation*}
$$

Therefore, for an even-even nucleus where I has integral values, s can take the values $\pm 1$, corresponding to the spin and parity sequences,

$$
\begin{array}{ll}
s=+1: & \mathrm{J}^{\pi}=0^{+}, 1^{-}, 2^{+}, 3^{-}, 4^{+} \ldots \\
s=-1: & \mathrm{J}^{\pi}=0^{-}, 1^{+}, 2^{-}, 3^{+}, 4^{-} \ldots
\end{array}
$$

For an odd-mass nucleus, with half-integral values of I , s can be $\pm i$, with,

$$
\begin{array}{ll}
s=+i: & \mathrm{J}^{\pi}=\frac{1}{2}^{+}, \frac{3}{2}^{-}, \frac{5}{2}^{+}, \frac{7^{-}}{}{ }^{-}, \frac{9}{2}^{+} \cdots \\
s=-i: & \mathrm{J}^{\pi}=\frac{1}{2}^{-}, \frac{3}{2}^{+}, \frac{5}{2}^{-}, \frac{7}{2}^{+}, \frac{9}{2}^{-} \cdots
\end{array}
$$



Figure 2.3: Schematic representation of the simplex operation. The leaves on the pear-shape are intended to differentiate between rotations about the symmetry axis.

To what extent a nucleus is octupole deformed is something which is difficult to quantify experimentally. Quadrupole moments, which give a direct measure of quadrupole deformation, are relatively easy to measure but octupole moments are more difficult, and can only be measured in a few cases, for example [Ibbotson 93] [White 90] [Wollersheim 93]. Usually
the degree of octupole deformation must be inferred from the nuclear energy-level spectrum. Figure 2.4 shows three different octupole potentials together with their associated level spectra.

| Octupole Vibrational | Intermediate Form | Octupole Deformed |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  | $\begin{aligned} & \text { even - even } \\ & \begin{array}{r} 4+ \\ -3- \\ - \\ =1+ \\ - \\ - \\ - \end{array}+ \end{aligned}$ |
|  | odd - A | odd - A $\qquad$ |

Figure 2.4: Octupole-deformed and octupole-vibrational nuclear potentials together with an intermediate case. Also shown are the level schemes associated with each potential.

The potential on the left applies to a nucleus which has a reflection-symmetric ground-state shape, but undergoes octupole vibrations. There is a $\mathrm{K}^{\pi}=0^{-}$octupole-vibrational band at an excitation energy of about 1 MeV . At the other extreme, on the right, there is an infinite potential barrier between the reflection-asymmetric shape and its mirror image. This nucleus would have a stable ground-state octupole shape. Theoretically, the rotational level-spectrum would be similar to that of a rotating reflection-asymmetric molecule, that is of a band of alternating positive- and negative-parity states with the $+/-$ parity states exactly midway between the adjacent $-/+$ states. In an odd-mass nucleus there would be exactly-degenerate parity-doublet bands. The potential shown in the centre is that
associated with a nucleus which has octupole correlations of an intermediate strength; a finite barrier exists between the shape and its mirror image, and tunnelling is possible between the two shapes. The negative-parity states are displaced upwards in energy.

Whether the low-lying negative-parity states form part of a rotational band, characterized by simplex, or whether they are an independent rotational band based on a $0^{-}$octupolevibrational state has been a subject of much debate since the 1950s. Even in the best cases of octupole deformation in the light actinides, the positive- and negative-parity states only start to interleave as the nucleus is rotated, at about spin $5 \hbar$ or $7 \hbar$. Empirically it appears that rotation acts to stabilize or even induce the octupole deformation, and this is discussed in section 2.4. In no case has a true 'quasi-molecular'1 rotational band been observed in which the energy of the $1^{-}$state is below that of the $2^{+}$state. It is generally believed that there are no nuclei which have permanent ground-state octupole deformation, and even in the most favourable cases, the situation is like that in the centre of figure 2.4

### 2.3.2 Parity Doublets

The positive-parity states in an even-even nucleus occupy single-particle states in pairs but the negative-parity states have at least one broken pair. The difference in pairing energy causes a shift in the energy of the $0^{-}$state relative to the $0^{+}$state. In odd-mass nuclei both positive- and negative-parity states have an unpaired nucleon blocking a level and the two states are equivalent. There is no energy shift, and the opposite parity states can be degenerate. This is the reason, according to Ahmad and Butler [Ahmad 93], that parity doublets exist in odd-mass nuclei.

If there were an infinite barrier between the pear-shape and its mirror image, then the parity-doublet states would be exactly degenerate. With a potential like that shown in the centre of figure 2.4, then there is a finite probability of tunnelling from one shape to the mirror-image and a displacement in energy arises. Since the positive- and negativeparity parts of the parity doublet are based on the same intrinsic orbital, and if the nucleus is sufficiently deformed, rotational bands with similar properties should be built on the parity doublet. Also the intrinsic $g$-factor values, $g_{K}$, (see section 3.2.2) of the parity-mixed band-heads and the states in each band should be very similar.

[^0]

Figure 2.5: The parity-doublet bands in ${ }^{223} \mathrm{Th}$, drawn as two bands of constant simplex, taken from reference [Dahlinger 88].


Figure 2.6: The parity-doublet bands in ${ }^{151} \mathrm{Pm}$, taken from reference [Urban 90].

Although there is some evidence for a parity doublet in the ground state of ${ }^{229} \mathrm{~Pa}$ [Grafen ${ }^{2} 1$ ], parity doublets are more commonly observed at high spin, as apparent parity-doublet bands in nuclei such as ${ }^{221,223,225} \mathrm{Th}$ [Dahlinger 88] [Hughes 90], ${ }^{221,223} \mathrm{Ra}$ [Fernandez-Niello 91], ${ }^{153} \mathrm{Eu}$ [Pearson 94] and ${ }^{151} \mathrm{Pm}$ [Vermeer 90]. More parity-doublet bands have been observed in the actinides than in the lanthanide region. Indeed, in the lanthanide region, no paritydoublet bands have been observed in the odd-neutron nuclei, the reason for which is not understood. Some examples of parity-doublet bands, ${ }^{223} \mathrm{Th}$ and ${ }^{151} \mathrm{Pm}$, are shown in figures 2.5 and 2.6.

### 2.3.3 Electric Dipole Moments

In addition to the alternating positive- and negative-parity states, a spectroscopic characteristic of an octupole-deformed nucleus is the presence of strong E1 transitions between the opposite-parity states [Leander 86]. With experimental strengths of about $10^{-2} \mathrm{~W} .1$. ., the E1 transitions are only a fraction of the single-particle estimate but this is still a few orders of magnitude larger than the E1 strength observed elsewhere on the nuclear chart. The E1 strength has been hypothesised by Bohr and Mottelson [Bohr 57], [Bohr 59], Strutinsky [Strutinsky 56] and Leander [Leander 82] to be due to an intrinsic electric dipole moment, induced by the tendency of protons to gather at the narrow end of the pear shape, where the radius of curvature of the equipotential surface is smallest. This causes the centre of charge to be displaced from the centre of mass giving rise to the dipole moment, as shown
in figure 2.7. The weakness of the Coulomb force, relative to the nuclear force ensures that the dipole moment remains small yet it is large enough to qualitatively account for the E1-transition strength.

A shell correction to this macroscopic effect has an important contribution. The electric dipole moment, $\mathrm{D}_{0}$, can be written as,

$$
\begin{equation*}
\mathrm{D}_{0}=\mathrm{D}_{0}^{\text {macr }}+\mathrm{D}_{0}^{\mathrm{sc}} \tag{2.4}
\end{equation*}
$$

In calculations [Leander 82] [Butler 91], it has been found that the shell-correction term, $\mathrm{D}_{0}^{\text {sc }}$ is about the same magnitude as the macroscopic term, $\mathrm{D}_{0}^{\text {macr }}$ and the two terms can add constructively or destructively. Nuclei with the same shape can therefore have very different dipole moments, and extraction of a $\beta_{3}$ deformation from the dipole moment is not easy. Leander et al. [Leander 82] have applied the macroscopic-microscopic approach and successfully reproduced the electric dipole moments in the actinide region. They used the liquid-drop model to calculate the macroscopic term and a Woods-Saxon single-particle potential to calculate the shell-correction term. This work was extended by Butler and Nazarewicz [Butler 91] who included high-order deformations ( $\leq \beta_{8}$ ) in their calculations and used the droplet model [Dorso 86] instead of the liquid-drop model to calculate the macroscopic term. In addition, they calculated dipole moments for the octupole-deformed lanthanides, and compared their results with the experimental systematics. The calculations using this approach show excellent agreement with the experimental data, and the method can explain the anomalously small dipole moments in ${ }^{224} \mathrm{Ra}$ and ${ }^{146} \mathrm{Ba}$ as a cancellation of the microscopic and macroscopic terms. In an alternative approach, microscopic calculations of the $\mathrm{B}(\mathrm{E} 1)$ rates have been performed for both actinides and lanthanides in references [Egido 89], [Egido 90], [Egido 91] and [Egido 92] and are found to be in very good agreement with the method described above. A compilation of experimental $D_{0}$ values is given in [Butler 95].


Figure 2.7: There is a higher proton density at the pointed end of the pear.

### 2.3.4 Electric Octupole Moments

The electric octupole moment is very collective, and less sensitive to single-particle effects than the electric dipole moment, and is therefore a much better measure of octupole collectivity. Unfortunately, the $\mathrm{B}(\mathrm{E} 3)$ values for high-spin states in octupole-deformed nuclei are difficult to extract. One method that can be applied is that of Coulomb excitation.
> 'E3 matrix elements are the most unambiguous and direct measure of the octupole collective shape degree of freedom. Coulomb excitation is the only viable way of measuring E3 matrix elements since real photon E3 emission is $10^{4}$ times weaker than competing E1 or E2 decay in well-deformed nuclei. The E3 excitation is typically an order of magnitude stronger than the E1 for population of negative-parity states in Coulomb excitation, whereas the E1 transition rate completely dominates the de-excitation gamma-ray decay.' [Cline 93]

Coulomb-excitation probabilities depend on the atomic number, energy and scatteringangle of the projectile, and also on the transition matrix elements. The probabilities can be measured indirectly by measuring de-excitation gamma-ray yields. If several experiments are performed in which different projectiles, energies and scattering angles are used then the transition matrix elements are the only unknown, and can be extracted using the method described in section 3.11. In this manner, the $\mathrm{B}(\mathrm{E} 3)$ rates can be extracted.

### 2.3.5 Alpha-decay Probabilities

Alpha-decay is favoured when the initial and final states have the same intrinsic singleparticle configurations. The alpha-decay rate to the different-parity members of a parity doublet should be almost equal [Leander 84], and larger than between the states in a reflection-symmetric nucleus. The alpha hindrance-factors, the reciprocal of which gives the alpha-decay probability [Preston 47], for alpha decays from ${ }^{231,229} \mathrm{Am} \rightarrow{ }^{227,225} \mathrm{~Pa} \rightarrow{ }^{233,221} \mathrm{Ra}$ show a decrease and, therefore, an increase in octupole collectivity. Enhancements in alphadecay probabilities are a good indication of octupole collectivity.

### 2.3.6 Inverted Differential Isotope Shifts

The measurement of the radii of long chains of isotopes, using collinear laser spectroscopy [Otten 89] [Billowes 95] [Aufmuth 87], has included sequences of barium, caesium, europium, radon, francium and radium isotopes where octupole deformation is expected to occur. The isotope shift $\langle\delta\rangle^{N, N-1}$, which is defined as the radius of a nucleus with
( $\mathrm{N}-1$ ) neutrons subtracted from the radius of the same isotope with N neutrons, usually staggers markedly so that $\langle\delta\rangle^{N, N-1}$ for an even-N isotope is bigger than that for odd-N. This is found to be inverted for nuclei which are octupole deformed in the actinides, for example in the radium isotopes with $133 \leq N \leq 141$. In the lanthanide region, where octupole correlations are weaker, the staggering is not inverted but is wiped-out or attenuated, over the range $86 \leq \mathrm{N} \leq 91$ for the caesium and barium isotopes. This effect and its relationship to octupole deformation [Ahmad 84] [Coc 85] [Sheline 88], is not completely understood. It is of most interest for nuclei where the octupole-deformed interpretation is disputed, such as in the $\mathrm{A} \simeq 153$ europium isotopes [Hühnermann 87].

### 2.4 High-frequency Rotation and Octupole Deformation

Many octupole bands have been observed in the light-actinide and lanthanide regions [Ahmad 93]. In all cases, however, there is a discrepancy between the experimental rotational band and that expected for a reflection-asymmetric rotor, in that the lowest negative-parity states, $\mathrm{I}^{-}$are displaced upwards from their expected positions, midway between the states $(\mathrm{I}-1)^{+}$and $(\mathrm{I}+1)^{+}$. As the nucleus rotates the bands become more like the reflection-asymmetric rotor at spins of about $10 \hbar$. In many of the bands, the displacement between the positive- and negative-parity states is reduced to zero. Much theoretical work has been devoted to the rotation of octupole-shaped nuclei, for example that by Nazarewicz in references [Nazarewicz 84], [Nazarewicz 84a], [Nazarewicz 85], [Nazarewicz 87] and [Nazarewicz 92].

The rotational properties can be conveniently illustrated using two quantities defined by Nazarewicz and Olanders [Nazarewicz 85]. The parity splitting, $\delta E$, is a measure of the energy displacement from the the midpoint of the two adjacent opposite-parity states and is given by,

$$
\begin{equation*}
\delta E=E(I)^{-}-\frac{1}{2} E(I+1)^{+}+E(I-1)^{+} . \tag{2.5}
\end{equation*}
$$

In the limit of stable octupole deformation, $\delta \mathrm{E}(\mathrm{I})$ should be close to zero. Also the rotational-frequency ratio, $R$, defined as,

$$
\begin{equation*}
R=\frac{\omega^{-}(I)}{\omega^{+}(I)}=2 \frac{E(I+1)^{-}-E(I-1)^{-}}{E(I+2)^{+}-E(I-2)^{+}} \tag{2.6}
\end{equation*}
$$

should approach unity, that is the positive- and negative-parity bands should have the same rotational frequencies. For a nucleus which is an octupole vibrator, $R$ has the value
of $(2 \mathrm{I}-5) /(2 \mathrm{I}+1)$. Plots of these quantities for the nuclei studied in this thesis are given in chapters 4 and 5.

The effects of rotation are probably most noticeable in the behaviour of nuclei such as ${ }^{220} \mathrm{Ra}$ and ${ }^{228} \mathrm{Th}$, which are soft with respect to octupole deformation, but have $\beta_{3}=0$ in their ground state. With increasing rotational frequency, their octupole bands develop, and, empirically, it appears that rotation induces and stabilizes the octupole shape in these nuclei. A possible explanation for this, put forward by Nazarewicz [Nazarewicz 87a], is that the single-particle level density decreases with increasing octupole deformation, thus decreasing the pairing correlations and increasing the moment of inertia. Also the rotation will lower the intruder states and may bring the octupole driving orbitals closer together, making the octupole correlation effects stronger. The effects of rotation are of increased importance in the ${ }^{146} \mathrm{Ba}$ region where octupole correlations are weaker than in the actinides.

### 2.5 Regions of Octupole Deformation

Nuclei in the light-actinide and lanthanide regions have the strongest octupole correlations and these regions are shown in figures 2.8 and 2.9 and discussed below. Some other nuclei, such as ${ }^{64} \mathrm{Ge},{ }^{72} \mathrm{Se},{ }^{96} \mathrm{Kr},{ }^{112} \mathrm{Xe}$ and ${ }^{190} \mathrm{Hg}$ are all predicted to exhibit octupole correlations, and there is some experimental evidence for this (see reference [Butler 95] and therein), but they are weaker than those seen in the two main regions.

### 2.5.1 The Light-actinide Region

The octupole-deformed nuclei in the light-actinide region are shown in figure 2.8. This is where the strongest octupole correlations occur. Many of these nuclei are the decay products of natural radioisotopes and were some of the first nuclei whose spectroscopy validated the collective model. For these reasons much theoretical interest has been devoted to these nuclei.

The nuclei in which an alternating-parity band has been observed are those to which a reference has been assigned in figure 2.8. Most of the nuclei around ${ }^{220} \mathrm{Ra}$ are populated using beams such as ${ }^{14} \mathrm{C}$ or ${ }^{18} \mathrm{O}$ on ${ }^{208} \mathrm{~Pb}$ or ${ }^{209} \mathrm{Bi}$ targets. The thoriums with $\mathrm{A}>223$ are populated using ( $\alpha, \mathrm{xn}$ ) on a ${ }^{226} \mathrm{Ra}$ target ( $\mathrm{t}_{\frac{1}{2}}=1600$ years). The lack of stable targets above ${ }^{209} \mathrm{Bi}$ means that the rest of the octupole-deformed nuclei (shaded grey) are very difficult to access. The cross-section for the ${ }^{208} \mathrm{~Pb}\left({ }^{18} \mathrm{O}, 4 \mathrm{n}\right){ }^{222} \mathrm{Th}$ reaction is of the order of tens of millibarns, and
such reactions can be used successfully to study these nuclei, if efficient detection systems are employed. With the available beams and targets, the production cross-sections for nuclei further to the top right of figure 2.8 become very small: ${ }^{208} \mathrm{~Pb}\left({ }^{20} \mathrm{Ne}, 4 \mathrm{n}\right){ }^{224} \mathrm{U}$ has a crosssection of 800 nanobarns [Yeremin 94]. In order to study such nuclei, channel-selection techniques must be employed or alternative methods of population must be used.

The predicted octupole bands in the radium and thorium nuclei were observed in the references given on figure 2.8 and those therein. It was a logical progression to trace the behaviour of these bands to higher spins [Nazarewicz 87]: the top-left corners of the radium and thorium nuclei with $218 \leq \mathrm{A} \leq 228$ indicate the results of this investigation. They can be divided into four groups. The white-cornered nuclei have only weak quadrupole deformation in their ground state. An octupole band develops with rotation but it remains very irregular. Striped corners indicate a nucleus with a spherical ground state, which becomes octupole and quadrupole deformed as it rotates. Black-cornered nuclei have octupole-deformed ground states that persist to intermediate spin values, and the greycornered nuclei only show octupole vibrations about a reflection-symmetric shape. These predictions are the basis of the experimental study recounted in chapter 4.

### 2.5.2 The Lanthanide Region

Compared with the actinides, the interpretation of lanthanide nuclei in terms of reflection asymmetry is a relatively recent idea [Phillips 86]. The lanthanide region of octupole deformation is shown on figure 2.9. The most octupole-deformed lanthanides, close to ${ }^{146} \mathrm{Ba}$, are too neutron rich to be populated via compound-nucleus reactions, but have been very successfully populated as fragments from the spontaneous fission of ${ }^{252} \mathrm{Cf}$ and ${ }^{248} \mathrm{Cf}$ by Phillips et al. [Phillips 86] [Phillips 88]. Moving towards the line of $\beta$-stability, the nuclei in this region can be populated with compound-nucleus reactions or by Coulomb excitation. References are given on figure 2.9.

Experimental studies indicate that the region of octupole deformation may be more extensive than that predicted. Several nuclei around ${ }^{153} \mathrm{Eu}$ have been shown to display level structures that can be interpreted as parity-doublet bands. This interpretation is, however, open to question, and is the subject matter of chapter 5 of this thesis.

Figure 2.8: The region of octupole deformation in the actinides. The shaded area is the region of octupole deformation defined by Sheline [Sheline 87], Nazarewicz [Nazarewicz 84] and Cwiok and Nazarewicz [Cwiok 87]. The triangle in the top-left hand corner of some radium and thorium nuclei, refers to the calculations of [Nazarewicz 87] and is described in the text. CN indicates population by a compound-nucleus reaction; CE - Coulomb excitation; NT - nucleon transfer. Also given is the reference of the most recent experimental study, for the nuclei which have alternating-parity bands.


Figure 2.9: The region of octupole deformation in the lanthanides. The shaded nuclei are stable $\left(\mathrm{t}_{\frac{1}{2}} \geq 10^{9}\right.$ years). The top-left corners indicate the predictions of Möller, Nix, Myers and Swiatecki [Möller 95] and Nazarewicz [Nazarewicz 92]: white, indicates octupole deformation at high spin, black in the ground state and grey in both the ground state and at high spin. CN indicates population by a compound-nucleus reaction; CE - Coulomb excitation; SF - spontaneous fission. Also given is the reference of the most recent experimental study, for the nuclei which have alternating-parity bands. Black squares indicate the nuclei that are not predicted to be octupole deformed but for which there is some experimental evidence [Afanasjev 95].

## CHAPTER 3

## Gamma-ray Spectroscopy

Gamma-ray spectroscopy has become one of the most useful types of nuclear spectroscopy, due to the ease of detection of highly-penetrating gamma radiation. Excited nuclear states often decay to ground by the emission of one or more gamma rays. Measurement of the gamma rays' properties such as their energies, intensities, coincidence relationships, relative and absolute transition rates, and angular distributions, can reveal much information about excited states and, therefore, about nuclear structure.

The importance of gamma-ray data has lead to the development of high-resolution germanium detectors, which have become the principal tool in most gamma-ray spectroscopy experiments. Subsequently, large arrays of such detectors have been developed to increase sensitivity. To complement advances in gamma-ray detection and hardware, sophisticated analysis techniques in the computing and software area have been developed, such as multidimensional data analysis.

The following chapter covers the subjects mentioned above and introduces some important concepts. The chapter begins with a description of gamma rays as electromagnetic radiation, and describes measurable properties and gamma-ray spectroscopic techniques. A brief outline of large state-of-the-art germanium-detector arrays and analysis methods are also given. The chapter ends with a description of the population mechanisms that are used in the experiments of this thesis; heavy-ion fusion-evaporation (chapter 4) and Coulomb excitation (chapter 5).

### 3.1 Introduction

Gamma rays are a type of electromagnetic radiation with wavelengths typically a million times smaller than visible light. When gamma rays are emitted between excited states in a nucleus, the recoil momentum given to the nucleus is usually about 1 part in $10^{5}$ and can be ignored. This approximation means that the gamma-ray energy corresponds to the difference in the energies of the nuclear states involved in the transition, usually between 0.1 and 10 MeV . A consideration of vector coupling gives the angular-momentum selection rule for gamma-ray emission,

$$
\begin{equation*}
\left|\mathrm{I}_{\mathrm{i}}-\mathrm{I}_{\mathrm{f}}\right| \leq \mathrm{L} \leq\left(\mathrm{I}_{\mathrm{i}}+\mathrm{I}_{\mathrm{f}}\right) \quad \forall \quad \mathrm{L} \neq 0, \tag{3.1}
\end{equation*}
$$

where $\mathrm{I}_{i}$ and $\mathrm{I}_{f}$ are the angular momenta of the initial and final states, respectively, and L is the multipolarity of the radiation. L cannot equal 0 because the photon has an intrinsic spin of 1 . If the gamma ray carries the difference in angular momenta of the initial and final states then the transition is said to be stretched. In addition, the parity selection rule states that,

$$
\begin{equation*}
\pi_{i} \pi_{f}=\pi_{L}, \tag{3.2}
\end{equation*}
$$

where $\pi_{i}, \pi_{f}$ and $\pi_{L}$ are the parities of the initial and final states and of the gamma ray involved, respectively. Even-multipole electric and odd-multipole magnetic transitions have $\pi_{L}=+1$, and odd-multipole electric and even-multipole magnetic transitions have $\pi_{L}=-1$.

### 3.2 Electromagnetic Moments and Transition Probabilities

Protons are charged, and moving charge constitutes an electric current. The charge and current distributions within the nucleus are described by the electric and magnetic multipole moments, respectively. When the nucleus emits electromagnetic radiation in the form of gamma rays, the radiation field can be expanded into various components depending on the multipole moments. Such a multipole expansion is performed in several references such as [Blatt 52], [Davydov 65], [De Shalit 74] and [Ring 80]. Considering the behaviour of the multipole moments in the presence of an external field, allows some expressions to be deduced relating the multipole moments to the transition rates between states. Applying Maxwell's equations and some angular-momentum algebra, and recognising that the electromagnetic moments are the expectation values of the multipole operators, the resulting
terms describing the charge-distributions lead to the electric multipoles, $Q_{\lambda \mu}$,

$$
Q_{\lambda \mu}=<I_{i} m_{i}\left|\hat{Q}_{\lambda \mu}\right| I_{f} m_{f}>=(-1)^{I_{i}-m_{i}}\left(\begin{array}{ccc}
I_{i} & \lambda & I_{f}  \tag{3.3}\\
-m_{i} & \mu & m_{f}
\end{array}\right)<I_{i}\left\|\hat{Q}_{\lambda}\right\| I_{f}>
$$

while those describing current-distributions lead to the magnetic multipoles, $M_{\lambda \mu}$

$$
M_{\lambda \mu}=<I_{i} m_{i}\left|\hat{M}_{\lambda \mu}\right| I_{f} m_{f}>=(-1)^{I_{i}-m_{i}}\left(\begin{array}{ccc}
I_{i} & \lambda & I_{f}  \tag{3.4}\\
-m_{i} & \mu & m_{f}
\end{array}\right)<I_{i}\left\|\hat{M}_{\lambda}\right\| I_{f}>
$$

The interaction between the electromagnetic field and the nucleus will cause a change in energy of a nuclear state. If the interaction is represented by $H_{\text {int }}$, then it will cause a transition from a state $\mid i>$ to a state $\mid f>$ at a rate,

$$
\begin{equation*}
T_{f i}=\frac{2 \pi}{\hbar}|<f| H_{\mathrm{int}}|i>|^{2} g\left(E_{f}\right) \tag{3.5}
\end{equation*}
$$

where $g\left(E_{f}\right)$ is the density of states at energy $E_{f}$. Substituting appropriate expressions for $g\left(E_{f}\right)$ and $H_{\text {int }}$, it can be shown [Ring 80] that,

$$
\begin{equation*}
T_{f i}=\frac{8 \pi(\lambda+1)}{\hbar \cdot \lambda .((2 \lambda+1)!!)^{2}}\left(\frac{E_{\gamma}}{\hbar c}\right)^{2 \lambda+1}|<f| M(\sigma \lambda, \mu)|i>|^{2}, \tag{3.6}
\end{equation*}
$$

where $M(\sigma \lambda)$ is a general multipole transition operator. In the electric case,

$$
\begin{equation*}
M(E \lambda, \mu)=\frac{(2 \lambda+1)!!}{k^{\lambda}(\lambda+1)} \int \mathbf{j}(\mathrm{r}) . \nabla \times(\mathrm{r} \times \nabla)\left\{\mathrm{j}_{\lambda}(\mathrm{kr}) \mathrm{Y}_{\lambda \mu}(\theta, \phi)\right\} \mathrm{d} \tau \tag{3.7}
\end{equation*}
$$

and in the magnetic case

$$
\begin{equation*}
M(M \lambda, \mu)=\frac{-(2 \lambda+1)!!}{c k^{\lambda}(\lambda+1)} \int \mathrm{j}(\mathrm{r}) \cdot(\mathrm{r} \times \nabla)\left\{\mathrm{j}_{\lambda}(\mathrm{kr}) \mathrm{Y}_{\lambda \mu}(\theta, \phi)\right\} \mathrm{d} \tau \tag{3.8}
\end{equation*}
$$

$\mathbf{j}(\mathbf{r})$ is the current density, not to be confused with the spherical Bessel functions $\mathrm{j}_{\lambda}$. In most discrete line gamma-ray spectroscopy applications, the wavelengths of the gamma rays are long compared with the dimensions of the nucleus, that is $\mathrm{kR}_{0} \ll 1$. In this case the spherical Bessel functions, $\mathrm{j}_{\lambda}$, are simplified [Ring 80] and the operators can be expressed as

$$
\begin{align*}
M(E \lambda, \mu) & =\int \rho(r) r^{\lambda} Y_{\lambda \mu} d \tau,  \tag{3.9}\\
M(M \lambda, \mu) & =\frac{1}{c(\lambda+1)} \int(\mathbf{r} \times \mathbf{j}) \cdot \nabla\left(\mathrm{r}^{\lambda} Y_{\lambda \mu}\right) \mathrm{d} \tau \tag{3.10}
\end{align*}
$$

where $\rho(\mathrm{r})$ is the charge density. Usually the different orientations of angular momentum are not distinguished so to obtain a total transition probability, an arithmetic average of the initial $m$-state values is taken and a sum over the final $m$-state values,

$$
\begin{equation*}
T_{f i}=\frac{1}{2 I_{i}+1} \sum_{m_{i}, m_{f}, \mu} T_{f i}(\sigma \lambda, \mu) . \quad[\sigma \equiv E, M] \tag{3.11}
\end{equation*}
$$

Substitution of equation 3.3 or 3.4 into equations 3.6 and 3.11 and applying the WignerEckhart theorem, which extracts the directional properties from the matrix element as a constant multiplicative factor, the total transition probability can be expressed as,

$$
\begin{equation*}
T_{f i}=\frac{8 \pi(\lambda+1)}{\hbar . \lambda .((2 \lambda+1)!!)^{2}}\left(\frac{E_{\gamma}}{\hbar c}\right)^{2 \lambda+1} B\left(\sigma \lambda, I_{i} \rightarrow I_{f}\right) \tag{3.12}
\end{equation*}
$$

And,

$$
\begin{align*}
B\left(E \lambda, I_{i} \rightarrow I_{f}\right) & =\frac{1}{2 I_{i}+1}|<f\|\hat{Q}\| i>|^{2}  \tag{3.13}\\
B\left(M \lambda, I_{i} \rightarrow I_{f}\right) & =\frac{1}{2 I_{i}+1}|<f\|\hat{M}\| i>|^{2} \tag{3.14}
\end{align*}
$$

The $\mathrm{B}(\sigma \lambda)$ values are known as the reduced transition probabilities and contain information about the nuclear wavefunctions while the other parts of equation 3.12 are kinematic factors.

### 3.2.1 Single-particle Transition Rates

Simple estimates of the transition rates have been calculated for the case where the transition is assumed to be due to a single proton changing from one shell model state to another [Ring 80] [Krane 88]. They can be written as,

$$
\begin{align*}
T(E \lambda) & =\frac{8 \pi(\lambda+1)}{\lambda[(2 \lambda+1)!!]^{2}} \frac{e^{2}}{4 \pi \varepsilon_{0} \hbar c}\left(\frac{E}{\hbar c}\right)^{2 L+1}\left(\frac{3}{\lambda+3}\right)^{2} c R^{2 \lambda}  \tag{3.15}\\
T(M \lambda) & =\frac{8 \pi(\lambda+1)}{\lambda[(2 \lambda+1)!!]^{2}}\left(\mu_{p}-\frac{1}{\lambda+1}\right)^{2}\left(\frac{\hbar}{m_{p} c}\right)^{2}\left(\frac{s e^{2}}{4 \pi \varepsilon_{0} \hbar c}\right)\left(\frac{E}{\hbar c}\right)^{2 \lambda+1}\left(\frac{3}{\lambda+2}\right)^{2} c R^{2 \lambda-2}
\end{align*}
$$

where $\mu_{p}$ and $m_{p}$ are the magnetic moment and the mass of the proton, respectively, $E$ is the gamma-ray energy and the other terms are in standard notation. With $R=R_{0} A^{\frac{1}{3}}$ some estimates can be made for the transition rates of the lower multipole orders, which are given in table 3.1. These are known as the Weisskopf estimates and are not meant to be accurate calculations, merely estimates to provide reasonable comparisons of transition rates. They are often used as the units of real collective transition rates.

| $\mathrm{T}(\mathrm{E} 1)=1.0 \times 10^{14} A^{\frac{2}{3}} E^{3}$ | $\mathrm{~T}(\mathrm{M} 1)=5.6 \times 10^{13} E^{3}$ |
| :--- | :--- |
| $\mathrm{~T}(\mathrm{E} 2)=7.3 \times 10^{7} A^{\frac{4}{3}} E^{5}$ | $\mathrm{~T}(\mathrm{M} 2)=3.5 \times 10^{7} A^{\frac{2}{3}} E^{5}$ |
| $\mathrm{~T}(\mathrm{E} 3)=3.4 \times 10^{1} A^{2} E^{7}$ | $\mathrm{~T}(\mathrm{M} 3)=1.6 \times 10^{1} A^{\frac{4}{3}} E^{7}$ |
| $\mathrm{~T}(\mathrm{E} 4)=1.1 \times 10^{-5} A^{\frac{8}{3}} E^{9}$ | $\mathrm{~T}(\mathrm{M} 4)=4.5 \times 10^{-6} A^{2} E^{9}$ |

Table 3.1: Weisskopf Estimates. The $T(\sigma \lambda)$ values are in seconds ${ }^{-1}$ when $E$ is expressed in MeV .

### 3.2.2 Collective Transition Rates

The $\mathrm{B}(\lambda \mathrm{L})$ transition probabilities have been calculated in terms of rotating multipoles by Bohr and Mottelson [Bohr 75] and are given as ,

$$
\begin{equation*}
B\left(E 2 ; I_{i} \rightarrow I_{f}\right)=\frac{5}{16 \pi} e^{2} Q_{0}^{2}\left\langle I_{i} K_{i} 20 \mid I_{f} K_{f}\right\rangle^{2} \tag{3.16}
\end{equation*}
$$

and,

$$
\begin{equation*}
\left.B\left(M 1 ; I_{i} \rightarrow I_{f}\right)=\frac{3}{4 \pi}\left(g_{K}-g_{R}\right)^{2}<I_{i} K_{i} 10 \right\rvert\, I_{f} K_{f}>^{2} \tag{3.17}
\end{equation*}
$$

where K is the projection of total nuclear angular momentum on the symmetry axis, and $\mathrm{Q}_{0}$ is the intrinsic quadrupole moment related to the measured quadrupole moment, Q, by,

$$
\begin{equation*}
Q=Q_{0} \frac{3 K^{2}-I(I+1)}{(I+1)(2 I+3)} . \tag{3.18}
\end{equation*}
$$

The difference in the intrinsic and rotational gyromagnetic ratios, $\left(g_{K}-g_{R}\right)$, is related to the magnetic moment by,

$$
\begin{equation*}
\mu(I)=\left[g_{R} I+\left(g_{K}-g_{R}\right) \frac{K^{2}}{I+1}\right] \mu_{n} \tag{3.19}
\end{equation*}
$$

In a nucleus which is reflection asymmetric, an intrinsic electric dipole moment, $D_{0}$ may arise (section 2.3.3). Values for the $\mathrm{B}(\mathrm{E} 1)$ reduced transition probabilities for a rotating electric dipole have been calculated [Bohr 75] to be,

$$
\begin{equation*}
B\left(E 1 ; I_{i} \rightarrow I_{f}\right)=\frac{3}{4 \pi} e^{2} D_{0}^{2}<I_{i} K_{i} 10\left|I_{f} K_{f}\right\rangle^{2} \tag{3.20}
\end{equation*}
$$

In the work described in this thesis, use was made of equations 3.3 and 3.4 in the forms,

$$
\begin{align*}
& \langle f\|E \lambda\| i\rangle=\sqrt{2 I_{i}+1} Q_{\lambda} a_{\lambda}\left\langle I_{i} K_{i} \lambda 0 \mid I_{f} K_{f}\right\rangle  \tag{3.21}\\
& \langle f\|M 1\| i\rangle=\sqrt{2 I_{i}+1}\left(g_{K}-g_{R}\right) K \sqrt{\frac{3}{4 \pi}}\left\langle I_{i} K_{i} 10 \mid I_{f} K_{f}\right\rangle \tag{3.22}
\end{align*}
$$

In these equations $Q_{\lambda}$ is the appropriate electric multipole moment, and,

$$
\begin{equation*}
a_{\lambda \neq 1}=\sqrt{(2 \lambda+1) /(16 \pi)} ; \quad a_{1}=\sqrt{3 / 4 \pi} \tag{3.23}
\end{equation*}
$$

Multiplying out the constant terms in the equation 3.12 yields the estimates given in table 3.2.

| $\mathrm{T}(\mathrm{E} 1)=1.59 \times 10^{15} B(E 1) E^{3}$ | $\mathrm{~T}(\mathrm{M} 1)=1.76 \times 10^{13} B(M 1) E^{3}$ |
| :--- | :--- |
| $\mathrm{~T}(\mathrm{E} 2)=1.22 \times 10^{9} B(E 2) E^{5}$ | $\mathrm{~T}(\mathrm{M} 2)=1.35 \times 10^{7} B(M 2) E^{5}$ |
| $\mathrm{~T}(\mathrm{E} 3)=5.67 \times 10^{2} B(E 3) E^{7}$ | $\mathrm{~T}(\mathrm{M} 3)=6.28 \times 10^{0} B(M 3) E^{7}$ |
| $\mathrm{~T}(\mathrm{E} 4)=1.69 \times 10^{-4} B(E 4) E^{9}$ | $\mathrm{~T}(\mathrm{M} 4)=1.87 \times 10^{-6} B(M 4) E^{9}$ |

Table 3.2: Transition rates expressed in terms of the reduced transition probabilities. The transition rates have units of seconds ${ }^{-1}$, the $\mathrm{B}(\mathrm{E} \lambda)$ are in $\mathrm{e}^{2} \mathrm{fm}^{2 \lambda}$, the $\mathrm{B}(\mathrm{M} \lambda)$ are in $\mu_{n}^{2} \mathrm{fm}^{2 \lambda-2}$ and E in MeV . Taken from [Ring 80].

