# Essays in Macroeconomics

*Understanding Capital-skill Complementarity and Business Cycle Dynamics*


By

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Thesis submitted in accordance with the requirements of the University of Liverpool for the degree of Doctor in Philosophy.

July 2023

THE UNIVERSITY
of LIVERPOOL

# Essays in Macroeconomics

## Understanding Capital-skill Complementarity and Business Cycle Dynamics

**Ruoxi Cao**

## Abstract

This thesis aims to provide a comprehensive analysis of the interplay between technological advancements, capital-skill complementarity, and labour market dynamics. The central focus is to explore the implications of these factors on business cycle dynamics. The research builds upon the existing literature while introducing novel elements, such as varying utilisation associated with different skill levels for labour. By employing empirical analyses and developing a stochastic dynamic general equilibrium (DSGE) model, this study seeks to offer insights into the intricate relationships among technology, labour composition, and economic outcomes.

Chapter 1 revisits the study conducted by Basu et al. (2006) on constructing a measure of aggregate technology change while accounting for aggregation effects, non-constant returns, varying utilisation of capital and labour, and imperfect competition. However, our study extends their work by incorporating varying utilisation associated with different skill levels for labour. Empirical findings demonstrate that technological improvements lead to an increase in both output and input, with high-skilled labour experiencing higher utilisation while low-skilled labour experiences decreased utilisation. Furthermore, the analysis reveals sector-specific variations in the responses of high-skilled and low-skilled labour utilisation to technological advancements.

Chapter 2 examines the capital-skill complementarity hypothesis across various industries in the U.S. economy. Applying Hansen's (2002) data-splitting methodology, the study identifies four primary findings. Firstly, industries with higher capital-to-output ratios exhibit substitution between capital and skilled labour, while those with lower ratios demonstrate complementarity. Secondly, skilled and unskilled labour display a complementary relationship in both industry groups, with a stronger association observed in industries characterized by a higher-educated workforce. Thirdly, a time break in capital-skill substitution is identified in industries with high capital-to-output ratios, indicating a higher degree of substitutability between capital and skilled labour after 1980. These findings are robust across various sensitivity analyses, aligning with the non-linearity observed in the time-series analysis by Goldin and Katz (1998) using U.S. manufacturing data. This suggests the possibility of a transitory nature in the observed capital-skill complementarity phenomenon.

Chapter 3 focuses on the development of a stochastic dynamic general equilibrium (DSGE) model that integrates capital-skill complementarity in production, Calvo prices, and nominal wage rigidity. The primary objective is to gain insights into the negative dynamics observed in the skilled to unskilled working hours ratio and the differential responses of hours worked by skilled and unskilled workers. Four distinct scenarios are simulated, including a generalized real business cycle (RBC) model, a frictionless monopolistic competition model, a generalized model incorporating Calvo (1983) pricing, and a model incorporating Calvo wage and Calvo price settings. The analysis reveals that with moderate capital-skill complementarity, both the generalized RBC model and the frictionless monopolistic competition model can explain the negative response of the skilled to unskilled hours ratio to technological advancements. However, these models fail to capture

the divergent responses of hours worked for the two skill levels. In contrast, a generalized model incorporating Calvo (1983) pricing provides a more satisfactory explanation. Additionally, the incorporation of Calvo wage and price settings further enriches the analysis, offering deeper insights into the interplay between technology, labour market dynamics, and economic outcomes.

*Declaration

I declare that the thesis has been composed by myself and that the work has not be submitted for any other degree or professional qualification. I confirm that the work submitted is my own, except where work which has formed part of jointly-authored papers has been included. My contribution and those of the other authors to this work have been explicitly indicated within the authorship declaration forms attached at the end of the thesis. I confirm that appropriate credit has been given within this thesis where reference has been made to the work of others.

*Acknowledgements

First and foremost, I would like to express my heartfelt appreciation to Dr. Lin Liu for her exceptional supervision and unwavering support throughout my PhD studies. Her guidance, patience, and encouragement have been instrumental in the successful completion of this thesis. I am also grateful to Dr. Alessandro Marchesiani for providing me with the opportunity to serve as a Teaching Assistant during my studies, and for his constant encouragement and invaluable guidance, which have provided me with valuable insights and advice. In addition to my gratitude towards Dr. Lin Liu and Dr. Alessandro Marchesiani, I would like to extend my sincere appreciation to Dr. Syed Hussian for his invaluable guidance and mentorship during my PhD journey.

Additionally, I would like to extend my heartfelt thanks to my friends Kaori, Yiming, Shu, Xinjiao, Xing, and Carol for their continuous encouragement and unwavering support throughout my academic journey.

I am also deeply appreciative of my family, especially my husband Hongliang, whose consistent support and encouragement have played an instrumental role in my success. Their belief in my abilities has been a constant source of motivation throughout my studies.

Lastly, I would like to acknowledge the adorable companionship and emotional support provided by my beloved pets, Piqiu, Xiaomi, Xiaohua, Xiaofu, and Cheechee. Their presence has brought comfort and joy during the challenging times of my research.

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Introduction

The business cycle, characterized by the fluctuation of gross domestic product (GDP) around its long-term growth trend, has been a subject of extensive research and analysis. One key characteristic of business cycles is the positive co-movement between economic conditions and various economic variables, including labour input. However, the factors driving these fluctuations and their implications for economic stability remain areas of ongoing investigation.

In existing studies, there has been significant attention given to the relationship between technology shocks, capital-labour utilisation, and their effects on business cycles. The impact of technology shocks on labour productivity is a well-documented feature of business fluctuations. Scholars have long recognized the importance of identifying the primary macroeconomic shocks that drive fluctuations in economic activity and determining the most effective policies for maintaining economic stability. Among the various models used to explain economic fluctuations, the Real Business Cycle (RBC) model stands out as a prominent framework. This model emphasizes the role of technology shocks as the primary driver of business cycles, building on the observation that detrended Solow residuals closely track detrended real GDP.

However, the use of Solow residuals as a reflection of stochastic movements in technology has been subject to criticism and challenges in the literature. Basu and Kimball (1997) highlights three main explanations for the observed pro-cyclicality of productivity. Firstly, variations in measured productivity may stem from exogenous changes in production technology. Secondly, productivity can exhibit a pro-cyclical pattern due to increasing returns to scale, leading to enhanced efficiency as the economy operates at higher activity levels. Lastly, inaccuracies in input measurements can cause measured productivity to display a pro-cyclical pattern even if true productivity remains constant. It is emphasized that the gap between actual and measured technology shocks most likely arises from mismeasurement of inputs, which refers to the inadequate consideration of unobserved fluctuations in capital utilisation and labour effort. When firms face significant adjustment costs in their decisions to hire employees and accumulate capital, there is a tendency for employment and capital to become quasi-fixed factors. Consequently, firms respond to technology advancement in production by intensively utilising labour and capital during economic booms and less during recessions in the short run. Therefore, disregarding cyclical variations in factors utilisations when analyzing the Solow residual can lead to the risk of cyclical mismeasurement and inaccurate assessment of technology shocks and result in misleading business cycle results

The importance of variable utilisation in understanding business cycles and their implications is widely acknowledged in the literature. Scholars have developed various approaches to measure unobservable labour effort and capital utilisation, including proxies such as accident rates, overtime hours, the ratio of production to non-production workers, material growth, and capital depreciation rate. These studies have shed light on the role of variable factor utilisation as a propagation mechanism in the economy, explaining lead-lag relationships in the data and providing insights into the cyclicality of measured productivity.

While existing literature has explored the importance of variable utilisation in the business cycle context, there is a notable gap in research specifically examining heterogeneity in utilisation, particularly in relation to the disparity in utilisation between high-skilled and low-skilled workers. Understanding the dynamics of capital and labour utilisation across different skill levels is crucial for a comprehensive understanding of business cycles and the factors driving them. In light of this research gap, the first chapter of this thesis is to contribute to the literature by estimating the utilisation-adjusted technology shock series that is robust across different skill levels. Moreover, a key focus of this chapter is to estimate the utilisation series specifically for high and low skilled workers. Furthermore, this thesis examines the implications of utilisation-adjusted technology shocks and capital-labour utilisation in business cycle models, with a specific focus on the response of utilisation among different skill levels of labour to utilisation-adjusted technology advancement. The insights gained from these findings shed light on the dynamics of capital-skill complementarity/substitution in various industry sectors of the U.S. economy. These insights serve as the inspiration for the subsequent exploration of this issue in the second chapter.

This thesis also provides insights into the intricate relationship between technology shocks and capital-skill complementarity. Over the past six decades, the labour market in the United States has undergone significant changes characterized by a consistent increase in the supply of both skilled and unskilled workers. Paralleling this trend, there has been a notable escalation in the skill premium, which pertains to the relative hourly wage disparity between skilled and unskilled workers. The growing body of literature in this field suggests that technological advancements tend to favor skilled workers by enhancing their productivity and substituting tasks traditionally performed by the less-skilled counterparts, thereby exacerbating wage inequality. The hypothesis of capital-skill complementarity emerges as one plausible explanation for the observed rise in the skill premium. It postulates that the elasticity of substitution between capital equipment and skilled labour is lower than that between capital and unskilled labour. In essence, this hypothesis suggests that capital and skilled workers exhibit a higher degree of complementarity compared to capital and unskilled workers. With capital-skill complementarity, technological progress can effectively drive an increase in the marginal productivity of skilled labour while diminishing that of unskilled labour. However, it is important to note that the relationship between technology shocks and capital-skill complementarity is complex and multifaceted. While the existing literature predominantly supports the hypothesis of capital-skill complementarity, there are also studies that yield null or weak evidence in this regard. Goldin and Katz (1998) argue that technological advancements may not always lead to an increased demand for skilled workers. In fact, certain technological developments have the potential to replace skilled labour, resulting in a decline in their demand. Such observations suggest that the phenomenon of capital-skill complementarity may not be a universally enduring characteristic but rather contingent on various factors, including the stage of economic development. To contribute to this ongoing discourse, the second chapter of this thesis endeavors to test the hypothesis of capital-skill complementarity by employing industry-level panel data encompassing a wide range of sectors within the U.S. economy.

Lastly, this thesis thoroughly investigates the role of technology factors in business-cycle models and specifically focuses on analyzing the implications of technology shocks on the response of hours worked among labour of different skill levels. The study builds upon the foundation laid in the first chapter, which extensively discussed the importance of accounting for variable utilisation in the context of business cycles. Basu (2006) utilizes the methodologies introduced by Basu (1997) and Basu, Fernald, and Kimball, who build upon the works of Solow (1957) and Hall (1990), to construct an index of aggregate technology change. This index effectively controls for the cyclical patterns observed in the Solow residual due to variations in both returns to scale and utilisation of inputs. Notably, Basu's findings suggest that technology improvements lead to a significant decrease in total hours worked in the short run, contradicting the conventional results of the real-business-cycle model (RBC) that propose increased input across all time horizons. Since then, the utilisation-adjusted technology shock has become commonplace in analyzing the dynamics of business cycles, leading to further expansions and investigations in this field. The relationship between hours worked, employment, and technology shocks has been extensively explored in the literature, with empirical studies providing evidence of a persistent decline in employment following positive technology shocks, challenging the real business cycle model's ability to explain business cycles. In contrast to previous literature, the third chapter of this thesis seeks to examine the response of hours worked by skilled and unskilled workers to utilisation-adjusted technology shocks.

The subsequent sections of this thesis are structured as follows:

Chapter 1 introduces an innovative methodology for measuring utilisation-controlled neutral technology shocks, ensuring robustness to factor heterogeneity. This approach estimates utilisation series separately for high-skilled and low-skilled labour, allowing for a more comprehensive analysis of cyclical fluctuations in utilisation at both the aggregate and disaggregate levels.

Chapter 2 utilizes the linear estimation technique proposed by Goldin and Katz (1998) and adopts the data-splitting methodology outlined by Hansen (2000) to examine the validity of Griliches (1969)'s capital-skill complementarity hypothesis across various industries in the U.S. economy.

In Chapter 3, a stochastic dynamic general equilibrium (DSGE) model is employed, integrating capital-skill complementarity in production, Calvo prices, and nominal wage rigidity. Through simulations and calibration to the U.S. economy, this chapter aims to provide insights into the observed negative dynamics in the skilled-to-unskilled working hours ratio and the differential responses of hours worked by skilled and unskilled workers to utilisation-adjusted technology shocks.

In summary, the primary objective of this thesis is to enhance and refine our understanding of how technology shocks, labour and capital utilisations, and capital-skill complementarity impact labour market dynamics. By delving into these areas of research, this study seeks to deepen our insights into the intricate relationships among these factors and their implications for the labour market. Furthermore, this thesis aims to contribute to the exploration of policy implications, particularly in addressing the changes related to hours worked and the evolving skill demands in the modern economic landscape. Through this research, valuable insights can be gained to inform policymakers and guide the formulation of effective policies that promote sustainable economic growth and address the evolving labour market needs in an ever-changing technological environment.

# 1 Revisiting Aggregate Technological Shocks While Controlling for Varying Utilisations of Different Skill Levels

## 1.1 Introduction

The business cycle refers to the fluctuation of gross domestic product (GDP) around its long-term growth trend. The key characteristic of business cycles is the positive co-movement between economic status and various economic variables, and the cyclical variation of labour productivity is a well-documented feature of business fluctuations. It is crucial to identify the primary macroeconomic shocks that drive fluctuations in economic activity and determine the most effective policies for maintaining economic stability. There exist various underlying factors that may contribute to the occurrence of business cycles. The RBC model, developed by Kydland and Prescott (1982), is among the most well-known models used to explain economic fluctuations. These scholars were motivated by the observation that detrended Solow residuals closely track detrended real GDP, leading them to suggest that technology shocks could be a potential explanation for business cycles.

The literature on the relationship between technology shocks and productivity cycles is extensive, with numerous studies challenging the use of the Solow residual as a reflection of stochastic movements in aggregate production technology. The mismeasurement of inputs is a common issue that is often discussed in this context, as it refers to the inadequate consideration of unobserved fluctuations in capital utilisation and labour effort. Scholars contend that firms incur significant adjustment costs in their decisions to hire employees and accumulate capital, resulting in a form of 'factor hoarding,' whereby employment and capital become quasi-fixed factors. Consequently, firms respond to short-run fluctuations in production by intensively utilising labour and capital during economic booms and less during recessions. Therefore, failing to consider cyclical factor utilisations can lead to the risk of cyclical miss-measurement. Understanding the extent to which the cyclicality of productivity reflects variable factor utilisation is crucial for several reasons. Firstly, if variable utilisation is present, it can serve as an important propagation mechanism in the economy. Secondly, if factor utilisation varies over the business cycle, it can help to explain some of the observed lead-lag relationships in the data. Finally, accounting for variable utilisation enables the cyclicality of measured productivity to be explained without invoking increasing return to scale or unobserved fluctuations in technology. Consequently, the importance of variable utilisation is widely recognized in the literature on economic fluctuations.

Factor hoarding has been extensively studied in the economic literature. A number of contributions were simulated by Hall (1989) who challenged the real business theorists' identification of Solow residuals as true technology shocks. However, empirical tests of the labour hoarding hypothesis face a challenge in observing capital and labour utilisations. Economists have developed different ways to measure the unobservable labour effort and capital utilisations, recognizing the importance of variable factor utilisations in examining cyclical movements. For instance, Shea et al. (1990) uses accident rates to proxy for variation in effort and finds that this can explain some but not all of the cyclical productivity. Caballero and Lyons (1992) use data on overtime hours and the ratio of production over non-production workers as proxies for factor utilisations and conclude that neither true increasing returns nor unmeasured factor utilisations can solely account for aggregate procyclical productivity. Basu (1996) use material growth as an indicator of cyclical factor utilisations and find that technology shocks are small and have low correlation with output or hours growth after controlling for factor utilisations. Other studies take a different approach to measure this issue by imposing optimality conditions on the behavior of economic agents. Bils and Cho (1994) introduce procyclical labour and capital utilisation, along with the costs of rapidly increasing employment into a general equilibrium model. They posit that both effort and utilisation rate of capital increase when workers work longer hours. Their study demonstrates that workweeks, labour effort, capital utilisation, and productivity all exhibit pronounced leading behavior in relation to the business cycle. Burnside and Eichenbaum (1994) assume the capital utilisation rate as a function of capital depreciation rate and show that by adding plausible cyclical variations in capital utilisations, their model can generate the same amount of aggregate output variability with smaller technology disturbances. Sbordone (1997) and Sbordone (1996) and Basu and Kimball (1997) derive effort within a cost minimization framework and confirm the role of cyclical labour utilisations in generating cyclical productivity.

Existing literature has explored the importance of variable utilisation in the business cycle context. However, there is a lack of research that specifically examines heterogeneity in utilisation, particularly with regard to the difference in utilisation between high-skilled and low-skilled workers. Notably, the work of Bocola et al. (2011) is among the most relevant literature in this area, it focuses on identifying technology shocks in the presence of heterogeneity in factor utilisations. Bocola argues that identifying neutral technology using structural vector autoregressions (SVAR) is challenging in models with heterogeneous capital and labour, as the key long-run restrictions required for identifying neutral technology are violated. The long-run restrictions implemented in the business cycle context by Gali (1999) assume that only technology shocks have a long-term effect on labour productivity. [1] It as been intensively discussed in existing literature, questioning the reliability of the results when imposing long-run restriction in relatively small sample (e.g., Erceg et al. (2005), Faust and Leeper (1997), Chari et al. (2008)). Additionally, the long-run restrictions have been also criticized for their weakness (e.g., Uhlig (2004), Shea (1998)). [2] Amidst the literature that has cast doubts on long-run restrictions, Bocola et al. (2011) question that in the model with heterogeneous labour or capital inputs most non-technology shocks can have a long-run effect on labour productivity as well, consequently violating long-run restrictions. Therefore, in steady of using SVAR, Bocola et al. (2011) propose an alternative method for estimating neutral technology shocks using filtering/smoothing techniques. However, this method cannot uncover the series of capital and labour utilisation for high-skilled and low-skilled workers which we focused in this paper.

This paper presents a novel approach for measuring utilisation-controlled neutral technology shocks that is robust to factor heterogeneity. More importantly, the approach estimates the utilisation series for both high-skilled and low-skilled labour, allowing for a more granular analysis of cyclical movements of utilisation at both the aggregate and disaggregate levels. To estimate neutral technology, we use a classic approach first proposed by Solow (1957), which involves estimating residual output growth - i.e., the growth of output not accounted for by the growth of inputs. Effort and capital utilisation associated with high-skilled and low-skilled labour are derived using a cost-minimization framework, building on Basu et al. (2006)'s construction of aggregated index of labour and capital utilisations controlled aggregate technology. [3] Our work differs from Basu's in two key ways. Firstly, we decompose the aggregate labour input into two types - high-skilled and low-skilled - which enables us to identify utilisation for different skill levels. Secondly, while Basu et al. (2006) used the standard Cobb-Douglas production function with fixed capital, labour, and intermediate input shares to demonstrate that hours-per-worker could be used as a proxy for capital's workweek, we extend this analysis by employing a more general production function with varying input ratios across industries. Specifically, our approach allows us to find that capital's workweek depends not only on hours-per-worker but also on the relative share of total expenditures in capital and labour compensations to the total cost of input factors. By adopting this more flexible approach, our analysis provides a more robust understanding of the factors influencing capital utilisation and its relationship with labour utilisation.

We make several critical assumptions in this paper, firstly, we assume that firms behave competitively in the factor market. Secondly, consistent with Basu et al. (2006), we adopt the assumption that increasing labour utilisation incurs a higher wage cost due to the increased disutility of effort. Moreover, we posit that the primary cost of increasing capital utilisation is the shift premium paid to employees for working during undesirable times, as workers have to be present to ensure the smooth operation of the production process. [4] However, unlike Basu, we introduce two skill types and thus the shift premium paid to workers must be adjusted to reflect these differences. For the sake of simplicity, we assume a uniform shift premium function for both high- and low-skilled labour. Nevertheless, the heterogeneity in the shift premium for different types of workers is accounted for by differences in the workweek of capital associated with different skill levels.

[1]The work of Blanchard and Quah (1988),King et al. (1991) highlighted a widespread interest in using vector autoregressions (VARs) to impose long-run restrictions for identifying the severity of the effects of shocks.

[2]According to Shea (1998), the bankruptcy of low-productivity companies during economic downturns could have a lasting impact on productivity. Meanwhile, Uhlig (2004) contends that a shift in societal attitudes toward work, in which individuals engage in leisure activities at work instead of at home, could lead to inaccuracies in measuring effective work hours and ultimately affect productivity measurements over an extended period of time.

[3]Basu et al. (2006)'s work follows Basu and Fernald (1997) and Basu and Kimball (1997). Basu and Fernald (1997) build on Solow (1957) and Hall (1990) and stress the role of sectoral heterogeneity and aggregation. And Basu and Kimball (1997) stress the role of variable capital and labour utilisation.

[4]Bils and Cho (1994) find evidence that capital utilisation does respond significantly to movements in weekly hours of work, which indicating the close relationship between capital utilisation and hours per worker.

The decision of a firm to increase the workweek of capital may have varying impacts on the working hours of different labour types, contingent on the proportion of their workload associated with managing machines and other capital. Workers whose job is closely related to managing machines and equipment may have their working hours extended to a greater extent when firms increase capital utilisation than those whose workload is less reliant on such equipment. This discrepancy arises because the former group of workers needs to be present during periods of capital utilisation to ensure the smooth and effective operation of the production process. To address this, we disaggregate the capital's workweek into high- and low-skilled workers to reflect the fact that an increase in capital's workweek corresponds to specific types of workers' working hours. We believe that the assumptions in our cost-minimization framework are more plausible than the assumptions required for solving a full general-equilibrium model.

Our study yields several main results. First, at the aggregate level, our model generates results that are consistent with the standard Real Business Cycle (RBC) model. However, we find that a utilisation-controlled technology shock leads to less business cycle fluctuations in output, inputs, employment, and total hours than the TFP shock. Secondly, our analysis indicates that at aggregate economy level, firms tend to motivate high-skilled workers to put in more effort and prolong high skills working hours when increasing capital utilisation in response to technological advancements, while simultaneously reducing utilisation of both labour and capital associated with low-skilled workers. Thirdly, we find that the IRFs of utilisation for high-skilled and low-skilled workers to a technology advancement differ across sectors. In response to technological advancements, firms in the durable manufacturing sector exhibit a decrease in the working intensity of high-skilled labour while maintaining the working intensity of low-skilled labour relatively constant. Conversely, in the nondurable goods sector, firms demonstrate an increase in the utilisation of high-skilled labour and a reduction in the utilisation of low-skilled labour. In the non-manufacturing sector, firms demonstrate a reduction in the utilisation of low-skilled labour while maintaining the utilisation of high-skilled labour relatively constant. The cyclical movements of utilisation provide some evidence supporting capital-skill complementarity hypothesis in the aggregate economy, the nondurable manufacturing sector, and the non-manufacturing sector. However, in the durable manufacturing sector, our results indicate a substitution relationship between capital and skilled labour.

The present study identified and analyzed the utilisation series for both high-skilled and low-skilled labour, thus providing a more nuanced perspective on the cyclical movements of utilisation at both the aggregate and sectoral levels. By doing so, this research sheds light on the importance of undertaking an industry-focused analysis in order to gain a comprehensive understanding of the diversity in utilisation patterns for workers with different skill levels. Furthermore, this paper contributes to the existing literature on the interplay between capital and skilled labour in the context of technological progress. Specifically, it provides insights into the differential effects of technology shocks on workers with different skill levels across various economic sectors. Overall, this research expands our understanding of the complex dynamics between technology, capital, and labour, and emphasizes the importance of considering the heterogeneity of labour utilisation patterns across different skill levels and economic sectors.

The paper is organized as follows. In Section 1.2, we review our methodology for identifying utilisation-controlled neutral technology shocks that is robust to different skill levels. Section 1.3 describes the data and econometric model used in the analysis. In Section 1.4, we present our main results for the aggregate economy, while Section 1.5 discusses the results for the disaggregate sectors. Section 1.6 examines the complementarity/substitution between capital and skill across sectors. Finally, in Section 1.7, we conduct robustness checks to test the sensitivity of our results to alternative specifications and model assumptions.

## 1.2 Model

We identify aggregate technology growth by estimating a regression equation with proxies for capital and labour utilisation in each disaggregated industry by GMM estimation. We then aggregate the industry-level technology growth as the weighted sum of the resulting residuals. Section 1.2.1 shows the decomposition of the production function; Section 1.2.2 discuss how we control for utilisations; and Section 1.2.3 discuss the aggregation.

### 1.2.1 Decompose Production Function

We assume each industry has a production function that firms use capital inputs $\tilde{K}_i$, labour inputs of high-skilled and low-skilled workers, $\tilde{L}_{i,H}$,$\tilde{L}_{i,L}$, and intermediate inputs of materials and energy $M_i$ to produce gross output $Y_i$ through the technology $Z_i$ and production technique $F$:

$$Y_{i,t} = F(\tilde{K_{i,t}}, \tilde{L}_{i,H,t}, \tilde{L}_{i,L,t}, M_{i,t}, Z_{i,t})$$

The labour and capital inputs depend on the number of total hours and capital stock and the intensity with which capital and labour are utilised. The labour inputs are the product of the number of employees, $N$, hours per worker $h$, and the labour effort, $\phi_H$ and $\phi_L$. And the capital inputs are the product of the workweek of capital, $\phi_{K,H}$ and $\phi_{K,L}$ and the capital stock, $K$. We eliminate the industry and time subscript for convenience. Thus, the inputs are defined as:

$$\tilde{L}_H = \phi_H h_H N_H$$

$$\tilde{L}_L = \phi_L h_L N_L$$

$$\tilde{K} = \phi_{K,H}\phi_{K,L}K$$

For a meaningful model with varying capital and labour utilisation, we assume the firm cannot change the number of employees and capital stock costless; thus, $K$ and $N$ are quasi-fixed. [5] We also assume the cost of increasing the capital utilisation is to pay a shift premium to compensate workers for working for a long time or at an undesirable time. And when labour utilisation increases, workers must be compensated with a higher wage. Basu et al. (2006) only consider the aggregate labour inputs in the production function; different from them, our model introduced two types of labour; high-skilled and low-skilled workers.

The impact of a firm's decision to increase the workweek of capital on the working hours of different types of workers is dependent on the proportion of their workload associated with managing machines and other capital. As such, the extent to which each type of worker will extend their working hours will vary. This has implications for the shift premium, which will be different for each type of labour. Workers whose workload is more closely tied to the management of machines and other capital are likely to extend their working hours to a greater degree than workers whose work is less reliant on capital. The reason for this is that the former group of workers will need to be present during the increased utilisation of capital in order to

[5] Basu et al. (2006) illustrated that the quasi-fixed number of capital stock and employees ensures a meaningful model with varying factor utilisations. If a firm can vary $K$ and $N$ freely, it would always keep the factor utilisation at the cost-minimizing level to avoid paying shift premiums and higher wages to compensate workers for working longer. Only if it is costly to change $K$ and $N$ is it reasonable for a firm to change factor utilisation and pay the costs of varying utilisations.

manage it effectively. Since the extent to which each type of worker extends their working hours is different, the shift premium will need to be adjusted accordingly in order to reflect this difference. For simplicity, we assume the shift premium function is the same for two types of labours, and the workweek of capital, i.e., capital utilisation $\phi_{K,H}$ and $\phi_{K,L}$ captures the heterogeneity in shift premium. That is, we use $V(.)$ to denote the shift premium function, then the shift premium for high and low-skilled workers are $V(\phi_{K,H})$ and $V(\phi_{K,L})$, respectively. Taking the log of both sides of the production function and then differentiating with respect to time gives,

$$\frac{dY/dt}{Y} = \frac{F_1 d\tilde{K}/dt}{F} + \frac{F_2 d\tilde{L}_H/dt}{F} + \frac{F_3 d\tilde{L}_L/dt}{F} + \frac{F_4 dM/dt}{F} + \frac{F_5 dZ/dt}{F} \tag{1}$$

Substitute the differentiation of capital and labour inputs $\tilde{K}, \tilde{L_H}$ and $\tilde{L_L}$ with respect to time, we can rewrite Equation 1 as (after some algebra):

$$\begin{aligned} \frac{dY/dt}{Y} = & \frac{F_1\tilde{K}}{F}\left(\frac{dK/dt}{K} + \frac{d\phi_{K,H}/dt}{\phi_{K,H}} + \frac{d\phi_{K,L}/dt}{\phi_{K,L}}\right) \\ & + \frac{F_2\tilde{L_H}}{F}\left(\frac{d\phi_H/dt}{\phi_H} + \frac{dh_H/dt}{h_H} + \frac{dN_h/dt}{N_h}\right) \\ & + \frac{F_3\tilde{L_L}}{F}\left(\frac{d\phi_L/dt}{\phi_L} + \frac{dh_L/dt}{h_L} + \frac{dN_L/dt}{N_L}\right) + \frac{F_4 M}{F}\frac{dM/dt}{M} + \frac{F_5 Z}{F}\frac{dZ/dt}{Z} \end{aligned} \tag{2}$$

Let $dj$ represent the growth rate of input factor $J$ (i.e., $dj = \frac{dJ/dt}{J}$), let $d\phi_{K,m}$ denote the growth rate of capital utilisation, and $d\phi_m$ denote the growth rate of labour utilisation for labour type $m \in (H, L)$. To simplify the analysis, we have normalized the output elasticity with respect to technology to one. As a result, Equation 2 can be expressed as follows:

$$\begin{aligned} dy = & \frac{F_1\tilde{K}}{F}(dk + d\phi_{K,H} + d\phi_{K,L}) \\ & + \frac{F_2\tilde{L_H}}{F}(d\phi_H + dh_H + dn_H) \\ & + \frac{F_3\tilde{L_L}}{F}(d\phi_L + dh_L + dn_L) + \frac{F_4 M}{F}dm + dz \end{aligned} \tag{3}$$

Suppose that firms take the price of all input $J$, $P_{Ji}$ as given by competitive markets, and denote the markup over marginal cost as $\mu_i$. According to Basu and Fernald (2007), the first order condition for costs-minimization implies the firm set the value of maginal product of factor $J$ equal to a markup over the factor's input price, that is:

$$P_i F_J^i = \mu_i P_{Ji} \tag{4}$$

Using equation 4, we can write the output elasticity with respect to factor $J$ as the markup times the total expenditure for factor $J$ over the total revenue,[6] that is,

[6] $S_J$ is defined as the total expenditure for input factor $J$ over the total revenue. When the firm earns zero profit, $S_J$ equals to the ratio of expenditure for input factor $J$ to the total costs for inputs factors; $S_J = \frac{P_J J}{\sum j P_{Jj} J_j}$.

$$\frac{F_1\tilde{K}}{F} = \frac{F_K K}{Y} = \mu\frac{P_K K}{PY} = \mu S_K$$

$$\frac{F_2\tilde{L_H}}{F} = \frac{F_{h_H} h_H}{Y} = \mu\frac{P_{h_H} h_H}{PY} = \mu S_{L_H}$$

$$\frac{F_3\tilde{L_L}}{F} = \frac{F_{h_L} h_L}{Y} = \mu\frac{P_{h_L} h_L}{PY} = \mu S_{L_L}$$

$$\frac{F_4 M}{F} = \frac{F_M M}{Y} = \mu\frac{P_M M}{PY} = \mu S_M$$

Now, substituting the output elasticities with respect to each input factors equations into Equation 3 gives our main estimation equation:

$$\begin{aligned} dy &= \mu(S_K dk + S_{L_H}(dh_H + dn_H) + S_{L_L}(dh_L + dn_L) + S_M dm) \\ &+ \mu(S_K d\phi_{K,H} + S_K d\phi_{K,L} + S_{L_H} d\phi_H + S_{L_L} d\phi_L) + dz \end{aligned} \tag{5}$$

Appendix C.1 details the derivations behind all the equations in this section.

### 1.2.2 Utilisations

To derive the proxies for capital and labour utilisation growth, we set up the cost-minimization problem for the representative firm within each industry. Basu and Kimball (1997) and Basu et al. (2006) provide micro-foundations for using changes in hours-per-worker as a simple proxy for capital and labour utilisation. It is derived from a firm's cost-minimizing problem with the underlying intuition that cost-minimizing firms operate on all margins simultaneously; change in observed factors can be potentially used as a proxy for unobserved utilisation change. The model in this paper follows Basu et al. (2006), but there are two main differences. The first difference is that rather than include the aggregate labour input, we decompose it and introduced two types of labour inputs; high-skilled labour and low-skilled labour. Secondly, Basu and Kimball used the standard Cobb-Douglas production function, which allows the hours-per-worker to be a proxy for capitals' workweek. In this paper, we extend the model to the general production function form and found that besides the hours-per-worker, the capitals' workweek also depends on the share of total expenditures in capital and labour compensations to the total costs of inputs factors. The critical assumptions of the approach are competitive behaviour in the factor market, shift premium function for high and low-skilled labour are equal, and wage functions depend on both labour effort and hours worked, which are more plausible than the assumptions needed for solving a full general-equilibrium model.

We have assumed that the firm can freely change labour effort $\phi_H, \phi_L$ and capital's workweek $\phi_{K,H}$ and $\phi_{K,L}$. Consider an industry's representative firm minimizes the present value of expected costs:

$$\begin{aligned} & min_{\phi_{k,H},\phi_{k,L},\phi_H,\phi_L,h_H,h_L,I,D} E_t \sum_{\tau=t}^{\infty}[\prod_{j=t}^{\tau-1}(1+r_j)^{-1}] \\ & \times [W_{H,\tau}G(h_{H,\tau},\phi_{H,\tau})V(\phi_{K,H,\tau})N_{H,\tau} + W_{L,\tau}G(h_{L,\tau},\phi_{L,\tau})V(\phi_{K,L,\tau})N_{L,\tau} \\ & + W_{H,\tau}N_{H,\tau}\Psi\left(\frac{D_{H,\tau}}{N_{H,\tau}}\right) + W_{L,\tau}N_{L,\tau}\Psi\left(\frac{D_{L,\tau}}{N_{L,\tau}}\right) + P_{I,\tau}K_\tau J_\tau\left(\frac{I_\tau}{K_\tau}\right) + P_M M_\tau] \end{aligned} \tag{6}$$

*s.t.*

$$Y_\tau = F[(\phi_{K,H,\tau}\phi_{K,L,\tau}K_\tau), (\phi_{H,\tau}h_{H,\tau}N_{H,\tau}), (\phi_{L,\tau}h_{L,\tau}N_{L,\tau}), M_\tau, Z_\tau] \tag{7}$$

$$K_{\tau+1} = I_\tau + (1-\delta)K_\tau$$

$$N_{\tau+1,H} = N_{\tau,H} + D_{\tau,H}$$

$$N_{\tau+1,L} = N_{\tau,L} + D_{\tau,L}$$

In each period, the firm's costs in Equation 6 encompass the total payments for two types of labour, capital, and materials, as well as the costs of new hiring denoted by $D$ and gross investment denoted by $I$. Specifically, $W$ represents the wage rate, and $G(.)$ is the wage function that determines how the hourly wage depends on effort and the hours worked per worker. $V(.)$ represents the shift premium function for capital utilisation. When a firm increases the capital workweek, it incurs additional costs in the form of shift premiums to compensate workers for working longer and harder. For simplicity, we assume that the shift premium functions $V(.)$ are the same for both types of workers, and $\phi_{K,H}$ and $\phi_{K,L}$ capture the heterogeneity in the shift premiums for the two types of workers. The term $WG(.)V(.)N$ represents the total labour compensation. Moreover, $\Psi(\frac{D}{N})$ represents the adjustment costs associated with changing the number of workers. It is important to note that, following Basu et al. (2006), for a meaningful model of capital and labour utilisation, it assumes that the number of machines, buildings, and workers is quasi-fixed. Thus, firms face adjustment costs when changing the number of production inputs. With these costs, it is sensible for a firm to consider the expenses of variable capital and labour utilisation. The term $P_I KJ(I/K)$ represents the cost of investment, and $\delta$ denotes the rate of depreciation. Furthermore, we assume that the cost functions $\Psi$, $G$, and $J$ are convex.

To save space, we only analyze the optimization conditions that affect our derivations, which are those for $\phi_{K,H}$, $\phi_{K,L}$, $\phi_H$, $\phi_L$, $h_H$, $h_L$, $N_H$, and $N_L$. Denoting the multiplier on constraint Equation 7 as $\lambda$, we use $F_J, J = 1, 2, 3$ to represent derivatives of the $F$ function with respect to input factor $J$. The intertemporal first-order conditions are as follows (we omit the time and industry subscripts except where needed for clarity) from where we can further derive the capital and labour utilisation in the following subsections.

$$\phi_{K,H}: \quad \lambda F_1 \phi_{K,L} K = W_H N_H G(h_H, \phi_H) V'(\phi_{K,H}) \tag{8}$$

$$\phi_{K,L}: \quad \lambda F_1 \phi_{K,H} K = W_L N_L G(h_L, \phi_L) V'(\phi_{K,L}) \tag{9}$$

$$h_H: \quad \lambda F_2 \phi_H N_H = W_H N_H G_{h_H}(h_H, \phi_H) V(\phi_{K,H}) \tag{10}$$

$$h_L: \quad \lambda F_3 \phi_L N_L = W_L N_L G_{h_L}(h_L, \phi_L) V(\phi_{K,L}) \tag{11}$$

$$\phi_H: \quad \lambda F_2 h_H N_H = W_H N_H G_{\phi_H}(h_H, \phi_H) V(\phi_{K,H}) \tag{12}$$

$$\phi_L: \quad \lambda F_3 h_L N_L = W_L N_L G_{\phi_L}(h_L, \phi_L) V(\phi_{K,L}) \tag{13}$$

$$N_H: \quad \lambda F_2 \phi_H h_H = W_H G(h_H, \phi_H) V(\phi_{K,H}) + W_H \Psi\left(\frac{D_H}{N_H}\right) - W_H N_H \left(\frac{D_H}{N_H^2}\right) \Psi_{N_H}\left(\frac{D_H}{N_H}\right) \tag{14}$$

$$N_L: \quad \lambda F_2 \phi_L h_L = W_L G(h_L, \phi_L) V(\phi_{K,L}) + W_L \Psi\left(\frac{D_L}{N_L}\right) - W_L N_L \left(\frac{D_L}{N_L^2}\right) \Psi_{N_L}\left(\frac{D_L}{N_L}\right) \tag{15}$$

#### 1.2.2.1 Labour Utilisation

Combining Equation 10 and Equation 12 gives an equation relating labour effort and hours worked for the high skilled workers:

$$\frac{\phi_H G_{\phi_H}(h_H, \phi_H)}{G(h_H, \phi_H)} = \frac{h_H G_{h_H}(h_H, \phi_H)}{G(h_H, \phi_H)} \tag{16}$$

This implies the equalisation of the elasticities of labour costs with respect to labour effort $\phi_H$ and hours $h_H$. As $G(.)$ is an increasing and convex cost function, Equation 16 implies an upward-sloping expansion path between labour effort and hours for high skill worker: $\phi_H = \phi_H(h_H)$, $\phi'_H(h_H) > 0$. Thus, the unobservable labour utilisation can be expressed as a monotonically increasing function of the observed number of hours per worker. In the long run, it can be shown that the elasticities of labour costs with respect to hours and effort are equal to the elasticity of labour costs with respect to employment which is 1. Only in the short run, with the adjustment cost, the elasticity of labour costs with respect to employment is different from the others. In the long run, employment reaches a steady state, and the terms with $D_H$ in Equation 14 disappear. [7] Combining Equation 14 with steady state version of Equation 10 shows the elasticity of labour costs with respect to $h_H$ equals to 1. [8] Moreover, combining the steady-state version of Equation 14 and Equation 12 shows the elasticity of labour costs with respect to the effort also is 1. [9] Combining the steady-state version of equation 15 with Equation 11 and 13 gives symmetrical results for low-skilled workers.

Log-linearize Equation 16 at the steady-state gives the equation relates hours-per-worker with labour effort for the high-skilled worker. In other words, the unobserved effort growth can be proxied by the observed growth of hours.

$$d\phi_H = \eta_H dh_H \tag{17}$$

Similarly, combining Equation 11 and Equation 13 gives the equation between hours-per-worker and labour effort for the low-skilled worker,

$$d\phi_L = \eta_L dh_L \tag{18}$$

[7] $\Psi(0) = \Psi'(0) = 0$

[8] Steady-state version of Equation 14 and Equation 10 are $\lambda F_2 \phi_H^* h_H^* = W_H^* G(h_H^*, \phi_H^*) V(\phi_{K,H}^*)$ and $\lambda F_2 \phi_H^* N_H^* = W_H^* N_H^* G_{h_H}(h_H^*, \phi_H^*) V(\phi_{K,H}^*)$, respectively. Combining two equations shows $\frac{h_H^*}{N_H^*} = \frac{G(h_H^*, \phi_H^*)}{N_H^* G_{h_H}(h_H^*, \phi_H^*)}$. Rearrange to $\frac{h_H^* N_H^* G_{h_H}(h_H^*, \phi_H^*)}{N_H^* G(h_H^*, \phi_H^*)} = \frac{N_H^* G(h_H^*, \phi_H^*)}{N_H^* G(h_H^*, \phi_H^*)} = 1$.

[9] Combining the steady-state version of Equation 14 and Equation 12 shows $\frac{\phi_H^* N_H^* G_{\phi_H}(h_H^*, \phi_H^*)}{N_H^* G(h_H^*, \phi_H^*)} = \frac{N_H^* G(h_H^*, \phi_H^*)}{N_H^* G(h_H^*, \phi_H^*)} = 1$.

Where $\eta_H = \frac{h_H^* \phi_H'(h_H^*)}{\phi_H(h_H^*)}$ and $\eta_L = \frac{h_L^* \phi_L'(h_L^*)}{\phi_L(h_L^*)}$ are the elasticity of labour effort with respect to hours, evaluated at the steady state.

#### 1.2.2.2 Capital Utilisation

To derive the proxy for capital utilisation, we combine Equation 8 and Equation 10, and after some algebra, it gives,

$$\frac{(F_1\tilde{K})/F}{(F_2\tilde{L_H})/F} = \frac{G(h_H, \phi_H)}{h_H G_{h_H}(h_H, \phi_H)} \cdot \frac{\phi_{K,H} V'(\phi_{K,H})}{V(\phi_{K,H})} \tag{19}$$

The left-hand side is the ratio of output elasticity with respect to $\tilde{K}$ and $\tilde{L_H}$. As mentioned in Section 1.2.1, this ratio is proportional to share of factor costs to total revenue, denoted as $\frac{S_K}{S_{L_H}}$. [10] Now, let's denote $g(h_H)$ and $v(\phi_{K,H})$ as the elasticity of labour costs function $G(.)$ with respect to hours for high-skilled workers and the elasticity of shift premium $V(.)$ with respect to capital's workweek associated with high-skilled workers, respectively. Note that $g(h_H)$ is positive because $G(\cdot) > 0$ and $G'(\cdot) > 0$, and $v(\phi_{K,H})$ is positive if the shift premium $V(\phi_{K,H})$ is positive. With this information, we can express the right-hand side of Equation 19 as the multiplication of $v(\phi_{K,H})$ and the inverse of $g(h_H)$. Therefore, we can rewrite Equation 19 as follows:

$$\frac{S_K}{S_{L_H}} = \frac{v(\phi_{K,H})}{g(h_H)} \tag{20}$$

Let $g(h_L)$ denote the elasticity of the labour costs function $G(\cdot)$ with respect to the hours of low-skilled workers, and $v(\phi_{K,L})$ represent the elasticity of the shift premium function $V(\cdot)$ with respect to capital's workweek associated with low-skilled workers. We define $\frac{(F_1\tilde{K})/F}{(F_3\tilde{L_L})/F}$ as the ratio of factor cost share, denoted as $\frac{S_K}{S_{L_L}}$. Then, by the same logic as embodied in Equation 20, combining Equation 9 and Equation 11 we find the similar equation for the low skilled worker. [11]

$$\frac{S_K}{S_{L_L}} = \frac{v(\phi_{K,L})}{g(h_L)} \tag{21}$$

Log-linearizing Equation 20 and Equation 21 and define $\epsilon = \frac{h^* g(h^*)_h}{g(h^*)}$ as elasticity of $g(h^*)$ with respect to $h$ and $\kappa = \frac{\phi_K^* v(\phi_K^*)_{\phi_K}}{v(\phi_K^*)}$ as elasticity of $v(\phi_K^*)$ with respect to $\phi_K$ to obtain:

$$d\phi_{K,H} = \frac{1}{\kappa_H} ds_k + \frac{\epsilon_H}{\kappa_H} dh_H - \frac{1}{\kappa_H} ds_{L_H} \tag{22}$$

$$d\phi_{K,L} = \frac{1}{\kappa_L} ds_k + \frac{\epsilon_L}{\kappa_L} dh_L - \frac{1}{\kappa_L} ds_{L_L} \tag{23}$$

Equation 22 and Equation 23 show that the growth rate of capital utilisation depends on the growth rate of hours-per-worker, the growth rate of the proportion of expenditure in capital and labour inputs over total revenue. In other words, the unobserved capital utilisation growth can be proxied by observed hours growth and factor costs share growth. Basu and Fernald (2006) employs a generalized Cobb-Douglas production

[10] Basu et al. (2006) assumes a generalized Cobb-Douglas function for the inputs of capital ($K$) and labor ($L$), given by: $Y = ZF\Big((\phi_K K)^{\alpha_K} (\phi_H HN)^{\alpha_L}, M\Big)$. This functional form implies that the ratio of the output elasticity with respect to capital and labour inputs is proportional to the factor cost shares $\alpha_K$ and $\alpha_L$.

[11] Combining Equation 9 and Equation 11 gives $\frac{(F_1\tilde{K})/F}{(F_3\tilde{L_L})/F} = \frac{G(h_L, \phi_L)}{h_L G_{h_L}(h_L, \phi_L)} \cdot \frac{\phi_{K,L} V'(\phi_{K,L})}{V(\phi_{K,L})}$.

function, assuming a constant share of capital and labour inputs across industries. Consequently, Equation 20 and Equation 21 simplifies to $v(\phi_K) = \frac{\alpha_K}{\alpha_L} g(h)$, where $\frac{\alpha_K}{\alpha_L}$ is assumed to be constant. The results obtained from log-linearization indicate that capital utilisation is solely proxied by the measure of hours worked per worker.

By substituting Equation 17, 18, 22, and 23 into Equation 5, we obtain the estimation equation that incorporates controls for both capital utilisation and labour effort for the two types of labour.

$$\begin{aligned} dy &= \mu dx + \left(\frac{\mu S_K}{\kappa_H} + \frac{\mu S_K}{\kappa_L}\right) ds_k + \left(\frac{\mu S_K \epsilon_H}{\kappa_H} + \mu S_{L_H} \eta_H\right) dh_H - \frac{\mu S_K}{\kappa_H} ds_{L_H} \\ &+ \left(\frac{\mu S_K \epsilon_L}{\kappa_L} + \mu S_{L_L} \eta_L\right) dh_L - \frac{\mu S_K}{\kappa_L} ds_{L_L} + dz \\ &= \mu dx + \alpha_1 ds_k + \alpha_2 dh_H + \alpha_3 ds_{L_H} + \alpha_4 dh_L + \alpha_5 ds_{L_L} + dz \end{aligned} \tag{24}$$

Where

$$dx = S_K dk + S_{L_H}(dn_H + dh_H) + S_{L_L}(dn_L + dh_L) + S_M dm$$

We do not need to explicitly estimate all of the parameters in Equation 24 that multiply $ds_k, dh_L, dh_H, ds_{L_L}$ and $ds_{L_H}$. Following Basu et al. (2006), to conserve parameters, we constrain the coefficients associated with capital and labour utilisation variables, namely $ds_k, dh_L, dh_H, ds_{L_L}$ and $ds_{L_H}$, within three groups: durables, nondurables and nonmanufacturing. However, we allow the return-to-scale coefficient $\mu$ to vary across industries. The industry-level utilisation-controlled technology change is quantified as the residuals $dz$. In the absence of capital and labour utilisation and under assumptions of constant returns and perfect competition, the technology change corresponds to the standard Solow residual or total factor productivity (TFP): : $tfp = dy - dx$.[12]

Appendix C.2 details the derivations behind all the equations in this section.

### 1.2.3 Aggregation

Estimating Equation 24 gives residuals $dz_i$ for each industry, from which we aggregate utilisation-controlled technology growth by Domar-weight, [13] that is a weighted sum of industry technology growth: [14]

$$dz = \sum_{i=1}^{30} \left(\frac{w_i}{1 - s\bar{sn}_i}\right) dz_i$$

The industry's share of the aggregate nominal added value, denoted as $w_i$, is defined as the ratio of industry $i$'s value added ($VA_i$) to the sum of value added across all 30 industries ($\sum_{i=1}^{30} VA_i$). Additionally, $s\bar{sn}_i$ represents the average time ratio of intermediate input expenses in total input costs for industry $i$. [15]

[12]Under the assumptions of constant returns, perfect competition, and no change in utilisation, Equation 5 simplifies to $dy = dx + dz$.

[13]This weighting scheme is from Domar (1961). Basu et al. (2006) illustrated that Domar-weighted series are more robust to mismeasurement.

[14]In nonmanufacturing sector, we include a break of 1972 into the regression; $d72$, and the regression function for nonmanufacturing sector is $dy = C + \alpha_1 dx + \alpha_4 dh_H + \alpha_6 dh_L + C72 * d72 + dz$. Thus, the aggregated growth of utilisation-controlled technology change is defined as $dz = \sum_{i=1}^{30}(\frac{w_i}{1-ssnbar})(dz_i + C + C72 * d72)$, where $C$ is the constant term and $C72$ is the coefficients multiplying break dummy variable.

[15]As mentioned in Section 1.2.1, in each industry $S_J$ refers to the total expenditure on input factor $J$ relative to total revenue. When a firm earns zero profit, $S_J$ is equivalent to the ratio of expenditure on input factor $J$ to the total costs of input factors, which can be expressed as $S_J = \frac{P_J J}{\sum J P_J J} = ssj$. So $\frac{P_M M}{PY} = S_M = ssn$.

Dividing by $(1-s\bar{sn}_i)$ allows us to convert gross-output technology shocks to a value-added basis. Converting to a value-added basis is more desirable because the final aggregate expenditure is accounted as aggregate value added in national accounts. And the growth of aggregate utilisation is calculated as a weighted average of industry-level utilisation growth.

$$du = \sum_{i=1}^{30} \left( \frac{w_i}{1 - s\bar{sn}_i} \right) du_i$$

where

$$du_i = \alpha_1 ds_{k,i} + \alpha_2 dh_{H,i} + \alpha_3 ds_{L_H,i} + \alpha_4 dh_{L,i} + \alpha_5 ds_{L_L,i}$$

## 1.3 Data and Method

The data used in this study is sourced from the April 2013 release of the World KLEMS database. This database, which was adapted from the database on U.S. productivity growth by industry created by Dale Jorgenson and associates, provides detailed labour and capital inputs for the U.S. using the North American Industry Classification System (NAICS). Our sample encompasses industry-level labour and capital inputs from 1949 to 2001, and includes data from 30 industries, including 6 durable, 7 non-durable, and 17 non-manufacturing industries. Table 46 provides a detailed classification of the industries. The sectoral accounts include gross industry output, capital inputs, labour, energy and materials, employment, and value-added. In this study, the labour data encompasses detailed information on labour compensation per hour, average weekly working hours, and employment. The data is further categorized into six skill levels: less than high school, some high school, some high school graduate, some college, college graduate, and more than college. Following the existing literature (such as Krusell et al. (2000), Kawaguchi et al. (2014), Parro (2013)), workers with a college education or higher (at least 16 years of schooling) are classified as skilled labour, while low-skilled labour is defined as workers without a college education. Workers who are aged under 16 or above 64 are excluded from the analysis.

To avoid the potential endogenous problem caused by the correlation of independent variables with technology growth $dz$ in Equation 24, we use instruments that are uncorrelated with technology growth. Specifically, we utilise three instruments from Basu et al. (2006): oil prices, growth in actual government defence spending and quarterly Federal Reserve 'monetary shocks'. [16] Further details about these instruments can be found in the appendix of Basu et al. (2006). The lagged values of the input factors' cost shares to the total costs of inputs serve as remaining three appropriate instruments.

As mentioned in Section 1.2.2.2, to conserve parameters, we constrain the utilisation coefficients (coefficients in front of the growth of capital share, labour input share, and hours-per-worker) to be the same within three groups: durable, nondurable, and nonmanufacturing (Without these constraints, the estimated technology variance increases, but the qualitative and quantitative results show little change). Therefore, for each group, we estimate,

$$dy_i = c_i + \mu_i dx_i + \alpha_1 ds_{k,i} + \alpha_2 dh_{H,i} + \alpha_3 ds_{L_H,i} + \alpha_4 dh_{L,i} + \alpha_5 ds_{L_L,i} + dz_i \tag{25}$$

The residual gives the industry-level utilisation controlled technology change, and the aggregate utilisation-controlled technology change is the weighted sum of the industry residuals. The industry level capital and

[16] For oil, it refers to the increase in the U.S. refiner acquisition price for crude oil. Additionally, monetary shock is defined as innovations to the 3-month Treasury bill rate. These three instruments are lagged by one year.

labour utilisation change are defined as the sum of hours-per-worker change, capital and labour share changes for two types of workers.

