DEVELOPMENT OF A MODEL TO ESTIMATE THE EFFECTIVE SECOND MOMENT OF AREA OF ONE-WAY REINFORCED CONCRETE FLEXURAL ELEMENTS

Thesis submitted in accordance with the requirements of the

University of Liverpool

for the degree of

Doctor in Philosophy

by

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JULY 1996

ABSTRACT

A model for the effective second moment of area, Ie, to be used in deflection calculations of one way reinforced concrete flexural elements is developed.

The model consists of an analytical expression developed using as boundary conditions the completely uncracked section when the loads are low and the completely cracked section when the loads are very high. For the conditions when the loads are moderately high to cause partial cracking of the section a probable curve was developed relating a fictitious steel area, taken to represent the concrete stiffening effect, and the applied moment. The curve was then used to formulate the model.

The analytical model developed as explained above involved an auxiliary coefficient which had to be empirically defined to fit results of tested beams. Because no experiments were conducted exclusively for the present study the coefficient was defined based on the results of experiments reported in the literature.

Also included in the study is the approximation of the second moment of area of the cracked transformed section, Icr, which is an integral part of the developed model. The approximation not only facilitated the computation of Icr but also made simple graphical representation of the proposed model of Ie possible. In the process of the approximation computer programs had to be developed to study the effect of combining the different parameters involved so that a simple and accurate model for the approximation of Icr is achieved. The result was a compact expression that led itself well into the final model proposed for the effective second moment of area and which made simple graphical representation of the model possible.

The model of Ie and the expression for the approximation of Icr described above were derived within the limitations and scope of the British and American practices as specified by the British and American codes which form the basis for most other codes in the world.

In the course of the study numerical examples are given where necessary to illustrate the application of the different methodologies and to explain their aspects of practicality as well as the limitations involved and to show the solution process used by the different computer programs in arriving at the concluding results.

Using the results of the tested beams the proposed model of Ie is compared with the expression currently used by the American code and the one most recently proposed as its substitute. The results of the comparison as included in the Appendices and summarized in the body of the thesis have shown that the errors obtained using the developed model are almost half of those from the other two expressions especially for low reinforcement ratios and the practical range of applied moments.

Finally the proposed model of Ie was also compared to the methods in the British code and Eurocode 2 for calculating deflections. Two numerical examples were furnished to demonstrate the ease and great simplicity offered by the proposed model.

ACKNOWLEDGEMENT

Grateful acknowledgements are due to my academic supervisor, Mr. C. Thomas, for his inspiration and indispensable advice during the course of the study. It is for his help and enlightening guidance that this work was completed.

Thanks are also extended to my parents, wife and daughter for their unfailing support through out the years of the study. It is their continuous patience and encouragement that made it all possible.

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LIST OF SYMBOLS

As	area of tension steel reinforcement
As′	area of compression steel reinforcement
b	width of a rectangular section
b′	width of an equivalent section
be	flange width of a flanged section
bw	web width of a flanged section
CF	correction factor as defined in Sec.4.3
C_t	creep coefficient ≡ ratio of creep strain to elastic strain
d	effective depth of the tension steel reinforcement. It is the distance from the centroid of the tension steel to the extreme fibre in compression
d′	effective depth of the compression steel reinforcement. It is the distance from the centroid of the compression steel to the extreme fibre in compression
Ec	short term elastic modulus of concrete
Es	elastic modulus of the reinforcing steel
f	tensile stress in the outer fibre of concrete
fc	compressive stress in the outer fibre of concrete
f_{ct}	tensile stress in the concrete at the level of the tension steel
fcu	cubic compressive strength of concrete
fc′	cylindrical compressive strength of concrete
fr	modulus of rupture of concrete. It is the tensile strength of concrete in bending
fs	stress in the tension steel
fs′	stress in the compression steel
h	total depth of a section
hf	flange depth of a flanged section
Icr	moment of inertia of a cracked transformed section considering only the compression concrete area and transforming all steel areas into equivalent concrete areas
Icre	equivalent or approximate value of Icr
Ie	effective moment of inertia of a concrete section or element
Ig	moment of inertia of the gross concrete area neglecting steel
I_{tr}	moment of inertia of the gross concrete area considering steel
L	span of a concrete element
Lcr	span of a concrete element in which cracks occur
Ma	applied service load moment
Mcr	cracking moment. It is the moment required to cause first cracking of a concrete section.
n	short term modular ratio, Es/Ec

N.A	neutral axis of a section
$n\rho$	the product of n times ρ
$n\rho e$	the product of n times ρ taken relative to the equivalent section
$n\rho'$	the product of n times ρ'
R	a factor defined in Chap.4 as $Ig/(bd^3/12)$ or $Ig/(b'd^3/12)$
$1/r_x$	curvature at point x
$1/r_b$	maximum curvature to be used in deflection calculations
$(1/r_{l.perm})_{pcr}$	long term curvature due to permanent loads considering the partially cracked section
$(1/r_{l.perm})_{tr}$	long term curvature due to permanent loads considering the uncracked section
$(1/r_{shr})_{pcr}$	curvature due to shrinkage effects considering the partially cracked section
$(1/r_{shr})_{tr}$	curvature due to shrinkage effects considering the uncracked section
$(1/r_{s.perm})_{pcr}$	short term curvature due to permanent loads considering the partially cracked section
$(1/r_{s.perm})_{tr}$	short term curvature due to permanent loads considering the uncracked section
$(1/r_{s.tot})_{pcr}$	short term curvature due to total loads considering the partially cracked section
$(1/r_{s.tot})_{tr}$	short term curvature due to total loads considering the uncracked section
s_s	moment of steel area about the centroid of the considered section
x	depth of the neutral axis relative to the outer compression fibre
y_t	distance from neutral axis to the extreme concrete fibre in tension
α	a factor used in evaluating Icre
α'	a converting factor used in case of compression reinforcement
αf	a converting factor used in case of flanged sections
β	a factor used in evaluating Icre
β_1 , β_2	factors used in deflection calculations according to Eurocode 2
δ	deflection at a point
δ_{long}	total long term deflection
$\delta_{l.perm}$	long term deflection due to permanent loads
δ_{shr}	deflection due to shrinkage effects
$\delta_{s.perm}$	short term deflection due to permanent loads
$\delta_{s.tot}$	short term deflection due to total loads
ε_c	compressive strain in the outer fibre of concrete
ε_s	strain in the tension steel
ε_{shr}	free shrinkage strain of plain concrete
ε_s'	strain in the compression steel
ξ	an interpolation factor used to find the average curvature according to Eurocode 2
ρ	ratio of tension reinforcement to effective concrete area $\equiv As/(bd)$
ρ'	ratio of compression reinforcement to effective concrete area $\equiv As'/(bd)$
Φ	coefficient developed as part of the proposed model of Ie

CHAPTER 1

INTRODUCTION

1.1 General

When the tensile stresses in concrete exceed its modulus of rupture the concrete cracks. These tension cracks form at a finite spacing as shown in the typical simply supported beam below.

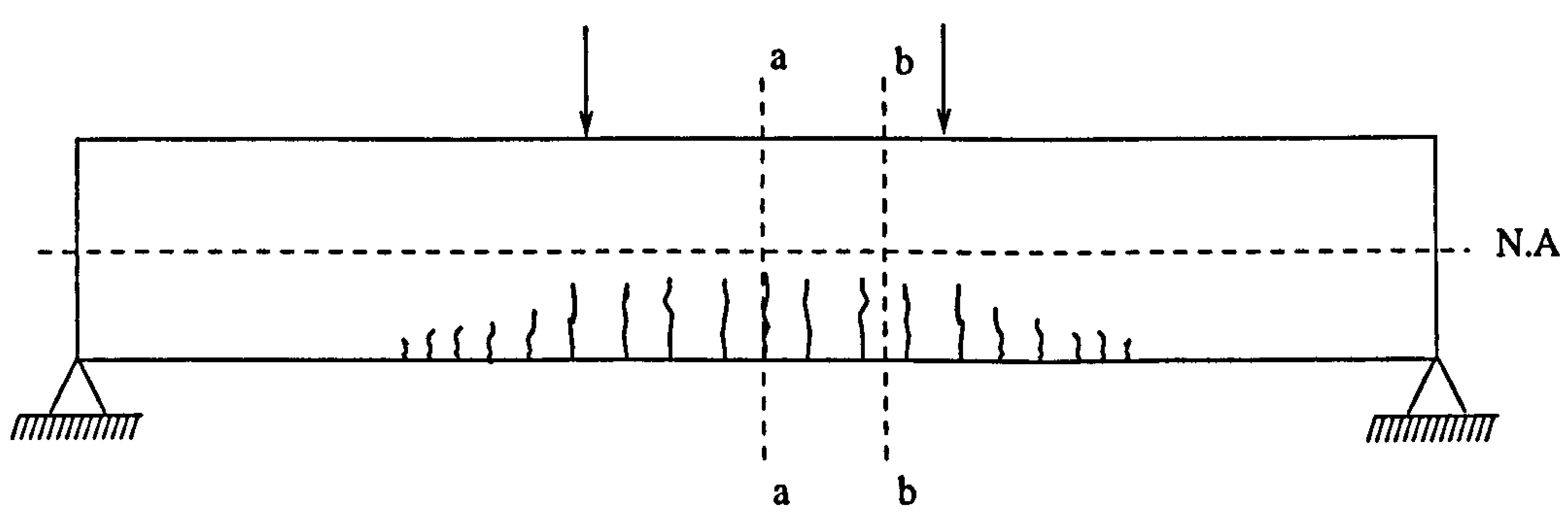


FIGURE 1.1.1 A typical flexurally cracked simply supported beam

Since a crack creates a gap in the concrete through which stresses obviously can not be transferred the tensile resistance of a section where the concrete in the tension face is completely cracked, for example at section a-a, must be provided for entirely by the steel reinforcement. However, between the cracks, for example at section b-b, or at sections where the cracks do not propagate deep into the element concrete will still be able to take some tension. This phenomenon of the ability of concrete to take some tension is usually referred to as " tension stiffening " of concrete.

While tension stiffening of concrete is conservatively ignored in flexural design it is most physically represented by the so called "effective moment of inertia, Ie" proposed by many codes for the purpose of deflection calculations (for ease of reference the term moment of inertia as used in these codes are retained through out this thesis. This is also more consistent with the references consulted than the term second moment of area which is more commonly used in U.K.). Expressing the effect of uncracked concrete in the tension face the effective moment of inertia assumes a value greater than the moment of inertia of a completely cracked section and less than that of a completely uncracked one. With such a moment of inertia assumed over the entire span deflection calculations can then proceed and the neutral axis position can be assumed fixed throughout as shown, denoted by N.A., in Fig.1.1.1.

Many empirical expressions for the evaluation of Ie have been proposed. The most widely recognized of these is the so called Branson's equation [1] where the effective moment of inertia, for the purpose of deflection calculations, is expressed as follows,

$$Ie = Icr + (Ig - Icr)(Mcr/Ma)^3 \leq Ig$$

where,

> $Icr \equiv$ moment of inertia of the completely cracked section with all steel areas transformed into equivalent concrete areas.
>
> $Ig \equiv$ moment of inertia of the completely uncracked section neglecting steel.
>
> $Mcr \equiv$ cracking moment
>
> $Ma \equiv$ applied service moment

Although the equation has been adopted by many codes (i.e.the American and the Canadian codes [2,3]) serious contentions regarding its nature and the accuracy of its results have been recently raised. Among these are :

1. The equation was proposed as an empirical expression based on results obtained from beams tested under uniform loads[1]. Because of this and that the equation does not account for other loading types it can be shown that for load types other than uniform the equation gives results that are inconsistent and with errors as high as 100 % [4,5].
2. The evaluation of the moment of inertia of the cracked transformed section,Icr, can be both complex and time consuming, especially in case of flanged sections, to the extent that some scholars tried to approximate the equation in an effort to eliminate the need for calculating Icr [6].
3. The equation can not easily be represented graphically through solution curves. These curves as provided in the different references [7,8] are not only complex but also fail to represent the phenomena involved and thus offer little as design aids[9].
4. In general, the accuracy of the results obtained from the equation does not justify the efforts involved.

Because of these disadvantages many scholars and designers are of the opinion that an alternative to Branson's equation must be found.

In 1981 and in an effort to eliminate the need for having to calculate Icr, Grossman [6] proposed simplifications of Branson's equation. Since no experimental results were consulted the study assumed the effective moment of inertia as given by Branson's equation to be exact. However, because the proposed expressions still did not represent the loading type and that their validity was drawn based on Branson's equation which itself bears the drawbacks discussed above there was no reason to accept the proposed expressions as a sound alternative.

Similarly, In 1993 and in their effort to provide a substitute for Branson's equation, Al-shaikh and Al-zaid [4] have proposed an empirical expression in which the loading type and the effect of reinforcement were accounted for. However, because it proved to give only slightly better accuracy than Branson's equation and that the detailed evaluation of Icr was still required the proposed expression did not actually offer many improvements.

Because of this inability of the expressions proposed so far to replace Branson's equation the need for an alternative still exists. In this study and for the purpose of calculating deflection in one way reinforced concrete elements an alternative to Branson's equation for the evaluation of the effective moment of inertia will be developed where all the forementioned drawbacks are eliminated.

1.2 Scope and Limitations of the Study

The goal set in this study is to develop an expression for the effective moment of inertia to be used in deflection calculation of one way reinforced concrete elements. The expression is to achieve the following objectives :

1. It has to be as general as to be applicable within the limitations and scopes of both the British and the American practices as set by the specifications of the British code BS 8110 [10] and the American code ACI 318 [2] and which form the basis for other codes used through out the world.
2. It has to be mainly analytically developed and that all the empirical parts be represented by a single coefficient which is independent of the form of the expression. This is particularly useful for future research since the expression being analytically developed will always be valid and any refinement or modification that may be felt necessary need only be applied to the empirical coefficient without having to derive a new expression or altering the existing one.
3. Following the same reasoning as in 2 above, any graphical representation of the developed model must also be independent of the expression of the empirical coefficient.
4. The expression should be derived such that all the drawbacks usually claimed to be associated with Branson's equation as outlined in Sec.1.1 are completely eliminated.
5. The expression should be of a simple format that leads itself well into a graphical representation that can be used as a design aid similar to the design charts usually provided for other design purposes in the respective codes. The graph should be simple, easy to read and representative of the phenomena involved.

As the study progresses the objectives outlined above will be fully elaborated upon and the different related aspects as well as the limitations of the proposed expression will be fully explained. Numerical examples will also be given in the respective chapters to help illustrate the different concepts involved.

For a problem of such great diversity as the deflection of reinforced concrete elements it was thought pointless to test a few beams in the laboratory and try to fit the best curve through the obtained results. This is because the best fit obtained for the limited number of beams tested is no guarantee that the proposed equation of the curve is the most accurate representation of the actual behaviour. In other words, the problem of deflection in concrete structures is of a statistical nature [page 5 of Ref.1] and the most accurate estimate of the actual behaviour is the one based on the greater number of population which is in this case the tested beams. For this and because the results of testing 10 to 20 beams that could have been cast in the laboratory within the time limit of this study will only be a small sample of the population as compared to the test results of over 340 beams found in literature, there were no independent tests conducted for the sake of developing the empirical coefficient that was an auxiliary part of the developed model of Ie.

1.3 Structure of Thesis

The current thesis is divided into six chapters. Each chapter is devoted to a major part of the study, the chapters successively develop the final model and ideas and concepts are progressively focused toward the final goal set for the study.

In order to give a broader understanding of the behaviour of reinforced concrete flexural elements as related to deflections as well as to lay down the scope and the range over which the parameters considered in the study are to be defined, Chap.2 was designed to give a short literature and parametric review explaining the different concepts of deflections in reinforced concrete and defining the limits and the boundaries of the respective parameters within the specifications of the codes.

Once the parameters are well defined and the limits are well drawn a model for the approximation of Icr is developed in Chap.3. Because of the complexity and the different aspects of the problem computer programs were developed to aid in the analysis and envelopes were constructed to help achieve the best accuracies. The result was a simple model by which Icr of all the sections considered can be easily evaluated.

In Chap.4, the model of Icr developed in Chap.3 is used to derive an expression for the effective moment of inertia, Ie. This integration of Icr into an expression of Ie was possible by assuming the concrete stiffening effect as a fictitious steel area added to a cracked transformed section for which Icr was then evaluated. The result is a compact expression of Ie where the requirement for the lengthy evaluation of Icr is eliminated and for which a simple graphical representation was presented. The model of Ie thus developed involved a coefficient which had to be determined empirically. Based on the experimental results of over 340 beams found in the literature an expression for this coefficient was also proposed. The results obtained using the proposed model of Ie were then compared with those from Branson's equation and the model proposed in Ref.4 using the results and the sectional properties of the tested beams. Because of the large number of data a computer program was developed to carry out the analysis. While the full computer output is included in Appendix C a summary of the results is shown in the chapter and conclusions are drawn confirming the accuracy of the proposed model of Ie.

The proposed model of Ie as developed in Chap.4 is intended for deflection calculations. However, the methods for deflection calculations in reinforced concrete elements can be broadly classified into two: the effective moment of inertia method and the curvature based approaches. Although the model is primarily proposed as a substitute for Branson's equation to be used in the former method, Chap.5 is devoted to show the great practicality and ease offered by the proposed model of Ie as compared to the curvature based methods in BS 8110 [10] and Eurocode 2 [11] for calculating deflections.

In the course of the study presented in Chaps.2 through 5 many aspects were reviewed and different concepts were considered. These are then narrowed and the different advantages of the proposed model are outlined in the summary presented in Chap.6. In addition, different possiblities for future research are discussed and suggestions are given.

CHAPTER 2

LITERATURE AND PARAMETRIC REVIEW

2.1 Introduction

Because of its introductory nature, Chap.1 was narrow and precise in outlining the reason for the present study as well as in setting up its goals and purpose. Detailed discussions related to the different aspects of deflections in reinforced concrete structures in general and to the proposed model in particular were omitted. Cracking as related to concrete stiffening effect, which forms the basis of the current study, as well as methods proposed as substitutes for Branson's equation [1] were only briefly introduced. These and other topics are detailed in this chapter.

Also given in this chapter is a review of the different parameters involved in deflection calculations. These are discussed and reviewed within the scope of the British and American practices and as set by the specifications of BS 8110 [10] and ACI 318 [2]. It should be noted however that the parametric review thus given is not merely a repetition of the specifications in the two codes but an overview of these specifications as applied to the current study. In fact, in the course of specifying the limits of certain parameters it was necessary to provide detailed analyses to serve the current study more specifically within the guidelines of the codes.

2.2 Deflections in Reinforced Concrete Elements

2.2.1 Significance of Deflections

Any reinforced concrete element must be designed not only against failure but also to provide the service for which it is built. This is called the serviceability aspect of the design of which deflection is an important part.

Aesthetics as well as economical factors usually require minimum depth of beams and slab thicknesses. This was reinforced in recent years by the availability of higher strength materials and advanced methods of design and resulted in structures that are more susceptible to excessive deflections and large deformations.

Excessive deflections of floor elements may cause damages to the non-structural elements attached to it like plastered ceilings, partitioning walls, drainage and piping systems as well as sensitive equipments. They may also cause separation of joints and damage of sealants in the structure as well as misalignment of doors and windows.

In flat roofs the consequences may be severely damaging. With excessive deflection in midspans water may accumulate. The accumulated water increases the deflection and attracts even more water to accumulate in the so called "ponding" phenomenon which is a very serious structural problem and has caused numerous collapses.

Also caused by excessive deflections is the dull and often unpleasant appearance of the affected elements. Visible sagging of ceiling and floor beams often causes discomfort and leads to loss of confidence in the integrity of the structure which may actually be structurally safe.

2.2.2 Estimation of Deflections

Due to the problems associated with excessive deflections as discussed above the design is therefore not only required to provide structural safety but also to minimize deflections. In doing so almost all design codes follow the common trend of requiring either to satisfy a set of span effective depth ratios or as an alternative require the detailed calculations of deflections. The span effective depth ratio rules are generally empirically based on past experience. They provide adequate safeguards when deflection is not of main concern. If these rules are violated or deflection is a major concern then detailed deflection calculations have to be made. These are no easy task due to the following:

1. Almost all the present methods of calculating deflections involve the computations of the gross moment of inertia, Ig, and the cracked transformed moment of inertia, Icr. While it can be argued that the computation of Ig is mostly straight forward the calculations involved in computing Icr are not.
2. While there are numerous charts available for flexural design that are easy and handy to use, the currently available graphical charts for deflection calculations are not. For example, simply for an estimate of the cracked moment of inertia, Icr, Ref.12 proposed four separate charts. Likewise, charts produced in Refs.7 and 8 are all complex and not easy to use.
3. Separate calculations are required for short and long term deflections. This is particularly so in the methods of BS 8110 [10] and Eurocode 2 [11] where the full calculations are repeated twice.
4. Methods recommended by some codes involve assumptions that lead to equations

for which there are no closed-form solutions. Iterative or graphical solutions are therefore resorted to which make the process too complicated and messy.

2.2.3 Factors Affecting Deflections

Despite the difficulties in calculating deflections as outlined above, it is always stressed that deflection calculations can never be exact. This is usually reasoned to:

1. Uncertainties are always involved in load estimates and in the prediction of loading history and duration as well as the possibility of load sharing between members of the same structure.
2. Assumptions used with regard to the supports and support restraints may not be exact.
3. The effect of cracking may yield deflection values that are substantially different than those anticipated.
4. The added stiffness due to screeds, plasters and other finishing materials are hard to assess.

The four main factors mentioned above are generally stated as making deflection calculations only estimates of the actual deflection values. The logical argument that follows from this is therefore that, if the calculated deflections are only estimates, why then the lengthy methods of the codes! How one can justify the lengthy deflection procedures that may even involve iterative as well as graphical solutions if these solutions are only estimates.

While it can be argued that factors 1, 2 and 4 are general design problems (as will be discussed later), cracking effect is directly related to deflection. Since internal force and moment distribution within structural elements is a function of their relative rather than absolute stiffness values, it makes no substantial difference whether a cracked or uncracked section is assumed in the elastic analysis. This is not so when deflection is considered. Since deflection is a function of the absolute stiffness a cracked beam will obviously deflect substantially more than one which is crack free. It is therefore logical to assume that loading types which induce different crack patterns and distributions will also produce different deflections. This phenomenon, however, is not recognized by the present methods of the codes. While the methods engage in lengthy calculations of parameters that can otherwise be easily approximated to a considerable accuracy they all ignore this potential factor.

2.2.4 Moment of Inertia Approach

Therefore, and in the light of the above argument, the present study is set to develop a model for the effective moment of inertia that will make deflection calculations more efficient. That is to put emphases where they should be in order to achieve more accuracy with less effort and thus a more justifiable method than the present methods of the codes.

The reason that an effective moment of inertia approach is used is that:

1. It is generally found simpler and more convenient for practical use [4,13].
2. It is more consistent with the common methods of structural analysis and engineering mechanics where the moment of inertia is usually involved.
3. Factors affecting deflections can be easily represented. For example, cracking and concrete stiffening effect.
4. The method is readily adaptable to computer programming. Powerful computer programs that use flexural rigidities and hence moment of inertia as part of the input data can be used to analyze deflections of concrete elements which are parts of a complete frame. This not only enables the study of complicated structures but also provides a more realistic analysis. In addition, the concept of semirigid joints as a more representative mean of the nature of different supports can be easily incorporated.

Because Branson's equation [1], being part of the ACI [2] and other design codes, is the most widely used expression of the effective moment of inertia, the developed model is proposed primarily as a substitute for the equation as already discussed in Chap.1. Once the accuracy of the proposed model is established and the elimination of the drawbacks usually claimed to be associated with Branson's equation is shown, the model is also compared with the methods in BS 8110 [10] and Eurocode 2 [11]. To make the model simplest to apply the study was first directed toward sectional geometries and reinforcement conditions. This led to the elimination of the detailed calculation of Icr and produced a simple form equation for which a simple graphical representation was possible. The mathematical development of the equation, however, involved a simplifying assumption for the concrete stiffening effect and produced a coefficient, Φ, the expression for which had to be empirically determined.

As will be shown in the next section the effect of concrete stiffening is a function of: loading type, loading intensity, compressive strength of concrete, amount and type of reinforcement and load duration. Reflecting the first four factors the expression of Φ included: Lcr/L which is the ratio of the cracked length over the total span and is a function of the loading type, Ma/Mcr represents the loading intensity and that Mcr is a function of the compressive strength, and the reinforcement ratio ρ. Because deformed bars are almost always used as main reinforcement and that long term deflections can always be obtained from the short term values (as will be discussed later) the corresponding factors were not considered in the expression of Φ.

One may argue however, that the empirical expression proposed for Φ as described above ignores load sharing and end restraint assumptions along with the stiffening effect of non-structural elements mentioned previously which may affect deflection. The answer is that unlike cracking these are general design problems and can be studied with a broader design sense. Load sharing, for example, can be reduced by a proper flexural design where the reinforcements are only provided where shown needed by the calculations and according to the assumptions used. This will make the structure follow the least energy path according to the so called "wisdom of structures" and behave in the way it is designed for and thus correspond to the same loading distribution assumed in the design phase. Alternatively and using the same philosophy, a construction joint may deliberately be created where needed (the idea is similar to saw cutting techniques used in in-situ testing) to force the structure into the required behavioural pattern and thus reduce load sharing.

With regard to the assumptions used for end restraints, supports and joints in reinforced concrete structures are usually assumed hinged or rigid (a rigid joint is one where the angles between its joining elements before and after deformation remain same). They are never assumed semirigid where the degree of rigidity can be assumed greater than 0 but less than 1. This is partly because such an assumption requires special elastic analysis theories and is not suitable for moment distribution techniques generally used in concrete designs. Also due to the nature of finite element methods, most package programs used in routine designs are not able to consider such conditions. However it is always possible to derive stiffness matrices that consider semirigid joints. These can then be incorporated into computer programs that can be used along with any model of moment of inertia to further understand the nature of joints in reinforced concrete structures. Such a program using the model of the effective moment of inertia as proposed in this study is currently under consideration in an effort to further develop the present research.

2.2.5 Measurement of Deflections and Strains

A brief review of load testing, deflection and strain measurement procedures will assist understanding of the theory presented in the thesis.

Mainly there are two types of load tests: in-situ and ultimate load tests. The in-situ tests involve testing elements that are part of an existing structure and the testing loads are therefore kept within the service and overload range. In the ultimate load tests, on the other hand, the test elements are either cast or removed from an already existing structure and then tested until failure.

In both types of tests, load deflection curves are used to monitor the behaviour of the element under test. If large increase of deflection is noticed at an almost constant load, loads are then quickly removed to avoid collapse in case of in-situ tests. In ultimate load tests, however, testing is continued with additional precautionary measures to avoid injuries during collapse of the element under test.

Two load deflection curves are shown in Fig.2.2.5.1. The ductile behaviour of underreinforced beams before failure can be clearly seen. This is in contrast to the overreinforced beam where an abrupt and brittle failure is shown. Due to this sudden failure of overreinforced beams careful precautions are therefore necessary during their

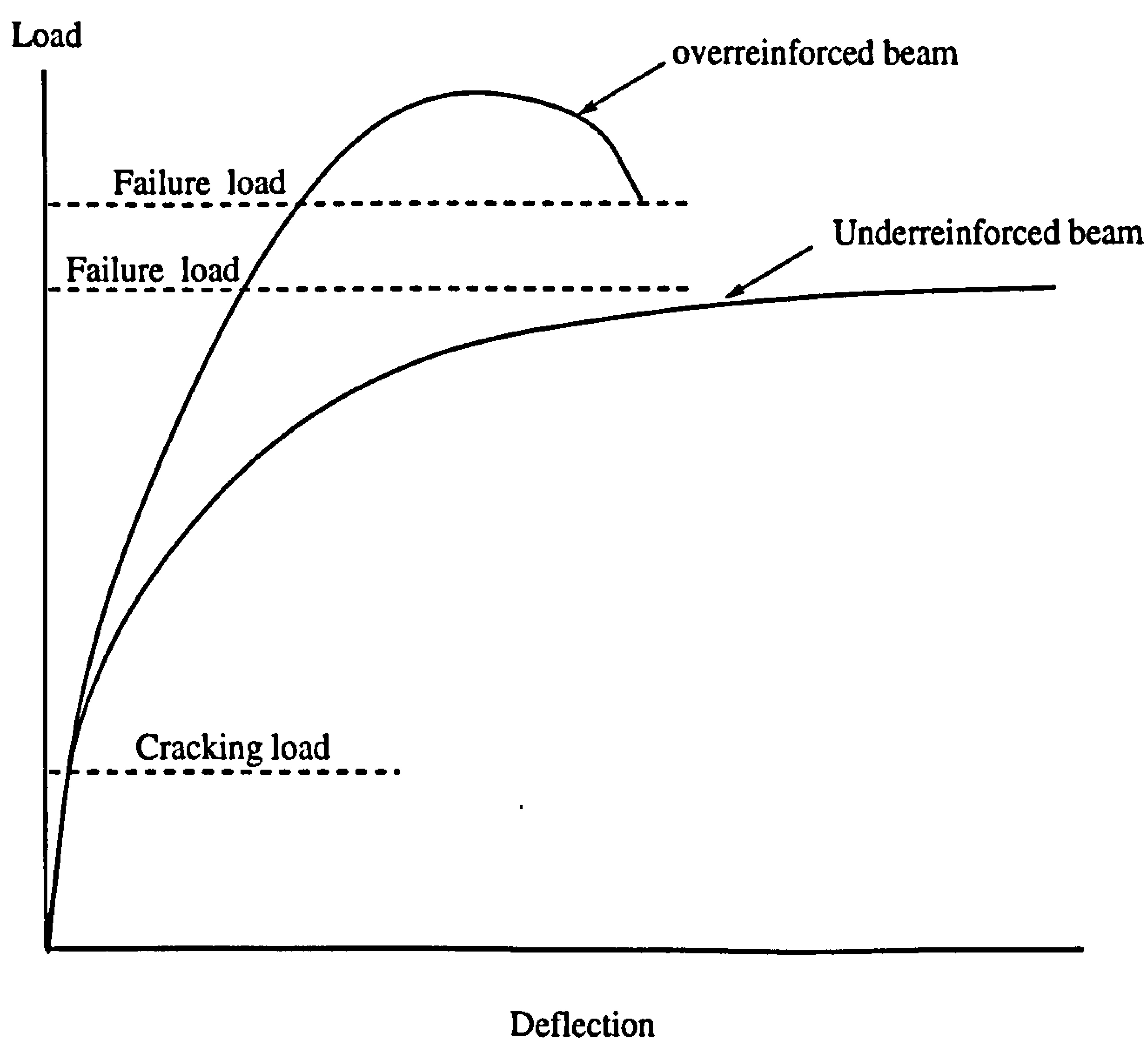


Figure 2.2.5.1 Load deflection curves for under and overreinforced beams

tests to avoid injuries as much as possible.

In testing concrete structures many forms of loadings can be used. Among these are bulk loads, hydraulically applied loads, water loads or even vacuum created loads.

Bulk loads are generally in the form of bricks, sand or cement bags and steel weights. These forms of loads are useful when uniform load distribution is required over short distances and are generally cheap and easy to handle. However, they can not be unloaded as quickly as may be desired when it is important to avoid collapse of the element under test or apply cyclic loadings.

Hydraulic jacks are used when large concentrated loads are desired. These are very easy to control and monitor and suitable for cyclic loadings. However, they usually require special equipment and thus are most suitable for laboratory tests.

Water loads are usually used to uniformly load large areas like roof slabs. They are cheap in labour and control and can be removed relatively quickly. However, they may cause great damage in case of leakage.

When loading slabs from above is not possible or not practical due to curves or steep slopes, vacuum loading is considered. In this case polythene lined partition walls and seals are constructed under the test area and a vacuum is created by suction pumps.

To monitor the behaviour of the tested element and to construct load deflection curves during the testing procedure deflection and sometimes strain readings are necessary.

Deflection measurements are usually taken using mechanical dial gages which are mounted on independent rigid frames. As an alternative and when retrieval of data is necessary electronic or electric displacement transducers are sometimes used.

Deflection gages are normally placed at points of maximum deflections as well as at midspan and 1/4 points to check symmetry of behaviour. If the tested element is less than 150 mm in width one gage is usually used at each measurement point along the axis of the element. For wider elements, however, and to minimize errors two gages are recommended at each measurement point.

To measure strain, strain gages are used. These generally include:

1. Mechanical gages

   The most popular of these gages are the so called Demec gages (demountable mechanical) which can be mounted on the surface of concrete for strain reading and then demounted for reuse. These are relatively cheap and available in a range of gage lengths. The disadvantage associated with these gages is the difficulty and lack of a remote readouts which can cause serious operational difficulties when a large number of measurements are involved.

2. Electrical resistance gages

   These are usually stuck to the test surface and strain is measured as the change in the resistance to the electric current as the gage stretches or compresses. Because they are difficult to use under site conditions and require considerable care they are usually used under controlled indoor locations. Also because their gage lengths are usually short relative to aggregate dimensions their applicability to concrete is thus limited. These gages, however, are good for dynamic as well as long term strain readings and give high accuracy. They are particularly useful if strain of the reinforcing steel is to be monitored.

3. Acoustic vibrating wire gages

   These gages are suitable for long term strain readings and are generally cast into the concrete. However, they are very sensitive to magnetic fields and thus care is required to avoid magnetic influences.

4. Inductive displacement transducers

   These are expensive and sophisticated gages and need considerable experience. They are however useful in reading diagonal and longitudinal strains as well as for dynamic tests.

5. Photoelastic gages:

   these are usually stuck to the surface of concrete and normally used to examine strain distributions or concentrations at localized critical points of a member.

6. Piezo-elastic gauges

   These are used to measure small, rapid strain changes and are more suitable for laboratory rather than site applications.

2.3 Concrete Stiffening Effect and the Methods for Calculating Deflection

In Chap.1 and in defining the effective moment of inertia the concept of the concrete stiffening effect was briefly introduced. To build a broader understanding of the concept and its related aspects a more thorough discussion on the subject is given in this section.

Consider Fig.2.3.1 which represents a portion of a reinforced concrete element. Because the bending moments are high enough to induce tensile stresses greater than the modulus of rupture of the concrete used, cracks have developed as shown. Because

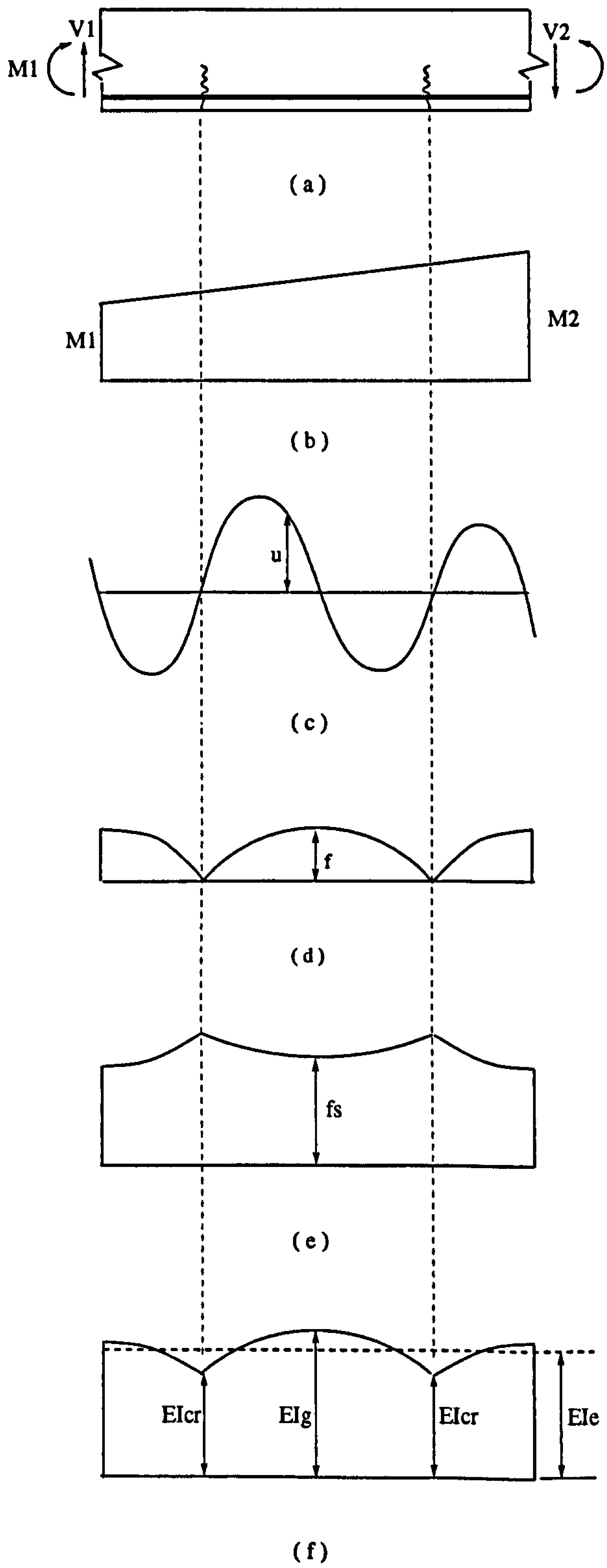


Figure 2.3.1 (part adpated from Ref.14) Effect of cracking of a reinforced concrete flexural element (a) segment of a beam (b) Bending moment distribution (c) Bond stress distribution (d) Concrete tensile stress distribution (e) steel tensile stress distribution (f) Flexural rigidity distribution

of the discontinuity in concrete that a crack creates all the tension at a cracked section has to be taken by the steel reinforcement. Between the cracks , however, concrete will still be able to take some tension which is transferred from the steel by bond stresses. The tension thus carried by the concrete between the cracks will stiffen the member resulting in a stiffness which is higher than that of a completely cracked section and hence the name concrete or tension stiffening. The magnitude and distribution of the bond stresses determine the distribution of the tensile stresses in the steel and the surrounding concrete between the cracks, and thus the magnitude of concrete stiffening. Figures 2.3.1 (c), (d), and (e) give schematic representations of the distribution of bond stress and concrete and steel tensile stresses at and between cracks.

From Fig.2.3.1 one can therefore summarize the factors that may influence the concrete stiffening as:

1. Loading type (different loading types induce cracks of different magnitude and distribution).
2. Loading intensity (higher loads induce more and deeper cracks).
3. Compressive strength of the concrete used (since it can be related to its tensile strength).
4. Type and magnitude of the steel reinforcement used (deformed or plain).

In addition to the above four factors that may influence the concrete stiffening effect a fifth and equally important factor that Fig.2.3.1 fails to depict is the effect of sustained loads. Because of the redistribution of internal stresses under sustained loads extra deflection is produced causing new cracks to develop. Due to this, and the downward movement of the neutral axis caused by the additional strains in the compressive concrete, the concrete stiffening is reduced. This phenomenon is called "loss" of concrete stiffening under sustained loads.

In incorporating the effect of concrete stiffening in deflection calculations in reinforced concrete flexural elements different approaches are followed. Broadly speaking, however, the approaches can be classified into two:

1. The effective moment of inertia method:

   This method as introduced in Chap.1 and represented in Fig.2.3.1 (f) assumes an effective moment of inertia, Ie, which is constant through out the span. The value of Ie thus assumed is greater than Icr but less than Ig with the difference Ie-Icr being due to the effect of concrete stiffening. The deflection equation in its simplest form can be written in this case as,

$$\delta = K\, Ma\, L^2/EcIe \qquad (2.3.1)$$

2. The curvature based approaches:

   In this case the curvature, 1/r is substituted for Ma/EcIe into the general deflection equation to obtain

$$\delta = K\, L^2\, (1/r) \qquad (2.3.2)$$

The effect of concrete stiffening is then incorporated in approximating the curvature, 1/r. The most comperhensive of such approaches is that of BS 8110 [10] where the curvature is obtained assuming a tensile stress in the concrete at the level of the tensile steel as will be discussed in Sec.2.6.

2.4 Loading Stages and the Moment Curvature Relationship

In an effort to provide basic grounds for the understanding of the behaviour of concrete elements as related to deflection and the concept of the effective moment of inertia, the present section is devoted to give an overview of the flexural behaviour of a reinforced concrete beam under different loading stages. These loading stages as related to the moment curvature relationship of a reinforced concrete beam will be used to explain the variation of the moment of inertia as load increases. It will also show the elastic and inelastic behaviour of the beam at the corresponding service and ultimate load stages that will help to better understand the problem of deflection.

The behaviour of reinforced concrete beams can be classified into three main stages:

1. Precracking stage
2. postcracking stage
3. Yield and ultimate load stage

It was shown in the previous section that methods of calculating deflections relate the effect of concrete stiffening to either the effective moment of inertia or curvature. To further enhance the understanding of the concrete stiffening effect as related to the effective moment of inertia and curvature as well as the applied load in the different loading stages mentioned above, the moment curvature curve of Fig.2.4.1 is provided (to better serve the present discussion and emphasize the effect of concrete stiffening the figure represents an average curvature over a finite length of a beam rather than curvature at a point).

In the figure, the different loading stages and the corresponding flexural rigidities, EI,

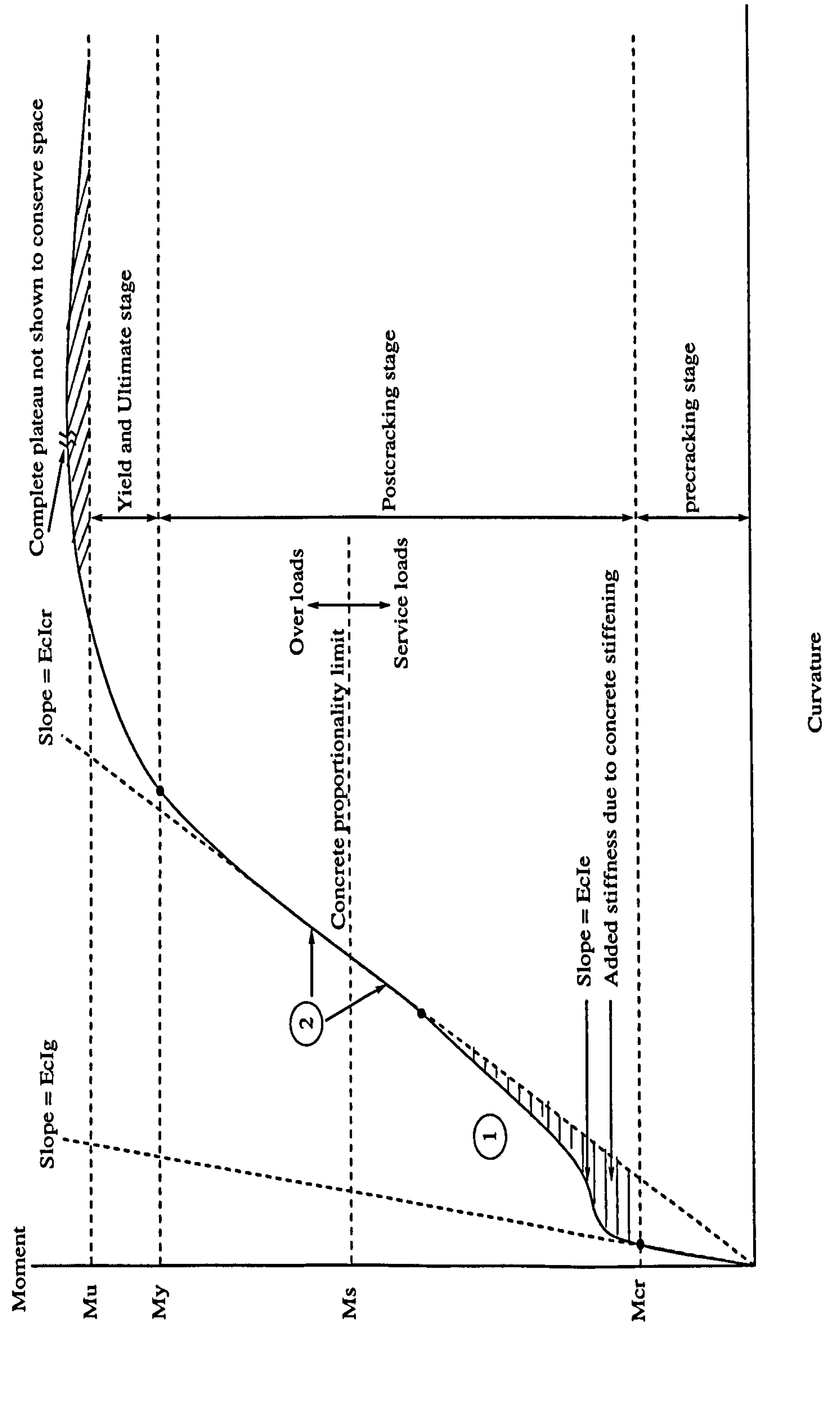


Figure 2.4.1 Moment-curvature curve for a concrete beam with low to normal reinforcement .

are shown. These are next discussed in further detail.

1. Precracking stage:

When the loads are not high enough to cause the tensile stresses in concrete to exceed its modulus of rupture concrete in both tension and compression will be effective. As shown in Fig.2.4.2 the stresses in both concrete and steel will be elastic. Because the beam at this early stage will be completely crack-free the moment of inertia at any section is taken as that of a completely uncracked section neglecting steel, Ig. Therefore, the corresponding flexural rigidity is equal to EcIg as shown in Fig.2.4.1 where Ec is the elastic modulus of concrete. As the section can be considered homogeneous (neglecting steel) and that the stresses are elastic the position of the neutral axis and that of the centroidal axis coincide and the straight line theory applies.

2. Postcracking stage:

When the load is further increased to cause a moment higher than the cracking moment, Mcr, tensile stresses higher than the concrete's modulus of rupture will be induced and cracks start to develop and propagate towards the compression zone of the beam. Due to this and the effect of concrete stiffening the stiffness of the beam will be less than that at the precracking stage but greater than or equal to that corresponding to a completely cracked section.

The relation between the curvature and the effective moment of inertia when the effect of concrete stiffening is considered is represented by the flexural rigidity, EcIe, shown as the slope of the moment curvature curve in Fig.2.4.1. The enhanced

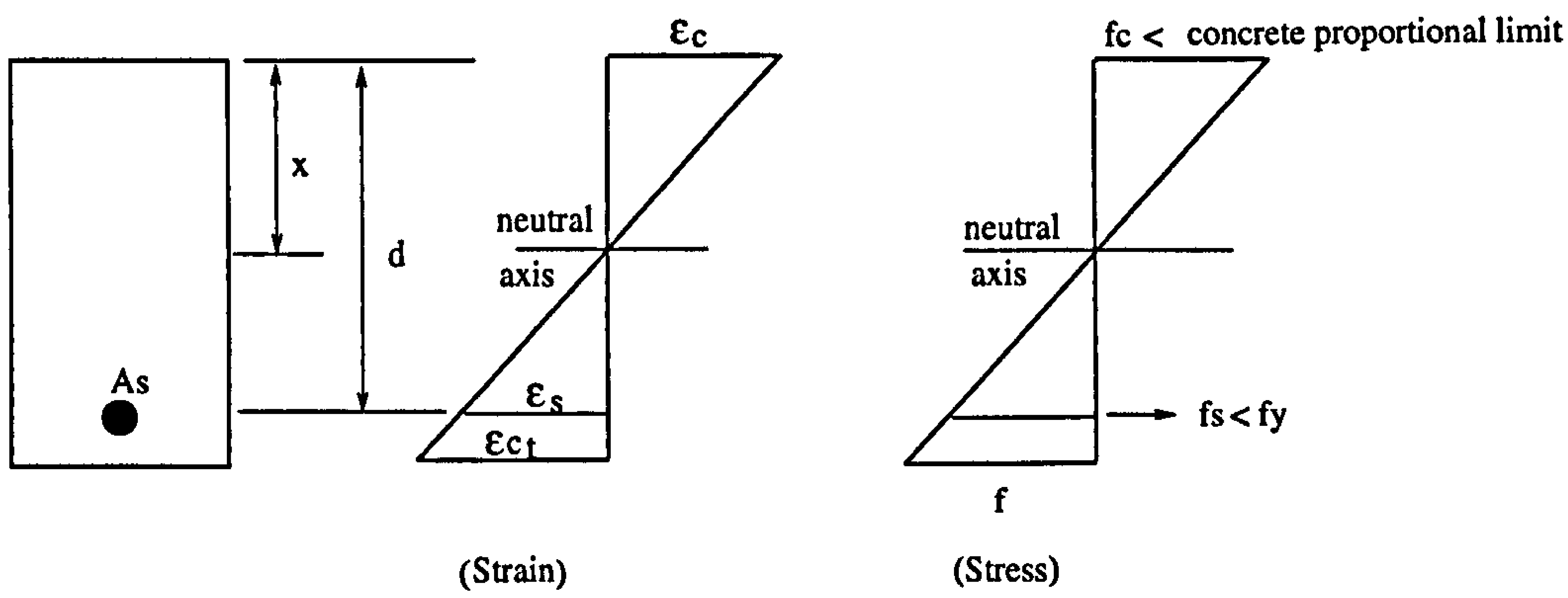


Figure 2.4.2 Stress and strain diagrams in the precracking stage

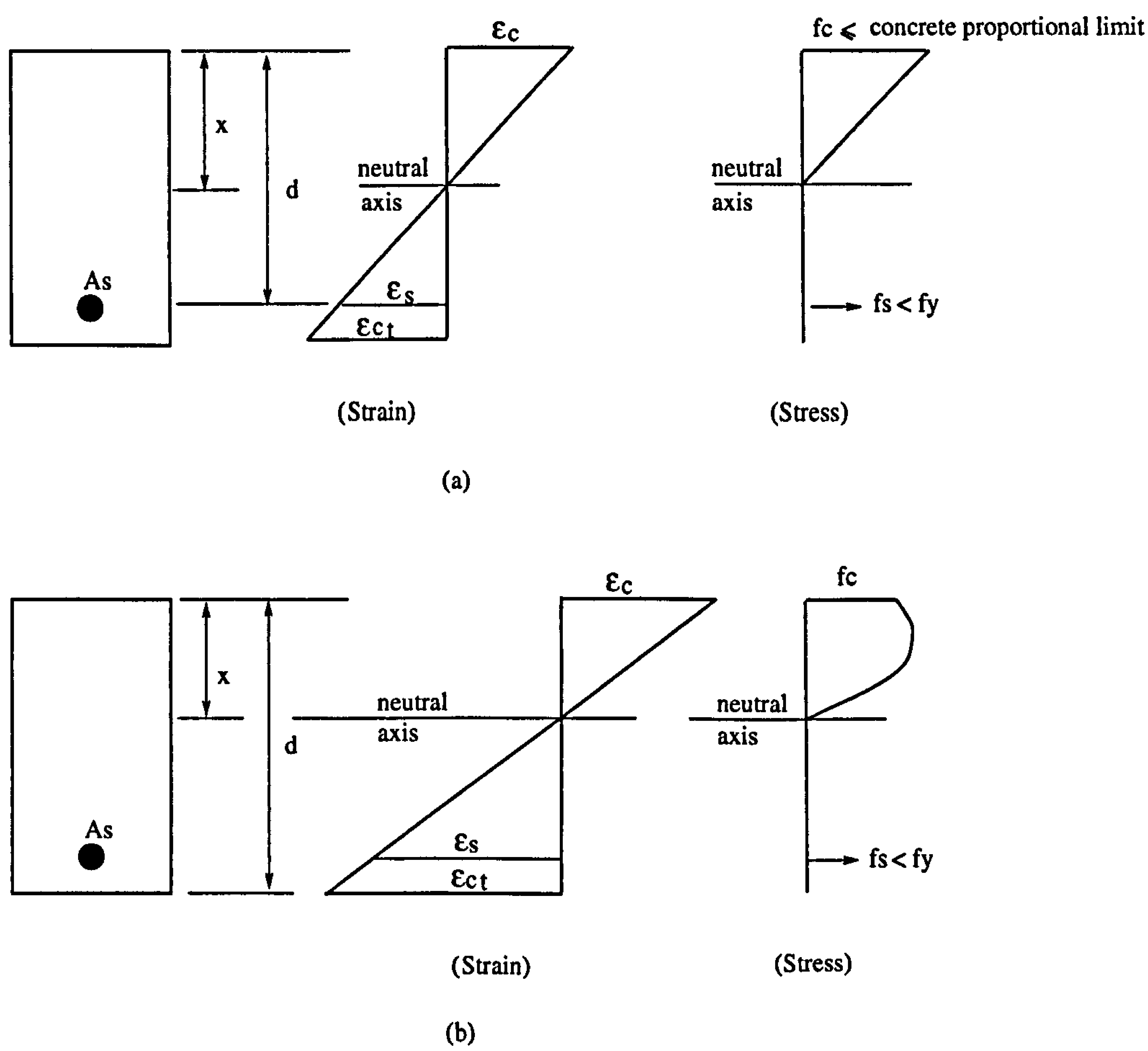


Figure 2.4.3 Stress and strain diagrams in the postcracking stage (a) Service load stage (b) Overload stage

stiffness due the contribution of the concrete stiffening can be clearly seen from the shaded area shown therein.

Figure 2.4.1 reveals that the postcracking stage consists of two regions. One in which the effective moment of inertia is less than Ig but greater than the cracked transformed moment of inertia, Icr, and another where the section can be considered as completely cracked and the moment of inertia is taken as Icr. Due to the continuous change in the effective moment of inertia with increasing moments the slope of the curve in the first region is not constant (had the curvature been taken as that at a point, rather than an average over a finite length, this region would have been represented by a horizontal straight line. Such a line would represent the immediate reduction in the stiffness to that corresponding to Icr right after cracking). In the second region however, the effective moment of inertia reaches a steady state marked by the constant slope of the curve shown as EcIcr where the moment of inertia can be considered as that of a completely cracked section.

The stress and strain diagrams for this loading stage are shown in Fig.2.4.3. As the figure reveals, the stress and strain diagrams in this stage can be of two types. One in which both the concrete and steel are elastic, Fig.2.4.3(a), and another in which the concrete compressive stresses are inelastic while those of the steel are still elastic, Fig.2.4.3(b). Since stresses in Fig.(a) are all elastic the position of the neutral axis and the centroidal axis coincide. However, because the section is inhomogeneous the straight line theory does not apply unless the area of the steel reinforcement is transformed into an equivalent concrete area. In Fig.(b), on the other hand, the stresses in concrete are shown to be inelastic. However, the behaviour in this case can be classified into two: One in which an elastic behaviour can be assumed and the elastic theory can still be applied. Another where such an assumption can not be used due to high inelastic concrete stresses and the straight line EcIcr and the deflection methods cease to apply (this corresponds to the slightly curved higher part of region 2 shown in Fig.2.4.1).

3. Yield and ultimate load stage:

In this stage the carrying capacity of the beam will eventually be reached as the load increases. Failure can be caused in one of two ways:

(a) When amount of reinforcement is low or normal:

In this case the steel will first yield. The yield of steel will be sudden after which it will stretch a large amount causing the cracks in concrete to widen and propagate towards the compressive face of the beam forcing the neutral axis to shift upwards. This will be associated with excessive deflections and puts the concrete in increasing compressive strain until it crushes at a load only slightly higher than that which caused the steel to yield. This crushing failure is known as "secondary compression failure" because it was caused by excessive deflection due to yield of steel without any significant increase of load above that caused yielding. This is shown in Fig.2.4.1 where the ultimate moment, Mu, is shown only slightly above the moment at first yield, My (a usual ratio of My to Mu is 0.9 to 0.95).

(b) When the amount of reinforcement is high:

In this case the compressive strain in concrete will reach that of failure prior to yielding of steel and the beam will fail by sudden crushing of concrete in an explosive nature (Figure 2.4.1 does not depict this kind of behaviour since it is produced for a beam of low to normal amount of reinforcement).

The stress and strain diagrams in this loading stage is similar to Fig.2.4.3(b) of the postcracking stage with fs=fy (steel yields) for case (a) and fs < fy for case (b) and that the stress distribution over the compression face of the beam resembling the stress-strain curve of the concrete used.

In Fig.2.4.1 two shaded areas are shown. The first is in the postcracking stage and is already discussed. The second is in the ultimate load stage and represents the strain hardening of the reinforcing steel prior to failure. This adds slightly to the moment capacity of the beam (ignored in design) but reduces ductility and is one of the reasons for the conservative ductility requirements imposed by most codes.

The classification of the loading stages discussed above was designed to best serve the purpose of the present study. However, loads can also be classified in terms of the stresses they produce into the concrete and the reinforcing steel as follows:

1. Service loads:

   These loads usually act in the precracking loading stage as well as in region 1 and part of region 2 of the postcracking loading stage. Represented by the maximum service moment, Ms, shown at the concrete proportionality limit in Fig.2.4.1, the concrete compressive stresses under these kind of loads are always elastic. Because of this, and because the stresses in the steel reinforcement are also elastic the straight line theory always applies. In lieu of a full analysis, Ms can be approximated at 0.6Mu. Reflecting this, is the usual approximation of the stress in the tension steel, fs, as 0.6fy in many deflection and crack control provisions in the different codes (i.e Table 3.11,pt.1 of BS 8110 [10] and Sec.10.6.4 of ACI 318 [2]).

2. Overloads:

   a. Low overloads:

      These loads act in region 2 of the post cracking loading stage. Although the concrete compressive stresses induced under these loads are inelastic, a perfectly elastic behaviour is usually assumed.

   b. High overloads:

      These loads usually act in the higher part of region 2 of the post cracking loading stage which is closer to the yield and ultimate loading stage. Although the tensile stresses in the reinforcing steel are elastic the compressive stresses induced into the concrete under these loads are very inelastic. Thus an elastic behaviour cannot be assumed and the straight line of EcIcr and the deflection methods cease to apply.

3. Ultimate loads:

   These loads correspond to the yield and ultimate loading stage. The stresses under these kind of loads are all inelastic and the elastic theory can not be applied.

2.5 Review of the Different Models for the Effective Moment of Inertia, Ie

Because the effective moment of inertia is the primary concern of the current study it was thought useful to give a chronological review of the most important models for the effective moment of inertia. These are then schematically represented on a moment curvature curve and followed by a discussion pertaining to the approximations involved.

1. Murashev's method [15]:

To account for the less stressed steel reinforcement between the cracks due to concrete stiffening (see Fig.2.3.1(e)) Murashev proposed in 1940 and based on numerous experiments, that for deflection calculations the effective moment of inertia, Ie, be taken as the cracked transformed moment of inertia, Icr, computed using an effective elastic modulus for the reinforcing steel. This effective elastic modulus was given as,

$$Es\ (effective) = Es/\Psi$$

where,

Es= is the elastic modulus of the reinforcing steel

$\Psi = 1 - (2/3)(Mcr/Ma)^2 \leq 1.0$

2. The gross moment of inertia suggested by Portland Cement Association [16]:

In 1947 Portland Cement Association, PCA, proposed that the moment of inertia of reinforced concrete flexural elements be taken as the gross moment of inertia, Ig, neglecting the area of steel reinforcement.

3. Methods A and B of Yu and Winter [17]:

Based on extensive studies, Yu and Winter suggested in 1960 two methods for representing the moment of inertia of reinforced concrete beams. These are:

a. Method A:

The effective moment of inertia can be taken as the moment of inertia of the completely cracked section at midspan of simple beams. For continuous beams an average of the cracked section moment of inertia values for the negative and positive regions can be taken.

b. Method B:

Based partly on an elastic theory approach the method estimates the effective moment of inertia as,

$$Ie = Icr / (1 - b' M1/ Ma)$$

where,

b' = width of beam at the tension side

$M1 = 0.1 (fc')^{2/3} (h)(h - x)$

h = total depth

x = depth of centroidal axis

Because the factor 0.1 involved in the expression of M1 is empirical the equation of Ie given in this method is therefore semi-empirical.

4. The 1963 ACI code method [18]:

Under a prescribed moment the depth of a reinforced concrete flexural element increases for lower reinforcement ratios and vice versa. Reflecting this and since deeper sections are less likely to crack, the ACI code in 1963 presented the following for deflection calculations:

a. When $\rho fy \leq 500$ psi (3.5 MPa) :

The effective moment of inertia can be taken as the moment of inertia of the gross concrete section neglecting steel, Ig.

b. When ρfy > 500 psi (3.5 MPa) :

The effective moment of inertia can be taken as the moment of inertia of the cracked transformed section, Icr.

5. Branson's equation [1]:

In 1963 and based on experimental results considering beams under uniform loads Branson proposed the following empirical expression for the effective moment of inertia:

$$Ie = Icr + (Ig - Icr)(Mcr/Ma)^{m}$$

He further suggested that a value of 4.0 be taken for the power m whenever numerical integrations are used to calculate Ie over the entire span. Alternatively, however, he recommended a power of 3 to be used for an average Ie over the entire span and reformed the equation to be,

$$Ie = Icr + (Ig - Icr)(Mcr/Ma)^{3}$$

Since for Ma/Mcr < 1 (the precracking stage) there is no guarantee that the equation will not yield Ie > Ig a limitation of Ie ≤ Ig was further imposed resulting in the final form of the equation shown below,

$$Ie = Icr + (Ig - Icr)(Mcr/Ma)^{3} \leq Ig$$

ACI committee 435 [19] and based on measured and computed deflections found the equation to be more accurate than all other methods of calculating deflections. This was further supported by Mirza and Sabnis [20] in connection with other experimental data.

Because of this, and that the equation satisfies the boundary conditions of Ig and Icr as well as recognizes the effect of concrete stiffening, it was adapted in 1971 as part of the ACI code and remained the most widely used expression of Ie ever since. It is currently part of the ACI code and other codes through out the world. Since the equation was recognized as part of the ACI code in 1971, several studies were launched to eliminate the drawbacks usual claimed to be associated with it as already pointed out in Chap.1. These are discussed below as part of the current chronological review.

6. Grossman [6]:

In 1981 and in an effort to eliminate the need for the cumbersome calculation of the cracked transformed moment of inertia, Icr, Grossman modified Branson's equation as follows,

a. When $Ma/Mcr \leq 1.6$:

$$Ie = (Ma/Mcr)^2 \, Ig$$

b. When $Ma/Mcr > 1.6$:

$$Ie = 0.1 \, (Mcr/Ma)$$

As his work was purely theoretical, Grossman did not conduct any experiments nor did he consult any experimental data and considered instead the results obtained from Branson's equation as exact. Because of this, the proposed equations did not offer much advantage over that of Branson's equation except for the elimination of Icr.

7. Al-Shaikh and Al-Zaid [4]:

In 1993 and in an effort to eliminate the inaccuracies involved in Branson's equation, Al-Shaik and Al-Zaid proposed the following empirical model for Ie,

$$Ie = Ig + (Icr - Ig)\,(Lcr/L)^{m} \qquad (2.5.1)$$

where,

Lcr = The length of the span over which the applied moment exceeds Mcr (usually known as the cracking length)

L = The span of the beam.

m = 0.8ρ (Mcr/Ma) with ρ in %

Because the equation was based on limited number of tested beams, it will be shown in this study that it does not actually offer much accuracy over Branson's equation and therefore can not be considered an improved substitute.

In the chronological review given in this section different models for the moment of inertia of reinforced concrete flexural elements were discussed. Some merely approximate the moment of inertia as that of the gross concrete section, Ig, or the cracked transformed one, Icr. Others more appropriately consider the effect of concrete stiffening and approximate the moment of inertia to reflect such an effect. In doing so however, and except for Eq.2.5.1, they fail to represent all the different factors discussed in Sec.2.3 as to influence the concrete stiffening effect. In an effort to recover such a fault Eq.2.5.1 was proposed. Although the equation uses the ingenious idea of Lcr/L to represent the different cracking effect of different loads and considers the effect of reinforcement, it will be shown to give only little accuracy over that of Branson's equation.

The different trends in the approximation of the moment of inertia discussed in this section can be more efficiently summarized on a moment curvature curve. Such a curve is shown in Fig.2.5.1.

It is clear from the figure that methods which approximate the effective moment of inertia as the moment of inertia of the gross uncracked section may grossly underestimate deflections. This is particularly so for shallower sections which are more susceptible to cracking. Therefore with the increasing use of higher strength materials and the ultimate design methods and the consequent use of shollower sections, the method of Portland Cement Association became increasingly unpopular. To avoid such overestimate of the effective moment of inertia of sections that are more susceptible to cracking the 1963 ACI code method was proposed. The method recognized that for lower reinforcement ratios such that $\rho fy \leq 3.5$ MPa the corresponding sections are usually deep. Since deeper sections are less likely to crack due to their higher Mcr values they can practically be assumed uncracked. On the other hand, for $\rho fy > 3.5$ MPa the corresponding sections are usually shallow. Because shallower sections have

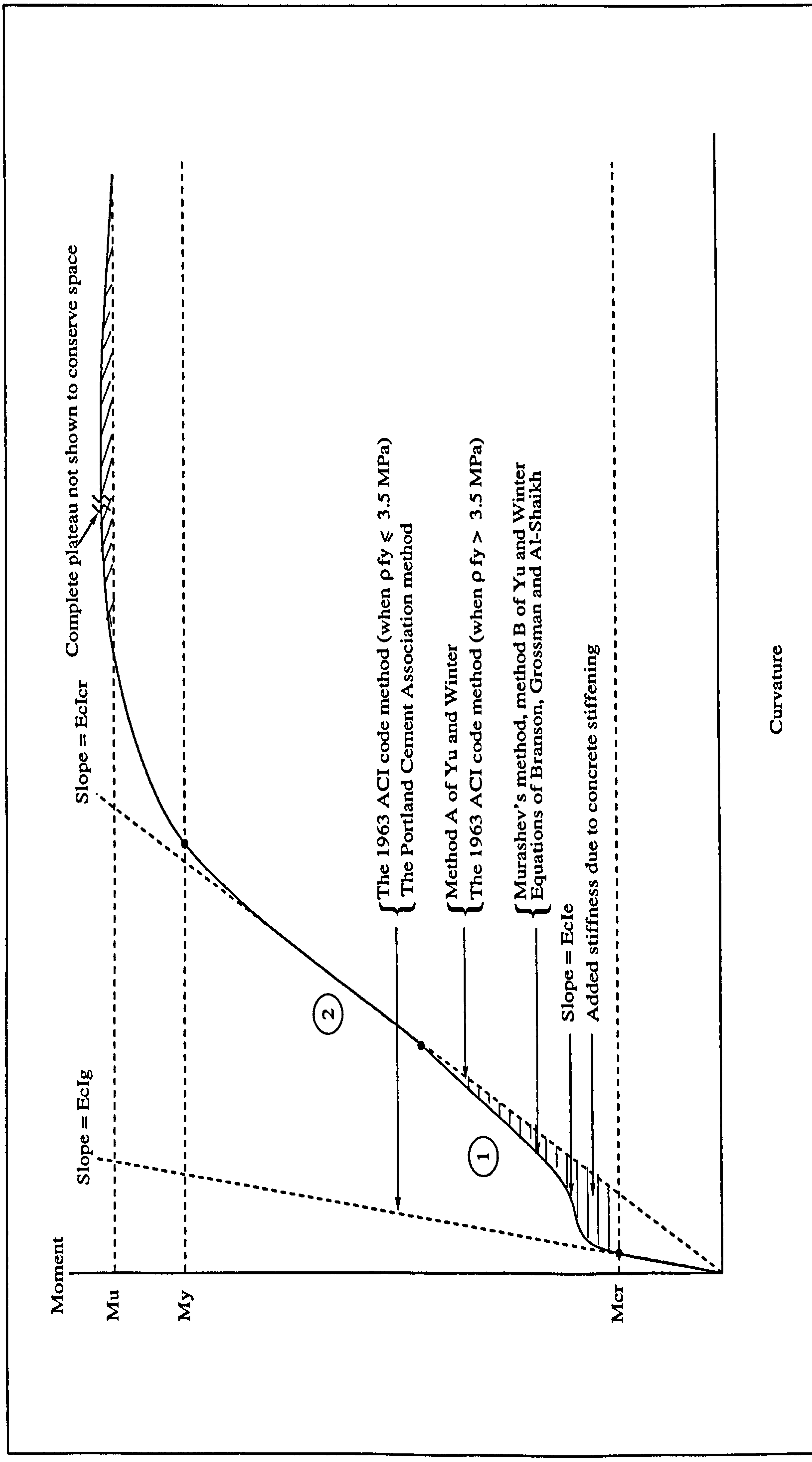


Figure 2.5.1 Moment-curvature curve and the different models of Ie

lower Mcr values, they are more likely to crack under loads and therefore can generally be assumed so. These two approximations of of the method are shown in Fig.2.5.1. The figure also reveals, however, that the method actually ignores the concrete stiffening effect marked by the shaded area and does not recognize the transition of Ie from Ig to Icr in the most practical range of loading. With the need to consider such effect of concrete stiffening the method was further reviewed by ACI committee 435 [19] and compared with method B of Yu and Winter and with Branson's equation. The committee concluded that the latter is the most accurate model of Ie for estimating deflections. This was further supported by other studies as was mentioned earlier. The most important of these, however, was a comparative study based on the results of experiments conducted by Base, Read, Beeby and Taylor [9,21,22] where the accuracy of Branson's equation was clearly shown. Despite this, however, and using these same experimental results as a common data base it will be shown that the accuracy of estimating Ie and thus deflections can be substantially improved over that of Branson's equation by the model of Ie proposed in this study.

2.6 Deflection calculations by the Curvature Based Approaches of BS 8110 and Eurocode 2

As stated previously, this study is aimed at proposing a model for calculating deflections using the effective moment of inertia method. Nevertheless, a comparative study of the proposed model and the curvature based approaches in BS 8110 [10] and

Eurocode 2 [11] is also given in Chap.5. For the purpose of such a study and to complete the review of the different aspects of deflections, deflection methods of BS8110 [10] and Eurocode 2 [11] are discussed in this section.

2.6.1 Deflection Calculations in BS 8110

The approach suggested by the code is to determine deflections from curvatures. Using small-deflection theory, the curvature at any point x along the span can be written as,

$$1/r_x = d^2\delta/dx^2 \qquad (2.6.1.1)$$

where,

$1/r_x \equiv$ the curvature at any point x along the span.

$\delta \equiv$ the deflection at the point considered.

Using the boundary conditions of the span Eq.2.6.1.1 is then double integrated by any convenient numerical integration technique to obtain the desired deflection.

The detailed method outlined above is usually complex and cannot be carried out without the aid of a computer. Because of this the code proposes an approximate method where the maximum deflection is evaluated as follows,

$$\delta(max) = KL^2\,(1/r_b) \qquad (2.6.1.2)$$

where,

$K \equiv$ a loading type factor (given in Table 3.1,pt.2 of the code)

$L \equiv$ the effective span

$1/r_b \equiv$ the curvature at the midspan of beams or at the support of cantilevers.

According to the code the curvatures used in deflection calculations should be the greater of those obtained from the uncracked and partially cracked sections as described below,

1. <u>The uncracked section</u>:

   In this case the gross concrete area with all steel areas (both tension and compression if any) transformed into an equivalent area of concrete is considered. The curvature is then calculated as,

$$(1/r)_{tr} = M/EcI_{tr} \qquad (2.6.1.3)$$

   where I_{tr} is the moment of inertia of the gross section thus assumed.

2. <u>The Partially Cracked Section:</u>

   The partially cracked section is called the cracked section in the code. However, the term partially is used herein to indicate the tension stiffening of concrete which is considered and to avoid confusion with the cracked section as defined in this study. This is a section in which the concrete in the tension zone below the neutral axis is assumed to sustain a triangular stress distribution. Unlike the concrete compressive stresses above the neutral axis and the tensile stress of the steel, these concrete tensile stresses are not related to the strains. In addition, the tensile stress in the concrete at the level of the tension steel, denoted by f_{ct}, is assumed to have values of 1 and 0.55 MPa for short and long term loadings, respectively. Figure 2.6.1.1 summarizes the above assumptions.

From the strain diagram in Fig.2.6.1.1 and for n=Es/Ec, fs′=ε_s′Es, fs=ε_sEs and f_c=ε_cEc it can be shown that ,

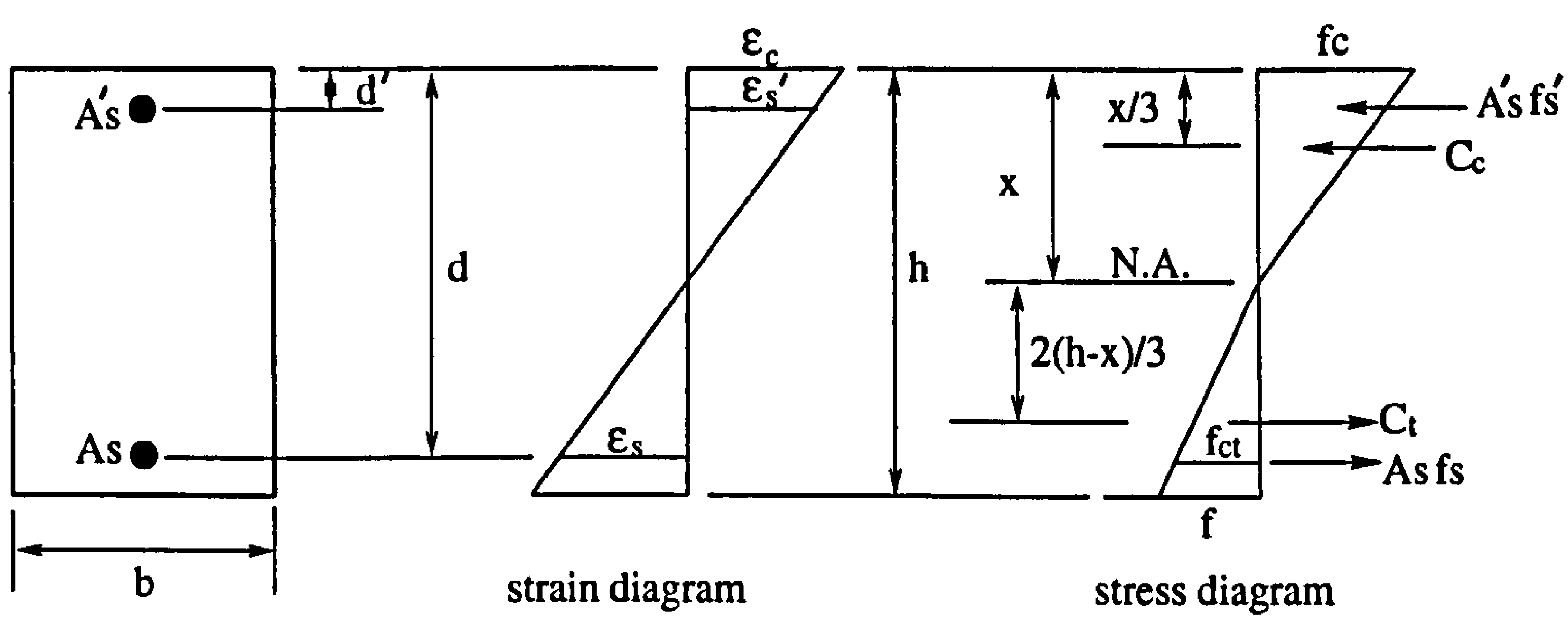


Figure 2.6.1.1 The assumptions of BS 8110 for the partially cracked section

$$fs' = nfc(x-d')/x \quad , \quad fs=nfc(d-x)/x$$

From the stress diagram it can be seen that f_{ct}=(d-x)f/(h-x). Thus the tensile stress in concrete at the soffit of the beam can be written as,

$$f = f_{ct}(h-x)/(d-x)$$

Therefore one can write,

$$Asfs = nAsfc(d-x)/x$$

$$As'fs' = nAs'fc(x-d')/x$$

C_t (the resultant concrete tension force)$=0.5(f_{ct}b)(h-x)^2/(d-x)$

C_c (the resultant concrete compressive force)$=0.5fcbx$

Force equilibrium requires that,

$$C_c + As'fs' - Asfs - C_t = 0$$

Moment equilibrium requires that,

$$(2/3)(C_c)(x) + As'fs'(x-d') + Asfs(d-x)+(2/3)(C_t)(h-x) = M$$

Substituting the relative expressions into the force and moment equilibrium equations and solving for fc gives,

$$fc=[bf_{ct}(x/2)(h-x)^2/(d-x)]/[bx^2/2+nAs'(x-d')-nAs(d-x)] \qquad (2.6.1.4)$$

$$fc=[M(x)-bf_{ct}(x/3)(h-x)^3/(d-x)]/[bx^3/3+nAs'(x-d')^2+nAs(d-x)^2] \qquad (2.6.1.5)$$

Equations 2.6.1.4 and 2.6.1.5 are derived for the rectangular section assumed in Fig.2.6.1.1. However, similar equations can also be derived for flanged sections. In order to obtain a unique value of fc from Eqs.2.6.1.4 and 2.6.1.5 for the same value of x the equations are solved iteratively or graphically as will be shown in Chap.5. Once x and fc are found the curvature at the section considered is calculated as,

$$(1/r)_{pcr} = fc/xEc \qquad (2.6.1.6)$$

With regard to the loading history and duration the following are defined :

$1/r_{s.perm}$ ≡ the curvature due to the short term effect of the permanent load.

$1/r_{l.perm}$ ≡ the curvature due to the long term effect of the permanent load.

$1/r_{s.tot}$ ≡ the curvature due to the short term effect of the total load.

$1/r_{shr}$ ≡ the curvature due to shrinkage effect.

Once the curvatures as defined above are found for the uncracked and partially cracked sections the final curvature, $1/r_b$, to be used in Eq.2.6.1.2 is obtained as follows:

a. For short term deflections :

$$1/r_b = \max [(1/r_{s.tot})_{pcr} , (1/r_{s.tot})_{tr}] \qquad (2.6.1.7)$$

b. For long term deflections :

$$1/r_b = 1/r_{l.perm} + 1/r_{s.tot} - 1/r_{s.perm} + 1/r_{shr} \qquad (2.6.1.8)$$

where each individual curvature is taken as the maximum of the values obtained for the uncracked and partially cracked sections.

While all the curvatures are found from either Eq.2.6.1.3 or 2.6.1.6 the shrinkage curvature to be used in Eq.2.6.1.8 is defined by the code as

$$1/r_{shr} = (n\ \varepsilon_{shr})(s_s / I) \qquad (2.6.1.9)$$

where,

ε_{shr} ≡ free shrinkage strain of plain concrete as defined in Sec.7.4,pt.2 and represented in Fig.7.2 of the code.

n ≡ long term modular ratio = $Es/E_{eff} = Es(1+c_t)/Ec$

c_t ≡ creep coefficient (referred to by the code as Φ) as defined in Sec.7.3,pt.2 and represented in Fig.7.1 of the code.

s_s ≡ moment of steel area about the centroid of the considered section (which is either the partially cracked or uncracked section).

I ≡ moment of inertia of the considered section (which is either the partially cracked or uncracked section).

2.6.2 Deflection Calculations in Eurocode 2

While the overall approach of calculating deflections from curvatures is similar to that in the British code [10, 23], Eurocode 2 [11] requires that the curvatures used in deflection calculations be evaluated as the average (rather than the maximum) of the curvatures corresponding to the cracked section (rather than the partially cracked section) and the uncracked one. Namely, using the previously defined notations,

$$1/r = \xi\,(1/r)_{cr} + (1 - \xi)(1/r)_{tr} \qquad (2.6.2.1)$$

where,

$(1/r)_{cr}$ = The curvature of the cracked section. For loading effects it is the service moment divided by flexural rigidity. For shrinkage effects it is evaluated from Eq.2.6.1.9

$(1/r)_{tr}$ = Same as above but with respect to the uncracked section.

$\xi = 1 - \beta_1 \beta_2 (Mcr/Ma)^2$

β_1 = 1 for high bond steel

= 0.5 for plain bars

β_2 = 1 for short term loadings

= 0.5 for long term loadings

For total deflections the curvatures according to Eq.2.6.2.1 under different effects are summed to obtain the final curvature, $1/r_b$. This is then substituted into Eq.2.6.1.2 to obtain the value of the final deflection.

The review given so far was general and designed to build a ground for understanding the different aspects of deflections and the philosophy of the present study. In the sections to come, the relative parameters will be discussed and limits will be deduced to serve the present study more specifically. For a more general study, both BS 8110 [10] and ACI 318 [2] will be consulted and the parameters will be reviewed as given in the two codes. When limits are to be drawn the extreme used in the British and American practices as set by the respective codes will be adapted.

2.7 The Elastic Modulus of Concrete and the Modular Ratio

Due to nonlinearity of the stress-strain curve for concrete a secant modulus is taken as the elastic modulus, denoted normally as Ec. The empirical expressions for the evaluation of Ec as given in the American and the British codes are discussed in this section.

Based on experimental data, Sec.8.5.1 of ACI 318 [2] proposes the following empirical equation for the elastic modulus of concrete

$$Ec=33w^{1.5}\sqrt{fc'} \qquad (2.7.1)$$

where Ec is the elastic modulus of concrete in psi,w is the density of concrete in pcf and fc′ is the cylindrical compressive strength in psi.

The SI version of Eq.2.7.1 which gives Ec in MPa (N/mm^2) for w in Kg/m^3 and fc′ in MPa is given as

$$Ec=0.043w^{1.5}\sqrt{fc'} \qquad (2.7.2)$$

The ACI Eq.2.7.1 and its metric version, Eq.2.7.2 (also adopted by other codes) give reasonably accurate results for fc′ in the range from 3000 psi (21MPa) to 6000 psi (41 MPa). For fc′ larger than 6000 psi the equations were seen to overestimate the elastic modulus by as much as 20%. Recent research conducted at Cornell University [24] suggests that the ACI equation of Ec be replaced by the following,

$$Ec=[10^6 + 4(10^4)\sqrt{fc'}](w/145)^{1.5} \qquad (2.7.3)$$

where terms and units are defined as those for Eq.2.7.1.

On the other hand, cl.7.2,pt.2 of BS 8110 [10] specifies the following for the elastic modulus of normal weight concrete,

$$Ec=(20+0.2fcu) \qquad (2.7.4)$$

where Ec is the elastic modulus in concrete in GPa (10^3 MPa) and fcu is the cube compressive strength in MPa. When the concrete considered is lightweight aggregate concrete Eq.2.7.4 must then be multiplied by $(w/2400)^2$ as specified by the code.

When the elastic modulus of the reinforcing steel, Es, is divided by the elastic modulus of the concrete defined above the so called modular ratio, n, is obtained. This ratio is used to transform the steel area into an equivalent concrete area when calculating the moment of inertia of a section and is therefore an important parameter for the current study.

When the concrete is not subjected to sustained loads its elastic modulus increases with time. This increase is ignored in most codes (i.e ACI 318 [2]). The British code, however, considers it in the serviceability and elastic deformation studies. In the current study the elastic modulus of concrete will be taken as that at 28 days and any increase in the value beyond that will be ignored. This is believed to give only minor errors as the increase in the compressive strength over that at 28 days has been seen to be slight. Thus the corresponding increase in the elastic modulus from Eqs.2.7.1-2.7.4 will also be small and therefore negligible.

If the concrete is subjected to sustained compressive loads its elastic modulus is then reduced. This is because sustained compressive stresses induce additional strains to those already caused according to Hooke's law (elastic strains). These additional strains are called creep strains and are usually found to be proportional to the sustained compressive stresses that are less than or equal to half of the compressive strength (concrete compressive stresses due to service loads are well within this range). Therefore, defining strain as ε,

$$\varepsilon(\text{total})=\varepsilon(\text{elastic})+\varepsilon(\text{creep})$$

Since ε(creep) is proportional to the sustained stresses to which ε(elastic) is also proportional according to Hooke's law, one can write,

$$\varepsilon(\text{total})=\varepsilon(\text{elastic})+[\varepsilon(\text{creep})/\varepsilon(\text{elastic})]\ \varepsilon(\text{elastic})$$

Defining the creep coefficient as $c_t=\varepsilon(\text{creep})/\varepsilon(\text{elastic})$ and denoting concrete stress as fc and elastic strain as ε_e , then

$$\varepsilon(\text{total})=\varepsilon_e(1+c_t)=(fc/Ec)(1+c_t)$$

or

$$fc/\varepsilon(\text{total})=Ec/(1+c_t)$$

Thus, the elastic modulus of concrete considering the effect of creep, usually referred to as the effective elastic modulus, can be written as,

$$Ec(effective)=Ec/(1+c_t) \quad (2.7.5)$$

This equation appears in Sec.3,pt.2 of BS 8110 as part of Eq.9 [10]. In Sec.7 of the code the values of c_t , denoted by the code as ϕ, are given by Fig.7.1. For long term deflection calculations according to the code, Eq.2.7.5 is used to determine the modular ratio needed for the evaluation of sectional properties and curvatures.
The ACI code [2], however, only recognizes the effect of creep on increasing the stresses of the compression steel and ignores its effect on the tension steel and the surrounding concrete when computing flexural stresses. This is because creep increases the stresses in the compression steel by almost 60% while those in the tension steel by only 3-4%. Accordingly, Sec.A5.5 of ACI 318 [2] (Appendix A of the code) specifies a modular ratio of 2Es/Ec, which corresponds to c_t=1.0 in Eq.2.7.5, only when transforming compression steel area into concrete for computing stresses. For deflection calculations, on the other hand, the code combines creep and shrinkage effects in a single factor such that when the short term deflection is multiplied by this factor the additional long term effects are obtained. By doing this Eq.2.7.5 is therefore bypassed and the modular ratio is only defined for short term calculations.
In this study the idea that long term effects can always be obtained by multiplying the short term effects by an appropriate factor will be adapted and thus only the short term elastic modulus and modular ratio need to be considered. This, however, should not be taken to mean that the effective elastic modulus can not be used in conjunction

with the proposed model of Ie. As will be shown in Chap.5, if the parameters involved are within the limits for which the model is proposed, the effective moment of inertia can be evaluated regardless of whether short or long term elastic modulus is used.

In the foregoing discussion different expressions of the elastic modulus were presented through Eqs.2.7.1-2.7.4. Equations 2.7.1-2.7.3 relate to cylindrical compressive strength, fc′, while Eq.2.7.4 is expressed in terms of the cube strength, fcu. Obviously conversion from fc′ to fcu and vice versa is always possible in which case any of these equations can be used. However, to keep the study as close as possible to the British and the American practices the respective equations as expressed in terms of fc′ and fcu will be used depending on the type of compressive strength specified. The units involved in such equations will also be retained as used in the two practices. However, since the modular ratio n and the reinforcement ratio ρ are dimensionless, units will eventually cancel out and the type of units will therefore remain immaterial in defining the limits and the boundaries in the current study.

In addition, because all test beams considered in the study will have fc′ values less than 6000 psi (41 MPa) there will be no need of Eq.2.7.3 and only the ACI equations will be applied. This is consistent with the usual practice where fc′ values higher than 6000 psi are usually used in compression elements which are rarely investigated for deflection and in which cracking effects are not predominant [8,24,25].

2.8 The Limits of nρ

Since the developed models will be expressed in terms of the product nρ, it is important to establish the range over which the product may vary. Both the British and the American codes will be considered and the extreme limits will be taken as the range of nρ for which the developed analysis will be assumed.

2.8.1 Upper Bound of nρ

Both the ultimate limit state (the British code) and the ultimate strength (the American code) design methods design for a ductile section where the reinforcement yields prior to the failure of the section by crushing of concrete in the so called " primary tension-secondary compression " failure. In doing so the codes specify limitations on either the depth of the neutral axis of the section, x, or the steel ratio, ρ, used. However, explicit expressions for the limiting maximum steel ratio ρ necessary to define the upper bound of nρ needed for the current study are not given and must therefore be derived.

If the condition when the steel strain is exactly at yield, ε_y, while that of the concrete at its crushing value is called the balanced condition and the corresponding depth of the neutral axis is denoted as x_b then the ductile failure described above will be ensured if $x \leq x_b$. This is because with the concrete crushing strain, ε_c, set at a prescribed value, smaller x values give steel strains, ε_s, larger than that at yield and thus ensures yield of steel prior to crushing of the section as shown in Fig.2.8.1.1 for a singly reinforced "general" section.

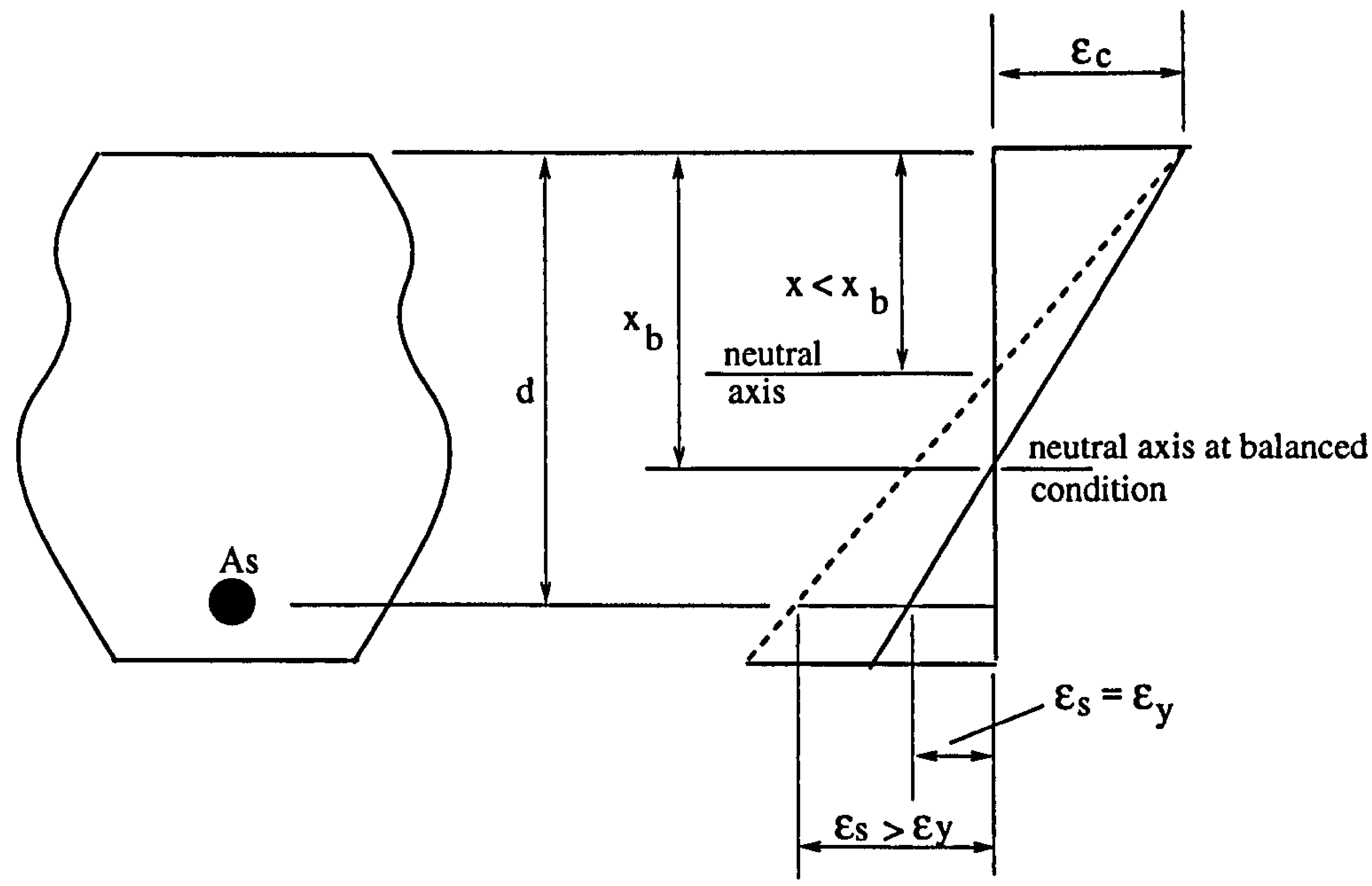


Figure 2.8.1.1 A general cross section and the corresponding strain diagram

From the geometry of the figure,

$$x_b=[\varepsilon_c / (\varepsilon_c + \varepsilon_y)](d)$$

multiplying the denominator and the numerator by Es gives,

$$x_b=[\varepsilon_c E_s/(\varepsilon_c E_s+fy)](d) \qquad (2.8.1.1)$$

The ACI code specifies the followings for ε_c and E_s,

ε_c=0.003 (Sec.10.2.3 of ACI 318 [2])

E_s=29x10^6 psi (200 KN/mm^2) (Sec.8.5.2 of ACI 318 [2])

Substituting these values into Eq.2.8.1.1 gives,

x_b=[87000/(87000+fy)](d), where fy is in psi, x_b and d in inches (2.8.1.2a)

or

x_b=[600/(600+fy)](d), where fy in MPa, x_b and d in mm (2.8.1.2b)

To ensure ductility Sec.10.3.3 of ACI 318 [2] requires that the tension reinforcement ratio is limited as follows,

$\rho \leq 0.75(\rho$ required at $x=x_b)$ (2.8.1.3)

The limitation, as can be seen, has therefore been imposed on the steel reinforcement ratio rather than the position of the neutral axis,x. It is only when the section is singly reinforced rectangular or doubly reinforced rectangular with yielding compression steel that the above limitation is equivalent to limiting the neutral axis depth. For flanged sections the limitation of the code provides more ductility than if x was limited to $0.75x_b$.

When the moments obtained by elastic analysis are to be redistributed the code requires extra ductility to allow for the development of plastic hinges at regions of maximum negative moments over continuous supports. This is expressed in Sec.8.4 of ACI 318 [2] where the limiting factor of 0.75 is further reduced to 0.5. Because for the current study the upper bound of $n\rho$ is sought, this extra restriction of the code is ignored and only the higher factor of 0.75 is considered.

The limitation set by Eq.2.8.1.3 is next applied to rectangular and flanged sections to obtain expressions for the maximum steel ratios.

(1) Rectangular Sections:

Using the assumption of Sec.10.2.7 of ACI 318 [2] for the stress block and that part of the tension steel will be in equilibrium with the concrete in compression while the remaining part is there to balance the compression steel area, the behaviour of a doubly reinforced rectangular section is shown in Fig.2.8.1.2.

Referring to the figure, Static equilibrium requires that,

$$0.85fc'\beta_1 xb+As'fs'=(As_1+As_2)fs=Asfs$$

where fs and fs′are the stress in the tension and compression steel, respectively.

At the balance condition, fs=fy, $x=x_b$ and if fs′ is denoted as fs_b' then,

$$0.85fc'\beta_1 x_b b+As'fs_b'=Asfy$$

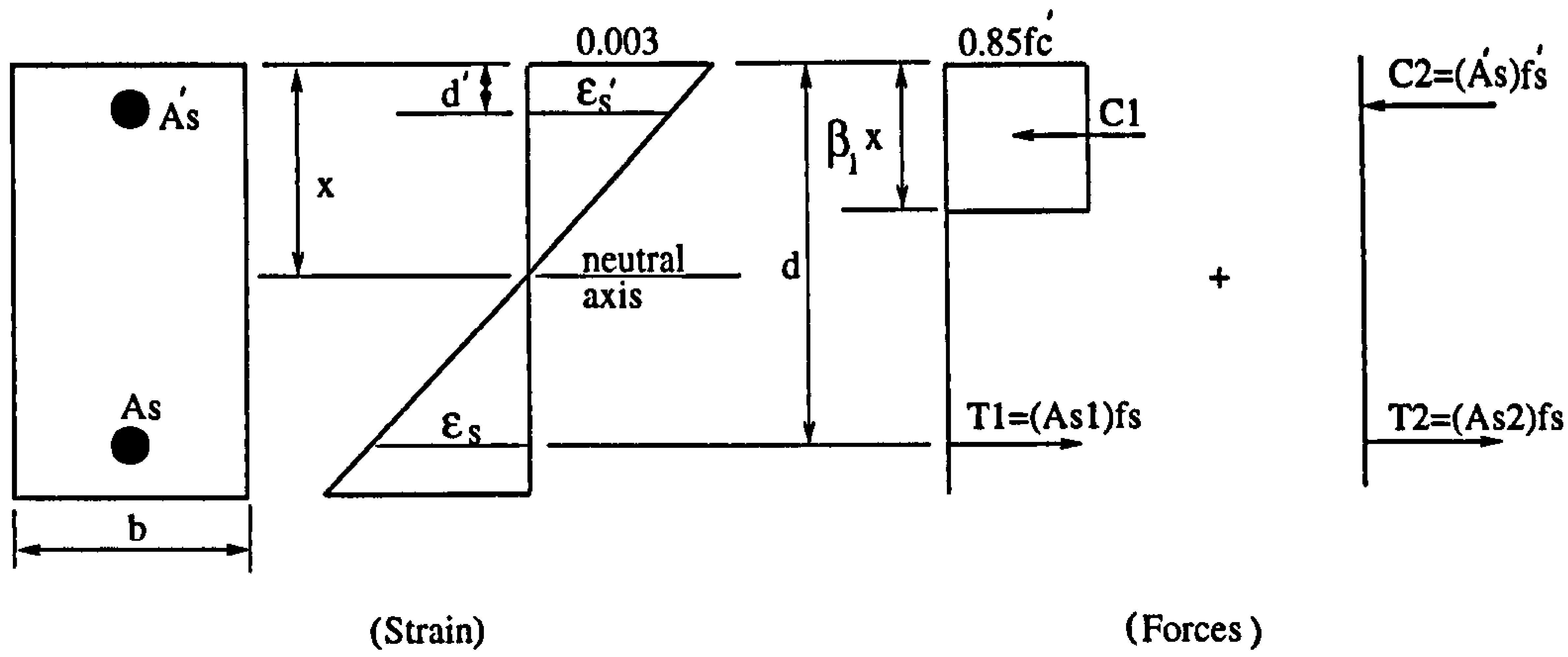


Figure 2.8.1.2 A doubly reinforced rectangular section and the correspondig strain and force diagrams according to ACI

However, with the reinforcement ratios ρ and ρ′ taken relative to bd

$$As=\rho bd \;\;,\;\; As'=\rho' bd$$

Thus,

$$0.85fc'\beta_1 x_b b+\rho' bdfs_b'=\rho bdfy$$

From the above equation the expression for ρ at $x=x_b$ can be written as,

$$\rho(\text{at } x=x_b)=(0.85fc'/fyd)(\beta_1 x_b)+\rho' fs_b'/fy$$

Thus, according to Sec.10.3.3 of ACI 318 [2],

$$\rho \le 0.75[(0.85fc'/fyd)(\beta_1 x_b)] + \rho' fs_b'/fy$$

where the 0.75 limitation on the compression steel is waived as allowed by the code. Defining,

$$\rho_b=[(0.85fc'/fyd)](\beta_1 x_b)$$

Then for doubly reinforced rectangular section,

$$\rho \le 0.75\rho_b + \rho' fs_b'/fy \qquad (2.8.1.4)$$

Using the strain diagram of Fig.2.8.1.2 and Eq.2.8.1.2 to obtain the strain in the compression steel at the balanced condition, ε_{sb}', and that $fs_b'=(Es)\varepsilon_{sb}'$ one obtains,

$$fs_b' = \min[87000-(d'/d)(87000+fy),\ fy] \quad \text{for fy and fc' in psi} \qquad (2.8.1.5a)$$

$$= \min[\ 600-(d'/d)(600+fy),\ fy] \quad \text{for fy and fc' in MPa} \qquad (2.8.1.5\ b)$$

When $\rho'=0$ is substituted into Eq.2.8.1.4 the limitation on the steel ratio for singly reinforced rectangular section is also obtained as

$$\rho \leq 0.75\rho_b \qquad (2.8.1.6)$$

The term ρ_b used in Eqs.2.8.1.4 and 2.8.1.6 was defined previously as

$$\rho_b = (0.85fc'/fyd)(\beta_1 x_b)$$

Substituting x_b from Eq.2.8.1.2 gives,

$$\rho_b = (0.85fc'\beta_1/ fy)[87000/(87000+fy)] \quad \text{for } fc' \text{ and } fy \text{ in psi} \qquad (2.8.1.7a)$$

$$= (0.85fc'\beta_1/fy)[600/(600+fy)] \quad \text{for } fc' \text{ and } fy \text{ in MPa} \qquad (2.8.1.7b)$$

where β_1 is given by the code as (Sec.10.2.7.3 of ACI 318 [2])

$$\beta_1=\min[0.85,\max(0.65,0.85-0.05(fc'-4000)/1000] \quad \text{for } fc' \text{ in psi} \qquad (2.8.1.8a)$$

$$=\min[0.85,\max(0.65,0.85-0.00725(fc'-28)] \quad \text{for } fc' \text{ in MPa} \qquad (2.8.1.8b)$$

(2) Flanged Sections :

As already explained above, according to Sec.10.2.7 of ACI 318 [2] the depth of the stress block is taken as $\beta_1 x$ where x is the depth of the neutral axis and β_1 is as given by Eq.2.8.1.8. When the depth of the neutral axis, x, is such that $\beta_1 x$ is less than or equal to the flange thickness, hf, and because the concrete area in tension is ignored the section can be treated as a perfectly rectangular section of width be (shown in Fig.2.8.1.3) and for which the equations derived above will still apply.

However, if the depth of the stress block falls within the web a T-section analysis must be carried out. Using the assumption of Sec.10.2.7 of ACI 318 [2] for the stress block and that part of the tension steel will be in equilibrium with the concrete in the web while the remainder will be there to balance the overhanging portions of the flange the behaviour of T-section can be assumed as shown in Fig.2.8.1.3

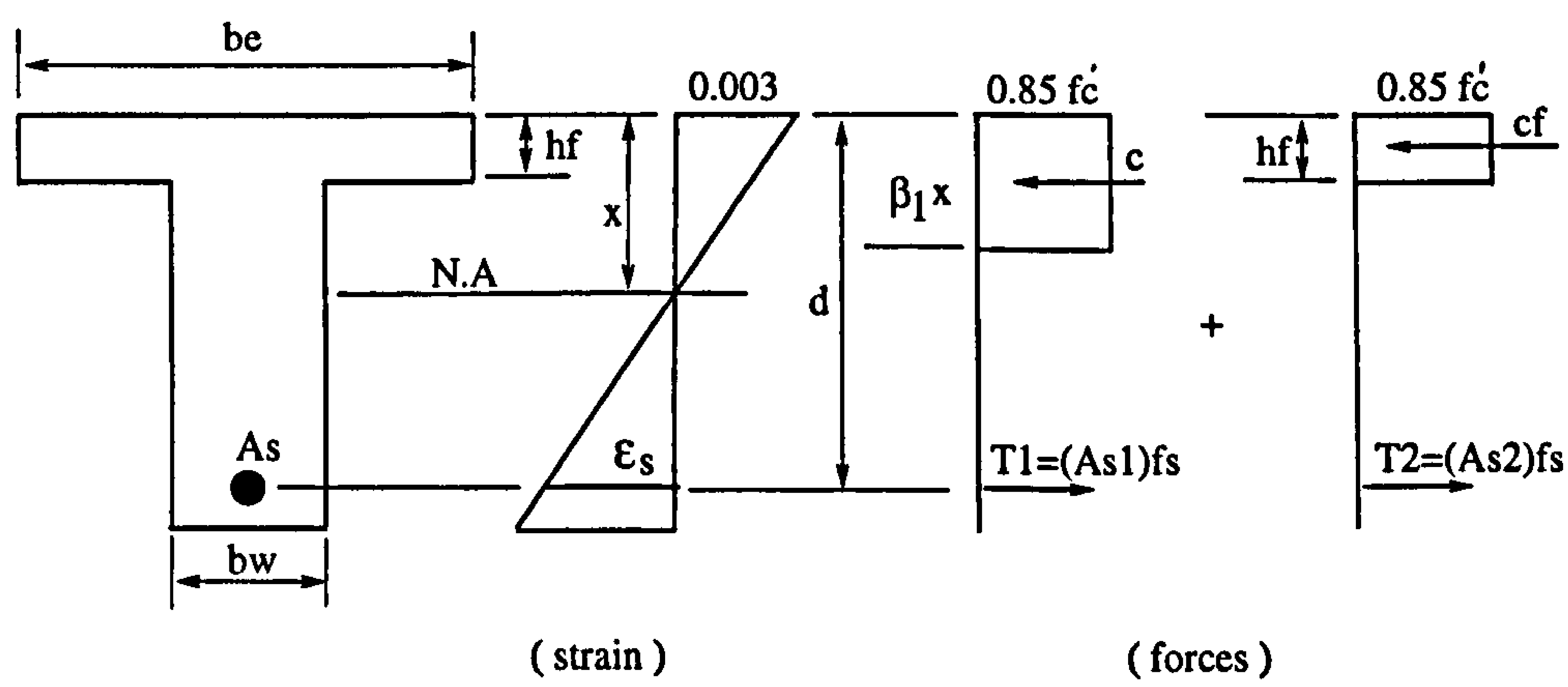


Figure 2.8.1.3 A singly reinforced flanged section and the corresponding strain and force diagrams according to ACI

Referring to the figure, Static equilibrium requires that

$$0.85fc' \, bw\beta_1 x + 0.85fc' hf(be-bw) = (As_1 + As_2)fs = Asfs$$

At the balanced condition, fs=fy and $x=x_b$. Thus, if $\beta_1 x > hf$,

$$0.85fc' \, bw\beta_1 x_b + 0.85fc' hf(be-bw) = Asfy$$

Writing the steel area, As, as ρbwd and rearranging gives,

$$\rho(\text{at } x=x_b) = (0.85fc'/fyd)(\beta_1 x_b) + (0.85fc'/fy)[hf(be-bw)/bwd)]$$

Substituting x_b from Eq.2.8.1.2 and applying Sec.10.3.3 of ACI 318 [2] gives the following limitation on the steel ratio of singly reinforced T-sections,

$$\rho \leq 0.75(\rho_b + \rho_f) \qquad (2.8.1.9)$$

where,

ρ_b (as given by Eq.2.8.1.7)

$\rho_f = 0.85fc' hf(be-bw)/fybwd$

Unlike the American code the British code restricts the depth of the neutral axis rather than the steel ratio. Clauses 3.2.2.1 and 3.4.4.3-4,pt.1 of BS 8110 [10] imply the following [26],

$$x \leq (\beta_b - 0.4)(d) \quad (2.8.1.10)$$

where β_b is given as:

(a) when the moment after redistribution is less than or equal to the moment before redistribution,

$$\beta_b = \min[\ 0.9\ ,\ 1 - (\ \%\ \text{redistribution}/100)\ ]$$

where % redistribution allowed by the code is 0 - 30%.

(b) when the moment after redistribution is greater than the moment before redistribution,

1.when the redistribution is $\leq 10\%$: $\beta_b = 0.9$

2.when the redistribution is $> 10\%$: $\beta_b = 1.0$

To show that Eq.2.8.1.10 does in fact produce $x < x_b$ and thus ensures an underreinforced section, Eq.2.8.1.2 for x_b are rederived using the code specifications of $\varepsilon_c = 0.0035$ (cls.2.5.3, 3.4.4.1 and Fig.3.1,pt.1 of BS 8110 [10]), Es=200 KN /mm^2 (cl.2.5.4,pt.1 of BS 8110 [10]) and of design material strengths (cl.2.4.2.2,pt.1 of BS 8110 [10]). Thus,

$x_b = [101500/(101500 + fy/\gamma_m)](d)$ for fy in psi,x_b and d in inches

$= [700/(700 + fy/\gamma_m)](d)$ for fy in MPa,x_b and d in mm

Comparing the above equation to Eq.2.8.1.10 gives,

$$x \leq [(101500+fy/\gamma_m)/101500](\beta_b-0.4)(x_b) \qquad \text{for fy in psi}$$

$$\leq [(700+fy/\gamma_m)/700](\beta_b-0.4)(x_b) \qquad \text{for fy in MPa}$$

Since the steel area required is highest at sections of maximum moments where the moment is always reduced by moment redistribution, condition (a) of β_b prevails. Thus for % redistribution ≤10%, β_b=0.9. Substituting this into the limitation of x gives,

for fy=250 MPa (and γ_m = 1.15 according to cl.2.4.4.1,pt.1 of BS 8110 [10]),

$$x \leq 0.655x_b$$

and for fy=460 MPa (and γ_m=1.15),

$$x \leq 0.786x_b$$

From the above two limitations it can be seen that the restriction of the code as represented by Eq.2.8.1.10 does in fact ensure an underreinforced section (it is interesting to note that the average of 0.655 and 0.786 is 0.72 which is almost same as the limiting factor of 0.75 used on the steel ratio by ACI 318 [2]).