### 3.3 Angular Distributions and Directional Correlations

A measurement of the probability with which a gamma ray is emitted in a specific direction can provide information on the gamma-ray multipolarity and the multipole mixing ratio for the transition. Many comprehensive publications concerning angular distributions and correlations are available by, for example, Gill [Gill 75], Ferguson [Ferguson 65], Frauenfelder and Steffen [Frauenfelder 65] and Rose and Brink [Rose 67].

### 3.3.1 The Angular Distribution of Gamma Rays

To observe an anisotropic gamma-ray distribution the decaying level must not have a uniform m-state population; it must be oriented ${ }^{1}$. The probability of gamma-ray emission at an angle $\theta$ with respect to the quantization axis is then,

$$
\begin{equation*}
W(\theta)=\sum_{k \text { even }} A_{k} P_{k}(\cos \theta) \tag{3.24}
\end{equation*}
$$

The $A_{k}$ coefficients are composed of two terms: those which describe the m-state population called the statistical tensors, $\rho_{\mathrm{k}}$; and those which depend on the initial and final spins of the states involved, $\mathrm{J}_{\mathrm{i}}$ and $\mathrm{J}_{f}$, and also on the competing multipolarities in a mixed transition, $L_{1}$ and $L_{2}$. They are written as,

$$
\begin{equation*}
\mathrm{A}_{\mathrm{k}}\left(\mathrm{~J}_{\mathrm{i}} \mathrm{~L}_{1} \mathrm{~L}_{2} \mathrm{~J}_{\mathrm{f}}\right)=\frac{\rho_{\mathrm{k}}(\mathrm{~J})}{1+\delta^{2}}\left[\mathrm{~F}_{\mathrm{k}}\left(\mathrm{~J}_{\mathrm{f}} \mathrm{~L}_{1} \mathrm{~L}_{1} \mathrm{~J}_{\mathfrak{i}}\right)+2 \delta \mathrm{~F}_{\mathrm{k}}\left(\mathrm{~J}_{\mathrm{f}} \mathrm{~L}_{1} \mathrm{~L}_{2} \mathrm{~J}_{\mathrm{i}}\right)+\delta^{2} \mathrm{~F}_{\mathrm{k}}\left(\mathrm{~J}_{\mathrm{f}} \mathrm{~L}_{2} \mathrm{~L}_{2} \mathrm{~J}_{\mathrm{i}}\right)\right] \tag{3.25}
\end{equation*}
$$

where $\delta$ is the multipole mixing ratio, and $\delta^{2}$ gives the ratio of the intensities of the competing multipolarities in a mixed transition. The $F_{k}$ coefficients are,

$$
\begin{equation*}
\mathrm{F}_{\mathrm{k}}\left(\mathrm{~J}_{\mathrm{f}} \mathrm{~L}_{1} \mathrm{~L}_{2} \mathrm{~J}_{\mathrm{i}}\right)=(-1)^{\mathrm{J}_{\mathrm{f}}-\mathrm{J}_{\mathrm{i}}-1}\left[\left(2 \mathrm{~L}_{1}+1\right)\left(2 \mathrm{~L}_{2}+1\right)\left(2 \mathrm{~J}_{\mathrm{i}}+1\right)\right]^{\frac{1}{2}}\left\langle\mathrm{~L}_{1} 1 \mathrm{~L}_{2}-1\right| \mathrm{k} 0>\mathrm{W}\left(\mathrm{~J}_{\mathrm{i}} \mathrm{~J}_{\mathrm{f}} \mathrm{~L}_{1} \mathrm{~L}_{2}: k \mathrm{~kJ}_{\mathrm{f}}\right), \tag{3.26}
\end{equation*}
$$

[^1]where the last two terms are Clebsch-Gordan and Racah coefficients, respectively. The statistical tensors for complete alignment [Yamazaki 67] may be written as,
\[

$$
\begin{equation*}
\left.\rho_{\mathrm{k}}(\mathrm{~J})=\mathrm{B}_{\mathrm{k}}(\mathrm{~J})=(2 \mathrm{~J}+1)^{\frac{1}{2}}(-1)^{\mathrm{J}}<\mathrm{J} 0 \mathrm{~J} 0 \right\rvert\, \mathrm{k} 0> \tag{3.27}
\end{equation*}
$$

\]

for integer spin values, and,

$$
\begin{equation*}
\rho_{\mathrm{k}}(\mathrm{~J})=\mathrm{B}_{\mathrm{k}}(\mathrm{~J})=(2 \mathrm{~J}+1)^{\frac{1}{2}}(-1)^{\mathrm{J}-\frac{1}{2}}<\mathrm{J} \frac{1}{2} \mathrm{~J} \frac{1}{2}|\mathrm{k} 0\rangle \tag{3.28}
\end{equation*}
$$

for half-integer spins. Usually the alignment is only partial and,

$$
\begin{equation*}
\mathrm{A}_{\mathrm{k}}\left(\mathrm{~J}_{\mathrm{i}} \mathrm{~L}_{1} \mathrm{~L}_{2} \mathrm{~J}_{\mathrm{f}}\right)=\alpha_{\mathrm{k}}\left(\mathrm{~J}_{\mathrm{i}}\right) \mathrm{A}_{\mathrm{k}}^{\max }\left(\mathrm{J}_{\mathrm{i}} \mathrm{~L}_{1} \mathrm{~L}_{2} \mathrm{~J}_{\mathrm{f}}\right) \tag{3.29}
\end{equation*}
$$

where $A_{k}^{\max }$ is the A coefficient for complete alignment and $\alpha_{k}=\rho_{k}\left(\mathrm{~J}_{\mathrm{i}}\right) / \mathrm{B}_{\mathrm{k}}\left(\mathrm{J}_{\mathbf{i}}\right)$. The degree of alignment depends on the method of population of the state. The $\mathrm{F}_{\mathrm{k}}$ coefficients and the statistical tensors for complete alignment are tabulated in [Yamazaki 67].

Experimentally, equation 3.24 describing a simple angular distribution must be modified, so that,

$$
\begin{equation*}
W(\theta)=\sum_{k \text { even }} A_{k} P_{k}(\cos \theta) Q_{k} G_{k} \tag{3.30}
\end{equation*}
$$

The $G_{k}$ coefficients account for the destruction of the alignment and, therefore, of the gamma-ray angular distribution, by extra-nuclear electromagnetic fields. The large hyperfine field due to rapidly-changing highly-excited electronic configurations as a recoiling nucleus flies through a vacuum is one possible source of such a field. This is discussed in more detail in chapter 5.

The $\mathrm{Q}_{\mathrm{k}}$ coefficients are the solid angle correction factors which correct for the finite size of the gamma-ray detectors [Yates 66]. The $Q_{k}$ value would be equal to 1 for a point-like detector and would be equal to 0 for a detector covering $4 \pi$. The $Q_{k}$ are obtained by integrating the Legendre polynomials, weighted by the gamma-ray detector efficiency, over the solid angle of the detector.

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{k}}=\frac{\mathrm{J}_{\mathrm{k}}}{\mathrm{~J}_{0}} \quad \text { where } \quad \mathrm{J}_{\mathrm{k}}=\int_{\theta=0}^{\theta=\pi} \varepsilon(\theta, \mathrm{E}) \mathrm{P}_{\mathrm{k}}(\cos \theta) \mathrm{d} \cos \theta \tag{3.31}
\end{equation*}
$$

and E is the gamma-ray energy.


Figure 3.1: A simple example of an angular correlation is given in the text. This figure shows the gamma-ray transitions and notation used in the example.

### 3.3.2 Angular Correlations and the DCO Method

A method often used to orient nuclear states is to observe a preceding gamma ray. This method is useful because it involves no extraneous equipment apart from a second gammaray detector. If the direction of the preceding transition is taken to be the quantization axis, then this places restrictions on the allowed transitions between magnetic substates and, therefore, biases the m-state population of the final state. The final state is then the initial state for a second transition. This is perhaps easiest to understand with the help of the simple example shown in figure 3.1, where observation of the transition $\gamma_{A B}$ orients the state B and therefore causes $\gamma_{B C}$ to be emitted with an intensity dependent upon the angle of observation, $\theta_{B C}$.

In the transition from state $\mathrm{A} \rightarrow \mathrm{B}$, the components $\mathrm{m}_{A}=0 \rightarrow \mathrm{~m}_{B}= \pm 1$ have intensities proportional to $\left[1+\cos ^{2}\left(\theta_{A B}\right)\right]$, while those from $\mathrm{m}_{A}=0 \rightarrow \mathrm{~m}_{B}=0$ are proportional to $\sin ^{2}\left(\theta_{A B}\right)$ [Krane 88], where $\theta_{A B}$ is the angle at which $\gamma_{A B}$ is observed. Since the quantization axis is defined by the direction of $\gamma_{A B}$, then $\theta_{A B}=0$ and the intensity of the transition $m_{A}=0 \rightarrow \mathrm{~m}_{B}=0$, which is proportional to $\sin ^{2} \theta_{A B}$, must also be zero. Hence when $\gamma_{B C}$ is observed after $\gamma_{A B}$ then the distribution of $\gamma_{B C}$ will be anisotropic. This type of measurement is known as an angular correlation. Usually, however, the situation is much more complicated.

The angular distribution from an oriented state or the angular correlation of an unoriented state can give information on the multipolarities of pure transitions, but it is difficult, if not impossible, to extract the multipole mixing ratio in cases where there is more than
one multipolarity present [Krane 73]. In such cases, the mixing ratio can be obtained by measuring the angular correlation of two gamma rays from an oriented state, or by observing the triple angular correlation from an unoriented state, where observation of the first gamma ray of the triple correlation defines the quantization axis and causes orientation, as in the simple example above. The theory of directional correlations from oriented states (DCO) and triple angular correlations can be complicated, and is dealt with in several references such as [Biedenharn 53], [Ferguson 65] and [Watson 67]. Practical aspects, such as detector geometries, are given in [Krane 73].

### 3.4 Internal Conversion

Internal conversion is an electromagnetic process which competes with gamma-ray emission, in which the nucleus imparts its excitation energy to an atomic electron as kinetic energy. The process knocks the atomic electron out of its orbit around the nucleus. The kinetic energy given to the electron $T$, will be the difference between the transition energy ( $\mathrm{E}_{i}-\mathrm{E}_{f}$ ) and the electron binding energy $\mathrm{B}_{e}$,

$$
\begin{equation*}
T=\left(E_{i}-E_{f}\right)-B_{e} . \tag{3.32}
\end{equation*}
$$

The probability of internal conversion relative to gamma-ray emission is given by the internal conversion coefficient, $\alpha$. The total transition rate, $\lambda_{t}$ is the sum of the gamma-ray emission rate, $\lambda_{\gamma}$, and internal-conversion rate, $\lambda_{e}$, and so,

$$
\begin{equation*}
\lambda_{t}=\lambda_{\gamma}+\lambda_{e}=\lambda_{\gamma}(1+\alpha) . \tag{3.33}
\end{equation*}
$$

The internal-conversion coefficients, $\alpha$, depend upon the atomic number, Z , the transition energy, $\mathrm{E}_{\gamma}$ and the electron shell given by the quantum number n (where $\mathrm{n}=1,2,3$, 4 corresponds to the K, L, M, N electron shells). The approximate dependence on these quantities is,

$$
\begin{equation*}
\alpha \propto \frac{Z^{3}}{n^{3} E_{\gamma}^{2.5}} \tag{3.34}
\end{equation*}
$$

The coefficients are larger for magnetic than electric transitions and increase with increasing multipolarity. Hence low-energy magnetic transitions, with high multipolarity in heavy nuclei internally-convert the most, with the largest contribution coming from the K-shell electrons.

Conversion-electron spectroscopy has advantages over gamma-ray spectroscopy in that there are no Compton-scattering effects (see section 3.6.1) which can cause large backgrounds in
gamma-ray spectra. The conversion coefficients are sensitive to multipolarity and so their measurement can help to make spin assignments. Conversion-electron spectroscopy also offers the possibility of studying the $0^{+} \rightarrow 0^{+}$transitions where a gamma-ray transition is forbidden.

To calculate the transition probabilities properly, the finite nuclear-size and its penetration by atomic electrons must be taken into account, for example see reference [Church 56]. Electron spectroscopy and its relation to gamma-ray spectroscopy has been covered in many references such as [Rose 55] [Rose 65] and [Marmier 69]. Conversion coefficients have been calculated and tabulated by Sliv and Band [Sliv 65], Hager and Seltzer [Hager 68] [Hager 69], Dragoun et al. [Dragoun 69], Trusov [Trusov 72] and Rösel, Alder, Fries and Pauli [Rösel 78].

### 3.5 Gamma-ray Multiplicity

The multiplicity ${ }^{2}$ is the number of gamma rays that are emitted following a reaction. What is often of interest in gamma-ray spectroscopy, is the multiplicity of a particular reaction channel which can provide information on the mean angular momenta given to the evaporation residues, and such a measurement is described in chapter 4 . The principle behind the measurement is given below.

The absolute efficiency of a gamma-ray detector is a measure of the likelihood that it can detect a gamma ray, that is, if the absolute efficiency is $\Omega$ then the probability of detecting a gamma ray is given by $\mathrm{P}=\Omega$. The probability of not detecting a gamma ray must therefore be given by $\bar{P}=1-\Omega$. Considering a cascade of multiplicity $M$, that is where $M$ gamma rays are simultaneously emitted, the probability that none of them are detected is $\bar{P}^{M}=(1-\Omega)^{M}$. Therefore it follows that the probability of detecting exactly one of the gamma rays is therefore $\mathrm{P}^{M}=1-(1-\Omega)^{M}$. Under the assumption that one gamma ray has already been detected, the probability of detecting one of the remaining ( $M-1$ ) gamma rays is $\mathrm{P}^{M-1}=1-(1-\Omega)^{M-1}$.

Extending this argument, when n gamma rays have been detected, the probability of detecting one of the remaining $(M-n)$, that is the $(n+1)^{\text {th }}$ gamma ray, is $\mathrm{P}^{M-n}=1-(1-\Omega)^{M-n}$. So it follows from this reasoning that for every $N_{n} \mathrm{n}$-fold coincidences, there will be $N_{n+1}$

[^2]$(n+1)$-fold coincidences, where,
\[

$$
\begin{equation*}
N_{n+1}=N_{n}\left[1-(1-\Omega)^{M-n}\right] . \tag{3.35}
\end{equation*}
$$

\]

If $\Omega \ll 1$ then $(1-\Omega)^{M-n}$ can be expressed in ascending powers of $\Omega$,

$$
\begin{equation*}
(1-\Omega)^{M-n}=1-(M-n) \Omega+(M-n)(M-n-1) \frac{\Omega^{2}}{2}+\ldots \tag{3.36}
\end{equation*}
$$

Taking a first-order approximation and combining equations 3.36 and 3.37 gives,

$$
\begin{equation*}
N_{n+1}=N_{n} \Omega(M-n), \tag{3.37}
\end{equation*}
$$

which has the implications that: (i) measurement of $N_{n}, N_{n+1}$ and $\Omega$ will enable $M$ to be deduced; and (ii) $N_{n+1} / N_{n}$ is proportional to $(M-n)$. Chapter 4 of this work makes use of equation 3.38 with $n=1$ and $n=2$ so that

$$
\begin{align*}
& N_{2}=N_{1} \Omega(M-1),  \tag{3.38}\\
& N_{3}=N_{2} \Omega(M-2), \tag{3.39}
\end{align*}
$$

where $N_{1}, N_{2}$ and $N_{3}$ are the number of singles and double and triple coincidences respectively.

### 3.6 The Detection of Gamma Rays

### 3.6.1 Gamma-ray Interactions

There are three main types of interaction between a gamma ray and an absorbing medium which play an important role in gamma-ray detection. All involve transfer of energy from the photon to an electron in the medium. The three processes are shown schematically in figure 3.2.


Figure 3.2: The three main types of gamma-ray interaction.

## Photoelectric Absorption

Photoelectric absorption is the process where the gamma ray ionizes one of the absorber atoms and loses all of its energy in doing so. The energetics involved in the process are given by,

$$
\begin{equation*}
T=h \nu-E_{b} \tag{3.40}
\end{equation*}
$$

where $\mathrm{h} \nu$ is the energy of the incident gamma ray, T is the kinetic energy imparted to the electron and $\mathrm{E}_{b}$ is the binding energy of the electron. Gamma rays of intermediate energies $\geq 200 \mathrm{keV}$ will nearly always remove a K-shell electron. In germanium, the binding energy of a K-shell electron is 11.03 eV so the photo-electron carries off practically all of the incident gamma-ray energy. The photoelectric process is the most likely interaction for gamma rays with energies $<200 \mathrm{keV}$, and is constrained to use bound electrons in order to conserve energy and momentum.

The probability of photoelectric absorption, $\tau$, increases rapidly with atomic number Z ; the more electrons in an absorber the greater the chance of a photon-electron interaction. However, as gamma-ray energy, $\mathrm{E}_{\gamma}$, increases the probability decreases. No rigid analytical expression exists which relates these three quantities, however a rough approximation is given by,

$$
\begin{equation*}
\tau=\text { constant } \times \frac{Z^{n}}{E_{\gamma}^{3.5}} \tag{3.41}
\end{equation*}
$$

where $\mathrm{n} \simeq 4$ or 5 depending on the gamma-ray energy, $\mathrm{E}_{\gamma}$. The Z dependence explains the use of high Z materials for gamma-ray shielding, such as lead, and for gamma-ray detectors, such as germanium.

## Compton Scattering

In the Compton scattering process, the incident gamma ray is scattered through an angle $\theta$ with respect to its original direction, by an interaction with a bound electron of an absorber atom. The gamma ray transfers some of its energy to the electron. A consideration of energy and momentum conservation leads to an expression for the scattered-photon energy, $h \nu^{\prime}$, in terms of the incident photon energy, $h \nu$, the scattering angle $\theta$ and the electron rest mass, $m_{o} c^{2}$ :

$$
\begin{equation*}
h \nu^{\prime}=\frac{h \nu}{1+\frac{h \nu}{m_{o} c^{2}}(1-\cos \theta)} . \tag{3.42}
\end{equation*}
$$

All scattering angles are possible and so the energy given to an electron can range from zero up to a large fraction of the gamma-ray energy. It can be seen from this relationship that
the photon always retains some of its energy, even in the limit $\theta=180^{\circ}$, which corresponds to maximum energy transfer.

The probability for Compton scattering depends on the number of electrons which are available to scatter from, and therefore increases with the atomic number Z. It is the most dominant process for gamma rays of intermediate energies with its effect decreasing in importance with increasing energy. Although Compton scattering is the most probable process for gamma rays with energies in the range $200-1000 \mathrm{keV}$, it is not the most useful. If the full energy of the gamma ray is to be measured in a detector, then it must undergo multiple Compton scatterings, yet still remain inside the detector. A possible process is that the gamma ray will scatter out of the detector only having deposited part of its energy, therefore giving a false reading of the incident gamma-ray energy. The measured energy in this case will increase the background as part of the Compton continuum, which will be discussed in section 3.6.3. In order to combat this problem the method of Compton or escape suppression has been introduced to gamma-ray detection and this will be discussed in section 3.6.4.

## Pair Production

In the presence of an atomic nucleus, a gamma ray can change into an electron-positron pair providing that its energy is greater that the rest mass of the pair, 1.022 MeV . This process is known as pair production and must take place in the Coulomb field of an absorber atom in order to conserve energy and momentum. Any excess energy, that is $\mathrm{h} \nu-1.022 \mathrm{MeV}$, is equally divided between the electron and the positron as kinetic energy. When the positron has slowed down to about the thermal velocity of an atomic electron, it will annihilate with the electron, creating two back-to-back photons of energy 511 keV . Escape peaks can arise in gamma-ray spectra at 511 and 1022 keV below the energy of the photopeak, corresponding to the escape from the detector of one or both of the annihilation photons (section 3.6.3). Pair production is only of importance for gamma rays of energy in the range $5-10 \mathrm{MeV}$, and its probability varies as the square of the absorber atomic number.

### 3.6.2 Germanium Detectors

Practically all present-day gamma ray spectroscopy experiments utilize the high resolution offered by germanium detectors. Basically, a germanium detector is a large reverse-biased, pn-junction diode which should be maximally depleted and slightly over-biased. The operation of semiconductor radiation detectors is described in [Knoll 89] and [Leo 94]. The depletion region at the pn-boundary is the active volume where interacting radiation is detected, so it is desirable to make this volume as large as possible. The width of the depletion region is proportional to $\left(\frac{V}{N}\right)^{\frac{1}{2}}$, where $V$ is the reverse-bias applied to the pn-junction and $N$ is the impurity concentration in the germanium. A large reverse-bias (high $V$ ) of approximately -3 kV across a crystal of high-purity (low $N$ ) germanium (HPGe) will provide a large active-volume.

Gamma rays will interact with the atoms in the active volume via the mechanisms described in section 3.6.1. Electron-hole pairs are created and swept towards the electrical contacts by the large reverse-bias where they constitute a current. The energy needed to create an electron-hole pair in germanium is only 3 eV . This means that many pairs are created, statistical fluctuations are low and the resolution is very good. Charge collection occurs at a speed of about 1 cm per 100 ns which is much slower than in scintillators, described later in section 3.6.5. Charge collection times depend on where in the crystal the electron-hole pair was created. Differing charge collection times cause different rise times of the pulses which means that they must be carefully shaped, electronically.

A planar germanium crystal can sustain an active volume of $10-30 \mathrm{~cm}^{3}$, but this can be increased to about $400 \mathrm{~cm}^{3}$ in a bulletized coaxial crystal shape. Bulletizing or 'rounding-off-the-corners' helps keep the electric field uniform inside the crystal. The $\mathrm{n}^{+}$contact is on the inside because it is much thicker than the $\mathrm{p}^{+}$contact on the outside through which the gamma rays must pass. This is one reason for using n-type germanium detectors in gamma-ray spectroscopy experiments. Another reason is that p-type germanium is more susceptible to neutron damage. Damage to the germanium crystal occurs when fast neutrons dislodge germanium atoms in the crystal causing trapping and recombination sites, which impair charge collection and therefore impair resolution. The effects of neutron damage can be reversed by annealing the detector, which is done by removing the outer aluminium can and the preamplifier, and wrapping the germanium crystal with heating tape at about $115^{\circ} \mathrm{C}$ for two or three days.

Due to the relatively small band-gap in germanium, of about 0.67 eV , the crystals must be
cooled, to prevent the thermal excitation of electrons across the band gap. In practice this is achieved by making thermal-contact between the crystal and a dewar of liquid nitrogen at 77 K , using a copper rod known as a cold-finger.

The absolute efficiencies of germanium detectors cannot be standardized as easily as those of scintillators because of the different shapes and sizes of the crystals involved, and the difficulty in defining the active volume. An approximation often used is that the efficiency, relative to a 3 inch $\times 3$ inch $\mathrm{NaI}(\mathrm{Tl})$ detector at 1.33 MeV , is [detector volume $\left.\left(\mathrm{cm}^{3}\right)\right] / 5$. The usual method of measuring relative efficiencies is to compare measured relative gamma-ray intensities from radioactive sources, with accurately measured known relative intensities. Gamma rays following the $\beta$ decay of ${ }^{152} \mathrm{Eu}$ are often used, because they span the energy range 0.1 to 1.5 MeV .

### 3.6.3 Germanium Detector Spectra

A spectrum of the gamma rays emitted following the $\beta^{-}$decay of ${ }^{60} \mathrm{Co}$ to ${ }^{60} \mathrm{Ni}$, recorded by a Eurogam phase-I type detector is presented in figure 3.3. Labelled on the figure are some of the prominent features of a germanium detector spectrum.

The backscatter peak arises when a gamma ray scatters from material outside of the detector. The annihilation peak and single- and double-escape peaks arise as a consequence of pair production and are only therefore prominent when gamma rays of several MeV are present, and are small on figure 3.3: the annihilation peak at 511 keV , arises when a positron annihilates with an electron in material outside of the germanium crystal; the single- and double-escape peaks at 511 and 1022 keV less than the photopeak, respectively, corresponds to the escape of one and both of the annihilation photons after pair production within the germanium crystal. The Compton edge and Compton continuum are due to Compton scattering; the Compton edge corresponds to the maximum energy transferred when $\theta=\pi$ in equation 3.43, while scatterings to other $\theta$ contribute to the Compton continuum. (The Compton continuum peaks at the edges and dips in the centre due to the scattering-angle dependence of the Compton-scattering cross-section which is given by the Klein-Nishina formula [Krane 88]).

It is the photo-peak which is the feature of most interest in discrete-line gamma-ray spectroscopy experiments. This is the peak that corresponds to all of the gamma-ray energy being deposited in the detector, and hence the position of the peak gives a measure of the transition energy. All of the other features of the spectrum are undesirable and when


Figure 3.3: A spectrum of the gamma rays emitted following the $\beta^{-}$decay of ${ }^{60} \mathrm{Co}$, which illustrates the prominent features of a germanium gamma-ray spectrum and also the effects of Compton suppression.
many transitions contribute to the spectrum, the unwanted features add up to give a large background and so they should be eliminated as much as possible.

### 3.6.4 Escape Suppression

Escape suppression offers a method of reducing the Compton continuum and thereby increasing the peak-to-total ratio. As its name suggests, the peak-to-total ratio is the ratio of counts in the photo-peak to the total number of counts in the total spectrum, and should be maximized. Table 4.1 shows the improvement in peak-to-total ratio for an escape-suppressed Eurogam phase-I detector.

The technique of escape suppression involves surrounding the germanium detector by an inorganic-scintillator detector, which detects the gamma rays that scatter out of the germanium. These gamma rays do not leave a measure of their full energy in the germanium detector, thus contributing to the Compton continuum. Inorganic scintillators, such as bismuth germanate are used because of their good timing properties, and are described in


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the next section. Figure 3.4 shows a schematic picture of a germanium detector and its suppression shield from the Eurogam phase-I array. The dimensions of the crystal are given in table 4.1 and the rest of the figure is scaled accordingly. The thin-lined spectrum on the lower half of figure 3.3 shows the effect of Compton suppression. The Compton continuum is clearly reduced compared with the case where escape suppression is not used, shown by the thick-lined spectrum. The number of counts in the photopeak remains approximately the same, which can be seen in the inset of the top half of figure 3.3.


Figure 3.4: A Eurogam phase-I escape suppressed spectrometer, comprising germanium detector and BGO escape suppression-shield.

### 3.6.5 Inorganic Scintillators

A gamma-ray incident on an inorganic scintillator, will excite electrons across a relatively large band gap of about 4 eV into the conduction band. These electrons then decay back to the valence band with the emission of a visible photon. The scintillator is opticallycoupled to a photomultiplier tube which can detect the photon. The addition of activator impurities such as thallium in sodium iodide, written $\mathrm{NaI}(\mathrm{Tl})$, reduces self absorption in that it creates states in the band gap, so that the energy of the visible photon is not sufficient to excite an electron from the valence band into the conduction band. Scintillation detectors have poor resolution because of fluctuations in photo-emission and because of the necessary use of a photomultiplier tube [Knoll 89]. However, the short decay times associated with luminescence mean that scintillators are useful when the gamma-ray energy is not as important as its presence, such as in a Compton suppression shield.

Bismuth Germanate, $\mathrm{Bi}_{4} \mathrm{Ge}_{3} \mathrm{O}_{12}$, or ' BGO ' is an inorganic scintillator commonly used in suppression shields. Its high density of $7.3 \mathrm{~g} / \mathrm{cm}^{3}$ and the high atomic number of bismuth, means that it has a high probability of stopping gamma rays, and that only a small volume need be used. This is important when used in a closely-packed $4 \pi$ germanium detector array. The only disadvantage of BGO is that it has a low light output, only $10-20 \%$ of that from $\mathrm{NaI}(\mathrm{Tl})$. In this work, two types of inorganic scintillator were used: BGO was used in the suppression shields throughout, and a 3 inch $\times 3$ inch $\mathrm{NaI}(\mathrm{Tl})$ detector was used in the multiplicity measurement described in chapter 4.

### 3.7 Large Arrays of Germanium Detectors

A 'third-generation' [Nolan 95] array consists of a large number of large-volume escapesuppressed high-purity germanium detectors which are packed closely in a $4 \pi$ arrangement. Three large arrays of germanium detectors are presently operational: Gammasphere in Berkeley, Gasp in Legnaro and Eurogam in Strasbourg. The arrays differ slightly in design, and in the types of germanium detectors that are used, but are all built to achieve the same aim. The relative merits of each array are discussed and compared in a recent review article by Nolan, Beck and Fossan [Nolan 95].

It has been shown, for example in reference [Nolan 95], that the peak-to-background ratio increases as the fold increases. Earlier detector arrays, such as TESSA [Nolan 85] and Nordball were optimized for two-fold coincidences and could observe gamma-ray sequences at the $1-0.1 \%$ population level. The new arrays take advantage of the increase in sensitivity that can be obtained with high-fold coincidences, and are optimal for four- or five-fold coincidences and can observe structures below the $0.01 \%$ level.

## Desirable Properties of The Arrays and Energy Resolution

The use of large-volume germanium detectors covering a large fraction of $4 \pi$ ensures a high total photopeak efficiency. The arrays must have good energy resolution, high peak-tototal ratios and high granularity. High granularity means that the the total solid angle coverage is made up of many detectors, each subtending a small solid angle. This reduces the probability of the same detector being struck by two coincident gamma rays, which will reduce the peak-to-total ratio. This property is usually given by the isolated hit probability, which should be as large as possible. High granularity also helps to reduce the Doppler broadening at of the measured gamma-ray energies in the detectors at $90^{\circ}$. The arrays
must have good efficiency for detecting gamma-rays in the range $0.1 \mathrm{MeV} \leq \mathrm{E}_{\gamma} \leq 10 \mathrm{MeV}$. They must also have good resolving power, R , which is defined as,

$$
\begin{equation*}
\mathrm{R}=\left[\frac{\mathrm{SE}_{\gamma}}{\Delta \mathrm{E}_{\gamma}} \mathrm{PT}\right] \tag{3.43}
\end{equation*}
$$

where $\mathrm{SE}_{\boldsymbol{\gamma}}$ is the mean energy separation of gamma rays in a cascade. $\Delta \mathrm{E}_{\boldsymbol{\gamma}}$ is the resolution of the detector, which is usually quoted for the 1.33 MeV gamma ray emitted from a ${ }^{60} \mathrm{Co}$ source placed 250 mm from the detector on the axis of the coaxial crystal. PT is the peak-to-total ratio. It is not easy to increase the peak-to-total ratio, so to improve the resolving power, $\Delta \mathrm{E}_{\gamma}$ must be decreased. $\Delta \mathrm{E}_{\gamma}$ is given by,

$$
\begin{equation*}
\Delta \mathrm{E}_{\gamma}^{2}=\Delta \mathrm{E}_{\mathrm{in}}^{2}+\Delta \mathrm{E}_{\mathrm{D}}^{2}+\Delta \mathrm{E}_{\mathrm{R}}^{2}+\Delta \mathrm{E}_{\mathrm{V}}^{2} \tag{3.44}
\end{equation*}
$$

$\Delta \mathrm{E}_{\mathrm{D}}, \Delta \mathrm{E}_{\mathrm{R}}$ and $\Delta \mathrm{E}_{\mathrm{V}}$ are Doppler effects resulting from the detector opening angle, angularspread of the recoils and velocity-spread of the recoils, respectively. $\Delta \mathrm{E}_{\text {in }}$ is the intrinsic resolution of the detector, which has three contributions,

$$
\begin{equation*}
\Delta \mathrm{E}_{\gamma}^{2}=\mathrm{W}_{\mathrm{D}}^{2}+\mathrm{W}_{\mathrm{T}}^{2}+\mathrm{W}_{\mathrm{E}}^{2} . \tag{3.45}
\end{equation*}
$$

$W_{D}^{2}$ is due to statistical fluctuations in the numbers of electron-hole pairs, $W_{T}^{2}$ is due to incomplete charge collection and $\mathrm{W}_{\mathrm{E}}^{2}$ is due to broadening within the electronic components.

## $3.8 \gamma \gamma$-coincidence and Escape-suppression Electronics

A block diagram of the electronics associated with an escaped-suppressed germaniumdetector array is shown in figure 3.5. The setup shown is representative of that of the TARDIS detector-array at the K-130 cyclotron laboratory in Jyväskylä, Finland. The genaral principles of the suppression-and-coincidence circuitry are universal and are the basis of the more complicated electronic setup of the Eurogam array, which is mentioned in chapter 4. A qualitative description of signal processing through the electronics is given below.