## 1.4 Estimation Results

### 1.4.1 Estimates Summary

The results of the GMM estimation are presented in Table 1. The average return-to-scale estimates are 0.96, 1.02, and 1.00 for durable, nondurable, and non-manufacturing industries, respectively, with an average estimate across all industries of 0.99. These estimates were calculated after excluding the estimates for the real estate activities industry, for which the coefficients were negative and statistically insignificant. There is no evidence of increasing returns after controlling for utilisation, and the return-to-scale estimates vary widely across industries during the sample period. This results align with Basu (1996), Basu and Kimball (1997) and Basu et al. (2006). In recent years, the declining labour income share in the U.S. has been a focal point, with increased markups and returns to scale drawing attention. Basu (2019) examined several empirical methodologies for gauging markups and returns to scale, all of which originate from a firm's cost-minimization problem with given inputs prices. For a firm producing output $Y$ with a production function $Y = F(K, L, Z)$, the cost-minimizing problem requires product of the markup and the cost of each input equal to the marginal product of that input, denoted by $PF_L = \mu W$ and $PF_K = \mu R$. Multiplying each condition by the corresponding input, dividing by the output $Y$, and then summing the conditions, gives the first approach for measuring markups and returns to scale. Specifically, in the first approach, the return to scales (the left-hand side) are equated to product of the markup and one minus the profit ratio $s_\pi$,

$$\frac{F_L L}{Y} + \frac{F_K K}{Y} = \mu \left[ \frac{WL}{PY} + \frac{RK}{PY} \right] = \mu \frac{TotalCost}{Revenue} = \mu(1 - s_\pi)$$

The alternative methodology, aligning with this thesis and sourced from Hall (1990), involves a log-linear approximation of the production function and taking differences over time of the resulting expression. By applying the cost-minimization condition to this differential expression where $\Delta x$ denotes the growth rate of variable $X$, then return to scale coefficient $\mu$ can be estimated by regressing the growth rate of output against the growth rate of inputs,

$$\Delta y = \mu \left[ \frac{WL}{PY} \Delta l + \frac{RK}{PY} \Delta k \right] + \Delta z$$

Barkai (2020) utilises the initial approach, observing an increase in the profit rate from 2.2 percent in 1984 to 15.7 percent in 2014, implying a rise in the markup ratio from 1.02 to 1.19, assuming constant returns to scale. This observation finds support in the work of Gutiérrez and Philippon (2017). However, Basu (2019) contends that for the markup increase to 1.2 to be consistent with the decline in labor's share, returns to scale would need to have increased by about 10 percent during the examined period. Adopting the second strategy, Hall (2018) analyzes data across 60 U.S. industries, determining that the weighted average return to scale was approximately 1.3 in 2015, with individual industry returns spanning from 1.0 to 1.8. The findings related to constant returns to scale presented in this thesis are based on data spanning from 1949 to 2001. The constraints imposed by the length of the data set precluded us from replicating the increasing returns to scale documented in the literature mentioned above. Nonetheless, our results for the period 1949-2001 are consistent with those from existing studies that utilized similar lengths of sample data. In section 1.9,

| Durable | | Nondurable | | Nonmanufacturing | |
|---|---|---|---|---|---|
| A. Return-to-scale estimates | | | | | |
| 5 Wood | 0.75*** | 3 Food & Tobacco | 0.56 | 1 agriculture hunting | 0.22 |
| | (0.10) | | (0.73) | | (0.41) |
| 11 Basic metals | 1.01*** | 4 Textiles | 0.60** | 2 mining and quarrying | 0.27 |
| | (0.11) | | (0.26) | | (0.37) |
| 12 Machinery nec | 1.50*** | 6 Printing & Publishing | 1.35*** | 16 electricity gas and water | 0.45 |
| | (0.24) | | (0.35) | | (0.41) |
| 13 Electrical & optical equip | 0.69 | 7 Apparel | 0.79** | 17 constructions | 0.97*** |
| | (0.48) | | (0.34) | | (0.05) |
| 14 Transport equip | 1.00*** | 8 Chemicals | 1.65*** | 18 wholesale trade | 2.37** |
| | (0.11) | | (0.24) | | (1.10) |
| 15 Manufacturing nec | 0.80*** | 9 Rubber & Plastics | 1.12*** | 19 motor vehicles | 0.99** |
| | (0.25) | | (0.12) | | (0.39) |
| | | 10 Other mineral | 1.10*** | 20 retail trade | 2.13*** |
| | | | (0.14) | | (0.76) |
| | | | | 21 hotels and restaurants | 1.24*** |
| | | | | | (0.31) |
| | | | | 22 transport and storage | 1.38*** |
| | | | | | (0.17) |
| | | | | 23 post and telecommunications | 0.78** |
| | | | | | (0.37) |
| | | | | 24 financial intermediations | 0.05 |
| | | | | | (1.03) |
| | | | | 25 real estate activities | -0.343 |
| | | | | | (1.20) |
| | | | | 26 other business activities | 1.24*** |
| | | | | | (0.35) |
| | | | | 27 public admin and defense | 0.84*** |
| | | | | | (0.09) |
| | | | | 28 educations | 0.87*** |
| | | | | | (0.21) |
| | | | | 29 health and social work | 0.714*** |
| | | | | | (0.17) |
| | | | | 30 social&personal services | 1.49*** |
| | | | | | (0.38) |
| Average | 0.96 | | 1.02 | | 1.00 |
| Median | 0.90 | | 1.10 | | 0.92 |
| B. Coefficients on hours per worker | | | | | |
| high skill ($dh_H$) | 0.95 | | 3.42** | | -0.17 |
| | (2.15) | | (1.61) | | (0.40) |
| low skill ($dh_L$) | -0.05 | | -3.20* | | -0.43 |
| | (2.20) | | (1.75) | | (0.45) |
| C. Coefficients on labour inputs share | | | | | |
| high skill ($ds_{L_H}$) | 0.04 | | 0.01 | | 0.03 |
| | (0.08) | | (0.06) | | (0.04) |
| low skill ($ds_{L_L}$) | 0.03 | | 0.18** | | 0.13* |
| | (0.11) | | (0.07) | | (0.07) |
| D. Coefficients on capital share ($ds_k$) | -0.02 | | 0.06** | | 0.01 |
| | (0.04) | | (0.03) | | (0.02) |

Table 1: Two-skills estimation results

Notes: Heteroskedasticity- and autocorrelation-robust standard errors are in parenthesis. Constant terms are not shown. Coefficients multiplying out-per-worker labour inputs share and capital share are constrained to be equal within three industry groups (durables, nondurables, and nonmanufacturing). $***$ represents significance at 1 percent significance level, $**$ and $*$ represents 5 percent and 10 percent significance level, respectively.

we discussed the exploration of more current data. In future research, with access to updated datasets, we aim to replicate the findings related to increasing returns to scale.

Returning to Table 1, in the nondurable manufacturing sector, the coefficient for the labour cost share of high-skilled workers (i.e., coefficient in front of $ds_{L_H}$) is insignificant. However, the coefficient for the labour cost share of low-skilled workers (i.e., coefficient in front of $ds_{L_L}$) is positive and statistically significantly different from zero. This positive coefficient suggests $\kappa_L < 0$ indicating a negative elasticity of $v(\phi_K^*)$ with respect to capital's workweek associated with low-skilled workers. [17] Furthermore, in the nondurable sector, the coefficient on the capital input share ($ds_k$) is both significant and positive. This, combined with the negative value of $\kappa_L$, suggests that $\kappa_H$ is greater than zero. This positive value of $\kappa_H$ indicates a positive elasticity of $v(\phi_K^*)$ with respect to the capital's workweek for high-skilled workers. [18] The coefficients for capital and labour cost shares ($ds_k, ds_{L_H}$, and $ds_{L_L}$) are not significant for the durable manufacturing and non-manufacturing sectors, except for the low-skilled labour cost share ($ds_{L_L}$) in the non-manufacturing sector, which is significant at a 90-percent confidence level. When the regression function eliminates the variables of capital and input shares, the estimation results become less accurate, with the coefficients for hours per worker losing statistical significance across all three industry sectors, and the return-to-scale coefficients for several industries becoming insignificant. (The results can be found in Table 11 in Appendix B.) [19]

The positive value of $\kappa_H$ and negative value of $\kappa_L$ in nondurable sector indicate that as the workweek of capital increases, the elasticity of $v(.)$ with respect to capital's workweek increases for high-skilled workers, while it decreases for low-skilled workers. [20] This can be interpreted as the shift premium increasing at an increasing rate for high-skilled workers as the workweek of capital increases, while it increases at a decreasing rate for low-skilled workers. In other words, the utilisation of additional capital leads to an increase in the demand for hours working from high-skilled workers, thereby elevating the elasticity of the shift premium in relation to the capital's workweek associated with high-skilled workers. This finding is consistent with the capital-skill complementarity hypothesis, which postulates that that capital is more complementary with high-skilled workers compared to low-skilled workers. Capital skill complementarity has been extensively discussed in the literature (e.g., for example, see Griliches (1969), Goldin and Katz (1998), Krusell et al. (2000), Duffy et al. (2004), Polgreen and Silos (2008), Hara et al. (2014), Balleer and Van Rens (2013)). However, the coefficients for the durable manufacturing and non-manufacturing sectors are not statistically significant, thus it is not possible to determine the complementarity or substitution relationship between capital and skill in these two sectors through this method. We will discuss this further in Section 1.6.

The coefficients for hours per worker in the nondurable manufacturing sector are significant for two types of workers, with the estimated coefficient for high-skilled workers at 3.42 and -3.2 for low-skilled workers. While, the coefficients in durable manufacturing and nonmanufacturing are not statistically significant. In contrast to the standard model that utilises aggregate labour inputs, our model incorporates two skill levels, allowing for the possibility of negative coefficients in front of hours per worker. This outcome depends on the uncertain signs of the elasticity of labour effort with respect to hours (i.e., $\eta_H, \eta_L$) and the signs of the

[17] As shown in in the regression function 24, the coefficient in front of $ds_{L_L}$ is given by $-\frac{\mu S_K}{\kappa_L}$. Given that the return-to-scale parameter $\mu$ and the capital cost share $S_K$ are positive, the positive coefficient in front of $ds_{L_L}$ for the nondurable sector implies that $\kappa_L$ must be negative.

[18] Considering the regression function 24, the coefficient in front of $ds_k$ is $\frac{\mu S_K}{\kappa_H} + \frac{\mu S_K}{\kappa_L}$, and taking into account the conditions $\kappa_L < 0$, $\mu > 0$, and $S_K > 0$, a positive coefficient on $ds_k$ implies that $\kappa_H$ must be greater than zero.

[19] Removing capital and labour costs share variables, the regression function is reduced to $dy_i = c_i + \mu_i dx_i + +\alpha_1 dh_{H,i} + \alpha_2 dh_{L,i} + dz_i$, where inputs growth is defined as $dx = S_K dk + S_{L_H}(dn_H + dh_H) + S_{L_L}(dn_L + dh_L) + S_M dm$.

[20] Recall $v(\phi_{K,H})$ and $v(\phi_{K,L})$ are the elasticity of shift premium $V(.)$ with respect to capital's workweek.

elasticity of the $v(.)$ function with respect to the capital's workweek associated with different skill levels (i.e.,$\kappa_H, \kappa_L$), which represents the rate at which the shift premium will change when the capital's workweek associated with two skills labour changes. [21]

Table 2 summarizes means and standard deviations for TFP (the Solow residual) and two-skills estimated utilisation-controlled technology for sample period of 1950-2001. [22] TFP, also known as the Solow residual, measures technology change under the assumptions of constant returns, perfect competition, and no utilisation changes. On the other hand, the two-skills utilisation-controlled technology controls for utilisation changes and nonconstant returns for two skill levels. The standard deviations of two-skills technology are slightly lower for the entire sector than for TFP, indicating that input utilisation and other non-technological effects not accounted for by TFP increase its volatility. Figure 1 displays a comparison of the logarithmic growth of TFP and the two-skills utilisation-controlled technology, both series are in logarithmic growth. The two series are positively correlated, but TFP exhibits greater fluctuations compared to the utilisation-controlled technology. The volatility of TFP is particularly pronounced prior to the 1980s.

| | Solow Residual | Two-skills utilisation-controlled technology |
|---|---|---|
| Mean | 0.79 | 1.11 |
| Std deviation | 1.35 | 1.34 |

Table 2: Means and standard deviations of Solow residual and two-skills estimated utilisation-controlled technology

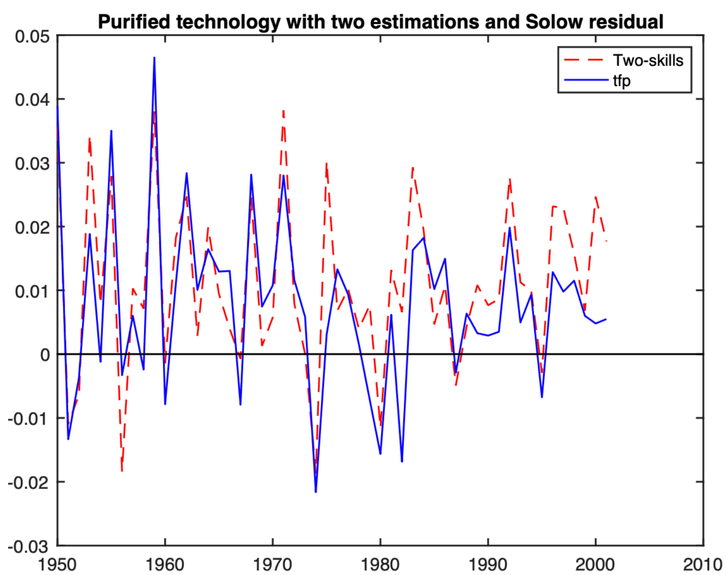


Figure 1: Utilisation-controlled technology improvement
Notes: The sample period is 1950-2001. This figure plots two-skills estimated utilisation-controlled technology series against TFP. All series are in logarithmic growth and are detrended by HP-filter.

[21] As shown in Equation 24, coefficients in front of hours per worker are given by $\frac{\mu S_K \epsilon_H}{\kappa_H} + \mu S_{L_H} \eta_H$ and $\frac{\mu S_K \epsilon_L}{\kappa_L} + \mu S_{L_L} \eta_L$ for high and low skilled workers, respectively.

[22] With constant returns, perfect competition, and no utilisation changes, technology change equals the standard Solow residual (TFP): $tfp = dy - dx$.

### 1.4.2 Business Cycle Results

#### 1.4.2.1 TFP Shocks and Utilisation Controlled Technology Shock

In this section, the dynamic impacts of the utilisation-controlled technology shock on various business-cycle related variables are summarized. These variables include logarithmic growth of output, inputs, TFP, utilisation, employment, and total hours worked. The impulse response functions were estimated through bivariate structural vector autoregressive models (SVARs). The utilisation-controlled technology series are considered stationary, as evidenced by the Augmented Dickey-Fuller test which strongly rejects the null hypothesis of a unit root at the conventional significance level, and also by a KPSS test, which fails to reject the null hypothesis of stationarity. Tests details can be found in Table 12 and Table 13 of the Appendix B. The results of the technology growth series analysis do not support the presence of autocorrelation, as indicated by panel data. Although negative correlation was observed at three lags, the associated p-values were large in the panel estimation. While examining the aggregated time series of technology, some evidence of the presence of autocorrelation was found, with a p-value of approximately 0.05. However, given the short sample period, the accuracy of these results may be compromised. Detailed point estimates can be found in Table 14 of the Appendix B.

Technology shocks are treated as exogenous in the bivariate SVAR system, as previously identified through the cost-minimization problem of the firm. In the first equation, the logarithmic growth of the variable $J$ ($dj$) is regressed on utilisation-controlled technology growth ($dz$) and two lags of itself and $dz$. The second equation is the equation of utilisation-controlled technology growth, and the parameters on $dj$ and its lagged terms are restricted to be zero, assuming that the technology change is not affected by the response variables; thus we regressed $dz$ on its two lags (this restriction has been shown to not affect the quality results, details are presented in the robustness check section).

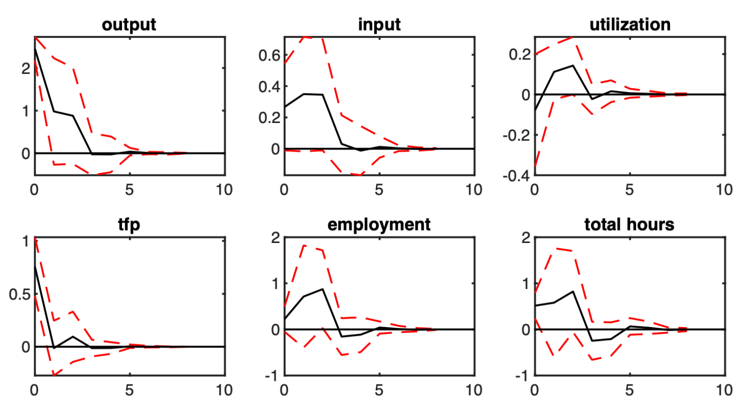


Figure 2: Impulse response to utilisation-controlled technology improvement-two skills estimation
Notes: The sample period is 1950-2001. The figure shows the Impulse responses of output, input, utilisation, tfp and total hours to one percent of the aggregate utilisation-controlled technology change. All series are in logarithmic growth, i.e., $dj = ln(J/J(-1))$. The TFP series are estimated by regressing $tfp = ty - dx$ and aggregating across industries. The horizontal scale shows the number of years after the initial technology shock. The dashed lines represent the 95-percent confidence intervals.

Figure 2 displays the impulse response results of the Two-skills estimation. The findings are generally consistent with the established predictions of the standard Real Business Cycle (RBC) model. Specifically, the model shows that the improvement in technology leads to an increase in both output and inputs. The response of inputs is characterized by a peak in the first period, followed by a gradual convergence to the steady state within a span of three periods. Surprisingly, the results of the technology shock indicate a mild and statistically insignificant impact decline in utilisation. However, utilisation exhibits a rapid increase following the initial shock, although its persistence is less pronounced compared to that of inputs; rises sharply within one year but remains flat with two lags. These findings warrant further investigation to fully understand the underlying dynamics of the utilisation response to technology shocks. The distinction in the magnitude and persistence of the response of utilisation compared to inputs in the aftermath of the initial technology shock can be attributed to the firms' prioritization of increasing working intensity in response to technology improvement in the short run. This increase in working intensity results in longer and harder work by the existing workforce and extended utilisation of existing equipment, and subsequently, the firms commence adjusting their production inputs. TFP exhibits a statistically significant and positive response to the utilisation-controlled technology shock, with an increase of 0.76 percent, reflecting the close correlation between TFP and utilisation-controlled technology. The results pertaining to employment and total hours demonstrate that while total hours experience an increase of 0.51 percent, employment experiences a comparatively lower increase of 0.23 percent in response to the initial technology shock. This disparity can be attributed to the initial increase in working intensity in the short run, which precedes adjustments in employment in response to the technology shock.

The results of the impulse responses to the two-skills utilisation-controlled technology shock and TFP shock are displayed in Figure 3 and Table 3 provides a summary of the point estimation of the impact responses to the utilisation-controlled technology shock and TFP shock. The results suggest that all of the variables respond positively and statistically significantly to both the utilisation-controlled technology shock and the TFP improvement, which aligns with the real business cycle facts. However, the responses of output, input, employment, and total hours are found to be more pronounced in response to the TFP shock as compared to the two-skills utilisation-controlled technology shock. This result is consistent with Bocola et al. (2011). The estimation of TFP shock assumes constant returns to scale, and it does not control for utilisations. From an economic perspective, these results are intuitive as utilisation and other non-technological factors contribute to the cyclicality of the response variables and amplify the business cycle fluctuations, leading to more profound positive responses to the TFP shock. The statistical insignificance of the utilisation response to both technology shocks has been observed, and subsequent sections will provide a detailed analysis of utilisation

| | Two skills | | TFP | |
|---|---|---|---|---|
| | Estimates | p-value | Estimates | p-value |
| Output | 2.45 | 0.000 | 3.660 | 0.000 |
| Input | 0.27 | 0.059 | 0.690 | 0.000 |
| Utilisation | -0.08 | 0.579 | 0.191 | 0.178 |
| TFP | 0.76 | 0.000 | | |
| Total hours | 0.51 | 0.000 | 2.110 | 0.000 |
| Employment | 0.23 | 0.110 | 1.970 | 0.000 |

Table 3: Point estimation for initial responses of business cycle related variables to the two skills utilisation-controlled technology shock and the TFP shock

Notes: The sample period is 1950-2001. This table shows the point estimates of the initial responses of output, inputs, utilisation, tfp, total hours worked, and employment to utilisation-controlled technology shock and TFP shock for aggregate economy. All series are in logarithmic growth.

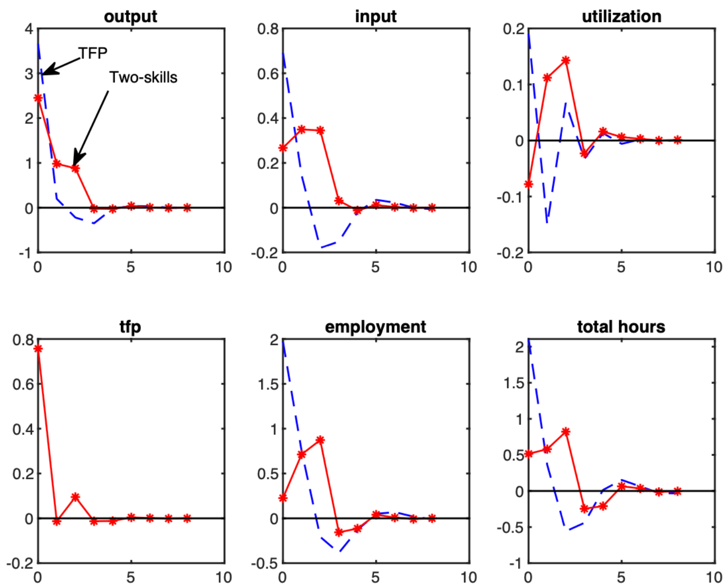


Figure 3: Impulse response to utilisation-controlled technology improvement and TFP shock
Notes: The sample period is 1950-2001. The figure shows the Impulse responses of output, input, utilisation, tfp and total hours to one percent of the aggregate utilisation-controlled technology change and TFP shock. All series are in logarithmic growth, i.e., $dj = ln(J/J(-1))$. The TFP series are estimated by regressing $tfp = ty - dx$ and aggregating across industries. The horizontal scale shows the number of years after the initial technology shock. The dashed lines represent the 95-percent confidence intervals.

#### 1.4.2.2 Differences in IRFs with Different Skill Levels

To gain a deeper understanding of how changes in technology affect the utilisation with differing skill levels, providing insights into the dynamic nature of the utilisation response to technology shock, we distinguish between high-skilled and low-skilled utilisation. In each sector, we define the utilisation growth for high and low skilled workers in industry $i$ as follows,

$$du_{H,i} = \alpha_1 ds_{k,i} + \alpha_2 dh_{H,i} + \alpha_3 ds_{L_H,i}$$

$$du_{L,i} = \alpha_1 ds_{k,i} + \alpha_4 dh_{L,i} + \alpha_5 ds_{L_L,i}$$

We follow the Domar-weight scheme and aggregate the utilisation growth for two skill levels. The results depicted in Figure 4 demonstrate the relationship between the utilisation-controlled technology series $dz$ and the utilisation series of both high-skilled and low-skilled workers in the aggregate economy. High-skilled utilisation shows a clearly positive contemporaneous comovement with technology series throughout the entire period, with the correlation coefficient between these two series of 0.56. Conversely, low-skilled utilisation covaries negatively with technology over the same period, with correlation coefficient of $-0.65$. Figure 5 shows the IRFs of utilisation, total hours and employment for workers of two skills levels to a positive utilisation-controlled technology shock. The utilisation of high-skilled workers increases by 0.37 percent, while that of low-skilled workers decreases by approximately 0.56 percent in response to a 1 percent technology improvement. In Section 1.2 we employed hours-per-worker and inputs cost shares as proxies for labour effort and capital utilisation to estimate the utilisation series. Consequently, the utilisation series comprises both capital utilisations and labour effort. Therefore, the response of utilisations in this study suggests that firms tend to encourage high-skilled workers to exert more effort and prolong their working hours in response to the increase in capital's workweek resulting from technological advancements. In contrast, firms tend to reduce the utilisation of both labour and capital associated with low-skilled workers. These results suggest that, as a response to technological progress, the workload of high-skilled workers becomes more closely tied to managing equipment and machinery. Consequently, when firms increase the workweek of capital, the working hours of high-skilled workers are extended to a greater extent than those of low-skilled workers, whose work is less reliant on capital. This disparity in the impact of increased capital utilisation on high- and low-skilled workers can be attributed to the fact that the former group of workers needs to be present during the increased utilisation of capital in order to effectively manage it effectively.

The relatively deep decline in the utilisation of low-skilled workers offsets the rise in the utilisation of high-skilled workers, resulting in an insignificant change in aggregate utilisation as depicted in Figure 2. The empirical evidence that firms tend to utilise high-skilled workers more than low-skilled workers in the short run responding to technological advancements align with the capital-skill complementarity hypothesis (CSC). This hypothesis posits that technology advances enhances the productivity of high-skilled workers while concurrently decreasing the productivity of low-skilled workers, thus induce deeper utilisation for high-skilled workers in the short run. More will be discussed in section 1.6.

The results of the total hours and employment demonstrate a comparable pattern. Responding to a technology advancement, in the short run, there is a decline in high-skilled total hours and employment of approximately 0.05-percent and 0.48-percent, respectively, while there is a corresponding increase of 0.6-percent in low-skilled total hours and 0.37-percent in low-skilled employment. All the responses are statistically significant. except for the high-skilled hours, which are demonstrated in Table 4. The total hours worked is determined by the combined effects of employment and hours per worker. In the short run, the rise in utilisation and decline in employment for high-skilled workers results in a mild, statistically insignificant decline in total hours. Conversely, the fall in utilisation and increase in employment for low-skilled workers in the short run drives a positive response of total hours. The confluence of technological advancements and CSC provide an explanation for the differential employment short run responses between skill levels, which will be further explicated in Section 6.

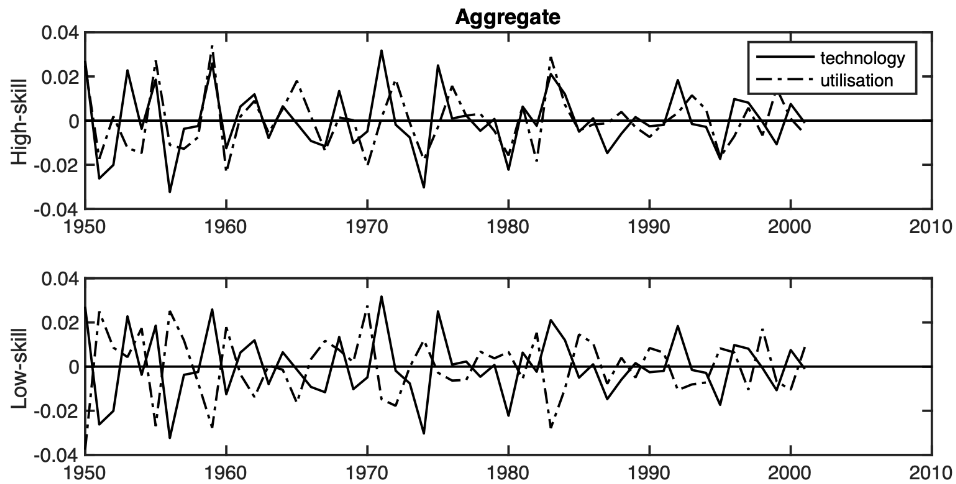


Figure 4: Utilisation-controlled technology and utilisation for high-skilled and low-unskilled workers in aggregate economy

Notes: The sample period is 1950-2001. This figure plots utilisation-controlled technology series against utilisation series for high-skilled and low-skilled workers. All series are in logarithmic growth and are detrended by HP-filter.

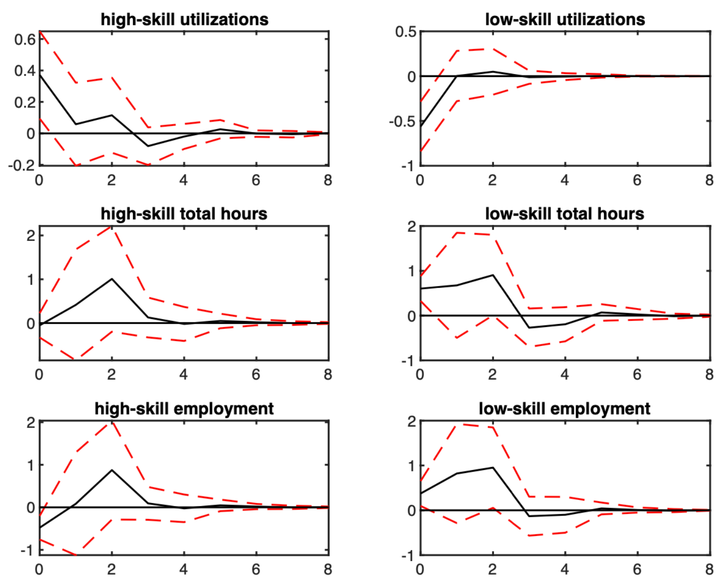


Figure 5: Impulse response to utilisation-controlled technology improvement-two skills estimation

Notes: The sample period is 1950-2001. The figure shows the Impulse responses of output, input, utilisation, tfp and total hours to one percent of the aggregate utilisation-controlled technology change. All series are in logarithmic growth, i.e., $dj = ln(J/J(-1))$. The TFP series are estimated by regressing $tfp = ty - dx$ and aggregating across industries. The horizontal scale shows the number of years after the initial technology shock. The dashed lines represent the 95-percent confidence intervals.

| Two skills estimation | | |
|---|---|---|
| | Estimates | p-value |
| High skill utilisation | 0.37 | 0.009 |
| Low skill utilisation | -0.56 | 0.000 |
| High skill total hours | -0.05 | 0.712 |
| Low skill total hours | 0.60 | 0.000 |
| High skill employment | -0.48 | 0.001 |
| Low skill employment | 0.37 | 0.008 |

Table 4: Point estimation for the initial responses

Notes: The sample period is 1950-2001. This table shows the point estimates of the initial responses of utilisations, employment and total hours for two skills to utilisation-controlled technology shock for aggregate economy. All series are in logarithmic growth.

### 1.5 Heterogeneity in Utilisation with Different Skills Across Sectors

Despite the aggregate level, we are able to identify the utilisation series for high-skilled and low-skilled workers at disaggregate levels by following the Domar-weight scheme and aggregating the utilisation growth for two skills level into durable manufacturing, nondurable manufacturing and non-manufacturing sectors.

The results depicted in Figure 6 demonstrate the relationship between the utilisation-controlled technology series $dz$ and the utilisation series of both high-skilled and low-skilled workers in the various sectors. These sectors include durable goods, nondurable goods, and nonmanufacturing. The analysis of the durable goods sector reveals that the utilisation series for low-skilled workers demonstrates significantly lower fluctuations in comparison to the utilisation series for high-skilled workers. This observation suggests that technological advancements do not have as profound an effect on high-skilled workers' utilisation as they do on low-skilled workers' utilisation in durable manufacturing. Moreover, a distinct contemporaneous negative correlation is observed between technology and both high-skilled and low-skilled utilisations, with correlation coefficients of -0.17 and -0.36, respectively. However, the correlations display a non-monotonic trend throughout the period. Specifically, the negative correlation between high-skilled utilisation and technology is profound in the period after 1980, with a correlation coefficient of -0.48, compared to a coefficient of 0.01 in the period prior to 1980. Conversely, the negative correlation between low-skilled utilisation and technology is pronounced in the period before 1980, with a correlation coefficient of -0.49, and the coefficient becomes positive, at 0.09, after 1980. Thus, our analysis indicates that in the period following 1980, technological improvements are negatively and significantly correlated with high-skilled utilisation, while they positively comove with low-skilled utilisation in durable goods sector.

The results in the nondurable goods sector indicate a clear contemporaneous positive correlation between technology and high-skilled utilisation, while a contemporaneous negative correlation is observed between technology and low-skilled utilisation for the entire period, with correlation coefficients of 0.21 and -0.41, respectively. In the nonmanufacturing sector, there is no clear contemporaneous comovement between high-skilled utilisation and technology. The correlation coefficient is only 0.02 for the entire period. However, the low-skilled utilisation exhibits a profound negative relationship with technology for the entire period, with a correlation coefficient of -0.45.

| | Durable | Nondurable | Nonmanufacturing |
|---|---|---|---|
| corr (dz,du_h) | -0.173 | 0.205 | 0.022 |
| corr (dz,du_l) | -0.365 | -0.412 | -0.449 |

Table 5: Correlation between utilisation-controlled technology series and utilisation series across sectors

Notes:All series are in logarithmic growth and are detrended by hp-filter.The sample period is 1950-2001

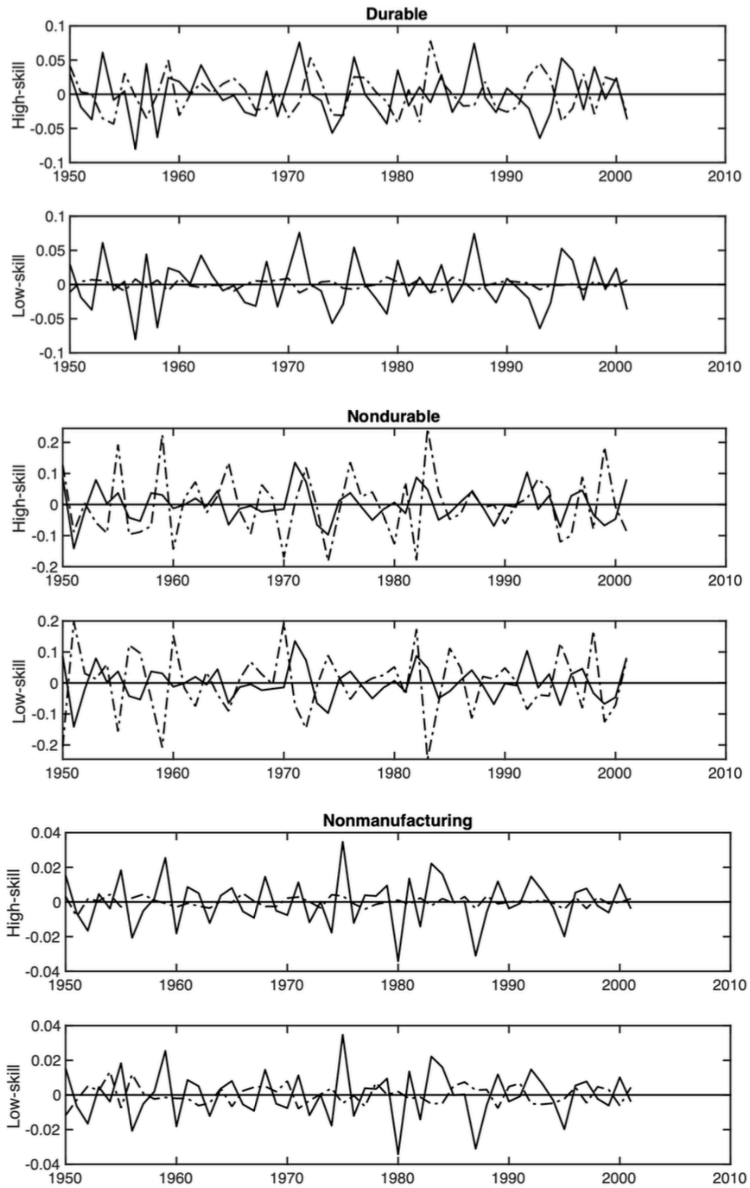


Figure 6: Relationship between utilisation-controlled technology series and utilisations for high-skilled and low-unskilled workers across sectors

Notes: The sample period is 1950-2001. This figure plots utilisation-controlled technology series against utilisation series for high-skilled and low-skilled workers. All series are in logarithmic growth and are detrended by HP-filter.

Then we estimate the IRFs of utilisations to sector-specified utilisation-controlled technology growth. The

IRFs results are shown in Figure 7, and the point estimates for impact responses are summarized in table 6. Notably, in the durable goods sector, a significant decrease in utilisation is observed among high-skilled workers, while a relatively insignificant decrease in utilisation is observed among low-skilled workers. The drop in low-skilled utilisation exhibits a strongly statistically insignificant result with a large p-value, while the p-value for high-skilled utilisation response is 0.143, indicating significance at an 85-percent confidence level. In the nondurable goods sector, the utilisation patterns among workers are consistent with the results observed at the aggregate level, with a rise in utilisation among high-skilled workers and a corresponding decrease in utilisation among low-skilled workers. As indicated in Table 6, the initial response of low-skilled workers is found to be statistically significant, while the response of high-skilled workers is statistically significant at an 85-percent confidence level. In contrast, the utilisation patterns in the nonmanufacturing sector are found to be statistically insignificant, with a substantial decrease in utilisation among low-skilled workers and only a minimal change in utilisation among high-skilled workers.

Table 7 summarized the results of the regression analysis of a FDL model regressing the utilisation growth of high-skilled and low-skilled workers on current and two lagged values of technology shocks. Using more lags has little effect on quality results; further regression results, including four lags of technology shocks can be found in Table 15 in Appendix B. The regression results are consistent with the Impulse Response Functions (IRFs) of utilisations displayed in Figure 5 and Figure 7. The coefficients on the current technology shock ($dz$) are comparable to the initial utilisation responses depicted in the IRFs. Moreover, the regression results provide additional statistical significance compared to the IRFs results, complementing the shortage in the IRF results. The decline of high-skilled utilisation in the durable manufacturing sector to technology shocks is significant at only the 85-percent confidence level (p-value is 0.143) as shown in in Table 6, whereas the significance of the decline in $du_h$ in the regression results is elevated to around 95-percent confidence level (p-value for coefficient on $dz$ is 0.051). Similarly, the decline in low-skilled utilisation in the nonmanufacturing sector is strongly significant at 99-percent confidence level (p-value for coefficient on $dz$ is 0.001), a result that was previously found to be insignificant in Table 6 (p-value is 0.314).

In the short term, given the constraints of quasi-fixed employment and capital stock, firms firstly respond to positive technological shocks by adjust the working intensity of existing workers and capital. Our findings indicate that, in response to technological advancements, firms in the durable manufacturing sector exhibit a decrease in the working intensity of high-skilled labour while maintaining the working intensity of low-skilled labour relatively constant. Conversely, in the nondurable goods sector, firms demonstrate an increase in their utilisation of high-skilled labour and a reduction in their utilisation of low-skilled labour. In the nonmanufacturing sector, firms demonstrate a reduction in the utilisation of low-skilled labour while maintaining the utilisation of high-skilled labour relatively constant.

| | Durable sector | | Nondurable sector | | Nonmanufacturing sector | |
|---|---|---|---|---|---|---|
| | estimates | p-value | estimates | p-value | estimates | p-value |
| high-skill utilisation | -0.207 | 0.143 | 0.215 | 0.129 | -0.023 | 0.867 |
| low-skill utilisation | -0.045 | 0.750 | -0.572 | 0.000 | -0.142 | 0.314 |

Table 6: Point estimations of the initial responses of utilisation for two types of skill levels to utilisation-controlled technology shock across sectors

Notes: The sample period is 1950-2001. This table shows the point estimates of the initial responses of utilisations for two skills to utilisation-controlled technology shock across sectors. All series are in logarithmic growth.

| | | Regressors | | | | |
|---|---|---|---|---|---|---|
| | | dz | dz(-1) | dz(-2) | R-square | Prob>F |
| Aggregate | du_h | **0.465** | 0.188 | 0.187 | 0.254 | 0.012 |
| | p-value | 0.001 | 0.183 | 0.085 | | |
| | du_l | **-0.538** | -0.099 | -0.036 | 0.324 | 0.001 |
| | p-value | 0.000 | 0.426 | 0.748 | | |
| Durable | du_h | **-0.199** | 0.073 | 0.090 | 0.086 | 0.208 |
| | p-value | **0.051** | 0.517 | 0.365 | | |
| | du_l | **-0.043** | 0.011 | 0.004 | 0.087 | 0.244 |
| | p-value | 0.107 | 0.611 | 0.874 | | |
| Nondurable | du_h | **0.377** | 0.735 | 0.203 | 0.187 | 0.050 |
| | p-value | 0.261 | 0.011 | 0.455 | | |
| | du_l | **-0.693** | -0.580 | -0.223 | 0.224 | 0.018 |
| | p-value | 0.026 | 0.023 | 0.283 | | |
| Nonmanufacturing | du_h | **-0.027** | -0.041 | -0.040 | 0.076 | 0.302 |
| | p-value | 0.482 | 0.165 | 0.161 | | |
| | du_l | **-0.154** | -0.004 | 0.034 | 0.349 | 0.000 |
| | p-value | **0.001** | 0.938 | 0.551 | | |

Table 7: Regression of utilisations for two types of skill levels on current and lagged utilisation-controlled technology

Note: Each row shows a separate OLS regression of the utilisations for high-skilled workers ($du_h$) and for low-skilled workers $du_l$ (in growth rates) on current and lagged values of utilisation-controlled technology growth, dz, plus a constant term and linear trend (not shown). Heteroskedasticity- and autocorrelation-robust standard errors in parentheses. All regressions are estimated from 1950 to 2001.

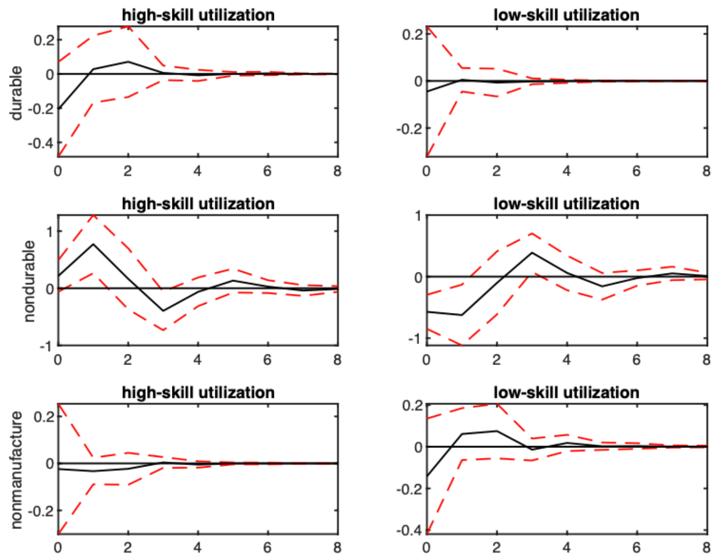


Figure 7: Impulse response of utilisations to sector specified utilisation-controlled technology improvement across sectors

Notes: The sample period is 1950-2001. The figure shows the Impulse responses of utilisations for high and low-skilled workers to the aggregate utilisation-controlled technology change in durable, nondurable, and nonmanufacturing sectors. All series are in logarithmic growth, i.e., $dj = ln(J/J(-1))$. The horizontal scale shows the number of years after the initial technology shock. The dashed lines represent the 95-percent confidence intervals.

## 1.6 Capital-Skill Complementarity/Substitution?

The concept of capital-skill complementarity refers to the phenomenon whereby the elasticity of substitution between capital equipment and skilled labour is lower than that between capital and unskilled labour. The key implication of the capital-skill complementarity; it is that the technology development induces lower prices of capital equipment, thus accumulating the stock of equipment, which further increases the marginal product of skilled workers, but decreases the marginal product of unskilled worker. Previous research has established that capital and skill are complementary at the aggregate level of the economy, as evidenced by numerous empirical studies, for example, see Griliches (1969), Goldin and Katz (1998), Krusell et al. (2000), Polgreen and Silos (2008), Hara et al. (2014). In this study, we extend the utilisation-controlled technology model to encompass two distinct types of labour with varying skill levels. This extension enables us to identify the utilisation series for two skill levels, providing some insights into the cyclical movement of utilisations for each skill level. The results can shed some light on the complementarity/substitution between capital and skill labour at both the aggregate and disaggregate levels, thus contributing to our understanding of how the capital-skill complementarity effect shapes the short-run dynamics of utilisations and the distribution of employment across different skill levels. And this information is crucial for understanding the implications of technology development and other economic changes for labour markets and overall economic performance. This study employs three main approaches to analyze the complementarity/substitution between capital and skilled labour.

### 1.6.1 Signs of $\kappa_H$ and $\kappa_L$

The first approach involves an analysis of the signs of $\kappa_H$ and $\kappa_L$, which are indicative of the elasticity of $v(.)$ with respect to the workweek of capital (as outlined in Section 1.2.2.2). A positive value of $\kappa_H$ and a negative value of $\kappa_L$ indicate that the shift premium for highly skilled workers increases at an increasing rate as the workweek of capital increases, while it increases at a decreasing rate for low-skilled workers. This suggests a complementarity between capital and skilled workers. However, it should be noted that the reliable signs of $\kappa_H$ and $\kappa_L$ can only be deduced in the nondurable sector. For detailed analyses, please see Section 1.4.1.

### 1.6.2 IRFs of Utilisation

A second approach to analyzing the complementarity/substitution between capital and skilled labour involves incorporating different skill levels to separate the utilisation (including labour and capital utilisation) of high-skilled and low-skilled workers. By analyzing the utilisation responses of workers with different skill levels to a utilisation-controlled technology shock, this approach provides insights into the complementarity/substitution between capital and skilled labour at both the aggregate economy and disaggregate sector levels. The paper presents Figure 5 in Section 1.4, which depicts the impulse response functions (IRFs) of utilisation, total hours, and employment for workers of two skill levels in response to a positive utilisation-controlled technology shock in the aggregate economy. The distinct responses of utilisation of high-skilled and low-skilled workers suggest that, in response to technological advancements, firms tend to encourage existing high-skilled workers to impose more effort and prolong high-skilled workers' working hours when increasing capital's workweek, while reducing that of the low-skilled workers. It should be noted that the analysis assumes that the number of employees and total hours cannot be changed costlessly. Thus, in the short run, when a positive technological shock occurs, firms primarily adjust the hours per worker to further utilise their existing labour and capital stock. When capital-skill complementarity is present, firms will tend to increase the hours-per-worker of existing high-skilled employees and reduce that of existing low-skilled employees due to the amplification of productivity of high-skilled labour through the aforementioned adjustments in response to advanced technology.

The responses of employment of high-skilled and low-skilled workers depicted in Figure 5 can also be explained by the interplay of the capital-skill complementarity (CSC) effect and technology advancement. The short-run employment dynamics can be analyzed through the lens of two interrelated factors: technological advancement and the CSC effect. The former drives firms to expand their labour force in response to rising demand, as technological improvement stimulates increased productivity and market demand. The CSC effect impacts the distribution of employment across different skill levels, where high-skilled workers experience heightened productivity and low-skilled workers face reduced productivity, this leads firm to utilise high-skilled workers more than low-skilled workers in the short run. When the CSC effect dominates, the short-run employment dynamics are characterized by a reduction in the demand for high-skilled labour and an increase in the demand for low-skilled labour. This trend is driven by the heightened efficiency of high-skilled workers, which allows firms to meet the rising demand with a smaller number of high-skilled workers and a larger number of low-skilled workers in the short run. However, as the utilisation of high-skilled labour reaches its limit, firms begin to increase the employment of both skill types, with a greater emphasis on hiring high-skilled workers. This trend continues as the demand for labour continues to rise, reflecting the interplay between technological progress and the CSC effect in shaping employment dynamics in the long run.

The analysis of the complementarity/substitution between capital and skilled labour has been extended to the disaggregate sectors. And the impulse response functions (IRFs) of utilisation of two skill levels at disaggregate sectors depicted in Figure 7 have been discussed in Section 1.5. The decline in high-skilled utilisation and relative unchanged low-skilled utilisation in the durable sector can be attributed to the interplay between capital and skills. When capital and skills are substitutes, technological advancements lead to the automation of tasks previously performed by skilled workers, resulting in a decrease in working intensity for high-skilled workers. On the contrary, the new technology might be creating roles or tasks that low-skilled workers can quickly adapt to or are better suited for. For instance, they may take on jobs related to operating, maintaining, or working alongside the new technology, or they might be filling roles not directly impacted by the new technology. The phenomenon of capital and skilled worker substitution has been examined by Goldin and Katz (1998), who propose that technological advancements may not always increase the demand for skills. In fact, technological progressions may replace skilled workers by 'deskilling' the production process. This involves simplifying complex tasks previously performed by skilled workers and breaking them down into smaller, less-skilled pieces, thereby reducing the demand for skilled labour. Similar conclusions have been reached by Braverman (1998) and Marglin (1974). More recently, Duffy et al. (2004) and Balleer and Van Rens (2013) found weak support for CSC. Previous literature has examined this issue at the aggregate economy level, Perez-Laborda and Perez-Sebastian (2020) used EU KELMS dataset found in manufacturing sector capital is relatively more complementary to unskilled labour. In this paper, the validity of capital-skill substitution in the disaggregate durable goods sector will be further supported by applying a third methodology discussed in Section 1.6.3.

In the nondurable sector, the increase in high-skilled utilisation and decrease in low-skilled utilisation indicate capital-skill complementarity. The improvement in technology increases the marginal product of high-skilled workers relative to low-skilled workers, leading to higher demand for high-skilled inputs. In the short term, facing with the quasi-fixed employment and capital, this increased demand results in a deeper utilisation of high-skilled workers by firms to make use of the higher high-skilled labour productivity. This conclusion aligns with the results obtained from the initial methodology outlined in Section 1.6.1. For the non-manufacturing sector, the deep decline in the utilisation of low-skilled workers and relative unchanged high-skilled utilisation are an indication of capital-skill complementarity. This phenomenon occurs as firms tend to substitute low-skilled hours for high-skilled hours in response to technological advancements due to the amplification of marginal labour productivity of high-skilled labour, resulting in a reduction of low-skilled labour utilisation in the short term. However, the lack of statistically significant IRFs in the non-manufacturing sector calls for further investigation. To address this issue, an additional analytical approach will be undertaken in Section 1.6.3 to shed further light on the situation.

### 1.6.3 IRFs of Skill Premium, K/S and S/U

A third approach for evaluating the complementarity/substitution between capital and skilled labour involves analyzing the reactions of the skill premium, the proportion of capital stock to the number of hours worked by skilled labour (K/S), and the ratio of skilled to unskilled relative hours worked (S/U). These measures provide valuable insights into the nature of capital-skill complementarity/substitution across sectors.

To illustrate the third method for illuminating the complementarity/substitution between capital and skill at disaggregate level, we consider an economy where firm uses two types of labour inputs, skilled ($S$) and unskilled ($U$), and capital ($K$) to produce final goods ($Y_i$) in each sector $i$. The production function is as follow

$$Y_{i,t} = Z_{i,t}\left[\mu_i(K_{i,t}^{\rho_i} + (1-\lambda_i)(S_{i,t})^{\rho_i})^{\sigma_i/\rho_i} + (1-\mu_i)(U_{i,t})^{\sigma_i}\right]^{1/\sigma_i}$$

where $Z_{i,t}$ is the sector-specific technology shock, $\mu_i$ and $\lambda_i$ control the input shares of capital and unskilled labour, respectively. And $\sigma_i$ and $\rho_i$ governs the elasticity of substitution between inputs. Specifically, $1/(1-\rho_i)$ is the elasticity of substitution between capital and skilled labour; and $1/(1-\sigma_i)$ is the elasticity of substitution between capital (or skilled labour) and unskilled labour. If $\sigma_i > \rho_i$, there is capital-skill complementarity; that is, the elasticity between capital and unskilled labour is larger than that between capital and skilled labour. Denote the wage of skilled and unskilled labour by $w_{s,i,t}$ and $w_{u,i,t}$. Then, under the assumption that worker's wages are proportional to their marginal product, we can calculate the skill premium by the ratio of the marginal product of each type of labour.

$$\frac{w_{s,i,t}}{w_{u,i,t}} = \frac{(1-\mu_i)(1-\lambda_i)}{\mu_i}\left[\lambda_i\left(\frac{K_{i,t}}{S_{i,t}}\right)^{\rho_i} + (1-\lambda_i)\right]^{\frac{\sigma_i-\rho_i}{\rho_i}}\left(\frac{S_{i,t}}{U_{i,t}}\right)^{\sigma_i-1} \tag{26}$$

Given that $\sigma < 1, \mu < 1, \lambda < 1$, an increase in the ratio of skilled to unskilled hours $(S/U)$ will, ceteris paribus, result in a decrease in the skill premium for any acceptable values of $\sigma$. [23] This is referred to as the relative supply effect by Krusell et al. (2000). Furthermore, if $\sigma > \rho$, an increase in the ratio of capital stock to skilled hours $(K/S)$ will, ceteris paribus, result in an increase in the skill premium, which is referred to as the capital-skill complementarity effect by Krusell et al. (2000). Analyzing the sign of the responses of skill premium, capital stock to skilled hours ratio and relative hours to the utilisation-controlled technology shock can shed light on the complementarity or substitution between capital and skilled labour across sectors. The results of the impulse response functions (IRFs) are shown in Figure 8 and the point estimates are shown in Table 8.

The integration of advanced technology leads to a decrease in the relative total hours (S/U) in all aggregate and disaggregate sectors. it indicate a positive relative supply effect across sectors given $\sigma < 1$. The responses are strongly statistically significant in the aggregate, durable, and non-manufacturing sectors and significant at a 75-percent confidence level in the non-durable sector as shown in Table 8. Therefore, the sign of the response of skill premium will be the same as the sign of the responses of capital-to-skilled hours ratio (K/S) when $\sigma > \rho$, that is when capital and skill are complementary, otherwise, capital and skill are substitution.

In the aggregate economy and non-manufacturing sectors, the capital to skilled hours ratio (K/S) responds positively and significantly to the technological advancement, thereby contributing to the rise in the skill premium and suggesting a positive complementarity effect (i.e., $\sigma > \rho$). In the durable manufacturing sector, the capital to skilled ratio (K/S) responds positively to the technology shock, though the skill premium decreases, indicating $\sigma < \rho$, that is skilled labour in this sector are likely to be substitutes rather than complements. However, the decrease in skill premium is insignificant. In the non-durable manufacturing sector, both the capital to skilled ratio (K/S) and skill premium respond negatively and significantly to the technology improvement, thereby indicating $\sigma > \rho$, and there are some evidence of capital-skill complementarity.

In conclusion, the results obtained from the three analytical approaches are consistent and provide some evidences of capital-skill complementarity in the aggregate economy, the nondurable manufacturing sector,

[23] A positive elasticity of substitution between capital (or skilled labour) and unskilled labour ensures $1/(1-\sigma_i) > 0$, and we have $\sigma_i < 1$. Similar logic applies to $\rho_i$.

and the nonmanufacturing sector. However, in the durable manufacturing sector, the results indicate a substitution relationship between capital and skilled labour. These findings contribute to our understanding of the complex interplay between capital and skilled labour and highlight the need for further research to refine our understanding of these relationships in different economic sectors.