Because for the current study the maximum limit of ρ is required to determine the upper bound of nρ, the limitation of the British code as specified above must be expressed in terms of the reinforcement ratio. This is next done where rectangular as well as flanged sections are studied to derive expressions for the maximum steel ratios.

(1) Rectangular Sections:

Using the simplified stress block of Fig.3.3,pt.1 of BS 8110 [10] with γ_m of 1.5 and that part of the tension steel will be in equilibrium with the concrete in compression while the remaining part is there to balance the compression steel, the behaviour of a doubly reinforced section is shown in Fig.2.8.1.4.

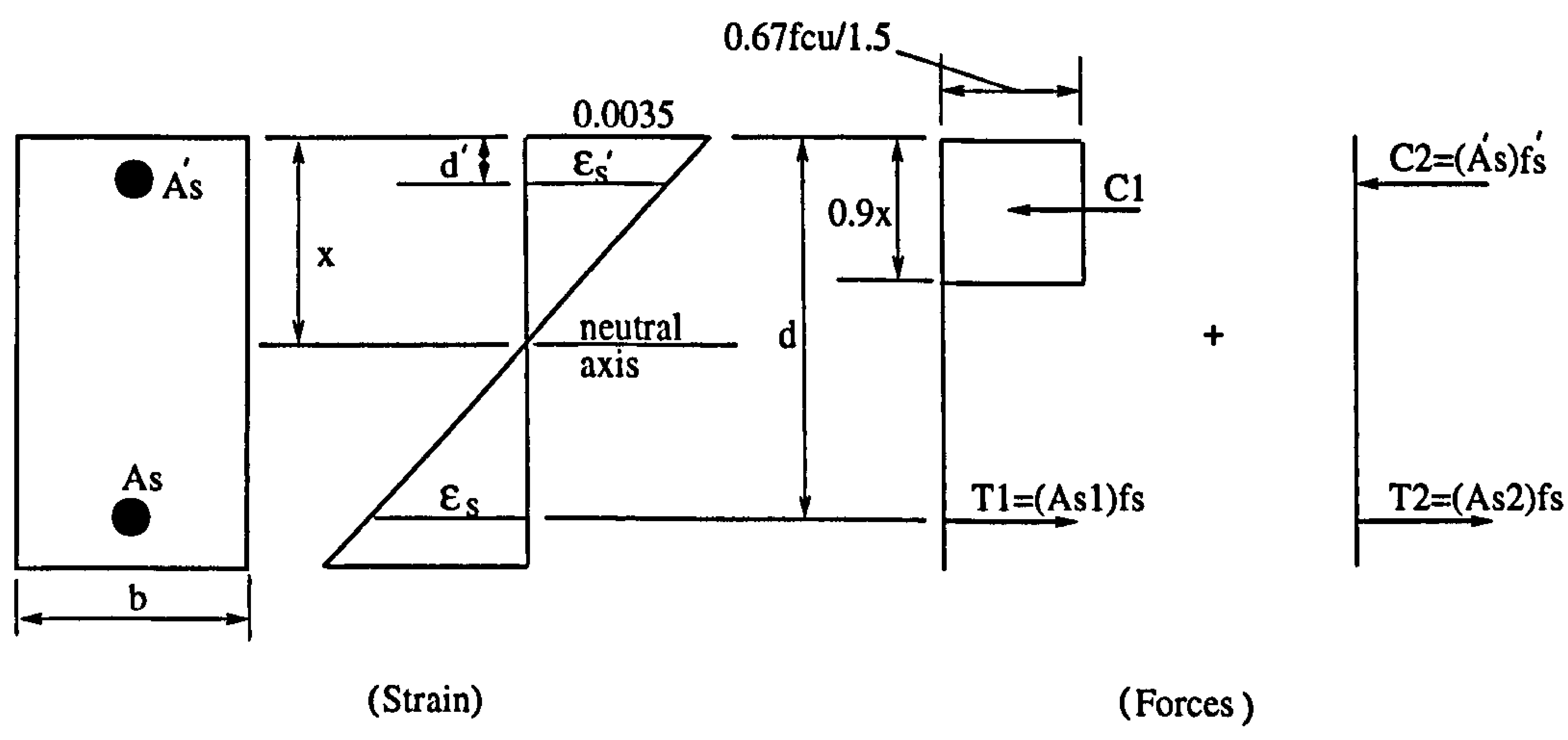


Figure 2.8.1.4 A doubly reinforced rectangular section and the corresponding strain and force diagrams according to BS

Referring to the figure, Static equilibrium requires that

$$(0.67fcu/1.5)(0.9x)(b)+As'fs'=(As_1+As_2)fs = Asfs$$

taking $x(max)=(\beta_b-0.4)(d)$ and noticing that fs at such a condition is equal to fy/γ_m as proved previously gives,

$$(0.9)(0.67fcu/1.5)(\beta_b-0.4)(bd)+As'fsm' = [As(max)](fy/\gamma_m)$$

where fs′ when x=x(max) has been denoted as fsm′.

Writing As(max) = $\rho(max)(bd)$, $As'=\rho'bd$ and taking γ_m as 1.15 the above equation simplifies to,

$$\rho(max)=0.4623(\beta_b-0.4)(fcu/fy) + (\rho'fsm')/(fy/1.15)$$

From the strain diagram,

$$\varepsilon_s'=0.0035(1-d'/x)$$

When $x(max) = (\beta_b-0.4)(d)$ is substituted into the above equation ε_s' at x(max) is obtained. Multiplying this ε_s' by Es will then give fsm′. Thus,

$$fsm'=\min\{ 0.0035Es[1-(d'/d) / (\beta_b-0.4)] \ , \ fy/1.15 \}$$

Therefore, to ensure that a doubly reinforced section is underreinforced the following must be satisfied,

$$\rho \leq 0.4623(\beta_b\text{-}0.4)(fcu/fy) + (\rho'fsm')/(fy/1.15) \qquad (2.8.1.11)$$

where,

$$fsm' = \min\{101500[1-(d'/d)/(\beta_b\text{-}0.4)], fy/1.15\} \quad \text{for fy and fsm' in psi} \qquad (2.8.1.12a)$$

$$= \min\{700[1-(d'/d)/(\beta_b\text{-}0.4)], fy/1.15\} \quad \text{for fy and fsm' in MPa} \qquad (2.8.1.12b)$$

When $\rho'=0$ is substituted into Eq.2.8.1.11 the limiting reinforcement ratio for a singly reinforced rectangular section is obtained as,

$$\rho \leq 0.4623(\beta_b\text{-}0.4)(fcu/fy) \qquad (2.8.1.13)$$

(2) Flanged sections:

As stated earlier when the depth of the stress block of a flanged section falls within the flange the section will behave as a rectangular section and all the pertaining equations will still be valid with b replaced by be.

If on the other hand the depth of the stress block falls within the web then a T-section analysis must be carried out. Using the assumption of Fig.3.3,pt.1 of BS 8110 [10] and that part of the tension steel will be in equilibrium with the concrete in the web while the remaining will be there to balance the overhanging portions of the flange the behaviour of a T-section can be assumed as shown in Fig.2.8.1.5.

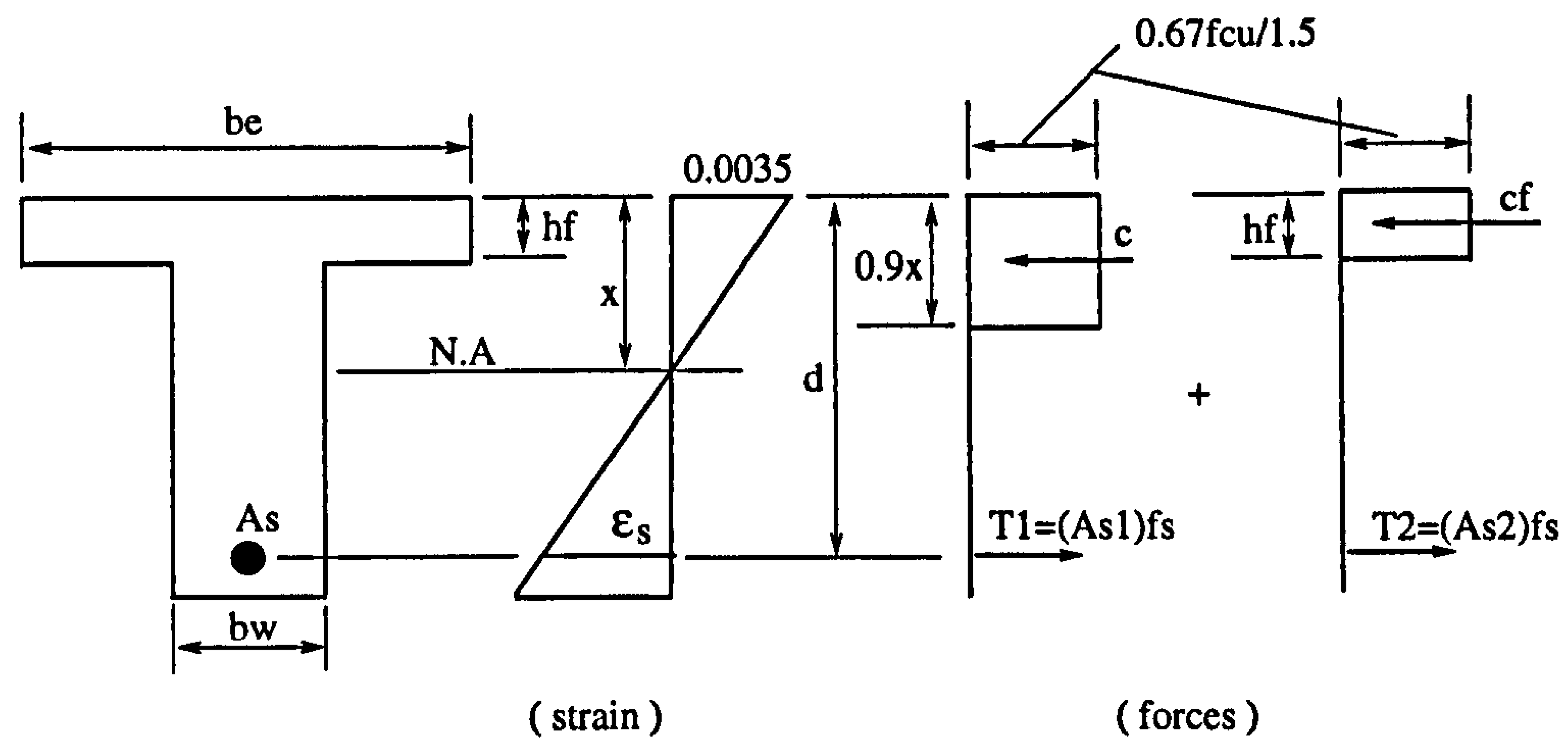


Figure 2.8.1.5 A singly reinforced flanged section and the corresponding strain and force diagrams according to BS

Static equilibrium requires that,

$$(0.67fcu/1.5)(0.9x)(bw)+(0.67fcu/1.5)(hf)(be-bw)=(As_1+ As_2)(fs)=Asfs$$

At the condition x(max)=(β_b -0.4)(d), fs is equal to fy/γ_m. Thus the above equation can be rewritten as,

$$\gamma_m(0.67fcu/1.5)(0.9)(\beta_b -0.4)(d)(bw)+\gamma_m(0.67fcu/1.5)(hf)(be-bw)=As(max)fy$$

Writing As(max) as ρ(max)bwd and taking γ_m =1.15 and rearranging gives,

$$\rho(max)=0.4623(\beta_b -0.4)(fcu/fy)+0.5137hf(be-bw)(fcu / bwdfy)$$

Therefore, to ensure that a singly reinforced T-section is underreinforced the following must be met,

$$\rho \leq 0.4623(\beta_b-0.4)(fcu/fy)+0.5137hf(be-bw)(fcu / bwdfy) \qquad (2.8.1.14)$$

where ρ= As/bwd.

The upper bound of the product nρ can now be determined as the extreme limit defined by the American code through Eqs.2.8.1.4-2.8.1.9 and by the British code through Eqs.2.8.1.11-2.8.1.14 where β_b is assumed at the maximum value of 0.9 according to condition(a) for β_b as given by Eq.2.8.1.10 and as explained previously which corresponds to an x(max) of 0.5d.

The limitations set by the two codes on the maximum ρ value for singly reinforced rectangular beams are used to determine the upper bound of nρ shown (in percentage) in Tables 2.8.1.1 and 2.8.1.2.

Because Eqs.2.8.1.6-7 and 2.8.1.13 give higher reinforcement ratios for lower values of fy, the minimum specified code values of fy were used to obtain the values of ρ shown in the tables.

Table 2.8.1.1. Values of n, ρ(max) and nρ(max) for different fc′ values according to ACI 318 [2] for singly reinforced rectangular beams

fc′(psi)	n	nfc′/fy	β1	ρ(max)%	nρ(max)%
3000	9	0.675	0.85	2.78	25.00
3500	8.5	0.744	0.85	3.25	27.63
4000	8	0.8	0.85	3.71	29.68
5000	7	0.875	0.8	4.37	30.59
5500	6.8	0.935	0.77	4.65	31.62
6000	6.5	0.975	0.75	4.91	31.92

Table 2.8.1.2. Values of n, ρ(max) and nρ(max) for different fcu values according to BS 8110 [10] for singly reinforced rectangular beams

fcu(MPa)	n	nfcu/fy	ρ(max)%	nρ(max)%
25	8	0.80	2.31	18.48
30	7.7	0.92	2.77	21.33
35	7.4	1.04	3.23	23.90
40	7.1	1.14	3.70	26.27
45	6.9	1.24	4.16	28.70
50	6.7	1.34	4.62	30.95

In accordance with ASTM A615-617 [27] which are part of the ACI code as declared in Secs.3.5.3 and 3.8 of ACI 318 a value of 40 ksi is used as the lowest fy value to obtain the reinforcement ratios in Table 2.8.1.1 from Eqs.2.8.1.6-7.

Likewise, based on cl.3.1.7,pt.1 of BS 8110 [10] the minimum fy value of 250 MPa is used to obtain the reinforcement ratios in Table 2.8.1.2 from Eq.2.8.1.13.

Because fc′ are cylindrical strengths, Eq.2.7.1 for Ec was used in determining the n values in Table 2.8.1.1. In Table 2.8.1.2, on the other hand, the values of n were computed using Eq.2.7.4 of Ec as fcu are cubic strengths.

The range of values of fc′ included in Table 2.8.1.1 is the one usually used in flexural elements like slabs, beams or girders. Values of fc′ higher than 6000 psi are usually used for columns on lower stories of high-rise buildings rather than in flexural elements [8, page 41 of Ref.24, page 12 of Ref.25]. Because such columns are mainly in compression the effect of cracking is not significant and is therefore not part of this study as was explained earlier.

In Table 2.8.1.2 concrete grades of C25 - C50 were included. While grades C30 - C50 are the ones usually used in normal practice as recommended by Table 3.4,pt.1 of BS 8110 [10], grade C25 is also considered based on the relaxation of cl.3.3.5.2,pt.1 of the code.

In addition to the ductility requirements of Eq.2.8.1.13 used to determine ρ values in Table 2.8.1.2 and to ensure proper placement and compaction of concrete, cl.3.12.6.1,pt.1 of BS 8110 [10] specifies 4% as the absolute maximum steel ratio of either the tension or the compression reinforcement based on the gross concrete area, bwh. For the extreme ratio of d/h of 0.72, as is assumed in most studies [i.e.7], such a limit corresponds to 4(1/0.72) or 5.5% relative to bwd. Because such an absolute limit exceeds the ductility requirements for the maximum ρ values for the common grades of concrete shown in the table the latter controls.

Comparing the results summarized in Tables 2.8.1.1 and 2.8.1.2 it can be seen that for singly reinforced rectangular sections the maximum nρ that can be taken as an upper limit is 31.92 or about 32%. Namely,

$n\rho$ (upper bound, singly reinforced rectangular section) = 32% (2.8.1.15)

When the rectangular section is doubly reinforced the compression steel area that is needed for extra moment capacity is almost always smaller than the maximum steel area permitted for a singly reinforced section. Namely As′ of Fig.2.8.1.2 or 2.8.1.4 is normally found to be smaller than As_1(max). Therefore, ρ'(max) can be taken as ρ(max) for a singly reinforced section. If the value of ρ'(max) thus assumed is substituted into Eq.2.8.1.4 or Eq.2.8.1.11 with fs_b'=fy or fsm′=fy/1.15 a value of ρ(max) twice that of a singly reinforced section will be obtained (a fact which is consistent with ρ(max) given for doubly reinforced sections in the design aids of Ref.8). Since the values of $n\rho$(max) of Table 2.8.1.1 are greater than those of Table 2.8.1.2 and because cl.3.12.6.1,pt.1 of BS 8110 [10] further restricts the amount of reinforcements to be allowed it is obvious that the ACI specifications give the highest reinforcement ratios for doubly reinforced rectangular sections. Thus and from Table 2.8.1.1,

$n\rho$(upper bound, doubly reinforced rectangular sections)=64% (2.8.1.16)

In the case of flanged sections the maximum ρ value as can be seen from Eqs.2.8.1.9 and 2.8.1.14 is not only a function of fc′ or fcu and fy but also of the sectional dimensions.

According to Eq.2.8.1.9 ρ(max) can be written as,

ρ(max)=ρ(max) for a singly reinforced rectangular section+0.6375[hf(be-

$bw)/bwd](fc'/fy)$

Thus from Table 2.8.1.1 with $n\rho$ in percentage and for the maximum value of nfc'/fy of 0.975,

$$n\rho(max)=31.92+62.16(hf/d)(be-bw)/bw$$

Or expressing $n\rho$ relative to the flange width be (leads to a better structure for the programs to be discussed in Sec.3.4 and reduces the do loop iterations involved),

$$n\rho(max)=31.92(bw/be)+62.16(hf/d)(1-bw/be) \qquad (2.8.1.17)$$

On the other hand the British code limits the amount of reinforcement based on the ductility requirement and the requirement of cl 3.12.6.1,pt.1 of BS 8110 [10] with the minimum of the two as the controlling limit. While the ductility requirement according to Eq.2.8.1.14 will give the highest value of $n\rho(max)$ at the maximum nfcu/fy, the value of n to be multiplied by ρ as obtained using cl 3.12.6.1,pt.1 of BS 8110 [10] is highest at the lowest nfcu/fy of Table 2.8.1.2.

Because of this inconsistency the two limits have to be checked at each case of Table 2.8.1.2 and the minimum taken as the corresponding $n\rho(max)$. The highest of the values of $n\rho(max)$ thus obtained for all cases will then be the $n\rho(max)$ for the particular ratios of be/bw and hf/d considered.

From Eq.2.8.1.14 and for $n\rho$ in percentage and relative to be,

$n\rho$(max,for ductility)=$n\rho$(max) for singly reinforced rectangular section(bw/be)

+51.37(nfcu/fy)(hf/d)(1-bw/be) (2.8.1.18)

The limitation of cl.3.12.6.1,pt.1 of BS 8110 [10], although it refers to the gross concrete area, is often taken as 4% of bwh even for flanged sections [page 143 of Ref.28]. With such a limitation being actually equivalent to ρ(max) of 5.5% relative to (bw)d, as was seen earlier, one can write, relative to be,

$n\rho$(max,due to cl.3.12.6.1,pt.1 of BS 8110 [10])=5.5n(bw/be) (2.8.1.19)

Combining Eqs.2.8.1.18 and 2.8.1.19,

$n\rho$(max)=min(Eq.2.8.1.18,Eq.2.8.1.19) (2.8.1.20)

Equation 2.8.1.20 is applied to all cases in Table 2.8.1.2. The highest value of $n\rho$(max) obtained is then compared with $n\rho$(max) from Eq.2.8.1.17 to determine the overall $n\rho$(max) for the particular ratios of be/bw and hf/d considered. To find the upper bound of $n\rho$ the value of $n\rho$(max) has to be determined for all combinations of be/bw and hf/d that can practically occur so that the highest of these values can be chosen. For this purpose a computer program has been developed in which the above argument was used to evaluate the upper bound of $n\rho$ for the practical ratios of be/bw from 1.1 to 10 and of hf/d from 0.1 to 0.60. The coding of the program along with the printed results are shown in Appendix A. Consistent with the notation used to define the equations the program, being the first in this section, is referred to as

Prog.2.8.1 (a practice used throughout this document). The program has been structured to printout for each combination of be/bw and hf/d the maximum nρ value found using the British and the American codes and the highest of these values as well the position of the neutral axis within the cracked transformed section.

The results obtained from the computer analysis indicated that for hf/d ratios greater than 0.55 the neutral axis always falls within the flange and the section can therefore be analyzed as a rectangle of width be. Hence the upper bound of nρ for T-section behaviour has to be determined from the results given for hf/d ratios of less than or equal to 0.55. By scanning the values of nρ(max) given for this range of hf/d it can be seen that the specifications of ACI 318 [2] as given by Eq.2.8.1.17 always control giving an upper bound of nρ of 33.96% (practical considerations will probably not allow such a high steel area into the web of a flanged section [page 335 of Ref.25]. However, since such a practical limitation is not part of the ACI code, it is not considered in this study).

For the unlikely condition of doubly reinforced flanged sections no separate analysis has been devoted. However, in Sec.3.5 it will be shown that the results drawn from studying doubly reinforced rectangular and singly reinforced flanged sections can be easily extended to include such cases.

2.8.2 Lower Bound of nρ

If the tension steel area provided is too little the section will suddenly fail when the tensile stresses exceed the tensile strength of the concrete.

To guard against such a sudden failure, Sec.10.5.1 of ACI 318 [2] requires that in case of beams the tension steel area is not to be taken less than ρ(min) as specified below,

$$\rho(min,beams)=200/fy \qquad \text{for fy in psi} \qquad (2.8.2.1a)$$

or, in SI units

$$\rho(min,beams)=1.4/fy \qquad \text{for fy in MPa} \qquad (2.8.2.1b)$$

The highest value of fy=60ksi [27] will therefore give a limiting value of ρ(min)=0.3%. Hence,

$$\rho(\text{min,rectangular section beams according to ACI 318 [2]})=0.3\% \qquad (2.8.2.2)$$

For slabs the ACI code simply restricts the minimum reinforcement ratio to those required to control shrinkage and temperature cracks. Thus, from Sec.7.12.2.1 of ACI 318 [2],

minimum steel ratio(relative to gross area of concrete) for slabs =0.0014

Because in this study ρ is taken with respect to the effective depth d the above limitation must therefore be accordingly modified. Taking d/h for slabs as 0.8, on average, then

$$\rho(min,slabs)=0.0014/0.8=0.0018$$

or, after expressing in percentage,

$$\rho(\text{min,slabs according to ACI 318 [2]})=0.18\% \qquad (2.8.2.3)$$

On the other hand the British code specifies empirical values for the minimum steel area for the same reason as above and to control thermal and shrinkage effects. These are given in Table 3.27, pt.1 and according to cl.3.12.5.3,pt.1. of BS 8110. From the table it can be seen that the smallest minimum percentage of reinforcement is 0.13% which is again based on the gross concrete area. Thus, dividing by the extreme ratio of d/h of 0.97 (consistent with most references) to express ρ relative to d,

$$\rho(\text{min}) = 0.13/0.97=0.134\%$$

Hence,it can be taken that,

$$\rho(\text{min,rectangular section according to BS 8110 [10]})=0.134\% \qquad (2.8.2.4)$$

From Eqs.2.8.2.2-2.8.2.4 and using the minimum n values from Tables 2.8.1.1-2 it can be concluded therefore that,

$$\text{lower bound of } n\rho(\text{rectangular section})=\min[\ 6.5(0.18)\ ,\ 6.7(0.134)\ ]$$

or,

$$\text{lower bound of } n\rho(\text{rectangular sections}) \approx 0.9\% \qquad (2.8.2.5)$$

For flanged sections $\rho(min)$ based on the American code is as given by Eq.2.8.2.2. However, to express ρ relative to the flange width be the equation has to be written as,

$$\rho(min)=0.3/(be/bw)\%$$

and for the minimum n value from Table 2.8.1.1 one can write,

$$n\rho(min)=6.5[0.3/(be/bw)]\%$$

or

$$n\rho(\text{min,flanged section according to ACI 318 [2]})=1.95/(be/bw) \qquad (2.8.2.6)$$

On the other hand, the specification of BS 8110 [10] as given in Table 3.27,pt.1 of the code can be written as,

$$\rho(min)=[0.13/(d/h)]/(be/bw)\% \qquad \text{when } be/bw \leq 2.5$$

$$=[0.18/(d/h)]/(be/bw)\% \qquad \text{when } be/bw > 2.5$$

where the division by d/h and then by be/bw was necessary to convert the given ρ to that relative to (be)d. Hence for the minimum n value of 6.7 from Table 2.8.1.2 and for d/h of 0.97 one can write,

$$n\rho(\text{min,flanged sections according to BS 8110 [10]}) = 0.9/(be/bw) \quad \text{for } be/bw \leq 2.5$$
$$= 1.24/(be/bw) \quad \text{for } be/bw > 2.5 \qquad (2.8.2.7)$$

Because Eq.2.8.2.6 gives higher $n\rho(\text{min})$ values than those from Eq.2.8.2.7 the latter controls. Therefore,

$$n\rho(\text{min,flanged sections}) = 0.9/(be/bw) \quad \text{for } be/bw \leq 2.5 \qquad (2.8.2.8)$$
$$= 1.24/(be/bw) \quad \text{for } be/bw > 2.5$$

From the above equations the lower bound of $n\rho$ can be obtained by setting be/bw at 2.5 and at 10 (as was previously mentioned the maximum be/bw considered in the current study is 10) in the two parts of the equation and then taking the minimum of the two values. Because 1.24/10 is obviously smaller than 0.9/2.5 the former controls.Thus,

$$\text{lower bound of } n\rho \text{ for flanged sections (relative to be)} = 0.124\% \qquad (2.8.2.9)$$

2.9 The Limits of d′/d

The limits of d′/d are usually controlled by the force and moment capacity provided by the compression steel as placed within the section as well as the concrete cover allowed by the different codes. References consulted in the British practice suggest d′/d ratios from 0.08 to 0.3 while in the American practice ratios vary from 0.03 to 0.37. Since this study considers the extreme limits in the two practices, d′/d ratios within the limits of 0.03 to 0.37 are adopted.

2.10 The Modulus of Rupture

Although the tensile strength of concrete is usually ignored in strength computations of flexural elements it forms an important factor in the serviceability considerations such as deflection calculations and in cases where the moment of inertia of the section is evaluated considering the effect of cracking. The type of concrete tensile strength considered in these cases is called the modulus of rupture.

The modulus of rupture of concrete is defined as its ability to withstand tensile stresses that are induced by bending. It is obtained by bending tests conducted on plain concrete beams where the flexural formula is used to evaluate the tensile strength of the concrete tested. Although the modulus of rupture is not an accurate function of the compressive strength it can always be approximated in terms of the cylindrical compressive strength of the concrete as,

$$fr=k\sqrt{fc'}$$

Experiments have shown that for normal weight concrete and for fc′ in psi k varies from 7 to 12 and for light weight concrete from 5 to 11. The smaller factors tend to apply to higher strength concretes and the larger to lower strength concretes.

Taking the lower limit of the possible values of k, Sec.9.5.2.3 of ACI 318 [2] specifies the following for the modulus of rupture, fr

$$fr=7.5\sqrt{fc'} \qquad \text{where fr and fc' in psi} \qquad (2.10.1a)$$

$$fr=0.62\sqrt{fc'} \qquad \text{where fr and fc' in MPa} \qquad (2.10.1b)$$

The code also specifies that 75% and 85% of Eq.2.10.1 can be used for the modulus of rupture of all-lightweight concrete and sand-lightweight concrete, respectively.

Since there is no expression for the flexural tensile strength of the concrete as a function of the compressive strength give in BS 8110 [10], Eq.2.10.1 is used throughout this study. For this the equation had to be modified for the cases where cube compressive strengths are specified. Dividing the cylindrical compressive strength by an average conversion factor of 0.8 [29] the equation is modified as follows,

$$fr=6.8\sqrt{fcu}, \qquad \text{where fr and fcu in psi} \qquad (2.10.2a)$$

or

$$fr=0.56\sqrt{fcu}\ , \qquad \text{where fr and fcu in MPa} \qquad (2.10.2b)$$

2.11 Summary

In the preceding sections different parameters and sectional ratios were considered. Based on the British and the American codes and as found in the references consulted limits for the different parameters were derived and equations for use in the proposed models were suggested. Because these limits define the scope of the study as well as the boundaries of the computer programs developed in the forecoming chapters, it was thought useful to provide, as a summary, Table 2.11.1 that can be used as a quick reference.

Table 2.11.1

Summary of the parameters and equations defining the study

parameters	parameters' value range or equations
nρ(singly reinforced rectangular sections)	0.9 to 32%
nρ(doubly reinforced rectangular sections	0.9 to 64%
nρ(singly reinforced flanged sections)*	0.124 to 33.96%
d′/d(doubly reinforced sections)	0.03 to 0.37
be/bw(flanged sections)	1.1 to 10
hf/d(flanged sections)	0.1 to 0.55
d/h	0.72 to 0.97
elastic modulus of concrete,Ec	Eqs.2.7.1,2.7.2,2.7.4
modulus of rupture,fr	Eqs.2.10.1 and 2.10.2

**nρ taken relative to the flange width be.*

CHAPTER 3

APPROXIMATION OF Icr

3.1 Introduction

For any concrete section the effective moment of inertia can be written as,

$$Ie = Icr + Is$$

where Is is the part contributed by the concrete that has not yet cracked and the symbol "s" is used to represent concrete stiffening effect.
Icr is, as previously defined, the moment of inertia of the section when all the concrete in tension has cracked. It is evaluated considering the concrete in compression and transforming the tension and compression steel (if any) into an equivalent concrete area.

It is the computation of Icr that makes Ie calculations lengthy especially for flanged sections or when Ie is to be evaluated at different sections. Thus it is useful to develop an approximation for Icr that will facilitate its calculation with the best accuracy possible. This is done in this chapter.

First an approximating equation will be developed for the case of rectangular sections with tension reinforcement only. This will then be used as the basic equation for the evaluation of Icr for sections that are doubly reinforced and for those that are flanged. In modelling the basic equation two criteria are observed. First the equation is made as simple as possible by avoiding higher degree polynomials and instead straight line fits were used in the approximations involved in the development of the equation. Second the equation is formulated in a way that its application to sections other than singly reinforced rectangles is possible by modifying only a single factor. This does not only make the equation simpler to apply but also makes it possible to develop a single compact model of the effective moment of inertia for all the sections considered in this study. In addition, and as will be shown in the next chapter, such a model will be easy to represent graphically such that the need for the separate evaluation of Icr is completely eliminated.

Because Icr will be part of the overall expression of Ie to be developed in the next chapter and which is required to maintain an acceptable level of accuracy, any expression of Icr had to bear the least error possible. To ensure this intensive computer analysis had to be conducted considering all possible combinations of the factors involved. Through such intensive studies it was possible to develop the basic equation for the approximation of Icr and its modifications as applied to doubly reinforced rectangular sections as well as singly and doubly reinforced flanged sections bearing a maximum error of only ±6%.

The models for the approximation of Icr as developed in this chapter were intended for use in the evaluation of the effective moment of inertia as will be discussed in the next chapter. However, it is also useful due to its simplicity and high accuracy in any analysis or design of concrete elements where Icr is involved.

3.2 Singly Reinforced Rectangular Sections

Consider the cracked transformed section shown below

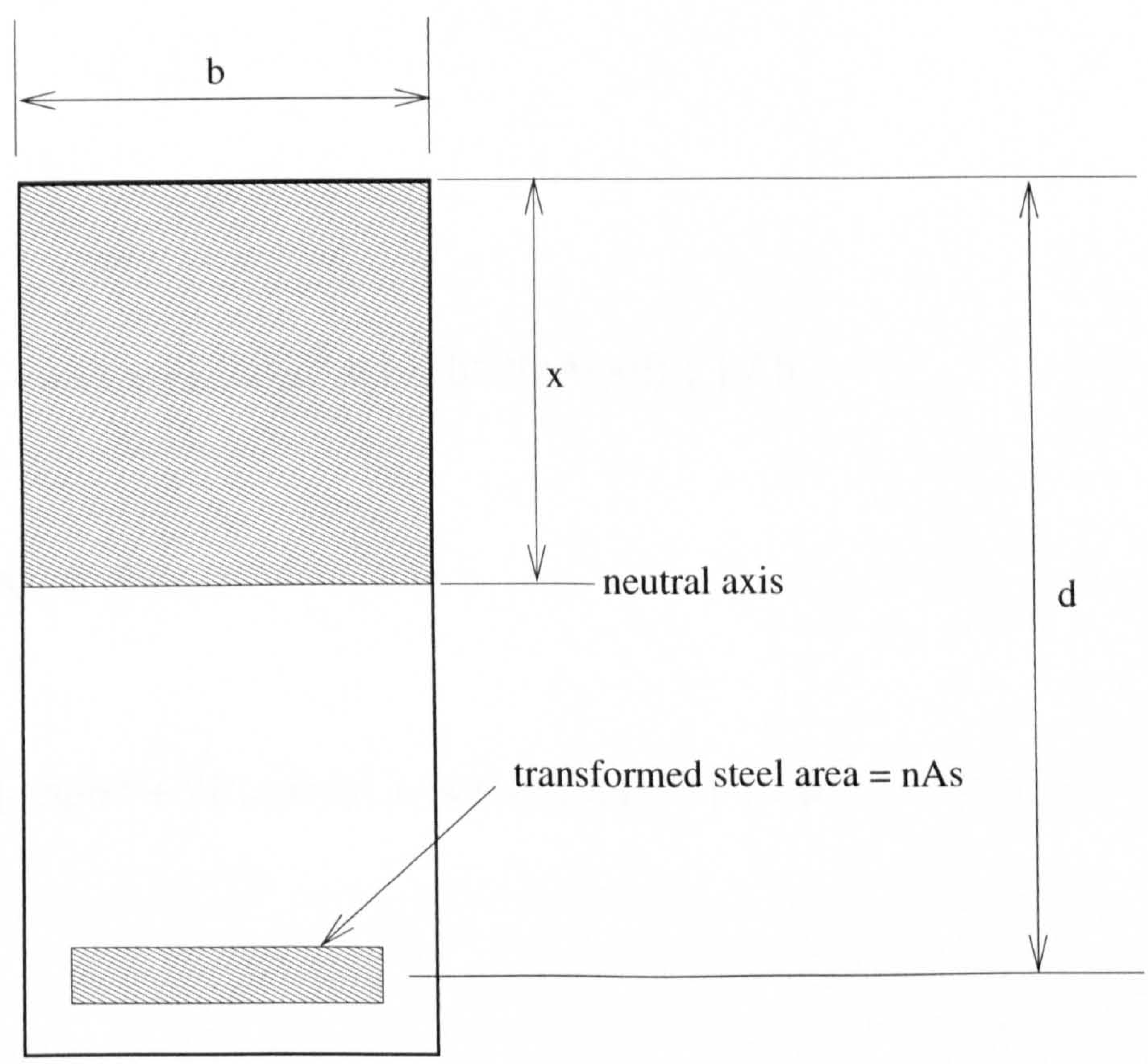


Figure 3.2.1 Transformed cracked section of a singly reinforced rectangular section.

The position of the neutral axis, which is same as that of the centroidal axis since stresses are elastic (as is always the case in serviceability considerations), is given by,

$$x = [\ (bx)(x/2) + (nAs)(d)]\ /\ [\ bx+nAs\ ]$$

Thus

$$bx^2/2 + (nAs)(x) - (nAs)(d) = 0$$

or

$$x = \{\ -nAs + \sqrt{[\ (nAs)^2 + (4)(b/2)(nAs)(d)\ ]}\ \}\ /\ b$$

Expressing As as ρbd,

$$x = \{\ -n\rho bd + \sqrt{[\ (n\rho bd)^2 + (4)(b/2)(n\rho bd^2)\ ]}\ \}\ /\ b$$

or

$$x = -n\rho d + d\sqrt{(n\rho^2 + 2n\rho)}$$

taking $k = -\ n\rho + \sqrt{(n\rho^2 + 2n\rho)}$, then

$$x = dk$$

On the other hand,

$$Icr = bx^3/12 + (bx)(x/2)^2 + n\rho bd(d-x)^2$$

Substituting dk for x,

$$Icr = bd^3k^3/12 + bd^3k^3/4 + n\rho bd(d - dk)^2$$

$$= 4bd^3k^3/12 + n\rho bd^3(1-k)^2$$

or

$$Icr = (bd^3/12)[\ 4k^3 + 12n\rho(1-k)^2\ ]$$

In order to preserve accuracy usual practice recommends ρ to be taken to four decimal places. However, because it is difficult to adhere to, this rule is commonly overlooked. Therefore, and to avoid loss of accuracy it is thought to be better if ρ is expressed in percentage where only two decimal places will be required. Thus,

$$Icr = (bd^3/12)[\ 4k^3 + 0.12n\rho(1-k)^2\ ]$$

or

$$Icr\ /\ (bd^3/12) = 4k^3 + 0.12n\rho(1-k)^2 \qquad (3.2.1)$$

where,

$$k = [\, - n\rho + \sqrt{(n^2\rho^2 + 200n\rho)}\ ] / 100$$

The development of Eq.3.2.1 was necessary to consolidate b, d, x and $n\rho$ as separate variables into the two variables $Icr/(bd^3/12)$ and $n\rho$ (since k is a function of $n\rho$) such that a curve can be produced in a two axes coordinate system. To reduce the complexity of Eq.3.2.1 this curve can then be approximated by a series of straight line intervals which can be represented by the simple expression of a straight line.

Figures 3.2.2(a)-(b) represent the curve of $Icr/(bd^3/12)$ as obtained from Eq.3.2.1 and for $n\rho$ values from 0.12% up to 64% encompassing all the lower and upper bounds discussed in Chap.2.

For the $n\rho$ values in the range discussed above different trials were attempted to approximate the curve of Figs.3.2.2(a) and (b) using straight line fits. Based on these trials it was found that the approximations shown in the figure give the accuracy desired with the least number of intervals and thus approximate Icr as Icre given by the following basic formula,

$$Icre = (\ \alpha + \beta n\rho\ )(\ bd^3/12\ ) \qquad (3.2.2)$$

where,

$\alpha = 0.003,\ \beta = 0.095$ for $n\rho \leq 1.9\%$

$\alpha = 0.05,\ \beta = 0.07$ for $1.9\% < n\rho \leq 5\%$

$\alpha = 0.16,\ \beta = 0.05$ for $5\% < n\rho \leq 17\%$

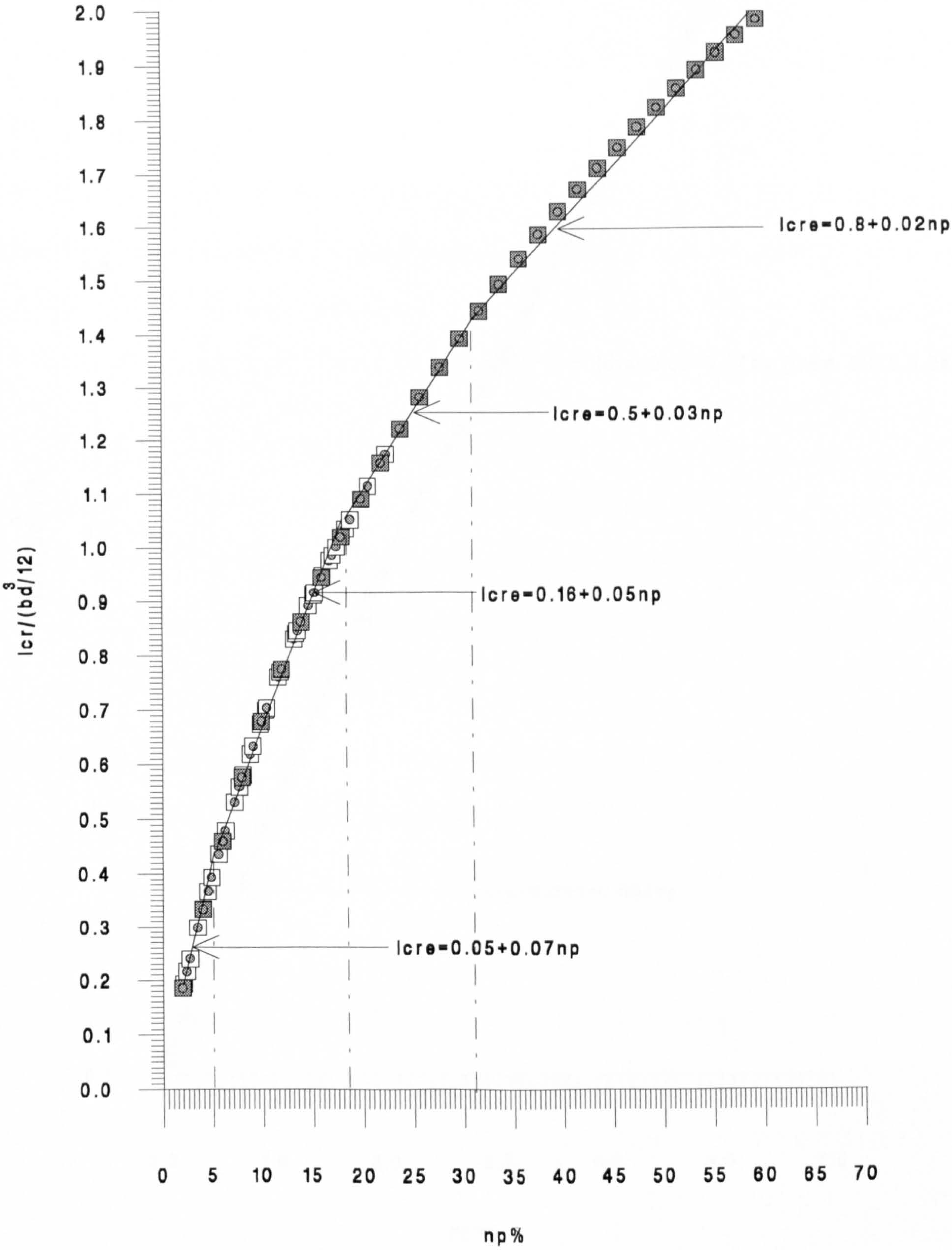


Figure 3.2.2(a) Approximation of Icr

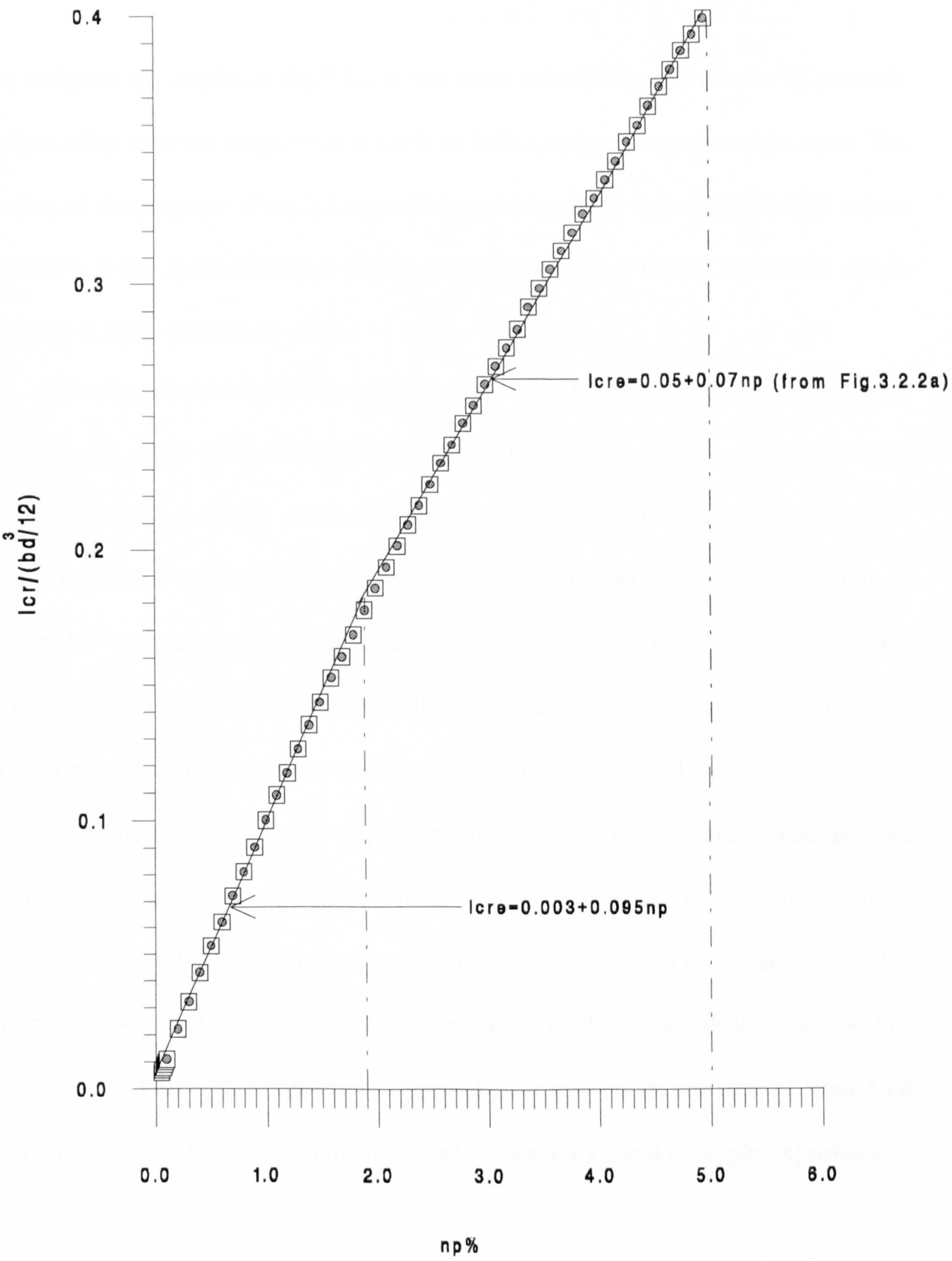


Figure 3.2.2(b) Approximation of Icr

α =0.50, β =0.03 for 17% < $n\rho \leq$ 32%

α =0.80, β =0.02 for $n\rho$ > 32%

To compare the results of Eq.3.2.2 to the exact value of Icr for almost all possible values of $n\rho$ over the range from 0.124% to 64% a computer program was used. The coding of the program (Prog.3.2.1) and its output appear in Appendix B1. The results as shown in the appendix reveal that the proposed model of Eq.3.2.2 gives Icr values that are at least accurate to ±6%.

In the sections to follow it will be shown that doubly reinforced rectangular sections and singly or doubly reinforced flanged sections can always be transformed into an equivalent singly reinforced rectangular section so that Eq.3.2.2 can be applied. Therefore, in this context and because the equation is seen to give results of acceptable accuracy over $n\rho$ values that encompass all the lower and upper bounds discussed in Chap.2 (0.124% to 64%), it can be taken as the basic expression for the approximation of Icr for all the sectional geometries considered in this study.

In Chap.4 a model of Ie will be developed as the cracked transformed moment of inertia of a rectangular section in which the effect of concrete stiffening is represented by a fictitious steel area using the basic expression of the approximation of Icr as given by Eq.3.2.2. Because the equation is of the simplest form possible, due to straight line approximation it represents, it will be shown to lead itself well into an easy analytical development that will eventually yield a simple expression of Ie.

3.3 Doubly Reinforced Rectangular Sections

Although it is sometimes claimed that compression reinforcement has little effect on Icr [30,31], it can be shown that neglecting such reinforcement often yields Icr values that are overconservative (conservative moment of inertia values are those lower than the actual ones for they predict conditions that are worse than otherwise). Because any Icr approximation will be part of the models to be developed in the forthcoming chapters it is important then to avoid undue safety that may lead to cumulative loss of accuray. It is thus thought necessary to consider the effect of compression reinforcement. As will be shown in this section such an effect can always be accounted for by modifying only the parameter "b" of the basic formula of Eq.3.2.2.

In order to evaluate Icr for a doubly reinforced rectangular section using Eq.3.2.2 the section must be transformed into another which can be assumed to behave as a singly reinforced rectangular section for which the equation is derived. The easiest way to do this is to transform the compression reinforcement into an equivalent concrete area that acts as a flange of sufficient thickness such that the resulting flanged section behaves as a rectangular section. Such transformation is shown in Fig.3.3.1 where the doubly reinforced rectangular section of Fig(a) has been transformed into the equivalent section of Fig(b). For the cracked section of this figure to behave as a singly reinforced rectangular section that is equivalent to the cracked section of Fig(a) the transformation factors β' and t must be properly chosen.

If in Fig.3.3.1(b), the flange depth t and the centroidal axis depth xe of the equivalent cracked section are made equal and equal to the depth x of the centroidal axis of the cracked section of Fig3.3.1(a), then to keep the moment of inertia of the

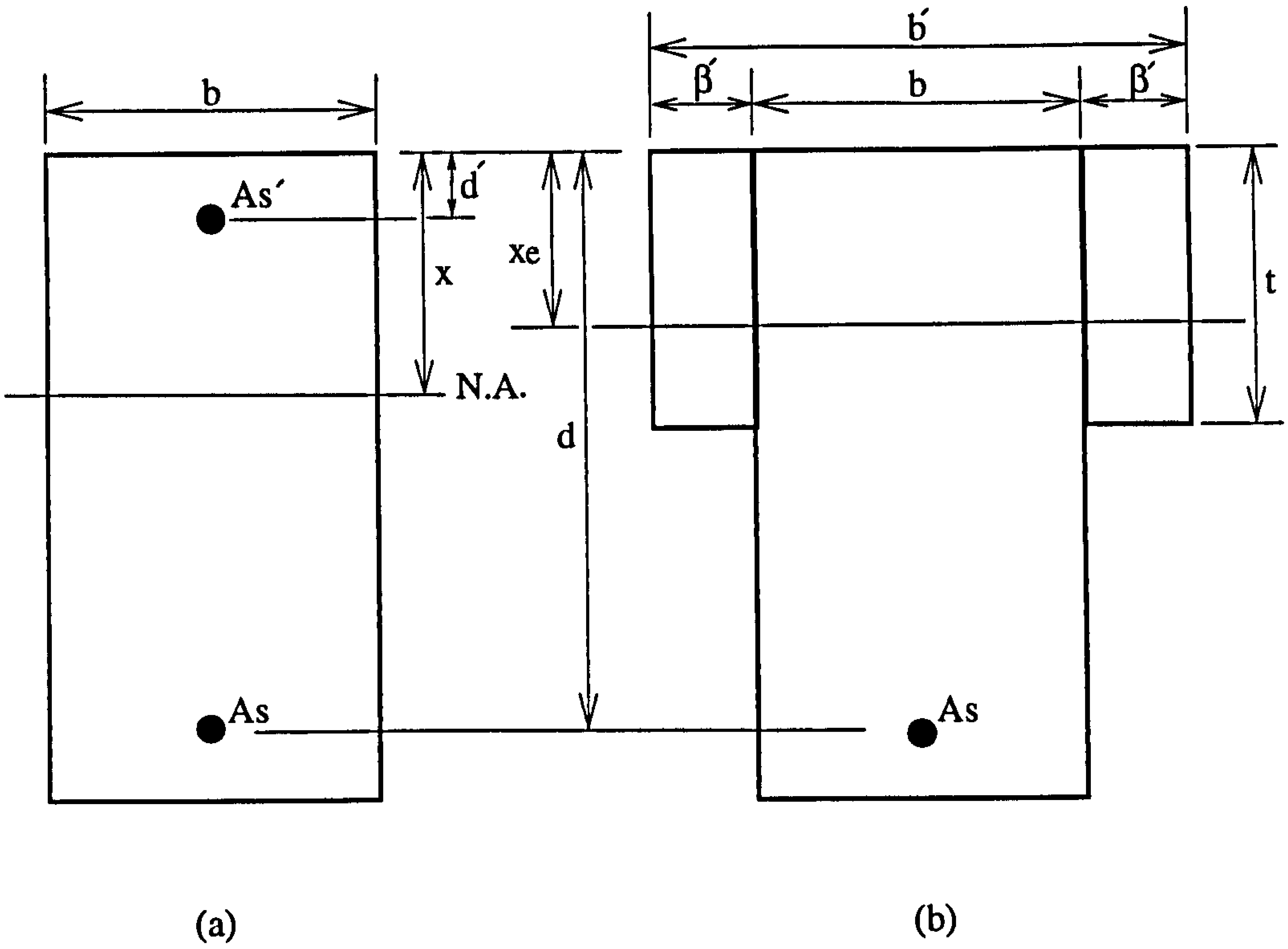


Figure 3.3.1
(a) a singly reinforced rectangular section
(b) an equivalent flanged section to that of (a)

two cracked sections equal requires that,

$$(n)(As'/2)(x - d')^2 = \beta' x^3/3$$

Hence,

$$\beta' = 3(n)(\rho' bd)(x - d')^2 / (2x^3)$$

But,

$$b' = b + 2\beta'$$

Thus,

$$b' = b + (3n\rho' bd)(x - d')^2 / x^3$$

or

$$b' = b + (3n\rho' b)(d/x)(1-d'/x)^2$$

and if $n\rho'$ is expressed in percentage,

$$b' = b + (0.03n\rho' b)(d/x)(1-d'/x)^2$$

It can be shown from statics that,

$$x = k_1 d$$

where,

$$k_1 = \{\sqrt{[(n\rho + n\rho')^2 + 200(n\rho + n\rho' d'/d)]} - (n\rho + n\rho')\} / 100$$

Substituting for x in the equation of b′ will therefore give,

$$b' = b + (0.03n\rho' b/k_1)(1 - d'/k_1 d)^2$$

which can also be written as,

$$b' = [\ 1 + \alpha' n\rho' d/d'\ ]\ (b) \qquad (3.3.1)$$

in which,

$$\alpha' = 0.03(d'/d)(1 - d'/k_1 d)^2 \ /\ k_1$$

The value of b′ obtained from Eq.3.3.1 along with t = x makes the moment of inertia of Fig.3.3.1(a) and (b) exactly equal. In addition and because the centroidal axis is exactly at the bottom of the flange of the equivalent section, a perfectly rectangular section of width b′ can be assumed to which Eq.3.2.2 can be applied. However, the expression of α' and k_1 involved in the evaluation of b′ was thought to be too complex for practical use. In addition, since Eq.3.2.2 will be eventually applied to the equivalent section any value of α' that is not derived on the basis of the equation itself or within its context may not actually produce exact results. In other words, because Eq.3.2.2 in itself may sometimes bear slight error the results may not be exact no matter how exact the value of b′ is unless it is derived on the basis of the equation or within its context. For that the complexity of Eq.3.3.1 is not really justified and a different method for obtaining α' should be adopted. This is done next where it will be shown that while retaining the format of Eq.3.3.1, the factor α', when derived within the context of Eq.3.2.2, can be greatly simplified.

If the centroidal axis of the equivalent section of Fig3.3.1(b) is assumed to fall within the flange, then the section will behave as a rectangle for which and in accordance with Eq.3.2.2 the equivalent cracked transformed moment of inertia, Icre, can be written as,

$$Icre = (\alpha + \beta n\rho e)(b'd^3/12)$$

in which,

$$n\rho e = (100nAs) / (b'd)$$

Using

$$n\rho = (100nAs) / (bd)$$

the above definition of $n\rho e$ can therefore be written as,

$$n\rho e = n\rho b/b'$$

Substituting this last expression of $n\rho e$ into the equation of Icre gives,

$$Icre = (\alpha + \beta n\rho b/b')(b'd^3/12) \quad (3.3.2)$$

or

$$12Icre/bd^3 = \alpha b'/b + \beta n\rho$$

Thus,

$$(b'/b) = [(12Icre/bd^3) - \beta n\rho] / \alpha$$

If Icre is set equal to Icr, as found on the basis of the general cracked transformed section of Fig.3.3.2 (on page 101) and the pertaining equations shown therein, the above equation will then give the exact (b′/b) value. Namely,

$$(b'/b)_{exact} = [(12Icr/bd^3) - \beta n\rho] / \alpha \quad (3.3.3)$$

Retaining the format of Eq.3.3.1, one can write

$$(b'/b) = 1 + \alpha' n\rho'(d/d') \quad (3.3.4)$$

or

$$\alpha' = [(b'/b) - 1] / (n\rho' d/d')$$

Thus,

$$\alpha'(\text{exact}) = [(b'/b)_{exact} - 1] / (n\rho' d/d') \tag{3.3.5}$$

Therefore, if the value of $(b'/b)_{exact}$ is known the exact value of α' can be determined. However, $(b'/b)_{exact}$ as given by Eq.3.3.3 is a function of the factors α and β which are themselves functions of $n\rho b/b'$. Due to this interrelation of the parameters involved the solution of Eq.3.3.3 for $(b'/b)_{exact}$ is best achieved iteratively. Since the value of $n\rho$ is known it will be easier to initially assume that $n\rho b/b'=n\rho$. From the value of $n\rho b/b'$ thus assumed the factors α and β will be obtained as given by the intervals specified in Eq.3.2.2 (since the original section has been transformed into an equivalent section of width b' the factors α and β will be determined on the basis of $n\rho b/b'$ rather than $n\rho$. Of course at this initial stage this makes no difference since $n\rho b'/b$ is assumed equal to $n\rho$). Substituting these factors into Eq.3.3.3 a value for (b'/b) will be found. Based on the value of b' thus found a new value of $n\rho b/b'$ will be calculated and based on which new values for α and β will be found. These factors will again be substituted into Eq.3.3.3 to obtain a new value for (b'/b). The process will be repeated until (b'/b) obtained from two successive iterations become exactly equal. The final (b'/b) will then be taken as the exact value of (b'/b) based on which $\alpha'(\text{exact})$ will be calculated using Eq.3.3.5.

Once the exact value of α' is determined at each combination of $n\rho$, $n\rho'$ and d'/d that is practically possible a preset percentage of error can be allowed for to obtain an upper and a lower bound of α' at each set of combination. The maximum of these lower bounds and the minimum of the upper bounds over the full range of $n\rho$, $n\rho'$ and

d'/d can then be found to construct an envelope of α'. From the envelope of α' thus constructed one can then approximate the value of α' such that the ratio of the approximate Icr to the exact Icr, namely Icre/Icr, at any combination of $n\rho$,$n\rho'$ and d'/d corresponds to an error which is less than or equal to the percent error allowed for. To determine the upper and lower bounds of α' at any combination of $n\rho$, $n\rho'$ and d'/d the exact value of α' has to be incremented either upwards or downwards. When the incremented α' (denoted hereafter as α_e') is larger than the exact α' the ratio of Icre/Icr will be greater than 1.0 which amounts to a "+" error or an over estimate of Icr. On the other hand, when α_e' is less than α'(exact) the ratio Icre/Icr will then be less than 1.0 which corresponds to a "-" error or an underestimate of Icr. To show why this is so let it be assumed, for the sake of simplicity, that α and β factors corresponding to $n\rho b/b'$ relative to the exact and the incremented α' are equal. Therefore, the difference between the exact and the approximate value of Icr, namely Icr - Icre, can be written as,

$$Icr - Icre = (\alpha + \beta n\rho b/b'_{exact})(b'_{exact} d^3/12) - (\alpha + \beta n\rho b/b_e')(b_e' d^3/12)$$

or

$$Icr - Icre = (\alpha b'_{exact} + \beta n\rho b - \alpha b_e' - \beta n\rho b)(d^3/12)$$

where b_e' is used to denote b' that corresponds to α_e'. Hence,

$$Icr - Icre = (\alpha d^3/12)(b'_{exact} - b_e')$$

or

$$Icr - Icre = (\alpha bd^3/12)[(b'/b)_{exact} - (b'/b)_e]$$

Substituting for b′/b from Eq.3.3.4,

$$Icr - Icre = (\alpha bd^3/12)[1 + \alpha'_{exact} n\rho'(d/d') - 1 - \alpha_e' n\rho'(d/d')]$$

or

$$Icr - Icre = [\alpha bd^3 n\rho'(d/d')/12]\ (\alpha'_{exact} - \alpha_e')$$

Dividing through by Icr,

$$1 - Icre/Icr = [\alpha n\rho'(d/d') \ / \ (12Icr/bd^3)]\ [\alpha'_{exact} - \alpha_e']$$

Hence,

$$\alpha'_{exact} - \alpha_e' = (1 - Icre/Icr)\ [(12Icr/bd^3)/(\alpha n\rho' d/d')]$$

or

$$\alpha_e' - \alpha'_{exact} = (Icre/Icr-1)\ [(12Icr/bd^3)/(\alpha n\rho'(d/d'))] \qquad (3.3.6)$$

If α' is incremented upwards then the difference α_e' - α'_{exact} will obviously be positive. This implies, according to Eq.3.3.6, that the term "Icre/Icr - 1" is also positive which means that Icre is greater than Icr. Likewise if α' is incremented downwards, the difference α_e' - α'_{exact} will be negative. Again from Eq.3.3.6 this will imply that the term "Icre/Icr -1" is also negative or Icre is less than Icr.

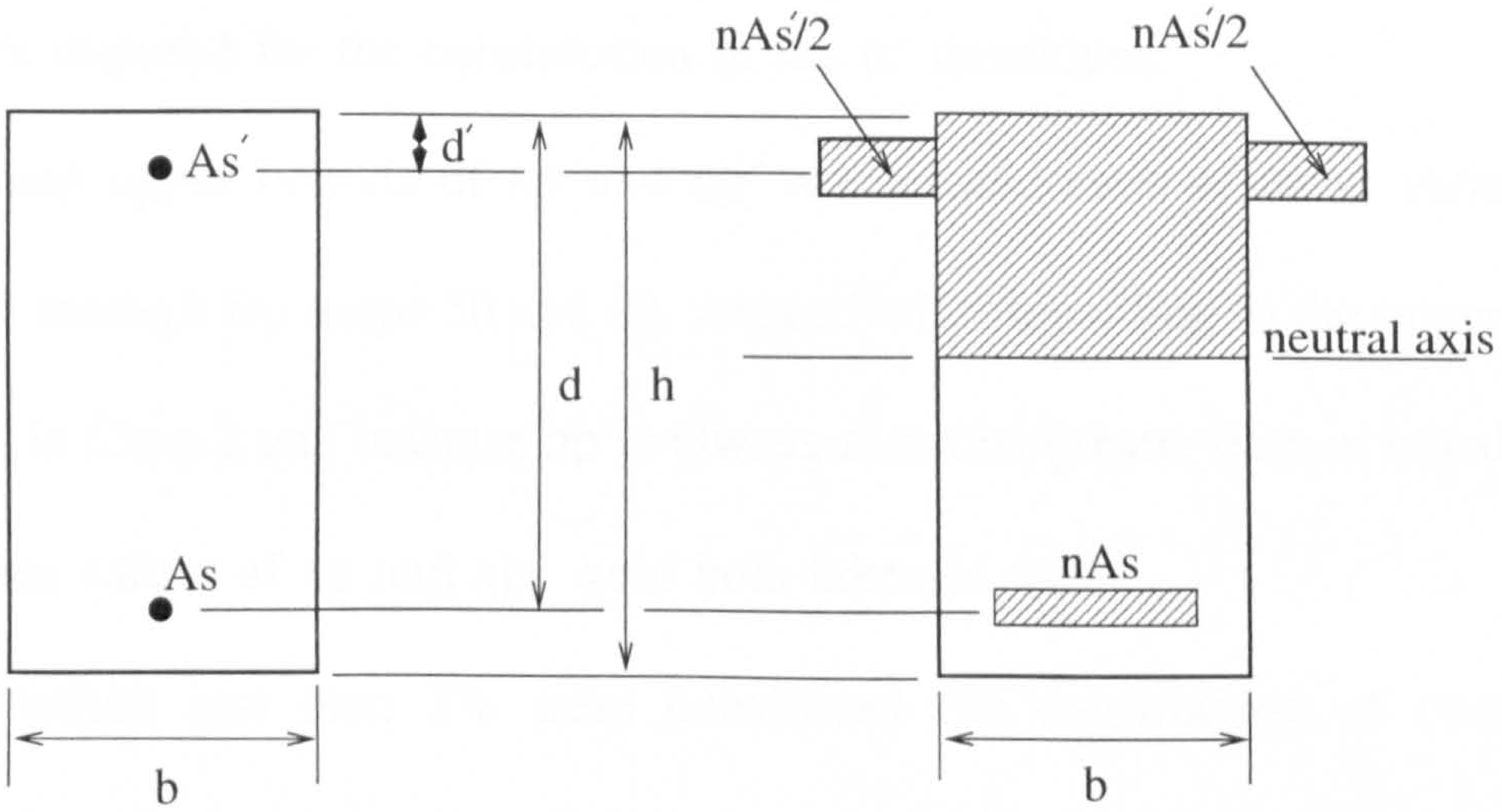
The above argument drawn from Eq.3.3.6 is actually equivalent to saying that any preset error value, "Icre/Icr - 1" will actually correspond to a value of α_e which is either greater or smaller than α'_{exact}. If the error was preset at a positive value the corresponding α_e' will then be greater than α'_{exact} and vice versa. The exact value of α_e' however will depend on the combination of $n\rho$, $n\rho'$ and d'/d. The set of α_e' greater than α'_{exact} for all $n\rho$, $n\rho'$ and d'/d values is the one referred to earlier as the upper bound of α' and the set of α_e' smaller than α'_{exact} is the one referred to as the lower bound of α'. The lowest of the upper bound of α' and the highest of the lower bound at each d'/d ratio will then represent the upper and the lower points of the α' envelope, respectively. By plotting these upper and lower points at each respective d'/d value upper and lower α' envelopes can therefore be constructed. If such envelopes are wide apart it will then be possible to approximate α' as simply a function of d'/d.

Although the above discussion was based on observing the format of Eq.3.3.6 the equation *per se* cannot be used to obtain α_e' directly. This is because it is based on the assumption that the factors α and β for $n\rho b/b'$ corresponding to α'_{exact} and α_e' are the same which is not necessarily true. Therefore, any attempt to obtain α_e' using

Eq.3.3.6 must be part of an iterative solution in which the factors α and β are evaluated at each iteration and compared with the previous values. For that it is thought to be simpler to follow and easier to automate if instead of using Eq.3.3.6 the α' bounds are found by directly incrementing α' from its exact value by adding positive increments if the upper bound is sought or negative increments when the lower bound is sought. Each incremented value of α' is then substituted into Eq.3.3.4 to obtain a value for b'. Using the value of b' thus found $n\rho b/b'$ will then be calculated and the factors α and β are found from Eq.3.2.2. These values are then substituted into Eq.3.3.2 to obtain a value for Icre. Using this Icre value the error "Icre/Icr - 1" is calculated. If the absolute value of the error thus obtained was found to be less than the absolute value of the error initially preset, the value of α' is then further incremented. The process is repeated until the calculated error eventually converges to the preset value and the resulting α' is then taken as either the upper or the lower bound of α' at the particular combination of $n\rho$, $n\rho'$ and d'/d considered.

Because of the many repetitive and iterative procedures involved it is obviously impractical to apply the above method using hand calculations and a computer program had to be developed exclusively for the purpose. In the program, Prog.3.3.1, all sections that can be encountered in practice were considered. In accordance with the limits discussed in Chap.2 the ratio of d'/d was allowed to vary from 0.03 to 0.37. In addition both tension and compression steel areas were allowed to vary within the limits of Sec.2.8 and that the latter is assumed to be always less than or equal to the former. This is in recognition of the fact that the compression steel area needed for flexure is always small and it is only when intended for deflection control (to reduce creep and shrinkage deflection) that it is made equal to the tension steel. Furthermore, because the effect of $n\rho'$ less than 1% on Icr was thought to be insignificant (as will be seen in case(a) of the example given at the end of the section), only $n\rho$ and $n\rho'$ greater than or equal to 1% were considered.

To generalize the analysis the study was conducted assuming a section of any width b and of any effective depth d. As indicated earlier the exact moment of inertia Icr required in the analysis was computed on the basis of the general cracked transformed section of Fig.3.3.2 and the pertaining equations shown therein.



$$\rho = 100As/bd \ , \qquad \rho' = 100As'/bd$$

$$x=[-(n\rho+n\rho') + \sqrt{(n\rho+n\rho')^2 + 200\,(n\rho+n\rho'\,d'/d\,)}\,]\,(d/100)$$

$$Icr = [100\,x^3/3 + n\rho'\,d\,(x-d')^2 + n\rho d(d-x)^2\,](b/100)$$

Figure 3.3.2 The cracked transformed section of a doubly reinforced rectangular section and the pertaining equations of x and Icr

The flow charts of Prog.3.3.1 used in the study are shown in Fig.3.3.3 while the program's listing is included in Appendix B2.

The flowcharts of Fig.3.3.3 give a clear and detailed schematic representation of the iteration process involved in the analysis and are thus an effective means of summarizing earlier discussions. In addition, they serve as an overall outline of the logic of Prog.3.3.1.

The moment the program is invoked it will prompt for the % error to be allowed where the preset error value desired has to be entered in percentage. This same percentage of error will then be assumed by the program in evaluating the upper and lower values required for the construction of the α' envelopes.

The lower and upper bounds of $n\rho$ and $n\rho'$ are set and each of these variables is incremented through Do loops 50 and 40, respectively. According to the upper bound of $n\rho$ found in Chap.2 and because $n\rho$ is always assumed greater than or equal to $n\rho'$ the maximum values of $n\rho$ and $n\rho'$ were both taken at 64%.

When $n\rho'$ values less than 2% were considered the construction of continuous envelopes of α' was not possible. This is because such low $n\rho'$ values (especially when d'/d is high) have negligible effect on Icre and thus correspond to α' values of different "order" than those for $n\rho' > 2\%$. Because of this and their negligible effect on Icre, $n\rho'$ (as will be seen in case (a) of the example given at the end of the section) values less than 2% were not considered in the study. In addition, for each combination of $n\rho$ and $n\rho'$ the ductility requirements of Chap.2 were also imposed to discard any section that is found nonductile. Because the ACI code limitations allow more sections to be considered such ductility requirements were applied using Eq.2.8.1.4 with f_{sb}' taken as f_y and the conditions of fc' and fy that correspond to

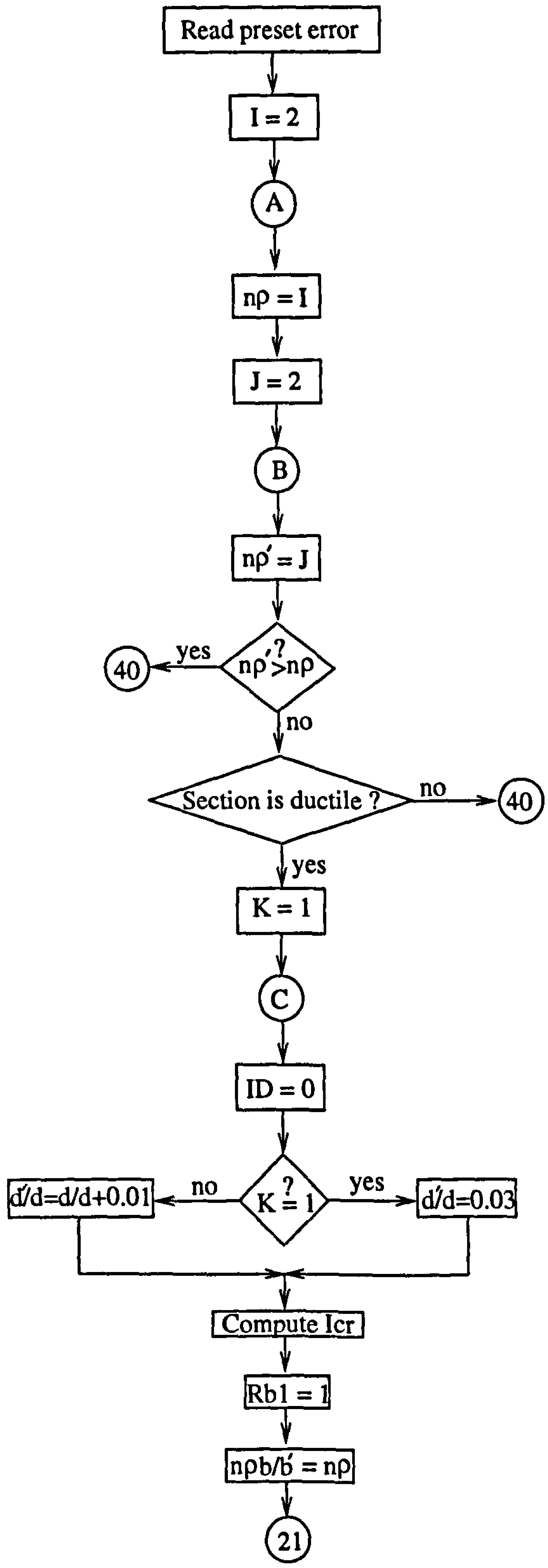


Figure 3.3.3 The flowcharts for Prog.3.3.1

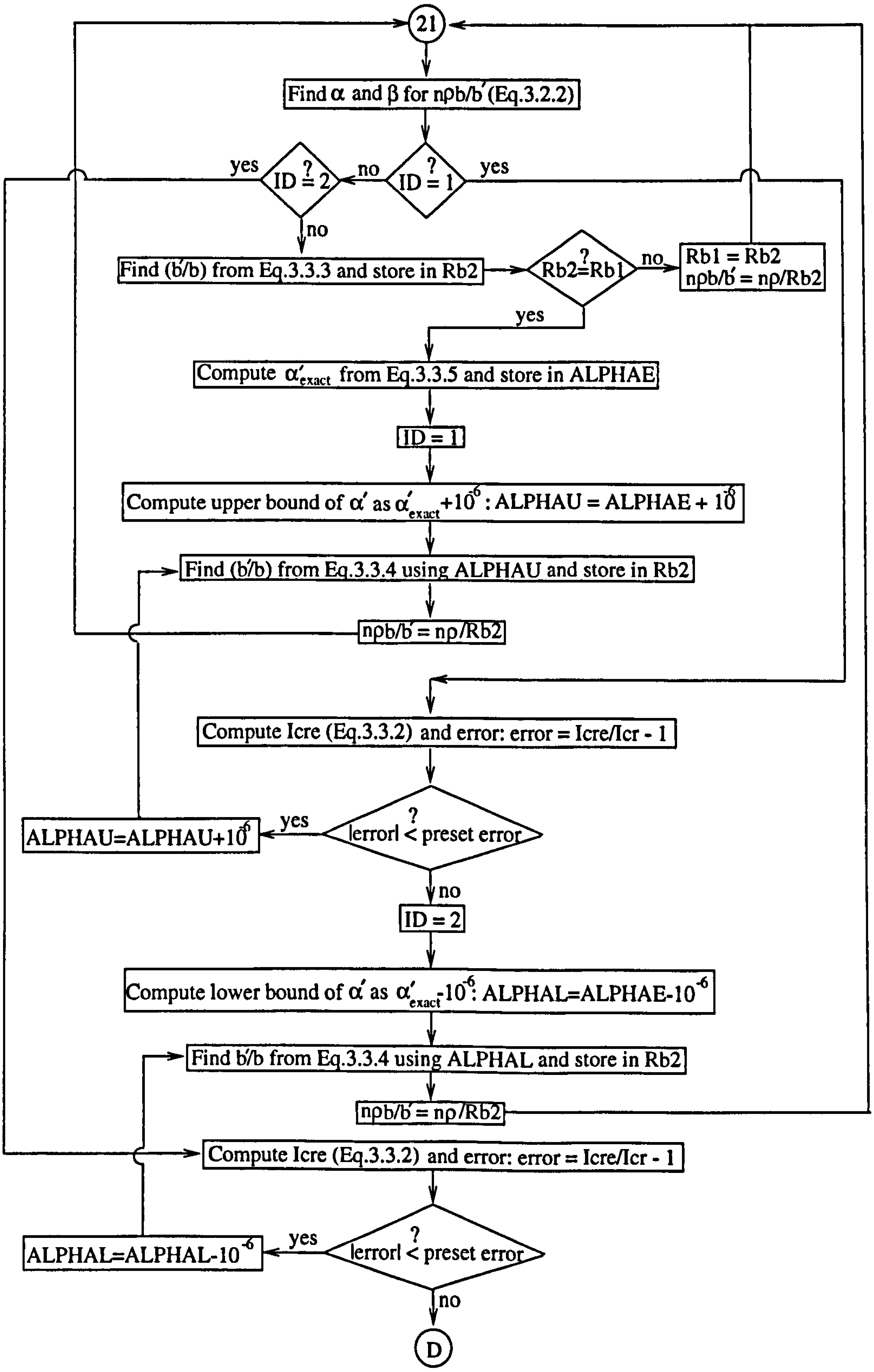


Figure 3.3.3 (cont'd)

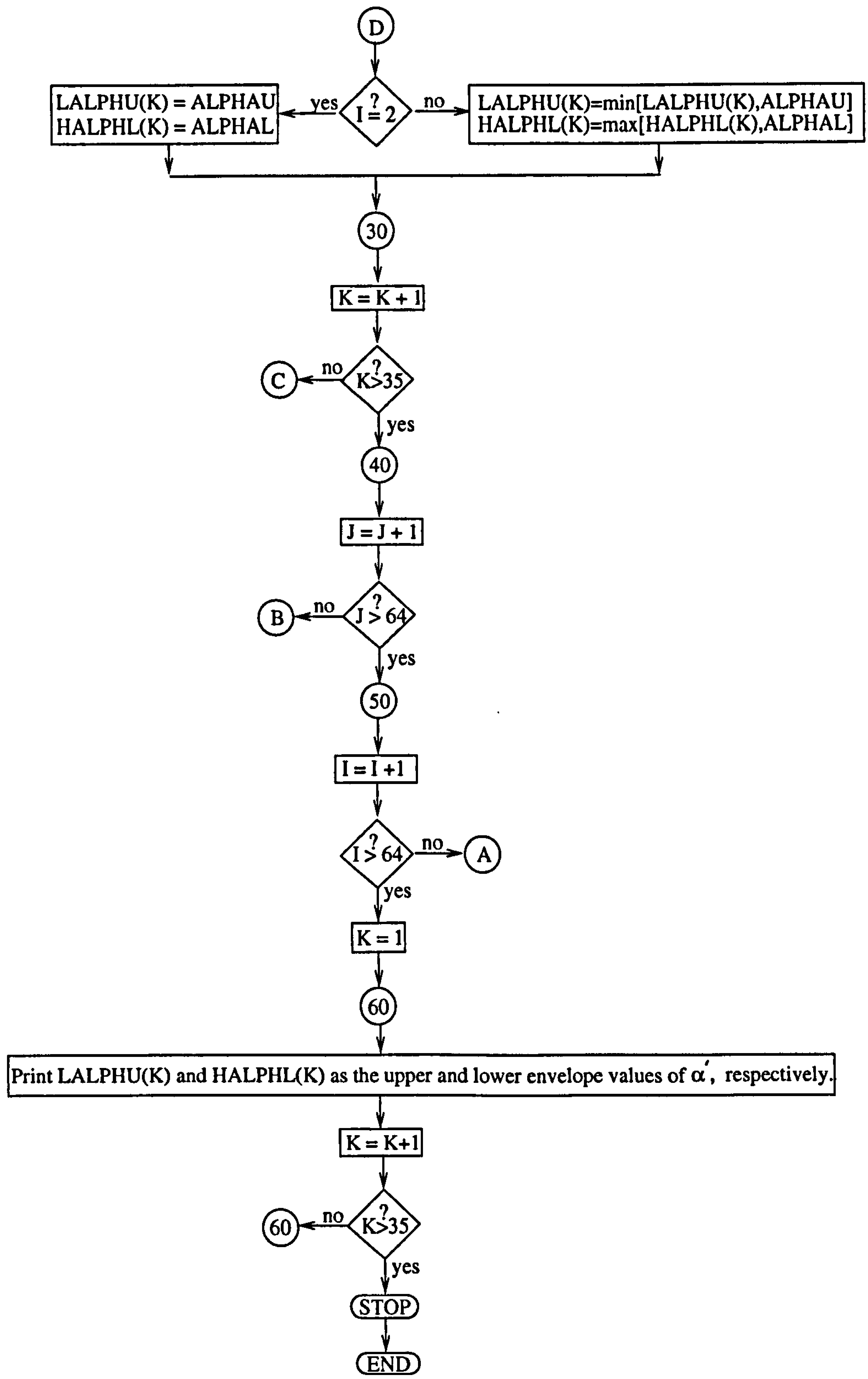


Figure 3.3.3 (cont'd)

$n\rho_{max}$ of Table 2.8.1.1. For each combination of $n\rho$, $n\rho'$ and d'/d the exact α' is calculated as the variable "ALPHAE". Successive increments of $\pm10^{-6}$ were then applied to the value of α'_{exact} to obtain the bounds of α'. The lowest of all the upper bounds and the highest of all the lower bounds relative to each particular d'/d and over the full range of $n\rho$ and $n\rho'$ considered were then printed as the upper and lower envelope values, respectively.

Different trials with errors less than 5% (starting with errors of ±1% upto ±4.5%) have shown that for such error values the construction of practical envelopes was impossible. This is because in such cases the upper and lower envelopes were close together leaving no range for any practical approximation of α'. For this greater error percentages had to be considered. Figure 3.3.4 shows the envelopes of α' for errors of ±5% and ±6.5% which are plots of the upper and lower envelope values printed by the program and shown in Table 3.3.1.

Within the range confined by the envelopes of Fig.3.3.4 two straight line approximations of α' were made as shown in the figure. Namely,

$$\alpha' = 0.0037 \quad \text{for } 0.065 \le d'/d \le 0.305 \qquad (3.3.7)$$

$$= 0.0025 \quad \text{otherwise}$$

To confirm the above approximation of α' and thus the envelopes of Fig.3.3.4 and to indicate numerically the magnitude of the errors involved a separate program had to be developed. The program is designated as Prog.3.3.2 and its flow charts are shown in Fig.3.3.5. The listing of the program appears in Appendix B3.

In the program the limits of $n\rho$, $n\rho'$ and d'/d remained same as in Prog.3.3.1 and the

Table 3.3.1. Values of the upper and lower envelopes of α'

d′/d	% error allowed in Icre			
	5%		6.5%	
	upper envelope values	lower envelope values	upper envelope values	lower envelope values
0.03	0.001829	0.001874	0.001913	0.001670
0.04	0.002334	0.002321	0.002441	0.002051
0.05	0.002788	0.002685	0.002919	0.002350
0.06	0.003187	0.002971	0.003346	0.002573
0.07	0.003549	0.003185	0.003715	0.002720
0.08	0.003854	0.003328	0.004050	0.002802
0.09	0.004117	0.003407	0.004328	0.002872
0.10	0.004335	0.003427	0.004562	0.003025
0.11	0.004527	0.003386	0.004744	0.003148
0.12	0.004684	0.003441	0.004928	0.003243
0.13	0.004801	0.003520	0.005073	0.003309
0.14	0.004882	0.003571	0.005179	0.003348
0.15	0.004934	0.003596	0.005249	0.003360
0.16	0.004941	0.003646	0.005294	0.003391
0.17	0.004935	0.003683	0.005323	0.003421
0.18	0.004892	0.003702	0.005283	0.003434
0.19	0.004805	0.003705	0.005236	0.003430
0.20	0.004738	0.003692	0.005206	0.003412
0.21	0.004572	0.003664	0.005087	0.003381
0.22	0.004447	0.003622	0.005001	0.003336
0.23	0.004294	0.003567	0.004892	0.003278
0.24	0.004112	0.003500	0.004758	0.003209
0.25	0.003984	0.003422	0.004599	0.003131
0.26	0.003827	0.003333	0.004414	0.003041
0.27	0.003570	0.003234	0.004308	0.002943
0.28	0.003462	0.003126	0.004130	0.002837
0.20	0.003188	0.003011	0.003891	0.002724
0.30	0.003095	0.002888	0.003803	0.002605
0.31	0.002798	0.002759	0.003622	0.002479
0.32	0.002661	0.002625	0.003440	0.002349
0.33	0.002569	0.002486	0.003361	0.002216
0.34	0.002361	0.002343	0.003152	0.002078
0.35	0.002103	0.002198	0.002977	0.001934
0.36	0.002023	0.002051	0.002909	0.001787
0.37	0.001950	0.001901	0.002847	0.001634

notes:

1. *Upper envelope values correspond to a maximum (+) error in Icre of 5 and 6.5%.*
2. *Lower envelope values correspond to a maximum (-) error in Icre of 5 and 6.5%*
3. *Error in Icre is defined as:*

(Icre/Icr -1)100

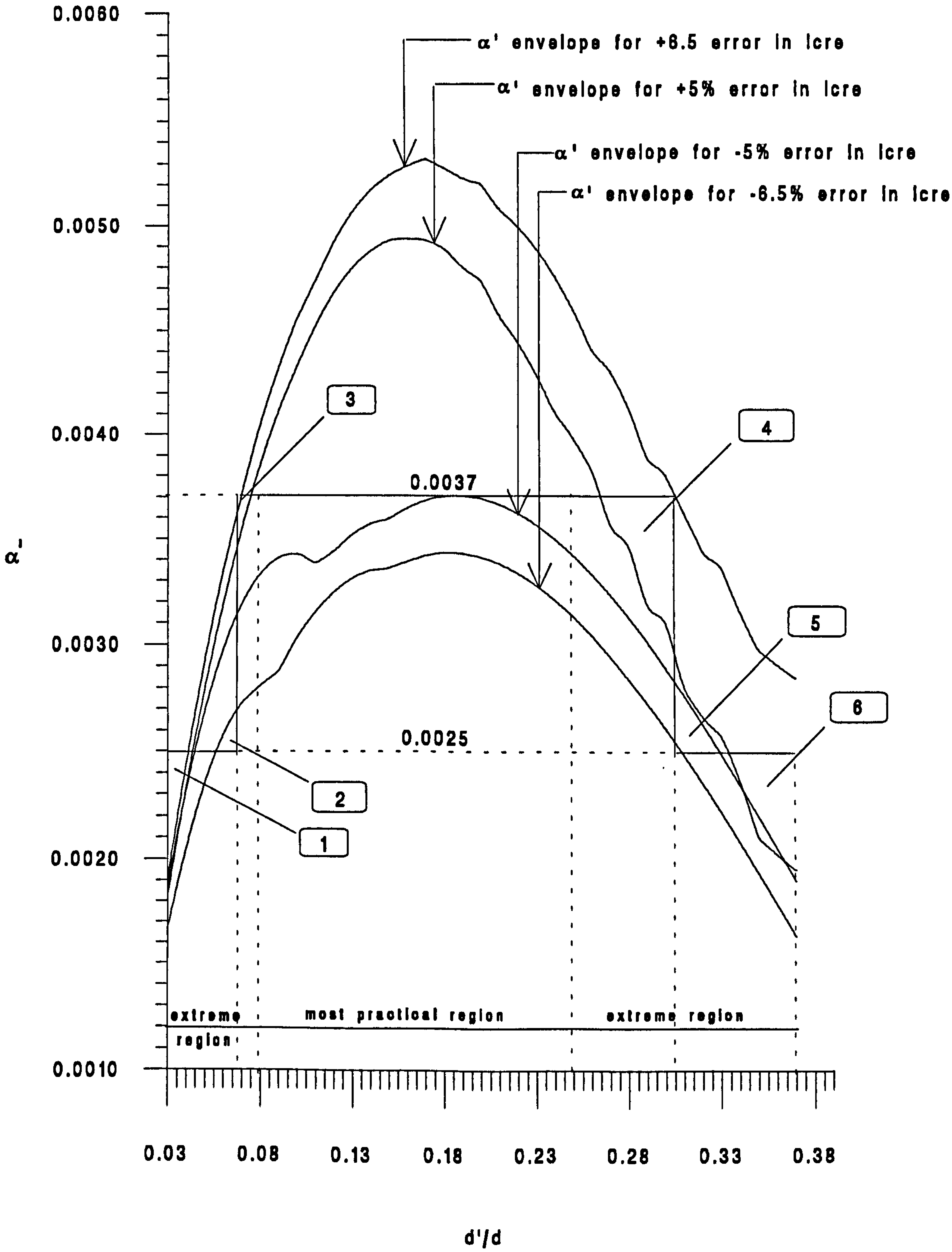


Figure 3.3.4 Envelopes of α' and the approximation of Eq.3.3.7

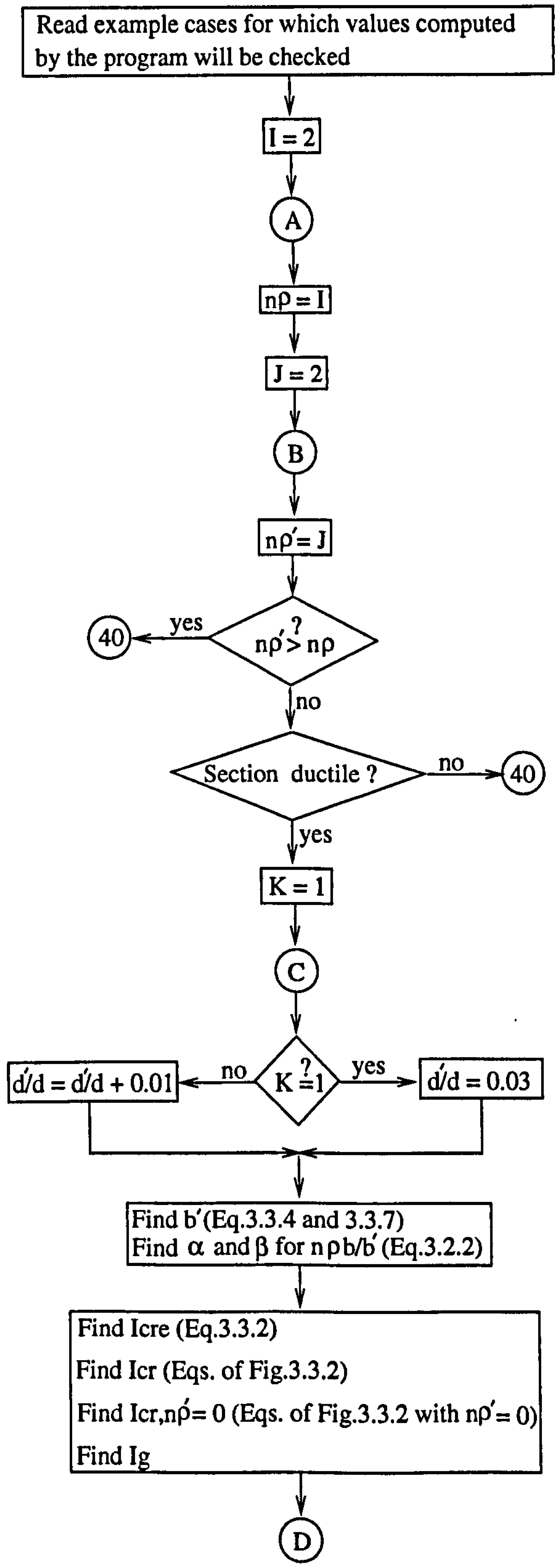


Figure 3.3.5 The flow charts of Prog.3.3.2

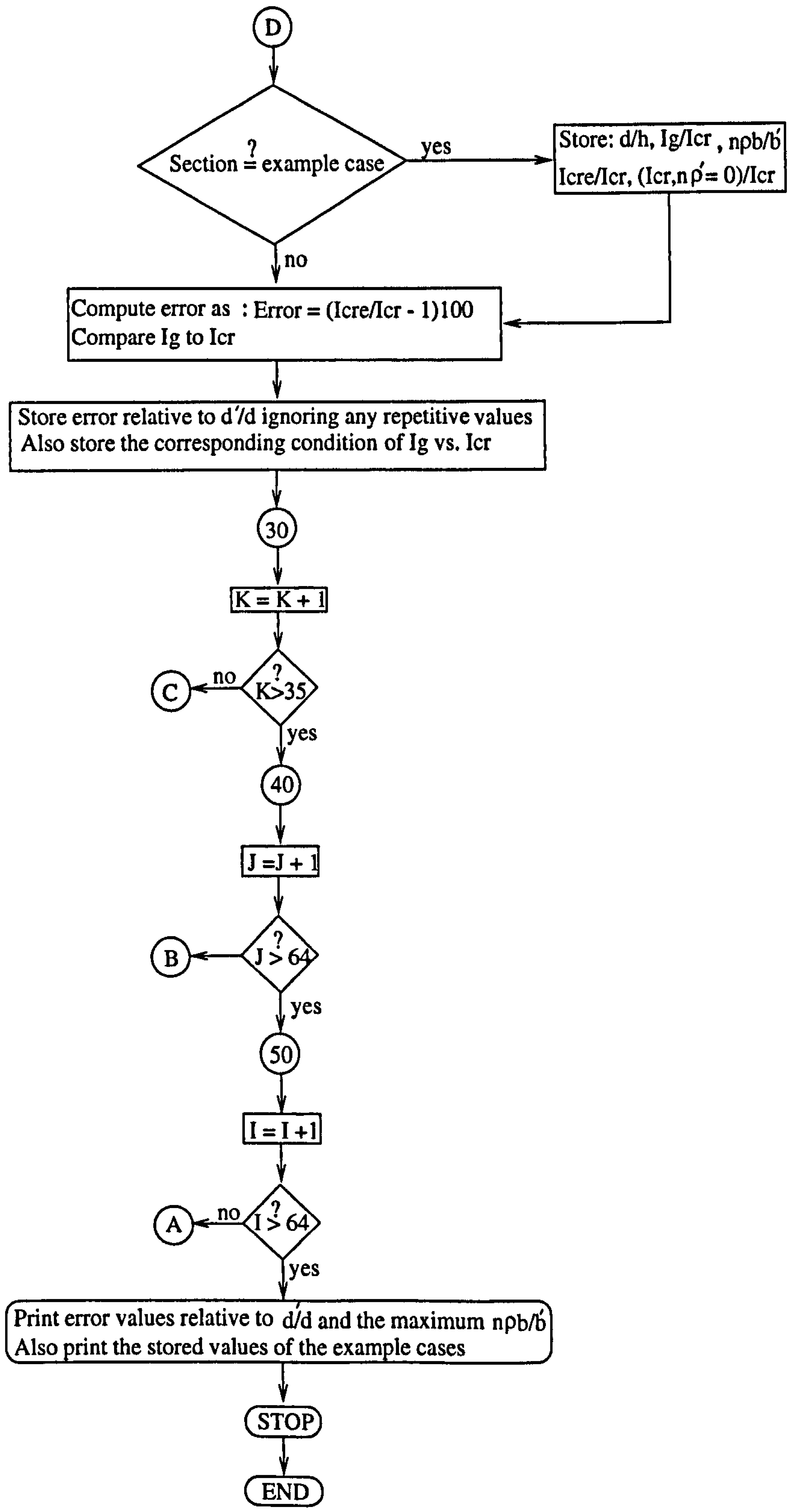


Figure 3.3.5 (cont'd)

computations were made for the general section of width b, effective depth d and overall depth h. As in the previous program this made the results applicable to any rectangular section as long as d'/d and the steel ratios are within the limits considered. The only exception to the generality of the results is the ratio Ig/Icr. This is because in order to compute Ig for comparison with Icr it was necessary to assume that the concrete cover on both the tension and compression steel were equal which made the computation possible by considering h=d+d'.

Using the above limits and the approximation of α' the program computes Icre from Eqs.3.3.2 and 3.3.4 and Icr from the equations of Fig.3.3.2 for every combination of $n\rho$, $n\rho'$ and d'/d. For each case the error is computed as Icre/Icr -1 and stored along with the condition of Ig vs. Icr. Because the approximation of α' and the envelopes from which it is drawn are independent of $n\rho$ and $n\rho'$ the errors were stored with reference to the different d'/d ratios and regardless of the values of $n\rho$ and $n\rho'$. In addition, at any particular d'/d ratio any error value that may happen to be encountered repeatedly (this is very much likely because of the large possible combination of $n\rho$, $n\rho'$ and d'/d) is only stored once. If such a repetitive error value was found to correspond to two different conditions of Ig vs. Icr it is then the condition of Ig > Icr that is stored. This is because Ig < Icr is thought to be extreme and an error is more emphasized if it corresponds to a non-extreme condition.

As an example two sample pages of the computer printouts are shown in the next two pages. The first page shows how all the different errors at every particular d'/d ratio are printed. Corresponding to each error the condition of Ig vs. Icr is shown by printing an answer "yes" or "no" to the question Ig > Icr?. Obviously "yes" implies that the particular error printed corresponds to a condition of Ig > Icr while "no"

When d'/d = 0.03

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.0	Yes	-0.3	Yes	0.0	Yes	-1.3	Yes	-1.4	Yes	-0.7	Yes	-1.1	Yes
-1.7	Yes	2.2	Yes	-0.1	Yes	-0.6	Yes	-1.0	Yes	0.8	Yes	1.8	Yes
-0.4	Yes	-0.9	Yes	0.7	Yes	1.5	Yes	2.4	Yes	-2.0	Yes	-1.6	Yes
-0.5	Yes	1.3	Yes	2.1	Yes	-2.3	Yes	-0.8	Yes	0.1	Yes	0.6	Yes
1.2	Yes	-2.2	Yes	-1.9	Yes	-0.2	Yes	-1.8	Yes	-1.5	Yes	0.2	Yes
-1.2	Yes	0.4	Yes	1.7	Yes	2.6	Yes	1.1	No	3.7	No	3.0	No
1.9	No	1.4	No	1.0	No	4.4	No	4.0	No	3.3	No	2.7	No
3.8	No	5.0	No	4.2	No	3.6	No	2.5	No	1.6	No	0.9	No
4.8	No	5.2	No	4.5	No	3.9	No	2.8	No	2.3	No	0.5	No
0.3	No	2.9	No	4.3	No	5.7	No	5.4	No	4.7	No	4.1	No
3.5	No	6.4	No	6.1	No	6.6	No	5.9	No	4.6	No	3.4	No
6.9	No	6.8	No	6.0	No	3.2	No	6.5	No	7.6	No	6.2	No
5.6	No	7.2	No	7.8	No	7.1	No	3.1	No	7.9	No	8.0	No
5.3	No	4.9	No	5.8	No	6.7	No	8.5	No	8.1	No	7.4	No
5.5	No	8.3	No	8.9	No	8.2	No	7.5	No	5.1	No	9.0	No
7.0	No	6.3	No	8.6	No	9.4	No	9.1	No	8.4	No	7.7	No
9.2	No	9.9	No	9.7	No	10.0	No	9.3	No	7.3	No	9.5	No
10.2	No	10.1	No	8.7	No	8.8	No	10.7	No	10.5	No	10.9	No
9.8	No	10.4	No	10.8	No	11.4	No	9.6	No	11.2	No	11.7	No
11.0	No	10.3	No	10.6	No	11.1	No	11.6	No	11.5	No	12.0	No
11.9	No	12.4	No	11.8	No	12.2	No	11.3	No	12.6	No	12.1	No
12.5	No	12.9	No	12.8	No	13.1	No	12.7	No	13.4	No	13.0	No
13.7	No	13.3	No	13.6	No	13.9	No	13.5	No	14.2	No	13.8	No
14.1	No	14.3	No	14.4	No	14.0	No	14.6	No	14.8	No	12.3	No
14.5	No	14.7	No	15.0	No	15.2	No	15.4	No	15.6	No	15.1	No

When d'/d = 0.04

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.2	Yes	-0.8	Yes	-1.6	Yes	-1.9	Yes	-2.1	Yes	-1.0	Yes	-1.5	Yes
-2.0	Yes	-2.3	Yes	1.0	Yes	1.7	Yes	-0.6	Yes	-0.1	Yes	0.3	Yes
0.9	Yes	-2.2	Yes	-1.7	Yes	-1.4	Yes	-0.5	Yes	-2.8	Yes	-2.7	Yes
-2.5	Yes	-1.3	Yes	-3.0	Yes	-3.1	Yes	-2.6	Yes	-2.4	Yes	-2.9	Yes
-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-1.8	Yes	-3.7	Yes
-1.1	Yes	-3.8	Yes	-3.9	Yes	-0.3	Yes	-0.9	Yes	1.2	Yes	0.5	Yes
-0.7	Yes	-1.2	Yes	-4.0	No	1.4	Yes	0.7	Yes	0.0	Yes	3.2	Yes
2.4	Yes	1.6	No	0.8	No	0.2	No	2.7	No	3.4	No	2.5	No
0.4	No	3.0	No	3.5	No	2.6	No	1.9	No	3.3	No	3.6	No
2.8	No	2.0	No	1.3	No	0.1	No	2.1	No	3.7	No	2.9	No
1.5	No	1.1	No	3.1	No	3.8	No	2.3	No	3.9	No	1.8	No
0.6	No	4.1	No	4.0	No	4.3	No	-0.4	No	4.5	No	4.2	No
4.7	No	4.8	No	4.4	No	5.0	No	5.1	No	5.2	No	4.6	No
5.4	No	-0.2	No	4.9	No	5.5	No	5.6	No	5.3	No	5.7	No
5.8	No	5.9	No	6.0	No	6.1	No	6.2	No	6.3	No	6.4	No
6.5	No	6.6	No	6.7	No	6.8	No	6.9	No	7.0	No	7.1	No
7.2	No												

The followings are printed 'only as typical cases ' that are used in the examples to illustrate the computaions involved in the program :

When NP = 2.00% , NP' = 2.00% and d'/d = 0.030 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.97	5.76	1.71	1.020	0.97

When NP = 15.00% , NP' = 10.00% and d'/d = 0.240 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.81	2.04	13.00	1.001	0.97

When NP = 25.00% , NP' = 25.00% and d'/d = 0.160 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.86	1.04	15.84	0.997	0.83

When NP = 40.00% , NP' = 40.00% and d'/d = 0.030 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.97	0.41	9.23	1.020	0.62

When NP = 45.00% , NP' = 35.00% and d'/d = 0.370 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.73	1.36	36.39	0.998	0.91

implies that Ig < Icr. The second page shows values for different sections that the program can be asked to print. These can be used as examples to illustrate the computations involved in the program as will be shown latter in this section.

A complete set of the program's output is included in Appendix B3 which will be referred to next in trying to correlate the envelopes of Fig.3.3.4 and the approximation of α' as shown therein to the actual error values obtained from the program.

It is shown in Fig.3.3.4 that within the most practical range of d'/d, α' of 0.0037 will give a maximum negative error of -5% and a maximum positive error of less than +5%. This is easily confirmed by scanning the printouts shown in Appendix B3 where it can be noticed that the maximum negative and positive errors occurring in the range of d'/d from 0.08 to 0.25 are -5% and +4.3%, respectively.

It may be noticed that the maximum negative error of -5% referred to above is shown by the envelopes to occur exactly at d'/d of 0.18 and 0.19. This observation is perfectly consistent with the computer output where it can be very easily noticed that the maximum negative errors of -5% within the range of d'/d from 0.08 to 0.25 does actually occur at d'/d of 0.18 and 0.19. This shows the accuracy and reliability of the envelopes as a reference for approximating α' and that they actually present on a single page what may otherwise require pages of computer printouts.

It may also be noticed that although for the d'/d ratios in the range from 0.13 to 0.23 α' of 0.0037 is closer to the envelopes of negative errors than it is to the envelopes of the positive errors, the computer printouts still show some, if not many, positive errors. The reason for this is that the envelopes represent the minimum of all the upper bounds of α' (upper bounds of α' correspond to positive error in Icre as was explained earlier) and the maximum of all the lower bounds of α' (lower bounds of

α' correspond to negative errors in Icre) for d'/d over the full range of $n\rho$ and $n\rho'$ and thus are actually the boundaries beyond which all the upper and lower bounds of α' for all $n\rho$, $n\rho'$ and d'/d combinations may occur. Therefore, there is always a possibility of a condition of $n\rho$, $n\rho'$ and d'/d for which the upper and the lower bounds of α' is such that α' of 0.0037 actually falls closer to the upper bound and thus will yield positive error values.

From the above argument it can be said that although the envelopes are able to predict the precise condition (in terms of d'/d) and the value of the maximum positive and negative errors they fail to show the actual fluctuation of the errors that are less than the maximum. Fortunately, however, for a proper approximation of α' all one needs is a prediction of the maximum positive and negative errors that a chosen approximation of α' may involve regardless of how the errors may fluctuate in between and thus the inability of the envelopes to predict such a fluctuation remains of no importance.

The above discussion has not only shown the errors obtained from the computer analysis and those predicted by the envelopes to be identical and provided a better understanding of the nature of these envelopes but also that such errors were low enough that the approximation of α' at 0.0037 in the range of d'/d from 0.08 to 0.25 is acceptable. It remains now to examine the extreme regions where higher errors are shown on the envelopes.

In the extreme regions, α' is approximated as either 0.0025 or 0.0037 depending on the ratio d'/d. As shown in Fig.3.3.4 the envelopes predict that α' of these values will give errors that are larger than ±5% in the regions of d'/d that are designated on the figure as areas 1-6.

In particular, the envelopes show that for area 1 the maximum error will greatly exceed +6.5% while for areas 3 and 4 it will very likely be at +6.5%. For areas 2 and 5 the expected maximum error is slightly larger than -6.5% and that of area 6 should be between +5% and +6.5%. Confirming these expectations drawn from the envelopes the printouts have shown the followings :

in area 1 : maximum error is +15.6% occurring at d′/d = 0.03

in area 2 : maximum error is -6.7% occurring at d′/d = 0.06

in area 3 : maximum error is +6.3% occurring at d′/d = 0.07

in area 4 : maximum error is +6.1% occurring at d′/d = 0.30

in area 5 : maximum error is -6.3% occurring at d′/d = 0.31

in area 6 : maximum error is +5.9% occurring at d′/d = 0.37

The slight difference that may be noticed between the errors predicted by the envelopes and those actually calculated is because the program prints errors for d′/d ratios that are incremented by 0.01 starting from 0.03. For example in area 4 the envelopes show the maximum error to be +6.5% at d′/d of 0.305. The program, however, was not instructed to print any error values for this d′/d ratio and thus the maximum error that was found from the printouts was +6.1% at d′/d of 0.30 and for which the envelopes show a maximum error of less than +6.5%.

In the above argument it was said that at d′/d of 0.30 the envelopes show a maximum error in Icre of less than +6.5%. This is obviously because the coordinates of d′/d of 0.30 and α' of 0.0037 correspond to a point which falls below the envelope of +6.5% error. This raises the immediate question, can one then scale the vertical distances of the point relative to the +5% and +6.5% envelopes to conclude the exact value of the error?. The answer is that direct scaling can not be used to obtain the magnitudes of errors that correspond to points which do not exactly fall on the envelope curves. The reason for this is that points on the envelopes which are on the same vertical line although are at the same d'/d ratio they may not correspond to the same combination of $n\rho$, $n\rho'$ and d'/d. Again this stems from the fact that the envelopes are actually the minimum of all the upper bounds and the maximum of all the lower bounds of α' obtained by scanning the full range of $n\rho$, $n\rho'$ and d'/d combinations.

The maximum positive error of +6.3% and negative error of -6.7% are the worst values occurring in areas 2 to 6 but are very much within tolerable limits. However, the maximum error of +15.6% which was found at d'/d of 0.03 in area 1 is thought to be relatively high. This is because Icre is an approximation of Icr that will be part of an overall model for the evaluation of the effective moment of inertia.. Therefore, to avoid cumulative loss of accuracy the error involved in Icre as an approximation of Icr should not be allowed to exceed ±6 to 7%. Nevertheless, one can argue that by scanning the errors printed for area 1 (that is for d'/d of 0.03 to 0.06) it can be noticed that all the errors which are higher than ±7% correspond to conditions of Ig < Icr and d'/d of 0.03 and 0.04 and can therefore be considered too extreme to be likely encountered in practice.

The above argument can be used as a legitimate reason for accepting the approximation of α' as given by Eq.3.3.7 and discussed above in order to find b' from Eq.3.3.4 and then evaluate Icre using Eq.3.3.2.

For the present study, however, and to avoid any uncertainties a different approach for the approximation of α' will be sought that gives errors of acceptable value regardless of the condition of Ig vs. Icr and for all regions of d'/d. With slight sacrifice of simplicity it will be shown next that such an approximation of α' is possible as a better and more general alternative to that of Eq.3.3.7.