Two similar signals come from theomania. detector: one is used for timing and the other as an energy signal. The energy signal goes through a spectroscopy amplifier where it is carefully shaped, in order to get an accurate energy measurement. The timing signal goes through a timing-filter amplifier (TFA) which is not concerned with the shaping of the signal, so much as the recording of its presence. Besides throwing away signals that do not reach a certain threshold, and, which are therefore probably noise, the constant-fraction


Figure 3.5: A block diagram of the escape-suppression and germanium-detector coincidence electronics. The round-cornered boxes represent the signal which an oscilloscope would display at those positions.
discriminator (CFD) converts the analogue signal from the TFA into a logic signal that can be read by coincidence box. Also fed into the same coincidence box is a signal from the BGO suppression shield. If the two signals arrive in coincidence then the event is thrown away, or vetoed and the coincidence box gives no output: this is the basis of Compton suppression. The width of the BGO signal is set to about 200 ns while the germanium signal is about 50 ns , and the two pulses are centred. This accommodates the poor timing properties of the germanium detectors.

Signals from the coincidence boxes of all twelve germanium detectors then go into the multiplicity logic unit (MLU). A multiplicity threshold can be set on this box, for instance $\mathrm{N} \geq 2$ means that 2 or more suppressed germanium signals must arrive at the box before it gives output. The output from the MLU is used to gate the analogue-to-digital converters (ADCs): that is, they do not read their inputs until the gate signal is received from the MLU.

The time-to-amplitude converter (TAC) shown in the figure, records the time interval between two gamma rays in coincidence, although several different TAC arrangements are used with the TARDIS array. The TAC in the figure, is started by a gamma-gamma coincidence signal from the MLU and is stopped by a delayed gamma signal from the first


Figure 3.6: The lowest $250 \mathrm{keV} \times 250 \mathrm{keV}$ of a quadruples matrix gated on the yrast band of ${ }^{222} \mathrm{Th}$. Black is no counts, white is maximum counts.
of the two germanium detectors that fired. (This prevents the TAC from being started unnecessarily by every single gamma event, in which case it is dead until about 200 ns have elapsed, when it resets itself).

The signals from the ADCs are passed on to the VME data acquisition cards and data acquisition system, which goes beyond the subject matter of this thesis.

### 3.9 Construction and Use of $\gamma \gamma$-correlation Matrices

Data must be presented in a manner such that useful information can be extracted. In this work, data were read from tape into one- and two-dimensional spectra using the Eurogam sort-package. The lifetime analysis of chapter 5 was carried out using only one-dimensional spectra but the analysis of high-fold multi-coincidence data in chapter 4 necessitated the use of two-dimensional spectra or gamma-gamma correlation matrices. The construction of two-dimensional matrices is described below. (In the data described in chapter 4 the mean gamma-ray fold is about 4. Gamma-gamma correlation matrices are therefore suitable for this analysis, but for higher-fold data it becomes necessary to use three-dimensional spectra or cubes)

An ungated matrix was constructed by decomposing each $n$-fold coincidence event into ${ }_{n} \mathrm{C}_{2}$ two-fold events, $\gamma_{1}$ and $\gamma_{2}$, and the matrix was incremented at ( $\gamma_{1}, \gamma_{2}$ ) and ( $\gamma_{2}, \gamma_{1}$ ). In order to cut down background and contaminants in the spectra, single-gated three-fold ('triples') and double-gated four-fold ('quadruples') matrices were also used, which had the advantage of being mostly composed of a certain set of events of interest. A gated $m$-fold matrix was constructed firstly by decomposing each $n$-fold event into ${ }_{n} \mathrm{C}_{m} m$-fold events. Each $m$-fold event was then checked against a list of energies or 'gates' which were desired to be in coincidence. If ( $m-2$ ) of the energies in the event satisfy gates, then the remaining two gamma rays, $\gamma_{i}$ and $\gamma_{k}$, are used to increment the matrix at the positions ( $\gamma_{i}, \gamma_{k}$ ) and $\left(\gamma_{k}, \gamma_{i}\right)$. For a triples matrix $m=3$ and the number of gates that must pass the gates list is $(m-2)=1$. For quadruples $(m-2)=2$. Matrices were also constructed consisting of coincidences between one ring of constant- $\theta$ detectors, with another, in order to extract angular-intensity ratios. This is described in section 4.4.2.

Mention should be made of a problem that arises with this method of matrix incrementation, in that if the gating conditions are met more often than the required number of times then false increments are made in the matrix. This is only a problem with data of very-high fold, $n=9$ or 10 . Since the data described in chapter 4 had a mean fold of about 4 , this effect was not a problem. A method of combating this problem has been devised by Beausang et al. [Beausang 95].

A section of a quadruples matrix, double-gated on the yrast band of ${ }^{222} \mathrm{Th}$ is shown in figure 3.6. The coincidences shown are mainly the thorium K X-rays around 90 keV and some of the low-energy interband dipoles. By summing projected slices through matrices such as these, spectra like those shown in figures 4.9 and 4.10 were obtained.

In this work, several methods of background subtraction were used which produced similar results. The method most often used was to subtract local background, that is to subtract from the original spectrum a background spectrum, obtained by gating close to the gate that generated the original spectrum. Other methods that were used were to subtract from the spectrum a fraction of the total projection of the matrix, to 'cut out' the background, visibly [Radford 95] from the total projection of the matrix (used in the 'Radware' programme Escl8r, described in appendix D) [Radford 95a], and the Palameta-Waddington method [Palameta 85].

### 3.10 Heavy-ion Fusion-evaporation

### 3.10.1 Introduction

Niels Bohr first suggested [Bohr 36] that an energetic collision between target and projectile nuclei would result in a fused compound-nuclear system. The fusion of heavy ions followed by the evaporation of particles, in order to populate high-spin states in the evaporation residues, was pioneered by Morinaga and Gugelot [Morinaga 63], and has become a widely used technique in nuclear-structure physics. The highly-excited compound nucleus will lose its excitation energy by evaporating particles and by emitting gamma rays. These reactions have become the most efficient method of populating the highest-spin states in heavy nuclei with the largest-cross section. For a successful reaction two criteria must be satisfied: (i) The projectile kinetic energy must be sufficiently large to overcome the Coulomb barrier; and (ii) The angular momentum transfer should be sufficiently small to ensure that the short-range nuclear force overcomes the centrifugal repulsion of the rapidly rotating compound nucleus.


Figure 3.7: A schematic representation of the de-excitation of a compound nucleus. In the case shown here an alpha particle is first emitted, followed by two neutrons. Once below the particle emission threshold (yrast energy +10 MeV ) the nucleus decays to ground by emitting gamma rays. The axis labels, particle-emission threshold and limit between continuous and discrete gamma emission are all approximate.

### 3.10.2 Cross-sections and Angular Momentum

Considering the two nuclei as classical charged black spheres, the cross-section for a reaction, $\sigma_{R}$ can be written as,

$$
\begin{equation*}
\sigma_{R}=\pi\left(R_{t}+R_{p}\right)^{2}\left[1-\frac{E_{C b}}{E_{c m}}\right]=\pi\left[\frac{\lambda}{2 \pi}\right]^{2} \ell_{\max }\left(\ell_{\max }+1\right) . \tag{3.46}
\end{equation*}
$$

The $R_{t}$ and $R_{p}$ terms, are the target and projectile radii, $E_{c m}$ is the centre of mass bombarding energy, $E_{C b}$ is the Coulomb barrier, $\lambda$ is the de Broglie wavelength and $\ell_{\max }$ is the maximum angular momentum that leads to compound nucleus formation. Hence,

$$
\begin{equation*}
\ell_{\max }=0.219\left(R_{t}+R_{p}\right)\left[\mu\left(E_{c m}-E_{C b}\right)\right]^{\frac{1}{2}} \tag{3.47}
\end{equation*}
$$

where $\mu$ is the reduced mass. The highest $\ell$ values will correspond to glancing collisions and not to compound nucleus formation. A more realistic value to form an evaporation residue is $\ell=\ell_{e r}$ where $\ell_{e r}$ is given by,

$$
\begin{equation*}
\ell_{e r}^{2}=1.5 \sigma_{e r} \mu E_{c m} \tag{3.48}
\end{equation*}
$$

The distribution of angular momentum is given by,

$$
\begin{equation*}
\sigma(\ell)=\pi \frac{\lambda}{2 \pi}(2 \ell+1) T_{\ell}, \tag{3.49}
\end{equation*}
$$

where $T_{\ell}$ is a transmission coefficient which has the value of 1 for $\ell \leq \ell_{\max }$ and 0 for $\ell>\ell_{\max }$.

### 3.10.3 Decay of the Compound Nucleus

The decay of the compound nucleus can be described statistically. The de-excitation process generally occurs in two steps, and is shown schematically in figure 3.7:
(i). Neutron, proton and alpha-particle evaporation first remove the majority of the excitation energy. Fission can also occur at this stage, particularly in heavier nuclei, and in some cases high-energy gamma rays are emitted as a consequence of a giant-dipole resonance; (ii). Gamma-ray emission then removes what excitation energy remains.

In the statistical model each state decays independently of its method of formation. Decay proceeds via one of the available channels (neutron emission, proton emission, alpha-particle emission or fission) according to the width of the channel. For a given channel the transition rate from a state with excitation energy, $\mathrm{E}_{\mathrm{i}}$, and spin, $\mathrm{J}_{\mathrm{i}}$, to that with $\mathrm{E}_{\mathrm{f}} \mathrm{J}_{\mathrm{f}}$ is related to the level density, $\rho$, and is given by,

$$
\begin{equation*}
\mathrm{R}\left(\mathrm{E}_{i} \mathrm{~J}_{\mathrm{i}} \mathrm{E}_{\mathrm{f}} \mathrm{~J}_{f}\right)=\frac{1}{\hbar} \frac{\rho\left(\mathrm{E}_{f} \mathrm{~J}_{f}\right)}{\rho\left(\mathrm{E}_{i} \mathrm{~J}_{i}\right)} \sum_{\text {all available } \ell} \mathrm{T}_{\ell} . \tag{3.50}
\end{equation*}
$$

$\mathrm{T}_{\ell}$ can be written as the product of the inverse cross-section, $\sigma_{c}$, and the particle kinetic energy, $\varepsilon$. The level density, $\rho$, is approximately

$$
\begin{equation*}
\rho(\mathrm{E}) \propto\left(\mathrm{E}-\mathrm{E}_{y}\right)^{-2} \exp \left[2 a\left(\mathrm{E}-\mathrm{E}_{y}\right)\right]^{\frac{1}{2}}, \tag{3.51}
\end{equation*}
$$

where $a$ is the level density parameter, which is about $\mathrm{A} / 8 \mathrm{MeV}^{-1}$. Therefore,

$$
\begin{equation*}
\mathrm{R}\left(\mathrm{E}_{f}\right) \propto\left(\mathrm{E}_{f}-\mathrm{E}_{y}\right)^{-2} \varepsilon \sigma_{c} \exp \left[2 a\left(\mathrm{E}_{f}-\mathrm{E}_{y}\right)\right]^{\frac{1}{2}} \tag{3.52}
\end{equation*}
$$

Using this relationship, the competition between reaction channels can be estimated. $\mathrm{E}_{\mathrm{y}}$ is the yrast energy related to the spin I and the moment of inertia, $\mathrm{J}, \mathrm{by}$,

$$
\begin{equation*}
\mathrm{E}_{y}=\frac{\hbar^{2}}{J}[\mathrm{I}(\mathrm{I}+1)] . \tag{3.53}
\end{equation*}
$$

At any point above the yrast line ${ }^{3}$ on figure 3.7 , the level density will be larger to the left (lower I) than to the right (larger I). This explains why evaporated particles carry away angular momentum rather than add it back.

### 3.10.4 Particle Evaporation

If the initial nuclear energy is $\mathrm{E}_{\mathrm{i}}$, the binding energy of the evaporated particle is B and the Coulomb barrier is $\mathrm{E}_{\mathrm{Cb}}$, then the final energy of the nucleus $\mathrm{E}_{\mathrm{f}}$ is given by,

$$
\begin{equation*}
\mathrm{E}_{\mathrm{f}}=\mathrm{E}_{\mathrm{i}}-\mathrm{B}-\mathrm{E}_{\mathrm{Cb}} \tag{3.54}
\end{equation*}
$$

For similar $\mathrm{A} \simeq 150$ product nuclei, around the line of $\beta$ stability, $\mathrm{E}_{\mathrm{f}}$ would be $\sim 8 \mathrm{MeV}$ lower after proton emission than after neutron emission, making neutron evaporation much more preferable. (Proton emission can however become more preferable in very neutron deficient nuclei). Alpha-particle binding energies vary with neutron number in the same manner as protons, so by making the compound nucleus as neutron rich as possible the evaporation of both protons and alpha particles, can be almost completely avoided. This conserves the large amount of angular momentum which alpha particles can remove, and reduces the number of available channels. However, because the line of stability tends towards $\mathrm{N}>\mathrm{Z}$ with increasing mass, heavy-ion fusion-evaporation reactions with the available beams and targets tend to produce neutron deficient nuclei.

[^3]

Figure 3.8: Below the threshold for particle emission, the evaporation residue decays by gamma-ray emission.

### 3.10.5 Gamma-ray Emission

When the nucleus has de-excited to within $\sim 8 \mathrm{MeV}$ of the yrast line, particle emission is no longer energetically possible and the evaporation residue continues to de-excite via gamma-ray emission. De-excitation gamma rays can be classified into two types: statistical and 'yrast-like', and are shown schematically on figure 3.8.

Just less than 8 MeV above the yrast line the density of states is still high, and there are many available paths via which gamma decay can proceed. Consequently transitions are weak and closely spaced in energy, and are therefore not resolvable in a spectrum. The emitted gamma rays form part of a continuum. If the nucleus is collective, yrastlike rotational cascades of E2 transitions can take place at high excitation ènergies. These yrast-like transitions remove angular momentum, but very little energy, and are shown as the near-horizontal arrows on figure 3.8. Competing at every step are the statistical gamma rays which conversely remove large amounts of energy, but very little angular momentum, and are shown by the near-vertical arrows on figure 3.8. Statistical gamma rays are predominantly dipole transitions. Nuclear structure information can be deduced by spectroscopy with the continuum gamma rays [Holzmann 89] [Henry 94].

As the de-excitation process continues the density of states becomes lower until there are only a few paths available to the decay. Intensity will therefore be concentrated and the transitions will eventually become strong enough to form discrete peaks in the spectra. It is
the discrete-line gamma-ray transitions which are of interest in the experiment described in chapter 4 . In order to be observable a transition must carry about $\geq 0.5 \%$ of the intensity, depending on the sensitivity of the gamma-ray detection system.

### 3.11 Coulomb Excitation

### 3.11.1 Introduction

Niels Bohr described the excitation and ionization of atoms by impinging charged particles in a completely classical manner in 1913 [Bohr 13]. In the 1930s it was realized that an analogous process could be used to excite atomic nuclei. In 1952, Bohr and Mottelson [Alder 66] pointed out that the time-dependent electromagnetic field of impinging charged particles could be used to excite rotational states in deformed nuclei, and this was confirmed experimentally a year later [Huus 53] [McClelland 53]. The process, which became known as Coulomb excitation, rapidly became the most important tool in nuclear spectroscopy for exciting low-lying nuclear states, and the early results strongly supported the collective nuclear model. The appeal of Coulomb excitation arose in the fact that it is a completely electromagnetic process, and was, therefore, very well understood. A comprehensive and refined theory was developed, which meant that the experimental results could be stated with much confidence. The early experiments used light ions as projectiles which could only excite few low-lying states but the development of heavy-ion accelerators meant that higher-Z projectiles were used. The resulting complex multiple-excitation patterns meant that the data became increasingly more difficult to analyse necessitating the use of Coulombexcitation computer codes. Many review articles and monographs have extensively covered the subject of Coulomb excitation and the material which forms the rest of this section is largely drawn from these sources: [Alder 56], [Alder 66], [Alder 75] and [Cline 86].

### 3.11.2 Basic Concepts

The principal use for Coulomb excitation is to determine electromagnetic transition matrix elements and moments. The matrix element involved in the Coulomb excitation of one state to another is the same as that responsible for the gamma-ray transition between the two states, and the same selection rules apply (section 3.1). The probability of Coulomb excitation to a level depends not only on the nuclear matrix element but also on the energy, Z and scattering-angle of the projectile. By changing these parameters and measuring the excitation probabilities (indirectly via a measure of the gamma-ray yields) the transition
matrix elements involved can be found with the help of computer codes. New technology has made it possible to accelerate even very heavy nuclear species above and beyond the Coulomb barrier. In theory, it is therefore possible to extract matrix elements as a function of spin for many nuclei. In practice, the nuclei that can be studied are restricted by the complexity of the analysis which, even with the most sophisticated computer codes can be very difficult and time consuming.

In order to be able to interpret Coulomb-excitation data, the interaction between the two colliding heavy ions must be purely electromagnetic. The energy of the projectile is chosen so that it is low enough to ensure that nuclear interactions do not take place, yet high enough to maximize the Coulomb-excitation interaction and populate high-spin states. The optimum beam energy is usually taken to be $80 \%$ of the Coulomb barrier, $\mathrm{E}_{C b}$ which is approximated by,

$$
\begin{equation*}
E_{C b}=1.44 \frac{Z_{t} Z_{p}}{R}\left(1+\frac{A t}{A p}\right) \mathrm{MeV} \tag{3.55}
\end{equation*}
$$

where,

$$
\begin{equation*}
R=1.16\left(A_{t}^{\frac{1}{3}}+A_{p}^{\frac{1}{3}}+2\right) \mathrm{fm} . \tag{3.56}
\end{equation*}
$$

Z and A are the charge and mass numbers (in atomic mass units) and the subscripts $t$ and $p$ represent target and projectile. It has been estimated that this projectile-energy results in a contribution of nuclear effects of $<0.1 \%$ [Lesser 72].

The usual method of computing Coulomb-excitation probabilities, is to adopt the semiclassical approach which is described in the next section. An improvement would be to include quantum-mechanical corrections although these have been shown to be of minor importance [Alder 75].

### 3.11.3 The Semi-classical Approximation

For the semi-classical picture to be applicable certain requirements must be met. Firstly, the projectile must not penetrate the target nucleus, so the wavelength of the projectile, $\lambda$, must be less than the distance of closest approach, $b$ for a head-on collision (see figure 3.9). This requirement is embodied in the definition of the Sommerfeld parameter, $\eta$,

$$
\begin{equation*}
\eta=\frac{b}{(2 \lambda / 2 \pi)}=\frac{Z_{p} Z_{t} e^{2}}{4 \pi \varepsilon_{0} \hbar v}, \tag{3.57}
\end{equation*}
$$

together with the condition that,

$$
\begin{equation*}
\eta \gg 1, \tag{3.58}
\end{equation*}
$$



Figure 3.9: The projectile with $\mathrm{Z}_{p}$ and $\mathrm{A}_{p}$ at an energy $\mathrm{E}_{p}$ travels with relative velocity $v$ towards the target which has $\mathrm{Z}_{t}$ and $\mathrm{A}_{t}$. The distance of closest approach is b and the projectile continues on its way at an angle $\theta$ to its original trajectory.
where $v$ is the relative velocity of the target and projectile, and the other terms are in the usual notation. Apart from use of the lightest projectiles, this condition is nearly always fulfilled, for instance, for ${ }^{58} \mathrm{Ni}$ at 220 MeV on a ${ }^{153} \mathrm{Eu}$ target, $\eta \simeq 119$.

A second requirement is that the energy transfer must be small so that the classical motion is not altered, that is,

$$
\begin{equation*}
\frac{\Delta E}{E} \ll 1 \tag{3.59}
\end{equation*}
$$

where $E$ is the centre of mass energy, $\frac{1}{2} m_{0} v^{2}$. It is a general principle [Alder 75] that in order to excite a state, the collision time, $\tau$, must be short compared to the period of the nuclear transition $\Delta \mathrm{t}$, where $\Delta \mathrm{t}=\Delta \mathrm{E} / \hbar$. This is expressed by defining the adiabaticity parameter, $\xi=\tau / \Delta \mathrm{t}$, and the requirement that $\xi \leq 1$. The collision time can be estimated as the time taken to travel a distance,

$$
\begin{equation*}
b(\theta)=a\left[1+\frac{1}{\sin \frac{\theta}{2}}\right], \tag{3.60}
\end{equation*}
$$

which is the distance of closest approach, when the scattering angle is $\theta$, and $a$ is half of the distance of closest approach in a head-on collision. Hence the collision time is given by,

$$
\begin{equation*}
\tau(\theta)=\frac{b(\theta)}{2 v}=\frac{a}{2 v}\left[1+\frac{1}{\sin \frac{\theta}{2}}\right] \tag{3.61}
\end{equation*}
$$

Now the adiabaticity parameter can be expressed as,

$$
\begin{equation*}
\xi=\frac{\tau}{\Delta t}=\frac{\Delta E a}{2 v \hbar}\left[1+\frac{1}{\sin \frac{\theta}{2}}\right]=\xi_{0} \frac{1}{2}\left[1+\frac{1}{\sin \frac{\theta}{2}}\right] \tag{3.62}
\end{equation*}
$$

where,

$$
\begin{equation*}
\xi_{0}=\xi(\pi)=\frac{\Delta E a}{v \hbar} \tag{3.63}
\end{equation*}
$$

And so this implies that,

$$
\begin{equation*}
\frac{\Delta E}{E} \equiv \frac{2 \xi}{\eta} . \tag{3.64}
\end{equation*}
$$

Since $\eta \gg 1$ and $\xi \leq 1$ are usually satisfied, then the requirement that $\Delta E / E \ll 1$ is also usually satisfied. $\xi$ can become large without violating this condition, but the excitation probabilities vanish exponentially with increasing $\xi$ [Alder 75].

### 3.11.4 Excitation Probabilities

Classically, the excitation is caused by the electromagnetic field sweeping over the target nucleus as the projectile passes by. In this picture, the excitation probabilities are found by solving the time-dependent Schrödinger equation for the target nucleus,

$$
\begin{equation*}
i \hbar \frac{\partial}{\partial t}\left|\psi(t)>=\left[H_{0}+V(t)\right]\right| \psi(t)> \tag{3.65}
\end{equation*}
$$

where $H_{0}$ is the Hamiltonian of the free nucleus and $V(t)$ is the electromagnetic interaction, which is a function of time due to its $r$ dependence. This equation should be solved with the initial condition that $|\psi(t=-\infty)\rangle=|0\rangle$, that is the nucleus is in its ground state before anything happens. The excitation amplitudes are the expansion coefficients $a_{n}$ of the state $\mid \psi(t=+\infty)>$ on the free nucleus state $|n\rangle$,

$$
\begin{equation*}
a_{n}=<n \mid \psi(t=+\infty)> \tag{3.66}
\end{equation*}
$$

where,

$$
\begin{equation*}
H_{0}\left|n>=E_{n}\right| n>. \tag{3.67}
\end{equation*}
$$

Once the amplitudes have been determined, evaluation of the cross sections, angular distributions, and polarizations is a matter of angular-momentum algebra. For example the excitation probability expressed in terms of the excitation amplitudes is,

$$
\begin{equation*}
P_{0 \rightarrow n}=\frac{1}{2 I_{0}+1} \sum_{m_{0}, m_{n}}\left|a_{n}\right|^{2}, \tag{3.68}
\end{equation*}
$$

where the sum is over the magnetic substates of the initial and final states. Using first-order perturbation theory, the excitation probabilities are calculated explicitly in [Alder 75] and are found to be,

$$
\begin{equation*}
\left.a_{n}=\frac{1}{i \hbar} \int_{-\infty}^{\infty}\langle n| V(t) \right\rvert\, 0>\exp \left(\frac{i \Delta E t}{\hbar}\right) d t . \tag{3.69}
\end{equation*}
$$

The electromagnetic interaction $V(r(t))$ has the form,

$$
\begin{equation*}
V(t)=V(r(t))=4 \pi Z_{t} e \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \frac{1}{(2 \lambda+1)} r_{p}^{(\lambda-1)} Y_{\lambda \mu}\left(\theta_{p}, \phi_{p}\right) M(E \lambda, \mu), \tag{3.70}
\end{equation*}
$$

where the electric multipole moments, $M(E \lambda, \mu)$ were defined in equation 3.10 . If the states $10>$ and $\mid n>$ are represented by their angular momenta, $I$, and magnetic quantum numbers $m\left(|0\rangle \equiv\left|I_{i} m_{i}\right\rangle,|n\rangle \equiv\left|I_{n} m_{n}\right\rangle\right)$ then the excitation amplitude is,

$$
\begin{equation*}
a_{n}=\frac{4 \pi Z_{t} e}{i \hbar} \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \frac{1}{(2 \lambda+1)}<I_{n} m_{n}|M(E \lambda, \mu)| I_{0} m_{0}>S_{E \lambda, \mu}, \tag{3.71}
\end{equation*}
$$

where,

$$
\begin{equation*}
S_{E \lambda, \mu}=\int_{-\infty}^{\infty} \exp (-i \omega t) Y_{\lambda \mu}\left(\theta_{p}(t), \phi_{p}(t)\right)\left[r_{p}^{-\lambda-1}(t)\right] d t \tag{3.72}
\end{equation*}
$$

That is, the amplitudes can be split into a term which depends on the nucleus through the matrix elements of the nuclear moments, and an integral which depends on $\Delta \mathrm{E}$ and the kinematics of the classical orbit.

If it is assumed that only target nuclei are excited, and that the excitation does not affect the motion of the nuclei (the semi-classical approximation), then if the probability of exciting from a state $\mid 0>$ to a state $\mid n>$ is $\mathrm{P}_{0 \rightarrow n}$, the differential scattering cross-section is given by,

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}=\left(\frac{d \sigma}{d \Omega}\right)_{R} P_{0 \rightarrow n}=\frac{1}{4} a^{2} \sin ^{-4}\left[\frac{\theta}{2}\right] P_{0 \rightarrow n}, \tag{3.73}
\end{equation*}
$$

where $\left(\frac{d \sigma}{d \Omega}\right)_{R}$ is the Rutherford scattering cross-section and $\mathrm{a}=\mathrm{b} / 2=Z_{p} Z_{t} e^{2} / m_{0} v^{2}$, which is half the distance of closest approach in a head-on collision, see figure 3.9. Using equations 3.3, 3.13 and 3.14, an expression can be derived for the total excitation cross-section, $\sigma_{E \lambda}$. The cross-section for electric excitations is,

$$
\begin{equation*}
\sigma_{E \lambda}=\left(\frac{Z_{1} e}{\hbar v}\right)^{2} a^{-2 \lambda+2} B(E \lambda) f_{E \lambda}(\xi) \tag{3.74}
\end{equation*}
$$

The equivalent expression for magnetic excitation is,

$$
\begin{equation*}
\sigma_{M \lambda}=\left(\frac{Z_{1} e}{\hbar c}\right)^{2} a^{-2 \lambda+2} B(M \lambda) f_{M \lambda}(\xi) \tag{3.75}
\end{equation*}
$$

Hence a measurement of the cross-section can be used to obtain the matrix element. The matrix elements are the same as those involved in gamma decay and the same selection rules apply. The $f_{E \lambda}$ coefficients contain the dependence on the trajectory, and they must be computed numerically. They are found to reduce with increasing multipolarity. Some typical values, taken from [Alder 75] are, $f_{E 1}=4, f_{E 2}=0.3, f_{E 3}=0.02$, and $f_{E 4}=0.0020$. In gamma decay the gamma-ray intensity falls off by a factor of $\left(\frac{\omega}{c} R_{0}\right)^{2}$ with each increment in multipolarity but the Coulomb excitation probabilities fall off much more slowly. Coulomb excitation, therefore, offers a possibility for measuring E3 and E4 matrix elements which is not possible with gamma decay. Despite the larger $f$ coefficient for E1 excitation, E2
matrix elements are often enhanced and so the dominant mode of excitation is E2. Also it can be seen from the expression for $\sigma_{M \lambda}$ that the cross-section for magnetic excitations is reduced by a factor of $(v / c)^{2}$, the $f$ coefficients being of similar magnitude for $\mathrm{E} \lambda$ and $\mathrm{M} \lambda$. A typical value of $(v / c)$ is 0.05 and hence magnetic excitation can usually be neglected.

## Realistic Situations

The approach outlined above has used first-order perturbation theory. A necessary condition for this approach to be valid is that the excitation probabilities must be small for all excited states. In real situations, with high-Z projectiles, excitation probabilities can be large, especially for collective states, and the condition is violated. When many states are populated there are a variety of ways that each can be reached through the virtual excitation of many intermediate states. There are usually several strongly-coupled, low-lying states that influence the excitation probabilities and a more accurate treatment would use second-order perturbation theory. To second order the excitation amplitude is,

$$
\begin{equation*}
a_{i f}=a_{i f}^{(1)}+\dot{a}_{i z f}^{(2)}, \tag{3.76}
\end{equation*}
$$

where $\mathrm{a}_{i f}^{(1)}$ is the amplitude to first order and $\mathrm{a}_{i z f}^{(2)}$ is the amplitude of exciting from $|i\rangle$ to $\mid f>$ via an intermediate state $\mid z>$. The implication is that the first-order excitation probabilities must be corrected for the second-order effects. For instance figure 3.10 shows a set of levels with all potential E2 and E4 couplings. The dotted lines represent 'first-order' excitation paths, and the solid lines represent 'second-order' excitation paths. (The semicircular arrows represent what is known as the re-orientation effect, whereby the interaction excites different magnetic substates of the same state). As an illustrative example, consider the $2^{+}$states. 'First-order' excitation to the first $2^{+}$state will have to be corrected for the interference effects with the 'second order' excitations from the ground state to the second $2^{+}$state, and between the $2^{+}$states themselves. The interference effects, and their form,


Figure 3.10: Typical excitation paths between a set of states. All potential E2 and E4 couplings are shown. First-order excitations are represented by dotted arrows and second-order excitations by solid arrows.
are discussed at length in reference [Alder 75]. The excitation amplitudes must be found by solving the set of coupled differential equations that describe the excitation. In practice, this is done numerically using computer codes.

### 3.11.5 Gosia - The Semi-classical Coulomb-excitation Analysis Code

Until recently the most widely used Coulomb-excitation code was that of Winther and De Boer which is described in reference [Alder 66]. Given the details of an experiment and a level scheme, together with a set of matrix elements, the code would output calculated gamma-ray yields which would be compared to those measured. The calculated yields would be altered by varying the model-dependent input parameters until they matched those measured. The use of this code had problems such as model dependencies and unknown sensitivity to assumed sets of matrix elements.

The Rochester semi-classical least-squares search code Gosia [Czosnyka 83] was developed to overcome these problems by fitting a set of matrix elements to a measured set of gammaray yields in a model-independent way. The code can also be used to calculate gamma-ray yields and level populations in the same manner as the Winther-De Boer code. It is designed to take in data from up to 50 experiments, for a nucleus with up to 50 excited states and a few hundred matrix elements between the states. Only the electric excitations E1, E2, E3 and E4 are considered in the excitation while M1 matrix elements are input to account for gamma-ray decay branches. In order to fit many matrix elements it is necessary to collect a large amount of data by performing many experiments (or 'sub-experiments') exploiting the excitation probability dependence on scattering angles, projectile Z and projectile-energy. The code constructs a $\chi^{2}$ value dependent on the difference between the calculated and observed gamma-ray yields. The matrix elements used to calculate the gamma-ray yields are altered to reduce the $\chi^{2}$, the changes in each matrix element being determined by the gradient of the $\chi^{2}$ function with respect to each matrix element, and the final set of matrix elements corresponding to a minimum in the $\chi^{2}$ function.

With the help of Gosia, E3 matrix elements have been measured, as a function of spin, in ${ }^{148} \mathrm{Nd}$ [White 90], [Ibbotson 93] ${ }^{150} \mathrm{Nd}$ [Clarkson 92], ${ }^{150} \mathrm{Sm}$ [Ibbotson 95] and ${ }^{226} \mathrm{Ra}$ [Wollersheim 93]. In the work described in chapter 5, Gosia was used in order to calculate level populations and angular distribution coefficients, which were necessary to extract lifetimes from the Coulomb-excited recoil-distance data.

## CHAPTER 4

## The High-spin Behaviour of ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$

The isotones ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ are only several nucleons away from ${ }^{224} \mathrm{Th}$ which is predicted to be the most strongly octupole-deformed nucleus [Nazarewicz 84]. The octupole-deformed radium and thorium nuclei have been the subject of several theoretical investigations, focussing on the interplay between octupole deformation and high-frequency rotation. In this chapter, a gamma-ray spectroscopy study is described which was performed using the Eurogam gamma-ray spectrometer, in order to investigate the high-spin behaviour of ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$. A brief description of the Eurogam gamma-ray spectrometer is first given, followed by an account of the experimental details and a presentation of the results. A discussion of results in terms of the octupole model, and their implications concludes the chapter.

### 4.1 Eurogam Phase-I

### 4.1.1 The Eurogam Phase-I Array

The European gamma-ray microscope project, or Eurogam, was made possible by advances in detector, software and electronic technology. Phase- $\mathrm{I}^{1}$ of the Eurogam array was operational at the Nuclear Structure Facility at the Daresbury Laboratory between October 1992 and March 1993.


Figure 4.1: Positions of the germanium detectors used with Eurogam phase-1. The numbers below are the approximate $\theta$ positions of each ring, and in brackets the number of detectors in that ring.

Eurogam was the first of the third generation of germanium-detector arrays, which use large-volume germanium crystals [Nolan 95]. The technology to grow such crystals has only recently become available. Specific details of the array are available in several sources [Nolan 90] [Beausang 92] [Beck 92] [Edocs@dl] and are summarized in table 4.1. Many of the concepts used in this section and an explanation of the quantitics of table 4.1 are given in section 3.6.2. In brief, the array consisted of 45 escape-suppressed large-volume high-purity coaxial germanium detectors packed closely in a $4 \pi$ arrangement. The angular positions of the detectors are given in table 4.1.

[^4]| Eurogam Phase-I : Specifications |  |  |
| :---: | :---: | :---: |
| $\mathrm{N}^{\circ}$. of detectors : 45 |  | Type : high-purity n-type |
| Resolving Power Parameter : $5.6{ }^{1}$ |  | Efficiency : $65 \%^{2}-85 \%^{2}$ at 1.33 MeV |
| Size : Bulletized coaxial crystal $\sim 70 \mathrm{~mm} \times \sim 75 \mathrm{~mm}$, tapered over front 3 cm to $5.7^{\circ}$ |  |  |
| Total Solid-Angle Coverage : $\sim 23 \%$ ( $\sim 0.51 \%$ by each detector) |  |  |
| Total Photo-peak Efficiency : $5.6 \%$ at $1.33 \mathrm{MeV}^{3}$ |  |  |
| Peak-to-Total Ratio : $58 \%$ individual supp., $65 \%$ shared supp., $25 \%$ unsupp. |  |  |
| Isolated Hit Probability : 0.88 for multiplicity ${ }^{4} 30,0.94$ for multiplicity ${ }^{4} 15$ |  |  |
| Detector Positions: ${ }^{\text {, } 6}$ |  |  |
| $\theta$ | $\phi$ |  |
| $157.60^{\circ}$ | $(36+\mathrm{n} 72)^{\circ} \mathrm{n} \leq 5$ |  |
| $133.57^{\circ}$ | $(54+\mathrm{n} 36)^{\circ} \mathrm{n} \leq 10$ |  |
| $107.94{ }^{\circ}$ | $20.0^{\circ}, 49.6^{\circ}, 94.6^{\circ}, 121.6^{\circ}, 16$ | $4^{\circ}, 193.6^{\circ}, 288.4^{\circ}, 265.4^{\circ}, 310.4^{\circ}, 340.0^{\circ}$ |
| 94.16 ${ }^{\circ}$ | $(72+\mathrm{n} 72)^{\circ} \mathrm{n} \leq 5$ |  |
| $85.84{ }^{\circ}$ | $(36+n 72)^{\circ} \mathrm{n} \leq 5$ |  |
| $72.05^{\circ}$ | $13.6^{\circ}, 54.8^{\circ}, 89.9^{\circ}, 130.4^{\circ}, 15$ | $6^{\circ}, 202.4^{\circ}, 229.6^{\circ}, 274.1^{\circ}, 301.6^{\circ}, 346.4^{\circ}$ |

Table 4.1: Specifications of the Eurogam Phase-I gamma-ray spectrometer. The superscripts are defined below: ${ }^{1}$ Assuming a mean $\mathrm{v} / \mathrm{c}$ of $2.5 \% ;{ }^{2}$ Relative to a 3 inch $\times 3$ inch $\mathrm{NaI}(\mathrm{Tl})$ crystal; ${ }^{3}$ Measured with a source at the target position, 189 mm from the front of the crystal; ${ }^{4}$ Summed gamma and neutron multiplicities; ${ }^{5}$ No detectors at forward angles due to the physical positioning of the Daresbury recoil mass separator; and, ${ }^{6}$ Notice there are many detectors at $\sim 90^{\circ}$, which will increase the Doppler broadening, on average. The detector positions are taken from reference [Cresswell 94].