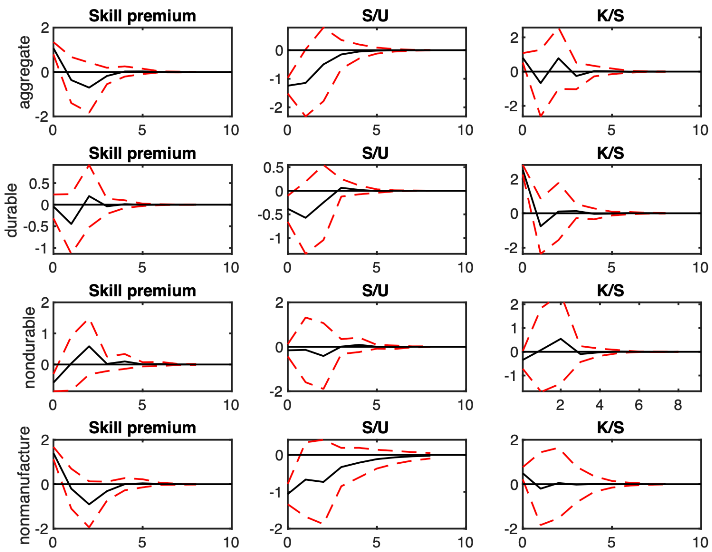


Figure 8: Cumulative impulse response functions of two estimations

Notes: The sample period is 1950-2001. The figure shows the impulse responses of skill premium, the ratio of capital to skilled hours (K/S), and the relative total hours ratio (S/U) to the aggregate utilisation-controlled technology change in disaggregate sectors. All series are in logarithmic growth, i.e., $dj = ln(J/J(-1))$. The horizontal scale shows the number of years after the initial technology shock. The dashed lines represent the 95-percent confidence intervals.

| | Skill premium | | S/U | | K/S | |
|---|---|---|---|---|---|---|
| | estimates | p-value | estimates | p-value | estimates | p-value |
| Aggregate | 1.07 | 0.000 | -1.24 | 0.000 | 0.8 | 0.000 |
| Durable | -0.04 | 0.771 | -0.38 | 0.007 | 2.53 | 0.000 |
| Nondurable | -0.58 | 0.000 | -0.16 | 0.265 | -0.37 | 0.009 |
| Nonmanufacturing | 1.40 | 0.000 | -1.06 | 0.000 | 0.49 | 0.001 |

Table 8: Point estimations for initial responses

Notes: The sample period is 1950-2001. This table shows the point estimates of the initial responses of skill premium, skilled to unskilled relative hours (S/U), capital to skilled hours ratio (K/S) to utilisation-controlled technology shock across sectors. All series are in logarithmic growth.

## 1.7 Robustness Check

The present study endeavors to identify the separate labour effort and capital's workweek for skilled and unskilled workers. Our findings indicate that the utilisation response to utilisation-controlled technology advancement differs between skilled and unskilled workers across various sectors. In this Section, we conduct robustness tests and present a range of SVAR specifications to examine the utilisation responses. The technology series are considered as either white noise or as autoregressive, and shocks to variable J are allowed to affect technology with a two-lag structure. Figures 9 to 12 in Appendix B demonstrate the robustness of our results through four different estimates of the utilisation response for both the aggregate economy and the disaggregate sectors.

The representation with boxes displays the inferred response derived through a direct regression of utilisation growth on growth in the current and eight lags of technology growth.This methodology employs a significant number of degrees of freedom, with the sample period spanning from 1950 to 2001. The relatively short sample period is the primary factor leading to the substantial magnitude of responses from direct regression across sectors. The representation with triangles illustrates the results obtained from considering technology as an exogenous white-noise process. The representation with circles represents our benchmark SVAR results, which accommodates for autocorrelations but does not allow for feedback of shocks to utilisation on technology, i.e., the lagged growth in utilisation is not incorporated into the technology equation. The representation with crosses demonstrates the results obtained from allowing for serial correlation and (lagged) feedback of shocks to utilisation on technology.

In summary, the results indicate a consistent pattern across all cases, namely that the responses of utilisation of high- and low-skilled workers to a technology improvement vary by sector. Specifically, In the aggregate economy, the utilisation of high-skilled workers increases on impact, while that of low-skilled workers decreases. In the durable sector, the utilisation rate of high-skilled workers exhibits a sharp decline, while the utilisation rate of low-skilled workers remains relatively stable. In contrast, in the nondurable sector, the utilisation rate of high-skilled workers increases, whereas the utilisation rate of low-skilled workers declines significantly. Finally, in the non-manufacturing sector, the utilisation rate of low-skilled workers experiences a significant reduction, while the utilisation rate of high-skilled workers remains relatively unchanged. Nevertheless, our estimates of the utilisation responses do not alter the significance of the initial impact responses, which remain similar to our benchmark results (the point estimates for initial responses are shown in Table 16 of Appendix B).

The sensitivity of the SVAR results was further examined by changing the order of variables in the SVAR system. The impulse response results, as shown in Figures 2, 5, and 7, were re-estimated accordingly. In the equation regressing utilisation-controlled technology growth ($dz$) on two lags of itself and growth of the responding variable $J$ ($dj$), the coefficient in front of $dj$ and two lags of $dj$ were restricted to zero, under the assumption that technology growth is exogenous and not affected by the responding variables in the contemporaneous period. Thus, when changing the order of variables in the SVAR, the coefficient in front of $dz$ in the equation of variable $dj$ is restricted to zero instead. The re-estimation results after changing the order were then compared with the original results from period 1 after the initial responses. The IRF results after changing the order, as depicted in Figures 13 to 15, were found to be very similar to the original SVAR results. Thus, the sensitivity analysis confirms the robustness of the original SVAR results, as changing the order of variables does not substantially alter the findings.

## 1.8 Conclusion

This paper presents a novel approach for measuring utilisation-controlled neutral technology shocks that is robust to factor heterogeneity. Specifically, the approach estimates the utilisation series for both high-skilled and low-skilled labour,enabling a more detailed analysis of cyclical movements of utilisation at both the aggregate and disaggregate levels.

Our study generates several key findings. First, at the aggregate level, our model generates results that are consistent with the standard Real Business Cycle (RBC) model. However, we find that a utilisation-controlled technology shock leads to less business cycle fluctuations in output, inputs, employment, and total hours than TFP. Secondly, our analysis indicates that firms tend to utilising existing high-skilled workers and prolong the capital's workweek associated with high-skilled workers in response to technological advancements, while simultaneously reducing utilisation of both labour and capital associated with low-skilled workers. Thirdly, we find that the responses of utilisation for high-skilled and low-skilled workers to a technology advancement differ across sectors. In the durable sector, high-skilled utilisation declines sharply while low-skilled utilisation remains relatively unchanged; in the nondurable sector, high-skilled utilisation increases and low-skilled utilisation decreases significantly. For the non-manufacturing sector, low-skilled utilisation falls significantly while high-skilled worker's utilisation remains relatively unchanged.

Futhermore, this paper proposed three methords to analyzing the complementarity or substitution between capital and skilled workers at aggregate and disaggregate level. The results obtained from the three analytical approaches are consistent and suggest that the cyclical movement of utilisation can provide evidence of capital-skill complementarity in the aggregate economy, the nondurable manufacturing sector, and the non-manufacturing sector. However, in the durable manufacturing sector, the results indicate a substitution relationship between capital and skilled labour. These findings underscore the sector-specific nature of the relationship between technology improvements and utilisation, highlighting the need for an industry-focused analysis in comprehending the diverse utilisation patterns among workers of varying skill levels.

In conclusion, this paper offers a nuanced perspective on the cyclical movements of utilisation at both the aggregate and sectoral levels, emphasizing the importance of an industry-focused analysis for a comprehensive understanding of the utilisation patterns of workers with different skill levels. Furthermore, this paper contributes to the understanding of the interplay between capital and skilled labour in the context of technological progress, providing insights into the differential effects of technology shocks on workers with varying skill levels across different economic sectors.

By shedding light on the impact of the complementarity/substitution between capital and skill on labour utilisation, this paper expands our understanding of the complex dynamics between technology, capital, and labour. In addition, our findings highlight how the CSC effect shapes the short-term responses of utilisation to technology improvements, providing a valuable foundation for further investigation in this critical area.

Overall, this paper's contribution to the literature on labour utilisation and technological progress provides important insights for policymakers and firms seeking to navigate the changing landscape of the modern economy.

## 1.9 Looking Ahead

The scope of analysis in this chapter is confined to the period between 1949 and 2001, a temporal boundary primarily determined by the availability of essential instrumental data, which is critical for the methodology employed. In alignment with the approach outlined in Basu et al. (2006), this study addresses endogeneity concerns that may arise due to possible correlations between the error term, defined here as utilisation-adjusted technological shocks, and the growth rate of production inputs along with labour hours per worker

To address these endogeneity issues, Basu (2006) incorporates Hall-Ramey instruments, specifically the oil prices and the growth in real government defense spending. These instruments are amenable to extension into more recent periods. Nevertheless, this study encounters a methodological limitation concerning the third instrumental variable, which involves updating the quarterly Federal Reserve 'monetary shocks' based on an identified VAR, as originally introduced by Burnside (1996). The availability of this particular instrument imposing a constraint on the temporal range of this chapter.

Moreover, diverging from Basu's methodology, this study introduces various types of labour into the model. This modification alters the regression function by integrating the growth rate of the ratio of capital and labour input costs to the total input costs. Given that these ratios could be influenced by technological shocks, the study necessitates the inclusion of three additional instruments. It is posited that the lagged values of the growth rates of these ratios serve as effective instruments, as technological shocks in the current period would not influence these ratios in preceding periods. These additional instruments are viable up to the year 2010, in line with the availability of our primary EU KLEMS data.

Looking ahead, future research will aim to extend the data to more recent periods, enabling an in-depth analysis of the cyclical fluctuations in the utilisation series for different types of workers. Instead of deriving monetary shocks from a VAR framework that includes GDP, the GDP deflator, an index of commodity prices, the 3-month T-bill rate, and M1, an alternative and more streamlined approach for extending the monetary shock data could be adopted, as suggested in Bu et al. (2021). Bu et al. (2021) develops a series of U.S. monetary policy shocks that seamlessly bridge periods of both conventional and unconventional policy-making. Notably, this series is characterized by minimal data requirements and is devoid of significant central bank information effects.

The methodology employed for constructing this measure of monetary policy shock draws upon the two-step regression framework pioneered by Fama (1971). The initial step entails conducting time-series regressions to ascertain the sensitivity of interest rates at varying maturities to Federal Open Market Committee (FOMC) announcements. This step is crucial for quantifying the reaction of interest rates to monetary policy decisions. To isolate the impact of monetary policy from other non-policy news, the approach incorporates the heteroskedasticity-based estimator techniques outlined by Rigobon (2003, 2004), which effectively filter out confounding variables. In the subsequent phase, the methodology involves regressing all relevant output variables against the sensitivity indices derived from the first step, for each respective time point. This process can generate a novel series of monetary shocks, represented as the array of estimated coefficients from the second-step regressions, reminiscent of the Fama-MacBeth regression approach. Adopting this methodology could significantly simplify the extension of the monetary shocks series to more contemporary periods, enhancing the robustness and applicability of the research.

# Appendix A

1. The data comprise 30 industries including 6 durable industries, 7 nondurable industries, and 17 non-manufacturing industries. Table 46 gives the industry list .

| | |
|---|---|
| **Durables** | |
| 1 | wood and cork (20) |
| 2 | basic metals and fabricated metal (27t28) |
| 3 | machinery nec (29) |
| 4 | electrical and optical equipment (30t33) |
| 5 | transport equipment (34t35) |
| 6 | manufacturing nec; recycling (36t37) |
| **Nondurables** | |
| 7 | food beverages and tobacco (15t16) |
| 8 | textiles textile leather and footwear (17t19) |
| 9 | pulp paper paper printing and publishing (21t22) |
| 10 | coke refined petroleum and nuclear fuel (23) |
| 11 | chemicals and chemical products (24) |
| 12 | rubber and plastics (25) |
| 13 | other nonmetallic mineral (26) |
| **Non-manufacture** | |
| 14 | sale maintenance and repair of motor vehicles (50) |
| 15 | wholesale trade and commission trade (51) |
| 16 | retail trade except of motor vehicles (52) |
| 17 | transport and storage (60t63) |
| 18 | post and telecommunications (64) |
| 19 | real estate activities (70) |
| 20 | renting of m&eq and other business activities (71t74) |
| 21 | agriculture hunting forestry and fishing (AtB) |
| 22 | mining and quarrying (C) |
| 23 | electricity gas and water supply € |
| 24 | construction (F) |
| 25 | hotels and restaurants (H) |
| 26 | financial intermediation (J) |
| 27 | public admin and defense; compulsory social security (L) |
| 28 | education (M) |
| 29 | health and social work (N) |
| 30 | other community social and personal services (O) |

Table 9: Industry list

Notes: The code used in parenthesis is in the World KLEMS data base.

2. For each sector accounts in the World KLEMS 2013 release data set, it contains data for industry gross output, inputs of capital, labour, energy and matrieals, employment, and value added, Table 10 gives the variables list.

3. In the regression function 5, the logarithmic growth of gross output is defined using the quantity index for output, denoted as GO_QI.We represent the logarithmic growth of gross output as $dy_i = \ln(\frac{GO\_QI_{i,t}}{GO\_QI_{i,t-1}})$. Similarly, the logarithmic growth of hours per worker is calculated as $dh_i = \ln(\frac{h_{i,t}}{h_{i,t-1}})$, where $h_i$ represents the hours per worker. We can also determine the logarithmic growth rates for employees, capital, and intermediate goods using $dn_i = \ln(\frac{LAB\_QI_{i,t}}{LAB\_QI_{i,t-1}})$, $dk_i = \ln(\frac{CAP\_QI_{i,t}}{CAP\_QI_{i,t-1}})$ and $dm_i = \ln(\frac{II\_QI_{i,t}}{II\_QI_{i,t-1}})$ respectively. The share-weighted input $dx_i$ is constructed by aggregating the growth rates of the three real inputs: capital, labour, and intermediate goods, as follows:

$$dx_i = S_{K,i}dk_i + S_{L_H,i}(dn_{H,i} + dh_{H,i}) + S_{L_L,i}(dn_{L,i} + dh_{L,i}) + S_{M,i}dm_i$$

| Variable Name (Code) | Note |
|---|---|
| Output (GO_QI) | Quantity index of output |
| Capital (CAP_QI) | Quantity index of capital |
| labour (LAB_QI) | Quantity index of labour |
| Intermediate (II_QI) | Quantity index of intermediate inputs |
| Value Added (VA_QI) | Quantity index of value added |
| Hours per worker (h) | Calculate these by dividing number of hours (H_EMP) by number of workers (EMP) |
| Nominal Capital (CAP) | Value of capital in current prices |
| Nominal Intermediate (II) | Value of intermediate inputs in current prices |
| Total Hours (HEMP) | Average number of hours worked per week |
| Employment (EMP) | Total persons engaged |
| Wage Compensation (COMP) | Labour compensation per hour worked (U.S. dollars) |

Table 10: Variables list

Notes: The code used in parentheses is from the KLEMS database.

The shares associated with the costs of each input are calculated by dividing the nominal value of each input by the sum of nominal values of all inputs:

$$s_{K,i} = \frac{CAP_i}{CAP_i + LAB_i + II_i}$$

$$S_{L_H,i} = \frac{LAB_{H,i}}{CAP_i + LAB_i + II_i}$$

$$S_{L_L,i} = \frac{LAB_{L,i}}{CAP_i + LAB_i + II_i}$$

$$s_{M,i} = \frac{II_i}{CAP_i + LAB_i + II_i}$$

where

$$LAB_i = LAB_{H,i} + LAB_{L,i}$$

$$LAB_{H,i} = COMP_{H,i} * EMP_{H,i} * HEMP_{H,i} * 48/1000000$$

$$LAB_{L,i} = COMP_{L,i} * EMP_{L,i} * HEMP_{L,i} * 48/1000000$$

24

[24] Multiplying by 48/1000000 is done to convert weekly labour compensation in U.S. dollars into annual labour compensation in millions.

# Appendix B

| Durable | | Nondurable | | Nonmanufacturing | |
|---|---|---|---|---|---|
| A. Return-to-scale estimates | | | | | |
| 5 Wood | 0.68*** | 3 Food & Tobacco | 0.26 | 1 agriculture hunting | 0.05 |
| | (0.14) | | (1.43) | | (0.42) |
| 11 Basic metals | 1.00*** | 4 Textiles | 0.57** | 2 mining and quarrying | 0.173 |
| | (0.13) | | (0.26) | | (0.25) |
| 12 Machinery nec | 1.50*** | 6 Printing & Publishing | 1.99* | 16 electricity gas and water | 0.308 |
| | (0.29) | | (1.19) | | (1.25) |
| 13 Electrical & optical equip | 0.61 | 7 Apparel | -0.19 | 17 constructions | 0.94*** |
| | (0.57) | | (2.02) | | (0.04) |
| 14 Transport equip | 1.00*** | 8 Chemicals | 1.71*** | 18 wholesale trade | 1.95** |
| | (0.11) | | (0.28) | | (0.85) |
| 15 Manufacturing nec | 0.85** | 9 Rubber & Plastics | 1.11*** | 19 motor vehicles | 1.14** |
| | (0.28) | | (0.16) | | (0.40) |
| | | 10 Other mineral | 0.94*** | 20 retail trade | 1.74* |
| | | | (0.25) | | (0.94) |
| | | | | 21 hotels and restaurants | 1.75* |
| | | | | | (0.99) |
| | | | | 22 transport and storage | 1.22*** |
| | | | | | (0.24) |
| | | | | 23 post and telecommunications | 0.21 |
| | | | | | (1.17) |
| | | | | 24 financial intermediations | -0.06 |
| | | | | | (1.24) |
| | | | | 25 real estate activities | 0.09 |
| | | | | | (1.16) |
| | | | | 26 other business activities | 1.63** |
| | | | | | (0.76) |
| | | | | 27 public admin and defense | 0.76*** |
| | | | | | (0.08) |
| | | | | 28 educations | 0.81* |
| | | | | | (0.45) |
| | | | | 29 health and social work | 0.47 |
| | | | | | (0.41) |
| | | | | 30 social&personal services | 1.47** |
| | | | | | (0.68) |
| Average | 0.94 | | 1.10 | | 0.92 |
| Median | 0.93 | | 1.03 | | 0.88 |
| Average (eliminate negative estimates) | 0.94 | | 0.91 | | 0.97 |
| Median (eliminate negative estimates) | 0.93 | | 0.94 | | 0.94 |
| B. Coefficients on hours per worker | | | | | |
| high skill | 1.11 | | -0.75 | | -0.37 |
| | (2.12) | | (3.52) | | (0.71) |
| low skill | -0.47 | | 2.57 | | 0.14 |
| | (2.11) | | (4.48) | | (0.71) |

Table 11: Two-skills estimation results without inputs costs share variables

Notes: Heteroskedasticity- and autocorrelation-robust standard errors are in parenthesis. Constant terms are not shown. Coefficients multiplying out-per-worker, capital and labour costs shares are constrained to be equal within three industry groups (durables, nondurables, and nonmanufacturing). *** represents significance at 1 percent significance level, ** and * represents 5 percent and 10 percent significance level, respectively.

| Dicky fuller | |
|---|---|
| statistic (Z(t)) | p-value for Z(t) |
| -9.168 | 0.000 |

Table 12: Augmented Dicky Fuller test for utilisation-controlled technology growth series

| KPSS | | |
|---|---|---|
| Lag order | Test statistic | Critical values for H0 |
| 0 | 0.0506 | 10%: 0.119 |
| 1 | 0.0649 | 5% : 0.146 |
| 2 | 0.0792 | 2.5%: 0.176 |
| 3 | 0.105 | 1% : 0.216 |

Table 13: KPSS test for utilisation-controlled technology growth series

| | dz(-1) | | dz(-2) | | dz(-3) | | dz(-4) | |
|---|---|---|---|---|---|---|---|---|
| | estimates | p-value | estimates | p-value | estimates | p-value | estimates | p-value |
| Panel | -0.07 | 0.384 | -0.01 | 0.646 | -0.02 | 0.451 | 0.03 | 0.330 |
| Time series | -0.23 | 0.063 | -0.11 | 0.320 | -0.30 | 0.080 | 0.00 | 1.000 |

Table 14: Autocorrelation test for utilisation-controlled technology growth series

Notes: This table summarized the autocorrelation tests for utilisation-controlled technolog growth series with panel data and times series data. The sample period is 1949-2001 with panel data and it is 1950-2001 with time series data. The time series utilisation-controlled technology growth series is obtained by aggregating industry-level technology data by Domar-weight.

| | | Regressors | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | dz | dz(-1) | dz(-2) | dz(-3) | dz(-4) | R-square | Prob>F |
| Aggregate | du_h | 0.491 | 0.153 | 0.064 | -0.140 | -0.080 | 0.318 | 0.014 |
| | p-value | 0.001 | 0.298 | 0.536 | 0.311 | 0.567 | | |
| | du_l | -0.576 | -0.076 | 0.042 | 0.047 | 0.060 | 0.351 | 0.002 |
| | p-value | 0.000 | 0.604 | 0.689 | 0.718 | 0.658 | | |
| Durable | | | | | | | | |
| | du_h | -0.192 | 0.086 | 0.076 | -0.231 | -0.139 | 0.187 | 0.038 |
| | p-value | 0.043 | 0.435 | 0.448 | 0.043 | 0.238 | | |
| | du_l | -0.045 | 0.015 | 0.006 | 0.044 | 0.004 | 0.155 | 0.114 |
| | p-value | 0.109 | 0.451 | 0.802 | 0.054 | 0.863 | | |
| Nondurable | | | | | | | | |
| | du_h | 0.377 | 0.843 | -0.042 | 0.055 | -0.249 | 0.229 | 0.026 |
| | p-value | 0.295 | 0.015 | 0.886 | 0.864 | 0.401 | | |
| | du_l | -0.766 | -0.697 | -0.096 | -0.261 | 0.095 | 0.247 | 0.034 |
| | p-value | 0.028 | 0.037 | 0.695 | 0.419 | 0.712 | | |
| Nonmanufacturing | | | | | | | | |
| | du_h | -0.027 | -0.050 | -0.051 | -0.021 | -0.027 | 0.098 | 0.496 |
| | p-value | 0.451 | 0.116 | 0.148 | 0.508 | 0.382 | | |
| | du_l | -0.155 | -0.006 | 0.049 | 0.019 | -0.018 | 0.369 | 0.002 |
| | p-value | 0.002 | 0.914 | 0.416 | 0.758 | 0.741 | | |

Table 15: Regression of utilisations for two types of skill levels on current and lagged utilisation-controlled technology

Note: Each row shows a separate OLS regression of the utilisation for high-skilled workers ($du_h$) and for low-skilled workers $du_l$ (in growth rates) on current and lagged values of utilisation-controlled technology growth, dz, plus a constant term and linear trend (not shown). Heteroskedasticity- and autocorrelation-robust standard errors in parentheses. All regressions are estimated from 1950 to 2001.

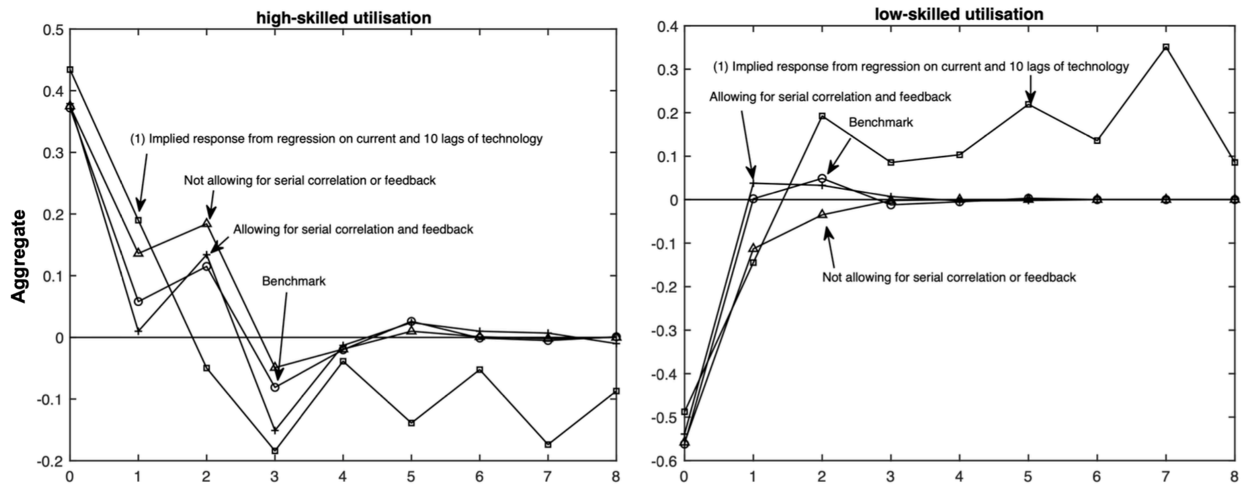


Figure 9: Alternative estimation of the utilisation for high skilled workers and low skilled workers responses to a utilisation-controlled technology improvement in aggregate economy

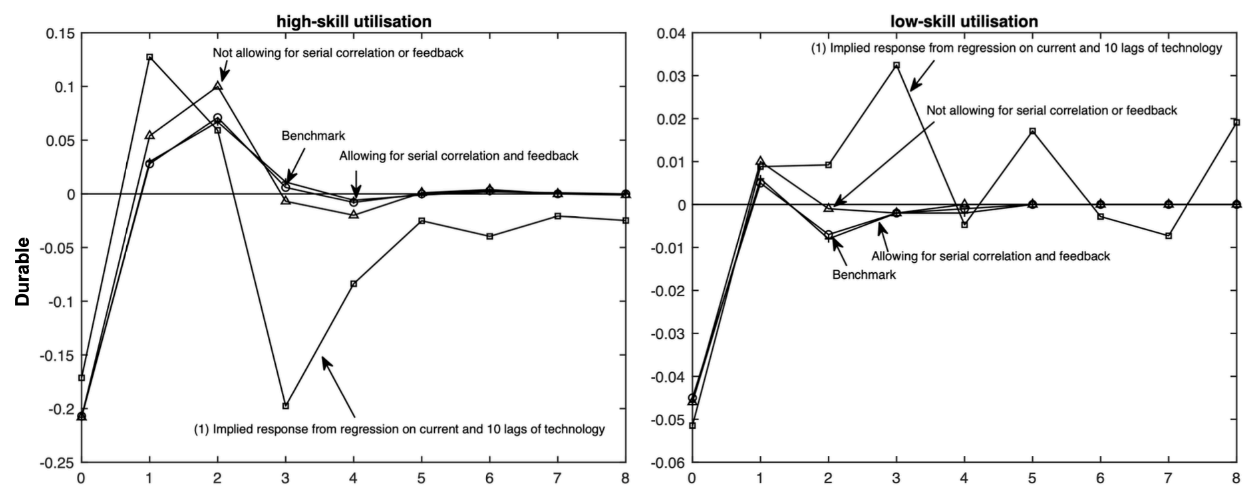


Figure 10: Alternative estimation of the responses of utilisation for high-skilled workers and low-skilled workers to a utilisation-controlled technology improvement in the durable manufacturing sector.

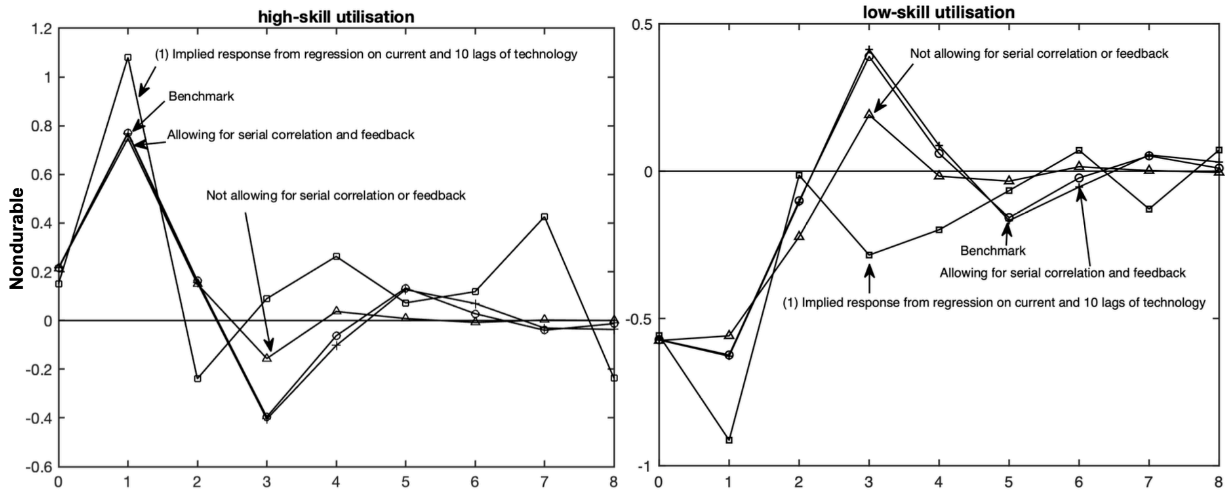


Figure 11: Alternative estimation of the utilisation for high skilled workers and low skilled workers responses to a utilisation-controlled technology improvement in nondurable manufacturing sector

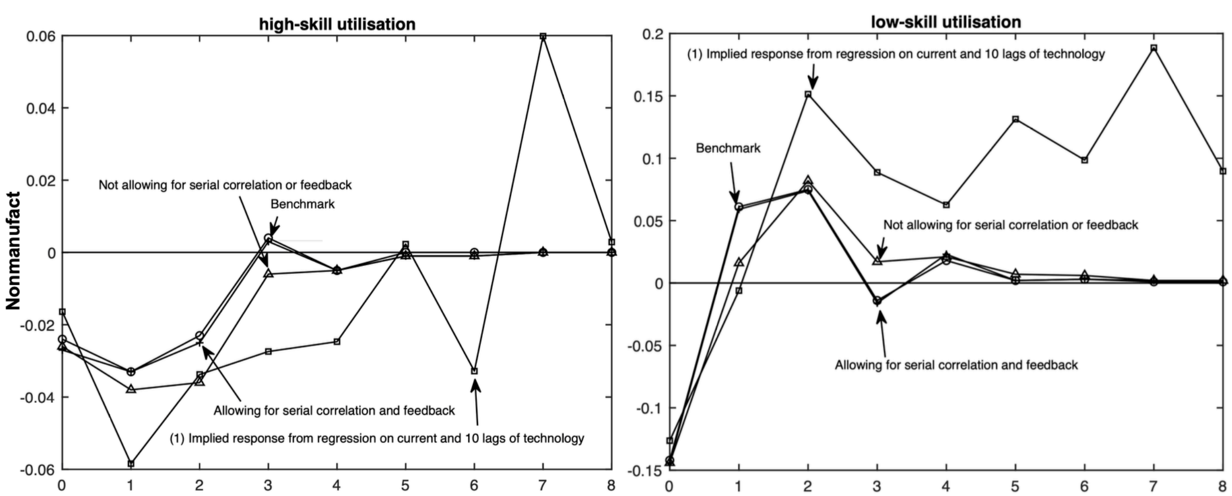


Figure 12: Alternative estimation of the utilisation for high skilled workers and low skilled workers responses to a utilisation-controlled technology improvement in non-manufacturing sector

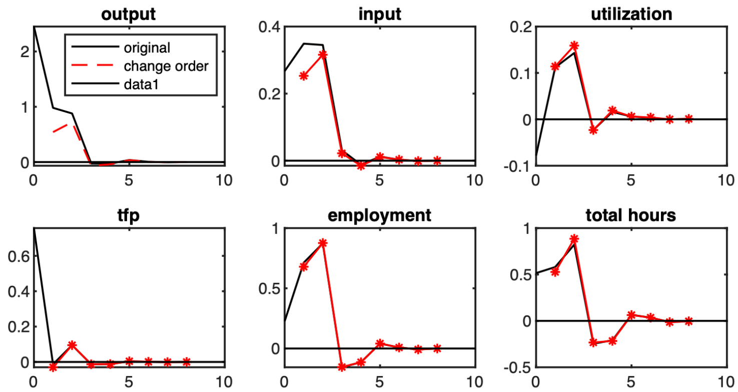


Figure 13: Impulse response to utilisation-controlled technology improvement-two skills estimation

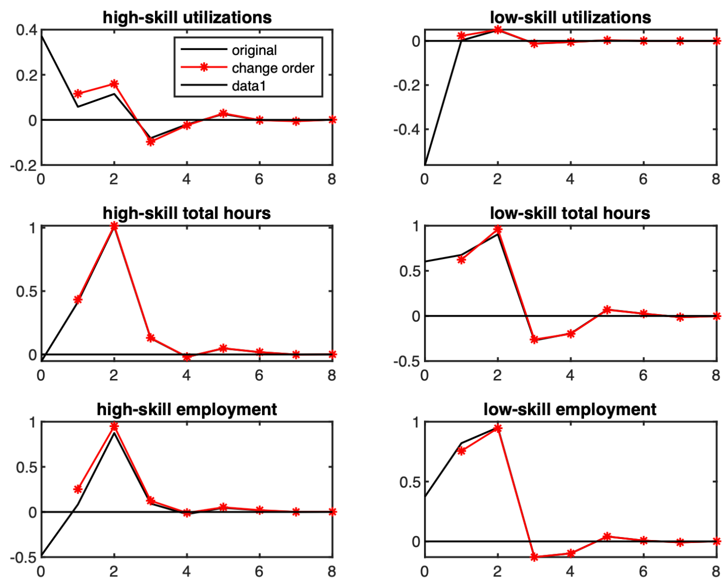


Figure 14: Impulse response to utilisation-controlled technology improvement-two skills estimation

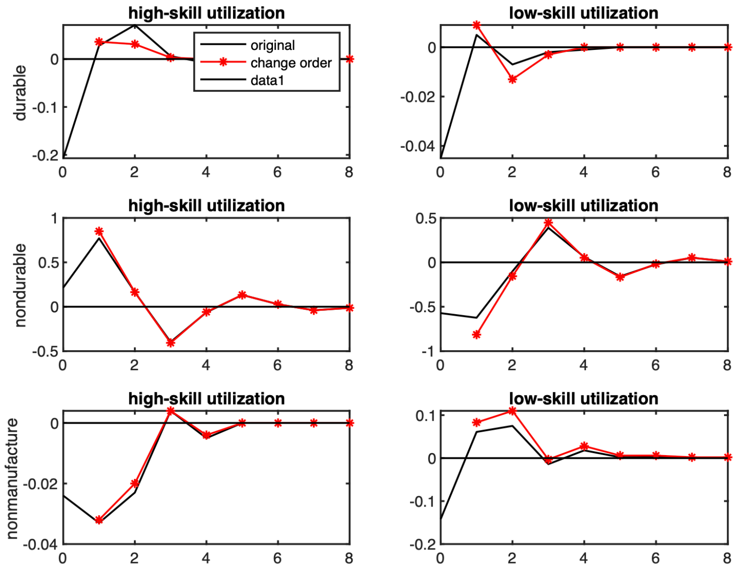


Figure 15: Impulse response of utilisations to sector specified utilisation-controlled technology improvement across Sectors

| | Aggregate | | | | Durable | | | |
|---|---|---|---|---|---|---|---|---|
| | High-skilled | | Low-skilld | | High-skilled | | Low-skilld | |
| | Initial response | p-value | Initial response | p-value | Initial response | p-value | Initial response | p-value |
| Auto & feedback | 0.378 | 0.007 | -0.539 | 0.000 | -0.208 | 0.142 | -0.046 | 0.746 |
| No auto&no feedback | 0.375 | 0.008 | -0.558 | 0.000 | -0.208 | 0.141 | -0.046 | 0.745 |
| Bechmark | 0.371 | 0.009 | -0.562 | 0.000 | -0.207 | 0.143 | -0.045 | 0.750 |
| Regression | 0.465 | 0.001 | -0.538 | 0.000 | -0.199 | 0.051 | -0.043 | 0.107 |

| | Non-durable | | | | Non-manufacture | | | |
|---|---|---|---|---|---|---|---|---|
| | High-skilled | | Low-skilld | | High-skilled | | Low-skilld | |
| | Initial response | p-value | Initial response | p-value | Initial response | p-value | Initial response | p-value |
| Auto & feedback | 0.216 | 0.126 | -0.572 | 0.000 | -0.027 | 0.85 | -0.144 | 0.308 |
| No auto&no feedback | 0.214 | 0.131 | -0.575 | 0.000 | -0.026 | 0.854 | -0.144 | 0.307 |
| Bechmark | 0.215 | 0.129 | -0.572 | 0.000 | -0.024 | 0.867 | -0.142 | 0.314 |
| Regression | 0.377 | 0.261 | -0.693 | 0.026 | -0.027 | 0.482 | -0.154 | 0.001 |

Table 16: Point estimation the initial responses of utilisations to utilisation-controlled technology shock across sectors

Notes: The sample period is 1950-2001. This table shows the point estimates of the initial responses of utilisations for two skills to utilisation-controlled technology shock across sectors. The responses are estimated by four different estimates; including our benchmark estimates, implied regression, treating technology series as white-noise process (i.e., with no autocorrelation and no feedback of response variables), and allowing for autocorrelation and feedback. All series are in logarithmic growth.

# Appendix C.1

Consider the aggregate production function,

$$Y_{i,t} = F(\tilde{K_{i,t}}, \tilde{L}_{i,H,t}, \tilde{L}_{i,L,t}, M_{i,t}, Z_{i,t})$$

Where

$$\tilde{L}_{i,H,t} = \phi_{i,H,t} h_{i,H,t} N_{i,H,t}$$

$$\tilde{L}_{i,L,t} = \phi_{i,L,t} h_{i,L,t} N_{i,L,t}$$

$$\tilde{K_{i,t}} = \phi_{i,K,H,t} \phi_{i,K,L,t} K_{i,t}$$

Taking log on both sides of production function gives (we eliminate industry and time subscript for simplicity except where needed for clarity):

$$logY = logF(\tilde{K}, \tilde{L_H}, \tilde{L_L}, M, Z)$$

Then differentiate with respect to time ,

$$\frac{dY_t/dt}{Y} = \frac{F_1 d\tilde{K_t}/dt}{F} + \frac{F_2 d\tilde{L_{H,t}}/dt}{F} + \frac{F_3 d\tilde{L_{L,t}}/dt}{F} + \frac{F_4 dM_t/dt}{F} + \frac{F_5 dZ_t/dt}{F}$$

Where

$$\frac{d\tilde{K}}{dt} = \frac{d(\phi_{K,H}\phi_{K,L}K)}{dt} = \frac{d\phi_{K,H}}{dt}\phi_{K,L}K + \frac{d\phi_{K,L}}{dt}\phi_{K,H}K + \frac{dK}{dt}\phi_{K,L}\phi_{K,H}$$

$$\frac{d\tilde{L_H}}{dt} = \frac{d(\phi_H h_H N_H)}{dt} = \frac{d\phi_H}{dt} h_H N_H + \frac{dh_H}{dt}\phi_H N_H + \frac{dN_H}{dt}\phi_H h_H$$

$$\frac{d\tilde{L_L}}{dt} = \frac{d(\phi_L h_L N_L)}{dt} = \frac{d\phi_L}{dt} h_L N_L + \frac{dh_L}{dt}\phi_L N_L + \frac{dN_L}{dt}\phi_L h_L$$

Combining those equations, we find:

$$\begin{aligned}\frac{dY/dt}{Y} &= \frac{F_1}{F}(\frac{d\phi_{K,H}}{dt}\phi_{K,L}K + \frac{d\phi_{K,L}}{dt}\phi_{K,H}K + \frac{dK}{dt}\phi_{K,L}\phi_{K,H}) \\ &+ \frac{F_2}{F}(\frac{d\phi_H}{dt}h_H N_H + \frac{dh_H}{dt}\phi_H N_H + \frac{dN_H}{dt}\phi_H h_H) \\ &+ \frac{F_3}{F}(\frac{d\phi_L}{dt}h_L N_L + \frac{dh_L}{dt}\phi_L N_L + \frac{dN_L}{dt}\phi_L h_L) \\ &+ \frac{F_4}{F}\frac{dM}{dt} + \frac{F_5}{F}\frac{dZ}{dt}\end{aligned}$$

Rearrange gives,

$$\begin{aligned}
\frac{dY/dt}{Y} &= \frac{F_1\phi_{K,L}\phi_{K,H}K}{F}(\frac{d\phi_{K,H}/dt}{\phi_{K,H}} + \frac{d\phi_{K,L}/dt}{\phi_{K,L}} + \frac{dK/dt}{K}) \\
&+ \frac{F_2\tilde{L_H}}{F}(\frac{d\phi_H/dt}{\phi_H} + \frac{dh_H/dt}{h_H} + \frac{dN_h/dt}{N_h}) \\
&+ \frac{F_3\tilde{L_L}}{F}(\frac{d\phi_L/dt}{\phi_L} + \frac{dh_L/dt}{h_L} + \frac{dN_L/dt}{N_L}) + \frac{F_4M}{F}\frac{dM/dt}{M} + \frac{F_5Z}{F}\frac{dZ/dt}{Z} \\
&\rightarrow \\
\frac{dY/dt}{Y} &= \frac{F_1\tilde{K}}{F}(\frac{dK/dt}{K} + \frac{d\phi_{K,H}/dt}{\phi_{K,H}} + \frac{d\phi_{K,L}/dt}{\phi_{K,L}}) \\
&+ \frac{F_2\tilde{L_H}}{F}(\frac{d\phi_H/dt}{\phi_H} + \frac{dh_H/dt}{h_H} + \frac{dN_h/dt}{N_h}) \\
&+ \frac{F_3\tilde{L_L}}{F}(\frac{d\phi_L/dt}{\phi_L} + \frac{dh_L/dt}{h_L} + \frac{dN_L/dt}{N_L}) \\
&+ \frac{F_4M}{F}\frac{dM/dt}{M} + \frac{F_5Z}{F}\frac{dZ/dt}{Z}
\end{aligned} \tag{27}$$

with

$$\frac{F_1\tilde{K}}{F} = \frac{F_1\phi_{K,H}\phi_{K,L}K}{F} = \frac{F_KK}{Y}$$

$$\frac{F_2\tilde{L_H}}{F} = \frac{F_2\phi_H h_H N_H}{F} = \frac{F_{h_H}h_H}{Y}$$

$$\frac{F_3\tilde{L_L}}{F} = \frac{F_3\phi_L h_L N_L}{F} = \frac{F_{h_L}h_L}{Y}$$

$$\frac{F_4M}{F} = \frac{F_MM}{Y}$$

Suppose that firms take the price of all input $J$, $P_{Ji}$ as given by competitive markets, and denote the markup over marginal cost as $\mu_i$. According to Basu and Fernald (2007), the first order condition for costs-minimization implies the firm set the value of marginal product of factor $J$ equal to a markup over the factor's input price, that is:

$$P_iF_J^i = \mu_iP_{Ji} \tag{28}$$

For each factor $J$, we multiply both side of Equation 28 by $K_i$ and divide both side by $P_iY_i$ gives:

$$\frac{F_KK}{Y} = \mu\frac{P_KK}{PY} = \mu S_K$$

$$\frac{F_{h_H}h_H}{Y} = \mu\frac{P_{h_H}h_H}{PY} = \mu S_{L_H}$$

$$\frac{F_{h_L}h_L}{Y} = \mu\frac{P_{h_L}h_L}{PY} = \mu S_{L_L}$$

$$\frac{F_M M}{Y} = \mu \frac{P_M M}{PY} = \mu S_M$$

Thus,

$$\mu S_K = \frac{F_1 \tilde{K}}{F} \tag{29}$$

$$\mu S_{L_H} = \frac{F_2 \tilde{L_H}}{F} \tag{30}$$

$$\mu S_{L_L} = \frac{F_3 \tilde{L_L}}{F} \tag{31}$$

$$\mu S_M = \frac{F_4 M}{F} \tag{32}$$

Substitute equations 58 to 61 into 27;

$$\begin{aligned}\frac{dY/dt}{Y} &= \mu S_K(\frac{dK/dt}{K} + \frac{d\phi_{K,H}/dt}{\phi_{K,H}} + \frac{d\phi_{K,L}/dt}{\phi_{K,L}}) \\ &+ \mu S_{L_H}(\frac{d\phi_H/dt}{\phi_H} + \frac{dh_H/dt}{h_H} + \frac{dN_h/dt}{N_h}) \\ &+ \mu S_{L_L}(\frac{d\phi_L/dt}{\phi_L} + \frac{dh_L/dt}{h_L} + \frac{dN_L/dt}{N_L}) \\ &+ \mu S_M \frac{dM/dt}{M} + \frac{F_5 Z}{F}\frac{dZ/dt}{Z}\end{aligned} \tag{33}$$

Use $dx$ to denote the growth rate of $X$ (so, $dx = \frac{dX/dt}{X}$), equation 33 can be rewritten as:

$$\begin{aligned}dy &= \mu S_K(dk + d\phi_{K,H} + d\phi_{K,L}) \\ &+ \mu S_{L_H}(d\phi_H + dh_H + dn_H) + \mu S_{L_L}(d\phi_L + dh_L + dn_L) + \mu S_M dm + \frac{F_5 Z}{F} dz \\ &= \mu(S_K dk + S_{L_H}(dh_H + dn_H) + S_{L_L}(dh_L + dn_L) + S_M dm) \\ &+ \mu(S_K d\phi_{K,H} + S_K d\phi_{K,L} + S_{L_H} d\phi_H + S_{L_L} d\phi_L) + \frac{F_5 Z}{F} dz\end{aligned}$$

# Appendix C.2

## Labour utilisation

An representative firm's cost-minimising problem is:

$$\begin{aligned}&min_{\phi_k,\phi_H,\phi_L,h_H,h_L,I,D} E_t \sum_{\tau=t}^{\infty} [\prod_{j=t}^{\tau-1} (1+r_j)^{-1}] \\ &\times [W_{H,\tau} G(h_{H,\tau}, \phi_{H,\tau}) V(\phi_{K,H,\tau}) N_{H,\tau} + W_{L,\tau} G(h_{L,\tau}, \phi_{L,\tau}) V(\phi_{K,L,\tau}) N_{L,\tau} \\ &+ W_{H,\tau} N_{H,\tau} \Psi(\frac{D_{H,\tau}}{N_{H,\tau}}) + W_{L,\tau} N_{L,\tau} \Psi(\frac{D_{L,\tau}}{N_{L,\tau}}) + P_{I,\tau} K_\tau J_\tau(\frac{I_\tau}{K_\tau}) + P_M M_\tau]\end{aligned}$$

*s.t.*

$$Y_\tau = F[\phi_{K,H,\tau}\phi_{K,L,\tau}K_\tau, \phi_{H,\tau}h_{H,\tau}N_{H,\tau}, \phi_{L,\tau}h_{L,\tau}N_{L,\tau}, M_\tau, Z_\tau]$$

$$K_{\tau+1} = I_\tau + (1-\delta)K_\tau$$

$$N_{\tau+1,H} = N_{\tau,H} + D_{\tau,H}$$

$$N_{\tau+1,L} = N_{\tau,L} + D_{\tau,L}$$

Optimally conditions are:

$$\lambda F_1 \phi_{K,L} K = W_H N_H G(h_H, \phi_H) V'(\phi_{K,H}) \tag{1}$$

$$\lambda F_1 \phi_{K,H} K = W_L N_L G(h_L, \phi_L) V'(\phi_{K,L}) \tag{2}$$

$$\lambda F_2 \phi_H N_H = W_H N_H G_{h_H}(h_H, \phi_H) V(\phi_{K,H}) \tag{3}$$

$$\lambda F_3 \phi_L N_L = W_L N_L G_{h_L}(h_L, \phi_L) V(\phi_{K,L}) \tag{4}$$

$$\lambda F_2 h_H N_H = W_H N_H G_{\phi_H}(h_H, \phi_H) V(\phi_{K,H}) \tag{5}$$

$$\lambda F_3 h_L N_L = W_L N_L G_{\phi_L}(h_L, \phi_L) V(\phi_{K,L}) \tag{6}$$

Combine Equation 3 and Equation 5 gives,

$$\phi_H G_{\phi_H}(h_H, \phi_H) = h_H G_{h_H}(h_H, \phi_H)$$

Denote $\phi_H^*$, $h_H^*$, $G_{\phi_H}^* = G_{\phi_H}(h_H^*, \phi_H^*)$ and $G_{h_H}^* = G_{h_H}(h_H^*, \phi_H^*)$ as steady state values, and in steady state we have $\phi_H^* G_{\phi_H}^* = h_H^* G_{h_H}^*$. Log-linearize above equation around the steady state gives:

$$\begin{aligned}&\phi_H^* G_{\phi_H}^* + G_{\phi_H}^*(\phi_H - \phi_H^*) + \phi_H^* G_{\phi_H\phi_H}^*(\phi_H - \phi_H^*) + \phi_H^* G_{\phi_H h_H}^*(h_H - h_H^*)\\ &= h_H^* G_{h_H}^* + G_{h_H}^*(h_H - h_H^*) + h_H^* G_{h_H h_H}^*(h_H - h_H^*) + h_H^* G_{h_H\phi_H}^*(\phi_H - \phi_H^*)\end{aligned}$$

Rearrange,

$$\frac{\phi_H - \phi_H^*}{\phi_H^*}\phi_H^*(G_{\phi_H}^* + \phi_H^* G_{\phi_H\phi_H}^* - h_H^* G_{h_H\phi_H}^*) = \frac{h_H - h_H^*}{h_H^*}h_H^*(G_{h_H}^* + h_H{}^* G_{h_H h_H}^* - \phi_H^* G_{\phi_H h_H}^*)$$

Define denote $dx$ as the growth rate of $X$ at the steady state $\frac{X-X^*}{X^*}$, then we can rearrange the equation to:

$$d\phi_H = \frac{h_H^*(G_{h_H}^* + h_H{}^* G_{h_H h_H}^* - \phi_H^* G_{\phi_H h_H}^*)}{\phi_H^*(G_{\phi_H}^* + \phi_H^* G_{\phi_H \phi_H}^* - h_H^* G_{h_H \phi_H}^*)} dh_H \tag{7}$$

In steady state, we can define $\Upsilon = \phi_H^* G_{\phi_H}(h_H^*, \phi_H^*) - h_H^* G_{h_H}(h_H^*, \phi_H^*) = 0$, then,

$$\frac{\partial \Upsilon}{\partial h_H^*} = \phi_H^* G_{\phi_H h_H}^* - G_{h_H}^* - h_H^* G_{h_H h_H}^*$$

$$\frac{\partial \Upsilon}{\partial \phi_H^*} = G_{\phi_H}^* + \phi_H^* G_{\phi_H \phi_H}^* - h_H^* G_{h_H \phi_H}^*$$

Using implicit function theorem, we find,

$$\frac{\partial \phi_H^*}{\partial h_H^*} = \phi_H'(h_H^*) = -\frac{\frac{\partial \Upsilon}{\partial h_H^*}}{\frac{\partial \Upsilon}{\partial \phi_H^*}} = \frac{G_{h_H}^* + h_H^* G_{h_H h_H}^* - \phi_H^* G_{\phi_H h_H}^*}{G_{\phi_H}^* + \phi_H^* G_{\phi_H \phi_H}^* - h_H^* G_{h_H \phi_H}^*}$$

Substitute into Equation 7 gives,

$$d\phi_H = \frac{h_H^* \phi_H'(h_H^*)}{\phi_H(h_H^*)} d_H$$

Define $\eta = \frac{h_H^* \phi_H'(h_H^*)}{\phi_H(h_H^*)}$ (note that $\phi_H^* = \phi_H(h_H^*)$), thus,

$$d\phi_H = \eta_H dh_H$$

Similarly, combining Equation 4 and Equation 6 gives,

$$\phi_L G_{\phi_L}(h_L, \phi_L) = h_L G_{h_L}(h_L, \phi_L)$$

Again, log-linearize around the steady state and define $\alpha = \frac{h_L^* \phi_L'(h_L^*)}{\phi_L(h_L^*)}$, we find,

$$d\phi_L = \eta_L dh_L$$

## Capital utilisation

For high skilled workers, combining Equation 1 and 3 gives,

$$\frac{\lambda F_1 \phi_{K,L} K}{\lambda F_2 \phi_H N_H} = \frac{W_H N_H G(h_H, \phi_H) V'(\phi_{K,H})}{W_H N_H G_{h_H}(h_H, \phi_H) V(\phi_{K,H})}$$

Multiply both sides of above equation by $\frac{\phi_{K,H}}{h_H}$ gives,

$$\frac{F_1 \phi_{K,L} \phi_{K,H} K}{F_2 \phi_H N_H h_H} = \frac{G(h_H, \phi_H)}{h_H G_{h_H}(h_H, \phi_H)} \cdot \frac{\phi_{K,H} V'(\phi_{K,H})}{V(\phi_{K,H})} \tag{8}$$

Define $g(h_H) = \frac{h_H G_{h_H}(h_H, \phi_H)}{G(h_H, \phi_H)}$ as the elasticity of labour costs with respect to skilled hours; and define $v(\phi_{K,H}) = \frac{\phi_{K,H} V'(\phi_{K,H})}{V(\phi_{K,H})}$ as the elasticity of shift premium with respect to capital's workweek associated to high skilled workers. And recall that $F_1 \phi_{K,L} \phi_{K,H} K = F_1 \tilde{K}$ and $F_2 \phi_H N_H h_H = F_2 \tilde{L_H}$. We can then write equation 8 as,

$$\frac{(F_1\tilde{K})/F}{(F_2\tilde{L_H})/F} = \frac{v(\phi_{K,H})}{g(h_H)}$$

Substitute equation 58 and 30 into Equation 8 gives,

$$\frac{S_K}{S_{L_H}} = \frac{v(\phi_{K,H})}{g(h_H)} \tag{9}$$

Similarly, for low skilled workers combining Equation 2 and 4 gives,

$$\frac{F_1\phi_{K,L}\phi_{K,H}K}{F_3\phi_L N_L h_L} = \frac{v(\phi_{K,L})}{g(h_L)}$$

Because.

$$\frac{F_1\phi_{K,L}\phi_{K,H}K}{F_3\phi_L N_L h_L} = \frac{(F_1\tilde{K})/F}{(F_3\tilde{L_L})/F} = \frac{\mu S_K}{\mu S_{L_L}}$$

Thus we have,

$$\frac{S_K}{S_{L_L}} = \frac{v(\phi_{K,L})}{g(h_L)} \tag{10}$$

The steady-state version of Equation 9 is $S_K^* g(h_H^*) = S_{L_H}^* v(\phi_{K,H}^*)$. Log-linearize Equation 9 around steady state:

$$\begin{aligned}
&S_K^* g(h_H^*) + g(h_H^*)(S_K - S_K^*) + S_K^* g(h_H^*)_{h_H}(h_H - h_H^*) \\
&= S_{L_H}^* v(\phi_{K,H}^*) + S_{L_H}^* v(\phi_{K,H}^*)_{\phi_{K,H}}(\phi_{K,H} - \phi_{K,H}^*) + v(\phi_{K,H}^*)(S_{L_H} - S_{L_H}^*)
\end{aligned}$$

Rearrange gives,

$$\begin{aligned}
&\frac{S_K - S_K^*}{S_K^*} S_K^* \cdot g(h_H^*) + \frac{h_H - h_H^*}{h_H^*} h_H^* \cdot S_K^* g(h_H^*)_{h_H} \\
&= \frac{S_{L_H} - S_{L_H}^*}{S_{L_H}^*} S_{L_H}^* \cdot v(\phi_{K,H}^*) + \frac{\phi_{K,H} - \phi_{K,H}^*}{\phi_{K,H}^*} \phi_{K,H}^* \cdot S_{L_H}^* v(\phi_{K,H}^*)_{\phi_{K,H}}
\end{aligned}$$

Define denote $dx$ as the growth rate of $X$ at the steady state $\frac{X-X^*}{X^*}$, then we can rearrange the equation to:

$$ds_k \cdot S_K^* g(h_H^*) + dh_H \cdot h_H^* S_K^* g(h_H^*)_{h_H} = ds_{L_H} \cdot S_{L_H}^* v(\phi_{K,H}^*) + d\phi_{K,H} \cdot \phi_{K,H}^* S_{L_H}^* v(\phi_{K,H}^*)_{\phi_{K,H}} \tag{11}$$

Divide both side of Equation 11 by the steady-state version of Equation 10, $S_K^* g(h_H^*) = S_{L_H}^* v(\phi_{K,H}^*)$, gives,

$$ds_k + dh_H \cdot \frac{h_H^* g(h_H^*)_{h_H}}{g(h_H^*)} = ds_{L_H} + d\phi_{K,H} \cdot \frac{\phi_{K,H}^* v(\phi_{K,H}^*)_{\phi_{K,H}}}{v(\phi_{K,H}^*)}$$

Define $\epsilon_H = \frac{h_H^* g(h_H^*)_{h_H}}{g(h_H^*)}$ as elasticity of $g(h_H^*)$ and $\kappa_H = \frac{\phi_{K,H}^* v(\phi_{K,H}^*)_{\phi_{K,H}}}{v(\phi_{K,H}^*)}$ as elasticity of $v(\phi_{K,H}^*)$, we have,

$$d\phi_{K,H} = \frac{1}{\kappa_H} ds_k + \frac{\epsilon_H}{\kappa_H} dh_H - \frac{1}{\kappa_H} ds_{L_H} \tag{12}$$

Similarly, define $\epsilon_L = \frac{h_L^* g(h_L^*)_{h_L}}{g(h_L^*)}$ as elasticity of $g(h_L^*)$ and $\kappa_L = \frac{\phi_{K,L}^* v(\phi_{K,L}^*)_{\phi_{K,L}}}{v(\phi_{K,L}^*)}$ as elasticity of $v(\phi_{K,L}^*)$. Then by the same logic as embodied in Equation 12, log-linearize Equation 10 and after some algebra gives,

$$d\phi_{K,L} = \frac{1}{\kappa_L} ds_k + \frac{\epsilon_L}{\kappa_L} dh_L - \frac{1}{\kappa_H} ds_{L_L} \tag{13}$$

## Regression function

Substituting $d\phi_H$, $d\phi_L$, $d\phi_{K,H}$ and $d\phi_{K,L}$ into the decomposed regression function:

$$\begin{aligned} dy = {} & \mu\Bigg(S_K dk + S_{L_H}(dh_H + dn_H) + S_{L_L}(dh_L + dn) + S_M dm\Bigg) \\ & + \mu\Bigg(S_K\left(\frac{1}{\kappa_H} ds_k + \frac{\epsilon_H}{\kappa_H} dh_H - \frac{1}{\kappa_H} ds_{L_H}\right) + S_K\left(\frac{1}{\kappa_L} ds_k + \frac{\epsilon_L}{\kappa_L} dh_L - \frac{1}{\kappa_L} ds_{L_L}\right) + S_{L_H}\eta_H dh_H + S_{L_L}\eta_L dh_L\Bigg) + \frac{F_4 Z}{F} dz \end{aligned}$$

Rearrange gives,

$$\begin{aligned} dy = {} & \mu dx + \left(\frac{\mu S_K}{\kappa_H} + \frac{\mu S_K}{\kappa_L}\right) ds_k + \left(\frac{\mu S_K \epsilon_H}{\kappa_H} + \mu S_{L_H}\eta_H\right) dh_H - \frac{\mu S_K}{\kappa_H} ds_{L_H} \\ & + \left(\frac{\mu S_K \epsilon_L}{\kappa_L} + \mu S_{L_L}\eta_L\right) dh_L - \frac{\mu S_K}{\kappa_L} ds_{L_L} + \frac{F_4 Z}{F} dz \end{aligned}$$

Where

$$dx = S_K dk + S_{L_H}(dn_H + dh_H) + S_{L_L}(dn_L + dh_L) + S_M dm$$

# 2 Examining the Nonlinear Relationship between Capital-Skill Complementarity and Skilled-Unskilled Complementarity across the U.S. Industries

## 2.1 Introduction

Over the last 60 years, the U.S. economy has experienced a steady increase in the supply of skilled workers and unskilled workers, while the wages have not deteriorated. However, the skill-premium, defined as the relative hourly wage of the skilled-to-unskilled workers, has been rising heavily since 1980. Many pieces of literature emerged to explain the steady increase in the skill premium over time. A recent consensus is that technology change favours skilled workers, makes the skilled workers more productive, and replaces the tasks performed by the less skilled, thus exacerbating wage inequality by higher demand for the skilled and lower demand for unskilled workers. This view is a company with the witness of rapid development in personal computers, new machines and robotic appears, Krueger (1993) summarized this view by the title of ”How computers have changed the wage structure”.