It has been explained earlier that points on the envelopes at the same d′/d ratio (on the same vertical line) do not necessarily correspond to the same combination of $n\rho$, $n\rho'$ and d′/d. With different combinations having different exact α' values it is therefore impossible to plot a curve for α'_{exact} as a function of d′/d alone.
However, one can always average two envelopes that correspond to the positive and negative errors of the same magnitude to obtain a curve of α' that will give an error in Icre that is always less than ± the error considered.
Following the procedure described above, the upper envelope values corresponding to +5% error in Icre and the lower envelope values corresponding to -5% error in Icre are averaged as shown in Table 3.3.2. These values are then plotted relative to d′/d as shown in Fig.3.3.6 and the best curve is fitted through the plotted points using the "cricket graph" package program. The program has given the following equation for the curve,

$$\alpha'=0.00062+0.05180(d'/d)-0.21563(d'/d)^2+0.23005(d'/d)^3$$

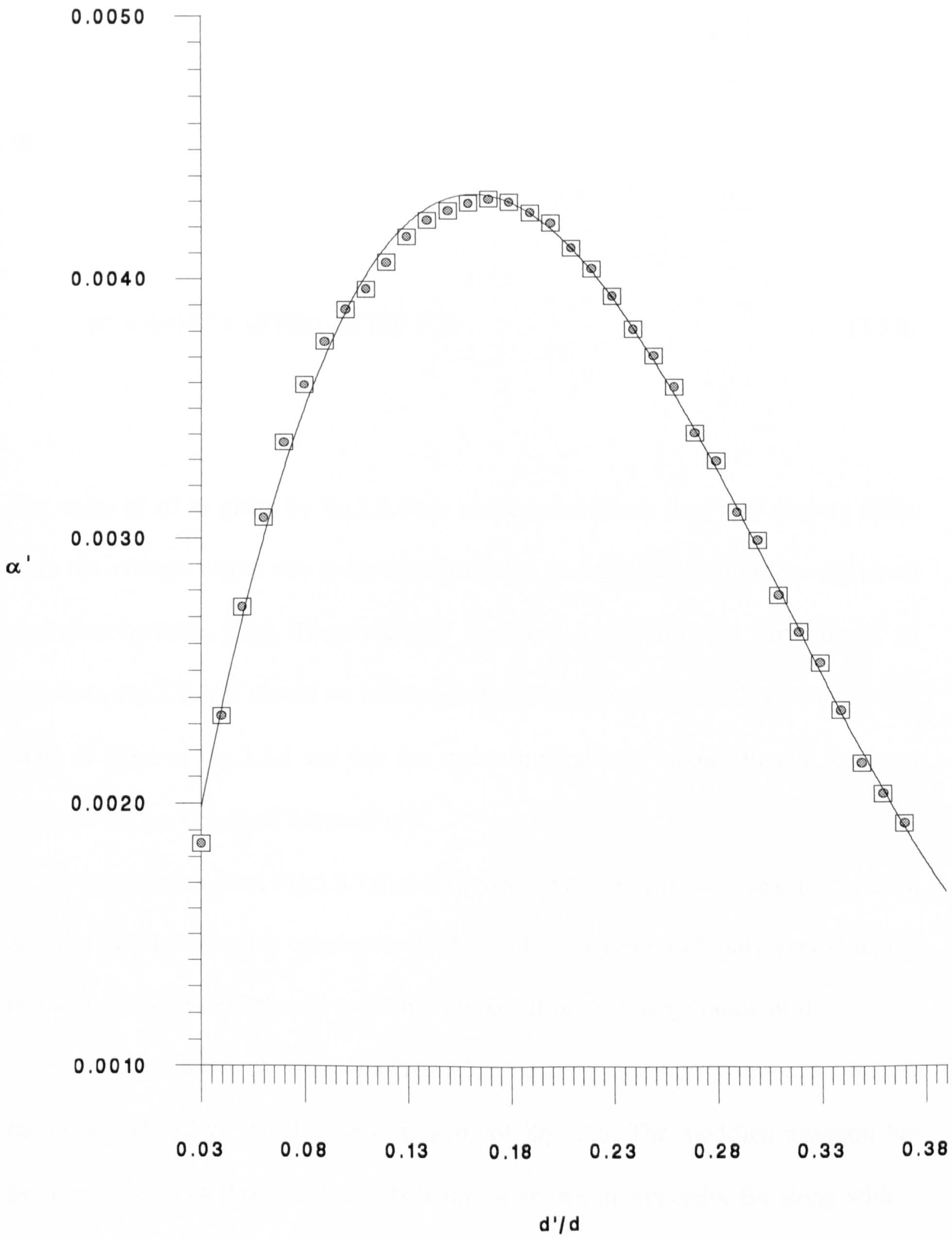
which can be approximated as,

$$\alpha'=0.0006 + 0.05(d'/d) - 0.2(d'/d)^2 + 0.2(d'/d)^3$$

writing in a more compact form,

Table 3.3.2 Values of α' used to plot the α' curves of Figs.3.3.6 and 3.3.7

d'/d	average envelope values of α' for 5% error	α'values from Eq.3.3.8
0.03	0.001850	0.0019254
0.04	0.002328	0.0022928
0.05	0.002737	0.0026250
0.06	0.003079	0.0029232
0.07	0.003367	0.0031886
0.08	0.003591	0.0034224
0.09	0.003762	0.0036258
0.10	0.003881	0.0038000
0.11	0.003957	0.0039462
0.12	0.004063	0.0040656
0.13	0.004161	0.0041594
0.14	0.004227	0.0042288
0.15	0.004265	0.0042750
0.16	0.004294	0.0042992
0.17	0.004309	0.0043026
0.18	0.004297	0.0042864
0.19	0.004255	0.0042518
0.20	0.004215	0.0042000
0.21	0.004118	0.0041322
0.22	0.004035	0.0040496
0.23	0.003931	0.0039534
0.24	0.003806	0.0038448
0.25	0.003703	0.0037250
0.26	0.003580	0.0035952
0.27	0.003402	0.0034566
0.28	0.003294	0.0033104
0.29	0.003099	0.0031578
0.30	0.002992	0.0030000
0.31	0.002779	0.0028382
0.32	0.002643	0.0026736
0.33	0.002528	0.0025074
0.34	0.002352	0.0023408
0.35	0.0021505	0.0021750
0.36	0.002037	0.0020112
0.37	0.001926	0.0018506



Figue 3.3.6 A curve fit through the average values of α' for 5% error

$$\alpha' = 0.0006 + 0.05(d'/d)(1-2d'/d)^2$$

or

$$\alpha' = 6x10^{-4} + (d'/d)(1-2d'/d)^2 / 20 \qquad (3.3.8)$$

The value of α' as given by Eq.3.3.8 are recalculated (since they may slightly differ from the average values due to the approximation assumed in obtaining the equation) as shown in Table 3.3.2. These are then used to plot the proposed curve for α' as shown in Fig.3.3.7. It should be noted that the envelopes of Fig.3.3.7 are exactly the same as those of Fig.3.3.4 and that the approximation of α' shown therein has been replaced by the proposed curve of α'.

It can be seen from Fig.3.3.7 that the proposed curve of α' as given by Eq.3.3.8 actually falls between the envelopes of - 5% and + 5% error with only very slim and in fact negligible deviation in cases of very small or very large ratios of d'/d .

To confirm the above observations Prog.3.3.2 has been modified in order to compute the errors when Icre is calculated using α' of Eq.3.3.8. The modified program has been referred to as Prog.3.3.3 and its listing is shown in Appendix B4 along with a complete set of printouts as obtained from the program.

The flow charts of the program, however, are omitted since they will be identical to

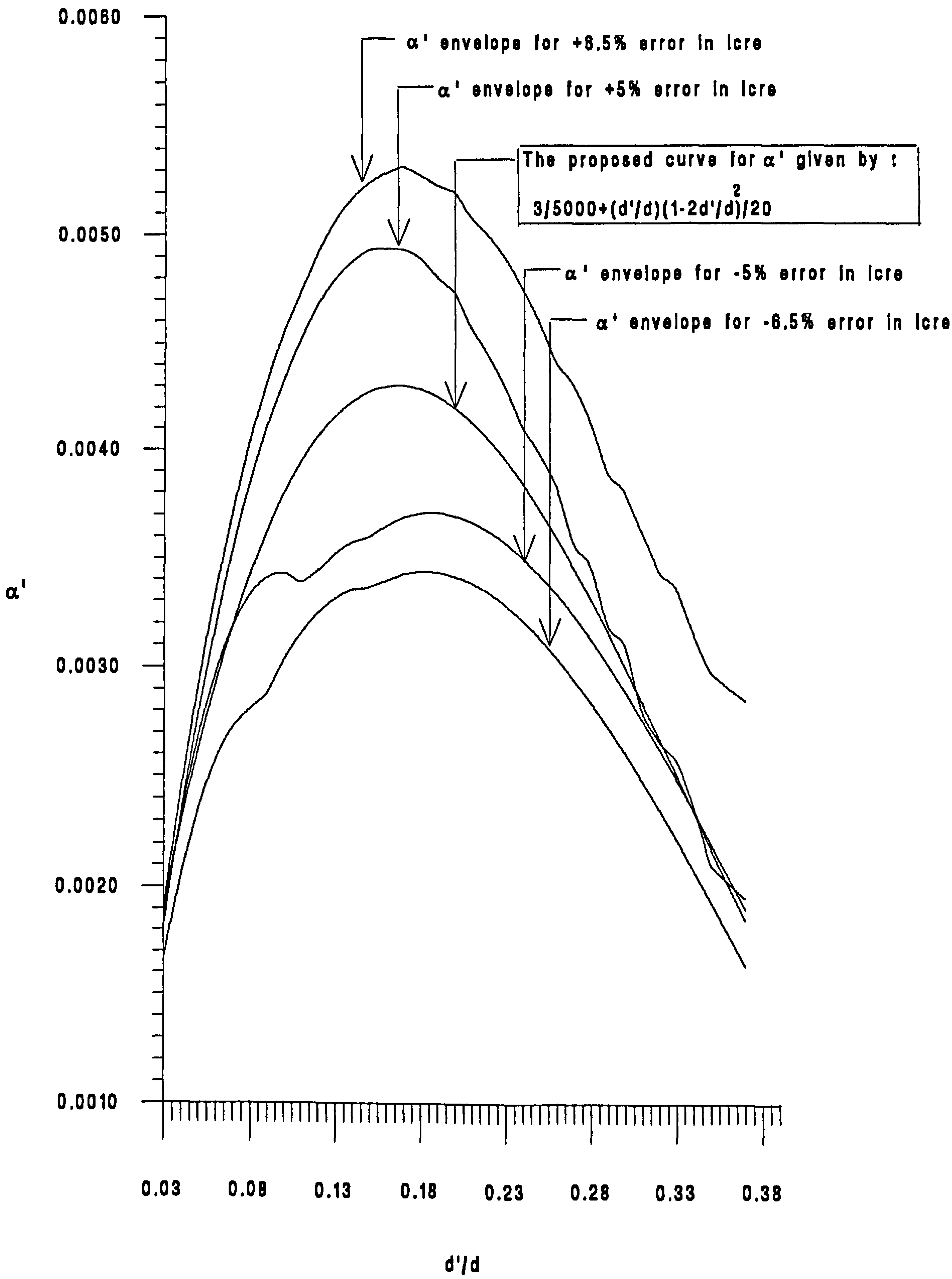


Figure 3.3.7 Envelopes of α' and the approximation of Eq.3.3.8

those of Fig.3.3.5.

Referring to the printouts in Appendix B4 it can be seen that except for d′/d of 0.03 where the maximum error was found to be +6.7%, for all d′/d ratios the maximum error can be said to be less than or equal to ±5% with some occasional deviations. These deviations as were expected from Fig.3.3.7 are negligible (the maximum deviated errors being ±5.2% as compared to ±5%). Such magnitude of maximum errors (including +6.7%) as calculated by the program using α' of Eq.3.3.8 are very much acceptable. In fact one would not expect an accuracy higher than this when approximating a parameter such as Icr which is dependent on many interrelated factors.

Because the envelopes of α' were produced as plots of upper and lower α' values calculated using $n\rho$ and $n\rho'$ increments of 1% (the very small increments of 10^{-6} used to obtain the envelope values of α' made it inconceivable to assume smaller increments of $n\rho$ and $n\rho'$ since the number of iterations will then be enormous) then in order to correlate the numerical values of the errors with these envelopes Progs.3.3.2 and 3.3.3 also had to assume $n\rho$ and $n\rho'$ increments of 1%. From the shape of the envelopes it is thought that increments of $n\rho$ and $n\rho'$ less than 1% will not have significant effect on the concluding results. To verify this numerically, however, Prog.3.3.3 was restructured to consider $n\rho$ and $n\rho'$ values in increments of 0.01%. As the reinforcement ratios, ρ and ρ', when expressed in decimals are always taken to four decimal places, their values in percentage will then be expressed to two decimal places. Therefore, an increment of 0.01% for $n\rho$ and $n\rho'$ is actually the smallest that one has to assume. The modification of Prog.3.3.3 to include such increments is referred to as Prog.3.3.3m and is shown in Appendix B5 along with the printed output. As was expected the results have shown that the maximum error values are the same as when increments of 1% were considered. Namely, except for d'/d of 0.03 where the maximum error value was +6.7%, for all d'/d ratios the maximum error can be said to be less than or equal to ±5% with some minor deviations as before.

Due to its generality and high accuracy Eq.3.3.8 will be adopted in the present study for obtaining α' necessary for the approximation of Icr, that is for the evaluation of Icre, of a doubly reinforced rectangular section. This should not be taken to mean that Eq.3.3.7 can not be used to evaluate α'. If one is considering the common range of d'/d from 0.08 to 0.25 or is certain that Ig can not be less than Icr, for the considered section, then Eq.3.3.7 will also yield equally accurate results.

Now that the study related to the approximation of Icr for a doubly reinforced rectangular section is complete it is important to summarize all what has been said above in order to avoid confusion and to put things in perspective :

The cracked transformed moment of inertia, Icr, for a doubly reinforced rectangular section can be approximated as Icre given below,

$$Icre = (\alpha + \beta n\rho b/b')(b'd^3/12) \qquad (3.3.9)$$

where,

$$b' = (\alpha' n\rho' d/d' + 1)(b) \quad , \quad \alpha' = 6x10^{-4} + (d'/d)(1-2d'/d)^2 / 20$$

$$n\rho = 100(nAs/bd) \quad , \quad n\rho' = 100(nAs'/bd)$$

α and β are factors determined for $n\rho b/b'$ from the intervals of Eq.3.2.2

Alternatively, for the most common cases of d'/d in the range from 0.08 to 0.25 α' can also be taken simply as 0.0037 given by Eq.3.3.7 which will reduce the expression of b' to,

$$b' = (0.0037n\rho' d/d' + 1)(b) \tag{3.3.10}$$

In all the discussions presented in this section there was no mention of the maximum value of $n\rho b/b'$ though it is purposely calculated and printed by the programs as can be seen in the print out sets of Appendices B3-5. The reason for calculating this value, however, is that it will be compared with the value that will be found in the flanged section analysis to determine the maximum $n\rho b/b'$ for which the solution curves (to be discussed in Chap.4) must be provided.

To bring the section to its conclusion a numerical example is next given. In addition to justifying some of the assumptions used in the study the example is meant to explain the solution process used by the program and how the developed equations are applied. It is also meant to show that within the limits considered in this study neglecting compression reinforcement not only gives errors that are significantly higher than those associated with the developed model of Icre but that it may actually result in Icr values that are grossly in error. This justifies the study undertaken to develop the model of approximating Icr for doubly reinforced sections rather than merely ignoring the compression reinforcement as suggested in some references.

Example 3.3.1

Prog.3.3.3m has been asked to print the different values computed during the solution process for the following cases :

(a) $n\rho = 2\%$, $n\rho' = 2\%$, $d'/d = 0.03$

(b) $n\rho = 15\%$, $n\rho' = 10\%$, $d'/d = 0.24$

(c) $n\rho = 25\%$, $n\rho' = 25\%$, $d'/d = 0.16$

(d) $n\rho = 40\%$, $n\rho' = 40\%$, $d'/d = 0.03$

(e) $n\rho = 45\%$, $n\rho' = 35\%$, $d'/d = 0.37$

The values printed for these cases appear in Appendix B5 as d/h, Ig/Icr, nρb/b′, Icre/Icr and (Icr,nρ′=0)/Icr where "Icr,nρ′=0" is the value of Icr computed neglecting compression reinforcement.

Required :

For each of these cases verify the program's printed values.

Solution :

Since the "implicit double precision statement" of the program retains the maximum number of decimal places the calculations to follow will also show as many decimal places as obtained using a special hand calculator. This will help to exactly confirm the printed results and is used in all the examples included in this chapter.

(a) The case of $n\rho = n\rho' = 2\%$ and $d'/d = 0.03$:

-For the exact Icr:

$$n\rho + n\rho' = 2 + 2 = 4\%$$

Substituting into the equations of Fig.3.3.2,

$$x = \{-4 + \sqrt{[4^2 + 200(2 + 2(0.03))]}\}\ (d/100) = 0.167d$$

Hence,

$$Icr = [100(0.167d)^3/3 + (2d)(0.167d - 0.03d)^2 + (2d)(d-0.167d)^2](b/100)$$

$$= 0.0158056\ bd^3$$

-For Icre:

Using Eq.3.3.9 and for the given d'/d and $n\rho'$,

$$b' = \{[6x10^{-4} + 0.03(1-2(0.03))^2/20](2/0.03) + 1\}(b) = 1.12836b$$

For $n\rho b/b'=2/1.12836=1.7725\%$, $\alpha + \beta n\rho b/b' = 0.17139$. Thus,

$$Icre=0.17139(1.12836bd^3/12)=0.0161155bd^3$$

-For Icr,$n\rho'=0$:

Substituting into the equations of Fig.3.3.2 with $n\rho'=0$,

$$x=[-2+\sqrt{(2^2 + 200(2))}](d/100) = 0.180998\ d$$

Hence,

$$(Icr,n\rho'=0) = [100(0.180998d)^3/3 + (2d)(d-0.180998d^2)](b/100)$$

$$=0.0153918\ bd^3$$

-For Ig:

$$Ig=bh^3/12$$

Prog.3.3.3m assumes that $h=d+d'$. Thus,

$$h=d+0.03d=1.03d$$

$$Ig=(b)(1.03d)^3/12=0.0910605\ bd^3$$

Therefore,

$d/h=d/1.03d=0.97$ (printed value:0.97)

$Ig/Icr=0.0910605bd^3/0.0158056bd^3=5.76$ (printed value:5.76)

$n\rho b/b' = 1.77$ (printed value:1.77)

$Icre/Icr=0.0161155bd^3/0.0158056bd^3=1.02$ (printed value:1.02)

$(Icr,n\rho'=0)/Icr=0.0153918bd^3/0.0158056bd^3=0.97$ (printed value:0.97)

Because the effect of compression reinforcement on the value of Icr is maximum at the minimum d′/d ratio of 0.03 and because (Icr,nρ′=0)/Icr of 0.97 implies an error of only 3%, the results obtained therefore show that neglecting nρ′ values of less than 2% is justifiable and thus the assumption that such nρ′ values have negligible effect on Icr, which is used in the study as one of the reasons for considering nρ and nρ′ values that are greater than or equal to 2%, is reasonable.

(b)The case of nρ=15%, nρ′=10% and d′/d=0.24:

-For the exact Icr:

$n\rho+n\rho'=15+10=25\%$

Substituting into the equations of Fig.3.3.2,

$$x=\{-25+\sqrt{[\,25^2 + 200(15 + 10(0.24))]}\}\,(d/100)=0.3907027\ d$$

Hence,

$$Icr=[100(0.3907027d)^3/3+(10d)(0.3907027d-0.24d)^2+(15d)(d-0.3907027d)^2](b/100) = 0.0778376\ bd^3$$

-For Icre:

From Eq.3.3.9 and for the given d′/d and nρ′,

$$b'=\{[6x10^{-4}+0.24(1-2(0.24))^2/20](10/0.24)+1\}(b) = 1.1602\ b$$

For $n\rho b/b'=15/1.1602=12.9288\%$, $\alpha + \beta n\rho b/b' = 0.80644$. Thus,

$$Icre=0.80644(1.1602bd^3/12)=0.0779693\ bd^3$$

-For Icr,nρ′=0:

Using the equations of Fig.3.3.2 with $n\rho'=0$,

$$x=[-15 + \sqrt{(15^2+200(15))}]\,(d/100) = 0.41789\ d$$

Hence,

$$(Icr,n\rho'=0) = [100(0.41789d)^3/3 + (15d)(d-0.41789d)^2](b/100)$$

$$=0.0751536\ bd^3$$

-For Ig:

Prog.3.3.3m assumes that $h = d'+d$. Thus,

$$h=d+0.24d=1.24d$$

$$Ig=bh^3/12=(b)(1.24d)^3/12=0.1588853\ bd^3$$

Therefor,

$d/h=d/1.24d=0.81$	(printed value:0.81)
$Ig/Icr=0.1588853bd^3/0.0778376bd^3=2.04$	(printed value:2.04)
$n\rho b/b' = 12.93\%$	(printed value:12.93)
$Icre/Icr=0.0779693bd^3/0.0778376bd^3=1.002$	(printed value:1.002)
$(Icr,n\rho'=0)/Icr=0.0751536bd^3/0.0778376bd^3=0.97$	(printed value:0.97)

Because d'/d of 0.24 is within the common range of 0.08 to 0.25 Icre could have also been computed using Eq.3.3.10 as follows,

$$b'=[0.0037(10/.24) + 1](b)=1.1541667b$$

Hence, $n\rho b/b' =15/1.1541667=12.99639$ for which $\alpha=0.16$, $\beta=0.05$. Thus,

$$Icre=[0.16 + 0.05(12.99639)](1.1541667bd^3/12)=0.0778888\ bd^3$$

or

$$Icre/Icr=0.0778888bd^3/0.0778376bd^3 = 1.001$$

which checks exactly with the value printed by Prog.3.3.2 (Appendix B3)

(c)The case of $n\rho = n\rho' = 25\%$ and $d'/d = 0.16$:

-For the exact Icr:

$$n\rho + n\rho' = 25 + 25 = 50\%$$

Substituting into the equations of Fig.3.3.2,

$$x = \{-50 + \sqrt{[50^2 + 200(25 + 25(0.16))]}\}\,(d/100) = 0.4110433\ d$$

Hence,

$$Icr = [100(0.4110433d)^3/3 + (25d)(0.4110433d - 0.16d)^2 + (25d)$$

$$(d - 0.4110433d)^2](b/100) = 0.1256226\ bd^3$$

-For Icre

From Eq.3.3.9 and for the given d'/d and $n\rho'$,

$$b' = \{[6x10^{-4} + (0.16)(1-2(0.16))^2/20](25/0.16) + 1]\}(b) = 1.67175\ b$$

For $n\rho b/b' = 25/1.67175 = 14.954389\%$, $\alpha + \beta n\rho b/b' = 0.90772$. Thus,

$$Icre = 0.90772(1.67175bd^3/12) = 0.1264566\ bd^3$$

-For Icr,$n\rho'$=0:

Using the equations of Fig.3.3.2 with $n\rho'=0$,

$$x = [-25 + \sqrt{(25^2 + 200(25))}](d/100) = 0.5d$$

Hence,

$$(Icr, n\rho'=0) = [100(0.5d)^3/3 + (25d)(d-0.5d)^2](b/100) = 0.1041666\ bd^3$$

-For Ig:

Prog.3.3.3m assumes that $h=d+d'$. Thus,

$$h = d + 0.16d = 1.16d$$

$$Ig = bh^3/12 = (b)(1.16d)^3/12 = 0.1300746\ bd^3$$

Therefore,

$d/h=d/1.16d=0.86$ (printed value:0.86)

$Ig/Icr=0.1300746bd^3/0.1256226bd^3=1.04$ (printed value:1.04)

$n\rho b/b'=14.95\%$ (printed value:14.95)

$Icre/Icr=0.1264566bd^3/0.1256226bd^3=1.007$ (printed value:1.007)

$(Icr,n\rho'=0)/Icr=0.1041666bd^3/0.12562bd^3=0.83$ (printed value:0.83)

Again and as in the previous case since d'/d of 0.16 is within the common range of 0.08 to 0.25, Icre could have also been computed using Eq.3.3.10 as follows,

$$b'=[0.0037(25/0.16)+1](b)=1.578125b$$

Hence, $n\rho b/b'=25/1.578125=15.841584\%$ for which $\alpha=0.16$, $\beta=0.05$.Thus,

$$Icre=[0.16+0.05(15.841584)](1.578125bd^3/12)=0.1252083bd^3$$

or

$$Icre/Icr=0.997$$

which checks exactly with the value printed by Prog.3.3.2(Appendix B3)

The above computations have shown that when compression reinforcement is ignored the Icr value was 17% in error as compared to an error of only 0.7% (or 0.3% when Eq.3.3.10 is used) when Icr was evaluated using the developed model. This not only justifies the effort of developing the model but also proves that neglecting compression reinforcement may have significant effect on the value of Icr which is contrary to what has been claimed in some references[30,31]. In fact in the extreme conditions of Ig<Icr the error resulting from neglecting compression reinforcement

may actually go up to 40%. The next case is an example of such a condition.

(d) The case of $n\rho=n\rho'=40\%$ and $d'/d=0.03$:

-For the exact Icr:

$$n\rho+n\rho'=40+40=80\%$$

Substituting into the equations of Fig.3.3.2

$$x=\{-80+\sqrt{[80^2+200(40+40(0.03))]}\}\ (d/100) = 0.4099586\ d$$

Hence,

$$Icr=[100(0.4099586)^3/3 + (40d)(0.4099586d-0.03d)^2 + (40d)(d-0.4099586d)^2](b/100) = 0.2199736\ bd^3$$

-For Icre;

From Eq.3.3.9 and for the given d'/d and $n\rho'$,

$$b'=\{[6x10^{-4} +0.03(1-2(0.03))^2/20](40/0.03) + 1](b) = 3.5672b$$

For $n\rho b/b'=40/3.5672=11.213277\%$, $\alpha + \beta n\rho b/b' = 0.72066$. Thus,

$$Icre=0.72066(3.5672bd^3/12) = 0.2142293\ bd^3$$

-For $Icr,n\rho'=0$:

Using the equations of Fig.3.3.2 with $n\rho'=0$,

$$x=[-40+\sqrt{(40^2 + 200(40))}](d/100) = 0.5797959d$$

Hence,

$$(Icr,n\rho'=0) = [100(0.5797959d)^3/3 + (40d)(d-0.5797959d)^2](b/100) =0.1355972\ bd^3$$

-For Ig:

Prog.3.3.3m assumes that $h=d+d'$. Thus,

$$Ig=bh^3/12=(b)(1.03d)^3/12=0.0910605\ bd^3$$

Therefore,

$d/h=d/1.03d=0.97$ (printed value:0.97)

$Ig/Icr=0.0910605bd^3/0.2199736bd^3=0.41$ (printed value:0.41)

$n\rho b/b'=11.21\%$ (printed value:11.21)

$Icre/Icr=0.2142293bd^3/0.2199736bd^3=0.974$ (printed value:0.974)

$(Icr,n\rho'=0)/Icr=0.1355972bd^3/0.2199736bd^3=0.62$ (printed value:0.62)

As one may expect the possibility of Ig<Icr increases as the member becomes more heavily reinforced or that the tension and compression reinforcements are placed further apart. The current case is a good example of such a condition where the value of Ig was found to be only 41% of that of Icr. Neglecting compression reinforcement in this case has given an error in the value of Icr of 38% which is far beyond tolerable limits. Using Eq.3.3.9, however, has underestimated the value of Icr by only 2.6%.

(e) The case of $n\rho=45\%$, $n\rho'=35\%$, $d'/d=0.37$:

-For the exact Icr:

$n\rho+n\rho'=45+35=80\%$

Substituting into the equations of Fig.3.3.2,

$$x=\{-80+\sqrt{[80^2+200(45+35(0.37))]}\}(d/100)=0.541268d$$

Hence,

$$Icr=[100(0.541268d)^3/3+(35d)(0.541268d-0.37d)^2+(45d)$$
$$(d-0.541268d)^2](b/100)=0.1578208\ bd^3$$

-For Icre:

From Eq.3.3.9 and for the given d′/d and nρ′,

$$b' = \{[6\times10^{-4} + 0.37(1-2(0.37))^2/20](35/0.37)+1\}(b) = 1.1750568b$$

For nρb/b′=45/1.1750568=38.296021%, $\alpha + \beta n\rho b/b' = 1.56592$. Thus,

$$Icre = 1.56592(1.1750568bd^3/12) = 0.1533371bd^3$$

-For Icr,nρ′=0:

From the equations of Fig.3.3.2 with nρ′=0,

$$x = [-45+\sqrt{(45^2 + 200(45))}]\ (d/100) = 0.6d$$

Hence,

$$(Icr, n\rho'=0) = [100(0.6d)^3/3 + (45d)(d-0.6d)^2](b/100) = 0.144\ bd^3$$

-For Ig:

Prog.3.3.3m assumes that h=d+d′. Thus,

$$h = d+0.37d = 1.37d$$

$$Ig = bh^3/12 = (b)(1.37d)^3/12 = 0.2142794\ bd^3$$

Therefore,

$d/h = d/1.37d = 0.73$	(printed value:0.73)
$Ig/Icr = 0.2142794bd^3/0.1578208bd^3 = 1.36$	(printed value:1.36)
$n\rho b/b' = 38.30\%$	(printed value:38.30)
$Icre/Icr = 0.1533371bd^3/0.1578208bd^3 = 0.972$	(printed value:0.972)
$(Icr, n\rho'=0)/Icr = 0.144bd^3/0.1578208bd^3 = 0.91$	(printed value:0.91)

Although in this case heavy reinforcement is used the tension and compression reinforcements are placed as close as possible. As a result Ig was found to be greater than Icr. However, the error resulting from neglecting compression reinforcement was still significantly higher than when Eq.3.3.9 was used.

Because Ig was greater than Icr, Eq.3.3.7 could also have been used to calculate Icre even though d′/d of 0.37 is not within the common range of 0.08 to 0.25. Thus when using Eq.3.3.7,

$$b'=[0.0025(35/0.37)+1](b)=1.2364865b$$

Hence, $n\rho b/b'=45/1.2364865=36.393443\%$ for which $\alpha=0.8$, $\beta=0.02$. Thus,

$$Icre=[0.8+0.02(36.393443)](1.2364865bd^3/12)=0.1574324\ bd^3$$

or

$$Icre/Icr=0.998$$

which compares exactly with that printed by Prog.3.3.2 (Appendix B3)

3.4 Singly Reinforced Flanged Sections

The evaluation of Icre for flanged sections is actually the reverse of the process used to analyse doubly reinforced rectangular sections. Figure 3.4.1 explains the concept where the flanged section shown in Fig(a) has been transformed into the equivalent doubly reinforced rectangular section of Fig(b).

Because the two overhanging concrete areas of the flange have been replaced by the area of compression steel, As′ , one can write,

$$As'=(be-bw)(hf)/n$$

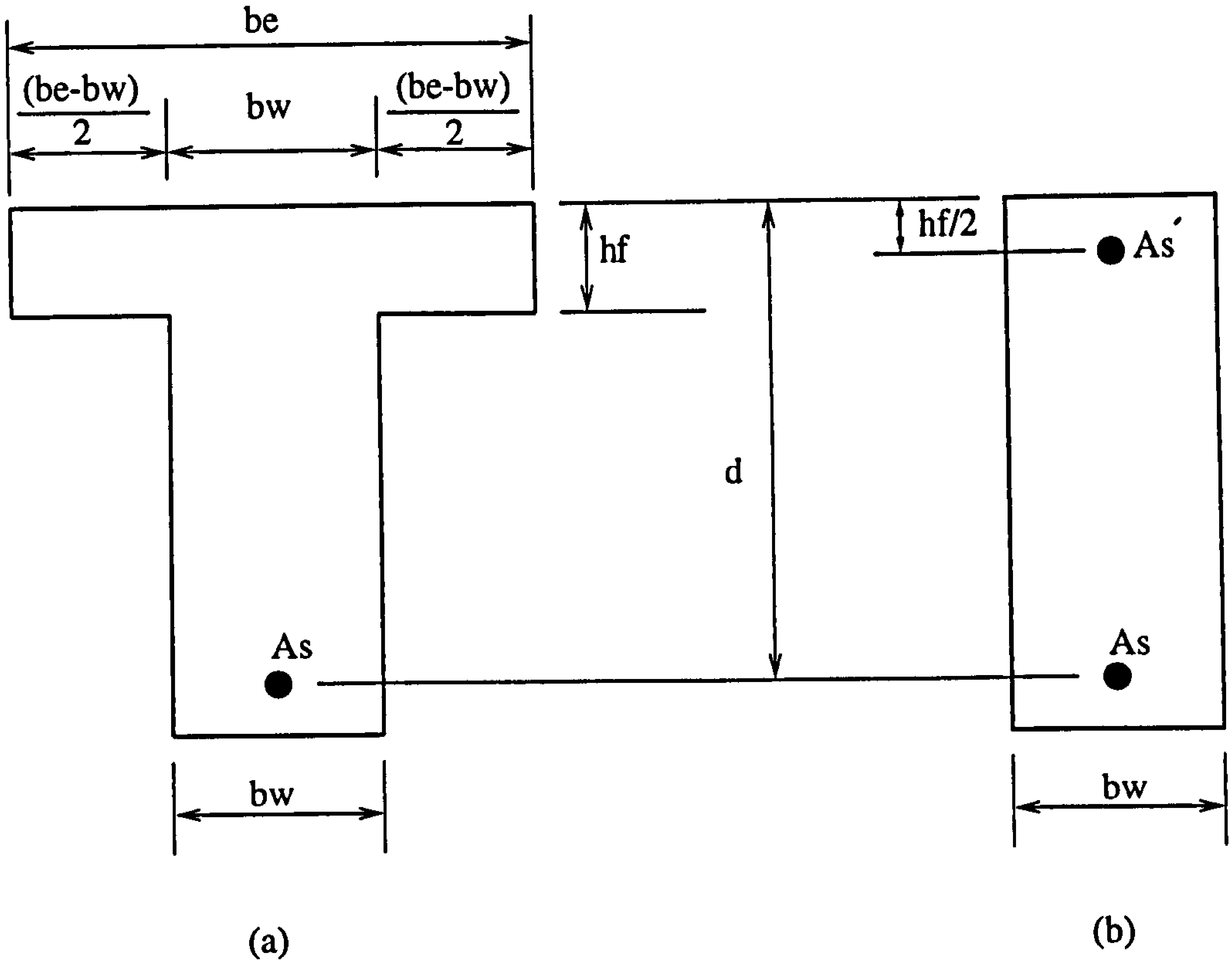


Figure 3.4.1 :
(a) a singly reinforced flanged section
(b) an equivalent section to that of (a)

Writing As´ as $\rho' bwd/100$ (taking ρ' in percentage),

$$\rho' bwd/100 = (be - bw)(hf)/n$$

Thus,

$$n\rho' bwd = 100(be - bw)(hf)$$

or

$$n\rho' = 100(be - bw)(hf) / bwd = 100(be/bw - 1)(hf/d)$$

Analogous to Eq.3.3.1 one can therefore write,

$$b' = [100\alpha_1(be/bw - 1)(hf/d)(d/d') + 1](bw)$$

$$= [100\alpha_1(be/bw - 1)(hf/d') + 1](bw)$$

where α_1 is used to avoid confusion with α'.

Substituting hf/2 for d',

$$b' = [200\alpha_1(be/bw - 1) + 1](bw)$$

Redefining $200\alpha_1$ as αf the above equation will then become,

$$b' = [\alpha f(be/bw - 1) + 1](bw) \qquad (3.4.1)$$

Because the equivalent $n\rho'$ defined above may not necessarily be smaller than the given value of $n\rho$ the result of the study of Sec.3.3 can not be used to evaluate αf for it was based on the assumption that $n\rho'$ is always less than or equal to $n\rho$. Therefore a separate analysis had to be carried out.

Following exactly the same procedure as that of Sec.3.3, Prog.3.4.1 was developed to compute the upper and lower envelope values of αf with $n\rho'$ and d'/d replaced by be/bw and hf/d, respectively.

While the program's listing is included in Appendix B6, the flowcharts outlining the logic of the program and the iteration procedures involved are shown in Fig.3.4.2. The value of $n\rho$ was set at the lower bound of 0.124% and is incremented by 0.01% up to the upper bound of 33.96%. Since the bounds of $n\rho$ as given in Secs.2.8.1 and 2.8.2 are expressed relative to the width be the program starts with $n\rho$ values relative to be and then converts the values to those relative to bw. Because the reinforcement ratio when expressed in percentage is usually taken to two decimal places, an increment smaller than 0.01% in the value of $n\rho$ was thought unnecessary. This is confirmed latter by the smoothness of the αf envelopes obtained.

For each value of $n\rho$, be/bw is also incremented by 0.1 starting at 1.1 up to the maximum value of 10. Again the range of be/bw from 1.1 to 10 was set in Chap.2 and is thought to encompass almost all the sections that may be encountered in practice and is consistent with the values used in the various references consulted [i.e page 427 of Ref.6, pages 212-213 of Ref.8, page 40 of Ref.12].

With each combination of $n\rho$ and be/bw different values of hf/d are considered. Starting at a value of 0.1, hf/d is successively incremented using increments of 0.1 up to the maximum value of 0.55 beyond which, and as was found in Chap.2, the section is assumed to behave as a rectangle of width be to which the analysis of Sec.3.2 applies. Consistent with the references consulted hf/d values smaller than 0.1were not

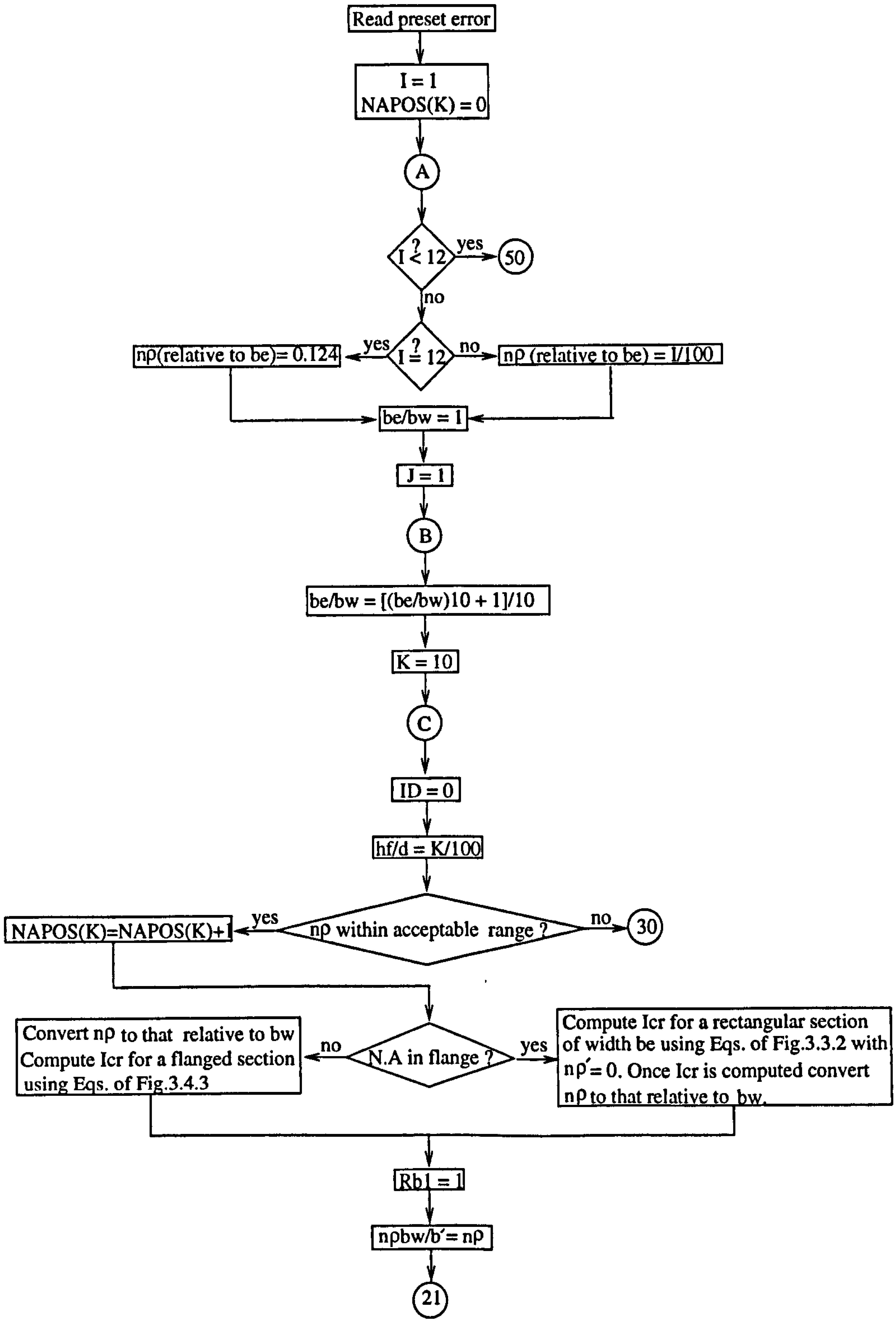


Figure 3.4.2 The flowcharts of Prog.3.4.1

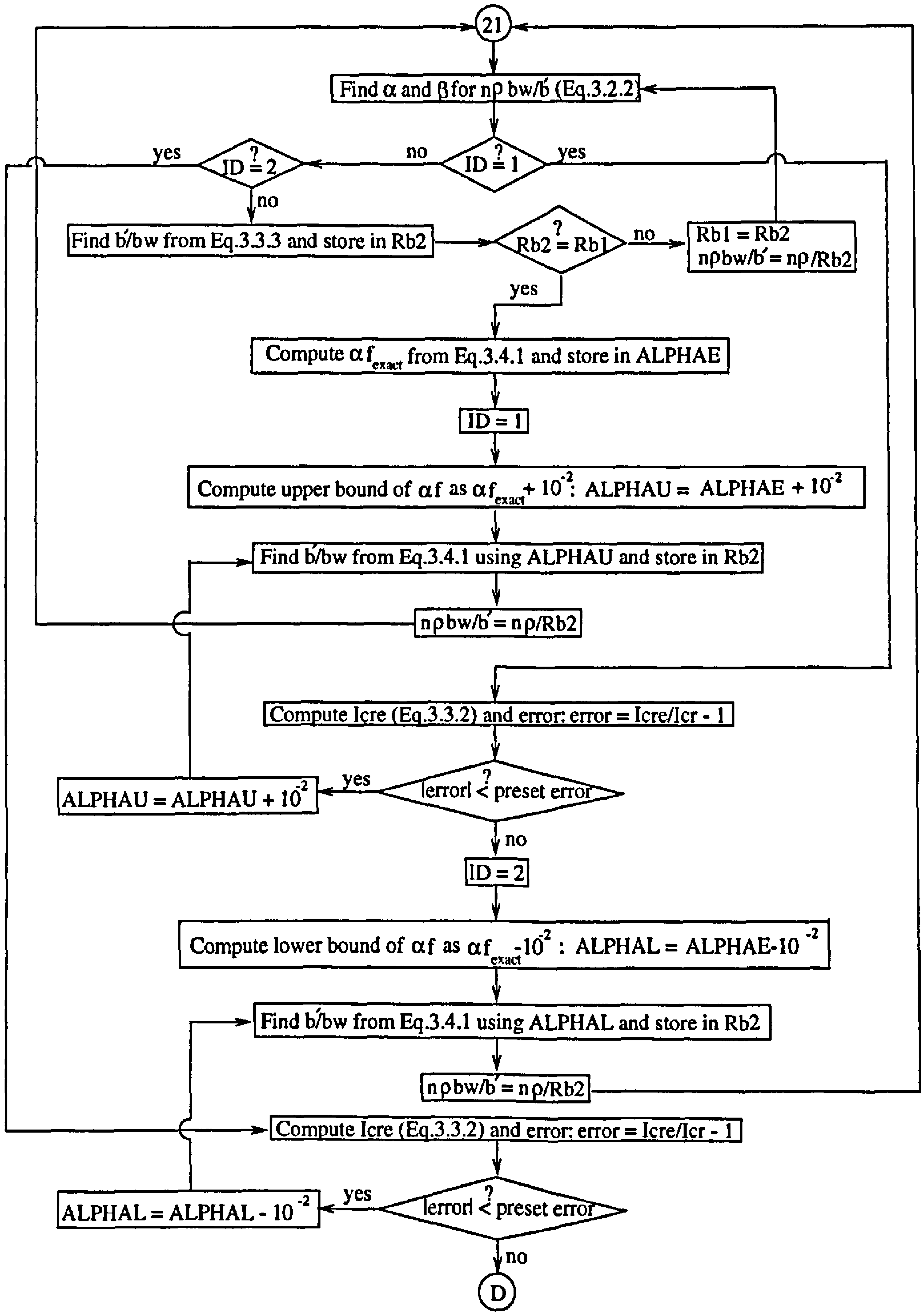


Figure 3.4.2 (cont'd)

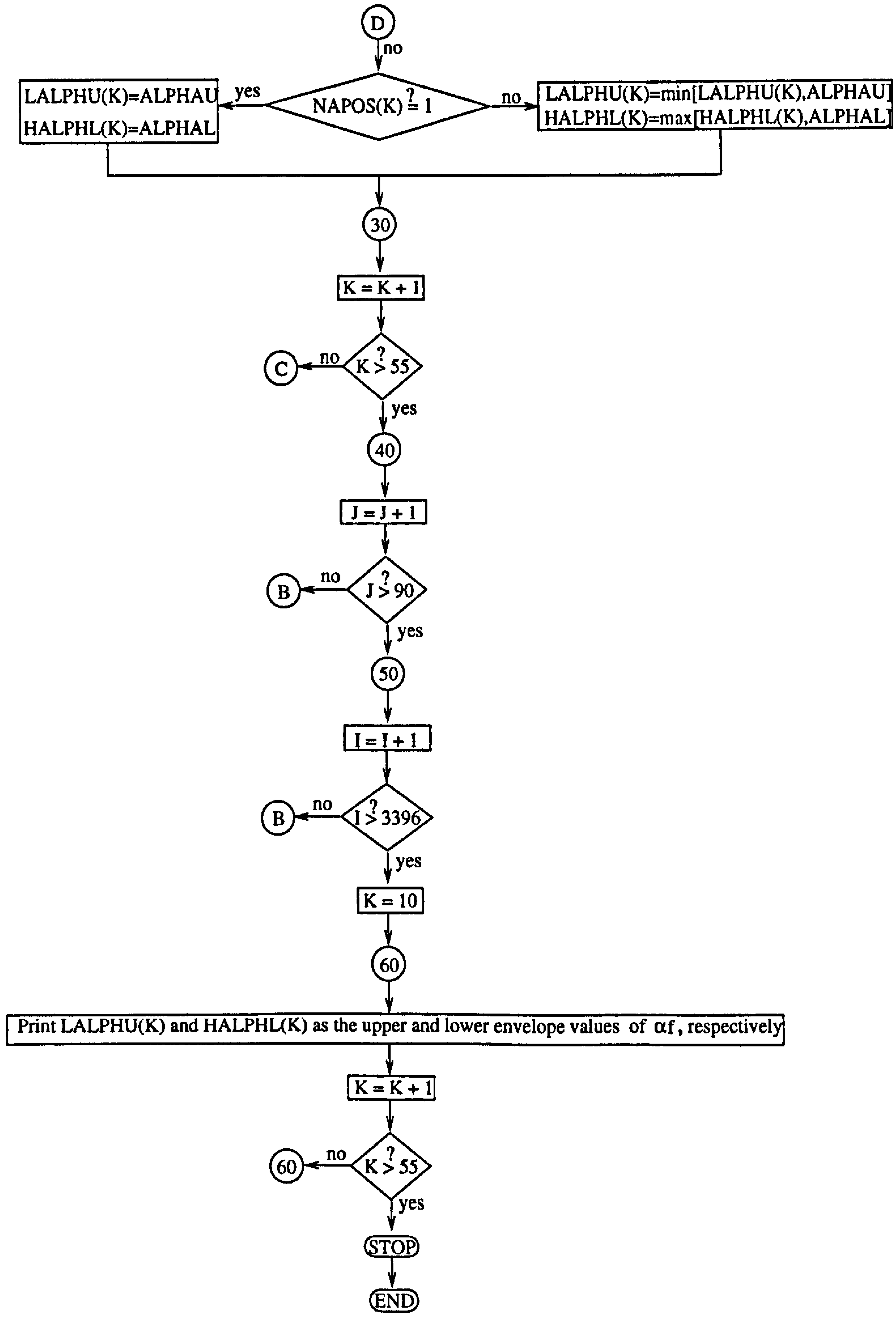


Figure 3.4.2 (cont'd)

considered for they were thought to be too extreme to be encountered in usual practice.

To discard any section that is outside the range allowed by the codes, the limitations discussed in Secs.2.8.1-2 are applied at each combination of nρ, be/bw and hf/d. If the section is found unacceptable it is then ignored and a new section is considered.

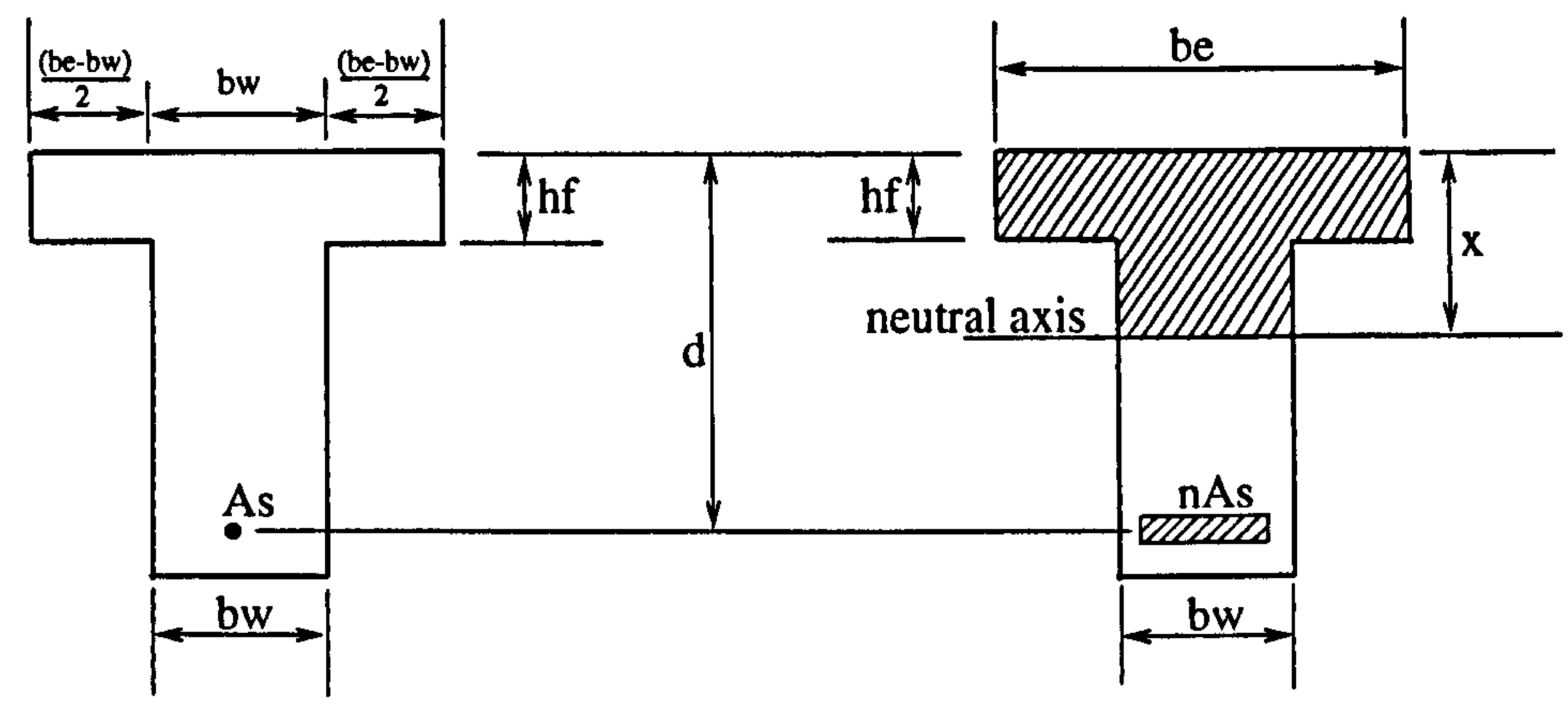
As in Prog.3.3.1, in order to obtain $(b'/bw)_{exact}$ using Eq.3.3.3 the exact values of Icr had to be determined. The method used to compute such values was based upon whether the neutral axis falls within the flange or in the web of the considered section. If the neutral axis is found to fall within the flange the section is then assumed as a rectangle of width be for which the equations of Fig.3.3.2 with b taken as be and $n\rho'=0$ are used to determine the exact value of Icr. If, on the other hand, the neutral axis is found to fall within the web the equations of Fig.3.4.3 are then used.

Once the exact value of Icr is determined the program proceeds in exactly the same manner as in Prog.3.3.1 except that b'/bw is now based on Eq.3.4.1 instead of Eq.3.3.4 and that αf is incremented by $\pm 10^{-2}$.

The upper and lower envelope values for errors of ±5.5 and ±6.5% as found by the program are shown in Table 3.4.1. When these values are plotted against hf/d the envelopes of Fig.3.4.5 are obtained. Within these envelopes an approximation of αf was then fitted. Therefore and from Fig.3.4.5,

$$\alpha f = (1 + 8hf/d)/3 \leq 0.9 \qquad (3.4.2)$$

To confirm the accuracy of the results obtained from Eq.3.4.2 and thus the envelopes of Fig.3.4.5, Prog.3.4.2 was developed. In the program Eq.3.4.2 is used to evaluate b'

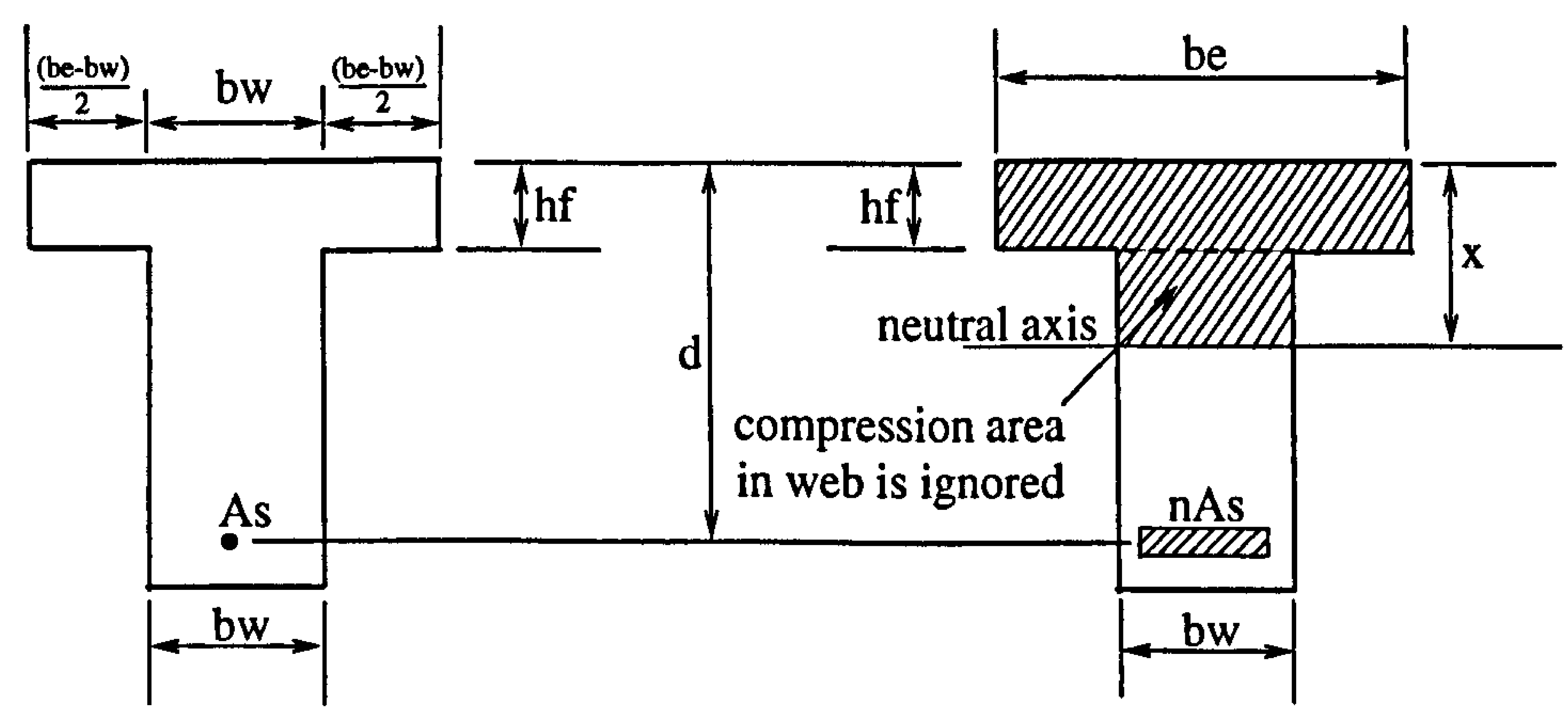


$n\rho = 100nAs/bwd$, $x = (-b + \sqrt{b^2 + 4c})/2$, $b = 2hf[\, be/bw - 1 + n\rho/(100hf/d)\,]$

$c = hf^2[\, be/bw - 1 + (n\rho/50)/(hf/d)^2\,]$

$Icr = [(100/3)(be/bw)(hf/d)^3 + (100/3)(x/d - hf/d)^3 + 100(be/bw)(hf/d)(x/d)(x/d - hf/d)$
$\quad + n\rho\,(1 - x/d)^2](bwd^3)/100$

Figure 3.4.3 The cracked transormed section of a singly reinforced flanged section and the pertaining equations of x and Icr



$n\rho = 100nAs/bwd$, $x = [\, n\rho + 50(be/bw)(hf/d)^2\,](d)/[n\rho + 100(be/bw)(hf/d)]$

$Icr = [\,(100/12)(be/bw)(hf/d)^3 + 100(be/bw)(hf/d)(x/d - hf/2d)^2$
$\quad + n\rho\,(1 - x/d)^2](bwd^3)/100$

Figure 3.4.4 The cracked transformed section of a singly reinforced flanged section and the pertaining equations of x and Icr when the compression area in the web is ignored

Table 3.4.1. Values of the upper and lower envelopes of αf

hf/d	% error allowed in Icre			
	5.5%		6.5%	
	upper envelope values	lower envelope values	upper envelope values	lower envelope values
0.10	0.574678	0.739201	0.595732	0.592477
0.11	0.620794	0.739201	0.642465	0.592477
0.12	0.656926	0.739201	0.678395	0.618663
0.13	0.691038	0.739201	0.721209	0.648672
0.14	0.731098	0.739201	0.753442	0.675904
0.15	0.761636	0.739201	0.783750	0.700892
0.16	0.790271	0.755715	0.820904	0.723101
0.17	0.817107	0.775722	0.847834	0.742495
0.18	0.849930	0.794719	0.873113	0.760044
0.19	0.873417	0.810519	0.896618	0.775413
0.20	0.895291	0.825045	0.918693	0.788311
0.21	0.915725	0.836563	0.939143	0.800007
0.22	0.934559	0.847601	0.958131	0.809674
0.23	0.951967	0.856035	0.975654	0.817836
0.24	0.967894	0.863151	0.991819	0.824602
0.25	0.974435	0.869385	1.006590	0.829521
0.26	0.974435	0.873896	1.020082	0.833896
0.27	0.974435	0.877213	1.024435	0.837213
0.28	0.974435	0.879693	1.024435	0.839657
0.29	0.974435	0.881925	1.024435	0.840965
0.30	0.974435	0.883157	1.024435	0.841869
0.31	0.974435	0.883944	1.024435	0.842351
0.32	0.974435	0.884303	1.024435	0.842667
0.33	0.974435	0.884405	1.024435	0.842749
0.34	0.974435	0.884411	1.024435	0.842752
0.35	0.974435	0.884411	1.024435	0.842752
0.36	0.974435	0.884411	1.024435	0.842752
0.37	0.974435	0.884411	1.024435	0.842752
0.38	0.974435	0.884411	1.024435	0.842752
0.39	0.974435	0.884411	1.024435	0.842752
0.40	0.974435	0.884411	1.024435	0.842752
0.41	0.974435	0.884411	1.024435	0.842752
0.42	0.974435	0.884411	1.024435	0.842752
0.43	0.974435	0.884411	1.024435	0.842752
0.44	0.974435	0.884411	1.024435	0.842752
0.45	0.974435	0.884411	1.024435	0.847600

Table 3.4.1 (cont'd)

hf/d	% error allowed in Icre			
	5.5%		6.5%	
	upper envelope values	lower envelope values	upper envelope values	lower envelope values
0.46	0.974435	0.884411	1.024435	0.848551
0.47	0.974435	0.884411	1.024435	0.854478
0.48	0.974435	0.884411	1.024435	0.859733
0.49	0.974435	0.884411	1.024435	0.864433
0.50	0.974435	0.885975	1.024435	0.865975
0.51	0.974435	0.886648	1.024435	0.866648
0.52	0.974435	0.886927	1.024435	0.866927
0.53	0.974435	0.889285	1.024435	0.869252
0.54	0.974435	0.892861	1.024435	0.872826
0.55	0.974435	0.896178	1.024435	0.876136

Notes:

1. *Upper envelope values correspond to a maximum (+) error in Icre of 5.5 and 6.5%.*
2. *Lower envelope values correspond to a maximum (-) error in Icre of 5.5 and 6.5%.*
3. *Error in Icre is defined as :*

(Icre/Icr -1)100

from Eq.3.4.1. The value of b′ thus found is then used to evaluate Icre using Eq.3.3.2 as applied to Fig.3.4.1(b). Namely,

$$Icre = (\alpha + \beta n\rho bw/b')(b'd^3/12) \qquad (3.4.3)$$

where nρ is taken relative to bw and α and β are determined for nρbw/b′.

Finally the errors are calculated in the same manner as in Progs.3.3.2 and 3.3.3 except that they are now given for each hf/d ratio rather than d′/d.

The exact value of Icr needed to calculate the errors are found from the equations of Fig.3.3.2 or 3.4.3 and as in Prog.3.4.1.

To correlate the results with the envelopes of Fig.3.4.5, the values of nρ, be/bw and

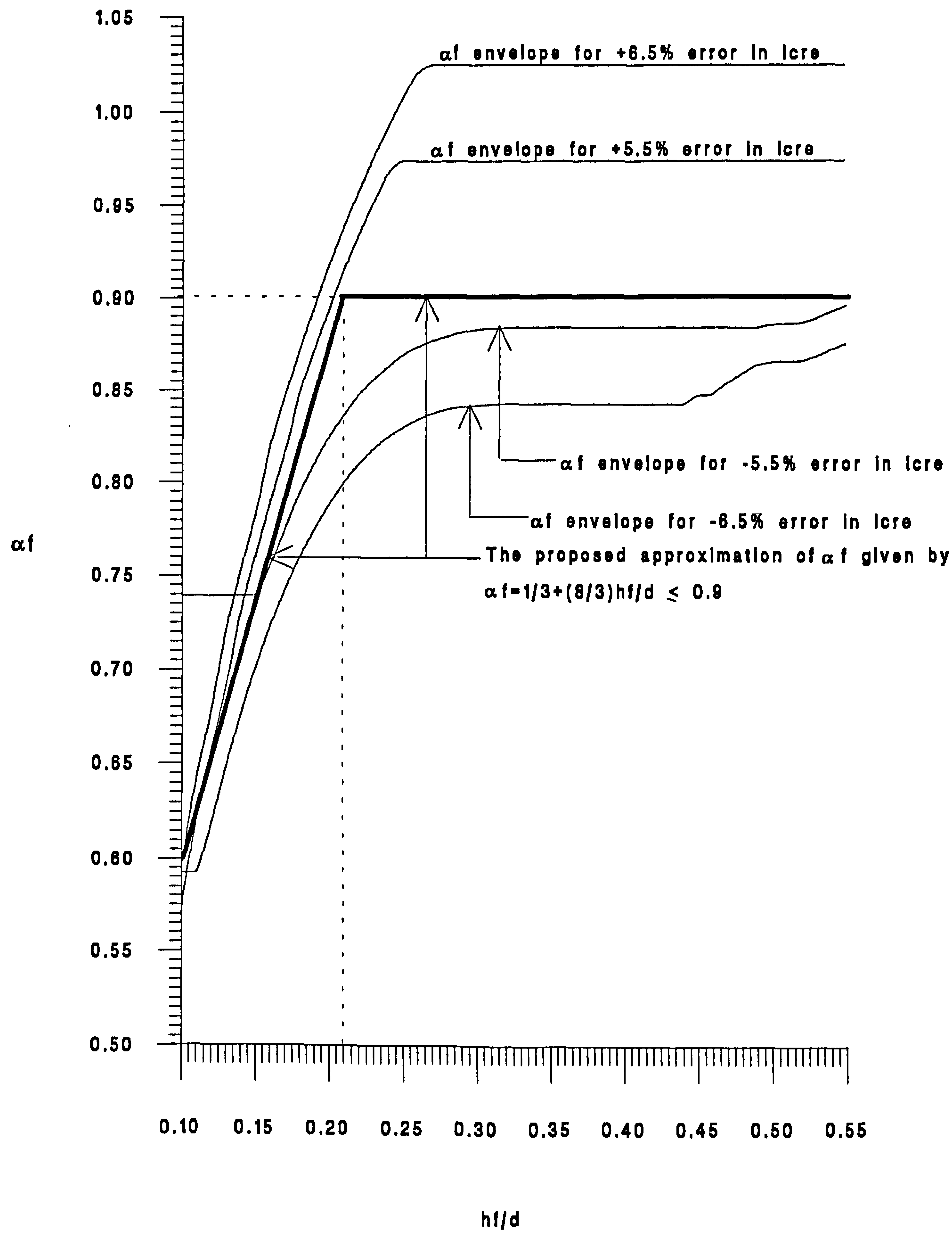


Figure 3.4.5 Envelopes of α_f and the approximation of Eq.3.4.2

hf/d are set and incremented in exactly the same way as that of Prog.3.4.1

The flow charts of the program are shown in Fig.3.4.6 while the program's listing along with a complete set of output are included in Appendix B7.

From the results shown in the Appendix it can be seen that the errors involved in Icre when Eq.3.4.2 is used vary within the range of ±6.4% (the only exception being at hf/d of 0.1 where the maximum error was found to be +6.8%)

As in Progs.3.3.2 and 3.3.3, Prog.3.4.2 was also structured to print the corresponding condition of Ig vs. Icr for all the different errors with Ig now evaluated using the equations of Fig.3.4.7. Because the condition of Ig < Icr is thought to be too extreme and that any error is more emphasized if it corresponds to a non-extreme condition, the parameters involved in the equations for Ig must therefore be chosen to favour the condition of Ig > Icr as much as possible. Because be/bw and hf/d are predefined through the respective do loops, d/h remains the only parameter to be chosen. From the equations it can be seen that in order to get the largest possible Ig the smallest d/h that is practically common should be used. Different references consulted in this regard has shown that such a ratio of d/h can be assumed at 0.72 (i.e page 211 of Ref.7). Using this ratio of d/h along with the combination of be/bw and hf/d, Ig values were computed and compared with the values of Icr as was done in Progs.3.3.2 and 3.3.3.

In addition to studying the accuracy of Eq.3.4.1, Prog.3.4.2 was also structured to investigate the effect of neglecting the compression area of the web on the value of Icr when the neutral axis falls below the flange. Although not part of the present study the investigation was useful in showing that within the limits considered in the current study the value of Icr computed as such using the equations of Fig.3.4.4 may actually

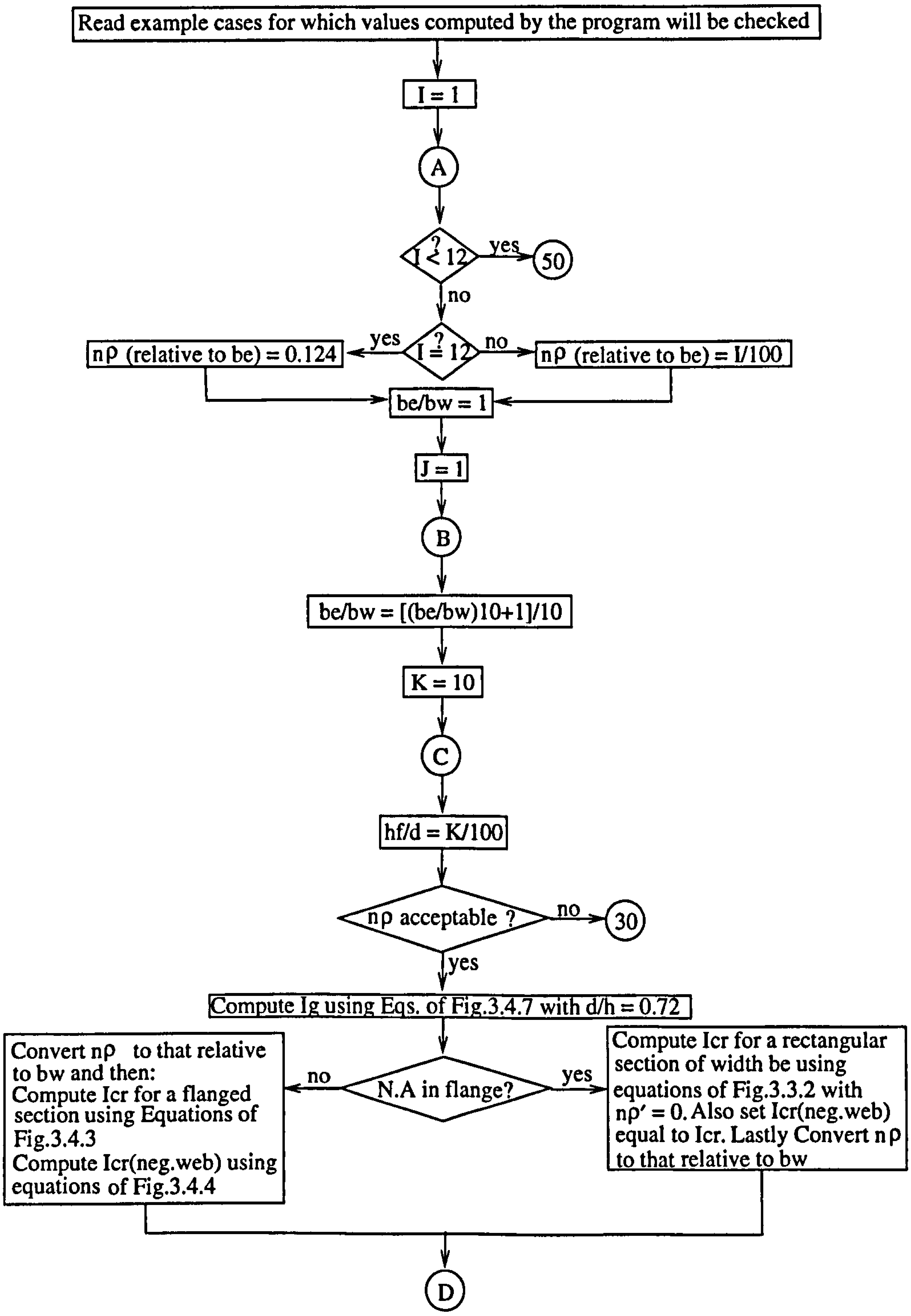


Figure 3.4.6 The flow charts of Prog.3.4.2

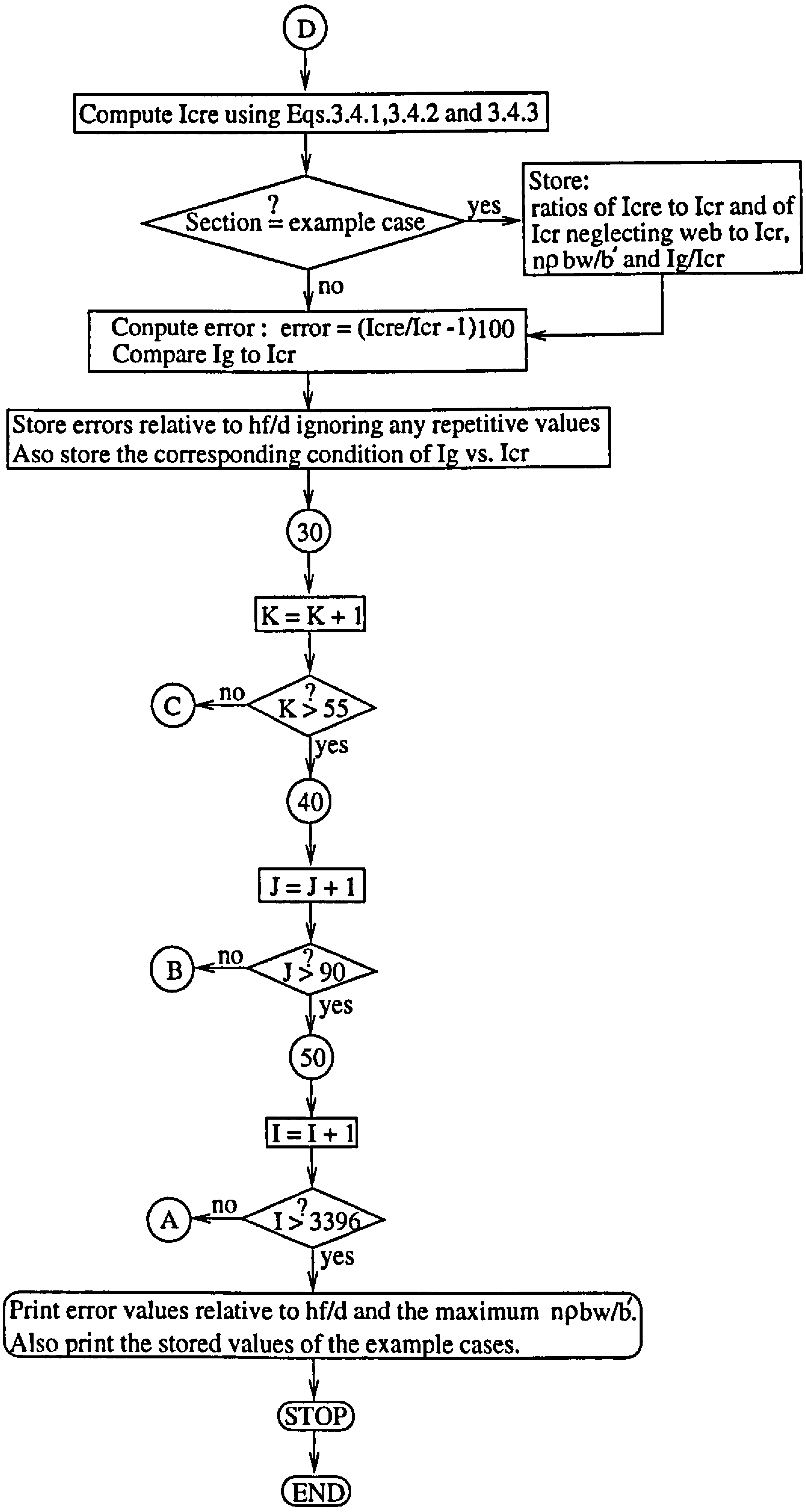
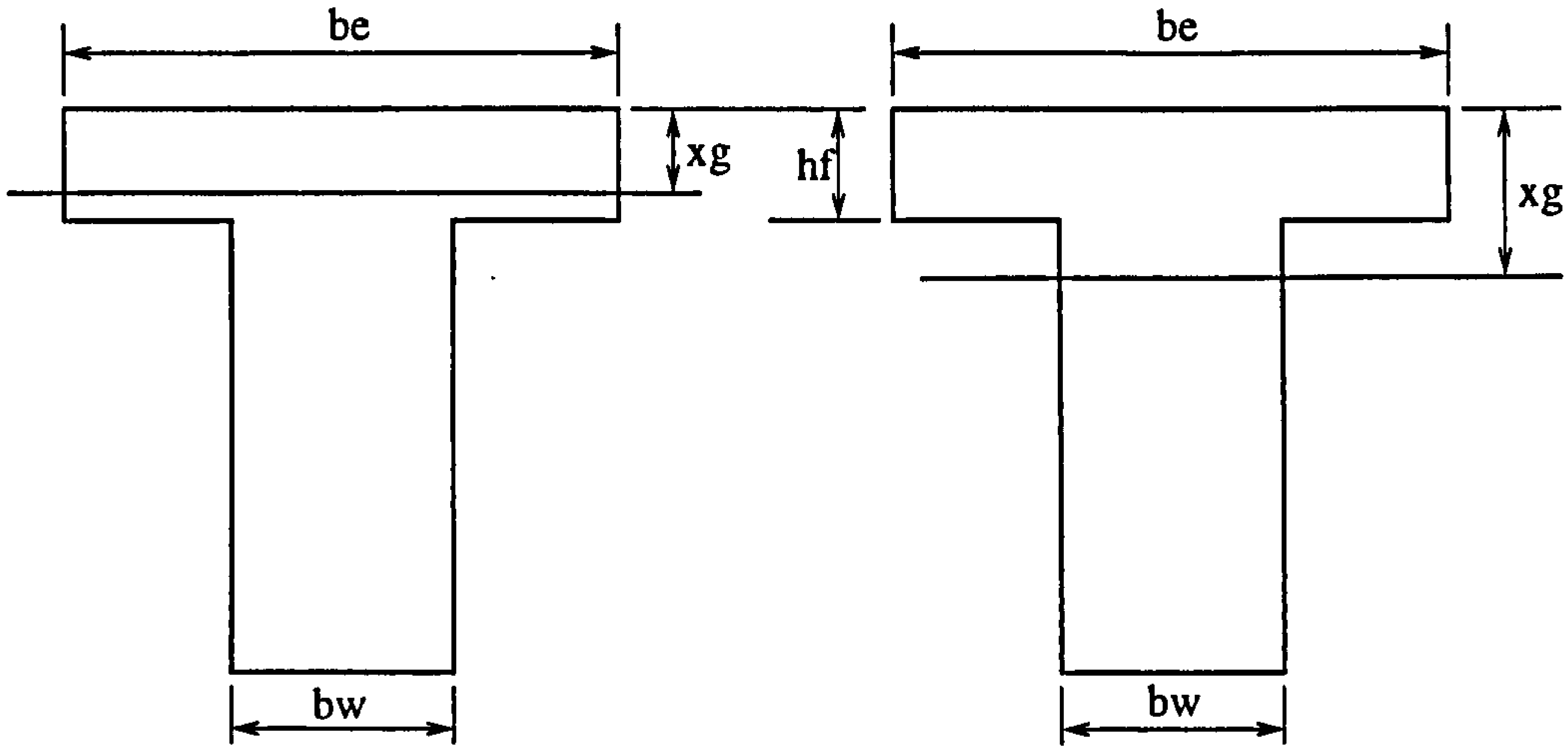


Figure 3.4.6 (cont'd)



(a) (b)

$$xg=\{0.5(be/bw)(hf/d)^2+0.5[(1/d/h)^2-(hf/d)^2]\}\quad (d)/[(be/bw)(hf/d)+(1/d/h)-(hf/d)]$$

$$Ig(Fig.a)=[(1/3)(be/bw)(xg/d)^3+(1/3)(1/d/h-xg/d)^3+(1/3)(be/bw-1)(hf/d-xg/d)^3](bwd^3)$$

$$Ig(Fig.b)=[(1/12)(be/bw)(hf/d)^3+(be/bw)(hf/d)(xg/d-0.5hf/d)^2$$
$$+(1/3)(1/d/h-xg/d)^3+(1/3)(xg/d-hf/d)^3]\ (bwd^3)$$

Figure 3.4.7 General flanged sections neglecting steel and the pertaining equations of xg and Ig.

be in error of as much as -30%. This contradicts the claim stated on Page 213 of Ref.7 that such an assumption leads to a negligible error in the value of Icr and shows that the graphical approximation given in the reference may not always be reliable for use in case of flanged sections.

Finally, to end the discussion a numerical example is given. The example is not only intended to explain the solution process followed by Prog.3.4.2 but also to provide numerical cases in support of the argument that neglecting the web compression area is not always permissible.

Example 3.4.1

Prog.3.4.2 has been asked to print the different values computed during the solution process for the following cases:

(a) $n\rho$ (based on be) = 0.5% , be/bw = 5.0 , hf/d = 0.35

(b) $n\rho$ (based on be) = 8% , be/bw = 10 , hf/d = 0.20

(c) $n\rho$ (based on be) = 14% , be/bw = 3 , hf/d = 0.11

(d) $n\rho$ (based on be) = 22% , be/bw = 1.5 , hf/d = 0.10

The values printed for these cases appear in Appendix B7 as Icre/Icr, (Icr,neg.web)/Icr, $n\rho b/b'$ (relative to bw) and Ig/Icr where "Icr,neg.web" is the value of Icr computed neglecting the compression area in the web using the equations of Fig.3.4.4

Required:

For each of these cases verify the program's printed value

Solution:

(a) The case of $n\rho$ = 0.5%, be/bw = 5, hf/d = 0.35

-For the exact Icr:

assume the neutral axis to fall in the flange. Thus,from the equations of Fig.3.3.2 with $n\rho$ relative to be,

$$x=[-0.5 + \sqrt{(0.5^2 + 200(0.5))}](d/100) = 0.0951249d$$

Because x/d < hf/d, the neutral axis therefore falls in the flange as assumed.

Hence,

$$Icr=[100(0.0951249d)^3/3 + 0.5d(d - 0.0951249d)^2](be/100)$$

$$=0.0043809151\ bed^3$$

$$=0.0043809151(5bwd^3)$$

or

$$Icr / bwd^3 = 0.0219045$$

-For Icr when the compression area in the web is ignored:

Because the neutral axis was found to fall in the flange,

$$Icr(neg.web) = Icr$$

or

$$Icr(neg.web) / bwd^3 = 0.0219045$$

-For Icre:

$$n\rho(\text{relative to bw}) = 0.5(5) = 2.5\%$$

$$\alpha f = \min[0.9,(1+8(0.35))/3] = 0.9. \qquad \text{(from Eq.3.4.2)}$$

$$b' = [0.9(4) + 1]bw = 4.6\ bw \qquad \text{(from Eq.3.4.1)}$$

For $n\rho bw/b'=2.5/4.6=0.543478\%$, $\alpha+\beta n\rho bw/b'=0.0546304$. Thus,

$$Icre = 0.0546304(4.6bwd^3/12) \qquad \text{(from Eq.3.4.3)}$$

or

$$Icre/bwd^3= 0.0209416$$

-For Ig:

Substituting into the equations of Fig.3.4.7 with d/h=0.72,

$$xg=\{0.5(5)0.35^2 + 0.5[(1/0.72)^2 - 0.35^2]\}d/\{5(0.35)+1/0.72 - 0.35\}$$

$$=0.4336874d$$

$$Ig=[(5/12)(0.35)^3+5(0.35)(0.4336874-0.5(0.35)^2)+(1/0.72-0.4336874)^3/3$$

$$+(0.4336874 - 0.35)^3/3](bwd^3) = 0.4256802\ bwd^3$$

-Therefore

Icre/Icr=0.956	(printed value:0.956)
Icr(neg.web)/Icr=1.0	(printed value:1.0)
nρb/b′ (relative to bw) =0.54%	(printed value:0.54)
Ig/Icr=19.43	(printed value:19.43)

(b)The case of nρ=8%, be/bw=10, hf/d=0.20

-For the exact Icr:

Assume the neutral axis to fall in the flange. Thus from the equations of Fig.3.3.2 with nρ relative to be,

$$x=[-8+\sqrt{(8^2 + 200(8))}](d/100)=0.3279215d$$

Because x/d > hf/d, the neutral axis therefore falls in the web and the above assumption is not valid. Hence the equations of Fig.3.4.3 must be used,

nρ (relative to bw) = 8(10) = 80%

$$b = 2(0.2d)[10 -1 + 80/(100(0.2))] = 5.2d$$

$$c = (0.2d)^2[10 -1 + (80/50)/(0.2^2)] = 1.96d^2$$

Hence,

$$x=[-5.2 + \sqrt{(5.2^2 + 4(1.96))}](d/2) = 0.3529646d$$

Thus,

$$Icr=[100(10)(0.2)^3/3 + (100/3)(0.3529646 - 0.2)^3 +100(10)(0.2)$$

$$(0.3529646)(0.3529646 - 0.2) + 80(1-0.35296646)^2](bwd^3/100)$$

or

$$Icr/bwd^3 = 0.4707657$$

-For Icr when compression area in the web is ignored:

Using the equations of Fig.3.4.4,

$$x=[80 + 50(10)0.2^2](d)/[80+100(10)0.2] = 0.3571428d$$

Hence,

$$Icr(neg.web) = [100(10)(0.2)^3/12 + 100(10)(0.2)(0.3571428 -0.2/2)^2$$

$$+80(1-0.3571428)^2](bwd^3/100) = 0.4695238\ bwd^3$$

-For Icre:

$$\alpha f=\min[(1+8(0.2))/3\ ,\ 0.9] = 0.8667 \qquad \text{(from Eq.3.4.2)}$$

$$b'=[0.8667(9)+1]bw=8.8bw \qquad \text{(from Eq.3.4.1)}$$

For $n\rho bw/b'=80/8.8=9.0909\%$, $\alpha + \beta n\rho bw/b'=0.614545$. Therefore,

$$Icre/bwd^3 = 0.614545(8.8/12) = 0.4506663 \qquad \text{(from Eq.3.4.3)}$$

-For Ig:

Substituting into the equations of Fig.3.4.7 with d/h=0.72,

$$xg=\{0.5(10)0.2^2+0.5[(1/0.72)^2 - 0.2^2\ ]\}(d)\ /\ \{10(0.2)+1/0.72 - 0.2\}$$

$$=0.3589044d$$

Thus,

$$Ig=[(10/12)(0.2)^3+(10)(0.2)(0.3589044-0.5(0.2))^2+(1/0.72-0.3589044)^3/3$$

$$+ (0.3589044-0.2)^3/3](bwd^3) = 0.506293\ bwd^3$$

-Therefore,

Icre/Icr=0.957 (printed value:0.957)

Icr(neg.web)/Icr = 0.997 (printed value:0.997)

$n\rho b/b'$ (relative to bw) =9.09% (printed value:9.09)

$Ig/Icr = 1.08$ (printed value:1.08)

(c) The case of $n\rho$=14%, be/bw=3, hf/d=0.11

-For the exact Icr:

Assume the neutral axis to fall in the flange. Thus from the equations of Fig.3.3.2 with $n\rho$ taken relative to be,

$$x = [-14+\sqrt{(14^2+200(14))}](d/100) = 0.4073572\ d$$

Because x/d > hf/d the neutral axis therefore falls in the web and the above assumption is not valid. Thus the equations of Fig.3.4.3 must be used,

$$n\rho \text{ (relative to bw)} = 14(3) = 42\%$$

$$b = 2(0.11d)[3 -1 + 42/(100(0.11))] = 1.28\ d$$

$$c = (0.11d)^2[3 -1 + 42/(50(0.11^2))] = 0.8642\ d^2$$

Thus,

$$x = [-1.28 + \sqrt{(1.28^2 + 4(0.8642))}](d / 2) = 0.4886274\ d$$

Hence,

$$Icr=[100(3/3)(0.11)^3+(100/3)(0.4886274 - 0.11)^3+100(3)(0.11)$$
$$(0.4886274)(0.4886274 - 0.11)+42(1-0.4886274)^2](bwd^3/100)$$

or

$$Icr/bwd^3 = 0.1903075$$

-For Icr when the compression area in the web is ignored:

Using the equations of Fig.3.4.4,

$$x=[42+50(3)0.11^2](d)/[42+100(3)0.11] = 0.5842\ d$$

Thus,

$$Icr(neg.web) = [100(3/12)(0.11)^3+100(3)(0.11)(0.5842-0.11/2)^2$$

$$+42(1-0.5842)^2](bwd^3/100) = 0.1653637\ bwd^3$$

-For Icre:

$$\alpha f = \min[(1+8(.11))/3\ ,\ 0.9] = 0.6267 \qquad \text{(from Eq.3.4.2)}$$

$$b' = [0.6267(2)+1]bw = 2.2533\ bw \qquad \text{(from Eq.3.4.1)}$$

For $n\rho bw/b'=42/2.2533=18.639\%$, $\alpha + \beta n\rho bw/b'=1.05917$. Therefore,

$$Icre/bwd^3 = 1.05917(2.2533/12) = 0.1988856 \qquad \text{(from Eq.3.4.3)}$$

-For Ig:

Using the equations of Fig.3.4.7 with d/h=0.72,

$$xg=\{0.5(3)0.11^2 + 0.5[(1/0.72)^2 - 0.11^2]\}(d)/\{3(0.11)+1/0.72-0.11\}$$

$$= 0.6070066\ d$$

$$Ig=[(3/12)(0.11)^3+3(0.11)(0.6070066-0.5(0.11))^2+(1/0.72-0.6070066)^3/3$$

$$+(0.6070066-0.11)^3/3](bwd^3) = 0.3011421\ bwd^3$$

-Therefore:

Icre/Icr = 1.045	(printed value:1.045)
Icr(neg.web)/Icr = 0.869	(printed value:0.869)
$n\rho b/b'$ (relative to bw) = 18.64%	(printed value:18.64)
Ig/Icr = 1.58	(printed value:1.58)

The ratio of Icr when the compression area in the web is ignored to the exact Icr that is found above amounts to an error of -13%. This error is relatively high which indicates that the neglect of the compression area in the web can not always be used to approximate Icr. In fact, in cases of high reinforcement ratios and low be/bw and hf/d ratios the errors may even be in the order of -30 to 40% which are unacceptably high. The next case is a typical example of such a condition.

(d) The case of $n\rho$=22%, be/bw=1.5 and hf/d=0.10:

-For the exact Icr:

Assume the neutral axis to fall in the flange. Thus from the equations of Fig.3.3.2 with $n\rho$ taken relative to be,

$$x=[-22 + \sqrt{(22^2 + 200(22))}](d/100) = 0.4788562\ d$$

Because x/d > hf/d the neutral axis therefore falls in the web and the above assumption is not valid. Thus the equations of Fig.3.4.3 must be used,

$$n\rho \text{ (relative to bw)} = 22(1.5) = 33\%$$

$$c = (0.1d)^2[1.5 -1 + (33/50)/(0.1)^2] = 0.665\ d^2$$

$$b = 2(0.1d)[1.5 - 1 + 33/(100(0.1))] = 0.76\ d$$

Hence,

$$x=[-0.76+\sqrt{(0.76^2+4(0.665))}](d/2) = 0.5196666\ d$$

Thus,

$$Icr = [100(1.5)(0.1)^3/3+(100/3)(0.5196666-0.1)^3+100(1.5)(0.1)(0.5196666)(0.5196666-0.1)+33(1-0.5196666)^2](bwd^3/100)$$

$$= 0.1339879\ bwd^3$$

-For Icr when the compression area in the web is ignored:

Using the equation Fig.3.4.4,

$$x=[33+50(1.5)(0.1)^2](d) / [33+100(1.5)0.1] = 0.703125\ d$$

Hence,

$$Icr(neg.web)=[100(1.5/12)(0.1)^3+100(1.5)(0.1)(0.703125-0.1/2)^2$$

$$+33(1-0.703125)^2](bwd^3/100) = 0.0931953\ bwd^3$$

-For Icre:

$\alpha f = \min[(1+8(0.1))/3\ ,\ 0.9] = 0.6$ (from Eq.3.4.2)

$b'=[0.6(0.5)+1](bw) = 1.3\ bw$ (from Eq.3.4.1)

For $n\rho bw/b'=33/1.3=25.384615$, $\alpha + \beta n\rho bw/b'=1.26154$. Therefore,

$Icre/bwd^3 =1.26154(1.3/12)=0.1366666$ (from Eq.3.4.3)

-For Ig:

Using the equations of Fig.3.4.7,

$$xg=\{0.5(1.5)(0.1)^2+0.5[(1/0.72)^2 - (0.1)^2](d) / [1.5(0.1)+1/0.72 - 0.1]$$

$$=0.6720506\ d$$

Hence,

$$Ig=[(1.5/12)(0.1)^3+1.5(0.1)(0.6720506-0.5(0.1))^2+(1/0.72-0.6720506)^3/3$$

$$+(0.6720506-0.1)^3/3](bwd^3) = 0.2433507\ bwd^3$$

-Therefore:

Icre/Icr = 1.02 (printed value:1.02)

Icr(neg.web)/Icr = 0.696 (printed value:0.696)

$n\rho b/b'$ (relative to bw) =25.38% (printed value:25.38)

Ig/Icr = 1.82 (printed value:1.82)

3.5 Doubly Reinforced Flanged Sections

Because of their large flange areas which are available to take compressive stresses flanged sections are usually capable of resisting large moments without the need for compression reinforcement. Nevertheless, one should not rule out the possibility of having a flanged section with steel bars running in the flange for reasons other than added capacity. For example in positive moment regions with bars from the negative moment regions extending throughout the flange or when bars are provided in the flange as a mean of supporting web reinforcements (shear reinforcement) or to reduce creep and shrinkage deflections. If these bars are well anchored such that they attain their full capacity at the section under consideration they must then be accounted for in the evaluation of Icr. This will be studied in this section.

As the expression of b' for doubly reinforced flanged sections must be compatible with those obtained previously for doubly reinforced rectangular sections and singly reinforced flanged sections, the following equation is proposed for b',

$$b'=[\alpha' n\rho'(d/d') + \alpha f(be/bw - 1) + 1]\,(bw) \qquad (3.5.1)$$

where α' and αf are as given in Eqs.3.3.7 or 3.3.8 and 3.4.2, respectively.
Once b' is obtained from Eq.3.5.1 Icre can then be evaluated using Eq.3.4.3 with α and β factors determined from Eq.3.2.2 for $n\rho bw/b'$. As with the previous cases $n\rho$ and $n\rho'$ used to evaluate Icre are taken relative to bw.

When $n\rho'=0$ Eq.3.5.1 reduces to that for singly reinforced flanged sections as given by Eq.3.4.1. On the other hand, if be=bw the equation will then reduce to that for doubly reinforced rectangular sections as discussed in Sec.3.3. Obviously when $n\rho'=0$ and be=bw then b′ as given by the equation reduces to the trivial condition of b′=bw.

Because of this compatibility with the previously proposed equations which were derived based upon intensive studies and their accuracy confirmed over the full range of $n\rho$, $n\rho'$, d′/d, be/bw and hf/d a separate detailed analysis for the confirmation of the accuracy of Eq.3.5.1 was thought unnecessary and only a numerical example is given instead. In the example sections with different combinations of $n\rho$, $n\rho'$, d′/d, be/bw and hf/d are considered. For each section the exact Icr is computed using the equations of Fig.3.5.1. The exact Icr thus found is then compared with the value of Icre found using Eq.3.4.3 with b′ evaluated from Eq.3.5.1.

Example 3.5.1

Determine the exact Icr and the approximate Icr, that is Icre using Eq.3.5.1, and compare the two for the following cases :

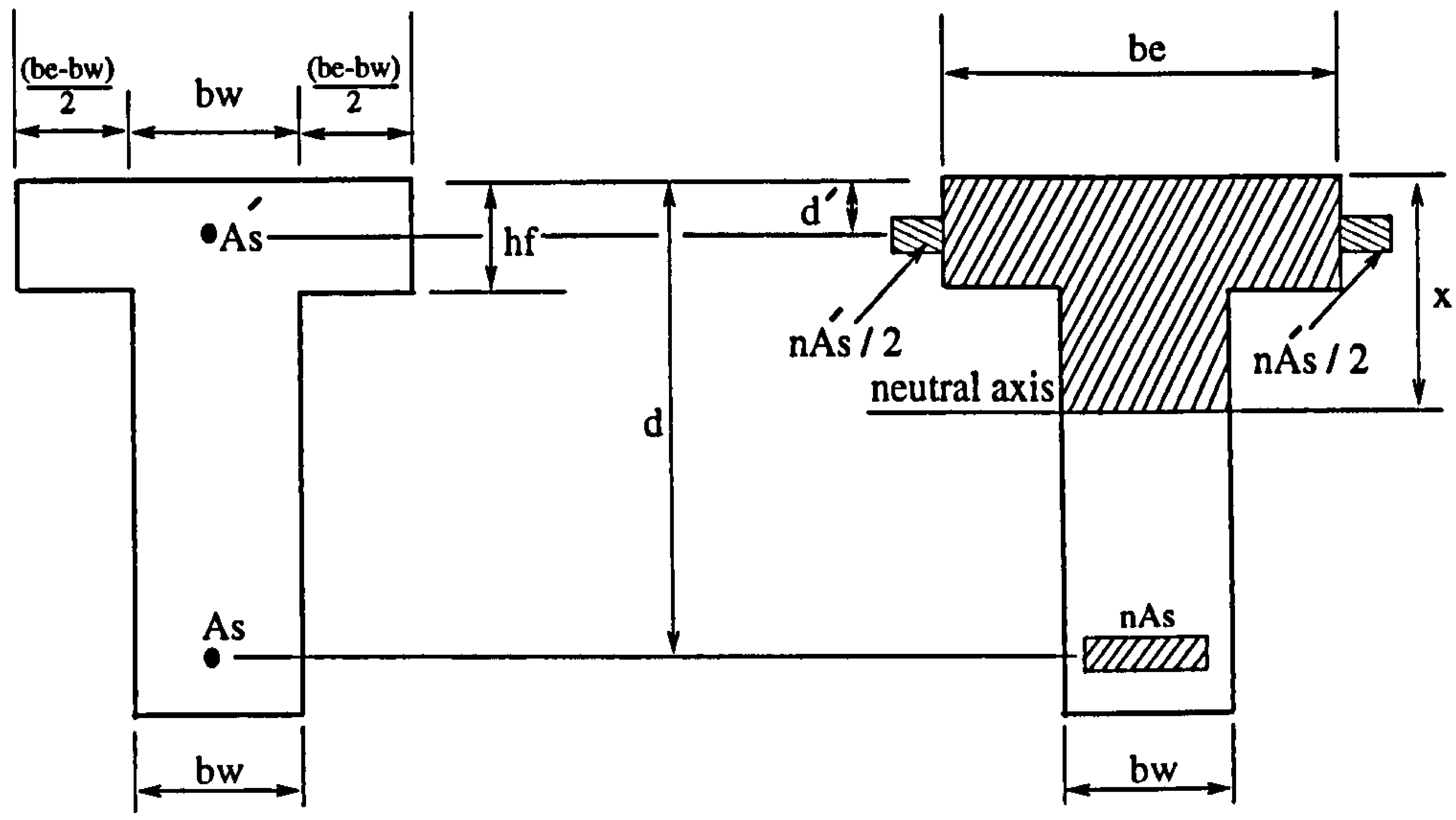
(a) $n\rho=15.09\%$, $n\rho'=4.12\%$. d′/d=0.15, be/bw=5.8 and hf/d=1/3

(b)$n\rho=4.5\%$, $n\rho'=4.5\%$, d′/d=0.05, be/bw=5 and hf/d=0.1

(c) $n\rho=48\%$, $n\rho'=36\%$, d′/d=0.10, be/bw=6 and hf/d=0.13

(d) $n\rho=16\%$, $n\rho'=8\%$,d′/d=0.08, be/bw=4 and hf/d=0.14

where all $n\rho$ and $n\rho'$ are relative to bw.



$n\rho = 100nA_s/b_wd$, $n\rho' = 100nA_s'/b_wd$

$b = 2h_f[b_e/b_w - 1 + (n\rho/100)/(h_f/d) + (n\rho'/100)/(h_f/d)]$

$c = h_f^2\{b_e/b_w - 1 + (2n\rho'/100)/[(h_f/d)(h_f/d')] + (2n\rho/100)/(h_f/d)^2\}$, $x = (-b + \sqrt{b^2 + 4c})/2$

$$I_{cr} = [(100/3)(b_e/b_w)(h_f/d)^3 + (100/3)(x/d - h_f/d)^3 + (100b_e/b_w)(h_f/d)(x/d)(x/d - h_f/d)$$
$$+ n\rho(1 - x/d)^2 + n\rho'(x/d - d'/d)^2](b_wd^3)/100$$

Figure 3.5.1 The cracked transormed section of a doubly reinforced flanged section and the pertaining equations

Solution

(a) The case of $n\rho$=15.09%, $n\rho'$=4.12%, d'/d=0.15, b_e/b_w=5.8 and h_f/d=1/3:

-For the exact Icr:

Assume the neutral axis to fall in the flange. Thus,

$n\rho$(relative to be) = 15.09/5.8=2.60%

$n\rho'$(relative to be)=4.12/5.8=0.71%

$n\rho+n\rho'$(relative to be)=2.60+0.71=3.31%

Hence, from the equations of Fig.3.3.2,

$$x=\{-3.31+\sqrt{[(3.31)^2+200(2.6+0.71(0.15))]}\}\,(d/100)=0.202d$$

Because x/d < hf/d the neutral axis therefore falls within the flange as assumed. Thus,

$$Icr=[100(0.202d)^3/3+0.71d(.202d-0.15d)^2+2.6d(d-0.202d)^2]$$

$$(5.8bw/100) = 0.1121\ bwd^3$$

-For Icre:

$\alpha'=6x10^{-4} + 0.15[1-2(0.15)]^2/20 = 0.0043$ (from Eq.3.3.8)

$\alpha f=\min[(1+8(0.333))/3\ ,\ 0.9] = 0.9$ (from Eq.3.4.2)

$b'=[0.0043(4.12)(1/0.15)+0.9(4.8)+1]bw=5.437bw$ (from Eq.3.5.1)

For $n\rho bw/b'$=15.09/5.437=2.77%, $\alpha + \beta n\rho bw/b'$=0.2439. Therefore,

$Icre=0.2439(5.437bwd^3/12) = 0.111\ bwd^3$ (from Eq.3.4.3)

-The error in calculating Icre:

% error = (Icre/Icr - 1)(100)

=(0.111/0.1121 - 1)(100) = -0.98%

In calculating Icre α' could have alternatively been taken as 0.0037. This is because d'/d of 0.15 is within the common range of 0.08 to 0.25. For such a value of α' Icre was found to be 0.111 bwd^3 which again amounts to an error of -0.98%.

(b) The case of $n\rho$=4.5%, $n\rho'$=4.5%, d'/d=0.05, be/bw=5 and hf/d=0.1:

-For the exact Icr:

Assume the neutral axis to fall within the flange. Thus,

$n\rho$(relative to be) = 4.5/5 = 0.9%

$n\rho'$(relative to be) = 4.5/5 = 0.9%

$n\rho+n\rho'$=0.9+0.9 = 1.8%

Substituting into the equations of Fig.3.3.2,

$$x=\{-1.8+\sqrt{[1.8^2 + 200(0.9+0.9(0.05))}\}(d/100) = 0.121\ d$$

Because x/d > hf/d the neutral axis therefore falls in the web and the above assumption is not valid. Thus the equations of Fig.3.5.1 must be used,

$$b=2(0.1d)[5-1+(4.5/100)/0.1+(4.5/100)/0.1] = 0.98\ d$$

$$c=(0.1d)^2[5-1+2(4.5/100)/(0.1(2))+2(4.5/100)/(0.1^2)] = 0.1345d^2$$

Thus,

$$x = [-0.98+\sqrt{(0.98^2 + 4(0.1345))}](d / 2) = 0.122d$$

Hence,

$$Icr = [100(5/3)(0.1)^3+(100/3)(0.122-0.1)^3+100(5)(0.1)(0.122)(0.122-0.1)+4.5(1-0.122)^2+4.5(0.122-0.05)^2](bwd^3/100)=0.0379\ bwd^3$$

-For Icre:

$\alpha'=6x10^{-4} + 0.05[1-2(0.05)]^2/20=0.0026$ (from Eq.3.3.8)

$\alpha f=\min[(1+8(.1))/3\ ,\ 0.9] = 0.6$ (from Eq.3.4.2)

$b'=[0.0026(4.5)(1/0.05)+0.6(4)+1]bw=3.636bw$ (from Eq.3.5.1)

For $n\rho bw/b'$=4.5/3.636=1.24%, $\alpha + \beta n\rho bw/b'$=0.1208. Therefore,

$Icre=0.1208(3.636bwd^3/12)=0.0366\ bwd^3$ (from Eq.3.4.3)

-For the % error in Icre:

% error=(Icre/Icr - 1)(100) = (0.0366/0.0379 - 1)(100) = -3.4%

(c) The case of $n\rho$=48%, $n\rho'$=36%, d'/d=0.10, be/bw=6 and hf/d=0.13:

-For the exact Icr:

Assume the neutral axis to fall in the flange. Thus,

$n\rho$(relative to be) = 48/6=8%

$n\rho'$(relative to be) = 36/6=6%

$n\rho+n\rho'$=8+6=14%

Substituting into the equations of Fig.3.3.2,

$$x=\{-14+\sqrt{[14^2+200(8+6(0.1))]}\}(d/100) = 0.298\ d$$

Because x/d > hf/d the neutral axis falls in the web and the above assumption is therefore not valid. Thus the equations of Fig.3.5.1 has to be used,

$$b=2(0.13d)[6-1+(48/100)/0.13+(36/100)/0.13]=2.98\ d$$

$$c=(0.13d)^2[6-1+2(36/100)/(0.13(1.3))+2(48/100)/(.13^2)] = 1.1165\ d^2$$

Thus,

$$x = [-2.98+\sqrt{(2.98^2+4(1.1165))}]/2 = 0.3366\ d$$

Hence,

$$Icr = [100(6/3)(0.13)^3 + (100/3)(0.3366-0.13)^3+600(0.13)(0.3366)(0.3366-0.13)+48(1-0.3366)^2+36(0.3366-0.10)^2](bwd^3/100)$$

$$=0.293\ bwd^3$$

-For Icre:

$$\alpha' = 6\text{x}10^{-4} +0.1[1-2(0.1)]^2/20 = 0.0038 \qquad \text{(from Eq.3.3.8)}$$

$$\alpha f = \min[(1+8(.13))/3\ ,\ 0.9] = 0.68 \qquad \text{(from Eq.3.4.2)}$$

$$b' = [0.0038(36/0.1)+0.68(5)+1]bw=5.768bw \qquad \text{(from Eq.3.5.1)}$$

For $n\rho bw/b'$=48/5.768=8.32%, $\alpha + \beta n\rho bw/b'$=0.576. Therefore,

$$Icre=0.576(5.768bwd^3/12) = 0.277\ bwd^3 \qquad \text{(from Eq.3.4.3)}$$

-For the % error in Icre:

$$\%\ error = (Icre/Icr - 1)(100)$$

$$= (0.277/0.293 - 1)(100) = -5.5\%$$

Because d′/d of 0.1 is within the common range of 0.08 to 0.25 α' could have also been taken as 0.0037. For such a value of α' Icre was found to be 0.276 bwd^3 which corresponds to an error of -5.8%.