A Eurogam escape-suppressed spectrometer is shown schematically in figure 3.4. The suppression shield consists of 10 optically-isolated BGO crystals each, with its own photomultiplier tube. The crystals are tapered, with the thickness varying from 20 mm to 3 mm , to achieve close packing, which is important when the 'shared-suppression' mode is used. Shared-suppression mode is the operation when gamma rays in a germanium detector are not only suppressed by the detector's own shield, but also by the suppression shield of neighbouring detectors, and results in an improvement in the peak-to-total as given in table 4.1. The mode of suppression used in this work, however, was that of traditional 'individualsuppression' when gamma rays are only suppressed by the detector's own shield. The efficiency of a Eurogam Phase-I detector is $\sim 70 \%$, compared with only $\sim 25 \%$ for a TESSA-type
detector (relative to a 3 inch $\times 3$ inch sodium-iodide detector). This increase in efficiency is due to the larger volume of the germanium crystal. The crystal is approximately 75 mm in diameter $\times 70 \mathrm{~mm}$ in length for a Eurogam detector compared to approximately 57 mm in diameter $\times 52 \mathrm{~mm}$ in length for a TESSA-type detector. The solid-angle coverage of a Eurogam detector is typically $0.51 \%$ for a Eurogam detector, compared with $0.26 \%$ on TESSA. The peak-to-total of $58 \%$ for individual suppression is approximately equal to that of a TESSA-type detector, due to the limited volume of BGO that can be used when the detectors are closely packed.

The array was designed with the considerations of section 3.7 in mind and has a dodecahedral structure, that is, a ball made up of 12 large regular-pentagonal sides. Each large pentagon is divided into five irregular pentagons and one regular pentagon, as shown in figure 4.2. These smaller pentagons form the collars to hold the BGO shields in place. The array therefore had space to hold 60 detectors in the small irregular pentagons and 10 detectors in the regular pentagons, with two of the regular pentagons accommodating the beam pipe.


Figure 4.2: One of the 12 pentagonal faces of the Eurogam phase-1 array. Each such pentagon is divided into one regular pentagon and five irregular pentagons. In the phase-1 configuration the irregular pentagons each served as a collar to hold one escape-suppressed germanium detector.

The large dodecahedral ball was suspended from an overhead frame, and was separable into two halves, to allow access to the target chamber. Only 45 detectors were used in Eurogam phase I. No detectors were placed in the regular pentagons at the centre of each large pentagon. Also, 15 of the detectors at forward angles had to be left out due to the positioning of the array with respect to the Daresbury recoil-mass separator, which was used in some experiments to provide additional mass selection [Paul 93] [Paul 95]. Therefore, the upstream half of the ball held 30 detectors, and the downstream half held 15.

Both the germanium detectors and the BGO shields require a high voltage which was supplied by computer-controllable units. For the BGO crystals, 16 -channel 2.5 kV cards
provided the necessary +1 kV , whilst the -3 kV required by the germanium detectors was provided by 8 -channel 5.6 kV cards. To prevent damage to the germanium crystal, the germanium high voltage was automatically turned off by a hardware link should the temperature of the germanium crystal rise (see section 3.6.2). The high-voltage cards were computer-controlled through a VME crate connected to the ethernet.

Filling of the germanium-detector liquid-nitrogen Dewars was done automatically by an 'autofill' system, using remotely-operated solenoid valves. The valves were operated using a purpose-built VME system allowing monitoring and operation from computer terminals [Edocs@dl].

### 4.1.2 Eurogam Electronics

The state-of-the-art electronic setup associated with the Eurogam array, has been developed and is maintained by electronic and data-acquisition specialists, and a thorough description of the system is very complex and would be unnecessary here. What follows is an overview of the electronics from the experimental nuclear physicist's point of view, described with respect to the discussion of section 3.8.

With so many detectors being used, a preliminary requirement was that the electronics associated Eurogam must be compact and capable of handling high event-rates, up to about 10000 events per second in each detector. Also, conventional NIM $^{2}$ electronics would be impractical due to the large volume of modules that would be needed and because of their susceptibility to bad connections. It was for these reasons that highly-integrated circuitry of VXI ${ }^{3}$ was chosen for signal processing. The VXI crates were also computer-controllable, allowing adjustments to the electronics to be made very easily.

The large size of the VXI card allows both both analogue and digital processing for six germanium detectors or six BGO shields on any one card. With twenty cards per VXI crate, only two crates were needed for the whole Eurogam array. The physical arrangement was that the germanium and BGO cards were placed side by side so the card to the right of any germanium card contained its suppression electronics. Each VXI crate also held a

[^5]cpu-card, and a readout-controller card. The master-trigger card was held in one of the crates.


Figure 4.3: A block diagram showing the associated components of Eurogam.

Signals from the VXI crates were passed into a VME crate known as the 'event builder' [Edocs@dl], where the hardware digital signals were turned into a software event according to a user-input list of commands known as an 'event builder file' which is read into the crate at the control workstation. As the data were passed into the event builder they were also passed into the 'histogrammer'. The histogrammer was another VME crate, where spectra were created so that the data could be checked and preliminary analysis could be carried out as the experiment was running. For speed of transfer the data were sent to the storage crate, not by the traditional ethernet cable, but by an optical fibre. The storage crate controlled the transfer of events onto two 8 mm data-tapes; one as a backup to protect against tape-drive failure. Also available for use at this stage was a sorter crate for the on-line creation of matrices and coincidence spectra.

The control of the array was carried out with a Sun 'Sparc-2' workstation using the 'Eurogam session' software which was specifically written for that purpose. The software was capable of controlling data acquisition, changing the data-tapes, monitoring event-rates, viewing the on-line spectra, and tuning the electronics, such as CFD thresholds, using a user-friendly window system.


Figure 4.4: Schematic layout of the electronics on a germanium VXI data-acquisition card.


Figure 4.5: Schematic layout of the electronics on a BGO VXI data-acquisition card.

### 4.1.3 Signal Processing

The general principles behind signal processing for a germanium detector array are given in section 3.8. Figures 4.4 and 4.5 show a schematic layout of the germanium detector and suppression-shield VXI data-acquisition cards.

## Germanium Signal

The $200 \mathrm{mV} / \mathrm{MeV}$ signal from the germanium-detector preamplifier is split into two parts. One is used as a timing signal and the other is used as an energy signal. Both need further amplification.

The energy signal passes through a shaping amplifier to reduce the length of the pulse and avoid pile-up. This is done with a quasi-triangular shaping filter of fixed gain and fixed time constant. Two amplifiers are used, one of low-gain so the resulting signal varies from $0-20 \mathrm{MeV}$ and the other of high-gain with the output signal having $0-4 \mathrm{MeV}$. The signals are then corrected for ballistic deficit before being put through a peak-detect-and-hold (PDH) circuit where the peak voltage is held until it has been converted into a digital signal.

The timing signal is passed through a TFA and CFD as described in section 3.8. The output of the CFD, a logic signal, is used to drive the germanium sumbus for germanium multiplicity which is used to provide the master trigger, and it is also used as the local trigger in each germanium card. It also starts a TAC which can be used to filter events in the off-line analysis.

## BGO Signal

The BGO signals are combined and shaped ( $\tau=2 \mu \mathrm{~s}$ ) in a sum amplifier so that only one energy output is taken per shield. The energy signal is of low gain and enables the total energy deposited by a gamma-ray to be calculated. The timing signal is hard-wired into the associated germanium card to perform Compton suppression.

### 4.2 Previous Studies of ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ : Motivation for Further Study

The nuclei ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ are particularly good examples of the evolution of octupole deformation to high spin. Cranked Woods-Saxon-Bogolyubov-Strutinsky calculations have been performed by Nazarewicz et al. [Nazarewicz 87] to investigate the shapes of the rotating radium and thorium nuclei with $130 \leq N \leq 138$. In view of the observation of octupole bands in all of these nuclei and of the empirical finding that the octupole deformation in these nuclei appears to reinforced at high spin, the calculations were performed under the assumption of octupole deformation. The results show the shapes of the octupole-deformed radium and thorium nuclei fall into four categories depending on the degree of the octupole and quadrupole deformation, as was mentioned in 2.5.1. The nucleus ${ }^{222} \mathrm{Th}$ which, theoretically, has an octupole-deformed ground state, is predicted to maintain its octupole deformation up to spin $\sim 26 \hbar$ when the octupole configuration becomes non-yrast following a band-crossing with a reflection-symmetric four-quasiparticle $\nu\left(j \frac{15}{2}\right)^{2} \pi\left(i \frac{13}{2}\right)^{2}$ band. The isotone ${ }^{220} \mathrm{Ra}$ has a spherical ground state and only becomes octupole deformed as it rotates. It is predicted to maintain its octupole deformation to higher spin. More detail about the calculations of Nazarewicz et al. will be given in section 4.6.

Being some of the most octupole-deformed nuclei with reasonably large cross-sections for formation in heavy-ion fusion-evaporation reactions ( $\sim$ tens of millibarns), ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ have been the subject of several high-spin studies. Both nuclei have been shown to display the characteristics of octupole deformation, that is alternating-parity bands and

| Year | Reference | Reaction | $\mathrm{E}_{l a b}$ <br> (MeV) | Maximum Yrast Spin |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{220} \mathrm{Ra}$ |  |  |  |  |
| 1984 | [Burrows 84] | ${ }^{208} \mathrm{~Pb}\left({ }^{18} \mathrm{O}, \alpha 2 \mathrm{n}\right){ }^{220} \mathrm{Ra}^{r}$ | 83 | $12^{+}$ |
| 1984 | [Cottle 84] | ${ }^{208} \mathrm{~Pb}\left({ }^{14} \mathrm{C}, 2 \mathrm{n}\right){ }^{220} \mathrm{Ra}$ | 63 | $21^{-}$ |
| 1985 | [Celler 84] | ${ }^{208} \mathrm{~Pb}\left({ }^{14} \mathrm{C}, 2 \mathrm{n}\right){ }^{220} \mathrm{Ra}$ | 61, 64 | $18^{+}$ |
| 1985 | [Shriner 85] | ${ }^{208} \mathrm{~Pb}\left({ }^{14} \mathrm{C}, 2 \mathrm{n}\right){ }^{220} \mathrm{Ra}$ | 60-78 | $21^{-}$ |
| 1991 | [Schulz 91] |  |  | $24^{+}$ |
| ${ }^{222} \mathrm{Th}$ |  |  |  |  |
| 1983 | [Bonin 83] | ${ }^{208} \mathrm{~Pb}\left({ }^{18} \mathrm{O}, 4 \mathrm{n}\right)^{222} \mathrm{Th}^{r}$ | 95 | 15- |
| 1983 | [Ward 83] | ${ }^{208} \mathrm{~Pb}\left({ }^{18} \mathrm{O}, 4 \mathrm{n}\right)^{222} \mathrm{Th}^{r}$ | 93 | 17- |
| 1985 | [Bonin 85] | ${ }^{208} \mathrm{~Pb}\left({ }^{18} \mathrm{O}, 4 \mathrm{n}\right)^{222} \mathrm{Th}^{r}$ | 95 | $15^{-}$ |
| 1986 | [Schwartz 86] | ${ }^{208} \mathrm{~Pb}\left({ }^{18} \mathrm{O}, 4 \mathrm{n}\right)^{222} \mathrm{Th}^{r}$ | 94, 95 | (25-) |
| 1987 | [Schwartz 87] | ${ }^{208} \mathrm{~Pb}\left({ }^{18} \mathrm{O}, 4 \mathrm{n}\right)^{222} \mathrm{Th}^{r}$ | 94 | $\left(26^{+}\right)$ |

Table 4.2: A summary of recent high-spin studies of ${ }^{222} \mathrm{Th}$ and ${ }^{220} \mathrm{Ra}$. The superscript r indicates that gamma rays were collected in coincidence with a residue detector.
enhanced E1 transitions. Details of recent high-spin studies of ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ are summarized in table 4.2. The right-hand column indicates that the known spins were not sufficiently high to test the predictions of Nazarewicz et al. A high-spin study with a very efficient, large germanium-detector array, such as Eurogam, would be the best method to observe higher-spin yrast states. Other structures may also be found in these nuclei, similar to the sideband recently reported in ${ }^{218} \mathrm{Ra}$ [Schulz 89].

### 4.3 Experimental Details

High-spin studies using heavy-ion fusion-evaporation reactions are ideally suited to nuclei with $\mathrm{A} \simeq 140$ [Stephens 80 ]. Actinide nuclei with $\mathrm{A} \geq 220$ are difficult to populate at high spin using such reactions because of the severe competition from fission, and because of the lack of beam-target combinations.

This work used a heavy-ion fusion-evaporation reaction with an ${ }^{18} \mathrm{O}$ beam incident on ${ }^{208} \mathrm{~Pb}$ targets, to produce ${ }^{222} \mathrm{Th}$ in the 4 n channel, ${ }^{220} \mathrm{Ra}$ in the ( $\alpha, 2 \mathrm{n}$ ) channel and ${ }^{219} \mathrm{Ra}$ in the ( $\alpha, 3 \mathrm{n}$ ) channel. The beam energy used was 95 MeV , which was found to be about the optimum beam energy for the production of ${ }^{222} \mathrm{Th}$ in an excitation-function measurement made by Ward et al. [Ward 83]. The beam was accelerated by the 20 MV FN tandem Van de Graaff accelerator at the Nuclear Structure Facility, Daresbury Laboratory [Lilley 82]. The beam intensity was maintained between 3 and 5 pnA for the duration of the experiment. The target consisted of two stacked self-supporting thin ( $500 \mu \mathrm{~g} / \mathrm{cm}^{2}$ ) foils of isotopically enriched ${ }^{208} \mathrm{~Pb}$. De-excitation gamma-ray coincidence data were collected using the Eurogam array, described in previous sections. Copper and tin absorbers were taped to the front of the germanium detectors in order to absorb unwanted X-rays which would make the event rate artificially high. Typical singles count-rates were about 8000 per second in each germanium detector. For this experiment, the Eurogam master-trigger was set to $\geq 5$ unsuppressed germanium detectors, that is 5 or more unsuppressed germanium signals had to register before an event was processed. The data are described quantitatively in the next section.

### 4.3.1 Data

After 48 hours of data collection, a total of $1.76 \times 10^{9}$ double, $1.08 \times 10^{9}$ triple and $0.425 \times 10^{9}$ quadruple gamma-ray coincidence events were written to magnetic tape. The fold, that is the number of gamma rays in each event, is plotted against the number events for both raw ('folded') and unfolded data in figure 4.6. In unfolding the data, each $m$ ( $>\mathrm{n}$ ) fold event is decomposed into ${ }_{m} \mathrm{C}_{\mathrm{n}}=\mathrm{m}!/(\mathrm{n}!(\mathrm{m}-\mathrm{n})!) \mathrm{n}$-fold events and the distribution is incremented accordingly. (For example in each 6 -fold event there are ${ }_{8} \mathrm{C}_{3}=203$-fold ('triple') coincidences). The distributions illustrate how unfolding the data increases the statistics, and it was from unfolded data, which the spectra used in this analysis were created. As with all heavy-ion fusion-evaporation experiments involving actinide nuclei, by far the largest crosssection is that of the fission channel - $95 \%$ in this case. Of the useful $5 \%$ of the cross-section corresponding to fusion-evaporation residues, $67 \%$ ( $3.35 \%$ of the total cross-section) were
${ }^{222} \mathrm{Th}, 25 \%(1.25 \%)$ were ${ }^{220} \mathrm{Ra}$ and $7 \%(0.35 \%)$ were ${ }^{219} \mathrm{Ra}$. Although the same beam and target combination is used to populate other nuclei such as ${ }^{221} \mathrm{Th}$ and ${ }^{223} \mathrm{Th}$, when used with a similar beam energy ( $\sim 85 \mathrm{MeV}$ ), no trace of these products could be found in the data.


Figure 4.6: Raw and unfolded data distributions for the complete set of data.

### 4.3.2 Analysis

The analysis involved reading the coincidence events from tape into 2-dimensional gammagamma correlation matrices as described in section 3.9. The total projections of some of the gamma-gamma correlation matrices are shown in figure 4.7. The uppermost spectrum shows the projection of an ungated gamma-gamma matrix which contains all $1.76 \times 10^{9}$ unfolded double gamma-ray coincidences. The two lower spectra are the projections of single-gated triples and double-gated quadruples matrices, which demanded coincidences with known, clean gamma transitions in ${ }^{222} \mathrm{Th}$ before they were incremented. The middle and lower spectra are overlaid on the upper spectrum as a comparison, and to illustrate how the technique of gating the matrices severely reduces the large fission background and selects individual reaction channels, but also reduces the overall number of counts. The analysis
described in the rest of this chapter was carried out predominantly by projecting slices through gated gamma-gamma correlation matrices and the spectra presented are obtained thus.

In order that the transition energies were independent of those previously measured, an energy calibration was made using ${ }^{152} \mathrm{Eu}$ source data, rather than using the energies of known transitions in ${ }^{222} \mathrm{Th}$ or ${ }^{220} \mathrm{Ra}$. Use of the source data meant that an explicit Doppler shift correction had to be applied using a recoil velocity calculated from reaction kinematics. To check that the energy calibration was correct, a spectrum was sorted for each detector to make sure that each peak was at the correct position.

### 4.4 Results

### 4.4.1 High-spin States

Obtaining coincidence relationships by slicing through gated gamma-gamma correlation matrices, and considering energy and intensity-balance arguments, the level schemes presented in figure 4.8 were deduced. There are many transition energies common to both nuclei, and also to ${ }^{219} \mathrm{Ra}$, so only the cleanest transitions were used to select each channel. The matrices were gated on the following lists of transitions.
${ }^{220}$ Ra gates: $128.1 \mathrm{keV}, 152.4 / 153.6 \mathrm{keV}$ (doublet), $156.7 \mathrm{keV}, 178.4 \mathrm{keV}, 215.1 \mathrm{keV}, 224.6$ $\mathrm{keV}, 231.6 \mathrm{keV}, 278.1 \mathrm{keV}, 313.3 \mathrm{keV}, 399.1 \mathrm{keV}$ and $454.6 / 455.8 \mathrm{keV}$ (doublet).
${ }^{222}$ Th gates: $98.7 \mathrm{keV}, 161.1 / 161.2 \mathrm{keV}$ (doublet), $170.1 \mathrm{keV}, 173.1 \mathrm{keV}, 182.9 \mathrm{keV}, 228.5$ $\mathrm{keV}, 244.6 \mathrm{keV}, 256.0 / 256.2 / 256.3 \mathrm{keV}$ (triplet), and 310.2 keV .

Care had to be taken in choosing the gates. For example it was possible to use the 278.1 keV transition to select the ${ }^{220} \mathrm{Ra}$ channel when constructing double-gated matrices, but in a single-gated matrix the 278.1 keV transition would bring in unwanted coincidences via the 278 keV transition in ${ }^{219} \mathrm{Ra}$. Figures 4.9 and 4.10 are summed slices through double-gated quadruples matrices, for both ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$.

The incrementing energy pattern of the E2 transitions gave an indication of where to look in the data for new transitions, in order to extend the yrast octupole bands. The octupole band in ${ }^{220}$ Ra has been observed up to spin $31 \hbar$ and all of the interband E1 transitions have been observed right up to the highest spin. In contrast, the yrast octupole band in


Figure 4.7: Projections of gamma-gamma correlation matrices.


Figure 4.8: The level schemes for (a) ${ }^{220} \mathrm{Ra}$ and (b) ${ }^{222} \mathrm{Th}$ deduced in this work. The values in parentheses to the right of the transition energies are the intensities normalized to the $4^{+} \rightarrow 2^{+}$in each nucleus, and then corrected for internal conversion. The level schemes were previously known up to $24^{+}$in both ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$.


Figure 4.9: Four-fold coincidence spectra gated on ${ }^{220} \mathrm{Ra}$.
${ }^{222} \mathrm{Th}$ can only be seen up to spin $25 \hbar$, despite the fact that it is by far the stronger reaction channel. Although the highest-energy transition that can be identified to belong to the yrast band in ${ }^{222} \mathrm{Th}$ is the $515 \mathrm{keV}\left(25^{-} \rightarrow 23^{-}\right)$transition, several transitions with energies $>500 \mathrm{keV}$ have been identified and assigned to ${ }^{222} \mathrm{Th}$ via their X -ray coincidences and coincidence with the low-lying states in the yrast band. Most notable of such transitions are the 533 keV and 575 keV transitions, which appear to feed into the yrast levels $18^{+}$and $11^{-}$respectively, and they can be seen in the gated quadruples-spectra of figure 4.10.

### 4.4.2 Angle-dependent Intensity Measurements

An angular-intensity measurement was made on these data, whereby the intensities of transitions in the rings of detectors at $\left(85.84^{\circ}+94.16^{\circ}\right)$ were measured in coincidence with those at $157.60^{\circ}$, and conversely the intensities at $157.60^{\circ}$ were measured in coincidence with $\left(85.84^{\circ}+94.16^{\circ}\right)$. The ratio of the two intensities showed a different value for stretcheddipole than for stretched-quadrupole transitions, allowing the two to be differentiated, and spin assignments to be made.

While the correlation between two gamma rays from an orientated state was used in this


Figure 4.10: Four-fold coincidence spectra gated on ${ }^{222} \mathrm{Th}$.
measurement, strictly speaking this was not a true DCO measurement as it was assumed that all transitions were of pure multipolarity and only one $\left(\theta_{1}, \theta_{2}\right)$ combination was used. Gating helped to 'clean up' the spectra so that the transition of interest could be accurately measured. This method was used because it had previously been shown [Paul 93] to distinguish clearly between the stretched-quadrupole and stretched-dipole cases.

For the purpose of this analysis, the detector rings at $\theta=85.84^{\circ}$ and $\theta=94.16^{\circ}$ were summed into one ring with $\theta \simeq 90^{\circ}$. Also the intensities were summed over azimuthal angle $\phi$. The intensity in the 5 detectors at $\theta=157.60^{\circ}$ (with azimuthal angles $\phi=0^{\circ}, \pm 72^{\circ}$, $\pm 144^{\circ}$ ) was measured in coincidence with the 10 detectors at $\theta \simeq 90^{\circ}$ (5 detectors at $\theta=85.84^{\circ}\left(\phi=180^{\circ}, \pm 108^{\circ}, \pm 36^{\circ}\right)$ and 5 detectors at $\left.\theta=94.16^{\circ}\left(\phi=0^{\circ}, \pm 72^{\circ}, \pm 144^{\circ}\right)\right)$, $\mathrm{I}(158,90)$, and similarly the intensity in the detectors at $\theta \simeq 90^{\circ}$ was measured in coincidence with those at $\theta=157.60^{\circ}, \mathrm{I}(90,158)$. As an independent check of the results, these measurements were repeated with the 10 detectors at $\theta=133.57^{\circ}\left(\phi= \pm 18^{\circ}, \pm 54^{\circ}, \pm 90^{\circ}\right.$, $\left.\pm 126^{\circ}, \pm 162^{\circ}\right)$, in place of the $157.60^{\circ}$ detectors to give the intensity values $I(134,90)$ and $\mathrm{I}(90,134)$. The angular-intensity ratios $\mathrm{I}_{90}^{158}=[\mathrm{I}(158,90)-\mathrm{I}(90,158)] /[\mathrm{I}(158,90)+\mathrm{I}(90,158)]$ and $\mathrm{I}_{90}^{134}=[\mathrm{I}(134,90)-\mathrm{I}(90,134)] /[\mathrm{I}(134,90)+\mathrm{I}(90,134)]$ were constructed.

| matrix | x axis | y axis |
| :---: | :---: | :---: |
| (i) | $\theta \simeq 90^{\circ}$ | $\theta=157.60^{\circ}$ |
| (ii) | $\theta \simeq 90^{\circ}$ | $\theta=133.57^{\circ}$ |

Table 4.3: Matrices used in the angular-intensity analysis.

In practice, the coincidence intensities between the different rings of detectors were extracted by constructing two matrices containing all coincidences between the detector rings, as shown in table 4.3. For example, the intensities $\mathrm{I}(90,158)$, that is the intensities of transitions in the ring of detectors at $\theta=\sim 90^{\circ}$ measured in coincidence with a gating transition in the ring of detectors at $\theta=157.60^{\circ}$, were measured by setting a gate on the $x$-axis of matrix (i). and measuring the transition intensities in the resulting spectrum. Conversely, the intensities $\mathrm{I}(158,90)$ were measured by gating on the y -axis and subsequently measuring the peak intensities.

Using transitions previously shown to be of stretched-dipole character [Bonin 85] as the gates, the results presented in figure 4.11, figure 4.12 , table C and table D were obtained. The values lie in the range $-0.03 \pm 0.03$ to $0.36 \pm 0.28$ for a stretched dipole and $-0.36 \pm 0.17$ to $-0.07 \pm 0.02$ for a stretched E 2 , with the respective weighted mean values $0.03 \pm 0.03$ and $-0.22 \pm 0.12$.

### 4.4.3 Intensities and $\mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2)$ Ratios

## Eurogam Detector Response

In order to measure the intensities of the gamma-ray transitions, the energy dependence of the relative efficiency of the gamma-ray detectors must be known. To this end, ${ }^{133} \mathrm{Ba}$, ${ }^{152} \mathrm{Eu}$ and ${ }^{182} \mathrm{Ta}$ sources were placed in the target-position before and after the in-beam experiment was carried out. The transition intensities were measured in the singles spectra, and were compared to the known actual relative intensities in order to deduce the relative efficiency. The curve shown in figure 4.13 is a 3rd-order polynomial fit made to the data points using the code GREMLIN (appendix D).

The finite time allowed between gamma-ray coincidences, means that low-energy gamma rays, which have poor timing, will be more likely to 'miss' the coincidence time window. There will be less chance of detecting these gamma rays, so the relative efficiency for low


Figure 4.11: Measured angle-dependent intensity ratios for ${ }^{220} \mathrm{Ra}$, plotted against gamma-ray energy.


Figure 4.12: Measured angle-dependent intensity ratios for ${ }^{222} \mathrm{Th}$, plotted against gamma-ray energy.
energies will be lower in coincidence than it is in singles. The usual method of calculating such coincidence efficiencies is to use a cascade of gamma-ray transitions in the level schemes of one of the nuclei populated, in which there is no feeding in or out or for which all the feeding can be accounted. This procedure is used and described in reference [Joyce 93].


Figure 4.13: A plot showing how Eurogam detector response varies with gamma-ray energy. The fit to the data is described in the text.

In the nuclei populated in this reaction, no such cascade of gamma-ray transitions exists from which such a measurement can be made. In order to measure the coincidence efficiency, use was made of the several coincident gamma rays following the ${ }^{152} \mathrm{Eu}$ and ${ }^{133} \mathrm{Ba}$ decay. For example, when two gamma rays, $A$ and $B$ directly feed each other, a gate on $A$ will see $B$ with an intensity given by

$$
\begin{equation*}
I_{B(\text { coinc })}=I_{A(\text { singles })} \varepsilon_{B} \tau_{B} \tag{4.1}
\end{equation*}
$$

where $I_{B(\text { coinc })}$ is the intensity of B in coincidence, $I_{A(\text { singles })}$ is the intensity of $A$ in singles, $\varepsilon_{B}$ is the singles efficiency at $B \mathrm{keV}$ and $\tau_{B}$ is the timing correction at $B \mathrm{keV}$ due to the coincidence time window. In this measurement it was assumed that the timing correction $>500 \mathrm{keV}$ was equal to unity.

Both singles efficiency and the efficiency corrected for coincidence-timing are plotted on
figure 4.13. A method to compare efficiency curves is to check the turnover point, that is the position of maximum efficiency, and the gradient beyond the turnover point. In these respects the efficiency curves measured here are similar to those obtained using coincident transitions in ${ }^{193} \mathrm{Hg}$ and ${ }^{152} \mathrm{Dy}$, from reference [Joyce 93].

## Intensity Measurements

The many contaminated transitions in this data set meant that measurement of the relative intensities was not an easy task. The relative intensities of the clean, uncontaminated transitions were measured in spectra gated on either the $2^{+} \rightarrow 0^{+}$or the $4^{+} \rightarrow 2^{+}$for each nucleus. In order to measure some of the higher lying transitions, some higher clean gates had to be set. In the case of doublets, it was necessary to gate on a transition which would eliminate one half of the doublet. In these cases appropriate normalizations were made.

As an illustrative example of the difficulties encountered with doublets and contaminants, consider one of the worst examples, that of the $367.8 \mathrm{keV}\left(11^{-} \rightarrow 9^{-}\right) / 368.1 \mathrm{keV}\left(10^{+} \rightarrow 8^{+}\right)$ doublet in ${ }^{222} \mathrm{Th}$. Gating on the $182.9 \mathrm{keV}\left(2^{+} \rightarrow 0^{+}\right)$would bring in contamination of the $367.6 \mathrm{keV}\left(13^{-} \rightarrow 11^{-}\right) / 368.2 \mathrm{keV}\left(12^{+} \rightarrow 10^{+}\right)$doublet in ${ }^{220} \mathrm{Ra}$, via the $184.9 \mathrm{keV}\left(7^{-} \rightarrow 6^{+}\right)$ and $182.4 \mathrm{keV}\left(19^{-} \rightarrow 18^{+}\right)$in ${ }^{220} \mathrm{Ra}$. A gate on the the $256 \mathrm{kev}\left(4^{+} \rightarrow 2^{+}\right)$would select the ${ }^{222} \mathrm{Th}$ channel, but would see both parts of the ' $367 / 368$ doublet'. A gate on the 331.6 keV $\left(9^{-} \rightarrow 11^{-}\right)$would not see the 361.8 keV part of the doublet, but would bring in contamination via the $332.7 \mathrm{keV}\left(11^{-} \rightarrow 9^{-}\right)$in ${ }^{220} \mathrm{Ra}$. The way in which this problem was resolved was to set an 'and' gate on the 256.3 keV , 'and' the $389.8 \mathrm{keV}\left(12^{+} \rightarrow 10\right)$, the former to select the reaction channel, the latter to eliminate unwanted coincidences, that is that of the $11^{-} \rightarrow 9^{-}$. By measuring the intensity of the 367.8 keV transition in the spectrum gated in such a manner, relative to some other clean transition nearby such as the 170.1 kev $\left(8^{+} \rightarrow 7^{-}\right)$the intensity of the 367.8 keV transition can be normalized to the intensities of transitions measured in other gates. Once the intensity of the 367.8 keV transition is known this can be subtracted from the intensity of the combined $367.8 \mathrm{keV} / 368.1 \mathrm{keV}$ doublet in order to give the intensity of the 367.8 keV part of the doublet.

The intensities measured here are presented on the level schemes of figure 4.8 and in tables $A$ and $B$ of the appendix. In cases where different gates and normalizations have been used, the error on the peak area is increased accordingly. Doublets and triplets in ${ }^{222} \mathrm{Th}$, ${ }^{220} \mathrm{Ra}$ and ${ }^{229} \mathrm{Ra}$ are indicated in tables A and B of the appendix. The intensities are also presented on figure 4.22 and are discussed in section 4.6.

## B(E1)/B(E2) Ratios and Electric Dipole Moments

A measure of the E1 strength in such octupole bands can be made using the ratio of reduced transition probabilities $\mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2)$. If the intensities of the E 1 and E 2 transitions are $\mathrm{I}(\mathrm{E} 1)$ and $\mathrm{I}(\mathrm{E} 2)$ respectively, then the $\mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2)$ ratios can be calculated using the expression,

$$
\begin{equation*}
\frac{I(E 1)}{I(E 2)}=\frac{1.587 \times 10^{15} E_{\gamma}^{3} B(E 1)}{1.223 \times 10^{9} E_{\gamma}^{5} B(E 2)} \tag{4.2}
\end{equation*}
$$

The values thus obtained are plotted on figure 4.14 and given in tables A and B. Assuming that the electric quadrupole moments are constant, and taking the weighted mean $B(E 1) / B(E 2)$ ratios, values for the electric dipole moment for each nucleus can be inferred, using equations given in section 3.2 in the form,

$$
\begin{equation*}
\frac{B(E 1)}{B(E 2)}=2.4\left[\frac{D_{0}}{Q_{0}} \frac{<I_{i} K_{i} 10 \mid\left(I_{i}-1\right) K_{f}>}{\left\langle I_{i} K_{i} 20\right|\left(I_{i}-2\right) K_{f}>}\right]^{2} . \tag{4.3}
\end{equation*}
$$

The values used, and the inferred electric dipole moments are presented in table 4.4 below and are discussed later in this chapter.

|  | $\frac{B(E 1)}{B(E 2)}\left(10^{-6} \mathrm{fm}^{-2}\right)$ | $\mathrm{Q}_{0}\left(\mathrm{efm}^{2}\right)$ | $\mathrm{D}_{0}(\mathrm{efm})$ |
| :---: | :---: | :---: | :---: |
| ${ }^{222} \mathrm{Th}$ | $3.809(0.185)$ | $548(23)^{a}$ | $0.55(0.03)$ |
| ${ }^{220} \mathrm{Ra}$ | $1.647(0.249)$ | $501(40)^{b}$ | $0.34(0.04)$ |

Table 4.4: The weighted mean $B(E 1) / B(E 2)$ ratios and inferred electric dipole moments. The quadrupole moments were assumed to be constant within each band: ${ }^{a}$ is measured in reference [Bonin 85]; and ${ }^{b}$ is a mean value of that measured for ${ }^{218} \mathrm{Ra}$ and ${ }^{222} \mathrm{Ra}$ [Raman 87] (Note that Grodzins systematics [Grodzins 62] are predicted to be unreliable in this mass region [Gai 88]).