The view that technology development favors skilled workers has led to a literature postulating the existence of capital-skill complementarity. Capital-skill complementarity refers to the elasticity of substitution between capital equipment and skilled labour being less than that between capital and unskilled labour. The key implication of capital-skill complementarity is that technology development induces lower prices of capital equipment, which increases the stock of equipment, further increasing the marginal product of skilled workers but decreasing the marginal product of unskilled workers. Griliches (1969) formalized the hypothesis of capital-skill complementarity, supported by their analysis of cross-sectional manufacturing data from the United States, which suggested that capital and skilled workers are more complementary than capital and unskilled workers. Since then, a significant amount of literature has emerged reinforce the existence of capital skill complementarity. Flug and Hercowitz (2000) uses a panel data set of a wide range of countries and finds a positive and strong effect of machinery investment on the relative demand for skilled labour. Krusell et al. (2000) develop a four-factor aggregate production function, including skilled, unskilled labours, capital equipment and capital structure, that allows for different elasticity of substitution among the input factors, arguing that capital-skill complementarity helps explain the observed change in the skill premium in the U.S. economy. Caselli and Coleman (2002) identify the efficiency embodied in unskilled labour, skilled labour and capital in aggregate production function, finding that the efficiencies of skilled labour and capital have risen over 1963-1992, while the efficiency of unskilled labour has been falling since 1970s. Polgreen and Silos (2008) re-examine the findings of Krusell et al. (2000), confirming the existence of capital-skill complementarity. Hara et al. (2014) use a two-sector neoclassical general equilibrium model with two types of labour, in which the CES production function allows for a degree of capital-skill complementarity, showing that the decline in capital-skill complementarity can account for the observed changes in the skill premium in the Japanese economy, with significant differences in the degree of capital-skill complementarity found in Japan. The hypothesis of capital-skill complementarity has been extensively documented in the literature, and as a result, there is a growing body of research focused on examining its empirical and quantitative implications (see, for example, Lindquist (2004), Caselli and Wilson (2004)). Additionally, another literature centers on the assumption of capital-skill complementarity as the primary driving force of the development process (see, for example, Galor and Moav (2000), Galor and Moav (2004)).

While there is a significant body of literature supporting the existence of capital-skill complementarity, other studies find null or weak support for it. Goldin and Katz (1998) argue that technological developments may not always increase the demand for skills; in fact, advances in technology may replace skilled workers. They suggest that capital-skill complementarity may be a transitory phenomenon, as companies progress through various stages of development, and skilled labour transitions from being more substitutable to being complementary. For example, in the early era, when the factory replaced the artisan shop, the new machines make the production process "deskilling", that is, the complex tasks previously performed by the skilled workers have been simplified by breaking them into smaller and less-skilled requiring pieces, declining the demand for skilled workers. Similar conclusions were made by Braverman (1998) and Marglin (1974). Inspired by the findings of Goldin and Katz (1998), Duffy et al. (2004) emphasize the importance of finding evidence of capital complementarity over long periods and across countries at different stages of development. Thus, they examine the capital-complementarity hypothesis using a panel data set of 73 countries over 25 years and various specifications of aggregate production function, concluding that that the evidence of the capital-skill complementarity hypothesis is weak. Ruiz-Arranz (2002) find that while capital-skill complementarity accounts for at most 40 percent of the rise in the skill premium but only a small fraction of equipment capital, information technology, is complementary to skilled labour. In the following years, Caselli and Coleman (2006) exams the cross-country differences in the aggregate production function and find that higher-income countries use skilled labour more efficiently than lower-income countries, while they use unskilled labour relatively less efficiently. They attribute this phenomenon to the technology adoption rather capital-skill complementarity. Balleer and Van Rens (2013) construct a quarterly skill premium series for the U.S. economy and identify various technology shocks using a structural vector autoregression framework. They find that in response to the investment-specific technology shocks, the skill premium responds negatively. Their finding rejects the possibility of capital-skill complementarity and favours the existence of capital-skill substitutability in the aggregate production technology.

Our work contributes to this literature by using industry level panel data in the U.S. economy to test the hypothesis across various industries. Recent papers use highly nonlinear nested constant-elasticity-of-substitution (CES) aggregate production function which are shown to be quite difficult to accurately estimate, we instead use simple linear economic estimation techniques and show that significant progress can be made in testing this hypothesis.[25]. Furthermore, previous research, typically based on aggregated data, may neglect significant differences in input structures and technology development across industries, leading to blurred relationships between input structures at the industry level. For example, Herrendorf et al. (2015) demonstrate the importance of accounting for differences in technology and input structures by estimating CES production functions for agriculture, manufacturing, and services on postwar US data. Their findings reveal significant heterogeneity in capital share and the degree of substitution between capital and labour across sectors. Additionally, Jorgenson and Timmer (2011) suggests an emphasis shift from manufacturing to the production of the services. They find the agricultural sector has become smaller while services comprise about three-quarters of GDP in advanced economies. Therefore, studies based only on manufacturing data neglect the crucial role of various services industries, which account for a significant portion of total value-added and hours worked in developed economies. In our paper, we estimate the model on a total of 26 industries that have been grouped into different categories using various threshold estimations, providing insight into a range of important economic issues.

[25]Using nonlinear estimation techniques to obtain reliable estimates of the relative size of elasticity parameters can still be cumbersome and problematic, as demonstrated by Duffy et al. (2004)

The research we are conducting has a strong connection to two previous studies, namely Goldin and Katz (1998) and Papageorgiou and Chemlarova (2005). Goldin and Katz used panel data from 1909 and 1919 at the industry level in the U.S. manufacturing sector to estimate the regression of skilled labour's share of total labour costs on the logarithm of skill premium and capital to output ratio. Their model was developed based on the assumption of constant returns to scale and aimed to minimize costs. Their analysis indicates that industries with a higher increase in the proportion of skilled workers in total labour costs experienced a corresponding increase in capital to output ratio, implying that skilled labour and capital are complementary. Similarly, Papageorgiou and Chemlarova utilized the same estimation technique on a cross-country dataset, examining the hypothesis among both OECD and non-OECD countries, and among three regimes that were endogenously determined through threshold estimations. In our study, we extended these works by applying the linear estimation technique to panel data that covered 26 industries in the U.S. economy, including non-manufacturing industries, durable goods, and non-durable goods, for 62 years spanning from 1949 to 2010. Furthermore, we classified the industries into several groups based on threshold estimations, and then applied the estimation technique to each of these industry groups that were endogenously determined.

The main findings of the paper are as follows: Firstly, we discovered that splitting the full sample of 26 industries into manufacturing and non-manufacturing sectors without a clear criteria can result in misleading results when testing the capital-skill complementarity hypothesis. Second, after employing the data-sorting method developed by Hansen (2000), that allows the data to endogenously select groups. Our analysis indicated that the threshold variable that could effectively cluster industries into two groups was the capital to output ratio. Specifically, we found that industries with a low capital to output ratio ($K/Y < 0.9897$) exhibited evidence of capital-skill complementarity, whereas industries with a high capital to output ratio ($K/Y > 0.9897$) did not show any evidence of capital-skill complementarity. Instead, our results revealed that capital and skilled labour were substitutes in these industries. Our third finding indicates that the complementarity between skilled and unskilled workers is stronger in industries with a high capital to output ratio compared to those with a low capital to output ratio. This can be attributed to the fact that most industries in the high capital to output ratio group have lower levels of education with a smaller proportion of skilled labour than industries in the low capital to output ratio group. Consequently, in the high capital to output ratio group (industries with lower education level) there is a larger skill gap between skilled and unskilled labour, leading to a more noticeable complementarity between the two types of workers. Finally, our study identified a notable time break in the year 1980 for industries with a high capital to output ratio. Our analysis showed that capital-skill substitution became more profound in this industry group from 1980 onwards compared to the period before 1980.

The paper is structured as follows: In Section 2.2, we present the approach utilized in this study and provide details on the data and estimation method. Section 2.3 presents the findings on the non-linearity in capital-skill complementarity/substitution across industries. Next, in Section 2.4, we discuss the non-linearity in skilled and unskilled complementarity/substitution across industries. Section 2.5 focuses on the time break in capital-skill complementarity/substitution. The robustness check is presented in Section 2.6. In Section 2.7, we discuss the implications of our findings for the existing literature, and finally, Section 2.9 presents the conclusions of the study.

## 2.2 Methodology and Data

The method used to estimate the correlation between capital and skilled labour is based on the approach presented in Brown and Christensen (1980). The estimation equation is inspired by a model where capital is considered the quasi-fixed factor and skilled and unskilled labour are the variable factors. If the total labour cost function for a particular industry $i$ has a translog form, then cost minimization under constant returns to scale generate the following 'share equation' for the skilled labour share of total costs in time $t$: [26]

$$S_{it} = \alpha_i + \phi_i t + \gamma_{11} ln\Big(\frac{w_s}{w_u}\Big)_{it} + \gamma_{1k} ln\Big(\frac{K}{Y}\Big)_{it} \tag{34}$$

The skilled labour share of the total wage bill, denoted by $S_{i,t}$, is a measure of the proportion of total wage expenditure allocated to skilled labour in a given period $t$ in a specific industry $i$. This measurement is represented by $S_{i,t} = \frac{w_{s,i,t}S_{i,t}}{w_{s,i,t}S_{i,t}+w_{u,i,t}U_{i,t}}$, where $w_{s,i,t}$ and $w_{u,i,t}$ represent the wages of skilled and unskilled labour, respectively. $\Big(\frac{w_s}{w_u}\Big)_{i,t}$ is the skill premium and $\Big(\frac{K}{Y}\Big)_{i,t}$ is the capital to output ratio. The key coefficients in this regression are $\gamma_{11}$ and $\gamma_{1k}$. The coefficient $\gamma_{11}$ reflects the elasticity of substitution between skilled and unskilled labour. If this elasticity is less than one, $\gamma_{11}$ is positive, indicating that an increase in the wage premium for skilled labour relative to unskilled labour leads to an increase in the skilled labour share of the wage bill, which implies skilled-unskilled labour complementarity. Conversely, if the elasticity of substitution is greater than one, $\gamma_{11}$ is negative, suggesting that an increase in the wage premium leads to a decrease in the skilled labour share of the wage bill, that is skilled-unskilled labour substitution.The coefficient $\gamma_{1k}$ represents the degree of complementarity between skilled labour and capital. The logic is the same as above; a positive value for this coefficient implies that an increase in the capital-output ratio leads to an increase in the skilled labour share of the wage bill. This result reflects the idea that capital and skilled labour are complementary in the production process, and an increase in the capital-output ratio leads to a higher demand for skilled labour. $\phi_i$ measures the rate of skill-biased technological change in industry $i$.

Our estimation of the 'share equation' (Equation 34) involves panel data on 26 industries with sample period of 1949-2010. The data used in this study is sourced from the April 2013 release of the World KLEMS database. This database, which was adapted from the database on U.S. productivity growth by industry created by Dale Jorgenson and associates, provides detailed labour and capital inputs for the U.S. using the North American Industry Classification System (NAICS). Our sample consists of industry-level data on labour, capital inputs, and output for 26 industries, including 6 durable, 7 non-durable, and 13 non-manufacturing industries. In our analysis, we use the capital services volume index (denoted by $K$) and the gross output volume index (denoted by $Y$).

This study's labour data includes detailed information on labour compensation per hour, average weekly working hours, and employment. The labour force is divided into six skill levels, including those with less than a high school education, some high school education, high school graduates, some college education, college graduates, and those with more than a college education. In line with established literature (such as Krusell et al. (2000), Kawaguchi et al. (2014), Parro (2013)), workers with a college education or higher, which is equivalent to at least 16 years of education, are classified as skilled labour, while unskilled labour refers to workers without a college education. Individuals under the age of 16 or over 64 are excluded from the analysis to ensure that the sample is consistent with the working-age population. To calculate skilled and unskilled workers' wage bills, we multiply their hourly wage by their weekly working hours and the number of employees. We then compute the skilled labour share of the total wage bill by dividing the skilled workers' wage bill by the total wage bill of both skilled and unskilled workers. The skill premium ($\frac{w_s}{w_u}$) data is derived by dividing the skilled labour hourly wage by the unskilled labour hourly wage. Lastly, the industry data are aggregated into manufacturing and non-manufacturing sectors. Table 46 provides a detailed classification of

[26] A detailed derivation can be found in the Appendix of Papageorgiou and Chemlarova (2005).

the industries.

## 2.3 Nonlinearity in Capital Skill Complementarity

### 2.3.1 Estimation Results for Manufacturing and Nonmanufacturing Sectors

In this section, we estimate the regression using the subsample, that is trivially divided into manufacturing goods and non-manufacturing goods sectors. The results are presented in Table 17. The Equation 34 (Model 3) is our baseline model, in which we regress the skilled labour share of the total wage bill ($S$) on the log of the skilled-to-unskilled labour wage rate ($\frac{w_s}{w_u}$), and the log of capital to output ratio ($\frac{K}{Y}$). To examine the robustness of the results from the baseline model, we follow Goldin and Katz (1998) and exam five modifications of Equation 34. We estimate a modification that drops the relative wage variable ($\frac{w_s}{w_u}$) (Model 1), because the cross-sectional wage variation could be endogenous as it largely reflect skill differences. Another modification is to drop the relative wage and add a logarithm of the output ($Y$) variable (Model 2). Adding output variable is because during different stages of a business cycle, the demand for skilled and unskilled labour may change. For instance, during an economic boom, there may be high demand for skilled workers to develop new products or services, while during a recession, there may be more demand for unskilled workers in industries such as construction and retail. Thus, adding output variable can account for cyclical differences in the extend to which skilled and unskilled labour are quasi-fixed factor. This also allows the possibility that the production function is non-homothetic. And the rest three models replace the variable ($Y$) by value added ($VA$) (Model 4 to Model 6).

For the manufacturing sector, in the baseline Model 3 (column 4), the coefficient $\gamma_{1k}$ is found to be -0.024, suggesting no evidence of capital-skill complementarity and instead indicating a preference for substitution between capital and skilled labour. Our findings are robust to alternative models, as shown in columns 2, 3 and 5 to 7, where the coefficient $\gamma_{1k}$ consistently exhibits a negative sign regardless of other controls. Specifically, our estimates reveal values of -0.030 in Model 1, -0.018 in Model 2, -0.030 in Model 4, -0.017 in Model 5, and -0.027 in Model 6, all of which are statistically significant at the 1% level. In the non-manufacturing sector, the results reveal a consistently positive and statistically significant coefficient $\gamma_{1k}$, indicating evidence of capital-skill complementarity in the production process. The coefficient in the baseline Model 3 is 0.043, which is significant at 1 % level. Again, dropping the relative wage variable and inclusion of output variable does not affect the key results; the coefficients are 0.057 in Model 1, 0.045 in Model 2, 0.051 in Model 4, 0.055 in Model 5, and 0.040 in Model 6, all of which are statistically significant at the 1% level. In both sectors, the coefficient estimates for the log of $Y$ is positive and statistically significant in Model 2 and 5, which provides evidence that the production function is non-homothetic.

| Specification | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|---|---|---|---|---|---|---|
| Manufacturing | | | | | | |
| ln(K/Y) | -0.0304*** | -0.0181*** | -0.0239*** | | | |
| (*std.*) | (0.0060) | (0.0080) | (0.0050) | | | |
| ln(Y) | | 0.0136*** | | | | |
| (*std.*) | | (0.0060) | | | | |
| ln(K/VA) | | | | -0.0297*** | -0.0167*** | -0.0268*** |
| (*std.*) | | | | (0.0040) | (0.0060) | (0.0030) |
| ln(VA) | | | | | 0.0133*** | |
| (*std.*) | | | | | (0.0050) | |
| ln(Ws/Wu) | | | 0.1389*** | | | 0.1337*** |
| (*std.*) | | | (0.0110) | | | (0.0100) |
| t | 0.0054*** | 0.0049*** | 0.0054*** | 0.0053*** | 0.0047*** | 0.0053*** |
| (*std.*) | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| R-square | 0.4605 | 0.4450 | 0.4467 | 0.3782 | 0.3649 | 0.3760 |
| Obs | 806 | 806 | 806 | 806 | 806 | 806 |
| Prob>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | | | | | |
| Nonmanufacturing | | | | | | |
| ln(K/Y) | 0.0573*** | 0.0451*** | 0.0432*** | | | |
| (*std.*) | (0.0050) | (0.0050) | (0.0040) | | | |
| ln(Y) | | 0.0518*** | | | | |
| (*std.*) | | (0.0050) | | | | |
| ln(K/VA) | | | | 0.0509*** | 0.0550*** | 0.0396*** |
| (*std.*) | | | | (0.0040) | (0.0040) | (0.0040) |
| ln(VA) | | | | | 0.0424*** | |
| (*std.*) | | | | | (0.0040) | |
| ln(Ws/Wu) | | | 0.1391*** | | | 0.1392*** |
| (*std.*) | | | (0.0080) | | | (0.0080) |
| t | 0.0040*** | 0.0025*** | 0.0044*** | 0.0040*** | 0.0026*** | 0.0044*** |
| (*std.*) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| R-square | 0.2491 | 0.1809 | 0.3392 | 0.2414 | 0.1756 | 0.3299 |
| Obs | 806 | 806 | 806 | 806 | 806 | 806 |
| Prob>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 17: Regression results for manufacturing and nonmanufacturing sectors

Notes: Sample period is 1949-2010. The dependent variable is skilled-labour share of the wage bill (S). $w_u/w_s$ is skilled-unskilled wage premium, $K/Y$ is capital–output ratio, and $Y$ is output. $VA$ is value added volume index. Robust standard error are listed in the parenthesis ***Significantly different from 0 at the 1 % level.

### 2.3.2 Threshold Estimation

Unlike the previous section in which we arbitrarily split the data into manufacturing and nonmanufacturing sectors, we follow Hansen (1999) and Hansen (2000) to endogenously determine the subsamples in the data using the 'share equation' (Equation 34). [27] Hansen develops a statistical theory of threshold estimation in the linear regression context in which the fixed-effect estimation is considered, and an asymptotic distribution theory for the regression estimates is developed. [28] In this study, we endogenously split the sectors using

[27] The threshold estimation analysis in this section is based on the baseline model (Model 3). In the robustness checking section, threshold estimations are done using the alternative models (Model 2 and Model 5, Model 6). Due to the potential inaccuracy of results from single-variable estimations, we did not perform threshold estimations based on Model 1 and Model 4.

[28] For a detailed discussion of the statistic theory for threshold estimation in linear regression, see Hansen (1999),Hansen (2000) and Caner and Hansen (2004).

capital to output ratio $(K/Y)$ as a potential threshold variable. We choose the capital to output ratio as the potential threshold variable because industries with a high capital to output ratio are more capital-intensive, implying that they may have a different pattern of substituting capital for labour from less capital-intensive industries. We postulate that the industries with a higher capital to output ratio typically employ less-skilled workers and rely more on capital-intensive technologies, implying less sophisticated production processes. Conversely, industries with a lower capital to output ratio generally require more sophisticated skills and a higher level of education among their workforce, as a result, the accumulation of capital necessitates the employment of more highly skilled labour to operate and manage machinery and equipment. [29] Table 18 represents the threshold estimation results.

The results reveal the existence of a sample division based on the capital to output ratio, which segregates our complete sample of 26 industries into two distinct groups: a low capital to output ratio group, characterised by a capital-output ratio below the estimated threshold of 0.9897 and a high capital to output ratio group with a capital-output ratio above this threshold. Notably, the estimated threshold value is statistically significant. Additionally, our results indicate that the coefficient $\gamma_{1k}$ (i.e., coefficient in front of $ln(K/Y)$) exhibits a positive sign for the low capital to output ratio group, suggesting that an increase in the capital to output ratio leads to an increase in the skilled labour share of the wage bill for this group. Conversely, for the high capital to output ratio group, the estimated coefficient is negative. Table 19 presents the classification of industries into two distinct groups based on the estimated threshold value. The categorization is based on the median value of the capital-output ratio $(K/Y)$ cross time for each industry, as shown in Table 37 in Appendix A. Specifically, in Table 19, industries with a median $K/Y$ value greater than the estimated threshold value of 0.9897 are classified into group 1, while industries with a median $K/Y$ value lower than the threshold are categorised into group 2. The industries that belong to the manufacturing sector are highlighted in bold in the table, while those not highlighted in bold are classified as non-manufacturing industries.

[29] We acknowledge that other potential threshold variables can be used further to investigate non-linearity in the capital–skill complementarity. We leave this investigation for future research.

| Threshold estimator (level = 95) | Estimates | Lower | Upper | |
|---|---|---|---|---|
| | 0.9897 | 0.9866 | 0.992 | |
| | | | | |
| Threshold effect test | RSS | MSE | Fstat | P-value |
| Single | 5.4248 | 0.0035 | 811.68 | **0.000** |
| | | | | |
| Fixed-effects (within) regression | | | | |
| ln(K/Y) | Estimates | P-value | | |
| 0 | **0.0744976** | 0.000 | | |
| 1 | **-0.6335534** | 0.000 | | |
| ln(Ws/Wu) | | | | |
| 0 | 0.0592849 | 0.000 | | |
| 1 | 0.1890651 | 0.000 | | |
| | | | | |
| R-squared | 0.1126 | | | |
| Obs | 1612 | | | |
| Prob>F | 0.000 | | | |

Table 18: Threshold estimation based on baseline model (Model 3)

Note: The sample period is 1949 to 2010. The threshold estimation is based on our baseline model (Model 3) with capital to output ratio ($K/Y$) as the threshold variable.

| | Group 1 | | | Group 2 | |
|---|---|---|---|---|---|
| | K/Y>0.9897 | | | K/Y<0.9798 | |
| 1 | AtB | agriculture hunting forestry and fishing | 2 | C | mining and quarrying |
| **9** | **25** | **rubber and plastics** | **3** | **15t16** | **food beverages and tobacco** |
| **13** | **30t33** | **electrical and optical equipment** | **4** | **17t19** | **textiles textile leather and footwear** |
| **15** | **36t37** | **manufacturing nec; recycling** | **5** | **20** | **wood and of wood and cork** |
| 21 | H | hotels and restaurants | **6** | **21t22** | **pulp paper paper printing and publishing** |
| 22 | 60t63 | transport and storage | **7** | **23** | **coke refined petroleum and nuclear fuel** |
| | | | **8** | **24** | **chemicals and chemical products** |
| | | | **10** | **26** | **other nonmetallic mineral** |
| | | | **11** | **27t28** | **basic metals and fabricated metal** |
| | | | **12** | **29** | **machinery nec** |
| | | | **14** | **34t35** | **transport equipment** |
| | | | 16 | E | electricity gas and water supply |
| | | | 17 | F | construction |
| | | | 18 | 51 | wholesale trade and commission trade |
| | | | 19 | 50 | sale maintenance and repair of motor vehicles |
| | | | 20 | 52 | retail trade except of motor vehicles |
| | | | 23 | 64 | post and telecommunications |
| | | | 24 | J | financial intermediation |
| | | | 26 | 71t74 | renting of m&eq and other business activities |
| | | | 30 | O | other community social and personal services |

Table 19: Industry classification based on threshold estimation

Note: The sample period is 1949 to 2010. This table displays the industry classification based on the threshold estimation on our baseline model (Model 3). Industries that are categorised as manufacturing industries are highlighted in bold, while those categorised as non-manufacturing industries are not highlighted.

In order to conduct a more in-depth investigation of the potential nonlinearity in the capital-skill complementarity relationship across industries, as determined by the endogenous threshold methodology, we direct our focus towards estimating the regression coefficients of Model 1 through Model 6 for two distinct groups. The resulting coefficient estimates are presented in Table 20. All models' estimated coefficients are strongly significant except for Model 2 in group 1. Specifically, in group 1, the coefficient $\gamma_{1k}$ consistently displays a negative sign across all six models analysed. On the other hand, for group 2, the coefficients $\gamma_{1k}$ for all models are positive and strongly significant. The findings from the analysis reject the capital-skill complementarity hypothesis for industries in group 1, and suggest that in industries with high capital to output ratios, capital and skill are substitutable factors of production, as evidenced by the consistently negative coefficients of $\gamma_{1k}$ across all six models in group 1. On the other hand, in industries with low capital to output ratios, the results favour the capital-skill complementary hypothesis, as supported by the consistently positive coefficients of all models in group 2.

Furthermore, in Table 19, only three of the six industries (i.e., industries with id of 9, 13, and 15) in group 1 are categorised as manufacturing, while the remaining belong to the non-manufacturing sector. Additionally, the majority of industries classified as manufacturing are placed in group 2 based on the threshold value. As shown in Table 17, manufacturing industries consistently exhibit a negative sign for the coefficient $\gamma_{1k}$. Moreover, only three industries in the manufacturing group are categorized into group 1, which indicates a negative sign for $\gamma_{1k}$ when using threshold estimation. Thus, the arbitrary categorization of industries into manufacturing and non-manufacturing sectors may lead to misleading findings. In order to further investigate this issue, we have removed the industries in group 1 from both the manufacturing and non-manufacturing categories and re-estimated Model 1 through Model 6. The results are presented in Table 21. Not surprisingly, after removing industries with id of 9, 13, and 15 from the manufacturing group, the coefficient $\gamma_{1k}$ becomes positive and statistically significant for Model 1 to Model 3, although the coefficients are generally small in magnitude. However, for Model 4 through Model 6, the coefficients remained negative and strongly insignificant. For the non-manufacturing group, after removing industries with id of 1, 21, and 22, the coefficients $\gamma_{1k}$ remain significant and positive for all six models. In the non-manufacturing sector, it was also observed that after removing these industries, the $\gamma_{1k}$ coefficients were larger than those estimated in Table 17, in which the industries were arbitrarily classified. Specifically, they were approximately 0.06, 0.05, 0.04, 0.05, 0.06, and 0.04 for Model 1 through Model 6 for industries arbitrarily classified as non-manufacturing, and approximately 0.06, 0.06, 0.04, 0.06, 0.07, and 0.04 for the industries after removing industries 9, 13, and 15 from non-manufacturing industries. This is because the arbitrary classification of industries into non-manufacturing includes industries 1, 21, and 22, which exhibit statistically negative coefficients, leading to a lower overall value of $\gamma_{1k}$ when estimating the entire non-manufacturing group. These findings suggest that the results obtained from simply splitting the entire sample into manufacturing and non-manufacturing sectors do not carry significant implications. This observation is consistent with our exploration of an endogenous grouping of the sample into subsets that adhere to distinct statistical models.

In Table 20, several points merit attention. Firstly, including the log of $Y$ and $VA$ in Model 2 and Model 5 yields positive and significant coefficients in both group 1 and group 2. However, when removing industries in group 1 from the manufacturing and non-manufacturing categories, the coefficients in front of $Y$ are positive in non-manufacturing industries but turn negative in manufacturing industries (see Table 21). This finding suggests that the production function in manufacturing industries may be homothetic after removing the high capital to output ratio industries, while it remains non-homothetic in other industries. Secondly, it was observed that in all models, the coefficient for $w_s/w_u$ (i.e., $\gamma_{11}$) is positive and significant, indicating that the elasticity of substitution between skilled and unskilled labour is less than unity, implying a complementary relationship between these labour inputs.

Finally, we observe that in Table 20, the coefficient in front of $(w_s/w_u)$ (i.e., $\gamma_{11}$) for group 2 is greater than that for group 1. Specifically, the coefficient is approximately 0.15 in Model 3 and Model 6 for group

2 and only 0.10 in both Models for group 1. This result suggests that skilled and unskilled labour exhibit more noticeable complementarity in the industries belonging to group 2. Table 22 presents a listing of industries in descending order of the skilled-to-unskilled employment ratio,[30] with industries belonging to group 1 being highlighted in bold. Notably, most of the industries in group 1 are characterised by relatively low education levels with a relatively small proportion of skilled labour. The electrical and optical industry is the only exception, as it exhibits a relatively higher fraction of skilled workers. The median skilled-to-unskilled employment ratio for industries in group 1 is approximately 0.12, while it is 0.16 for industries in group 2 (detail see Table 38 in Appendix A). Therefore, our findings indicate that in the industry group with higher median levels of education (group 2) with a greater proportion of skilled workers, the complementarity between skilled and unskilled labour is more noticeable compared to industries with lower median levels of education (group 1). For a more detailed discussion on this topic, please refer to Section 2.4 of our study.

[30]We aggregated the skilled-to-unskilled employment ratio by summing the values across time for each year. Using the median value of this ratio for each year did not affect the quality of the results.

| Specification | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|---|---|---|---|---|---|---|
| Group 1 | | | | | | |
| ln(K/Y) | -0.0627*** | -0.0011 | -0.0535*** | | | |
| (*std.*) | (0.0138) | (0.0149) | (0.0133) | | | |
| ln(Y) | | 0.0575*** | | | | |
| (*std.*) | | (0.0072) | | | | |
| ln(K/VA) | | | | -0.0678*** | -0.0321*** | -0.0650*** |
| (*std.*) | | | | (-0.0043) | (-0.0089) | (0.0041) |
| ln(VA) | | | | | 0.0290*** | |
| (*std.*) | | | | | (0.0064) | |
| ln(Ws/Wu) | | | **0.1090*****| | | **0.0936*** ** |
| (*std.*) | | | (0.0183) | | | (0.0143) |
| t | 0.0048*** | 0.0028*** | 0.0048*** | 0.0043*** | 0.0033*** | 0.0043*** |
| (*std.*) | (0.0001) | (0.0003) | (0.0001) | (0.0001) | (0.0002) | (0.0001) |
| R-square | 0.4329 | 0.2893 | 0.4945 | 0.1805 | 0.1578 | 0.2231 |
| Obs | 372 | 372 | 372 | 372 | 372 | 372 |
| Prob>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | | | | | |
| Group 2 | | | | | | |
| ln(K/Y) | 0.0446*** | 0.0444*** | 0.0307*** | | | |
| (*std.*) | (0.0050) | (0.0049) | (0.0043) | | | |
| ln(Y) | | 0.0299*** | | | | |
| (*std.*) | | (0.0037) | | | | |
| ln(K/VA) | | | | 0.0186*** | 0.0365*** | 0.0137*** |
| (*std.*) | | | | (0.0032) | (0.0035) | (0.0027) |
| ln(VA) | | | | | 0.0343*** | |
| (*std.*) | | | | | (0.0033) | |
| ln(Ws/Wu) | | | **0.1513*****| | | **0.1559*****|
| (*std.*) | | | (0.0070) | | | (0.0070) |
| t | 0.0040*** | 0.0032*** | 0.0044*** | 0.0045*** | 0.0033*** | 0.0048*** |
| (*std.*) | (0.0001) | (0.0001) | (0.0001) | (0.0001) | (0.0001) | (0.0001) |
| R-square | 0.3862 | 0.3221 | 0.4134 | 0.4151 | 0.3208 | 0.4308 |
| Obs | 1240 | 1240 | 1240 | 1240 | 1240 | 1240 |
| Prob>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 20: Regression results for industries in group 1 and group 2

Notes: Sample period is 1949-2010. The dependent variable is skilled-labour share of the wage bill (S). $w_s/w_u$ is skilled-unskilled wage premium, K/Y is capital–output ratio and Y is output. VA is value added volume index. Robust standard error are listed in the parenthesis ***Significantly different from 0 at the 1 % level.

| Specification | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|---|---|---|---|---|---|---|
| Manufacturing (excluding industries in group 1) | | | | | | |
| ln(K/Y) | 0.017** | 0.011 | 0.014** | | | |
| (*std.*) | (0.007) | (0.008) | (0.006) | | | |
| ln(Y) | | -0.023*** | | | | |
| (*std.*) | | (0.007) | | | | |
| ln(K/VA) | | | | -0.002 | -0.008 | -0.001 |
| (*std.*) | | | | (0.004) | (0.006) | (0.003) |
| ln(VA) | | | | | -0.008 | |
| (*std.*) | | | | | (0.007) | |
| ln(Ws/Wu) | | | 0.138*** | | | 0.139*** |
| (*std.*) | | | (0.010) | | | (0.010) |
| t | 0.004*** | 0.005*** | 0.005*** | 0.005*** | 0.005*** | 0.005*** |
| (*std.*) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| R-square | 0.469 | 0.491 | 0.416 | 0.462 | 0.471 | 0.412 |
| Obs | 620 | 620 | 620 | 620 | 620 | 620 |
| Prob>F | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Nonmanufacturing (excluding industries in group 1) | | | | | | |
| ln(K/Y) | 0.061*** | 0.055*** | 0.039*** | | | |
| (*std.*) | (0.007) | (0.006) | (0.006) | | | |
| ln(Y) | | 0.052*** | | | | |
| (*std.*) | | (0.005) | | | | |
| ln(K/VA) | | | | 0.057*** | 0.069*** | 0.040*** |
| (*std.*) | | | | (0.006) | (0.005) | (0.005) |
| ln(VA) | | | | | 0.043*** | |
| (*std.*) | | | | | (0.004) | |
| ln(Ws/Wu) | | | 0.159*** | | | 0.157*** |
| (*std.*) | | | (0.010) | | | (0.010) |
| t | 0.004*** | 0.002*** | 0.005*** | 0.004*** | 0.002*** | 0.004*** |
| (*std.*) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| R-square | 0.325 | 0.231 | 0.459 | 0.317 | 0.227 | 0.450 |
| Obs | 620 | 620 | 620 | 620 | 620 | 620 |
| Prob>F | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Table 21: Regression results for manufacturing and nonmanufacturing sectors excluding the industries in group 1

Notes: Sample period is 1949-2010. The dependent variable is skilled-labour share of the wage bill (S). $w_s/w_u$ is skilled-unskilled wage premium, K/Y is capital–output ratio. VA is value added volume index. Robust standard error are listed in the parenthesis. *** denotes significantly different from 0 at the 1 % level and ** denotes significantly different from 0 at the 5 %.

| code | High-educationn industries (from high to low) | N_h/N_l | code | Low-education industries (from high to low) | N_h/N_l |
|---|---|---|---|---|---|
| 71t74 | renting of m&eq and other business activities | 0.53898735 | 64 | post and telecommunications | 0.1937005 |
| J | financial intermediation | 0.43572977 | **36t37** | **manufacturing nec; recycling** | **0.16915695** |
| 24 | chemicals and chemical products | 0.34472345 | 34t35 | transport equipment | 0.16527641 |
| **30t33** | **electrical and optical equipment** | 0.25687565 | C | mining and quarrying | 0.15018373 |
| 50 | sale maintenance and repair of motor vehicles | 0.24410074 | 29 | machinery nec | 0.13274616 |
| O | other community social and personal services | 0.23813289 | 52 | retail trade except of motor vehicles | 0.12841383 |
| 23 | coke refined petroleum and nuclear fuel | 0.23772318 | **60t63** | **transport and storage** | **0.12588585** |
| 21t22 | pulp paper paper printing and publishing | 0.20493153 | **25** | **rubber and plastics** | **0.11646072** |
| E | electricity gas and water supply | 0.20355606 | 51 | wholesale trade and commission trade | 0.10645388 |
| | | | 15t16 | food beverages and tobacco | 0.10210159 |
| | | | 27t28 | basic metals and fabricated metal | 0.09297724 |
| | | | 26 | other nonmetallic mineral | 0.0926741 |
| | | | F | construction | 0.08507863 |
| | | | **H** | **hotels and restaurants** | **0.08228209** |
| | | | 20 | wood and of wood and cork | 0.06163423 |
| | | | **AtB** | **agriculture hunting forestry and fishing** | **0.05525503** |
| | | | 17t19 | textiles textile leather and footwear | 0.04317118 |

Table 22: Skilled-to-unskilled employment ratio

Notes: The sample period is 1949 to 2010. This figure lists the industries with skilled-to-unskilled employment ratio $N_h/N_l$. The industries in group 1 are listed in bold.

## 2.4 Nonlinearity in Skilled and Unskilled Complementarity

This section focuses on investigating the nonlinearity in the complementarity between skilled and unskilled workers. As shown in Table 22, the majority of industries belonging to group 1 are characterized by relatively lower levels of education and a smaller proportion of skilled workers compared to the majority of industries in group 2. Moreover, Table 20 demonstrates that the coefficients in front of $(w_s/w_u)$ (i.e., $\gamma_{11}$) are greater for group 2 than for group 1, indicating a more substantial complementarity between skilled and unskilled workers in the industries belonging high education group (group 2) than the low education group (group 1). The categorization of group 1 and group 2 in Table 20 is determined by the threshold estimation procedure conducted with our baseline model (Model 3). However, in the robustness check section, we performed threshold estimations for alternative models (Model 2, Model 5, and Model 6), and the results indicate that the nonlinearity in the complementarity between skilled and unskilled labour across industries with different education levels remains robust to different group classifications based on alternative threshold estimations. For detailed information, please refer to Section 2.6.

To gain deeper insights into the relationship between the complementarity of the two types of workers and education levels in industries, we conduct an alternative threshold estimation following the methodology proposed by Hansen (2000), using the skilled-to-unskilled employment ratio in each industry as the threshold variable. The results of the threshold estimation are presented in Table 23. The outcomes reveal the existence of a threshold value based on the skilled-to-unskilled employment ratio, which effectively segregates the complete sample of 26 industries into two distinct groups: industries with low education characterised by a skilled-to-unskilled employment ratio below the estimated threshold of 0.1108 and industries with high education characterised by a skilled-to-unskilled employment ratio above this threshold. Specifically, for industries with a skilled-to-unskilled employment ratio larger than the threshold value, the coefficient in front of $w_s/w_u$ (i.e., $\gamma_{11}$) is approximately 0.20, which is about four times greater than the coefficient for industries with a skilled-to-unskilled employment ratio less than the threshold value, approximately 0.05. The classification of industries based on the estimated threshold value is presented in Table 24. This classification is based on the value of the aggregated skilled-to-unskilled employment ratio $(S/U)$ for each industry, as indicated in Table 41 in Appendix A. [31]

Through out Model 1 to Model 6, only our baseline model (Model 3) and Model 6 contains $(w_s/w_u)$ variable, we then re-estimate our baseline model (Model 3) and Model 6 for two groups split according to threshold estimation using $(S/U)$ as threshold variable, and the coefficient estimates obtained are presented in Table 25. The results show that in both models the $\gamma_{11}$ coefficients are positive and statistically significant in both groups, indicating a complementary relationship between skilled and unskilled labour inputs, with the elasticity of substitution between these labour inputs being less than unity. Furthermore, we observe that the coefficient $\gamma_{11}$ is approximately 0.11 for industries with relatively low education levels and about 0.15 for industries with high education levels in both models. These findings provide evidence of the disparity in the complementarity between skilled and unskilled labour in industries with varying education levels. Moreover, the results are consistent with the observations from Table 22 and Table 20, suggesting that industries with higher levels of education display a more noticeable complementarity between skilled and unskilled workers than industries with a relatively lower proportion of skilled employees.

| Threshold estimator (level = 95) | Estimates | Lower | Upper | |
|---|---|---|---|---|
| | 0.1108 | 0.1097 | 0.1115 | |
| | | | | |
| Threshold effect test | RSS | MSE | Fstat | p-value |
| Single | 1.831 | 0.001 | 1350.34 | **0.000** |
| | | | | |
| Fixed-effects (within) regression | | | | |
| ln(K/Y) | estimates | p-value | | |
| 0 | 0.0279667 | 0.000 | | |
| 1 | 0.0467715 | 0.000 | | |
| ln(Ws/Wu) | | | | |
| 0 | **0.0523841** | 0.000 | | |
| 1 | **0.1998515** | 0.000 | | |
| | | | | |
| R-squared | 0.9126 | | | |
| Obs | 1860 | | | |
| Prob>F | 0.000 | | | |

Table 23: Threshold estimation based on baseline model (Model 3)

Note: The sample period is 1949 to 2010. The threshold estimation is based on our baseline model (Model 3) with skilled-to-unskilled employment ratio as threshold variable.

[31] For each industry, we aggregated the skilled-to-unskilled employment ratio by summing the values across time for each year. Using the median value of this ratio for each year did not affect the quality results.

| High education levels S/U>0.1108 | | | Low education levels S/U<0.1108 | | |
|---|---|---|---|---|---|
| id | Code | Industry | id | Code | Industry |
| 26 | 71t74 | Renting of m&eq and other business activities | 18 | 51 | Sale maintenance and repair of motor vehicles and motorcycles; retail sale of fuel |
| 24 | J | Financial intermediation | 3 | 15t16 | Food products beverages and tobacco |
| 8 | 24 | Chemicals and chemical products | 11 | 27t28 | Basic metals and fabricated metal products |
| 13 | 30t33 | Electrical and optical equipment | 10 | 26 | Other non-metallic mineral products |
| 19 | 50 | Wholesale trade and commission trade except of motor vehicles and motorcycles | 17 | F | Construction |
| 30 | O | Other community social and personal services | 21 | H | Hotels and restaurants |
| 7 | 23 | Coke refined petroleum products and nuclear fuel | 5 | 20 | Wood and products of wood and cork |
| 6 | 21t22 | Pulp paper paper products printing and publishing | 1 | AtB | Agriculture hunting forestry and fishing |
| 16 | E | Electricity gas and water supply | 4 | 17t19 | Textiles textile products leather and footwear |
| 23 | 64 | Post and telecommunications | | | |
| 15 | 36t37 | Manufacturing nec; recycling | | | |
| 14 | 34t35 | Transport equipment | | | |
| 2 | C | Mining and quarrying | | | |
| 12 | 29 | Machinery nec | | | |
| 20 | 52 | Retail trade except of motor vehicles and motorcycles; repair of household goods | | | |
| 22 | 60t63 | Transport and storage | | | |
| 9 | 25 | Rubber and plastics products | | | |

Table 24: Industry classification based on skilled-to-unskilled employment ratio

Note: The sample period is 1949 to 2010. This table displays the industry classification based on the threshold estimation on our baseline model (Model 3).

| | Model 3 | | Model 6 | |
|---|---|---|---|---|
| Variable | High education group | Low education group | High education group | Low education group |
| ln(K/Y) | 0.0102*** | 0.0029*** | -0.0079*** | -0.0066** |
| (*std.*) | (0.0035) | (0.0037) | (0.0035) | (0.0032) |
| ln(Ws/Wu) | **0.1532***** | **0.1099***** | **0.1554***** | **0.1093***** |
| (*std.*) | (0.0070) | (0.0060) | (0.0070) | (0.0060) |
| t | 0.0049*** | 0.0033*** | 0.0052*** | 0.0035*** |
| (*std.*) | (0.0001) | (0.0001) | (0.0001) | (0.0001) |
| R-square | 0.450 | 0.750 | 0.458 | 0.745 |
| Obs | 1550 | 620 | 1550 | 620 |
| Prob>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 25: Regression results for high-education group and low-education group

Notes: Sample period is 1949-2010. The dependent variable is skilled-labour share of the wage bill (S). $w_s/w_u$ is skilled-unskilled wage premium, K/Y is capital–output ratio. VA is value added volume index. Robust standard error are listed in the parenthesis. *** denotes significantly different from 0 at the 1 % level and ** denotes significantly different from 0 at the 5 %.

## 2.5 Time Break in Capital-Skill Complementarity

In Figure 16, we aggregate the skilled-to-unskilled relative employment ratio and the capital-to-output ($K/Y$) ratio for industries in group 1 and group 2 and plot the series over the sample period. For industries in group 2, the ratio of skilled to unskilled employment and the capital to output ratio both exhibited consistent growth throughout the sample period. Nonetheless, there was a slight decline in the growth rate of the capital to output ratio after 1980. In contrast, for the industries in group 1, the capital to output ratio remained relatively flat between 1949 and 1980, and it started to decline from 1980 onwards. Additionally, the rate of increase in the skilled-to-unskilled employment ratio was lower during the period around 1980 onwards. These observations suggest the possibility of a time break in the non-linearity of the relationship between capital stock and skilled labour throughout the sample period.

Motivated by the observations presented in Figure 16, in this section, we conduct a more detailed investigation of the potential non-linearity in the capital-to-skill complementarity/substitution relationship for industries in group 1 and group 2 by re-estimating the threshold estimation using the year as the threshold variable. The results of this analysis are presented in Table 26. The findings indicate that there exists a threshold split for the period before and after 1979 for group 1, although the p-value is relatively large (about 0.2). The coefficients in front of $ln(K/Y)$, i.e., $\gamma_{1k}$, are approximately -0.06 for the period before 1980, and it increases to about -0.08 from 1980 onwards, and these estimates are statistically significant. On the other hand, for group 2, the p-value for the threshold estimation is 0.543, suggesting that there is strong evidence for no split based on the year for the industries in group 2.

Table 27 presents the estimation results for industries in group 1 for two distinct periods based on the time horizon threshold estimation. Our analysis revealed several key findings that are worth highlighting. Firstly, the models consistently show negative coefficients ($\gamma_{1k}$) in front of $ln(K/Y)$, which indicates that capital and skilled labour are relatively more substitutive in group 1. This finding aligns with the consistently negative coefficients observed in group 1 in Table 20. Moreover, the coefficient in Model 2 is not statistically significant, while they are statistically significant at the 1% level in the remaining models (Model 1 and Model 3 to Model 6). Secondly, the magnitude of the estimates for $\gamma_{1k}$ is very similar to those obtained in group 1 from Table 20. In addition, Additionally, similar to the findings in Table 20, in our baseline model (Model 3) and Model 6, the coefficient for $w_s/w_u$ ($\gamma_{11}$) is positive and statistically significant for both periods, implying a complementary relationship between skilled and unskilled labour across the entire sample period. Furthermore, the coefficient for $w_s/w_u$ is smaller from 1980 onwards for both models (Model 3 and Model 6), indicating a potentially less profound complementarity between skilled and unskilled labour. Thirdly, the coefficients in front of $Y$ (Model 2) and front of $VA$ (Model 5) are positive and significant. This finding suggests that the production function in industries in group 1 may be non-homothetic, and this trend persists throughout the entire sample period, and again, it is consistent with results in Table 20. Lastly, and most importantly, our results indicate that the capital-skill substitution for industries in group 1 is more pronounced between 1980 to 2010 than in the period before 1980. The coefficients $\gamma_{1k}$ are larger in absolute value for the period from 1980 onwards than before 1980 for all models except for Model 4 [32]. The results of our analysis, which indicate a more pronounced capital-skill substitution for industries in group 1 from 1980 onwards, are consistent with the more profound negative relationship between the capital to output ratio and skilled-to-unskilled employment ratio observed in Figure 16. Additionally, this finding aligns with the results obtained from the threshold estimation analysis presented in Table 26 for industries in group 1.

[32] Since the coefficients in Model 2 are not statistically significant, we disregard it for this analysis.