(d) The case of $n\rho$=16%, $n\rho'$=8%, d′/d=0.08, be/bw=4 and hf/d=0.14:

-For the exact Icr:

Assume the neutral axis falls in the flange. Thus,

$$n\rho(\text{relative to be}) = 16/4 = 4\%$$

$$n\rho'(\text{relative to be}) = 8/4 = 2\%$$

$$n\rho+n\rho'=4+2 = 6\%$$

Substituting into the equations of Fig.3.3.2,

$$x=\{-6+\sqrt{[6^2+200(4+2(0.08))]}\}(d/100) = 0.2346\ d$$

Because x/d > hf/d the neutral axis therefore falls in the web and the above assumption is not valid. Thus the equations of Fig.3.5.1 have to be used,

$$b=2(0.14d)[4-1+(16/100)/0.14+(8/100)/0.14] = 1.32\ d$$

$$c=(0.14d)^2[4-1+2(8/100)/(0.14(1.75))+2(16/100)/(0.14^2)]=0.3916\ d^2$$

Hence,

$$x=[-1.32+\sqrt{(1.32^2+4(0.3916))}](d / 2) = 0.2495\ d$$

Thus,

$$Icr = [(400/3)(0.14)^3+(100/3)(0.2495-0.14)^3+400(0.14)(0.2495)$$

$$(0.2495-0.14)+16(1-0.2495)^2+8(0.2495-0.08)^2](bwd^3/100)$$

$$= 0.112\ bwd^3$$

-For Icre:

$\alpha'=6x10^{-4} + 0.08[1-2(0.08)]^2/20 = 0.0034$ (from Eq.3.3.8)

$\alpha f=\min[(1+8(0.14))/3\ ,\ 0.9] = 0.71$ (from Eq.3.4.2)

$b'=[0.0034(8/0.08)+0.71(3)+1]bw = 3.47bw$ (from Eq.3.5.1)

Thus, $n\rho bw/b'= 16/3.47=4.61\%$, $\alpha + \beta n\rho bw/b'=0.3727$. Therefore,

$Icre = 0.3727(3.47bwd^3/12) = 0.108\ bwd^3$ (from Eq.3.4.3)

-For the % error in Icre:

$$\%\ error = (Icre/Icr - 1)(100)$$

$$= (0.108/0.112 - 1)(100) = -3.6\%$$

Again for d'/d of 0.08 α' could have been alternatively taken as 0.0037. For such a value of α' Icre was found to be 0.108 bwd^3 which amounts to an error of -3.6%

3.6 Summary

In the preceding sections different equations have been developed as part of the model for the approximation of Icr. Examples were also given to illustrate the applications of these equations.

Although the model was intended to be used in the evaluation of the effective moment of inertia as will be discussed in the next chapter it can also be used in any other conditions where the cracked transformed moment of inertia is involved.

For ease of reference and to provide a general framework through which the equations involved can be easily understood an overall outline of the developed model is presented in this section in the form of the following summary:

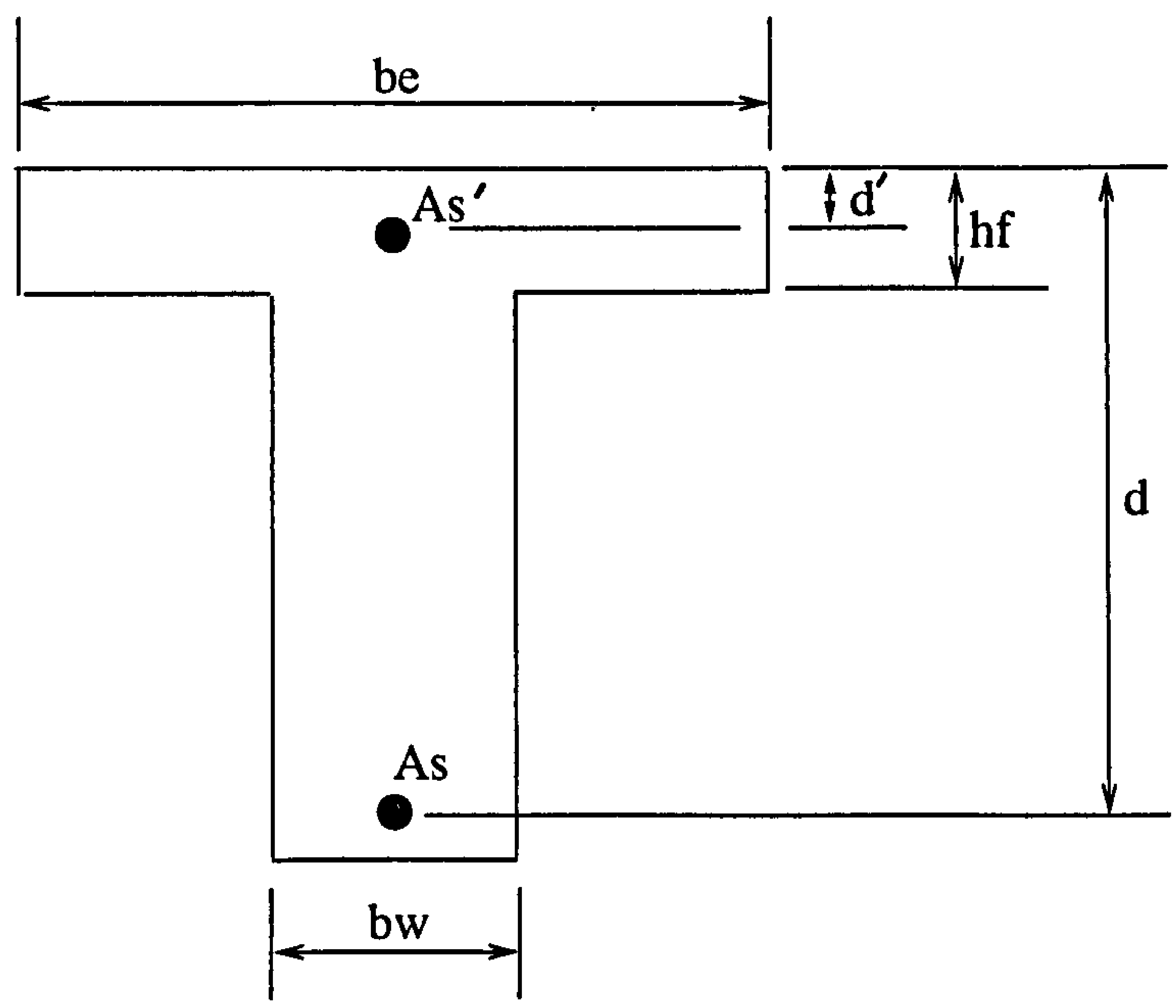


Figure 3.6.1. A general beam section :
- for rectangular sections be=bw
- for singly reinforced sections Ás=0

For the general cross section shown in Fig.3.6.1, Icr can be approximated as Icre given by Eq.3.6.1 below

$$Icre = (\alpha + \beta n\rho e) (b'd^3/12) \qquad (3.6.1)$$

where,

$n\rho e = n\rho bw/b'$

$b' = [\, \alpha' n\rho'(d/d') + \alpha f(be/bw - 1) + 1 \,] bw$

$\alpha' = 6x10^{-4} + (d'/d)(1-2d'/d)^2/20$

$\alpha f = \min[(1+8hf/d)/3 \, , 0.9]$

$n\rho = (nAs/bwd)(100)$, $n\rho' = (nAs'/bwd)(100)$

The factors α and β are as given below,

$\alpha = 0.003, \ \beta = 0.095$ for $n\rho e \leq 1.9\%$

$\alpha = 0.05, \ \beta = 0.07$ for $1.9\% < n\rho e \leq 5\%$

$\alpha = 0.16, \ \beta = 0.05$ for $5\% < n\rho e \leq 17\%$

$\alpha = 0.5, \ \beta = 0.03$ for $17\% < n\rho e \leq 32\%$

$\alpha = 0.80, \ \beta = 0.02$ for $n\rho e > 32\%$

For the usual range of d′/d from 0.08 to 0.25 α′ can also be taken simply as 0.0037 for which the expression of b′ becomes,

$$b'[0.0037n\rho'(d/d') + \alpha f(be/bw - 1) + 1]bw \quad (3.6.2)$$

When be/bw is greater than 0.55 the section should be treated as a rectangle of width be.

CHAPTER 4

THE EFFECTIVE MOMENT OF INERTIA, Ie

4.1 Introduction

When the tensile stresses in concrete exceed its modulus of rupture the concrete cracks. These tension cracks form at a finite spacing as shown in the typical simply supported beam of Fig.4.1.1

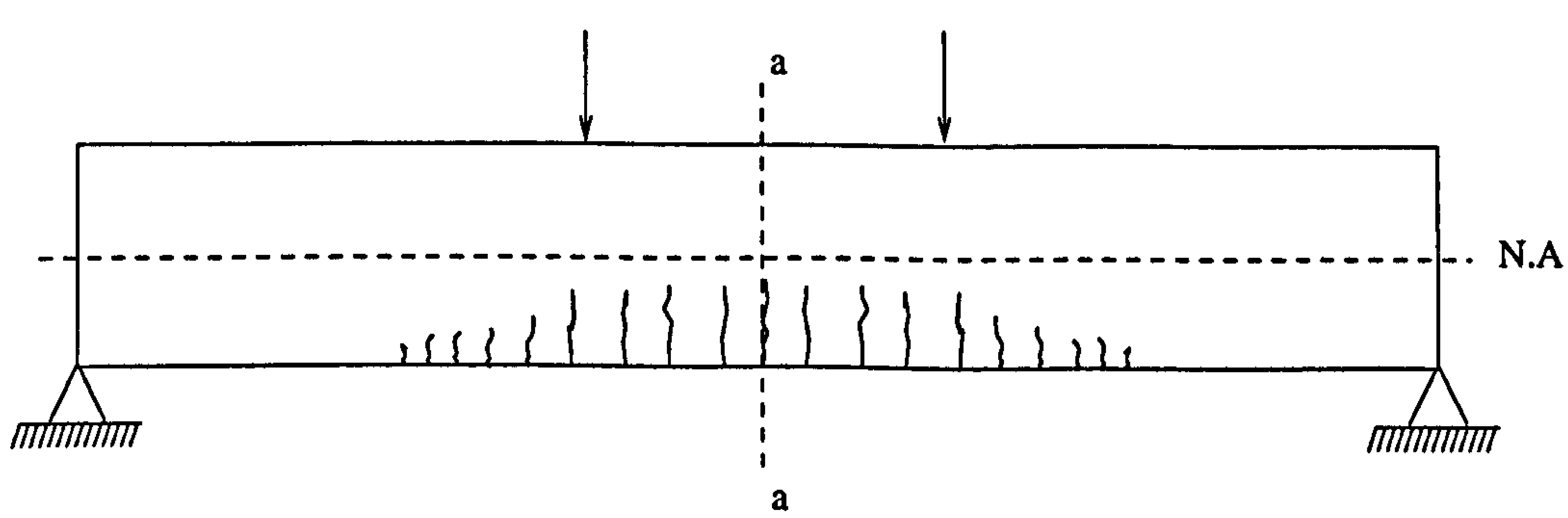


FIGURE 4.1.1 A typical flexurally cracked simply supported beam

At sections where the concrete in the tension face is completely cracked, for exampleat section a-a, tensile resistance of the section will be provided for entirely by the steel reinforcement and the moment of inertia of the section is Icr. However, between the cracks or at sections where the cracks do not propagate deep into the element concrete will still be able to take some tension and the tensile resistance of the section is therefore provided for not only by the steel but also by the uncracked concrete in the tension face. In such sections the moment of inertia will have a value greater than Icr but less than or equal to Ig. This phenomenon of the ability of concrete to take some tension is usually referred to as " tension stiffening " of concrete.

The effect of tension stiffening on the variation of the moment of inertia of a concrete element along its span is a function of many factors and is difficult to correctly predict. Therefore, it is always easier to assume a constant value for the moment of inertia throughout the span. The moment of inertia thus assumed is referred to as the effective moment of inertia and is denoted by Ie. Many empirical expressions for the evaluation of Ie have been proposed. The most widely used of these is the so called Branson's equation which has been adopted by many codes (i.e the American and the Canadian codes) where the effective moment of inertia, for deflection calculations, is expressed as follows,

$$Ie = Icr + (Ig - Icr) (Mcr/Ma)^3 \leq Ig \qquad (4.1.1)$$

in which

Mcr ≡ cracking moment = $f_r Ig/y_t$

$y_t \equiv$ distance from the neutral axis of the gross area neglecting steel to the extreme concrete fibre in tension .

Ma $\equiv$ maximum moment acting within the span or between points of inflection of the element for which Ie is evaluated.

Although Eq.4.1.1 has been the widely recognized equation for the evaluation of Ie for many years it has the following serious drawbacks :

1. It can be shown that for load types other than uniform and for $\rho < 1$ % and Ma/Mcr < 3 the equation can give results that are grossly in error. These errors can be as high as 80-100 % [4,5].
2. The evaluation of the moment of inertia of the cracked transformed section, Icr, has been a major source of contention with designers throughout the years since the equation was recognized. This is because the task of evaluating Icr is both involved and time consuming and one is likely to make mistakes especially in case of flanged sections [6].
3. Due to the complexities and nature of the expressions necessary to evaluate Icr the equation cannot easily be represented graphically through solution curves. These curves as shown in the different references [7,8] also fail to represent the phenomena involved and thus offer little as design aids [9].

Because of these disadvantages many scholars and designers are of the opinion that the equation is of minimum practical use and an alternative must therefore be found.

In 1993 and in an attempt to provide a substitute for Branson's equation, AL-Shaikh and AL-Zaid [4] have proposed the following empirical equation for Ie,

$$Ie = Ig + (Icr - Ig) (Lcr/L)^m \quad (4.1.2)$$

where,

Lcr ≡ The length of the span over which the applied moment exceeds Mcr (usually known as the cracking length)

L ≡ The span of the beam

$m = 0.8\rho Mcr/Ma$ with ρ in %

It will be shown in this study that although Eq.4.1.2 may prove to be slightly more accurate than Branson's equation it still gives errors as high as 80 % in cases of low reinforcement ratios and Ma/Mcr < 4. In addition, because no attempt is made to approximate Icr the equation remains unpractical and can not lead itself into a simple graphical representation and thus does not actually offer much advantages over Branson's equation.

In this chapter and for the purpose of calculating deflection in reinforced concrete beams an alternative to Branson's equation will be developed for the evaluation of Ie where the forementioned drawbacks are eliminated.

The background philosophy of the developed equation is that :

1. The models of Icre obtained in Chap. 3 should be easily incorporated to eliminate the need for detailed calculation of Icr.
2. All the parts that may have to be empirically determined should be consolidated into a single coefficient the expression of which is independent of the equation's format. This has the following advantages :
   a. The form of the equation remains the result of a pure theoretical development and will therefore be always valid. Thus, if a new study is launched in the

future one need not start from scratch but instead only the expression of this coefficient can be modified.

b. Any graphical representation of the equation will be independent of how this coefficient is expressed.

3. The equation should provide a quick and efficient way of evaluating Ie and should lead itself well into a "single page" graphical representation that can be used as a design aid similar to those usually provided for other design purposes in the respective codes. The gragh should be simple and yet applicable to all usual cases.

4.2 The Expression of Ie

In this section a general expression for the effective moment of inertia will be developed considering first the basic case of a simply supported beam with a singly reinforced rectangular section throughout the span. The expression thus found will then be extended to include other sectional geometries and span conditions.

Consider the simply supported beam of Fig.4.1.1 with a prismatic singly reinforced rectangular section. If the moment of inertia is assumed constant throughout the span and always greater than or equal to the value of Icr at the section of maximum moment the effect of concrete stiffening discussed earlier can be thought of as a fictitious steel area which has been added to the steel reinforcement actually provided at the section of maximum moment such that the cracked transformed moment of inertia of the equivalent section is equal to the effective moment of inertia.

Figure 4.2.1 explains the idea where the cracked transformed section shown is assumed throughout the span.

Based on the above assumption and in accordance with Eq.3.2.2 one can write,

$$Ie = (\alpha + \beta n\tilde{\rho}) (bd^3 /12) \qquad (4.2.1)$$

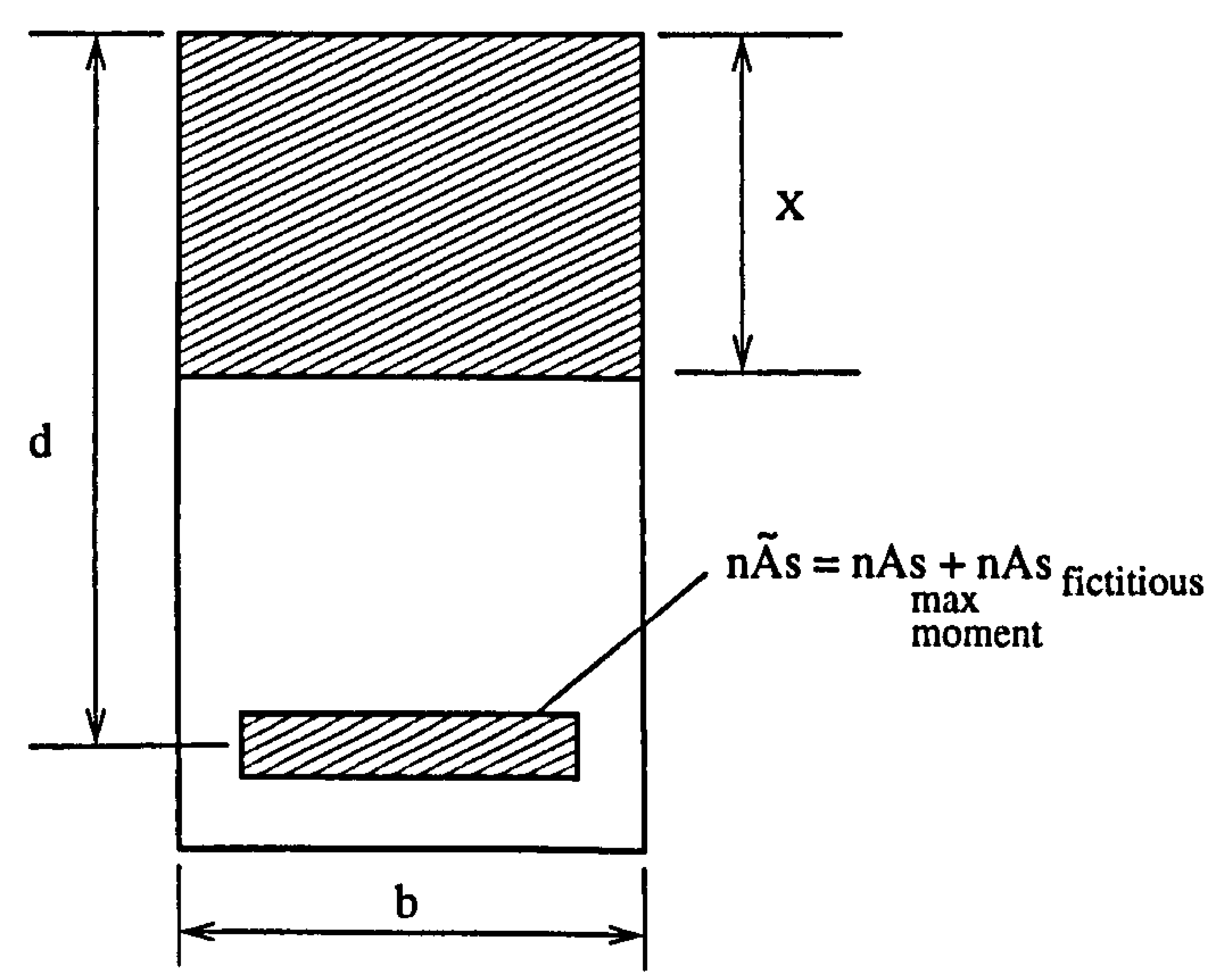


Figure 4.2.1 The cracked transfomed section representing concrete stiffening effect

where,

$\rho^{\sim} = \rho$ at the section of maximum moment + fictitious steel area representing concrete stiffening effect

When Ma/Mcr = 1 the element at the section of maximum moment will be on the verge of cracking but cracks will not have yet developed. At this stage $\rho^{\sim}$ must be such that Eq.4.2.1 yields an effective moment of inertia equal to Ig. Such a value of $\rho^{\sim}$ will be defined as $\rho_0^{\sim}$. As Ma/Mcr increases cracks start to develop and the effective moment of inertia starts to decrease. Because $bd^3/12$ remains constant and that α and β are uniquely defined for the same $n\rho$, Eq.4.2.1 implies that any decrease in Ie must also correspond to a decrease in the value of $\rho^{\sim}$. Therefore, as Ie starts to decrease with increasing Ma/Mcr values so does $\rho^{\sim}$. When eventually large values of Ma/Mcr are reached the effective moment of inertia will gradually converge to Icr which means that $\rho^{\sim}$ must also converge to ρ as required by the equation.

A probable curve representing the above relation between Ma/Mcr and $\rho^{\sim}$ is shown in Fig.4.2.2. As can be seen from the figure the slope of the $\rho^{\sim}$ curve is not constant but varies as $\rho^{\sim}$ changes with respect to Ma/Mcr to give a smooth and gradual convergence to ρ at larger values of Ma/Mcr and as required to correspond to the convergence of Ie to Icr. Therefore , and if Ma/Mcr is referred to for simplicity as t on can write,

$$d\rho^{\sim}/dt = k_1 + k_2\rho^{\sim} \qquad (4.2.2)$$

where $\rho^{\sim} = f(\text{Ma/Mcr}) = f(t)$. As $\rho^{\sim}$ converges to ρ at larger values of Ma/Mcr the

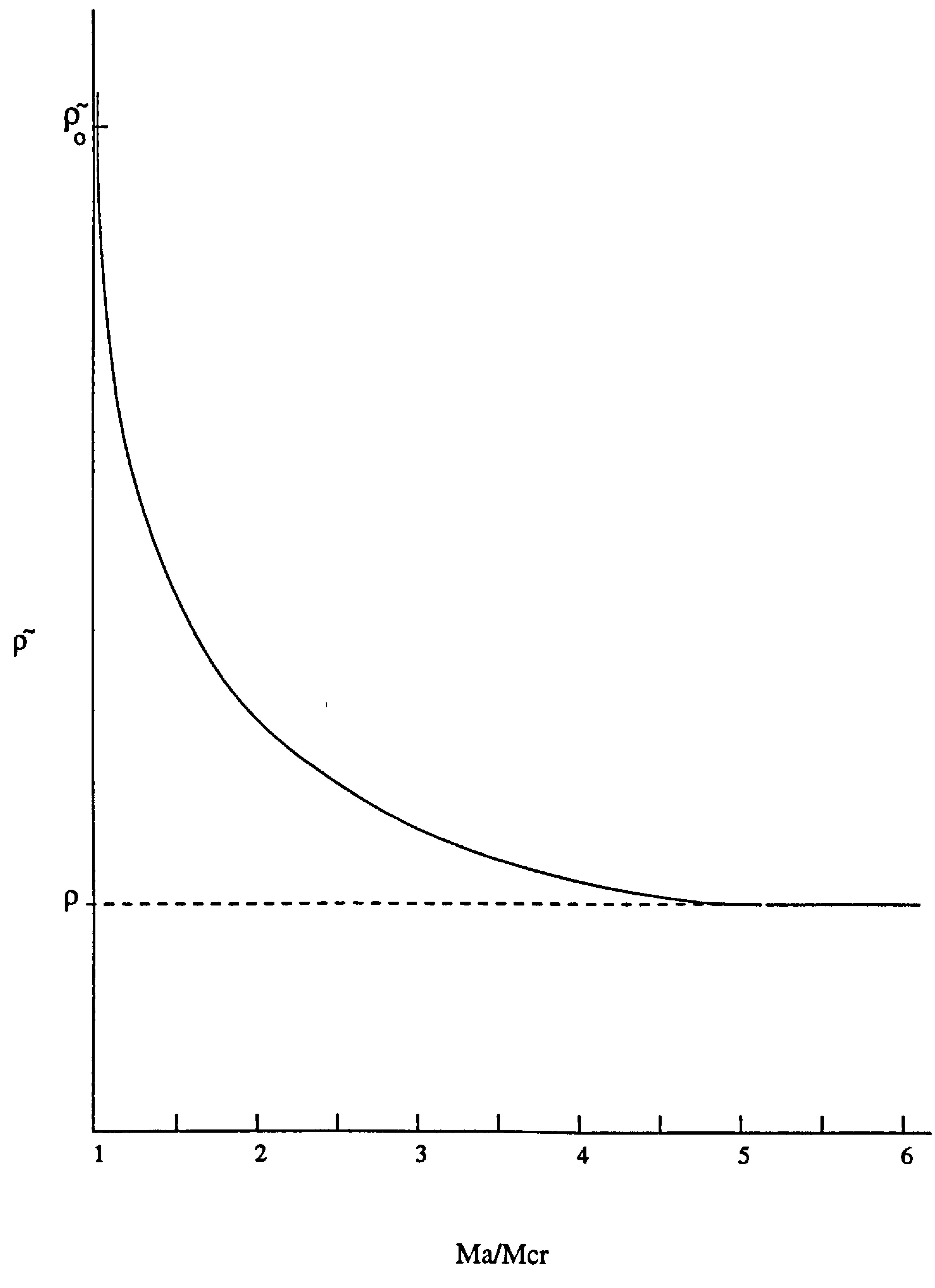


Figure 4.2.2 Variation of $\rho^{\sim}$ with Ma/Mcr

slope of the $\rho^{\sim}$ curve approaches zero. Thus,

$$d\rho^{\sim}/dt = k_1 + k_2\,\rho = 0$$

or,

$$k_1 = -\,k_2\,\rho$$

Back substituting into Eq.4.2.2 will therefore give

$$d\rho^{\sim}/dt = -\,k_2\,\rho + k_2\,\rho^{\sim} = k_2\,(\,\rho^{\sim} - \rho\,)$$

separating the variables and integrating,

$$\int_{\rho_o^{\sim}}^{\rho^{\sim}} [\,1\,/\,(\,\rho^{\sim} - \rho\,)\,]\,d\rho^{\sim} = \int_{1}^{t} k_2\,dt$$

or

$$\ln(\,\rho^{\sim} - \rho\,)\Big|_{\rho_o^{\sim}}^{\rho^{\sim}} = k_2\,(\,t\,)\Big|_{1}^{t}$$

Thus,

$$\ln(\,\rho^{\sim} - \rho\,) - \ln(\,\rho_o^{\sim} - \rho\,) = k_2\,(\,t-1\,)$$

or,

$$\text{In}[\,(\,\rho^{\sim} - \rho\,)\,/\,(\,\rho_o^{\sim} - \rho\,)\,] = k_2\,(\,t - 1\,)$$

Taking the exponential " e " of both sides gives,

$$(\,\rho^{\sim} - \rho\,)\,/\,(\,\rho_o^{\sim} - \rho\,) = e^{k2(t-1)}$$

or,

$$\rho^{\sim} = \rho + (\,\rho_o^{\sim} - \rho\,)\,e^{k2(t-1)}$$

Redefining k_2 as c and restoring t as Ma/Mcr the above equation will therefore become

$$\rho^{\sim} = \rho + (\,\rho_o^{\sim} - \rho\,)\,e^{c(\,Ma/Mcr - 1\,)} \qquad (\,4.2.3\,)$$

By introducing the term $\Phi = c\,(\,Ma/Mcr - 1\,)$, Eq.4.2.3 will therefore simplify to,

$$\rho^{\sim} = \rho + (\,\rho_o^{\sim} - \rho\,)\,e^{\Phi} \qquad (\,4.2.4\,)$$

Because of the descending or negative slope of $\rho^{\sim}$ curves the factor c and hence the coefficient Φ must always be negative for all values of Ma/Mcr grater than 1.0 as is dictated by Eqs.4.2.3 and 4.2.4. This is also consistent with the trivial condition that when Ma/Mcr is large $\rho^{\sim}$ converges to ρ.

From Eqs.4.2.3 it can be noticed that when Ma = Mcr , $\rho^{\sim}$ becomes equal to $\rho_o^{\sim}$. However, when Ma = Mcr the value of $\rho^{\sim}$ must be such that Eq.4.2.1 gives an effective moment of inertia equal to Ig. This means that

$$Ig = (\alpha + \beta n \rho_o^{\sim}) (bd^3/12)$$

or,

$$\rho_o^{\sim} = (12\, Ig / bd^3 - \alpha) / \beta n$$

Defining,

$$R = Ig / (bd^3/12)$$

With this definition the expression for $\rho_o^{\sim}$ will therefore become

$$\rho_o^{\sim} = (R - \alpha) / \beta n$$

Substituting Eq.4.2.4 along with the above expression of $\rho_o^{\sim}$ into Eq.4.2.1 gives,

$$Ie = (\alpha + \beta n \rho) (bd^3/12) + (R - \alpha - \beta n \rho) (bd^3/12)\, e^{\Phi} \qquad (4.2.5)$$

The expression of Eq.4.2.5 gives the effective moment of inertia for a beam with the following basic conditions :

1. The beam is simply supported

2. The beam is prismatic with singly reinforced rectangular section

The bending moment diagram of any beam can be thought of as consisting of subdiagrams depending on the number and location of the inflection points within the span. For example, the bending moment diagram for a simple beam is only one diagram since the points of inflection are located right at the supports. On the other hand, the bending moment diagram for an end span of a continuous beam can be assumed to consist of two subdiagrams while that for an interior span can be thought of as three subdiagrams and so on. If then to each of these subdiagrams the concept of concrete stiffening as used above is applied Eq.4.2.5 can be taken to represent the effective moment of inertia in the span of each subdiagram. In other words, Eq.4.2.5 can be generalized to apply not only to simply supported beams but to any beam under any loading condition

In addition, the condition of singly reinforced rectangular section can also be generalized to include all the sections that are considered in this study by simply incorporating the equivalent width method of Chap.3 into Eq.4.2.5. Doing so results in the following general form of the equation,

$$Ie = (\alpha + \beta n\rho e) (b'd^3/12) + (R - \alpha - \beta n\rho e) (b'd^3/12) e^{\Phi} \quad (4.2.6)$$

with R now defined as

$R = Ig / (b'd^3/12)$

The first part of Eq.4.2.6 is actually the expression of Icre as given by Eq.3.6.1. Realizing this and substituting the above expression of R into Eq.4.2.6 and simplifying leads to the following compact form of the equation

$$Ie = Icre + (Ig - Icre) e^{\Phi} \qquad (4.2.7)$$

Equation 4.2.7 is the expression proposed by the current study for the evaluation of Ie. Although the equation is similar in format to Branson's equation (Eq.4.1.1), except for the exponential term, it has the following advantages :

1. To calculate the value of the exact Icr used in Branson's equation involves two lengthy steps. In the first step one has to find the position of the neutral axis and then calculate the value of Icr as a second step. In flanged sections the calculations are even more complicated because the way Icr is calculated depends on whether the neutral axis falls in the flange or in the web and one has to start by assuming the position of the neutral axis and then modify the assumption if necessary.

   In the proposed equation the difficulties involved in computing Icr are eliminated by using Icre instead. As was shown in Chap.3 the task of evaluating Icre from Eq.3.6.1 is a simple matter and the equations involved are much easier to use as compared to the lengthy equations of Figs.3.3.2, 3.4.3 or 3.5.1

2. It will be shown in the next section that graphical representation of the detailed form of the proposed equation (Eq.4.2.6) makes it possible to determine Ie directly from curves that can be included on a single plot which is easy to read. This is a clear advantage over the graphical representation available for Branson's equation where one has to work through different plots in order to obtain a value for Ie. These plots as given in Refs.7 and 8 are complex as compared to the plot that will be proposed in the current study on the basis of the developed equation. In addition, as already mentioned in Chap.3 and demonstrated by the concluding example given therein the plot given for the evaluation of Icr in Ref.7 can give values that are in error of as much as 30% in case of flanged sections and thus can be regarded as being unreliable for the limits considered in this study.
3. Except for the exponential factor Φ which is yet to be determined the equation is the result of an analytical development as compared to Branson's equation which is purely empirical.
4. Based on conditions dictated by Eqs.4.2.3 and 4.2.4 and as related to Fig.4.2.2 an expression for Φ is proposed in Sec.4.4. Besides expressing the intensity of the applied load the expression will also consider the loading type and the effect of the reinforcement which Branson's equation does not consider. This expression when used into Eq.4.2.7 will prove to give results that are more accurate and consistent than when Branson's equation is applied.

Equation 3.6.1 used to evaluate Icre in the proposed equation of Ie can be applied to sections that are either rectangular or flanged. Most elements of concrete structures that are usually investigated for deflection are of these geometries. If, however, an element of unusual sectional geometry is encountered then Eq.3.6.1 can not be used. In this case the proposed equation can be applied using Icr instead of Icre. Although this will eliminate the advantages of not having to calculate Icr the equation will still retain its superiority over Branson's for considering the effect of the loading type and reinforcement incorporated in the expression of Φ.

The limitation of Ig imposed on Branson's equation is intended to prevent Ie>Ig when Ma<Mcr. For the proposed equation, however, such a limitation is not necessary as will be seen from the expression of Φ to be developed in Sec.4.4.

The trend in the behaviour of $\rho^{\sim}$ vs. Ma/Mcr shown in Fig.4.2.2 which is the base for the development of the proposed equation assumes that Icr<Ig. For that and because Icre is an approximation of Icr, Eq.4.2.7 applies only to conditions of Icre<Ig. At the unlikely condition of Icre>Ig it is suggested that Ie be taken equal to Icre (or Icr as has normally been the practice when Branson's equation is used).

4.3 Graphical Representation of Ie

Graphical representation is always found useful in physically explaining the trends in the behaviour of structural elements from which concrete members are no exception. Therefore for any developed equation to be fully useful it must be transformable into plots that are easy to read and can best represent the phenomena for which the equation has been developed. In this section it will be shown that for Eq.4.2.6 which is the detailed version of the equation that has been proposed for the evaluation of Ie

(Eq.4.2.7) such a graphical representation is easily obtained.

Dividing through by $b'd^3/12$ in Eq.4.2.6,

$$Ie / (b'd^3/12) = \alpha + \beta n\rho e + (R - \alpha - \beta n\rho e) e^{\Phi}$$

If for ease of reference the term $Ie / (b'd^3/12)$ is referred to as γ the above equation will then become

$$\gamma = \alpha + \beta n\rho e + (R - \alpha - \beta n\rho e) e^{\Phi} \qquad (4.3.1)$$

The task remaining now is to plot the values of γ vs. Φ in accordance with the above equation .

In Eq.4.3.1 four variables are involved : the nondimensional quantity γ, the product of the modular ratio multiplied by the reinforcement ratio represented by $n\rho e$, the variable R relating sectional dimensions and finally the exponent Φ which is supposed to relate the type and intensity of loading. Despite these four variables, however, a graphical representation of γ vs. Φ in two axes rectangular coordinate system is sought.

If different curves were chosen for different $n\rho e$ values each of these curves will then represent a constant value of $n\rho e$ and the number of variables left to be considered will therefore be three. However, it will still not be possible to represent the equation by a two coordinate axes system because of the third variable unless multiaxes system is used. For example if Φ and γ were represented along the horizontal and vertical axes respectively, then different vertical axes will be required to represent different values of the third variable R.

To avoid the complexities associated with such multiaxes systems a reference condition with a specific R, referred to hereafter as R_{ref} , will be chosen for which a plot of Φ vs. γ is produced in a two axes coordinate system. Any other condition with different R value is then converted to its corresponding reference condition with R equal to R_{ref} whereby the reference plot can be used.

Because the reference plot will have different curves for different $n\rho e$ values any condition i will be converted to a corresponding condition where $n\rho e$ is kept the same. This leaves only the corresponding reference value of Φ, denoted as Φ_{ref} , to be determined such that γ read from the reference plot is exactly equal to γ value found using the parameters at condition i , namely $n\rho e$, R_i and Φ_i

Applying Eq.4.3.1 the above argument can be represented in an equation form as,

$$\alpha + \beta n\rho e + (R_i - \alpha - \beta n\rho e) e^{\Phi i} = \alpha + \beta n\rho e + (R_{ref} - \alpha - \beta n\rho e) e^{\Phi ref}$$

solving for Φ_{ref} ,

$$e^{\Phi ref} / e^{\Phi i} = (R_i - \alpha - \beta n\rho e) / (R_{ref} - \alpha - \beta n\rho e)$$

or

$$e^{\Phi ref} = (e^{\Phi i})[(R_i - \alpha - \beta n\rho e) / (R_{ref} - \alpha - \beta n\rho e)]$$

Taking the natural log of both sides,

$$\text{In}\,(e^{\Phi ref}) = \text{In}\,\{\,(e^{\Phi i})\,[\,(R_i - \alpha - \beta n\rho e)\,/\,(R_{ref} - \alpha - \beta n\rho e)\,]\,\}$$

$$= \text{In}\,(e^{\Phi i}) + \text{In}\,[\,(R_i - \alpha - \beta n\rho e)\,/\,(R_{ref} - \alpha - \beta n\rho e)\,]$$

Thus ,

$$\Phi_{ref} = \Phi_i + \text{In}\,[\,(R_i - \alpha - \beta n\rho e)\,/\,(R_{ref} - \alpha - \beta n\rho e)\,]$$

or

$$\Phi_{ref} = \Phi_i + \text{In}\,[\,(\alpha + \beta n\rho e - R_i)\,/\,(\alpha + \beta n\rho e - R_{ref})\,]$$

If for the sake of simplicity the second part of the above equation is referred to as the correction factor, CF , one can then write,

$$\Phi_{ref} = \Phi_i + CF \qquad (4.3.2)$$

where

$$CF = \text{In}\,[\,(\alpha + \beta n\rho e - R_i)\,/\,(\alpha + \beta n\rho e - R_{ref})\,]$$

If R_{ref} is properly chosen, Eq.4.3.2 can be used to transform Φ at any condition i to its corresponding reference value where the reference plot can then be entered to read the value of γ. It will be shown next that this proper selection of R_{ref} is actually a function of the condition of Icre vs. Ig and the nature of the coefficient Φ.

It was stated in the previous section that at conditions of Icre>Ig, Ie will be taken equal to Icre given be Eq.3.6.1 and thus the expression of Ie as proposed by Eq.4.2.7 will not be used. Therefore the graphical representation of the equation must be provided for the condition of Icre < Ig. Because any such graphical representation will be useless without Eq.4.3.2 and that the expression of CF involved in the equation is in terms of $\alpha + \beta n\rho e$ -R it will therefore be useful if a relationship between the condition of Icre vs.Ig and the value of $\alpha + \beta n\rho e$ - R can be drawn. To do that one may recall from Eq.3.6.1 that

$$Icre = (\alpha + \beta n\rho e) (b'd^3/12)$$

or

$$\alpha + \beta n\rho e = Icre / (b'd^3 /12) \qquad (4.3.3)$$

While from the definition of R,

$$R = Ig / (b'd^3/12) \qquad (4.3.4)$$

Thus, if Icre < Ig then from Eqs.4.3.3 and 4.3.4 it follows that

$$\alpha + \beta n\rho e < R$$

or

$\alpha + \beta n\rho e - R < 0$

Therefore, the graphical representation of Eq.4.2.7 must be provided for any condition i where $\alpha + \beta n\rho e - R_i < 0$.

Because the natural log of a negative value is undefined then for the expression of CF to be valid the condition of $\alpha + \beta n\rho e - R_i < 0$ will also require that $\alpha + \beta n\rho e - R_{ref} < 0$. To ensure this, R_{ref} must be selected to be greater than the maximum value of $\alpha + \beta n\rho e$. According to the analysis presented in chapter 3 and from the results included in Appendices B3-7 it can be seen that the maximum value of $n\rho e$ for the sections to which Eq.3.6.1 is applicable is 55.17. Thus from the intervals of α and β of Eq.3.6.1 and using the maximum value of $n\rho e$ of each intervals it is not hard to prove that,

$$(\alpha + \beta n\rho e)_{max} = 0.8 + 0.02(55.17) = 1.9$$

Therefore to ensure that $\alpha + \beta n\rho e - R_{ref} < 0$, R_{ref} must always be greater than 1.9. In addition to having to satisfy the criterion $\alpha+\beta n\rho e - R_{ref} < 0$ explained above the choice of R_{ref} must also ensure that Φ_{ref} obtained from Eq.4.3.2 is always negative. This is because positive values of Φ will not be compatible with the nature of the coefficient Φ as defined in Sec. 4.2. For this and since $\alpha + \beta n\rho e$ values for condition i and its corresponding reference condition are equal, R_{ref} must always be chosen as being greater than or equal to the maximum value possible for R_i so that the value for which the natural log is taken is always ≤ 1 and thus CF is ≤ 0 which ensures that Φ will always be negative. If one assumes the basic condition of a singly reinforced rectangular section, the definition of R will therefore simplify as follows,

$$R = Ig / (b'd^3/12) = Ig / (bd^3/12) = (bh^3/12) / (bd^3/12) = (h/d)^3$$

If then the smallest d/h ratio of 0.72 (as previously assumed) is used in the above equation, the maximum value of R_i will be obtained as,

$$(R_i)_{max} = (1/0.72)^3 = 2.7$$

In the light of the above discussion and as dictated by the requirements of the condition of Icre vs. Ig and the nature of Φ it is therefore reasonable to set $R_{ref} = 3$. Substituting this value of R_{ref} into Eq.4.3.2 gives

$$\Phi_{ref} = \Phi_i + CF \qquad (4.3.5)$$

where,

$$CF = In [(\alpha + \beta n\rho e - R_i) / (\alpha + \beta n\rho e - 3)]$$

It is worth noticing that because the maximum value of $\alpha + \beta n\rho e$ is 1.9, as previously found, the denominator of Eq.4.3.5 is always negative. It follows therefore that whenever $\alpha + \beta n\rho e - R_i > 0$, Eq.4.3.5 becomes undefined and thus will automatically signalize that for the condition considered Icre > Ig and Ie should be taken as Icre.

Prog.4.3.1 shown in Appendix C1 was developed to calculate γ values using

Eq.4.3.1 for R_{ref} of 3 and for the range of nρe from 0.12 to 56% and of Φ from 0 to -10. The results as obtained from the program are included in the appendix. The values of γ thus obtained vs. Φ are then used to produce the reference plot of Fig.4.3.1.

The values of nρe for which the curves of Fig.4.3.1 are produced cover the range over which nρe may vary. The value of 0.12% is the minimum limit as described in chap.3. The value of 56%, on the other hand, is slightly beyond the maximum nρe of 55.17% referred to earlier. For any value of nρe between 0.12 and 56% but for which no independent curve is produced interpolation relative to the curves of nρe below and above the given value should be used.

Because the curves are produced for R value of the reference condition one therefore needs to convert the given value of Φ to its corresponding value relative to the reference condition by applying the correction factor as explained earlier and as shown in the figure. Namely, the figure is entered with $\Phi_{ref} = \Phi + CF$.

It can be seen from the figure that for values of Φ_{ref} beyond -7 the curves become almost horizontally steady. As Φ will be shown to be a direct function of Ma/Mcr, the steady portions of the curves represent the condition of a constant Ie at higher values of Ma/Mcr. Because Ie is known to approach Icr as Ma/Mcr increases, the constant Ie values corresponding to the steady portions of the curves represent Icre as integrated into Eq.4.3.1. It follows therefore that although the curves are produced to determine Ie, the steady portions of these curves can also be used to find Icre. Clearly, when Icr<Ig the curves can be entered directly to read Ie and for higher values of Ma/Mcr the Ie thus read may in fact be equal to Icre. However, when Icr>Ig the correction factor, CF , will be undefined and the value of Φ_{ref} required to enter

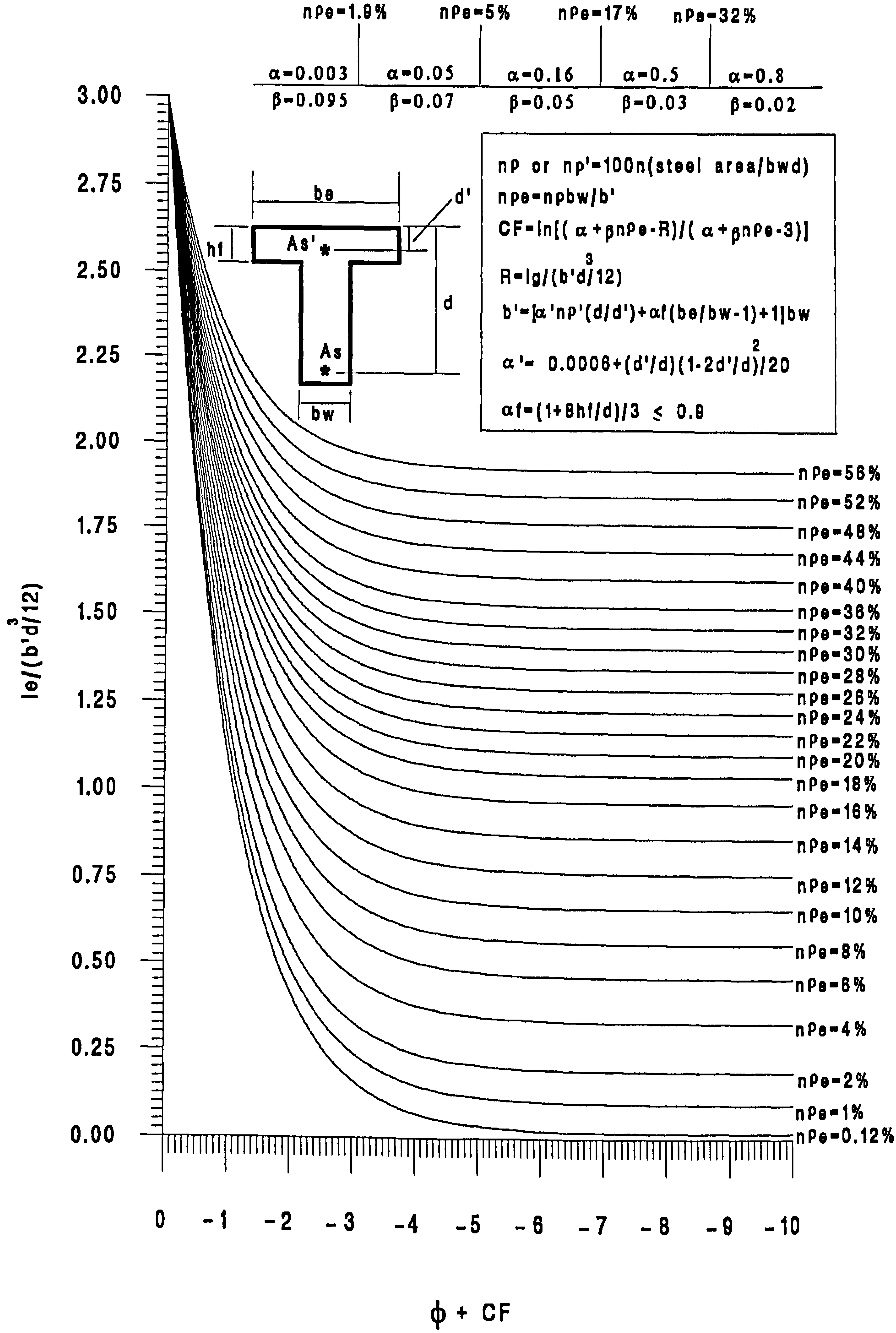


Figure 4.3.1 Graphical representation of the proposed model of Ie

Fig.4.3.1 can not be determined. In this case and in cases where only the value of Icre is needed for whatever purpose the steady portions of the curves can be entered regardless of the value of Φ_{ref} to read off the value of Icre (which will be taken as Ie in case of Icr>Ig as discussed earlier) corresponding to the respective nρe.

In the light of the above the applicability of Fig.4.3.1 can therefore be summarized as follows:

1. If the correction factor is defined the figure can be entered to read off the value of Ie for the value of $\Phi_{ref} = \Phi + CF$.
2. If the correction factor is undefined the steady portions of the curves of the figure can be entered to read off the value of Ie (which in this case is equal to Icre) without regard to Φ.
3. If for purposes other than deflection calculations the value of Icre is required (to be taken as an approximation of Icr as discussed in chap.3) the steady portions of the curves of the figure can be entered to read off the value of Icre and of course without reference to Φ.

Obviously all Ie or Icre values read from the figure are in terms of $b'd^3/12$.

For ease of reference all the equations necessary to evaluate b′ and the correction factor are provided in the figure. Along with these equations, the general section to which the equation of b′ is applicable is also shown. If the section considered is rectangular, be/bw=1 and if singly reinforced nρ′=0.

The reference plot as produced and described above can be said to provide the following advantages:

1. The value of Ie can be read directly without having to calculate Icr or Icre.
2. For conditions with Icr > Ig where Ie is to be taken equal to Icre or when Icre is needed for whatever purposes and without regard to Φ, the steady portions of the curves can be entered to read off the value of Icre directly.
3. From 1 and 2 above, it can be said that except for the coefficient Φ the reference plot can be considered as a summary of all that has been proposed in the current study. That is the approximation of Icr and the evaluation of Ie.
4. The curves are independent of how the value of Φ is obtained. Although the model proposed for Φ in the next section will prove to give results that are more accurate and consistent than if Branson's equation is used it is however fair to say that in the field of science there is always room for improvement. If in the future the proposed model of Φ is somehow improved or replaced by other models the curves can still be used.
5. The curves can be used for either singly or doubly reinforced rectangular or flanged sections which are the most commonly investigated sections for deflection.

While any example intending to explain the use of Fig.4.3.1 for the determination of Ie has to be postponed until an expression of Φ is proposed and discussed, an example pertaining to the evaluation of Icre from the curves of Fig.4.3.1 is next given.

Example 4.3.1

The intervals of α and β shown in Fig.4.3.1 can be used to evaluate Icre according to,

$$Icre = (\alpha + \beta n\rho e)(b'd^3/12)$$

Alternatively, the steady portions of the curves of the figure can be entered to read off the value of Icre directly.

Show how this alternative can be applied to the following cases:

(a) Case (d) of example 3.3.1

(b) Case (c) of example 3.4.1

(c) Case (d) of example 3.5.1

Solution:

(a) In example 3.3.1 the following were given:

$n\rho=40\%$, $n\rho'=40\%$, $d'/d=0.03$

The section is rectangular

Using the above values and that be/bw=1, since the section is rectangular, into the equations of Fig.4.3.1 gives,

$b'=3.57bw$, $n\rho e=11.2\%$

entering the steady portions of the curves of the figure with $n\rho e$ of 11.2% one reads,

$$Ie/(b'd^3/12)=Icre/(b'd^3/12)\cong 0.72$$

Thus,

$$Icre=(0.72/12)(3.57bwd^3)=0.2142bwd^3$$

If for the sake of illustration the correction factor is evaluated, then using Ig of

$0.0910605\ bwd^3$ as was found in the original example along with the equations of Fig.4.3.1,

$R = 0.306$, $\alpha + \beta n\rho e = 0.7205$

$CF=In[(0.7205-0.306)/(0.7205-3)]=In[-0.1818]$

and the correction factor is therefore undefined. This implies that Icre (or Icr) is greater than Ig which is evident from the respective values of Icre and Ig and is consistent with the findings of the original example. In cases like these Ie is taken equal to Icre as already has been discussed.

(b) In example 3.4.1 the following were given:

$n\rho$(relative to be)=14% , be/bw=3 , hf/d=0.11

Converting the given $n\rho$ to its corresponding value relative to bw,

$n\rho=14(3)=42\%$

Using the above $n\rho$ value along with the given ratios of be/bw and hf/d into the equations of Fig.4.3.1 gives,

$b'=2.25\ bw$, $n\rho e=18.66\%$

entering the steady portions of the curves of the figure with $n\rho e$ of 18.66% one reads,

$Ie/(b'd^3/12) = Icre/(b'd^3/12)\cong 1.06$

Thus,

$Icre = (\ 1.06/12\ )(\ 2.25\ bwd^3\ ) = 0.19875\ bwd^3$

(c) In example 3.5.1 the following were given:

$n\rho$(relative to bw) = 16% , $n\rho'$(relative to bw) = 8%

$d'/d = 0.08$, $be/bw = 4$, $hf/d = 0.14$

Using the above values into the equations of Fig.4.3.1 gives,

$b' = 3.46\ bw$, $n\rho e = 4.62\%$

entering the steady portions of the curves with $n\rho e$ of 4.62% one reads,

$Ie/(b'd^3/12) = Ie/(b'd^3/12) \cong 0.375$

Thus,

$Icre = (\ 0.375\ /\ 12)(\ 3.46\ bwd^3\ ) = 0.108125\ bwd^3$

4.4 The Expression of Φ

4.4.1 The General Expression of Φ

The discussion pertaining to Eqs.4.2.3-4 requires that any expression of Φ used in conjunction with the proposed model of Ie must be a direct function of Ma/Mcr and must always yield a negative value. Bearing this in mind and based on the results of over 340 tested beams found in literature [17,21,22,32] the following empirical expression for Φ is proposed

$$\Phi = -\ (\ Ma/Mcr\ )(Lcr\ /L)\ \rho \qquad \text{for } \rho > 1\% \qquad (\ 4.4.1\ )$$

$$= -\ (\ Ma/Mcr\ )(Lcr\ /L) \qquad \text{for } \rho \leq 1\%$$

In the expression of Eq.4.4.1, Ma/Mcr is as defined previously and represents the loading intensity. The reinforcement ratio ρ is in percentage and is taken relative to bw of the general section shown in Fig.4.3.1. The term Lcr/L is the ratio of the length over which cracking occurs, Lcr , to the length of the span considered, L. The cracking length Lcr can always be scaled from the bending moment diagram by defining the region over which the bending moment exceeds the value of the cracking moment, Mcr. Because the shape of the bending moment diagram is a function of the loading condition, different loading types will have different ratios of Lcr/L.

When Ma $\leq$ Mcr the cracked length Lcr reduces to zero and so does the value of Φ according to Eq.4.4.1. Substituting a value of $\Phi = 0$ into Eq.4.2.7 gives Ie=Ig. This is a trivial condition that any proposed expression of Ie should satisfy. It is interesting to note, however, that Branson's equation without the limitation of Ie $\leq$ Ig does not satisfy this condition. This is because when Mcr/Ma > 1 there is no guarantee that Eq.4.1.1 will not yield a value of Ie greater than Ig [6, 19]. For this the limiting condition of Ie $\leq$ Ig had to be imposed. Because the proposed equation of Ie satisfies the trivial condition as explained above there is no such inconsistency to be avoided and the limitation of Ie $\leq$ Ig is therefore not required.

Although the proposed expression of Φ may seem to be the simplest way of representing all the relative factors , it will be shown in later sections to give results that are more accurate and consistent than if Branson's equation was applied.

4.4.2 The Expression of Φ for Some Typical Loading Conditions

Because it is always possible to scale Lcr from the bending moment diagram, Eq.4.4.1 is the general expression which can be used to evaluate Φ under the effect of any loading type and span condition.

However, it will be shown in this section that from the bending moment diagram and using the principles of elementary structural analysis it is possible in the case of simple spans to derive expressions of Lcr/L for the most common loading types in terms of Ma/Mcr. When these expressions are then substituted into Eq.4.4.1 they give simpler expressions of Φ for each of the respective loading type considered.

A. The case of a uniformly distributed load over a simple span :

Consider Fig.4.4.2.1 (a). The bending moment diagram shown therein can be represented by the following equation

$$M_b = wLx/2 - wx^2/2$$

At distance x where Mcr is exactly equal to M_b one can write

$$Mcr = wLx/2 - wx^2/2$$

or

$$wx^2/2 - wLx/2 + Mcr = 0$$

solving the above equation for x gives

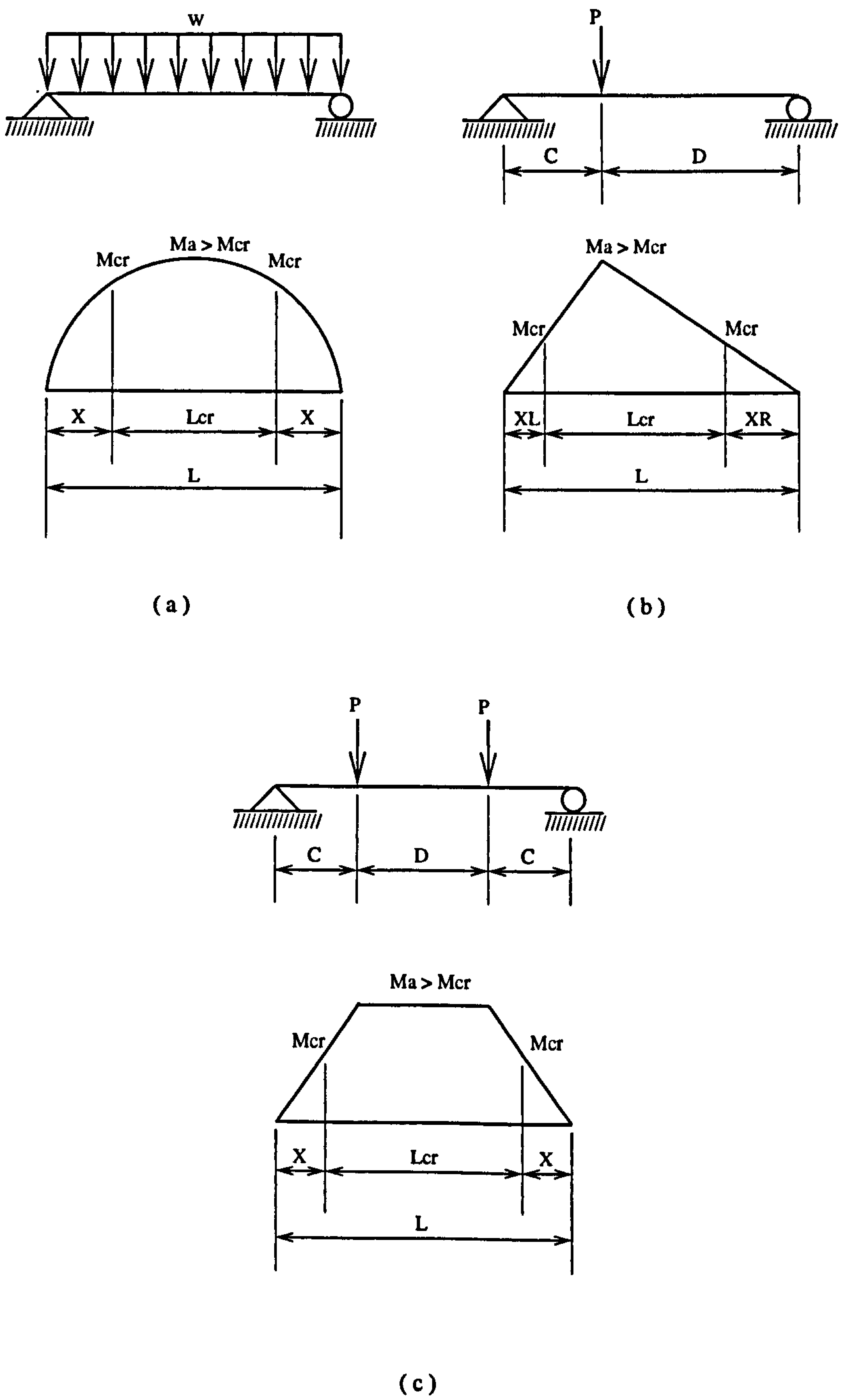


Figure 4.4.2.1 The Loading condititions for which expressions of Lcr/L are derived

$$x = \{ L - \sqrt{[L^2 - 4(2Mcr / w)]}\} / 2$$

or

$$x = \{L - L \sqrt{[1 - (Mcr / wL^2/8)]}\} / 2$$

and for $Ma = wL^2/8$,

$$x =\{ L - L \sqrt{[1 - Mcr /Ma]}\} / 2$$

Because the bending moment diagram is symmetric the cracked length Lcr is

$$Lcr = L - 2x$$

Substituting for x as determined above and simplifying

$$Lcr / L = \sqrt{(1 - Mcr / Ma)}$$

Substituting the above into the general expression of Φ as given by Eq.4.4.1 yields

$$\Phi = - (Ma / Mcr)[\sqrt{(1 - Mcr / Ma)}] \rho \quad \text{for } \rho > 1\% \qquad (4.4.2.1)$$

$$= - (Ma / Mcr)[\sqrt{(1 - Mcr / Ma)}] \quad \text{for } \rho \leq 1\%$$

B. The case of a simple span with a concentrated load acting between the supports :

Referring to Fig.4.4.2.1 (b) one can write

$$Lcr = L - (x_L + x_R)$$

and from the shape of the bending moment diagram it can also be written that

x_L=CMcr/Ma , x_R=DMcr/Ma

Therefore,

$$Lcr = L - (CMcr/Ma + DMcr/Ma)$$

or

$$Lcr = L - (Mcr/Ma)(C + D)$$

However, C + D = L. Thus,

$$Lcr = L(1 - Mcr/Ma)$$

or

$Lcr/L = 1 - Mcr/Ma$

Substituting the above into the general expression of Φ gives

$$\Phi = -(Ma/Mcr - 1)\rho \qquad \text{for } \rho > 1\% \qquad (4.4.2.2)$$

$$= -(Ma/Mcr - 1) \qquad \text{for } \rho \leq 1\%$$

It is important to notice that the expression of Φ obtained above is independent of the position of the concentrated load applied. This is because as the position of the load changes so does the position of the cracked length within the span. However, the value of the cracked length and thus its ratio relative to the total length of the span remains a function of only Mcr/Ma as can be seen from the equation of Lcr/L derived above.

<u>C. The case of two equal concentrated loads acting on a simple span and equally spaced from the supports (usually referred to as two point loads):</u>

Referring to the bending moment diagram shown in Fig.4.4.2.1 (c),

$$Lcr = L - 2x$$

and from the geometry of the diagram

$$x = CMcr / Ma$$

Therefore,

$$Lcr = L - 2CMcr/Ma$$

or

$$Lcr/L = 1 - 2CMcr/LMa$$

Substituting the above into the general expression of Φ gives

$$\Phi = -\,(Ma/Mcr - 2C/L)\rho \qquad \text{for } \rho > 1\% \qquad (4.4.2.3)$$
$$= -\,(Ma/Mcr - 2C/L) \qquad \text{for } \rho \leq 1\%$$

As an alternative to the general expression the above equation can be used to evaluate Φ in the case of a two concentrated load acting on a simple span and equally spaced from the supports

It may be noticed that when the two concentrated loads converge to a single load at midspan C will be equal to L/2 and the above expression will reduce to that of Eq.4.4.2.2 derived for the case of a single concentrated load acting anywhere within the span. This means that Eq.4.4.2.3 can also be assumed to apply to the case of a midspan concentrated load.

The expressions of Φ as given by Eqs.4.4.2.1-3 for the three loading conditions considered above were derived assuming Ma > Mcr. When Ma ≤ Mcr the expressions do not apply. In this case Lcr/L and thus Φ are automatically taken equal to zero as was pointed out in Sec.4.4.1.

In the next section a computer program will be used to compute deflection values for comparison with test results. The tested beams were subject to the three loading conditions considered above. For such special purpose programs Eqs.4.4.2.1-3 will always be found easier to use and simpler to automate than if Lcr was scaled from the bending moment diagram. However, it is fair to say that because the equations are only applicable to the loading conditions for which they are derived, they are of limited use if one has to write a general purpose program wherein any loading condition can be considered.

For hand calculations, although scaling Lcr from the bending moment diagram is always easy, the equations do provide an alternative whereby one can evaluate Φ directly without referring to the bending moment diagrams if the case considered is one of the three loading conditions for which the equations were derived.

4.4.3 Numerical Verification of the Expression of Φ

In this section the proposed expression of Φ as given in Secs.4.4.1 and 4.4.2 and as used in the model of Ie given by Eq.4.2.7 is checked against over 340 test data found in literature and is compared with the equation of Branson and that proposed in Ref.4 (namely Eqs.4.1.1 and 4.1.2). The data used are for simply supported beams tested under central point loads, two point loads and uniform loads and are taken from a variety of references. The data as quoted from the references include sectional geometry, percent and type of reinforcement, the concrete compressive strength, the total span length, the applied moment and the corresponding measured deflection at midspan.

Using the equations of Ec given in Sec.2.7 the modular ratio n and the product nρ were computed. Based on the values of nρ thus found and using the sectional geometries given the respective values of Icr and Icre along with Ig were calculated in accordance with the equations in Chap.3.

The values of Mcr, taken as $f_r Ig/y_t$, necessary to calculate the ratios of Ma/Mcr were found using the equations of f_r given in Sec.2.10.

According to the discussion of previous sections Ie is taken as equal to Ig whenever Ma ≤ Mcr and is assumed equal to Icr if Icr > Ig. As there will be no ground for comparison, the total number of 340 test data exclude all cases of Ma ≤ Mcr and Icr > Ig.

The deflections at midspan of the beams were calculated as δ given by

$$\delta = 5MaL^2 / 48EcIe \qquad \text{(for U.D.L)}$$

$$= (3MaL^2 - 4MaX^2) / 24\ EcIe \qquad \text{(for C.P.L or X.P.L)}$$

where U.D.L stands for uniformly distributed loads, C.P.L stands for central point load and X.P.L stands for two point loads equally spaced at X distance from each support. For central point loads X is taken equal to L/2.

By substituting the respective values of Ie into the above expressions the deflection values as given by the different equations of Ie were computed and the errors involved in each case were calculated as follows,

% error=[(computed deflection-measured deflection)/measured deflection](100)

Because of the large number of data the procedure described above was automated through Prog.4.4.3.1 shown in Appendix C2. The program was structured to print the data as read from the references and the results of the analysis in two separate tables. The first table could be used to check whether the data have been entered correctly into the program. The second table contains the computed as well as the measured values of the deflection, the errors involved in each method, the reinforcement ratio, the ratio of Ma/Mcr and finally the loading type for each case of the tested data examined.

Two sets of data were examined. The first set, which stored in data file "CERA" shown in Appendix C3, corresponds to Tables C2.1 and C2.2 of the printouts included in Appendix C2 and pertains to the experiments conducted by BASE, READ,BEEBY and TAYLOR as described and summarized in Table 2 of Ref.21 and Table 1 of Ref.22 comprising a total of 258 test data. The beams considered in this set were all singly reinforced with 243 of them being rectangular (rectangular sections are those having bw=be and hf/d=0 in the printout tables) and 15 as flanged. The reinforcement ratios varied from the minimum of 0.86% to the maximum of 2.67% (or 5.23% relative to bw in case of flanged sections). The sectional geometries were also varied along with the effective depths. The beams were simply supported and loaded with two point loads at a distance of 0.28 of the span from each support.
The results of the analysis obtained for this first set of data have indicated that despite the equal accuracy observed otherwise, for $\rho < 1\%$ and $Ma/Mcr \leq 3.4$ the errors obtained from the proposed model are consistently almost half of those from Branson's equation or the equation of Ref.4. This can be seen from the results summarized in Table 4.4.3.1 for the cases where $\rho < 1\%$ and $Ma/Mcr \leq 3.4$.

Table 4.4.3.1. Summary of results considering beams of data file CERA

Bm#	% error Branson	% error Ref.4	% error Model	ρ (%)	Ma/Mcr	Load Type
203	13.2	11.3	6.9	0.86	3.15	0.28 p.l
205	14.9	12.5	7.1	0.93	2.92	"
207	15.9	13.3	8.3	"	2.98	"
209	10.7	9.5	-1.6	"	2.37	"
210	8.6	6.1	4.0	"	3.31	"
211	19.9	18.4	7.1	"	2.43	"
212	11.6	9.1	7.4	"	3.4	"
213	24.8	22.1	17.8	"	3.08	"
215	15.4	12.8	8.2	"	3.02	"
217	19.8	18.1	7.4	"	2.46	"
218	10.7	8.3	6.9	"	3.44	"
219	29.7	28.5	15.0	"	2.34	"
220	18.6	16.0	13.2	"	3.27	"
221	12.0	9.5	7.1	"	3.23	"
223	15.4	12.9	9.4	"	3.12	"
225	23.8	22.0	12.2	"	2.53	"
227	13.6	11.7	3.2	"	2.59	"
229	12.9	10.6	5.5	"	2.93	"
231	15.9	13.4	8.7	"	3.00	"
233	19.3	17.5	7.6	"	2.51	"
235	14.8	13.2	2.8	"	2.44	"
236	7.2	4.8	3.4	"	3.42	"
237	22.6	20.0	13.7	"	2.89	"
239	13.1	11.8	0.5	"	2.37	"

Table 4.4.3.1 (cont'd)

Bm #	% error Branson	% error Ref.4	% error Model	ρ (%)	Ma/Mcr	Load Type
240	8.2	5.7	3.6	"	3.32	0.28 p.l
241	23.1	22.0	8.5	"	2.31	"
242	16.1	13.4	10.2	"	3.23	"
243	18.9	16.3	12.8	"	3.12	"
245	14.1	11.7	6.9	"	2.98	"
247	11.8	10.4	0.5	"	2.45	"
248	5.6	3.3	2.0	"	3.43	"
249	18.9	17.2	7.0	"	2.49	"
250	8.1	5.7	4.7	"	3.49	"
251	5.5	3.2	0.0	"	3.10	"
253	6.1	3.8	1.2	"	3.20	"
255	14.2	11.7	8.8	"	3.20	"
257	7.9	5.6	2.9	"	3.2	"
Mean error	14.7	12.5	6.8	-	-	-

Because the ratio of the errors for the proposed model to those for the other equations is consistent and that there is no reason to assume that the errors are normally distributed there was no statistical analysis performed. However, since the errors are almost all positive the mean errors calculated in the table are believed to be enough to indicate the accuracy of the different methods used.

To confirm the above observations the second set of test data, which is stored in data file "INF" shown in Appendix C3 was considered. These test data were the results of independent experiments conducted by different parties. The loading conditions considered were central point loads and two point loads at a distance of 0.33 of the span from each support [given in Table 4 of Ref.22 as supplementary data] as well as uniformly distributed loads [17,32]. The beams were simply supported and varied in sectional geometries and reinforcement.

Tables C2.3 and C2.4 shown in Appendix C2 give the details read by the program and the computed results. Despite the almost equal accuracy of the results observed for $\rho \geq 1\%$, the proposed model is again noticed to give better accuracy than the other equations for $\rho < 1\%$ and $Ma/Mcr \leq 4$ as is summarized in Table 4.4.3.2.

For the central point loads the errors shown in the table indicate that the equation of Branson and that of Ref.4 may grossly overestimate the deflection. Out of the 23 beams considered Branson's equation has shown 11 cases of gross error (errors greater than ± 30%) ranging from +31.7 up to 101.1% while the equation of Ref.4 has shown 10 of such cases ranging from 38.5 to 94%. The proposed model, on the other hand, has shown only 3 cases of gross error which are -33.8%, -51.3% and 39.5%.

Because the errors found for Branson's equation and that of Ref.4 are predominantly positive their mean values shown in the table give some indication of the accuracy of these equations. In the case of the proposed model, however, the errors are not predominantly of the same sign which required that the mean of the positive errors and that of the negative errors be calculated separately. These were found to be +13% and -17% which were then approximated as ±15% shown in the table.

In the case of the 0.33L point loads only 4 cases were considered with $\rho < 1\%$ and the

Table 4.4.3.2. Summary of results considering beams of data file INF

Bm #	% error Branson	% error Ref.4	% error Model	ρ (%)	Ma/Mcr	Load Type
1	15.1	38.5	-1.9	0.48	1.12	c.p.l
2	1.9	7.8	-33.8	0.48	1.62	"
5	33.5	20.0	-1.6	0.82	1.85	"
6	23.4	13.5	3.1	0.82	2.71	"
7	17.5	4.1	-18.1	0.82	1.63	"
8	11.3	-1.0	-15.9	0.82	2.38	"
17	54.1	46.4	8.7	0.55	2.40	"
18	31.7	25.4	13.2	0.55	3.48	"
21	57.9	49.2	1.7	0.55	2.24	"
22	52.9	43.0	12.5	0.55	2.85	"
23	73.3	72.9	14.6	0.48	2.01	"
24	14.0	9.9	-4.1	0.48	3.29	"
27	23.2	16.1	11.6	0.79	3.38	"
29	40.9	28.2	17.5	0.82	2.89	"
31	5.7	-6.3	-15.3	0.90	2.85	"
35	17.0	4.0	-5.0	0.88	2.87	"
37	-20.8	-23.2	-51.3	0.59	1.71	"
38	-0.7	-8.2	-27.4	0.59	2.85	"
47	66.7	71.8	11.5	0.53	1.58	"
48	101.1	94.0	39.5	0.53	2.23	"
49	70.5	87.7	18.7	0.53	1.35	"
50	68.9	65.9	3.8	0.53	1.90	"
53	26.0	18.3	15.4	0.84	3.65	"
Mean error	34	29.5	±15			

Table 4.4.3.2 (cont'd)

Bm #	% error Branson	% error Ref.4	% error Model	ρ (%)	Ma/Mcr	Load Type
11	39.8	50.5	20.9	0.82	1.52	0.33 p.l
12	32.5	30.6	13.7	"	2.25	"
13	9.3	16.8	-9.6	"	1.60	"
14	16	14.3	-4.7	"	2.18	"
Mean error	25.4	28	±12			

results are summarized in Table 4.4.3.2. As can be seen from the table the proposed model has shown no gross error in these cases and the error's mean value was found to be ±12%. The other equations, on the other hand, have given gross error for half of the cases considered which indicates once again the tendency of these equations to overestimate the deflection in cases of $\rho < 1\%$ as was noticed previously.

The mean values of the errors as discussed above and shown in Table 4.4.3.2 again indicate that the errors obtained from the proposed model are almost half of those from the equation of Branson or of Ref.4 for cases of $\rho < 1\%$ and $Ma/Mcr \leq 4$.

Unlike Branson's equation which is purely empirical, the proposed model of Ie as given by Eq.4.2.7 is the result of an analytical development. This not only makes the equation always valid but also consolidates all the empirical factors in the coefficient Φ the expression of which is independent of the format of the equation.

Hence, instead of proposing a different expression of Ie each time a deflection study is launched (as was the case in Ref.4) one only needs to look at the expression of Φ and thus whatever graphical representations have been produced to represent Ie or however the approximation of Icr has been incorporated into the equation remain valid and one need not start from the scratch. This gives more freedom and flexibility for future research that can be aimed at refining or improving the expression of Φ. In the light of this, the expression of Φ proposed in this study can be thought of as a first step toward the more proper expression that may be developed in the future without actually sacrificing the time and effort spent on approximating Icr or developing the expression of Ie and its graphical representation. The fact that the current expression gives errors that are half of those resulted from Branson's equation which has long been used in structural design indicates that this first step is at least in the right direction.

In the remaining of the section two numerical examples will be given . The first example is intended to explain the numerical procedure used in prog.4.4.3.1 and to show how the printed results included in Appendix C2 were calculated. The second example is meant to describe how to use the solution curves of Fig.4.3.1 to determine Ie once an appropriate expression of Φ, like the one proposed in this study, is available.

Example 4.4.3.1

The following were specified according to the notation of the general section of Fig.4.3.1 and using Ma for the maximum moment at midspan, fc′ as the cylindrical compressive strength, fcu as the cubic compressive strength and δ for deflection :-

(a) for beam # 15 of Tables C2.1 and C2.2 of Appendix C2 :

the section is singly reinforced rectangular (be = bw , hf=0 , As′=0 and d′=0)

h=15.125", d=13.125", bw=8", As=2.45 in^2, fcu =4520 psi, δ (measured)=0.457"

span=180" (simply supported) , Ma=773000 lb.1" due to two point loads at 51" from each support

(b) for beam#239 of Tables C2.1 and C2.2 of Appendix C2 :

the section is singly reinforced rectangular (be=bw , hf=0 , As′=0 , and d′=0)

h=15.25" , d=13.5" , bw=7" , As=0.88 in^2 , fcu=4600 psi , δ (measured)=0.293"

span=180" (simply supported) , Ma=297000 lb.1" due to two point loads at 51" from each support .

(c) for beam#21 of Tables C2.3 and C2.4 of Appendix C2 :

section is doubly reinforced rectangular (be=bw, hf=0), h=11", d=9.58", d′=1.42", bw=5.9, As=0.312 in^2, As′=0.088 in^2, fc′=5242psi,δ (measured)=0.12", span=110"(simple supported), Ma=145000 lb.1" due to a central point load.

(d) for beam#84 of Tables C2.3 and C2.4 of Appendix C2 :

section is singly reinforced flanged (As′=0, d′=0). h=12", d=10.19", bw=6",be=12",hf=2.5",As=0.62in^2,fc′=3680psi,δ(measured)=1.34", span=240"(simply supported), Ma=264000 lb.1" due to uniformly distributed load

Required :

Knowing that the data for the above beams are the same as those given in the data files considered by Prog.4.4.3.1, verify the values printed by the program in Tables C2.2 and C2.4 of Appendix C2

Solution:

Since the computer retains the maximum number of decimal places as allowed by the "implicit double precision" statement declared in the program, the computations to follow will also show the maximum number of decimal places that are obtained during the calculations using a special hand calculator. This will help to retain the accuracy as much as possible in order to exactly confirm the printed values.

(a) For beam#15:

-determine Ma/Mcr:

The cracking moment Mcr was previously defined as

$$Mcr=f_r Ig/y_t$$

For any rectangular section the gross moment of inertia is given by

$Ig=bh^3/12$

Thus, for the section at hand,

$Ig= (8)(15.125)^3/12 = 2306.720052\ in^4$

According to Sec2.10, when the cube compressive strength is specified, as the case here, the modulus of rupture,fr, is given by Eq.2.10.2. Hence

$fr = 6.8\sqrt{fcu} = 6.8\sqrt{4520} = 457.1704277$ psi

Substituting the above values of Ig and fr along with y_t=15.125/2 into the expression of Mcr gives

Mcr=(457.1704277)(2306.720052)/(15.125/2)=139446.5048 lb.1"

Hence,

Ma/Mcr=773000/139446.5048=5.543344389

-determine nρ and Icr:

In accordance with Sec2.7 and because the cube compressive strength is specified, Ec is evaluated as,

$Ec=(20+0.2fcu)=[20+0.2(4520/145)](10^3)(145)=3804000$ psi

For As=2.45 in^2,

ρ=2.45(100)/[8(13.125)]=2.33%

Using Es=$29x10^6$ psi (According to Sec.2.8) nρ is calculated as,

$n\rho=29(10^6)(2.33)/3804000=17.76288118\%$

Substituting nρ found above into the equations of Fig.3.2.2 (with nρ′=0 since the section is singly reinforced),

$x=[-17.76288118+\sqrt{17.76288118^2+200(17.76288118)}](13.125/100)$

=5.831587033"

Hence,

$$Icr=[100(5.831587033^3/3)+17.76288118(13.125)(13.125-5.831587033)^2](8/100)=1520.96601\ in^4$$

-determine Icre:

From the expression of b′ in Eq.3.6.1 with $n\rho'=0$ and be/bw=1,b′=bw. Thus,

$n\rho e=n\rho=17.76288118\%$ for which $\alpha+\beta n\rho e=1.032886435$. Hence,

$$Icre=1.032886435(8)(13.125^3/12)=1556.894739\ in^4$$

-determine Ie:

According to Eq.4.1.1,

$$Ie(Branson)=Icr+(Ig-Icr)(Mcr/Ma)^3\leq Ig$$

$$=1520.96601+(2306.720052-1520.966)(1/5.543344389)^3$$

$$=1525.578878\ in^4$$

From Eq.4.1.2,

$$Ie(Ref.4)=Ig+(Icr-Ig)(Lcr/L)^m$$

where,

$$m=0.8\rho Mcr/Ma=0.8(2.33)(1/5.543344389)=0.3362591009$$

Because the case is of a two point loads acting on a simple span,

$$Lcr/L=1-2(51)(1/5.543344389)/180=0.8977753091$$

Hence,

$$Ie(Ref.4)=2306.720052+(1520.966-2306.720052)(0.897775)^{0.336259}$$

$$=1548.947584\ in^4$$

The proposed model of Ie was given by Eq.4.2.7 as,

$$Ie(model)=Icre+(Ig-Icre)e^{\Phi}$$

From Eq.4.4.1,

Φ=-(5.543344389)(0.8977753091)(2.33)=-11.59565909

Hence,

$$Ie(model)=1556.894739+(2306.720052-1556.894739)e^{-11.59565909}$$

$$=1556.901642 \text{ in}^4$$

-determine δ and % error:

For two point loads on a simple span

$$\delta=(3MaL^2 - 4MaX^2)/(24EcIe)$$

Thus for the case at hand,

$$\delta=[3(773000)180^2 - 4(773000)(51^2)]/[24(3804000)Ie]$$

$$=734.8986593/Ie$$

Substituting the different values of Ie into the above equation gives,

δ(Branson)=0.4817179039" (printed value:0.482)

δ(Ref.4)=0.4744503086" (printed value:0.474)

δ(model)=0.4720263885 (printed value:0.472)

For the % error involved in any computed deflection value the following was previously defined,

% error={[δ(computed)-δ(measured)]/δ(measured)}(100)

Thus, using the deflection values computed by the different methods and the measured deflection of 0.457" the followings were obtained,

% error(Branson)=5.408731707 (printed value:5.41)

% error(Ref.4)=3.818448271 (printed value:3.82)

% error(model)=3.28805 (printed value:3.29) (b)For beam#239:

-determine Ma/Mcr:

Following the same argument as in case(a),

$Ig=bh^3/12=7(15.25^3/12)=2068.83724$

$fr=6.8\sqrt{fcu}=6.8\sqrt{4600}=461.1984389$ psi

Substituting the above values along with y_t=15.25/2 into the expression of Mcr gives,

Mcr=[461.1984389(2068.83724)]/(15.25/2)=125133.7056 lb.1"

Thus,

Ma/Mcr=297000/125133.7056=2.37346124

-determine nρ and Icr:

Because the cube compressive strength is specified,

$Ec=20+0.2fcu=[20+0.2(4600/145)](10^3)(145)=3820000$ psi

For As=0.88 in^2,

ρ=0.88(100)/[(7(13.5)]=0.93%

Thus,

$n\rho=29(10^6)(0.93)/3820000=7.060209424\%$

Substituting the above value of nρ into the equations of Fig.3.2.2 (with nρ′=0 since the section is singly reinforced),

$x=[-7.060209424+\sqrt{7.060209424^2+200(7.060209424)}](13.5/100)$

$=4.208549446"$

Hence,

$Icr=[100(4.208549446^3/3)+7.060209424(13.5)(13.5$

$-4.20849446)^2](7/100)=749.9286277$ in^4

-determine Icre:

From the expression of b′ of Eq.3.6.1 with nρ′=0 and be/bw=1,b′=bw. Thus,

$n\rho e=n\rho=7.060209424\%$ or $\alpha+\beta n\rho e=0.5130104712$. Hence,

$Icre=0.5130104712(7)(13.5^3/12)=736.2822472\ in^4$

-determine Ie:

According to Eq.4.1.1,

$Ie(Branson)=Icr+(Ig-Icr)(Ma/Mcr)^3\leq Ig$

$=749.9286277+(2068.83724-749.9286277)$

$(1/2.37346124)^3=848.5720821\ in^4$

From Eq.4.1.2,

$Ie(Ref.4)=Ig+(Icr-Ig)(Lcr/L)^m$

where,

$m=0.8\rho Mcr/Ma=0.8(0.93)(1/2.37346124)=0.3134662524$

Because the case is of two point loads acting on a simple span,

$Lcr/L=1-2(51)(1/2.237346124)/180=0.7612488221$

Hence,

$Ie(Ref.4)=2068.83724+(749.9286277-2068.83724)$

$(0.7612488221)^{0.3134662524} = 858.023608\ in^4$

According to the proposed model of Ie,

$Ie(model)=Icre+(Ig-Icre)e^{\Phi}$

From Eq.4.4.1 with ρ taken as 1 (since the given ρ is less than 1%),

$\Phi=-(Ma/Mcr)(Lcr/L)=-2.37346124(0.7612488221)=-1.806794573$

Hence,

$Ie(model)=736.2822472+(2068.83724-736.2822472)e^{-1.806794573}$

$=955.0605394 \text{ in}^4$

-determine δ:

For the loading condition considered,

$$\delta=(3MaL^2 - 4MaX^2)/(24EcIe)$$

$$=[3(297000)180^2 - 4(297000)51^2]/[24(3820000)Ie]$$

$$=281.1781414/Ie$$

Substituting the different values of Ie into the above equation gives,

δ(Branson)=0.3313569198"	(printed value:0.331)
δ(Ref.4)=0.3277044347"	(printed value:0.328)
δ(model)=0.29440871"	(printed value:0.294)

Based on the expression of the % error defined previously and using the deflection values computed by the different methods and the measured deflection value of 0.293" the followings were obtained,

% error(Branson)=13.09109891	(printed value:13.09)
% error(Ref.4)=11.84451696	(printed value:11.84)
% error(model)=0.4807883959	(printed value:0.48)

(c)For beam#21

-determine Ma/Mcr:

Because the section is rectangular, the gross moment of inertia, Ig , is determined as,

$$Ig=bh^3/12=5.9(11^3/12)=654.4083333 \text{ in}^4$$

According to Sec.2.10, when the cylindrical compressive strength is specified,as the case here, themodulus of rupture,fr, is given by Eq.2.10.1.

Hence,

$$fr=7.5\sqrt{fc'}=7.5\sqrt{5242}=543.0124308 \text{ psi}$$

Substituting the above values of Ig and fr along with y_t=11/2 into the expression of Mcr gives,

$$Mcr=frIg/yt=543.0124308(654.4083333)/(11/2)$$

$$=64609.42905 \text{ lb.1"}$$

Hence,

$$Ma/Mcr=145000/64609.42905=2.244254471$$

-determine nρ, nρ′ and Icr:

In accordance with Sec.2.7 and because the cylindrical compressive strength is specified, Ec is evaluated as,

$$Ec=33(w^{1.5})\sqrt{fc'}=33(145^{1.5})\sqrt{5242}=4171713.276 \text{ psi}$$

For As=0.312 in^2 and As′=0.088 in^2,

$$\rho=0.312(100)/[5.9(9.58)]=0.5519974523\%$$

$$\rho'=0.088(100)/[5.9(9.58)]=0.1556915891\%$$

Thus,

$$n\rho=29(10^6)0.5519974523 \,/\, 4171713.276=3.837254638\%$$

$$n\rho'=29(10^6)0.1556915891 \,/\, 4171713.276=1.08230259\%$$

$$n\rho+n\rho'=4.919557228\%$$

Substituting the above values of nρ, nρ′and nρ+nρ′ into the equations of Fig.3.2.2,

$$x=\{\sqrt{[4.919557228^2+200(3.837254638+1.08230259(1.42/9.58))]}$$

$$-4.919557228\}(9.58/100)=2.278246416"$$

Thus,

$$Icr=[100(2.278246416^3/3)+1.08230259(9.58)(2.278246416$$

$$-1.42)^2+3.837254638(9.58)(9.58-2.278246416)^2](5.9/100)$$

$$=139.3423294\ in^4$$

-determine Icre:

Using Eq.3.6.1 with be/bw=1,

$\alpha'= 0.004268441641.$ (for d′/d=1.42/9.58)

$b'=[0.004268441641(1.08230259)9.58/1.42+1](5.9)=6.083885389$

Hence, $n\rho e=3.721273646$ for which $\alpha+\beta n\rho e=0.3104891552$. Thus,

$$Icre=0.3104891552(6.083885389)(9.58^3/12)=138.4021195\ in^4$$

-determine Ie:

According to Eq.4.1.1,

$$Ie(Branson)=Icr+(Ig-Icr)(Mcr/Ma)^3\leq Ig$$

$$=139.3423294+(654.4083333-139.3423294)$$

$$(1/2.244254471)^3=184.9089257\ in^4$$

From Eq.4.1.2,

$$Ie(Ref.4)=Ig+(Icr-Ig)(Lcr/L)^m$$

where,

$$m=0.8\rho Mcr/Ma=0.8(0.5519974523)(1/2.244254471)$$

$$=0.1967682219$$

Because the case is of a midspan concentrated load,

$$Lcr/L=1-Mcr/Ma=1-1/2.244254471=0.5544177307$$

Hence,

$$Ie(Ref.4)=654.4083333+(139.3423294-654.4083333)$$

$$(0.5544177307)^{0.1967682219}=195.7828634\ in^4$$

According to the proposed model of Ie,

$$Ie(model)=Icre+(Ig-Icre)e^{\Phi}$$

using the proposed expression of Φ with ρ taken as 1(since the given ρ is less than 1%),

$$\Phi=-2.244254471(0.5544177307)=-1.244254471$$

Thus,

$$Ie(model)=138.4021195+(654.4083333-138.4021195)e^{-1.244254471}$$

$$=287.0922288$$

-determine δ:

$$\delta=(3MaL^2 - 4MaX^2)/24EcIe$$

Substituting X=L/2 for central point loading,

$$\delta=[(3MaL^2 - 4Ma(L/2)^2] /24EcIe=MaL^2/12EcIe$$

Thus for the case at hand,

$$\delta=145000(110^2)/[12(4171713.276)Ie]=35.04755089/Ie$$

Substituting the different values of Ie into the above equation gives,

δ(Branson)=0.1895395301" (printed value:0.19)

δ(Ref.4)=0.1790123522" (printed value:0.179)

δ(model)=0.1220776718" (printed value:0.122)

Based on the expression of the % error defined previously and using the deflection values computed by the different methods and the measured deflection of 0.12" the followings were obtained,

% error(Branson)=57.94960842 (printed value:57.95)

% error(Ref.4)=49.17696017 (printed value:49.18)

% error(model)=1.731393167 (printed value:1.73)

(d) For beam#84:

-determine Ma/Mcr:

using the equations of Fig.3.3.7 the gross moment of inertia of the current flanged section is determined as follows,

$$xg=\{0.5(12/6)(2.5/10.19)^2+0.5[(12/10.19)^2-(2.5/10.19)^2]\}$$
$$(10.19)/[(12/6)(2.5/10.19)+(12/10.19)-(2.5/10.19)]$$
$$=5.181034483''$$

Thus,

$$Ig=[((12/6)/12)(2.5/10.19)^3+(12/6)(2.5/10.19)(5.181034483/10.19$$
$$-0.5(2.5)/10.19)^2+(12/10.19-5.181034483/10.19)^3\ /3$$
$$+(5.181034483/10.19-2.5/10.19)^3\ /3](6)(10.19^3)$$
$$=1151.898706\ in^4$$

Because the cylindrical compressive strength is specified,

$$fr=7.5\sqrt{fc'}=7.5\sqrt{fc'}=7.5\sqrt{3680}=454.9725266\ psi$$

For the tension is at the bottom face of the beam,

$$y_t=h-xg=12-5.181034483=6.818965517''$$

Substituting the above values of fr,Ig and y_t into the expression of Mcr,

$$Mcr=frIg/yt$$
$$=454.9725266(1151.898706)/6.818965517$$
$$=76856.56474\ lb.1''$$

Thus,

$$Ma/Mcr=264000/76856.56474=3.434970076$$

-determine $n\rho$ and Icr:

Because the cylindrical compressive strength is specified,

$$Ec=33(w^{1.5})\sqrt{fc'}=33(145^{1.5})\sqrt{3680}=3495343.425 \text{ psi}$$

For $As=0.62$ in^2,

$$\rho(\text{relative to the web})=0.62(100)/[6(10.19)]=1.014066078\%$$

$$\rho(\text{relative to the flange})=0.62(100)/[12(10.19)]=0.5070330389\%$$

Hence,

$$n\rho(\text{relative to the web})=29(10^6)1.014066078 \,/\, 3495343.425$$
$$=8.413455471\%$$

$$n\rho(\text{relative to the flange})=29(10^6)0.5070330389 \,/\, 3495343.425$$
$$=4.206727735\%$$

To determine the position of the neutral axis assume the axis to fall in the flange. According to this assumption the section must behave as a rectangle of width be. Thus from the equations of Fig.3.3.2 and using $n\rho$ relative to the flange (with $n\rho'=0$ since the section is singly reinforced),

$$x=[\sqrt{(4.206727735^2+200(4.206727735))} - 4.2067277](10.19/100)$$
$$=2.55796435''$$

Because this exceeds the flange width, hf, the assumption is therefore not valid and the neutral axis definitely falls within the web whereby the equations of Fig.3.4.3 must be applied using $n\rho$ relative to the web. Therefore and from the equations of Fig.3.4.3,

$$b=2(2.5)\{(12/6)-1+8.413455471/[100(2.5)/10.19]\}=6.714662225$$

$$c=(2.5^2)[(12/6)-1+(8.413455471/50)/(2.5/10.19)^2]=23.72240807$$

Thus,

$$x=[\sqrt{(6.714662225^2+4(23.72240807))} - 6.714662225]/2$$
$$=2.558248343''$$

Hence,

$$Icr=[(100(12/6))(2.5/10.19)^3/3 + (100/3)(2.558248343/10.19 -2.5/10.19)^3+100(12/6)(2.5/10.19)(2.558248343/10.19)(2.558248343/10.19-2.5/10.19)+8.413455471(1-2.558248343/10.19)^2](6)(10.19)^3/100=366.575281 \text{ in}^4$$

-determine Icre:

Using Eq.3.6.1 with $n\rho'=0$,

$$\alpha_f=\min[(1+8(2.5/10.19))/3 , 0.9]=0.9$$

$$b'=[\alpha_f(be/bw-1)+1]bw=[0.9(12/6-1)+1]bw=1.9bw$$

Hence, $n\rho e=8.413455471/1.9=4.4281345$ for which $\alpha=0.05,\beta=0.07$

Thus,

$$Icre=[0.05+0.07(4.4281345)](1.9)(6)(10.19)^3/12=361.8359853 \text{ in}^4$$

-determine Ie:

According to Eq.4.1.1,

$$Ie(Branson)=Icr+(Ig-Icr)(Mcr/Ma)^3$$

$$=366.575281+(1151.898706-366.575281)(1/3.434970076)^3=385.9519749 \text{ in}^4$$

From Eq.4.1.2,

$$Ie(Ref.4)=Ig+(Icr-Ig)(Lcr/L)^m$$

where,

$$m=0.8\rho Mcr/Ma=0.8(1.014066078)(1/3.434970076)$$

$$=0.236174652$$

Because the case is of a uniformly distributed load over a simple span,

$$Lcr/L = \sqrt{(1-Mcr/Ma)} = \sqrt{(1-1/3.434970076)} = 0.8419481271$$

Thus,

$$Ie(Ref.4)=1151.898706+(366.575281-1151.898706)$$

$$(0.8419481271)^{0.2361746652}=397.844022 \text{ in}^4$$

According to the proposed model of Ie,

$$Ie(model)=Icre+(Ig-Icre)e^{\Phi}$$

using the proposed expression of Φ,

$$\Phi=-3.434970076(0.8419481271)1.014066078=-2.932746657$$

Hence,

$$Ie(model)=361.8359853+(1151.898706-361.8359853)e^{-2.932746657}$$

$$=403.9072803 \text{ in}^4$$

-determine δ and % errors:

Since the loading considered is uniformly distributed,

$$\delta=5MaL^2/48EcIe$$

$$=5(264000)(240)^2 / [48(3495343.425)Ie]$$

$$=453.1743544/Ie$$

Substituting the different values of Ie into the above equation gives,

δ(Branson)=1.174172912"	(printed value:1.174)
δ(Ref.4)=1.13907544"	(printed value:1.139)
δ(model)=1.121976197"	(printed value:1.122)

Based on the above deflection values and the measured deflection of 1.34" the following % errors were computed

% error(Branson)=-12.37515582 (printed value:-12.38)

% error(Ref.4)=-14.99437015 (printed value:-14.99)

% error(model)=-16.27043306 (printed value:-16.27)

Example 4.4.3.2

As part of the solution in example 4.4.3.1 the followings were obtained:

(a)For beam#15 of Tables C2.1 and C2.2 of Appendix C2 :

$Ig=2306.72\ in^4$, $n\rho=17.76\%$, $\Phi=-11.59$

(b)For beam#239 of Tables C2.1 and C2.2 of Appendix C2 :

$Ig=2068.84\ in^4$, $n\rho=7.06\%$ $\Phi=-1.81$

(c)For beam#21 of Tables C2.3 and C2.4 of Appendix C2 :

$Ig=654.41\ in^4$, $n\rho=3.84\%$, $n\rho'=1.08\%$, $\Phi=-1.24$

(d)For beam#84 of Tables C2.3 and C2.4 of Appendix C2 :

$Ig=1151.90\ in^4$, $n\rho$(relative to the web)$=8.41\%$, $\Phi=-2.93$

Required :

Using the above values and the sectional properties given as data in example 4.4.3.1 determine Ie from Fig.4.3.1 and the equations shown therein.

Solution :

(a) For beam# 15 of Tables C2.1 and C2.2:

Using the equations of Fig.4.3.1 with $n\rho'=0$ and be/bw=1 (since the section is singly reinforced rectangular),

$b'=bw$ and thus $b'd^3/12 = 8(13.125^3/12) = 1507.32$

$n\rho e = n\rho = 17.76\%$

From Fig.4.3.1, it can be seen that for $n\rho e$ of 17.76%, $Ie/(b'd^3/12)$ assumes a constant value of almost 1.03 for any value of $\Phi < -7$. Because the given value of Φ is -11.59 and knowing that the correction factor,CF, is always negative,the value of $\Phi_{ref} = \Phi + CF$ will always be <-7. Thus,

$Ie=1.03(1507.32)=1553\ in^4$

(b) For beam#239 of Tables C2.1 and C2.2:

Using the equations of Fig.4.3.1 with $n\rho'=0$ and be/bw=1 (since the section is singly reinforced rectangular),

$b' = bw$ and thus $b'd^3/12 = 7(13.5^3/12) = 1435.22$

$n\rho e = n\rho = 7.06\%$ and thus $\alpha = 0.16$, $\beta = 0.05$ or $\alpha+\beta n\rho e = 0.513$

Hence, for R = 2068.84/1435.22 = 1.44,

$\Phi_{ref} = \Phi+CF = -1.81+In[(0.513-1.44)/(0.513-3)] = -2.8$

For $n\rho e$ of 7.06% and Φ_{ref} of -2.8 the figure gives $Ie/(b'd^3/12) \cong 0.6625$. Thus,

$Ie=0.6625(1435.22)=951\ in^4$

(c) For beam#21 of Tables C2.3 and C2.4:

Using the equations of Fig.4.3.1 with be/bw = 1 (since the section is rectangular),

$\alpha'=0.0006+(1.42/9.58)[1-2(1.42/9.58)]^2/20=0.0043$

Thus,

$b'=[0.0043(1.08)(9.58/1.42) + 1]bw=1.03bw$

Hence,

$b'd^3/12=1.03(5.9)(9.58)^3/12=445.25$, $n\rho e = 3.84/1.03 = 3.73\%$. Thus,

$\alpha = 0.05$, $\beta = 0.07$ or $\alpha + \beta n\rho e = 0.311$

Therefore, and for $R = 654.41/445.25 = 1.47$,

$$\Phi_{ref} = \Phi+CF = -1.24+\ln[(0.311-1.47)/(0.311-3)] = -2.08$$

For $n\rho e$ of 3.73 and Φ_{ref} of -2.08 the figure gives $Ie/(b'd^3/12) \cong 0.65$. Thus,

$Ie=0.65(445.25)=289\ in^4$

(d) For beam#84 of Tables C2.3 and C2.4:

Using the equations of Fig.4.3.1 with $n\rho' = 0$ (since the section is singly reinforced flanged),

$\alpha_f=\min[(1+8(2.5/10.19))/3\ ,\ 0.9]=0.9$

Thus,

$b'=[0.9(12/6 - 1)+1]bw=1.9bw$. Hence,

$b'd^3/12=1.9(6)(10.19)^3/12=1005.19$, $n\rho e = 8.41/1.9 = 4.43\%$ for which,

$\alpha = 0.05$, $\beta = 0.07$ or $\alpha + \beta n\rho e = 0.36$

Therefore and for $R=1151.90/1005.19=1.15$,

$$\Phi_{ref}= \Phi + CF=-2.93+\ln[(0.36-1.15)/(0.36-3)] = -4.14$$

For $n\rho e$ of 4.426 and Φ_{ref} of -4.14 the figure gives $Ie/(b'd^3/12) \cong 0.4$. Thus,

$Ie\cong0.4(1005.19)\cong402\ in^4$

4.5 The Case of Continuous Beams

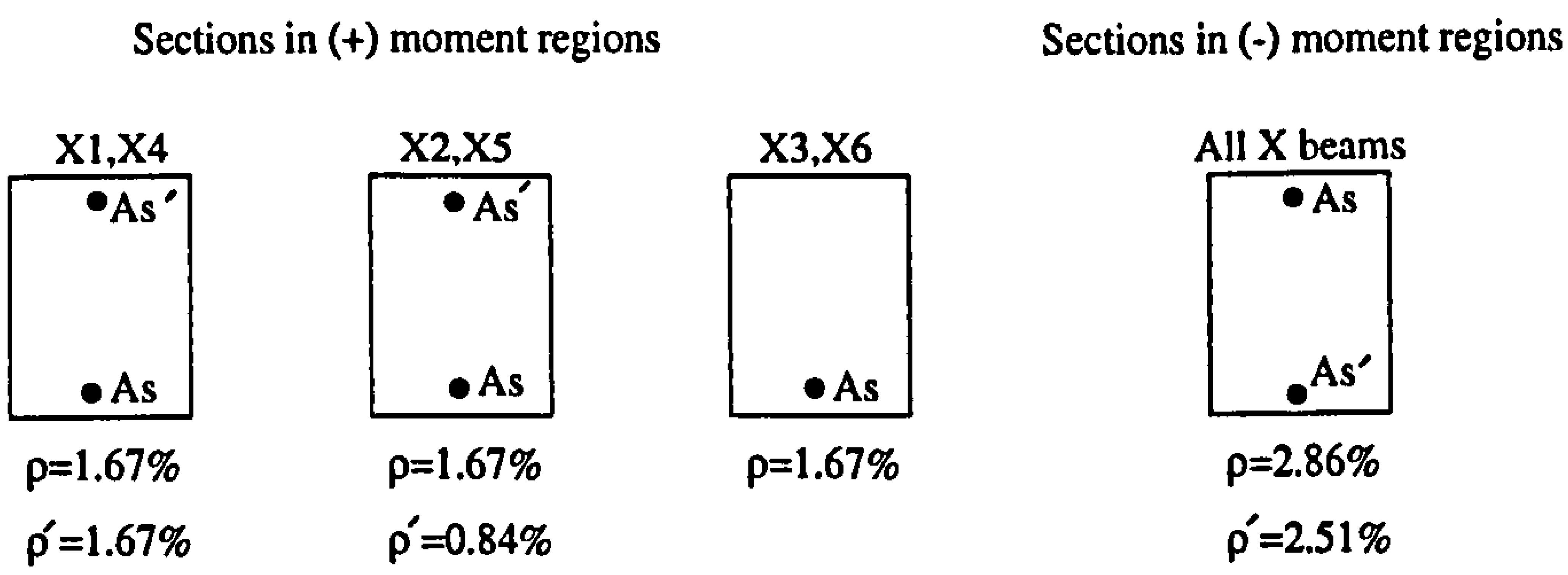
The numerical verification of the proposed models of Ie and Φ given in the pervious section considered beams that are simply supported. However, to support the argument of Sec.4.2 that the proposed model of Ie is equally applicable to continuous beams numerical verification must also be given for such beams. This is carried out in this section using, once again, experimental results found in literature[33].

In Ref.33 the details and results of testing 18 beams were given [also summarized in Ref.19]. Every two beams had identical sectional geometries, steel ratios, supports and loading conditions and thus comprising 9 pairs of tested beams. The beams were all rectangular in section and loaded as two equal continuous spans and the deflections at 0.42 of each span (which is the point of maximum deflection for two equal span continuous beams) were read and averaged to give a single measured deflection for each pair of beams.

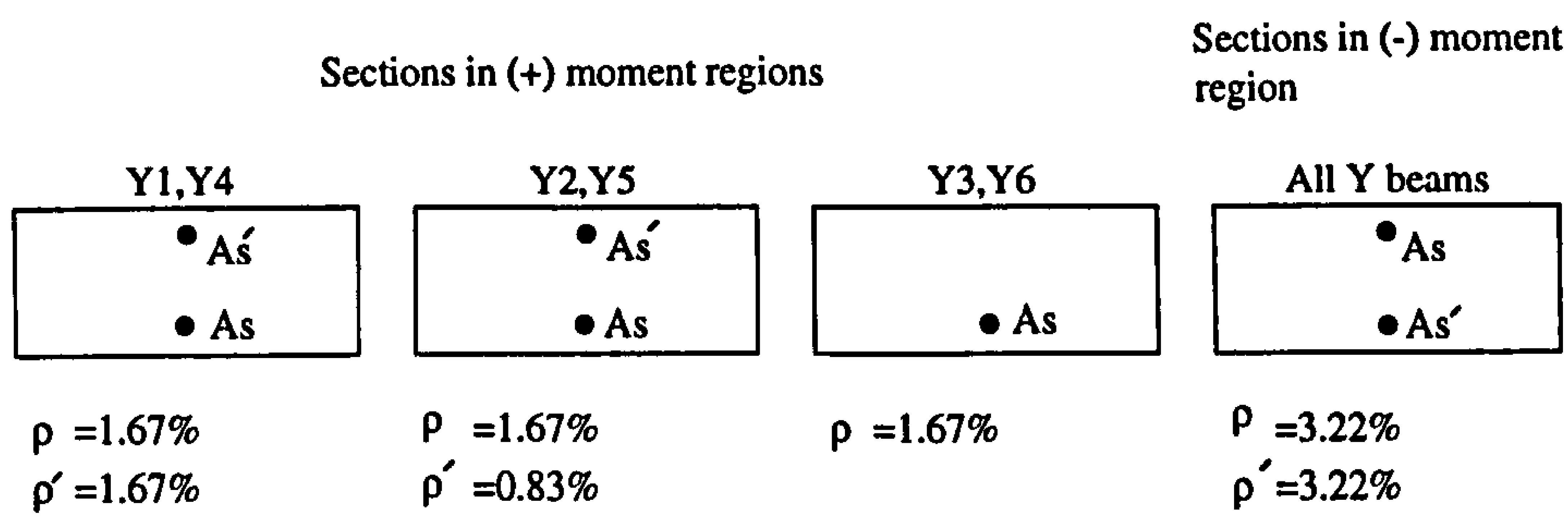
Figure 4.5.1 summarizes the different properties of the tested beams while Figs.4.5.2-4 give the loading conditions and the corresponding bending moment diagrams for the beams as grouped in X,Y and Z beams.

Based on the data presented in the given figures the deflection values are computed using the proposed model of Ie as represented by the curves and equations of Fig.4.3.1.