### 4.5 Multiplicity Measurement

### 4.5.1 Motivation

It is clear from section 4.4.1 and figure 4.8, that the yrast octupole band in ${ }^{220} \mathrm{Ra}$ is observed to continue to significantly higher spins than that in ${ }^{222} \mathrm{Th}$, despite ${ }^{222} \mathrm{Th}$ being the stronger of the two reaction channels. Two possible explanations for this observation are discussed in section 4.6, and are outlined below.
(i). The band-crossing mentioned in section 4.2 causes the octupole band to become nonyrast so that the intensity is shared over several states, and depopulating transitions are


Figure 4.14: $\mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2)$ ratios plotted against spin for ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$.
too weak to be observed.
or (ii). The probability of fission on the path from the compound nucleus to the evaporation residues, being proportional to $\mathrm{Z}^{2} / \mathrm{A}$, will be larger for ${ }^{222} \mathrm{Th}$. The mean angular momentum populated in ${ }^{222} \mathrm{Th}$ would therefore be lower than that in ${ }^{220} \mathrm{Ra}$.

A measurement of the multiplicities of the two reaction channels would provide an indication of the mean angular momenta of the states that are populated, and help resolve this problem. A method of extracting multiplicities from the ratio of $n$-fold to $(n+1)$-fold gamma-gamma coincidence data was described in section 3.5. Unfortunately, the mastertrigger of 5 unsuppressed germanium coincidences, which was used when collecting the data with Eurogam, meant that it was not possible to extract multiplicities using those data. This prompted the need for a second experiment to be performed.

### 4.5.2 The Multiplicity Experiment

An experiment to measure the multiplicities of the two reaction channels was performed using the K-130 cyclotron at the Accelerator Laboratory of the University of Jyväskylä in

Finland. To be able to infer anything about the results from the Eurogam data set, the reaction used had to be identical to that used for the experiment with Eurogam, that is ${ }^{18} \mathrm{O}$ ions at 95 MeV bombarding two stacked targets of ${ }^{208} \mathrm{~Pb}$, each of thickness $500 \mu \mathrm{~g} / \mathrm{cm}^{2}$. For this experiment, de-excitation gamma rays were collected with the array called TARDIS ${ }^{4}$ and a 3 inch $\times 3$ inch sodium-iodide detector. In brief, TARDIS consisted of $12 \sim 20 \%$ efficiency TESSA-type detectors placed $\sim 150 \mathrm{~mm}$ from the target, arranged in two rings of 6 with a cone angle of $54.7^{\circ}$. The sodium-iodide detector was placed downstream of the TARDIS array, approximately 350 mm from the targets, at the polar position $\theta=22.7^{\circ}$, $\phi=1.43^{\circ}$. The arrangement of detectors is shown schematically in figure 4.15.


Figure 4.15: A schematic representation of the positions of the germanium detectors and the sodium-iodide detector with respect to the beam and target. The angles and distances are scaled, although the detectors are not drawn to scale.

A block diagram of the electronics associated with the TARDIS array and the sodium-iodide detector is shown in figure 4.16. For clarity the germanium and suppression-shield electronics, which were described in section 3.8, are omitted. Three TACs were used. One TAC was started by the sodium-iodide detector and stopped by any germanium timing signal. This TAC was used in order to count the number of germanium-sodium iodide coincidences. Another TAC was started by the RF signal from the cyclotron and stopped by any germanium-sodium iodide coincidence. (This was used to eliminate unwanted coincidences with background gamma rays from a Faraday cup, placed about a metre up-stream of the array).

### 4.5.3 Data

Data were collected until sufficient numbers of germanium and germanium-sodium iodide coincidences had been collected to deduce information about the multiplicities, that is for

[^6]

Figure 4.16: A block diagram of the electronics used in the multiplicity-measurement experiment. The TARDIS germanium and BGO-suppression electronics are curtailed for clarity.
about 24 hours. In total 390 million singles, 9 million double, 0.8 million triple unfolded germanium coincidences, and 5.5 million germanium-sodium iodide coincidences were collected. As mentioned earlier the number of double and triple coincidences could be increased by unfolding the higher-fold events.

### 4.5.4 Analysis

The initial intention of the experiment was to use the equation,

$$
\begin{equation*}
N_{2}=N_{1} \Omega(M-1), \tag{4.4}
\end{equation*}
$$

to determine the absolute multiplicities in each reaction channel. This involves measuring the number of singles events in the germanium detectors, $N_{1}$, and the number of germaniumsodium iodide coincidences, $N_{2}$. Using source data to measure the absolute efficiency, $\Omega$, of the sodium-iodide detector then allows the multiplicity $M$ to be evaluated. If $N_{2}$ and $N_{1}$ are measured using known gamma rays in a particular reaction channel, then M is the multiplicity of that particular channel.

The number of coincidences between the sodium-iodide detector and the germanium detectors, was measured by gating the germanium-sodium iodide TAC by a transition of interest and measuring the number of counts recorded by the TAC. The absolute efficiency of the sodium-iodide detector was measured using some of the coincident transitions in the source calibration data, in a similar method to that described in section 4.4.3.

Unfortunately, due to poor peak shapes and contaminants in the spectra, it was only possible to determine the absolute multiplicity for the ${ }^{222} \mathrm{Th}$ channel, and even in that case the associated errors were very large.

It was not possible to measure the absolute efficiency of the germanium detectors accurately, so the absolute multiplicities could not be determined from the germanium-detector data alone. It was, however, possible to extract the relative multiplicities of the reaction channels using equation 3.40 in the form,

$$
\begin{equation*}
\frac{N_{3}}{N_{2}}=k(M-2) \tag{4.5}
\end{equation*}
$$

where $k$ is a constant of proportionality, $N_{2}$ is the number of double germanium-detector coincidences, $N_{3}$ is the number of triple germanium-detector coincidences, and $M$ is the multiplicity. This method had the advantage that the double and triple coincidences could be 'double-gated' in order to define the reaction channel. This meant that the numbers of coincidences could be measured with high accuracy because the spectra were less contaminated and had less background than when using singles and double coincidences in the in the absolute multiplicity measurement. In practice the numbers of double and triple germanium-detector coincidences were measured by constructing two matrices as follows.

Doubles Matrix: All gamma-ray coincidences of fold $\geq 2$ were decomposed into doubles, $\gamma_{1}$ and $\gamma_{2}$, and the matrix was incremented at $\gamma_{1} \gamma_{2}$ and $\gamma_{2} \gamma_{1}$ as described in section 3.9. By gating on the matrix and integrating peaks in the projected spectra, the number of double coincidences involving both the gate and the integrated peak is obtained.

Triples Matrix: All gamma-ray coincidences of fold $\geq 3$ were decomposed into triples $\gamma_{1}, \gamma_{2}$ and $\gamma_{3}$ and the matrix was incremented at $\gamma_{1} \gamma_{2}, \gamma_{1} \gamma_{3}, \gamma_{2} \gamma_{3}, \gamma_{2} \gamma_{1}, \gamma_{3} \gamma_{1}$ and $\gamma_{3} \gamma_{2}$. Similarly, the numbers of triples is measured.

The gating transitions for ${ }^{220} \mathrm{Ra}$ were $128.1 \mathrm{keV}, 152.4 / 153.6 \mathrm{keV}$ (doublet), 162.5 keV , $178.4 \mathrm{keV}, 184.9 \mathrm{keV}, 215.1 \mathrm{keV}, 224.6 \mathrm{keV}, 231.6 \mathrm{keV}$ and 278.1 keV .

The gating transitions for ${ }^{222}$ Th were $161.1 / 161.2 \mathrm{keV}$ (doublet), $170.1 \mathrm{keV}, 173.1 \mathrm{keV}$, $206.4 / 206.9 \mathrm{keV}$ (doublet), $211.1 \mathrm{keV}, 228.5 \mathrm{keV}, 244.6 \mathrm{keV}, 256.0 / 256.2 / 256.3 \mathrm{keV}$ (triplet) and 310.2 keV .

In this analysis it was necessary to correct for the effect of evaporation neutrons which cause spurious counts in the gamma-ray detectors from ( $\mathrm{n}, \mathrm{n}^{\prime}$ ) reactions on, for example, germanium, bismuth (of the BGO) and aluminium (of the detector can). In refer-


Figure 4.17: An example of one of the germanium singles spectra used to determine the absolute multiplicity of the ${ }^{222} \mathrm{Th}$ reaction channel.
ence [Holzmann 87], the neutron-induced counts for the reaction ${ }^{112} \mathrm{Cd}\left({ }^{36} \mathrm{~S}, 4 \mathrm{n}\right){ }^{148} \mathrm{Gd}$ are found to constitute $3.1 \%$ of the total spectrum recorded by a BGO-suppressed HPGe detector at $34^{\circ}$ to the beam line, $1.2 \%$ of the total spectrum recorded at $90^{\circ}$, and $0.8 \%$ at $146^{\circ}$. The multiplicity of the ${ }^{112} \mathrm{Cd}\left({ }^{36} \mathrm{~S}, 4 \mathrm{n}\right){ }^{148} \mathrm{Gd}$ reaction-channel is about 25 [Holzmann 89], and so the ratio of neutrons ( 4 n channel) to gamma rays (multiplicity 25 ) is about $1 / 6$. In this multiplicity measurement, the ratio of evaporation neutrons to gamma rays is about $1 / 3$ for ${ }^{222} \mathrm{Th}$ and $1 / 6$ for ${ }^{220} \mathrm{Ra}$. Taking the $90^{\circ}$ result from [Holzmann 87] as an average ${ }^{5}$ the correction for evaporation neutrons was about $2 \%$ for ${ }^{222} \mathrm{Th}$ and $1 \%$ for ${ }^{220} \mathrm{Ra}$.

### 4.5.5 Results

## Absolute Multiplicities

Due to poor peak shapes and contaminants in the singles spectra, it was only possible to obtain a value of the absolute multiplicity for ${ }^{222} \mathrm{Th}$. Taking a weighted mean of the values obtained when using the $182.9 \mathrm{keV}\left(2^{+} \rightarrow 4^{+}\right)$and the $256.3 \mathrm{keV}\left(4^{+} \rightarrow 2^{+}\right)$transitions as gates, the multiplicity was found to be $12.7 \pm 1.6$. The absolute efficiency of the sodium

[^7]|  | Doubles | Triples | $\frac{\text { Triples }}{\text { Doubles }}$ |
| :---: | :---: | :---: | :---: |
| ${ }^{222} \mathrm{Th}$ | $61015 \pm 848$ | $7406 \pm 312$ | $8.24 \pm 0.36$ |
| ${ }^{220} \mathrm{Ra}$ | $10277 \pm 368$ | $1256 \pm 75$ | $8.18 \pm 0.57$ |

Table 4.5: Relative multiplicities from the doubles and triples analysis
iodide detector was measured to be $1.006(0.024) \times 10^{-3}$. The largest sources of error were poor peak shapes in the singles spectra causing difficulty in determining the numbers of events recorded by the sodium iodide-germanium TAC.

## Relative Multiplicities

The results obtained from the triples and doubles matrices are summarized in the table 4.5. The ratio of multiplicities ${ }^{222} \mathrm{Th} /{ }^{220} \mathrm{Ra}$ is $1.01 \pm 0.07$. The largest contribution to this $\sim 7 \%$ error arising from fits to the weak peaks in the triples spectra. It was possible to obtain the relative multiplicities with small errors because of the high quality of the spectra that were obtained using double gates.

### 4.6 Discussion

### 4.6.1 The Level Scheme

The yrast states of ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ form bands of alternating parity. Interband E1 transitions connect the states of $\mathrm{I}^{\pi}=\mathrm{I}^{+/-}$to states of $\mathrm{I}^{\pi}=(\mathrm{I}-1)^{-/+}$and they compete very strongly with the intraband E 2 transitions which connect $\mathrm{I}^{\pi}=\mathrm{I}^{+/-}$to $\mathrm{I}^{\pi}=(\mathrm{I}-2)^{+/-}$. These observations are characteristic of the rotation of a nucleus which is octupole deformed (section 2.3.1). The octupole band-structure has long been firmly established: it is the purpose of this work to present an extension of the bands and to discuss their behaviour with respect to cranked reflection-asymmetric mean-field calculations of [Nazarewicz 87].

The level schemes of ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ are shown in figure 4.8. There are three arguments supporting the spin and parity assignments of the levels identified in this work:
(i). Opposite-parity bands of stretched-E2 transitions connected by stretched-E1 transitions, have been observed and confirmed in ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ previously. The 'new' levels identified in this work appear to continue this band structure.
(ii). A consideration of internal conversion supports the assignment of electric, rather than magnetic, character to the interband transitions. Correcting the intensities using the E1
conversion coefficients leads to more plausible relative intensities than if the M1 coefficient is used. As an example consider transitions feeding in and out of the $18^{+}$state. The E1 internal-conversion coefficients, given by reference [Rösel 78] for the populating (206.9 keV ) and depopulating ( 260.9 keV ) dipole transitions are 0.0880 and 0.0516 respectively. The corresponding M1 internal-conversion coefficients being 2.55 and 1.34 respectively, also from [Rösel 78]. Considering the dipoles to be of E1 character, the total internal-conversion corrected populating intensity would be 30 , relative to a $4^{+}$to $2^{+}$intensity of 128 (see figure 4.8 and its caption for explanation). The total depopulating intensity would be 28. However, if the dipoles are assumed to be M1 then the populating intensity would be 85 and the depopulating intensity would be 55 , that is there is a larger discrepancy. This argument in itself however, is not conclusive due to the very large errors on the gamma-ray intensities, and the possibility of feeding out of the band. All of the measured intensities are tabulated in appendices A and B.
(iii). The angular-intensity analysis, supported the assignment of stretched-dipole and stretched-quadrupole nature to the transitions. The angular-intensity ratios showed the transitions which were already established to be of stretched-dipole or stretched-quadrupole character to lie in two distinct regions. Using E1 transitions as gates, the values of the angular-intensity ratios were found to be approximately given by $I_{90}^{134}, I_{90}^{158}>1$ for a stretched dipole, and, $\mathrm{I}_{90}^{134}, \mathrm{I}_{90}^{158}<1$ for a stretched quadrupole. Hence the multipole character of the new transitions was assigned according to the region in which its angular-intensity ratio fell. All spin and parity assignments that were guessed using argument (i) above, were confirmed using the angular-intensity analysis up to $20^{+}$in ${ }^{222} \mathrm{Th}$ and $26^{+}$in ${ }^{220} \mathrm{Ra}$.

### 4.6.2 Properties of the Octupole Bands

A nucleus which is purely vibrational will have $\mathrm{E}\left(4^{+}\right) / \mathrm{E}\left(2^{+}\right)=2$. This value is 2.3 for ${ }^{220} \mathrm{Ra}$ and 2.4 for ${ }^{222} \mathrm{Th}$, indicating that ${ }^{220} \mathrm{Ra}$ is more vibrational than ${ }^{222} \mathrm{Th}$ at low spins. At high-spins the bands behave very similarly, and show spin and parity patterns characteristic of the rotation of an octupole-shaped nucleus.

## Parity Splitting and Rotational Frequency Ratios

In an octupole-deformed even-even nucleus, the ground-state band and the low-lying negative parity band should should form a single $s=1$ band. The properties of this band can be conveniently illustrated by plotting the parity splitting and frequency ratios, in a manner suggested by Nazarewicz and Olanders [Nazarewicz 85], as described in chapter 2. These


Figure 4.18: The parity splitting, $\delta \mathrm{E}$, for the nuclei studied in this work, and for the even thorium isotopes. For an octupole rotor $\delta \mathrm{E}$ should tend towards zero.


Figure 4.19: The ratio of rotational frequencies plotted for the nuclei studied in this work, compared to neighbouring even thorium isotopes. The ratio should equal 1 for an octupole rotor, that is the rotational frequencies in the positive and negative parity bands should be the same.
quantities are plotted for ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ in figures 4.18 and 4.19 , respectively. Shown on the plots for comparison are the even thorium isotopes with $130 \leq \mathrm{N} \leq 142,(\neq 140)$. The nuclei ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ appear to behave very similarly. On figure 4.18 , the stable octupole deformed limit is crossed at $\sim 9 \hbar$, by ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$. The reason for the parabolic shape for ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ is not understood. It may arise as a consequence of band-mixing with the reflection-symmetric four quasi-particle band (section 4.6.5). On the frequency ratio plot, ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ cross the stable octupole-deformed limit in the region of $13 \hbar$, beyond which the values follow the stable octupole-deformed limit. In contrast, the frequency ratios for the octupole-vibrational nucleus, ${ }^{232} \mathrm{Th}$, tend towards the octupole-vibrational limit.

## Moments of Inertia

The static moments of inertia are shown in figure 4.20. Also shown for comparison are the moments of inertia for ${ }^{218} \mathrm{Ra}$ and ${ }^{220} \mathrm{Th}$. Clearly the addition of two neutrons causes an increase in collectivity, and ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ have smoother moments of inertia. Contrary to what may be expected of octupole bands the moments of inertia of ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ do not merge at high rotational frequencies, but remain distinguishable.


Figure 4.20: The static moments of inertia plotted against rotational frequency for both the positive and negative parity bands in $\mathrm{Z}=88,90, \mathrm{~N}=130,132$ nuclei.

### 4.6.3 $B(E 1) / B(E 2)$ Ratios and Electric Dipole Moments

A measure of the E1 strength in octupole deformed nuclei is usually made by comparing $B(E 1) / B(E 2)$ ratios. Using the equations given in section 3.2, the $B(E 1) / B(E 2)$ ratios plotted on figure 4.14 and given in appendices A and B were obtained. Using the weighted mean of these values, average electric dipole moments were deduced.

The values of $0.34(4)$ efm for ${ }^{220} \mathrm{Ra}$, and $0.55(3)$ efm for ${ }^{222} \mathrm{Th}$, are larger than both the previously measured and theoretical values. In fact, the value of $0.55(3)$ efm for ${ }^{222} \mathrm{Th}$ is the largest electric dipole moment ever measured, ever larger than that of $0.44(9)$ efm for ${ }^{223} \mathrm{Th}$ reported by Dahlinger et al. [Dahlinger 88]. Table 4.6 compares theoretical and experimental electric dipole moments for some even-even octupole-deformed radium and thorium isotopes.

|  | Theoretical |  |  |  |  | Experimental |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | [Denisov 89] | [Leander 83] | [Butler 91] | [Egido 90] | Previous | This Work |  |
| Radiums |  |  |  |  |  |  |  |
| ${ }^{218} \mathrm{Ra}$ | 0.207 | -0.07 | 0.28 |  | $0.23(5)^{a}$ |  |  |
| ${ }^{220} \mathrm{Ra}$ | 0.250 | 0.19 | 0.17 | 0.37 | $0.27(7)^{b}$ | $0.34(4)$ |  |
| ${ }^{222} \mathrm{Ra}$ | 0.291 | 0.18 | 0.09 | 0.17 | $0.38(6)^{c}$ |  |  |
| ${ }^{224} \mathrm{Ra}$ | 0.271 | 0.12 | 0.01 | 0.046 | $0.028(4)^{d}$ |  |  |
| ${ }^{226} \mathrm{Ra}$ | 0.066 | 0.05 | -0.09 | 0.27 | $0.103(12)^{e}$ |  |  |


| Thoriums |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| ${ }^{220} \mathrm{Th}$ |  | 0.20 | 0.34 |  | $0.25(3)^{f}$ |  |
| ${ }^{222} \mathrm{Th}$ |  | 0.32 | 0.30 | 0.48 | $0.38(7)^{g}$ | $0.55(3)$ |
| ${ }^{224} \mathrm{Th}$ |  | 0.33 | 0.27 |  | $0.41(5)^{h}$ |  |
| ${ }^{226} \mathrm{Th}$ |  | 0.29 | 0.16 |  | $0.30(3)^{h}$ |  |
| ${ }^{228} \mathrm{Th}$ |  | 0.10 | 0.07 |  | $0.04(1)^{h}$ |  |

Table 4.6: Comparing experimental and theoretical values for the electric-dipole moments. All electric dipole moment values are in efm. The superscripts on the previously-measured values indicate the references from which they were taken: $a$ [Gai 88a]; $b$ [Burrows 84]; $c$ [Ruchowska 92]; $d$ [Poynter 89]; e [Wollersheim 93]; $f$ [Bonin 83]; $g$ [Bonin 85]; and $h$ [Schüler 86]. The spin ranges for which the values apply are given in the publications.

The theoretical values given in table 4.6 are calculated using different methods:
(i). Denisov [Denisov 89] has applied the droplet model, and only taken into account macroscopic properties;
(ii). Leander et al. [Leander 82] have used the liquid-drop model plus a shell-correction term;
(iii). Butler and Nazarewicz [Butler 91] have used the droplet model with a shell-correction term; and
(iv). Egido and Robledo [Egido 89] have calculated the $\mathrm{B}(\mathrm{E} 1)$ values using a microscopic Hartree Fock/BCS theory with Gogny forces.

The droplet model and the liquid-drop model-plus-shell correction both underestimate the experimental $D_{0}$ values. Even the recent calculations of Butler and Nazarewicz, which successfully reproduce the small $\mathrm{D}_{0}$ in ${ }^{224} \mathrm{Ra}$, via a cancellation of macroscopic and microscopic terms, are not as successful in reproducing the values measured here. It can be seen from the table that their droplet model-plus-shell correction approach underestimates all the experimental $D_{0}$ values in the $\mathrm{N} \simeq 132$ isotopes of radium and thorium.

The $\mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2)$ ratios are re-plotted on figure 4.21 in a different manner in order that they might be compared to the $D_{0} / Q_{0}$ values from a recent systematic study of neighbouring octupole deformed actinides by Ackerman et al. [Ackerman 93]. The relationship between $B(E 1) / B(E 2)$ and $D_{0} / Q_{0}$ is simply,

$$
\begin{equation*}
\frac{B(E 1)}{B(E 2)}=\frac{16}{5} \frac{(2 I-1)}{(2 I-2)}\left[\frac{D_{0}}{Q_{0}}\right]^{2} . \tag{4.6}
\end{equation*}
$$

The values measured in this work for ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ are of the same order as the other nuclei plotted, with ${ }^{222} \mathrm{Th}$ having on average larger values than the other actinides as may be expected from the discussion of the previous paragraphs. The $D_{0} / Q_{0}$ values in ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ appear to be independent of spin. This is in agreement with the work of Leander et al. [Leander 82] who, despite reporting a calculated drop in $D_{0}$ from 0.19 efm to 0.13 efm as the nuclear spin drops from $\sim 6 \hbar$ to $17 \hbar$, also report a drop in $Q_{0} 4.6 \mathrm{eb}$ to 4.3 eb , concluding that the ratio $D_{0} / Q_{0}$ would stay approximately constant.

### 4.6.4 Multiplicities and Mean Angular Momenta

Despite the very similar behaviour of these two yrast octupole bands, as described in the previous two sections, it is apparent from the level schemes that the bands behave differently at high spin. In the stronger reaction channel, ${ }^{222} \mathrm{Th}$, no states are observable above $25 \hbar$


Figure 4.21: $\mathrm{D}_{0}$ to $\mathrm{Q}_{0}$ ratios. All the values apart from the nuclei studied in this work were taken from [Ackerman 93].
while the band in ${ }^{220} \mathrm{Ra}$ is observed to continue to $31 \hbar$. One possible reason for this might be that the band in ${ }^{222} \mathrm{Th}$ is more-weakly populated than the band in ${ }^{220} \mathrm{Ra}$, and hence is not observable as high in spin. Figure 4.22 shows that this is not the case. In figure 4.22 the intensities depopulating the levels above $12 \hbar$, normalized to the $4^{+}$to $2^{+}$in ${ }^{222} \mathrm{Th}$ are plotted against spin. The depopulating intensity is calculated by summing the E1 and E2 intensities. On this plot the ${ }^{222} \mathrm{Th}$ intensities are higher and fall to zero much faster than the ${ }^{220} \mathrm{Ra}$ values, which fall off more gradually, although the error bars on the plot are large. There must be an alternative reason for the behaviour of the bands. Two possible explanations are:
(i). The band-crossing with the reflection-symmetric four quasi-particle band mentioned in section 4.2, and discussed in section 4.6.5, may cause the octupole band to become non-yrast. The intensity will therefore be shared over several states and any depopulating transitions would be too weak to be observed. Or,
(ii). The probability of fission on the path from the compound nucleus to each evaporation residue, being proportional to $\mathrm{Z}^{2} / \mathrm{A}$, will be larger for ${ }^{222} \mathrm{Th}$. The mean angular momentum


Figure 4.22: A comparison of the intensities depopulating the high-spin states in ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$. The intensities are normalized to the $4^{+} \rightarrow 2^{+}$in ${ }^{222} \mathrm{Th}$.
in ${ }^{222} \mathrm{Th}$ would therefore be lower than that in ${ }^{220} \mathrm{Ra}$. Considering the formation of ${ }^{220} \mathrm{Ra}$, if during the particle-evaporating stage the alpha particle is emitted before either of the neutrons, then the probability of fission will be reduced at the remaining neutron emitting steps, and the evaporation residue will, on average, hold more angular momentum. For the final evaporation residue to be ${ }^{222} \mathrm{Th}$, it must retain the alpha particle, but emit four neutrons. There are more particle emission steps, with fission competing at each step and the probability of fission is larger because of the higher $\mathrm{Z}^{2} / \mathrm{A}$ value. To survive fission the neutrons must remove more angular momentum on average and hence the mean angular momentum in ${ }^{222} \mathrm{Th}$ will be lower than that in ${ }^{220} \mathrm{Ra}$.

The absolute value of the multiplicity measured for ${ }^{222} \mathrm{Th}$, of $12.7(16)$, is in good agreement with the $13.0(5)$ measured by Ward et al. [Ward 83] for the same reaction with a beam energy of 93 MeV . The relative multiplicities of the channels were measured and the ratio of relative multiplicities for ${ }^{220} \mathrm{Ra}$ to ${ }^{222} \mathrm{Th}$ was found to be $1.01(7)$, that is the multiplicities of the reaction channels are very nearly equal. If the intensity values on the level schemes are normalized so that the $4^{+}$to $2^{+}$transition in each nucleus is 1 , and the intensities are
summed, then sums are $13.9(9)$ for ${ }^{220} \mathrm{Ra}$ and $9.9(2)$ for ${ }^{222} \mathrm{Th}$. This takes into account the fact that the E1 transitions are stronger in ${ }^{222} \mathrm{Th}$.

There are, therefore, less gamma rays observed in ${ }^{222} \mathrm{Th}$ than there are in ${ }^{220} \mathrm{Ra}$, yet the multiplicities are very nearly the same. This implies that there are more gamma rays feeding the yrast band in ${ }^{222} \mathrm{Th}$ but they are of insufficient intensity to be observed. The very similar multiplicities, and therefore similar mean angular momenta rule out the possibility of different fission limits. The difference in behaviour can therefore be construed as evidence for the crossing with the reflection-symmetric band, which causes the octupole configuration to become non-yrast.

### 4.6.5 Comparison with Cranking Calculations

The rotation of an octupole-shaped nucleus was first investigated by Nazarewicz et al. [Nazarewicz 84a] who performed Woods-Saxon cranking calculations for the nucleus ${ }^{222} \mathrm{Th}$. They chose ${ }^{222} \mathrm{Th}$ for their calculations, because, already in 1984, it had experimental data [Ward 83] suggestive of octupole deformation, and they performed calculations both with ( $\beta_{3}=0.096$ ) and without ( $\beta_{3}=0$ ) octupole deformation.

Solution of the cranking-equations [Szymanski 83] gave results like those presented in figure 4.23 which shows quasi-neutron Routhians for $\beta_{3}=0,0.1$ and 0.14 . There is an obvious difference between the calculations with $\beta_{3}=0$ and those with $\beta_{3} \neq 0$. At $\beta_{3}=0$, with no octupole deformation, the $j_{\frac{1}{2}}$ intruder states cannot mix with the normal-parity states of the $\mathrm{N}=126-182$ shell because they are of opposite parity. These orbitals slope down steeply which indicates that they acquire a large alignment (the alignment being proportional to the negative-slope of the Routhian). An upbend or a backbend is predicted at low rotational frequency ( $\simeq 0.2 \mathrm{MeV} / \hbar$ ). With $\beta_{3} \neq 0$, no orbital slopes down more than any of the others and all orbits have about the same alignment, and in this case a backbend is not predicted. This is due to a fundamental property of the octupole potential in that it can mix high-j opposite-parity intruder states from the next shell up, with the normal parity states of a given shell. The high-j intruders are therefore fragmented over several orbits by the octupole interaction. Hence, an early success of the octupole model was that it explained why a backbend predicted at low spin in ${ }^{222} \mathrm{Th}$ [Dudek 82] was not observed [Nazarewicz 84a].

Using the same cranking approach, Nazarewicz, Leander and Dudek [Nazarewicz 87] went on to systematically study the shapes of the radium and thorium nuclei with $130 \leq N \leq 138$


Figure 4.23: Quasi-neutron Routhians for $N=134$ calculated for $\beta_{3}=0,0.1$ and 0.14 . Taken from [Nazarewicz 85].
at high spin. Detailed specification of the model is given in the reference. In brief, they used a Woods-Saxon potential, with central, spin-orbit and Coulomb terms together with a BCS pairing field that was gradually decreased with rotational frequency, rather than disappearing at a critical value. Deformation parameters $\hat{\beta}=\beta_{2} \rightarrow \beta_{6}$ and pairing parameters $\Delta_{n}$ and $\Delta_{p}$ were allowed to vary and the Routhian $\mathrm{H}^{\omega}=\mathrm{H}-\omega \mathrm{I}_{x}$ was minimized with respect to variations in the mean field. At a certain $\hat{\beta}$ this is achieved by solving the cranked HFB [Szymanski 83] (see chapter 1) equations and from there it is possible to obtain the angular momentum and energy.

Total Routhians were calculated in order to look for minima in the $\beta_{2}-\beta_{3}$ plane. The shapes of the radium and thorium nuclei into four regions depending on the degree of octupole and
quadrupole deformation. The results are summarized in figure 4.24 which was taken from the publication, and were also presented in figure 2.8.


Figure 4.24: Theoretical predictions of the yrast shapes of doubly-even (a) ${ }^{218-226} \mathrm{Ra}$ and (b) ${ }^{220-228} \mathrm{Th}$ nuclei at rotational frequencies going from 0 to $0.30 \mathrm{MeV} / \hbar$ in steps of $0.05 \mathrm{MeV} / \hbar$.

In the context of rotation ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ are interesting nuclei to study and compare, because they are isotonic even-even neighbours, with well established octupole bands, and they are predicted to behave differently at high spin. Theoretically, ${ }^{220} \mathrm{Ra}$ has a spherical ground-state shape, which is predicted to become octupole deformed at low rotational frequency, and remain octupole deformed up to very high spins. The isotone ${ }^{222} \mathrm{Th}$, however is predicted to lose its octupole band at about spin $26 \hbar$ due to a crossing with a reflectionsymmetric four-quasiparticle band.

In [Nazarewicz 87], more detailed calculations were carried out for ${ }^{222} \mathrm{Th}$, both at the reflection-asymmetric shape $\left(\beta_{2}=0.116, \beta_{3}=0.104\right)$ and at the reflection-symmetric shape $\left(\beta_{2}=0.120, \beta_{3}=0\right)$ associated with the aligned four-quasiparticle configuration. The calculated aligned angular momenta for the different configurations are presented in figure 4.25. At about $\hbar \omega=0.25 \mathrm{MeV}$ the octupole band is predicted to cross the four-quasiparticle $\nu\left(\mathrm{j}_{\frac{12}{2}}\right)^{2} \pi\left(\mathrm{i}_{\frac{13}{2}}\right)^{2}$ highly-aligned band which then becomes yrast, causing a shape transition to reflection symmetry.


Figure 4.25: Aligned angular momenta calculated as a function of rotational frequency for ${ }^{222} \mathrm{Th}$, both with (thick black curve) and without (dashed curves) octupole deformation.


Figure 4.26: The level structure of ${ }^{150} \mathrm{Sm}$ : an example of the crossing of an octupole band by a reflectionsymmetric band. Taken from [Urban 87].

The experimental data from this work are plotted on figure 4.25 for both ${ }^{222} \mathrm{Th}$ and ${ }^{220} \mathrm{Ra}$, along with the aligned angular momenta calculated for ${ }^{222} \mathrm{Th}$. The ${ }^{222} \mathrm{Th}$ data are in very good agreement with the prediction for the $\beta_{3}=0$ configuration (thick black line), but do not extend beyond the region of the predicted shape transition; the data for ${ }^{220} \mathrm{Ra}$ go to higher rotational frequencies.

The only other observation of the crossing of an octupole band with a reflection-symmetric band is that in ${ }^{150} \mathrm{Sm}$, observed by Urban et al. [Urban 87]. In that case, an octupole band develops at $\sim 7 \hbar$ but is lost at $\sim 15 \hbar$ due to a crossing with a reflection-symmetric band. The levels beyond the band-crossing have also been observed; the level scheme is presented in figure 4.26. Presumably, a similar level structure would be observed in ${ }^{222} \mathrm{Th}$ were it possible to observe the states above the backbend. The octupole band structure is lost and the population is fractionated over a number of states. The depopulating transitions are too weak to be observed in the present data set.

### 4.7 Summary

The established octupole bands in ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ have been studied by gamma-gamma coincidence techniques, using the Eurogam gamma-ray spectrometer. The bands have been observed to $31 \hbar$ in ${ }^{220} \mathrm{Ra}$ and $25 \hbar$ in ${ }^{222} \mathrm{Th}$. Measurements of the intensities of the transitions at different angles have helped assign spins to the new levels. The intrinsic electric dipole moments for these nuclei have been inferred from $\mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2)$ values, and have the values $0.34(4)$ efm for ${ }^{220} \mathrm{Ra}$, and $0.55(3)$ efm for ${ }^{222} \mathrm{Th}$. The value measured for ${ }^{222} \mathrm{Th}$ is the largest electric dipole moment known.

In a separate experiment, the multiplicities of the ${ }^{220} \mathrm{Ra}$ and ${ }^{222} \mathrm{Th}$ reaction channels have been measured to be very nearly the same, although more gamma rays are observed in ${ }^{220} \mathrm{Ra}$. This is construed as evidence that the octupole band becomes non-yrast in ${ }^{222} \mathrm{Th}$ at $25 \hbar$, presumably due to a predicted band-crossing.

The octupole model appears to adequately describe the behaviour of the two nuclei studied in this work. It would, of course, be interesting, and useful, to observe states above the band-crossing in ${ }^{222} \mathrm{Th}$, although at such high spins these nuclei are very near their fission limits. It remains an experimental challenge to observe evidence for the band-crossing in neighbouring nuclei for which it is predicted, such as ${ }^{224} \mathrm{Th}$ and ${ }^{222} \mathrm{Ra}$.