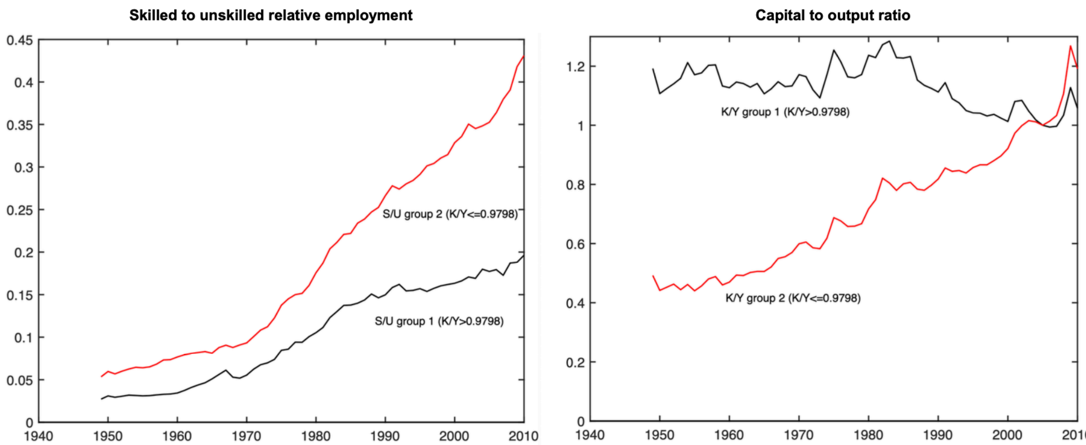


Figure 16: Skilled-to-unskilled relative employment ratio and capital-to-output ratio for group 1 and group 2

Notes: The sample period is 1949-2010. This figure plots the skilled-to-unskilled relative employment ratio and capital to output ratio for group 1 (industries with $K/Y > 0.9897$) and group 2 (industries with $K/Y <= 0.9897$).

| | Group 1 | | | | Group 2 | | | |
|---|---|---|---|---|---|---|---|---|
| Threshold estimator (level = 95) | Estimates | Lower | Upper | | Estimates | Lower | Upper | |
| | 1979 | 1976 | 1980 | | 1980 | 1978 | 1981 | |
| Threshold effect test | RSS | MSE | Fstat | P-value | RSS | MSE | Fstat | P-value |
| Single | 0.6476 | 0.0021 | 49.76 | **0.197** | 2.0321 | 0.0015 | 36.73 | **0.543** |
| Fixed-effects (within) regression | | | | | | | | |
| ln(K/Y) | estimates | P-value | | ln(K/Y) | estimates | P-value | | |
| 0 | **-0.0558492** | 0.000 | | 0 | 0.0125871 | 0.001 | | |
| 1 | **-0.080821** | 0.000 | | 1 | -0.0015471 | 0.824 | | |
| ln(Ws/Wu) | | | | ln(Ws/Wu) | | | | |
| 0 | 0.0487648 | 0.013 | | 0 | 0.141738 | 0.000 | | |
| 1 | 0.1962876 | 0.000 | | 1 | 0.1829981 | 0.000 | | |
| R-squared | 0.5017 | | | R-squared | 0.3165 | | | |
| Obs | 372 | | | Obs | 1426 | | | |
| Prob>F | 0.000 | | | Prob>F | 0.000 | | | |

Table 26: Threshold estimation based on baseline model (Model 3)

Note: The sample period is 1949 to 2010. The process of threshold estimation is founded on our baseline model (Model 3) with the year as the threshold variable. Group1 and group 2 are splited based on threshold estimation results with $K/Y$ as threshold variable. Group 1 contains the industries with the aggregated value of $K/Y$ larger than 0.9798, the rest of industries are classfied into group 2.

| Specification | Model 1 | | Model 2 | | Model 3 | |
|---|---|---|---|---|---|---|
| | 1949-1979 | 1980-2010 | 1949-1979 | 1980-2010 | 1949-1979 | 1980-2010 |
| ln(K/Y) | -0.0630*** | -0.0720*** | 0.0100 | -0.0018 | -0.0537*** | -0.0588*** |
| (*std.*) | (0.0145) | (0.0002) | (0.0163) | (0.0164) | (0.0139) | (0.0143) |
| ln(Y) | | | 0.0619*** | 0.0586*** | | |
| (*std.*) | | | (0.0079) | (0.0079) | | |
| ln(K/VA) | | | | | | |
| (*std.*) | | | | | | |
| ln(VA) | | | | | | |
| (*std.*) | | | | | | |
| ln(Ws/Wu) | | | | | 0.1136*** | 0.0990*** |
| (*std.*) | | | | | (0.0193) | (0.0217) |
| t | 0.0048*** | 0.0049*** | 0.0025*** | 0.0027*** | 0.0049*** | 0.0049*** |
| (*std.*) | (0.0002) | (0.0002) | (0.0003) | (0.0003) | (0.0001) | (0.0002) |
| R-square | 0.4365 | 0.3915 | 0.2735 | 0.2630 | 0.4975 | 0.4483 |
| Obs. | 341 | 341 | 341 | 341 | 341 | 341 |
| Prob>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Specification | Model 4 | | Model 5 | | Model 6 | |
| | 1949-1979 | 1980-2010 | 1949-1979 | 1980-2010 | 1949-1979 | 1980-2010 |
| ln(K/Y) | | | | | | |
| (*std.*) | | | | | | |
| ln(Y) | | | | | | |
| (*std.*) | | | | | | |
| ln(K/VA) | -0.0680*** | -0.0678*** | -0.0280*** | -0.0310*** | -0.0652*** | -0.0653*** |
| (*std.*) | (0.0043) | (0.0044) | (0.0095) | (0.0096) | (0.0041) | (0.0042) |
| ln(VA) | | | 0.0325*** | 0.0300*** | | |
| (*std.*) | | | (0.0069) | (0.0070) | | |
| ln(Ws/Wu) | | | | | 0.0967*** | 0.0886*** |
| (*std.*) | | | | | (0.0149) | (0.0167) |
| t | 0.0043*** | 0.0043*** | 0.0031*** | 0.0033*** | 0.0044*** | 0.0043*** |
| (*std.*) | (0.0001) | (0.0001) | (0.0003) | (0.0003) | (0.0001) | (0.0001) |
| R-square | 0.1700 | 0.1607 | 0.1402 | 0.1395 | 0.2144 | 0.1928 |
| Obs. | 341 | 341 | 341 | 341 | 341 | 341 |
| Prob>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 27: Regression results for group 1 in periods 1949 to 1979 and 1980 to 2010.

Notes: Sample period is 1949-2010. The dependent variable is skilled-labour share of the wage bill (S). $w_s/w_u$ is skilled-unskilled wage premium, K/Y is capital–output ratio and Y is output. VA is value added volume index. Robust standard error are listed in the parenthesis. *** denotes significantly different from 0 at the 1 % level.

The regression results for group 2 for both distinct periods are summarized in Table 39 in the Appendix A. Again, all estimates are statistically significant, and the magnitude of estimates is very similar to the estimates obtained in group 2 from Table 20. Similar to the results of the time-horizon threshold estimation, there is no apparent non-linearity in capital-skill complementarity for periods before and after 1980. Specifically, Model 1, Model 2, and Model 5 show that the coefficients $\gamma_{1k}$ are smaller in absolute value during the period from 1980 to 2010 than the period before 1980. On the other hand, in our baseline model (Model 3), Model 4, and Model 6, the coefficients $\gamma_{1k}$ are larger for periods after 1980, with larger $\gamma_{1k}$. However, the differences in $\gamma_{1k}$ between the two periods are minimal across all models, indicating that there is no apparent non-linearity in capital-skill complementarity for both periods.

## 2.6 Robustness Check

This section is dedicated to investigating the robustness of the results. To achieve this, we employ several alternative models to conduct threshold estimations, which enable us to scrutinize the non-linearity in capital-skill complementarity across industries. We then perform a Chow test to check for structural breaks. Furthermore, we examine the robustness of the non-linearity in skilled and unskilled complementarity using alternative threshold estimations.

### 2.6.1 Threshold Estimations Using Alternative Models

In this section, we investigate the robustness of our threshold results by considering several alternative specifications in addition to our baseline model (Model 3). Specifically, we analyse Model 2, Model 5, and Model 6. Due to the potential inaccuracy of results from single-variable estimations, we did not perform threshold estimations based on Model 1 and Model 4. The results of these analyses are presented in Table 28. The key finding is that regardless of the model specifications employed, the outcomes imply a straightforward division based on the ratio of capital to output. We obtain capital to output ratio threshold values of 0.8185, 0.8640, and 0.8721 for Model 2, Model 5, and Model 6, respectively. Table 40 (in Appendix A) displays the classification of industries based on the threshold estimation results obtained from the alternative models. As before, the industries listed in bold represent manufacturing industries.

For the classification based on Model 2, we observe that the high capital to output ratio group 1 now comprises ten industries, five of which are manufacturing industries and the remaining are non-manufacturing. Conversely, the low capital to output ratio group 2 includes 16 industries, with eight of them being manufacturing industries. Notably, industries with ID numbers 10, 11, 29, and 30 move from group 2 to group 1 when we use Model 2 to conduct the threshold estimation compared using the benchmark Model 3. The classification based on the threshold estimation obtained from Model 5 and Model 6 is the same, as these models' threshold values are very similar. Compared to the classification in Model 2, only the industry with id number 11 moves out of group 1. To assess the robustness of our results for the high capital to output ratio and low-intensity groups, we re-estimated Model 1 through Model 6 for group 1 (industries with high capital to output ratio) and group 2 (industries with low capital to output ratio) using the threshold estimations obtained from the alternative model specifications. The results of the estimations, with groups classified based on Model 2, are presented in Table 29, while the results with groups classified based on Model 5 and Model 6 are shown in Table 30.

The findings indicate that the coefficient estimates for both classifications bear a remarkable resemblance to those provided in Table 20. Notably, in Table 29, the estimates for $\gamma_{1k}$ are consistently negative for group 1 and positive for group 2 across all six models. In group 1, the coefficients in all models but Model 2 and Model 6 are statistically significant at the 1% level; the coefficient is significant at 5% level for Model 6, and it is strongly insignificant in Model 2. And coefficients in group 2 are significant at 1% level across all models. Furthermore, Table 30 reveals that for all six models, $\gamma_{1k}$ estimates are consistently positive for group 2, while for group 1, they are negative for all models except Model 2 where it is positive but strongly statistically insignificant. The coefficients are significant at the 1% level in the rest of the models for both groups, except for Model 6 in group 1, which is significant at the 10% level. These findings reinforce our primary result, which indicates that in industries with high capital to output ratios, capital and skill are substitutable factors of production, while in industries with low capital to output ratios, capital and skill are complementary. These robustness checks provide additional evidence supporting our key findings and underscore the reliability and validity of our results. In addition, the significance of the findings suggests that the threshold estimation derived from our baseline model (Model 3) outperforms the estimations obtained from Model 2, Model 5, and Model 6.

| | Model 2 | | | |
|---|---|---|---|---|
| Threshold estimator (level = 95) | Estimates | Lower | Upper | |
| | 0.8185 | 0.8068 | 0.8200 | |
| Threshold effect test | RSS | MSE | Fstat | P-value |
| Single | 3.1011 | 0.002 | 239.22 | 0.007 |
| Fixed-effects (within) regression | | | | |
| ln(K/Y) | estimates | P-value | | |
| 0 | 0.1028105 | 0.000 | | |
| 1 | 0.2491137 | 0.000 | | |
| ln(Y) | | | | |
| 0 | 0.0866742 | 0.000 | | |
| 1 | 0.1071001 | 0.000 | | |
| R-squared | 0.1427 | | | |
| Obs | 1612 | | | |
| Prob>F | 0.000 | | | |

| | Model 5 | | | | | Model 6 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Threshold estimator (level = 95) | Estimates | Lower | Upper | | Threshold estimator (level = 95) | Estimates | Lower | Upper | |
| | 0.864 | 0.8565 | 0.8681 | | | 0.8721 | 0.8652 | 0.874 | |
| Threshold effect test | RSS | MSE | Fstat | P-value | Threshold effect test | RSS | MSE | Fstat | P-value |
| Single | 3.1067 | 0.002 | 210 | 0.033 | Single | 6.3456 | 0.0041 | 654.22 | 0.000 |
| Fixed-effects (within) regression | | | | | Fixed-effects (within) regression | | | | |
| ln(K/VA) | estimates | P-value | | | ln(K/VA) | estimates | P-value | | |
| 0 | 0.0793396 | 0.000 | | | 0 | 0.1323592 | 0.000 | | |
| 1 | -0.0024052 | 0.000 | | | 1 | -0.132715 | 0.000 | | |
| ln(VA) | | | | | ln(Ws/Wu) | | | | |
| 0 | 0.0855148 | 0.000 | | | 0 | 0.0373797 | 0.002 | | |
| 1 | 0.0944679 | 0.000 | | | 1 | 0.1352968 | 0.000 | | |
| R-squared | 0.0976 | | | | R-squared | 0.0419 | | | |
| Obs | 1612 | | | | Obs | 1612 | | | |
| Prob>F | 0.000 | | | | Prob>F | 0.000 | | | |

Table 28: Threshold estimations based on alternative models (Model 2, Model 5 and Model 6)

Note: The sample period is 1949 to 2010. The threshold estimations are based on alternative models (Model 2, Model 5 and Model 6) with capital to output ratio ($K/Y$) as threshold variable.

| Group 1 K/Y>0.8185 | | | | | | |
|---|---|---|---|---|---|---|
| Specification | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
| ln(K/Y) | -0.0445*** | -0.0053 | -0.0264*** | | | |
| (*std.*) | (0.0073) | (0.0078) | (0.0067) | | | |
| ln(Y) | | 0.0475*** | | | | |
| (*std.*) | | (0.0047) | | | | |
| ln(K/VA) | | | | -0.0503*** | -0.0130** | -0.0444*** |
| (*std.*) | | | | (0.0029) | (0.0052) | (0.0026) |
| ln(VA) | | | | | 0.0345*** | |
| (*std.*) | | | | | (0.0041) | |
| ln(Ws/Wu) | | | **0.1298*** ** | | | **0.1124*** ** |
| (*std.*) | | | (0.0102) | | | (0.0086) |
| t | 0.0048*** | 0.0031*** | 0.0048*** | 0.0045*** | 0.0033*** | 0.0046*** |
| (*std.*) | (0.0001) | (0.0002) | (0.0001) | (0.0001) | (0.0002) | (0.0001) |
| R-square | 0.4883 | 0.3670 | 0.5149 | 0.2863 | 0.2352 | 0.3318 |
| Obs | 744 | 744 | 744 | 744 | 744 | 744 |
| Porb.>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Group 2 K/Y<0.8185 | | | | | | |
| Specification | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
| ln(K/Y) | 0.0465*** | 0.0471*** | 0.0207*** | | | |
| (*std.*) | (0.0062) | (0.0061) | (0.0057) | | | |
| ln(Y) | | 0.0261*** | | | | |
| (*std.*) | | (0.0046) | | | | |
| ln(K/VA) | | | | 0.0467*** | 0.0533*** | 0.0295 |
| (*std.*) | | | | (0.0047) | (0.0048) | (0.0043) |
| ln(VA) | | | | | 0.0201*** | |
| (*std.*) | | | | | (0.0043) | |
| ln(Ws/Wu) | | | **0.1499*** ** | | | **0.1436*** ** |
| (*std.*) | | | (0.0094) | | | (0.0092) |
| t | 0.0041*** | 0.0034*** | 0.0048*** | 0.0040*** | 0.0033*** | 0.0046*** |
| (*std.*) | (0.0002) | (0.0002) | (0.0001) | (0.0001) | (0.0002) | (0.0001) |
| R-square | 0.3645 | 0.3080 | 0.4102 | 0.3407 | 0.3001 | 0.3860 |
| Obs | 868 | 868 | 868 | 868 | 868 | 868 |
| Porb.>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 29: Regression results for group 1 and group 2 based on thereshold estimation of Model 2

Notes: Sample period is 1949-2010. The dependent variable is skilled-labour share of the wage bill (S). $w_s/w_u$ is skilled-unskilled wage premium, K/Y is capital–output ratio and Y is output. VA is value added volume index. Robust standard error are listed in the parenthesis. *** denotes significantly different from 0 at the 1 % level. ** denotes significant at the 5 % level.

| Group 1 K/Y>0.8721 | | | | | | |
|---|---|---|---|---|---|---|
| Spesification | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
| ln(K/Y) | -0.0307*** | 0.0035 | -0.0142*** | | | |
| (*std.*) | (0.0080) | (0.0083) | (0.0073) | | | |
| ln(Y) | | 0.0460*** | | | | |
| (*std.*) | | (0.0049) | | | | |
| ln(K/VA) | | | | -0.0473*** | -0.0100* | -0.0421*** |
| (*std.*) | | | | (0.0030) | 0.0054 | (0.0028) |
| ln(VA) | | | | | 0.0349*** | |
| (*std.*) | | | | | (0.0043) | |
| ln(Ws/Wu) | | | **0.1289*** ** | | | **0.1117*** ** |
| (*std.*) | | | (0.0105) | | | (0.0090) |
| t | 0.0049*** | 0.0033*** | 0.0049*** | 0.0046*** | 0.0033*** | 0.0047*** |
| (*std.*) | (0.0001) | (0.0002) | (0.0001) | (0.0001) | (0.0002) | (0.0001) |
| R-square | 0.5088 | 0.3963 | 0.5326 | 0.3171 | 0.2631 | 0.3619 |
| Obs | 682 | 682 | 682 | 682 | 682 | 682 |
| Porb.>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Group 2 K/Y<0.8640 | | | | | | |
| Spesification | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
| ln(K/Y) | 0.0465*** | 0.0493*** | 0.0246*** | | | |
| (*std.*) | (0.0061) | (0.0059) | (0.0056) | | | |
| ln(Y) | | 0.0323*** | | | | |
| (*std.*) | | (0.0044) | | | | |
| ln(K/VA) | | | | 0.0470*** | 0.0555*** | 0.0302*** |
| (*std.*) | | | | (0.0047) | (0.0048) | (0.0043) |
| ln(VA) | | | | | 0.0272*** | |
| (*std.*) | | | | | (0.0041) | |
| ln(Ws/Wu) | | | **0.1534*** ** | | | **0.1491*** ** |
| (*std.*) | | | (0.0093) | | | (0.0090) |
| t | 0.0039*** | 0.0030*** | 0.0046*** | 0.0039*** | 0.0030*** | 0.0044*** |
| (*std.*) | (0.0002) | (0.0002) | (0.0001) | (0.0001) | (0.0002) | (0.0001) |
| R-square | 0.3448 | 0.2736 | 0.3914 | 0.3248 | 0.2669 | 0.3696 |
| Obs | 930 | 930 | 930 | 930 | 930 | 930 |
| Porb.>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 30: Regression results for group 1 and group 2 based on thereshold estimation of Model 5 and Model 6

Notes: Sample period is 1949-2010. The dependent variable is skilled-labour share of the wage bill (S). $w_s/w_u$ is skilled-unskilled wage premium, K/Y is capital–output ratio and Y is output. VA is value added volume index. Robust standard error are listed in the parenthesis. *** denotes significantly different from 0 at the 1 % level. * denotes significant at the 10 % level.

### 2.6.2 Chow Test for Structural Change

Secondly, we perform Chow tests for our six models to examine whether our arbitrarily chosen subsamples (manufacturing and non-manufacturing) and endogenously determined groups (Group 1 and Group 2) are structurally different. Table 31 displays the outcomes of the structural change test conducted for both the manufacturing and non-manufacturing sectors. The test rejects no structural change for all six models. Moreover, the coefficients $\gamma_{1k}$ for the manufacturing sector are negative across all six models and are statistically significant when we use Model 1, Model 3, Model 4, and Model 6. At the same time, it is insignificant when we use Model 2 and Model 5. Conversely, the coefficients for the non-manufacturing industries are statistically positive for all six models. Upon examining the results presented in Table 17, it is noteworthy that both Model 2 and Model 5 perform relatively poorly compared to the other models in both sectors, with the lowest adjusted R-squared values.

We have also tested the structural change between endogenously determined groups using Hansen (2000) methodology based on our baseline model (Model 3) and alternative models (Model 2, Model 5 and Model 6). Table 32 displays the outcomes of a structural change test conducted on two groups, namely Group 1 and Group 2. The results are presented in three panels. The first panel depicts the test results when the classification is determined based on the threshold estimation of the baseline model, which is Model 3. The second panel presents the test results when the classification is based on Model 2. The third panel illustrates the test results when the classification is based on Model 5 and Model 6. In all panels, the Chow test rejects no structural change for all models. Furthermore, the coefficients $\gamma_{1k}$ exhibit consistent statistical significance and negativity for group 1 (i.e., industries with high capital to output ratio) and statistical significance and positive coefficients for group 2 (i.e., industries with low capital to output ratio). These findings align with our baseline results and provide robust evidence of non-linearity in the capital-skill complementarity between our endogenously determined groups.

| Specification | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|---|---|---|---|---|---|---|
| Manufacturing & Non-manufacturing | | | | | | |
| Test statistic | 47.750*** | 24.710*** | 38.980*** | 142.010*** | 66.390*** | 127.890*** |
| P-value | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| ln(K/Y) for Manufacturing | -0.009* | -0.002 | -0.009* | -0.026*** | -0.005 | -0.024*** |
| P-value | (0.083) | (0.747) | (0.067) | (0.000) | (0.183) | (0.000) |
| ln(K/Y) for Non-manufacturing | 0.033*** | 0.035*** | 0.025*** | 0.030*** | 0.037*** | 0.024*** |
| P-value | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |

Table 31: Chow tests testing structural change between manufacturing and nonmanufacturing sectors

Notes: The sample period is 1949-2010. This table shows the test results for structural change in manufacturing and nonmanufacturing sectors. *** denotes significantly different from 0 at the 1 % level; ** denotes significant at the 5 % level, and * denotes significant at the 10 % level.

### 2.6.3 LR-Tests for Non-nested Linear Regression Models

A careful look at the results from Table 20, Table 27, and Table 39 (in Appendix A), the results for group 1 whose that Model 3 performs best fit than the rest of the models with the largest adjusted R-squared. However, the results for group 2 show that Model 6 has the best fit among all models. We employed a likelihood ratio test to test for non-nested linear regression models to compare among models. [33] The likelihood ratio test compares the fit of the two models by calculating the difference between their log-likelihoods. This difference is known as the likelihood ratio statistic, and it is calculated as the log-likelihood of the Model $i$ minus the log-likelihood of the Model $j$. A small p-value indicates that the difference in fit between the two models is statistically significant and that Model $i$ is a better fit than Model $j$.

The likelihood ratio test was conducted on the industries belonging to group 1 and group 2, and the test outcomes are presented in Table 33. The results for group 1 reveal that the test repudiates Model 2, Model 5, and Model 6, which aligns with our assertion that our baseline model (Model 3) dominates the rest of the

[33] Notice that out of six models, Model 2, Model 3, Model 5, and Model 6 are non-nested.

| Group 1 & Group 2 (Baseline Model) | | | | | | |
|---|---|---|---|---|---|---|
| Test statistic | 45.930*** | 13.440*** | 36.220*** | 198.760*** | 88.920*** | 206.850*** |
| P-value | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| ln(K/Y) for Group 1 | -0.068*** | -0.025* | -0.056*** | -0.064*** | -0.053*** | -0.060*** |
| P-value | (0.000) | (0.077) | (0.000) | (0.000) | (0.000) | (0.000) |
| ln(K/Y) for Group 2 | 0.026*** | 0.028*** | 0.020*** | 0.018*** | 0.022*** | 0.015*** |
| P-value | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| **Group 1 & Group 2 (Model 2)** | | | | | | |
| Test statistic | 75.960*** | 51.970*** | 39.030*** | 310.350*** | 183.800*** | 249.170*** |
| P-value | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| ln(K/Y) for Group 1 | -0.043*** | -0.030*** | -0.028*** | -0.050*** | -0.034*** | -0.044*** |
| P-value | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| ln(K/Y) for Group 2 | 0.027*** | 0.032*** | 0.018*** | 0.033*** | 0.038*** | 0.026*** |
| P-value | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| **Group 1 & Group 2 (Model 5 & Model 6)** | | | | | | |
| Test statistic | 30.830*** | 20.270*** | 11.870*** | 229.820*** | 119.580*** | 187.840*** |
| P-value | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| ln(K/Y) for Group 1 | -0.027*** | -0.014*** | -0.014* | -0.048*** | -0.027*** | -0.042*** |
| P-value | (0.000) | (0.000) | (0.076) | (0.000) | (0.000) | (0.000) |
| ln(K/Y) for Group 2 | 0.022*** | 0.028*** | 0.014*** | 0.028*** | 0.035*** | 0.021*** |
| P-value | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |

Table 32: Chow tests testing structural change between group 1 and group 2 based on various threshold estimations

Notes: The sample period is 1949-2010. The table displays the outcomes of structural change tests conducted on group 1 and group 2. The first panel depicts the test results when the classification is determined based on the threshold estimation of the baseline model, which is Model 3. The second panel presents the test results when the classification is based on Model 2. The third panel illustrates the test results when the classification is based on Model 5 and Model 6. *** denotes significantly different from 0 at the 1 % level; ** denotes significant at the 5 % level, and * denotes significant at the 10 % level.

models. However, the test results for group 2 contradict our hypothesis that the test indicates Model 5 as the superior non-nested model.

| Group 1 K/Y>0.9897 | Model 3 v.s. Model 2 | Model 3 v.s. Model 5 | Model 3 v.s. Model 6 |
|---|---|---|---|
| Test Statistic | 24.96 | 160.05 | 181.18 |
| P-value | 0.000 | 0.000 | 0.000 |
| | | | |
| Group 2 K/Y<=0.9897 | Model 5 v.s. Model 2 | Model 5 v.s. Model 3 | Model 5 v.s. Model 6 |
| Test Statistic | 5.21 | 347.35 | 322.180 |
| P-value | 0.023 | 0.000 | 0.000 |

Table 33: Likelihood ratio test for non-nested models

Notes: Models 2,3,5, and Model 6 are non-nested models. The likelihood ratio statistic is calculated by, for example, for Model 3 v.s. Model 2, the log-likelihood of Model 3 minums the log-likelihood of Model 2. A small p-vale indicates that the difference in fit between the two models is statistically significant and that Model 3 is better than Model 2.

### 2.6.4 Nonlinearity in Skilled and Unskilled Complementarity under Alternative Threshold Estimations

In this section, we aim to investigate the robustness of the nonlinearity in skilled and unskilled complementarity across industries with different education levels under alternative threshold estimations.

In section 2.6.1, we conducted threshold estimations based on Model 2, Model 5, and Model 6 using the capital to output ratio as the threshold variable. Based on the estimation outcomes, industries were re-classified, and the new classifications are presented in Table 40. Furthermore, Table 29 and Table 30 show the estimation results from Model 1 to Model 6 based on the updated classification of industries.

The outcomes from both tables (Table 29 and Table 30) indicate that the coefficient in front of $(w_s/w_u)$ (i.e., $\gamma_{11}$) is positive and statistically significant. Moreover, the coefficient $\gamma_{11}$ is larger in group 2 than in group 1. Specifically, in Table 29, the $\gamma_{11}$ coefficient for group 1 (industries with a capital to output ratio larger than the threshold value) is approximately 0.13 and 0.11 for Model 3 and Model 6, respectively. Meanwhile, for group 2 (industries with a capital to output ratio less than the threshold value), the $\gamma_{11}$ coefficients are about 0.15 in Model 3 and 0.14 in Model 6. Similarly, in Table 30, the coefficients $\gamma_{11}$ for group 1 are about 0.13 and 0.11 for the two models and approximately 0.15 for group 2.

To assess the education levels of employees in the two groups based on alternative threshold estimations, we calculated the median of aggregated skilled-to-unskilled employment ratio for each group under each classification, and the results are presented in Table 34. The top panel of the table shows the outcomes for groups based on the threshold estimation of Model 2. In group 1, the median skilled-to-unskilled employment ratio is approximately 0.12, while the median for industries in group 2 is around 0.16. These findings suggest that the employees in the industries belonging to group 1 have relatively lower education levels than those in the industries belonging to group 2. The table's bottom panel displays the outcomes for groups based on the threshold estimation of Model 5 and Model 6. Like the top panel, the results demonstrate a notable resemblance, where employees in group 1 have relatively lower education levels than those in group 2. Specifically, the median value of the skilled-to-unskilled employment ratio is about 0.13 and 0.15 in group 1 and group 2, respectively.

These results suggest that the nonlinearity in skilled and unskilled complementarity across industries with different education levels is robust to different group classifications based on alternative threshold estimations.

That is, an industrial group with higher levels of education display a more noticeable complementarity between skilled and unskilled workers than an industry group with a relatively lower proportion of skilled employees.

| Model 2 | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | Group 1 K/Y>0.8185 | | | | Group 2 K/Y<0.8185 | |
| id | code | industry | Median (S/U) | | code | industry | Median (S/U) |
| 1 | AtB | agriculture hunting forestry and fishing | 0.0553 | 2 | C | mining and quarrying | 0.1502 |
| 3 | 15t16 | food beverages and tobacco | 0.1021 | 4 | 17t19 | textiles textile leather and footwear | 0.0432 |
| 7 | 23 | coke refined petroleum and nuclear fuel | 0.2377 | 5 | 20 | wood and of wood and cork | 0.0616 |
| 9 | 25 | rubber and plastics | 0.1165 | 6 | 21t22 | pulp paper paper printing and publishing | 0.2049 |
| 10 | 26 | other nonmetallic mineral | 0.0927 | 8 | 24 | chemicals and chemical products | 0.3447 |
| 11 | 27t28 | basic metals and fabricated metal | 0.0930 | 12 | 29 | machinery nec | 0.1327 |
| 13 | 30t33 | electrical and optical equipment | 0.2569 | 14 | 34t35 | transport equipment | 0.1653 |
| 15 | 36t37 | manufacturing nec; recycling | 0.1692 | 16 | E | electricity gas and water supply | 0.2036 |
| 19 | 50 | sale maintenance and repair of motor vehicles | 0.2441 | 17 | F | construction | 0.0851 |
| 21 | H | hotels and restaurants | 0.0823 | 18 | 51 | wholesale trade and commission trade | 0.1065 |
| 22 | 60t63 | transport and storage | 0.1259 | 20 | 52 | retail trade except of motor vehicles | 0.1284 |
| 30 | O | other community social and personal services | 0.2381 | 23 | 64 | post and telecommunications | 0.1937 |
| | | **median** | **0.1212** | 24 | J | financial intermediation | 0.4357 |
| | | | | 26 | 71t74 | renting of m&eq and other business activities | 0.5390 |
| | | | | | | **median** | **0.1577** |

| Model 5 & Model 6 | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | Group 1 K/Y>0.8721 | | | | Group 2 K/Y<0.8640 | |
| id | code | industry | Median (S/U) | | code | industry | Median (S/U) |
| 1 | AtB | agriculture hunting forestry and fishing | 0.0553 | 2 | C | mining and quarrying | 0.1502 |
| 3 | 15t16 | food beverages and tobacco | 0.1021 | 4 | 17t19 | textiles textile leather and footwear | 0.0432 |
| 7 | 23 | coke refined petroleum and nuclear fuel | 0.2377 | 5 | 20 | wood and of wood and cork | 0.0616 |
| 9 | 25 | rubber and plastics | 0.1165 | 6 | 21t22 | pulp paper paper printing and publishing | 0.2049 |
| 10 | 26 | other nonmetallic mineral | 0.0927 | 8 | 24 | chemicals and chemical products | 0.3447 |
| 13 | 30t33 | electrical and optical equipment | 0.2569 | 11 | 27t28 | basic metals and fabricated metal | 0.0930 |
| 15 | 36t37 | manufacturing nec; recycling | 0.1692 | 12 | 29 | machinery nec | 0.1327 |
| 19 | 50 | sale maintenance and repair of motor vehicles | 0.2441 | 14 | 34t35 | transport equipment | 0.1653 |
| 21 | H | hotels and restaurants | 0.0823 | 16 | E | electricity gas and water supply | 0.2036 |
| 22 | 60t63 | transport and storage | 0.1259 | 17 | F | construction | 0.0851 |
| 30 | O | other community social and personal services | 0.2381 | 18 | 51 | wholesale trade and commission trade | 0.1065 |
| | | **median** | **0.1259** | 20 | 52 | retail trade except of motor vehicles | 0.1284 |
| | | | | 23 | 64 | post and telecommunications | 0.1937 |
| | | | | 24 | J | financial intermediation | 0.4357 |
| | | | | 26 | 71t74 | renting of m&eq and other business activities | 0.5390 |
| | | | | | | **median** | **0.1502** |

Table 34: Median of skilled-to-unskilled employment ratio

Note: The sample period is 1949 to 2010. This table displays the median of the skilled-to-unskilled employment ratio for industries belonging to group 1 and group 2. The group classification is based on the threshold estimation of Model 2, Model 5 and Model 6.

### 2.6.5 Time Break

Finally, we assess the reliability of our findings that the substitution of capital for skilled labour in industries in group 1 is more pronounced in the period after 1980 than before. To achieve this, we create a dummy variable that takes on a value of unity for the period after 1980 and zero otherwise. By multiplying this dummy variable by the logarithm of the ratio of capital-to-output ($ln(K/Y)$) and the logarithm of the ratio of capital to value-added ($ln(K/VA)$), we can investigate whether the coefficients in front of these variables differ before and after 1980. We then re-estimate Model 1 to Model 6 using these modified variables. The results of these robustness tests are presented in Table 35.

In general, including the dummy variables does not alter the main findings; the results reported in Table 20 for group 1 remain robust. Specifically, the coefficients in front of the logarithm of the capital-output ratio ($ln(K/Y)$) and the logarithm of the capital-value added ratio ($ln(K/VA)$) (i.e., $\gamma_{1k}$) continue to be consistently negative and statistically significant for all models except for Model 2, indicating that capital and skilled labour are substitutes. Furthermore, the coefficients in front of the logarithm of the ratio of skilled-to-unskilled wages ($ln(w_s/w_u)$) (i.e., $\gamma_{11}$) remain positive and statistically significant, indicating that skilled and unskilled labour are complementary. Additionally, the coefficients in front of the logarithm of output ($ln(Y)$) and the logarithm of value-added ($ln(VA)$) continue to be statistically significant and positive, suggesting that the production function is non-homothetic.

To investigate the time break in the effect of the logarithm of the capital-output ratio ($ln(K/Y)$) on variable the dependent variable $S$, we examine the coefficients of the interaction terms between $ln(K/Y)$ or $ln(K/VA)$ and the time dummy. Only Model 3 shows positive but insignificantly estimated coefficients in front of the interaction term out of the six models. For the other five models, the coefficients are negative and strongly significant. These findings suggest that the negative relationship between skilled-labour share of the wage bill ($S$) and the log of capital-output ratio $ln(K/Y)$ is more pronounced after 1980 than before, indicating that capital-skill substitution is more noticeable in the later period.

| Specification | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|---|---|---|---|---|---|---|
| 1980 | 0.0451*** | 0.0580*** | 0.0318*** | 0.0437*** | 0.0505*** | 0.0328*** |
| (*std.*) | (0.0111) | (0.0102) | (0.0110) | (0.0084) | (0.0082) | (0.0082) |
| ln(K/Y) | **-0.0634******* | 0.0182 | **-0.0603******* | **-0.0679******* | | |
| (*std.*) | (0.0143) | (0.0160) | (0.0138) | (0.0041) | | |
| ln(K/Y)*1980 | **-0.0124******* | **-0.0720******* | 0.0172 | **-0.0031******* | | |
| (*std.*) | (0.0231) | (0.0221) | (0.0228) | (0.0051) | | |
| ln(Y) | | 0.0643*** | | | | |
| (*std.*) | | (0.0073) | | | | |
| ln(K/VA) | | | | | **-0.0247*** ** | **-0.0658*** ** |
| (*std.*) | | | | | (0.0092) | (0.0040) |
| ln(K/VA)*1980 | | | | | **-0.0132******* | 0.0046 |
| (*std.*) | | | | | (0.0053) | (0.0050) |
| ln(VA) | | | | | 0.0344*** | |
| (*std.*) | | | | | (0.0066) | |
| ln(Ws/Wu) | | | 0.1041*** | | | 0.0892*** |
| (*std.*) | | | (0.0186) | | | (0.0144) |
| t | 0.0037*** | 0.0013*** | 0.0040*** | 0.0032*** | 0.0020*** | 0.0035*** |
| (*std.*) | (0.0003) | (0.0004) | (0.0003) | (0.0002) | (0.0003) | (0.0002) |
| R-square | 0.434 | 0.266 | 0.497 | 0.182 | 0.1468 | 0.229 |
| Obs. | 372 | 372 | 372 | 372 | 372 | 372 |
| Prob>F | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Table 35: Time break analysis

Notes: The sample period is 1949-2010. The dependent variable is the skilled-labour share of the wage bill (S). $w_s/w_u$ is skilled-unskilled wage premium, K/Y is capital–output ratio, and Y is output. VA is a value-added volume index. 1980 is a dummy variable equal to one if the period is from 1980 onward. Robust standard errors are listed in the parenthesis. *** denotes significantly different from 0 at the 1 % level. * denotes significant at the 10 % level.

## 2.7 Discussion

Our main results can be summarized as follows: First, when we arbitrarily divide the entire sample into manufacturing and nonmanufacturing sectors, the results indicate that manufacturing industries exhibit capital-skill substitution while nonmanufacturing industries exhibit capital-skill complementarity. However, when we employ Hansen (2000) data-sorting methodology, we find evidence in favour of non-linearity in capital-skill complementarity and the presence of two industry groups endogenously determined based on a capital to output ratio. In addition, we find most of the industries in manufacturing sectors belong to the relatively low capital to output ratio group. And when we remove the industries in which the capital to output ratio is higher than the threshold value from the manufacturing sector, the results suggest that we cannot reject the null hypothesis of capital-skill complementarity. Therefore, arbitrarily splitting industries into manufacturing and nonmanufacturing sectors can lead to misleading results. This could be due to the sensitivity of fixed effect estimation to the outliers and influential observations. The existence of capital-skill complementarity in manufacturing sectors which is after refining is qualitatively consistent with existing literature which provides evidence of capital-skill complementarity in the manufacturing sector in the U.S. economy (for example, see Goldin and Katz (1998), Berman et al. (1994)).

Second, when we apply the estimation on to two groups which are endogenously determined, we find in the industry groups with high capital to output ratio, the null hypothesis of capital-skill complementarity hypothesis is rejected, but is not rejected in the industry group with low capital to output ratio. And we also find that the industries belonging to the low capital to output ratio group have relatively higher average education levels than the industries belonging to the high capital to output ratio group.

These results make sense because industries with relatively high capital to output ratios are those whose production processes are less sophisticated and may rely more on capital-intensive technologies. These industries predominantly include the production of rubber and plastics, hotels and restaurants, transport and storage, agriculture, hunting, and optical equipment, as outlined in Table 19. When we employ the threshold estimation on alternative models (Model 2, Model 5 and Model 6), the industries of food, beverages and tobacco, coke refined petroleum, nonmetallic mineral, basic metals, maintenance, and personal services are included in the high capital to output ratio industries (Table 40). These industries can be operated by less-skilled workers, which explains why the average education levels of their workforce tend to be lower and also explains the substitution between capital and skilled labour in those industries. Conversely, industries with a relatively low capital to output ratio are generally associated with production processes that require more sophisticated skills and thus require a higher level of education among their workforce. This leads to the accumulation of capital and necessitates the employment of more highly skilled labour to operate and manage machinery and equipment. As a result, a complementary relationship exists between capital and skilled labour. In summary, technological advancements are associated with an increased demand for skilled workers in industries characterized by lower capital-output ratios, and a heightened demand for unskilled workers in industries exhibiting relatively higher capital-output ratios. The results of our study add to the body of evidence supporting the concept of labour market polarization, which suggests that technology is contributing to a growing polarization in the labour market, with employment opportunities becoming concentrated at the higher and lower ends of the skill spectrum (see, for example, Autor et al. (2006), Autor and Dorn (2013), Vom Lehn (2020)).

The third finding of our study indicates that skilled and unskilled workers are complementary for both high and low capital to output ratio groups. This finding is consistent with the research conducted by Goldin and Katz (1998) and Papageorgiou and Chmelarova (2005). Goldin and Katz analyse industry-level panel data from 1909 and 1919 and found statistically significant positive coefficients in front of the natural logarithm of the ratio of skilled-to-unskilled wages ($ln(w_s/w_u)$), which indicates skilled-unskilled complementarity. Papageorgiou and Chmelarova conducted a cross-country analysis and found that skilled and unskilled workers are complementary in both OECD and non-OECD countries. These results hold when samples are reclassified into three regimes based on literacy rate and initial per-capita output thresholds. Our study also reveals that the complementarity between skilled and unskilled workers is more pronounced in industries with low capital to output ratios compared to those with high ratios. This finding may be attributed to the fact that, on average, the education levels of the workforce in high capital to output ratio industries are relatively lower than those in low capital to output ratio industries in our dataset. Therefore, the more significant discrepancy in skill levels between skilled and unskilled workers in industries with higher education levels makes it more challenging to replace one type of worker with another, resulting in a more profound complementarity between skilled and unskilled workers. This finding aligns with Mollick (2011) who showed that the higher the threshold for defining skilled labour, the less likely is the switch between types of labour.

Finally, inspired by the facts that in industries with high capital to output ratios, the capital to output ratio remained stable until 1980, when it started to decline, and the rate of increase in skilled-to-unskilled employment ratio slowed during this period. We estimate the six models before and after 1980 for two industry groups. We find that there is no precise time break in capital-skill complementarity in industries belonging to the group with the relatively low capital to output ratio; however, our analysis reveals that the substitution between capital and a skilled workforce is more pronounced after 1980 than before in the high capital to output ratio industry group. This finding aligns with the research of Goldin and Katz (1998), who demonstrated that capital-skill complementarity was equally substantial during the periods of 1909-1919 and 1979-1989 in the U.S. manufacturing industry, but notably less so during 1959-1979 (refer to Goldin and Katz, 1998, pp. 722–723). These results and ours imply that it could be erroneous to view capital-skill complementarity/substitution as a universal phenomenon. Instead, our study suggests that the relationship between capital and labour may differ over time and across various industries.

To ensure the reliability of our findings, we conducted extensive robustness analyses. Firstly, we employed an alternative model to assess the robustness of the threshold estimations derived from our baseline model. Secondly, we tested the robustness of the capital-skill substitution/complementarity by performing estimations using an alternative classification based on alternative threshold estimations. Additionally, we conducted several Chow tests based on different models to examine the structural change in our samples. Finally, we also tested for a time break in our subsample of group 1, which comprises industries with high capital to output ratios. These analyses indicate that our empirical findings were shown to be quite robust to alternative function forms, and the Chow test shows there is indeed a structure change between industries with high and low capital to output ratios.

## 2.8 Intangible Capital

Intangible capital is primarily understood as the amassed expertise and knowledge derived from investments in research and development, branding, and organizational structures. Typically, companies record this type of capital as an expense, rather than as an asset. Intangible capital has increasingly been acknowledged as playing a significant role in several macroeconomic trends, such as the declining labour share and a slowdown in productivity growth. Solow (1957) famously noted that the impact of the computer revolution is apparent in many areas, but not in productivity statistics. Since the 1980s, with the momentum of the IT revolution, numerous studies have assessed the impact of IT capital on output growth using the Solow-Jorgenson-Griliches Sources-of-Growth (SOG) framework (Jorgenson and Stiroh (2017), Oliner and Sichel (2000), Jorgenson et al. (2002), Stiroh (2002)). However, intangible capital has traditionally been treated as an expense on inputs rather than as an investment, which has led to the understatement of intangibles' significant contribution to economic growth. The work of Corrado et al. (2009) has shown that when incorporating intangibles into the Sources-of-Growth (SOG) framework results in noticeably higher output growth rates. Additionally, their research revealed that including intangibles leads to an increased share of Gross Domestic Income (GDI) accruing to capital, while simultaneously reducing the share allocated to labour income. Complementing these findings, Mitra (2019) demonstrated that an increase in the proportion of intangible capital within production amplifies its effect on output. She also noted that the output in the final goods sector initially decreases when there is a positive shock in intangible capital. This is attributed to the redirection of resources towards the generation of more productive intangible capital, which temporarily reduces the output of final goods following the shock in intangible capital.

The literature provides extensive support for the growing importance of intangible capital, which often comes at the expense of labour and physical capital in production processes since the 1980s. Hall et al. (2000) links the surge in stock values of firms that have adopted new technologies based on computer software to a shift from traditional costs of inventories and equipment towards the accumulation of intangible assets, introducing this shift as an increase in what is termed 'E-capital'. Nakamura et al. (2001) conducted an estimation of intangible investment in the U.S. economy from three distinct perspectives. Each estimate strongly suggests that annually, between 6 percent to 10 percent of the U.S. GDP is allocated to intangible capital, with the potential for even higher figures. This investment in intangible assets began to increase significantly around 1980. The study also indicates a lower-bound estimate for U.S. gross investment in intangibles at 1.1 trillion dollars. Corrado et al. (2005) estimated that during the 1990s, business expenditure on intangible assets increased in comparison to other significant components of overall aggregate demand. Moreover, Corrado et al. (2009) observed a significant increase in intangible assets, which rose from 103.4 billion dollars in the 1970s to 349.3 billion dollars in the 1980s, with a continuing upward trend thereafter. The ratio of intangible to tangible investment was 0.60 prior to 1980 and increased to 0.82 in the 1980s.

The shift from physical capital investment towards intangible capital has also had significant effects on the labour market. Hall et al. (2000) notes that a sizeable portion of E-capital is related to the creation and application of computers and software, tasks predominantly carried out by individuals with college degrees. This has led to a notable increase in both the number and earnings of college-educated workers. As a result, the earnings ratio comparing all college graduates to non-college-educated workers increased from 0.61 in 1990 to 0.89 in 1998. Mitra (2019), utilizing occupational data from the Current Population Survey (CPS), discovered a significant rise in the proportion of jobs mainly linked with the production of intangibles, relative to total employment, beginning around 1980. This observation aligns with the findings of McGrattan and Prescott (2012), who demonstrated a notable shift in employment towards IT sectors during the 1990s.

This thesis reveals that in industries characterized by a relatively low capital to output ratio, capital and skilled labor are complementary. Conversely, in capital-intensive industries, they tend to be substitutes. It also highlights a decrease in the growth rate of the capital-to-output (K/Y) ratio in sectors with lower physical capital intensity, alongside a downward trend in this ratio in highly capital-intensive industries since the 1980s. These observations align with the emergence of the intangible capital revolution around the same period, as previously documented. The observed change in the physical capital to output ratio may indicate an escalating significance of intangible capital. This is particularly relevant for industries that rely on sophisticated production processes, which are more profoundly influenced by the IT revolution and the increasing emphasis on intangible investments. These trends suggest a shift in the nature of capital that drives industry productivity, moving away from physical assets towards more intangible forms.

The complementarity between capital and skill in less capital-intensive industries post-1980s becomes more understandable when considering intangible capital. The result is linked to the escalating significance of intangible capital, which necessitates a workforce with higher education levels, particularly for the development of software and the operation of computers. As a result, there has been an increase in the share of skilled labour income relative to total labor costs in these sectors. These shifts in labour market dynamics underscore the direct impact of the rise of intangible capital on the demand for and compensation of skilled labour.

Future studies could expand on the regression model used in this thesis to include the effects of intangible investment on the share of skilled labour income in total labor costs. This would enable a more comprehensive analysis of how intangible capital affects the relationship between the physical capital to output ratio and the skilled labour income share. Additionally, it would be valuable to examine whether the inclusion of intangible capital alters the observed patterns of capital-skill complementarity across different industry groups. Such research could also quantify the extent to which increases in intangible and physical capital contribute to the rise in the skilled labour income share, offering deeper insights into the evolving dynamics of labour markets and capital investment.

## 2.9 Conclusion

The objective of this paper is to examine the validity of the Griliches' capital-skill complementarity hypothesis across various industries in the U.S. economy. Our study yields four primary results. Firstly, we employ Hansen's (2002) data-splitting methodology and discover evidence supporting parameter heterogeneity and non-linearity in the capital-skill complementarity/substitution relationship. Specifically, industries with a higher capital to output ratio demonstrate substitution between capital and skilled labour. In contrast, we find that capital and skilled labour are complementary in the group with a lower capital to output ratio. Our second finding is that skilled and unskilled labour are complementary in both industry groups. However, the level of complementarity is more significant in the group with a relatively lower capital to output ratio than the other group. Moreover, We observe that the average education levels of the workforce in the industries belonging to the group with a lower capital to output ratio are higher than the other group in our dataset. This suggests that the complementarity between skilled and unskilled labour is more pronounced in industries with a higher-educated workforce. Furthermore, we confirmed this result by conducting a threshold estimation using the skilled-to-unskilled relative employment ratio and an estimation based on the industry classification according to the threshold value. Our final discovery was motivated by the observation that in industries with high capital to output ratios, the ratio remained stable until 1980 before declining, and the rate of skilled-to-unskilled employment ratio increase slowed during this period. This observation implies that there may be non-linearity in the capital-skill substitution relationship within the high capital to output ratio group over time. By re-estimating both industry groups before and after 1980, we confirm a time break at the year 1980. We find that capital and skilled labour are more substitutable after 1980 than before. There is no clear time break in the capital-skill complementarity relationship in the group with low capital to output ratios. These results are quite robust to a range of sensitivity analyses.

Our research highlights the need for a theoretical framework to better comprehend the behavior of the nonlinear relationship between capital-skill complementarity, skilled-unskilled complementarity, and the industry structure. We propose a model where capital-skill complementarity non-linearity, similar to those identified in our study, arise naturally within the economic system. Furthermore, our findings emphasize the importance of further investigation into capital-skill complementarity, as confirming the hypothesis would have significant implications in determining the aggregate production function and re-evaluating existing empirical and theoretical research. These findings have important implications for understanding the aggregate production function and for further empirical and theoretical research in this area.

# Appendix A

| | Manufacturing | | | Non-manufacturing | |
|---|---|---|---|---|---|
| id | code | Industry | id | code | Industry |
| 5 | 20 | wood and of wood and cork | 19 | 50 | sale maintenance and repair of motor vehicles |
| 11 | 27t28 | basic metals and fabricated metal | 18 | 51 | wholesale trade and commission trade |
| 12 | 29 | machinery nec | 20 | 52 | retail trade except of motor vehicles |
| 13 | 30t33 | electrical and optical equipment | 22 | 60t63 | transport and storage |
| 14 | 34t35 | transport equipment | 23 | 64 | post and telecommunications |
| 15 | 36t37 | manufacturing nec; recycling | 26 | 71t74 | renting of m&eq and other business activities |
| 3 | 15t16 | food beverages and tobacco | 1 | AtB | agriculture hunting forestry and fishing |
| 4 | 17t19 | textiles textile leather and footwear | 2 | C | mining and quarrying |
| 6 | 21t22 | pulp paper paper printing and publishing | 16 | E | electricity gas and water supply |
| 7 | 23 | coke refined petroleum and nuclear fuel | 17 | F | construction |
| 8 | 24 | chemicals and chemical products | 21 | H | hotels and restaurants |
| 9 | 25 | rubber and plastics | 24 | J | financial intermediation |
| 10 | 26 | other nonmetallic mineral | 30 | O | other community social and personal services |

Table 36: Industry list

Notes: The dataset used in this study includes 26 industries, consisting of 6 durable industries, 7 non-durable industries, and 13 non-manufacturing industries. The complete list of these industries is provided in this table. The code enclosed in parentheses corresponds to the industry classification used in the World KLEMS database.

| id | Industry | Median(K/Y) |
|---|---|---|
| 13 | Electrical and optical equipment | 1.8241 |
| 1 | Agriculture hunting forestry and fishing | 1.2829 |
| 22 | Transport and storage | 1.1483 |
| 21 | Hotels and restaurants | 1.0233 |
| 15 | Manufacturing nec; recycling | 1.0226 |
| 9 | Rubber and plastics products | 1.0148 |
| 10 | Other non-metallic mineral products | 0.98225 |
| 3 | Food products beverages and tobacco | 0.92932 |
| 7 | Coke refined petroleum products and nuclear fuel | 0.92323 |
| 19 | Wholesale trade and commission trade except of motor vehicles and motorcycles | 0.90817 |
| 30 | Other community social and personal services | 0.90769 |
| 11 | Basic metals and fabricated metal products | 0.85078 |
| 23 | Post and telecommunications | 0.78463 |
| 14 | Transport equipment | 0.7482 |
| 8 | Chemicals and chemical products | 0.72644 |
| 4 | Textiles textile products leather and footwear | 0.7249 |
| 20 | Retail trade except of motor vehicles and motorcycles; repair of household goods | 0.70386 |
| 2 | Mining and quarrying | 0.67298 |
| 16 | Electricity gas and water supply | 0.65183 |
| 18 | Sale maintenance and repair of motor vehicles and motorcycles; retail sale of fuel | 0.58122 |
| 17 | Construction | 0.50626 |
| 12 | Machinery nec | 0.50431 |
| 5 | Wood and products of wood and cork | 0.4795 |
| 24 | Financial intermediation | 0.42704 |
| 6 | Pulp paper paper products printing and publishing | 0.32645 |
| 26 | Renting of m&eq and other business activities | 0.31842 |

Table 37: Median of capital-to-ouput ratio across industries

Notes: The sample period is 1949 to 2010. For each industry, we calculate the median value of $K/Y$ where $K$ and $Y$ are volume indexes with 2005 as the base year. The industries are listed with the median value from high to low.

| Model 3 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Group 1 K/Y>0.9897 | | | | Group 2 K/Y<0.9798 | | | |
| id | code | industry | Median (S/U) | id | code | industry | Median (S/U) |
| 1 | AtB | agriculture hunting forestry and fishing | 0.05525503 | 2 | C | mining and quarrying | 0.15018373 |
| 9 | 25 | rubber and plastics | 0.11646072 | 3 | 15t16 | food beverages and tobacco | 0.10210159 |
| 13 | 30t33 | electrical and optical equipment | 0.25687565 | 4 | 17t19 | textiles textile leather and footwear | 0.04317118 |
| 15 | 36t37 | manufacturing nec; recycling | 0.16915695 | 5 | 20 | wood and of wood and cork | 0.06163423 |
| 21 | H | hotels and restaurants | 0.08228209 | 6 | 21t22 | pulp paper paper printing and publishing | 0.20493153 |
| 22 | 60t63 | transport and storage | 0.12588585 | 7 | 23 | coke refined petroleum and nuclear fuel | 0.23772318 |
| | | **median** | **0.12117329** | 8 | 24 | chemicals and chemical products | 0.34472345 |
| | | | | 10 | 26 | other nonmetallic mineral | 0.0926741 |
| | | | | 11 | 27t28 | basic metals and fabricated metal | 0.09297724 |
| | | | | 12 | 29 | machinery nec | 0.13274616 |
| | | | | 14 | 34t35 | transport equipment | 0.16527641 |
| | | | | 16 | E | electricity gas and water supply | 0.20355606 |
| | | | | 17 | F | construction | 0.08507863 |
| | | | | 18 | 51 | wholesale trade and commission trade | 0.10645388 |
| | | | | 19 | 50 | sale maintenance and repair of motor vehicles | 0.24410074 |
| | | | | 20 | 52 | retail trade except of motor vehicles | 0.12841383 |
| | | | | 23 | 64 | post and telecommunications | 0.1937005 |
| | | | | 24 | J | financial intermediation | 0.43572977 |
| | | | | 26 | 71t74 | renting of m&eq and other business activities | 0.53898735 |
| | | | | 30 | O | other community social and personal services | 0.23813289 |
| | | | | | | **median** | **0.15773007** |

Table 38: Skilled-to-unskilled employment ratio

Note: The sample period is 1949 to 2010. This table displays median of skilled-to-unskilled employment ratio for industries belonging to group 1 and group 2. The group classification based on our baseline model (Model 3). We aggregated the skilled-to-unskilled employment ratio by summing the values across time for each year.

| Specification | Model 1 | | Model 2 | | Model 3 | |
|---|---|---|---|---|---|---|
| | 1949-1979 | 1980-2010 | 1949-1979 | 1980-2010 | 1949-1979 | 1980-2010 |
| ln(K/Y) | 0.0466*** | 0.0444*** | 0.0463*** | 0.0442*** | 0.0295*** | 0.0298*** |
| | (0.0052) | (0.0051) | (0.0051) | (0.0050) | (0.0045) | (0.0045) |
| ln(Y) | | | 0.0298*** | 0.0301*** | | |
| | | | (0.0037) | (0.0037) | | |
| ln(K/VA) | | | | | | |
| ln(VA) | | | | | | |
| ln(Ws/Wu) | | | | | 0.1522*** | 0.1507*** |
| | | | | | (0.0071) | (0.0074) |
| t | 0.0040*** | 0.0041*** | 0.0032*** | 0.0033*** | 0.0045*** | 0.0045*** |
| | (0.0001) | (0.0001) | (0.0002) | (0.0002) | (0.0001) | (0.0001) |
| R-square | 0.3735 | 0.3885 | 0.3096 | 0.3237 | 0.4062 | 0.4161 |
| Obs. | 1209 | 1209 | 1209 | 1209 | 1209 | 1209 |
| Prob>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Specification | Model 4 | | Model 5 | | Model 6 | |
| | 1949-1979 | 1980-2010 | 1949-1979 | 1980-2010 | 1949-1979 | 1980-2010 |
| ln(K/Y) | | | | | | |
| ln(Y) | | | | | | |
| ln(K/VA) | 0.0185*** | 0.0186*** | 0.0370*** | 0.0366*** | 0.0128*** | 0.0130*** |
| | (0.0033) | (0.0032) | (0.0036) | (0.0035) | (0.0028) | (0.0028) |
| ln(VA) | | | 0.0346*** | 0.0339*** | | |
| | | | (0.0034) | (0.0033) | | |
| ln(Ws/Wu) | | | | | 0.1574*** | 0.1553*** |
| | | | | | (0.0071) | (0.0074) |
| t | 0.0045*** | 0.0046*** | 0.0032*** | 0.0033*** | 0.0048*** | 0.0048*** |
| | (0.0001) | (0.0001) | (0.0002) | (0.0002) | (0.0001) | (0.0001) |
| R-square | 0.4056 | 0.4188 | 0.3091 | 0.3236 | 0.4235 | 0.4349 |
| Obs. | 1209 | 1209 | 1209 | 1209 | 1209 | 1209 |
| Prob>F | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 39: Regression results for group 2 in 1949 to 1979 and 1980 to 2010.