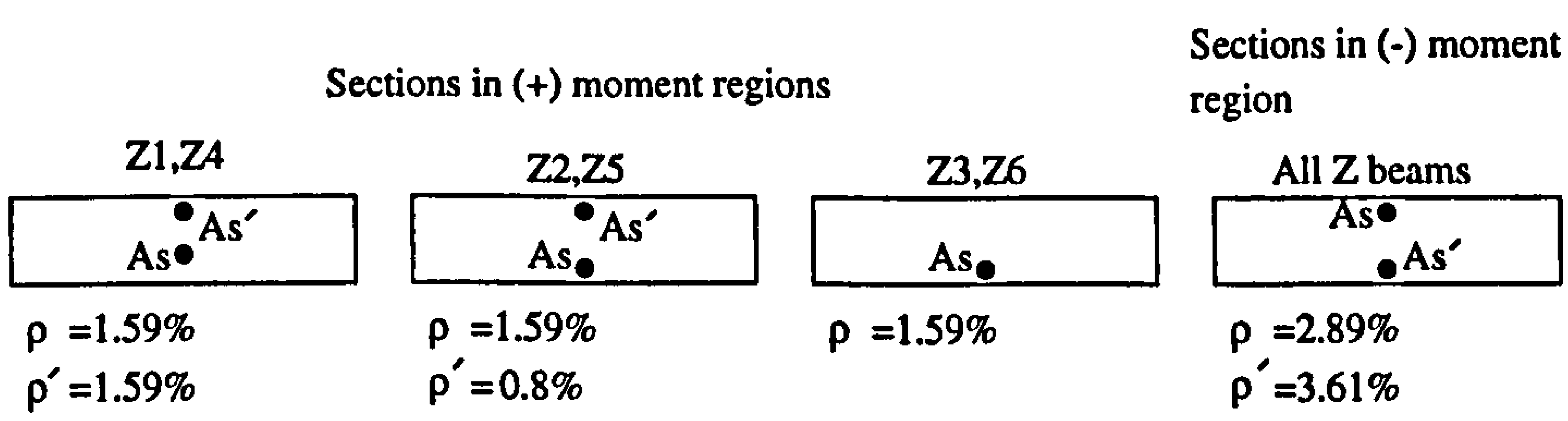
It was shown in the previous section that as the applied load deviates from being uniform Branson's equation loses accuracy as compared to the proposed model. This is because the equation was proposed based on beams tested under uniform loads[1] and is thus most accurate when such loading type is considered. If the proposed model



(a) The X beams . fc' = 3230 psi , h = 8" , bw = 6" , d = 6.18" , d' = 1.82"



(b) The Y beams. fc' = 3360 psi , h = 5" , bw = 12" , d = 4" , d' = 1"



(c) The Z beams . fc' = 3295 psi , h = 3" , bw =1 2" , d = 2.31" , d' = 0.69"

Figure 4.5.1 The sections of the continuous beams tested

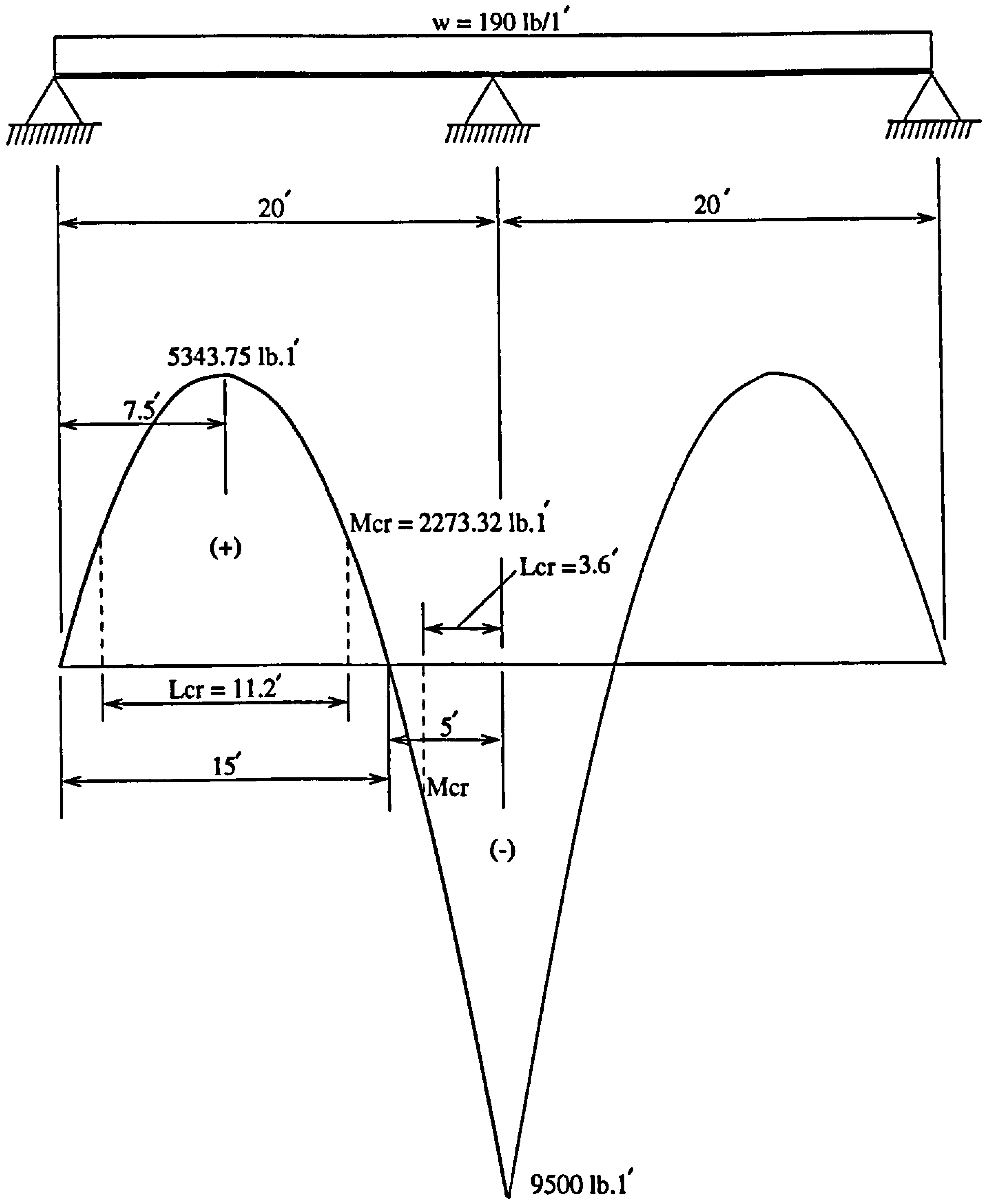


Figure 4.5.2 The loading condition and the bending moment diagram for the X beams

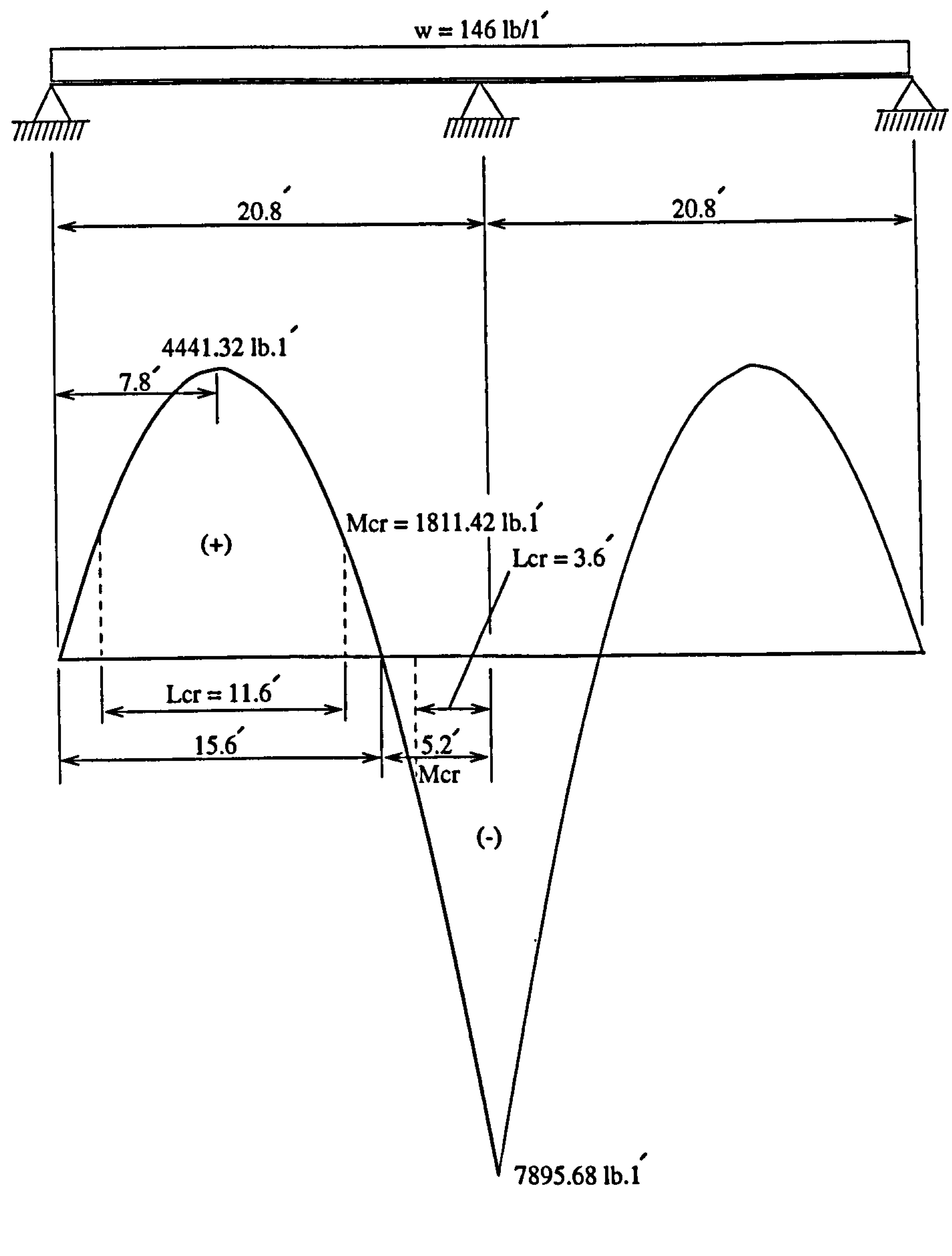


Figure 4.5.3 The loading condition and the bending moment diagram for the Y beams

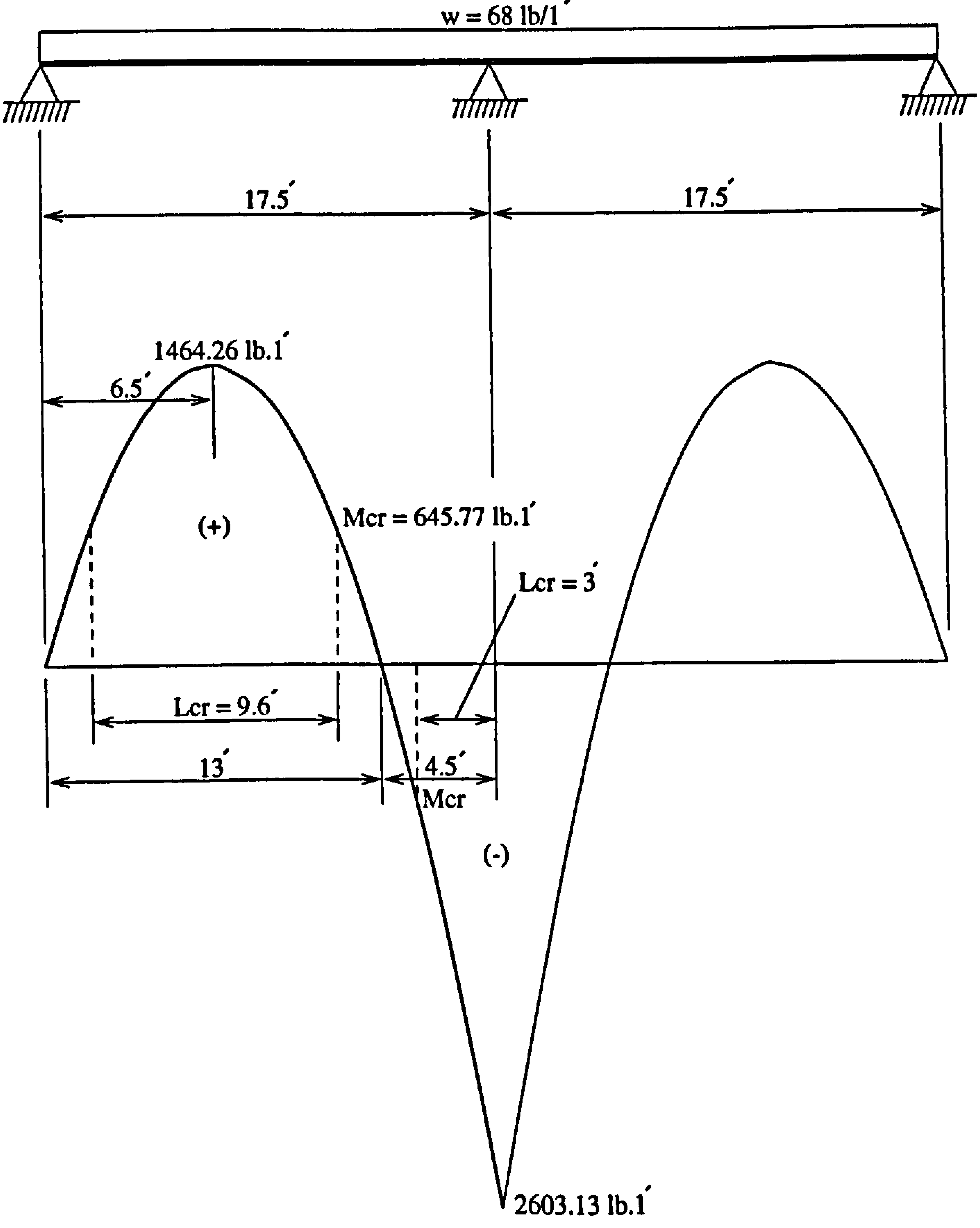


Figure 4.5.4 The loading condition and the bending moment diagram for the Z beams

is shown to be as accurate as Branson's equation when the equation is most accurate and more accurate when the equation losses accuracy then the model can be considered to be more representative of the effective moment of inertia than Branson's equation. As this has already been shown for simply supported beams in the previous section, it remains to be confirmed for continuous beams. For this Branson's equation is also used to compute the deflections for the continuous beams considered and results are compared with those obtained using the proposed model.

Because the beams are symmetric about the interior support only one span of each continuous beam is considered. This is shown in Figs.4.5.2-4 where only the bending moment diagrams over the left spans have been detailed.

According to the discussion in Sec.4.2 the bending moment diagrams over each of the considered spans have been assumed to consist of two subdiagrams. The effective moment of inertia for the regions covered by each subdiagram is then found and the results were averaged to obtain a single value of Ie over the entire span. The value of Ie thus found using the proposed model and the equation of Branson are then used to calculate the deflection, δ, at 0.42 point of the span according to the following expression as derived using the principles of structural analysis,

$$\delta = 8wL^4 / 1477EI$$

where,

w ≡ The magnitude of the distributed load

L ≡ The length of the span

Once the deflection values are obtained the percent error are then calculated as before.

The results are summarized in Table 4.5.1

Table 4.5.1. Summary of results considering beams of Ref.33

Beams	Measure deflection (in)	Computed deflection (in)		% error	
		Model	Branson	Model	Branson*
X1&X4	0.56	0.62	0.62	10.7	10.7
X2&X5	0.57	0.63	0.63	10.5	10.5
X3&X6	0.62	0.63	0.63	1.6	1.6
Y1&Y4	0.89	0.95	0.95	6.7	6.7
Y2&Y5	0.93	0.96	0.96	3.2	3.2
Y3&Y6	1.0	0.96	0.97	-4	-3
Z1&Z4	1.04	1.22	1.24	17.3	19.2
Z2&Z5	1.13	1.23	1.25	8.8	10.6
Z3&Z6	1.20	1.24	1.24	3.3	3.3

* *Branson's equation is accurate for the cases considered since loads are uniformly distributed and* ρ *>1%. The proposed model on the other hand shows same accuracy for all loading types and reinforcement conditions.*

To numerically describe the procedure outlined above and to show how the values of Table 4.5.1 are obtained the following example is furnished,

Example 4.5.1

Knowing that the deflection values of Table 4.5.1 are those at 0.42 of the spans of the continuous beams of Fig.4.5.2-4 and using the sectional properties of the beams as given in Fig.4.5.1, verify the results shown in the table for the following beams using Branson's equation as given by Eq.4.1.1 and the proposed model as represented by Fig.4.3.1:

(a) beams X1 and X4

(b) beams Y3 and Y6

(c) beams Z2 and Z5

Solution

(a) beams X1 and X4:

-determine Ma/Mcr:

Using $fr=7.5\sqrt{fc'}$, $Ig=bwh^3/12$ and $y_t=h/2$, the cracking moment, Mcr, for the X beams of Fig.4.5.1(a) is determine as,

$$Mcr=frIg/y_t=(7.5\sqrt{3230})(6)(8^3/12)/(8/2)=27279.9 \text{ lb.1"}$$

$$(1'/12")=2273.32 \text{ lb.1}'$$

Thus and using the maximum moment values shown on the bending moment diagram of Fig.4.5.2,

$Ma/Mcr=5343.75/2273.32=2.35$ (for the (+) moment region)

$=9500/2273.32=4.18$ (for the (-) moment region)

-determine $n\rho$ and $n\rho'$:

Using Es of $29x10^6$ psi and Ec of Eq.2.7.1 with the fc′ value given in Fig.4.5.1(a),

$$n\rho=(Es/Ec)\rho=29(10^6)\rho/[(33)(145^{1.5})\sqrt{3230}]=8.856\rho$$

Likewise,

$$n\rho'=8.856\rho'$$

Therefore for the given values of ρ and ρ′,

$n\rho=8.856(1.67)=14.8\%$	(in the (+) moment region)
$=8.856(2.86)=25.33\%$	(in the (-) moment region)
$n\rho'=8.856(1.67)=14.8\%$	(in the (+) moment region)
$=8.856(2.51)=22.23\%$	(in the (-) moment region)

-determine Ie using the proposed model (Fig.4.3.1) :

For d′/d=1.82/6.18=0.2945,

$$\alpha'=0.0006+(0.2945)[1-2(0.2945)]^2/20=0.0031$$

Thus for $n\rho'=14.8\%$ in the positive moment region,

$b'=[(0.0031)(14.8)/(0.2945)+1]bw=1.156bw$ for which,

$R=(h/d)^3/1.156=1.9$, $n\rho e=14.8/1.156=12.80\%$ or $\alpha+\beta n\rho e=0.8$

$CF=\ln[(0.8-1.9)/(0.8-3)]=-0.693$

Likewise, for $n\rho'=22.23$ in the negative moment region,

$b'=[(0.0031)(22.23)/(0.2945)+1]bw=1.234bw$ for which,

$R=(h/d)^3/1.234=1.8$, $n\rho e=25.33/1.234=20.53\%$ or $\alpha+\beta n\rho e=1.116$

$CF=\ln[(1.116-1.8)/(1.116-3)]=-1.013$

Thus,

$$\Phi_{ref} = -(Ma/Mcr)(Lcr/L)(\rho) + CF$$

$=-(2.35)(11.2/15)(1.67) - 0.693=-3.63$ (in the (+) moment region)

$=-(4.18)(3.6/5)(2.86) - 1.013=-9.62$ (in the (-) moment region)

where Lcr values were scaled from the respective moment subdiagrams of Fig.5.4.2 and L was the span of each subdiagram.

Entering Fig.4.3.1 with the respective values of Φ_{ref} and nρe one reads,

$Ie/(b'd^3/12)=0.86$ (in the (+) moment region)

$=1.125$ (in the (-) moment region)

Substituting in for the respective values of b′ and averaging gives Ie for the entire span as,

$Ie(model)=[0.86(1.156)+1.125(1.234)](6)(6.18^3/12)/2=140.6\ in^4$

-determine Ie using Branson's equation:

For the positive moment region:

$n\rho+n\rho'=14.8+14.8=29.6\%$

Substituting the above values of nρ+nρ′ along with nρ of 14.8%, nρ′ of 14.8% and d′/d of 0.2945 into the equations of Fig.3.3.2,

$x=\{-29.6+\sqrt{[29.6^2+200(14.8+14.8(0.2945))]}\}(6.18/100)=2.41"$

Thus,

$Icr=[100(2.41^3/3)+14.8(6.18)(2.41-1.82)^2+14.8(6.18)(6.18-2.41)^2](6/100)$

$=107.9\ in^4$

Hence, for $Ig=bwh^3/12=256\ in^4$,

$Ie=Icr+(Ig-Icr)(Mcr/Ma)^3$

$=107.9+(256-107.9)(1/2.35)^3=119.3\ in^4$

For the negative moment region:

$$n\rho+n\rho'=25.33+22.23=47.56\%$$

Substituting the above values of $n\rho+n\rho'$ along with $n\rho$ of 25.33%, $n\rho'$ of 22.23% and d'/d of 0.2945 into the equations of Fig.3.3.2,

$$x=\{-47.56+\sqrt{[47.56^2+200(25.33+22.23(0.2945))]}\}(6.18/100)=2.8"$$

Thus,

$$Icr=[100(2.8^3/3)+22.23(6.18)(2.8-1.82)^2+25.33(6.18)(6.18-2.8)^2](6/100)$$

$$=159.12 \text{ in}^4$$

Hence,

$$Ie=Icr+(Ig-Icr)(Mcr/Ma)^3$$

$$=159.12+(256-159.12)(1/4.18)^3=160.45 \text{ in}^4$$

Averaging Ie of the positive and negative moment regions one obtains Ie for the entire span as,

$$Ie(\text{Branson})=(119.3+160.45)/2=139.9 \text{ say } 140 \text{ in}^4$$

-determine δ:

$$\delta=8wL^4/1477EI$$

Using Ec of Eq.2.7.1 along with w and L (of a span) as given in Fig.4.5.2 into the above expression gives,

$$\delta=86.888/I$$

Substituting the respective values of Ie into the above gives,

$$\delta(\text{Model})=0.62"$$

$$\delta(\text{Branson})=0.62"$$

Thus, and for the measured deflection of 0.56",

% error(Model)=[(0.62-0.56)/0.56]100=10.7%

% error(Branson)=10.7%

(b) beams Y3 and Y6:

-determine Ma/Mcr:

Using $fr=7.5\sqrt{fc'}$, $Ig=bwh^3/12$ and $y_t=h/2$, the cracking moment, Mcr, for the Y beams of Fig.4.5.1(b) is determined as,

$$Mcr=frIg/y_t=(7.5\sqrt{3360})(12)(5^3/12)/(5/2)=21737.1 \text{ lb.1"}$$

$$(1'/12")=1811.4 \text{ lb.1'}$$

Thus, From Fig.4.5.3,

Ma/Mcr=4441.32/1811.4=2.45	(for the (+) moment region)
=7895.68/1811.4=4.36	(for the (-) moment region)

-determine nρ and nρ':

Using Es of $29x10^6$ psi and Ec of Eq.2.7.1 with the fc' value given in Fig.4.5.1(b),

$$n\rho=(Es/Ec)\rho=29(10^6)\rho/[(33)(145^{1.5})\sqrt{3360}]=8.683\rho$$

Likewise,

$$n\rho'=8.683\rho'$$

Therefore for the given values of ρ and ρ',

nρ=8.683(1.67)=14.5%	(in the (+) moment region)
=8.683(3.22)=27.96%	(in the (-) moment region)
nρ'=8.683(3.22)=27.96%	(in the (-) moment region)

-determine Ie using the proposed model (Fig.4.3.1) :

Because $n\rho'=0$ in the (+) moment region,

$b'=bw$ and thus $n\rho e=n\rho=14.5\%$ or $\alpha+\beta n\rho e=0.885$, $R=(h/d)^3=1.953$

$CF=\ln[(0.885-1.953)/(0.885-3)]=-0.683$

For $d'/d=1/4=0.25$ and $n\rho'=27.96\%$ in the negative moment region,

$b'=\{[0.0006+0.25(1-2(0.25))^2/20](27.96/0.25)+1\}bw=1.42bw$ for which,

$R=(h/d)^3/1.42=1.373$, $n\rho e=27.96/1.42=19.69\%$ or $\alpha+\beta n\rho e=1.09$

$CF=\ln[(1.09-1.373)/(1.09-3)]=-1.91$

Hence, for the respective Ma/Mcr and CF and scaling Lcr/L from Fig.4.5.3,

$\Phi_{ref}=-(Ma/Mcr)(Lcr/L)(\rho)+CF$

$=-(2.45)(11.6/15.6)(1.67)-0.683=-3.73$ (in the (+) moment region)

$=-(4.36)(3.6/5.2)(3.22)-1.91=-11.63$ (in the (-) moment region)

Entering Fig.4.3.1 with the respective values of Φ_{ref} and $n\rho e$ one reads,

$Ie/(b'd^3/12)=0.925$ (for the (+) moment region)

$=1.1$ (for the (-) moment region)

Taking Ie of the entire span as the average of the above Ie values,

$Ie(Model)=[0.925(1)+1.1(1.42)](12)(4^3/12)/2=79.58\ in^4$

-determine Ie using Branson's equation :

For the positive moment region $n\rho=14.5\%$, $n\rho'=0$. Thus, using the equations of Fig.3.3.2,

$x=[-14.5+\sqrt{(14.5^2+200(14.5))}](4/100)=1.65"$

Hence,

$Icr=[100(1.65^3/3)+14.5(4)(4-1.65)^2](12/100)=56.41\ in^4$

For the negative moment region :

$n\rho+n\rho'=27.96+27.96=55.92\%$

Substituting the above value of $n\rho+n\rho'$ along with $n\rho$ of 27.96%, $n\rho'$ of 27.96% and d′/d of 0.25 into the equations of Fig.3.3.2 gives,

$$x=\{-55.92+\sqrt{[55.92^2+200(27.96+27.96(0.25))]}\}(4/100)=1.79"$$

Hence,

$$Icr=[100(1.79^3/3)+27.96(4)(1.79-1)^2+27.96(4)(4-1.79)^2](12/100)$$

$$=96.87 \text{ in}^4$$

From Eq.4.1.1,

$$Ie=Icr+(Ig-Icr)(Mcr/Ma)^3$$

Using the respective values of Icr and Ma/Mcr along with $Ig=12(5^3/12)=125$,

$Ie=56.41+(125-56.41)(1/2.45)^3=61.07 \text{ in}^4$ (for the (+) moment region)

$=96.87+(125-96.87)(1/4.36)^3=97.21 \text{ in}^4$ (for the (-) moment region)

Taking Ie of the entire span as the average of the above Ie values,

$$Ie(Branson)=(61.07+97.21)/2=79.14 \text{ in}^4$$

-determine δ :

$$\delta=8wL^4/1477\ EI$$

Using Ec of Eq.2.7.1 along with w and L (of a span) as given in Fig.4.5.3 into the above expression gives,

$$\delta=76.582/I$$

Substituting the respective values of Ie into the above gives,

δ(Model)=0.96"

δ(Branson)=0.97"

Thus and for the measured deflection of 1",

% error(Model)=[(0.96-1)/1]100=-4%

%error(Branson)=[(0.97-1)/1]100=-3%

(c) beams Z2 and Z5 :

-determine Ma/Mcr :

Using $fr=7.5\sqrt{fc'}$, $Ig=bwh^3/12$ and $y_t=h/2$, the cracking moment,Mcr, for the Z beams of Fig.4.5.1(c) is determined as,

$$Mcr=frIg/y_t=(7.5\sqrt{3295})(12)(3^3/12)/(3/2)=7749.28 \text{ lb.1"}$$

$$(1'/12")=645.77 \text{ lb.1'}$$

Thus, from Fig.4.5.4,

Ma/Mcr=1464.26/645.77=2.77 (for the (+) moment region)

=2603.13/645.77=4.03 (for the (-) moment region)

-determine nρ and nρ′ :

Using Es of $29x10^6$ psi and Ec of Eq.2.7.1 with the fc′ value given in Fig.4.5.1(c),

$$n\rho=(Es/Ec)\rho=29(10^6)\rho/[(33)(145^{1.5})(\sqrt{3295})]=8.768\rho$$

Likewise,

$$n\rho'=8.768\rho'$$

Therefore for the given ρ and ρ′,

nρ=8.768(1.59)=13.94% (in the (+) moment region)

=8.768(2.89)=25.34% (in the (-) moment region)

nρ′=8.768(0.8)=7.01% (in the (+) moment region)

$=8.768(3.61)=31.65\%$ (in the (-) moment region)

-determine Ie using the proposed model (Fig.4.3.1) :

For $d'/d=0.69/2.31=0.3$,

$$\alpha'=0.0006+0.3[1-(2)(0.3)]^2 / 20 = 0.003$$

Thus, for $n\rho'=7.01\%$ in the (+) moment region,

$b'=[(0.003)(7.01)/0.3+1]bw=1.07bw$ for which,

$R=(h/d)^3/1.07=2.05$, $n\rho e=13.94/1.07=13.03$ or $\alpha+\beta n\rho e=0.812$,

$CF=\text{In}[(0.812-2.05)/(0.812-3)]=-0.57$

Likewise, for $n\rho'=31.65\%$ in the (-) moment region,

$b'=[(0.003)(31.65)/0.3+1]bw=1.3165bw$ for which,

$R=(h/d)^3/1.3165=1.66$, $n\rho e=25.34/1.3165=19.25\%$ or $\alpha+\beta n\rho e=1.078$

$CF=\text{In}[(1.078-1.66)/(1.078-3)]=-1.19$

Thus,

$\Phi_{ref}=-(Ma/Mcr)(Lcr/L)(\rho)+CF$

$=-(2.27)(9.6/13)(1.59)-0.57=-3.23$ (for the (+) moment region)

$=-(4.03)(3/4.5)(2.89)-1.19=-8.95$ (for the (-) moment region)

Entering Fig.4.3.1 with the respective Φ_{ref} and $n\rho e$ one reads,

$Ie/(b'd^3/12)=0.9$ (for the (+) moment region)

$=1.08$ (for the (-) moment region)

Therefore, Ie for the entire span is,

$Ie(\text{Model})=[0.9(1.07)+1.08(1.3165)](12)(2.31^3/12)/2=14.7\ in^4$

-determine Ie using Branson's equation :

For the positive moment region :

$$n\rho+n\rho'=13.94+7.01=20.95\%$$

Substituting the above $n\rho+n\rho'$, $n\rho$ of 13.94%, $n\rho'$ of 7.01% and d'/d of 0.3 into the equations of Fig.3.3.2,

$$x=\{-20.95+\sqrt{[20.95^2+200(13.94+7.01(0.3))]}\}(2.31/100)=0.91''$$

Hence,

$$Icr=[100(0.91^3/3)+7.01(2.31)(0.91-0.69)^2+13.94(2.31)(2.31-0.91)^2]$$

$$(12/100)=10.682\ in^4$$

For the negative moment region :

$$n\rho+n\rho'=25.34+31.65=56.99\%$$

Substituting the above $n\rho+n\rho'$ value along with $n\rho$ of 25.34%, $n\rho'$ of 31.65% and d'/d of 0.3 into the equations of Fig.3.3.2,

$$x=\{-56.99+\sqrt{[56.99^2+200(25.34+31.65(0.3))]}\}(2.31/100)=1.02''$$

Hence,

$$Icr=[100(1.02/3)+31.65(2.31)(1.02-0.69)^2+25.34(2.31)(2.31-1.02)^2]$$

$$(12/100)=16.724\ in^4$$

Using the above values of Icr and the respective values of Ma/Mcr along with $Ig=12(3^3/12)=27$ into Eq.4.1.1,

$$Ie=Icr+(Ig-Icr)(Mcr/Ma)^3$$

$$=10.682+(27-10.682)(1/2.27)^3=12.08\ in^4 \quad \text{(for the (+) moment region)}$$

$$=16.724+(27-16.724)(1/4.03)^3=16.88\ in^4 \quad \text{(for the (-) moment region)}$$

Taking Ie of the entire span as the average of the above Ie values,

$$Ie(Branson)=(12.08+16.88)/2=14.48\ in^4$$

-determine δ :

$$\delta=8wL^4/1477EI$$

Using Ec of Eq.2.7.1 along with w and L(of a span) as given in Fig.4.5.4 into the above expression gives,

$$\delta=18.05/I$$

Substituting the respective values of Ie into the above gives,

δ(Model)=1.23"

δ(Branson)=1.25"

Thus and for the measured deflection of 1.13",

% error(Model)=[(1.23-1.13)/1.13]100=8.8%

% error(Branson)=[(1.25-1.13)/1.13]100=10.6%

The continuous beams considered in this section involved singly and doubly reinforced rectangular sections. In real designs, however, continuous beams almost always involve flanged sections in the positive moment regions. To curb creep and shrinkage deflection such sections may be doubly reinforced. To determine Icr for singly or doubly reinforced flanged sections the equations of Fig.3.4.3 or 3.5.1 must be used. It is not hard to see that the computations involved in these equations are lengthy and complex. For that the approximation of Icr as given by Eq.3.6.1 or integrated into the curves of Fig.4.3.1 does provide a simpler alternative where the complicated equations of Figs.3.4.3 and 3.5.1 have been replaced by the simple expressions of α' and α_f. Because continuous beams are very common in concrete design such a simplification should be useful in practical design.

The errors shown in Table 4.5.1 are well within tolerable limits. For Branson's equation this was expected since the equation was actually based on beams tested under uniform loads as is already mentioned. For the proposed model, on the other hand, these results are consistent with the findings of the previous section that the model is as accurate as Branson's equation when the equation is most accurate. Because of this consistent accuracy of the proposed model as compared to Branson's equation which looses accuracy as loads deviate from being uniform and because of the gross errors that the equation has been shown (in the previous section) to produce as loads are more concentrated towards the centre of the spans one can conclude therefore that the model of Ie proposed in this study provides not only a simpler but also a more accurate alternative to Branson's equation for all types of span and loading conditions.

4.6 Summary

In the development of the model for the effective moment of inertia proposed in this chapter different ideas were considered and many aspects were discussed and the study was detailed and elaborate. However, for a concise and short review of the proposed model and its related aspects as discussed in the different sections of the chapter the following summary is furnished :

The effective moment of inertia, Ie, can be evaluated from the graphical representation of the proposed model as given in Fig.4.3.1.

With values of Φ and CF the value of Ie is directly read. When CF is undefined or the cracked transformed moment of inertia, Icr, is sought the steady portion of the curves are entered to read off the value of Ie which in this case is equal to Icre or the approximation of Icr.

While all the necessary parameters are given in the figure, the coefficient Φ can be evaluated from any proper expression. The expression proposed in this study is,

$$\Phi = -(Ma/Mcr)(Lcr/L)\rho \qquad \text{for } \rho > 1\%$$

$$= -(Ma/Mcr)(Lcr/L) \qquad \text{for } \rho \leq 1\%$$

where,

- Representing the loading intensity, Ma/Mcr is the ratio of the maximum applied service moment to the cracking moment in the region for which Ie is evaluated.
- Representing the loading type, Lcr/L is the ratio of the cracked length to the total length of the region for which Ie is evaluated.
- Representing the effect of reinforcement, ρ is the reinforcement ratio, in percentage, taken relative to the web width bw at the section corresponding to Ma.

Alternative to Fig.4.3.1, Ie can also be evaluated from the proposed model as given by,

$$Ie = Icre + (Ig - Icre)\, e^{\Phi}$$

with all terms as previously defined.

The region over which Ie is evaluated depends on the support and span condition of the element considered. When the element is simply supported the whole span is considered. If, on the other hand, the element is continuous Ie is then evaluated for every region of the bending moment diagram that is confined by two inflection points or an inflection point and a support or an exterior end. The different values of Ie obtained are then averaged to arrive at a single value for the entire element.

CHAPTER 5

DEFLECTION CALCULATIONS USING THE PROPOSED MODEL OF Ie

VS.

THE METHODS OF BS 8110 AND EUROCODE 2

5.1 Introduction

The proposed model of Ie was developed to be used in deflection calculations whenever it is felt that such calculations are best carried out using the effective moment of inertia method. As discussed in Chap.2, it can be argued that the method is relatively simpler and more convenient for practical use as compared to the curvature based approaches adapted in some codes. Those who argue against the use of the effective moment of inertia do so on the ground of the difficulties and the drawbacks usually associated with Branson's equation being the widely used model for Ie [5]. However, since these limitations no longer exist in the proposed model for Ie the argument should be to use the effective moment of inertia approach.

In this chapter, the proposed model of the effective moment of inertia as used in deflection calculations is compared with the curvature methods in the British code [10] and Eurocode 2 [11].

Pertaining to the methods in the two codes, two numerical examples are provided as a means of comparison between such methods and the proposed model of Ie in calculating deflections. Since different concepts are involved each example is preceded by an overview of the methods in the respective codes. These have already been discussed in Chap.2 but repeated nevertheless for convenience.

Because of the approximations involved, the methods in the British code and Eurocode 2 give only an estimate of the deflection values [34, 35, 36] and thus can not be used to judge the accuracy of the proposed model. Since the accuracy of the proposed model has already been established using test results only the simplicity and practical aspects of the model need to be shown.

As part of the comparison long term deflections will also be considered. It is known that long term deflections can always be obtained by multiplying the short term values by a proper magnification factor. Alternatively, however, the sustained elastic modulus can also be used to carry out such deflection calculations. The method in the British code and Eurocode 2 use the sustained elastic modulus to obtain the curvature under permanent loads and thus the long term deflections. To keep the analysis parallel to such practice the sustained elastic modulus will also be used in determining the long term deflections using the effective moment of inertia approach. It will be demonstrated that the simplicity and ease of application of the proposed model for Ie, though derived considering short term effects, as represented in Fig.4.3.1 can be useful in calculating both the short and long term deflections. In doing so it will be shown that even when using the sustained elastic modulus Fig.4.3.1 can be an efficient design aid that offers simpler and easier mean for deflection calculations.

5.2 Deflection Calculations in BS 8110

The approach suggested by the code is to determine deflections from curvatures. Using small-deflection theory, the curvature at any point x along the span can be written as,

$$1/r_x = d^2\delta/dx^2 \tag{5.2.1}$$

where,

$1/r_x \equiv$ the curvature at any point x along the span.

$\delta \equiv$ the deflection at the point considered.

Using the boundary conditions of the span Eq.5.2.1 is then double integrated by any convenient numerical integration technique to obtain the desired deflection.

The detailed method outlined above is usually complex and can not be carried out without the aid of a computer. Because of this the code proposes an approximate method where the maximum deflection is evaluated as follows,

$$\delta(max) = KL^2\ (1/r_b) \tag{5.2.2}$$

where,

$K \equiv$ a loading type factor (given in Table 3.1,pt.2 of the code)

$L \equiv$ the effective span

$1/r_b \equiv$ the curvature at the midspan of beams or at the support of cantilevers.

According to the code the curvatures used in deflection calculations should be the greater of those obtained from the uncracked and partially cracked sections as described below,

1. <u>The uncracked section</u>:

   In this case the gross concrete area with all steel areas (both tension and compression if any) transformed into an equivalent area of concrete is considered. The curvature is then calculated as,

   $$(1/r)_{tr} = M/EcI_{tr} \qquad (5.2.3)$$

   where I_{tr} is the moment of inertia of the gross section thus assumed.

2. <u>The Partially Cracked Section:</u>

   The partially cracked section is called the cracked section in the code. However, the term partially is used herein to indicate the tension stiffening of concrete which is considered and to avoid confusion with the cracked section as defined in this study. This is a section in which the concrete in the tension zone below the neutral axis is assumed to sustain a triangular stress distribution. Unlike the concrete compressive stresses above the neutral axis and the tensile stress of the steel, these concrete tensile stresses are not related to the strains. In addition, the tensile stress in the concrete at the level of the tension steel, denoted by f_{ct}, is assumed to have values of 1 and 0.55 MPa for short and long term loadings, respectively. Figure 5.2.1 summarizes the above assumptions.

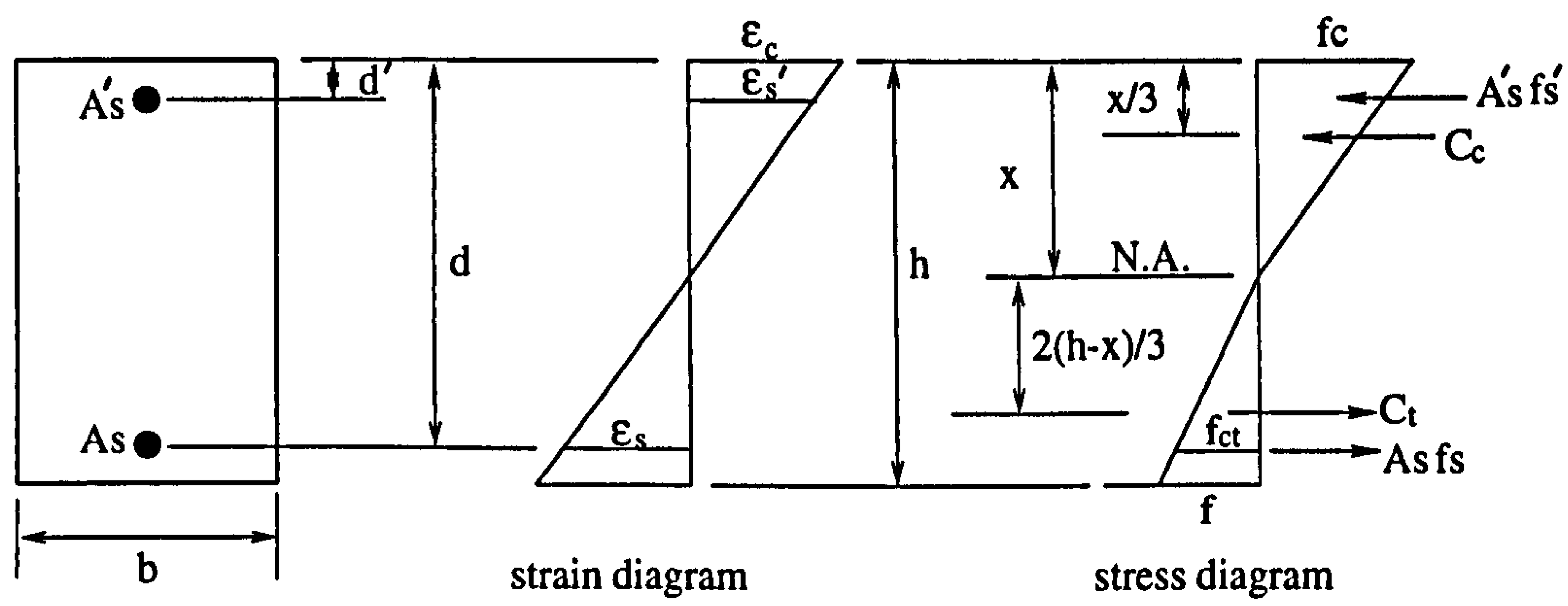


Figure 5.2.1 The assumptions of BS 8110 for the partially cracked section

From the strain diagram in Fig.5.2.1 and for n=Es/Ec, $fs'=\varepsilon_s'Es$, $fs=\varepsilon_sEs$ and $f_c=\varepsilon_cEc$ it can be shown that ,

$fs' = nfc(x-d')/x$, $fs=nfc(d-x)/x$

From the stress diagram it can be seen that $f_{ct}=(d-x)f/(h-x)$. Thus the tensile stress in concrete at the soffit of the beam can be written as,

$f = f_{ct}(h-x)/(d-x)$

Therefore one can write,

$Asfs = nAsfc(d-x)/x$

$As'fs' = nAs'fc(x-d')/x$

C_t (the resultant concrete tension force)$=0.5(f_{ct}b)(h-x)^2/(d-x)$

C_c (the resultant concrete compressive force)$=0.5fcbx$

Force equilibrium requires that,

$C_c + As'fs' - Asfs - C_t = 0$

Moment equilibrium requires that,

$(2/3)(C_c)(x) + As'fs'(x-d') + Asfs(d-x)+(2/3)(C_t)(h-x) = M$

Substituting the relative expressions into the force and moment equilibrium equations and solving for fc gives,

$$fc=[bf_{ct}(x/2)(h-x)^2/(d-x)]/[bx^2/2+nAs'(x-d')-nAs(d-x)] \qquad (5.2.4)$$

$$fc=[M(x)-bf_{ct}(x/3)(h-x)^3/(d-x)]/[bx^3/3+nAs'(x-d')^2+nAs(d-x)^2] \qquad (5.2.5)$$

Equations 5.2.4 and 5.2.5 are derived for the rectangular section assumed in Fig.5.2.1. However, similar equations can also be derived for flanged sections. Since the numerical example to be given will only consider a rectangular section these are not needed for the current discussion.

In order to obtain a unique value of fc from Eqs.5.2.4 and 5.2.5 for the same value of x the equations are solved iteratively or graphically as will be shown in the example.

Once x and fc are found the curvature at the section considered is calculated as,

$$(1/r)_{pcr} = fc/xEc \qquad (5.2.6)$$

With regard to the loading history and duration the following are defined :

$1/r_{s.perm} \equiv$ the curvature due to the short term effect of the permanent load.

$1/r_{l.perm}$ ≡ the curvature due to the long term effect of the permanent load.

$1/r_{s.tot}$ ≡ the curvature due to the short term effect of the total load.

$1/r_{shr}$ ≡ the curvature due to shrinkage effect.

Once the curvatures as defined above are found for the uncracked and partially cracked sections the final curvature, $1/r_b$, to be used in Eq.5.2.2 is obtained as follows:

a. For short term deflections :

$$1/r_b = \max \left[(1/r_{s.tot})_{pcr} , (1/r_{s.tot})_{tr} \right] \tag{5.2.7}$$

b. For long term deflections :

$$1/r_b = 1/r_{l.perm} + 1/r_{s.tot} - 1/r_{s.perm} + 1/r_{shr} \tag{5.2.8}$$

where each individual curvature is taken as the maximum of the values obtained for the uncracked and partially cracked sections.

While all the curvatures are found from either Eq.5.2.3 or 5.2.6 the shrinkage curvature to be used in Eq.5.2.8 is defined by the code as

$$1/r_{shr} = (n\ \varepsilon_{shr})(s_s / I) \tag{5.2.9}$$

where,

ε_{shr} ≡ free shrinkage strain of plain concrete as defined in cl.7.4,pt.2 and represented in Fig.7.2 of the code.

n ≡ long term modular ratio = Es/E_{eff} = $Es(1+c_t)/Ec$

c_t ≡ creep coefficient (referred to by the code as Φ) as defined in cl.7.3,pt.2 and represented in Fig.7.1 of the code.

s_s ≡ moment of steel area about the centroid of the considered section (which is either the partially cracked or uncracked section).

I ≡ moment of inertia of the considered section (which is either the partially cracked or uncracked section).

5.3 The Proposed Model of Ie vs. BS 8110

Now that the background information for deflection calculations in the British code is given and the necessary equations for carrying out deflection calculations are provided a numerical example is presented. The example is intended to show the great simplicity offered by the proposed model for Ie in deflection calculations as compared to the approximate method in the code.

Example 5.3.1 (adapted from Ref.35)

Given:

-A simply supported beam with an effective span, L, of 12m has a prismatic rectangular cross section of width b=300 mm, overall depth h=700 mm and effective depth d=600 mm.

-The beam can be considered as singly reinforced with As = 2450 mm^2 and fy = 460 MPa.

-The loads acting on the beam are all uniformly distributed with 10KN/m as dead load (permanent) and 5 KN/m live load (transitory).

-The beam is made of normal weight aggregate concrete with fcu=30 MPa and props are removed at 28 days.

Required:

Calculate the maximum short and long term deflections using :

(1) The approximate method of BS 8110

(2) The "effective moment of inertia method" with Ie taken as proposed in this study and represented in Fig.4.3.1.

Discuss the results obtained in (1) and (2).

Solutions

-Determine the moment at midspan (maximum for simply supported beams) :

$$M\ (\text{total}) = wL^2/8$$

$$= (10+5)(12)^2/8 = 270 \text{ KN.m} = 270\text{x}10^6 \text{ N.mm}$$

$$M\ (\text{permanent}) = wL^2/8 = (10)(12)^2/8 = 180 \text{ KN.m} = 180\text{x}10^6 \text{ N.mm}$$

-Determine the elastic modulus:

for normal weight concrete,

$$Ec = 20 + 0.2fcu$$

$$= 20 + 0.2(30) = 26 \text{ GPa} = 26\text{x}10^3 \text{ MPa} \qquad \text{(from Eq.2.7.4)}$$

-Determine the modular ratio n, nAs, ρ and nρ (short term) :

n=Es/Ec

= 200/26 = 7.69

Hence,

nAs = 7.69(2450) = 18840.5 mm^2, ρ = 100(2450)/(300x600) = 1.36%

nρ= 7.69(1.36)= 10.46%

(1) <u>Using the BS 8110 approximate method:</u>

(a) Consider the partially cracked section:-

- Under total load - short term :

Substituting f_{ct}=1 (short term effects), nAs′=0, nAs=18840.5, b=300, h=700, d=600 and different values of x into Eq.5.2.4 gives,

for x=220 mm, fc=198.87 MPa

for x=225 mm, fc=38.42 MPa

for x=230 mm, fc=21.37 MPa

for x=235 mm, fc=14.84 MPa

for x=240 mm, fc=11.39 MPa

Similarly, substituting f_{ct}=1, nAs′=0, nAs=18840.5, b=300, h=700, d=600 and M=270x10^6 into Eq.5.2.5 gives,

for x=220 mm, fc=14.00 MPa

for x=225 mm, fc=14.34 MPa

for x=230 mm, fc=14.66 MPa

for x=235 mm, fc=14.96 MPa

for x=240 mm, fc=15.25 MPa

To obtain a unique solution for x and fc the above x values and their corresponding fc values were plotted as shown in Fig.5.3.1. The point of intersection of the two curves is then taken to correspond to the values of x and fc sought. Thus and from Fig.5.3.1,

$x \cong 234$ mm , $fc \cong 15$ MPa

Therefore, substituting into Eq.5.2.6 one obtains,

$(1/r_{s.tot})_{pcr} = 15 / (26x10^3)(234) = 2.5x10^{-6}/mm$

- Under permanent load - short term :

Substituting $f_{ct}=1$, $nAs'=0$, $nAs=18840.5$, $b=300$, $h=700$, $d=600$ and different values of x into Eq.5.2.4 gives,

for x=230 mm, fc=21.37 MPa

for x=235 mm, fc=14.84 MPa

for x=240 mm, fc=11.39 MPa

for x=245 mm, fc=9.26 MPa

for x=250 mm, fc=7.802 MPa

Similarly, substituting the same values of $f_{ct,}$ nAs′, nAs, b, h, and d along with M= $180x10^6$ into Eq.5.2.5 gives,

for x=230 mm, fc=9.21 MPa

for x=235 mm, fc=9.41 MPa

for x=240 mm, fc=9.60 MPa

for x=245 mm, fc=9.78 MPa

for x=250 mm, fc=9.94 MPa

Plotting the values of x and fc as found above Fig.5.3.2 was obtained.

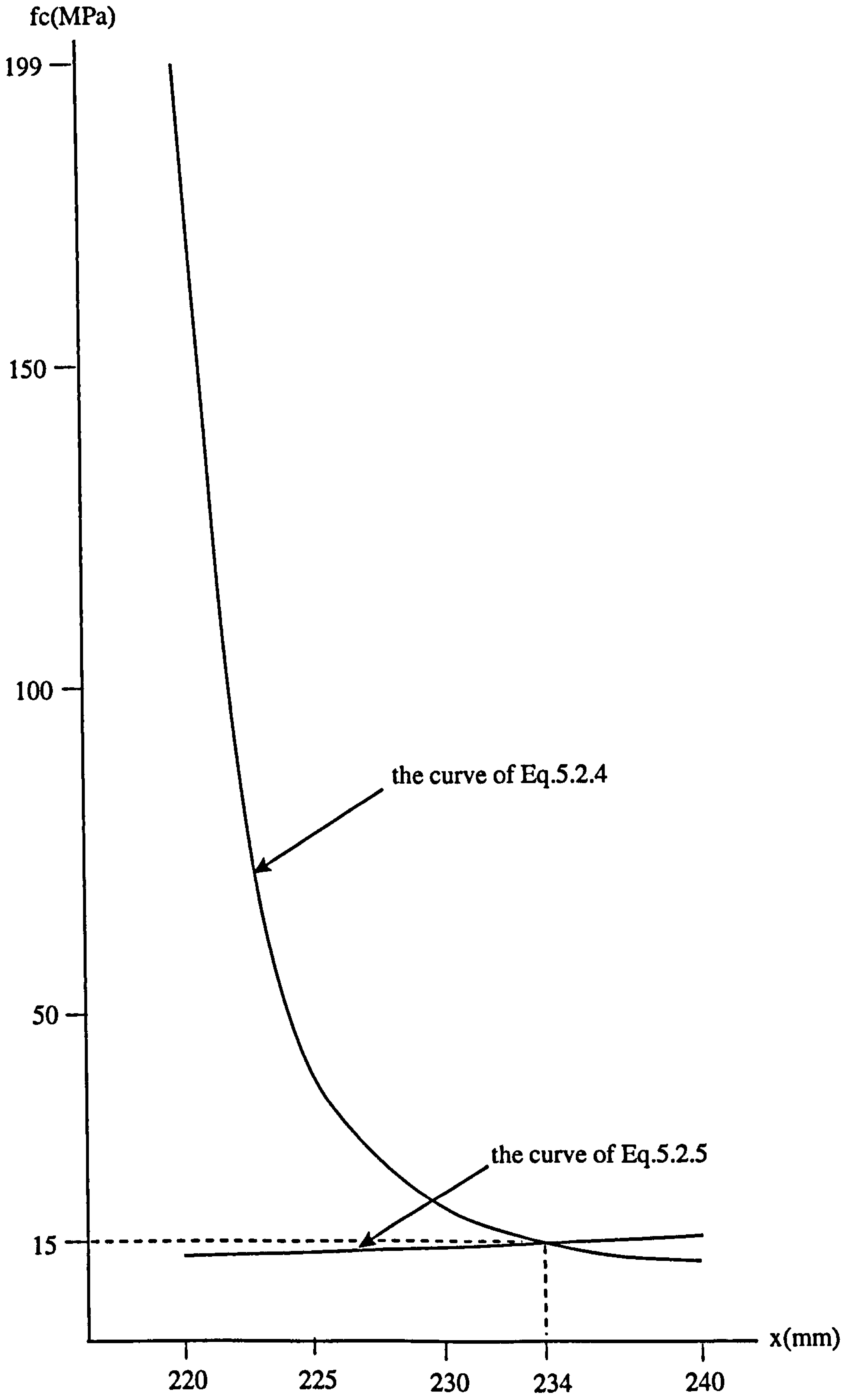


Figure 5.3.1 The curves of Eqs.5.2.4 and 5.2.5 considering total load-short term

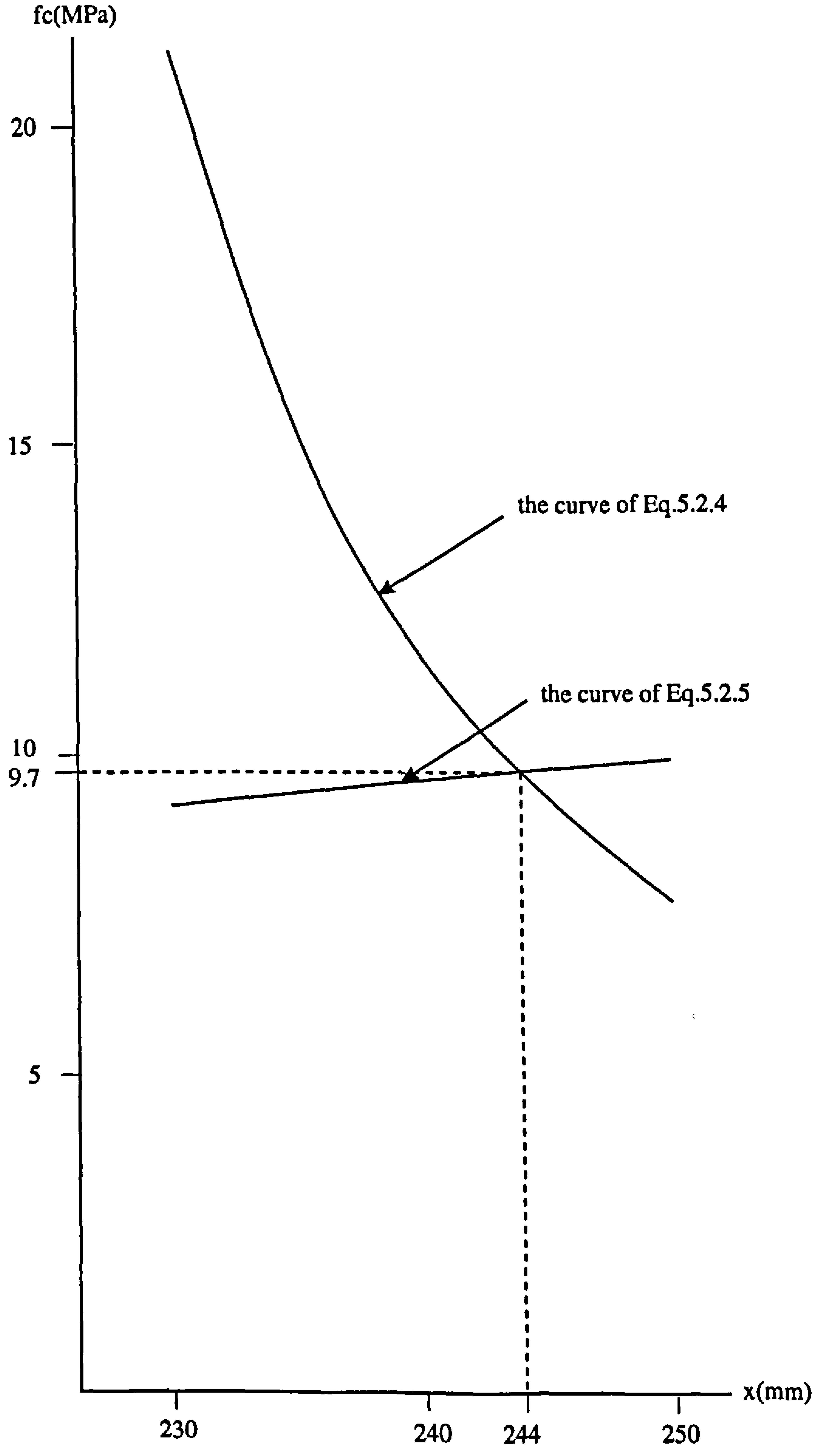


Figure 5.3.2 The curves of Eqs.5.2.4 and 5.2.5 considering permanent load-short term

From the figure,

$x \cong 244$ mm, fc=9.7 MPa

Therefore, from Eq.5.2.6

$$(1/r_{s.perm})_{pcr} = 9.7 \;/\; (26x10^3)(244) \cong 1.5x10^{-6}/mm$$

- Under permanent load - long term :

the modular ratio, n, for long term effects is defined as

$$n \text{ (long term)} = Es/E_{eff} = Es(1+c_t)/Ec$$

where E_{eff} =Ec/(1+c_t). For a value of the effective thickness of 2(700x300)/2(300+700)=210 mm, as defined in the code, and from Fig.7.1 of cl.7.3,pt.2 and for 45% relative humidity, the creep coefficient is found to be,

$$c_t \cong 2.75$$

Thus,

$$n = 200(1+2.75)/26 = 28.846$$

$$nAs = 28.85x2450 = 70672.7 \text{ mm}^2$$

Substituting f_{ct}=0.55 MPa (long term effects), nAs′=0, nAs=70672.7 b=300, h=700, d=600 and different values of x into Eq.5.2.4 gives,

for x=350 mm, fc=20.02 MPa

for x=355 mm, fc=8.95 MPa

for x=360 mm, fc=5.77 MPa

for x=365 mm, fc=4.26 MPa

for x=370 mm, fc=3.38 MPa

Similarly, substituting same values of f_{ct}, nAs′, nAs, b, h and d along with

$M=180x10^6$ into Eq.5.2.5 gives,

for x=350 mm, fc=6.86 MPa

for x=355 mm, fc=6.96 MPa

for x=360 mm, fc=7.05 MPa

for x=365 mm, fc=7.13 MPa

for x=370 mm, fc=7.20 MPa

Plotting the above values of x and fc, Fig.5.3.3 was obtained. From the figure,

$x \cong 358$ mm, $fc \cong 6.9$ MPa

Therefore,

$$(1/r_{l.perm})_{pcr} = fc/(E_{eff}x) = 6.9(1+2.75) / (26x10^3x358) = 2.8x10^{-6}/mm$$

- Under shrinkage effect :

Substituting the x value obtained for the partially cracked section under the effect of the permanent load (x=358 mm) into the Equation of Icr of Fig.3.3.2 along with nρ of 39.23% (which is 28.846x1.36),

$$I=I_{pcr}=[100(358^3/3) + 39.23(600)(600-358)^2](300/100)=8.73x10^9mm^4$$

using x of 358 mm the moment of the steel area about the centroid of the section is found as,

$$s_s = As\ (d - x) = 2450(600 - 358) = 593x10^3\ mm^3$$

For 45% humidity, Fig.7.2 of cl.7.4,pt.2 of the code gives

$$\varepsilon_{shr} \cong 390x10^{-6}$$

Therefore,

$$(1/r_{shr})_{pcr} = [(390x10^{-6})(28.846)(593x10^3)]/(8.73x10^9) = 0.8x10^{-6}$$

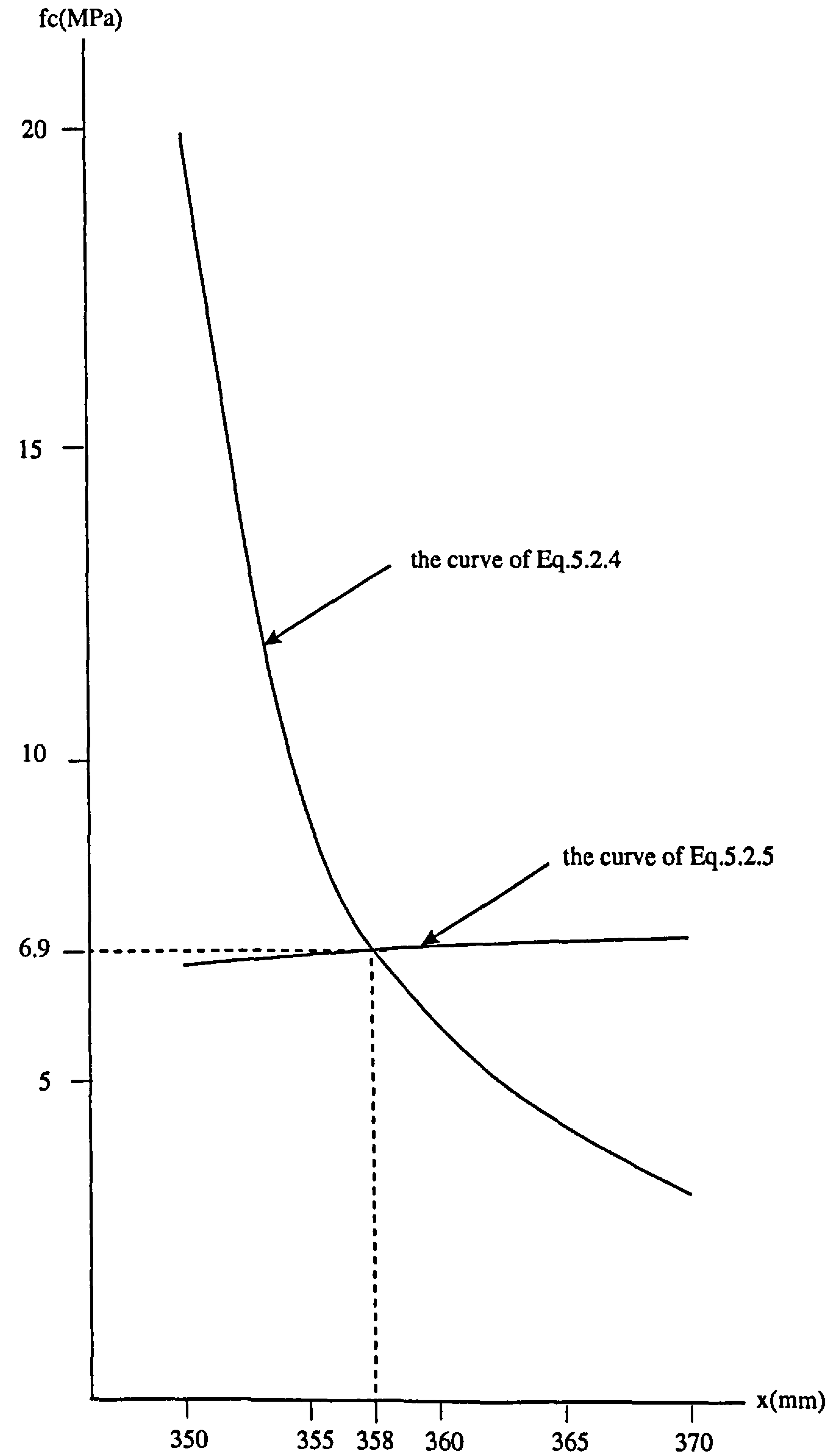


Figure 5.3.3 The curves of Eqs.5.2.4 and 5.2.5 considering permanent load-long term

(b) Consider the uncracked section :

- Under total load - short term :

From Eq.5.2.3,

$$(1/r_{s.tot})_{tr} = M/EcI_{tr}$$

Substituting I_g=$bh^3/12$ for I_{tr} (due to lower n values under short term effects $I_{tr} \cong I_g$), 270x10^6 N.mm for M and 26x10^3 MPa for Ec into the equation,

$$(1/r_{s.tot})_{tr} = [12(270x10^6)]/[(26x10^3)(300x700)^3 = 1.2x10^{-6}/mm$$

- Under permanent load - short term :

Substituting I_g for I_{tr} and 180x10^6 N.mm for M along with Ec of 26x10^3 MPa into Eq.5.2.3 gives,

$$(1/r_{s.perm})_{tr} = 0.8x10^{-6}/mm$$

- Under permanent load - long term :

Because of the higher n values under long term effects I_{tr} has to be evaluated considering the transformed area of steel. Thus, from Fig.5.3.4

x=[210000x350+(2x35336.35)(600)]/[210000+2x35336.35] = 413 mm

I_{tr}=300$(700)^3$/12+210000$(413-350)^2$+70673$(600-413)^2$=1.188x10^{10} mm^4

Substituting the above I_{tr}, 180x10^6 for M and E_{eff} into Eq.5.2.3 gives

$$(1/r_{l.perm})_{tr} = 2.19x10^{-6}/mm$$

- Under shrinkage effect :

Using the above x and I_{tr}, n=28.846 and ε_{shr} of 390x10^{-6} into Eq.5.2.9,

$$(1/r_{shr})_{tr} = 28.846x390x10^{-6}[2450(600-413)]/(1.188x10^{10})=0.434x10^{-6}/mm$$

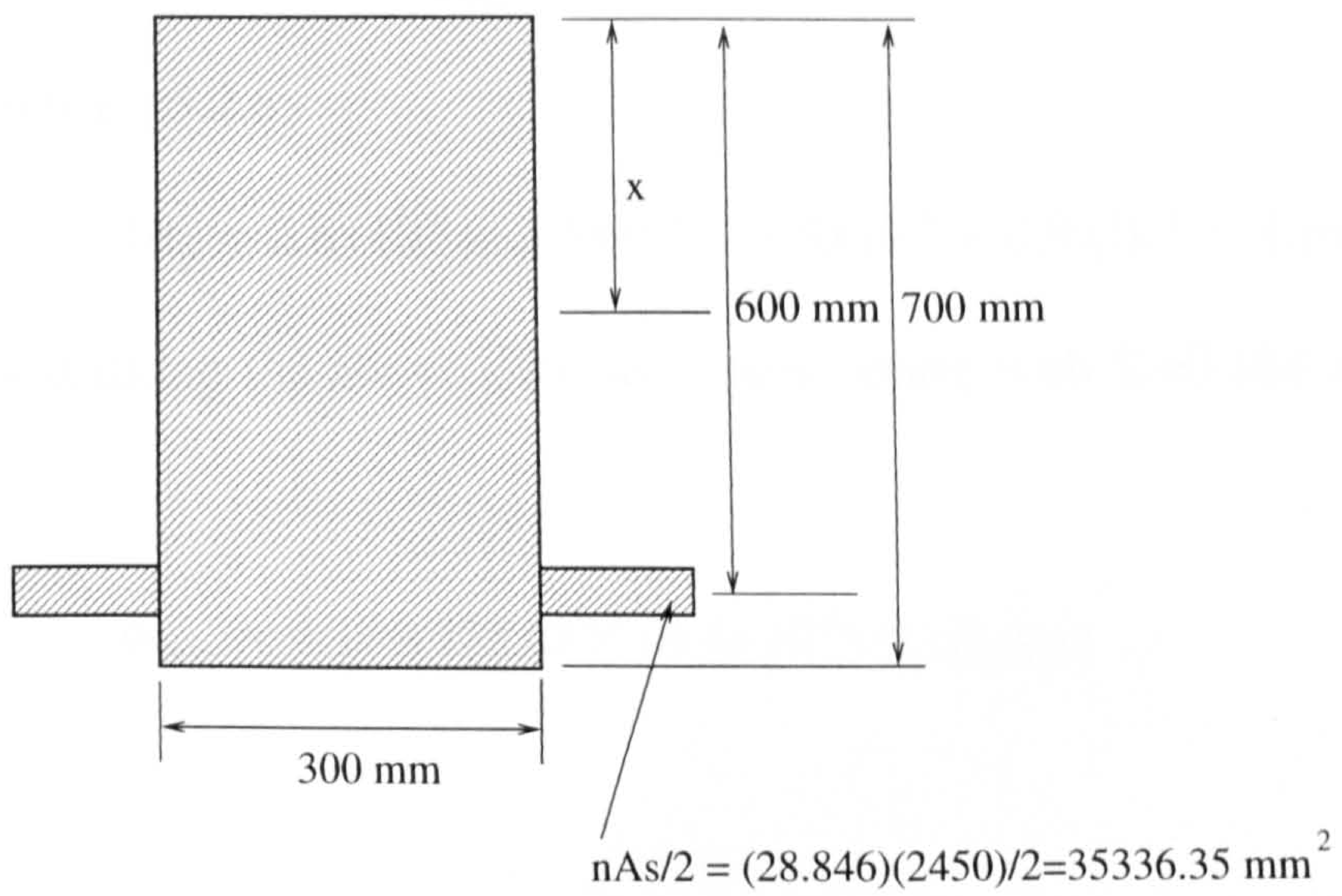


Figure 5.3.4 The transformed section considered in evaluating I_{tr} in example 5.3.1.

(c) Summarizing all results :

$$1/r_{s.tot} = \max\{(1/r_{s.tot})_{pcr}\ ,\ (1/r_{s.tot})_{tr}\} = 2.5\text{x}10^{-6}/\text{mm}$$

$$1/r_{s.perm} = \max\{(1/r_{s.perm})_{pcr}\ ,\ (1/r_{s.perm})_{tr}\} = 1.5\text{x}10^{-6}/\text{mm}$$

$$1/r_{l.perm} = \max\{(1/r_{l.perm})_{pcr}\ ,\ (1/r_{l.perm})_{tr}\} = 2.8\text{x}10^{-6}/\text{mm}$$

$$1/r_{shr} = \max\{(1/r_{shr})_{pcr}\ ,\ (1/r_{shr})_{tr}\} = 0.8\text{x}10^{-6}/\text{mm}$$

(d) Deflection calculations :

- Short term deflection due to total load, $\delta_{s.tot}$:

From Eq.5.2.7,

$$1/r_b = 1/r_{s.tot.} = \max\ \{2.5\text{x}10^{-6}\ ,\ 1.2\text{x}10^{-6}\} = 2.5\text{x}10^{-6}/\text{mm}$$

Substituting the above curvature along with K=0.104 into Eq.5.2.2,

$$\delta_{s.tot} = 0.104(12\text{x}10^3)^2\ (2.5\text{x}10^{-6}) = \underline{37\ \text{mm}}$$

- Long term deflection, δ_{long} :

From Eq.5.2.8,

$$1/r_b = 2.8x10^{-6} + 2.5x10^{-6} - 1.5x10^{-6} + 0.8x10^{-6} = 4.6x10^{-6}/mm$$

Substituting the above curvature value along with K=0.104 into Eq.5.2.2,

$$\delta_{long} = 0.104(12x10^3)^2 (4.6x10^{-6}) = \underline{69\ mm}$$

(2) Using Fig.4.3.1 :

- Determine Ma/Mcr, Φ and Ie :

Using Mcr as defined in Chap.4, f_r from Eq.2.10.2.b and y_t=h/2,

$$Ma/Mcr = Ma/(f_r I_g/y_t) = Ma(350)/[(0.56\sqrt{30})(8575x10^6)]$$

Thus, for Ma as the maximum moment within the span,

Ma/Mcr (total load) =3.59,

Ma/Mcr (permanent load) = 2.4

As the section is singly reinforced and rectangular, b′=b, nρe=nρ and $R=(h/d)^3$ = 1.59. Thus

for nρ of 10.46% (short term), α+βnρe=0.683 or CF=-0.938

for nρ of 39.23% (long term), α+βnρe=1.585 or CF=-5.645

Substituting the respective Ma/Mcr into the expression of Φ of Eq.4.4.2.1 given for uniformly loaded simple spans,

$$\Phi_{ref} = \Phi + CF$$

$$= -(Ma/Mcr)[\sqrt{(1-Mcr/Ma)}]\ \rho + CF$$

$$=-(3.59)[\sqrt{(1-1/3.59)}](1.36)-0.938=-5.1 \quad \text{(total load-short term)}$$

$=-(2.4)[\sqrt{(1-1/2.4)}](1.36)-0.938=-3.43$ (permanent load-short term)

$=-(2.4)[\sqrt{(1-1/2.4)}](1.36)-5.645=-8.14$ (permanent load-long term)

Entering Fig.4.3.1 with the respective Φ_{ref} and $n\rho e$ one reads,

$Ie = 0.7(300)(600)^3/12 = 378x10^7 \text{ mm}^4$ (total load-short term)

$= 0.8(300)(600)^3/12 = 432x10^7 \text{ mm}^4$ (permanent load-short term)

$= 1.595(300)(600)^3/12=8613x10^6 \text{ mm}^4$ (permanent load-long term)

- Determine deflections :

The deflection at midspan of a simply supported beam is given by

$\delta = 5MaL^2 / 48EcIe$

- Short term deflection due to total load :

Substituting the respective values of Ma, Ie and Ec, the short term deflection due to total load, $\delta_{s.tot}$, is determined as,

$\delta_{s.tot} = 5(270x10^6)(12x10^3)^2/48(26000)(378x10^7) = \underline{41 \text{ mm}}$

- Long term deflection:

Similar to Eq.5.2.8 the total long term deflection, δ_{long} , is given by

$\delta_{long} = \delta_{l.perm} + \delta_{s.tot} - \delta_{s.perm} + \delta_{shr}$

Using the respective Ie, Ma, Ec and c_t as previously found with E_{eff}=Ec/1+c_t,

$\delta_{l.perm} = 5(180x10^6)(12000)^2 (1+2.75)/48(26000)(8613x10^6) = 45 \text{ mm}$

$\delta_{s.perm} =5(180x10^6)(12000)^2/48(26000)(432x10^7) = 24 \text{ mm}$

When using the effective moment of inertia approach, the shrinkage deflection, δ_{shr} , is best obtained using the following as proposed and discussed in Refs.1 and 9 (useful related discussions are also given in Refs.14, 25, 37, 38 and 39)

$$\delta_{shr} = 0.7(L^2/h)(\gamma\,\varepsilon_{shr})(\rho-\rho')^{1/3}[(\rho-\rho')/\rho]^{1/2} \qquad \text{for } (\rho-\rho') \leq 3\%$$

$$= (L^2/h)(\gamma\,\varepsilon_{shr}) \qquad \text{for } (\rho-\rho') > 3\%$$

with γ given as 0.125 for simply supported beams (0.5 for cantilevers, 0.086 for one end continuous and 0.063 for both end continuous),

$$\delta_{shr} = 0.7(12000^2/700)(0.125)(390\text{x}10^{-6})(1.36)^{1/3} = 7.7 \text{ mm}$$

Therefore,

$$\delta_{long} = 45 + 41 - 24 + 7.7 = 69.7 \text{ or say } \underline{70 \text{ mm}}$$

- Discussion of Results:

Judging from the accuracies obtained when the proposed model of Ie is used to calculate short term deflections of beams under uniform loads in Chap.4 (i.e. see Table 4.5.1) the deflection of 41 mm found using Ie as obtained from Fig.4.3.1 is thought to be more representative of the real value than 37 mm as determined by the method in the code.

Different references consulted suggest that methods for calculating shrinkage curvatures as that in the British code tend to overestimate the deflection values while the method used in part (2) generally provides the closest agreement with test results [1, 19, 25]. As the short term deflection of 37 mm is thought to be on the low side while the total long term deflection values obtained in parts (1) and (2) are almost equal, these observations seem to be consistent with the results of the present example.

It follows therefore that the almost equal end results do not imply that the code's method is as accurate as the proposed model of Ie in calculating deflections as established in Chap.4 but that in this particular example the errors involved using the code's method tend to offset each other.

The results as obtained from the example and discussed above show that the lengthy and elaborate process involved in finding deflections using the present code method is not justifiable. By comparing part (1) to part (2) of the solution it is not hard to see that Fig.4.3.1 provided an efficient and a quick way of obtaining values of Ie which resulted in deflection values that are at least as accurate as those obtained using the code's method. With no iterations or plotting required the calculations are simple, straightforward and easy.

Should flanged sections be involved the solution by the method in the code will become even more elaborate as the corresponding equilibrium and compatibility equations required in the analysis of the partially cracked section will become more complicated. In addition, with the sustained elastic modulus giving higher modular ratios the neutral axis will almost always fall in the web and the complicated equations of Figs.3.4.3 and 3.5.1 will then be required to evaluate the moment of inertia of the partially cracked section.

Therefore, although not intended as an alternative for the method in the code but rather as a substitute for Branson's equation, the proposed model of Ie as represented in Fig.4.3.1 seems to be simpler and a more reliable mean for calculating deflections.

5.4 Deflection Calculations in Eurocode 2

While the overall approach of calculating deflections from curvatures is similar to that in the British code [10, 23], Eurocode 2 [11] requires that the curvatures used in deflection calculations be evaluated as the average (rather than the maximum) of the curvatures corresponding to the cracked section (rather than the partially cracked section) and the uncracked one. Namely, using the previously defined notations,

$$1/r = \xi\ (1/r)_{cr} + (1 - \xi)(1/r)_{ur} \qquad (5.4.1)$$

where,

$(1/r)_{cr}$ = The curvature of the cracked section. For loading effects it is the service moment divided by flexural rigidity. For shrinkage effects it is evaluated from Eq.5.2.9.

$(1/r)_{ur}$ = Same as above but with respect to the uncracked section.

ξ = $1 - \beta_1\ \beta_2\ (Mcr/Ma)^2$

β_1 = 1 for high bond steel

= 0.5 for plain bars

β_2 = 1 for short term loadings

= 0.5 for long term loadings

For total deflections the curvatures according to Eq.5.4.1 under different effects are summed to obtain the final curvature, $1/r_b$. This is then substituted into Eq.5.2.2 to obtain the value of the final deflection.

5.5 The Proposed Model of Ie vs. Eurocode 2

As Eurocode 2 is the anticipated future code of the European countries, it was thought useful to provide a numerical example in which the proposed model of Ie as used in deflection calculations is compared with the method in Eurocode 2. The example is meant to show the reliability as well as the simplicity of the proposed model as compared to the method in the code.

Example 5.5.1 (adapted from Ref.36)

Given:

- A simply supported beam with an effective span, L, of 9.5m has a prismatic rectangular cross section of width b=300 mm, overall depth h=700 mm and an effective depth d=600 mm.
- The beam can be considered as singly reinforced with As=2450 mm^2
- The concrete used is normal weight with a cubic characteristic strength of 30 MPa and props are removed at 28 days.
- The beam carries a uniformly distributed load giving rise to a quasi-permanent moment of 200 KN.m. such that Ma/Mcr=3.1

  The effective elastic modulus and the free shrinkage strain were found to be,

$$E_{eff} = 8710 \text{ MPa}$$

$$\varepsilon_{shr} = 590 \times 10^{-6}$$

Required:

Calculate the long term deflection due to the quasi-permanent load using,

(1) The method in Eurocode 2

(2) The "effective moment of inertia method" with Ie taken as proposed in this study and represented in Fig.4.3.1

Discuss the results obtained in (1) and (2)

Solution

-Determine nAs, ρ and nρ:

$nAs = (Es/E_{eff})As = (200/8.71)(2450) = 56257.2\ mm^2$

$\rho = 100(As/bd) = 100(2450)/(300x600) = 1.36\%$

$n\rho = (200/8.71)(1.36) = 31.25\%$

(1) Using the method in Eurocode 2:

(a) Consider the cracked section:

- Determine Icr:

Substituting nρ of 31.25% into the equations of Fig.3.3.2 with nρ′ and d′/d taken as zero,

$$x = \{-31.25+\sqrt{[(31.25)^2 + 200(31.25)]}\}(600/100) = 322.6\ mm$$

Hence,

$$Icr=[100(322.6^3/3)+31.25(600)(600-322.6)^2](300/100)=7686x10^6\ mm^4$$

- Determine $(1/r)_{cr}$ under loading effects:

$$(1/r)_{cr} = M/E_{eff}Icr = (200x10^3)/(8.71x7686x10^6) = 2.99x10^{-6}/mm$$

- Determine $(1/r_{shr})_{cr}$:

From Eq.5.2.9,

$(1/r_{shr})_{cr} = (n\ \varepsilon_{shr})(s_s / Icr)$

Using the value of x as determined above the moment of the steel area about the centroid of the section is found as,

$s_s = As(d\text{-}x) = 2450(600\text{-}322.6) = 679630\ mm^3$

Using the above values of s_s and Icr and the given value of ε_{shr} one obtains,

$(1/r_{shr})_{cr} = (200/8.71)(590x10^{-6})[679630/(7686x10^6)] = 1.2x10^{-6}/mm$

(b) Consider the uncracked section:

- Determine I_{tr} :

From Fig.5.3.4 with nAs now taken as 56257.2 mm^2,

x=(210000x350+56257.2x600)/(210000+56257.2)=402.8 mm

Hence,

$I_{tr} = 300(700)^3/12 + 210000(402.8\text{-}350)^2 + 56257.2(600\text{-}402.8)^2$

$=1.135x10^{10}\ mm^4$

- Determine $(1/r)_{tr}$ under loading effects:

$(1/r)_{tr} = M/E_{eff}I_{tr} = (200x10^3)/(8.71x1.135x10^{10}) = 2.02x10^{-6}/mm$

- Determine $(1/r_{shr})_{tr}$:

Using the value of x as found above the moment of the steel area about the centroid of the section is found as,

$s_s = As(d\text{-}x) = 2450(600\text{-}402.8) = 483140\ mm^3$

Substituting the above value of s_s , I_{tr} and ε_{shr} into Eq.5.2.9,

$(1/r_{shr})_{tr} = (200/8.71)(590x10^{-6})[483140/(1.135x10^{10})]=0.6x10^{-6}/mm$

(c) Calculate the total average curvature, $1/r_b$:

- Determine ξ :

$$\xi = 1 - \beta_1\beta_2(Mcr/Ma)^2 = 1 - 1x0.5x(1/3.1)^2 = 0.95$$

- Determine the average curvature due to loading, $1/r_{load}$:

$$1/r_{load} = \xi\ (1/r)_{cr} + (1 - \xi)(1/r)_{tr}$$

$$= 0.95(2.99x10^{-6}) + (1\text{-}0.95)(2.02x10^{-6})$$

$$= 2.94x10^{-6}/mm$$

- Determine the average curvature due to shrinkage, $1/r_{shr}$:

$$1/r_{shr} = \xi(1/r_{shr})_{cr} + (1 - \xi)(1/r_{shr})_{tr}$$

$$= 0.95(1.2x10^{-6}) + (1 - 0.95)(0.6x10^{-6})$$

$$= 1.17x10^{-6}/mm$$

Therefore,

$$1/r_b = 1/r_{load} + 1/r_{shr}$$

$$= 2.94x10^{-6} + 1.17x10^{-6}$$

$$= 4.11x10^{-6}/mm$$

(d) Calculate the required long term deflection, δ_{long} :

Substituting $1/r_b$ into Eq.5.2.2 with K of 0.104,

$\delta_{long} = 0.104(9500)^2(4.11x10^{-6}) = 38.6$ or say <u>39 mm</u>

(2) Using Fig.4.3.1

- Determine Φ and Ie:

As the section is singly reinforced rectangular, $b'=b$ and $R=(h/d)^3=1.59$. Thus,

for $n\rho e = n\rho$ of 31.25%, $\alpha+\beta n\rho e = 1.44$ or CF = -2.34

Using Eq.4.4.2.1 for Φ, Φ_{ref} is obtained as

$$\Phi_{ref} = \Phi + CF = -(Ma/Mcr)[\sqrt{(1-Mcr/Ma)}]\rho + CF$$

$$= -(3.1)[\sqrt{(1-1/3.1)}](1.36) -2.34$$

$$= -5.81$$

From Fig.4.3.1 and for $n\rho e$ of 31.25% and Φ_{ref} of -5.81,

$$Ie = 1.44(300)(600^3/12) = 7776\text{x}10^6 \text{ mm}^4$$

- Determine the required long term deflection, δ_{long} :

deflection due to loading,

$$\delta_{load} = 5MaL^2/48E_{eff}Ie$$

$$= 5(200\text{x}10^6)(9500)^2/(48\text{x}8710\text{x}7776\text{x}10^6)$$

$$= 27.76 \text{ mm}$$

deflection due to shrinkage,

$$\delta_{shr} = 0.7(L^2/h)(\gamma\,\varepsilon_{shr})(\rho-\rho')^{1/3}[(\rho-\rho')/\rho]^{1/2}$$

$$= 0.7(9500^2/700)(0.125)(590\text{x}10^{-6})(1.36)^{1/3}$$

$$= 7.37 \text{ mm}$$

Therefore,

$$\delta_{long} = \delta_{load} + \delta_{shr} = 27.76 + 7.37 = \underline{35 \text{ mm}}$$

- Discussion of Results:

Using $1/r_{load}$ of 2.94×10^{-6} as obtained in part (1) into Eq.5.2.2 the deflection due to loading effects using the method in Eurocode 2 can be shown to be 27.6 mm. Since this value is almost equal to that obtained in part (2), the difference of 4 mm between the total deflections obtained in the two parts is due to the different methods used in calculating shrinkage deflections. Because the method used in part (2), as discussed in Sec.5.3, is believed to be more accurate the deflection value of 35 mm is thought to be more representative of the real value.

The ability of Fig.4.3.1 to represent the variation of the effective moment of inertia under load provides a clear explanation to the almost identical results obtained for δ_{load}. The figure shows that for the present values of Φ and $n\rho e$ the effective moment of inertia approaches almost a steady value equal to Icre. In other words, for such high values of Φ and $n\rho e$ the effect of concrete stiffening can virtually be ignored. It follows therefore that for the present case any proper method for evaluating deflections under loading effects should give almost identical results regardless of how the method reflects the effect of concrete stiffening. This ability of visual explanation is one of the unique advantages of the graphical representation of the effective moment of inertia in Fig.4.3.1.

Because the values of Ma/Mcr and E_{eff} used in the example are not based on the equations of Chap.2 one can therefore conclude from this example that the validity of the proposed model of Ie is independent of the expressions for fr and Ec given in Chap.2. That is, for any properly evaluated Ma/Mcr and Ec the proposed model of Ie is valid.

By comparing parts (1) and (2) of the present example, it is fair to say that using Fig.4.3.1 to obtain Ie and thus deflection offers a clear advantage over the method in Eurocode 2 in simplicity, accuracy and graphical representation. It can be seen from the example that even for the simplest case of a singly reinforced rectangular section the solution based on Fig.4.3.1 was direct, straightforward and easy as compared to the method in Eurocode 2. For the common case of flanged sections the method in Eurocode 2 will become even more complicated by the evaluation of Icr and I_{tr} while Fig.4.3.1 will only require the simple evaluation of the factor αf.

From the above discussion and that pertaining to example 5.3.1 it can be said that the proposed model possesses many advantages over the methods of BS 8110 and Eurocode 2. Among these is the simpler and more reliable means for calculating deflections offered by Fig.4.3.1 as was seen in the examples. Another is the simplicity of Fig.4.3.1 in visually representing the variation of the moment of inertia under load. Since visual presentations are more effective than others, the figure can be considered as a useful tool for presenting the different phenomena involved.

A third and equally important point that can be argued against the methods in the codes is that they fail to represent the different concrete stiffening effects under different loading types. Equation 5.2.2 used by the codes is the general deflecion equation $\delta=(M/EI)(KL^2)$ with M/EI taken as the curvature. Equivalent to M/EIe, the curvature is estimated considering the effect of concrete stiffening. However, in doing so either by the triangular tensile stress distribution of concrete as in BS 8110 or by the use of the factor ξ in Eurocode 2, the codes fail to reflect the effect of loading types. This can be considered a drawback as compared to the proposed model where the ratio Lcr/L used to evaluate Ie clearly reflects the different cracking effects and thus concrete stiffening under different loading types.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

In the preceding chapters a model for the effective moment of inertia for calculating deflection was developed, a graphical representation of the model was presented and illustrative examples were given confirming the many aspects involved.

Initially a model for the approximation of Icr was obtained. To arrive at a simple and compact expression, this model was then used to derive the required model of Ie assuming the concrete stiffening effect as a fictitious steel area added to a general cracked transformed section assumed. The study involved an analytical development in defining the overall form of the model as well as an empirical auxiliary coefficient to fit the results of over 340 tested beams found in literature. During the course of the study different possibilities were considered and conclusions were drawn as appropriate to suit the most probable conditions and design situation.

In this chapter, the results of the study presented in the previous chapters as to the nature of the proposed model are summarized and conclusions and suggestions for possible future research are given.

6.2 Nature of the Proposed Model

The proposed model of Ie was developed with a set of objectives which are not cumulatively possible using the presently common expression of Branson or the models proposed so far as its substitute. Reflecting these objectives the following can be said pertaining to the model for Ie proposed in this study:

1. The elimination of the detailed calculation of Icr

The nature of the computations involved in the two step process of calculating the position of the neutral axis and then the cracked transformed moment of inertia, Icr, has always been referred to as being cumbersome. This is particularly so for flanged sections where the equations used are more complex and the method of calculation depends on whether the neutral axis is within the flange or in the web. Therefore, In eliminating the difficulties involved in the evaluation of Icr as mentioned above, the approximation of Icr by the use of Icre as an integral part of the proposed model of Ie can be said to offer the following:

a. The elimination of the need for determining the position of the neutral axis.

b. Much simpler equations are required to carry out the full calculations where the complex equations of Fig.3.3.2, 3.4.3 and 3.5.1 are replaced by the simple expressions of α' and αf.

c. The method of approximation and the nature of the equations involved made it possible to produce a single graphical representation where Ie can be directly determined. This is particularly important in completely eliminating the need for the separate evaluation of Icr and is unique for the present study.

2. Good graphical representation

The graphical representation of the proposed model of Ie shown in Fig.4.3.1 can be said to offer the following :

a. Simplicity

It is not hard to see that the solution curves of Fig.4.3.1 are simple and easy to read. All sectional geometries and reinforcement conditions generally encountered in normal design situations are included in a single plot. This single plot can be used to read not only the values of Ie but also Icre if needed for the approximation of Icr for other purposes.

Without resorting to complicated trends the simple factors of be, bw and $n\rho'$ completely define the section. Setting be=bw automatically converts the section into a rectangular section while taking $n\rho'=0$ makes the section singly reinforced. By such simple manipulation of the factors the plot can therefore be used to estimate Ie for different sections and reinforcement conditions.

Because the proposed model of Ie was developed so as to be applicable within the scope of both the British and the American codes which form the basis for most other codes used through out the world, the graphical representation of Fig.4.3.1 can be considered as the general form valid for the widest range of parameters defined in most codes.

However, should Fig.4.3.1 be limited to a particular code it can be further simplified. For example, if the figure is chosen as a design aid in the British practice the expression of α' can be replaced by the simple value of 0.0037 (see Sec.2.9 and Eq.3.3.7). This is because the expression of α' as given in the figure was proposed to avoid the uncertainties involved when the extreme limits of d′/d as used in the American practice (encompassing those of the British practice) were considered. Likewise, because of the higher restriction on the maximum steel ratio required by BS 8110 [10] or Eurocode 2 [11] less intervals of $n\rho e$ will need be considered requiring less number of $n\rho e$ curves. As compared to the currently used method of the code, or that of Eurocode 2, for calculating deflections the superb simplicity and ease of application offered by the proposed model as represented by Fig.4.3.1 and demonstrated in Chap.5 can not be overlooked.

b. Good representation of the phenomena involved

Because the coefficient Φ was seen to be a direct function of Ma/Mcr, the gradual decay of the values of Ie with increasing value of the applied moment, Ma, is well represented by the shape of the curves. All curves initiate at a value of Ie corresponding to Ig when the applied moment is so low as to cause no cracking in concrete (Φ=0 corresponding to Lcr/L =0). As the moment increases the curves start to descend until the limiting value of Icr, as approximated by Icre, is finally reached at large values of Ma (large values of Φ) and as represented by the steady portions characteristic of all the curves. The gradual decay of the curves with increasing Ma values discussed above can be seen to decrease for higher $n\rho e$ values. This is consistent with the well known phenomenon that sections with higher percentages of reinforcement exhibit less changes in rigidity under increased loads than those with low percentages.

The direct relation between Ma/Mcr and Φ can also be used to explain the phenomenon related to the tension steel magnification factor of cl.3.4.6.5,pt.1 of BS 8110 [10] as represented by the curves. Because of the descending nature of the curves with increasing Ma, an increase in the applied moment does not correspond to an equal increase in the effective moment of inertia (and thus the stiffness) caused by the additional tension steel, nρe, required to withstand the moment at a prescribed stress. Due to this less proportionate increase in Ie as compared to that of Ma (or Φ), increasing moments cause increasing deflections. As the slopes of the curves decay with increasing moments the phenomenon is more emphasized. This is representative of the decreasing value of the tension steel magnification factor, by which the basic span effective depth ratios are multiplied, with increasing moments as given in Table 3.11,pt.1 of BS 8110 [10].

The addition of the presence of compression reinforcement to the stiffness of elements in deflection controls (the compression steel magnification factor of cl.3.4.6.6,pt.1 of BS 8110 [10]) can also be understood from Fig.4.3.1. According to the equations shown in the figure, to represent compression reinforcement b is increased to b′. Because Ie is expressed in terms of $b'd^3/12$, this increase will be magnified by $d^3/12$. As $d^3/12$ is usually large, the corresponding increase in Ie will often overshadow the slight decrease in nρe taken as nρb/b′. Nevertheless, since the two trends are not cumulative (a decrease in nρe and an increase in b to b′ magnified by $d^3/12$) one may argue that the net effect of compression reinforcement as represented by Fig.4.3.1 may not always be large. However, because Ie was taken with respect to short term behaviour, the resulting deflection will still have to be multiplied by the long term magnification factor to account for creep and shrinkage. This usually magnifies the effect of compression steel to a considerable degree.

Because b′ increases as the compression steel area increased, it is then logical to assume in the light of the above argument that with more compression steel deflection is less. This is consistent with the trend in the compression steel magnification factors of Table 3.12,pt.1 of BS 8110 [10].

c. Independency of empirical expressions

The discussion pertaining to Eqs.4.2.3-4 requires that any expression of Φ used in conjunction with the proposed model of Ie must be a direct function of Ma/Mcr and must always yield a negative value. However, the form of the proposed model and its graphical representation as given by Fig.4.3.1 are nevertheless independent of the detailed expression of Φ. In other words, for any different empirical expression of Φ that may be proposed in the future, satisfying the above two criteria, the figure will stand valid.

3. Representation of the different factors affecting deflections

As discussed in Chap.2, while end restraints and load sharing of structural elements are general design problems, cracking is directily related to deflections through concrete stiffening effect. This has been shown to be a function of:

- Loading type.

- Loading intensity.
- Compressive strength of the concrete used.
- Type and magnitude of the steel reinforcement used.
- Load duration.

Reflecting the first four factors the expression of Φ as given by Eq.4.4.1 includes:

(a) The moment ratio Ma/Mcr:

According to Eqs.2.10.1-2, the modulus of rupture of concrete, fr, is a function of its compressive strength. Since the cracking moment, Mcr, is determined from the sectional geometry and the modulus of rupture, it is then an indirect representation of the effect of the compressive strength on concrete stiffening. Also because Mcr for a prismatic beam is constant through out the span and that Ma is the maximum moment acting within the span, the ratio of Ma/Mcr reflects the loading intensity. As the loading varies so does Ma/Mcr.

(b) The ratio of the length over which cracking occurs to the total length, Lcr/L:

The length over which cracking occurs, Lcr, is a function of the variation of the bending moment over the span and can be scaled from the corresponding bending moment diagram. Because different types of loading have different bending moment diagrams, the ratio Lcr/L therefore reflects the loading type. Also reflected through the ratio Lcr/L is the convergence to the gross moment of inertia, Ig, under lower loads. When the maximum applied moment is less than the cracking moment, Mcr, the total length considered will be crack free. Therefore, Lcr/L and thus Φ will be equal to zero and Ie will correspond to Ig as dictated by the proposed model of Ie given by Eq.4.2.7 and represented in Fig.4.3.1.

(c) The reinforcement ratio, ρ

Since the main reinforcing bars in reinforced concrete flexural elements are almost always of the deformed type, the expression of Φ does not account for the type of reinforcement and only considers the reinforcement ratio, ρ.

In the many trials attempted to include the effect of ρ in the expression of Φ, a marked change of behaviour has always been noticed for reinforcement ratios less than 1%. The simplest way of representing this, which improved the accuracy substantially, was to set the effect of $\rho < 1\%$ as equivalent to that at $\rho = 1\%$ as shown in expresion of Φ.

Although the model of Ref.4 recognizes the effect of reinforcement, it fails to account for such change of behaviour for $\rho < 1\%$. This change of behaviour at lower reinforcement ratios has also been noted by other scholars in pointing out that Branson's equation actually ceases to apply at such reinforcement ratios [i.e. 5]. In fact recognizing such an effect is believed to be one of the reasons for the substantial improvement in the accuracy achieved by the proposed model as compared to the equation of Branson and that of Ref.4.

For higher reinforcement ratios it was found in Chap.4 that the results obtained using the different models of Ie were almost same. This, however, can be reasoned to the higher Ma/Mcr ratios (Ma/Mcr > 3.0) that most cases of $\rho > 1\%$ considered had corresponded to. Since for Ma/Mcr > 3 Ie is closer to Icr, the effect of concrete stiffening is thus minimum and the resulting deflection will be same regardless of the methods used in the calculations. In the light of this it can be said therefore that, though has not been shown, the proposed model of Ie should give substantial improvement of accuracy over the other models for $\rho > 1\%$ and the most practical range of Ma/Mcr < 3 as it does for the lower values of ρ.

The last factor given in the list of factors to affect concrete stiffening is load duration. As concrete creeps under sustained loads its compressive strains are substantially increased. This forces a downward movement of the neutral axis and thus reduces the concrete tension area. Because of this and the additional cracks created by the increased deflections under such loads, the concrete stiffening effect is reduced. However, a proper estimate of such an effect is a function of many factors and is therefore difficult to incorporate into any expression designed to be simple and easy to use. Because of this and that such an effect can always be accounted for by using an effective elastic modulus or by multiplying the short term effects by a proper factor, it is not considered in the expression of Φ.

4. <u>Proper integration of the different parts representing Ie</u>

In order to appreciate the nature of the proposed model of Ie it is helpful to recognize that any proper representation of the effective moment of inertia, Ie, is of two parts,

1. Representation of the general variation of Ie under increasing loads as affected by concrete stiffening and the boundary conditions of Ig and Icr.
2. Representation of the different factors affecting cracking and thus concrete stiffening.

These two parts, however, have to be properly related for any model of Ie to be sufficiently accurate.

As Branson's equation does not consider the effects of loading type and reinforcement which are known to affect concrete stiffening, the equation is found inaccurate for loading types other than uniform. For conditions of $\rho < 1\%$ the equation was found to be grossly erroneous and inconsistent as is repeately remarked by other scholars as mentioned before.

The expression of Ie proposed in Ref.4, on the other hand, recognizes both of the two parts discussed above and considers all the factors known to affect concrete stiffening. Yet, it proved to give only slightly better accuracy than Branson's equation. The probable reason for this is that due to its purely empirical nature that was based on limited number of tested beams, the two parts required for a proper representation of Ie, though recognized, were poorly related. In fact, this poor integration of the parts has resulted in the unusual form of the equation where instead of adding to Icr, Ie is obtained by subtracting from Ig as can be seen from Eq.4.1.2.

Unlike the expression of Ie of Ref.4, the proposed model was derived based on establishing a clear analytical relationship between the two parts in representing Ie. That is, the representation of the different factors known to affect concrete stiffening was expressed through the coefficient Φ which was found as a direct output of the mathematical development involved in deriving the general form of the proposed model of Ie. Reflecting this proper integration of the parts, it was found through the different trials attempted for the expression of Φ the clear and steady convergence toward the simple expression given by Eq.4.4.1. In fact, this proper integration of the parts involved is believed to be a major reason for the substantial accuracy of the proposed model as compared to that of Ref.4 and the equation of Branson.

5. Accuracy

Because of the nature of the proposed model of Ie as discussed above and the representation of all the factors known to affect cracking and concrete stiffening as well as depicting the behaviour under lower reinforcement ratios, the proposed model was seen to give results that are not only consistent but also of considerable accuracy. In particular, for reinforcement ratios of less than 1% and/or non-uniform loads the proposed model proved to give errors that are almost half of those resulted from Branson's equation and the model presented in Ref.4 as its substitute. In fact, from the inconsistancy and gross errors noticed for the equations of Branson and Ref.4 in cases of lower reinforcement ratios, it can be concluded that contrary to the high accuracy of the proposed model these equations are actually invalid for conditions of $\rho < 1\%$.

In addition to the above, It can be said from the results of all the cases considered that with the expression of Φ recommended and the model of Ie proposed in this study the errors in calculating deflections can be generally expected to range within ±11 %. This is substantially lower than those of Branson's equation which are known to range from -20 to +30%. In fact, due to uncertainties associated with the evaluation of the elastic modulus of concrete and the modulus of rupture, errors in calculating deflection of reinforced concrete elements lower than 10 % should not be expected.

6.3 Conclusions

The study presented in this thesis was concerned with the development of a model for the effective moment of inertia in which the limitations usually claimed to be associated with Branson's equation are eliminated. Reflecting this, the objectives described in Sec.1.2 were set.

For a problem of such great complexity as the evaluation of the effective moment of inertia of reinforced concrete elements these objectives are by no mean easy to meet. Nevertheless, it can be seen from the discussions presented in Sec.6.2 pertaining to the nature of the proposed model that all the objectives set in Sec.1.2 are well satisfied. Because Branson's equation and all the recent efforts for providing a substitute are short of satisfying these objectives the proposed model can therefore be considered an improved alternative.

Reflecting these objectives, and in a concise summary of the discussion presented in Sec.6.2 the proposed model of Ie and its auxiliary coefficient Φ can be said to offer the followings :

1. General applicability as the model was developed within the scope of both the British and the American codes which form the basis for most other codes used through out the world.
2. Ease of computations involved since the detailed evaluation of Icr is eliminated.
3. Simple and good graphical representation through Fig.4.3.1.
4. Representation of loading type and intensity through Lcr/L and Ma/Mcr.
5. Results of good accuracy where the average error is generally within 11%. In particular, substantial improvement of accuracy can be expected, as compared to

other models of Ie, when the reinforcement ratio is less than 1% and/or loads are not uniform.

Either the equation form or graphical representation of the model can be used to evaluate the effective moment of inertia, Ie. For computer application, a simple subroutine can be developed to automate the model of Ie as given by Eq.4.2.7 with Φ and Icre evaluated from Eqs.4.4.1 and 3.6.1, respectively. For practical design, however, Fig.4.3.1 is more suitable where Ie can be directly determined.

Although derived considering short term effects, the proposed model of Ie is nevertheless all that is needed to carry out a complete evaluation of the deflection of a concrete element. This is because long term deflections can always be obtained from the values calculated for the short term effects. Numerous methods for such evaluation of the long term deflections are available in literature. Alternatively, the long term effects represented by the sustained elastic modulus can also be used to determine Ie and thus long term deflections using the proposed model as demonstrated in Chap.5.

6.4 Recommendations for Future Research

Despite the many advantages offered by the proposed model of Ie it is fair to say however that in the field of science there is always room for improvements. Pertaining to the current study possible improvements may include :

1. Approximation of Ig

As defined, Ig is the moment of inertia of the completely uncracked section neglecting steel. For rectangular sections the computation of Ig is obviously simple. For flanged sections, such a calculation may be argued to be sufficiently complex as to require a method of approximation. However, should such a method be developed it must be compatible with the general framework of Ie such that the form of Fig.4.3.1 is not disturbed.

2. Approximation of Icr for sections other than those considered

The approximation of Icr as given by the model of Icre and its related expressions of α' and αf were presented for rectangular and flanged sections only. Although these sections are the most encountered in design, expressions similar to those presented can be developed for other sections following the procedure of Chap.3.

3. Application to flexural rigidity and stiffness

The model of the effective moment of inertia proposed in this study was developed for the purpose of deflection calculations. However, subject to further studies it may be possible to extend the model for use in evaluating the flexural rigidities, EI, of concrete frame elements for the purpose of elastic analysis.

4. Refinement of the expression of Φ

The expression of Φ proposed in this study is based on the results of over 340 beams tested by different parties under different conditions. Because of this and the very good accuracy obtained using the expression into the proposed model of Ie it is not believed that a different expression is needed. However, since the form of the model of Ie and its graphical representation are independent of the detailed expression of Φ any possible future refinement of Φ will not affect the analytical development presented in this thesis. This encourages future research to try to refine the expression of Φ if possible by studying parameters that are not considered in this study. Since any expression of Φ will eventually be used with the model of Ie proposed in this study, it is therefore important to understand such parameters within the prespective of the proposed model of Ie.

The proposed model of Ie can be written as,

$$Ie = Icre + Is$$

where Icre is an approximation of Icr and Is is the part contributed by the concrete stiffening effect. It follows therefore that parameters that may affect the accuracy of the proposed model of Ie can be classified into,

- Geometric parameters involved in the approximation of Icr and includes d′/d, be/bw, hf/d, nρ and nρ′.
- Prameters related to concrete stiffening effect and involve Ma/Mcr, Lcr/L, and ρ as previously discussed.

The accuracy of the approximation of Icr by the equivalent width method of Chap.3 which is then integrated into the proposed model of Ie and its graphical representation has been established for almost all possible combination of the parameters used in the British and American practices as set by BS 8110 [10] and ACI 318 [2] and summarized in Table 2.11.1. By constructing the respective envelopes discussed in

Chap.3, the errors in using such approximations and for almost all combinations possible can never exceed the errors assumed in the approximations shown in the envelopes. Because of this and that these parameters are purely geometric properties that are not known to affect concrete stiffening, the fact that the tested beams considered in this study do not cover all possible ranges of these parameters remains of no consequence. Therefore, these parameters should not be considered as the primary variables in any future experimental study.

On the other hand, the parameters related to the concrete stiffening are those represented in the expression of Φ as derived based on the tested beams considered. Therefore, it is important for any future program to study conditions that are not considered in the tested beams reviewed in this work. To summarize, these may include:

- Simple spans with loads other than uniform, third point loads or central point loads.
- Reinforcement ratios greater than 1% and Ma/Mcr less than 4.0
- Two or more span continuous beams with non-uniform loads.
- Beams that are parts of a concrete structural frame.

It should be noted however that any conclusions should not be drawn based upon a limited number of tested beams but rather on as many beams as practically possible. This is beacause the problem of deflections in concrete elements is of a statistical nature and the results pertaining to only limited number of beams may be greatly misleading.

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APPENDICES

APPENDIX A

The Listing of Prog.2.8.1 and its Output

PROG.2.8.1

This program computes the maximum value of np for singly reinforced flanged sections in the practical range of be/bw from 1.1 to 10 and hf/d from 0.1 to 0.6. The program is also structured to find and print out whether the neutral axis of the cracked transformed section falls within the flange or in the web.