## CHAPTER 5

## Recoil-distance Measurements Following Coulomb Excitation of ${ }^{153} \mathrm{Eu}$

The interpretation of the level schemes of some lanthanide nuclei, with $\mathrm{N} \simeq 88, \mathrm{Z} \simeq 56$, in terms of octupole deformation, has recently come into question due to alternative explanations of their structure. One such nucleus is ${ }^{153} \mathrm{Eu}$. Despite having the candidate paritydoublet bands and large, collective electric dipole moments, which are suggestive of reflection asymmetry, this nucleus has recently been interpreted using the reflection-symmetric Nilsson model. This chapter recounts an experiment in which some of the lifetimes of the excited states in ${ }^{153} \mathrm{Eu}$ were measured in order to extract electromagnetic transition matrix elements and moments, and hence information on octupole collectivity. The chapter begins with an introduction to the recoil-distance method which was employed. A description of the experimental details, analysis procedures and results follows, and a discussion of the results ends the chapter.

### 5.1 The Recoil-distance Method of Lifetime Measurement

### 5.1.1 The Principles

The recoil-distance method (RDM) is a standard technique to measure nuclear lifetimes in the range between picoseconds and nanoseconds, and is described in detail in reference [Nolan 79]. In essence, the method uses a measurement of the number of decays in a given time interval, in conjunction with an exponential decay law, to allow a lifetime to be deduced.

Following a beam-on-target reaction, highly excited nuclei recoil out of the target with a high velocity, typically 3 or 4 percent of the speed of light. In the usual arrangement, a thick stopper foil is placed downstream of the target which stops the recoiling nuclei. Deexcitation gamma rays are emitted either while the nucleus is in flight, and therefore with a large Doppler shift, or after the nucleus has been stopped in the foil, and therefore with no Doppler shift. The lifetime is obtained by varying the target-to-stopper distance and measuring the relative proportion of gamma rays emitted in flight to those emitted after the nucleus has been stopped.


Figure 5.1: The target and shifter foil: nuclei recoiling from the target are slowed by the shifter foil.
The method adopted in the present work is an adapted version of the standard recoildistance measurement, and the set-up is shown schematically in figure 5.1. Instead of using a stopper foil to stop the recoils, a 'shifter' foil was used to slow them down, so the gamma rays had either a large Doppler shift if they were emitted before the shifter foil, or a small Doppler shift if they were emitted afterwards. Use of a shifter foil rather than a stopper foil has several advantages. For instance, the two Doppler shifted components are closer together, which simplifies the spectra when many states have been excited. Also the measured gamma-ray peaks will have better resolution because the finite stopping time, of
about 4 ps , will cause Doppler broadening when using the 'stopper' method.

### 5.1.2 Extraction of the Lifetime

The number of counts in the component of the gamma-ray peak with the large Doppler shift (called henceforth the 'fast' peak), $\mathrm{I}_{\text {fast }}$, is equal to the number of decays which occur in flight between the target and shifter foil. If the recoil distance is $d$, the recoil velocity is $v$ and the total number of decaying nuclei is $N_{0}$ then application of the exponential decay law gives,

$$
\begin{equation*}
I_{\text {fast }}=N_{0}\left[1-\exp \left(\frac{-d}{v \tau}\right)\right] \tag{5.1}
\end{equation*}
$$

where $\tau$ is the lifetime of the decaying state. The number of counts in the peak with small Doppler shift (the 'slow' peak), $\mathrm{I}_{\text {slow }}$, will be whatever is left, that is,

$$
\begin{equation*}
\mathrm{I}_{\text {slow }}=\mathrm{N}_{0} \exp \left(\frac{-\mathrm{d}}{\mathrm{v} \tau}\right) . \tag{5.2}
\end{equation*}
$$

Defining the ratio R to be,

$$
\begin{equation*}
\mathrm{R}=\frac{\mathrm{I}_{\text {slow }}}{\mathrm{I}_{\text {fast }}+\mathrm{I}_{\text {slow }}}=\exp \left(\frac{-\mathrm{d}}{\mathrm{v} \tau}\right), \tag{5.3}
\end{equation*}
$$

a plot of $\ln \mathrm{R}$ against d will be a straight line with gradient $-(1 / \mathrm{v} \tau)$. The recoil velocity can easily be measured, and so the lifetime can be extracted. The method is essentially the same as that described in reference [Nolan 79].

An illustration of how the relative intensities of the fast and slow peaks vary with the recoil distance, d , is given in figure 5.2. At $\mathrm{d}=16 \mu \mathrm{~m}$ most of the nuclei are decaying downstream of the shifter foil and hence the slow peak is largest. At $\mathrm{d}=200 \mu \mathrm{~m}$ the situation is reversed, and nearly all the decays happen before the nucleus has reached the shifter foil.

### 5.1.3 Perturbing Effects

Once the fast and slow peak intensities have been measured, they must be corrected for several perturbing effects in order that an accurate result can be obtained. The perturbing effects and their corrections are outlined in the ensuing paragraphs, but are treated in more detail in references [Nolan 79] and [Sturm 76].

## Geometric Solid-angle Effects

The gamma-ray detector is normally placed at $0^{\circ}$ with respect to the beam axis. The solid angle opening of the detector, as seen by the recoiling nucleus, will get larger as the nucleus


Figure 5.2: The fast and slow peaks for the $406.9 \mathrm{keV}\left(\frac{19}{2}^{+} \rightarrow \frac{15}{2}^{+}\right)$transition serve to illustrate the principle of the recoil distance method.
travels towards the detector. The mean solid angle is therefore larger for gamma rays in the slow peaks than for those in the fast peaks.

## Relativistic Solid-angle Effects

The recoil velocities are typically a few percent of the speed of light. An isotropic gammaray distribution, in the rest frame of the nucleus, becomes forward peaked when transferred to the lab frame. This effect will cause the gamma rays in the fast peaks to see a larger solid angle at the detector than those in the slow peaks.

## Gamma-ray Detector Response

The fast and slow peaks will be separated by about 7 keV at about 400 keV . In order to be able to resolve the components, a high-resolution germanium detector is normally used. The efficiency for gamma-ray detection of a germanium detector varies with gamma-ray energy, so the detection efficiency for gamma rays in the fast and slow peaks will be different.

## Feeding from Higher-lying States

The relationship $\ln \mathrm{R}=-(\mathrm{d} / \mathrm{v} \tau)$ only holds for the decay of a state which is populated by
direct excitation, but in general a given state will also be populated by the decay of higherlying states. This effect will cause the state to appear to be longer lived. The populations can be corrected for this effect, provided that the direct population of a state can be exactly calculated and the approximate lifetimes of the higher-lying states are known. Coulomb excitation offers the possibility of calculating the direct excitation probabilities exactly and therefore is often used as the excitation mechanism in such experiments. The effect of feeding can be corrected by using the Bateman [Bateman 10] equations, given below.

$$
\begin{gather*}
N_{i}(t)=N_{i}^{0}+\sum_{j} N_{j}^{0} \sum_{j \neq k} C_{j k} e^{-\lambda k t} .  \tag{5.4}\\
C_{j k}=\frac{\substack{i=1 \\
l=1} \lambda_{\ell}}{i_{l=j} \Pi\left(\lambda_{l}-\lambda_{k}\right)} . \tag{5.5}
\end{gather*}
$$

In these equations, $N_{i}(t)$ is the number of nuclei in the state $i$ due to direct excitation and feeding. The values $N_{i}^{0}$ and $N_{j}^{0}$ are the number of nuclei in states $i$ and $j$ respectively, due to direct excitation at time $t=0$. The decay constant for the state $n$ is $\lambda_{n}$

## The Nuclear Deorientation Effect

In the experiment described in this thesis, gamma rays were detected in coincidence with backscattered beam-ions, so the recoiling target-nuclei emerged in a highly-aligned state. Nuclei recoiling into a vacuum are subject to very large hyperfine fields produced by the atomic electrons, which act to reduce the alignment and therefore reduce the anisotropy of the gamma-ray angular distribution as described in section 3.3. The time dependence of the gamma-ray angular distribution is then described by,

$$
\begin{equation*}
W(\theta, t)=\sum_{k \text { even }} A_{k} Q_{k}^{p} Q_{k}^{\gamma} G_{k}(t) P_{k}(\cos \theta) \tag{5.6}
\end{equation*}
$$

where the $Q_{k}^{\gamma}$ and $Q_{k}^{p}$ coefficients correct for the finite size of the gamma-ray and particle detectors (section 3.3), and the $\mathrm{G}_{\mathrm{k}}$ are the attenuation coefficients due to the hyperfine interaction. This angular-distribution attenuation must be taken into account when correcting the measured fast and slow peak intensitics. There are several models that provide expressions for the $G_{k}(t)$.

The static model assumes a stable distribution of electrons in states with lifetimes long compared to the observation time, so the nucleus feels a static field. The form of the attenuation coefficients are given in several references such as [Billowes 86]. In contrast, in the time-dependent model discussed by Abragam and Pound [Abragam 53], the angular distribution is perturbed by electric and magnetic fields which fluctuate randomly. The
basic assumption of this model is that deorientation arises entirely from this process. The attenuation coefficients have an exponential time dependence, $G(t)=\exp \left(-\lambda_{k} t\right)$, where $\lambda_{k}$ is a parameter of the model described in [Abragam 53].

An intermediate model is the 'Two-state Model' developed by Brenn et al. [Brenn 77], where it is assumed that the atoms are either in a fluctuating state during the atomic de-excitation, or the static state corresponding to the equilibrium configuration. A pure M1 interaction is still assumed. Brenn et al. describe static, $\mathbf{G}_{\mathbf{k}}^{\text {stat }}$, and fluctuating, $\mathrm{G}_{\mathrm{k}}^{\text {fuct }}$, attenuation coefficients in a similar manner to those described above. If an ensemble of atoms makes a transition from the fluctuating to the static state at a rate $\Lambda$, then the full attenuation coefficients are given by,

$$
\begin{equation*}
G_{k}(t)=\exp (\Lambda t) G_{k}^{\text {fuct }}(t)+\int_{0}^{t} \lambda G_{k}^{\text {fuct }}\left(t^{\prime}\right) \exp \left(-\Lambda t^{\prime}\right) G_{k}^{\text {stat }}\left(t-t^{\prime}\right) d t^{\prime} \tag{5.7}
\end{equation*}
$$


#### Abstract

'A static model with a wide distribution of static fields can give a good description of the attenuations'. This statement from [Billowes 86] specifically refers to some osmium, platinum and tungsten isotopes, although several static models have been developed which have accurately reproduced the measured time-dependent deorientation coefficients for many nuclei. However, it was the 'Two-state Model' of Brenn et al. which was used by the analysis codes Gosia and ORACLE that were used in this work. A more detailed description of the model and the parameters used can be found in reference [Czosnyka 83]. The attenuation of the angular distribution will be larger for gamma rays in the slow peaks than those in the fast peaks, as they experience a longer flight time.


### 5.2 Previous Gamma-ray Spectroscopy Studies of ${ }^{153} \mathrm{Eu}$ : Motivation for Lifetime Measurements

Being a stable isotope, many of the carliest studies of ${ }^{153} \mathrm{Eu}$ were by Coulomb excitation. In 1970, Lewis and Graetzer [Lewis 70] observed states up to $\frac{9}{2}^{+}$; extended up to $\frac{15}{2}^{+}$ in 1972 by Thun and Miller [Thun 72]. In 1975 Dracoulis ct al. [Dracoulis 75] used the ${ }^{154} \mathrm{Sm}(\mathrm{d}, 3 \mathrm{n})$ reaction to arrange the excited states into rotational bands. They found two bands of opposite-parity states, one based on the $\frac{5}{2}^{+}$ground state, the other based on a $\frac{5^{-}}{}{ }^{-}$ band-head at 97.4 keV . Within the bands, E2 and M1 transitions depopulated the states while E1 transitions connected the states with $\mathrm{I}^{+/-}$to those with ( $\left.\mathrm{I}-1\right)^{-/+}$. In that work a third band was also identified based on a $\frac{3}{2}^{+}$state at 103.2 keV . In 1994, Pearson et
al. [Pearson 94], [Pearson 94a] used the ${ }^{150} \mathrm{Nd}\left({ }^{7} \mathrm{Li}, 4 \mathrm{n}\right)$ reaction to extend the band structure still further: up to $\frac{45}{2}^{+}$in the ground-state band, $\frac{45}{2}^{-}$in the negative-parity band and $\frac{25}{2}{ }^{+}$ in the $\frac{3}{2}^{+}$sideband.

Nuclei with $\mathrm{Z} \simeq 56, \mathrm{~N} \simeq 88$, are susceptible to octupole deformation as was discussed in chapter 2. The level structure of ${ }^{153} \mathrm{Eu}$, that is, of almost degenerate states with the same spin but opposite parity, can be interpreted as parity-doublet bands which are characteristic of a nucleus with octupole deformation. Furthermore, from a measurement of E1 to E2 intensity ratios, Pearson et al. showed that the electric dipole moments of the states in the bands were larger than those in nuclei known to be reflection symmetric, giving more validity to the octupole deformed interpretation. However, contrary to this interpretation, Pearson et al. also inferred magnetic moment values from gamma-ray intensity ratios. The inferred magnetic moments were found to be different for the two bands, which is not what would be expected if the bands were a parity-doublet pair based on the same intrinsic configuration. The conclusion of their work was that the structure can be adequately described with the reflection-symmetric Nilsson model, without the need to introduce the idea of a reflection-asymmetric mean field.

Prior to the work described in this thesis, only the lifetimes of the states with $\mathrm{I}^{\pi}=\frac{7}{2}^{+}, \frac{9}{2}^{+}, \frac{5}{2}, \frac{7}{2}^{-}, \frac{3}{2}^{+}$and $\left(\frac{5}{2}\right) 2^{+}$had been measured. The remainder of this chapter describes an experiment in which a recoil-distance measurement following Coulomb excitation was performed, in order to determine the lifetimes of some of the higher-lying states in the positive- and negative-parity bands. A measurement of lifetimes yields electromagnetic transition matrix elements and moments, which provide useful information on the collective nature of nuclei.

### 5.3 Apparatus

The experiment was performed at the Nuclear Structure Research Laboratory of the University of Rochester in New York. A ${ }^{58} \mathrm{Ni}$ beam was used to Coulomb excite a ${ }^{153} \mathrm{Eu}$ target. The Rochester recoil-distance apparatus consists of a specially designed plunger device, germanium detectors and a parallel-plate avalanche counter (PPAC) to detect backscattered beam ions. The apparatus is shown in figure 5.4 and represented schematically by figure 5.3. The various components of the apparatus are described individually below.


Figure 5.3: A schematic picture of the apparatus.

### 5.3.1 Ion Beam

The ${ }^{58} \mathrm{Ni}$ beam at 220 MeV was provided by the 16 MV MP-tandem van de Graaff accelerator at the NSRL, which is described in reference [Purser 74], and is shown in figure 5.5. The beam energy, at $80 \%$ of the Coulomb barrier, ensured that nuclear effects play no part in the excitation. Nickel is a frequently used projectile in Coulomb excitation experiments because, having 'magic' number $\mathrm{Z}=28$, it has high-lying excited states and therefore projectile excitation does not occur easily ${ }^{1}$.

### 5.3.2 Targets

Two thin targets, of $350 \mu \mathrm{~g} / \mathrm{cm}^{2}$ and $328 \mu \mathrm{~g} / \mathrm{cm}^{2}$ of isotopically-enriched curopium oxide, ${ }^{153} \mathrm{Eu}_{2} \mathrm{O}_{3}$, were used, each on a $500 \mu \mathrm{~g} / \mathrm{cm}^{2}$ nickel backing. Thin targets were used in order that the recoil velocity would be sufficiently high to cause the fast and slow gamma-ray peaks to be well separated, and to minimize the velocity spread of the recoils. The targets were manufactured at Rochester using a sputtering technique, which is described in detail by Sletten and Knudsen in [Sletten 72]. The sputtering process is illustrated in figure 5.6. A beam of 10 keV argon ions, incident on an isotope sample placed on an angled graphite beam-stop causes the isotope sample to sputter onto the target backing. After sputtering the targets are stretched in their frames to provide a smooth surface. The process took 8 to 10 hours to sputter a target of thickness $\sim 350 \mu \mathrm{~g} / \mathrm{cm}^{2}$. The thickness of the targets was measured using alpha-particle attenuation.

[^8]

Figure 5.4: The Rochester plunger target chamber and germanium detectors.


Figure 5.5: The Rochester 16 MV MP-tandem Van de Graaff accelerator.


Figure 5.6: An argon beam incident on an isotope sample causes the isotope to 'sputter' onto the target backing.

### 5.3.3 The Rochester Plunger Apparatus

A commercially available 'Inchworm' device was used to alter the separation of the target and shifter foil, by moving the shifter foil in 6 nm steps over a range of 25 mm .

The target-to-shifter foil separation was measured in two ways. For large separations, an optical measurement was used which gave the separation to an accuracy of $\pm 2 \mu \mathrm{~m}$. At short separations, that is $<50 \mu \mathrm{~m}$, a capacitance measurement was used which gave the separation to $\pm 0.5 \mu \mathrm{~m}$. The target and shifter foil in a vacuum, will act as an infinite parallel plate capacitor, provided that their separation is much less than their dimensions. The capacitance C is therefore proportional to $1 / \mathrm{d}$, where d is the separation. A voltage V applied to the target will cause an output on the shifter foil which is proportional to CV, that is, proportional to $1 / \mathrm{d}$. Hence a measurement of the capacitance gives the separation.

The target and shifter foil were aligned by use of the beam from a helium-neon laser. The laser beam was in turn reflected from the surface of the target and from the shifter foil. When the targets were aligned, the reflected laser beam hit the same spot on the laboratory wall several metres away. Once aligned, the calibration of the target-to-shifter foil distance was carried out using a micrometer. The entire Rochester plunger apparatus is described in reference [Cline 81].

### 5.3.4 Detectors

Four detectors were used, three high-purity escape-suppressed germanium detectors for detection of gamma rays and one annular parallel-plate avalanche counter (PPAC) for the

|  | Position <br> $(\theta, \phi)$ | Crystal Size (mm) <br> (length $\times$ radius) | Efficiency | Target-to-Ge crystal <br> Distance (mm) |
| :---: | :---: | :---: | :---: | :---: |
| 1. | $0^{\circ}, 0^{\circ}$ | $78.0 \times 35.6$ | $\sim 70 \%$ | 123 |
| 2. | $55^{\circ}, 0^{\circ}$ | $51.3 \times 27.0$ | $\sim 20 \%$ | 150 |
| 3. | $90^{\circ}, 0^{\circ}$ | $47.8 \times 26.0$ | $\sim 20 \%$ | 210 |

Table 5.1: Summarized information concerning the germanium detectors. The crystal sizes are taken from the detector specification sheets. The efficiencies are given relative to a 3 inch $\times 3$ inch sodium-iodide detector.
detection of backscattered beam particles. Information about the germanium detectors is summarized in table 5.1. The large-volume germanium detector at $0^{\circ}$ was on loan from the Eurogam phase-I array. Such detectors are described in chapter 4, and in table 4.1.

The annular position sensitive PPAC was placed approximately 126 mm from the target and covered the angular range $\theta=138^{\circ}-168^{\circ}, \phi=0^{\circ}-360^{\circ}$. The detector is similar to those used in other Coulomb excitation experiments, which are described in references such as [White 90], [Ibbotson 91] and [Ibbotson 93]. The detector is sectored into 4 separate PPACs. Each PPAC has a front window of $50 \mu \mathrm{~g} / \mathrm{cm}^{2}$ polypropylene coated with a thin layer of aluminium acting as the anode. This is separated by a few millimetres from a copper-covered board which forms the cathode. Each quadrant was filled with 5 Torr of isobutane gas contained by a second window. The operation of the PPAC is based on the ionization of the isobutane by the incident scattered ions. Primary ionization is followed by rapid amplification of charge carriers, caused by the application of 400 V across the PPAC. The anode provided the timing signal, while the cathode was able to provide position sensitivity, allowing for Doppler corrections to be made to the gamma-ray spectra. Detecting the backscattered beam ions over $138^{\circ}-168^{\circ}$ corresponds to a recoil cone of $4^{\circ}-13^{\circ}$, and ensured that highest spin states were populated.

### 5.3.5 Electronics

A block diagram of the electronics is given in figure 5.7. A particle-gamma ray event was required as the mastergate, with an 'any-particle' signal being taken from the anodes on the PPAC. Also included in the mastergate were scaled down particle singles, to get an idea of the cross-section of the reaction, and also to make sure that the PPAC was working


Figure 5.7: A block diagram of the electronics associated with the germanium detectors and the PPAC. The germanium-BGO section was repeated for each of the three escape suppressed germanium detectors.
properly. The mastergate was used as an external gate to the coincidence register, and as the start signal to the TACs. Data were written onto ' 3 M ' 8 mm data-tapes using a data acquisition programme from Michigan State University, on the Rochester VAX computer. The parameters which were recorded on the data-tapes were:
(i). An escape-suppressed germanium-energy signal from each of the germanium detectors.
(ii). The input and output voltages, measuring the capacitance of the target and shifter foil.
(iii). Timing measurements from a TAC, to give particle position information and particlegamma ray timing.
(iv). A coincidence register, which could be used to distinguish between gas counter sectors and particle-gamma and particle-singles events.

### 5.4 Experimental Details

In total, the experiment ran for approximately 200 hours. The beam intensity was maintained at about 20 nA , or about 1.5 pnA and typical detector rates were: 3000 counts per second in the germanium detector at $\theta=0^{\circ}, \phi=0^{\circ} ; 700$ counts per second in the germanium detector at $\theta=55^{\circ}, \phi=0^{\circ} ; 400$ counts per second germanium detector at $\theta=90^{\circ}, \phi=0^{\circ}$; and 300 counts per second in the PPAC. Altogether, about $8 \times 10^{5}$ particle-gamma coincidence events were collected, which involved the germanium detector at $0^{\circ}$. These were the data used for the determination of lifetimes.

Fourteen recoil distances were used, one of which was repeated with both of the two targets that were used in order to test for differences between the two targets. The distances used, in $\mu \mathrm{m}$, were: $16,24,38,62,100(\times 2), 200,320,450,800,1500,3000,5000,8000,10000$. Several of the short distances were run for almost 24 hours, in order that enough events were collected to measure the shortest lifetimes at the top of the bands, of a few picoseconds.

In addition, a run was made with a thick stopper in place of the shifter foil, in order to obtain a value for the recoil velocity. Gamma rays in this case were either fully shifted or unshifted, and the recoil velocity, $\mathrm{v} / \mathrm{c}$, was obtained from the relation,

$$
\begin{equation*}
\frac{E-E_{0}}{E_{0}}=\frac{v}{c}\left(\frac{1+\cos \theta_{\mathrm{g}}}{2}\right), \tag{5.8}
\end{equation*}
$$

where E and $\mathrm{E}_{0}$ are the energies of the shifted and unshifted peaks and $\theta_{\mathrm{g}}$ is the half-angle subtended by the germanium detector.

Following the in-beam experiment, energy and relative-efficiency calibrations were carried out using sources of ${ }^{152} \mathrm{Eu},{ }^{60} \mathrm{Co}$ and a mixed source of ${ }^{125} \mathrm{Sb}$ and ${ }^{154,155} \mathrm{Eu}$.

### 5.5 Analysis

The data-tapes were scanned using the analysis programme 'Daphne'. Germanium-detector spectra were sorted in coincidence with backscattered beam ions, and a correction was made for random coincidences by gating on the particle-gamma TAC. A spectrum was sorted for each germanium detector and each distance, resulting in a total of 48 spectra. The lifetime analysis was, however, carried out using only the 16 spectra from the detector at $0^{\circ}$.

To illustrate the quality of the data, figure 5.8 shows a sum of the spectra taken at $\mathrm{d}=8000 \mu \mathrm{~m}$ and $\mathrm{d}=10000 \mu \mathrm{~m}$ in the detector at $0^{\circ}$. The peaks are predominantly fast and


Figure 5.8: Summed spectra for $\mathrm{d}=8000 \mu \mathrm{~m}$ and $\mathrm{d}=10000 \mu \mathrm{~m}$, shown on logarithmic (upper) and linear (lower) scales. The peaks at such long recoil distances are predominantly fast. The dispersion is 1 keV per channel.
each transition has a peak at $\left(1+\frac{v}{c}\right)=1.04$ times its actual energy. Figure 5.9 shows some of the fast and slow pairs for three recoil distances: $\mathrm{d}=16 \mu \mathrm{~m}, 100 \mu \mathrm{~m}$ and $450 \mu \mathrm{~m}$; at short distances the slow peaks dominate, whereas at long distances the intensity lies mainly in the fast peaks.

The intensities of the fast and slow components were measured by fitting Gaussian peak shapes using the codes 'Silvia', 'GF2', and 'Eurogam-session Spectrum-analyser', described in appendix D. Different codes were used as a matter of convenience when using different computers (a VAX in Rochester and SUN workstations in Liverpool). This served as a check for consistency in the fitting routines. Once the fast and slow peak intensities were measured in the spectra, they had to be corrected for the effects outlined in section 5.1.3. The corrections were implemented using the code ORACLE [Sturm 76], to which several modifications had been made [Kotlinski 89] to account for experiments in which a slowing foil is used and the recoils are not completely stopped. This acts to reduce some of the


Figure 5.9: Some sample spectra for $\mathrm{d}=16 \mu \mathrm{~m}, \mathrm{~d}=100 \mu \mathrm{~m}$ and $\mathrm{d}=450 \mu \mathrm{~m}$, with 1 keV per channel dispersion.
corrections. A summary of the modifications made to the code is:

- Correction for the loss of intensity as the recoil slows in the foil, as a result of which the gamma-ray energy will be between that of the fast and the slow peaks.
- Inclusion of the germanium-detector solid-angle correction coefficients $\mathrm{Q}_{k}$, as described in section 3.3.
- Change of the model describing the deorientation of nuclei recoiling in vacuum from the Abragam-Pound [Abragam 53] model to that of Brenn et al. [Brenn 77].
- The adaptation of the feeding correction to use a numerical time propagation method, which could account for complicated side-feeding [Kotlinski 89].

Besides the fast and slow peak intensities, the code ORACLE requires other data to be input about the detectors and the experiment. This includes the Z and A of the projectile and target, the beam energy, detector positions and efficiencies, and also the recoil distances that were used. In order to correct for the effects of feeding from higher-lying states, ORACLE needs to know the initial populations of all the states, and also their approximate lifetimes.

These are both calculated using the Coulomb-excitation code Gosia, which was described in section 3.11.5 and in appendix D. Given information about the states in the level scheme, and the matrix elements of all the potentially important couplings, Gosia will calculate the Coulomb-excitation probabilities using the approach outlined in section 3.11. On the first pass the excitation probabilities and lifetimes are calculated with approximate rotor-model matrix elements. If, after completion of the analysis, the matrix elements are found to be significantly different, the excitation probabilities and lifetimes are re-calculated using the measured values, and the feeding correction is iterated.

### 5.6 Results

### 5.6.1 The Recoil Velocity

The recoil velocity was calculated using equation 5.8 , from the cleanest and most-intense transitions in the ground state band, namely the $288.1 \mathrm{keV}, 330.0 \mathrm{keV}, 370.8 \mathrm{keV}$ and 406.9 keV transitions. The weighted mean value of $\mathrm{v} / \mathrm{c}$ was found to be $0.0435 \pm 0.0007$. The recoil time corresponding to a recoil distance of $d \mu \mathrm{~m}$, was therefore ( $0.07 d$ ) ps. From the positions of the fast and slow peaks, it was found that the shifter foil reduced the recoil velocity to 0.0257 c .

### 5.6.2 Germanium-detector Response

Energy and relative-efficiency calibrations of the germanium detectors were carried out using ${ }^{152,154,155} \mathrm{Eu}$ and ${ }^{125} \mathrm{Sb}$ sources. Relative-efficiency curves are given for each detector in figure 5.10 where a third-order polynomial has been fitted to the data using the code GREMLIN described in appendix D. The higher relative efficiency at high energies for the large-volume Eurogam phase-I germanium detector at $0^{\circ}$ is very apparent from the figure.

### 5.6.3 Coulomb-excited States

The level scheme of ${ }^{153} \mathrm{Eu}$, given in figure 5.11 , shows the levels which were populated in this experiment. The transition energies measured in this work were in good agreement with those of [Pearson 94], at most differing by $\sim 0.2 \mathrm{keV}$. Their work, using gamma-gamma coincidence techniques, provided the opportunity of resolving the many doublets and triplets by 'gating out' unwanted contaminant peaks. The transition energies are therefore adopted from [Pearson 94].


Figure 5.10: Relative-efficiency curves for the germanium detectors.
In total, 38 levels were included as input to the code Gosia: $16\left(\frac{5}{2}^{+}\right.$to $\left.\frac{35}{2}^{+}\right)$in the groundstate band; $16\left(\frac{5}{2}^{-}\right.$to $\left.\frac{35}{2}^{-}\right)$in the negative-parity band; and $6\left(\frac{3}{2}^{+}\right.$to $\left.\frac{13}{2}^{+}\right)$in the $\frac{3}{2}^{+}$band. The uppermost one or two levels in each band were included as 'buffering levels' which, while not actually populated, were included to prevent the code from over-estimating the population of the top levels [Czosnyka 83]. Connecting the 38 levels, 529 matrix elements were input, including all potentially significant E1 (57 matrix elements), E2 (115), E3 (124), E4 (170) and M1 (63) couplings. The matrix elements were the rigid-rotor values calculated using the equations,

$$
\begin{align*}
<\|E \lambda\|> & =\sqrt{\left(2 I_{i}+1\right)} a_{\lambda} Q_{\lambda 0}<I_{i} K_{i} \lambda 0\left|I_{f} K_{f}\right\rangle  \tag{5.9}\\
<\|M \lambda\|> & =\sqrt{\left(2 I_{i}+1\right)} a_{\lambda}\left(g_{K}-g_{R}\right) K<I_{i} K_{i} \lambda 0 \mid I_{f} K_{f}>,  \tag{5.10}\\
a_{\lambda \neq 1} & =\sqrt{\frac{2 \lambda+1}{16 \pi}}, \quad a_{1}=\sqrt{\frac{3}{4 \pi}} \tag{5.11}
\end{align*}
$$

where $g_{K}$ and $g_{R}$ are the intrinsic and rotational gyromagnetic ratios and $Q_{\lambda 0}$ are the $2^{\lambda}$-th order electric multipole moments. The terms $<I_{i} K_{i} \lambda 0 \mid I_{f} K_{f}>$ are the Clebsch-Gordan coefficients. The electric dipole moments used were those obtained from [Pearson 94], oth-


Figure 5.11: Levels populated by the multiple Coulomb excitation of ${ }^{153} \mathrm{Eu}$ with $220 \mathrm{MeV}{ }^{58} \mathrm{Ni}$ ions. The level energies are taken from reference [Pearson 94].
erwise a mean value of 0.075 efm was used. The intrinsic electric quadrupole moment used was that of 6.75 eb , from the measurement of the spectroscopic quadrupole moment in reference [Tanaka 83]. The octupole moment was assumed to be $1.136 \mathrm{eb}^{\frac{3}{2}}$, equal to that of ${ }^{150} \mathrm{Nd}$ measured by Clarkson [Clarkson 92], and the hexadecapole matrix elements used were an average of the values of $<4^{+}\|E 4\| 0^{+}>$measured for the isotonic neighbours ${ }^{152} \mathrm{Sm}$ and ${ }^{154} \mathrm{Gd}$ [Wollersheim 77]. This rough approximation is valid because the dependence of the excitation probabilities fall off very quickly with increasing multipolarity, and E4 excitation plays only a very small part. The signs of all matrix elements were given according to the signs of the Clebsch-Gordan coefficients.

### 5.6.4 Intensity Measurements

To extract lifetimes, the intensities of the fast and slow peaks were measured in the spectra recorded by the detector at $0^{\circ}$. With about a hundred transitions in the level scheme below 500 keV , it is not surprising that many of the transition energies are degenerate. Furthermore, in the recoil-distance spectra, each transition has a fast and a slow component which amplifies the problem. This meant that it was not possible to measure the lifetimes of some of the states, due to problems with contaminated doublets and triplets. In order to check that the measured peaks were not contaminated the sum of the fast and slow peaks measured in each spectrum was divided by the sum of the 288.1 keV fast-plus-slow sum measured in the same spectrum which was known to be clean and not contaminated. If the peaks are not contaminated this ratio should be the same for all spectra. However, this would not be of any use with near-degenerate transitions.

For some transitions, it was possible to measure just one or the other of the fast or slow components. In such cases the lifetime was extracted by normalizing to a clean transition. For instance, consider the following; if it is possible to measure the the slow peak of a transition at a recoil distance $d, I_{\text {slow }}^{(d)}$, but the fast peak is obscured, then the intensity of the fast peak $\mathrm{I}_{\text {fast }}^{(d)}$ can be constructed using the expression,

$$
\begin{equation*}
I_{\text {fast }}^{(d)}=\left[\frac{I_{s l o w}^{(10000)}+I_{\text {fast }}^{(10000)}}{I_{\text {norm }}^{(1000)}}\right] I_{\text {norm }}^{(d)}-I_{s l o w}^{(d)} . \tag{5.12}
\end{equation*}
$$

where $\mathrm{I}_{\text {norm }}^{(x)}$ is the sum of the fast and slow components of an uncontaminated transition at distance $x \mu \mathrm{~m}$, and $\mathrm{I}_{s l o w}^{(10000)}$ and $\mathrm{I}_{\text {fast }}^{(10000)}$ are the slow and fast intensities of the transition of interest at the recoil distance is $d=10000 \mu \mathrm{~m}$. The $10000 \mu \mathrm{~m}$ spectrum is used because at this distance, only transitions from states with lifetimes of more than about 300 ps will
have significant slow components. Equation 5.12 then approximates to,

$$
\begin{equation*}
I_{\text {fast }}^{(d)} \simeq\left[\frac{I_{\text {fast }}^{(10000)}}{I_{\text {norm }(\text { fast }]}^{(1000)}}\right] I_{\text {norm }}^{(d)}-I_{\text {slow }}^{(d)} . \tag{5.13}
\end{equation*}
$$

The normalizing transition most often used was the $288.1 \mathrm{keV}\left(\frac{13}{2}^{+} \rightarrow \frac{9}{2}^{+}\right)$. The approximate lifetime for this transition, as calculated by Gosia, was 32 ps , that is a half-life of about 20 ps . At $\mathrm{d}=10000 \mu \mathrm{~m}$ the flight time is 700 ps , so during flight, 35 half-lives will elapse and the fraction of the intensity that goes into the slow peak will be $2^{-35}$. It can therefore be assumed that the transition is all fast. When such normalizations and estimations are used, the errors on the intensities such as $\mathrm{I}_{\text {fast }}^{(d)}$ above are increased accordingly.

### 5.6.5 Lifetimes and Matrix Elements

From the data, it was possible to extract the lifetimes of thirteen excited states: nine in the ground-state band; three in the negative-parity band; and one in the $\frac{3}{2}^{+}$band. The lifetimes are given in table 5.2. The fits made to the data by the code ORACLE are presented in figures 5.12 to 5.16. The errors on the lifetimes include contributions from the error on the linear fit made by the code ORACLE, the $\mathrm{v} / \mathrm{c}$ measurement, the measurement of the recoil distance and the feeding correction. The latter error was estimated by varying the lifetimes of the feeding levels with lifetimes $\tau_{f} \pm \delta \tau_{f}$, between the limits $\tau_{f}+\delta \tau_{f}$ and $\tau_{f}-\delta \tau_{f}$ and investigating the subsequent effect on the lifetime being evaluated. The corrections made by ORACLE have all been tested to determine their effects on the extracted lifetimes [Ibbotson 91]. The largest correction is found to be that due to feeding from the higher-lying states, which produced effects up to $16 \%$ in the worst cases. Variation of the deorientation parameters by $50 \%$ results in less than $1 \%$ change in the lifetime. The correction due to radiation lost in the shifter foil is found to be about 0.1 ps . The offset in the distance was found by fitting one of the strongest transitions in the ground state band, and was fixed for all other measurements.