Notes: The sample period is 1949-2010. The dependent variable is skilled-labour share of the wage bill (S). $w_s/w_u$ is skilled-unskilled wage premium, K/Y is capital–output ratio and Y is output. VA is value added volume index. Robust standard errors are given in parentheses. *** denotes significantly different from 0 at the 1 % level.

| Model 2 | | | | | |
|---|---|---|---|---|---|
| Group 1 K/Y>0.8185 | | | Group 2 K/Y<0.8185 | | |
| id | code | industry | | code | industry |
| 1 | AtB | agriculture hunting forestry and fishing | 2 | C | mining and quarrying |
| 3 | 15t16 | food beverages and tobacco | 4 | 17t19 | textiles textile leather and footwear |
| 7 | 23 | coke refined petroleum and nuclear fuel | 5 | 20 | wood and of wood and cork |
| 9 | 25 | rubber and plastics | 6 | 21t22 | pulp paper paper printing and publishing |
| 10 | 26 | other nonmetallic mineral | 8 | 24 | chemicals and chemical products |
| 11 | 27t28 | basic metals and fabricated metal | 12 | 29 | machinery nec |
| 13 | 30t33 | electrical and optical equipment | 14 | 34t35 | transport equipment |
| 15 | 36t37 | manufacturing nec; recycling | 16 | E | electricity gas and water supply |
| 19 | 50 | sale maintenance and repair of motor vehicles | 17 | F | construction |
| 21 | H | hotels and restaurants | 18 | 51 | wholesale trade and commission trade |
| 22 | 60t63 | transport and storage | 20 | 52 | retail trade except of motor vehicles |
| 30 | O | other community social and personal services | 23 | 64 | post and telecommunications |
| | | | 24 | J | financial intermediation |
| | | | 26 | 71t74 | renting of m&eq and other business activities |

| Model 5 & Model 6 | | | | | |
|---|---|---|---|---|---|
| Group 1 K/Y>0.8721 | | | Group 2 K/Y<0.8640 | | |
| id | code | industry | | code | industry |
| 1 | AtB | agriculture hunting forestry and fishing | 2 | C | mining and quarrying |
| 3 | 15t16 | food beverages and tobacco | 4 | 17t19 | textiles textile leather and footwear |
| 7 | 23 | coke refined petroleum and nuclear fuel | 5 | 20 | wood and of wood and cork |
| 9 | 25 | rubber and plastics | 6 | 21t22 | pulp paper paper printing and publishing |
| 10 | 26 | other nonmetallic mineral | 8 | 24 | chemicals and chemical products |
| 13 | 30t33 | electrical and optical equipment | 11 | 27t28 | basic metals and fabricated metal |
| 15 | 36t37 | manufacturing nec; recycling | 12 | 29 | machinery nec |
| 19 | 50 | sale maintenance and repair of motor vehicles | 14 | 34t35 | transport equipment |
| 21 | H | hotels and restaurants | 16 | E | electricity gas and water supply |
| 22 | 60t63 | transport and storage | 17 | F | construction |
| 30 | O | other community social and personal services | 18 | 51 | wholesale trade and commission trade |
| | | | 20 | 52 | retail trade except of motor vehicles |
| | | | 23 | 64 | post and telecommunications |
| | | | 24 | J | financial intermediation |
| | | | 26 | 71t74 | renting of m&eq and other business activities |

Table 40: Industry classification based on threshold estimations of Model 2, Model 5 and Model 6

Note: The sample period is 1949 to 2010. This table displays the industry classification based on the threshold estimations on alternative models (Model 2, Model 5 and Model 6). Industries that are categorised as manufacturing industries are highlighted in bold, while those categorised as non-manufacturing industries are not highlighted.

| id | Industry | S/U |
|---|---|---|
| 26 | Renting of m&eq and other business activities | 0.53898735 |
| 24 | Financial intermediation | 0.43572977 |
| 8 | Chemicals and chemical products | 0.34472345 |
| 13 | Electrical and optical equipment | 0.25687565 |
| 19 | Wholesale trade and commission trade except of motor vehicles and motorcycles | 0.24410074 |
| 30 | Other community social and personal services | 0.23813289 |
| 7 | Coke refined petroleum products and nuclear fuel | 0.23772318 |
| 6 | Pulp paper paper products printing and publishing | 0.20493153 |
| 16 | Electricity gas and water supply | 0.20355606 |
| 23 | Post and telecommunications | 0.1937005 |
| 15 | Manufacturing nec; recycling | 0.16915695 |
| 14 | Transport equipment | 0.16527641 |
| 2 | Mining and quarrying | 0.15018373 |
| 12 | Machinery nec | 0.13274616 |
| 20 | Retail trade except of motor vehicles and motorcycles; repair of household goods | 0.128413825 |
| 22 | Transport and storage | 0.12588585 |
| 9 | Rubber and plastics products | 0.11646072 |
| 18 | Sale maintenance and repair of motor vehicles and motorcycles; retail sale of fuel | 0.10645388 |
| 3 | Food products beverages and tobacco | 0.10210159 |
| 11 | Basic metals and fabricated metal products | 0.09297724 |
| 10 | Other non-metallic mineral products | 0.0926741 |
| 17 | Construction | 0.08507863 |
| 21 | Hotels and restaurants | 0.08228209 |
| 5 | Wood and products of wood and cork | 0.06163423 |
| 1 | Agriculture hunting forestry and fishing | 0.05525503 |
| 4 | Textiles textile products leather and footwear | 0.04317118 |

Table 41: Skilled-to-unskilled employment ratio for industries

Notes: The sample period is 1949 to 2010. For each industry, we aggregated the skilled-to-unskilled employment ratio by summing the values across time for each year.

# 3 A Business Cycle Model with CES Production Function and Calvo Price and Wage Setting

## 3.1 Introduction

When analyzing business cycles, it becomes apparent that there are discernible patterns of similarity in the co-movements of macroeconomic time series. One prominent characteristic of business cycles is the tendency for economic conditions and labour input to exhibit positive co-movement. However, there exists a multitude of underlying factors that contribute to the occurrence of business cycles, each with varying degrees of influence.

When technology improves, does employment of capital and labour rise in the short run, rely heavily on the appropriate measure of aggregate technology. The Solow residual, proposed by Solow (1957), is one of the well-known proxy for capturing the stochastic movements in the aggregate production technology process. However, this measure has received a range of critiques. As highlighted by Basu and Kimball (1997), there are three main explanations for the observed procyclicality of productivity. First, variations in measured productivity may be driven by exogenous changes in production technology. Second, productivity can exhibit a procyclical pattern due to increasing returns to scale, which lead to improved efficiency as the economy operates at higher activity levels. Finally, if inputs are inaccurately measured, measured productivity may display a procyclical pattern even if true productivity remains constant. It is emphasized that the gap between actual and measured technology shocks most likely arises from unobserved changes. Therefore, failing to consider the cyclical fluctuations of utilisation can lead to measurement errors when using Solow residuals as a reflection of technology shocks and lead to mismeasured business cycle results. The importance of accounting for variable utilisation in business cycle contexts has been widely recognized and extensively discussed in Chapter 1.

Aligning with this topic, Basu et al. (2006) employees tools from Basu and Fernald (1997) and Basu and Kimball (1997), who inturn build on Solow (1957) and Hall (1990) and construct an index of aggregate technology change by controlling for cyclicality of the Solow residual coming from both non-constant returns to scale and variable utilisations. Furthermore, their findings indicate that in the short run, technology improvements lead to a substantial reduction in total hours worked. This observation stands in contrast to the conventional parameterizations of the Real Business Cycle model, which suggest that technology advancements should increase input utilisation across all time horizons. [34] Subsequent to the pioneering work of Basu, Fernald, and Kimball, the adoption of a utilisation-adjusted series as a measure of total factor productivity (TFP) has become customary in the analysis of the U.S. economy. Building upon this framework, numerous scholarly works have undertaken extensive expansions and investigations in this field.

A body of literature has focused on examining the relationship between hours worked or employment and positive technology shocks. For example, Collard and Dellas (2007) illustrate that empirical results have suggested a persistent decline in employment in response to a positive technology shock, thereby questioning the ability of the Real Business Cycle (RBC) model to adequately account for business cycles. However,

[34] Basu and Fernald (1997) emphasize the importance of considering sectoral heterogeneity and aggregation. They argue that due to factors such as variations in market power across industries, the marginal productivity of an input may differ across different uses. Consequently, the aggregate Solow residual, which represents the growth in aggregate total factor productivity (TFP), depends on the sectors that experience the most significant changes in input utilisation during the business cycle. The identification of aggregate technology is carried out by Basu et al. (2006) through the estimation of a regression equation, following the approach outlined by Hall (1990), which incorporates a proxy for utilisation within each disaggregate industry. Subsequently, aggregate technology change is defined as a weighted sum of the resulting residuals.

they demonstrate that a standard, open economy, flexible price model can generate a negative employment response to a positive technology shock and effectively capture the negative conditional correlation between productivity and employment, particularly when trade elasticities are low. Khan and Tsoukalas (2013) examine the short-run effects of productivity shocks on total hours worked at the industry level in the United Kingdom, providing evidence that positive productivity shocks at the industry level lead to a reduction in hours worked in the short term within the UK economy. Expanding upon this line of inquiry, Thomet and Wegmueller (2021) extend the analysis to a wide range of countries and analyze how the response of aggregate hours to permanent technology shocks compares across industrialized nations. Their study reveals a remarkably similar short-run response of aggregate hours to a positive technology shock across countries, with a significant decrease observed in 13 out of 14 OECD countries. Further exploration of the relationship between total hours and technology shocks can be found in the works of Christiano et al. (2004), Cantore et al. (2012, 2015, 2017), Uhlig (2004). Canova et al. (2010), Galí (2005),Cacciatore et al. (2020), among others.

Our study makes a contribution to the existing literature by analyzing the implications of technology factors in business-cycle models, focusing specifically on examining the response of hours worked among different skill levels of labour following a technology shock. To achieve this, we introduce Constant Elasticity of Substitution (CES) production techniques. The primary objective of this research is to develop a stochastic dynamic general equilibrium (DSGE) model that incorporates capital-skill complementarity in production, Calvo prices, and nominal wage rigidity. The model is designed to provide insights into the observed negative dynamics in the skilled to unskilled working hours ratio and the differential responses of hours worked by skilled and unskilled workers to utilisation-adjusted technology shocks. We conduct simulations to calibrate to the U.S. economy for four distinct scenarios, including a generalized Real Business Cycle (RBC) model, a frictionless monopolistic competition model, a generalized model incorporating Calvo pricing (Calvo (1983)), and a model incorporating both Calvo wage and price settings.

The study of capital-labour substitution in business cycle models is grounded in robust theoretical and empirical justifications. The dynamics of hours worked are intricately linked to essential elements of the production process, specifically the degree of substitutability or complementarity observed between different factors. For instance, Cantore et al. (2014) incorporate Constant Elasticity of Substitution (CES) production technologies into both Real Business Cycle (RBC) and New Keynesian (NK) models. Their findings demonstrate that the response of hours worked is contingent upon the factor-augmenting nature of shocks and the elasticity of capital-labour substitution in both model frameworks. Cantore et al. (2017) observe that the response of hours worked to technology shocks in the postwar US economy has exhibited an upward trend over time. They identify that the changes observed in the response of hours worked can be attributed to the escalating magnitude of capital-labour substitution.

Our study differentiates itself from the existing literature illustrated above by explicitly distinguishing labour inputs into skilled and unskilled categories, in contrast to previous works that typically consider labour as an aggregate input. Our model shares similarities with the works of Vasilev (2022) and Lin and Weise (2019). Vasilev (2022) develops a model with a richer government sector, monopolistic competition in the product market, and price and wage rigidity based on the Calvo (1983) framework, aiming to replicate observed variability and correlations within the Bulgarian labour market. On the other hand, Lin and Weise (2019) introduces a New Keynesian model incorporating CES production technologies with both traditional and robot capital. In contrast, our contribution lies in the development of a stochastic dynamic general equilibrium (DSGE) model that integrates capital-skill complementarity in production, Calvo prices, and nominal wage rigidity. To the best of our knowledge, this is the first study to specifically focus on the short-run responses of hours worked for skilled and unskilled workers, aiming to explain both the negative responses of the skilled-to-unskilled hours ratio and the distinct direction of skilled and unskilled hours responses to utilisation-adjusted technology advancements.

Our findings reveal that, with moderate capital-skill complementarity, both the generalized Real Business Cycle (RBC) model and the frictionless monopolistic competition model can account for the negative response of the skilled-to-unskilled hours ratio to technological advancements. However, these models fall short in explaining the disparate responses of hours worked for different skill levels in the short run. In contrast, a generalized model incorporating Calvo (1983) pricing successfully offers an explanation. Furthermore, we enhance our analysis by considering a model that incorporates Calvo wage and price settings.

The remainder of this paper is structured as follows: Section 3.2 presents a comprehensive discussion of the data and empirical results. In Section 3.3, the model frameworks are outlined and described. The calibration procedure is elaborated upon in Section 3.4. Subsequently, in Sections 3.5, the principal findings for the three models are presented. Section 5 encompasses sensitivity tests and an exploration of the role of capital-skill complementarity in generating the results. Finally, Section 6 concludes the paper.

## 3.2 The Data and Empirical Results

The primary source of data for this study is the World KLEMS data 2013 release. This dataset provides comprehensive industry-level measures of labour and capital inputs. Specifically, it includes information on labour compensation per hour, the average number of hours worked per week, and employment figures. The data is categorized into six skill levels, namely: less than high school, some high school, high school graduate, some college, college graduate, and more than college. In line with existing literature (see Krusell et al. (2000), Kawaguchi et al. (2014), Parro (2013), Perez-Laborda and Perez-Sebastian (2020)), we classify workers who graduated from college and more than college (at least 16 years of school) as skilled labour; low skilled labour is defined as workers without a college education. We drop the workers who are aged under 16 or above 64. Our sample period is from 1950 to 2001, and the data consist of 30 industries. Furthermore, for each industry, the dataset provides labour compensation per hour, employment figures, and the average number of hours worked per week for both skilled. These variables are utilised to derive aggregate measures by aggregating the data across industries. Additional information regarding industry classification can be found in in Appendix B, and aggregation can be found in Chapter 1 Section 1.2.

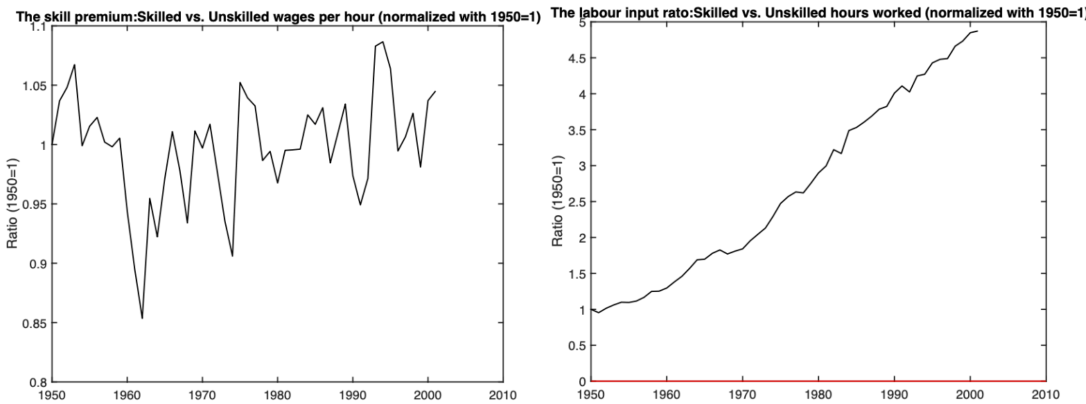


Figure 17: Skill premium and skilled to unskilled hours worked ratio
Notes: The sample period is 1949-2001, and both ratios are normalized with 1950 = 1.

The left panel depicted in Figure 17 presents discernible patterns in the skill premium throughout the observation period from 1950 to 2001. Initially, there was a substantial decline in the skill premium during the 1950s, followed by a modest increase in the 1960s. Subsequently, the skill premium experienced a period of decline throughout much of the 1970s, followed by an increase with some fluctuations since 1980. In aggregate, the skill premium exhibited an overall upward trend over the entire duration of the study. Conversely, the right panel of Figure 17 portrays the ratio of skilled labour hours to unskilled labour hours. This ratio displays a clear ascending trend throughout the entire observation period, indicating a substantial increase in the relative hours worked by skilled labour in comparison to unskilled labour. Notably, the relative hours worked ratio of skilled to unskilled labour witnessed a growth of more than 100 percent.

In addition to analyzing the long-term trend, this study investigates the short-term dynamics by estimating the impulse response functions (IRFs) of several variables in response to a utilisation-controlled neutral technology shock. The key variables of interest encompass hours worked, employment, and wages for both skilled and unskilled workers, as well as the ratio of skilled to unskilled labour hours and the ratio of capital inputs to the skilled labour hours.

The Solow residual, initially introduced by Solow (1957), has become the most widely recognized proxy for measuring neutral technology progress. Solow utilised the Cobb-Douglas production function with constant returns to scale to calculate the Solow residual, which represents the difference between output and the combined inputs of capital and labour. However, the accuracy of the Solow residual as a measure of technology progress has been subject to criticism in various literature. Notable studies such as Gali (1999),Basu and Kimball (1997),Basu and Fernald (1997),Basu et al. (2006) have highlighted several shortcomings of the Solow residual. The main criticisms revolve around the neglect of varying degrees of returns to scale, the role of input utilisation, and sectoral shifts within production functions. These limitations can lead to misleading results when studying business cycles. To address these concerns, this paper adopts the concept of utilisation-adjusted technology data, as introduced by Basu et al. (2006). The utilisation-adjusted TFP shock, originally developed by Basu and Kimball (1997) and Basu et al. (2006), focused on the U.S. economy during the period from 1949 to 1996. Basu's work specifically covers this time frame for constructing the utilisation-adjusted TFP series. In line with Basu's identification method, this study extends the utilisation-adjusted TFP series for the U.S. economy, encompassing the period from 1949 to 2001. The estimation

| | Response | p-value |
|---|---|---|
| Skill wage | 0.270 | 0.056 |
| Uskill wage | 0.051 | 0.716 |
| Skill hours worked | -0.897 | 0.000 |
| Uskill hours worked | 0.449 | 0.001 |
| Skill premium | 0.686 | 0.000 |
| Ratio of skilled to unskilled hours worked | -1.391 | 0.000 |
| Ratio of capital to skilled hours worked | 0.637 | 0.000 |

Table 42: Point estimates of impact responses of variables to an utilisation-controlled technology shock

details and methodology are elaborated upon in the first chapter of this thesis. [35]

To estimate the IRFs, we estimate a small bivariate SVAR. To begin, the utilisation-controlled technology series appears to be stationary; in the Augmented Dicky-Fuller test, we strongly reject the null hypothesis of a unit root at the conventional significance level for all six countries considered. Besides, by a KPSS test, we fail to reject the null hypothesis of stationary. In the bivariate SVAR system, the equation of response variables involves regressing the growth response variable, i.e., $dx$, on utilisation-controlled technology change, i.e., $dz$, and two lags of itself and $dz$. (For any variable $X$, we define $dx$ as its logarithmic growth rate, namely $ln(X_t/X_{t-1})$.) In the equation of utilisation-controlled technology change, we restricted the parameters on the response variable and its lags to be zero and regressed utilisation-controlled technology growth on only its two lags. This restriction consists of the assumption that the technology change is not affected by the response variables.

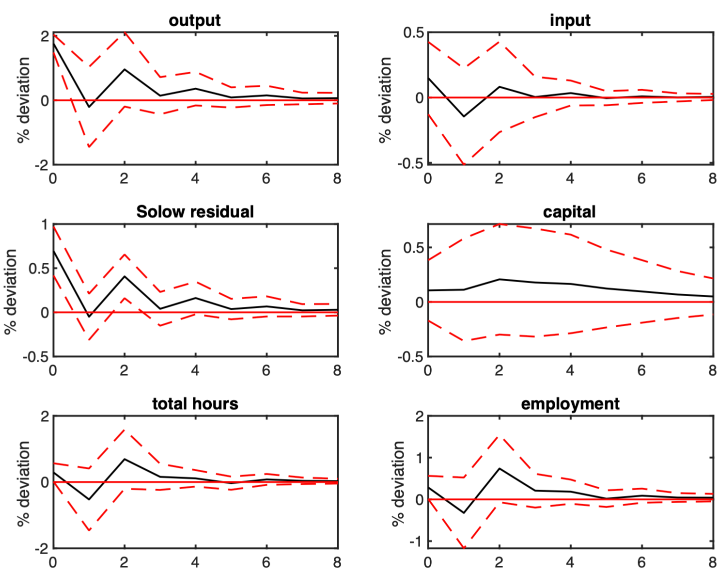


Figure 18: Impulse response of aggregate variables to an utilisation-controlled technology shock
Notes: The sample period is 1949-2001. The figure shows the impulse responses of aggregate variables to an positive utilisation-controlled technology shock. All responses variables and technology shock are in logarithmic growth rate. The dotted lines represent the 95-percent confidence intervals.

When observing the impulse response functions (IRFs) of the aggregate variables to a positive utilisation-controlled technology shock, as depicted in Fig 18, the findings align with the standard Real Business

[35] Huo et al. (2020) prolonged the utilisation-adjusted TFP data for the U.S. to a more recent period from 1978 to 2005.

Cycle (RBC) model. All of the aggregate variables exhibit an immediate and positive response. Next, the analysis focuses on the IRFs of specific variables, namely the skill premium, the ratio of skilled to unskilled labour hours worked, and the ratio of capital inputs to skilled labour hours. These IRFs are presented in Figure 19. In response to a positive utilisation-controlled technology shock, a sharp decrease is observed in the skilled-to-unskilled hours ratio. Conversely, the capital-to-skilled hours ratio demonstrates a positive response. Importantly, both responses exhibit strong statistical significance (refer to Table 43 for detailed point estimates). Additionally, the skill premium experiences a statistically significant increase upon impact.

Subsequently, the response functions of hours worked, employment, and wages for both skilled and unskilled workers were further decomposed, and the outcomes are displayed in Figure 20. It is evident that a positive utilisation-controlled technology shock induces opposite responses in skilled and unskilled hours worked. Specifically, such a shock leads to an increase in total hours worked by unskilled workers, while simultaneously decreasing the total hours worked by skilled workers in the short run. The statistical analysis presented in Table 43 confirms that the responses in hours worked for both types of workers are highly significant. Furthermore, the positive technological change shock generates a rise in wages for skilled workers, while wages for unskilled workers exhibit minimal changes. The increase in skilled wages is statistically significant at a 90% confidence level, whereas the rise in unskilled wages is found to be insignificant. These outcomes align with the observed increase in the skill premium. Additionally, the employment responses for skilled and unskilled workers demonstrate divergent patterns. The technology advancement shock initially reduces skilled employment, while simultaneously increasing the employment of unskilled workers. In subsequent periods, the demand for skilled workers surpasses that for unskilled workers, reaching its peak two periods after the initial shock. It is worth noting that the impact responses of employment for both skill levels are statistically significant, as indicated in Table 43.

In order to assess the robustness of the empirical findings, we conducted additional analyses using the utilisation-controlled technology shock series proposed by Huo et al. (2020). By incorporating this alternative measure, we re-estimated the impulse response functions (IRFs) for the aforementioned variables to technological advancements. The results of these re-estimations are presented in Figure 21. Remarkably, the findings obtained from this alternative approach closely align with those reported in Figure 20. The consistency between the two sets of results supports the robustness of the empirical findings. The observed contrasting responses of hours worked for skilled and unskilled workers, along with the significant decrease in the relative hours worked by skilled workers compared to unskilled workers in response to a utilisation-controlled technology shock in the short run, serves as motivation for constructing a stochastic dynamic general equilibrium (DSGE) model. This proposed model incorporates elements such as capital-skill complementarity in production, Calvo prices, and nominal wage rigidity. The primary objective of this paper is to provide an explanation for the negative response observed in the skilled-to-unskilled working hours ratio and the divergent reactions of hours worked for skilled and unskilled workers.

By employing the DSGE framework, the paper seeks to offer a comprehensive understanding of the underlying mechanisms driving these empirical findings. The model's inclusion of capital-skill complementarity allows for the analysis of how changes in technology affect the relative demand for skilled and unskilled labour inputs. Additionally, the incorporation of Calvo prices and nominal wage rigidity accounts for the sticky nature of prices and wages in the economy, enabling an examination of their impact on labour market outcomes. Through the DSGE model, this study aims to contribute to the theoretical understanding of the observed dynamics in the skilled-to-unskilled working hours ratio and the contrasting responses of hours worked for skilled and unskilled workers. By providing a rigorous framework, the paper aims to shed light on

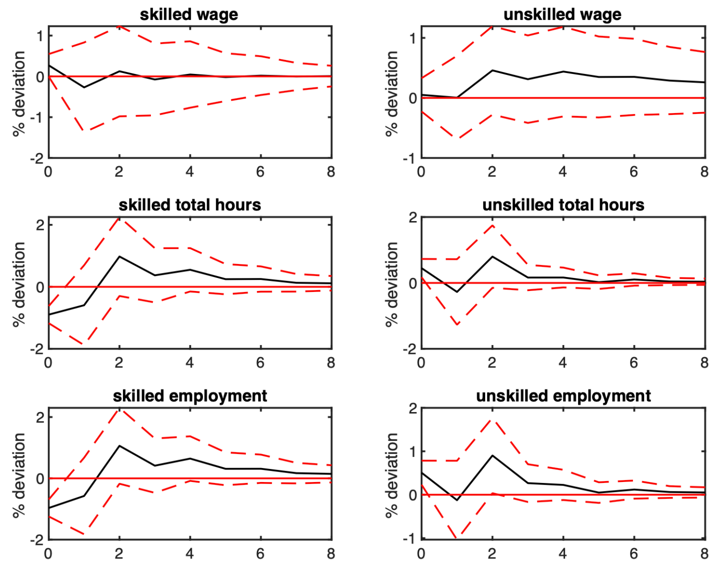


Figure 20: Impulse response of wages, hours and employment for skilled and unskilled workers to an utilisation-controlled technology shock

Notes: The sample period is 1949-2001. All responses variables and technology shock are in logarithmic growth rate. The dotted lines represent the 95-percent confidence intervals.

the mechanisms driving these phenomena and offer insights into the broader implications for labour market dynamics.

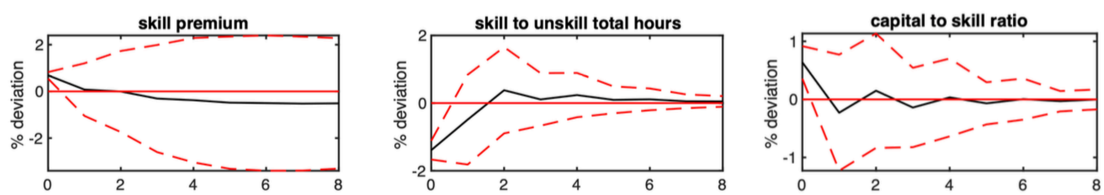


Figure 19: Impulse response of skilled to unskilled labour input share and skill premium to an utilisation-controlled technology shock

Notes: The sample period is 1949-2001. All responses variables and technology shock are in logarithmic growth rate. The dotted lines represent the 95-percent confidence intervals.

## 3.3 Model Setup

The economic framework employed in this study comprises a continuum of infinitely-lived households that provide two distinct types of labour services: skilled labour and unskilled labour. Furthermore, the model assumes the existence of a continuum of wholesalers, each specializing in the production of a single intermediate good. These wholesalers employ both skilled and unskilled labour, as well as capital, as inputs. The production technology employed by these wholesalers follows a constant elasticity of substitution (CES)

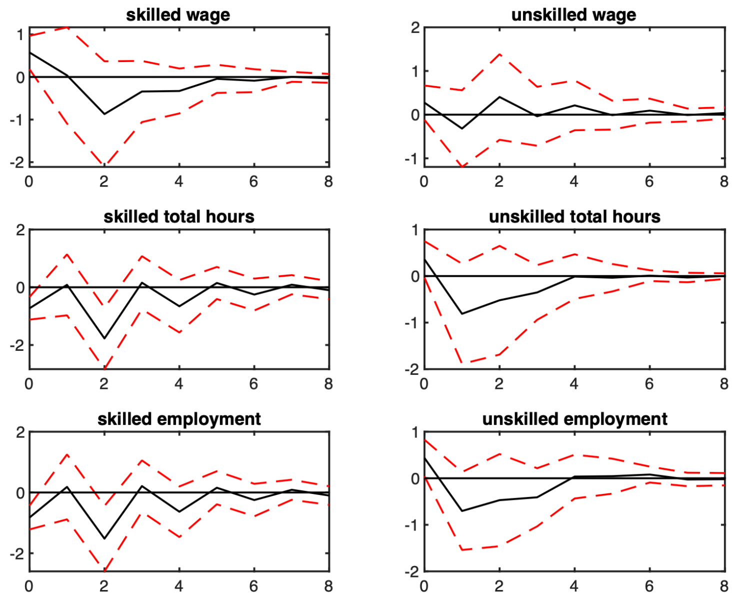


Figure 21: Impulse response of wages and employment for skilled and unskilled workers to an utilisation-controlled technology shock estimated by Huo et al. (2020)

Notes: The sample period is 1949-2001. All responses variables and technology shock are in logarithmic growth rate. The dotted lines represent the 95-percent confidence intervals.

| | Response | p-value |
|---|---|---|
| Output | 1.761 | 0.000 |
| Input | 0.149 | 0.292 |
| Solow residual | 0.695 | 0.000 |
| Total hours | 0.280 | 0.051 |
| Employment | 0.285 | 0.044 |
| Capital | 0.106 | 0.454 |
| | | |
| High skill wage | 0.270 | 0.056 |
| Low skill wage | 0.051 | 0.716 |
| High skill hours | -0.897 | 0.000 |
| Low skill hours | 0.449 | 0.001 |
| High skill employment | -0.967 | 0.000 |
| Low skill employment | 0.507 | 0.000 |
| Skill premium | 0.686 | 0.000 |
| Ratio of skilled to unskilled hours worked | -1.391 | 0.000 |
| Ratio of capital to skilled hours worked | 0.637 | 0.000 |

Table 43: Point estimates

Note: This table summarizes the estimates of the impact responses for various variables to utilisation-controlled technology shock.

function. [36] Within this framework, each monopolistic wholesale firm possesses market power and exercises control over the pricing of the intermediate good it produces. These firms set prices at a markup above their marginal product, thereby operating with a degree of market power. Additionally, a representative retailer exists, who combines the various intermediate goods to manufacture a homogeneous final good. Notably, the market for final goods operates under conditions of perfect competition. The final goods produced by retailers serve various purposes, including household consumption and investment. This allows for the allocation of final goods to both meet the consumption needs of households and support investment activities.

### 3.3.1 Households

There are two types of households, indexed by $H \in (S, U)$, representing skilled and unskilled households, respectively, distinguished by their skill levels. In addition to their roles in the production process, households share similar preferences and characteristics. At the outset, each household possesses a positive endowment of physical capital, which is rented to the firm at the nominal rental rate $Q_t$, resulting in capital income equivalent to $Q_t \times K_{H,t}$. Consequently, households have the option to augment their capital stock through investment decisions guided by a predetermined law of motion,

$$K_{H,t+1} = I_{H,t} + (1-\delta)K_{H,t}$$

where $0 < \delta < 1$ is the depreciation rate of physical capital.

In addition to the rental income from capital, each household of type $H$ receives a wage $W_{H,t}$ in exchange for the hours worked. Furthermore, they own an equal share of the firm responsible for producing the final goods, entitling them to dividend payments $D_t$. Each household engages in consumption represented by $C_{H,t}$ and savings by investing in risk-free government bonds denoted as $B_{H,t}$. These bonds offer a gross nominal return of $R_t$. The aggregate price index for period $t$ is denoted as $P_t$. Consequently, the budget constraint for household type $H$ in period $t$ can be expressed as follows:

$$P_t C_{H,t} + P_t K_{H,t+1} + B_{H,t+1} = W_{H,t} H_t + Q_t K_{H,t} + P_t(1-\delta)K_{H,t} + R_t B_{H,t} + D_t$$

And households' utility function are given by

$$U = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_{H,t}^{1-\sigma_c}}{1-\sigma_c} - \frac{H_t^{1+\phi}}{1+\phi} \right\}$$

The operator $E_t$ denotes the expectations operator as of period $t$. The parameter $0 < \beta < 1$ represents the time discount factor. The parameter $\sigma_c$ represents the relative risk aversion parameter. Additionally, $\phi$ denotes the curvature of the function that captures the disutility of hours worked, reflecting the household's preference for leisure.

The objective for each household is to maximize utility by making choices regarding consumption ($C_{H,t}$), hours worked ($H_t$), capital accumulation ($K_{H,t+1}$), and bond holdings ($B_{H,t+1}$), subject to the budget constraint. [37] The first-order conditions (FOCs) associated with this optimization problem can be expressed as follows:

[36] Adoption of CES production technology is in line with numerous studies, see Duffy et al. (2004), Krusell et al. (2000), Hara et al. (2014), Lindquist (2004), Maliar et al. (2020), Skaksen and Sorensen (2005), Maliar et al. (2020), Belianska (2020), Vasilev (2018). Cantore et al. (2015) shows the model with CES production function fits U.S. data better than a standard Cobb-Douglas production function.

[37] The discussion of optimal wage-setting is deferred to the next section.

$$C_{H,t} : C_{H,t}^{-\sigma_c} = \lambda_t P_t$$

$$H_t : H_t^{\phi} = \lambda_t W_{H,t}$$

$$K_{H,t+1} : \lambda_{t+1}(Q_{t+1} + P_{t+1}(1-\delta)) = \lambda_t P_t$$

$$B_{H,t+1} : \lambda_{t+1} R_{t+1} = \lambda_t$$

Upon rearranging the FOCs for households supplying skilled labour, we obtain the following expressions:

$$C_{s,t}^{-\sigma_c} = \beta E_t \left\{ C_{s,t+1}^{-\sigma_c} R_{t+1} \frac{P_t}{P_{t+1}} \right\}$$

$$C_{s,t}^{-\sigma_c} = \beta E_t \left\{ C_{s,t+1}^{-\sigma_c} \Big( \frac{Q_{t+1}}{P_{t+1}} + (1-\delta) \Big) \right\}$$

$$\frac{W_{s,t}}{P_t} = \frac{S_t^{\phi}}{C_{s,t}^{-\sigma_c}}$$

Symmatrically, the FOCs for household who supplies unskilled labour are

$$C_{u,t}^{-\sigma_c} = \beta E_t \left\{ C_{u,t+1}^{-\sigma_c} R_{t+1} \frac{P_t}{P_{t+1}} \right\}$$

$$C_{u,t}^{-\sigma_c} = \beta E_t \left\{ C_{u,t+1}^{-\sigma_c} \Big( \frac{Q_{t+1}}{P_{t+1}} + (1-\delta) \Big) \right\}$$

$$\frac{W_{u,t}}{P_t} = \frac{U_t^{\phi}}{C_{u,t}^{-\sigma_c}}$$

### 3.3.2 Optimal Wage Setting

It is assumed that each type of household, denoted by $H \in (S, U)$, supplies differentiated labour services: $S_{i,t}$ and $U_{j,t}$, where $i, j \in [0, 1]$. These differentiated labour services are traded in a monopolistic competitive market, implying that households possess some degree of market power when determining their wages. Following the approach of Vasilev (2018) and Junior (2016), the model assumes that the differentiated labour services supplied by households are rented to a representative labour-aggregating firm. This firm aggregates the differentiated labour inputs $S_{i,t}$ and $U_{j,t}$ to derive the aggregate labour inputs $S_t$ and $U_t$, employing a constant-elasticity-of-substitution (CES) technology,

$$S_t = \Big( \int_0^1 S_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} di \Big)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

$$U_t = \Big( \int_0^1 U_{j,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \Big)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

It is assumed that the elasticity of substitution ($\epsilon_w$) between the differentiated labour services is the same for both types of labour. Each labour service, denoted by $i$ and $j$, receives a nominal wage: $W_{s,i,t}$ for skilled labour and $W_{u,j,t}$ for unskilled labour. The labour-firm is then faced with the task of optimizing its profit by choosing the quantities of each differentiated labour service ($S_{i,t}$ and $U_{j,t}$). This optimization problem can be formulated as follows:

$$\max_{S_{i,t}} W_{s,t}S_t - \int_0^1 W_{s,i,t}S_{i,t}di$$

$$\max_{S_{i,t}} W_{s,t}\Big(\int_0^1 S_{i,t}^{\frac{\epsilon_w-1}{\epsilon_w}}di\Big)^{\frac{\epsilon_w}{\epsilon_w-1}} - \int_0^1 W_{s,i,t}S_{i,t}di$$

Symmetrically, labour-aggregating firm aggregate differentiated unskilled labour service:

$$\max_{U_{j,t}} W_{u,t}\Big(\int_0^1 U_{j,t}^{\frac{\epsilon_w-1}{\epsilon_w}}dj\Big)^{\frac{\epsilon_w}{\epsilon_w-1}} - \int_0^1 W_{u,j,t}U_{j,t}dj$$

The FOCs for each type of differentiated labour services are,

$$S_{i,t}: W_{s,t}\Big(\int_0^1 S_{i,t}^{\frac{\epsilon_w-1}{\epsilon_w}}di\Big)^{\frac{1}{\epsilon_w-1}} S_{i,t}^{\frac{-1}{\epsilon_w}} - W_{s,i,t} = 0$$

$$U_{j,t}: W_{u,t}\Big(\int_0^1 U_{j,t}^{\frac{\epsilon_w-1}{\epsilon_w}}dj\Big)^{\frac{1}{\epsilon_w-1}} U_{j,t}^{\frac{-1}{\epsilon_w}} - W_{u,j,t} = 0$$

Combining above first-order conditions with the aggregation constraints for both types of labour services respectively, we can express aggregate hours for each types of labour as follows:

$$S_{i,t}: W_{s,t}S_t^{\frac{1}{\epsilon_w}} S_{i,t}^{\frac{-1}{\epsilon_w}} - W_{s,i,t} = 0$$

$$U_{j,t}: W_{u,t}U_t^{\frac{1}{\epsilon_w}} U_{j,t}^{\frac{-1}{\epsilon_w}} - W_{u,j,t} = 0$$

After some algebra, the demand for $S_{i,t}$ and $U_{j,t}$ can be expressed as:

$$S_{i,t} = \Big(\frac{W_{s,i,t}}{W_{s,t}}\Big)^{-\epsilon_w} S_t$$

$$U_{j,t} = \Big(\frac{W_{u,j,t}}{W_{u,t}}\Big)^{-\epsilon_w} U_t$$

Now substitute above expression into labour-aggregating firm's zero profit conditions and solve for the aggregate wage rates as a function of differentiated labour wage rates,

$$W_{s,t} = \Big(\int_0^1 W_{s,i,t}^{1-\epsilon_w}di\Big)^{\frac{1}{1-\epsilon_w}}$$

$$W_{u,t} = \Big(\int_0^1 W_{u,j,t}^{1-\epsilon_w}dj\Big)^{\frac{1}{1-\epsilon_w}}$$

In terms of wage rigidity, this study follows the framework introduced by Calvo (1983). According to this framework, it is assumed that, in each period, only a fraction $1-\theta_w^H, H \in (S,U)$ of households, randomly selected from the population, have the opportunity to re-optimize their posted nominal wages. [38] The remaining households, $\theta_w^H$ keep the same wage level as the previous period, i.e., $W_{s,i,t} = W_{s,i,t-1}$ and $W_{u,j,t} = W_{u,j,t-1}$. For a household $i$ supplying skilled labour services, when resetting its wage in period $t$, let $W^*_{s,i,t}$ represent the newly set wage. The household chooses $W^*_{s,i,t}$ to maximize the expected discounted sum of utilities generated over the period during which the wage remains unchanged at $W_{s,i,t}$, which was set in the current period. This maximization is subject to the sequence of labour demand schedules and budget constraints that are in effect while $W^*_{s,i,t}$ remains unchanged. [39]

$$\max_{W^*_{s,i,t}} E_t\Big\{\sum_{k=0}^{\infty}(\beta\theta_w^s)^k U(C_{s,i,t+k}, S_{i,t+k})\Big\}$$

s.t.

$$S_{i,t+k} = \Big(\frac{W^*_{s,i,t}}{W_{s,t+k}}\Big)^{-\epsilon_w} S_{t+k}$$

$$P_{t+k}C_{s,i,t+k}+P_{t+k}K_{s,i,t+k+1}+B_{s,i,t+k+1} \leq Q_{t+k}K_{s,i,t+k}+W^*_{s,i,t}S_{i,t+k}+P_{t+k}(1-\delta)K_{s,i,t+k}+R_{t+k}B_{s,i,t+k}+D_t$$

The FOC associated with the problem above is given by

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_{i,t+k}\Big[U_s(C_{s,i,t+k}, S_{i,t+k})M_w + \frac{W^*_{s,i,t}}{P_{t+k}}U_c(C_{s,i,t+k}, S_{i,t+k})\Big]\Big\} = 0$$

where $M_w = \frac{\epsilon_w}{\epsilon_w - 1}$.

Letting $MRS_{s,i,t+k} = -\frac{U_s(C_{s,i,t+k},S_{i,t+k})}{U_c(C_{s,i,t+k},S_{i,t+k})}$ denote the marginal rate of substitution between consumption and skilled labour hours in period $t+k$ for the household supplying skilled labour and resetting the wage in period $t$, we can rewrite the optimality condition as follows:

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_{i,t+k}U_c(C_{s,i,t+k}, S_{i,t+k})\Big(\frac{W^*_{s,i,t}}{P_{t+k}} - M_w MRS_{s,i,t+k}\Big)\Big\} \tag{35}$$

In the case of full wage flexibility, where $\theta_w^s = 0$, the markup between the real wage and the marginal rate of substitution is given by $M_w = \frac{\epsilon_w}{\epsilon_w - 1}$, representing the desired gross wage markup. By log-linearizing Equation 35 around the steady state and performing some algebraic manipulations, we obtain the following approximate wage-setting rule (using lowercase letters to denote logarithmic variables):

$$w^*_{s,i,t} = \mu^w + (1-\beta\theta_w^s)\sum_{k=0}^{\infty}E_t\{mrs_{s,i,t+k} + p_{t+k}\} \tag{36}$$

where $\mu^w = logM_w = log\big(\frac{\epsilon_w}{\epsilon_w - 1}\big)$

Letting $mrs_{t+k}$ define the economy's average marginal rate of substitution. The form of assumed utility function combined with the aggregation constraint of household, implies the ($log$) marginal rate of substitution in period $t+k$ for household $i$ can be written as a function of average marginal rate of substitution as follow:

[38] Under the assumption of full consumption risk sharing across households, all households resetting their wage in any given period will choose the same wage because they face an identical problem.

[39] The maximization problem for households who supply unskilled labour services is symmetric to the households that supply skilled labour.

$$mrs_{s,i,t+k} = mrs_{s,t+k} - \epsilon_w \phi(w^*_{s,i,t+k} - w_{s,t+k})$$

Hence, Equation 36 can be rewritten as (since a fraction of $1-\theta^s_w$ of households choose the same nominal wage, i.e., $w^*_{s,i,t} = w_{s,t}$, the subscript $i$ is eliminated thereafter),

$$w^*_{s,t} = \beta\theta^s_w E_t\{w^*_{s,t+1}\} + (1-\beta\theta^s_w)(w_{s,t} - (1+\epsilon_w\phi)^{-1}\hat{\mu_t}^w) \quad (37)$$

where $\hat{\mu_t}^s = \mu^s_t - \mu^w$ denotes the deviation of the economy's ($log$) average wage markup, i.e., $\mu^s_t = (w_{s,t} - p_t) - mrs_{s,t}$ from its steady state $\mu^w$.

Given the assumed Calvo wage setting rules, the evolution of the aggregate wage can be written as:

$$W^{1-\epsilon_w}_{s,t} = \int_0^{\theta^s_w} W^{1-\epsilon_w}_{s,t-1} di + \int_{\theta^s_w}^1 (W^*_{s,t})^{1-\epsilon_w} di$$

$$W^{1-\epsilon_w}_{s,t} = [W^{1-\epsilon_w}_{s,t-1}]_0^{\theta^s_w} + [(W^*_{s,t})^{1-\epsilon_w}]^1_{\theta^s_w}$$

$$W^{1-\epsilon_w}_{s,t} = \theta^s_w W^{1-\epsilon_w}_{s,t-1} + (1-\theta^s_w)(W^*_{s,t})^{1-\epsilon_w}$$

$$W_{s,t} = \left[\theta^s_w W^{1-\epsilon_w}_{s,t-1} + (1-\theta^s_w)(W^*_{s,t})^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$$

Log-linearize the above equation around the zero wage inflation steady state yield:

$$w_{s,t} = \theta^s_w w_{s,t-1} + (1-\theta^s_w) w^*_{s,t} \quad (38)$$

Combining Equation 38 and 37 and letting $\pi^s_t = w_{s,t} - w_{s,t-1}$ denote the skilled labour wage inflation yields the wage inflation equation,

$$\pi^s_t = \beta E_t\{\pi^s_{t+1}\} - \lambda_s \hat{\mu_t}^s$$

where $\lambda_s = \frac{(1-\theta^s_w)(1-\beta\theta^s_w)}{\theta^s_w(1+\epsilon_w\phi)}$

Symmetrically, for household supplies unskilled labour service the wage inflation equation is,

$$\pi^u_t = \beta E_t\{\pi^u_{t+1}\} - \lambda_u \hat{\mu_t}^u$$

where $\lambda_u = \frac{(1-\theta^u_w)(1-\beta\theta^u_w)}{\theta^u_w(1+\epsilon_w\phi)}$

### 3.3.3 Retailer (Final goods producer)

The industry structure setup follows Dixit and Stiglitz (1977); assumes a continuum of wholesaler (intermediate goods producer), each produces a single intermediate goods with identical CES production technology and sell each intermediate good to retailer. Assume a retailer (final good producer) uses intermediate goods as input to produce final good. The final good producer produce in perfect competitive market thus takes prices as given. In contrast, intermediate good producers produce in the environment of monopolistic competition and have market power over setting their prices.

Final good firm's production function is

$$Y_t = \Big( \int_0^1 Y_{m,t}^{\frac{\epsilon-1}{\epsilon}} dm \Big)^{\frac{\epsilon}{\epsilon-1}}$$

where $Y_t$ represents the output of the final good in period $t$, and $Y_{m,t}$, where $m \in [0,1]$, denotes the output of the wholesaler producing the differentiated intermediate goods. The parameter $\epsilon > 1$ represents the elasticity of substitution between the differentiated intermediate wholesale goods.

The final goods producer operates in a perfectly competitive market and seeks to maximize its profits, taking the prices of intermediate goods $P_{m,t}$ and the price of the final good $P_t$ as given. Thus, the maximization problem faced by the final goods producer can be expressed as follows:

$$\max_{Y_{m,t}} P_t Y_t - \int_0^1 P_{m,t} Y_{m,t} dm$$

Plug the aggregate production technology expression back into the objective function yields,

$$\max_{Y_{m,t}} P_t \Big( \int_0^1 Y_{m,t}^{\frac{\epsilon-1}{\epsilon}} di \Big)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_{m,t} Y_{m,t} dm$$

By taking the first-order condition and performing some algebraic manipulations, we can derive the expression for wholesale goods demand, which is proportional to the demand for final goods and the relative price level.

$$Y_{m,t} = \Big( \frac{P_{m,t}}{P_t} \Big)^{-\epsilon} Y_t$$

By substituting this expression back into the zero-profit condition, we can derive the expression for the price of the retail goods (aggregate price) in terms of the prices of the intermediate goods.

$$P_t = \Big( \int_0^1 P_{m,t}^{1-\epsilon} dm \Big)^{\frac{1}{1-\epsilon}}$$

### 3.3.4 Wholesaler (Intermediate Goods Producer)

Assuming a continuum of intermediate goods firms indexed by $m \in [0,1]$, each firm produces an intermediate good using identical constant-elasticity-of-substitution (CES) technology.

$$Y_{m,t} = A_t \Big\{ \mu U_{m,t}^{\sigma} + (1-\mu) \Big[ \lambda K_{m,t}^{\rho} + (1-\lambda) S_{m,t}^{\rho} \Big]^{\frac{\sigma}{\rho}} \Big\}^{\frac{1}{\sigma}}$$

$$\mu, \lambda \in (0,1); \sigma, \rho \in (-\infty, 1), \sigma, \rho \neq 0$$

The chosen form of the production function allows for capturing capital-skill complementarity, as the elasticity of substitution between capital and skilled labour and the elasticity of substitution between skilled and unskilled labour can be separated. The degree of elasticity of substitution between unskilled labour, physical capital, and skilled labour is determined by parameters $\sigma$ and $\rho$. Specifically, $\frac{1}{1-\sigma}$ represents the elasticity of substitution between capital (or skilled labour) and unskilled labour, while $\frac{1}{1-\rho}$ represents the elasticity of substitution between capital and skilled labour. Consistent with the existing literature on capital-skill complementarity, such as Krusell et al. (2000), Lindquist (2004), Johnson (1997), and Duffy et al. (2004), this study assumes $\frac{1}{1-\sigma} > \frac{1}{1-\rho}$ and $\sigma > \rho$, indicating that capital and skill are complementary.