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      REAL NP,NP1,NP2,NRAU,II
      CHARACTER SAND*12
      OPEN(3,FILE='OUT231',STATUS='UNKNOWN')
C The program will now assign values for be/be and for hf/d and the value
C of the maximum np will be computed using the British and the American
C codes.
      DO 20 I=1,19
      IF(I.EQ.1) THEN
      II=1+0.1
      ELSE IF(I.EQ.2) THEN
      II=II+0.4
      ELSE
      II=II+0.5
      END IF
      WRITE(3,'(10X,A,3X,F4.1,2X,A)')'When be/bw =',II,':'
      WRITE(3,'(10X,5(A,2X))')'hf/d','max.NP based on BS','max.NP based
     *on ACI','max.NP','N.A.pos.based on max.NP'
      DO 10 J=10,65,5
      hfd=J/100.
      IF(J.EQ.60) hfd=0.56
      IF(J.EQ.65) hfd=0.60
      NP1=31.92/II+62.16*hfd*(1-1/II)
      DUM1=DMIN1(8*5.5/II,18.48/II+41.096*hfd*(1-1/II))
      DUM2=DMIN1(7.7*5.5/II,21.33/II+47.26*hfd*(1-1/II))
      DUM3=DMIN1(7.4*5.5/II,23.90/II+53.42*hfd*(1-1/II))
      DUM4=DMIN1(7.1*5.5/II,26.27/II+58.56*hfd*(1-1/II))
      DUM5=DMIN1(6.9*5.5/II,28.70/II+63.70*hfd*(1-1/II))
      DUM6=DMIN1(6.7*5.5/II,30.95/II+68.84*hfd*(1-1/II))
      NP2=DMAX1(DUM1,DUM2,DUM3,DUM4,DUM5,DUM6)
      NP=DMAX1(NP1,NP2)
      NRAU=hfd**2./(2*(1-hfd))*100.
      IF(NP.LE.NRAU) SAND='in flange'
      IF(NP.GT.NRAU) SAND='in web'
      WRITE(3,'(10X,F4.2,10X,F5.2,12X,F5.2,11X,F5.2,10X,A)')hfd,NP2,NP1,
     *NP,SAND
   10 CONTINUE
   20 CONTINUE
      WRITE(3,'(//10X,A)') '*Note: all NP are relative to be'
      STOP
      END
```

OUTPUT OF PROG.2.8.1

When be/bw = 1.1 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	28.76	29.58	29.58	in web
0.15	29.08	29.87	29.87	in web
0.20	29.39	30.15	30.15	in web
0.25	29.70	30.43	30.43	in web
0.30	30.01	30.71	30.71	in web
0.35	30.33	31.00	31.00	in web
0.40	30.64	31.28	31.28	in web
0.45	30.95	31.56	31.56	in web
0.50	31.27	31.84	31.84	in web
0.55	31.58	32.13	32.13	in flange
0.56	31.64	32.18	32.18	in flange
0.60	31.89	32.41	32.41	in flange

When be/bw = 1.5 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	22.93	23.35	23.35	in web
0.15	24.08	24.39	24.39	in web
0.20	24.57	25.42	25.42	in web
0.25	24.57	26.46	26.46	in web
0.30	25.30	27.50	27.50	in web
0.35	25.30	28.53	28.53	in web
0.40	25.32	29.57	29.57	in web
0.45	26.03	30.60	30.60	in web
0.50	26.03	31.64	31.64	in web
0.55	26.03	32.68	32.68	in flange
0.56	26.03	32.88	32.88	in flange
0.60	26.62	33.71	33.71	in flange

When be/bw = 2.0 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	18.42	19.07	19.07	in web
0.15	18.98	20.62	20.62	in web
0.20	18.99	22.18	22.18	in web
0.25	19.52	23.73	23.73	in web
0.30	19.96	25.28	25.28	in web
0.35	20.35	26.84	26.84	in web
0.40	20.35	28.39	28.39	in web
0.45	21.17	29.95	29.95	in web
0.50	21.17	31.50	31.50	in web
0.55	21.17	33.05	33.05	in flange
0.56	21.17	33.36	33.36	in flange
0.60	21.57	34.61	34.61	in flange

When be/bw = 2.5 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	15.18	16.50	16.50	in web
0.15	15.62	18.36	18.36	in web
0.20	15.97	20.23	20.23	in web
0.25	16.28	22.09	22.09	in web
0.30	16.94	23.96	23.96	in web
0.35	16.94	25.82	25.82	in web
0.40	17.26	27.69	27.69	in web

0.45	17.60	29.55	29.55	in web
0.50	17.60	31.42	31.42	in web
0.55	17.60	33.28	33.28	in flange
0.56	17.60	33.65	33.65	in flange
0.60	17.60	35.15	35.15	in flange

When be/bw = 3.0 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	12.66	14.78	14.78	in web
0.15	13.31	16.86	16.86	in web
0.20	13.57	18.93	18.93	in web
0.25	14.12	21.00	21.00	in web
0.30	14.38	23.07	23.07	in web
0.35	14.67	25.14	25.14	in web
0.40	14.67	27.22	27.22	in web
0.45	14.67	29.29	29.29	in web
0.50	14.67	31.36	31.36	in web
0.55	14.67	33.43	33.43	in flange
0.56	14.67	33.85	33.85	in flange
0.60	14.67	35.50	35.50	in flange

When be/bw = 3.5 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	11.16	13.56	13.56	in web
0.15	11.63	15.78	15.78	in web
0.20	12.10	18.00	18.00	in web
0.25	12.57	20.22	20.22	in web
0.30	12.57	22.44	22.44	in web
0.35	12.57	24.66	24.66	in web
0.40	12.57	26.88	26.88	in web
0.45	12.57	29.10	29.10	in web
0.50	12.57	31.32	31.32	in web
0.55	12.57	33.54	33.54	in flange
0.56	12.57	33.98	33.98	in flange
0.60	12.57	35.76	35.76	in flange

When be/bw = 4.0 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	09.98	12.64	12.64	in web
0.15	10.59	14.97	14.97	in web
0.20	10.78	17.30	17.30	in web
0.25	11.00	19.64	19.64	in web
0.30	11.00	21.97	21.97	in web
0.35	11.00	24.30	24.30	in web
0.40	11.00	26.63	26.63	in web
0.45	11.00	28.96	28.96	in web
0.50	11.00	31.29	31.29	in web
0.55	11.00	33.62	33.62	in web
0.56	11.00	34.09	34.09	in flange
0.60	11.00	35.95	35.95	in flange

When be/bw = 4.5 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	09.04	11.93	11.93	in web
0.15	09.41	14.35	14.35	in web
0.20	09.78	16.76	16.76	in web
0.25	09.78	19.18	19.18	in web
0.30	09.78	21.60	21.60	in web
0.35	09.78	24.01	24.01	in web
0.40	09.78	26.43	26.43	in web

0.45	09.78	28.85	28.85	in web
0.50	09.78	31.27	31.27	in web
0.55	09.78	33.68	33.68	in web
0.56	09.78	34.17	34.17	in flange
0.60	09.78	36.10	36.10	in flange

When be/bw = 5.0 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	08.14	11.36	11.36	in web
0.15	08.63	13.84	13.84	in web
0.20	08.80	16.33	16.33	in web
0.25	08.80	18.82	18.82	in web
0.30	08.80	21.30	21.30	in web
0.35	08.80	23.79	23.79	in web
0.40	08.80	26.28	26.28	in web
0.45	08.80	28.76	28.76	in web
0.50	08.80	31.25	31.25	in web
0.55	08.80	33.73	33.73	in web
0.56	08.80	34.23	34.23	in flange
0.60	08.80	36.22	36.22	in flange

When be/bw = 5.5 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	07.70	10.89	10.89	in web
0.15	08.00	13.43	13.43	in web
0.20	08.00	15.98	15.98	in web
0.25	08.00	18.52	18.52	in web
0.30	08.00	21.06	21.06	in web
0.35	08.00	23.60	23.60	in web
0.40	08.00	26.15	26.15	in web
0.45	08.00	28.69	28.69	in web
0.50	08.00	31.23	31.23	in web
0.55	08.00	33.78	33.78	in web
0.56	08.00	34.28	34.28	in flange
0.60	08.00	36.32	36.32	in flange

When be/bw = 6.0 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	07.06	10.50	10.50	in web
0.15	07.33	13.09	13.09	in web
0.20	07.33	15.68	15.68	in web
0.25	07.33	18.27	18.27	in web
0.30	07.33	20.86	20.86	in web
0.35	07.33	23.45	23.45	in web
0.40	07.33	26.04	26.04	in web
0.45	07.33	28.63	28.63	in web
0.50	07.33	31.22	31.22	in web
0.55	07.33	33.81	33.81	in web
0.56	07.33	34.33	34.33	in flange
0.60	07.33	36.40	36.40	in flange

When be/bw = 6.5 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	06.52	10.17	10.17	in web
0.15	06.77	12.80	12.80	in web
0.20	06.77	15.43	15.43	in web
0.25	06.77	18.06	18.06	in web
0.30	06.77	20.69	20.69	in web
0.35	06.77	23.32	23.32	in web
0.40	06.77	25.95	25.95	in web

0.45	06.77	28.58	28.58	in web
0.50	06.77	31.21	31.21	in web
0.55	06.77	33.84	33.84	in web
0.56	06.77	34.37	34.37	in flange
0.60	06.77	36.47	36.47	in flange

When be/bw = 7.0 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	06.16	09.89	09.89	in web
0.15	06.29	12.55	12.55	in web
0.20	06.29	15.22	15.22	in web
0.25	06.29	17.88	17.88	in web
0.30	06.29	20.54	20.54	in web
0.35	06.29	23.21	23.21	in web
0.40	06.29	25.87	25.87	in web
0.45	06.29	28.54	28.54	in web
0.50	06.29	31.20	31.20	in web
0.55	06.29	33.86	33.86	in web
0.56	06.29	34.40	34.40	in flange
0.60	06.29	36.53	36.53	in flange

When be/bw = 7.5 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	05.87	09.64	09.64	in web
0.15	05.87	12.34	12.34	in web
0.20	05.87	15.03	15.03	in web
0.25	05.87	17.72	17.72	in web
0.30	05.87	20.42	20.42	in web
0.35	05.87	23.11	23.11	in web
0.40	05.87	25.80	25.80	in web
0.45	05.87	28.50	28.50	in web
0.50	05.87	31.19	31.19	in web
0.55	05.87	33.89	33.89	in web
0.56	05.87	34.42	34.42	in flange
0.60	05.87	36.58	36.58	in flange

When be/bw = 8.0 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	05.50	09.43	09.43	in web
0.15	05.50	12.15	12.15	in web
0.20	05.50	14.87	14.87	in web
0.25	05.50	17.59	17.59	in web
0.30	05.50	20.31	20.31	in web
0.35	05.50	23.03	23.03	in web
0.40	05.50	25.75	25.75	in web
0.45	05.50	28.47	28.47	in web
0.50	05.50	31.18	31.18	in web
0.55	05.50	33.90	33.90	in web
0.56	05.50	34.45	34.45	in flange
0.60	05.50	36.62	36.62	in flange

When be/bw = 8.5 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	05.18	09.24	09.24	in web
0.15	05.18	11.98	11.98	in web
0.20	05.18	14.72	14.72	in web
0.25	05.18	17.47	17.47	in web
0.30	05.18	20.21	20.21	in web
0.35	05.18	22.95	22.95	in web
0.40	05.18	25.69	25.69	in web

0.45	05.18	28.44	28.44	in web
0.50	05.18	31.18	31.18	in web
0.55	05.18	33.92	33.92	in web
0.56	05.18	34.47	34.47	in flange
0.60	05.18	36.66	36.66	in flange

When be/bw = 9.0 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	04.89	09.07	09.07	in web
0.15	04.89	11.83	11.83	in web
0.20	04.89	14.60	14.60	in web
0.25	04.89	17.36	17.36	in web
0.30	04.89	20.12	20.12	in web
0.35	04.89	22.89	22.89	in web
0.40	04.89	25.65	25.65	in web
0.45	04.89	28.41	28.41	in web
0.50	04.89	31.17	31.17	in web
0.55	04.89	33.94	33.94	in web
0.56	04.89	34.49	34.49	in flange
0.60	04.89	36.70	36.70	in flange

When be/bw = 9.5 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	04.63	08.92	08.92	in web
0.15	04.63	11.70	11.70	in web
0.20	04.63	14.48	14.48	in web
0.25	04.63	17.26	17.26	in web
0.30	04.63	20.05	20.05	in web
0.35	04.63	22.83	22.83	in web
0.40	04.63	25.61	25.61	in web
0.45	04.63	28.39	28.39	in web
0.50	04.63	31.17	31.17	in web
0.55	04.63	33.95	33.95	in web
0.56	04.63	34.51	34.51	in flange
0.60	04.63	36.73	36.73	in flange

When be/bw = 10.0 :

hf/d	max.NP based on BS	max.NP based on ACI	max.NP	N.A.pos.based on max.NP
0.10	04.40	08.79	08.79	in web
0.15	04.40	11.58	11.58	in web
0.20	04.40	14.38	14.38	in web
0.25	04.40	17.18	17.18	in web
0.30	04.40	19.98	19.98	in web
0.35	04.40	22.77	22.77	in web
0.40	04.40	25.57	25.57	in web
0.45	04.40	28.37	28.37	in web
0.50	04.40	31.16	31.16	in web
0.55	04.40	33.96	33.96	in web
0.56	04.40	34.52	34.52	in flange
0.60	04.40	36.76	36.76	in flange

*Note: all NP are relative to be

APPENDIX B1

The Listing of Prog.3.2.1 and its Output

PROG.3.2.1

This program evaluates Icre, using Eq.3.2.2, for a singly reinforced rectangular section for nρ values within the limits of 0.124% and 64%. Results are compared to the exact Icr as Icre/Icr.

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION DNP(641),RNP(641)
      REAL NP,Icre
      OPEN(3,FILE='OUTPUT',STATUS='UNKNOWN')
      DO 10 I=1,640
      NP=I/10.
      IF(I.EQ.1) NP=0.124
      A1=0.003
      B1=0.095
      A2=0.05
      B2=0.07
      A3=0.16
      B3=0.05
      A4=0.5
      B4=0.03
      A5=0.8
      B5=0.02
      IF(NP.LE.1.9) THEN
      Icre=(A1+B1*NP)/12.
      ELSE IF((NP.GT.1.9).AND.(NP.LE.5.0)) THEN
      Icre=(A2+B2*NP)/12.
      ELSE IF((NP.GT.5.0).AND.(NP.LE.17.0)) THEN
      Icre=(A3+B3*NP)/12.
      ELSE IF((NP.GT.17.0).AND.(NP.LE.32.0)) THEN
      Icre=(A4+B4*NP)/12.
      ELSE
      Icre=(A5+B5*NP)/12.
      END IF
      NP=NP/100.
      X=-NP+SQRT(NP**2.+2.*NP)
      XICR=(X**3.)/3.+NP*(1-X)**2.
      RATICR=Icre/XICR
      NP=NP*100.
      DNP(I)=NP
      RNP(I)=RATICR
   10 CONTINUE
      WRITE(3,155)
  155 FORMAT(11X,'NP%',3X,'Icre/Icr',3(4X,'NP%',3X,'Icre/ICR'))
      ICOUNT=0.0
      DO 20 I=1,637,4
      ICOUNT=ICOUNT+1
      IF(ICOUNT.EQ.66) THEN
      WRITE(3,155)
      ICOUNT=0
      END IF
      WRITE(3,'(10X,F5.2,3X,F5.2,3(5X,F5.2,3X,F5.2))') DNP(I),RNP(I),DNP
     *(I+1),RNP(I+1),DNP(I+2),RNP(I+2),DNP(I+3),RNP(I+3)
   20 CONTINUE
      STOP
      END
```

OUTPUT OF PROG.3.2.1

NP%	Icre/Icr	NP%	Icre/Icr	NP%	Icre/Icr	NP%	Icre/Icr
0.12	1.06	0.20	1.00	0.30	0.97	0.40	0.96
0.50	0.96	0.60	0.96	0.70	0.97	0.80	0.97
0.90	0.98	1.00	0.98	1.10	0.99	1.20	1.00
1.30	1.00	1.40	1.01	1.50	1.01	1.60	1.02
1.70	1.03	1.80	1.03	1.90	1.04	2.00	1.03
2.10	1.02	2.20	1.02	2.30	1.01	2.40	1.01
2.50	1.00	2.60	1.00	2.70	1.00	2.80	1.00
2.90	1.00	3.00	0.99	3.10	0.99	3.20	0.99
3.30	0.99	3.40	0.99	3.50	0.99	3.60	0.99
3.70	0.99	3.80	0.99	3.90	0.99	4.00	0.99
4.10	0.99	4.20	0.99	4.30	1.00	4.40	1.00
4.90	1.00	5.00	1.00	5.10	1.03	5.20	1.02
5.30	1.02	5.40	1.01	5.50	1.01	5.60	1.01
5.70	1.01	5.80	1.00	5.90	1.00	6.00	1.00
6.10	1.00	6.20	0.99	6.30	0.99	6.40	0.99
6.50	0.99	6.60	0.99	6.70	0.99	6.80	0.98
6.90	0.98	7.00	0.98	7.50	0.98	7.60	0.98
7.70	0.98	7.80	0.98	7.90	0.97	8.00	0.97
8.10	0.97	8.20	0.97	8.30	0.97	8.40	0.97
8.50	0.97	8.60	0.97	8.70	0.97	8.80	0.97
8.90	0.97	9.00	0.97	9.10	0.97	9.20	0.97
9.30	0.97	9.40	0.97	9.50	0.97	9.60	0.97
9.70	0.97	9.80	0.97	9.90	0.97	10.00	0.97
10.10	0.97	10.20	0.97	10.30	0.97	10.40	0.97
10.50	0.97	10.60	0.98	10.70	0.98	10.80	0.98
10.90	0.98	11.00	0.98	11.10	0.98	11.20	0.98
11.30	0.98	11.40	0.98	11.50	0.98	11.60	0.98
11.70	0.98	11.80	0.98	11.90	0.98	12.00	0.98
12.10	0.98	12.20	0.98	12.30	0.99	12.40	0.99
12.50	0.99	12.60	0.99	12.70	0.99	12.80	0.99
12.90	0.99	13.00	0.99	13.10	0.99	13.20	0.99
13.30	0.99	13.40	0.99	13.50	1.00	13.60	1.00
13.70	1.00	13.80	1.00	13.90	1.00	14.00	1.00
14.10	1.00	14.20	1.00	14.30	1.00	14.40	1.00
14.50	1.00	14.60	1.01	14.70	1.01	14.80	1.01
14.90	1.01	15.00	1.01	15.10	1.01	15.20	1.01
15.30	1.01	15.40	1.01	15.50	1.01	15.60	1.02
15.70	1.02	15.80	1.02	15.90	1.02	16.00	1.02
16.10	1.02	16.20	1.02	16.30	1.02	16.40	1.02
16.50	1.02	16.60	1.03	16.70	1.03	16.80	1.03
16.90	1.03	17.00	1.03	17.10	1.03	17.20	1.03
17.30	1.03	17.40	1.03	17.50	1.03	17.60	1.02
17.70	1.02	17.80	1.02	17.90	1.02	18.00	1.02
18.10	1.02	18.20	1.02	18.30	1.02	18.40	1.02
18.50	1.02	18.60	1.02	18.70	1.02	18.80	1.02
18.90	1.02	19.00	1.02	19.10	1.01	19.20	1.01
19.30	1.01	19.40	1.01	19.50	1.01	19.60	1.01
19.70	1.01	19.80	1.01	19.90	1.01	20.00	1.01
20.10	1.01	20.20	1.01	20.30	1.01	20.40	1.01

NP%	Icre/Icr	NP%	Icre/Icr	NP%	Icre/Icr	NP%	Icre/Icr
20.50	1.01	20.60	1.01	20.70	1.01	20.80	1.01
20.90	1.01	21.00	1.01	21.10	1.01	21.20	1.01
21.30	1.01	21.40	1.00	21.50	1.00	21.60	1.00
21.70	1.00	21.80	1.00	21.90	1.00	22.00	1.00
22.10	1.00	22.20	1.00	22.30	1.00	22.40	1.00
22.50	1.00	22.60	1.00	22.70	1.00	22.80	1.00
22.90	1.00	23.00	1.00	23.10	1.00	23.20	1.00
23.30	1.00	23.40	1.00	23.50	1.00	23.60	1.00
23.70	1.00	23.80	1.00	23.90	1.00	24.00	1.00
24.10	1.00	24.20	1.00	24.30	1.00	24.40	1.00
24.50	1.00	24.60	1.00	24.70	1.00	24.80	1.00
24.90	1.00	25.00	1.00	25.10	1.00	25.20	1.00
25.30	1.00	25.40	1.00	25.50	1.00	25.60	1.00
25.70	1.00	25.80	1.00	25.90	1.00	26.00	1.00
26.10	1.00	26.20	1.00	26.30	1.00	26.40	1.00
26.50	1.00	26.60	1.00	26.70	1.00	26.80	1.00
26.90	1.00	27.00	1.00	27.10	1.00	27.20	1.00
27.30	1.00	27.40	1.00	27.50	1.00	27.60	1.00
27.70	1.00	27.80	1.00	27.90	1.00	28.00	1.00
28.10	1.00	28.20	1.00	28.30	1.00	28.40	1.00
28.50	1.00	28.60	1.00	28.70	1.00	28.80	1.00
28.90	1.00	29.00	1.00	29.10	1.00	29.20	1.00
29.30	1.01	29.40	1.01	29.50	1.01	29.60	1.01
29.70	1.01	29.80	1.01	29.90	1.01	30.00	1.01
30.10	1.01	30.20	1.01	30.30	1.01	30.40	1.01
30.50	1.01	30.60	1.01	30.70	1.01	30.80	1.01
30.90	1.01	31.00	1.01	31.10	1.01	31.20	1.01
31.30	1.01	31.40	1.01	31.50	1.01	31.60	1.01
31.70	1.01	31.80	1.01	31.90	1.01	32.00	1.01
32.10	1.00	32.20	1.00	32.30	1.00	32.40	1.00
32.50	1.00	32.60	1.00	32.70	1.00	32.80	1.00
32.90	1.00	33.00	1.00	33.10	0.99	33.20	0.99
33.30	0.99	33.40	0.99	33.50	0.99	33.60	0.99
33.70	0.99	33.80	0.99	33.90	0.99	34.00	0.99
34.10	0.99	34.20	0.99	34.30	0.99	34.40	0.99
34.50	0.99	34.60	0.99	34.70	0.99	34.80	0.99
34.90	0.99	35.00	0.99	35.10	0.99	35.20	0.99
35.30	0.99	35.40	0.99	35.50	0.99	35.60	0.99
35.70	0.99	35.80	0.99	35.90	0.99	36.00	0.99
36.10	0.99	36.20	0.99	36.30	0.99	36.40	0.99
36.50	0.99	36.60	0.99	36.70	0.99	36.80	0.99
36.90	0.99	37.00	0.99	37.10	0.99	37.20	0.99
37.30	0.99	37.40	0.99	37.50	0.99	37.60	0.99
37.70	0.99	37.80	0.99	37.90	0.99	38.00	0.98
38.10	0.98	38.20	0.98	38.30	0.98	38.40	0.98
38.50	0.98	38.60	0.98	38.70	0.98	38.80	0.98
38.90	0.98	39.00	0.98	39.10	0.98	39.20	0.98
39.30	0.98	39.40	0.98	39.50	0.98	39.60	0.98
39.70	0.98	39.80	0.98	39.90	0.98	40.00	0.98
40.10	0.98	40.20	0.98	40.30	0.98	40.40	0.98
40.50	0.98	40.60	0.98	40.70	0.98	40.80	0.98
40.90	0.98	41.00	0.98	41.10	0.98	41.20	0.98
41.30	0.98	41.40	0.98	41.50	0.98	41.60	0.98
41.70	0.98	41.80	0.98	41.90	0.98	42.00	0.98
42.10	0.98	42.20	0.98	42.30	0.98	42.40	0.98

NP%	Icre/Icr	NP%	Icre/Icr	NP%	Icre/Icr	NP%	Icre/Icr
42.50	0.98	42.60	0.98	42.70	0.98	42.80	0.98
42.90	0.98	43.00	0.98	43.10	0.98	43.20	0.98
43.30	0.98	43.40	0.98	43.50	0.98	43.60	0.98
43.70	0.98	43.80	0.98	43.90	0.98	44.00	0.98
44.10	0.98	44.20	0.98	44.30	0.98	44.40	0.98
44.50	0.98	44.60	0.98	44.70	0.98	44.80	0.98
44.90	0.98	45.00	0.98	45.10	0.98	45.20	0.98
45.30	0.98	45.40	0.98	45.50	0.98	45.60	0.98
45.70	0.98	45.80	0.98	45.90	0.98	46.00	0.98
46.50	0.98	46.60	0.99	46.70	0.99	46.80	0.99
46.90	0.99	47.00	0.99	47.10	0.99	47.20	0.99
47.30	0.99	47.40	0.99	47.50	0.99	47.60	0.99
47.70	0.99	47.80	0.99	47.90	0.99	48.00	0.99
48.10	0.99	48.20	0.99	48.30	0.99	48.40	0.99
48.50	0.99	48.60	0.99	48.70	0.99	48.80	0.99
48.90	0.99	49.00	0.99	49.10	0.99	49.20	0.99
49.30	0.99	49.40	0.99	49.50	0.99	49.60	0.99
49.70	0.99	49.80	0.99	49.90	0.99	50.00	0.99
50.10	0.99	50.20	0.99	50.30	0.99	50.40	0.99
50.50	0.99	50.60	0.99	50.70	0.99	50.80	0.99
50.90	0.99	51.00	0.99	51.10	0.99	51.20	0.99
51.30	0.99	51.40	0.99	51.50	0.99	51.60	0.99
51.70	0.99	51.80	0.99	51.90	0.99	52.00	0.99
52.10	0.99	52.20	0.99	52.30	0.99	52.40	0.99
52.50	0.99	52.60	0.99	52.70	0.99	52.80	0.99
52.90	0.99	53.00	0.99	53.10	0.99	53.20	0.99
53.30	0.99	53.40	0.99	53.50	1.00	53.60	1.00
53.70	1.00	53.80	1.00	53.90	1.00	54.00	1.00
54.10	1.00	54.20	1.00	54.30	1.00	54.40	1.00
54.50	1.00	54.60	1.00	54.70	1.00	54.80	1.00
54.90	1.00	55.00	1.00	55.10	1.00	55.20	1.00
55.30	1.00	55.40	1.00	55.50	1.00	55.60	1.00
55.70	1.00	55.80	1.00	55.90	1.00	56.00	1.00
56.10	1.00	56.20	1.00	56.30	1.00	56.40	1.00
56.50	1.00	56.60	1.00	56.70	1.00	56.80	1.00
56.90	1.00	57.00	1.00	57.10	1.00	57.20	1.00
57.30	1.00	57.40	1.00	57.50	1.00	57.60	1.00
57.70	1.00	57.80	1.00	57.90	1.00	58.00	1.00
58.10	1.01	58.20	1.01	58.30	1.01	58.40	1.01
58.50	1.01	58.60	1.01	58.70	1.01	58.80	1.01
58.90	1.01	59.00	1.01	59.10	1.01	59.20	1.01
59.30	1.01	59.40	1.01	59.50	1.01	59.60	1.01
59.70	1.01	59.80	1.01	59.90	1.01	60.00	1.01
60.10	1.01	60.20	1.01	60.30	1.01	60.40	1.01
60.50	1.01	60.60	1.01	60.70	1.01	60.80	1.01
60.90	1.01	61.00	1.01	61.10	1.01	61.20	1.01
61.30	1.01	61.40	1.01	61.50	1.01	61.60	1.01
61.70	1.01	61.80	1.01	61.90	1.01	62.00	1.02
62.10	1.02	62.20	1.02	62.30	1.02	62.40	1.02
62.50	1.02	62.60	1.02	62.70	1.02	62.80	1.02
62.90	1.02	63.00	1.02	63.10	1.02	63.20	1.02
63.30	1.02	63.40	1.02	63.50	1.02	63.60	1.02
63.70	1.02	63.80	1.02	63.90	1.02	64.00	1.02

APPENDIX B2

The Listing of Prog.3.3.1

PROG.3.3.1

This program evaluates the upper and lower envelope values of α' required for the construction of Fig.3.3.4

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION HALPHL(35)
      REAL NP,NPC,NEWNP,LALPHU(35)
      OPEN(3,FILE='OUTPUT',STATUS='UNKNOWN')
* The program will now ask for the percentage of error to be allowed :
      WRITE(*,'(1X,A)')'Please enter % error to be allowed :'
      READ*, Seterr
      Seterr=Seterr/100.
* The program will next assign values for np , np' and d'/d in a way
* that  np' is kept less than  or equal to np  and that the  section
* remains ductile :
      DO 50 I=2,64
      NP=I/100.
      DO 40 J=2,64
      NPC=J/100.
      IF(NPC.GT.NP) GO TO 40
      IF(I.GT.(32.0+J)) GO TO 40
      DO 30 K=1,35
      ID=0
      IF(K.EQ.1) THEN
      DRATIO = 0.03
      ELSE
      DRATIO=DRATIO+0.01
      END IF
*The program will now commence processing the solution :
      X=-(NP+NPC)+SQRT((NP+NPC)**2+2*NP+2*NPC*DRATIO)
      XICR=(X**3)/3.+NPC*(X-DRATIO)**2+NP*(1-X)**2
      Rb1=1.0
      A1=0.0
      B1=0.0
      NEWNP=100.*NP
      ICOUNT=0
   20 ICOUNT=ICOUNT+1
   21 IF(NEWNP.LE.1.9) THEN
      A=0.003
      B=0.095
      ELSE IF((NEWNP.GT.1.9).AND.(NEWNP.LE.5.0)) THEN
      A=0.05
      B=0.07
      ELSE IF((NEWNP.GT.5.0).AND.(NEWNP.LE.17.0)) THEN
      A=0.16
      B=0.05
      ELSE IF((NEWNP.GT.17.0).AND.(NEWNP.LE.32.0)) THEN
      A=0.50
      B=0.03
      ELSE
      A=0.8
      B=0.02
      END IF
```

```
      IF(ID.EQ.1) GO TO 23
      IF(ID.EQ.2) GO TO 25
      Rb2=(12.*XICR-B*100.*NP)/A
      IF(ICOUNT.EQ.10) THEN
      A=(A+A1)/2.
      B=(B+B1)/2.
      Rb2=(12.*XICR-B*100.*NP)/A
      ELSE
      IF(ABS(Rb2-Rb1).GT.0.0) THEN
      A1=A
      B1=B
      NEWNP=100.*NP/Rb2
      Rb1=Rb2
      GO TO 20
      END IF
      END IF
      ALPHAE=(Rb2-1)/(100.*NPC/DRATIO)
      ID=1
      ALPHAU=ALPHAE
   22 ALPHAU=ALPHAU+0.000001
      Rb2=1+ALPHAU*100.*NPC/DRATIO
      NEWNP=100.*NP/Rb2
      GO TO 21
   23 XICRe=(A+B*100.*NP/Rb2)*Rb2/12.
      error=XICRe/XICR-1
      IF(DABS(error).LT.Seterr) GO TO 22
      ID=2
      ALPHAL=ALPHAE
   24 ALPHAL=ALPHAL-0.000001
      Rb2=1+ALPHAL*100.*NPC/DRATIO
      NEWNP=100.*NP/Rb2
      GO TO 21
   25 XICRe=(A+B*100.*NP/Rb2)*Rb2/12.
      error=XICRe/XICR-1
      IF(DABS(error).LT.Seterr) GO TO 24
      IF(I.EQ.2) THEN
      LALPHU(K)=ALPHAU
      HALPHL(K)=ALPHAL
      ELSE
      LALPHU(K)=DMIN1(LALPHU(K),ALPHAU)
      HALPHL(K)=DMAX1(HALPHL(K),ALPHAL)
      END IF
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
* Now that the solution processing is over the values of the upper and the
* lower envelopes will be determined and stored in a file called "OUTPUT":
      Seterr=Seterr*100.
      WRITE(3,'(5(/),T23,A,//,T26,A,1X,F4.1,1X,A)')'Values of The Upper
     *And Lower Envelope of','% Error Allowed in Icre =',Seterr,'%'
      WRITE(3,'(//T20,3(A,3X))')'d''/d','Upper Envelope Value','Lower En
     *velope Value'
      DO 60 K=1,35
      IF(K.EQ.1) THEN
      DRATIO=0.03
      ELSE
```

```
      DRATIO=DRATIO+0.01
      END IF
      WRITE(3,'(T20,F4.2,9X,F8.6,15X,F8.6)')DRATIO,LALPHU(K),HALPHL(K)
   60 CONTINUE
      WRITE(3,'(//T15,A/T15,A//T15,A//T16A,F4.1,A//T15,A//T16A,F4.1,A//T
     *15,A//T35,A)')'Notes :','-------','1.Upper envelope values corresp
     *ond to a max. (+) error',' in Icre of',Seterr,'%','2.Lower envelop
     *e values correspond to a max. (-) error',' in Icre of',Seterr,'%',
     *'3.Error in Icre is defined as :','(Icre/Icr-1)*100'
       STOP
       END
```

APPENDIX B3

The Listing of Prog.3.3.2 and its Output

PROG.3.3.2

This program evaluates the cracked transformed moment of inertia of the equivalent section for a doubly reinforced rectangular section using α' of Eq.3.3.7 and compares the results with the exact value. It also studies the effect of neglecting compression reinforcement.

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION SAND(35,700),error(35,700),M(35),ID(35,700),IDerr(35,700
     *),DR(10),DEPTH(10),RIGICR(10),RICRe(10),RNPCO(10),IFLAG(10),JFLAG(
     *10),INDEX(35)
      CHARACTER SAND*5
      REAL NP,NPC,NWRICR,NEWNP,NPe,MODNP(64),IFLAG,JFLAG
      OPEN(3,FILE='OUTPUT',STATUS='UNKNOWN')
C The program will now read np , np' and d'/d . Any section that is
C found unductile is ignored.
      NPe=0.0
      DO 15 I=1,700
      DO 10 K=1,35
      M(K)=0
      ID(K,I)=0
      error(K,I)=0
      INDEX(K)=0
   10 CONTINUE
   15 CONTINUE
      WRITE(*,'(1X,A)')'How many cases are to be given as examples (max.
     * of 10)?'
      READ*,MXCASE
      WRITE(*,'(1X,A/8X,A,7(/12X,A))')'For each case please give np,np''
     * and d''/d . Enter one combination per line','Notes :','1. np shou
     *ld always be greater than or equal to np''','2.Subject  to  the co
     *ndition above , np and np'' can',' have any value (in an icrement
     * of 1) from 2 to 64','3.d''/d should be  chosen using icrements of
     * 0.01 in',' the range from 0.03 to 0.37 . Values outside this','
     * range must not be used'
      DO 16 ICASE=1,MXCASE
      READ*,IFLAG(ICASE),JFLAG(ICASE),DR(ICASE)
   16 CONTINUE
      WRITE(*,'(A)')'*=================== The program is running pleas
     *e wait ===================*'
      DO 50 I=2,64
      NP=I/100.
      DO 40 J=2,64
      NPC=J/100.
      IF(NPC.GT.NP) GO TO 40
      IF(I.GT.(32.0+J)) THEN
C*****This section is not ductile and thus will not be considered .
      GO TO 4
      END IF
      DO 30 K=1,35
      IF(K.EQ.1) THEN
      DRATIO = 0.03
      ELSE IF(K.EQ.8) THEN
      DRATIO=0.1
      ELSE
```

```
      DRATIO=DRATIO+0.01
      END IF
C Now that all the parameters have been read  b' will be evaluated  and
C npb/b' will be calculated as NEWNP. The parameters  required  for the
C evaluation of Icre  will also be found :
      IF((DRATIO.GE.0.065).AND.(DRATIO.LE.0.305)) THEN
      APRIME=0.37
      ELSE
      APRIME=0.25
      END IF
      BPRIME=1+APRIME*NPC/DRATIO
      NEWNP=100.*NP/BPRIME
      IF(NEWNP.LE.1.9) THEN
      A=0.003
      B=0.095
      ELSE IF((NEWNP.GT.1.9).AND.(NEWNP.LE.5.0)) THEN
      A=0.05
      B=0.07
      ELSE IF((NEWNP.GT.5.0).AND.(NEWNP.LE.17.0)) THEN
      A=0.16
      B=0.05
      ELSE IF((NEWNP.GT.17.0).AND.(NEWNP.LE.32.0)) THEN
      A=0.50
      B=0.03
      ELSE
      A=0.8
      B=0.02
      END IF
C The program will  now evaluate Icr , Icre and  Icr  when  compression
C reinforcement is ignored (stored in "Icr,np'=0") and will print the
C ratios of the values relative to Icr .
      X=-(NP+NPC)+SQRT((NP+NPC)**2+2*NP+2*NPC*DRATIO)
      XO=-NP+SQRT(NP**2+2*NP)
      XICR=(X**3)/3.+NPC*(X-DRATIO)**2+NP*(1-X)**2
      OICR=(XO**3)/3.+NP*(1-XO)**2
      XIcre=(A+B*NEWNP)*BPRIME/12.
      NWRICR=XIcre/XICR
      ROICR=OICR/XICR
      DH=1/(1+DRATIO)
      XIG=((1+DRATIO)**3)/12
      XIGRAT=XIG/XICR
C*****The following  13  lines relate  only to the  sections  used  in the
C*****examples which are provided to illustrate the computations involoved
C***** in the program :
      DO 17 ICASE=1,MXCASE
      III=NP*100000.
      JJJ=NPC*100000.
      IIFLG=IFLAG(ICASE)*1000.
      JJFLG=JFLAG(ICASE)*1000.
      IF((III.EQ.IIFLG).AND.(JJJ.EQ.JJFLG)) THEN
      IDRAT=DRATIO*1000.
      IDR=DR(ICASE)*1000.
      IF(IDRAT.EQ.IDR) THEN
      INDEX(ICASE)=1
      DEPTH(ICASE)=DH
      RIGICR(ICASE)=XIGRAT
```

```
      MODNP(ICASE)=NEWNP
      RICRe(ICASE)=NWRICR
      RNPCO(ICASE)=ROICR
      END IF
      END IF
   17 CONTINUE
C***********************************************************************C
      Ierror=(NWRICR-1)*1000.
      Ierror=Ierror+400
      NPe=DMAX1(NEWNP,NPe)
      DO 20 IJ=1,700
      IF(Ierror.NE.IJ) GO TO 20
      IF(ID(K,IJ).EQ.0) THEN
      M(K)=M(K)+1
      IDerr(K,IJ)=M(K)
      error(K,M(K))=(Ierror-400)/10.
      IF(XIGRAT.GE.1.0) THEN
      ID(K,IJ)=1
      SAND(K,M(K))='Yes'
      ELSE
      ID(K,IJ)=2
      SAND(K,M(K))='No'
      END IF
      ELSE IF(ID(K,IJ).EQ.2) THEN
      IF(XIGRAT.GE.1.0) THEN
      ID(K,IJ)=1
      SAND(K,IDerr(K,IJ))='Yes'
      END IF
      END IF
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
      DO 70 K=1,35
      IF(K.EQ.1) THEN
      WRITE(3,'(7(/))')
      ICOUNT=0
      DRATIO=0.03
      END IF
      IF(K.NE.1) DRATIO=DRATIO+0.01
      ICOUNT=ICOUNT+1
      WRITE(3,'(2(/),65X,A,1X,F4.2,/)')'When d''/d =',DRATIO
      WRITE(3,55)
   55 FORMAT(10X,7('% error  Ig>Icr?',3X))
      DO 65 I=1,M(K)
      IF(I.EQ.1) II=1
      IF(I.NE.1) II=II+7
      IJ=I*7
      IF(IJ.GT.M(K))IJ=M(K)
      IF(II.GT.M(K)) GO TO 65
      WRITE(3,60)(error(K,J),SAND(K,J),J=II,IJ)
   60 FORMAT(11X,7(F5.1,4X,A,5X))
   65 CONTINUE
      IF(K.EQ.35) GO TO 70
      IF((ICOUNT.EQ.3).OR.(K.EQ.2)) THEN
      ICOUNT=0
```

```
      WRITE(3,'(1H1,7(/))')
      END IF
   70 CONTINUE
      WRITE(3,'(2(/),64X,A,F5.2)')'Max NPb/b'' is ',NPe
      WRITE(3,75)
   75 FORMAT(T64,21('-'))
      WRITE(3,'(1H1,7(/))')
      WRITE(3,'(15X,A,//15X,A,//15X,A)')'The followings are printed ''on
     *ly as typical cases '' that are ','used in the examples to illustr
     *ate the computaions involved ','in the program :'
      DO 80 I=1,MXCASE
      WRITE(3,'(//15X,A,F5.2,A,F5.2,A,F5.3,A)')'When NP = ',IFLAG(I),'%
     *, NP'' = ',JFLAG(I),'%  and d''/d  = ',DR(I),' :'
      IF(INDEX(I).EQ.1) THEN
      WRITE(3,'(/25X,5(A,3X))')'d/h','Ig/Icr','NPb/b''','Icre/Icr','(Icr
     *,NP''=0)/Icr'
      WRITE(3,'(/25X,F4.2,2X,F5.2,4X,F5.2,6X,F5.3,10X,F4.2)')DEPTH(I),RI
     *GICR(I),MODNP(I),RICRe(I),RNPCO(I)
      ELSE
      IF(IFLAG(I).GT.(32.0+JFLAG(I))) THEN
      WRITE(3,'(/25X,A)')'This section is not ductile and thus ignored'
      ELSE
      WRITE(3,'(/25X,A)')'The program does not consider such a section'
      END IF
      END IF
   80 CONTINUE
      WRITE(3,'(///)')
      STOP
      END
```

OUTPUT OF PROG.3.3.2

When d'/d = 0.03

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.0	Yes	-0.3	Yes	0.0	Yes	-1.3	Yes	-1.4	Yes	-0.7	Yes	-1.1	Yes
-1.7	Yes	2.2	Yes	-0.1	Yes	-0.6	Yes	-1.0	Yes	0.8	Yes	1.8	Yes
-0.4	Yes	-0.9	Yes	0.7	Yes	1.5	Yes	2.4	Yes	-2.0	Yes	-1.6	Yes
-0.5	Yes	1.3	Yes	2.1	Yes	-2.3	Yes	-0.8	Yes	0.1	Yes	0.6	Yes
1.2	Yes	-2.2	Yes	-1.9	Yes	-0.2	Yes	-1.8	Yes	-1.5	Yes	0.2	Yes
-1.2	Yes	0.4	Yes	1.7	Yes	2.6	Yes	1.1	No	3.7	No	3.0	No
1.9	No	1.4	No	1.0	No	4.4	No	4.0	No	3.3	No	2.7	No
3.8	No	5.0	No	4.2	No	3.6	No	2.5	No	1.6	No	0.9	No
4.8	No	5.2	No	4.5	No	3.9	No	2.8	No	2.3	No	0.5	No
0.3	No	2.9	No	4.3	No	5.7	No	5.4	No	4.7	No	4.1	No
3.5	No	6.4	No	6.1	No	6.6	No	5.9	No	4.6	No	3.4	No
6.9	No	6.8	No	6.0	No	3.2	No	6.5	No	7.6	No	6.2	No
5.6	No	7.2	No	7.8	No	7.1	No	3.1	No	7.9	No	8.0	No
5.3	No	4.9	No	5.8	No	6.7	No	8.5	No	8.1	No	7.4	No
5.5	No	8.3	No	8.9	No	8.2	No	7.5	No	5.1	No	9.0	No
7.0	No	6.3	No	8.6	No	9.4	No	9.1	No	8.4	No	7.7	No
9.2	No	9.9	No	9.7	No	10.0	No	9.3	No	7.3	No	9.5	No
10.2	No	10.1	No	8.7	No	8.8	No	10.7	No	10.5	No	10.9	No
9.8	No	10.4	No	10.8	No	11.4	No	9.6	No	11.2	No	11.7	No
11.0	No	10.3	No	10.6	No	11.1	No	11.6	No	11.5	No	12.0	No
11.9	No	12.4	No	11.8	No	12.2	No	11.3	No	12.6	No	12.1	No
12.5	No	12.9	No	12.8	No	13.1	No	12.7	No	13.4	No	13.0	No
13.7	No	13.3	No	13.6	No	13.9	No	13.5	No	14.2	No	13.8	No
14.1	No	14.3	No	14.4	No	14.0	No	14.6	No	14.8	No	12.3	No
14.5	No	14.7	No	15.0	No	15.2	No	15.4	No	15.6	No	15.1	No

When d'/d = 0.04

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.2	Yes	-0.8	Yes	-1.6	Yes	-1.9	Yes	-2.1	Yes	-1.0	Yes	-1.5	Yes
-2.0	Yes	-2.3	Yes	1.0	Yes	1.7	Yes	-0.6	Yes	-0.1	Yes	0.3	Yes
0.9	Yes	-2.2	Yes	-1.7	Yes	-1.4	Yes	-0.5	Yes	-2.8	Yes	-2.7	Yes
-2.5	Yes	-1.3	Yes	-3.0	Yes	-3.1	Yes	-2.6	Yes	-2.4	Yes	-2.9	Yes
-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-1.8	Yes	-3.7	Yes
-1.1	Yes	-3.8	Yes	-3.9	Yes	-0.3	Yes	-0.9	Yes	1.2	Yes	0.5	Yes
-0.7	Yes	-1.2	Yes	-4.0	No	1.4	Yes	0.7	Yes	0.0	Yes	3.2	Yes
2.4	Yes	1.6	No	0.8	No	0.2	No	2.7	No	3.4	No	2.5	No
0.4	No	3.0	No	3.5	No	2.6	No	1.9	No	3.3	No	3.6	No
2.8	No	2.0	No	1.3	No	0.1	No	2.1	No	3.7	No	2.9	No
1.5	No	1.1	No	3.1	No	3.8	No	2.3	No	3.9	No	1.8	No
0.6	No	4.1	No	4.0	No	4.3	No	-0.4	No	4.5	No	4.2	No
4.7	No	4.8	No	4.4	No	5.0	No	5.1	No	5.2	No	4.6	No
5.4	No	-0.2	No	4.9	No	5.5	No	5.6	No	5.3	No	5.7	No
5.8	No	5.9	No	6.0	No	6.1	No	6.2	No	6.3	No	6.4	No
6.5	No	6.6	No	6.7	No	6.8	No	6.9	No	7.0	No	7.1	No
7.2	No												

When d'/d = 0.05

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.5	Yes	-1.0	Yes	-1.1	Yes	-1.7	Yes	-2.1	Yes	-2.4	Yes	-1.6	Yes
-2.6	Yes	0.4	Yes	0.8	Yes	-2.2	Yes	-1.4	Yes	-0.9	Yes	-0.5	Yes
-0.1	Yes	-2.7	Yes	-2.5	Yes	-1.9	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes
-3.1	Yes	-3.6	Yes	-3.8	Yes	-4.0	Yes	-4.1	Yes	-3.2	Yes	-3.9	Yes
-4.3	Yes	-4.5	Yes	-4.6	Yes	-2.8	Yes	-3.7	Yes	-4.7	Yes	-4.8	Yes
-4.9	Yes	-5.0	Yes	-4.4	Yes	-5.2	Yes	-5.3	Yes	-1.5	Yes	-5.1	Yes
-5.4	Yes	-5.5	Yes	-0.7	Yes	-5.7	Yes	0.1	Yes	-0.6	Yes	-1.3	Yes
-2.0	Yes	-5.6	Yes	-5.8	Yes	1.0	Yes	0.2	Yes	-1.2	Yes	-3.0	Yes
-4.2	Yes	2.0	Yes	1.1	Yes	0.3	Yes	-0.4	Yes	-2.3	Yes	2.1	Yes
1.2	Yes	-0.3	No	1.8	Yes	2.3	Yes	1.3	No	0.5	No	-0.2	No
-0.8	No	1.7	No	2.2	No	1.4	No	0.6	No	0.0	No	-2.9	No
0.9	No	1.6	No	1.5	No	0.7	No	-1.8	No	1.9	No		

When d'/d = 0.06

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.8	Yes	-1.0	Yes	-1.1	Yes	-1.7	Yes	-2.1	Yes	-2.4	Yes	-1.6	Yes
-2.5	Yes	0.1	Yes	0.3	Yes	0.6	Yes	-1.8	Yes	-1.4	Yes	-3.0	Yes
-3.1	Yes	-2.9	Yes	-3.5	Yes	-3.7	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes
-4.2	Yes	-3.6	Yes	-4.5	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes
-3.4	Yes	-3.8	Yes	-4.6	Yes	-5.1	Yes	-5.3	Yes	-5.4	Yes	-5.5	Yes
-5.6	Yes	-4.4	Yes	-5.8	Yes	-5.9	Yes	-6.0	Yes	-5.2	Yes	-6.2	Yes
-6.3	Yes	-2.3	Yes	-6.1	Yes	-6.5	Yes	-0.8	Yes	-6.4	Yes	-6.6	Yes
0.0	Yes	-1.5	Yes	-2.2	Yes	-6.7	Yes	0.9	Yes	-0.7	Yes	-2.8	Yes
1.9	Yes	1.0	Yes	-0.6	Yes	-2.0	Yes	-2.7	Yes	-3.3	Yes	-4.3	Yes
2.0	Yes	0.2	Yes	-0.5	Yes	-1.3	Yes	-1.9	Yes	-2.6	No	1.3	Yes
1.5	Yes	1.7	Yes	1.1	Yes	-0.4	No	-1.2	No	-5.7	No	0.7	Yes
1.2	No	-0.3	No	0.4	No	-0.2	No	-0.9	No	0.8	No	0.5	No
-0.1	No	-3.2	No										

When d'/d = 0.07

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.1	Yes	-0.4	Yes	-0.2	Yes	-1.1	Yes	-1.3	Yes	-0.5	Yes	-0.9	Yes
-1.4	Yes	1.1	Yes	1.8	Yes	-0.6	Yes	-1.0	Yes	0.0	Yes	0.4	Yes
1.0	Yes	1.6	Yes	-2.1	Yes	-1.9	Yes	-1.6	Yes	-1.2	Yes	-0.8	Yes
-2.7	Yes	-2.6	Yes	-2.5	Yes	-2.3	Yes	-2.0	Yes	-1.8	Yes	-1.5	Yes
-2.9	Yes	-2.8	Yes	-2.4	Yes	-2.2	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-0.3	Yes	-3.3	Yes	0.5	Yes	-1.7	Yes	1.4	Yes	0.8	Yes	0.2	Yes
-0.7	Yes	2.4	Yes	1.7	Yes	2.7	Yes	2.0	Yes	1.3	Yes	3.2	Yes
2.9	Yes	2.2	Yes	1.9	Yes	2.5	Yes	3.3	Yes	0.7	No	-0.1	No
3.4	No	2.1	No	1.5	No	2.8	No	3.6	No	2.3	No	1.2	No
0.9	No	3.5	No	3.8	No	4.0	No	0.6	No	3.7	No	4.2	No
0.1	No	2.6	No	0.3	No	3.0	No	4.3	No	3.9	No	4.1	No
4.4	No	4.5	No	4.6	No	4.7	No	4.9	No	5.0	No	4.8	No
5.1	No	5.2	No	5.4	No	5.3	No	5.5	No	5.6	No	5.7	No
5.8	No	5.9	No	6.0	No	6.1	No	6.2	No	6.3	No		

When d'/d = 0.08

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.3	Yes	-0.4	Yes	-0.2	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-0.8	Yes
0.8	Yes	1.4	Yes	2.0	Yes	0.0	Yes	0.4	Yes	0.9	Yes	-2.3	Yes
-2.1	Yes	-1.9	Yes	-1.7	Yes	-1.4	Yes	-1.0	Yes	-0.7	Yes	-2.9	Yes
-2.8	Yes	-2.6	Yes	-2.5	Yes	-1.8	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-2.4	Yes	-2.7	Yes	-3.5	Yes	-2.2	Yes	-3.6	Yes
-3.7	Yes	-1.6	Yes	-2.0	Yes	-3.8	Yes	-0.3	Yes	-0.9	Yes	-3.9	Yes
0.5	Yes	-0.1	Yes	-0.6	Yes	-1.5	Yes	0.7	Yes	0.1	Yes	2.4	Yes
1.7	Yes	1.0	Yes	2.7	Yes	1.9	Yes	1.3	Yes	0.6	Yes	2.6	Yes
2.9	Yes	2.2	Yes	1.5	Yes	0.3	Yes	2.5	Yes	3.0	Yes	1.1	Yes
2.3	No	2.8	No	1.6	No	3.1	No	1.8	No	-0.5	No	1.2	No
0.2	No	3.2	No	2.1	No	3.4	No	3.5	No	3.6	No	3.7	No

When d'/d = 0.09

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.6	Yes	-0.3	Yes	-0.2	Yes	-1.1	Yes	-1.2	Yes	-0.4	Yes	-0.7	Yes
-0.9	Yes	0.7	Yes	1.2	Yes	1.7	Yes	-0.6	Yes	-1.3	Yes	-1.0	Yes
0.0	Yes	0.4	Yes	-2.4	Yes	-2.3	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes
-1.4	Yes	-2.9	Yes	-3.0	Yes	-2.8	Yes	-2.6	Yes	-2.2	Yes	-3.2	Yes
-3.3	Yes	-3.1	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes
-3.9	Yes	-4.0	Yes	-4.1	Yes	-1.6	Yes	-4.2	Yes	-1.8	Yes	0.5	Yes
-0.1	Yes	-2.0	Yes	-2.7	Yes	1.4	Yes	0.1	Yes	-2.5	Yes	2.4	Yes
1.0	Yes	-1.5	Yes	2.7	Yes	1.9	Yes	1.3	Yes	0.6	Yes	2.1	Yes
2.6	Yes	2.2	Yes	1.5	Yes	0.9	Yes	0.3	Yes	2.3	Yes	1.8	Yes
1.1	Yes	-0.8	No	0.8	Yes	2.5	Yes	2.0	Yes	1.6	Yes	0.2	Yes
-0.5	No												

When d'/d = 0.10

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.8	Yes	-0.3	Yes	-0.1	Yes	-1.0	Yes	-1.1	Yes	-0.5	Yes	-0.8	Yes
0.6	Yes	1.0	Yes	1.5	Yes	2.0	Yes	-1.3	Yes	0.1	Yes	-2.5	Yes
-2.4	Yes	-2.2	Yes	-2.1	Yes	-1.9	Yes	-1.6	Yes	-1.4	Yes	-3.0	Yes
-2.9	Yes	-2.8	Yes	-2.6	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes
-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-2.3	Yes
-4.1	Yes	-4.2	Yes	-4.3	Yes	-0.9	Yes	-1.8	Yes	0.5	Yes	0.0	Yes
-0.6	Yes	-2.0	Yes	-2.7	Yes	1.4	Yes	0.8	Yes	-1.7	Yes	2.4	Yes
1.7	Yes	0.4	Yes	-1.5	Yes	2.1	Yes	2.5	Yes	1.3	Yes	0.7	Yes
-1.2	Yes	1.8	Yes	2.3	Yes	1.6	Yes	1.2	Yes	2.2	Yes	1.9	Yes
-0.7	Yes	0.9	Yes	0.3	Yes	-0.2	No	-0.4	No	1.1	No	0.2	No

When d'/d = 0.11

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.9	Yes	-0.2	Yes	0.0	Yes	-0.9	Yes	-1.0	Yes	-0.4	Yes	-0.6	Yes
-0.8	Yes	0.5	Yes	1.0	Yes	1.4	Yes	1.9	Yes	-0.1	Yes	-1.4	Yes
-1.1	Yes	-0.3	Yes	-2.5	Yes	-2.4	Yes	-2.3	Yes	-2.1	Yes	-2.0	Yes
-1.7	Yes	-1.5	Yes	-3.0	Yes	-2.9	Yes	-2.8	Yes	-2.7	Yes	-2.6	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-1.8	Yes	-2.2	Yes	-4.2	Yes
-1.6	Yes	-4.3	Yes	-1.3	Yes	-0.5	Yes	-1.9	Yes	1.5	Yes	0.8	Yes
0.2	Yes	-0.7	Yes	-1.2	Yes	2.5	Yes	1.8	Yes	1.1	Yes	2.0	Yes
2.3	Yes	2.1	Yes	0.3	Yes	1.3	Yes	1.6	Yes	1.7	Yes	0.6	Yes
1.2	Yes	0.4	Yes	0.7	Yes	0.9	Yes	0.1	Yes				

When d'/d = 0.12

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	-0.2	Yes	0.0	Yes	-0.8	Yes	-0.1	Yes	-0.3	Yes	-0.4	Yes
-0.5	Yes	0.5	Yes	0.9	Yes	1.3	Yes	1.8	Yes	2.3	Yes	-1.4	Yes
-1.2	Yes	-0.9	Yes	-0.6	Yes	-2.5	Yes	-2.4	Yes	-2.3	Yes	-2.1	Yes
-2.0	Yes	-1.8	Yes	-1.5	Yes	-3.0	Yes	-2.9	Yes	-2.8	Yes	-2.7	Yes
-2.6	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-2.2	Yes	-3.1	Yes	-4.1	Yes	-1.1	Yes
-1.9	Yes	-4.2	Yes	-0.7	Yes	-1.6	Yes	0.6	Yes	-1.3	Yes	1.5	Yes
0.3	Yes	-1.0	Yes	2.6	Yes	1.9	Yes	0.7	Yes	0.1	Yes	2.1	Yes
2.2	Yes	1.6	Yes	1.0	Yes	0.4	Yes	-1.7	Yes	1.2	Yes	1.4	Yes
0.8	Yes	0.2	Yes	1.7	Yes	1.1	Yes						

When d'/d = 0.13

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.2	Yes	-0.1	Yes	0.1	Yes	-0.7	Yes	-0.6	Yes	0.0	Yes	-0.2	Yes
-0.3	Yes	0.5	Yes	0.9	Yes	1.3	Yes	1.8	Yes	2.2	Yes	-1.4	Yes
-1.1	Yes	-0.9	Yes	-2.4	Yes	-2.2	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes
-1.5	Yes	-2.9	Yes	-3.0	Yes	-2.8	Yes	-2.7	Yes	-2.5	Yes	-3.1	Yes
-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-2.3	Yes	-2.6	Yes
-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-1.0	Yes	-1.8	Yes	0.6	Yes
-0.8	Yes	-1.2	Yes	-1.6	Yes	1.6	Yes	1.0	Yes	-0.4	Yes	-1.3	Yes
-2.0	Yes	2.6	Yes	2.0	Yes	1.4	Yes	0.8	Yes	0.3	Yes	-0.5	Yes
1.7	Yes	1.2	Yes	0.7	Yes	0.2	Yes	1.5	Yes	1.9	Yes	0.4	Yes
1.1	Yes												

When d'/d = 0.14

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.3	Yes	0.0	Yes	0.1	Yes	-0.6	Yes	-0.5	Yes	-0.1	Yes	0.5	Yes
0.9	Yes	1.3	Yes	1.8	Yes	2.2	Yes	-1.3	Yes	-1.1	Yes	-0.9	Yes
-0.3	Yes	-2.4	Yes	-2.3	Yes	-2.2	Yes	-2.0	Yes	-1.9	Yes	-1.7	Yes
-1.5	Yes	-2.9	Yes	-2.8	Yes	-2.7	Yes	-2.6	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-2.5	Yes	-3.0	Yes	-3.6	Yes	-3.7	Yes
-1.0	Yes	-1.4	Yes	0.7	Yes	0.2	Yes	-0.2	Yes	1.7	Yes	1.1	Yes
0.6	Yes	2.5	Yes	2.1	Yes	1.5	Yes	1.0	Yes	-0.7	Yes	1.9	Yes
1.4	Yes	0.4	Yes	1.2	Yes	1.6	Yes	0.8	Yes	0.3	Yes	-0.4	Yes
-0.8	Yes	-1.2	No	-1.6	No	-1.8	No	-2.1	No	-3.8	No	-3.9	No
-4.0	No	-4.1	No										

When d'/d = 0.15

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.0	Yes	0.0	Yes	0.2	Yes	-0.6	Yes	-0.5	Yes	-0.4	Yes	0.1	Yes
0.6	Yes	1.0	Yes	1.4	Yes	1.8	Yes	2.3	Yes	-1.3	Yes	-1.1	Yes
-0.8	Yes	-0.2	Yes	-2.4	Yes	-2.2	Yes	-2.1	Yes	-1.9	Yes	-1.8	Yes
-1.6	Yes	-1.4	Yes	-2.8	Yes	-2.7	Yes	-2.6	Yes	-2.3	Yes	-2.9	Yes
-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-0.9	Yes
-1.2	Yes	-1.5	Yes	-2.5	Yes	0.8	Yes	0.3	Yes	-2.0	Yes	1.2	Yes
-1.7	Yes	2.5	Yes	2.2	Yes	1.7	Yes	0.7	Yes	1.9	Yes	2.1	Yes
1.6	Yes	1.1	Yes	-0.7	Yes	1.3	Yes	1.5	Yes	-1.0	Yes	0.9	Yes
0.4	Yes	0.5	Yes	-0.3	Yes	-0.1	Yes	-3.6	No	-3.7	No	-3.8	No
-3.9	No	-4.0	No	-4.1	No	-4.2	No	-4.3	No				

When d'/d = 0.16

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.0	Yes	0.0	Yes	0.3	Yes	-0.5	Yes	-0.4	Yes	-0.2	Yes	0.2	Yes
0.6	Yes	1.0	Yes	1.4	Yes	1.9	Yes	2.3	Yes	-1.3	Yes	-1.0	Yes
-0.7	Yes	-0.1	Yes	0.1	Yes	-2.3	Yes	-2.2	Yes	-2.0	Yes	-1.9	Yes
-1.7	Yes	-1.5	Yes	-1.2	Yes	-2.8	Yes	-2.7	Yes	-2.6	Yes	-2.5	Yes
-2.4	Yes	-2.1	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-1.8	Yes
-0.8	Yes	-1.1	Yes	-1.4	Yes	-0.3	Yes	-1.6	Yes	0.9	Yes	0.4	Yes
-0.6	Yes	-0.9	Yes	0.5	Yes	2.5	Yes	2.4	Yes	1.8	Yes	1.7	Yes
2.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes	1.5	Yes	0.7	Yes	0.8	Yes
-3.3	No	-3.4	No	-3.5	No	-3.6	No	-3.7	No	-3.8	No	-3.9	No
-4.0	No	-4.1	No	-4.2	No	-4.3	No	-4.4	No	-4.5	No	-4.6	No

When d'/d = 0.17

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.0	Yes	0.0	Yes	0.3	Yes	-0.4	Yes	-0.3	Yes	-0.1	Yes	0.4	Yes
0.6	Yes	1.0	Yes	1.5	Yes	1.9	Yes	2.4	Yes	-1.2	Yes	-0.9	Yes
-0.7	Yes	0.2	Yes	-2.3	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes	-1.5	Yes
-1.3	Yes	-1.1	Yes	-2.7	Yes	-2.6	Yes	-2.5	Yes	-2.4	Yes	-2.0	Yes
-2.8	Yes	-2.9	Yes	-2.2	Yes	-1.4	Yes	-1.0	Yes	-0.2	Yes	-0.5	Yes
-0.8	Yes	-1.6	Yes	-1.8	Yes	1.1	Yes	0.7	Yes	2.5	Yes	2.0	Yes
1.6	Yes	1.2	Yes	0.8	Yes	0.1	Yes	1.7	Yes	2.2	Yes	2.1	Yes
0.9	Yes	0.5	Yes	1.3	Yes	1.8	Yes	1.4	Yes	-0.6	Yes	-3.0	No
-3.1	No	-3.2	No	-3.3	No	-3.4	No	-3.5	No	-3.6	No	-3.7	No
-3.8	No	-3.9	No	-4.0	No	-4.1	No	-4.2	No	-4.3	No	-4.4	No
-4.5	No	-4.6	No	-4.7	No	-4.8	No	-4.9	No				

When d'/d = 0.18

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.9	Yes	0.0	Yes	0.4	Yes	-0.4	Yes	-0.2	Yes	0.5	Yes	0.6	Yes
0.7	Yes	1.1	Yes	1.5	Yes	2.0	Yes	2.4	Yes	-1.2	Yes	-0.9	Yes
-0.6	Yes	-0.3	Yes	0.3	Yes	-2.2	Yes	-2.0	Yes	-1.8	Yes	-1.6	Yes
-1.4	Yes	-2.7	Yes	-2.6	Yes	-2.5	Yes	-2.4	Yes	-2.1	Yes	-1.9	Yes
-2.3	Yes	-1.1	Yes	-1.3	Yes	-1.5	Yes	-1.7	Yes	0.1	Yes	-0.1	Yes
-0.7	Yes	1.0	Yes	1.6	Yes	1.2	Yes	0.9	Yes	0.2	Yes	-0.5	Yes
2.5	Yes	2.6	Yes	2.2	Yes	1.8	Yes	1.4	Yes	1.7	Yes	1.9	Yes
2.1	Yes	2.3	Yes	-0.8	Yes	1.3	Yes	0.8	Yes	-1.0	Yes	-2.8	No
-2.9	No	-3.0	No	-3.1	No	-3.2	No	-3.4	No	-3.3	No	-3.5	No
-3.6	No	-3.7	No	-3.9	No	-3.8	No	-4.0	No	-4.1	No	-4.2	No
-4.3	No	-4.4	No	-4.5	No	-4.6	No	-4.7	No	-4.8	No	-4.9	No
-5.0	No												

When d'/d = 0.19

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.9	Yes	0.0	Yes	0.4	Yes	-0.3	Yes	-0.1	Yes	0.5	Yes	0.6	Yes
0.7	Yes	1.1	Yes	1.6	Yes	2.0	Yes	2.5	Yes	-1.1	Yes	-0.8	Yes
-0.5	Yes	-0.2	Yes	0.1	Yes	-2.2	Yes	-2.0	Yes	-1.8	Yes	-1.5	Yes
-1.3	Yes	-1.0	Yes	-2.6	Yes	-2.5	Yes	-2.4	Yes	-2.1	Yes	-1.9	Yes
-1.6	Yes	-2.3	Yes	-1.2	Yes	-1.7	Yes	0.2	Yes	-0.7	Yes	-0.9	Yes
-1.4	Yes	0.8	Yes	-0.4	Yes	-0.6	Yes	2.1	Yes	1.7	Yes	1.4	Yes
2.7	Yes	2.3	Yes	1.3	Yes	1.0	Yes	1.9	Yes	2.2	Yes	1.8	Yes
1.5	Yes	1.2	Yes	0.9	Yes	0.3	Yes	-2.7	No	-2.8	No	-2.9	No
-3.1	No	-3.0	No	-3.2	No	-3.4	No	-3.3	No	-3.5	No	-3.6	No
-3.7	No	-3.9	No	-3.8	No	-4.0	No	-4.1	No	-4.2	No	-4.3	No
-4.4	No	-4.5	No	-4.6	No	-4.7	No	-4.8	No	-4.9	No	-5.0	No

When d'/d = 0.20

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.8	Yes	0.0	Yes	0.4	Yes	-0.3	Yes	-0.1	Yes	0.1	Yes	0.5	Yes
0.6	Yes	0.7	Yes	0.8	Yes	1.2	Yes	1.6	Yes	2.1	Yes	2.6	Yes
-1.1	Yes	-0.8	Yes	-0.4	Yes	0.2	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes
-1.4	Yes	-1.2	Yes	-0.9	Yes	-0.6	Yes	-2.5	Yes	-2.4	Yes	-2.3	Yes
-2.0	Yes	-1.8	Yes	-1.6	Yes	-2.6	Yes	-2.2	Yes	-1.5	Yes	-0.5	Yes
-0.7	Yes	-1.0	Yes	-1.3	Yes	0.3	Yes	0.9	Yes	2.2	Yes	1.9	Yes
1.5	Yes	1.3	Yes	1.0	Yes	-0.2	Yes	2.5	Yes	2.8	Yes	1.8	Yes
2.3	Yes	2.4	Yes	1.1	Yes	1.4	Yes	2.0	Yes	1.7	Yes	-2.7	No
-2.9	No	-2.8	No	-3.0	No	-3.2	No	-3.1	No	-3.3	No	-3.4	No
-3.6	No	-3.5	No	-3.7	No	-3.8	No	-3.9	No	-4.0	No	-4.1	No
-4.2	No	-4.3	No	-4.4	No	-4.5	No	-4.6	No	-4.7	No	-4.8	No
-4.9	No												

When d'/d = 0.21

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.7	Yes	0.0	Yes	0.3	Yes	-0.2	Yes	0.1	Yes	0.6	Yes	0.7	Yes
0.8	Yes	1.0	Yes	1.2	Yes	1.7	Yes	2.1	Yes	2.6	Yes	-1.1	Yes
-0.7	Yes	-0.4	Yes	0.2	Yes	-2.1	Yes	-1.8	Yes	-1.6	Yes	-1.3	Yes
-0.8	Yes	-0.5	Yes	-2.5	Yes	-2.3	Yes	-2.2	Yes	-2.0	Yes	-1.4	Yes
-1.2	Yes	-2.4	Yes	-1.9	Yes	-1.7	Yes	-1.5	Yes	-0.6	Yes	-0.9	Yes
-1.0	Yes	0.4	Yes	-0.1	Yes	-0.3	Yes	1.3	Yes	2.3	Yes	2.0	Yes
1.4	Yes	2.8	Yes	2.7	Yes	2.4	Yes	1.8	Yes	1.6	Yes	1.1	Yes
0.9	Yes	2.2	Yes	1.9	Yes	1.5	Yes	0.5	Yes	2.5	Yes	-2.7	No
-2.6	No	-2.8	No	-3.0	No	-2.9	No	-3.1	No	-3.2	No	-3.4	No
-3.3	No	-3.5	No	-3.6	No	-3.7	No	-3.8	No	-3.9	No	-4.0	No
-4.1	No	-4.2	No	-4.3	No	-4.4	No	-4.5	No	-4.6	No	-4.7	No
-4.8	No												

When d'/d = 0.22

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.5	Yes	0.0	Yes	0.3	Yes	-0.2	Yes	0.1	Yes	0.6	Yes	0.8	Yes
0.9	Yes	1.1	Yes	1.2	Yes	1.7	Yes	2.2	Yes	2.6	Yes	-1.0	Yes
-0.7	Yes	-0.3	Yes	0.7	Yes	-2.0	Yes	-1.8	Yes	-1.5	Yes	-1.2	Yes
-0.4	Yes	-2.4	Yes	-2.3	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes	-1.3	Yes
-2.5	Yes	-1.4	Yes	-2.2	Yes	-1.6	Yes	-1.1	Yes	-0.6	Yes	-0.8	Yes
-0.9	Yes	0.4	Yes	-0.1	Yes	-0.5	Yes	1.4	Yes	0.2	Yes	2.4	Yes
2.1	Yes	1.9	Yes	1.6	Yes	1.0	Yes	0.5	Yes	2.8	Yes	2.5	Yes
2.3	Yes	1.8	Yes	1.5	Yes	1.3	Yes	2.0	Yes	2.7	Yes	3.0	Yes
2.9	Yes	-2.6	No	-2.7	No	-2.8	No	-3.0	No	-2.9	No	-3.1	No
-3.2	No	-3.3	No	-3.4	No	-3.5	No	-3.6	No	-3.7	No	-3.8	No
-3.9	No	-4.0	No	-4.1	No	-4.2	No	-4.3	No	-4.4	No	-4.5	No
-4.6	No												

When d'/d = 0.23

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.4	Yes	0.0	Yes	0.2	Yes	-0.2	Yes	0.6	Yes	0.8	Yes	1.0	Yes
1.1	Yes	1.2	Yes	1.7	Yes	2.2	Yes	2.7	Yes	-1.0	Yes	-0.6	Yes
-0.3	Yes	0.4	Yes	-2.0	Yes	-1.7	Yes	-1.4	Yes	-1.1	Yes	-0.9	Yes
-2.4	Yes	-2.2	Yes	-1.8	Yes	-1.6	Yes	-1.3	Yes	-2.3	Yes	-1.9	Yes
-1.5	Yes	-1.2	Yes	-2.1	Yes	-0.4	Yes	-0.5	Yes	-0.7	Yes	0.5	Yes
0.1	Yes	-0.1	Yes	1.4	Yes	1.3	Yes	0.9	Yes	0.7	Yes	0.3	Yes
2.4	Yes	2.0	Yes	1.8	Yes	1.6	Yes	1.5	Yes	2.6	Yes	2.9	Yes
3.0	Yes	2.5	Yes	2.3	Yes	2.1	Yes	1.9	Yes	2.8	Yes	3.1	Yes
3.2	Yes	3.3	Yes	3.5	Yes	-0.8	Yes	-2.6	No	-2.5	No	-2.7	No
-2.8	No	-2.9	No	-3.1	No	-3.0	No	-3.2	No	-3.3	No	-3.4	No
-3.5	No	-3.6	No	-3.7	No	-3.8	No	-3.9	No	-4.0	No	-4.1	No
-4.2	No	-4.3	No										

When d'/d = 0.24

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.2	Yes	0.0	Yes	0.2	Yes	-0.2	Yes	0.1	Yes	0.7	Yes	0.8	Yes
1.0	Yes	1.2	Yes	1.3	Yes	1.7	Yes	2.2	Yes	2.7	Yes	-1.0	Yes
-0.6	Yes	0.5	Yes	-1.9	Yes	-1.7	Yes	-1.4	Yes	-1.1	Yes	-0.8	Yes
-0.5	Yes	-0.1	Yes	-2.4	Yes	-2.1	Yes	-1.5	Yes	-1.2	Yes	-0.7	Yes
-2.2	Yes	-1.6	Yes	-2.0	Yes	-1.8	Yes	-1.3	Yes	-0.9	Yes	-0.3	Yes
-0.4	Yes	0.6	Yes	0.4	Yes	0.3	Yes	1.5	Yes	1.4	Yes	1.1	Yes
0.9	Yes	2.5	Yes	2.3	Yes	2.1	Yes	2.0	Yes	1.8	Yes	1.6	Yes
2.6	Yes	2.9	Yes	3.1	Yes	2.4	Yes	1.9	Yes	3.3	Yes	2.8	Yes
3.4	Yes	3.0	Yes	3.6	Yes	3.5	Yes	3.8	Yes	3.7	Yes	4.0	Yes
-2.3	Yes	-2.5	No	-2.7	No	-2.6	No	-2.8	No	-2.9	No	-3.0	No
-3.1	No	-3.2	No	-3.3	No	-3.4	No	-3.5	No	-3.6	No	-3.7	No
-3.8	No	-3.9	No	-4.0	No								

When d'/d = 0.25

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.0	Yes	-0.1	Yes	0.1	Yes	-0.2	Yes	0.0	Yes	0.7	Yes	0.9	Yes
1.0	Yes	1.2	Yes	0.8	Yes	1.3	Yes	1.7	Yes	2.2	Yes	2.7	Yes
-0.9	Yes	-0.6	Yes	0.5	Yes	-1.9	Yes	-1.6	Yes	-1.3	Yes	-1.0	Yes
-0.7	Yes	-0.4	Yes	-2.3	Yes	-2.1	Yes	-1.8	Yes	-1.4	Yes	-1.1	Yes
-2.0	Yes	-1.2	Yes	-0.8	Yes	-1.7	Yes	-1.5	Yes	-0.5	Yes	-0.3	Yes
0.6	Yes	0.4	Yes	0.3	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.1	Yes
2.6	Yes	2.4	Yes	2.3	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes
3.3	Yes	3.1	Yes	2.9	Yes	2.8	Yes	2.5	Yes	3.5	Yes	3.4	Yes
3.2	Yes	3.8	Yes	3.7	Yes	4.0	Yes	3.9	Yes	3.6	Yes	4.3	Yes
4.1	Yes	0.2	Yes	-2.2	Yes	-2.4	Yes	-2.5	No	-2.6	No	-2.7	No
-2.8	No	-2.9	No	-3.0	No	-3.1	No	-3.2	No	-3.3	No	-3.4	No
-3.5	No	-3.6	No	-3.7	No								

When d'/d = 0.26

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.8	Yes	-0.2	Yes	0.0	Yes	-0.3	Yes	-0.1	Yes	0.7	Yes	0.9	Yes
1.0	Yes	1.2	Yes	0.8	Yes	1.3	Yes	1.7	Yes	2.2	Yes	2.7	Yes
-0.9	Yes	-0.5	Yes	0.1	Yes	0.5	Yes	-1.9	Yes	-1.6	Yes	-1.3	Yes
-0.6	Yes	-2.3	Yes	-2.0	Yes	-1.8	Yes	-1.5	Yes	-1.0	Yes	-0.7	Yes
-2.1	Yes	-1.7	Yes	-1.1	Yes	-1.4	Yes	-0.8	Yes	-1.2	Yes	-0.4	Yes
0.2	Yes	0.6	Yes	1.6	Yes	1.5	Yes	1.4	Yes	2.5	Yes	2.4	Yes
2.3	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	3.0	Yes	3.3	Yes
3.1	Yes	2.9	Yes	2.6	Yes	3.4	Yes	3.7	Yes	3.5	Yes	3.2	Yes
4.0	Yes	3.9	Yes	3.8	Yes	3.6	Yes	1.1	Yes	4.3	Yes	4.4	Yes
4.2	Yes	4.1	Yes	4.5	Yes	4.7	Yes	4.6	Yes	0.3	Yes	0.4	Yes
-2.2	Yes	-2.4	Yes	-2.5	No	-2.6	No	-2.7	No	-2.8	No	-2.9	No
-3.0	No	-3.1	No	-3.2	No	-3.3	No						

When d'/d = 0.27

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.6	Yes	-0.3	Yes	-0.1	Yes	0.0	Yes	0.7	Yes	0.8	Yes	1.0	Yes
1.2	Yes	1.7	Yes	2.2	Yes	-0.9	Yes	-0.5	Yes	0.1	Yes	0.5	Yes
0.9	Yes	-1.9	Yes	-1.6	Yes	-1.2	Yes	-0.6	Yes	-0.2	Yes	-2.2	Yes
-2.0	Yes	-1.7	Yes	-1.5	Yes	-0.7	Yes	-0.4	Yes	-1.8	Yes	-1.4	Yes
-1.0	Yes	-1.3	Yes	-1.1	Yes	-0.8	Yes	0.2	Yes	0.3	Yes	0.4	Yes
1.6	Yes	1.5	Yes	2.7	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.1	Yes
3.1	Yes	3.4	Yes	3.3	Yes	3.2	Yes	3.0	Yes	2.9	Yes	2.8	Yes
2.0	Yes	3.5	Yes	3.8	Yes	3.9	Yes	3.7	Yes	3.6	Yes	1.4	Yes
1.9	Yes	4.2	Yes	4.4	Yes	4.3	Yes	4.1	Yes	4.0	Yes	1.1	Yes
1.3	Yes	1.8	Yes	4.5	Yes	4.8	Yes	4.7	Yes	4.6	Yes	0.6	Yes
5.1	Yes	5.2	Yes	-2.1	Yes	-2.3	Yes	-2.4	No	-2.5	No	-2.6	No
-2.7	No	-2.8	No	-2.9	No								

When d'/d = 0.28

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.3	Yes	-0.4	Yes	-0.3	Yes	-0.2	Yes	0.0	Yes	0.7	Yes	0.8	Yes
1.0	Yes	1.1	Yes	1.2	Yes	1.7	Yes	2.1	Yes	2.6	Yes	-0.9	Yes
-0.5	Yes	-0.1	Yes	0.1	Yes	0.5	Yes	0.9	Yes	-1.9	Yes	-1.5	Yes
-1.2	Yes	-2.2	Yes	-2.0	Yes	-1.7	Yes	-1.4	Yes	-1.1	Yes	-0.6	Yes
-1.8	Yes	-1.3	Yes	-1.6	Yes	-1.0	Yes	-0.8	Yes	-0.7	Yes	0.2	Yes
0.3	Yes	0.4	Yes	0.6	Yes	1.3	Yes	1.8	Yes	1.9	Yes	2.8	Yes
2.7	Yes	2.5	Yes	2.4	Yes	3.1	Yes	3.5	Yes	3.6	Yes	3.4	Yes
3.3	Yes	3.2	Yes	2.0	Yes	2.9	Yes	3.9	Yes	4.2	Yes	4.1	Yes
4.0	Yes	3.8	Yes	1.4	Yes	2.2	Yes	3.7	Yes	4.3	Yes	4.6	Yes
4.7	Yes	4.5	Yes	4.4	Yes	5.0	Yes	5.2	Yes	5.1	Yes	1.6	Yes
5.3	Yes	5.6	Yes	3.0	Yes	4.8	Yes	1.5	Yes	-2.1	Yes	-2.3	No
-2.4	No	-2.5	No										

When d'/d = 0.29

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.1	Yes	-0.5	Yes	-0.4	Yes	-0.3	Yes	-0.2	Yes	0.6	Yes	0.8	Yes
0.9	Yes	1.1	Yes	0.7	Yes	1.2	Yes	1.6	Yes	2.5	Yes	-0.9	Yes
0.1	Yes	0.5	Yes	-1.8	Yes	-1.5	Yes	-1.2	Yes	0.0	Yes	-2.2	Yes
-1.9	Yes	-1.7	Yes	-1.4	Yes	-1.1	Yes	-0.8	Yes	-1.0	Yes	-1.3	Yes
-0.7	Yes	-0.1	Yes	-0.6	Yes	0.3	Yes	0.4	Yes	0.2	Yes	1.0	Yes
1.3	Yes	1.4	Yes	1.5	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.2	Yes
2.9	Yes	2.8	Yes	2.7	Yes	3.2	Yes	3.5	Yes	3.7	Yes	3.6	Yes
2.4	Yes	3.0	Yes	3.3	Yes	4.0	Yes	4.4	Yes	4.3	Yes	4.2	Yes
1.7	Yes	2.3	Yes	2.6	Yes	3.8	Yes	4.1	Yes	4.5	Yes	4.8	Yes
5.1	Yes	5.0	Yes	4.9	Yes	3.1	Yes	3.4	Yes	4.6	Yes	5.2	Yes
5.5	Yes	5.6	Yes	4.7	Yes	5.3	Yes	5.9	Yes	3.9	Yes	-1.6	Yes
-2.0	No												

When d'/d = 0.30

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.8	Yes	-0.7	Yes	-0.5	Yes	-0.4	Yes	-0.3	Yes	0.6	Yes	0.7	Yes
0.9	Yes	1.0	Yes	1.1	Yes	1.5	Yes	2.0	Yes	2.4	Yes	-0.9	Yes
-0.6	Yes	-0.2	Yes	0.1	Yes	0.5	Yes	0.8	Yes	-1.8	Yes	-1.5	Yes
-1.2	Yes	0.0	Yes	-2.2	Yes	-1.9	Yes	-1.6	Yes	-1.3	Yes	-1.1	Yes
-0.8	Yes	-1.7	Yes	-1.4	Yes	-1.0	Yes	0.3	Yes	0.2	Yes	0.4	Yes
1.2	Yes	1.3	Yes	1.4	Yes	1.6	Yes	1.7	Yes	1.9	Yes	2.1	Yes
2.2	Yes	2.3	Yes	2.5	Yes	2.6	Yes	2.9	Yes	3.0	Yes	3.1	Yes
3.2	Yes	2.8	Yes	3.6	Yes	3.8	Yes	3.9	Yes	2.7	Yes	3.4	Yes
4.1	Yes	4.5	Yes	4.7	Yes	4.6	Yes	3.3	Yes	4.0	Yes	4.3	Yes
5.0	Yes	5.3	Yes	5.4	Yes	3.5	Yes	4.8	Yes	5.1	Yes	5.7	Yes
6.1	Yes	3.7	Yes	4.9	Yes	5.2	Yes	5.5	Yes	5.8	Yes	4.2	Yes
5.6	Yes	4.4	Yes	-0.1	Yes								

When d'/d = 0.31

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.3	Yes	-1.0	Yes	-1.1	Yes	-0.7	Yes	0.4	Yes	0.5	Yes	0.6	Yes
0.7	Yes	0.9	Yes	1.2	Yes	1.5	Yes	-1.2	Yes	-0.5	Yes	-0.2	Yes
0.0	Yes	-2.1	Yes	-1.8	Yes	-1.6	Yes	-1.4	Yes	-0.8	Yes	-2.4	Yes
-2.2	Yes	-2.0	Yes	-1.3	Yes	-2.3	Yes	-1.9	Yes	-1.7	Yes	-1.5	Yes
-0.9	Yes	-0.6	Yes	-0.4	Yes	-0.3	Yes	-0.1	Yes	0.1	Yes	0.2	Yes
0.3	Yes	0.8	Yes	1.0	Yes	1.1	Yes	1.8	Yes	1.7	Yes	2.8	Yes
2.7	Yes	2.6	Yes	2.5	Yes	2.9	Yes	3.1	Yes	3.3	Yes	3.5	Yes
3.4	Yes	3.2	Yes	1.9	Yes	2.1	Yes	2.2	Yes	2.4	Yes	3.0	Yes
3.6	Yes	3.8	Yes	4.0	Yes	4.1	Yes	3.9	Yes	1.4	Yes	1.6	Yes
2.0	Yes	4.2	Yes	2.3	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes
-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes
-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes
-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes
-5.0	Yes	-5.1	Yes	-5.2	Yes	-5.3	No	-5.4	No	-5.5	No	-5.6	No
-5.7	No	-5.8	No	-5.9	No	-6.0	No	-6.1	No	-6.2	No	-6.3	No

When d'/d = 0.32

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.9	Yes	-1.2	Yes	-1.4	Yes	-0.8	Yes	-0.9	Yes	0.4	Yes	0.5	Yes
0.3	Yes	0.6	Yes	1.1	Yes	1.4	Yes	-1.0	Yes	-0.7	Yes	-0.5	Yes
-0.3	Yes	0.0	Yes	-2.1	Yes	-1.9	Yes	-1.6	Yes	-2.4	Yes	-2.2	Yes
-2.0	Yes	-1.8	Yes	-2.3	Yes	-1.7	Yes	-1.5	Yes	-1.3	Yes	-1.1	Yes
-0.4	Yes	-0.6	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.7	Yes
1.0	Yes	1.2	Yes	1.3	Yes	1.9	Yes	1.8	Yes	2.0	Yes	2.1	Yes
2.9	Yes	2.8	Yes	2.5	Yes	2.7	Yes	3.1	Yes	3.4	Yes	3.6	Yes
3.7	Yes	2.2	Yes	2.3	Yes	3.3	Yes	3.5	Yes	4.0	Yes	4.2	Yes
4.4	Yes	4.3	Yes	1.5	Yes	1.7	Yes	3.9	Yes	4.1	Yes	4.5	Yes
0.8	Yes	1.6	Yes	2.4	Yes	3.2	Yes	3.8	Yes	3.0	Yes	-2.5	Yes
-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes
-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	No	-5.1	No	-5.2	No	-5.3	No
-5.4	No	-5.5	No	-5.6	No								

When d'/d = 0.33

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.6	Yes	-1.3	Yes	-1.7	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	0.3	Yes
0.5	Yes	0.8	Yes	1.0	Yes	1.2	Yes	-1.2	Yes	-0.8	Yes	-0.6	Yes
-0.3	Yes	-0.1	Yes	-2.1	Yes	-1.9	Yes	-1.4	Yes	-2.4	Yes	-2.2	Yes
-2.0	Yes	-1.8	Yes	-1.6	Yes	-2.3	Yes	-1.5	Yes	-0.7	Yes	-0.4	Yes
-0.2	Yes	-0.5	Yes	0.0	Yes	0.1	Yes	0.2	Yes	0.4	Yes	0.7	Yes
0.9	Yes	1.1	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.9	Yes
2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes	2.9	Yes	3.0	Yes
3.1	Yes	2.6	Yes	2.8	Yes	3.2	Yes	3.5	Yes	3.7	Yes	3.9	Yes
1.8	Yes	3.3	Yes	4.1	Yes	4.4	Yes	4.6	Yes	4.7	Yes	2.5	Yes
2.7	Yes	4.3	Yes	4.5	Yes	4.8	Yes	1.7	Yes	-2.5	Yes	-2.6	Yes
-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes
-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes
-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	No	-4.7	No
-4.8	No	-4.9	No										

When d'/d = 0.34

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.2	Yes	-1.5	Yes	-2.0	Yes	-1.1	Yes	-1.2	Yes	-1.4	Yes	0.4	Yes
0.6	Yes	0.8	Yes	1.1	Yes	-1.3	Yes	-0.9	Yes	-0.7	Yes	-0.4	Yes
-0.2	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes	-2.4	Yes	-2.2	Yes	-1.8	Yes
-1.6	Yes	-2.3	Yes	-1.0	Yes	-0.6	Yes	-0.5	Yes	-0.1	Yes	-0.3	Yes
0.0	Yes	0.1	Yes	0.3	Yes	0.5	Yes	0.7	Yes	0.9	Yes	1.0	Yes
1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes
1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes
2.6	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	2.8	Yes
3.5	Yes	3.8	Yes	4.0	Yes	4.1	Yes	4.2	Yes	2.7	Yes	2.9	Yes
3.6	Yes	4.3	Yes	4.5	Yes	4.8	Yes	5.0	Yes	5.1	Yes	3.7	Yes
3.9	Yes	4.6	Yes	4.4	Yes	-0.8	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes
-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes
-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	No
-4.2	No												

When d'/d = 0.35

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-0.1	Yes	-1.8	Yes	-2.3	Yes	-1.2	Yes	-1.4	Yes	-1.6	Yes	0.2	Yes
0.1	Yes	0.0	Yes	0.3	Yes	0.5	Yes	0.7	Yes	0.9	Yes	-1.3	Yes
-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.6	Yes	-0.4	Yes	-2.1	Yes	-1.9	Yes
-1.7	Yes	-1.0	Yes	-2.4	Yes	-2.2	Yes	-2.0	Yes	-1.5	Yes	-0.7	Yes
-0.3	Yes	-0.2	Yes	-0.5	Yes	0.4	Yes	0.6	Yes	0.8	Yes	1.0	Yes
1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.7	Yes
1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes
2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	3.0	Yes	3.1	Yes	3.2	Yes
3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	2.9	Yes	3.9	Yes
4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes	4.0	Yes	4.7	Yes
4.9	Yes	5.2	Yes	5.3	Yes	5.4	Yes	3.8	Yes	4.8	Yes	5.0	Yes
4.6	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes						

When d'/d = 0.36

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-0.5	Yes	-2.0	Yes	-2.6	Yes	-1.3	Yes	-1.6	Yes	-1.9	Yes	0.0	Yes
-0.1	Yes	-0.2	Yes	0.1	Yes	0.2	Yes	0.4	Yes	0.5	Yes	0.7	Yes
-1.4	Yes	-1.2	Yes	-1.0	Yes	-0.9	Yes	-0.7	Yes	-2.2	Yes	-1.8	Yes
-1.5	Yes	-1.1	Yes	-2.4	Yes	-2.1	Yes	-1.7	Yes	-2.3	Yes	-0.6	Yes
-0.8	Yes	-0.4	Yes	-0.3	Yes	0.6	Yes	0.3	Yes	0.8	Yes	0.9	Yes
1.0	Yes	1.1	Yes	1.2	Yes	1.4	Yes	1.3	Yes	1.5	Yes	1.6	Yes
1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes
2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes
3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes
3.8	Yes	3.9	Yes	4.2	Yes	4.4	Yes	4.5	Yes	4.6	Yes	4.7	Yes
4.8	Yes	4.1	Yes	4.3	Yes	5.1	Yes	5.4	Yes	5.6	Yes	5.7	Yes
4.0	Yes	4.9	Yes	5.2	Yes	-2.5	Yes	-2.7	Yes				

When d'/d = 0.37

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-0.9	Yes	-2.2	Yes	-3.0	Yes	-1.5	Yes	-1.9	Yes	0.0	Yes	-0.1	Yes
-0.3	Yes	-0.4	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes	-1.4	Yes
-1.3	Yes	-1.2	Yes	-1.0	Yes	-0.7	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes
-2.4	Yes	-2.3	Yes	-2.1	Yes	-1.8	Yes	-0.6	Yes	-1.1	Yes	-0.8	Yes
-0.5	Yes	-0.2	Yes	0.5	Yes	0.6	Yes	0.7	Yes	0.8	Yes	0.9	Yes
1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes
1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes
2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes
3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes
3.8	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.3	Yes	4.5	Yes	4.6	Yes
4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	4.2	Yes	4.4	Yes	5.2	Yes
5.5	Yes	5.7	Yes	5.9	Yes	5.1	Yes	5.4	Yes	5.6	Yes		

Max NPb/b' is 52.62

The followings are printed 'only as typical cases ' that are used in the examples to illustrate the computaions involved in the program :

When NP = 2.00% , NP' = 2.00% and d'/d = 0.030 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.97	5.76	1.71	1.020	0.97

When NP = 15.00% , NP' = 10.00% and d'/d = 0.240 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.81	2.04	13.00	1.001	0.97

When NP = 25.00% , NP' = 25.00% and d'/d = 0.160 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.86	1.04	15.84	0.997	0.83

When NP = 40.00% , NP' = 40.00% and d'/d = 0.030 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.97	0.41	9.23	1.020	0.62

When NP = 45.00% , NP' = 35.00% and d'/d = 0.370 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.73	1.36	36.39	0.998	0.91

APPENDIX B4

The Listing of Prog.3.3.3 and its Output

PROG.3.3.3

This program is same as PROG.3.3.2 except that α' is now based on Eq.3.3.8

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION SAND(35,700),error(35,700),M(35),ID(35,700),IDerr(35,700
     *),DR(10),DEPTH(10),RIGICR(10),RICRe(10),RNPCO(10),IFLAG(10),JFLAG(
     *10),INDEX(35)
      CHARACTER SAND*5
      REAL NP,NPC,NWRICR,NEWNP,NPe,MODNP(64),IFLAG,JFLAG
      OPEN(3,FILE='OUTPUT',STATUS='UNKNOWN')
C The program will now read np , np' and d'/d . Any section that is
C found unductile is ignored.
      NPe=0.0
      DO 15 I=1,700
      DO 10 K=1,35
      M(K)=0
      ID(K,I)=0
      error(K,I)=0
      INDEX(K)=0
   10 CONTINUE
   15 CONTINUE
      WRITE(*,'(1X,A)')'How many cases are to be given as examples (max.
     * of 10)?'
      READ*,MXCASE
      WRITE(*,'(1X,A/8X,A,7(/12X,A))')'For each case please give np,np''
     * and d''/d . Enter one combination per line','Notes :','1. np shou
     *ld always be greater than or equal to np''','2.Subject  to  the co
     *ndition above , np and np'' can',' have any value (in an icrement
     * of 1) from 2 to 64','3.d''/d should be  chosen using icrements of
     * 0.01 in',' the range from 0.03 to 0.37 . Values outside this','
     * range must not be used'
      DO 16 ICASE=1,MXCASE
      READ*,IFLAG(ICASE),JFLAG(ICASE),DR(ICASE)
   16 CONTINUE
      WRITE(*,'(A)')'*==================== The program is running pleas
     *e wait ===================*'
      DO 50 I=2,64
      NP=I/100.
      DO 40 J=2,64
      NPC=J/100.
      IF(NPC.GT.NP) GO TO 40
      IF(I.GT.(32.0+J)) THEN
C*****This section is not ductile and thus will not be considered .
      GO TO 40
      END IF
      DO 30 K=1,35
      IF(K.EQ.1) THEN
      DRATIO = 0.03
      ELSE IF(K.EQ.8) THEN
      DRATIO=0.1
      ELSE
      DRATIO=DRATIO+0.01
      END IF
```

```
C Now that all the parameters have been read  b' will be evaluated  and
C npb/b' will be calculated as NEWNP. The parameters  required  for the
C evaluation of Icre  will also be found :
      APRIME=(3./5000.+(1/20.)*(DRATIO*(1-2*DRATIO)**2.))*100.
      BPRIME=1+APRIME*NPC/DRATIO
      NEWNP=100.*NP/BPRIME
      IF(NEWNP.LE.1.9) THEN
      A=0.003
      B=0.095
      ELSE IF((NEWNP.GT.1.9).AND.(NEWNP.LE.5.0)) THEN
      A=0.05
      B=0.07
      ELSE IF((NEWNP.GT.5.0).AND.(NEWNP.LE.17.0)) THEN
      A=0.16
      B=0.05
      ELSE IF((NEWNP.GT.17.0).AND.(NEWNP.LE.32.0)) THEN
      A=0.50
      B=0.03
      ELSE
      A=0.8
      B=0.02
      END IF
C The program will  now evaluate Icr , Icre and  Icr  when  compression
C reinforcement is ignored (stored in "Icr,np'=0") and will print the
C ratios of the values relative to Icr .
      X=-(NP+NPC)+SQRT((NP+NPC)**2+2*NP+2*NPC*DRATIO)
      XO=-NP+SQRT(NP**2+2*NP)
      XICR=(X**3)/3.+NPC*(X-DRATIO)**2+NP*(1-X)**2
      OICR=(XO**3)/3.+NP*(1-XO)**2
      XIcre=(A+B*NEWNP)*BPRIME/12.
      NWRICR=XIcre/XICR
      ROICR=OICR/XICR
      DH=1/(1+DRATIO)
      XIG=((1+DRATIO)**3)/12
      XIGRAT=XIG/XICR
C*****The following  13  lines relate  only to the  sections  used  in the
C*****examples which are provided to illustrate the computations involoved
C***** in the program :
      DO 17 ICASE=1,MXCASE
      III=NP*100000.
      JJJ=NPC*100000.
      IIFLG=IFLAG(ICASE)*1000.
      JJFLG=JFLAG(ICASE)*1000.
      IF((III.EQ.IIFLG).AND.(JJJ.EQ.JJFLG)) THEN
      IDRAT=DRATIO*1000.
      IDR=DR(ICASE)*1000.
      IF(IDRAT.EQ.IDR) THEN
      INDEX(ICASE)=1
      DEPTH(ICASE)=DH
      RIGICR(ICASE)=XIGRAT
      MODNP(ICASE)=NEWNP
      RICRe(ICASE)=NWRICR
      RNPCO(ICASE)=ROICR
      END IF
      END IF
   17 CONTINUE
```

```
C***************************************************************************C
      Ierror=(NWRICR-1)*1000.
      Ierror=Ierror+400
      NPe=DMAX1(NEWNP,NPe)
      DO 20 IJ=1,700
      IF(Ierror.NE.IJ) GO TO 20
      IF(ID(K,IJ).EQ.0) THEN
      M(K)=M(K)+1
      IDerr(K,IJ)=M(K)
      error(K,M(K))=(Ierror-400)/10.
      IF(XIGRAT.GE.1.0) THEN
      ID(K,IJ)=1
      SAND(K,M(K))='Yes'
      ELSE
      ID(K,IJ)=2
      SAND(K,M(K))='No'
      END IF
      ELSE IF(ID(K,IJ).EQ.2) THEN
      IF(XIGRAT.GE.1.0) THEN
      ID(K,IJ)=1
      SAND(K,IDerr(K,IJ))='Yes'
      END IF
      END IF
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
      DO 70 K=1,35
      IF(K.EQ.1) THEN
      WRITE(3,'(7(/))')
      ICOUNT=0
      DRATIO=0.03
      END IF
      IF(K.NE.1) DRATIO=DRATIO+0.01
      ICOUNT=ICOUNT+1
      WRITE(3,'(2(/),65X,A,1X,F4.2,/)')'When d''/d =',DRATIO
      WRITE(3,55)
   55 FORMAT(10X,7('% error  Ig>Icr?',3X))
      DO 65 I=1,M(K)
      IF(I.EQ.1) II=1
      IF(I.NE.1) II=II+7
      IJ=I*7
      IF(IJ.GT.M(K))IJ=M(K)
      IF(II.GT.M(K)) GO TO 65
      WRITE(3,60)(error(K,J),SAND(K,J),J=II,IJ)
   60 FORMAT(11X,7(F5.1,4X,A,5X))
   65 CONTINUE
      IF(K.EQ.35) GO TO 70
      IF(ICOUNT.EQ.3) THEN
      ICOUNT=0
      WRITE(3,'(1H1,7(/))')
      END IF
   70 CONTINUE
      WRITE(3,'(2(/),64X,A,F5.2)')'Max NPb/b'' is ',NPe
      WRITE(3,75)
   75 FORMAT(T64,21('-'))
```