Given the lifetime of a state and the branching ratios of its decay, it is possible to calculate the reduced matrix elements for the decays of the state by use of the formulae given in section 3.2. In a case such as this, with mixed M1/E2 transitions, it is necessary to know mixing ratios, $\delta$, in order to know the relative amounts of M1 and E2.

In this work, the $\delta$ used was that defined in reference [Czosnyka 83] as,

$$
\begin{equation*}
\delta=0.835 \mathrm{E}_{\gamma}(\mathrm{MeV}) \frac{<\|\mathrm{E} 2\|>}{<\|\mathrm{M} 1\|>} \tag{5.14}
\end{equation*}
$$



Figure 5.12: Fits to the corrected peak intensities made by the code ORACLE: (a). the $504.9 \mathrm{keV}\left(\frac{25}{2}^{+} \rightarrow\right.$ $\left.\frac{21}{2}^{+}\right)$transition gives $\tau\left(\frac{25}{2}^{+}\right)=1.80(14) \mathrm{ps} ;(\mathrm{b})$. the $473.7 \mathrm{keV}\left(\frac{23}{2}^{+} \rightarrow \frac{19}{2}^{+}\right)$transition gives $\tau\left(\frac{23}{2}^{+}\right)=2.48(10)$ $\mathrm{ps} ;(\mathrm{c})$. the $442.6 \mathrm{keV}\left(\frac{21}{2}^{+} \rightarrow \frac{17}{2}^{+}\right)$transition gives $\tau\left(\frac{21}{2}^{+}\right)=3.38(11) \mathrm{ps}$.


Figure 5.13: Fits to the corrected peak intensities made by the code ORACLE: (a). the $406.9 \mathrm{keV}\left(\frac{19}{2}^{+} \rightarrow\right.$ $\left.\frac{15}{2}^{+}\right)$transition gives $\tau\left(\frac{19}{2}^{+}\right)=5.12(13) \mathrm{ps} ;(\mathrm{b})$. the $210.2 \mathrm{keV}\left(\frac{19}{2}^{+} \rightarrow \frac{17}{2}^{+}\right)$transition gives $\tau\left(\frac{19}{2}^{+}\right)=4.29(62)$ $\mathrm{ps} ;(\mathrm{c})$. the $370.8 \mathrm{keV}\left(\frac{17}{2}^{+} \rightarrow \frac{13}{2}^{+}\right)$transition gives $\tau\left(\frac{17}{2}^{+}\right)=8.35(38) \mathrm{ps}$.


Figure 5.14: Fits to the corrected peak intensities made by the code ORACLE: (a). the 262.4 keV $\left(\frac{17}{2}^{+} \rightarrow \frac{15}{2}^{-}\right)$transition gives $\tau\left(\frac{17}{2}^{+}\right)=9.91(57) \mathrm{ps} ;(\mathrm{b})$. the $330.0 \mathrm{keV}\left(\frac{15}{2}^{+} \rightarrow \frac{11}{2}^{+}\right)$transition gives $\tau\left(\frac{15}{2}^{+}\right)=14.52(34) \mathrm{ps} ;(\mathrm{c})$. the $288.1 \mathrm{keV}\left(\frac{13}{2}^{+} \rightarrow \frac{9}{2}^{+}\right)$transition gives $\tau\left(\frac{13}{2}^{+}\right)=28.49(65) \mathrm{ps}$.


Figure 5.15: Fits to the corrected peak intensities made by the code ORACLE: (a). the 132.1 keV $\left(\frac{11}{2}^{+} \rightarrow \frac{9}{2}^{+}\right)$transition gives $\tau\left(\frac{11}{2}^{+}\right)=75.00(495) \mathrm{ps} ;(\mathrm{b})$. the $193.1 \mathrm{keV}\left(\frac{9}{2}^{+} \rightarrow \frac{5}{2}^{+}\right)$transition gives $\tau\left(\frac{9}{2}^{+}\right)=249.73(820) \mathrm{ps} ;(\mathrm{c})$. the $319.8 \mathrm{keV}\left(\left(\frac{11}{2}\right)_{2}^{+} \rightarrow\left(\frac{7}{2}\right)_{2}^{+}\right)$transition gives $\tau\left(\left(\frac{11}{2}\right)_{2}^{+}\right)=5.92(107) \mathrm{ps}$.


Figure 5.16: Fits to the corrected peak intensities made by the code ORACLE: (a). the $437.3 \mathrm{keV}\left(\frac{21}{2}^{-} \rightarrow\right.$ $\left.\frac{17}{2}^{-}\right)$transition gives $\tau\left(\frac{21}{2}^{-}\right)=2.72(57) \mathrm{ps} ;(\mathrm{b})$. the $365.3 \mathrm{keV}\left(\frac{19}{2}^{-} \rightarrow \frac{15}{2}^{-}\right)$transition gives $\tau\left(\frac{19}{2}^{-}\right)=6.60(60)$ $\mathrm{ps} ;(\mathrm{c})$. the $347.9 \mathrm{keV}\left(\frac{17}{2}^{-} \rightarrow \frac{13}{2}^{-}\right)$transition gives $\tau\left(\frac{17}{2}^{-}\right)=7.20(50) \mathrm{ps}$.

| $\mathrm{k}=\frac{5}{2}^{+}$ |  |  |
| :---: | :---: | :---: |
| $\mathrm{I}^{\pi}$ | $\tau^{\text {rotor }}$ (ps) | $\tau^{e x p t}(\mathrm{ps})$ |
| $\frac{25}{2}+$ | 1.33 | 1.80(0.14) |
| $\frac{23}{2}^{+}$ | 2.27 | 2.48 (0.10) |
| $\frac{21}{2}+$ | 3.35 | 3.38(0.11) |
| $\frac{19}{2}^{+}$ | 4.96 | M1: $4.29(0.62)$ <br> E2: $5.12(0.13)$ |
| $\frac{17_{2}}{}{ }^{+}$ | 7.47 | $\begin{array}{ll} \hline \text { E1: } & 8.91(0.57) \\ \text { E2: } & 8.35(0.38) \\ \hline \end{array}$ |
| $\frac{15}{2}^{+}$ | 14.7 | 14.5(0.3) |
| $\frac{13}{2}+$ | 32.3 | 28.5(0.7) |
| $\frac{112}{2}^{+}$ | 81.1 | 75.0(5.0) |
| $\frac{9}{2}+$ | 252.5 | 249.7(8.2) |
| $\mathrm{k}=\frac{5}{2}^{-}$ |  |  |
| $\frac{21}{2}$ | 2.07 | 2.72(0.57) |
| $\frac{19}{2}^{-}$ | 7.60 | $6.60(0.57)$ |
| $\frac{17^{-}}{}{ }^{-}$ | 5.43 | 7.20(0.50) |
| $\mathrm{k}=\frac{3}{2}^{+}$ |  |  |
| $\frac{112}{2}^{+}$ | 7.04 | 5.92(1.07) |

Table 5.2: Thirteen lifetime values measured in this work, compared to values calculated with the matrix elements that were used as input to the code Gosia. For the $\frac{19}{2}+$ and $\frac{17}{2}+$ states it was possible to extract a lifetime from two decay branches. Both the decay branch and the lifetime obtained are indicated in the right-hand column for these states.

The E2 component of the mixed transitions is $\left[\delta^{2} /\left(1+\delta^{2}\right)\right]$ and the M1 component is $\left[1 /\left(1+\delta^{2}\right)\right]$. The matrix elements used in equation 5.14 , to evaluate $\delta$, were the rotor values described in section 5.6.3. In principle, it should have been possible to determine $\delta$ using the angular-distribution information from the detectors at $55^{\circ}$ and $90^{\circ}$. In practice however this was not possible due to problems with contaminated transitions and low transition intensities. Despite these problems it was possible to extract intensities at each angle for some of the M1/E2 transitions, namely the transitions at $132.1 \mathrm{keV}\left(\frac{11}{2}^{+} \rightarrow \frac{9}{2}^{+}\right), 210.2 \mathrm{keV}$ $\left(\frac{19}{2}^{+} \rightarrow \frac{17}{2}^{+}\right), 232.5 \mathrm{keV}\left(\frac{21}{2}^{+} \rightarrow \frac{19}{2}^{+}\right), 86.8 \mathrm{keV}\left(\frac{11}{2}^{-} \rightarrow \frac{9}{2}^{-}\right), 308.2 \mathrm{keV}\left(\frac{21}{2}^{-} \rightarrow \frac{19}{2}^{-}\right)$, $69.7 \mathrm{keV}\left(\left(\frac{5}{2}^{\frac{5}{2}} \rightarrow_{\frac{3}{2}^{+}}{ }^{+}\right)\right.$and $141.5 \mathrm{keV}\left(\frac{11}{2}^{+} \rightarrow \frac{9}{2}^{+}\right)$. The ratios of measured intensities $\mathrm{I}\left(55^{\circ}\right) / \mathrm{I}\left(0^{\circ}\right)$ and $\mathrm{I}\left(90^{\circ}\right) / \mathrm{I}\left(0^{\circ}\right)$ were compared to those calculated using the code Gosia with

| $\mathrm{I}_{i}^{\pi i}$ | $\gamma$-ray Energy (keV) |  |  | Branching Ratios |  |  |  | $\delta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | E1 | E2 | M1/E2 | $\mathrm{I}(\mathrm{E} 1) / \mathrm{I}(\mathrm{E} 2)$ |  | $\mathrm{I}(\mathrm{M} 1 / \mathrm{E} 2) / \mathrm{I}(\mathrm{E} 2)$ |  |  |
|  |  |  |  | Pearson et al. | This work | Pearson et al. | This work |  |
| $\frac{9}{2}+$ | 41.5 | 193.1 | 109.7 |  |  | 0.294(23) |  | 0.7073 |
| $\frac{11}{2}+$ | 89.4 | 241.8 | 132.1 | 0.025(6) |  | 0.164(15) |  | 0.6910 |
| $\frac{13}{2}+$ | 159.3 | 288.1 | 155.9 | 0.166(25) |  | 0.145(17) |  | 0.6870 |
| $\frac{15}{2}+$ | 177.2 | 330.0 | 174.0 | 0.099(6) |  | 0.082(9) |  | 0.6622 |
| $\frac{17}{2}^{+}$ | 262.4 | 370.8 | 197.0 | 0.146(10) |  | 0.078(6) | 0.078(6) | 0.6603 |
| $\frac{19}{2}+$ | 236.3 | 406.9 | 210.2 | 0.135(7) | 0.135(17) | 0.055(8) | 0.034(10) | 0.6294 |
| $\frac{21}{2}+$ | 339.6 | 442.6 | 232.5 | 0.105(11) | 0.110(20) | 0.026(8) | 0.038(8) | 0.6293 |
| $\frac{23}{2}+$ | 272.6 | 473.7 | 241.2 | 0.121(51) | 0.089(27) | 0.051(46) |  | 0.5956 |
| $\frac{25}{2}+$ | 393.9 | 504.9 | 263.2 | 0.121(23) |  |  |  | 0.5976 |
| $\frac{17}{2}^{-}$ | 170.3 | 347.9 | 236.2 | 0.203(52) |  | 0.990(78) |  | 0.2536 |
| $\frac{19}{2}-$ | 103.7 | 365.3 | 129.0 | 0.012(8) |  | 0.162(23) |  | 0.1238 |
| $\frac{21}{2}^{-}$ | 201.0 | 437.3 | 308.2 | 0.069(12) |  | 0.621(50) |  | 0.6725 |
| $\frac{11}{2}^{+}$ | 302.7 | 268.2 | 141.5 | 0.76(14) |  | 1.81(27) |  | 0.1702 |

Table 5.3: The branching ratios and mixing ratios used in this analysis.
the rotor matrix elements. Due to large errors on the measured intensities, however, it was only possible to conclude that the measured and calculated intensity ratios did not contradict each other. Hence, it was deemed acceptable to calculate the mixing ratios using the rotor matrix elements. Similarly, due to problems with contaminated transitions, it was not possible to extract intensity ratios for all the gamma-branches that were observed. Where the branch could not be measured, the ratios from [Pearson 94] were adopted. The branching and mixing ratios used are given in table 5.3. The intensities were measured in the detector at $55^{\circ}$, where the Legendre polynomial $\mathrm{P}_{2}$ is close to zero $\left(\mathrm{P}_{2}\left(54.7^{\circ}\right)=0\right)$.

The reduced matrix elements deduced from the lifetimes are given in tables 5.4 to 5.7. Also given are the electric dipole moments, $\mathrm{D}_{0}$, the electric quadrupole moments, $\mathrm{Q}_{0}$ and the $\left(g_{K}-g_{R}\right)$ values which were deduced from the reduced matrix elements. The errors associated with the $\Delta \mathrm{I}=1<\|E 2\|>$ matrix elements are less than those for the $\Delta \mathrm{I}=2$ matrix elements due to the uncertainty attributed to the mixing ratio.

| $\mathrm{I}^{\pi}$ | $\mathrm{B}(\mathrm{E} 2)(\mathrm{eb})^{2}$ | $<\\|E 2\\|>\left(\mathrm{efm}^{2}\right)$ | $\mathrm{Q}_{0}(\mathrm{eb})$ |
| :---: | :---: | :---: | :---: |
| $\frac{9}{2}^{+}$ | $0.94(0.12)$ | $306(20)$ | $9.72(0.64)$ |
| $\frac{11}{2}^{+}$ | $1.11(0.16)$ | $364(27)$ | $8.10(0.59)$ |
| $\frac{13}{2}^{+}$ | $1.10(0.14)$ | $392(25)$ | $7.16(0.46)$ |
| $\frac{15}{2}^{+}$ | $1.22(0.16)$ | $442(30)$ | $7.05(0.47)$ |
| $\frac{17}{2}^{+}$ | $1.14(0.16)$ | $452(31)$ | $6.52(0.45)$ |
| $\frac{19}{2}^{+}$ | $1.22(0.19)$ | $494(38)$ | $6.56(0.50)$ |
| $\frac{\frac{21}{2}^{+}}{}$ | $1.24(0.17)$ | $521(36)$ | $6.25(0.43)$ |
| $\frac{\frac{23}{2}^{+}}{}$ | $1.21(0.18)$ | $538(39)$ | $6.28(0.46)$ |
| $\frac{25}{2}$ | $1.23(0.20)$ | $566(44)$ | $6.27(0.49)$ |
| $\frac{17}{2}^{-}$ | $1.01(0.14)$ | $427(29)$ | $6.15(0.42)$ |
| $\frac{19}{2}^{-}$ | $1.62(0.26)$ | $569(45)$ | $7.54(0.60)$ |
| $\frac{21}{2}^{-}$ | $1.11(0.27)$ | $494(60)$ | $5.92(0.83)$ |
| $\left(\frac{11}{2}_{2}+\right.$ |  |  |  |
|  | $2.78(0.63)$ | $578(65)$ | $10.49(1.18)$ |

Table 5.4: E2 matrix elements and quadrupole moments extracted from the $\Delta \mathrm{I}=2$ transitions.

| $\mathrm{I}^{\pi}$ | $\mathrm{B}(\mathrm{E} 2)(\mathrm{eb})^{2}$ | $<\\|E 2\\|>\left(\mathrm{efm}^{2}\right)$ | $\mathrm{Q}_{0}(\mathrm{eb})$ |
| :---: | :---: | :---: | :---: |
| $\frac{9}{2}^{+}$ | $1.56(0.28)$ | $395(35)$ | $7.19(0.95)$ |
| $\frac{11}{2}^{+}$ | $1.21(0.24)$ | $380(37)$ | $7.21(1.05)$ |
| $\frac{13}{2}^{+}$ | $1.10(0.21)$ | $393(39)$ | $7.85(1.16)$ |
| $\frac{15}{2}^{+}$ | $0.67(0.15)$ | $331(35)$ | $6.98(1.11)$ |
| $\frac{17}{2}^{+}$ | $0.64(0.12)$ | $339(31)$ | $7.51(1.04)$ |
| $\frac{19}{2}^{+}$ | $0.32(0.11)$ | $253(44)$ | $5.88(1.54)$ |
| $\frac{21}{2}^{+}$ | $0.33(0.09)$ | $271(37)$ | $6.59(1.34)$ |
| $\frac{23}{2}$ | $0.47(0.43)$ | $336(155)$ | $8.53(3.92)$ |
| $\frac{17}{2}^{-}$ | $0.42(0.09)$ | $275(31)$ | $6.10(1.03)$ |
| $\frac{19}{2}$ | $0.72(0.20)$ | $380(52)$ | $8.83(1.79)$ |
| $\frac{21}{2}^{-}$ | $0.27(0.08)$ | $241(36)$ | $5.87(1.42)$ |
| $\left(\frac{11}{2}^{+}\right.$ |  |  |  |

Table 5.5: E2 matrix elements and quadrupole moments extracted from the $\Delta \mathrm{I}=1$ transitions.

| $\mathrm{I}^{\pi}$ | $\mathrm{B}(\mathrm{E} 1) 10^{-4}(\mathrm{efm})^{2}$ | $<\\|E 1\\|>(\mathrm{efm})$ | $D_{0}(\mathrm{efm})$ |
| :---: | :---: | :---: | :---: |
| $\frac{11}{2}^{+}$ | $2.44(0.64)$ | $0.054(0.007)$ | $0.053(0.007)$ |
| $\frac{13}{2}^{+}$ | $6.84(1.00)$ | $0.098(0.007)$ | $0.085(0.006)$ |
| $\frac{15}{2}^{+}$ | $6.50(0.70)$ | $0.102(0.005)$ | $0.081(0.004)$ |
| $\frac{17}{2}^{+}$ | $4.93(0.57)$ | $0.094(0.005)$ | $0.070(0.004)$ |
| $\frac{19}{2}^{+}$ | $10.7(1.8)$ | $0.146(0.012)$ | $0.101(0.009)$ |
| $\frac{21}{2}^{+}$ | $4.51(1.00)$ | $0.100(0.011)$ | $0.065(0.007)$ |
| $\frac{23}{2}^{+}$ | $9.67(3.11)$ | $0.152(0.025)$ | $0.094(0.015)$ |
| $\frac{25}{2}^{+}$ | $6.12(1.37)$ | $0.126(0.014)$ | $0.075(0.008)$ |
| $\frac{17}{2}^{-}$ | $1.62(0.44)$ | $0.171(0.023)$ | $0.125(0.017)$ |
| $\frac{19}{2}$ |  |  |  |
| $\frac{21}{2}$ | $8.65(5.87)$ | $0.132(0.045)$ | $0.099(0.031)$ |
| $\left(\frac{11}{2}^{-}\right.$ | + | $8.08(2.21)$ | $0.098(0.013)$ |

Table 5.6: E1 matrix elements and dipole moments.

| $\mathrm{I}^{\pi}$ | $\mathrm{B}(\mathrm{M} 1) 10^{-2}\left(\mu_{N}^{2}\right)$ | $<\\|M 1\\|>\left(\mu_{n}\right)$ | $\left(g_{K}-g_{R}\right)$ |
| :---: | :---: | :---: | :---: |
| $\frac{9}{2}^{+}$ | $2.61(0.46)$ | $0.511(0.045)$ | $0.237(0.021)$ |
| $\frac{11}{2}^{+}$ | $3.07(0.60)$ | $0.607(0.059)$ | $0.238(0.023)$ |
| $\frac{13}{2}^{+}$ | $3.95(0.78)$ | $0.745(0.073)$ | $0.259(0.026)$ |
| $\frac{15}{2}^{+}$ | $3.30(0.70)$ | $0.736(0.007)$ | $0.234(0.022)$ |
| $\frac{17}{2}^{+}$ | $3.95(0.73)$ | $0.727(0.078)$ | $0.214(0.021)$ |
| $\frac{19}{2}^{+}$ | $2.47(0.86)$ | $0.844(0.123)$ | $0.232(0.022)$ |
| $\frac{21}{2}^{+}$ | $3.17(0.86)$ | $0.835(0.114)$ | $0.217(0.002)$ |
| $\frac{23}{2}^{+}$ | $5.39(0.50)$ | $1.14(0.52)$ | $0.354(0.165)$ |
| $\frac{17}{2}^{-}$ | $25.4(5.7)$ | $2.14(0.24)$ | $1.623(0.182)$ |
| $\frac{19}{2}^{-}$ | $54.6(14.9)$ | $3.30(0.45)$ | $1.183(0.158)$ |
| $\frac{21}{2}^{-}$ | $24.5(7.4)$ | $2.32(0.35)$ | $1.101(0.165)$ |
| $\left(\frac{11}{2}_{2}^{+}\right.$ | $170(55)$ | $4.52(0.72)$ | $2.669(0.427)$ |

Table 5.7: M1 matrix elements and $\left(g_{K}-g_{R}\right)$ values.

### 5.7 Discussion

### 5.7.1 Nuclear Structure Deduced from the Level Scheme

It is not the purpose of this work to present the level structure of ${ }^{153} \mathrm{Eu}$, merely to report a measurement of the lifetimes of the excited states. Nevertheless, in order to understand the relevance of the lifetime results it is first necessary to examine the level scheme and outline the structure of ${ }^{153} \mathrm{Eu}$, and its potential parity-doublet structure. In the following discussion, comparisons naturally arise with the nucleus, ${ }_{61}^{151} \mathrm{Pm}_{90}$, an isotonic neighbour of ${ }^{153} \mathrm{Eu}$, with very similar band structure that was reported in references [Vermeer 90] and [Urban 90]. Further comparison is made with the actinide nucleus ${ }_{90}^{223} \mathrm{Th}_{133}$, which is probably the best known example of a suspected parity-doublet band structure, and was reported in reference [Dahlinger 88].

The level structure of ${ }^{153} \mathrm{Eu}$ was shown in figure 5.11. The levels were drawn as two bands of constant parity, rather than two bands of constant simplex like that of ${ }^{223} \mathrm{Th}$ in figure 2.5. To recapitulate what was stated in section 2.3.2, the levels in an odd-mass octupole-deformed nucleus can be characterized by their simplex quantum number, $s$, which can have the values $\pm i$ that correspond to the spin and parity sequences,

$$
\begin{aligned}
& \mathrm{s}=+i: \mathrm{J}^{\pi}=\frac{1}{2}^{+}, \frac{3}{2}^{-}, \frac{5}{2}^{+}, \frac{7}{2}^{-}, \ldots . \\
& \mathrm{s}=-i: \mathrm{J}^{\pi}=\frac{1}{2}^{-}, \frac{3}{2}^{+}, \frac{5}{2}^{-}, \frac{7}{2}^{+}, \ldots .
\end{aligned}
$$

Higher-spin states than those shown were observed by Pearson et al: up to $\frac{45}{2} \hbar$ in the ground-state and negative parity bands; and up to $\frac{25}{2} \hbar$ in the $\frac{3}{2}^{+}$sideband. The $\left(\frac{5}{2}+2 \mathrm{n}\right)^{ \pm}$ levels (where n is a positive integer) are very closely spaced in energy and are almost degenerate at $\frac{25}{2} \hbar$. The $\left(\frac{7}{2}+2 n\right)^{ \pm}$levels are not as closely spaced due to a signature splitting effect which causes the obvious staggering in the negative-parity band. The interband E1 transitions have been observed to link the bands up to the $\frac{35}{2}^{ \pm}$levels, and have not been observed any higher due to lack of population of the levels.

According to the parity-doublet theory, the $\frac{5}{2}^{+}$and $\frac{5}{2}^{-}$bands, with, $s= \pm i$, originate from the same intrinsic orbital which occurs in a reflection-asymmetric potential as a result of the mixing of $\Delta \mathrm{j}=\Delta \ell=3$ orbitals by the octupole interaction. The octupole-driving orbitals in the lanthanide region are the $d_{\frac{8}{2}}$ and $h_{\frac{11}{2}}$ for protons and $f_{\frac{7}{2}}$ and $i_{\frac{13}{2}}$ for neutrons. The specific proton orbitals for ${ }^{153} \mathrm{Eu}$, and ${ }^{151} \mathrm{Pm}$, have the $\frac{5}{2}[413]$ and the $\frac{5}{2}[532]$ Nilsson configurations (see chapter 1). In the case of octupole deformation, these two orbitals merge into a single
intrinsic orbital of mixed parity. What should therefore be observed are two close-lying levels of $\frac{5}{2}^{+}$and $\frac{5}{2}^{-}$. In a well-deformed nucleus this doublet should continue to high spins, since rotational bands with similar properties will be built on the $\frac{5}{2}^{-}$and $\frac{5}{2}^{+}$states. The observation of two such bands in ${ }^{153} \mathrm{Eu}$ can therefore be interpreted in terms of octupole deformation, as can the similar structures which have been seen in several other nuclei, most notably, ${ }^{151} \mathrm{Pm}$ and ${ }^{223} \mathrm{Th}$. The most recent studies [Pearson 94] however have put forward an alternative explanation whereby the $\frac{5}{2}^{+}$and $\frac{5}{2}^{-}$bands arise from the accidental degeneracy of the $\frac{5}{2}[413]$ and the $\frac{5}{2}[532]$ Nilsson orbits. These are discussed below.

The behaviour of the rotational-frequency ratio, R (section 2.4), for ${ }^{153} \mathrm{Eu}$ is displayed in figure 5.17. R approaches the octupole-deformed limit at about $\frac{21}{2} \hbar$, but at the highest rotational frequencies, R tends to deviate away from this limit. The $\mathrm{s}=+i$ band approaches the limit at smaller spin values than the $s=-i$ band. This simplex dependence is also observed in ${ }^{151} \mathrm{Pm}$, although this is not shown. The simplex dependence cannot be explained within the framework of the octupole model. It could, however, be a consequence of the marked signature splitting in the negative-parity band. The signature splitting effects can clearly be seen in figure 5.18 where the difference in energy between two neighbouring spin states of the same parity is plotted against the square of the spin. The negative-parity band shows much larger signature splitting than the positive-parity band, in both ${ }^{153} \mathrm{Eu}$ and ${ }^{223} \mathrm{Th}$. The behaviour of ${ }^{151} \mathrm{Pm}$, although not shown, is very similar to that of ${ }^{153} \mathrm{Eu}$.

With such strong signature splitting effects, $\delta \mathrm{E}$, as defined in equation 2.5 is no longer simply a measure of the parity splitting. An expression is derived in reference [Dahlinger 88] to calculate a value of the parity splitting which is independent of signature splitting effects and is given by,

$$
\begin{equation*}
\delta E_{\pi}=\frac{1}{2}[E(\pi=-1, s=+i, I)-E(\pi=+1, s=+i, I)+E(\pi=-1, s=-i, I)-E(\pi=+1, s=-i, I)] . \tag{5.15}
\end{equation*}
$$

A plot of $\delta \mathrm{E}_{\pi}$ for both ${ }^{153} \mathrm{Eu}$ and ${ }^{223} \mathrm{Th}$ is given in figure 5.19 . The energy of levels with forbidden quantum numbers are interpolated between their neighbours. At intermediate spins of about $\frac{21}{2} \hbar, \delta \mathrm{E}_{\pi}$ appears to achieve stabilization close to the octupole deformed limit in both nuclei. However, for the case of ${ }^{153} \mathrm{Eu}$ at the highest spins, deviations away from the octupole-deformed limit are seen, as in the behaviour of $R$.

Finally, the effective moments of inertia $\Im_{\text {eff }}^{(1)}$ for ${ }^{153} \mathrm{Eu}$ are plotted in figure 5.20 where,

$$
\begin{equation*}
\Im_{\mathrm{eff}}^{(1)}=\frac{I_{x}}{\omega}, \tag{5.16}
\end{equation*}
$$



Figure 5.17: The rotational-frequency ratios for the simplex $= \pm i$ bands in ${ }^{153} \mathrm{Eu}$ and the actinide paritydoublet nucleus ${ }^{223} \mathrm{Th}$.


Figure 5.18: Illustration of the effects of signature splitting in the bands of ${ }^{153} \mathrm{Eu}$ and ${ }^{223} \mathrm{Th}$. The effect is larger for ${ }^{153} \mathrm{Eu}$ than for ${ }^{223} \mathrm{Th}$, and larger for the negative rather than positive parity bands.


Figure 5.19: The parity splitting for ${ }^{153} \mathrm{Eu}$ compared to that for a parity doublet from the actinide region, ${ }^{223} \mathrm{Th}$.


Figure 5.20: The effective moment of inertia, $\Im_{e f f}^{(1)}$ for the simplex $= \pm i$, parity $= \pm 1$ bands in ${ }^{153} \mathrm{Eu}$.

$$
\begin{align*}
\omega & =\frac{E(I+1)-E(I-1)}{I_{x}(I+1)-I_{x}(I+1)}  \tag{5.17}\\
I_{x} & =\sqrt{\left(I+\frac{1}{2}\right)^{2}-K^{2}} . \tag{5.18}
\end{align*}
$$

These quantities turn out to be similar to those discussed for the octupole-deformed actinides, discussed in reference [Dahlinger 88]. Namely,
(i). The moment of inertia is larger in the negative-parity band than the positive-parity band as is usually observed for octupole bands.
(ii). The behaviour is very similar for bands of equal parity but opposite simplex which would be expected for octupole-deformed nuclei, and would represent a common intrinsic structure.
(iii). The moment of inertia for different signatures and parities come together at intermediate rotational frequencies which is what may be expected if the octupole shape was being stabilized by rotation.

However, it is clear that at higher-rotational frequencies there is a backbend or sharp upbend in the moments of inertia of the negative-parity bands which is not as pronounced in the positive-parity bands. This suggests that the positive- and negative-parity bands have different structures and are not based on the same intrinsic state.

### 5.7.2 Lifetimes and Moments

The measured lifetimes are in excellent agreement with those calculated by the code Gosia, using the rotor matrix elements described in section 5.6.3. Prior to this experiment, only the lifetimes of the $\mathrm{I}^{\pi}=\frac{7^{+}}{2}, \frac{9^{+}}{2}, \frac{5_{2}^{-}}{2}, \frac{7}{2}^{-}, \frac{3^{+}}{2}$ and $\left(\frac{5}{2}\right)_{2}^{+}$states had been measured, by Thun and Miller in reference [Thun 72]. The only lifetime measured in this experiment that had previously been measured, was that of the $\frac{9}{2}^{+}$state at 193.1 keV . The previously measured value of $290(20) \mathrm{ps}$, is in agreement with the value measured here of $249.7(8.2) \mathrm{ps}$. The discrepancy may arise from the fact that, in [Thun 72], no mention is made of a feeding correction in extracting the lifetime.

## Quadrupole Moments

The electric quadrupole moments deduced from the lifetimes are presented in figure 5.21 and table 5.4. The spectroscopic quadrupole moment of the ground state has been measured using 'the hyperfine-splitting energies of muonic M X-rays' in reference [Tanaka 83] and shown to have a value of 2.41 eb , which corresponds to an intrinsic quadrupole mo-


Figure 5.21: The electric quadrupole moments deduced in this work.
ment of 6.75 eb . The values measured in this work are consistent with this measurement. The large values for the low-spin states in the ground-state band may be due to lack of knowledge about E1 branches out of the ground state band, which were not reported in references [Pearson 94] and [Pearson 94a].

## Electric Dipole Moments

The intrinsic dipole moments, $\mathrm{D}_{0}$, were presented in table 5.6 and are plotted on figure 5.22. No dependence on the spin, parity or simplex is observed. The weighted mean $D_{0}$ is comparable to the large collective $D_{0}$ values seen in the actinide region, and is much larger than known E1 rates outside the region of strong octupole correlations. This would be expected if the $\frac{5}{2}^{+}$and $\frac{5}{2}^{-}$bands constituted a parity doublet. In the recent study by Pearson et al., $\mathrm{D}_{0}$ values were deduced from the $\mathrm{I}(\mathrm{E} 1) / \mathrm{I}(\mathrm{E} 2)$ intensity ratio, assuming a constant quadrupole moment of 6.75 eb . The $\mathrm{D}_{0}$ values were found to show little variation and were scattered about 0.09 efm , in agreement with the values measured in this work. The values for the negative-parity band measured here, however, appear to be larger than those of the positive-parity band. These values are similar to those measured in reference [Vermeer 90] for ${ }^{151} \mathrm{Pm}$. Indeed Vermeer et al. calculate a B(E1) value for ${ }^{151} \mathrm{Pm}$ using a reflection-


Figure 5.22: The electric dipole moments deduced in this work.


Figure 5.23: The experimental values of the dipole moments for some lanthanides with $56 \leq \mathrm{Z} \leq 65,86 \leq \mathrm{N} \leq 90$.


Figure 5.24: The $\left(g_{K}-g_{R}\right)$ values deduced in this work.
symmetric model, and their calculated value turns out to about a thousand times smaller than the measured value.

It is interesting to look at the proton dependence of $\mathrm{D}_{0}$ which has recently been discussed for $\mathrm{Z} \leq 62$ in reference [Mowbray 89]. Figure 5.23 shows the dipole moments for a number of nuclei with $56 \leq \mathrm{Z} \leq 65$. The systematic trend suggests that octupole collectivity is a maximum for $\mathrm{Z}=60, \mathrm{~N}=88$ although the discussion of section 2.3.3 concerning shell effects should be born in mind. While the absolute magnitudes of the $D_{0}$ values support the interpretation in terms of a parity-doublet band, the apparently larger values for the negative-parity states would seem to indicate a difference in structure of the opposite-parity bands.

## Magnetic Dipole Moments and ( $g_{\mathrm{K}}-g_{\mathrm{R}}$ ) Values

The $\left(g_{K}-g_{R}\right)$ values deduced from the M1 reduced matrix elements are plotted in figure 5.24, and presented in table 5.7. The values are consistently larger in the negative-parity band than in the positive-parity band. The magnetic moments of the band-heads calculated using weighted mean $\left(g_{K}-g_{R}\right)$ values of $0.218(57)$ for the $\frac{5}{2}^{+}$band and 1.263(137) for
the $\frac{5}{2}^{-}$band, are presented in table 5.8, in comparison to those measured and predicted elsewhere. The interpretation of these bands as being built on a the same intrinsic paritydoublet state, requires that the magnetic moments for states of the same spin must be the same. The results of this work therefore contradict the parity-doublet interpretation and, by interpolation, they reproduce well the magnetic moment values calculated assuming reflection-symmetric configurations. All of the values are compared in table 5.8.

| Magnetic moments $\left(\mu_{N}\right)$ | $\mathrm{I}^{\pi}=\frac{5}{2}$ | $\mathrm{I}^{\pi}=\frac{5}{2}$ | $\mathrm{I}^{\pi}=\frac{3^{+}}{}{ }^{+}$ |
| :--- | :---: | :---: | :---: |
| This work | $1.41(37)$ | $3.28(36)$ | $3.01(48)$ |
| Measured [Lee 90] | $1.533(1)$ | $3.22(23)$ | $2.04(1)$ |
| Predicted [Ekström 73], [Bengtsson 89] | 1.5 | 2.6 | - |
| [Pearson 94] | $1.41(14)$ | $2.24(18)$ | $2.25(18)$ |

Table 5.8: A comparison of magnetic moments, for the band-heads. The model predictions are calculated using a reflection-symmetric mean-field approach, as described in the text.