#### 3.3.4.1 Aggregate Price Dynamics

Following the framework proposed in Calvo (1983), each wholesale firm has the opportunity to reset its price with a probability of $(1-\theta)$ in any given period, regardless of the time elapsed since the last adjustment. Consequently, in each period, a fraction of $(1-\theta)$ of wholesale firms, randomly drawn from the population, are able to reset their prices, while the remaining fraction $\theta$ of firms will maintain their prices unchanged. The aggregate price is shown as follows:[40]

$$P_t^{1-\epsilon} = \int_0^{\theta} P_{t-1}^{1-\epsilon} dm + \int_{\theta}^{1} (P_t^*)^{1-\epsilon} dm$$

$$P_t^{1-\epsilon} = [P_{t-1}^{1-\epsilon}]_0^{\theta} + [(P_t^*)^{1-\epsilon}]_{\theta}^{1}$$

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}$$

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Log-linearize above aggregate wage dynamic expression around the zero inflation steady state to obtain,

$$\hat{\pi}_t^p = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1})^{41} \tag{39}$$

where $\hat{x_t}$ denotes the deviation of $X_t$ from its steady state, i.e., $\hat{x_t} = log(X_t/X)$

#### 3.3.4.2 Optimal Inputs Demand

As pointed out earlier, each wholesale firm produces a single intermediate good with identical constant-return-to-scale CES production technology. Thus, the marginal cost function coincides with the average cost function, and total total cost equals the product of marginal cost times quantity. The first stage of the wholesale firm's problem is to choose the quantity of capital and labour (i.e., both skilled and unskilled labour) inputs to minimize real total cost subject to the production constraint. Because each wholesale firm meets the same cost minimization problem, the choice of capital and labour inputs will be the same among wholesale firms, thus it eliminates the individual firm footscript $m$ in the following equations.

$$\min_{S_t,U_t,K_t} \frac{Q_t}{P_t}K_t + \frac{W_{s,t}}{P_t}S_t + \frac{W_{u,t}}{P_t}U_t$$

s.t.

$$Y_t = A_t\Big\{\mu U_t^{\sigma} + (1-\mu)\Big[\lambda K_t^{\rho} + (1-\lambda)S_t^{\rho}\Big]^{\frac{\sigma}{\rho}}\Big\}^{\frac{1}{\sigma}}$$

The first-order conditions, after some algebraic manipulation are

$$\frac{W_{s,t}}{Q_t} = \frac{(1-\lambda)S_t^{\rho-1}}{\lambda K_t^{\rho-1}} \tag{40}$$

[40] Note that since all wholesale firms resetting prices will choose an identical price, the subscript $m$ is omitted in the following expression.

[41] In zero inflation steady state, $\Pi^p = \frac{P_t}{P_{t-1}} = \frac{P}{P} = 1$, thus $\hat{\pi}_t^p = log(\frac{\Pi_t^p}{\Pi^p}) = log\Pi_t^p = \pi_t^p$

$$\frac{W_{s,t}}{W_{u,t}} = \frac{(1-\mu)[\lambda K_t^{\rho} + (1-\lambda)S_t^{\rho}]^{\frac{\sigma-\rho}{\rho}}(1-\lambda)S_t^{\rho-1}}{\mu U_t^{\sigma-1}} \tag{41}$$

Above equation can be rewritten as

$$\frac{W_{s,t}}{W_{u,t}} = \frac{(1-\mu)(1-\lambda)}{\mu}\Big[\lambda\Big(\frac{K_t}{S_t}\Big)^{\rho} + (1-\lambda)\Big]^{\frac{\sigma-\rho}{\rho}}\Big(\frac{U_t}{S_t}\Big)^{1-\sigma}$$

Krusell et al. (2000) call the first component $\frac{(1-\mu)(1-\lambda)}{\mu}\Big[\lambda\Big(\frac{K_t}{S_t}\Big)^{\rho} + (1-\lambda)\Big]^{\frac{\sigma-\rho}{\rho}}$ as the capital-skill complementarity effect, that is when $\sigma > \rho$ (i.e., capital-skill are complementary), a rise in the ratio of capital to skilled labour input will raise the skill premium. And the second component $\Big(\frac{U_t}{S_t}\Big)^{1-\sigma}$ is called the relative supply effect; that is, a rise in the unskilled to skilled labour input ratio will raise the skill premium for any acceptable values of $\sigma$ and $\rho$.

To derive the real marginal cost function, use Equations 40 and 41 to solve for $K_t$ and $U_t$ as a function of $S_t$, then substitute into production function and let output to be unit to obtain the skilled labour input that produces one unit of output $S_t^*$. Substitute it back to the expression of $K_t$ and $U_t$ to obtain $K_t^*$ and $U_t^*$. Finally, the real marginal cost for each wholesale firm can be expressed as,

$$MC_t = w_{s,t}S_t^* + w_{u,t}U_t^* + q_t K_t^*$$

where

$$w_{s,t} = W_{s,t}/P_t, w_{u,t} = W_{u,t}/P_t, q_t = Q_t/P_t$$

$$S_t^* = A_t^{-1}\Big\{\Omega H_t^{\frac{(\sigma-\rho)\sigma}{\rho(\sigma-1)}}\Big(\frac{w_{u,t}}{w_{s,t}}\Big)^{\frac{\sigma}{\sigma-1}} + (1-\mu)H_t^{\frac{\sigma}{\rho}}\Big\}^{-\frac{1}{\sigma}}$$

$$U_t^* = S_t^*\Big\{\frac{1-\mu}{\mu}H_t^{\frac{\sigma-\rho}{\rho}}(1-\lambda)\frac{w_{u,t}}{w_{s,t}}\Big\}^{\frac{1}{\sigma-1}}$$

$$K_t^* = S_t^*\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{1}{1-\rho}}$$

$$\Omega = \mu\Big(\frac{1-\mu}{\mu}\Big)^{\frac{\sigma}{\sigma-1}}(1-\lambda)^{\frac{\sigma}{\sigma-1}}$$

$$H_t = (1-\lambda) + \lambda\Big(\frac{\lambda w_{s,t}}{q(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}}$$

The optimal demand for each input can be derived from the dual problem; the profit maximization one:

$$\max_{S_{m,t},U_{m,t},K_{m,t}} P_{m,t}Y_{m,t} - W_{s,t}S_{m,t} - W_{u,t}U_{m,t} - Q_t K_{m,t}$$

With $Y_{m,t} = \Big(\frac{P_{m,t}}{P_t}\Big)^{-\epsilon}Y_t$, we can obtain $P_{m,t} = P_t\Big(\frac{Y_t}{Y_{m,t}}\Big)^{\frac{1}{\epsilon}}$, and then the above equation can be expressed as,

$$\max_{S_{m,t},U_{m,t},K_{m,t}} P_t\Big(\frac{Y_t}{Y_{m,t}}\Big)^{\frac{1}{\epsilon}}Y_{m,t} - W_{s,t}S_{m,t} - W_{u,t}U_{m,t} - Q_t K_{m,t}$$

Dividing both sides of the first-order conditions by aggregate price level yield,

$$\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{W_{s,t}/P_t}{MPN_{s,m,t}}$$

$$\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{W_{u,t}/P_t}{MPN_{u,m,t}}$$

$$\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{Q_t/P_t}{MPK_{m,t}}$$

Since $\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon-1} MC_t$, the first-order conditions can be rewritten as follow, that is the real wages and rental costs are simply given by the multiplication product of marginal cost and the marginal product of labour and capital.

$$\frac{W_{s,t}}{P_t} = MC_t A_t \Big\{ \mu U^{\sigma}_{m,t} + (1-\mu) \Big[ \lambda K^{\rho}_{m,t} + (1-\lambda) S^{\rho}_{m,t} \Big]^{\frac{\sigma}{\rho}} \Big\}^{\frac{1-\sigma}{\sigma}} (1-\mu) \Big[ \lambda K^{\rho}_{m,t} + (1-\lambda) S^{\rho}_{m,t} \Big]^{\frac{\sigma-\rho}{\rho}} (1-\lambda) S^{\rho-1}_{m,t} \tag{42}$$

$$\frac{W_{u,t}}{P_t} = MC_t A_t \Big\{ \mu U^{\sigma}_{m,t} + (1-\mu) \Big[ \lambda K^{\rho}_{m,t} + (1-\lambda) S^{\rho}_{m,t} \Big]^{\frac{\sigma}{\rho}} \Big\}^{\frac{1-\sigma}{\sigma}} \mu U^{\sigma-1}_{m,t} \tag{43}$$

$$\frac{Q_t}{P_t} = MC_t A_t \Big\{ \mu U^{\sigma}_{m,t} + (1-\mu) \Big[ \lambda K^{\rho}_{m,t} + (1-\lambda) S^{\rho}_{m,t} \Big]^{\frac{\sigma}{\rho}} \Big\}^{\frac{1-\sigma}{\sigma}} (1-\mu) \Big[ \lambda K^{\rho}_{m,t} + (1-\lambda) S^{\rho}_{m,t} \Big]^{\frac{\sigma-\rho}{\rho}} \lambda K^{\rho-1}_{m,t} \tag{44}$$

#### 3.3.4.3 Optimal Price Setting

Similar to optimal wage setting, each monopolistic wholesale firm is assumed to possess market power when determining the prices for the intermediate goods it produces. In this context, a wholesale firm indexed by $m$ chooses the price $P_{m,t}$ in period $t$ to maximize the present value of real profits generated while the price remains in effect.

$$\max_{P_{m,t}} \sum_{k=0}^{\infty} (\beta\theta)^k E_t \Big\{ \Big( \frac{P_{m,t}}{P_{t+k}} - MC_{t+k} \Big) Y_{m,t+k} \Big\}$$

s.t.

$$Y_{m,t+k} = \Big( \frac{P_{m,t}}{P_{t+k}} \Big)^{-\epsilon} Y_{t+k}$$

Denote the solution as $P^*_{m,t}$ and the FOC is,

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \Big\{ \Big( \frac{P^*_{m,t}}{P_{t+k}} - \frac{\epsilon}{\epsilon - 1} MC_{t+k} \Big) Y_{m,t+k} \Big\}^{42}$$

Note that all wholesale firms that reset their prices apply the same markup on the same marginal cost. Therefore, in all periods, $P^*_{m,t}$ represents the identical price for the $1-\theta$ firms that reset their prices. In the subsequent derivations, the subscript $m$ is omitted. By log-linearizing the aforementioned first-order

[42] In basic monopolist competition, there is no price stickiness, all terms with $k > 0$ are zero, the equation is reduced to $\frac{P^*_{m,t}}{P_t} = \frac{\epsilon}{\epsilon-1} MC_t$. Without price stickiness, all wholesale firms can freely adjust prices, thus $P^*_{m,t} = P_t$, the first-order condition can be further reduced to $MC = \frac{\epsilon-1}{\epsilon}$.

condition around the steady state of flexible prices (i.e., the basic monopolistic competition case), we obtain the following expression (after some algebraic manipulation):

$$\hat{p}_t^* - \hat{p}_{t-1} = \beta\theta Et(\hat{p}_{t+1}^* - \hat{p}_t) + \pi_t^p + (1-\beta\theta)\hat{mc}_t$$

Combining with equation 39 yields the price inflation equation,

$$\hat{\pi}_t = \beta E_t\{\hat{\pi}_{t+1}\} + \frac{(1-\beta\theta)(1-\theta)}{\theta}\hat{mc}_t$$

### 3.3.5 Stochastic Process

The total factor productivity $A_t$ is assumed to follow $AR(1)$ process in logs,

$$ln(A_t) = \rho_A ln(A_{t-1}) + e_t$$

where $e_t \sim \mathcal{N}(\mu, , \sigma_e^2)$ represents random shocks to the total factor productivity process, which induces unexpected changes. The parameter $\rho_A$ captures the persistence associated with these random shocks. It is assumed that the productivity shocks are stationary, satisfying the condition $|\rho_A| < 1$.

### 3.3.6 Monetary Policy

Monetary authority adjust short term nominal interest rates in accordance with the standard Taylor rule:

$$\frac{R_{t+1}}{R} = \left(\frac{R_t}{R}\right)^{\gamma_R}\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y}\right)^{\gamma_y} \tag{45}$$

### 3.3.7 Closing the Model

In the model setup, final good produced by the retailer is used for private consumption and investment of skilled and unskilled households, thus,

$$Y_t = C_{s,t} + C_{u,t} + I_{s,t} + I_{u,t} \tag{46}$$

where $C_{s,t}$ and $C_{u,t}$ are aggregate consumption index given by,

$$C_{s,t} = \left(\int_0^1 C_{s,i,t}di\right)^{\frac{\epsilon}{\epsilon-1}} \tag{47}$$

$$C_{u,t} = \left(\int_0^1 C_{u,j,t}dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{48}$$

And $I_{s,t}$ and $I_{u,t}$ are aggregate investment index with the same as above aggregation technology. The associated capital law of motion defined as,

$$K_{s,t+1} = (1-\delta)K_{s,t} + I_{s,t} \tag{49}$$

$$K_{u,t+1} = (1-\delta)K_{u,t} + I_{u,t} \tag{50}$$

And the aggregate capital input used by intermediate good firm is the sum of capital rented by skilled and unskilled households

$$K_t = K_{s,t} + K_{u,t} \tag{51}$$

The complete derivation of the models can be found in Appendix A.

## 3.4 Calibration

The model is calibrated using annual US data. The parameter values associated with the model are either commonly referred to in existing literature or set to replicate empirical findings at the steady state. The time discount factor is set to $\beta = 0.99$, which corresponds to an approximate 4 percent steady-state return on financial assets. The risk aversion parameter is set to $\sigma_c = 1$, and the Frisch elasticity of labour supply is set to $\phi = 1$, following Galí (2007). Based on data from the CPS MORG survey, it is observed that approximately 21 percent of workers have a college education, while the remaining 69 percent are unskilled workers. Therefore, the input share of unskilled workers is calibrated as $\mu = 0.62$, and the input share of capital is set to $\lambda = 0.8$, as in Belianska (2020). The depreciation rate of equipment is chosen as $\delta = 0.125$, following Greenwood et al. (1997).

For the substitution parameters, the elasticity of substitution between skilled labour and capital is set to $\rho = -0.495$, while the elasticity of substitution between unskilled labour and capital (or skilled labour) is set to $\sigma = 0.401$. These parameter values are estimated by Krusell et al. (2000) and are commonly used in the existing literature. With these substitution parameters, the elasticity of capital to skilled labour is $\frac{1}{1-\rho} = 0.67$, and the elasticity of capital to unskilled labour is $\frac{1}{1-\sigma} = 1.67$. Regarding the interest rate rule coefficients, an interest rate smoothing parameter $\gamma_R$ is set to 0.9, and we set $\gamma_y = 0.3$, and $\gamma_\Pi = 1.5$, as in Belianska (2020).

Next, the elasticity of substitution between differentiated intermediate goods is set to $\epsilon = 6$, following Galí (2007). The elasticity of substitution between differentiated labour services is set to $\epsilon_w = 5$. The degree of price stickiness is generally high, aligning with the range of estimates found in U.S. data (see, e.g., Galı and Gertler (1999), Galı et al. (2001), Sbordone (2002), Eichenbaum and Fisher (2004), Ravenna and Walsh (2006), Dennis (2006)). Consistent with empirical evidence and following Galí (2007), the price stickiness parameter is assumed to be $\theta = 0.698$. According to Parker and Vissing-Jorgensen (2010), the wages of top-earning workers tend to be more sensitive to technology shocks. Therefore, in line with this finding, we assume asymmetric wage rigidity, where unskilled wages are more sticky. Specifically, we set $\theta_w^s = 0.65$ and $\theta_w^u = 0.8$, following the parameter values used in the calibration conducted by Belianska (2020). For the exogenous technology process, a persistence value of 0.95 is chosen, with an average standard deviation set to 0.01248. The calibrated values of the model parameters are summarized in Table 44.

## 3.5 Results

This paper's goal is to develop a stochastic dynamic general equilibrium model that includes capital-skill complementarity in production, Calvo pricing, and nominal wage rigidity. The model's primary purpose is to elucidate the observed negative response in the skilled-to-unskilled working hours ratio and the differing responses in working hours between skilled and unskilled labour. We simulate four distinct scenarios: a generalized Real Business Cycle (RBC) model, a frictionless monopolistic competition model, a generalized

| Parameter | Value | Description |
|---|---|---|
| $\sigma$ | 0.41 | Elasticity substitution between capital and unskill |
| $\rho$ | -0.495 | Elasticity substitution between capital and skill |
| $\beta$ | 0.99 | Time discount factor |
| $\lambda$ | 0.8 | Input share of capital |
| $\mu$ | 0.62 | Input share of unskill |
| $\delta$ | 0.125 | Capital depreciation rate |
| $\phi$ | 1 | Frisch lasticity of labour supply |
| $\sigma_c$ | 1 | Risk aversion parameter |
| $\epsilon$ | 6 | Elasticity of substitution between intermediate goods |
| $\epsilon_w$ | 5 | Elasticity of substitution between labour services |
| $\theta$ | 0.698 | Price rigidities |
| $\theta_w^s$ | 0.65 | Degree of wage rigidities for skilled workers |
| $\theta_w^u$ | 0.8 | Degree of wage rigidities for unskilled workers |
| $\rho_A$ | 0.95 | AR(1) parameter |
| $\gamma_R$ | 0.9 | Interest rate smooting |
| $\gamma_\pi$ | 1.5 | Taylor-coefficient on inflation |
| $\gamma_y$ | 0.3 | Taylor-coefficient on output |

Table 44: Parameter calibration

model with Calvo pricing (based on Calvo, 1983), and a model that integrates both Calvo wages and price settings.

Our findings indicate that the generalized model with Calvo pricing effectively explains both the negative reaction in the skilled-to-unskilled hours ratio and the divergent responses in hours worked by the two labour types. The other three models, while capable of accounting for the negative hours ratio response, do not fully address the distinct directional responses of skilled and unskilled labour hours.

Given the limited literature on incorporating a Constant Elasticity of Substitution (CES) production function with different types of labour, our analysis of the generalized RBC and monopolistic competition models provides benchmark results for our proposed model. Furthermore, we explore a model that combines a CES production function with differentiated labour types, Calvo pricing, and wage rigidity. To our knowledge, this is the first instance of constructing a model with these specific features.

### 3.5.1 Calvo Pricing

Figure 22 presents the results for the generalized model with Calvo pricing. One notable distinction from the standard model with Calvo pricing is the adoption of a CES production function with three factors instead of the Cobb-Douglas. In addition, we incorporate the Calvo pricing mechanism for intermediate goods. The absence of continuous price adjustments by firms implies that the average markup between the real prices of inputs and the marginal products of inputs in the economy will vary over time. Consequently, the markup will generally deviate from the constant frictionless markup observed in the monopolistic competition case. [43]. When considering the distortion caused by sticky prices, the markup is defined as the product of the real marginal cost and the inverse of the frictionless markup. This inefficiency can result in either higher or lower

[43] In the case of monopolistic competition, the frictionless markup depends on the elasticity of substitution between the differentiated intermediate goods, given by $-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{\epsilon-1}{\epsilon} MPN_t$. It is important to note that in equilibrium, the marginal rate of substitution $-\frac{U_{n,t}}{U_{c,t}}$ and the marginal product of labour exhibit increasing and decreasing (or non-increasing) patterns, respectively, with respect to hours. The presence of markup distortion in this setting results in an inefficiently low level of hours and output, i.e., $-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{\epsilon-1}{\epsilon} MPN_t < MPN_t$

levels of output and employment compared to the efficient levels observed in the RBC model, depending on the specific value of the markup. [44]

In Figure 22, advancements in technology trigger an initial period of heightened investment, subsequently resulting in an increase in capital. This increase in investment offsets the decrease in consumption and contributes to a boost in output. When examining the impact of technological improvements on consumption patterns across different skill levels, we observed a distinct divergence. Specifically, in response to technological advancements, consumption among highly skilled individuals increases, while it decreases for those with lower skills in the short term. This phenomenon can be largely attributed to the effects of wealth redistribution. Due to capital-skill complementarity, new technologies often replace tasks traditionally performed by unskilled labour, while simultaneously enhancing the efficiency of skilled workers. Consequently, firms increasingly prefer skilled labour, as the marginal productivity of skilled workers is amplified. This shift leads to higher wages for skilled workers and a decrease in wages for their unskilled counterparts. Unskilled workers, facing an immediate drop in income, might also contend with uncertainties regarding their future earnings. This fear, especially if it extends to concerns about long-term unemployment or further wage cuts, can result in a temporary reduction in their consumption levels. Conversely, skilled workers benefit from a boost in income, enhancing their financial security and expectations about future earnings. This increased sense of financial stability, in turn, leads to an increase in their consumption in the short run.

The response of hours worked for skilled and unskilled workers is in opposite directions. Skilled hours experience a sharp decline immediately and subsequently display a steady increase over time before reverting to steady-state level. Conversely, unskilled hours initially increase and then revert back to the steady-state level. This short-term decline in skilled hours and increase in unskilled hours lead to a pronounced decrease in the ratio of skilled to unskilled hours. The short-run responses of hours can be explained by the combined effects of three factors. Firstly, the Capital-Skill Complementarity (CSC) effect plays a role: a positive technology shock enhances the productivity of skilled workers while reducing that of unskilled workers. This leads skilled workers to complete their workload more efficiently, resulting in a decrease in skilled hours in the short run. Conversely, the lower productivity of unskilled workers leads to an increase in unskilled hours. Secondly, the higher marginal productivity of skilled workers relative to unskilled workers incentivizes firms to prefer skilled workers over unskilled workers, leading to a substitution effect from unskilled to skilled workers. This effect contributes to an increase in skilled wages and a decrease in unskilled wages in the short run. Thirdly, with heightened productivity, firms are capable of manufacturing at reduced costs, potentially resulting in lower prices. However, due to nominal rigidity, there might be a delay or incomplete adjustment in prices to reflect this increased productivity. As a consequence, firms require fewer labour inputs to maintain the same level of output. In the short run, this situation could lead to the layoff of skilled workers, as they are not only more productive but also command higher wages, making them costlier for firms to employ. The proposition of a decline in skilled employment in the initial period is consistent with empirical findings that demonstrate a decrease in the employment of skilled workers, as depicted in Figure 20. The negative impact of price stickiness and CSC on skilled hours offsets the positive substitution effect, resulting in a decrease in total hours worked by skilled workers in the initial period. Conversely, the positive CSC

[44] As demonstrated in Galí (2007), with the presence of sticky prices, we have $-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{\epsilon}{\epsilon-1} MC_t MPN_t$, which violates the efficiency condition in the RBC case, i.e., $-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t$, as long as $\frac{1}{MC_t} \neq \frac{\epsilon}{\epsilon-1}$. Once again, in equilibrium, the marginal rate of substitution $-\frac{U_{n,t}}{U_{c,t}}$ and the marginal product of labour exhibit increasing and decreasing (or non-increasing) patterns, respectively, with respect to hours. The level of output and employment can be either larger or smaller than the efficient level, depending on the value of $\frac{\epsilon}{\epsilon-1} MC_t$. This discrepancy could lead to either excessively low or excessively high levels of aggregate hours and output.

and negative substitution effect on unskilled hours contribute to an increase in unskilled hours in the short run. In the subsequent periods, the initial impact of the CSC effects diminishes as the change in marginal productivity of labour approaches its limit. The substitution effect gradually becomes the dominant factor, leading to an increase in skilled hours and a decrease in unskilled hours. As skilled hours increase, the marginal product of skilled labour declines, resulting in a decrease in the skilled wage, while the unskilled wage experiences the opposite response.

In our findings, the initial decrease in skilled hours and rise in unskilled hours lead to a rise in the capital-to-skilled labour input ratio and a decline in the ratio of skilled to unskilled hours. These observations indicate the presence of both the capital-skill complementarity effect and the relative supply effect, both acting in tandem to enhance the skill premium. Table 45 presents a summary of the point estimates for the impact responses of labour market variables obtained from the data compared to the responses computed from our generalized model with Calvo pricing. Overall, the model aligns closely with the data. However, there are some discrepancies observed. Specifically, the model tends to overestimate the increase in skill wage while underestimating the decline in unskilled wage. Consequently, this leads to an overestimation of the rise in the skill premium.

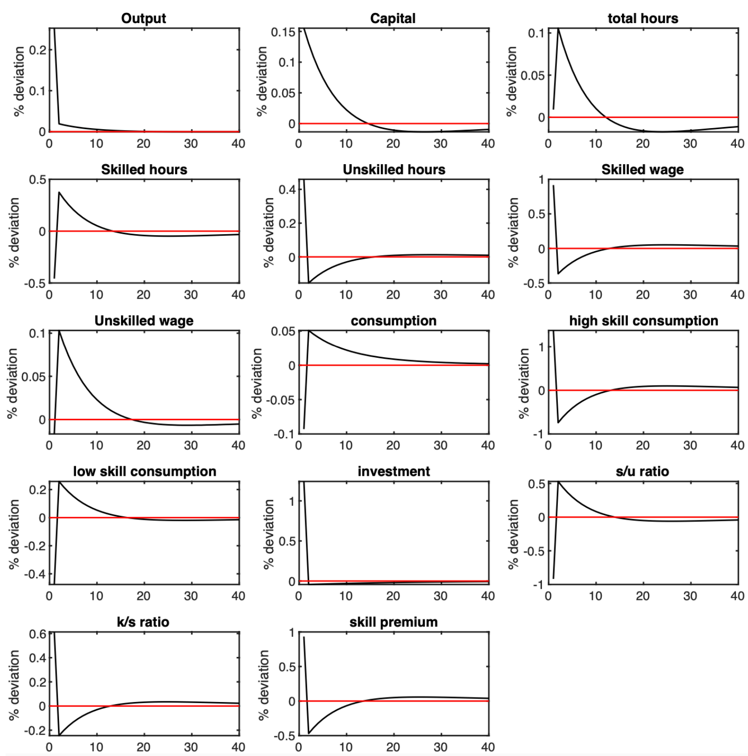


Figure 22: Impulse response functions to utilisation-controlled technology shock in the generalized model with Calvo (1983) pricing

| | data | model |
|---|---|---|
| Skill wage | 0.270 | 0.917 |
| Unskill wage | 0.051 | -0.017 |
| Skill hours worked | -0.897 | -0.458 |
| Unskill hours worked | 0.449 | 0.460 |
| Skill premium | 0.686 | 0.933 |
| Ratio of skilled to unskilled hours worked | -1.391 | -0.917 |
| Ratio of capital to skilled hours worked | 0.637 | 0.613 |

Table 45: Point estimates; model compared to the data

In a standard New Keynesian model featuring Calvo pricing, the response to a positive technology shock may present complex dynamics for key macroeconomic variables. Hours worked often initially decline due to increased productivity allowing the same level of output with less labour. Output typically increases as the economy becomes more productive. However, due to sticky prices, the immediate response may be muted compared to models without such frictions. Consumption generally rises as households anticipate higher future incomes from the productivity increase. Investment can also increase as firms anticipate greater profitability from more efficient production processes and may invest in new technologies and capital. However, these responses can vary based on the specific assumptions of the model, particularly how expectations are formed and the degree of price rigidity. These responses have been discussed in the literature, including in Ireland (2004), Jung (2022), Dedola and Neri (2007).

Our model's results largely align with the standard New Keynesian model's predictions, except it diverges in the initial muted response of output to a technology shock. The standard model suggests firms may reduce their workforce due to lower equipment costs and higher productivity if demand does not increase sufficiently. In contrast, our results show that firms may lay off some high-skilled workers due to price stickiness, while the employment of low-skilled workers is not as affected. Furthermore, the increased productivity of high-skilled workers may counterbalance the initial muted output. This response could be compared to findings in existing works such as those by Gali (1999), Peersman and Straub (2006), Smets and Wouters (2003), Christiano et al. (2005), which analyze the implications of technology shocks in New Keynesian frameworks. Furthermore, our model exhibits a divergence in the initial aggregate consumption response compared to that of the standard New Keynesian model. As described in the previous paragraph, our analysis examines in detail how consumption among skilled and unskilled workers responds to technological improvements. The initial aggregate responses of consumption are influenced by the consumption reactions of each type of labour and also depend on the proportion of skilled to unskilled labour within the total employment. Therefore, incorporating different skills level into the model may result in a difference in aggregate consumption response in the initial period compared to the standard model.

# 4 Extensions

## 4.1 Generalized RBC and Monopolist Competition Models

Considering the scarce research on integrating a Constant Elasticity of Substitution (CES) production function that includes various kinds of labour, our examination of the extended Real Business Cycle (RBC) and monopolistic competition models delivers foundational outcomes that set the standard for our suggested model. In the generalized RBC model, we assume two representative households that maximize the weighted sum of expected utilities by choosing consumption, labour supply, and capital given prices and wages. Additionally, we consider a representative firm that maximizes its real profit by selecting labour demand for two types of workers and capital, given the price level. This frictionless model does not incorporate distortions associated with monopolistic competition or staggered price and wage settings. Consequently, the marginal rate of substitution is equal to the marginal product of labour, and the marginal product of capital corresponds to the real rental costs. The key distinction between the RBC model used in this paper and the standard Real Business Cycle model is the extension of the production function. Instead of employing a Cobb-Douglas production function, our model incorporates a more general three-factor production function that accounts for capital-skill complementarity.

Figure 23 displays the impulse response results of a positive technology shock in the generalized RBC model. The findings suggest that the shock results in higher wages for both skilled and unskilled workers. This wage boost, in turn, encourages workers to contribute more hours, leading to elevated levels of output and an increased capital stock. With the rise in income, both skilled and unskilled workers have increased their consumption, and the technological improvements have further stimulated investment. Notably, the response of skilled hours is relatively smaller than that of unskilled hours in the initial period, leading to a slight decrease in the skilled to unskilled hours ratio.

These outcomes can be attributed to three key factors. Firstly, the positive technology shock stimulates higher demand for production inputs, which consequently raises the demand for both skilled and unskilled hours. Secondly, the presence of capital-skill complementarity (CSC) influences the response. Specifically, the shock enhances the productivity of skilled workers but reduces that of unskilled workers. This productivity differential leads to a decrease in skilled hours in the short run, as skilled workers can accomplish their workload more efficiently with fewer hours. Conversely, unskilled workers require more hours to complete their tasks. Thirdly, the higher marginal productivity of skilled workers relative to unskilled workers prompts firms to substitute unskilled workers with skilled workers.

In the short run, the dominance of the capital-skill complementarity (CSC) effect over the substitution effect is observed due to the inefficiency in reallocating employment between the two skill types. The negative CSC effect on skilled hours and the positive CSC effect on unskilled hours are unable to fully offset the positive impact of the technology shock. Consequently, the combined effect leads to a smaller increase in skilled hours compared to unskilled hours, ultimately resulting in a mild decline in the skilled to unskilled hours ratio in the initial period. Over time, the initial impact of the CSC effects diminishes as the changes in marginal productivity approach their limits. The substitution effect gradually becomes the dominant factor, resulting in an increase in skilled hours and a decrease in unskilled hours in subsequent periods. Additionally, the higher demand for skilled workers leads to a greater increase in their wages compared to unskilled workers, contributing to an overall rise in the skill premium.

Additionally, as shown by Krusell et al. (2000), the skill premium can be expressed as follows:

$$\frac{w_{s,i,t}}{w_{u,i,t}} = \frac{(1-\mu_i)(1-\lambda_i)}{\mu_i}\left[\lambda_i\left(\frac{K_{i,t}}{S_{i,t}}\right)^{\rho_i} + (1-\lambda_i)\right]^{\frac{\sigma_i-\rho_i}{\rho_i}}\left(\frac{S_{i,t}}{U_{i,t}}\right)^{\sigma_i-1}$$

Given that $\sigma < 1, \mu < 1, \lambda < 1$, an increase in the ratio of skilled to unskilled hours ($S/U$) will, ceteris paribus, result in a decrease in the skill premium for any acceptable values of $\sigma$. [45] This is referred to as the relative supply effect by Krusell et al. (2000). Furthermore, if $\sigma > \rho$, an increase in the ratio of capital

[45]Positive elasticity of substitution between capital (or skilled labour) and unskilled labour ensures $1/(1-\sigma_i) > 0$, and we have $\sigma < 1$.

stock to skilled hours $K/S$ will, ceteris paribus, result in an increase in the skill premium, which is referred to as the capital-skill complementarity effect. Our findings indicate that the positive response of the ratio of capital to skilled hours worked and the negative skilled to unskilled hours ratio align with the relative supply effect and the capital-skill complementarity effect, respectively. These effects work together in the same direction and contribute to an increase in the skill premium.

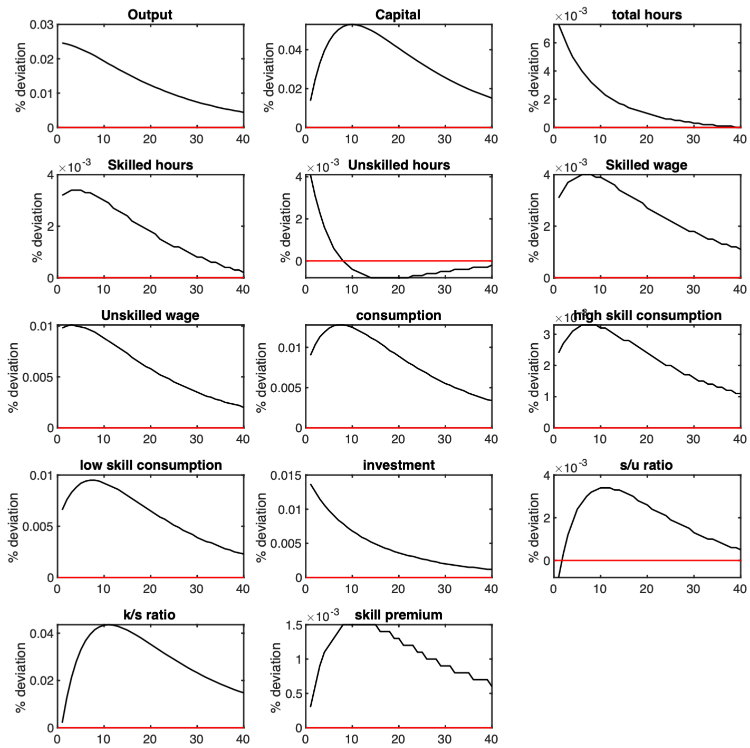


Figure 23: Impulse response functions to utilisation-controlled technology shock in generalized RBC model

The impulse response functions for the monopolistic competition model are presented in Figure 24. In this model, the firm side is characterized by a continuum of wholesalers and a retailer. The retailer operates in a perfectly competitive market and utilises intermediate goods as inputs to produce final goods. On the other hand, wholesalers operate in a monopolistic competition market, with each wholesaler producing a unique intermediate good and selling it to the retailer. We keep assuming there is no staggered price setting. Consequently, each intermediate goods firm can freely adjust the prices of its goods in each period. The presence of market power gives rise to a markup between the real wages and the marginal product of labour, as well as between the real costs of capital and the marginal product of capital. This markup distortion results in an inefficient level of output. In Figure 24, we observe that the responses of all variables exhibit similar patterns to those in Figure 23. However, due to the inefficiency induced by market power distortion, the level of output and capital in the monopolistic competition model is lower compared to the RBC model.

Our findings correspond closely with the predictions of the standard Real Business Cycle (RBC) model and the monopolistic competition framework, with the distinctive aspect being our consideration of the dynamics across two distinct skill levels. These two models do not adequately explain the differing responses of hours worked by various skill levels to the technology shock, in contrast to the Calvo pricing model discussed in the previous section. The main objective of analyzing these two models is to establish benchmark results for the DSGE model that incorporates varying skill levels.

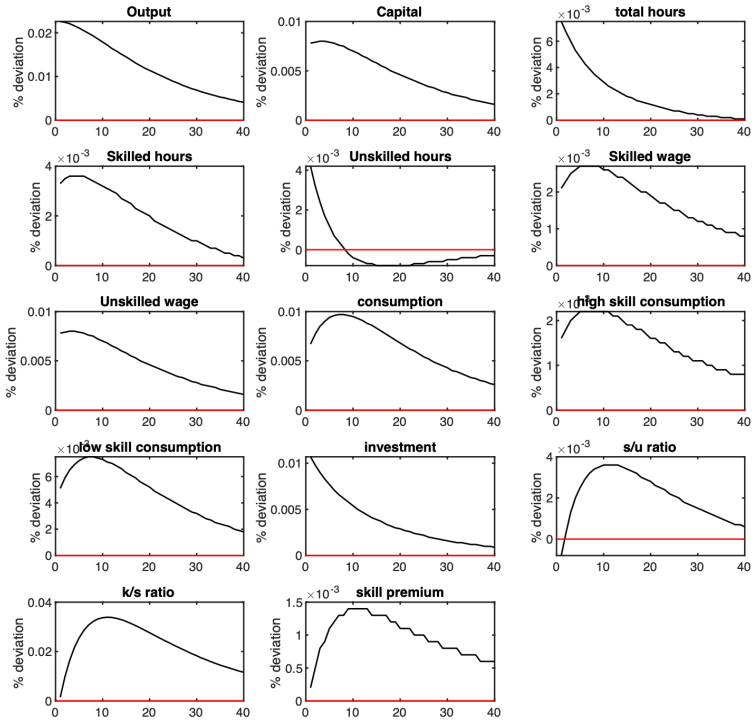


Figure 24: Impulse response functions to utilisation-controlled technology shock in the monopolist competition model

## 4.2 Calvo Pricing and Wages

Next, we investigate a model that merges a CES production function with differentiated labour types, Calvo pricing, and inflexible wages. As far as we are aware, this represents the initial effort to develop a model encompassing these particular characteristics. Instead of the representative household assumed in the generalized model with Calvo pricing, we assume two types of infinitely lived agents supply differentiated labour services, and workers can reset their wages following the Calvo-wage setting. The primary distinction between our model and the prevailing DSGE model incorporating sticky prices and nominal wage rigidity (see, for example, Lin and Weise (2019), Calvo (1983)) lies in the explicit differentiation of labour supply into two distinct skill levels. Additionally, we introduce varying degrees of wage stickiness for each type of worker. Figure 25 represents the impulse response results.

As the shocks propagate throughout the economy, the staggered wage adjustments create delays in the impact responses of both skilled and unskilled wages to the technology shock. In comparison to the sticky price model, the initial period of adjustment reveals a mild increase in skilled wages, while unskilled wages largely remain at the steady-state level. Notably, due to the relatively lower stickiness of skilled wages compared to unskilled wages, the former exhibit relatively higher sensitivity to the shocks. This discrepancy in wage stickiness considerably impacts the influence on skilled wages, exceeding the effects observed on unskilled wages. Specifically, skilled wages exhibit a substantial reduction, while unskilled wages experience minimal changes during the initial period, in contrast to the levels observed in the sticky price model. In the subsequent periods, wages begin to adjust, with skilled wages surging to their peak and unskilled wages declining towards their lowest point, before the responses converge towards patterns similar to those observed in the sticky price model.

The staggered nature of wage adjustments contributes to a delay in the increase of output and investment, as firms may hesitate to expand production or invest in new projects until they have a clearer understanding of labour costs and overall economic conditions. Furthermore, there is an observable decline in aggregate consumption in the short run, attributed to reduced spending among both skilled and unskilled workers. For low-skilled workers, the lack of initial wage change heightens income uncertainty, leading to a reduction in consumption. In the case of high-skilled workers, the staggered nature of wage adjustments results in only a modest increase in their wages. Our model suggests that this leads to a decrease in their consumption in the initial period. This prediction, however, may not fully align with standard New-Keynesian thinking. Typically, a modest wage increase is not expected to result in decreased consumption among skilled workers. Instead, it might lead to a slower growth in consumption due to uncertainties about future income and the stability of these wage increases.

In the standard New-Keynesian framework, the consumption response of high-skilled workers to wage changes typically does not entail a decrease, especially when wages increase, albeit modestly. The usual expectation is for their consumption to either rise, albeit less than anticipated, or remain steady. The divergence of our model's predictions from these standard expectations suggests an area for further exploration. Therefore, the nuances in the consumption patterns of high-skilled workers in response to staggered wage adjustments, as predicted by our model, present an interesting avenue for future research.

The staggered nature of wage also have impact on hours worked. In the initial period, there is a notable reduction in the wage gap between skilled and unskilled workers compared to the previous model. As a result, skilled workers become relatively more affordable, leading to a more pronounced substitution of unskilled workers with skilled workers. This increased positive substitution effect on skilled hours counteracts the negative impacts of sticky prices and the capital-skill complementarity (CSC) effect on skilled hours. Consequently, the net effect is a mild increase in skilled hours in the short run instead of a substantial decline as observed in the previous sticky price model. On the other hand, as more unskilled workers have been substituted away, the level of unskilled hours decreases in comparison to the level observed in the sticky price model. The presence of nominal wage rigidity diminishes the disparity in hours worked and wages between skilled and unskilled workers in the short run. Consequently, the decline in the ratio of skilled to unskilled hours worked and the increase in the skill premium are less pronounced compared to the outcomes observed in the sticky price model.

Our study parallels Lin and Weise (2019), who developed a New Keynesian model incorporating a CES production function that includes both human labour and robotics. While our model mirrors Lin's in examining the impact of wage rigidity on the allocation between diverse production inputs, our findings diverge significantly in their implications. Lin and Weise (2019) observed that wage rigidity, alongside a reduction in overall price levels, leads to higher real wages and lower rental rates, thus making human labour more expensive and incentivizing a shift towards robotics, thereby increasing the robot-to-human labour ratio. Contrarily, our analysis reveals that technological improvements, when coupled with short-term wage rigidity, do not elevate skilled labour wages as expected, rendering skilled labour more cost-effective relative to its prior state. This dynamic fosters a pronounced transition from unskilled to skilled labour, unveiling a distinct mechanism of labour substitution. Despite these discrepancies, a conceptual connection exists between our results and Lin's, underscoring how wage adjustments can influence labour substitution across different scenarios and contexts.

To the best of our knowledge, we are the pioneers in developing a DSGE model that integrates a CES production function with two types of labor skills, along with Calvo pricing and wage setting. The establishment of this model enables a more thorough analysis of business cycle outcomes, particularly regarding the responses of labor hours across different skill levels. However, this model falls short of replicating the empirical results it seeks to explain in this chapter.

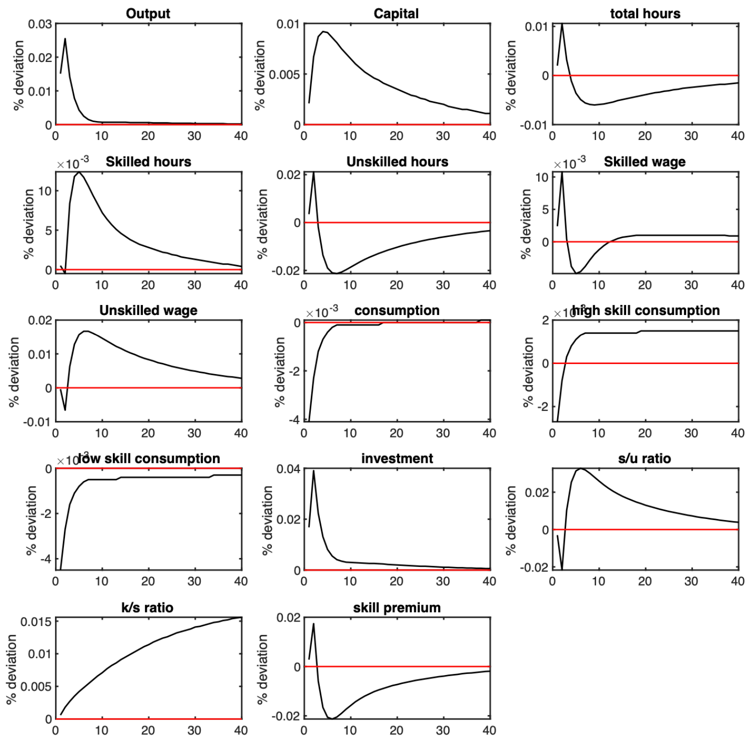


Figure 25: Impulse response functions to utilisation-controlled technology shock in the model with Calvo wages and Calvo prices

# 5 The Role of Capital-Skill Complementarity

The model incorporates capital-skill complementarity by representing it through the elasticity of substitution between capital and skilled labour, denoted as $\frac{1}{1-\rho}$. Figure 26 illustrates the responses of skilled and unskilled hours worked, as well as the skilled to unskilled hours ratio, to technological advancements in sticky price model. These responses are examined by varying the aforementioned elasticity. The benchmark calibration of the elasticity of capital and skilled labour is $\frac{1}{1-\rho} = 0.668$, with $\rho = -0.495$, while the elasticity of capital and unskilled labour is $\frac{1}{1-\sigma} = 1.69$, with $\sigma = 0.41$. We examine two additional cases: strong complementarity with $\frac{1}{1-\rho} = 0.37$ and $\rho = -1.7$ (indicated by the red dotted line with cross), and weak complementarity with $\frac{1}{1-\rho} = 1.14$ and $\rho = 0.12$ (represented by the blue dashed line). The corresponding impulse response functions (IRFs) for these cases are plotted in Figure 26 alongside the benchmark model for comparison.

A higher degree of capital-skill complementarity i.e., weak elasticity of substitution between capital and skilled labour, induces a more substantial substitution effect from unskilled to skilled workers, reflecting firms' preference for skilled workers over unskilled workers. This heightened positive substitution effect on skilled hours counteracts the negative influence of sticky prices and increased working efficiency on skilled hours, as observed in the benchmark case. Consequently, the initial period following the technology shock witnesses a mild increase in skilled hours, deviating from the initial decline observed in the benchmark case. Conversely, the higher degree of capital-skill complementarity leads firms to substitute more unskilled workers away, resulting in a smaller increase in unskilled hours compared to the benchmark case in the initial period. The narrowing gap in the initial responses between hours worked for the two skill levels results in a reduced decline in the skilled to unskilled hours ratio during the initial period following the technology shock. Similarly, the case with a lower degree of capital-skill complementarity exhibits notable similarities. Specifically, in response to technological advancement, a lower degree of substitution effect from unskilled hours to skilled hours leads to a more pronounced decrease in skilled hours and a greater increase in unskilled hours on impact. The widened gap in the initial responses of hours worked for the two labour types further contributes to a more substantial decline in the skilled to unskilled hours ratio, contrasting with the benchmark model.

In the subsequent period, the magnitude of the increase in skilled hours is more pronounced for the case of weak capital-skill complementarity and comparatively weaker for the case of strong capital-skill complementarity. One possible explanation is that stronger capital-skill complementarity enhances the productivity of skilled workers. Consequently, firms require a lesser number of skilled workers to accomplish production tasks compared to the benchmark case. Conversely, in situations with lower capital-skill complementarity, firms necessitate a greater number of skilled workers to fulfill the workload, as skilled workers are not as productive as in the benchmark case. This leads to a more significant increase in skilled hours relative to the levels observed in the benchmark case.

Figure 27 displays the impulse response functions (IRFs) obtained by varying the elasticity of substitution between capital and skilled labour in the model with sticky prices and sticky wages. Again, the results for strong complementarity are denoted by the red line with a cross, while the results for weak complementarity are represented by the blue dashed line. Notably, the patterns exhibited in the IRFs demonstrate remarkable similarity, as observed in Figure 26. For instance, in the case of a higher degree of capital-skill complementarity, the intensified substitution effects from unskilled workers to skilled workers resulting from stronger capital-skill complementarity lead to a more pronounced increase in skilled hours during the initial period, compared to the benchmark case. Conversely, due to more unskilled hours have been substituted away, unskilled hours experience a smaller increase. Moreover, the narrowing gap in the initial responses between hours worked for the two skill levels contributes to a reduced decline in the skilled to unskilled hours ratio during the initial period. Additionally, the deeper increase in skilled hours observed in the case of weak capital-skill complementarity in the subsequent period, compared to both the benchmark and strong capital-skill complementarity cases, can be explained by similar factors outlined in Figure 26.

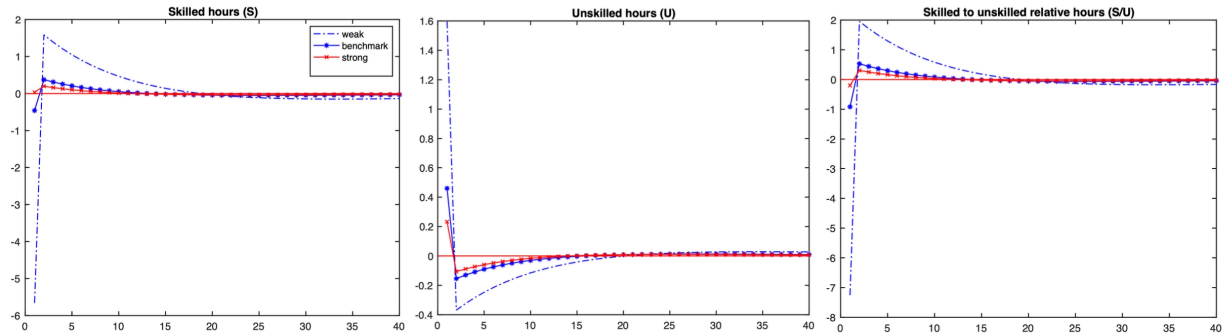


Figure 26: The role of CSC in Calvo prices model

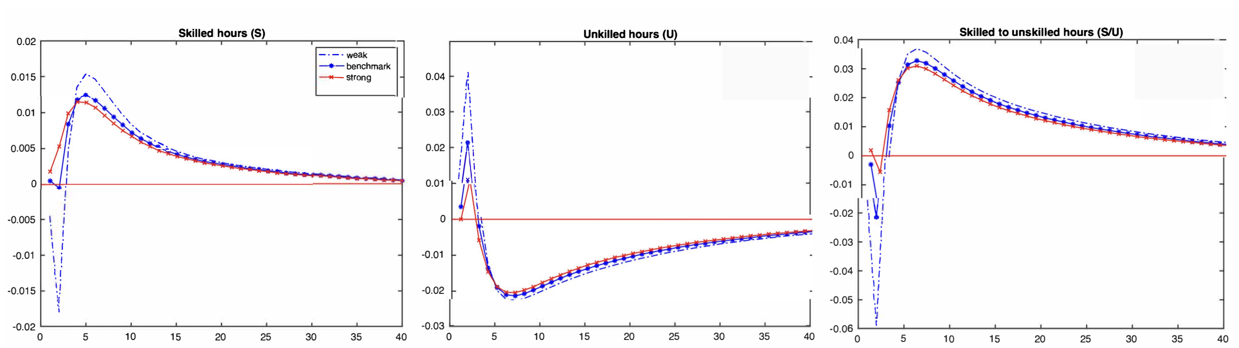


Figure 27: The role of CSC in Calvo prices and Calvo wages model

# 6 Conclusion

The aim of this study is to develop a stochastic dynamic general equilibrium (DSGE) model that integrates capital-skill complementarity in production, Calvo prices, and nominal wage rigidity. The model is designed to offer insights into the negative dynamics observed in the skilled to unskilled working hours ratio and the differential responses of hours worked by skilled and unskilled workers. Four distinct scenarios are simulated, including a generalized Real Business Cycle (RBC) model, a frictionless monopolistic competition model, a generalized model incorporating Calvo (1983) pricing, and a model incorporating Calvo wage and price settings.

We discovered that, with moderate capital-skill complementarity, both the generalized Real Business Cycle (RBC) model and the frictionless monopolistic competition model can elucidate the negative response of the skilled to unskilled hours ratio to technological advancements. The primary explanation lies in the fact that technological advancements are accompanied by increased productivity among skilled workers relative to unskilled workers. As a result, skilled workers can fulfill workloads more efficiently, leading to a smaller increase in skilled hours compared to unskilled hours in response to technological advancements and subsequently resulting in a decline in the skilled to unskilled hours ratio during the initial period. However, we found that these two models fail to explain the divergent responses of hours worked for the two skill levels. In contrast, a generalized model incorporating Calvo (1983) pricing is capable of providing an explanation. Sticky prices induce firms to reduce skilled workers due to both cost considerations and the higher efficiency of skilled workers. In addition, we also consider a model that incorporates Calvo wage and price settings to further enrich our analysis.

By exploring different modeling scenarios, we uncover the varying impacts of technological advancements on hours worked by different skill levels and highlight the importance of sticky prices and wages in explaining these dynamics.

# 7 Appendix A

## 7.1 RBC Model with CES Production Function

### 7.1.1 Representative Households

The maximization of a homogeneous utility function subject to a budget constraint is undertaken by two representative households, representing skilled and unskilled individuals, indexed by $H \in (S, U)$,

$$Max_{H_t, C_{H,t}, K_{H,t+1}, B_{H,t+1}} \left\{ E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_{H,t}^{1-\sigma_c}}{1-\sigma_c} - \frac{H_t^{1+\phi}}{1+\phi} \right\} \right\}$$

$$s.t.$$

$$P_t C_{H,t} + P_t K_{H,t+1} + B_{H,t+1} = W_{H,t} H_t + Q_t K_{H,t} + P_t(1-\delta) K_{H,t} + R_t B_{H,t} + D_t$$

F.O.Cs:

$$C_{H,t} : C_{H,t}^{-\sigma_c} = \lambda_t P_t$$

$$H_t : H_t^{\phi} = \lambda_t W_{H,t}$$

$$K_{H,t+1} : \lambda_{t+1}(Q_{t+1} + P_{t+1}(1-\delta)) = \lambda_t P_t$$

$$B_{H,t+1} : \lambda_{t+1} R_{t+1} = \lambda_t$$

Rearrange above first order conditions for skilled worker gives, i.e., $H = S$,

$$C_{s,t}^{-\sigma_c} = \beta E_t \left\{ C_{s,t+1}^{-\sigma_c} R_{t+1} \frac{P_t}{P_{t+1}} \right\}$$

$$C_{s,t}^{-\sigma_c} = \beta E_t \left\{ C_{s,t+1}^{-\sigma_c} \left( \frac{Q_{t+1}}{P_{t+1}} + (1-\delta) \right) \right\}$$

$$\frac{W_{s,t}}{P_t} = \frac{S_t^{\phi}}{C_{s,t}^{-\sigma_c}}$$

Symmetrically, the first-order conditions for household who supplies unskilled labour are, i.e., $H = U$,

$$C_{u,t}^{-\sigma_c} = \beta E_t \Big\{ C_{u,t+1}^{-\sigma_c} R_{t+1} \frac{P_t}{P_{t+1}} \Big\}$$

$$C_{u,t}^{-\sigma_c} = \beta E_t \Big\{ C_{u,t+1}^{-\sigma_c} \Big( \frac{Q_{t+1}}{P_{t+1}} + (1-\delta) \Big) \Big\}$$

$$\frac{W_{u,t}}{P_t} = \frac{U_t^{\phi}}{C_{u,t}^{-\sigma_c}}$$

### 7.1.2 Representative Firm

A representative firm chooses $S_t, K_t, U_t$ to maximize its profit,

$$Max_{S_t,K_t,U_t} \Big\{ A_t \Big\{ \mu U_t^{\sigma} + (1-\mu) \Big[ \lambda K_t^{\rho} + (1-\lambda) S_t^{\rho} \Big]^{\frac{\sigma}{\rho}} \Big\}^{\frac{1}{\sigma}} - \frac{Q_t}{P_t} K_t - \frac{W_{s,t}}{P_t} S_t - \frac{W_{u,t}}{P_t} U_t \Big\}$$

F.O.Cs:

$$S_t : \frac{W_{s,t}}{P_t} = A_t \Big\{ \mu U_t^{\sigma} + (1-\mu) \Big[ \lambda K_t^{\rho} + (1-\lambda) S_t^{\rho} \Big]^{\frac{\sigma}{\rho}} \Big\}^{\frac{1-\sigma}{\sigma}} \cdot (1-\mu) \Big[ \lambda K_t^{\rho} + (1-\lambda) S_t^{\rho} \Big]^{\frac{\sigma-\rho}{\rho}} \cdot (1-\lambda) S_t^{\rho-1}$$

$$U_t : \frac{W_{s,t}}{P_t} = A_t \Big\{ \mu U_t^{\sigma} + (1-\mu) \Big[ \lambda K_t^{\rho} + (1-\lambda) S_t^{\rho} \Big]^{\frac{\sigma}{\rho}} \Big\}^{\frac{1-\sigma}{\sigma}} \cdot \mu U_t^{\sigma-1}$$

$$K_t : \frac{W_{s,t}}{P_t} = A_t \Big\{ \mu U_t^{\sigma} + (1-\mu) \Big[ \lambda K_t^{\rho} + (1-\lambda) S_t^{\rho} \Big]^{\frac{\sigma}{\rho}} \Big\}^{\frac{1-\sigma}{\sigma}} \cdot (1-\mu) \Big[ \lambda K_t^{\rho} + (1-\lambda) S^{\rho} \Big]^{\frac{\sigma-\rho}{\rho}} \cdot \lambda K_t^{\rho-1}$$

### 7.1.3 Market Clearing and Stochastic Process

$$ln(A_t) = \rho_A ln(A_{t-1}) + e_t$$

$$K_{s,t+1} = (1-\delta) K_{s,t} + I_{s,t}$$

$$K_{u,t+1} = (1-\delta) K_{u,t} + I_{u,t}$$

$$K_t = K_{s,t} + K_{u,t}$$

$$Y_t = C_{s,t} + C_{u,t} + I_{s,t} + I_{u,t}$$

## 7.2 Monopolist Competition with CES production Function

### 7.2.1 Representative Households

The model for representative households in this context aligns with the structure outlined in the previously discussed Real Business Cycle (RBC) model.