```
      WRITE(3,'(1H1,7(/))')
      WRITE(3,'(15X,A,//15X,A,//15X,A)')'The followings are printed ''on
     *ly as typical cases '' that are ','used in the examples to illustr
     *ate the computaions involved ','in the program :'
      DO 80 I=1,MXCASE
      WRITE(3,'(//15X,A,F5.2,A,F5.2,A,F5.3,A)')'When NP = ',IFLAG(I),'%
     *, NP'' = ',JFLAG(I),'%  and d''/d  = ',DR(I),' :'
      IF(INDEX(I).EQ.1) THEN
      WRITE(3,'(/25X,5(A,3X))')'d/h','Ig/Icr','NPb/b''','Icre/Icr','(Icr
     *,NP''=0)/Icr'
      WRITE(3,'(/25X,F4.2,2X,F5.2,4X,F5.2,6X,F5.3,10X,F4.2)')DEPTH(I),RI
     *GICR(I),MODNP(I),RICRe(I),RNPCO(I)
      ELSE
      IF(IFLAG(I).GT.(32.0+JFLAG(I))) THEN
      WRITE(3,'(/25X,A)')'This section is not ductile and thus ignored'
      ELSE
      WRITE(3,'(/25X,A)')'The program does not consider such a section'
      END IF
      END IF
   80 CONTINUE
      WRITE(3,'(///)')
      STOP
      END
```

OUTPUT OF PROG.3.3.3

When d'/d = 0.03

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.9	Yes	-1.0	Yes	-1.1	Yes	-1.8	Yes	-2.2	Yes	-2.5	Yes	-1.2	Yes
-2.4	Yes	-2.8	Yes	0.9	Yes	1.5	Yes	-1.4	Yes	-2.0	Yes	-2.6	Yes
-0.8	Yes	-0.3	Yes	0.1	Yes	0.6	Yes	-2.1	Yes	-1.7	Yes	-1.3	Yes
-0.9	Yes	-0.5	Yes	-3.0	Yes	-2.9	Yes	-2.7	Yes	-2.3	Yes	-3.1	Yes
-3.3	Yes	-3.2	Yes	-3.4	Yes	-3.6	Yes	-3.5	Yes	-3.7	Yes	-3.8	Yes
-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-0.6	Yes	-4.4	No
0.2	Yes	-4.5	No	1.1	Yes	0.3	Yes	-1.5	Yes	-4.6	No	2.1	Yes
1.2	Yes	0.4	No	-0.2	No	-1.9	No	3.1	No	2.2	No	1.3	No
0.5	No	-0.1	No	-0.7	No	2.7	No	3.2	No	2.3	No	1.4	No
0.7	No	0.0	No	-1.6	No	3.0	No	0.8	No	-0.4	No	1.7	No
2.4	No	3.3	No	1.6	No	2.0	No	3.4	No	2.5	No	1.0	No
3.6	No	2.6	No	1.8	No	3.8	No	3.5	No	2.8	No	4.0	No
4.2	No	4.3	No	3.7	No	2.9	No	4.5	No	3.9	No	4.6	No
4.1	No	4.4	No	4.8	No	4.7	No	4.9	No	5.0	No	5.1	No
5.2	No	5.3	No	5.4	No	5.5	No	5.6	No	5.7	No	5.8	No
5.9	No	6.0	No	6.1	No	6.2	No	6.3	No	6.4	No	6.5	No
6.6	No	6.7	No										

When d'/d = 0.04

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.2	Yes	-1.0	Yes	-1.1	Yes	-1.8	Yes	-2.1	Yes	-2.4	Yes	-1.7	Yes
-2.2	Yes	-2.7	Yes	0.7	Yes	1.2	Yes	-1.2	Yes	-1.3	Yes	-0.7	Yes
-0.3	Yes	0.1	Yes	-2.5	Yes	-2.3	Yes	-2.0	Yes	-1.5	Yes	-3.1	Yes
-3.2	Yes	-3.0	Yes	-2.8	Yes	-2.6	Yes	-3.4	Yes	-3.6	Yes	-3.5	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-3.7	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes
-4.4	Yes	-4.5	Yes	-4.6	Yes	-1.4	Yes	-4.7	Yes	-4.8	Yes	-1.9	Yes
-2.9	Yes	-3.3	Yes	-4.9	Yes	-5.0	Yes	-0.5	Yes	-5.1	No	1.1	Yes
0.2	Yes	-0.4	Yes	2.0	Yes	0.4	Yes	-0.9	No	3.0	Yes	2.1	Yes
1.3	No	0.5	No	-0.1	No	-0.8	No	2.3	No	1.4	No	0.6	No
0.0	No	1.7	No	2.4	No	1.5	No	1.9	No	3.1	No	1.6	No
0.8	No	1.0	No	2.5	No	0.9	No	0.3	No	2.6	No	1.8	No
2.7	No	3.2	No	-0.6	No	-1.6	No	2.8	No	2.9	No	3.3	No
-0.2	No	3.4	No	3.5	No	3.6	No	3.7	No	3.8	No	3.9	No
4.0	No	4.1	No	4.2	No	4.3	No	4.4	No				

When d'/d = 0.05

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.5	Yes	-0.9	Yes	-1.6	Yes	-2.0	Yes	-2.2	Yes	-1.0	Yes	-1.5	Yes
-2.4	Yes	0.6	Yes	1.1	Yes	-2.1	Yes	-1.4	Yes	-1.2	Yes	-0.8	Yes
-0.5	Yes	-0.1	Yes	0.3	Yes	-2.6	Yes	-2.5	Yes	-1.7	Yes	-3.1	Yes
-3.2	Yes	-3.0	Yes	-2.8	Yes	-3.3	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.4	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-2.7	Yes	-3.8	Yes
-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes
-5.0	Yes	-0.6	Yes	-1.3	Yes	-1.9	Yes	-2.9	Yes	-5.1	Yes	0.1	Yes
-1.8	Yes	-2.3	Yes	-5.2	Yes	-0.4	Yes	2.1	Yes	1.2	Yes	0.4	Yes
-0.2	Yes	2.7	Yes	2.2	Yes	1.3	Yes	0.5	Yes	-0.7	No	2.0	Yes
2.6	Yes	2.3	No	1.4	No	0.7	No	0.0	No	1.5	No	2.4	No
1.6	No	0.8	No	1.7	No	0.9	No	0.2	No	-0.3	No	1.8	No
1.9	No	-1.1	No	2.8	No	1.0	No	2.9	No	3.0	No		

When d'/d = 0.06

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.8	Yes	-0.8	Yes	-0.7	Yes	-1.5	Yes	-1.8	Yes	-2.0	Yes	-1.3	Yes
-2.1	Yes	0.6	Yes	1.0	Yes	1.5	Yes	-1.7	Yes	-1.4	Yes	-1.2	Yes
-0.9	Yes	-0.6	Yes	-0.2	Yes	0.2	Yes	-2.6	Yes	-2.5	Yes	-2.4	Yes
-2.2	Yes	-3.1	Yes	-3.2	Yes	-3.0	Yes	-2.9	Yes	-2.7	Yes	-3.4	Yes
-3.6	Yes	-3.7	Yes	-3.5	Yes	-3.8	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes
-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-3.3	Yes
-4.9	Yes	-5.0	Yes	-2.8	Yes	-3.9	Yes	-5.1	Yes	-0.4	Yes	-1.1	Yes
1.1	Yes	0.4	Yes	-0.3	Yes	2.1	Yes	1.3	Yes	0.5	Yes	-0.1	Yes
-1.9	Yes	2.6	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.0	Yes	1.9	Yes
2.4	Yes	1.6	Yes	0.8	No	0.1	No	-1.0	No	-1.6	No	1.8	Yes
2.5	No	1.7	No	0.3	No	-2.3	No	0.9	No	2.0	No	-0.5	No
1.2	No	2.2	No										

When d'/d = 0.07

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.1	Yes	-0.6	Yes	-1.4	Yes	-1.6	Yes	-1.8	Yes	-0.7	Yes	-1.1	Yes
-1.5	Yes	0.6	Yes	1.0	Yes	1.5	Yes	-0.9	Yes	-1.2	Yes	-0.5	Yes
-0.2	Yes	0.1	Yes	-2.5	Yes	-2.4	Yes	-2.2	Yes	-2.0	Yes	-1.7	Yes
-3.1	Yes	-3.2	Yes	-3.0	Yes	-2.8	Yes	-2.6	Yes	-3.4	Yes	-3.5	Yes
-3.6	Yes	-3.7	Yes	-3.3	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes
-4.3	Yes	-4.4	Yes	-2.9	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-1.3	Yes
-1.9	Yes	-3.8	Yes	-4.8	Yes	-2.7	Yes	-4.9	Yes	0.3	Yes	-0.3	Yes
-1.0	Yes	-2.1	Yes	1.2	Yes	0.5	Yes	-0.1	Yes	-0.8	Yes	2.2	Yes
1.4	Yes	0.0	Yes	2.4	Yes	1.6	Yes	0.8	Yes	-0.4	Yes	1.8	Yes
2.5	Yes	1.7	Yes	2.0	Yes	1.9	No	1.1	Yes	2.3	No	2.1	No
0.7	No	-2.3	No	0.9	No	1.3	No	0.4	No	0.2	No		

When d'/d = 0.08

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.3	Yes	-0.5	Yes	-0.4	Yes	-1.2	Yes	-1.4	Yes	-1.5	Yes	-0.9	Yes
0.6	Yes	1.0	Yes	1.5	Yes	-0.6	Yes	-1.0	Yes	-1.1	Yes	-0.8	Yes
-0.1	Yes	0.2	Yes	-2.5	Yes	-2.4	Yes	-2.3	Yes	-2.1	Yes	-1.9	Yes
-1.7	Yes	-3.0	Yes	-3.1	Yes	-2.9	Yes	-2.7	Yes	-3.3	Yes	-3.4	Yes
-3.5	Yes	-3.6	Yes	-3.2	Yes	-3.8	Yes	-3.9	Yes	-3.7	Yes	-4.0	Yes
-4.1	Yes	-4.2	Yes	-2.0	Yes	-2.8	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes
-1.3	Yes	-1.8	Yes	-4.6	Yes	-1.6	Yes	-4.7	Yes	0.3	Yes	-0.2	Yes
1.3	Yes	0.0	Yes	2.3	Yes	0.8	Yes	0.1	Yes	2.5	Yes	1.7	Yes
-0.7	Yes	2.1	Yes	1.9	Yes	1.2	Yes	1.4	Yes	-0.3	No	-2.2	No
0.7	Yes	1.6	No	0.4	Yes	0.9	No	2.2	No	1.8	No	1.1	No
2.0	No	0.5	No	-2.6	No								

When d′/d = 0.09

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.6	Yes	-0.4	Yes	-0.2	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-0.7	Yes
-1.0	Yes	0.6	Yes	1.1	Yes	1.6	Yes	-0.3	Yes	-0.6	Yes	-0.8	Yes
-0.1	Yes	0.2	Yes	-2.5	Yes	-2.4	Yes	-2.2	Yes	-2.0	Yes	-1.8	Yes
-1.6	Yes	-3.0	Yes	-2.9	Yes	-2.7	Yes	-2.6	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-2.8	Yes
-3.9	Yes	-4.0	Yes	-1.9	Yes	-2.3	Yes	-4.1	Yes	-4.2	Yes	-1.7	Yes
-2.1	Yes	-4.3	Yes	-4.4	Yes	-0.9	Yes	-1.4	Yes	0.4	Yes	1.4	Yes
0.7	Yes	0.1	Yes	2.4	Yes	0.9	Yes	0.3	Yes	2.3	Yes	2.6	Yes
1.9	Yes	1.2	Yes	0.0	Yes	-0.5	Yes	2.0	Yes	2.1	Yes	0.8	Yes
1.8	Yes	1.7	Yes	0.5	Yes	1.3	No	1.5	No	2.2	No	-1.5	No
1.0	No												

When d′/d = 0.10

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.8	Yes	-0.3	Yes	0.0	Yes	-1.0	Yes	-0.2	Yes	-0.5	Yes	-0.7	Yes
-0.9	Yes	0.7	Yes	1.1	Yes	1.6	Yes	2.2	Yes	-1.3	Yes	0.3	Yes
-2.4	Yes	-2.3	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes	-1.5	Yes	-1.2	Yes
-2.9	Yes	-2.8	Yes	-2.7	Yes	-2.6	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-1.8	Yes
-2.2	Yes	-3.9	Yes	-4.0	Yes	-1.1	Yes	-1.6	Yes	-2.0	Yes	-4.1	Yes
-0.8	Yes	-2.5	Yes	0.5	Yes	1.5	Yes	0.8	Yes	0.2	Yes	2.5	Yes
1.8	Yes	-1.4	Yes	2.7	Yes	2.1	Yes	1.4	Yes	1.9	Yes	2.3	Yes
1.7	Yes	-0.4	Yes	1.3	Yes	2.0	Yes	2.4	Yes	-0.1	Yes	-0.6	Yes
0.6	Yes	0.9	Yes	1.2	Yes	1.0	Yes	0.1	Yes	0.4	Yes		

When d′/d = 0.11

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.9	Yes	-0.1	Yes	0.0	Yes	-0.8	Yes	-0.3	Yes	-0.5	Yes	-0.6	Yes
0.7	Yes	1.2	Yes	1.7	Yes	2.2	Yes	-1.2	Yes	-0.9	Yes	0.4	Yes
-2.3	Yes	-2.2	Yes	-2.0	Yes	-1.8	Yes	-1.6	Yes	-1.3	Yes	-1.0	Yes
-2.9	Yes	-2.8	Yes	-2.7	Yes	-2.6	Yes	-2.5	Yes	-2.1	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-1.7	Yes
-2.4	Yes	-3.7	Yes	-1.5	Yes	-3.8	Yes	-0.2	Yes	-0.7	Yes	-1.9	Yes
0.6	Yes	-0.4	Yes	1.6	Yes	0.9	Yes	-1.4	Yes	2.6	Yes	1.9	Yes
1.3	Yes	0.2	Yes	-1.1	Yes	1.0	Yes	0.5	Yes	1.5	Yes	2.5	Yes
0.8	Yes	0.3	Yes	2.3	Yes	1.1	Yes	0.1	Yes	1.4	Yes	2.0	Yes
1.8	No	2.1	No										

When d'/d = 0.12

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	-0.1	Yes	0.1	Yes	-0.7	Yes	-0.6	Yes	0.0	Yes	-0.3	Yes
-0.4	Yes	0.7	Yes	1.2	Yes	1.8	Yes	2.3	Yes	0.3	Yes	-1.2	Yes
-0.9	Yes	-0.5	Yes	-0.2	Yes	0.5	Yes	-2.3	Yes	-2.1	Yes	-1.9	Yes
-1.7	Yes	-1.5	Yes	-2.8	Yes	-2.7	Yes	-2.6	Yes	-2.5	Yes	-2.9	Yes
-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-2.0	Yes	-3.4	Yes	-1.3	Yes
-3.5	Yes	-1.0	Yes	-1.4	Yes	0.2	Yes	-1.1	Yes	1.6	Yes	1.1	Yes
-1.8	Yes	2.7	Yes	2.0	Yes	1.4	Yes	0.9	Yes	0.4	Yes	-0.8	Yes
2.1	Yes	2.5	Yes	2.4	Yes	1.3	Yes	1.5	Yes	2.2	Yes	0.6	Yes
1.0	Yes	1.9	Yes	0.8	Yes	1.7	Yes	-1.6	No				

When d'/d = 0.13

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.2	Yes	0.0	Yes	0.3	Yes	-0.6	Yes	-0.5	Yes	-0.4	Yes	-0.1	Yes
0.8	Yes	1.3	Yes	1.8	Yes	2.4	Yes	2.9	Yes	-1.1	Yes	-0.8	Yes
0.2	Yes	0.6	Yes	-2.2	Yes	-2.1	Yes	-1.9	Yes	-1.6	Yes	-1.4	Yes
-2.7	Yes	-2.6	Yes	-2.5	Yes	-2.3	Yes	-2.0	Yes	-1.8	Yes	-2.8	Yes
-2.9	Yes	-2.4	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-0.9	Yes	-1.2	Yes
-1.5	Yes	1.7	Yes	1.2	Yes	0.7	Yes	-1.7	Yes	2.8	Yes	2.2	Yes
1.6	Yes	1.1	Yes	2.1	Yes	2.5	Yes	2.6	Yes	2.0	Yes	1.5	Yes
1.0	Yes	0.1	Yes	1.9	Yes	1.4	Yes	0.9	Yes	0.5	Yes	-0.2	Yes
0.4	Yes	-1.3	Yes	2.3	Yes	-1.0	No	-0.3	No				

When d'/d = 0.14

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.3	Yes	0.0	Yes	0.4	Yes	-0.5	Yes	-0.4	Yes	-0.3	Yes	0.1	Yes
0.8	Yes	1.3	Yes	1.9	Yes	2.4	Yes	3.0	Yes	-1.1	Yes	-0.8	Yes
0.3	Yes	0.7	Yes	-2.2	Yes	-2.0	Yes	-1.8	Yes	-1.5	Yes	-1.3	Yes
-1.0	Yes	-0.7	Yes	-2.7	Yes	-2.6	Yes	-2.5	Yes	-2.4	Yes	-1.6	Yes
-2.8	Yes	-2.1	Yes	-2.3	Yes	-2.9	Yes	-1.4	Yes	-1.7	Yes	-1.9	Yes
-0.6	Yes	-1.2	Yes	1.8	Yes	-0.2	Yes	-0.9	Yes	2.8	Yes	2.3	Yes
2.1	Yes	2.7	Yes	2.2	Yes	1.7	Yes	1.5	Yes	2.6	Yes	1.2	Yes
0.5	Yes	-0.1	Yes	0.9	Yes	1.4	Yes	2.0	Yes	2.5	Yes	1.6	Yes
0.6	Yes	1.0	Yes	0.2	Yes	1.1	No						

When d'/d = 0.15

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.4	Yes	0.1	Yes	0.4	Yes	-0.4	Yes	-0.3	Yes	-0.1	Yes	0.2	Yes
0.8	Yes	1.4	Yes	1.9	Yes	2.5	Yes	3.0	Yes	-1.1	Yes	-0.7	Yes
0.0	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes	-1.4	Yes	-0.8	Yes	-0.5	Yes
-2.6	Yes	-2.5	Yes	-2.4	Yes	-2.3	Yes	-1.5	Yes	-2.7	Yes	-2.2	Yes
-2.0	Yes	-1.0	Yes	-1.3	Yes	-0.2	Yes	-0.6	Yes	-0.9	Yes	-1.6	Yes
-1.8	Yes	0.9	Yes	0.5	Yes	-1.2	Yes	1.0	Yes	0.6	Yes	2.9	Yes
2.4	Yes	1.5	Yes	1.1	Yes	0.7	Yes	0.3	Yes	2.1	Yes	2.8	Yes
1.7	Yes	2.0	Yes	2.3	Yes	2.7	Yes	1.2	Yes	2.6	Yes	1.6	Yes
2.2	Yes	1.3	Yes	1.8	Yes								

When d'/d = 0.16

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.5	Yes	0.1	Yes	0.5	Yes	-0.4	Yes	-0.2	Yes	0.0	Yes	0.3	Yes
0.4	Yes	0.8	Yes	1.4	Yes	1.9	Yes	2.5	Yes	3.1	Yes	-1.0	Yes
-0.7	Yes	-0.3	Yes	0.9	Yes	-2.1	Yes	-1.9	Yes	-1.6	Yes	-1.3	Yes
-1.1	Yes	-2.6	Yes	-2.5	Yes	-2.3	Yes	-2.2	Yes	-2.0	Yes	-1.8	Yes
-1.5	Yes	-2.7	Yes	-2.4	Yes	-1.4	Yes	-0.6	Yes	-0.9	Yes	-1.7	Yes
-0.1	Yes	1.0	Yes	0.6	Yes	-0.5	Yes	2.0	Yes	1.5	Yes	1.1	Yes
-0.8	Yes	2.8	Yes	2.1	Yes	1.7	Yes	1.3	Yes	2.4	Yes	2.7	Yes
2.6	Yes	2.2	Yes	1.8	Yes	0.7	Yes	2.3	Yes	1.2	Yes	1.6	Yes
0.2	Yes	-1.2	No										

When d'/d = 0.17

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.2	Yes	0.1	Yes	0.5	Yes	-0.3	Yes	-0.1	Yes	0.0	Yes	0.4	Yes
0.6	Yes	0.9	Yes	1.4	Yes	2.0	Yes	2.5	Yes	3.1	Yes	-1.0	Yes
-0.6	Yes	-0.2	Yes	-2.1	Yes	-1.8	Yes	-1.5	Yes	-1.3	Yes	-0.7	Yes
-2.5	Yes	-2.4	Yes	-2.2	Yes	-1.9	Yes	-1.6	Yes	-1.4	Yes	-1.2	Yes
-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-0.8	Yes	0.2	Yes	-0.5	Yes
-0.9	Yes	-1.1	Yes	1.1	Yes	0.7	Yes	2.1	Yes	1.7	Yes	1.3	Yes
1.0	Yes	2.8	Yes	2.6	Yes	2.2	Yes	1.8	Yes	1.5	Yes	0.8	Yes
0.3	Yes	-0.4	Yes	2.4	Yes	2.7	Yes	2.3	Yes	2.9	Yes	1.2	Yes
1.6	Yes	1.9	Yes										

When d'/d = 0.18

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	0.1	Yes	0.5	Yes	-0.3	Yes	0.0	Yes	0.6	Yes	0.7	Yes
0.8	Yes	0.9	Yes	1.4	Yes	2.0	Yes	2.5	Yes	3.1	Yes	-1.0	Yes
-0.6	Yes	-0.2	Yes	1.0	Yes	-2.0	Yes	-1.8	Yes	-1.5	Yes	-1.2	Yes
-0.9	Yes	-2.5	Yes	-2.3	Yes	-2.2	Yes	-1.3	Yes	-1.1	Yes	-2.4	Yes
-1.9	Yes	-1.7	Yes	-2.1	Yes	-1.4	Yes	-1.6	Yes	-0.5	Yes	-0.7	Yes
0.2	Yes	-0.4	Yes	1.2	Yes	0.3	Yes	-0.1	Yes	2.1	Yes	1.8	Yes
1.1	Yes	-0.8	Yes	2.8	Yes	2.4	Yes	1.7	Yes	0.4	Yes	2.3	Yes
2.7	Yes	3.0	Yes	2.6	Yes	2.2	Yes	1.9	Yes	1.6	Yes	1.3	Yes
1.5	Yes	2.9	Yes										

When d'/d = 0.19

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.0	Yes	0.1	Yes	0.5	Yes	-0.2	Yes	0.0	Yes	0.2	Yes	0.6	Yes
0.8	Yes	0.9	Yes	1.4	Yes	2.0	Yes	2.5	Yes	3.1	Yes	-1.0	Yes
-0.6	Yes	1.0	Yes	-2.0	Yes	-1.7	Yes	-1.4	Yes	-1.1	Yes	-0.8	Yes
-0.5	Yes	-2.5	Yes	-2.3	Yes	-2.1	Yes	-1.9	Yes	-1.2	Yes	-0.9	Yes
-2.4	Yes	-2.2	Yes	-1.6	Yes	-1.8	Yes	-1.3	Yes	-1.5	Yes	-0.4	Yes
0.3	Yes	-0.7	Yes	1.2	Yes	0.7	Yes	0.4	Yes	-0.3	Yes	2.2	Yes
1.9	Yes	1.6	Yes	1.3	Yes	2.8	Yes	2.9	Yes	1.1	Yes	-0.1	Yes
2.3	Yes	2.6	Yes	3.0	Yes	2.7	Yes	2.4	Yes	1.7	Yes	1.5	Yes
1.8	Yes	2.1	Yes	3.2	Yes								

When d'/d = 0.20

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.9	Yes	0.1	Yes	0.5	Yes	-0.2	Yes	0.0	Yes	0.2	Yes	0.6	Yes
0.7	Yes	0.9	Yes	1.0	Yes	1.4	Yes	2.0	Yes	2.5	Yes	3.1	Yes
-0.9	Yes	-0.5	Yes	-0.1	Yes	-2.0	Yes	-1.7	Yes	-1.4	Yes	-1.1	Yes
-0.8	Yes	-0.4	Yes	-2.4	Yes	-2.2	Yes	-1.8	Yes	-1.6	Yes	-1.3	Yes
-2.5	Yes	-2.1	Yes	-1.2	Yes	-1.9	Yes	-1.5	Yes	-0.7	Yes	-1.0	Yes
0.4	Yes	-0.6	Yes	1.3	Yes	0.8	Yes	-0.3	Yes	2.3	Yes	1.7	Yes
1.2	Yes	0.3	Yes	2.8	Yes	3.0	Yes	2.7	Yes	2.4	Yes	2.1	Yes
1.8	Yes	1.6	Yes	2.6	Yes	2.9	Yes	2.2	Yes	1.9	Yes	1.5	Yes
1.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	-2.3	No				

When d'/d = 0.21

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.8	Yes	0.1	Yes	0.5	Yes	-0.2	Yes	0.0	Yes	0.2	Yes	0.6	Yes
0.8	Yes	0.9	Yes	1.1	Yes	1.4	Yes	1.9	Yes	2.5	Yes	3.0	Yes
-0.9	Yes	-0.5	Yes	-0.1	Yes	1.0	Yes	-1.9	Yes	-1.7	Yes	-1.4	Yes
-1.0	Yes	-0.7	Yes	-0.4	Yes	-2.4	Yes	-2.2	Yes	-2.0	Yes	-1.5	Yes
-1.3	Yes	-0.8	Yes	-2.3	Yes	-1.1	Yes	-2.1	Yes	-1.8	Yes	-1.6	Yes
-1.2	Yes	-0.3	Yes	-0.6	Yes	0.4	Yes	0.3	Yes	0.7	Yes	2.4	Yes
2.1	Yes	1.8	Yes	1.6	Yes	1.2	Yes	2.8	Yes	3.1	Yes	2.3	Yes
2.0	Yes	1.3	Yes	2.6	Yes	2.9	Yes	3.2	Yes	2.7	Yes	2.2	Yes
1.7	Yes	1.5	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-2.5	No	-2.6	No	-2.7	No	-2.8	No						

When d'/d = 0.22

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.6	Yes	0.0	Yes	0.4	Yes	-0.2	Yes	0.2	Yes	0.7	Yes	0.8	Yes
1.0	Yes	1.2	Yes	0.9	Yes	1.4	Yes	1.9	Yes	2.4	Yes	3.0	Yes
-0.9	Yes	-0.5	Yes	-0.1	Yes	0.6	Yes	-1.9	Yes	-1.6	Yes	-1.3	Yes
-1.0	Yes	-0.7	Yes	-0.4	Yes	-2.4	Yes	-2.1	Yes	-1.7	Yes	-1.5	Yes
-1.2	Yes	-2.3	Yes	-1.8	Yes	-1.4	Yes	-2.0	Yes	-1.1	Yes	-0.3	Yes
-0.6	Yes	0.5	Yes	0.3	Yes	0.1	Yes	2.2	Yes	2.0	Yes	1.8	Yes
1.6	Yes	1.1	Yes	2.7	Yes	3.1	Yes	2.9	Yes	1.7	Yes	1.5	Yes
1.3	Yes	2.5	Yes	2.8	Yes	2.3	Yes	2.1	Yes	2.6	Yes	3.2	Yes
3.3	Yes	3.5	Yes	3.4	Yes	3.7	Yes	3.8	Yes	-0.8	Yes	-2.2	No
-2.5	No	-2.6	No	-2.7	No	-2.8	No	-2.9	No	-3.0	No		

When d'/d = 0.23

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.5	Yes	0.0	Yes	0.3	Yes	-0.2	Yes	0.2	Yes	0.7	Yes	0.8	Yes
1.0	Yes	1.2	Yes	1.4	Yes	1.9	Yes	2.4	Yes	2.9	Yes	-0.9	Yes
-0.5	Yes	-0.1	Yes	0.6	Yes	-1.9	Yes	-1.6	Yes	-1.3	Yes	-1.0	Yes
-0.7	Yes	-0.3	Yes	-2.3	Yes	-2.1	Yes	-1.7	Yes	-1.4	Yes	-1.2	Yes
-0.6	Yes	-2.4	Yes	-2.2	Yes	-2.0	Yes	-1.5	Yes	-1.1	Yes	-1.8	Yes
-0.8	Yes	-0.4	Yes	0.4	Yes	0.1	Yes	1.5	Yes	1.3	Yes	0.9	Yes
0.5	Yes	2.5	Yes	2.3	Yes	2.1	Yes	1.7	Yes	1.6	Yes	1.1	Yes
2.7	Yes	3.0	Yes	2.8	Yes	2.6	Yes	2.0	Yes	1.8	Yes	2.2	Yes
3.1	Yes	3.3	Yes	3.2	Yes	3.4	Yes	3.6	Yes	3.8	Yes	3.7	Yes
3.9	Yes	4.1	Yes	-2.5	No	-2.6	No	-2.7	No	-2.8	No	-2.9	No
-3.0	No	-3.1	No	-3.2	No								

When d'/d = 0.24

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.3	Yes	0.0	Yes	0.2	Yes	-0.2	Yes	0.7	Yes	0.9	Yes	1.0	Yes
1.2	Yes	0.8	Yes	1.3	Yes	1.8	Yes	2.3	Yes	2.8	Yes	-0.9	Yes
-0.5	Yes	-0.1	Yes	0.6	Yes	-1.9	Yes	-1.6	Yes	-1.3	Yes	-1.0	Yes
-0.7	Yes	-0.3	Yes	-2.3	Yes	-2.1	Yes	-1.4	Yes	-1.1	Yes	-0.6	Yes
-2.2	Yes	-2.0	Yes	-1.8	Yes	-1.5	Yes	-1.7	Yes	-1.2	Yes	-0.8	Yes
0.5	Yes	0.4	Yes	0.3	Yes	0.1	Yes	1.5	Yes	1.4	Yes	1.1	Yes
2.5	Yes	2.4	Yes	2.2	Yes	2.0	Yes	1.9	Yes	1.6	Yes	2.7	Yes
3.0	Yes	3.2	Yes	2.6	Yes	2.1	Yes	1.7	Yes	3.4	Yes	2.9	Yes
3.1	Yes	3.7	Yes	3.5	Yes	3.8	Yes	4.0	Yes	3.9	Yes	3.6	Yes
4.2	Yes	4.1	Yes	-0.4	Yes	-2.4	No	-2.5	No	-2.6	No	-2.7	No
-2.8	No	-2.9	No	-3.0	No	-3.1	No	-3.2	No	-3.3	No	-3.4	No

When d'/d = 0.25

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.0	Yes	-0.1	Yes	0.1	Yes	-0.2	Yes	0.0	Yes	0.7	Yes	0.9	Yes
1.0	Yes	1.2	Yes	0.8	Yes	1.3	Yes	1.7	Yes	2.2	Yes	2.7	Yes
-0.9	Yes	-0.6	Yes	0.5	Yes	-1.9	Yes	-1.6	Yes	-1.3	Yes	-1.0	Yes
-0.7	Yes	-0.4	Yes	-2.3	Yes	-2.1	Yes	-1.8	Yes	-1.1	Yes	-0.8	Yes
-2.0	Yes	-1.4	Yes	-1.2	Yes	-1.7	Yes	-1.5	Yes	-0.5	Yes	-0.3	Yes
0.6	Yes	0.4	Yes	0.3	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.1	Yes
2.6	Yes	2.4	Yes	2.3	Yes	2.0	Yes	1.9	Yes	1.8	Yes	3.3	Yes
3.1	Yes	2.9	Yes	2.8	Yes	2.5	Yes	2.1	Yes	3.5	Yes	3.4	Yes
3.6	Yes	3.8	Yes	3.7	Yes	3.2	Yes	3.9	Yes	4.1	Yes	4.3	Yes
4.2	Yes	0.2	Yes	-2.2	Yes	-2.4	No	-2.5	No	-2.6	No	-2.7	No
-2.8	No	-2.9	No	-3.0	No	-3.1	No	-3.2	No	-3.3	No	-3.4	No
-3.5	No	-3.6	No										

When d'/d = 0.26

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.8	Yes	-0.2	Yes	0.0	Yes	-0.3	Yes	-0.1	Yes	0.7	Yes	0.8	Yes
1.0	Yes	1.2	Yes	1.7	Yes	2.1	Yes	2.6	Yes	-1.0	Yes	-0.6	Yes
0.1	Yes	0.4	Yes	-1.9	Yes	-1.6	Yes	-1.3	Yes	-0.7	Yes	-0.4	Yes
-2.3	Yes	-2.1	Yes	-1.8	Yes	-1.1	Yes	-0.8	Yes	-1.7	Yes	-1.4	Yes
-1.2	Yes	-2.0	Yes	-1.5	Yes	-0.9	Yes	-0.5	Yes	0.6	Yes	0.5	Yes
1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.1	Yes	2.7	Yes	2.5	Yes
2.4	Yes	2.3	Yes	2.2	Yes	2.0	Yes	1.9	Yes	1.8	Yes	3.0	Yes
3.3	Yes	3.2	Yes	3.1	Yes	3.5	Yes	3.6	Yes	3.4	Yes	2.9	Yes
3.8	Yes	4.0	Yes	3.9	Yes	3.7	Yes	0.9	Yes	4.3	Yes	4.2	Yes
4.1	Yes	4.5	Yes	0.2	Yes	0.3	Yes	-2.2	Yes	-2.4	Yes	-2.5	Yes
-2.6	Yes	-2.7	No	-2.8	No	-2.9	No	-3.0	No	-3.1	No	-3.2	No
-3.3	No	-3.4	No	-3.5	No	-3.6	No	-3.7	No				

When d'/d = 0.27

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.6	Yes	-0.3	Yes	-0.2	Yes	0.0	Yes	0.7	Yes	0.8	Yes	1.0	Yes
1.1	Yes	1.6	Yes	2.0	Yes	2.4	Yes	-1.0	Yes	-0.6	Yes	0.4	Yes
-1.9	Yes	-1.6	Yes	-1.3	Yes	-0.7	Yes	-0.4	Yes	-0.1	Yes	-2.3	Yes
-2.1	Yes	-1.8	Yes	-1.1	Yes	-0.8	Yes	-1.7	Yes	-1.5	Yes	-0.9	Yes
-2.0	Yes	-1.4	Yes	-1.2	Yes	-0.5	Yes	0.1	Yes	0.2	Yes	0.3	Yes
0.6	Yes	1.7	Yes	1.5	Yes	1.4	Yes	1.3	Yes	2.7	Yes	2.5	Yes
2.3	Yes	2.2	Yes	2.1	Yes	1.9	Yes	2.9	Yes	3.2	Yes	3.3	Yes
3.1	Yes	3.0	Yes	2.8	Yes	3.5	Yes	3.7	Yes	3.6	Yes	3.4	Yes
3.9	Yes	4.1	Yes	4.0	Yes	3.8	Yes	0.9	Yes	1.2	Yes	1.8	Yes
4.4	Yes	4.5	Yes	4.3	Yes	0.5	Yes	4.6	Yes	-2.2	Yes	-2.4	Yes
-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	No	-3.1	No
-3.2	No	-3.3	No	-3.4	No	-3.5	No	-3.6	No	-3.7	No	-3.8	No
-3.9	No												

When d'/d = 0.28

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.3	Yes	-0.4	Yes	-0.3	Yes	-0.1	Yes	0.6	Yes	0.8	Yes	0.9	Yes
1.1	Yes	0.7	Yes	1.5	Yes	1.9	Yes	-1.0	Yes	-0.7	Yes	0.0	Yes
0.3	Yes	-1.9	Yes	-1.6	Yes	-1.4	Yes	-1.1	Yes	-0.8	Yes	-0.5	Yes
-0.2	Yes	-2.3	Yes	-2.1	Yes	-1.8	Yes	-1.3	Yes	-0.9	Yes	-0.6	Yes
-1.7	Yes	-1.5	Yes	-2.0	Yes	-1.2	Yes	0.1	Yes	0.2	Yes	0.4	Yes
1.0	Yes	1.7	Yes	1.6	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes
2.2	Yes	2.1	Yes	2.9	Yes	3.2	Yes	3.5	Yes	3.4	Yes	3.3	Yes
3.1	Yes	3.0	Yes	2.8	Yes	3.7	Yes	3.9	Yes	3.8	Yes	3.6	Yes
1.3	Yes	4.1	Yes	4.4	Yes	4.3	Yes	4.2	Yes	4.0	Yes	1.2	Yes
4.5	Yes	4.8	Yes	4.7	Yes	0.5	Yes	1.4	Yes	1.8	Yes	2.0	Yes
-2.2	Yes	-2.5	Yes	-2.4	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes
-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	No	-3.4	No	-3.5	No	-3.6	No
-3.7	No	-3.8	No	-3.9	No	-4.0	No	-4.1	No				

When d'/d = 0.29

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.0	Yes	-0.6	Yes	-0.5	Yes	-0.4	Yes	-0.3	Yes	0.6	Yes	0.7	Yes
0.8	Yes	1.0	Yes	1.4	Yes	1.7	Yes	2.1	Yes	-1.0	Yes	-0.7	Yes
-0.1	Yes	0.2	Yes	0.5	Yes	-2.0	Yes	-1.7	Yes	-1.4	Yes	-1.1	Yes
-0.8	Yes	-2.3	Yes	-2.1	Yes	-1.8	Yes	-1.6	Yes	-0.9	Yes	-1.9	Yes
-1.5	Yes	-1.3	Yes	-1.2	Yes	-0.2	Yes	0.0	Yes	0.1	Yes	0.3	Yes
0.4	Yes	0.9	Yes	1.1	Yes	1.8	Yes	1.6	Yes	2.8	Yes	2.7	Yes
2.6	Yes	2.5	Yes	2.4	Yes	2.9	Yes	3.2	Yes	3.4	Yes	3.5	Yes
3.3	Yes	3.1	Yes	3.0	Yes	1.9	Yes	2.3	Yes	3.6	Yes	3.8	Yes
4.1	Yes	4.0	Yes	3.9	Yes	3.7	Yes	1.3	Yes	1.5	Yes	4.2	Yes
4.5	Yes	4.4	Yes	1.2	Yes	4.6	Yes	4.9	Yes	2.2	Yes	-2.2	Yes
-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	No	-3.6	No	-3.7	No
-3.8	No	-3.9	No	-4.0	No	-4.1	No	-4.2	No	-4.3	No		

When d'/d = 0.30

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.7	Yes	-0.8	Yes	-0.6	Yes	-0.5	Yes	0.5	Yes	0.6	Yes	0.7	Yes
0.8	Yes	0.9	Yes	1.2	Yes	1.6	Yes	1.9	Yes	-1.1	Yes	-0.2	Yes
0.0	Yes	0.3	Yes	-2.0	Yes	-1.7	Yes	-1.4	Yes	-1.2	Yes	-0.9	Yes
-0.7	Yes	-0.4	Yes	-2.3	Yes	-2.1	Yes	-1.9	Yes	-1.6	Yes	-1.0	Yes
-1.5	Yes	-1.3	Yes	-1.8	Yes	-0.3	Yes	-0.1	Yes	0.1	Yes	0.2	Yes
0.4	Yes	1.0	Yes	1.1	Yes	1.3	Yes	1.8	Yes	2.8	Yes	2.7	Yes
2.6	Yes	2.9	Yes	3.1	Yes	3.4	Yes	3.6	Yes	3.5	Yes	3.3	Yes
2.1	Yes	2.3	Yes	2.5	Yes	3.0	Yes	3.8	Yes	4.0	Yes	4.2	Yes
4.1	Yes	1.5	Yes	3.9	Yes	4.4	Yes	4.6	Yes	4.8	Yes	4.7	Yes
2.0	Yes	2.2	Yes	2.4	Yes	3.7	Yes	3.2	Yes	1.4	Yes	-2.2	Yes
-2.5	Yes	-2.4	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	No
-3.8	No	-3.9	No	-4.0	No	-4.1	No	-4.2	No	-4.3	No	-4.4	No

When d'/d = 0.31

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.3	Yes	-0.9	Yes	-1.1	Yes	-0.7	Yes	0.5	Yes	0.6	Yes	0.7	Yes
0.8	Yes	1.1	Yes	1.4	Yes	1.7	Yes	-0.6	Yes	-0.3	Yes	0.0	Yes
0.2	Yes	-2.0	Yes	-1.8	Yes	-1.5	Yes	-1.3	Yes	-1.0	Yes	-0.8	Yes
-0.5	Yes	-2.3	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes	-1.2	Yes	-1.4	Yes
-1.6	Yes	-0.4	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.3	Yes	0.4	Yes
0.9	Yes	1.0	Yes	1.2	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes	3.1	Yes	3.3	Yes	3.6	Yes
3.7	Yes	3.5	Yes	2.3	Yes	2.5	Yes	3.0	Yes	3.2	Yes	3.4	Yes
3.9	Yes	4.2	Yes	4.4	Yes	4.3	Yes	1.5	Yes	1.6	Yes	2.2	Yes
2.4	Yes	4.0	Yes	4.7	Yes	4.9	Yes	5.0	Yes	3.8	Yes	4.5	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes
-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes
-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	No	-4.1	No	-4.2	No	-4.3	No
-4.4	No	-4.5	No	-4.6	No								

When d'/d = 0.32

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.0	Yes	-1.1	Yes	-1.4	Yes	-0.8	Yes	-0.9	Yes	0.4	Yes	0.5	Yes
0.7	Yes	0.9	Yes	1.2	Yes	1.5	Yes	-1.2	Yes	-0.7	Yes	-0.4	Yes
-0.2	Yes	0.0	Yes	-2.0	Yes	-1.8	Yes	-1.6	Yes	-1.3	Yes	-2.3	Yes
-2.1	Yes	-1.9	Yes	-1.7	Yes	-1.5	Yes	-1.0	Yes	-0.6	Yes	-0.3	Yes
-0.5	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.6	Yes	0.8	Yes
1.1	Yes	1.3	Yes	1.4	Yes	1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes
2.3	Yes	2.9	Yes	2.8	Yes	3.0	Yes	2.6	Yes	3.3	Yes	3.5	Yes
3.8	Yes	3.7	Yes	2.5	Yes	2.7	Yes	3.1	Yes	3.6	Yes	4.0	Yes
4.3	Yes	4.5	Yes	1.6	Yes	1.8	Yes	2.4	Yes	3.2	Yes	3.4	Yes
4.1	Yes	4.7	Yes	5.0	Yes	3.9	Yes	1.7	Yes	-2.2	Yes	-2.4	Yes
-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes
-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes
-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	No	-4.4	No	-4.5	No
-4.6	No	-4.7	No										

When d'/d = 0.33

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.6	Yes	-1.3	Yes	-1.7	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	0.3	Yes
0.5	Yes	0.8	Yes	1.0	Yes	1.2	Yes	-1.2	Yes	-0.8	Yes	-0.6	Yes
-0.3	Yes	-0.1	Yes	-2.1	Yes	-1.9	Yes	-1.4	Yes	-2.4	Yes	-2.2	Yes
-2.0	Yes	-1.8	Yes	-1.6	Yes	-2.3	Yes	-1.5	Yes	-0.7	Yes	-0.4	Yes
-0.2	Yes	-0.5	Yes	0.0	Yes	0.1	Yes	0.2	Yes	0.4	Yes	0.7	Yes
0.9	Yes	1.1	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.9	Yes
2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes	2.9	Yes	3.0	Yes
3.1	Yes	2.6	Yes	2.8	Yes	3.2	Yes	3.5	Yes	3.7	Yes	3.9	Yes
1.8	Yes	3.3	Yes	4.1	Yes	4.4	Yes	4.6	Yes	4.7	Yes	2.5	Yes
2.7	Yes	4.3	Yes	4.8	Yes	1.7	Yes	3.4	Yes	-2.5	Yes	-2.6	Yes
-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes
-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes
-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	No	-4.6	No	-4.7	No
-4.8	No												

When d'/d = 0.34

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.2	Yes	-1.6	Yes	-2.0	Yes	-1.1	Yes	-1.2	Yes	-1.4	Yes	0.1	Yes
0.4	Yes	0.6	Yes	0.8	Yes	1.0	Yes	-1.3	Yes	-0.9	Yes	-0.7	Yes
-0.5	Yes	-0.3	Yes	-2.1	Yes	-1.9	Yes	-1.7	Yes	-1.0	Yes	-2.4	Yes
-2.2	Yes	-1.5	Yes	-2.3	Yes	-1.8	Yes	-0.8	Yes	-0.6	Yes	-0.4	Yes
-0.2	Yes	-0.1	Yes	0.0	Yes	0.3	Yes	0.5	Yes	0.7	Yes	0.9	Yes
1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.7	Yes
1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	2.8	Yes	3.4	Yes	3.6	Yes
3.9	Yes	4.0	Yes	4.1	Yes	1.8	Yes	2.6	Yes	3.8	Yes	4.2	Yes
4.5	Yes	4.7	Yes	4.9	Yes	4.4	Yes	4.6	Yes	2.7	Yes	3.7	Yes
2.9	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	No	-4.9	No	-5.0	No		

When d'/d = 0.35

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-0.1	Yes	-1.8	Yes	-2.3	Yes	-1.2	Yes	-1.5	Yes	-1.7	Yes	0.1	Yes
0.0	Yes	0.2	Yes	0.4	Yes	0.5	Yes	0.7	Yes	-1.4	Yes	-1.1	Yes
-0.9	Yes	-0.7	Yes	-0.6	Yes	-2.2	Yes	-2.0	Yes	-2.4	Yes	-2.1	Yes
-1.6	Yes	-1.3	Yes	-1.9	Yes	-1.0	Yes	-0.8	Yes	-0.5	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	0.3	Yes	0.6	Yes	0.8	Yes	0.9	Yes	1.0	Yes
1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.7	Yes
1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes
2.5	Yes	2.6	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes
2.7	Yes	2.9	Yes	3.6	Yes	3.8	Yes	4.0	Yes	4.1	Yes	4.2	Yes
4.3	Yes	3.5	Yes	3.7	Yes	3.9	Yes	4.5	Yes	4.7	Yes	4.9	Yes
5.1	Yes	4.4	Yes	2.8	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes
-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes
-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes
-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes
-5.0	Yes	-5.1	No										

When d'/d = 0.36

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-0.6	Yes	-2.0	Yes	-2.7	Yes	-1.4	Yes	-1.7	Yes	0.0	Yes	-0.1	Yes
-0.3	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes	-1.5	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-1.0	Yes	-0.8	Yes	-2.2	Yes	-2.1	Yes	-1.8	Yes
-2.5	Yes	-2.3	Yes	-1.9	Yes	-1.6	Yes	-2.4	Yes	-0.9	Yes	-0.5	Yes
-0.4	Yes	-0.7	Yes	-0.2	Yes	0.5	Yes	0.6	Yes	0.7	Yes	0.8	Yes
0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes
1.6	Yes	1.7	Yes	1.8	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes
2.4	Yes	2.5	Yes	2.6	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes
3.4	Yes	3.5	Yes	2.7	Yes	2.9	Yes	3.7	Yes	3.9	Yes	4.1	Yes
4.2	Yes	4.3	Yes	4.4	Yes	2.8	Yes	3.6	Yes	3.8	Yes	4.5	Yes
4.7	Yes	4.9	Yes	4.0	Yes	1.9	Yes	-2.6	Yes	-2.8	Yes	-2.9	Yes
-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes
-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes
-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes
-5.1	Yes	-5.2	Yes										

When d'/d = 0.37

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-1.0	Yes	-2.3	Yes	-3.1	Yes	-1.6	Yes	-1.9	Yes	0.0	Yes	-0.2	Yes
-0.4	Yes	-0.5	Yes	0.1	Yes	-1.5	Yes	-1.4	Yes	-1.3	Yes	-1.2	Yes
-1.1	Yes	-2.2	Yes	-2.1	Yes	-2.0	Yes	-1.7	Yes	-2.5	Yes	-2.4	Yes
-1.8	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.3	Yes	-0.1	Yes
0.2	Yes	0.3	Yes	0.4	Yes	0.5	Yes	0.6	Yes	0.7	Yes	0.8	Yes
0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes
1.6	Yes	1.7	Yes	1.8	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes
2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	3.0	Yes	3.1	Yes	3.2	Yes
3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	2.8	Yes	3.8	Yes	4.0	Yes
4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes	1.9	Yes	2.9	Yes	4.7	Yes
3.7	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes
-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	Yes		

Max NPb/b' is 55.17

The followings are printed 'only as typical cases ' that are used in the examples to illustrate the computaions involved in the program :

When NP = 2.00% , NP' = 2.00% and d'/d = 0.030 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.97	5.76	1.77	1.020	0.97

When NP = 15.00% , NP' = 10.00% and d'/d = 0.240 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.81	2.04	12.93	1.002	0.97

When NP = 25.00% , NP' = 25.00% and d'/d = 0.160 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.86	1.04	14.95	1.007	0.83

When NP = 40.00% , NP' = 40.00% and d'/d = 0.030 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.97	0.41	11.21	0.974	0.62

When NP = 45.00% , NP' = 35.00% and d'/d = 0.370 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.73	1.36	38.30	0.972	0.91

APPENDIX B5

The Listing of Prog.3.3.3m and its Output

PROG.3.3.3m

This program is same as PROG.3.3.3 except that np and np′ are now incremented by 0.01%.

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION SAND(35,300),error(35,300),M(35),ID(35,300),IDerr(35,300
     *),DR(10),DEPTH(10),RIGICR(10),RICRe(10),RNPCO(10),IFLAG(10),JFLAG(
     *10),INDEX(35)
      CHARACTER SAND*5
      REAL NP,NPC,NWRICR,NEWNP,NPe,MODNP(64),IFLAG,JFLAG
      OPEN(3,FILE='OUTPUT',STATUS='UNKNOWN')
C The program will now read np , np' and d'/d . Any section that is
C found unductile is ignored.
      NPe=0.0
      DO 15 I=1,300
      DO 10 K=1,35
      M(K)=0
      ID(K,I)=0
      error(K,I)=0
      INDEX(K)=0
   10 CONTINUE
   15 CONTINUE
      WRITE(*,'(1X,A)')'How many cases are to be given as examples (max.
     * of 10)?'
      READ*,MXCASE
      WRITE(*,'(1X,A/8X,A,7(/12X,A))')'For each case please give np,np''
     * and d''/d . Enter one combination per line','Notes :','1.np shoul
     *d always be greater than or equal to np''','2.Subject  to  the con
     *dition above , np and np'' can',' have any value (in an icrement
     *of 0.01) from 2 to 64','3.d''/d should be  chosen using icrements
     *of 0.01 in',' the range from 0.03 to 0.37 . Values outside this',
     *' range must not be used'
      DO 16 ICASE=1,MXCASE
      READ*,IFLAG(ICASE),JFLAG(ICASE),DR(ICASE)
   16 CONTINUE
      WRITE(*,'(A)')'*==================== The program is running pleas
     *e wait ====================*'
      DO 50 I=200,6400
      NP=I/10000.
      DO 40 J=200,6400
      NPC=J/10000.
      IF(NPC.GT.NP) GO TO 40
      IF((100.*NP).GT.((32.0+(100.*NPC)))) THEN
C*****This section is not ductile and thus will not be considered .
      GO TO 40
      END IF
      DO 30 K=1,35
      IF(K.EQ.1) THEN
      DRATIO = 0.03
      ELSE IF(K.EQ.8) THEN
      DRATIO=0.1
      ELSE
      DRATIO=DRATIO+0.01
      END IF
```

```
C Now that all the parameters have been read  b' will be evaluated  and
C npb/b' will be calculated as NEWNP. The parameters  required  for the
C evaluation of Icre  will also be found :
      APRIME=(3./5000.+(1/20.)*(DRATIO*(1-2*DRATIO)**2.))*100.
      BPRIME=1+APRIME*NPC/DRATIO
      NEWNP=100.*NP/BPRIME
      IF(NEWNP.LE.1.9) THEN
      A=0.003
      B=0.095
      ELSE IF((NEWNP.GT.1.9).AND.(NEWNP.LE.5.0)) THEN
      A=0.05
      B=0.07
      ELSE IF((NEWNP.GT.5.0).AND.(NEWNP.LE.17.0)) THEN
      A=0.16
      B=0.05
      ELSE IF((NEWNP.GT.17.0).AND.(NEWNP.LE.32.0)) THEN
      A=0.50
      B=0.03
      ELSE
      A=0.8
      B=0.02
      END IF
C The program will  now evaluate Icr , Icre and  Icr  when  compression
C reinforcement is ignored (stored in "Icr,np'=0") and will print the
C ratios of the values relative to Icr .
      X=-(NP+NPC)+SQRT((NP+NPC)**2+2*NP+2*NPC*DRATIO)
      XO=-NP+SQRT(NP**2+2*NP)
      XICR=(X**3)/3.+NPC*(X-DRATIO)**2+NP*(1-X)**2
      OICR=(XO**3)/3.+NP*(1-XO)**2
      XIcre=(A+B*NEWNP)*BPRIME/12.
      NWRICR=XIcre/XICR
      ROICR=OICR/XICR
      DH=1/(1+DRATIO)
      XIG=((1+DRATIO)**3)/12
      XIGRAT=XIG/XICR
C*****The following  13  lines relate  only to the  sections  used  in the
C*****examples which are provided to illustrate the computations involoved
C***** in the program :
      DO 17 ICASE=1,MXCASE
      III=NP*100000.
      JJJ=NPC*100000.
      IIFLG=IFLAG(ICASE)*1000.
      JJFLG=JFLAG(ICASE)*1000.
      IF((III.EQ.IIFLG).AND.(JJJ.EQ.JJFLG)) THEN
      IDRAT=DRATIO*1000.
      IDR=DR(ICASE)*1000.
      IF(IDRAT.EQ.IDR) THEN
      INDEX(ICASE)=1
      DEPTH(ICASE)=DH
      RIGICR(ICASE)=XIGRAT
      MODNP(ICASE)=NEWNP
      RICRe(ICASE)=NWRICR
      RNPCO(ICASE)=ROICR
      END IF
      END IF
   17 CONTINUE
```

```
C**********************************************************************C
      Ierror=(NWRICR-1)*1000.
      Ierror=Ierror+100
      NPe=DMAX1(NEWNP,NPe)
      DO 20 IJ=1,300
      IF(Ierror.NE.IJ) GO TO 20
      IF(ID(K,IJ).EQ.0) THEN
      M(K)=M(K)+1
      IDerr(K,IJ)=M(K)
      error(K,M(K))=(Ierror-100)/10.
      IF(XIGRAT.GE.1.0) THEN
      ID(K,IJ)=1
      SAND(K,M(K))='Yes'
      ELSE
      ID(K,IJ)=2
      SAND(K,M(K))='No'
      END IF
      ELSE IF(ID(K,IJ).EQ.2) THEN
      IF(XIGRAT.GE.1.0) THEN
      ID(K,IJ)=1
      SAND(K,IDerr(K,IJ))='Yes'
      END IF
      END IF
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
      DO 70 K=1,35
      IF(K.EQ.1) THEN
      WRITE(3,'(7(/))')
      ICOUNT=0
      DRATIO=0.03
      END IF
      IF(K.NE.1) DRATIO=DRATIO+0.01
      ICOUNT=ICOUNT+1
      WRITE(3,'(2(/),65X,A,1X,F4.2,/)')'When d''/d =',DRATIO
      WRITE(3,55)
   55 FORMAT(10X,7('% error  Ig>Icr?',3X))
      DO 65 I=1,M(K)
      IF(I.EQ.1) II=1
      IF(I.NE.1) II=II+7
      IJ=I*7
      IF(IJ.GT.M(K))IJ=M(K)
      IF(II.GT.M(K)) GO TO 65
      WRITE(3,60)(error(K,J),SAND(K,J),J=II,IJ)
   60 FORMAT(11X,7(F5.1,4X,A,5X))
   65 CONTINUE
      IF(K.EQ.35) GO TO 70
      IF(ICOUNT.EQ.3) THEN
      ICOUNT=0
      WRITE(3,'(1H1,7(/))')
      END IF
   70 CONTINUE
      WRITE(3,'(2(/),64X,A,F5.2)')'Max NPb/b'' is ',NPe
      WRITE(3,75)
   75 FORMAT(T64,21('-'))
```

```
      WRITE(3,'(1H1,7(/))')
      WRITE(3,'(15X,A,//15X,A,//15X,A)')'The followings are printed ''on
     *ly as typical cases '' that are ','used in the examples to illustr
     *ate the computaions involved ','in the program :'
      DO 80 I=1,MXCASE
      WRITE(3,'(//15X,A,F5.2,A,F5.2,A,F5.3,A)')'When NP = ',IFLAG(I),'%
     *, NP'' = ',JFLAG(I),'%  and d''/d  = ',DR(I),' :'
      IF(INDEX(I).EQ.1) THEN
      WRITE(3,'(/25X,5(A,3X))')'d/h','Ig/Icr','NPb/b''','Icre/Icr','(Icr
     *,NP''=0)/Icr'
      WRITE(3,'(/25X,F4.2,2X,F5.2,4X,F5.2,6X,F5.3,10X,F4.2)')DEPTH(I),RI
     *GICR(I),MODNP(I),RICRe(I),RNPCO(I)
      ELSE
      IF(IFLAG(I).GT.(32.0+JFLAG(I))) THEN
      WRITE(3,'(/25X,A)')'This section is not ductile and thus ignored'
      ELSE
      WRITE(3,'(/25X,A)')'The program does not consider such a section'
      END IF
      END IF
   80 CONTINUE
      WRITE(3,'(///)')
      STOP
      END
```

When d'/d = 0.03

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes
2.6	Yes	2.7	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes
1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes
0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes
-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes
-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes
-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes
-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes
-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes
-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes
-4.3	Yes	-4.4	Yes	-4.5	No	-4.6	No	2.8	Yes	2.9	Yes	3.0	Yes
3.1	No	3.2	No	3.3	No	3.4	No	3.5	No	3.6	No	3.7	No
3.8	No	3.9	No	4.0	No	4.1	No	4.2	No	4.3	No	4.4	No
4.5	No	4.6	No	4.7	No	4.8	No	4.9	No	5.0	No	5.1	No
5.2	No	5.3	No	5.4	No	5.5	No	5.6	No	5.7	No	5.8	No
5.9	No	6.0	No	6.1	No	6.2	No	6.3	No	6.4	No	6.5	No
6.6	No	6.7	No										

When d'/d = 0.04

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes
2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes
1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes
0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes
0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes
-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes
-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes
-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes
-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes
-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes
-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes
-4.9	Yes	-5.0	Yes	-5.1	No	2.9	Yes	3.0	Yes	3.1	No	3.2	No
3.3	No	3.4	No	3.5	No	3.6	No	3.7	No	3.8	No	3.9	No
4.0	No	4.1	No	4.2	No	4.3	No	4.4	No				

When d'/d = 0.05

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes	3.1	Yes
2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes
1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes
1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes
0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes
-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes
-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes
-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes
-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes
-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes
-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	Yes

When d'/d = 0.06

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.8	Yes	2.9	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	2.7	Yes
2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes
1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes
1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes
0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes
-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes
-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes
-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes
-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes
-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes
-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes
-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes
-5.1	Yes												

When d'/d = 0.07

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.0	Yes	2.9	Yes
2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes
2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes
1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes
0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes
0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes
-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes
-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes
-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes
-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes
-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes
-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes
-4.9	Yes												

When d'/d = 0.08

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.2	Yes	3.1	Yes
3.0	Yes	2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes
2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes
1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes
0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes
0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes
-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes
-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes
-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes
-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes
-4.7	Yes												

When d'/d = 0.09

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.6	Yes	3.7	Yes	3.8	Yes	3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes
3.1	Yes	3.0	Yes	2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes
2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes
1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes
1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes
0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes
-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes
-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes
-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes
-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes
-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes
-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes		

When d'/d = 0.10

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.8	Yes	3.9	Yes	4.0	Yes	3.7	Yes	3.6	Yes	3.5	Yes	3.4	Yes
3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes	2.8	Yes	2.7	Yes
2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes
1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes
1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes
0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes
-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes
-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes
-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes
-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes
-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes
-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes				

When d'/d = 0.11

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.9	Yes	4.0	Yes	4.1	Yes	3.8	Yes	3.7	Yes	3.6	Yes	3.5	Yes
3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes	2.8	Yes
2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes
2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes
1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes
0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes
-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes
-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes
-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes
-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes
-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes
-3.6	Yes	-3.7	Yes	-3.8	Yes								

When d'/d = 0.12

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	4.2	Yes	4.0	Yes	4.3	Yes	3.9	Yes	3.8	Yes	3.7	Yes
3.6	Yes	3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes
2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes
2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes
1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes
0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes
0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes
-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes
-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes
-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes
-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes
-3.4	Yes	-3.5	Yes										

When d'/d = 0.13

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.2	Yes	4.3	Yes	4.0	Yes	3.9	Yes	3.8	Yes	3.7	Yes	3.6	Yes
3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes
2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes
2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes
1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes
0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes
0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes
-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes
-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes
-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes
-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes				

When d'/d = 0.14

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.3	Yes	4.4	Yes	4.1	Yes	4.0	Yes	3.9	Yes	3.8	Yes	3.7	Yes
3.6	Yes	3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes
2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes
2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes
1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes
0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes
0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes
-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes
-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes
-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes
-2.7	Yes	-2.8	Yes	-2.9	Yes								

When d'/d = 0.15

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.4	Yes	4.2	Yes	4.1	Yes	4.0	Yes	3.9	Yes	3.8	Yes	3.7	Yes
3.6	Yes	3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes
2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes
2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes
1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes
0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes
0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes
-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes
-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes
-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes
-2.7	Yes												

When d'/d = 0.16

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.5	Yes	4.1	Yes	4.0	Yes	3.9	Yes	3.8	Yes	3.7	Yes	3.6	Yes
3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes
2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes
2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes
1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes
0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes
0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes
-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes
-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes
-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes

When d'/d = 0.17

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.2	Yes	4.1	Yes	4.0	Yes	3.9	Yes	3.8	Yes	3.7	Yes	3.6	Yes
3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes
2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes
2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes
1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes
0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes
0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes
-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes
-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes
-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes		

When d'/d = 0.18

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	4.0	Yes	3.9	Yes	3.8	Yes	3.7	Yes	3.6	Yes	3.5	Yes
3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes	2.8	Yes
2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes
2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes
1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes
0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes
-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes
-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes
-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes
-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes				

When d'/d = 0.19

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.0	Yes	3.9	Yes	3.8	Yes	3.7	Yes	3.6	Yes	3.5	Yes	3.4	Yes
3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes	2.8	Yes	2.7	Yes
2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes
1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes
1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes
0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes
-0.2	Yes	-0.3	Yes	4.1	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes
-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes
-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes
-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes						

When d'/d = 0.20

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.9	Yes	3.8	Yes	3.7	Yes	3.6	Yes	3.5	Yes	3.4	Yes	3.3	Yes
3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes
2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes
1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes
1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes
0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes
-0.3	Yes	4.0	Yes	4.1	Yes	4.2	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes
-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes
-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes
-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes				

When d'/d = 0.21

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.8	Yes	3.7	Yes	3.6	Yes	3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes
3.1	Yes	3.0	Yes	2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes
2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes
1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes
1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes
0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	3.9	Yes
4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes
-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes
-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes
-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	No	-2.6	No
-2.7	No	-2.8	No										

When d'/d = 0.22

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.6	Yes	3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes
2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes
2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes
1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes
0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes
0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	3.7	Yes	3.8	Yes
3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	-0.4	Yes	-0.5	Yes
-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes
-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes
-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	No	-2.6	No
-2.7	No	-2.8	No	-2.9	No	-3.0	No						

When d'/d = 0.23

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes
2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes
2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes
1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes
0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes
0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	3.6	Yes	3.7	Yes	3.8	Yes
3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	-0.4	Yes
-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes
-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes
-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	No
-2.6	No	-2.7	No	-2.8	No	-2.9	No	-3.0	No	-3.1	No	-3.2	No

When d'/d = 0.24

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.3	Yes	3.2	Yes	3.1	Yes	3.0	Yes	2.9	Yes	2.8	Yes	2.7	Yes
2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes
1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes
1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes
0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes
-0.2	Yes	-0.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes
3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	-0.4	Yes
-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes
-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes
-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	No
-2.6	No	-2.7	No	-2.8	No	-2.9	No	-3.0	No	-3.1	No	-3.2	No
-3.3	No	-3.4	No										

When d'/d = 0.25

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.0	Yes	2.9	Yes	2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes
2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes
1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes
0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes
0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes
3.4	Yes	3.5	Yes	3.3	Yes	3.6	Yes	3.2	Yes	3.1	Yes	3.7	Yes
3.8	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	-0.5	Yes
-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes
-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes
-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	4.4	Yes	4.5	Yes	-2.4	Yes
-2.5	No	-2.6	No	-2.7	No	-2.8	No	-2.9	No	-3.0	No	-3.1	No
-3.2	No	-3.3	No	-3.4	No	-3.5	No	-3.6	No				

When d'/d = 0.26

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.8	Yes	2.7	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes
2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes
1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes
0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes
0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	3.4	Yes	3.3	Yes
3.5	Yes	3.2	Yes	3.6	Yes	3.1	Yes	3.7	Yes	3.0	Yes	3.8	Yes
2.9	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	-0.5	Yes
-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes
-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes
-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes
-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	No	-2.8	No	-2.9	No	-3.0	No
-3.1	No	-3.2	No	-3.3	No	-3.4	No	-3.5	No	-3.6	No	-3.7	No

When d'/d = 0.27

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes
1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes
1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes
0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes
-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	3.4	Yes	3.3	Yes	3.2	Yes
3.5	Yes	3.6	Yes	3.1	Yes	3.7	Yes	3.0	Yes	3.8	Yes	2.9	Yes
3.9	Yes	2.8	Yes	4.0	Yes	2.7	Yes	4.1	Yes	4.2	Yes	-0.6	Yes
-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes
-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes
-2.1	Yes	-2.2	Yes	-2.3	Yes	4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes
4.7	Yes	4.8	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes
-2.9	Yes	-3.0	No	-3.1	No	-3.2	No	-3.3	No	-3.4	No	-3.5	No
-3.6	No	-3.7	No	-3.8	No	-3.9	No						

When d'/d = 0.28

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes
1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes
0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes
0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes
-0.5	Yes	-0.6	Yes	3.3	Yes	3.4	Yes	3.2	Yes	3.5	Yes	3.1	Yes
3.6	Yes	3.0	Yes	3.7	Yes	2.9	Yes	3.8	Yes	2.8	Yes	3.9	Yes
2.7	Yes	4.0	Yes	2.6	Yes	4.1	Yes	2.5	Yes	2.4	Yes	-0.7	Yes
-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes
-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes
-2.2	Yes	-2.3	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes
4.7	Yes	4.8	Yes	4.9	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes
-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	No	-3.4	No
-3.5	No	-3.6	No	-3.7	No	-3.8	No	-3.9	No	-4.0	No	-4.1	No

When d'/d = 0.29

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes
1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes
0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes
-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes
3.3	Yes	3.2	Yes	3.4	Yes	3.1	Yes	3.5	Yes	3.0	Yes	3.6	Yes
2.9	Yes	3.7	Yes	2.8	Yes	3.8	Yes	2.7	Yes	3.9	Yes	2.6	Yes
4.0	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	-0.8	Yes
-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes
-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes
-2.3	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes
4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes
-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes
-3.4	Yes	-3.5	No	-3.6	No	-3.7	No	-3.8	No	-3.9	No	-4.0	No
-4.1	No	-4.2	No	-4.3	No								

When d'/d = 0.30

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes
1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes
0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes
-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	3.2	Yes
3.1	Yes	3.3	Yes	3.4	Yes	3.0	Yes	2.9	Yes	3.5	Yes	2.8	Yes
3.6	Yes	3.7	Yes	2.7	Yes	3.8	Yes	2.6	Yes	2.5	Yes	2.4	Yes
2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	-1.0	Yes
-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes
-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	3.9	Yes
4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes
4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes
-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes
-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	No	-3.8	No	-3.9	No	-4.0	No
-4.1	No	-4.2	No	-4.3	No	-4.4	No						

When d'/d = 0.31

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes
0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes
-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes
-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	3.1	Yes	3.2	Yes	3.0	Yes
3.3	Yes	2.9	Yes	3.4	Yes	2.8	Yes	3.5	Yes	2.7	Yes	3.6	Yes
2.6	Yes	3.7	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes
2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes
-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes
-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	3.8	Yes	3.9	Yes
4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes
4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	5.1	Yes	-2.4	Yes	-2.5	Yes
-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	No	-4.1	No	-4.2	No	-4.3	No	-4.4	No	-4.5	No	-4.6	No

When d'/d = 0.32

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes
0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes
-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes
-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes	3.0	Yes	3.1	Yes	2.9	Yes
3.2	Yes	2.8	Yes	3.3	Yes	2.7	Yes	3.4	Yes	2.6	Yes	2.5	Yes
2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes
1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes
-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes
-2.2	Yes	-2.3	Yes	-2.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes
3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes
4.6	Yes	4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	5.1	Yes	-2.5	Yes
-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	No	-4.4	No	-4.5	No	-4.6	No
-4.7	No												

When d'/d = 0.33

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes	0.0	Yes
-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes
-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes
-1.5	Yes	-1.6	Yes	-1.7	Yes	2.9	Yes	3.0	Yes	2.8	Yes	3.1	Yes
2.7	Yes	2.6	Yes	3.2	Yes	0.7	Yes	2.5	Yes	2.4	Yes	2.3	Yes
2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes
1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes
0.8	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes
-2.4	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes
3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes
4.6	Yes	4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	5.1	Yes	-2.5	Yes
-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	No	-4.6	No
-4.7	No	-4.8	No										

When d'/d = 0.34

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.2	Yes	0.1	Yes	0.0	Yes	-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes
-0.5	Yes	-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes
-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes
-1.9	Yes	-2.0	Yes	0.3	Yes	2.8	Yes	2.9	Yes	0.4	Yes	2.7	Yes
2.6	Yes	2.5	Yes	3.0	Yes	0.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes
2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes
1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes
0.7	Yes	0.6	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	3.1	Yes
3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes
3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes
4.6	Yes	4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	5.1	Yes	-2.5	Yes
-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes
-4.7	Yes	-4.8	No	-4.9	No	-5.0	No						

When d'/d = 0.35

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-0.1	Yes	-0.2	Yes	-0.3	Yes	-0.4	Yes	-0.5	Yes	-0.6	Yes	-0.7	Yes
-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes
-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes
-2.2	Yes	-2.3	Yes	0.0	Yes	0.1	Yes	0.2	Yes	2.7	Yes	2.6	Yes
2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes	2.0	Yes	1.9	Yes
1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes	1.3	Yes	1.2	Yes
1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes	0.6	Yes	0.5	Yes
0.4	Yes	0.3	Yes	-2.4	Yes	2.8	Yes	2.9	Yes	3.0	Yes	3.1	Yes
3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes
3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes
4.6	Yes	4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	5.1	Yes	-2.5	Yes
-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes
-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	No				

When d'/d = 0.36

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-0.6	Yes	-0.7	Yes	-0.8	Yes	-0.9	Yes	-1.0	Yes	-1.1	Yes	-1.2	Yes
-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes	-1.7	Yes	-1.8	Yes	-1.9	Yes
-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes
-2.7	Yes	-0.5	Yes	-0.4	Yes	-0.3	Yes	-0.2	Yes	-0.1	Yes	0.0	Yes
0.1	Yes	2.6	Yes	2.5	Yes	2.4	Yes	2.3	Yes	2.2	Yes	2.1	Yes
2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes	1.4	Yes
1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes	0.7	Yes
0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	2.7	Yes	2.8	Yes
2.9	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes
3.6	Yes	3.7	Yes	3.8	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes
4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes	4.7	Yes	4.8	Yes	4.9	Yes
5.0	Yes	5.1	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes
-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	Yes		

When d'/d = 0.37

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-1.0	Yes	-1.1	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.5	Yes	-1.6	Yes
-1.7	Yes	-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.2	Yes	-2.3	Yes
-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.5	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.0	Yes	2.4	Yes	2.3	Yes	2.2	Yes
2.1	Yes	2.0	Yes	1.9	Yes	1.8	Yes	1.7	Yes	1.6	Yes	1.5	Yes
1.4	Yes	1.3	Yes	1.2	Yes	1.1	Yes	1.0	Yes	0.9	Yes	0.8	Yes
0.7	Yes	0.6	Yes	0.5	Yes	0.4	Yes	0.3	Yes	0.2	Yes	0.1	Yes
2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes	3.1	Yes
3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes
3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes
4.6	Yes	4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	-3.2	Yes	-3.3	Yes
-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes
-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes
-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	Yes				

Max NPb/b' is 55.17

The followings are printed 'only as typical cases ' that are used in the examples to illustrate the computaions involved in the program :

When NP = 2.00% , NP' = 2.00% and d'/d = 0.030 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.97	5.76	1.77	1.020	0.97

When NP = 15.00% , NP' = 10.00% and d'/d = 0.240 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.81	2.04	12.93	1.002	0.97

When NP = 25.00% , NP' = 25.00% and d'/d = 0.160 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.86	1.04	14.95	1.007	0.83

When NP = 40.00% , NP' = 40.00% and d'/d = 0.030 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.97	0.41	11.21	0.974	0.62

When NP = 45.00% , NP' = 35.00% and d'/d = 0.370 :

d/h	Ig/Icr	NPb/b'	Icre/Icr	(Icr,NP'=0)/Icr
0.73	1.36	38.30	0.972	0.91

<u>APPENDIX B6</u>

The Listing of Prog.3.4.1

PROG.3.4.1

This program evaluates the upper and lower envelope values of αf required for the construction of Fig.3.4.5.

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION HALPHL(55),NAPOS(55)
      REAL NP,NPMAX,NPMIN,NRAU,NEWNP,JJ,II,LALPHU(55),hfd
      OPEN(3,FILE='OUT341',STATUS='UNKNOWN')
C The program will now ask for the percentage of error to be allowed :
      WRITE(*,'(1X,A)')'Please enter % error to be allowed :'
      READ*,Seterr
      Seterr=Seterr/100.
C The program will next assign values for np , be/bw and hf/d such
C that np is kept within the limits allowed by the codes :
      DO 15 K=1,55
      NAPOS(K)=0
   15 CONTINUE
      DO 50 I=1,3396
      IF(I.LT.12) THEN
      GO TO 50
      ELSE IF(I.EQ.12) THEN
      II=0.122
      ELSE
      II=I/100.
      END IF
      JJ=1.0
      DO 40 J=1,90
      JJ=(JJ*10.+1.)/10.
      IF(JJ.GT.2.5) NPMIN=1.22/JJ
      IF(JJ.LE.2.5) NPMIN=0.9/JJ
      DO 30 K=10,55
      ID=0
      hfd=K/100.
      NRAU=hfd**2/(2*(1-hfd))
      NPMAX=31.92/JJ+62.16*hfd*(1-1/JJ)
      IF((NPMIN.GT.II).OR.(NPMAX.LT.II)) GO TO 30
C The program will now commence processing the solution :
      NAPOS(K)=NAPOS(K)+1
      NP=II/100.
      IF(NP.LE.NRAU) THEN
      X=-NP+SQRT(NP**2.+2.*NP)
      XICR=((X**3)/3.+NP*(1-X)**2)*JJ
      NP=NP*JJ
      ELSE
      NP=NP*JJ
      B=2*hfd*(JJ-1+NP/hfd)
      C=(hfd**2.)*(JJ-1+2.*NP/(hfd**2.))
      X=(-B+SQRT(B**2.+4.*C))/2.
      XICR=(JJ/3.)*hfd**3+(X-hfd)**3/3.+JJ*hfd*X*(X-hfd)+NP*(1-X)**2
      END IF
      Rb1=1.0
      A1=0.0
      B1=0.0
```

```
      NEWNP=100.*NP
      ICOUNT=0
   20 ICOUNT=ICOUNT+1
   21 IF(NEWNP.LE.1.9) THEN
      A=0.003
      B=0.095
      ELSE IF((NEWNP.GT.1.9).AND.(NEWNP.LE.5.0)) THEN
      A=0.05
      B=0.07
      ELSE IF((NEWNP.GT.5.0).AND.(NEWNP.LE.17.0)) THEN
      A=0.16
      B=0.05
      ELSE IF((NEWNP.GT.17.0).AND.(NEWNP.LE.32.0)) THEN
      A=0.50
      B=0.03
      ELSE
      A=0.8
      B=0.02
      END IF
      IF(ID.EQ.1) GO TO 23
      IF(ID.EQ.2) GO TO 25
      Rb2=(12.*XICR-B*100.*NP)/A
      IF(ICOUNT.EQ.10) THEN
      A=(A+A1)/2.
      B=(B+B1)/2.
      Rb2=(12.*XICR-B*100.*NP)/A
      ELSE
      IF(ABS(Rb2-Rb1).GT.0.0) THEN
      A1=A
      B1=B
      NEWNP=100.*NP/Rb2
      Rb1=Rb2
      GO TO 20
      END IF
      END IF
      IF(Rb2.LT.1) Rb2=1.0
      ALPHAE=(Rb2-1)/(JJ-1)
      ID=1
      ALPHAU=ALPHAE
   22 ALPHAU=ALPHAU+0.01
      Rb2=1+ALPHAU*(JJ-1)
      NEWNP=100.*NP/Rb2
      GO TO 21
   23 XICRe=(A+B*100.*NP/Rb2)*Rb2/12.
      error=XICRe/XICR-1
      IF(DABS(error).LT.Seterr) GO TO 22
      ID=2
      ALPHAL=ALPHAE
   24 ALPHAL=ALPHAL-0.01
      Rb2=1+ALPHAL*(JJ-1)
      NEWNP=100.*NP/Rb2
      GO TO 21
   25 XICRe=(A+B*100.*NP/Rb2)*Rb2/12.
      error=XICRe/XICR-1
      IF(DABS(error).LT.Seterr) GO TO 24
      IF(NAPOS(K).EQ.1) THEN
```

```
      LALPHU(K)=ALPHAU
      HALPHL(K)=ALPHAL
      ELSE
      LALPHU(K)=DMIN1(LALPHU(K),ALPHAU)
      HALPHL(K)=DMAX1(HALPHL(K),ALPHAL)
      END IF
 30   CONTINUE
 40   CONTINUE
 50   CONTINUE
C Now that the solution processing is complete the values of the upper
C and lower envelopes will be stored in file "OUTPUT" which can then be
C printed
      Seterr=Seterr*100.
      WRITE(3,'(2(/),T23,A,/,T27,A,1X,F4.1,1X,A)')'Values of the Upper A
     *nd Lower Envelope of','% Error Allowed in Icre =',Seterr,'%'
      WRITE(3,'(//T20,3(A,3X))')'hf/d','Upper Envelope Value','Lower Env
     *elope Value'
      DO 60 K=10,55
      hfd=K/100.
      WRITE(3,'(T20,F4.2,9X,F10.6,15X,F8.6)')hfd,LALPHU(K),HALPHL(K)
 60   CONTINUE
      WRITE(3,'(/T15,A/T15,A/T15,A/T16A,F4.1,A/T15,A/T16A,F4.1,A/T15,A/T
     *35,A)')'Notes :','-------','1.Upper envelope values correspond to
     *a max. (+) error',' in Icre of',Seterr,'%','2.Lower envelope value
     *s correspond to a max. (-) error',' in Icre of',Seterr,'%','3.Erro
     *r in Icre is defined as :','(Icre/Icr-1)*100'
      STOP
      END
```

APPENDIX B7

The Listing of Prog.3.4.2 and its Output

PROG.3.4.2

This program evaluates the cracked transformed moment of inertia of the equivalent section for a singly reinforced flanged section and compares the results with the exact values. It also considers the effect of neglecting the web compression area.

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION ANSWER(55,700),error(55,700),M(55),ID(55,700),IDerr(55,7
     *00),DR(10),RIGICR(10),RICRe(10),RhflCR(10),CASAND(10),INDEX(55)
      CHARACTER SAND*5,ANSWER*5,CASAND*5
      REAL NP,NPMAX,NPMIN,NWRICR,NRAU,NEWNP,JJ,II,NPe,MODNP(10),
     *IFLAG(10),JFLAG(10),hfd
      OPEN(3,FILE='OUT342',STATUS='UNKNOWN')
      NPe=0.0
      DO 15 I=1,700
      DO 10 K=10,55
      M(K)=0
      ID(K,I)=0
      error(K,I)=0
      INDEX(K)=0
   10 CONTINUE
   15 CONTINUE
C*****The program will now ask for the sections to be given as examples.
      WRITE(*,'(1X,A)')'How many cases are to be given as examples (max.
     * of 10)?'
      READ*,MXCASE
      WRITE(*,'(1X,A/8X,A,10(/12X,A))')'For each  case please give np,be
     */bw and hf/d . Enter one combination per line','Notes :','1.np sho
     *uld be chosen in the range from 0.13 to 33.96 with  0.01',' increm
     *ents ','3.be/bw should be chosen using increments of 0.1 starting
     *at 1.1','  upto the maximum ratio of 10','4.hf/d should be chosen
     *using increments  of  .01 in  the range','  from .10 to .55 '
      DO 16 ICASE=1,MXCASE
      READ*,IFLAG(ICASE),JFLAG(ICASE),DR(ICASE)
   16 CONTINUE
      WRITE(*,'(A)')'*==================== The program is running pleas
     *e wait ===================*'
C*****The values of np , be/bw and hf/d will next be assigned.Any section
C*****that is  found  unacceptable  according to the  limitations of  the
C*****codes will be ignored.
      DO 50 I=1,3396
      IF(I.LT.12) THEN
      GO TO 50
      ELSE IF(I.EQ.12) THEN
      II=0.124
      ELSE
      II=I/100.
      END IF
      JJ=1.0
      DO 40 J=1,90
      JJ=(JJ*10+1)/10.
      IF(JJ.GT.2.5) NPMIN=1.24/JJ
      IF(JJ.LE.2.5) NPMIN=0.9/JJ
      DO 30 K=10,55
```

```
      hfd=K/100.
      NRAU=hfd**2/(2*(1-hfd))
      NPMAX=31.92/JJ+62.16*hfd*(1-1/JJ)
      IF((NPMIN.GT.II).OR.(NPMAX.LT.II)) THEN
C*****The given np is outside acceptable range.section is ignored
      GO TO 30
      END IF
      dh=0.72
C*****Now that all the parameters have been assigned the program will
C*****commence processing the solution.
      XG=(0.5*JJ*hfd**2+0.5*((1/dh)**2-hfd**2))/(JJ*hfd-hfd+(1/dh))
      IF(XG.LE.hfd)THEN
      XIG=(JJ/3.)*XG**3+(1/3.)*(1/dh-XG)**3+((JJ-1)/3.)*(hfd-XG)**3
      ELSE
      XIG=JJ/12.*hfd**3.+JJ*hfd*(XG-0.5*hfd)**2.+(1/12.)*(1/dh-hfd)**3.+
     1(1/dh-hfd)*((1/dh+hfd)/2.-XG)**2.
      END IF
      NP=II/100.
      IF(NP.LE.NRAU) THEN
      X=-NP+SQRT(NP**2.+2.*NP)
      XICR=((X**3)/3.+NP*(1-X)**2)*JJ
      hfICR=XICR
      NP=NP*JJ
      SAND='YES'
      ELSE
      SAND='NO'
      NP=NP*JJ
      B=2*hfd*(JJ-1+NP/hfd)
      C=(hfd**2.)*(JJ-1+2.*NP/(hfd**2.))
      X=(-B+SQRT(B**2.+4.*C))/2.
      XICR=(JJ/3.)*hfd**3+(X-hfd)**3/3.+JJ*hfd*X*(X-hfd)+NP*(1-X)**2
      X=(NP+0.5*JJ*hfd**2.)/(NP+JJ*hfd)
      hfICR=JJ*hfd**3./12.+JJ*hfd*(X-hfd/2.)**2.+NP*(1-x)**2.
      END IF
      a1=DMIN1((1./3.+(8./3.)*hfd),0.9)
      BPRIME=a1*(JJ-1)+1
      NEWNP=100.*NP/BPRIME
      IF(NEWNP.LE.1.9) THEN
      XICRe=(0.003+0.095*NEWNP)*BPRIME/12.
      ELSE IF((NEWNP.GT.1.9).AND.(NEWNP.LE.5.0)) THEN
      XICRe=(0.05+0.07*NEWNP)*BPRIME/12.
      ELSE IF((NEWNP.GT.5.0).AND.(NEWNP.LE.17.0)) THEN
      XICRe=(0.16+0.05*NEWNP)*BPRIME/12.
      ELSE IF((NEWNP.GT.17).AND.(NEWNP.LE.32.0)) THEN
      XICRe=(0.5+0.03*NEWNP)*BPRIME/12.
      ELSE
      XICRe=(0.8+0.02*NEWNP)*BPRIME/12.
      END IF
      NWRICR=XICRe/XICR
      ROICR=hfICR/XICR
      RIG=XIG/XICR
C*****The following 13 lines relate only to the sections used in the examples
C*****which are provided to illustrate the computations involved in the
C*****program :
      DO 17 ICASE=1,MXCASE
      III=II*1000.
```

```
      JJJ=JJ*1000.
      IIFG=IFLAG(ICASE)*1000.
      JJFG=JFLAG(ICASE)*1000.
      IF((III.EQ.IIFG).AND.(JJJ.EQ.JJFG)) THEN
      Ihfd=hfd*1000.
      Ihfcse=DR(ICASE)*1000.
      IF(Ihfd.EQ.Ihfcse) THEN
      INDEX(ICASE)=1
      RIGICR(ICASE)=RIG
      MODNP(ICASE)=NEWNP
      RICRe(ICASE)=NWRICR
      RhfICR(ICASE)=ROICR
      CASAND(ICASE)=SAND
      END IF
      END IF
   17 CONTINUE
C*************************************************************************C
      Ierror=(NWRICR-1)*1000.
      Ierror=Ierror+400
      NPe=DMAX1(NEWNP,NPe)
      DO 20 IJ=1,700
      IF(Ierror.NE.IJ) GO TO 20
      IF(ID(K,IJ).EQ.0) THEN
      M(K)=M(K)+1
      IDerr(K,IJ)=M(K)
      error(K,M(K))=(Ierror-400)/10.
      IF(RIG.GE.1.0) THEN
      ID(K,IJ)=1
      ANSWER(K,M(K))='Yes'
      ELSE
      ID(K,IJ)=2
      ANSWER(K,M(K))='No'
      END IF
      ELSE IF(ID(K,IJ).EQ.2) THEN
      IF(RIG.GE.1.0) THEN
      ID(K,IJ)=1
      ANSWER(K,IDerr(K,IJ))='Yes'
      END IF
      END IF
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
      ICOUNT=0
      DO 70 K=10,55
      hfd=K/100.
      IF(K.EQ.10) THEN
      WRITE(3,'(11(/))')
      ICOUNT=ICOUNT+1
      ELSE
      ICOUNT=ICOUNT+1
      END IF
      WRITE(3,'(2(/),65X,A,1X,F4.2,/)')'When hf/d =',hfd
      WRITE(3,55)
   55 FORMAT(10X,7('% error  Ig>Icr?',3X))
      DO 65 I=1,M(K)
```

```
      IF(I.EQ.1) II=1
      IF(I.NE.1) II=II+7
      IJ=I*7
      IF(IJ.GT.M(K))IJ=M(K)
      IF(II.GT.M(K)) GO TO 65
      WRITE(3,60)(error(K,J),ANSWER(K,J),J=II,IJ)
   60 FORMAT(11X,7(F5.1,4X,A,5X))
   65 CONTINUE
      IF(K.EQ.55) GO TO 70
      IF(ICOUNT.EQ.2) THEN
      ICOUNT=0
      WRITE(3,'(1H1,12(/))')
      END IF
   70 CONTINUE
      WRITE(3,'(2(/),60X,A,F5.2)')'Max NPb/b'' (relative to bw) is ',NPe
      WRITE(3,75)
   75 FORMAT(T61,36('-'))
      WRITE(3,'(1H1,7(/))')
      WRITE(3,'(15X,A,//15X,A,//15X,A)')'The followings are printed ''on
     *ly as typical cases '' that are ','used in the examples to illustr
     *ate the computations involved ','in the program :'
      DO 80 I=1,MXCASE
      IF(JFLAG(I).GT.2.5) NPMIN=1.24/JFLAG(I)
      IF(JFLAG(I).LE.2.5) NPMIN=0.9/JFLAG(I)
      NP1=31.92/JFLAG(I)+62.16*DR(I)*(1-1/JFLAG(I))
      NP2=30.95/JFLAG(I)+68.84*DR(I)*(1-1/JFLAG(I))
      NP3=40./JFLAG(I)
      NP2=DMIN1(NP2,NP3)
      NPMAX=DMAX1(NP1,NP2)
      WRITE(3,'(//15X,A,F6.3,A,F4.1,A,F4.2,A)')'When NP (based on be) ='
     *,IFLAG(I),' % , be/bw = ',JFLAG(I),' and hf/d =',DR(I),' : '
      IF(INDEX(I).EQ.1) THEN
      WRITE(3,'(/25X,3(A,3X),1X,A,2X,A)')'Icre/Icr','(Icr,neg.web)/Icr',
     *'Is N.A. in flange?','NPb/b''','Ig/Icr'
      WRITE(3,'(/26X,F5.3,11X,F5.3,16X,A,10X,F5.2,3X,F5.2)')RICRe(I),Rhf
     *ICR(I),CASAND(I),MODNP(I),RIGICR(I)
      ELSE
      IF((NPMIN.GT.IFLAG(I)).OR.(NPMAX.LT.IFLAG(I))) THEN
      WRITE(3,'(/25X,A)')'np given was outside acceptable range.Case is
     *ignored'
      ELSE
      WRITE(3,'(/25X,A)')'The program does not consider such a section'
      END IF
      END IF
   80 CONTINUE
      WRITE(3,'(///)')
      STOP
      END
```

OUTPUT OF PROG.3.4.2

When hf/d = 0.10

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-1.6	Yes	-2.0	Yes	-2.1	Yes	-2.7	Yes	-2.8	Yes	-3.2	Yes	-3.3	Yes
-3.6	Yes	-3.7	Yes	-3.8	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes
-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes
-5.1	Yes	-5.2	Yes	-5.3	Yes	-5.4	Yes	-5.5	Yes	-5.6	Yes	-5.7	Yes
-5.8	Yes	-5.9	Yes	-6.0	Yes	-6.1	Yes	-6.2	Yes	-6.3	Yes	-6.4	Yes
-3.9	Yes	-3.5	Yes	-3.4	Yes	-3.1	Yes	-3.0	Yes	-2.9	Yes	-2.6	Yes
-2.5	Yes	-2.4	Yes	-2.3	Yes	-2.2	Yes	-1.9	Yes	-1.8	Yes	-1.7	Yes
-1.5	Yes	-1.4	Yes	-1.3	Yes	-1.2	Yes	-1.1	Yes	-1.0	Yes	-0.9	Yes
-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.5	Yes	-0.4	Yes	-0.3	Yes	-0.2	Yes
-0.1	Yes	0.0	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes	0.5	Yes
0.6	Yes	0.7	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes
1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes
2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes	2.6	Yes
2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes
3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes	3.9	Yes	4.0	Yes
4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes	4.7	Yes
4.8	Yes	4.9	Yes	5.0	Yes	5.1	Yes	5.2	Yes	5.3	Yes	5.4	Yes
5.5	Yes	5.6	Yes	5.7	Yes	5.8	Yes	5.9	Yes	6.0	Yes	6.1	Yes
6.2	Yes	6.3	Yes	6.4	Yes	6.5	Yes	6.6	Yes	6.7	Yes	6.8	Yes

When hf/d = 0.11

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-1.1	Yes	-1.5	Yes	-1.6	Yes	-2.2	Yes	-2.3	Yes	-2.8	Yes	-2.9	Yes
-3.3	Yes	-3.4	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes
-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes
-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	Yes	-5.3	Yes	-5.4	Yes
-5.5	Yes	-5.6	Yes	-5.7	Yes	-5.8	Yes	-5.9	Yes	-6.0	Yes	-6.1	Yes
-6.2	Yes	-3.5	Yes	-3.2	Yes	-3.1	Yes	-3.0	Yes	-2.7	Yes	-2.6	Yes
-2.5	Yes	-2.4	Yes	-2.1	Yes	-2.0	Yes	-1.9	Yes	-1.8	Yes	-1.7	Yes
-1.4	Yes	-1.3	Yes	-1.2	Yes	-1.0	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes
-0.6	Yes	-0.5	Yes	-0.4	Yes	-0.3	Yes	-0.2	Yes	-0.1	Yes	0.0	Yes
0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes	0.5	Yes	0.6	Yes	0.7	Yes
0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes
2.9	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes
3.6	Yes	3.7	Yes	3.8	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes
4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes	4.7	Yes	4.8	Yes	4.9	Yes
5.0	Yes	5.1	Yes	5.2	Yes	5.3	Yes	5.4	Yes	5.5	Yes	5.6	Yes
5.7	Yes	5.8	Yes	5.9	Yes	6.0	Yes	6.1	Yes				

When hf/d = 0.12

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
-0.6	Yes	-1.1	Yes	-1.8	Yes	-2.4	Yes	-2.5	Yes	-2.9	Yes	-3.0	Yes
-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes
-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes
-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	Yes	-5.3	Yes
-5.4	Yes	-5.5	Yes	-5.6	Yes	-5.7	Yes	-5.8	Yes	-5.9	Yes	-6.0	Yes
-3.2	Yes	-3.1	Yes	-2.7	Yes	-2.8	Yes	-2.6	Yes	-2.3	Yes	-2.2	Yes
-2.1	Yes	-2.0	Yes	-1.9	Yes	-1.7	Yes	-1.6	Yes	-1.5	Yes	-1.4	Yes
-1.3	Yes	-1.2	Yes	-1.0	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.5	Yes
-0.4	Yes	-0.3	Yes	-0.2	Yes	-0.1	Yes	0.0	Yes	0.1	Yes	0.2	Yes
0.3	Yes	0.4	Yes	0.5	Yes	0.6	Yes	0.7	Yes	0.8	Yes	0.9	Yes
1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes
1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes
2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes
3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes
3.8	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes
4.5	Yes	4.6	Yes	4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes	5.1	Yes
5.2	Yes	5.3	Yes	5.4	Yes	5.5	Yes						

When hf/d = 0.13

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.0	Yes	-0.6	Yes	-1.3	Yes	-1.4	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes
-2.5	Yes	-2.6	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.3	Yes	-3.4	Yes
-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes
-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes
-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	Yes	-5.3	Yes	-5.4	Yes	-5.5	Yes
-5.6	Yes	-5.7	Yes	-5.8	Yes	-3.2	Yes	-2.7	Yes	-2.8	Yes	-2.4	Yes
-2.3	Yes	-2.2	Yes	-1.8	Yes	-1.7	Yes	-1.6	Yes	-1.5	Yes	-1.2	Yes
-1.1	Yes	-1.0	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.5	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.6	Yes	0.7	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes
1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes
1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes
2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes	3.1	Yes	3.2	Yes
3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes	3.9	Yes
4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes
4.7	Yes	4.8	Yes	4.9	Yes	5.0	Yes						

When hf/d = 0.14

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.4	Yes	-0.1	Yes	-0.9	Yes	-1.5	Yes	-1.6	Yes	-2.1	Yes	-2.2	Yes
-2.5	Yes	-2.6	Yes	-2.7	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes
-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes
-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes
-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	Yes	-5.3	Yes	-5.4	Yes
-5.5	Yes	-5.6	Yes	-5.7	Yes	-2.9	Yes	-2.8	Yes	-2.4	Yes	-2.3	Yes
-2.0	Yes	-1.9	Yes	-1.8	Yes	-1.7	Yes	-1.4	Yes	-1.3	Yes	-1.2	Yes
-1.1	Yes	-1.0	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.5	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	0.0	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.5	Yes
0.6	Yes	0.7	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes
1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes
2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes	2.6	Yes
2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes
3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes	3.9	Yes	4.0	Yes
4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes	4.6	Yes	4.7	Yes

When hf/d = 0.15

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
0.9	Yes	0.3	Yes	-0.4	Yes	-0.5	Yes	-1.1	Yes	-1.2	Yes	-1.7	Yes
-1.8	Yes	-2.2	Yes	-2.3	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes
-5.2	Yes	-5.3	Yes	-5.4	Yes	-5.5	Yes	-2.9	Yes	-2.5	Yes	-2.4	Yes
-2.1	Yes	-2.0	Yes	-1.9	Yes	-1.6	Yes	-1.5	Yes	-1.4	Yes	-1.3	Yes
-1.0	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.3	Yes	-0.2	Yes
-0.1	Yes	0.0	Yes	0.1	Yes	0.2	Yes	0.4	Yes	0.5	Yes	0.6	Yes
0.7	Yes	0.8	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes
2.9	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes
3.6	Yes	3.7	Yes	3.8	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes
4.3	Yes	4.4	Yes										

When hf/d = 0.16

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.4	Yes	0.8	Yes	0.0	Yes	-0.7	Yes	-0.8	Yes	-1.3	Yes	-1.4	Yes
-1.8	Yes	-1.9	Yes	-2.0	Yes	-2.3	Yes	-2.4	Yes	-2.7	Yes	-2.8	Yes
-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes
-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes
-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes
-5.1	Yes	-5.2	Yes	-5.3	Yes	-2.9	Yes	-2.6	Yes	-2.5	Yes	-2.2	Yes
-2.1	Yes	-1.7	Yes	-1.6	Yes	-1.5	Yes	-1.2	Yes	-1.1	Yes	-1.0	Yes
-0.9	Yes	-0.6	Yes	-0.5	Yes	-0.4	Yes	-0.3	Yes	-0.2	Yes	-0.1	Yes
0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes	0.5	Yes	0.6	Yes	0.7	Yes
0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes	1.5	Yes	1.6	Yes
1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes
2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes
3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes
3.8	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes		

When hf/d = 0.17

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
1.9	Yes	1.3	Yes	0.4	Yes	-0.3	Yes	-0.9	Yes	-1.0	Yes	-1.5	Yes
-1.6	Yes	-1.9	Yes	-2.0	Yes	-2.1	Yes	-2.3	Yes	-2.4	Yes	-2.5	Yes
-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes
-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes
-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes
-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	Yes	-2.6	Yes	-2.2	Yes
-1.8	Yes	-1.7	Yes	-1.4	Yes	-1.3	Yes	-1.2	Yes	-1.1	Yes	-0.8	Yes
-0.7	Yes	-0.6	Yes	-0.5	Yes	-0.4	Yes	-0.2	Yes	-0.1	Yes	0.0	Yes
0.1	Yes	0.2	Yes	0.3	Yes	0.5	Yes	0.6	Yes	0.7	Yes	0.8	Yes
0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.4	Yes	1.5	Yes	1.6	Yes
1.7	Yes	1.8	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes
2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes	3.1	Yes
3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes
3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes						

When hf/d = 0.18

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
2.4	Yes	1.8	Yes	0.9	Yes	0.8	Yes	0.1	Yes	0.0	Yes	-0.5	Yes
-0.6	Yes	-1.1	Yes	-1.2	Yes	-1.6	Yes	-1.7	Yes	-2.0	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-2.6	Yes
-2.3	Yes	-1.9	Yes	-1.8	Yes	-1.5	Yes	-1.4	Yes	-1.3	Yes	-1.0	Yes
-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.4	Yes	-0.3	Yes	-0.2	Yes	-0.1	Yes
0.2	Yes	0.3	Yes	0.4	Yes	0.5	Yes	0.6	Yes	0.7	Yes	1.0	Yes
1.1	Yes	1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.7	Yes
1.9	Yes	2.0	Yes	2.1	Yes	2.2	Yes	2.3	Yes	2.5	Yes	2.6	Yes
2.7	Yes	2.8	Yes	2.9	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes
3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes	3.8	Yes	3.9	Yes	4.0	Yes
4.1	Yes	4.2	Yes	4.3	Yes								

When hf/d = 0.19

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.0	Yes	2.3	Yes	1.3	Yes	0.5	Yes	0.4	Yes	-0.1	Yes	-0.2	Yes
-0.7	Yes	-0.8	Yes	-1.2	Yes	-1.3	Yes	-1.4	Yes	-1.7	Yes	-1.8	Yes
-2.1	Yes	-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes
-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes
-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes
-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-4.8	Yes	-2.3	Yes
-2.0	Yes	-1.9	Yes	-1.6	Yes	-1.5	Yes	-1.1	Yes	-1.0	Yes	-0.9	Yes
-0.6	Yes	-0.5	Yes	-0.4	Yes	-0.3	Yes	0.0	Yes	0.1	Yes	0.2	Yes
0.3	Yes	0.6	Yes	0.7	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes
1.2	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes
2.0	Yes	2.1	Yes	2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes
2.8	Yes	2.9	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.5	Yes
3.6	Yes	3.7	Yes	3.8	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes
4.3	Yes	4.4	No										

When hf/d = 0.20

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
3.5	Yes	2.8	Yes	1.8	Yes	1.7	Yes	0.9	Yes	0.2	Yes	0.1	Yes
-0.4	Yes	-0.9	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes
-2.1	Yes	-2.2	Yes	-2.3	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes
-2.9	Yes	-3.0	Yes	-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes
-3.6	Yes	-3.7	Yes	-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes
-4.3	Yes	-4.4	Yes	-4.5	Yes	-4.6	Yes	-4.7	Yes	-2.4	Yes	-2.0	Yes
-1.7	Yes	-1.6	Yes	-1.3	Yes	-1.2	Yes	-1.1	Yes	-0.8	Yes	-0.7	Yes
-0.6	Yes	-0.5	Yes	-0.3	Yes	-0.2	Yes	-0.1	Yes	0.0	Yes	0.3	Yes
0.4	Yes	0.5	Yes	0.6	Yes	0.7	Yes	0.8	Yes	1.0	Yes	1.1	Yes
1.2	Yes	1.3	Yes	1.4	Yes	1.5	Yes	1.6	Yes	1.9	Yes	2.0	Yes
2.1	Yes	2.2	Yes	2.3	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes
2.9	Yes	3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.4	Yes	3.6	Yes
3.7	Yes	3.8	Yes	3.9	Yes	4.0	Yes	4.1	Yes	4.2	Yes	4.3	Yes
4.4	Yes	4.5	Yes	4.6	No								

When hf/d = 0.21

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.0	Yes	3.3	Yes	2.2	Yes	1.3	Yes	0.6	Yes	0.5	Yes	0.0	Yes
-0.1	Yes	-0.6	Yes	-1.1	Yes	-1.5	Yes	-1.6	Yes	-1.9	Yes	-2.0	Yes
-2.2	Yes	-2.3	Yes	-2.5	Yes	-2.6	Yes	-2.7	Yes	-2.8	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.9	Yes	-2.4	Yes	-2.1	Yes	-1.8	Yes	-1.7	Yes	-1.4	Yes
-1.3	Yes	-1.2	Yes	-1.0	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.5	Yes
-0.4	Yes	-0.3	Yes	-0.2	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.7	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.4	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.3	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.4	Yes	3.5	Yes	3.6	Yes	3.7	Yes
3.8	Yes	3.9	Yes	4.1	Yes	4.2	Yes	4.3	Yes	4.4	Yes	4.5	Yes
4.6	Yes	4.7	No	4.8	No								

When hf/d = 0.22

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
3.8	Yes	3.9	Yes	4.0	Yes	4.2	No	4.3	No				

When hf/d = 0.23

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes

When hf/d = 0.24

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes

When hf/d = 0.25

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes										

When hf/d = 0.26

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes								

When hf/d = 0.27

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes						

When hf/d = 0.28

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes				

When hf/d = 0.29

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes				

When hf/d = 0.30

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.31

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.32

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.33

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.34

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.35

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.36

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.37

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.38

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.39

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.40

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.41

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.42

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.43

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.44

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.45

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.46

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.47

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.48

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.49

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.50

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.51

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes		

When hf/d = 0.52

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	No

When hf/d = 0.53

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	No
-5.3	No												

When hf/d = 0.54

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	No
-5.3	No	-5.4	No										

When hf/d = 0.55

% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?	% error	Ig>Icr?
4.1	Yes	3.4	Yes	2.3	Yes	1.4	Yes	0.7	Yes	0.6	Yes	0.0	Yes
-0.5	Yes	-1.0	Yes	-1.4	Yes	-1.5	Yes	-1.8	Yes	-1.9	Yes	-2.1	Yes
-2.2	Yes	-2.4	Yes	-2.5	Yes	-2.7	Yes	-2.8	Yes	-2.9	Yes	-3.0	Yes
-3.1	Yes	-3.2	Yes	-3.3	Yes	-3.4	Yes	-3.5	Yes	-3.6	Yes	-3.7	Yes
-3.8	Yes	-3.9	Yes	-4.0	Yes	-4.1	Yes	-4.2	Yes	-4.3	Yes	-4.4	Yes
-4.5	Yes	-2.6	Yes	-2.3	Yes	-2.0	Yes	-1.7	Yes	-1.6	Yes	-1.3	Yes
-1.2	Yes	-1.1	Yes	-0.9	Yes	-0.8	Yes	-0.7	Yes	-0.6	Yes	-0.4	Yes
-0.3	Yes	-0.2	Yes	-0.1	Yes	0.1	Yes	0.2	Yes	0.3	Yes	0.4	Yes
0.5	Yes	0.8	Yes	0.9	Yes	1.0	Yes	1.1	Yes	1.2	Yes	1.3	Yes
1.5	Yes	1.6	Yes	1.7	Yes	1.8	Yes	1.9	Yes	2.0	Yes	2.1	Yes
2.2	Yes	2.4	Yes	2.5	Yes	2.6	Yes	2.7	Yes	2.8	Yes	2.9	Yes
3.0	Yes	3.1	Yes	3.2	Yes	3.3	Yes	3.5	Yes	3.6	Yes	3.7	Yes
-4.6	Yes	-4.7	Yes	-4.8	Yes	-4.9	Yes	-5.0	Yes	-5.1	Yes	-5.2	No
-5.3	No	-5.4	No	-5.5	No	-5.6	No						

Max NPb/b' (relative to bw) is 37.32

The followings are printed 'only as typical cases ' that are used in the examples to illustrate the computations involved in the program :

When NP (based on be) = 0.500 % , be/bw = 5.0 and hf/d =0.35 :

Icre/Icr	(Icr,neg.web)/Icr	Is N.A. in flange?	NPb/b'	Ig/Icr
0.956	1.000	YES	0.54	19.43

When NP (based on be) = 8.000 % , be/bw = 10.0 and hf/d =0.20 :

Icre/Icr	(Icr,neg.web)/Icr	Is N.A. in flange?	NPb/b'	Ig/Icr
0.957	0.997	NO	9.09	1.08

When NP (based on be) =14.000 % , be/bw = 3.0 and hf/d =0.11 :

Icre/Icr	(Icr,neg.web)/Icr	Is N.A. in flange?	NPb/b'	Ig/Icr
1.045	0.869	NO	18.64	1.58

When NP (based on be) =22.000 % , be/bw = 1.5 and hf/d =0.10 :

Icre/Icr	(Icr,neg.web)/Icr	Is N.A. in flange?	NPb/b'	Ig/Icr
1.020	0.696	NO	25.38	1.82

APPENDIX C1

The Listing of Prog.4.3.1 and its Output

PROG.4.3.1

This program evaluates γ for the values of Φ from 0 to -10 and for the referance condition of R=3. These values are then used to plot the solution curves of Fig.4.3.1 as explained in Sec.4.3.

```
      DIMENSION GAMA(22,16)
      REAL NP(16)
      OPEN(3,FILE='OUTPUT',STATUS='UNKNOWN')
C***** The program will now assign a value 3 for Id and commence processing
C***** It will also assign values for npb/b' and phai
      XID=3.0
      WRITE(3,'(3(//),69X,A,1X,F3.1,/)')'When R = ',XID
      DO 40 K=1,3
      M=16
      IF(K.EQ.3)M=7
      DO 40 I=1,22
      IF(I.EQ.1) THEN
      PHI=0.0
      ELSE IF(I.GT.3) THEN
      PHI=PHI-0.5
      ELSE
      PHI=PHI-0.25
      END IF
      DO 10 J=1,M
      IF(K.EQ.1) THEN
        IF(J.EQ.1) THEN
        NP(J)=0.12
        ELSE IF(J.EQ.2) THEN
        NP(J)=0.15
        ELSE IF(J.EQ.3) THEN
        NP(J)=0.20
        ELSE IF((J.GT.3).AND.(J.LE.11)) THEN
        NP(J)=NP(J-1)+0.1
        ELSE IF(J.EQ.12) THEN
        NP(J)=2.0
        ELSE
        NP(J)=NP(J-1)+2
        END IF
      ELSE IF(K.EQ.2) THEN
        IF(J.EQ.1) THEN
        NP(J)=12.0
        ELSE
        NP(J)=NP(J-1)+2.0
        END IF
      ELSE IF(K.EQ.3) THEN
        IF(J.EQ.1) THEN
        NP(J)=44.0
        ELSE
        NP(J)=NP(J-1)+2.0
        END IF
      END IF
C***** The values of gama will now be evaluated . The results will be
C***** stored in a file called OUTPUT
      IF(I.EQ.1) THEN
```

```
      GAMA(I,J)=XID
      ELSE
        IF(NP(J).LE.1.9) THEN
        ALPHA=0.003
        BETA=0.095
        ELSE IF((NP(J).GT.1.9).AND.(NP(J).LE.5.0)) THEN
        ALPHA=0.05
        BETA=0.07
        ELSE IF((NP(J).GT.5.0).AND.(NP(J).LE.17.0)) THEN
        ALPHA=0.16
        BETA=0.05
        ELSE IF((NP(J).GT.17.0).AND.(NP(J).LE.32.0)) THEN
        ALPHA=0.50
        BETA=0.03
        ELSE
        ALPHA=0.80
        BETA=0.02
        END IF
        GAMA(I,J)=ALPHA+BETA*NP(J)+(XID-ALPHA-BETA*NP(J))*EXP(PHI)
      END IF
10    CONTINUE
      IF(I.NE.1) GO TO 25
      IF(K.EQ.1) THEN
      WRITE(3,15)(NP(J),J=1,M)
15    FORMAT(23X,'PHI',2X,'NPb/b'':',1X,2(F4.2,2X),F4.1,13(2X,F4.1))
      ELSE
      WRITE(3,20)(NP(J),J=1,M)
20    FORMAT(23X,'PHI',2X,'NPb/b'':',1X,F4.1,15(2X,F4.1))
      END IF
25    WRITE(3,30) PHI,(GAMA(I,J),J=1,M)
30    FORMAT(21X,F6.2,1X,'GAMA  :',16(1X,F5.3))
40    CONTINUE
      STOP
      END
```

OUTPUT OF PROG.4.3.1

When R = 3.0

PHI	NPb/b':	0.12	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	2.0	4.0	6.0	8.0	10.0
0.00	GAMA :	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
-0.25	GAMA :	2.340	2.340	2.341	2.343	2.345	2.348	2.350	2.352	2.354	2.356	2.358	2.378	2.409	2.438	2.460	2.482
-0.50	GAMA :	1.825	1.826	1.828	1.832	1.836	1.839	1.843	1.847	1.851	1.854	1.858	1.894	1.949	2.001	2.040	2.079
-1.00	GAMA :	1.113	1.115	1.118	1.124	1.130	1.136	1.142	1.148	1.154	1.160	1.166	1.224	1.312	1.394	1.458	1.521
-1.50	GAMA :	0.681	0.683	0.686	0.694	0.701	0.709	0.716	0.723	0.731	0.738	0.746	0.817	0.926	1.027	1.104	1.182
-2.00	GAMA :	0.418	0.421	0.425	0.433	0.441	0.450	0.458	0.466	0.474	0.483	0.491	0.570	0.691	0.804	0.890	0.977
-2.50	GAMA :	0.259	0.262	0.266	0.275	0.284	0.293	0.301	0.310	0.319	0.327	0.336	0.421	0.549	0.668	0.760	0.852
-3.00	GAMA :	0.163	0.166	0.170	0.179	0.188	0.197	0.206	0.215	0.224	0.233	0.242	0.330	0.463	0.586	0.681	0.777
-3.50	GAMA :	0.105	0.107	0.112	0.121	0.130	0.140	0.149	0.158	0.167	0.176	0.186	0.275	0.411	0.537	0.634	0.731
-4.00	GAMA :	0.069	0.072	0.077	0.086	0.095	0.105	0.114	0.123	0.132	0.142	0.151	0.241	0.379	0.507	0.605	0.703
-4.50	GAMA :	0.048	0.050	0.055	0.064	0.074	0.083	0.093	0.102	0.111	0.121	0.130	0.221	0.360	0.488	0.587	0.686
-5.00	GAMA :	0.035	0.037	0.042	0.052	0.061	0.070	0.080	0.089	0.099	0.108	0.118	0.209	0.348	0.477	0.576	0.676
-5.50	GAMA :	0.027	0.029	0.034	0.044	0.053	0.063	0.072	0.081	0.091	0.100	0.110	0.201	0.341	0.470	0.570	0.670
-6.00	GAMA :	0.022	0.025	0.029	0.039	0.048	0.058	0.067	0.077	0.086	0.096	0.105	0.197	0.337	0.466	0.566	0.666
-6.50	GAMA :	0.019	0.022	0.026	0.036	0.045	0.055	0.064	0.074	0.083	0.093	0.102	0.194	0.334	0.464	0.564	0.664
-7.00	GAMA :	0.017	0.020	0.025	0.034	0.044	0.053	0.063	0.072	0.082	0.091	0.101	0.193	0.332	0.462	0.562	0.662
-7.50	GAMA :	0.016	0.019	0.024	0.033	0.043	0.052	0.062	0.071	0.081	0.090	0.100	0.192	0.331	0.461	0.561	0.661
-8.00	GAMA :	0.015	0.018	0.023	0.032	0.042	0.051	0.061	0.070	0.080	0.089	0.099	0.191	0.331	0.461	0.561	0.661
-8.50	GAMA :	0.015	0.018	0.023	0.032	0.042	0.051	0.061	0.070	0.080	0.089	0.099	0.191	0.331	0.461	0.560	0.660
-9.00	GAMA :	0.015	0.018	0.022	0.032	0.041	0.051	0.060	0.070	0.079	0.089	0.098	0.190	0.330	0.460	0.560	0.660
-9.50	GAMA :	0.015	0.017	0.022	0.032	0.041	0.051	0.060	0.070	0.079	0.089	0.098	0.190	0.330	0.460	0.560	0.660
-10.00	GAMA :	0.015	0.017	0.022	0.032	0.041	0.051	0.060	0.070	0.079	0.089	0.098	0.190	0.330	0.460	0.560	0.660

PHI	NPb/b':	12.0	14.0	16.0	18.0	20.0	22.0	24.0	26.0	28.0	30.0	32.0	34.0	36.0	38.0	40.0	42.0
0.00	GAMA :	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
-0.25	GAMA :	2.505	2.527	2.549	2.566	2.580	2.593	2.606	2.620	2.633	2.646	2.659	2.664	2.673	2.681	2.690	2.699
-0.50	GAMA :	2.119	2.158	2.197	2.229	2.252	2.276	2.300	2.323	2.347	2.370	2.394	2.402	2.418	2.433	2.449	2.465
-1.00	GAMA :	1.584	1.647	1.710	1.761	1.799	1.837	1.875	1.913	1.951	1.989	2.027	2.039	2.064	2.090	2.115	2.140
-1.50	GAMA :	1.260	1.337	1.415	1.477	1.524	1.571	1.617	1.664	1.710	1.757	1.804	1.819	1.850	1.881	1.912	1.943
-2.00	GAMA :	1.063	1.150	1.236	1.305	1.357	1.409	1.461	1.513	1.565	1.617	1.668	1.686	1.720	1.755	1.789	1.824
-2.50	GAMA :	0.944	1.036	1.127	1.201	1.256	1.311	1.366	1.421	1.476	1.531	1.586	1.605	1.641	1.678	1.715	1.752
-3.00	GAMA :	0.872	0.967	1.062	1.138	1.195	1.252	1.309	1.366	1.423	1.480	1.537	1.556	1.594	1.632	1.670	1.708
-3.50	GAMA :	0.828	0.925	1.022	1.099	1.157	1.216	1.274	1.332	1.390	1.448	1.507	1.526	1.565	1.603	1.642	1.681
-4.00	GAMA :	0.801	0.899	0.997	1.076	1.135	1.194	1.253	1.312	1.370	1.429	1.488	1.508	1.547	1.586	1.626	1.665
-4.50	GAMA :	0.785	0.884	0.983	1.062	1.121	1.180	1.240	1.299	1.358	1.418	1.477	1.497	1.536	1.576	1.616	1.655
-5.00	GAMA :	0.775	0.874	0.974	1.053	1.113	1.172	1.232	1.292	1.351	1.411	1.470	1.490	1.530	1.570	1.609	1.649
-5.50	GAMA :	0.769	0.869	0.968	1.048	1.108	1.168	1.227	1.287	1.347	1.407	1.466	1.486	1.526	1.566	1.606	1.646
-6.00	GAMA :	0.766	0.865	0.965	1.045	1.105	1.165	1.224	1.284	1.344	1.404	1.464	1.484	1.524	1.564	1.603	1.643
-6.50	GAMA :	0.763	0.863	0.963	1.043	1.103	1.163	1.223	1.283	1.342	1.402	1.462	1.482	1.522	1.562	1.602	1.642
-7.00	GAMA :	0.762	0.862	0.962	1.042	1.102	1.162	1.222	1.282	1.342	1.401	1.461	1.481	1.521	1.561	1.601	1.641
-7.50	GAMA :	0.761	0.861	0.961	1.041	1.101	1.161	1.221	1.281	1.341	1.401	1.461	1.481	1.521	1.561	1.601	1.641
-8.00	GAMA :	0.761	0.861	0.961	1.041	1.101	1.161	1.221	1.281	1.341	1.401	1.461	1.481	1.520	1.560	1.600	1.640
-8.50	GAMA :	0.760	0.860	0.960	1.040	1.100	1.160	1.220	1.280	1.340	1.400	1.460	1.480	1.520	1.560	1.600	1.640
-9.00	GAMA :	0.760	0.860	0.960	1.040	1.100	1.160	1.220	1.280	1.340	1.400	1.460	1.480	1.520	1.560	1.600	1.640
-9.50	GAMA :	0.760	0.860	0.960	1.040	1.100	1.160	1.220	1.280	1.340	1.400	1.460	1.480	1.520	1.560	1.600	1.640
-10.00	GAMA :	0.760	0.860	0.960	1.040	1.100	1.160	1.220	1.280	1.340	1.400	1.460	1.480	1.520	1.560	1.600	1.640

PHI	NPb/b':	44.0	46.0	48.0	50.0	52.0	54.0	56.0
0.00	GAMA :	3.000	3.000	3.000	3.000	3.000	3.000	3.000
-0.25	GAMA :	2.708	2.717	2.726	2.735	2.743	2.752	2.761
-0.50	GAMA :	2.481	2.496	2.512	2.528	2.544	2.559	2.575
-1.00	GAMA :	2.166	2.191	2.216	2.241	2.267	2.292	2.317
-1.50	GAMA :	1.975	2.006	2.037	2.068	2.099	2.130	2.161
-2.00	GAMA :	1.859	1.893	1.928	1.962	1.997	2.032	2.066
-2.50	GAMA :	1.788	1.825	1.862	1.899	1.935	1.972	2.009
-3.00	GAMA :	1.746	1.784	1.822	1.860	1.898	1.936	1.974
-3.50	GAMA :	1.720	1.759	1.797	1.836	1.875	1.914	1.953
-4.00	GAMA :	1.704	1.743	1.783	1.822	1.861	1.901	1.940
-4.50	GAMA :	1.695	1.734	1.774	1.813	1.853	1.892	1.932
-5.00	GAMA :	1.689	1.729	1.768	1.808	1.848	1.888	1.927
-5.50	GAMA :	1.685	1.725	1.765	1.805	1.845	1.885	1.924
-6.00	GAMA :	1.683	1.723	1.763	1.803	1.843	1.883	1.923
-6.50	GAMA :	1.682	1.722	1.762	1.802	1.842	1.882	1.922
-7.00	GAMA :	1.681	1.721	1.761	1.801	1.841	1.881	1.921
-7.50	GAMA :	1.681	1.721	1.761	1.801	1.841	1.881	1.921
-8.00	GAMA :	1.680	1.720	1.760	1.800	1.840	1.880	1.920
-8.50	GAMA :	1.680	1.720	1.760	1.800	1.840	1.880	1.920
-9.00	GAMA :	1.680	1.720	1.760	1.800	1.840	1.880	1.920
-9.50	GAMA :	1.680	1.720	1.760	1.800	1.840	1.880	1.920
-10.00	GAMA :	1.680	1.720	1.760	1.800	1.840	1.880	1.920

APPENDIX C2

The Listing of Prog.4.4.3.1 and its Output

PROG.4.4.3.1

This program evaluates the deflection of a simply supported beams using the effective moment of inertia as given by Branson's equation (Eq.4.1.1), the equation proposed in Ref.4 (Eq.4.1.2) and the proposed model. The deflection values thus computed along with the measured deflection as well as the corresponding errors found in the values computed by the different methods are then printed. The loading types considered by this program are: Two point loads that are equally spaced from the supports, uniformly distributed loads and central point loads

Notes:

This program reads data from a data file. The terms read from the data files (see comment statement c1 below) are defined as follows:

Asc: As′, dc: d′, MESDEF: measured deflection, ILOAD: variable defining load type (1 for distributed loads, 2 otherwise), XX: distance fom a point load to the near support (half the span for central point loads, zero for distributed loads)

Other terms are as defined in Chaps.1-4.