### 5.7.3 Calculations

States in the potentially-octupole deformed promethium and europium isotopes have been investigated theoretically in two recent studies, the salient points of which are outlined below.

Nosek, Sheline et al. [Nosek 93] have performed microscopic calculations using a quasiphonon nuclear model Hamiltonian, with residual pairing and quadrupole-quadrupole and octupole-octupole interactions, to study octupole correlations between intrinsic states in ${ }^{151} \mathrm{Pm}$ and ${ }^{153,155} \mathrm{Eu}$. The model used is described in detail in the publication. The results state that there should be two pairs of $K^{\pi}=\frac{1}{2}^{ \pm}$bands at an excitation energy greater than 500 keV , in each of these nuclei. These are built on the quasi-particle configurations with Nilsson quantum numbers $\frac{1}{2}^{+}[420], \frac{1}{2}^{-}[550], \frac{1}{2}^{+}[411]$ and $\frac{1}{2}^{-}[541]$. These are the only states calculated to have sufficient octupole correlations to be interpreted as parity doublets. They also calculate that the $\mathrm{K}^{\pi}=\frac{3}{2}^{ \pm}$states are octupole-vibrational states in ${ }^{153,155} \mathrm{Eu}$, where the states are widely spaced in excitation energy, although it is possible that they form a parity doublet in ${ }^{151} \mathrm{Pm}$ where the states are more close-lying. Their most interesting result is that the $\mathrm{K}^{\pi}=\frac{5}{2}^{ \pm}$intrinsic states, which are the ground and first-excited states in both ${ }^{153} \mathrm{Eu}$ and ${ }^{151} \mathrm{Pm}$, cannot be said to constitute a parity doublet. The only $\frac{5}{2}{ }^{+}$state
with octupole collectivity is that based on a mixed state of $\frac{5}{2}^{+}[402]$ and $\frac{5}{2}^{-}[532]-Q_{30}$ at an excitation energy of about 1.5 MeV .

In another study, Afanasjev and Ragnarsson [Afanasjev 95] have performed calculations for the low-spin states in the promethium and europium nuclei around $A=153$, using a WoodsSaxon potential and taking into account Coriolis mixing. Again, detailed specification of the model is given in the reference. In this study, the polarization effects of the unpaired nucleons outside a reflection-symmetric even-even core are calculated. It is found that the $\frac{5}{2}[532]$ and $\frac{5}{2}[413]$ orbitals polarize the shape towards reflection symmetry, and the $\frac{1}{2}[411]$ and $\frac{1}{2}[420]$ orbitals polarize the shape towards octupole deformation. They conclude that the polarization effects tend towards octupole softness, but not to an octupole deformed shape. In addition the decoupling parameters, magnetic moments and energy splitting between the $\frac{5}{2}[532]$ and $\frac{5}{2}[413]$ band-heads are calculated. The experimental values of these quantities are reproduced without introducing octupole deformation.

In addition to the calculations described above, the large electric dipole moment values have also been interpreted by Alikov et al. in reference [Alikov 88] to arise from a contribution of Coriolis-coupling and shell-correction contributions.

### 5.8 Summary

Using the recoil-distance method following the multiple Coulomb excitation of ${ }^{153} \mathrm{Eu}$, the lifetimes of thirteen excited states have been measured. From the lifetimes and decay branching ratios, electromagnetic matrix elements and moments have been extracted. The quadrupole and dipole moments are observed to be independent of spin, parity and simplex and are scattered about mean values of $6.67(48) \mathrm{eb}$ and $0.077(7)$ efm respectively. The large electric dipole moments are consistent with the $\frac{5}{2}^{ \pm}$bands being interpreted as a pair of parity-doublet bands based on an intrinsic parity-mixed state. The ( $g_{K}-g_{R}$ ) values have also been extracted for the states and are found to be larger in the $\frac{5}{2}^{-}$band than in the $\frac{5}{2}^{+}$band. This observation implies a difference in structure of the bands which is therefore contradictory to the parity-doublet interpretation. The difference in band structures is also indicated in the much larger signature splitting in the negative-parity band. The parity splitting, frequency ratios and moments of inertia all tend away from the values that would be expected for an octupole deformed nucleus at the highest rotational frequencies.

With the observation of alternating parity bands connected by strong E1 transitions, it was natural to assume this to be a parity-doublet structure and interpret it in terms of reflection asymmetry or octupole deformation. However in the light of these and other recent experimental results, together with calculations explaining all the low-spin spectroscopic features without reflection asymmetry, it must be concluded that it is unlikely that static intrinsic reflection asymmetry exists in ${ }^{153} \mathrm{Eu}$. Furthermore, noting the similarity of the level structures of ${ }^{153} \mathrm{Eu}$ and ${ }^{151} \mathrm{Pm}$, the same conclusions may also apply to that nucleus. This also, therefore, draws into question the interpretation of parity-doublet bands in the actinide region, such as the famous case of ${ }^{223} \mathrm{Th}$.

The chance of many of the spectroscopic features of an octupole rotor occurring in a region known to be susceptible to octupole deformation seems rather too remote to rule out octupole deformation at the first stage. Further experimental studies should be performed to collect extra information on the structure in these nuclei. The calculations predict a $\mathrm{K}^{\pi}=\frac{1}{2}^{ \pm}$parity-doublet pair to occur at an excitation energy above 500 keV , in ${ }^{153} \mathrm{Eu}$, the observation of which would be a very interesting insight into octupole collectivity in this nucleus. Also, the $B(E 1)$ values should not be used to the exclusion of $B(E 3)$ values, which are the only true measure of octupole collectivity. An experimental challenge would be to measure the $\mathrm{B}(\mathrm{E} 3)$ values in ${ }^{153} \mathrm{Eu}$ using the Coulomb excitation method which has recently been used to extract E3 matrix elements in some rare-earth octupole-deformed nuclei in [Clarkson 92], [Ibbotson 93], [Ibbotson 95] and [White 90]. However, a full Coulombexcitation analysis in an odd-mass nucleus such as this would, at the very least, be difficult and time consuming, if not impossible.

Unlike the actinide region, where the effects of octupole deformation have been extensively treated theoretically, calculations concerning the relatively new island of octupole deformation in the lanthanides are scarce. Although the low-spin features of ${ }^{153} \mathrm{Eu}$ have been explained without invoking stable octupole deformation, this does not rule out the possibility of octupole deformation developing at high spin; calculations which trace the behaviour of nuclei like ${ }^{153} \mathrm{Eu}$ to high spin are needed. For the time being, it must be concluded that ${ }^{153} \mathrm{Eu}$ is not reflection asymmetric and that the island of reflection asymmetry in the lanthanides probably does not extend beyond $\mathrm{Z}=62$.

## APPENDIX A

${ }^{220}$ Ra Intensities

| $\mathrm{I}_{i}^{\pi_{i}}$ | $\mathrm{I}_{f}^{\pi_{f}}$ | Mult. | $\mathrm{E}_{\gamma}(\mathrm{keV})$ | Intensity | Intensity <br> [int.conv.cor.] | $\begin{array}{r} \mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2) \\ \left(\times 10^{-6} \mathrm{fm}^{-2}\right) \end{array}$ | Comments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2^{+}$ | $0^{+}$ | E2 | 178.4 | $95(6)$ | 182(11) |  | $\dagger$ |
| $4^{+}$ | $2^{+}$ | E2 | 231.6 | 100(8) | 136(10) |  |  |
| $5^{-}$ | $4^{+}$ | E1 | 224.6 | 48(5) | 51(6) |  |  |
| $6^{+}$ | $4^{+}$ | E2 | 278.1 | 89(8) | 106(10) | $0.386(0.103)$ | § |
| $6^{+}$ | $5^{-}$ | E1 | 53.0 | 4(1) | $6(2)$ |  |  |
| $7^{-}$ | $5^{-}$ | E2 | 237.9 | $7(3)$ | $9(4)$ | $1.115(0.487)$ | § |
| $7^{-}$ | $6^{+}$ | E1 | 184.9 | 84(7) | 93(8) |  | * |
| $8^{+}$ | $6^{+}$ | E2 | 313.3 | $30(1)$ | $34(1)$ | $1.950(0.161)$ |  |
| $8^{+}$ | $7{ }^{-}$ | E1 | 128.1 | $53(4)$ | $67(5)$ |  |  |
| $9^{-}$ | $7^{-}$ | E2 | 291.5 | 8(2) | $9(2)$ | 2.979 (0.797) | $\dagger$ § |
| $9^{-}$ | $8^{+}$ | E1 | 162.5 | $63(6)$ | $73(7)$ |  |  |
| $10^{+}$ | $8^{+}$ | E2 | 341.5 | 13(1) | 14(1) | $1.906(0.453)$ |  |
| $10^{+}$ | $9^{-}$ | E1 | 178.6 | 40(9) | $45(10)$ |  | $\dagger$ |
| 11- | $9^{-}$ | E2 | 332.7 | 20(2) | $22(2)$ | $2.128(0.332)$ | * |
| 11- | $10^{+}$ | E1 | 153.6 | $50(6)$ | $59(7)$ |  |  |
| $12^{+}$ | $10^{+}$ | E2 | 368.2 | 21(5) | $23(5)$ | $1.508(0.401)$ | $\dagger$ ¢ |
| $12^{+}$ | $11^{-}$ | E1 | 215.1 | 48(4) | 52(4) |  |  |
| $13^{-}$ | $11^{-}$ | E2 | 367.6 | $38(12)$ | 41(13) | $1.884(0.606)$ | $\dagger$ |
| $13^{-}$ | $12^{+}$ | E1 | 152.4 | $49(3)$ | 58(4) |  |  |
| $14^{+}$ | $12^{+}$ | E2 | 394.4 | 16(2) | 17(2) | $2.485(0.324)$ | * |
| $14^{+}$ | $13^{-}$ | E1 | 241.9 | $40(2)$ | $42(2)$ |  |  |
| $15^{-}$ | $13^{-}$ | E2 | 399.0 | 46(3) | $49(3)$ | $1.427(0.127)$ |  |
| $15^{-}$ | $14^{+}$ | E1 | 156.7 | $33(2)$ | $38(2)$ |  |  |
| $16^{+}$ | $14^{+}$ | E2 | 417.6 | 18(3) | $20(3)$ | $1.791(0.426)$ |  |
| $16^{+}$ | $15^{-}$ | E1 | 261.1 | $59(10)$ | $55(10)$ |  | * |
| $17^{-}$ | $15^{-}$ | E2 | 427.6 | $35(2)$ | $37(2)$ | $2.886(0.260)$ | * |
| $17^{-}$ | $16^{+}$ | E1 | 166.6 | 43(3) | 44(3) |  |  |

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| $\mathrm{I}_{i}^{\pi_{i}}$ | $\mathrm{If}_{f}^{\pi_{f}}$ | Mult. | $\mathrm{E}_{\gamma}(\mathrm{keV})$ | Intensity | $\begin{gathered} \text { Intensity } \\ \text { [int.conv.cor.] } \end{gathered}$ | $\begin{gathered} \mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2) \\ \left(\times 10^{-6} \mathrm{fm}^{-2}\right) \end{gathered}$ | Comments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $18^{+}$ | $16^{+}$ | E2 | 438.4 | 21(3)) | 23(3) | 1.646 (0.450) |  |
| $18^{+}$ | $17^{-}$ | E1 | 271.6 | $24(6)$ | 25(6) |  | $\dagger *$ |
| $19^{-}$ | $17^{-}$ | E2 | 455.2 | 24(8) | 25(8) | 2.181(0.958) | $\dagger$ |
| $19^{-}$ | $18^{+}$ | E1 | 182.4 | 21(6) | 23(7) |  | * |
| $20^{+}$ | $18^{+}$ | E2 | 454.4 | 11(3) | 12(3) | $1.007(0.434)$ | $\dagger$ |
| $20^{+}$ | $19^{-}$ | E1 | 273.2 | 15(5) | 25(5) |  | * |
| $21^{-}$ | $19^{-}$ | E2 | 479.4 | 19(1) | 20(1) | 1.892 (0.362) |  |
| $21^{-}$ | $20^{+}$ | E1 | 206.4 | 16(3) | 23(3) |  | * |
| $22^{+}$ | $20^{+}$ | E2 | 471.0 | 14(1) | 15(1) | 1.441(0.124) |  |
| $22^{+}$ | $21^{-}$ | E1 | 264.4 | 21(1) | 22(1) |  |  |
| $23^{-}$ | $21^{-}$ | E2 | 498.9 | 10(2) | 10(2) | $1.658(0.946)$ | $\dagger$ |
| $23^{-}$ | $22^{+}$ | E1 | 233.8 | $9(2)$ | 17(2) |  |  |
| $24^{+}$ | $22^{+}$ | E2 | 486.3 | 12(1) | 13(1) | $1.962(0.366)$ |  |
| $24^{+}$ | $23^{-}$ | E1 | 252.0 | 18(3) | 22(3) |  |  |
| $25^{-}$ | $23^{-}$ | E2 | 513.6 |  |  |  | * |
| $25^{-}$ | $24^{+}$ | E1 | 261.9 |  |  |  |  |
| $26^{+}$ | $24^{+}$ | E2 | 498.7 | 6 (2) | 6 (2) | $2.660(1.066)$ |  |
| $26^{+}$ | $25^{-}$ | E1 | 237.6 | $9(2)$ | 10(2) |  | $\dagger$ |
| $27^{-}$ | $25^{-}$ | E2 | 527.7 |  |  |  |  |
| $27^{-}$ | $26^{+}$ | E1 | 290.7 |  |  |  | $\dagger$ |
| $28^{+}$ | $26^{+}$ | E2 | 510.8 |  |  |  |  |
| $28^{+}$ | $27^{-}$ | E1 | 220.5 |  |  |  |  |
| $29^{-}$ | $27^{-}$ | E2 | 538.9 |  |  |  |  |
| $29^{-}$ | $28^{+}$ | E1 | 318.2 |  |  |  | $\dagger$ |
| $30^{+}$ | $28^{+}$ | E2 | 527.0 |  |  |  |  |
| $30^{+}$ | $29^{-}$ | E1 | 209.7 |  |  |  | $\dagger$ |
| $31^{-}$ | $29^{-}$ | E2 | 552.5 |  |  |  |  |
| $31^{-}$ | $30^{+}$ | E1 | 344.7 |  |  |  |  |
| $32^{+}$ | $30^{+}$ | E2 | 540.6 |  |  |  | $\dagger$ |
| $32^{+}$ | $31^{-}$ | E1 | 194.5 |  |  |  |  |

Table A: Intensities of transitions in ${ }^{220} \mathrm{Ra}$. † indicates a doublet in ${ }^{220} \mathrm{Ra}$; * is a doublet in ${ }^{222} \mathrm{Th},{ }^{220} \mathrm{Ra}$; § is a doublet in ${ }^{220} \mathrm{Ra},{ }^{219} \mathrm{Ra}$; and $\boldsymbol{\top}$ is a doublet in ${ }^{222} \mathrm{Th}$.

## APPENDIX B

${ }^{222} \mathrm{Th}$ Intensities

| $I_{i}^{\pi_{i}}$ | $\mathrm{I}_{f}^{\pi_{f}}$ | Mult. | $\mathrm{E}_{\gamma}(\mathrm{keV})$ | Intensity | Intensity <br> [int.conv.cor.] | $\begin{gathered} \mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2) \\ \left(\times 10^{-6} \mathrm{fm}^{-2}\right) \end{gathered}$ | Comments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2^{+}$ | $0^{+}$ | E2 | 182.9 | 90(6) | 174(12) |  |  |
| $4^{+}$ | $2^{+}$ | E2 | 256.3 | 100(8) | 128(8) |  | $\ddagger$ |
| $5^{-}$ | $4^{+}$ | E1 | 211.1 | $67(13)$ | 73(14) |  |  |
| $6^{+}$ | $4^{+}$ | E2 | 310.2 | 40(2) | 47(3) | $1.862(0.106)$ |  |
| $6^{+}$ | $5^{-}$ | E1 | 98.7 | 33(2) | 37(3) |  |  |
| $7^{-}$ | $5^{-}$ | E2 | 272.4 | $1(0.2)$ | 1 (0.2) | 14.23(3.01) | * 1 |
| $7{ }^{-}$ | $6^{+}$ | E1 | 173.1 | 60(4) | 68(4) |  |  |
| $8^{+}$ | $6^{+}$ | E2 | 343.6 | 11(2) | 13(3) | $4.464(0.910)$ |  |
| $8^{+}$ | $7^{-}$ | E1 | 170.1 | 68(4) | 77(4) |  |  |
| $9^{-}$ | $7^{-}$ | E2 | 331.6 | 13(1) | 15(1) | $3.180(0.415)$ | * |
| $9^{-}$ | $8^{+}$ | E1 | 161.2 | 57(8) | 66(9) |  |  |
| $10^{+}$ | $8^{+}$ | E2 | 368.1 | 9(2) | 10(2) | $3.874(0.619)$ | $\dagger$ |
| $10^{+}$ | $9^{-}$ | E1 | 206.4 | 62(4) | 67(4) |  |  |
| $11^{-}$ | $9^{-}$ | E2 | 367.8 | 17(1) | 18(1) | 2.190 (0.551) | $\dagger$ ¢ |
| $11^{-}$ | $10^{+}$ | E1 | 161.1 | $30(7)$ | 35(9) |  |  |
| $12^{+}$ | $10^{+}$ | E2 | 389.8 | $5(0.4)$ | $5(0.5)$ | 4.163(0.446) |  |
| $12^{+}$ | $11^{-}$ | E1 | 228.5 | $35(3)$ | 38(3) |  | * |
| $13^{-}$ | $11^{-}$ | E2 | 393.2 | 15(1) | 16(1) | 3.451 (0.170) | * |
| $13^{-}$ | $12^{+}$ | E1 | 164.4 | 33(2) | 38(2) |  |  |
| $14^{+}$ | $12^{+}$ | E2 | 409.2 | $3(1)$ | $3(0.5)$ | 5.849(1.092) |  |
| $14^{+}$ | $13^{-}$ | E1 | 244.3 | $27(2)$ | 29(2) |  | 9 |
| $15^{-}$ | $13^{-}$ | E2 | 416.6 | 10(1) | 10(1) | 3.678 (0.375) |  |
| $15^{-}$ | $14^{+}$ | E1 | 171.7 | 19(1) | 22(2) |  |  |
| $16^{+}$ | $14^{+}$ | E2 | 428.5 | $5(0.3)$ | $5(0.4)$ | $2.285(0.182)$ | * |
| $16^{+}$ | $15^{-}$ | E1 | 256.2 | 18(1) | 19(1) |  | $\ddagger$ |
| $17^{-}$ | $15^{-}$ | E2 | 441.3 | $6(0.3)$ | 7 (0.4) | $6.237(0.182)$ |  |
| $17^{-}$ | $16^{+}$ | E1 | 185.1 | 19(1) | 21(1) |  | * |

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| $\mathrm{I}_{i}^{\pi_{t}}$ | $\mathrm{I}_{f}^{\pi_{f}}$ | Mult. | $\mathrm{E}_{\gamma}(\mathrm{keV})$ | Intensity | Intensity <br> [int.conv.cor.] | $\begin{gathered} \mathrm{B}(\mathrm{E} 1) / \mathrm{B}(\mathrm{E} 2) \\ \left(\times 10^{-6} \mathrm{fm}^{-2}\right) \end{gathered}$ | Comments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $18^{+}$ | $16^{+}$ | E2 | 446.0 | $6(0.2)$ | $8(0.2)$ | $1.899(0.065)$ |  |
| $18^{+}$ | $17^{-}$ | E1 | 260.6 | 20 (2) | $20(2)$ |  | * |
| $19^{-}$ | $17^{-}$ | E2 | 468.0 | $8(0.5)$ | $9(0.5)$ | $5.337(0.531)$ |  |
| $19^{-}$ | $18^{+}$ | E1 | 206.9 | $22(2)$ | $23(2)$ |  | 4 |
| $20^{+}$ | $18^{+}$ | E2 | 462.8 | $6(0.4)$ | $7(0.4)$ | $2.736(0.424)$ |  |
| $20^{+}$ | $19^{-}$ | E1 | 256.2 | 18(3) | $19(3)$ |  | $\ddagger$ |
| $21^{-}$ | $19^{-}$ | E2 | 494.9 | $3(0.2)$ | $3(0.2)$ | $6.168(0.298)$ |  |
| $21^{-}$ | $20^{+}$ | E1 | 239.2 | 11(1) | 11(1) |  |  |
| $22^{+}$ | $20^{+}$ | E2 | 482.0 |  |  |  |  |
| $22^{+}$ | $21^{-}$ | E1 | 242.3 |  |  |  |  |
| $23^{-}$ | $21^{-}$ | E2 | 512.6 |  |  |  | 4 |
| $23^{-}$ | $22^{+}$ | E1 | 269.8 |  |  |  |  |
| $24^{+}$ | $22^{+}$ | E2 | 500.71 |  |  |  |  |
| $24^{+}$ | $23^{-}$ | E1 | 229.17 |  |  |  | * |
| $25^{-}$ | $23^{-}$ | E2 | 514.96 |  |  |  | * |
| $25^{-}$ | $24^{+}$ | E1 | 283.8 |  |  |  |  |
| $26^{+}$ | $24^{+}$ | E2 | 525 |  |  |  |  |

Table B: Intensities of transitions in ${ }^{222} \mathrm{Th}: *$ indicates a doublet in ${ }^{222} \mathrm{Th},{ }^{220} \mathrm{Ra}$; $\ddagger$ is a triplet in ${ }^{222} \mathrm{Th}$; $\boldsymbol{q}$ is a doublet in ${ }^{222} \mathrm{Th}$; and $\dagger$ is doublet in ${ }^{220} \mathrm{Ra}$.

## APPENDIX C

Angle-dependent Intensity Ratios

| $\Delta \mathrm{I}=1$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{222} \mathrm{Th}$ |  |  | ${ }^{220} \mathrm{Ra}$ |  |  |
| $\mathrm{E}_{\gamma}(\mathrm{keV})$ | $\frac{I\left(158^{\circ}\right)-I\left(90^{\circ}\right)}{l\left(158^{\circ}\right)+I\left(90^{\circ}\right)}$ | $\frac{I\left(134^{\circ}\right)-I\left(90^{\circ}\right)}{I\left(134^{\circ}\right)+I\left(90^{\circ}\right)}$ | $\mathrm{E}_{\gamma}(\mathrm{keV})$ | $\frac{I\left(158^{\circ}\right)-I\left(90^{\circ}\right)}{I\left(158^{\circ}\right)+I\left(90^{\circ}\right)}$ | $\frac{I\left(134^{\circ}\right)-I\left(90^{\circ}\right)}{I\left(134^{\circ}\right)+I\left(90^{\circ}\right)}$ |
| 98.7 | -0.005(0.039) | -0.042(0.030) | 53.0 |  |  |
| $161.1^{a}$ | -0.026(0.030) | 0.001(0.020) | 128.1 | -0.004(0.097) | 0.013(0.078) |
| $161.2^{a}$ |  |  | $152.4{ }^{\text {c }}$ | 0.073(0.092) | 0.031(0.080) |
| 164.4 | 0.017(0.068) | 0.013(0.044) | $153.6^{\text {c }}$ |  |  |
| 170.1 | 0.108(0.048) | -0.020(0.037) | 156.7 | -0.009(0.090) | $0.074(0.082)$ |
| 171.7 |  |  | 162.5 | 0.062(0.069) | 0.076(0.040) |
| 173.1 | $0.007(0.036)$ | 0.011(0.032) | 166.6 | 0.022(0.112) | 0.027(0.065) |
| $185.1^{\text {b }}$ | 0.028(0.246) | -0.014(0.074) | $178.6^{\text {c,* }}$ |  |  |
| $206.4^{\text {b,c }}$ | 0.029(0.040) | 0.033(0.025) | $182.4{ }^{\text {b }}$ | 0.102(0.208) | $0.139(0.147)$ |
| $206.9^{\text {b,c }}$ |  |  | $184.9^{\text {b }}$ | 0.013(0.110) | 0.050(0.43) |
| $211.1^{\text {b }}$ | $-0.030(0.045)$ | $0.057(0.030)$ | 194.5 |  |  |
| $228.2^{\text {c }}$ | $0.066(0.059)$ | 0.039(0.033) | $206.4{ }^{\text {b,c }}$ | 0.108(0.244) | $0.017(0.153)$ |
| 228.5 |  |  | $209.7^{\text {b }}$ |  |  |
| 239.2 | $0.017(0.142)$ | 0.040(0.151) | 215.1 | 0.045(0.059) | 0.072(0.081) |
| 242.3 |  |  | 220.5 | 0.358(0.282) | $0.132(0.170)$ |
| 244.3 | $0.052(0.108)$ | $0.038(0.034)$ | 224.6 | $0.207(0.115)$ | 0.014(0.073) |
| 256.0 ${ }^{e, *}$ |  |  | 233.8 |  |  |
| $256.2^{e, *}$ |  |  | $237.6^{\text {c,* }}$ | $0.105(0.209)$ | -0.038(0.173) |
| 260.6 | 0.059(0.137) | 0.094(0.130) | 241.9 | $0.050(0.113)$ | 0.148(0.059) |
| 269.8 |  |  | 252.0 | $0.164(0.164)$ | 0.177(0.135) |
|  |  |  | 261.9 | $0.082(0.130)$ | 0.026(0.062) |
|  |  |  | 264.4 | $0.010(0.227)$ | $0.221(0.112)$ |
|  |  |  | 271.6 | $0.249(0.081)$ | 0.095(0.063) |
|  |  |  | 273.6 |  |  |
|  |  |  | $290.7^{c, *}$ | 0.022(0.066) | -0.122(0.247) |

Table C: Angle-dependent intensity ratios for the stretched-dipole transitions.

| $\Delta \mathrm{I}=2$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{222} \mathrm{Th}$ |  |  | ${ }^{220} \mathrm{Ra}$ |  |  |
| $\mathrm{E}_{\gamma}(\mathrm{keV})$ | $\frac{I\left(158^{\circ}\right)-I\left(90^{\circ}\right)}{I\left(158^{\circ}\right)+I\left(90^{\circ}\right)}$ | $\frac{I\left(134^{\circ}\right)-I\left(90^{\circ}\right)}{I\left(134^{\circ}\right)+I\left(90^{\circ}\right)}$ | $\mathrm{E}_{\gamma}(\mathrm{keV})$ | $\frac{I\left(158^{\circ}\right)-I\left(90^{\circ}\right)}{I\left(158^{\circ}\right)+I\left(90^{\circ}\right)}$ | $\frac{I\left(134^{\circ}\right)-I\left(90^{\circ}\right)}{I\left(134^{\circ}\right)+I\left(90^{\circ}\right)}$ |
| $182.9^{\text {b,* }}$ | -0.710(0.047) | -0.078(0.019) | $178.4^{c, *}$ | $-0.135(0.045)$ | $0.013(0.024)$ |
| $256.3^{e, *}$ | -0.086(0.024) | $-0.007(0.012)$ | 231.6 | -0.152(0.072) | $-0.063(0.039)$ |
| $272.4{ }^{\text {b,*}}$ |  |  | $237.9^{\text {c,* }}$ |  |  |
| 310.2 | $-0.317(0.048)$ | -0.027(0.028) | $278.1^{\text {d,* }}$ | $-0.278(0.066)$ | $-0.076(0.049)$ |
| $331.6^{\text {b }}$ | -0.355(0.172) | $-0.235(0.076)$ | $291.5^{\text {c,* }}$ |  |  |
| 343.6 | -0.207(0.135) | -0.084(0.116) | 313.3 | $-0.289(0.174)$ | -0.096(0.130) |
| $367.8^{a, c}$ | -0.200(0.056) | -0.102(0.045) | $332.7{ }^{\text {b }}$ | -0.67(0.195) | $0.049(0.137)$ |
| $368.1^{\text {a,c }}$ |  |  | 341.5 | -0.253(0.187) | -0.052(0.121) |
| 389.8 | -0.267(0.186) | -0.127(0.134) | $367.6^{a, c}$ | -0.180(0.186) | $-0.073(0.082)$ |
| $393.2{ }^{\text {b }}$ | -0.264(0.096) | -0.113(0.057) | $368.2^{\text {a,c }}$ |  |  |
| 409.2 | $-0.307(0.253)$ | -0.191(0.127) | $394.4{ }^{\text {b }}$ | $-0.217(0.200)$ | $-0.316(0.277)$ |
| $416.6^{\text {b }}$ | -0.162(0.152) | -0.094(0.102) | 397.6 | -0.095(0.117) | $-0.113(0.066)$ |
| $428.5{ }^{\text {b }}$ | -0.169(0.205) | -0.068(0.137) | $417.6^{\text {b }}$ | -0.150(0.210) | -0.179(0.207) |
| 441.3 | -0.245(0.168) | -0.222(0.078) | $427.6^{\text {b }}$ | -0.320(0.141) | -0.089(0.082) |
| 446.0 | -0.224(0.177) | -0.103(0.284) | 438.4 | -0.111 (0.290) | -0.101(0.176) |
| 462.8 | -0.132(0.147) | -0.032(0.188) | $454.4^{c}$ | -0.220(0.133) | -0.088(0.085) |
| 468.0 | -0.149(0.129) | -0.229(0.193) | $455.2^{\text {c }}$ |  |  |
| 482.0 |  |  | 471.0 | $-0.210(0.163)$ | $-0.169(0.276)$ |
| 494.9 |  |  | 479.4 | -0.145(0.114) | $-0.115(0.145)$ |
| 500.7 |  |  | 486.3 | -0.214(0.335) | -0.168(0.246) |
| 512.6 |  |  | $498.7^{\text {c }}$ | -0.144(0.146) | -0.228(0.092) |
| 515.0 |  |  | $498.9^{\text {c }}$ |  |  |
|  |  |  | 510.8 |  |  |
|  |  |  | 513.6 |  |  |
|  |  |  | $527.0^{c}$ |  | $-0.337(0.163)$ |
|  |  |  | $527.7^{\text {c }}$ |  |  |

Table D: Angle-dependent intensity ratios for the stretched-quadrupole transitions.

## Comments on the tables:

(i). Doublets and triplets are indicated
a doublet in ${ }^{222} \mathrm{Th}$.
b doublet in ${ }^{222} \mathrm{Th},{ }^{220} \mathrm{Ra}$.
c doublet in ${ }^{220} \mathrm{Ra}$.
d doublet in ${ }^{220} \mathrm{Ra}$, ${ }^{219} \mathrm{Ra}$.
e triplet in ${ }^{222} \mathrm{Th}$.
(ii). Transitions marked with a superscript * are suspected to be stretched-dipole/stretchedquadrupole doublets.
(iii). $\mathrm{I}\left(158^{\circ}\right)$ represents the intensity at $158^{\circ}$ seen by a gate on $90^{\circ}$, and vise-versa.
(iv). $\mathrm{I}\left(134^{\circ}\right)$ represents the intensity at $134^{\circ}$ seen by a gate on $90^{\circ}$, and vise-versa.

## APPENDIX D

## Computer Codes: Description and Acknowledgement

- Scana

A code to analyse gamma-gamma correlation matrices, written by Waldek Urban at the University of Manchester. The code allows gates to be set of the projection of a two-dimensional matrix, and performs fits on the peaks in the resulting spectra.

## - Eurogam Analysis Session

A data-acquisition and analysis package developed by the Eurogam support team at Daresbury, Liverpool and Manchester. The package includes the Eurogam sort programme, written by Janet Sampson, which was used extensively in this work.

- ORACLE - the Oak Ridge Analysis Code for Lifetime Evaluation

Written by Sturm and Guidry at Oak Ridge National Laboratory. Modified by Bohdan Kotlinski, Ching-Yen Wu and Mike Simon at the University of Rochester.

## - Gosia

The semi-classical Coulomb-excitation analysis code written by Tomek Czosnyka of the University of Warsaw, and Douglas Cline and Ching-Yen Wu of the University of Rochester.

## - eg2ana2d

One of a series of programmes written by Pete Jones of the University of Liverpool in order to convert matrices and spectra from one format to another, so that they may be read by different analysis programmes.

## - GF2

Part of the comprehensive 'Radware' package of analysis programmes, written by Dave Radford at Chalk River, for the analysis of high-fold gamma-ray spectroscopy data. The package includes the code GF2 for fitting peak shapes in one-dimensional spectra, which was used in this thesis.

- Silvia \& GREMLIN -Gamma Ray Efficiency Measurement and Line Intensity programme Written by Alex Kavka of the Universities of Rochester and Uppsala. GREMLIN fits various degrees of polynomials to source data in order to produce a relative efficiency curve for a gamma-ray detector. Silvia is an analysis code for one-dimensional spectra.


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[^0]:    ${ }^{1}$ So-called because of the similarity of the rotational level spectrum to that of a reflection-asymmetric molecule.

[^1]:    ${ }^{1} \mathrm{~A}$ state is oriented when the relative populations, $\mathrm{P}(\mathrm{m})$, of the angular-momentum substates are unequal, that is $\mathrm{P}\left(\mathrm{m}_{i}\right) \neq \mathrm{P}\left(\mathrm{m}_{j}\right)$. When $\mathrm{P}\left(\mathrm{m}_{i}\right)=\mathrm{P}\left(-\mathrm{m}_{i}\right)$ the state is aligned.

[^2]:    ${ }^{2}$ Note the difference between fold and multiplicity: fold usually refers to the number of gamma rays that are detected while multiplicity refers to the number of gamma rays that are emitted.

[^3]:    ${ }^{3}$ The yrast line is the locus of points which have the lowest excitation energy for a given value of angular momentum.

[^4]:    ${ }^{1}$ An elaborated array known as Eurogam Phase-II [Nolan 95] is currently operational at the Vivitron tandem accelerator in Strasbourg, France. Henceforth in this work, Eurogam shall refer to the Eurogam phase-I array.

[^5]:    ${ }^{2}$ A Nuclear Instrument Module (NIM) is a standard electronics box designed to fit into a standard 19 inch NIM crate, along with 11 other modules. The NIM standard is described in appendix A of reference [Knoll 89].
    ${ }^{3} \mathrm{VXI}$ is an electronic standard, similar to NIM, except with larger cards, for use with integrated circuitry.

[^6]:    ${ }^{4}$ TARDIS is an acronym for Tessa ARray Deployed at ISolde. The array was originally designed to be used for radioactive-decay studies at the ISOLDE on-line-isotope separator facility at CERN.

[^7]:    ${ }^{5}$ The neutrons will be more evenly distributed in the oxygen on lead reaction because there is less kinematic focussing with the asymmetric beam and target combination.

[^8]:    ${ }^{1}$ For the same reason nickel is used in target backings and the shifter and stopper foils.