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION DEF1(500),DEF3(500),DEFBR(500),RSPAN(500),Rho(500),RM(50
     10)
      REAL Ma,Mcr,MESDEF(500),Ie1,Ie3,N,NRho,NRhoc,NEWICR
      CHARACTER  ASAD1*2 , ASAD2*3 , ASAD3*5 , ASAD4*3 , ASAD5*5 ,ASAD6*
     13
      OPEN(3,FILE='OUTPUT',STATUS='UNKNOWN')
      OPEN(4,FILE='CERA',ACCESS='SEQUENTIAL',STATUS='OLD')
      WRITE(*,'(1X,A,//T10,A,/T15,A,//T10,A,/T15,A,/T10,A,/T15,A,/T15,A,
     1/T15,A,/T18,A,///T20,A)')'Please note :','1.When using metric unit
     1s :','enter 1 for ID and use units of MPa,N.mm,mm and SQ.mm','2.Wh
     1en using English units :','enter 2 for ID and use units of psi,lb.
     1in,in and SQ.in','3.enter :','a."0" for As'' and d'' if no compres
     1sion reinforcement is used','b."0" for hf and take be=bw if the se
     1ction is rectangular','c."0" for fc'' if  cubic strength is used a
     1nd "0" for  fcu if','cylindrical strength is used','* ========= Hi
     1t return to continue ======== *'
      READ(*,'(A4)')
C*****Identify type of units and give sectional properties ;
      WRITE(*,'(//,1X,A\)')'Please give the units identification number,
     1ID:'
      READ*,ID
      WRITE(*,'(1X,A\)')'How many elements are you considering ?'
      READ*,I
      ICOUNT=0
      DO 10 J=1,I
      ICOUNT=ICOUNT+1
c1----The following statement reads data from a data file
      READ(4,*)fc,fcu,Ma,As,Asc,bw,be,hf,h,d,dc,span,MESDEF(J),ILOAD,XX
      RSPAN(J)=XX/SPAN
      bbw=bw
C*****Compute the ratios of the sectional dimensions :
      Rb=be/bw
      hfd=hf/d
      dh=d/h
C*****Since the values of hf and hfd will be altered somewhere in the
```

```
C*****program they have to be stored for printing purposes:
      Xhf=hf
      Xhfd=hf/d
C*****Compute the gross moment of inertia :
      IF(be.EQ.bw) THEN
      XG=h/2
      XIG=bw*h**3./12.
      ELSE
      R1=(0.5*Rb*hfd**2.+0.5*((1./dh)**2.-hfd**2.))*d
      R2=Rb*hfd+1/dh-hfd
      XG=R1/R2
      RXG=XG/d
      IF(XG.LE.hf) THEN
      XIG=(Rb*RXG**3./3.+(1/dh-RXG)**3./3.+(Rb-1)*(hfd-RXG)**3./3.)*bw*d
     1**3.
      ELSE
      XIG=(Rb*hfd**3./12.+Rb*hfd*(RXG-0.5*hfd)**2.+(1/dh-RXG)**3./3.+(RX
     1G-hfd)**3./3.)*bw*d**3.
      END IF
      END IF
C*****COMPUTE Ec and n :
      IF(ID.EQ.1) THEN
      ASAD1='mm'
      ASAD2='MPa'
      ASAD3='N.mm'
      IF(fcu.EQ.0.0) THEN
      Ec = 5000.*SQRT(fc)
      ASAD4='fc'''
      ELSE
      Ec = (20.+0.2*fcu)*1000.0
      ASAD4='fcu'
      END IF
      N=200*10**3./Ec
      ELSE
      ASAD1='in'
      ASAD2='psi'
      ASAD3='lb.in'
      IF(fcu.EQ.0) THEN
      Ec=33.*145.0**1.5*SQRT(fc)
      ASAD4='fc'''
      ELSE
      Ec=(2900.+0.2*fcu)*1000.
      ASAD4='fcu'
      END IF
      N=29.*10.**6./Ec
      END IF
C*****Compute the equivalent width b' ,np and np' :
      Rho(J)=As*100./(bw*d)
      NRhoc=N*Asc*100.0/(bw*d)
      ALPHAF=DMIN1(0.9,(1./3.)*(1.+8.*Xhfd))
      IF(Asc.EQ.0.0) THEN
      EQb=(ALPHAF*(Rb-1)+1)*bw
      ELSE
      ALPHA=0.0006+0.05*(dc/d)*(1-2*dc/d)**2.
      EQb=(ALPHA*NRhoc*d/dc + ALPHAF*( Rb-1)+1)*bw
      END IF
```

```
      EQNRho=N*As*100./(EQb*d)
C*****Compute Icr using the approximate method :
      IF(EQNRho.LE.1.9) THEN
      A=0.003
      B=0.095
      ELSE IF((EQNRho.GT.1.9).AND.(EQNRho.LE.5.)) THEN
      A=0.05
      B=0.07
      ELSE IF((EQNRho.GT.5.).AND.(EQNRho.LE.17)) THEN
      A=0.16
      B=0.05
      ELSE IF((EQNRho.GT.17.).AND.(EQNRho.LE.32.)) THEN
      A=0.5
      B=0.03
      ELSE
      A=0.80
      B=0.02
      END IF
      NEWICR=(A+B*EQNRho)*EQb*d**3./12.
C*****Compute exact Icr to use in Branson's equation :
      NRho=N*As*100./(be*d)
      IF(be.NE.bw) THEN
      Rhfd=((N*Asc*100./(be*d))*(hfd-dc/d)+50.*hfd**2.)/(1-hfd)
      IF(NRho.GT.Rhfd) THEN
      NRho=N*As/(bw*d)
      NRhoc=N*Asc/(bw*d)
      ELSE
      hf=0.0
      hfd=0.0
      bw=be
      NRho=NRho/100.
      NRhoc=N*Asc/(be*d)
      END IF
      ELSE
      NRho=N*As/(bw*d)
      NRhoc=N*Asc/(bw*d)
      END IF
      BB=2*(hf*Rb-hf+d*NRho+d*NRhoc)
      C=Rb*hf**2.-hf**2.+2.*d*dc*NRhoc+2.*NRho*d**2.
      X=(-BB+SQRT(BB**2.+4.*C))/2.
      XICR=(Rb*hfd**3./3.+(X/d-hfd)**3./3.+Rb*hfd*(X/d)*(X/d-hfd)+NRho*(
     11-X/d)**2.+NRhoc*(X/d-dc/d)**2.)*(bw*d**3.)
C*****Compute Ie :
      IF(XICR.GT.XIG) THEN
      BRIe=XICR
      Ie1=XICR
      Ie3=XICR
      ELSE
      IF(ID.EQ.1) THEN
      IF(fcu.EQ.0.0) Fr=0.62*SQRT(fc)
      IF(fc.EQ.0.0) Fr=0.56*SQRT(fcu)
      ELSE
      IF(fcu.EQ.0.0) Fr=7.5*SQRT(fc)
      IF(fc.EQ.0.0) Fr=6.8*SQRT(fcu)
      END IF
      Mcr=Fr*XIG/(h-XG)
```

```
      RM(J)=Ma/Mcr
      IF(RM(J).LE.1.0) THEN
      BRIe=XIG
      Ie1=XIG
      Ie3=XIG
      GO TO 15
      END IF
      BR=XICR+(XIG-XICR)*(1/RM(J))**3.
      BRIe=DMIN1(BR,XIG)
      IF(ILOAD.EQ.1) RL=SQRT(1-1/RM(J))
      IF(ILOAD.NE.1) RL=1-(2*XX/SPAN)*(1/RM(J))
      IF(Rho(J).LT.1) THEN
      PHAI1=-RL*RM(J)
      ELSE
      PHAI1=-RM(J)*RL*Rho(J)
      END IF
      Ie1=NEWICR*(1-EXP(PHAI1))+XIG*EXP(PHAI1)
      Ie1=DMIN1(Ie1,XIG)
      SUADI=0.8*Rho(J)/RM(J)
      Ie3=(RL**SUADI)*XICR+(1-RL**SUADI)*XIG
      Ie3=DMIN1(Ie3,XIG)
      END IF
C*****Compute deflection :
   15 IF(ILOAD.EQ.1) THEN
      DEFBR(J)=(5./48.)*Ma*SPAN**2./(Ec*BRIe)
      DEF1(J)=(5./48.)*Ma*SPAN**2./(Ec*Ie1)
      DEF3(J)=(5./48.)*Ma*SPAN**2./(Ec*Ie3)
      ELSE
      DEFBR(J)=(3.*Ma*SPAN**2.-4.*Ma*XX**2.)/(24.*Ec*BRIe)
      DEF1(J)=(3.*Ma*SPAN**2.-4.*Ma*XX**2.)/(24.*Ec*Ie1)
      DEF3(J)=(3.*Ma*SPAN**2.-4.*Ma*XX**2.)/(24.*Ec*Ie3)
      END IF
C*****Because Ma will be entered as integer it is recommended that it is
C*****printed as so . It was previously  declared as  real to avoid  mix
C*****Mode computation . Also fc and Fcu are usually given as whole numb
C*****ers and thus should be printed as integers
      MMa=Ma
      Mfc=fc
      MFcu=fcu
C*****PRINT OUTPUT :
      IF((J.EQ.1).OR.(ICOUNT.EQ.48)) THEN
      ICOUNT=0
      WRITE(3,'(1H1)')
      WRITE(3,21)ASAD4
   21 FORMAT(///T7,'.',72('-'),'.',/T7,'|',T11,'|',1X,'d',T16,'|',T23,'|
     *',1X,'d''',T28,'|',T35,'|',1X,'bw',T40,'|',1X,'hf',T45,'|',T52,'|'
     1,'As''',T57,'|',T63,'|',T80,'|',/T7,'|','BM',T11,'|',1X,'--',T16,'
     1|',2X,'d',T23,'|',1X,'--',T28,'|',2X,'bw',T35,'|',1X,'--',T40,'|',
     11X,'--',T45,'|',2X,'As',T52,'|','--',T57,'|',1X,A3,T63,'|',1X,'spa
     1n',2X,'|',3X,'Ma',T80,'|')
      WRITE(3,22)ASAD1,ASAD1,ASAD1,ASAD2,ASAD1,ASAD3
   22 FORMAT(T7,'|','#',T11,'|',1X,'h',T16,'|',2X,A2,T23,'|',1X,'d',T28,
     1'|',2X,A2,T35,'|',1X,'be',T40,'|',1X,'d',T45,'|','SQ.',A2,T52,'|',
     1'As',T57,'|',1X,A3,T63,'|',2X,A2,3X,'|',1X,A5,T80,'|')
      WRITE(3,24)
   24 FORMAT(T7,'|'3('-'),'|',4('-'),'|',6('-'),'|',4('-'),'|',6('-'),'|
```

```
     1',4('-'),'|',4('-'),'|',6('-'),'|',4('-'),'|',5('-'),'|',7('-'),'|
    *',8('-'),'|')
     END IF
     IF(fc.EQ.0.0) THEN
     WRITE(3,25)J,dh,d,dc/d,bbw,1/Rb,Xhfd,As,Asc/As,MFcu,span,MMa
     ELSE
     WRITE(3,25)J,dh,d,dc/d,bbw,1/Rb,Xhfd,As,Asc/As,Mfc,span,MMa
     END IF
25   FORMAT(T7,'|',I3,T11,'|',F4.3,'|',F6.3,'|',F4.2,'|',F6.2,'|',F4.2,
    *'|',F4.2,'|',F6.3,'|',F4.2,'|',I5,'|',F7.2,'|',I8,'|')
     IF((ICOUNT.EQ.47).OR.(J.EQ.I)) THEN
     WRITE(3,26)
26   FORMAT(T7,'.',72('_'),'.')
     END IF
10   CONTINUE
     ICOUNT=0
     DO 30 J=1,I
     ICOUNT=ICOUNT+1
     difBR=((DEFBR(J)-MESDEF(J))/MESDEF(J))*100.
     dif1=((DEF1(J)-MESDEF(J))/MESDEF(J))*100.
     dif3=((DEF3(J)-MESDEF(J))/MESDEF(J))*100.
     IF(RSPAN(J).EQ.0.5)ASAD5='C.P.L'
     IF(RSPAN(J).NE.0.5)ASAD6='P.L'
     IF(RSPAN(J).EQ.0)ASAD5='U.D.L'
     IF((J.EQ.1).OR.(ICOUNT.EQ.48)) THEN
     ICOUNT=0
     WRITE(3,'(1H1)')
     WRITE(3,27) ASAD1,ASAD1,ASAD1,ASAD1
27   FORMAT(///,T6,'.',73('-'),'.',/T6,'|',T10,'|','meas''d','|',3('def
    1.by','|'),3(2X,'%',3X,'|'),6X,'|',2X,'Ma',2X,'|',T80,'|',/T6,'|','
    1BM',1X,'|',1X,'def.',1X,'|','Brnson','|','Ref.4',1X,'|','Model',1X
    1,'|',3('error',1X,'|'),2X,'P',3X,'|',1X,'----',1X,'|',1X,'load',1X
    1,'|',/T6,'|','#',2X,'|',4(2X,A2,2X,'|'),'Brnson','|','Ref.4',1X,'|
    1','Model',1X,'|',1X,'(%)',2X,'|',2X,'Mcr',1X,'|',1X,'type',1X,'|',
    1/T6,'|',3('-'),'|',10(6('-'),'|'))
     END IF
     IF((RSPAN(J).EQ.0.5).OR.(RSPAN(J).EQ.0))THEN
     WRITE(3,'(T6,A,I3,T10,A,4(F6.3,A),3(F6.2,A),F6.2,A,F6.2,A,A5,T80,A
    *)')'|',J,'|',MESDEF(J),'|',DEFBR(J),'|',DEF3(J),'|',DEF1(J),'|',di
    1fBR,'|',dif3,'|',dif1,'|',Rho(J),'|',RM(J),'|',ASAD5,'|'
     ELSE
     WRITE(3,'(T6,A,I3,T10,A,4(F6.3,A),3(F6.2,A),F6.2,A,F6.2,A,F3.2,A3,
    1T80,A)')'|',J,'|',MESDEF(J),'|',DEFBR(J),'|',DEF3(J),'|',DEF1(J),'
    *|',difBR,'|',dif3,'|',dif1,'|',Rho(J),'|',RM(J),'|',RSPAN(J),ASAD6
    *,'|'
     END IF
     IF((ICOUNT.EQ.47).OR.(J.EQ.I)) THEN
     WRITE(3,28)
28   FORMAT(T6,'.',73('_'),'.')
     END IF
30   CONTINUE
     STOP
     END
```

OUTPUT OF PROG.4.4.3.1

Table C2.1. Data read from data file "CERA"

BM #	d -- h	d in	d' -- d	bw in	bw -- be	hf -- d	As SQ.in	As' -- As	fcu psi	span in	Ma lb.in
1	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	5320	180.00	616000
2	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	5320	180.00	862400
3	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	5400	180.00	829000
4	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	5400	180.00	1160600
5	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	5350	180.00	868000
6	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	5350	180.00	1215200
7	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4980	180.00	862500
8	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4980	180.00	1207500
9	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4600	180.00	554500
10	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4600	180.00	776300
11	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4580	180.00	761500
12	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4580	180.00	1066100
13	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4500	180.00	672000
14	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4500	180.00	940800
15	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4520	180.00	773000
16	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4520	180.00	1082200
17	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4550	180.00	714000
18	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4550	180.00	999600
19	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4600	180.00	795000
20	.868	13.125	0.00	8.00	1.00	0.00	2.446	0.00	4600	180.00	1113000
21	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4940	180.00	560000
22	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4940	180.00	784000
23	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4840	180.00	756000
24	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4840	180.00	1058400
25	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4910	180.00	795000
26	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4910	180.00	1113000
27	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4810	180.00	795000
28	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4810	180.00	1113000
29	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4880	180.00	756000
30	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4880	180.00	1058400
31	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4910	180.00	560000
32	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4910	180.00	784000
33	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4910	180.00	817500
34	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4910	180.00	1144500
35	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4780	180.00	773000
36	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4780	180.00	1082200
37	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4750	180.00	823000
38	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4750	180.00	1152200
39	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4870	180.00	806500
40	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4870	180.00	1129100
41	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4830	180.00	817500
42	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4830	180.00	1144500
43	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4440	180.00	661000
44	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4440	180.00	925400
45	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4380	180.00	801000
46	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4380	180.00	1121400
47	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4410	180.00	806500
48	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4410	180.00	1129100

Table C2.1 (cont'd)

BM #	d/h	d in	d'/d	bw in	bw/be	hf/d	As SQ.in	As'/As	fcu psi	span in	Ma lb.in
49	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4300	180.00	829000
50	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4300	180.00	1160600
51	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4340	180.00	817500
52	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4340	180.00	1144500
53	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4440	180.00	717000
54	.844	13.500	0.00	8.00	1.00	0.00	2.354	0.00	4440	180.00	1003800
55	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5620	180.00	689000
56	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5620	180.00	964600
57	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5490	180.00	840000
58	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5490	180.00	1176000
59	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5540	180.00	829000
60	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5540	180.00	1160600
61	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5450	180.00	834500
62	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5450	180.00	1168300
63	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5580	180.00	806500
64	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5580	180.00	1129100
65	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5740	180.00	761500
66	.867	13.875	0.00	8.00	1.00	0.00	2.364	0.00	5740	180.00	1066100
67	.868	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3843	180.00	532000
68	.868	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3843	180.00	744800
69	.868	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3843	180.00	526500
70	.868	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3843	180.00	737100
71	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3843	180.00	605000
72	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3843	180.00	847000
73	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3843	180.00	549000
74	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3843	180.00	768600
75	.868	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4175	180.00	756000
76	.868	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4175	180.00	1058400
77	.868	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4190	180.00	795000
78	.868	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4190	180.00	1113000
79	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4203	180.00	717000
80	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4203	180.00	1003800
81	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4216	180.00	767000
82	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4216	180.00	1073800
83	.833	13.125	0.00	3.50	0.44	0.50	2.405	0.00	4820	180.00	761500
84	.833	13.125	0.00	3.50	0.44	0.50	2.405	0.00	4820	180.00	1066100
85	.833	13.125	0.00	5.50	0.69	0.50	2.405	0.00	4870	180.00	795000
86	.833	13.125	0.00	5.50	0.69	0.50	2.405	0.00	4870	180.00	1113000
87	.833	13.125	0.00	7.75	0.97	0.50	2.405	0.00	4900	180.00	829000
88	.833	13.125	0.00	7.75	0.97	0.50	2.405	0.00	4900	180.00	1160600
89	.820	13.125	0.00	6.25	1.00	0.00	2.190	0.00	4320	180.00	554500
90	.820	13.125	0.00	6.25	1.00	0.00	2.190	0.00	4320	180.00	776300
91	.822	13.150	0.00	8.00	1.00	0.00	2.367	0.00	4550	180.00	633000
92	.822	13.150	0.00	8.00	1.00	0.00	2.367	0.00	4550	180.00	886200
93	.820	13.125	0.00	9.75	1.00	0.00	2.355	0.00	4500	180.00	644000
94	.820	13.125	0.00	9.75	1.00	0.00	2.355	0.00	4500	180.00	901600
95	.833	13.125	0.00	3.50	0.44	0.50	2.405	0.00	5270	180.00	739000
96	.833	13.125	0.00	3.50	0.44	0.50	2.405	0.00	5270	180.00	1034600

Table C2.1 (cont'd)

BM #	d / h	d in	d' / d	bw in	bw / be	hf / d	As SQ.in	As' / As	fcu psi	span in	Ma lb.in
97	.833	13.125	0.00	5.50	0.69	0.50	2.405	0.00	5320	180.00	795000
98	.833	13.125	0.00	5.50	0.69	0.50	2.405	0.00	5320	180.00	1113000
99	.833	13.125	0.00	7.75	0.97	0.50	2.405	0.00	5050	180.00	795000
100	.833	13.125	0.00	7.75	0.97	0.50	2.405	0.00	5050	180.00	1113000
101	.820	13.125	0.00	6.25	1.00	0.00	2.190	0.00	5210	180.00	750500
102	.820	13.125	0.00	6.25	1.00	0.00	2.190	0.00	5210	180.00	1050700
103	.820	13.125	0.00	8.00	1.00	0.00	2.362	0.00	5300	180.00	750500
104	.820	13.125	0.00	8.00	1.00	0.00	2.362	0.00	5300	180.00	1050700
105	.820	13.125	0.00	9.75	1.00	0.00	2.355	0.00	5340	180.00	873500
106	.820	13.125	0.00	9.75	1.00	0.00	2.355	0.00	5340	180.00	1222900
107	.766	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3800	180.00	823000
108	.766	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3800	180.00	1152200
109	.766	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3880	180.00	717000
110	.766	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3880	180.00	1003800
111	.814	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3950	180.00	784000
112	.814	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3950	180.00	1097600
113	.814	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4000	180.00	722500
114	.814	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4000	180.00	1011500
115	.882	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4190	180.00	756000
116	.882	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4190	180.00	1058400
117	.882	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4240	180.00	733500
118	.882	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4240	180.00	1026900
119	.882	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4600	180.00	722500
120	.882	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4600	180.00	1011500
121	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4800	180.00	728000
122	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4800	180.00	1019200
123	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4850	180.00	728000
124	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4850	180.00	1019200
125	.882	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4700	180.00	644000
126	.882	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4700	180.00	901600
127	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4950	180.00	700000
128	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4950	180.00	980000
129	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	5160	180.00	817500
130	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	5160	180.00	1144500
131	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	2950	180.00	610500
132	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	2950	180.00	854700
133	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	2985	180.00	627000
134	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	2985	180.00	877800
135	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4960	180.00	728000
136	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4960	180.00	1019200
137	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	2830	180.00	582500
138	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	2830	180.00	815500
139	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3475	180.00	661000
140	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3475	180.00	925400
141	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4900	180.00	733500
142	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4900	180.00	1026900
143	.820	13.125	0.00	8.00	1.00	0.00	2.362	0.00	4770	180.00	638500
144	.820	13.125	0.00	8.00	1.00	0.00	2.362	0.00	4770	180.00	893900

Table C2.1 (cont'd)

BM #	d/h	d in	d'/d	bw in	bw/be	hf/d	As SQ.in	As'/As	fcu psi	span in	Ma lb.in
145	.820	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4980	180.00	515000
146	.820	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4980	180.00	721000
147	.820	13.125	0.00	8.00	1.00	0.00	1.176	0.00	5050	180.00	330500
148	.820	13.125	0.00	8.00	1.00	0.00	1.176	0.00	5050	180.00	462700
149	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4700	180.00	554500
150	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4700	180.00	776300
151	.833	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4500	180.00	425500
152	.833	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4500	180.00	595700
153	.833	13.125	0.00	8.00	1.00	0.00	1.228	0.00	4940	180.00	325000
154	.833	13.125	0.00	8.00	1.00	0.00	1.228	0.00	4940	180.00	455000
155	.820	13.125	0.00	8.00	1.00	0.00	2.362	0.00	4850	180.00	806500
156	.820	13.125	0.00	8.00	1.00	0.00	2.362	0.00	4850	180.00	1129100
157	.820	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4775	180.00	750500
158	.820	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4775	180.00	1050700
159	.820	13.125	0.00	8.00	1.00	0.00	1.176	0.00	5020	180.00	498500
160	.820	13.125	0.00	8.00	1.00	0.00	1.176	0.00	5020	180.00	697900
161	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	5190	180.00	795000
162	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	5190	180.00	1113000
163	.833	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4960	180.00	616000
164	.833	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4960	180.00	862400
165	.833	13.125	0.00	8.00	1.00	0.00	1.228	0.00	4900	180.00	403000
166	.833	13.125	0.00	8.00	1.00	0.00	1.228	0.00	4900	180.00	564200
167	.820	13.125	0.00	8.00	1.00	0.00	2.362	0.00	4340	180.00	683000
168	.820	13.125	0.00	8.00	1.00	0.00	2.362	0.00	4340	180.00	956200
169	.820	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4150	180.00	644000
170	.820	13.125	0.00	8.00	1.00	0.00	1.764	0.00	4150	180.00	901600
171	.820	13.125	0.00	8.00	1.00	0.00	1.176	0.00	4540	180.00	493000
172	.820	13.125	0.00	8.00	1.00	0.00	1.176	0.00	4540	180.00	690200
173	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4085	180.00	717000
174	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4085	180.00	1003800
175	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4048	180.00	761500
176	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4048	180.00	1066100
177	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4150	180.00	638500
178	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4150	180.00	893900
179	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4279	180.00	834500
180	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4279	180.00	1168300
181	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3410	180.00	717000
182	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	3410	180.00	1003800
183	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4400	180.00	750500
184	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4400	180.00	1050700
185	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4670	180.00	817500
186	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4670	180.00	1144500
187	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4360	180.00	776000
188	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4360	180.00	1086400
189	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4360	180.00	752500
190	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4360	180.00	1053500
191	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4360	180.00	718000
192	.833	13.125	0.00	8.00	1.00	0.00	2.405	0.00	4360	180.00	1005200

Table C2.1 (cont'd)

BM #	d/h	d in	d'/d	bw in	bw/be	hf/d	As SQ.in	As'/As	fcu psi	span in	Ma lb.in
193	.889	14.000	0.00	3.50	0.44	0.46	1.176	0.00	4080	180.00	493000
194	.889	14.000	0.00	3.50	0.44	0.46	1.176	0.00	4080	180.00	690200
195	.889	14.000	0.00	5.50	0.69	0.46	1.176	0.00	4060	180.00	521000
196	.889	14.000	0.00	5.50	0.69	0.46	1.176	0.00	4060	180.00	729400
197	.889	14.000	0.00	7.75	0.97	0.46	1.176	0.00	3920	180.00	504000
198	.889	14.000	0.00	7.75	0.97	0.46	1.176	0.00	3920	180.00	705600
199	.898	14.375	0.00	6.25	1.00	0.00	1.204	0.00	4040	180.00	493000
200	.898	14.375	0.00	6.25	1.00	0.00	1.204	0.00	4040	180.00	690200
201	.898	14.375	0.00	8.00	1.00	0.00	1.208	0.00	4110	180.00	504000
202	.898	14.375	0.00	8.00	1.00	0.00	1.208	0.00	4110	180.00	705600
203	.898	14.375	0.00	9.75	1.00	0.00	1.205	0.00	3940	180.00	560000
204	.898	14.375	0.00	9.75	1.00	0.00	1.205	0.00	3940	180.00	784000
205	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4420	180.00	358500
206	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4420	180.00	501900
207	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4640	180.00	375000
208	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4640	180.00	525000
209	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4440	180.00	291000
210	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4440	180.00	407400
211	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4550	180.00	302500
212	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4550	180.00	423500
213	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4360	180.00	375000
214	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4360	180.00	525000
215	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4540	180.00	375000
216	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4540	180.00	525000
217	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4440	180.00	302500
218	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4440	180.00	423500
219	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4560	180.00	291000
220	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4560	180.00	407400
221	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	3950	180.00	375000
222	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	3950	180.00	525000
223	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4120	180.00	369500
224	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4120	180.00	517300
225	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	3900	180.00	291000
226	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	3900	180.00	407400
227	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4170	180.00	308000
228	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4170	180.00	431200
229	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4250	180.00	353000
230	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4250	180.00	494200
231	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4320	180.00	364000
232	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4320	180.00	509600
233	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4260	180.00	302500
234	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4260	180.00	423500
235	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4340	180.00	297000
236	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4340	180.00	415800
237	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4670	180.00	364000
238	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4670	180.00	509600
239	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4600	180.00	297000
240	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4600	180.00	415800

Table C2.1 (cont'd)

BM #	d -- h	d in	d' -- d	bw in	bw -- be	hf -- d	As SQ.in	As' -- As	fcu psi	span in	Ma lb.in
241	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4870	180.00	297000
242	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4870	180.00	415800
243	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	3990	180.00	364000
244	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	3990	180.00	509600
245	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4370	180.00	364000
246	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4370	180.00	509600
247	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4140	180.00	291000
248	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4140	180.00	407400
249	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4320	180.00	302500
250	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4320	180.00	423500
251	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4040	180.00	364000
252	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4040	180.00	509600
253	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4040	180.00	375000
254	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4040	180.00	525000
255	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4040	180.00	375000
256	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4040	180.00	525000
257	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4040	180.00	375000
258	.885	13.500	0.00	7.00	1.00	0.00	0.879	0.00	4040	180.00	525000

Table C2.2. Results obtained considering beams of data file "CERA"

BM #	meas'd def. in	def.by Brnson in	def.by Ref.4 in	def.by Model in	% error Brnson	% error Ref.4	% error Model	P (%)	Ma ---- Mcr	load type
1	0.377	0.377	0.366	0.369	-0.13	-2.90	-2.23	2.33	4.07	.28P.L
2	0.544	0.530	0.522	0.516	-2.60	-4.11	-5.13	2.33	5.70	.28P.L
3	0.516	0.508	0.500	0.495	-1.47	-3.14	-4.00	2.33	5.44	.28P.L
4	0.759	0.713	0.707	0.693	-6.02	-6.88	-8.63	2.33	7.61	.28P.L
5	0.451	0.533	0.525	0.519	18.19	16.37	15.09	2.33	5.72	.28P.L
6	0.674	0.748	0.741	0.727	10.93	10.01	7.82	2.33	8.01	.28P.L
7	0.380	0.533	0.526	0.521	40.34	38.35	37.00	2.33	5.89	.28P.L
8	0.592	0.748	0.742	0.729	26.32	25.36	23.11	2.33	8.25	.28P.L
9	0.318	0.343	0.334	0.338	7.92	4.97	6.24	2.33	3.94	.28P.L
10	0.483	0.483	0.476	0.473	0.01	-1.53	-2.05	2.33	5.52	.28P.L
11	0.451	0.474	0.466	0.464	5.08	3.43	2.95	2.33	5.42	.28P.L
12	0.677	0.665	0.659	0.650	-1.79	-2.63	-3.99	2.33	7.59	.28P.L
13	0.389	0.418	0.410	0.411	7.53	5.46	5.54	2.33	4.83	.28P.L
14	0.596	0.587	0.581	0.575	-1.46	-2.49	-3.56	2.33	6.76	.28P.L
15	0.457	0.482	0.474	0.472	5.41	3.82	3.29	2.33	5.54	.28P.L
16	0.673	0.676	0.670	0.661	0.40	-0.42	-1.81	2.33	7.76	.28P.L
17	0.446	0.444	0.437	0.436	-0.38	-2.12	-2.32	2.33	5.10	.28P.L
18	0.578	0.624	0.618	0.610	7.89	6.86	5.53	2.33	7.14	.28P.L
19	0.386	0.495	0.488	0.484	28.18	26.30	25.51	2.33	5.65	.28P.L
20	0.616	0.694	0.688	0.678	12.66	11.76	10.11	2.33	7.91	.28P.L
21	0.206	0.345	0.330	0.341	67.72	60.12	65.57	2.29	3.54	.28P.L
22	0.359	0.489	0.476	0.478	36.18	32.63	33.11	2.29	4.96	.28P.L
23	0.448	0.472	0.459	0.462	5.37	2.51	3.12	2.29	4.83	.28P.L
24	0.668	0.664	0.654	0.647	-0.65	-2.13	-3.18	2.29	6.76	.28P.L
25	0.434	0.496	0.484	0.485	14.33	11.43	11.74	2.29	5.04	.28P.L
26	0.705	0.694	0.684	0.676	-1.57	-2.96	-4.16	2.29	7.06	.29P.L
27	0.499	0.497	0.485	0.486	-0.37	-2.82	-2.57	2.29	5.10	.28P.L
28	0.743	0.699	0.689	0.681	-5.99	-7.25	-8.39	2.29	7.14	.28P.L
29	0.474	0.472	0.459	0.461	-0.49	-3.22	-2.64	2.29	4.81	.28P.L
30	0.708	0.663	0.653	0.646	-6.33	-7.74	-8.74	2.29	6.74	.28P.L
31	0.327	0.346	0.330	0.341	5.73	0.98	4.38	2.29	3.55	.28P.L
32	0.496	0.489	0.477	0.478	-1.37	-3.93	-3.58	2.29	4.97	.28P.L
33	0.502	0.510	0.498	0.499	1.68	-0.77	-0.67	2.29	5.19	.28P.L
34	0.755	0.717	0.708	0.698	-5.02	-6.27	-7.53	2.29	7.26	.28P.L
35	0.477	0.483	0.471	0.473	1.35	-1.25	-0.82	2.29	4.97	.28P.L
36	0.713	0.679	0.670	0.662	-4.71	-6.05	-7.11	2.29	6.96	.28P.L
37	0.549	0.516	0.504	0.504	-6.09	-8.24	-8.18	2.29	5.31	.28P.L
38	0.799	0.724	0.715	0.706	-9.38	-10.50	-11.68	2.29	7.43	.28P.L
39	0.503	0.504	0.492	0.492	0.17	-2.27	-2.10	2.29	5.14	.28P.L
40	0.767	0.708	0.698	0.689	-7.70	-8.94	-10.11	2.29	7.19	.28P.L
41	0.460	0.511	0.499	0.500	11.14	8.52	8.63	2.29	5.23	.28P.L
42	0.691	0.718	0.709	0.700	3.93	2.59	1.24	2.29	7.32	.28P.L
43	0.362	0.394	0.382	0.387	8.73	5.46	6.85	2.18	4.27	.28P.L
44	0.519	0.554	0.545	0.542	6.79	4.99	4.36	2.18	5.98	.28P.L
45	0.462	0.479	0.469	0.469	3.78	1.56	1.53	2.18	5.21	.28P.L
46	0.708	0.673	0.666	0.657	-4.89	-6.00	-7.25	2.18	7.30	.28P.L
47	0.490	0.483	0.472	0.472	-1.53	-3.63	-3.64	2.18	5.23	.28P.L
48	0.729	0.678	0.670	0.661	-7.04	-8.12	-9.32	2.18	7.33	.28P.L

Table C2.2 (cont'd)

BM #	meas'd def. in	def.by Brnson in	def.by Ref.4 in	def.by Model in	% error Brnson	% error Ref.4	% error Model	P (%)	Ma / Mcr	load type
49	0.483	0.497	0.487	0.486	2.95	0.93	0.58	2.18	5.45	.28P.L
50	0.734	0.698	0.691	0.680	-4.89	-5.91	-7.34	2.18	7.63	.28P.L
51	0.502	0.490	0.480	0.479	-2.42	-4.41	-4.61	2.18	5.35	.28P.L
52	0.806	0.688	0.680	0.670	-14.66	-15.61	-16.82	2.18	7.48	.28P.L
53	0.462	0.428	0.417	0.420	-7.40	-9.84	-9.17	2.18	4.64	.28P.L
54	0.688	0.602	0.593	0.588	-12.55	-13.82	-14.61	2.18	6.49	.28P.L
55	0.383	0.376	0.364	0.375	-1.95	-5.00	-2.16	2.13	3.96	.28P.L
56	0.561	0.529	0.520	0.525	-5.64	-7.30	-6.44	2.13	5.54	.28P.L
57	0.470	0.461	0.451	0.458	-1.88	-4.01	-2.65	2.13	4.88	.28P.L
58	0.708	0.648	0.640	0.641	-8.48	-9.57	-9.52	2.13	6.84	.28P.L
59	0.524	0.455	0.444	0.451	-13.25	-15.20	-13.86	2.13	4.80	.28P.L
60	0.777	0.639	0.631	0.632	-17.78	-18.80	-18.67	2.13	6.72	.28P.L
61	0.464	0.458	0.448	0.455	-1.20	-3.35	-2.00	2.13	4.87	.28P.L
62	0.688	0.644	0.636	0.637	-6.38	-7.50	-7.47	2.13	6.82	.28P.L
63	0.450	0.442	0.431	0.439	-1.85	-4.18	-2.46	2.13	4.65	.28P.L
64	0.669	0.621	0.613	0.615	-7.18	-8.40	-8.13	2.13	6.51	.28P.L
65	0.441	0.415	0.404	0.414	-5.84	-8.39	-6.16	2.13	4.33	.28P.L
66	0.656	0.584	0.576	0.579	-10.91	-12.26	-11.66	2.13	6.06	.28P.L
67	0.370	0.338	0.330	0.334	-8.59	-10.76	-9.86	2.29	4.14	.28P.L
68	0.580	0.476	0.469	0.467	-18.00	-19.08	-19.48	2.29	5.79	.28P.L
69	0.339	0.335	0.327	0.330	-1.29	-3.66	-2.63	2.29	4.09	.28P.L
70	0.555	0.471	0.464	0.462	-15.20	-16.34	-16.73	2.29	5.73	.28P.L
71	0.341	0.384	0.373	0.379	12.63	9.25	11.23	2.29	4.34	.28P.L
72	0.567	0.541	0.532	0.531	-4.66	-6.25	-6.34	2.29	6.07	.28P.L
73	0.351	0.348	0.335	0.344	-0.99	-4.47	-1.96	2.29	3.94	.28P.L
74	0.530	0.490	0.480	0.482	-7.54	-9.38	-9.07	2.29	5.51	.28P.L
75	0.547	0.480	0.473	0.470	-12.32	-13.58	-14.09	2.29	5.64	.28P.L
76	0.851	0.673	0.667	0.658	-20.96	-21.57	-22.69	2.29	7.90	.28P.L
77	0.547	0.504	0.498	0.494	-7.79	-9.00	-9.69	2.29	5.92	.28P.L
78	0.816	0.707	0.702	0.692	-13.33	-13.94	-15.25	2.29	8.29	.28P.L
79	0.465	0.453	0.442	0.445	-2.54	-4.97	-4.22	2.29	4.92	.28P.L
80	0.829	0.637	0.628	0.624	-23.17	-24.23	-24.79	2.29	6.88	.28P.L
81	0.469	0.485	0.474	0.476	3.46	1.15	1.55	2.29	5.25	.28P.L
82	0.671	0.681	0.673	0.667	1.55	0.32	-0.63	2.29	7.35	.28P.L
83	0.450	0.479	0.477	0.479	6.39	6.08	6.41	5.23	9.26	.28P.L
84	0.687	0.670	0.669	0.670	-2.43	-2.58	-2.42	5.23	12.96	.28P.L
85	0.569	0.499	0.492	0.493	-12.35	-13.53	-13.35	3.33	6.73	.28P.L
86	0.712	0.699	0.694	0.690	-1.85	-2.54	-3.06	3.33	9.43	.28P.L
87	0.481	0.518	0.506	0.507	7.72	5.30	5.32	2.36	5.39	.28P.L
88	0.735	0.728	0.719	0.709	-1.02	-2.23	-3.51	2.36	7.55	.28P.L
89	0.360	0.406	0.394	0.405	12.65	9.34	12.50	2.67	4.65	.28P.L
90	0.558	0.570	0.561	0.567	2.15	0.51	1.61	2.67	6.51	.28P.L
91	0.362	0.397	0.382	0.391	9.76	5.45	7.97	2.25	4.04	.28P.L
92	0.548	0.561	0.548	0.547	2.31	0.04	-0.12	2.25	5.66	.28P.L
93	0.344	0.376	0.357	0.385	9.40	3.78	11.82	1.84	3.39	.28P.L
94	0.509	0.536	0.520	0.542	5.35	2.09	6.39	1.84	4.75	.28P.L
95	0.539	0.461	0.459	0.460	-14.48	-14.82	-14.74	5.23	8.59	.28P.L
96	0.749	0.645	0.644	0.643	-13.83	-14.01	-14.10	5.23	12.03	.28P.L

Table C2.2 (cont'd)

BM #	meas'd def. in	def.by Brnson in	def.by Ref.4 in	def.by Model in	% error Brnson	% error Ref.4	% error Model	P (%)	Ma / Mcr	load type
97	0.478	0.495	0.487	0.488	3.49	1.91	2.00	3.33	6.44	.28P.L
98	0.722	0.693	0.688	0.683	-3.97	-4.75	-5.46	3.33	9.02	.28P.L
99	0.465	0.495	0.483	0.484	6.48	3.80	4.08	2.36	5.09	.28P.L
100	0.711	0.696	0.686	0.678	-2.17	-3.51	-4.70	2.36	7.13	.28P.L
101	0.458	0.541	0.529	0.537	18.14	15.57	17.20	2.67	5.73	.28P.L
102	0.709	0.759	0.750	0.751	7.09	5.84	5.99	2.67	8.03	.28P.L
103	0.463	0.469	0.452	0.462	1.24	-2.40	-0.19	2.25	4.44	.28P.L
104	0.714	0.660	0.647	0.647	-7.51	-9.36	-9.38	2.25	6.22	.28P.L
105	0.494	0.510	0.490	0.520	3.23	-0.83	5.25	1.84	4.23	.28P.L
106	0.748	0.721	0.705	0.729	-3.62	-5.80	-2.56	1.84	5.92	.28P.L
107	0.497	0.522	0.502	0.517	5.13	1.04	3.94	2.29	5.02	.28P.L
108	0.743	0.736	0.720	0.723	-0.97	-3.10	-2.66	2.29	7.03	.28P.L
109	0.453	0.452	0.430	0.449	-0.19	-5.18	-0.88	2.29	4.33	.28P.L
110	0.690	0.639	0.621	0.629	-7.42	-10.03	-8.87	2.29	6.06	.28P.L
111	0.482	0.498	0.486	0.490	3.35	0.74	1.70	2.29	5.29	.28P.L
112	0.731	0.700	0.690	0.686	-4.26	-5.58	-6.12	2.29	7.41	.28P.L
113	0.481	0.458	0.444	0.451	-4.81	-7.62	-6.21	2.29	4.85	.28P.L
114	0.724	0.644	0.633	0.632	-11.06	-12.50	-12.76	2.29	6.78	.28P.L
115	0.493	0.480	0.474	0.470	-2.69	-3.82	-4.72	2.29	5.82	.28P.L
116	0.761	0.673	0.668	0.658	-11.61	-12.17	-13.58	2.29	8.15	.28P.L
117	0.469	0.465	0.459	0.455	-0.87	-2.11	-2.95	2.29	5.62	.28P.L
118	0.724	0.652	0.648	0.637	-9.95	-10.56	-11.98	2.29	7.86	.28P.L
119	0.513	0.455	0.448	0.444	-11.37	-12.64	-13.41	2.29	5.31	.28P.L
120	0.766	0.638	0.633	0.622	-16.73	-17.38	-18.81	2.29	7.43	.28P.L
121	0.614	0.455	0.442	0.445	-25.96	-28.08	-27.48	2.29	4.67	.28P.L
122	0.771	0.639	0.629	0.623	-17.07	-18.38	-19.14	2.29	6.54	.28P.L
123	0.292	0.454	0.441	0.445	55.52	51.01	52.30	2.29	4.65	.28P.L
124	0.390	0.639	0.629	0.623	63.79	61.17	59.65	2.29	6.51	.28P.L
125	0.392	0.404	0.397	0.395	3.05	1.19	0.75	2.29	4.68	.28P.L
126	0.587	0.567	0.562	0.553	-3.38	-4.33	-5.81	2.29	6.56	.28P.L
127	0.499	0.435	0.422	0.427	-12.76	-15.53	-14.53	2.29	4.42	.28P.L
128	0.750	0.613	0.602	0.597	-18.29	-19.73	-20.38	2.29	6.19	.28P.L
129	0.510	0.508	0.495	0.497	-0.40	-2.95	-2.63	2.29	5.06	.28P.L
130	0.729	0.714	0.704	0.695	-2.08	-3.46	-4.63	2.29	7.08	.28P.L
131	0.406	0.396	0.387	0.392	-2.51	-4.64	-3.47	2.29	5.00	.28P.L
132	0.643	0.556	0.549	0.549	-13.55	-14.58	-14.67	2.29	7.00	.28P.L
133	0.484	0.406	0.398	0.402	-16.05	-17.82	-16.91	2.29	5.10	.28P.L
134	0.732	0.571	0.564	0.563	-22.05	-22.95	-23.09	2.29	7.14	.28P.L
135	0.464	0.453	0.440	0.443	-2.35	-5.26	-4.42	2.29	4.60	.28P.L
136	0.598	0.637	0.627	0.621	6.60	4.84	3.83	2.29	6.43	.28P.L
137	0.513	0.378	0.370	0.375	-26.23	-27.90	-26.87	2.29	4.87	.28P.L
138	0.753	0.532	0.525	0.525	-29.40	-30.27	-30.25	2.29	6.82	.28P.L
139	0.455	0.424	0.414	0.418	-6.83	-8.97	-8.03	2.29	4.99	.28P.L
140	0.689	0.596	0.588	0.586	-13.57	-14.65	-14.97	2.29	6.98	.28P.L
141	0.512	0.457	0.444	0.448	-10.71	-13.30	-12.59	2.29	4.66	.28P.L
142	0.768	0.643	0.633	0.627	-16.27	-17.61	-18.42	2.29	6.52	.28P.L
143	0.366	0.401	0.385	0.395	9.59	5.07	7.86	2.25	3.98	.28P.L
144	0.558	0.566	0.553	0.553	1.50	-0.88	-0.92	2.25	5.58	.28P.L

Table C2.2 (cont'd)

BM #	meas'd def. in	def.by Brnson in	def.by Ref.4 in	def.by Model in	% error Brnson	% error Ref.4	% error Model	P (%)	Ma ---- Mcr	load type
145	0.251	0.383	0.361	0.397	52.54	43.85	58.19	1.68	3.14	.28P.L
146	0.389	0.550	0.530	0.564	41.34	36.29	45.06	1.68	4.40	.28P.L
147	0.296	0.273	0.267	0.247	-7.77	-9.82	-16.71	1.12	2.00	.28P.L
148	0.463	0.439	0.420	0.420	-5.14	-9.29	-9.30	1.12	2.81	.28P.L
149	0.297	0.344	0.329	0.340	15.80	10.77	14.42	2.29	3.60	.28P.L
150	0.560	0.486	0.474	0.476	-13.15	-15.32	-14.99	2.29	5.03	.28P.L
151	0.360	0.316	0.298	0.327	-12.27	-17.33	-9.22	1.68	2.82	.28P.L
152	0.502	0.456	0.439	0.468	-9.14	-12.56	-6.84	1.68	3.95	.28P.L
153	0.266	0.270	0.262	0.252	1.66	-1.51	-5.10	1.17	2.06	.28P.L
154	0.396	0.425	0.407	0.418	7.41	2.75	5.51	1.17	2.88	.28P.L
155	0.471	0.509	0.495	0.498	8.12	5.03	5.83	2.25	4.99	.28P.L
156	0.730	0.716	0.705	0.698	-1.91	-3.45	-4.40	2.25	6.99	.28P.L
157	0.624	0.576	0.557	0.589	-7.75	-10.71	-5.60	1.68	4.68	.28P.L
158	0.911	0.812	0.797	0.826	-10.85	-12.48	-9.36	1.68	6.55	.28P.L
159	0.538	0.482	0.461	0.468	-10.42	-14.27	-12.95	1.12	3.03	.28P.L
160	0.778	0.706	0.683	0.720	-9.30	-12.24	-7.51	1.12	4.24	.28P.L
161	0.512	0.493	0.480	0.483	-3.63	-6.24	-5.70	2.29	4.91	.28P.L
162	0.771	0.694	0.683	0.676	-10.03	-11.37	-12.33	2.29	6.87	.28P.L
163	0.451	0.468	0.449	0.481	3.69	-0.42	6.66	1.68	3.89	.28P.L
164	0.674	0.663	0.647	0.676	-1.65	-3.94	0.33	1.68	5.44	.28P.L
165	0.425	0.366	0.350	0.352	-13.86	-17.61	-17.07	1.17	2.56	.28P.L
166	0.638	0.546	0.526	0.553	-14.36	-17.56	-13.25	1.17	3.58	.28P.L
167	0.487	0.434	0.420	0.427	-10.84	-13.80	-12.42	2.25	4.47	.28P.L
168	0.553	0.611	0.600	0.597	10.57	8.53	7.99	2.25	6.25	.28P.L
169	0.448	0.498	0.480	0.508	11.05	7.16	13.46	1.68	4.31	.28P.L
170	0.709	0.703	0.689	0.713	-0.82	-2.83	0.59	1.68	6.03	.28P.L
171	0.415	0.484	0.464	0.475	16.66	11.85	14.42	1.12	3.15	.28P.L
172	0.696	0.705	0.683	0.721	1.26	-1.80	3.62	1.12	4.41	.28P.L
173	0.413	0.454	0.443	0.447	10.01	7.35	8.17	2.29	4.99	.28P.L
174	0.662	0.638	0.630	0.625	-3.57	-4.84	-5.52	2.29	6.98	.28P.L
175	0.397	0.483	0.473	0.475	21.75	19.14	19.63	2.29	5.32	.28P.L
176	0.858	0.679	0.671	0.665	-20.90	-21.82	-22.51	2.29	7.45	.28P.L
177	0.559	0.403	0.391	0.397	-27.90	-30.06	-28.96	2.29	4.41	.28P.L
178	0.764	0.567	0.558	0.556	-25.75	-26.99	-27.22	2.29	6.17	.28P.L
179	0.553	0.528	0.517	0.517	-4.55	-6.42	-6.45	2.29	5.67	.28P.L
180	0.830	0.741	0.733	0.724	-10.75	-11.69	-12.74	2.29	7.94	.28P.L
181	0.435	0.461	0.452	0.455	5.98	3.92	4.53	2.29	5.46	.28P.L
182	0.795	0.647	0.640	0.637	-18.60	-19.46	-19.93	2.29	7.64	.28P.L
183	0.488	0.473	0.461	0.464	-3.11	-5.48	-4.96	2.29	5.03	.28P.L
184	0.727	0.664	0.655	0.649	-8.62	-9.84	-10.68	2.29	7.04	.28P.L
185	0.481	0.513	0.501	0.502	6.63	4.22	4.31	2.29	5.32	.28P.L
186	0.759	0.720	0.711	0.702	-5.10	-6.27	-7.45	2.29	7.45	.28P.L
187	0.481	0.528	0.522	0.518	9.75	8.52	7.63	2.29	5.23	.17P.L
188	0.735	0.741	0.737	0.725	0.88	0.22	-1.39	2.29	7.32	.17P.L
189	0.470	0.493	0.484	0.483	4.85	2.88	2.87	2.29	5.07	.23P.L
190	0.739	0.692	0.685	0.677	-6.32	-7.29	-8.41	2.29	7.09	.23P.L
191	0.429	0.432	0.418	0.424	0.59	-2.62	-1.23	2.29	4.83	.33P.L
192	0.675	0.607	0.596	0.593	-10.14	-11.69	-12.11	2.29	6.77	.33P.L

Table C2.2 (cont'd)

BM #	meas'd def. in	def.by Brnson in	def.by Ref.4 in	def.by Model in	% error Brnson	% error Ref.4	% error Model	P (%)	Ma ---- Mcr	load type
193	0.449	0.447	0.442	0.468	-0.37	-1.59	4.18	2.40	6.52	.28P.L
194	0.670	0.627	0.623	0.655	-6.41	-7.02	-2.26	2.40	9.12	.28P.L
195	0.395	0.470	0.460	0.490	18.90	16.35	24.10	1.53	4.83	.28P.L
196	0.592	0.661	0.653	0.687	11.69	10.32	16.10	1.53	6.77	.28P.L
197	0.381	0.447	0.436	0.450	17.24	14.38	18.00	1.08	3.66	.28P.L
198	0.570	0.636	0.626	0.654	11.65	9.82	14.80	1.08	5.13	.28P.L
199	0.448	0.438	0.429	0.451	-2.33	-4.35	0.71	1.34	4.28	.28P.L
200	0.689	0.617	0.610	0.636	-10.39	-11.50	-7.74	1.34	5.99	.28P.L
201	0.380	0.409	0.399	0.405	7.58	5.01	6.52	1.05	3.39	.28P.L
202	0.604	0.585	0.575	0.598	-3.16	-4.81	-1.03	1.05	4.74	.28P.L
203	0.377	0.427	0.420	0.403	13.17	11.27	6.94	0.86	3.15	.28P.L
204	0.603	0.618	0.608	0.616	2.46	0.83	2.20	0.86	4.42	.28P.L
205	0.369	0.424	0.415	0.395	14.91	12.48	7.10	0.93	2.92	.28P.L
206	0.579	0.619	0.607	0.615	6.95	4.80	6.18	0.93	4.09	.28P.L
207	0.383	0.444	0.434	0.415	15.85	13.32	8.29	0.93	2.98	.28P.L
208	0.603	0.647	0.634	0.643	7.26	5.11	6.64	0.93	4.18	.28P.L
209	0.294	0.325	0.322	0.289	10.67	9.49	-1.57	0.93	2.37	.28P.L
210	0.453	0.492	0.481	0.471	8.56	6.13	3.98	0.93	3.31	.28P.L
211	0.284	0.341	0.336	0.304	19.91	18.35	7.10	0.93	2.43	.28P.L
212	0.459	0.512	0.501	0.493	11.58	9.08	7.37	0.93	3.40	.28P.L
213	0.359	0.448	0.438	0.423	24.84	22.10	17.77	0.93	3.08	.28P.L
214	0.597	0.651	0.638	0.650	8.98	6.89	8.91	0.93	4.31	.28P.L
215	0.386	0.445	0.436	0.418	15.37	12.84	8.19	0.93	3.02	.28P.L
216	0.619	0.648	0.635	0.646	4.71	2.64	4.30	0.93	4.22	.28P.L
217	0.286	0.343	0.338	0.307	19.78	18.13	7.37	0.93	2.46	.28P.L
218	0.464	0.514	0.502	0.496	10.73	8.27	6.88	0.93	3.44	.28P.L
219	0.249	0.323	0.320	0.286	29.74	28.51	14.97	0.93	2.34	.28P.L
220	0.413	0.490	0.479	0.468	18.63	15.97	13.22	0.93	3.27	.28P.L
221	0.406	0.455	0.445	0.435	11.96	9.54	7.07	0.93	3.23	.28P.L
222	0.631	0.656	0.645	0.660	4.00	2.14	4.65	0.93	4.53	.28P.L
223	0.385	0.444	0.435	0.421	15.40	12.89	9.39	0.93	3.12	.28P.L
224	0.603	0.644	0.632	0.645	6.76	4.76	6.97	0.93	4.37	.28P.L
225	0.271	0.336	0.331	0.304	23.82	21.96	12.17	0.93	2.53	.28P.L
226	0.440	0.500	0.489	0.486	13.59	11.17	10.56	0.93	3.54	.28P.L
227	0.313	0.356	0.350	0.323	13.63	11.74	3.20	0.93	2.59	.28P.L
228	0.497	0.528	0.517	0.515	6.27	3.99	3.69	0.93	3.62	.28P.L
229	0.371	0.419	0.410	0.391	12.95	10.57	5.51	0.93	2.93	.28P.L
230	0.574	0.611	0.599	0.608	6.52	4.40	5.90	0.93	4.11	.28P.L
231	0.374	0.433	0.424	0.407	15.88	13.39	8.75	0.93	3.00	.28P.L
232	0.582	0.631	0.618	0.629	8.39	6.27	8.03	0.93	4.20	.28P.L
233	0.290	0.346	0.341	0.312	19.28	17.49	7.57	0.93	2.51	.28P.L
234	0.466	0.516	0.505	0.501	10.82	8.40	7.52	0.93	3.52	.28P.L
235	0.293	0.336	0.332	0.301	14.76	13.25	2.82	0.93	2.44	.28P.L
236	0.471	0.505	0.494	0.487	7.17	4.80	3.37	0.93	3.42	.28P.L
237	0.349	0.428	0.419	0.397	22.56	19.95	13.66	0.93	2.89	.28P.L
238	0.554	0.626	0.613	0.620	13.00	10.66	11.85	0.93	4.04	.28P.L
239	0.293	0.331	0.328	0.294	13.09	11.84	0.48	0.93	2.37	.28P.L
240	0.463	0.501	0.490	0.479	8.18	5.74	3.56	0.93	3.32	.28P.L

Table C2.2 (cont'd)

BM #	meas'd def. in	def.by Brnson in	def.by Ref.4 in	def.by Model in	% error Brnson	% error Ref.4	% error Model	P (%)	Ma ---- Mcr	load type
241	0.265	0.326	0.323	0.288	23.09	22.05	8.49	0.93	2.31	.28P.L
242	0.428	0.497	0.485	0.472	16.08	13.42	10.24	0.93	3.23	.28P.L
243	0.369	0.439	0.429	0.416	18.88	16.32	12.82	0.93	3.12	.28P.L
244	0.584	0.635	0.624	0.637	8.79	6.78	9.09	0.93	4.37	.28P.L
245	0.379	0.433	0.423	0.405	14.14	11.69	6.93	0.93	2.98	.28P.L
246	0.591	0.630	0.618	0.627	6.62	4.52	6.16	0.93	4.18	.28P.L
247	0.296	0.331	0.327	0.297	11.85	10.36	0.46	0.93	2.45	.28P.L
248	0.470	0.496	0.485	0.480	5.59	3.28	2.05	0.93	3.43	.28P.L
249	0.290	0.345	0.340	0.310	18.90	17.16	7.01	0.93	2.49	.28P.L
250	0.477	0.516	0.504	0.499	8.08	5.71	4.68	0.93	3.49	.28P.L
251	0.415	0.438	0.428	0.415	5.51	3.24	-0.04	0.93	3.10	.28P.L
252	0.639	0.635	0.623	0.636	-0.68	-2.53	-0.50	0.93	4.35	.28P.L
253	0.427	0.453	0.443	0.432	6.13	3.82	1.19	0.93	3.20	.28P.L
254	0.658	0.655	0.643	0.658	-0.46	-2.26	0.02	0.93	4.48	.28P.L
255	0.397	0.453	0.443	0.432	14.15	11.67	8.84	0.93	3.20	.28P.L
256	0.640	0.655	0.643	0.658	2.34	0.49	2.84	0.93	4.48	.28P.L
257	0.420	0.453	0.443	0.432	7.90	5.55	2.88	0.93	3.20	.28P.L
258	0.648	0.655	0.643	0.658	1.08	-0.76	1.57	0.93	4.48	.28P.L

Table C2.3 Data read from data file "INF"

BM #	d/h	d in	d'/d	bw in	bw/be	hf/d	As SQ.in	As'/As	fc' psi	span in	Ma lb.in
1	.909	10.000	0.10	9.90	1.00	0.00	0.477	1.00	4072	110.00	107400
2	.909	10.000	0.10	9.90	1.00	0.00	0.477	1.00	4072	110.00	155000
3	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.06	4017	110.00	251000
4	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.06	4017	110.00	361000
5	.909	10.000	0.10	5.90	1.00	0.00	0.486	0.18	4072	110.00	105200
6	.909	10.000	0.10	5.90	1.00	0.00	0.486	0.18	4072	110.00	154500
7	.909	10.000	0.10	5.90	0.50	0.39	0.486	0.36	3939	110.00	107100
8	.909	10.000	0.10	5.90	0.50	0.39	0.486	0.36	3939	110.00	156500
9	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.06	3986	110.00	249000
10	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.06	3986	110.00	370000
11	.909	10.000	0.10	5.90	1.00	0.00	0.486	0.18	3986	110.00	85700
12	.909	10.000	0.10	5.90	1.00	0.00	0.486	0.18	3986	110.00	126500
13	.909	10.000	0.10	5.90	0.50	0.39	0.486	0.36	4033	110.00	106500
14	.909	10.000	0.10	5.90	0.50	0.39	0.486	0.36	4033	110.00	144800
15	.871	9.580	0.15	5.90	1.00	0.00	1.051	0.08	3292	110.00	400000
16	.871	9.580	0.15	5.90	1.00	0.00	1.051	0.08	3292	110.00	547000
17	.871	9.580	0.15	5.90	1.00	0.00	0.312	0.28	3346	110.00	124000
18	.871	9.580	0.15	5.90	1.00	0.00	0.312	0.28	3346	110.00	179500
19	.871	9.580	0.15	5.90	1.00	0.00	1.051	0.08	4937	110.00	413000
20	.871	9.580	0.15	5.90	1.00	0.00	1.051	0.08	4937	110.00	606000
21	.871	9.580	0.15	5.90	1.00	0.00	0.312	0.28	5242	110.00	145000
22	.871	9.580	0.15	5.90	1.00	0.00	0.312	0.28	5242	110.00	184000
23	.909	10.000	0.10	9.85	1.00	0.00	0.477	1.00	3767	110.00	184000
24	.909	10.000	0.10	9.85	1.00	0.00	0.477	1.00	3767	110.00	301000
25	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.06	3923	110.00	448000
26	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.06	3923	110.00	539000
27	.909	10.000	0.10	5.90	1.00	0.00	0.466	0.19	3845	110.00	187000
28	.909	10.000	0.10	5.90	1.00	0.00	0.466	0.19	3845	110.00	243500
29	.909	10.000	0.10	5.90	0.50	0.39	0.486	0.36	3713	110.00	184500
30	.909	10.000	0.10	5.90	0.50	0.39	0.486	0.36	3713	110.00	273000
31	.818	4.500	0.22	11.80	1.00	0.00	0.477	1.00	3775	110.00	78000
32	.818	4.500	0.22	11.80	1.00	0.00	0.477	1.00	3775	110.00	125000
33	.742	4.080	0.35	11.80	1.00	0.00	1.434	0.06	3455	110.00	153000
34	.742	4.080	0.35	11.80	1.00	0.00	1.434	0.06	3455	110.00	212000
35	.818	4.500	0.22	11.80	1.00	0.00	0.468	0.19	3682	110.00	77600
36	.818	4.500	0.22	11.80	1.00	0.00	0.468	0.19	3682	110.00	112200
37	.818	4.500	0.22	11.80	1.00	0.00	0.312	0.28	3822	110.00	47200
38	.818	4.500	0.22	11.80	1.00	0.00	0.312	0.28	3822	110.00	78700
39	.871	9.580	0.15	5.90	1.00	0.00	1.865	0.25	3713	110.00	575000
40	.871	9.580	0.15	5.90	1.00	0.00	1.865	0.25	3713	110.00	806000
41	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.24	3736	110.00	545000
42	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.24	3736	110.00	814000
43	.871	9.580	0.15	5.90	1.00	0.00	1.051	0.26	4462	110.00	364000
44	.871	9.580	0.15	5.90	1.00	0.00	1.051	0.26	4462	110.00	549000
45	.871	9.580	0.15	5.90	1.00	0.00	1.051	0.08	3900	110.00	228000
46	.871	9.580	0.15	5.90	1.00	0.00	1.051	0.08	3900	110.00	328000
47	.909	10.000	0.10	5.90	1.00	0.00	0.312	0.28	3900	110.00	88000
48	.909	10.000	0.10	5.90	1.00	0.00	0.312	0.28	3900	110.00	124500

Table C2.3 (cont'd)

BM #	d/h	d in	d'/d	bw in	bw/be	hf/d	As SQ.in	As'/As	fc' psi	span in	Ma lb.in
49	.909	10.000	0.10	5.90	0.50	0.39	0.312	0.56	3900	110.00	88000
50	.909	10.000	0.10	5.90	0.50	0.39	0.312	0.56	3900	110.00	124500
51	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.06	4056	110.00	420000
52	.871	9.580	0.15	5.90	1.00	0.00	1.434	0.06	4056	110.00	570000
53	.909	10.000	0.10	5.90	1.00	0.00	0.494	0.18	4906	110.00	228000
54	.909	10.000	0.10	5.90	1.00	0.00	0.494	0.18	4906	110.00	328000
55	.909	10.000	0.10	5.90	0.50	0.39	0.494	0.36	4056	110.00	270000
56	.909	10.000	0.10	5.90	0.50	0.39	0.494	0.36	4056	110.00	371000
57	.871	9.580	0.15	5.90	1.00	0.00	1.865	0.05	5226	110.00	422000
58	.871	9.580	0.15	5.90	1.00	0.00	1.865	0.05	5226	110.00	630000
59	.871	9.580	0.15	5.90	1.00	0.00	1.865	0.05	4056	110.00	465000
60	.871	9.580	0.15	5.90	1.00	0.00	1.865	0.05	4056	110.00	696000
61	.871	9.580	0.15	5.90	1.00	0.00	1.865	0.26	4056	110.00	376000
62	.871	9.580	0.15	5.90	1.00	0.00	1.865	0.26	4056	110.00	556000
63	.909	10.000	0.10	5.90	1.00	0.00	0.724	0.14	5086	110.00	200000
64	.909	10.000	0.10	5.90	1.00	0.00	0.724	0.14	5421	110.00	200000
65	.909	10.000	0.10	5.90	1.00	0.00	0.724	0.14	4680	110.00	200000
66	.873	9.600	0.15	5.90	1.00	0.00	1.498	0.07	4641	110.00	400000
67	.873	9.600	0.15	5.90	1.00	0.00	1.498	0.07	5156	110.00	400000
68	.873	9.600	0.15	5.90	1.00	0.00	1.841	0.05	4937	110.00	400000
69	.844	10.130	0.19	8.00	1.00	0.00	1.320	1.00	3630	240.00	226800
70	.844	10.130	0.18	8.00	1.00	0.00	1.320	0.47	3630	240.00	226800
71	.844	10.130	0.00	8.00	1.00	0.00	1.320	0.00	3630	240.00	226800
72	.774	6.190	0.29	6.00	1.00	0.00	0.620	1.00	3020	240.00	64200
73	.774	6.190	0.29	6.00	1.00	0.00	0.620	0.50	3020	240.00	64200
74	.774	6.190	0.00	6.00	1.00	0.00	0.620	0.00	3020	240.00	64200
75	.800	4.000	0.25	12.00	1.00	0.00	0.800	1.00	2940	250.00	53215
76	.800	4.000	0.25	12.00	1.00	0.00	0.800	0.50	2940	250.00	53215
77	.800	4.000	0.00	12.00	1.00	0.00	0.800	0.00	2940	250.00	53215
78	.800	4.000	0.25	12.00	1.00	0.00	0.800	1.00	2915	150.00	53670
79	.800	4.000	0.25	12.00	1.00	0.00	0.800	0.50	2915	150.00	53670
80	.800	4.000	0.00	12.00	1.00	0.00	0.800	0.00	3215	150.00	53670
81	.770	2.310	0.30	12.00	1.00	0.00	0.440	1.00	2990	210.00	17456
82	.770	2.310	0.30	12.00	1.00	0.00	0.440	0.50	2990	210.00	17456
83	.770	2.310	0.00	12.00	1.00	0.00	0.440	0.00	2990	210.00	17456
84	.849	10.190	0.00	6.00	0.50	0.25	0.620	0.00	3680	240.00	264000
85	.849	10.190	0.15	6.00	0.50	0.25	0.620	0.50	3880	240.00	264600
86	.849	10.190	0.15	6.00	0.50	0.25	0.620	1.00	3530	240.00	263400
87	.807	9.680	0.00	6.00	0.25	0.26	1.200	0.00	3680	240.00	482400
88	.818	9.810	0.00	6.00	0.50	0.25	0.620	0.00	4260	168.00	247548
89	.774	6.190	0.00	6.00	0.50	0.32	0.620	0.00	4260	240.00	156000

Table C2.4 Results obtained considering beams of data file "INF"

BM #	meas'd def. in	def.by Brnson in	def.by Ref.4 in	def.by Model in	% error Brnson	% error Ref.4	% error Model	P (%)	Ma ---- Mcr	load type
1	0.030	0.035	0.042	0.029	15.10	38.48	-1.93	0.48	1.12	C.P.L
2	0.090	0.092	0.097	0.060	1.89	7.79	-33.75	0.48	1.62	C.P.L
3	0.120	0.144	0.139	0.143	20.00	15.83	19.05	2.54	4.44	C.P.L
4	0.190	0.208	0.204	0.205	9.30	7.39	8.14	2.54	6.38	C.P.L
5	0.070	0.093	0.084	0.069	33.47	20.04	-1.56	0.82	1.85	C.P.L
6	0.130	0.160	0.148	0.134	23.36	13.54	3.05	0.82	2.71	C.P.L
7	0.060	0.071	0.062	0.049	17.52	4.15	-18.05	0.82	1.63	C.P.L
8	0.120	0.134	0.119	0.101	11.31	-1.00	-15.90	0.82	2.38	C.P.L
9	0.170	0.183	0.179	0.181	7.51	5.21	6.71	2.54	4.42	.33P.L
10	0.270	0.272	0.270	0.270	0.88	-0.16	-0.16	2.54	6.57	.33P.L
11	0.060	0.084	0.090	0.073	39.81	50.53	20.90	0.82	1.52	.33P.L
12	0.120	0.159	0.157	0.136	32.48	30.61	13.70	0.82	2.25	.33P.L
13	0.080	0.087	0.093	0.072	9.26	16.80	-9.62	0.82	1.60	.33P.L
14	0.130	0.151	0.149	0.124	15.96	14.31	-4.66	0.82	2.18	.33P.L
15	0.280	0.292	0.288	0.287	4.14	2.76	2.66	1.86	7.81	C.P.L
16	0.390	0.399	0.396	0.393	2.32	1.58	0.79	1.86	10.68	C.P.L
17	0.120	0.185	0.176	0.130	54.14	46.38	8.73	0.55	2.40	C.P.L
18	0.230	0.303	0.289	0.260	31.70	25.45	13.16	0.55	3.48	C.P.L
19	0.240	0.283	0.275	0.286	17.74	14.71	19.29	1.86	6.59	C.P.L
20	0.380	0.415	0.410	0.420	9.32	7.98	10.55	1.86	9.66	C.P.L
21	0.120	0.190	0.179	0.122	57.95	49.18	1.73	0.55	2.24	C.P.L
22	0.180	0.275	0.257	0.202	52.86	43.02	12.46	0.55	2.85	C.P.L
23	0.080	0.139	0.138	0.092	73.33	72.89	14.58	0.48	2.01	C.P.L
24	0.250	0.285	0.275	0.240	13.95	9.89	-4.11	0.48	3.29	C.P.L
25	0.250	0.259	0.256	0.256	3.59	2.46	2.53	2.54	8.02	C.P.L
26	0.330	0.312	0.309	0.308	-5.55	-6.27	-6.55	2.54	9.64	C.P.L
27	0.170	0.210	0.197	0.190	23.24	16.08	11.56	0.79	3.38	C.P.L
28	0.250	0.279	0.268	0.271	11.72	7.29	8.34	0.79	4.40	C.P.L
29	0.120	0.169	0.154	0.141	40.90	28.23	17.48	0.82	2.89	C.P.L
30	0.240	0.265	0.251	0.256	10.38	4.49	6.64	0.82	4.28	C.P.L
31	0.390	0.412	0.365	0.330	5.68	-6.32	-15.34	0.90	2.85	C.P.L
32	0.660	0.710	0.668	0.679	7.60	1.18	2.80	0.90	4.56	C.P.L
33	0.440	0.535	0.503	0.537	21.54	14.26	22.05	2.98	5.83	C.P.L
34	0.820	0.743	0.719	0.744	-9.37	-12.32	-9.26	2.98	8.08	C.P.L
35	0.360	0.421	0.374	0.342	17.04	4.01	-4.97	0.88	2.87	C.P.L
36	0.580	0.649	0.605	0.617	11.95	4.26	6.36	0.88	4.14	C.P.L
37	0.280	0.222	0.215	0.136	-20.81	-23.23	-51.28	0.59	1.71	C.P.L
38	0.560	0.556	0.514	0.406	-0.65	-8.22	-27.43	0.59	2.85	C.P.L
39	0.260	0.270	0.269	0.270	3.70	3.53	3.76	3.30	10.57	C.P.L
40	0.400	0.378	0.378	0.378	-5.52	-5.59	-5.47	3.30	14.82	C.P.L
41	0.290	0.306	0.304	0.301	5.57	4.97	3.69	2.54	9.99	C.P.L
42	0.490	0.457	0.456	0.449	-6.66	-6.90	-8.34	2.54	14.92	C.P.L
43	0.210	0.248	0.241	0.251	17.88	14.80	19.73	1.86	6.11	C.P.L
44	0.360	0.374	0.370	0.379	3.94	2.70	5.35	1.86	9.21	C.P.L
45	0.120	0.161	0.152	0.161	33.77	26.94	34.32	1.86	4.09	C.P.L
46	0.180	0.232	0.226	0.232	29.11	25.71	29.08	1.86	5.89	C.P.L
47	0.050	0.083	0.086	0.056	66.72	71.83	11.45	0.53	1.58	C.P.L
48	0.080	0.161	0.155	0.112	101.10	93.96	39.46	0.53	2.23	C.P.L

Table C2.4 (cont'd)

BM #	meas'd def. in	def.by Brnson in	def.by Ref.4 in	def.by Model in	% error Brnson	% error Ref.4	% error Model	P (%)	Ma ---- Mcr	load type
49	0.030	0.051	0.056	0.036	70.49	87.68	18.70	0.53	1.35	C.P.L
50	0.070	0.118	0.116	0.073	68.92	65.89	3.84	0.53	1.90	C.P.L
51	0.250	0.241	0.238	0.239	-3.47	-4.76	-4.58	2.54	7.39	C.P.L
52	0.370	0.328	0.325	0.324	-11.43	-12.08	-12.51	2.54	10.03	C.P.L
53	0.190	0.239	0.225	0.219	26.02	18.29	15.38	0.84	3.65	C.P.L
54	0.210	0.353	0.341	0.347	68.13	62.27	65.46	0.84	5.25	C.P.L
55	0.210	0.255	0.240	0.242	21.54	14.09	15.42	0.84	4.05	C.P.L
56	0.320	0.358	0.344	0.360	11.79	7.65	12.52	0.84	5.56	C.P.L
57	0.210	0.196	0.193	0.196	-6.47	-7.93	-6.59	3.30	6.54	C.P.L
58	0.330	0.293	0.291	0.293	-11.09	-11.72	-11.26	3.30	9.77	C.P.L
59	0.270	0.227	0.226	0.227	-15.85	-16.39	-15.79	3.30	8.18	C.P.L
60	0.450	0.340	0.339	0.340	-24.41	-24.63	-24.37	3.30	12.25	C.P.L
61	0.180	0.174	0.173	0.173	-3.59	-4.16	-3.68	3.30	6.62	C.P.L
62	0.300	0.257	0.256	0.256	-14.44	-14.67	-14.54	3.30	9.78	C.P.L
63	0.160	0.156	0.143	0.153	-2.62	-10.64	-4.44	1.23	3.14	C.P.L
64	0.170	0.154	0.140	0.149	-9.53	-17.64	-12.09	1.23	3.04	C.P.L
65	0.180	0.158	0.147	0.157	-12.04	-18.49	-12.73	1.23	3.28	C.P.L
66	0.260	0.217	0.213	0.214	-16.60	-18.11	-17.77	2.64	6.58	C.P.L
67	0.250	0.213	0.208	0.209	-14.84	-16.75	-16.52	2.64	6.24	C.P.L
68	0.210	0.188	0.186	0.188	-10.28	-11.61	-10.36	3.25	6.38	C.P.L
69	0.530	0.597	0.571	0.616	12.64	7.72	16.31	1.63	2.61	U.D.L
70	0.620	0.613	0.585	0.633	-1.06	-5.65	2.17	1.63	2.61	U.D.L
71	0.670	0.636	0.604	0.651	-5.10	-9.85	-2.88	1.63	2.61	U.D.L
72	0.920	1.004	0.931	1.022	9.10	1.15	11.06	1.67	2.43	U.D.L
73	0.980	1.013	0.938	1.035	3.40	-4.26	5.57	1.67	2.43	U.D.L
74	1.040	1.026	0.948	1.048	-1.35	-8.82	0.75	1.67	2.43	U.D.L
75	1.580	1.700	1.607	1.729	7.57	1.69	9.42	1.67	2.62	U.D.L
76	1.710	1.730	1.632	1.760	1.18	-4.54	2.93	1.67	2.62	U.D.L
77	1.880	1.771	1.666	1.793	-5.80	-11.38	-4.65	1.67	2.62	U.D.L
78	0.470	0.619	0.586	0.629	31.75	24.69	33.91	1.67	2.65	U.D.L
79	0.560	0.630	0.595	0.641	12.59	6.34	14.43	1.67	2.65	U.D.L
80	0.700	0.627	0.587	0.640	-10.41	-16.21	-8.55	1.67	2.52	U.D.L
81	2.340	2.062	1.911	2.090	-11.90	-18.35	-10.68	1.59	2.36	U.D.L
82	2.200	2.077	1.923	2.114	-5.58	-12.59	-3.91	1.59	2.36	U.D.L
83	2.480	2.097	1.939	2.138	-15.44	-21.82	-13.78	1.59	2.36	U.D.L
84	1.340	1.174	1.139	1.122	-12.38	-14.99	-16.27	1.01	3.43	U.D.L
85	1.240	1.158	1.122	1.098	-6.59	-9.48	-11.44	1.01	3.35	U.D.L
86	1.190	1.167	1.134	1.116	-1.96	-4.74	-6.18	1.01	3.50	U.D.L
87	1.270	1.287	1.247	1.318	1.34	-1.83	3.80	2.07	5.49	U.D.L
88	0.510	0.555	0.531	0.513	8.77	4.20	0.60	1.05	2.99	U.D.L
89	2.200	2.102	1.991	2.205	-4.47	-9.49	0.23	1.67	4.23	U.D.L

APPENDIX C3

The Data Files Considered by Prog.4.4.3.1 Discussed in Sec.4.4.3

DATA FILE "CERA"

0.	5320	616000	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.377	2	51
0.	5320	862400	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.544	2	51
0.	5400	829000	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.516	2	51
0.	5400	1160600	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.759	2	51
0.	5350	868000	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.451	2	51
0.	5350	1215200	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.674	2	51
0.	4980	862500	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.380	2	51
0.	4980	1207500	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.592	2	51
0.	4600	554500	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.318	2	51
0.	4600	776300	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.483	2	51
0.	4580	761500	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.451	2	51
0.	4580	1066100	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.677	2	51
0.	4500	672000	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.389	2	51
0.	4500	940800	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.596	2	51
0.	4520	773000	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.457	2	51
0.	4520	1082200	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.673	2	51
0.	4550	714000	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.446	2	51
0.	4550	999600	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.578	2	51
0.	4600	795000	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.386	2	51
0.	4600	1113000	2.4465	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.616	2	51
0.	4940	560000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.206	2	51
0.	4940	784000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.359	2	51
0.	4840	756000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.448	2	51
0.	4840	1058400	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.668	2	51
0.	4910	795000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.434	2	51
0.	4910	1113000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.705	2	52
0.	4810	795000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.499	2	51
0.	4810	1113000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.743	2	51
0.	4880	756000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.474	2	51
0.	4880	1058400	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.708	2	51
0.	4910	560000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.327	2	51
0.	4910	784000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.496	2	51
0.	4910	817500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.502	2	51
0.	4910	1144500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.755	2	51
0.	4780	773000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.477	2	51
0.	4780	1082200	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.713	2	51
0.	4750	823000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.549	2	51
0.	4750	1152200	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.799	2	51
0.	4870	806500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.503	2	51
0.	4870	1129100	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.767	2	51
0.	4830	817500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.460	2	51
0.	4830	1144500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.691	2	51
0.	4440	661000	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.362	2	51
0.	4440	925400	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.519	2	51
0.	4380	801000	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.462	2	51
0.	4380	1121400	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.708	2	51
0.	4410	806500	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.490	2	51
0.	4410	1129100	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.729	2	51
0.	4300	829000	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.483	2	51
0.	4300	1160600	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.734	2	51
0.	4340	817500	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.502	2	51
0.	4340	1144500	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.806	2	51
0.	4440	717000	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.462	2	51
0.	4440	1003800	2.3544	0.0	8.0	8.0	0.0	16.00	13.500	0.0	180	0.688	2	51
0.	5620	689000	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.383	2	51
0.	5620	964600	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.561	2	51
0.	5490	840000	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.470	2	51
0.	5490	1176000	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.708	2	51
0.	5540	829000	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.524	2	51
0.	5540	1160600	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.777	2	51
0.	5450	834500	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.464	2	51
0.	5450	1168300	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.688	2	51
0.	5580	806500	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.450	2	51
0.	5580	1129100	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.669	2	51
0.	5740	761500	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.441	2	51
0.	5740	1066100	2.3643	0.0	8.0	8.0	0.0	16.00	13.875	0.0	180	0.656	2	51
0.	3843	532000	2.4045	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.370	2	51
0.	3843	744800	2.4045	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.580	2	51
0.	3843	526500	2.4045	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.339	2	51
0.	3843	737100	2.4045	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.555	2	51
0.	3843	605000	2.4045	0.0	8.0	8.0	0.0	15.750	13.125	0.0	180	0.341	2	51
0.	3843	847000	2.4045	0.0	8.0	8.0	0.0	15.750	13.125	0.0	180	0.567	2	51
0.	3843	549000	2.4045	0.0	8.0	8.0	0.0	15.750	13.125	0.0	180	0.351	2	51
0.	3843	768600	2.4045	0.0	8.0	8.0	0.0	15.750	13.125	0.0	180	0.530	2	51
0.	4175	756000	2.4045	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.547	2	51
0.	4175	1058400	2.4045	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.851	2	51
0.	4190	795000	2.4045	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.547	2	51
0.	4190	1113000	2.4045	0.0	8.0	8.0	0.0	15.125	13.125	0.0	180	0.816	2	51
0.	4203	717000	2.4045	0.0	8.0	8.0	0.0	15.750	13.125	0.0	180	0.465	2	51
0.	4203	1003800	2.4045	0.0	8.0	8.0	0.0	15.750	13.125	0.0	180	0.829	2	51
0.	4216	767000	2.4045	0.0	8.0	8.0	0.0	15.750	13.125	0.0	180	0.469	2	51
0.	4216	1073800	2.4045	0.0	8.0	8.0	0.0	15.750	13.125	0.0	180	0.671	2	51
0.	4820	761500	2.4045	0.0	3.5	8.0	6.5	15.750	13.125	0.0	180	0.450	2	51
0.	4820	1066100	2.4045	0.0	3.5	8.0	6.5	15.750	13.125	0.0	180	0.687	2	51
0.	4870	795000	2.4045	0.0	5.5	8.0	6.5	15.750	13.125	0.0	180	0.569	2	51

0.	4870	1113000	2.4045	0.0	5.5	8.0	6.5	15.750	13.125	0.0	180	0.712	2	51
0.	4900	829000	2.4045	0.0	7.75	8.0	6.5	15.750	13.125	0.0	180	0.481	2	51
0.	4900	1160600	2.4045	0.0	7.75	8.0	6.5	15.750	13.125	0.0	180	0.735	2	51
0.	4320	554500	2.1902	0.0	6.25	6.25	0.0	16.000	13.125	0.0	180	0.360	2	51
0.	4320	776300	2.1902	0.0	6.25	6.25	0.0	16.000	13.125	0.0	180	0.558	2	51
0.	4550	633000	2.367	0.0	8.0	8.00	0.0	16.000	13.150	0.0	180	0.362	2	51
0.	4550	886200	2.367	0.0	8.0	8.00	0.0	16.000	13.150	0.0	180	0.548	2	51
0.	4500	644000	2.3546	0.0	9.75	9.75	0.0	16.000	13.125	0.0	180	0.344	2	51
0.	4500	901600	2.3546	0.0	9.75	9.75	0.0	16.000	13.125	0.0	180	0.509	2	51
0.	5270	739000	2.4045	0.0	3.5	8.00	6.5	15.750	13.125	0.0	180	0.539	2	51
0.	5270	1034600	2.4045	0.0	3.5	8.00	6.5	15.750	13.125	0.0	180	0.749	2	51
0.	5320	795000	2.4045	0.0	5.5	8.00	6.5	15.750	13.125	0.0	180	0.478	2	51
0.	5320	1113000	2.4045	0.0	5.5	8.00	6.5	15.750	13.125	0.0	180	0.722	2	51
0.	5050	795000	2.4045	0.0	7.75	8.00	6.5	15.750	13.125	0.0	180	0.465	2	51
0.	5050	1113000	2.4045	0.0	7.75	8.00	6.5	15.750	13.125	0.0	180	0.711	2	51
0.	5210	750500	2.1902	0.0	6.25	6.25	0.0	16.000	13.125	0.0	180	0.458	2	51
0.	5210	1050700	2.1902	0.0	6.25	6.25	0.0	16.000	13.125	0.0	180	0.709	2	51
0.	5300	750500	2.3625	0.0	8.00	8.00	0.0	16.000	13.125	0.0	180	0.463	2	51
0.	5300	1050700	2.3625	0.0	8.00	8.00	0.0	16.000	13.125	0.0	180	0.714	2	51
0.	5340	873500	2.3546	0.0	9.75	9.75	0.0	16.000	13.125	0.0	180	0.494	2	51
0.	5340	1222900	2.3546	0.0	9.75	9.75	0.0	16.000	13.125	0.0	180	0.748	2	51
0.	3800	823000	2.4045	0.0	8.00	8.00	0.0	17.125	13.125	0.0	180	0.497	2	51
0.	3800	1152200	2.4045	0.0	8.00	8.00	0.0	17.125	13.125	0.0	180	0.743	2	51
0.	3880	717000	2.4045	0.0	8.00	8.00	0.0	17.125	13.125	0.0	180	0.453	2	51
0.	3880	1003800	2.4045	0.0	8.00	8.00	0.0	17.125	13.125	0.0	180	0.690	2	51
0.	3950	784000	2.4045	0.0	8.0	8.00	0.0	16.125	13.125	0.0	180	0.482	2	51
0.	3950	1097600	2.4045	0.0	8.0	8.00	0.0	16.125	13.125	0.0	180	0.731	2	51
0.	4000	722500	2.4045	0.0	8.0	8.00	0.0	16.125	13.125	0.0	180	0.481	2	51
0.	4000	1011500	2.4045	0.0	8.0	8.00	0.0	16.125	13.125	0.0	180	0.724	2	51
0.	4190	756000	2.4045	0.0	8.0	8.00	0.0	14.875	13.125	0.0	180	0.493	2	51
0.	4190	1058400	2.4045	0.0	8.0	8.00	0.0	14.875	13.125	0.0	180	0.761	2	51
0.	4240	733500	2.4045	0.0	8.0	8.00	0.0	14.875	13.125	0.0	180	0.469	2	51
0.	4240	1026900	2.4045	0.0	8.0	8.00	0.0	14.875	13.125	0.0	180	0.724	2	51
0.	4600	722500	2.4045	0.0	8.0	8.00	0.0	14.875	13.125	0.0	180	0.513	2	51
0.	4600	1011500	2.4045	0.0	8.0	8.00	0.0	14.875	13.125	0.0	180	0.766	2	51
0.	4800	728000	2.4045	0.0	8.0	8.00	0.0	15.750	13.125	0.0	180	0.614	2	51
0.	4800	1019200	2.4045	0.0	8.0	8.00	0.0	15.750	13.125	0.0	180	0.771	2	51
0.	4850	728000	2.4045	0.0	8.0	8.00	0.0	15.750	13.125	0.0	180	0.292	2	51
0.	4850	1019200	2.4045	0.0	8.0	8.00	0.0	15.750	13.125	0.0	180	0.390	2	51
0.	4700	644000	2.4045	0.0	8.0	8.00	0.0	14.875	13.125	0.0	180	0.392	2	51
0.	4700	901600	2.4045	0.0	8.0	8.00	0.0	14.875	13.125	0.0	180	0.587	2	51
0.	4950	700000	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.499	2	51
0.	4950	980000	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.750	2	51
0.	5160	817500	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.510	2	51
0.	5160	1144500	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.729	2	51
0.	2950	610500	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.406	2	51
0.	2950	854700	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.643	2	51
0.	2985	627000	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.484	2	51
0.	2985	877800	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.732	2	51
0.	4960	728000	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.464	2	51
0.	4960	1019200	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.598	2	51
0.	2830	582500	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.513	2	51
0.	2830	815500	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.753	2	51
0.	3475	661000	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.455	2	51
0.	3475	925400	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.689	2	51
0.	4900	733500	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.512	2	51
0.	4900	1026900	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.768	2	51
0.	4770	638500	2.3625	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.366	2	51
0.	4770	893900	2.3625	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.558	2	51
0.	4980	515000	1.764	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.251	2	51
0.	4980	721000	1.764	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.389	2	51
0.	5050	330500	1.176	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.296	2	51
0.	5050	462700	1.176	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.463	2	51
0.	4700	554500	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.297	2	51
0.	4700	776300	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.560	2	51
0.	4500	425500	1.764	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.360	2	51
0.	4500	595700	1.764	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.502	2	51
0.	4940	325000	1.2285	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.266	2	51
0.	4940	455000	1.2285	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.396	2	51
0.	4850	806500	2.3625	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.471	2	51
0.	4850	1129100	2.3625	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.730	2	51
0.	4775	750500	1.764	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.624	2	51
0.	4775	1050700	1.764	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.911	2	51
0.	5020	498500	1.176	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.538	2	51
0.	5020	697900	1.176	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.778	2	51
0.	5190	795000	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.512	2	51
0.	5190	1113000	2.4045	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.771	2	51
0.	4960	616000	1.764	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.451	2	51
0.	4960	862400	1.764	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.674	2	51
0.	4900	403000	1.2285	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.425	2	51
0.	4900	564200	1.2285	0.0	8.0	8.00	0.0	15.75	13.125	0.0	180	0.638	2	51
0.	4340	683000	2.3625	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.487	2	51
0.	4340	956200	2.3625	0.0	8.0	8.00	0.0	16.00	13.125	0.0	180	0.553	2	51
0.	4150	644000	1.764	0.0	8.0	8.0	0.0	16.00	13.125	0.0	180	0.448	2	51
0.	4150	901600	1.764	0.0	8.0	8.0	0.0	16.00	13.125	0.0	180	0.709	2	51

0.	4540	493000	1.176	0.0	8.0	8.0	0.0	16.00	13.125	0.0	180	0.415	2	51
0.	4540	690200	1.176	0.0	8.0	8.0	0.0	16.00	13.125	0.0	180	0.696	2	51
0.	4085	717000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.413	2	51
0.	4085	1003800	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.662	2	51
0.	4048	761500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.397	2	51
0.	4048	1066100	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.858	2	51
0.	4150	638500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.559	2	51
0.	4150	893900	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.764	2	51
0.	4279	834500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.553	2	51
0.	4279	1168300	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.830	2	51
0.	3410	717000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.435	2	51
0.	3410	1003800	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.795	2	51
0.	4400	750500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.488	2	51
0.	4400	1050700	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.727	2	51
0.	4670	817500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.481	2	51
0.	4670	1144500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.759	2	51
0.	4360	776000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.481	2	30
0.	4360	1086400	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.735	2	30
0.	4360	752500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.470	2	42
0.	4360	1053500	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.739	2	42
0.	4360	718000	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.429	2	60
0.	4360	1005200	2.4045	0.0	8.0	8.0	0.0	15.75	13.125	0.0	180	0.675	2	60
0.	4080	493000	1.176	0.0	3.5	8.0	6.5	15.75	14.000	0.0	180	0.449	2	51
0.	4080	690200	1.176	0.0	3.5	8.0	6.5	15.75	14.000	0.0	180	0.670	2	51
0.	4060	521000	1.176	0.0	5.5	8.0	6.5	15.75	14.000	0.0	180	0.395	2	51
0.	4060	729400	1.176	0.0	5.5	8.0	6.5	15.75	14.000	0.0	180	0.592	2	51
0.	3920	504000	1.176	0.0	7.75	8.0	6.5	15.75	14.000	0.0	180	0.381	2	51
0.	3920	705600	1.176	0.0	7.75	8.0	6.5	15.75	14.000	0.0	180	0.570	2	51
0.	4040	493000	1.2039	0.0	6.25	6.25	0.0	16.00	14.375	0.0	180	0.448	2	51
0.	4040	690200	1.2039	0.0	6.25	6.25	0.0	16.00	14.375	0.0	180	0.689	2	51
0.	4110	504000	1.2075	0.0	8.00	8.00	0.0	16.00	14.375	0.0	180	0.380	2	51
0.	4110	705600	1.2075	0.0	8.00	8.00	0.0	16.00	14.375	0.0	180	0.604	2	51
0.	3940	560000	1.2053	0.0	9.75	9.75	0.0	16.00	14.375	0.0	180	0.377	2	51
0.	3940	784000	1.2053	0.0	9.75	9.75	0.0	16.00	14.375	0.0	180	0.603	2	51
0.	4420	358500	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.369	2	51
0.	4420	501900	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.579	2	51
0.	4640	375000	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.383	2	51
0.	4640	525000	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.603	2	51
0.	4440	291000	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.294	2	51
0.	4440	407400	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.453	2	51
0.	4550	302500	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.284	2	51
0.	4550	423500	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.459	2	51
0.	4360	375000	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.359	2	51
0.	4360	525000	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.597	2	51
0.	4540	375000	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.386	2	51
0.	4540	525000	0.8789	0.0	7.00	7.00	0.0	15.25	13.500	0.0	180	0.619	2	51
0.	4440	302500	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.286	2	51
0.	4440	423500	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.464	2	51
0.	4560	291000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.249	2	51
0.	4560	407400	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.413	2	51
0.	3950	375000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.406	2	51
0.	3950	525000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.631	2	51
0.	4120	369500	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.385	2	51
0.	4120	517300	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.603	2	51
0.	3900	291000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.271	2	51
0.	3900	407400	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.440	2	51
0.	4170	308000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.313	2	51
0.	4170	431200	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.497	2	51
0.	4250	353000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.371	2	51
0.	4250	494200	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.574	2	51
0.	4320	364000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.374	2	51
0.	4320	509600	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.582	2	51
0.	4260	302500	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.290	2	51
0.	4260	423500	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.466	2	51
0.	4340	297000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.293	2	51
0.	4340	415800	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.471	2	51
0.	4670	364000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.349	2	51
0.	4670	509600	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.554	2	51
0.	4600	297000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.293	2	51
0.	4600	415800	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.463	2	51
0.	4870	297000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.265	2	51
0.	4870	415800	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.428	2	51
0.	3990	364000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.369	2	51
0.	3990	509600	0.8789	0.0	7.0	7.0	0.0	15.25	13.50	0.0	180	0.584	2	51
0.	4370	364000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.379	2	51
0.	4370	509600	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.591	2	51
0.	4140	291000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.296	2	51
0.	4140	407400	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.470	2	51
0.	4320	302500	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.290	2	51
0.	4320	423500	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.477	2	51
0.	4040	364000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.415	2	51
0.	4040	509600	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.639	2	51
0.	4040	375000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.427	2	51
0.	4040	525000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.658	2	51
0.	4040	375000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.397	2	51
0.	4040	525000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.640	2	51
0.	4040	375000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.420	2	51
0.	4040	525000	0.8789	0.0	7.0	7.0	0.0	15.25	13.500	0.0	180	0.648	2	51

DATA FILE "INF"

4072	0.	107400	0.477	0.477	9.9	9.9	0	11	10	1.	110	.03	2 55
4072	0.	155000	0.477	0.477	9.9	9.9	0	11	10	1.	110	.09	2 55
4017	0.	251000	1.434	0.088	5.9	5.9	0	11	9.58	1.42	110	.12	2 55
4017	0.	361000	1.434	0.088	5.9	5.9	0	11	9.58	1.42	110	.19	2 55
4072	0.	105200	0.486	0.088	5.9	5.9	0	11	10	1.	110	.07	2 55
4072	0.	154500	0.486	0.088	5.9	5.9	0	11	10	1.	110	.13	2 55
3939	0.	107100	0.486	0.176	5.9	11.8	3.9	11	10	1.	110	.06	2 55
3939	0.	156500	0.486	0.176	5.9	11.8	3.9	11	10	1.	110	.12	2 55
3986	0.	249000	1.434	0.088	5.9	5.9	0	11	9.58	1.42	110	.17	2 36.667
3986	0.	370000	1.434	0.088	5.9	5.9	0	11	9.58	1.42	110	.27	2 36.667
3986	0.	085700	0.486	0.088	5.9	5.9	0	11	10	1.	110	.06	2 36.667
3986	0.	126500	0.486	0.088	5.9	5.9	0	11	10	1.	110	.12	2 36.667
4033	0.	106500	0.486	0.176	5.9	11.8	3.9	11	10	1.	110	.08	2 36.667
4033	0.	144800	0.486	0.176	5.9	11.8	3.9	11	10	1.	110	.13	2 36.667
3292	0.	400000	1.051	0.088	5.9	5.9	0	11	9.58	1.42	110	.28	2 55
3292	0.	547000	1.051	0.088	5.9	5.9	0	11	9.58	1.42	110	.39	2 55
3346	0.	124000	0.312	0.088	5.9	5.9	0	11	9.58	1.42	110	.12	2 55
3346	0.	179500	0.312	0.088	5.9	5.9	0	11	9.58	1.42	110	.23	2 55
4937	0.	413000	1.051	0.088	5.9	5.9	0	11	9.58	1.42	110	.24	2 55
4937	0.	606000	1.051	0.088	5.9	5.9	0	11	9.58	1.42	110	.38	2 55
5242	0.	145000	0.312	0.088	5.9	5.9	0	11	9.58	1.42	110	.12	2 55
5242	0.	184000	0.312	0.088	5.9	5.9	0	11	9.58	1.42	110	.18	2 55
3767	0.	184000	0.477	0.477	9.85	9.85	0	11	10	1.	110	.08	2 55
3767	0.	301000	0.477	0.477	9.85	9.85	0	11	10	1.	110	.25	2 55
3923	0.	448000	1.434	0.088	5.9	5.9	0	11	9.58	1.42	110	.25	2 55
3923	0.	539000	1.434	0.088	5.9	5.9	0	11	9.58	1.42	110	.33	2 55
3845	0.	187000	0.466	0.088	5.9	5.9	0	11	10	1.	110	.17	2 55
3845	0.	243500	0.466	0.088	5.9	5.9	0	11	10	1.	110	.25	2 55
3713	0.	184500	0.486	0.176	5.9	11.8	3.9	11	10	1.	110	.12	2 55
3713	0.	273000	0.486	0.176	5.9	11.8	3.9	11	10	1.	110	.24	2 55
3775	0.	078000	0.477	0.477	11.8	11.8	0	5.5	4.5	1.	110	.39	2 55
3775	0.	125000	0.477	0.477	11.8	11.8	0	5.5	4.5	1.	110	.66	2 55
3455	0.	153000	1.434	0.088	11.8	11.8	0	5.5	4.08	1.42	110	.44	2 55
3455	0.	212000	1.434	0.088	11.8	11.8	0	5.5	4.08	1.42	110	.82	2 55
3682	0.	077600	0.468	0.088	11.8	11.8	0	5.5	4.5	1.	110	.36	2 55
3682	0.	112200	0.468	0.088	11.8	11.8	0	5.5	4.5	1.	110	.58	2 55
3822	0.	047200	0.312	0.088	11.8	11.8	0	5.5	4.5	1.	110	.28	2 55
3822	0.	078700	0.312	0.088	11.8	11.8	0	5.5	4.5	1.	110	.56	2 55
3713	0.	575000	1.865	0.468	5.9	5.9	0	11	9.58	1.42	110	.26	2 55
3713	0.	806000	1.865	0.468	5.9	5.9	0	11	9.58	1.42	110	.40	2 55
3736	0.	545000	1.434	0.351	5.9	5.9	0	11	9.58	1.42	110	.29	2 55
3736	0.	814000	1.434	0.351	5.9	5.9	0	11	9.58	1.42	110	.49	2 55
4462	0.	364000	1.051	0.273	5.9	5.9	0	11	9.58	1.42	110	.21	2 55
4462	0.	549000	1.051	0.273	5.9	5.9	0	11	9.58	1.42	110	.36	2 55
3900	0.	228000	1.051	0.088	5.9	5.9	0	11	9.58	1.42	110	.12	2 55
3900	0.	328000	1.051	0.088	5.9	5.9	0	11	9.58	1.42	110	.18	2 55
3900	0.	088000	0.312	0.088	5.9	5.9	0	11	10	1.	110	.05	2 55
3900	0.	124500	0.312	0.088	5.9	5.9	0	11	10	1.	110	.08	2 55
3900	0.	088000	0.312	0.176	5.9	11.8	3.9	11	10	1.	110	.03	2 55
3900	0.	124500	0.312	0.176	5.9	11.8	3.9	11	10	1.	110	.07	2 55
4056	0.	420000	1.434	0.088	5.9	5.9	0	11	9.58	1.42	110	.25	2 55
4056	0.	570000	1.434	0.088	5.9	5.9	0	11	9.58	1.42	110	.37	2 55
4906	0.	228000	0.494	0.088	5.9	5.9	0	11	10	1.	110	.19	2 55
4906	0.	328000	0.494	0.088	5.9	5.9	0	11	10	1.	110	.21	2 55
4056	0.	270000	0.494	0.176	5.9	11.8	3.9	11	10	1.	110	.21	2 55
4056	0.	371000	0.494	0.176	5.9	11.8	3.9	11	10	1.	110	.32	2 55
5226	0.	422000	1.865	0.088	5.9	5.9	0	11	9.58	1.42	110	.21	2 55
5226	0.	630000	1.865	0.088	5.9	5.9	0	11	9.58	1.42	110	.33	2 55
4056	0.	465000	1.865	0.088	5.9	5.9	0	11	9.58	1.42	110	.27	2 55
4056	0.	696000	1.865	0.088	5.9	5.9	0	11	9.58	1.42	110	.45	2 55
4056	0.	376000	1.865	0.478	5.9	5.9	0	11	9.58	1.42	110	.18	2 55
4056	0.	556000	1.865	0.478	5.9	5.9	0	11	9.58	1.42	110	.30	2 55
5086	0.	200000	0.724	0.098	5.9	5.9	0	11	10	1.	110	.16	2 55
5421	0.	200000	0.724	0.098	5.9	5.9	0	11	10	1.	110	.17	2 55
4680	0.	200000	0.724	0.098	5.9	5.9	0	11	10	1.	110	.18	2 55
4641	0.	400000	1.498	0.098	5.9	5.9	0	11	9.6	1.4	110	.26	2 55
5156	0.	400000	1.498	0.098	5.9	5.9	0	11	9.6	1.4	110	.25	2 55
4937	0.	400000	1.841	0.098	5.9	5.9	0	11	9.6	1.4	110	.21	2 55
3630	0.	226800	1.320	1.320	8.0	8.0	0	12	10.13	1.88	240	.53	1 0
3630	0.	226800	1.32	0.62	8.0	8.0	0	12	10.13	1.81	240	.62	1 0
3630	0.	226800	1.32	0.	8.	8.	0	12	10.13	0.	240	.67	1 0
3020	0.	64200	.62	0.62	6.0	6.0	0	8	6.19	1.81	240	.92	1 0
3020	0.	64200	.62	0.31	6.0	6.0	0	8	6.19	1.81	240	.98	1 0
3020	0.	64200	.62	0.0	6.0	6.0	0	8	6.19	0.0	240	1.04	1 0
2940	0.	53215	.8	0.8	12.0	12.0	0	5	4.0	1.0	250	1.58	1 0
2940	0.	53215	.8	0.4	12.0	12.0	0	5	4.0	1.0	250	1.71	1 0
2940	0.	53215	.8	0.	12.0	12.0	0	5	4.0	0.0	250	1.88	1 0
2915	0.	53670	.8	0.8	12.0	12.0	0	5	4.0	1.0	150	0.47	1 0
2915	0.	53670	.8	0.4	12.0	12.0	0	5	4.0	1.0	150	0.56	1 0
3215	0.	53670	.8	0.0	12.0	12.0	0	5	4.0	0.0	150	0.70	1 0
2990	0.	17456	.44	0.44	12.0	12.0	0	3	2.31	0.69	210	2.34	1 0
2990	0.	17456	.44	0.22	12.0	12.0	0	3	2.31	0.69	210	2.20	1 0
2990	0.	17456	.44	0.00	12.0	12.0	0	3	2.31	0.00	210	2.48	1 0
3680	0.	264000	.62	0.00	6.0	12.0	2.5	12	10.19	0.00	240	1.34	1 0
3880	0.	264600	.62	.31	6.0	12.0	2.5	12	10.19	1.56	240	1.24	1 0
3530	0.	263400	.62	.62	6.0	12.0	2.5	12	[illegible]	[illegible]	[illegible]	1.19	1 0
3680	0.	482400	1.20	0.0	6.0	24.0	2.5	12	9.68	0.0	240	1.27	1 0
4260	0.	247548	0.62	0.0	6.0	12.0	2.5	12	[illegible]	[illegible]	168	0.51	1 0
4260	0.	156000	0.62	0.0	6.0	12.0	[illegible]	[illegible]	6.19	[illegible]	[illegible]	2.2	1 0