### 7.2.2 Final Goods Producer

Final good producer is perfectly competitive and maximize its profits, taking as given intermediate goods prices $P_{m,t}$ and the final good price $P_t$. Thus, its maximization problem is,

$$\max_{Y_{m,t}} P_t Y_t - \int_0^1 P_{m,t} Y_{m,t} dm$$

$$s.t.$$

$$Y_t = \Big( \int_0^1 Y_{m,t}^{\frac{\epsilon-1}{\epsilon}} dm \Big)^{\frac{\epsilon}{\epsilon-1}}$$

where $Y_t$ is final good output in period $t$, and $Y_{m,t}, m \in [0,1]$ is the output of wholesaler. $\epsilon > 1$ is the elasticity of substitution between differentiated intermediate wholesale goods. Plug the aggregate production technology expression back into the objective function yields

$$\max_{Y_{m,t}} P_t \Big( \int_0^1 Y_{m,t}^{\frac{\epsilon-1}{\epsilon}} dm \Big)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_{m,t} Y_{m,t} dm$$

F.O.C:

$$P_t \left( \int_0^1 Y_{m,t}^{\frac{\epsilon-1}{\epsilon}} dm \right)^{\frac{\epsilon}{\epsilon-1}-1} Y_{m,t}^{\frac{\epsilon-1}{\epsilon}-1} = P_{m,t}$$

Substitute $\int_0^1 Y_{m,t}^{\frac{\epsilon-1}{\epsilon}} dm = Y_t^{\frac{\epsilon-1}{\epsilon}}$ in gives,

$$P_t \left( Y_t^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}-1} Y_{m,t}^{\frac{\epsilon-1}{\epsilon}-1} = P_{m,t}$$

Rearrange gives,

$$Y_{m,t} = \Big( \frac{P_{m,t}}{P_t} \Big)^{-\epsilon} Y_t$$

Zero profit condition for final goods producer is,

$$P_t Y_t = \int_0^1 P_{m,t} Y_{m,t} dm$$

Substitute $Y_{m,t} = \Big( \frac{P_{m,t}}{P_t} \Big)^{-\epsilon} Y_t$ in,

$$P_t Y_t = \int_0^1 P_{m,t} \Big( \frac{P_{m,t}}{P_t} \Big)^{-\epsilon} Y_t dm$$

Rearrange leads to,

$$P_t = \Big(\int_0^1 P_{m,t}^{1-\epsilon} dm\Big)^{\frac{1}{1-\epsilon}}$$

### 7.2.3 Wholesalers (Intermediate Goods Producers)

Assume a continuum of intermediate goods firms indexed by $m \in [0,1]$, each firm produce a intermediate good using identical constant-elasticity-substitution (CES) technology,

$$Y_{m,t} = A_t \Big\{\mu U_{m,t}^{\sigma} + (1-\mu)\Big[\lambda K_{m,t}^{\rho} + (1-\lambda) S_{m,t}^{\rho}\Big]^{\frac{\sigma}{\rho}}\Big\}^{\frac{1}{\sigma}}$$

Intermediate goods firms maximize the profit function taking $W_{s,t}, W_{u,t}$ and $Q_t$ as given,

$$P_{m,t}Y_{m,t} - W_{s,t}S_{m,t} - W_{u,t}U_{m,t} - Q_t K_{m,t}$$

From $Y_{m,t} = \Big(\frac{P_{m,t}}{P_t}\Big)^{-\epsilon} Y_t$ we could obtain $P_{m,t} = P_t\Big(\frac{Y_t}{Y_{m,t}}\Big)^{\frac{1}{\epsilon}}$, substitute in maximization problem gives,

$$Max_{S_{m,t},U_{m,t},K_{m,t}} \left(P_t\Big(\frac{Y_t}{Y_{m,t}}\Big)^{\frac{1}{\epsilon}} Y_{m,t} - W_{s,t}S_{m,t} - W_{u,t}U_{m,t} - Q_t K_{m,t}\right)$$

F.O.Cs:

$$S_{m,t} : P_t Y_t^{\frac{1}{\epsilon}} \left(1 - \frac{1}{\epsilon}\right) Y_{m,t}^{-\frac{1}{\epsilon}} \frac{\partial Y_{m,t}}{\partial S_{m,t}} = W_{s,t}$$

$$U_{m,t} : P_t Y_t^{\frac{1}{\epsilon}} \left(1 - \frac{1}{\epsilon}\right) Y_{m,t}^{-\frac{1}{\epsilon}} \frac{\partial Y_{m,t}}{\partial U_{m,t}} = W_{u,t}$$

$$K_{m,t} : P_t Y_t^{\frac{1}{\epsilon}} \left(1 - \frac{1}{\epsilon}\right) Y_{m,t}^{-\frac{1}{\epsilon}} \frac{\partial Y_{m,t}}{\partial K_{m,t}} = Q_t$$

Denote $\frac{\partial Y_{m,t}}{\partial U_{m,t}} = MPN_{u,m,t}$, $\frac{\partial Y_{m,t}}{\partial S_{m,t}} = MPN_{s,m,t}$, $\frac{\partial Y_{m,t}}{\partial K_{m,t}} = MPK_{m,t}$, and substitute $P_{m,t} = P_t\left(\frac{Y_t}{Y_{m,t}}\right)^{\frac{1}{\epsilon}}$ in gives,

$$P_{m,t} = \frac{\epsilon}{\epsilon - 1}\frac{W_{s,t}}{MPN_{s,m,t}}$$

$$P_{m,t} = \frac{\epsilon}{\epsilon - 1}\frac{W_{u,t}}{MPN_{u,m,t}}$$

$$P_{m,t} = \frac{\epsilon}{\epsilon - 1}\frac{Q_t}{MPK_{m,t}}$$

Dividing both sides by $P_t$,

$$\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon - 1}\frac{W_{s,t}/P_t}{MPN_{s,m,t}}$$

$$\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon - 1}\frac{W_{u,t}/P_t}{MPN_{u,m,t}}$$

$$\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon - 1}\frac{Q_t/P_t}{MPK_{m,t}}$$

Without price stickiness, each intermediate firm can freely adjust the price, thus the profit maximization problems are identical to all intermediate firms,

$$P_t = \Big(\int_0^1 P_{m,t}^{1-\epsilon} dm\Big)^{\frac{1}{1-\epsilon}} = \Big(\frac{1}{N}\sum_{i=1}^{N} P_{m,t}^{1-\epsilon}\Big)^{\frac{1}{1-\epsilon}} = \Big(\frac{1}{N} \cdot NP_{m,t}^{1-\epsilon}\Big)^{\frac{1}{1-\epsilon}} = P_{m,t}$$

Substitute $P_t = P_{m,t}$ in the first order conditions,

$$1 = \frac{\epsilon}{\epsilon - 1}\frac{W_{s,t}/P_t}{MPN_{s,m,t}}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\frac{W_{u,t}/P_t}{MPN_{u,m,t}}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\frac{Q_t/P_t}{MPK_{m,t}}$$

Rearranging gives,

$$\frac{W_{s,t}}{P_t} = \frac{\epsilon - 1}{\epsilon}MPN_{s,m,t}$$

$$\frac{W_{u,t}}{P_t} = \frac{\epsilon - 1}{\epsilon}MPN_{u,m,t}$$

$$\frac{Q_t}{P_t} = \frac{\epsilon - 1}{\epsilon}MPK_{m,t}$$

### 7.2.4 Market Clearing and Stochastic Process

$$ln(A_t) = \rho_A ln(A_{t-1}) + e_t$$

$$K_{s,t+1} = (1-\delta)K_{s,t} + I_{s,t}$$

$$K_{u,t+1} = (1-\delta)K_{u,t} + I_{u,t}$$

$$K_t = K_{s,t} + K_{u,t}$$

$$Y_t = C_{s,t} + C_{u,t} + I_{s,t} + I_{u,t}$$

## 7.3 Model with Calvo pricing and CES Production Function

### 7.3.1 Representative Households

The model for representative households in this context aligns with the structure outlined in the previously discussed monopolist competition model.

### 7.3.2 Final Goods Producer

The model for the final goods producer in this context aligns with the structure outlined in the previously discussed monopolist competition model.

### 7.3.3 Wholesalers (Intermediate Goods Producers)

Assume a continuum of intermediate goods firms indexed by $m \in [0,1]$, each firm produce a intermediate good using identical constant-elasticity-substitution (CES) technology and each intermediate goods firm maximize the profit function,

$$Max_{S_{m,t},U_{m,t},K_{m,t}} \left( P_t \Big( \frac{Y_t}{Y_{m,t}} \Big)^{\frac{1}{\epsilon}} Y_{m,t} - W_{s,t} S_{m,t} - W_{u,t} U_{m,t} - Q_t K_{m,t} \right)$$

F.O.Cs are,

$$\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{W_{s,t}/P_t}{MPN_{s,m,t}}$$

$$\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{W_{u,t}/P_t}{MPN_{u,m,t}}$$

$$\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{Q_t/P_t}{MPK_{m,t}}$$

Since $\frac{P_{m,t}}{P_t} = \frac{\epsilon}{\epsilon-1} MC_t$, the first-order conditions can be rewritten as follow,

$$\frac{W_{s,t}}{P_t} = MC_t MPN_{s,m,t}$$

$$\frac{W_{u,t}}{P_t} = MC_t MPN_{u,m,t}$$

$$\frac{Q_t}{P_t} = MC_t MPK_{m,t}$$

To derive the real marginal costs, we set up the minimization problem for each wholesale firm (because each wholesale firm meets the same cost minimization problem, the choice of capital and labour inputs will be the same among wholesale firms, thus it eliminates the individual firm footscript $m$ in the following equations),

$$\min_{S_t,U_t,K_t} \frac{Q_t}{P_t} K_t + \frac{W_{s,t}}{P_t} S_t + \frac{W_{u,t}}{P_t} U_t$$

s.t.

$$Y_t = A_t \Big\{ \mu U_t^{\sigma} + (1-\mu) \Big[ \lambda K_t^{\rho} + (1-\lambda) S_t^{\rho} \Big]^{\frac{\sigma}{\rho}} \Big\}^{\frac{1}{\sigma}}$$

The first-order conditions, after some algebraic manipulation are

$$\frac{w_{s,t}}{q_t} = \frac{(1-\lambda) S_t^{\rho-1}}{\lambda K_t^{\rho-1}}$$

$$\frac{w_{s,t}}{w_{u,t}} = \frac{(1-\mu)[\lambda K_t^\rho + (1-\lambda)S_t^\rho]^{\frac{\sigma-\rho}{\rho}}(1-\lambda)S_t^{\rho-1}}{\mu U_t^{\sigma-1}}$$

Where

$$w_{s,t} = W_{s,t}/P_t, w_{u,t} = W_{u,t}/P_t, q_t = Q_t/P_t$$

From the f.o.c.s we can solve $K_t$ and $U_t$ as function of $S_t$,

$$K_t = S_t\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{1}{1-\rho}}$$

$$U_t = S_t\Big\{\frac{1-\mu}{\mu}\Big[(1-\lambda)+\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}}\Big]^{\frac{\sigma-\rho}{\rho}}\cdot(1-\lambda)\frac{w_{u,t}}{w_{s,t}}\Big\}^{\frac{1}{\sigma-1}}$$

Substitute $K_t$, $U_t$ in production function and set it to unit, and let $K_t^*$,$U_t^*$, $S_t^*$ denote the capital, unskilled and skilled inputs that produce one unit of product,

$$Y_t = A_t\Big\{\mu(U_t^*)^\sigma + (1-\mu)\Big[\lambda(K_t^*)^\rho + (1-\lambda)(S_t^*)^\rho\Big]^{\frac{\sigma}{\rho}}\Big\}^{\frac{1}{\sigma}} = 1$$

$$\begin{aligned}Y_t = A_t\Bigg\{\mu\Bigg\{S_t^*\Big\{\frac{1-\mu}{\mu}\Big[(1-\lambda)+\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}}\Big]^{\frac{\sigma-\rho}{\rho}}\cdot(1-\lambda)\frac{w_{u,t}}{w_{s,t}}\Big\}^{\frac{1}{\sigma-1}}\Bigg\}^\sigma \\ +(1-\mu)\Big[\lambda\Big(S_t^*\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{1}{1-\rho}}\Big)^\rho + (1-\lambda)(S_t^*)^\rho\Big]^{\frac{\sigma}{\rho}}\Bigg\}^{\frac{1}{\sigma}} = 1\end{aligned}$$

Soving for $S_t^*$:

$$\begin{aligned}A_t\Bigg\{\mu(S_t^*)^\sigma\Big\{\frac{1-\mu}{\mu}\Big[(1-\lambda)+\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}}\Big]^{\frac{\sigma-\rho}{\rho}}\cdot(1-\lambda)\frac{w_{u,t}}{w_{s,t}}\Big\}^{\frac{\sigma}{\sigma-1}} \\ +(1-\mu)(S_t^*)^\sigma\Big[\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}} + (1-\lambda)\Big]^{\frac{\sigma}{\rho}}\Bigg\}^{\frac{1}{\sigma}} = 1\end{aligned}$$

$$\begin{aligned}S_t^*A_t\Bigg\{\mu\Big\{\frac{1-\mu}{\mu}\Big[(1-\lambda)+\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}}\Big]^{\frac{\sigma-\rho}{\rho}}\cdot(1-\lambda)\frac{w_{u,t}}{w_{s,t}}\Big\}^{\frac{\sigma}{\sigma-1}} \\ +(1-\mu)\Big[\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}} + (1-\lambda)\Big]^{\frac{\sigma}{\rho}}\Bigg\}^{\frac{1}{\sigma}} = 1\end{aligned}$$

$$\begin{aligned}S_t^* = A_t^{-1}\Bigg\{\mu\Big\{\frac{1-\mu}{\mu}\Big[(1-\lambda)+\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}}\Big]^{\frac{\sigma-\rho}{\rho}}\cdot(1-\lambda)\frac{w_{u,t}}{w_{s,t}}\Big\}^{\frac{\sigma}{\sigma-1}} \\ +(1-\mu)\Big[\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}} + (1-\lambda)\Big]^{\frac{\sigma}{\rho}}\Bigg\}^{-\frac{1}{\sigma}}\end{aligned}$$

$$S_t^* = A_t^{-1}\Big\{\mu\Big(\frac{1-\mu}{\mu}\Big)^{\frac{\sigma}{\sigma-1}}\Big[(1-\lambda)+\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}}\Big]^{\frac{(\sigma-\rho)\sigma}{\rho(\sigma-1)}}\cdot(1-\lambda)^{\frac{\sigma}{\sigma-1}}\Big(\frac{w_{u,t}}{w_{s,t}}\Big)^{\frac{\sigma}{\sigma-1}}$$
$$+(1-\mu)\Big[\lambda\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}}+(1-\lambda)\Big]^{\frac{\sigma}{\rho}}\Big\}^{-\frac{1}{\sigma}}$$

Denote

$$\Omega = \mu\Big(\frac{1-\mu}{\mu}\Big)^{\frac{\sigma}{\sigma-1}}(1-\lambda)^{\frac{\sigma}{\sigma-1}}$$

$$H_t = (1-\lambda)+\lambda\Big(\frac{\lambda w_{s,t}}{q(1-\lambda)}\Big)^{\frac{\rho}{1-\rho}}$$

Thus,

$$S_t^* = A_t^{-1}\Big\{\Omega H_t^{\frac{(\sigma-\rho)\sigma}{\rho(\sigma-1)}}\Big(\frac{w_{u,t}}{w_{s,t}}\Big)^{\frac{\sigma}{\sigma-1}}+(1-\mu)H_t^{\frac{\sigma}{\rho}}\Big\}^{-\frac{1}{\sigma}}$$

Substitute $S_t^*$ into $K_t$ and $U_t$ we will obtain $K_t^*$, $U_t^*$,

$$U_t^* = S_t^*\Big\{\frac{1-\mu}{\mu}H_t^{\frac{\sigma-\rho}{\rho}}(1-\lambda)\frac{w_{u,t}}{w_{s,t}}\Big\}^{\frac{1}{\sigma-1}}$$

$$K_t^* = S_t^*\Big(\frac{\lambda w_{s,t}}{q_t(1-\lambda)}\Big)^{\frac{1}{1-\rho}}$$

The real marginal cost for each wholesale firm is,

$$MC_t = w_{s,t}S_t^* + w_{u,t}U_t^* + q_tK_t^*$$

### 7.3.4 Optimal Price Setting

Consider a wholesale firm $m$ choose price $P_{m,t}$ in period $t$ to maximize the current market value of the real profits generated while the price remains effective,

$$\max_{P_{m,t}}\sum_{k=0}^{\infty}(\beta\theta)^k E_t\Big\{\Big(\frac{P_{m,t}}{P_{t+k}}-MC_{t+k}\Big)Y_{m,t+k}\Big\}$$

$$s.t.$$

$$Y_{m,t+k} = \Big(\frac{P_{m,t}}{P_{t+k}}\Big)^{-\epsilon}Y_{t+k}$$

where $MC_t$ is the real marginal cost and it is the same among wholesale firms.
Denote the solution as $P_{m,t}^*$ and the first-order condition is,

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \Big\{ \frac{1}{P_{t+k}} Y_{m,t+k} + \frac{P^*_{m,t}}{P_{t+k}} \cdot (-\epsilon) \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{-\epsilon-1} \frac{1}{P_{t+k}} Y_{t+k}$$
$$- MC_{t+k} \cdot (-\epsilon) \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{-\epsilon-1} \frac{1}{P_{t+k}} Y_{t+k} \Big\} = 0$$

Substituting $Y_{m,t+k} = \left( \frac{P^*_{m,t}}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}$ in gives,

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \Big\{ \frac{1}{P_{t+k}} \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{-\epsilon} Y_{t+k} + \frac{P^*_{m,t}}{P_{t+k}} \cdot (-\epsilon) \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{-\epsilon-1} \frac{1}{P_{t+k}} Y_{t+k}$$
$$- MC_{t+k} \cdot (-\epsilon) \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{-\epsilon-1} \frac{1}{P_{t+k}} Y_{t+k} \Big\} = 0$$

Rearranging,

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \Big\{ \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{1-\epsilon} (P^*_{m,t})^{-1} Y_{t+k} + (-\epsilon) \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{1-\epsilon} (P^*_{m,t})^{-1} Y_{t+k}$$
$$- MC_{t+k} \cdot (-\epsilon) \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{-\epsilon} (P^*_{m,t})^{-1} Y_{t+k} \Big\} = 0$$

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \Bigg\{ \Bigg( (1-\epsilon) \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{1-\epsilon} (P^*_{m,t})^{-1} + \epsilon \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{-\epsilon} (P^*_{m,t})^{-1} MC_{t+k} \Bigg) Y_{t+k} \Bigg\} = 0$$

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \Bigg\{ \Bigg( \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{1-\epsilon} - \frac{\epsilon}{\epsilon - 1} \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{-\epsilon} MC_{t+k} \Bigg) Y_{t+k} \Bigg\} = 0 \tag{52}$$

Again, substituting $Y_{m,t+k} = \left( \frac{P^*_{m,t}}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}$ in gives,

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \Big\{ \Big( \frac{P^*_{m,t}}{P_{t+k}} - \frac{\epsilon}{\epsilon - 1} MC_{t+k} \Big) Y_{m,t+k} \Big\} = 0$$

In basic monopolist competition, there is no price stickiness, all terms with $k > 0$ are zero, the equation is reduced to $\frac{P^*_{m,t}}{P_t} = \frac{\epsilon}{\epsilon-1} MC_t$. Without price stickiness, all wholesale firms can freely adjust prices, thus $P^*_{m,t} = P_t$, the first-order condition can be further reduced to $MC = \frac{\epsilon-1}{\epsilon}$.

Next, rewrite Equation 52 as,

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \Big\{ \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{1-\epsilon} Y_{t+k} \Big\} = E_t \sum_{k=0}^{\infty} (\beta\theta)^k \Big\{ \frac{\epsilon}{\epsilon - 1} \Big( \frac{P^*_{m,t}}{P_{t+k}} \Big)^{-\epsilon} MC_{t+k} Y_{t+k} \Big\}$$

Log-linearized above equation around the flexible prices steady state (i.e., basic monopolistic competition case) gives (note that in the flexible prices steady state, each wholesale firm can freely adjust their prices, thus we have $P^*_{m,t} = P_t = P_{t+k} = P$), [46]

[46] Without price stickiness, each intermediate firm can freely adjust their prices, thus the profit maximization problems are identical to all intermediate firms, thus, $P_t = \left( \int_0^1 P_{m,t}^{1-\epsilon} dm \right)^{\frac{1}{1-\epsilon}} = \left( \frac{1}{N} \sum_{i=1}^{N} P_{m,t}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} = \left( \frac{1}{N} \cdot N P_{m,t}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} = P_{m,t}$.

$$
\begin{aligned}
&E_t\sum_{k=0}^{\infty}(\beta\theta)^k\Big(\frac{P}{P}\Big)^{1-\epsilon}(Y_{t+k}-Y)+E_t\sum_{k=0}^{\infty}(\beta\theta)^kY(1-\epsilon)\Big(\frac{P}{P}\Big)^{-\epsilon}\frac{1}{P}(P^*_{m,t}-P)\\
&+E_t\sum_{k=0}^{\infty}(\beta\theta)^kY(1-\epsilon)\Big(\frac{P}{P}\Big)^{-\epsilon}\Big(-\frac{P}{P^2}\Big)(P_{t+k}-P)\\
&=E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{\epsilon}{\epsilon-1}\Big(\frac{P}{P}\Big)^{-\epsilon}Y(MC_{t+k}-MC)+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{\epsilon}{\epsilon-1}\Big(\frac{P}{P}\Big)^{-\epsilon}MC(Y_{t+k}-Y)\\
&+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{\epsilon}{\epsilon-1}MC\cdot Y\cdot(-\epsilon)\Big(\frac{P}{P}\Big)^{-\epsilon-1}\frac{1}{P}(P^*_{m,t}-P)\\
&+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{\epsilon}{\epsilon-1}MC\cdot Y\cdot(-\epsilon)\Big(\frac{P}{P}\Big)^{-\epsilon-1}\Big(-\frac{P}{P^2}\Big)(P_{t+k}-P)
\end{aligned}
$$

Rearranging,

$$
\begin{aligned}
&E_t\sum_{k=0}^{\infty}(\beta\theta)^k(Y_{t+k}-Y)+E_t\sum_{k=0}^{\infty}(\beta\theta)^kY(1-\epsilon)\frac{1}{P}(P^*_{m,t}-P)\\
&+E_t\sum_{k=0}^{\infty}(\beta\theta)^kY(1-\epsilon)\Big(-\frac{1}{P}\Big)(P_{t+k}-P)\\
&=E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{\epsilon}{\epsilon-1}Y(MC_{t+k}-MC)+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{\epsilon}{\epsilon-1}MC(Y_{t+k}-Y)\\
&+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{\epsilon}{\epsilon-1}MC\cdot Y\cdot(-\epsilon)\frac{1}{P}(P^*_{m,t}-P)\\
&+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{\epsilon}{\epsilon-1}MC\cdot Y\cdot(-\epsilon)\Big(-\frac{1}{P}\Big)(P_{t+k}-P)
\end{aligned}
$$

$$
\begin{aligned}
&E_t\sum_{k=0}^{\infty}(\beta\theta)^kY\Big\{\frac{Y_{t+k}-Y}{Y}+(1-\epsilon)\frac{P^*_{m,t}-P}{P}-(1-\epsilon)\frac{P_{t+k}-P}{P}\Big\}\\
&=E_t\sum_{k=0}^{\infty}(\beta\theta)^k\frac{\epsilon}{\epsilon-1}MC\cdot Y\Big\{\frac{MC_{t+k}-MC}{MC}+\frac{Y_{t+k}-Y}{Y}-\epsilon\cdot\frac{P^*_{m,t}-P}{P}+\epsilon\cdot\frac{P_{t+K}-P}{P}\Big\}
\end{aligned}
$$

In steady state $\frac{\epsilon}{\epsilon-1}MC_t=1$, above equation can be rewritten as,

$$
\begin{aligned}
&E_t\sum_{k=0}^{\infty}(\beta\theta)^k\Big\{\frac{Y_{t+k}-Y}{Y}+(1-\epsilon)\frac{P^*_{m,t}-P}{P}-(1-\epsilon)\frac{P_{t+k}-P}{P}\Big\}\\
&=E_t\sum_{k=0}^{\infty}(\beta\theta)^k\Big\{\frac{MC_{t+k}-MC}{MC}+\frac{Y_{t+k}-Y}{Y}-\epsilon\cdot\frac{P^*_{m,t}-P}{P}+\epsilon\cdot\frac{P_{t+K}-P}{P}\Big\}
\end{aligned}
$$

Use $\hat{x}_t$ to denote $\frac{X_t-X}{X}$, then,

$$E_t\sum_{k=0}^{\infty}(\beta\theta)^k\left\{\hat{y_{t+k}}+(1-\epsilon)\hat{p_{m,t}^*}-(1-\epsilon)\hat{p_{t+k}}\right\}=E_t\sum_{k=0}^{\infty}(\beta\theta)^k\left\{\hat{mc_{t+k}}+\hat{y_{t+k}}-\epsilon\cdot\hat{p_{m,t}^*}+\epsilon\cdot\hat{p_{t+k}}\right\}$$

Rearranging,

$$E_t\sum_{k=0}^{\infty}(\beta\theta)^k(\hat{p_{m,t}^*}-\hat{p_{t+k}})=E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{mc_{t+k}}$$

$$E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{p_{m,t}^*}=E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{mc_{t+k}}+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{p_{t+k}}$$

Substitute $E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{p_{m,t}^*}=\frac{1}{1-\beta\theta}\hat{p_{m,t}^*}$ in gives,

$$\frac{1}{1-\beta\theta}\hat{p_{m,t}^*}=E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{mc_{t+k}}+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{p_{t+k}}$$

$$\hat{p_{m,t}^*}=(1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{mc_{t+k}}+(1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{p_{t+k}} \tag{53}$$

And note that,

$$\begin{aligned}(1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{p_{t+k}}&=(1-\beta\theta)\hat{p_t}+(1-\beta\theta)(\beta\theta)E_t\hat{p_{t+1}}+(1-\beta\theta)(\beta\theta)^2E_t\hat{p_{t+2}}...\\&=\hat{p_t}-(\beta\theta)\hat{p_t}+(\beta\theta)E_t\hat{p_{t+1}}-(\beta\theta)^2E_t\hat{p_{t+1}}+(\beta\theta)^2E_t\hat{p_{t+2}}-(\beta\theta)^3E_t\hat{p_{t+2}}...\\&=\hat{p_{t-1}}+\hat{p_t}-\hat{p_{t-1}}+(\beta\theta)Et(\hat{p_{t+1}}-\hat{p_t})+(\beta\theta)^2Et(\hat{p_{t+2}}-\hat{p_{t+1}})+...\end{aligned}$$

Since $\hat{\pi_t}=\hat{p_t}-\hat{p_{t-1}}$, thus we have, [47]

$$(1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{p_{t+k}}=\hat{p_{t-1}}+\hat{\pi_t}+(\beta\theta)Et(\hat{\pi_{t+1}})+(\beta\theta)^2Et(\hat{\pi_{t+2}})+...$$

$$(1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{p_{t+k}}=\hat{p_{t-1}}+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{\pi_{t+k}}$$

Substitute above equation into Equation 53, (subsequently, we eliminate the index $m$ because of the symmetric equilibrium assumption),

$$\hat{p_t^*}=\hat{p_{t-1}}+E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{\pi_{t+k}}+(1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{mc_{t+k}}$$

Above equation can be rewritten as,

$$\hat{p_t^*}=\hat{p_{t-1}}+\hat{\pi_t}+(1-\beta\theta)\hat{mc_t}+E_t\sum_{k=1}^{\infty}(\beta\theta)^k\hat{\pi_{t+k}}+(1-\beta\theta)E_t\sum_{k=1}^{\infty}(\beta\theta)^k\hat{mc_{t+k}} \tag{54}$$

[47] Define $\Pi_t=\frac{P_t}{P_{t-1}}$, log-linearize around the steady state gives, $\Pi_t-\Pi=\frac{1}{P}(P_t-P)-\frac{P}{P^2}(P_{t-1}-P)$. In steady state we have $\Pi=\frac{P}{P}=1$, thus we have, $\frac{\Pi_t-\Pi}{\Pi}=\frac{1}{P}(P_t-P)-\frac{1}{P}(P_{t-1}-P)$. And we use $\hat{x_t}$ to denote $\frac{X_t-X}{X}$, thus, $\hat{\pi_t}=\hat{p_t}-\hat{p_{t-1}}$.

Forward the above equation one term gives,

$$E_t\hat{p^*_{t+1}} = \hat{p_t} + E_t\hat{\pi_{t+1}} + (1-\beta\theta)E_t\hat{mc_{t+1}} + E_t\sum_{k=1}^{\infty}(\beta\theta)^k\hat{\pi_{t+k+1}} + (1-\beta\theta)E_t\sum_{k=1}^{\infty}(\beta\theta)^k\hat{mc_{t+k+1}}$$

It can be rewritten as,

$$E_t\hat{p^*_{t+1}} = \hat{p_t} + E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{\pi_{t+k+1}} + (1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k\hat{mc_{t+k+1}}$$

Multiply both sides by $\beta\theta$ gives,

$$\beta\theta E_t\hat{p^*_{t+1}} = \beta\theta\hat{p_t} + E_t\sum_{k=0}^{\infty}(\beta\theta)^{k+1}\hat{\pi_{t+k+1}} + (1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^{k+1}\hat{mc_{t+k+1}}$$

$$\rightarrow$$

$$\beta\theta E_t\hat{p^*_{t+1}} = \beta\theta\hat{p_t} + E_t\sum_{k=1}^{\infty}(\beta\theta)^k\hat{\pi_{t+k}} + (1-\beta\theta)E_t\sum_{k=1}^{\infty}(\beta\theta)^k\hat{mc_{t+k}} \tag{55}$$

From Equation 54, we can obtain,

$$E_t\sum_{k=1}^{\infty}(\beta\theta)^k\hat{\pi_{t+k}} + (1-\beta\theta)E_t\sum_{k=1}^{\infty}(\beta\theta)^k\hat{mc_{t+k}} = \hat{p^*_t} - \hat{p_{t-1}} - \hat{\pi}_t - (1-\beta\theta)\hat{mc_t} \tag{56}$$

Then, substituting Equation 56 into Equation 55,

$$\beta\theta E_t\hat{p^*_{t+1}} = \beta\theta\hat{p_t} + \hat{p^*_t} - \hat{p_{t-1}} - \hat{\pi}_t - (1-\beta\theta)\hat{mc_t}$$

$$\hat{p^*_t} - \hat{p_{t-1}} = \beta\theta E_t(\hat{p^*_{t+1}} - \hat{p_t}) + \hat{\pi}_t + (1-\beta\theta)\hat{mc_t} \tag{57}$$

Following the formalism proposed in Calvo (1983), each wholesale firm may reset its price only with probability $(1-\theta)$ in any given period, independent of the time elapsed since the last adjustment. Thus, in each period a measure $(1-\theta)$ of wholesale firms drawn randomly from the population can reset their prices, while the remaining fraction $\theta$ of firms will keep their prices unchanged. The aggregate price is [48],

$$\begin{aligned} P_t^{1-\epsilon} &= \int_0^{\theta} P_{t-1}^{1-\epsilon}dm + \int_{\theta}^{1}(P_t^*)^{1-\epsilon}dm \\ &= [P_{t-1}^{1-\epsilon}]_0^{\theta} + [(P_t^*)^{1-\epsilon}]_{\theta}^{1} \\ &= \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \end{aligned}$$

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

[48]Given the fact that all wholesale firms re-setting prices will choose an identical price $P_t^*$, it eliminated $m$ footscript in the following expression.

$$\frac{P_t}{P_{t-1}} = \Big[\theta + (1-\theta)\Big(\frac{P_t^*}{P_{t-1}}\Big)^{1-\epsilon}\Big]^{\frac{1}{1-\epsilon}}$$

$$\Big(\frac{P_t}{P_{t-1}}\Big)^{1-\epsilon} = \theta + (1-\theta)\Big(\frac{P_t^*}{P_{t-1}}\Big)^{1-\epsilon}$$

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\Big(\frac{P_t^*}{P_{t-1}}\Big)^{1-\epsilon}$$

Where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate between $t-1$ and $t$. Log-linearze the above expression around the flexible prices steady state to obtain,

$$(1-\epsilon)\Pi^{-\epsilon}(\Pi_t - \Pi) = (1-\theta)(1-\epsilon)\Big(\frac{P}{P}\Big)^{-\epsilon}\frac{1}{P}(P_t^* - P) - (1-\theta)(1-\epsilon)\Big(\frac{P}{P}\Big)^{-\epsilon}\frac{P}{P^2}(P_{t-1} - P)$$

$$\Pi^{-\epsilon}(\Pi_t - \Pi) = (1-\theta)\Big(\frac{P}{P}\Big)^{-\epsilon}\frac{1}{P}(P_t^* - P) - (1-\theta)\Big(\frac{P}{P}\Big)^{-\epsilon}\frac{P}{P^2}(P_{t-1} - P)$$

In steady state $\Pi = \frac{P}{P} = 1$, thus,

$$\frac{\Pi_t - \Pi}{\Pi} = (1-\theta)\frac{1}{P}(P_t^* - P) - (1-\theta)\frac{1}{P}(P_{t-1} - P)$$

Use $\hat{x_t}$ to denote $\frac{X_t - X}{X}$, then,

$$\hat{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1}) \tag{58}$$

Combine Equation 57 with Equation 58 yeild the price inflation equation,

$$\frac{\hat{\pi}_t}{1-\theta} = \beta\theta E_t\Big(\frac{\hat{\pi}_{t+1}}{1-\theta}\Big) + \hat{\pi}_t + (1-\beta\theta)\hat{mc}_t \tag{59}$$

$$\hat{\pi}_t = \beta\theta E_t\{\hat{\pi}_{t+1}\} + (1-\theta)\hat{\pi}_t + (1-\beta\theta)(1-\theta)\hat{mc}_t$$

$$\hat{\pi}_t = \beta E_t\{\hat{\pi}_{t+1}\} + \frac{(1-\beta\theta)(1-\theta)}{\theta}\hat{mc}_t$$

#### 7.3.5 Market Clearing, Stochastic Process

$$ln(A_t) = \rho_A ln(A_{t-1}) + e_t$$

$$Y_t = C_{s,t} + C_{u,t} + I_{s,t} + I_{u,t}$$

$$K_{s,t+1} = (1-\delta)K_{s,t} + I_{s,t}$$

$$K_{u,t+1} = (1-\delta)K_{u,t} + I_{u,t}$$

$$K_t = K_{s,t} + K_{u,t}$$

## 7.4 Calvo Pricing and Calvo Wages with CES Production Function

### 7.4.1 Households

The labour-aggregating firm chooses each types of differentiated labour service $S_{i,t}$, $U_{j,t}$, for $i, j \in [0,1]$ to maximize its profit. For skilled labour,

$$\max_{S_{i,t}} W_{s,t}S_t - \int_0^1 W_{s,i,t}S_{i,t}di$$

$$s.t.$$

$$S_t = \Big(\int_0^1 S_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} di\Big)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

$$U_t = \Big(\int_0^1 U_{j,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\Big)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

The objective function can be rewritten as,

$$\max_{S_{i,t}} W_{s,t}\Big(\int_0^1 S_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} di\Big)^{\frac{\epsilon_w}{\epsilon_w - 1}} - \int_0^1 W_{s,i,t}S_{i,t}di$$

Symetrically, labour-aggregating firm aggregate differentiated unskilled labour service,

$$\max_{U_{j,t}} W_{u,t}\Big(\int_0^1 U_{j,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\Big)^{\frac{\epsilon_w}{\epsilon_w - 1}} - \int_0^1 W_{u,j,t}U_{j,t}dj$$

The first-order conditions for each type of differentiated labour services are,

$$S_{i,t} : W_{s,t}\Big(\int_0^1 S_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} di\Big)^{\frac{1}{\epsilon_w - 1}} S_{i,t}^{\frac{-1}{\epsilon_w}} - W_{s,i,t} = 0$$

$$U_{j,t} : W_{u,t}\Big(\int_0^1 U_{j,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\Big)^{\frac{1}{\epsilon_w - 1}} U_{j,t}^{\frac{-1}{\epsilon_w}} - W_{u,j,t} = 0$$

From $S_t = \Big(\int_0^1 S_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} di\Big)^{\frac{\epsilon_w}{\epsilon_w - 1}}$ to obtain $\Big(\int_0^1 S_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} di\Big)^{\frac{1}{\epsilon_w - 1}} = S_t^{\frac{1}{\epsilon_w}}$, thus above equations can be rewritten as,

$$S_{i,t} : W_{s,t}S_t^{\frac{1}{\epsilon_w}} S_{i,t}^{\frac{-1}{\epsilon_w}} - W_{s,i,t} = 0$$

$$U_{j,t} : W_{u,t}U_t^{\frac{1}{\epsilon_w}} U_{j,t}^{\frac{-1}{\epsilon_w}} - W_{u,j,t} = 0$$

After some algebra, the demand for $S_{i,t}$ and $U_{j,t}$ can be expressed as:

$$S_{i,t} = \Big(\frac{W_{s,i,t}}{W_{s,t}}\Big)^{-\epsilon_w} S_t$$

$$U_{j,t} = \Big(\frac{W_{u,j,t}}{W_{u,t}}\Big)^{-\epsilon_w} U_t$$

Labour-aggregating firm behave competitively, thus zero profit condition gives,

$$W_{s,t}S_t - \int_0^1 W_{s,i,t}S_{i,t}di = 0$$

$$W_{u,t}U_t - \int_0^1 W_{u,j,t}U_{j,t}dj = 0$$

$$\rightarrow$$

$$W_{s,t}S_t - \int_0^1 W_{s,i,t}\Big(\frac{W_{s,i,t}}{W_{s,t}}\Big)^{-\epsilon_w} S_t di = 0$$

$$W_{u,t}U_t - \int_0^1 W_{u,j,t}\Big(\frac{W_{u,j,t}}{W_{u,t}}\Big)^{-\epsilon_w} U_t dj = 0$$

$$\rightarrow$$

$$W_{s,t}^{1-\epsilon_w} = \int_0^1 W_{s,i,t}^{1-\epsilon_w} di = 0$$

$$W_{u,t}^{1-\epsilon_w} = \int_0^1 W_{u,j,t}^{1-\epsilon_w} dj = 0$$

$$\rightarrow$$

$$W_{s,t} = \Big(\int_0^1 W_{s,i,t}^{1-\epsilon_w} di\Big)^{\frac{1}{1-\epsilon_w}}$$

$$W_{u,t} = \Big(\int_0^1 W_{u,j,t}^{1-\epsilon_w} dj\Big)^{\frac{1}{1-\epsilon_w}}$$

Consider a household $i$ who supplies skilled labour services resetting its wage in period $t$, and let $W^*_{s,i,t}$ denote the newly set wage. The household will choose $W^*_{s,i,t}$ to maximize the expected discounted sum of utilities. The maximization is subject to the sequence of labour demand schedules and budget constraints that are effective while $W^*_{s,i,t}$ remains in place, [49]

$$\max_{W^*_{s,i,t}} E_t \Bigg\{ \sum_{k=0}^{\infty} (\beta\theta_w^s)^k U(C_{s,i,t+k}, S_{i,t+k}) \Bigg\}$$

$$s.t.$$

[49] The maximization problem for households who supply unskilled labour services is symmetric to the households that supply skilled labour.

$$S_{i,t+k} = \Big(\frac{W^*_{s,i,t}}{W_{s,t+k}}\Big)^{-\epsilon_w} S_{t+k}$$

$$P_{t+k}C_{s,i,t+k}+P_{t+k}K_{s,i,t+k+1}+B_{s,i,t+k+1} \le Q_{t+k}K_{s,i,t+k}+W^*_{s,i,t}S_{i,t+k}+P_{t+k}(1-\delta)K_{s,i,t+k}+R_{t+k}B_{s,i,t+k}+D_{s,i,t+k}$$

F.O.Cs:

$$C_{s,i,t+k} : U_c(C_{s,i,t+k}, S_{i,t+k}) = \lambda_{t+k}P_{t+k} \tag{60}$$

$$W^*_{s,i,t} : \sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{U_s(C_{s,i,t+k}, S_{i,t+k}) \cdot \frac{\partial S_{i,t+k}}{\partial W^*_{s,i,t}} + \lambda_{t+k}\Big(S_{i,t+k} + W^*_{s,i,t}\frac{\partial S_{i,t+k}}{\partial W^*_{s,i,t}}\Big)\Big\} = 0 \tag{61}$$

From $S_{i,t+k} = \Big(\frac{W^*_{s,i,t}}{W_{s,t+k}}\Big)^{-\epsilon_w} S_{t+k}$ we can obtain,

$$\frac{\partial S_{i,t+k}}{\partial W^*_{s,i,t}} = -\epsilon_w\Big(\frac{W^*_{s,i,t}}{W_{s,t+k}}\Big)^{-\epsilon_w-1}\Big(\frac{1}{W_{s,t+k}}\Big)S_{t+k} = -\frac{\epsilon_w}{W^*_{s,i,t}}S_{i,t+k} \tag{62}$$

Substitute Equation 62 and Equation 60 into Equation 61 to obtain,

$$\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{U_s(C_{s,i,t+k}, S_{i,t+k}) \cdot \frac{\partial S_{i,t+k}}{\partial W^*_{s,i,t}}$$
$$+\frac{1}{P_{t+k}}U_c(C_{s,i,t+k}, S_{i,t+k})\Big(S_{i,t+k} + W^*_{s,i,t}\frac{\partial S_{i,t+k}}{\partial W^*_{s,i,t}}\Big)\Big\} = 0$$

$$\rightarrow$$

$$\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{U_s(C_{s,i,t+k}, S_{i,t+k}) \cdot \Big(-\frac{\epsilon_w}{W^*_{s,i,t}}\Big)S_{i,t+k}$$
$$+\frac{1}{P_{t+k}}U_c(C_{s,i,t+k}, S_{i,t+k})\Big(S_{i,t+k} + W^*_{s,i,t}\Big(-\frac{\epsilon_w}{W^*_{s,i,t}}\Big)S_{i,t+k}\Big)\Big\} = 0$$

$$\rightarrow$$

$$\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{U_s(C_{s,i,t+k}, S_{i,t+k}) \cdot \Big(-\frac{\epsilon_w}{W^*_{s,i,t}}\Big)S_{i,t+k}$$
$$+\frac{1}{P_{t+k}}U_c(C_{s,i,t+k}, S_{i,t+k})\Big(S_{i,t+k}(1-\epsilon_w)\Big)\Big\} = 0$$

$$\rightarrow$$

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_{i,t+k}\Big[U_s(C_{s,i,t+k},S_{i,t+k})\Big(-\frac{\epsilon_w}{W^*_{s,i,t}}\Big)+\frac{1-\epsilon_w}{P_{t+k}}U_c(C_{s,i,t+k},S_{i,t+k})\Big]\Big\}=0$$

$$\rightarrow$$

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_{i,t+k}\Big[U_s(C_{s,i,t+k},S_{i,t+k})\Big(\frac{\epsilon_w}{\epsilon_w-1}\Big)+\frac{W^*_{s,i,t}}{P_{t+k}}U_c(C_{s,i,t+k},S_{i,t+k})\Big]\Big\}=0$$

Letting $MRS_{s,i,t+k}=-\frac{U_s(C_{s,i,t+k},S_{i,t+k})}{U_c(C_{s,i,t+k},S_{i,t+k})}$, the optimality condition above can be rewritten as,

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_{i,t+k}\Big[-MRS_{s,i,t+k}U_c(C_{s,i,t+k},S_{i,t+k})\Big(\frac{\epsilon_w}{\epsilon_w-1}\Big)+\frac{W^*_{s,i,t}}{P_{t+k}}U_c(C_{s,i,t+k},S_{i,t+k})\Big]\Big\}=0$$

$$\rightarrow$$

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_{i,t+k}U_c(C_{s,i,t+k},S_{i,t+k})\Big(\frac{W^*_{s,i,t}}{P_{t+k}}-\Big(\frac{\epsilon_w}{\epsilon_w-1}\Big)MRS_{s,i,t+k}\Big)\Big\}=0 \tag{63}$$

In the flexible price steady state, all terms with $k>0$ are zeros, thus,

$$\frac{W^*_{s,i,t}}{P_t}=\frac{W_{s,i}}{P}=\frac{\epsilon_w}{\epsilon_w-1}MRS_{s,i}$$

Taking log on both sides of above equation, and denote $x=log(X),\mu_w=log(\frac{\epsilon_w}{\epsilon_w-1})$,

$$w_s-p=\mu_w+mrs_{s,i} \tag{64}$$

Denote $M_w=\frac{\epsilon_w}{\epsilon_w-1}$, and log-linearize Equation 63 around the steady state gives,

$$\begin{aligned}
&\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{U_c(C_{s,i},S_i)\Big(\frac{W_{s,i}}{P}-M_wMRS_{s,i}\Big)(S_{i,t+k}-S_i)\Big\}\\
&+\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_i\Big(\frac{W_{s,i}}{P}-M_wMRS_{s,i}\Big)\Big(U_c(C_{s,i,t+k},S_{i,t+k})-U_c(C_{s,i},S_i)\Big)\Big\}\\
&+\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_iU_c(C_{s,i},S_i)\frac{1}{P}(W^*_{s,i,t}-W_{s,i})\Big\}\\
&-\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_iU_c(C_{s,i},S_i)\frac{W_{s,i}}{P^2}(P_{t+k}-P)\Big\}\\
&-\sum_{k=0}^{\infty}(\beta\theta_w^s)^k E_t\Big\{S_iU_c(C_{s,i},S_i)\Big(M_wMRS_{s,i,t+k}-M_wMRS_{s,i}\Big)\Big\}=0
\end{aligned}$$

Substitute $\frac{W_{s,i}}{P}=\frac{\epsilon_w}{\epsilon_w-1}MRS_{s,i}=M_wMRS_{s,i}$ in above equation,

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{S_iU_c(C_{s,i},S_i)\frac{1}{P}(W^*_{s,i,t}-W_{s,i})\Big\}-\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{S_iU_c(C_{s,i},S_i)\frac{W_{s,i}}{P^2}(P_{t+k}-P)\Big\}$$
$$-\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{S_iU_c(C_{s,i},S_i)\Big(M_wMRS_{s,i,t+k}-M_wMRS_{s,i}\Big)\Big\}=0$$

Cancle the constant term,

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{\frac{1}{P}(W^*_{s,i,t}-W_{s,i})\Big\}-\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{\frac{W_{s,i}}{P^2}(P_{t+k}-P)\Big\}$$
$$-\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big(M_wMRS_{s,i,t+k}-M_wMRS_{s,i}\Big)=0$$

$$\rightarrow$$

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{\frac{W_{s,i}}{P}\frac{W^*_{s,i,t}-W_{s,i}}{W_{s,i}}\Big\}=\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{\frac{W_{s,i}}{P}\frac{P_{t+k}-P}{P}\Big\}$$
$$+\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big(M_wMRS_{s,i}\frac{M_wMRS_{s,i,t+k}-M_wMRS_{s,i}}{M_wMRS_{s,i}}\Big)=0$$

Since $\frac{W_{s,i}}{P}=M_wMRS_{s,i}$, canceling the terms on both sides,

$$\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{\frac{W^*_{s,i,t}-W_{s,i}}{W_{s,i}}\Big\}=\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{\frac{P_{t+k}-P}{P}\Big\}+\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big(\frac{M_wMRS_{s,i,t+k}-M_wMRS_{s,i}}{M_wMRS_{s,i}}\Big)=0$$

$$\rightarrow$$

$$\frac{1}{1-\beta\theta_w^s}\frac{W^*_{s,i,t}-W_{s,i}}{W_{s,i}}=\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{\frac{P_{t+k}-P}{P}+\frac{M_wMRS_{s,i,t+k}-M_wMRS_{s,i}}{M_wMRS_{s,i}}\Big\}$$

$$\rightarrow$$

$$\frac{W^*_{s,i,t}-W_{s,i}}{W_{s,i}}=(1-\beta\theta_w^s)\sum_{k=0}^{\infty}(\beta\theta_w^s)^kE_t\Big\{\frac{P_{t+k}-P}{P}+\frac{M_wMRS_{s,i,t+k}-M_wMRS_{s,i}}{M_wMRS_{s,i}}\Big\}$$

Since $\frac{W^*_{s,i,t}-W_{s,i}}{W_{s,i}}\approx log\Big(\frac{W^*_{s,i,t}}{W_{s,i}}\Big)=w^*_{s,i.t}-w_{s,i}$, similarly, $\frac{P_{t+k}-P}{P}\approx p_{t+k}-p$, $\frac{M_wMRS_{s,i,t+k}-M_wMRS_{s,i}}{M_wMRS_{s,i}}\approx$

$mrs_{s,i,t+k} - mrs_{s,i}$, and then,

$$w^*_{s,i.t} - w_{s,i} \approx (1-\beta\theta^s_w)\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{p_{t+k} - p + mrs_{s,i,t+k} - mrs_{s,i})\Big\}$$

$$= (1-\beta\theta^s_w)\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{p_{t+k} + mrs_{s,i,t+k}\Big\} + (1-\beta\theta^s_w)\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{-mrs_{s,i} - p\Big\}$$

$$= (1-\beta\theta^s_w)\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{p_{t+k} + mrs_{s,i,t+k}\Big\} + (1-\beta\theta^s_w)\frac{1}{(1-\beta\theta^s_w)}(-mrs_{s,i} - p)$$

$$\rightarrow$$

$$w^*_{s,i.t} = (1-\beta\theta^s_w)\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{p_{t+k} + mrs_{s,i,t+k}\Big\} + w_{s,i} - mrs_{s,i} - p$$

Taking log on both sides of $\frac{W_{s,i}}{P} = M_w MRS_{s,i}$ gives $w_{s,i} - p = log(M_w) + mrs_{s,i}$. Substitute in above equation to obtain,

$$w^*_{s,i.t} = (1-\beta\theta^s_w)\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{p_{t+k} + mrs_{s,i,t+k}\Big\} + log(M_w)$$

Denote $\mu_w = log(M_w) = log(\frac{\epsilon_w}{\epsilon_w - 1})$, then,

$$w^*_{s,i.t} = \mu_w + (1-\beta\theta^s_w)\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{p_{t+k} + mrs_{s,i,t+k}\Big\} \tag{65}$$

Assuming utility function is specialized to be of the form.

$$U(C_{s,i,t}, S_{i,t}) = \frac{C^{1-\sigma_c}_{s,i,t}}{1-\sigma_c} - \frac{S^{1+\phi}_{i,t}}{1+\phi}$$

Thus,

$$MRS_{s,i,t} = -\frac{U_s(C_{s,i,t}, S_{i,t})}{U_c(C_{s,i,t}, S_{i,t})} = \frac{S^{\phi}_{i,t}}{C^{-\sigma}_{s,i,t}}$$

Taking log on both sides, and setting $t = t + k$,

$$mrs_{s,i,t+k} = \sigma_c c_{s,i,t+k} + \phi s_{i,t+k}$$

The assumption of complete asset markets and separability between consumption and hours imply that consumption is independent of the wage history of a household, i.e., $C_{i,t+k} = C_{t+k}$ for $\forall k$, thus,

$$mrs_{s,i,t+k} = \sigma_c c_{s,t+k} + \phi s_{i,t+k}$$

Define the economy's average marginal rate of substitution as,

$$mrs_{s,t+k} = \sigma_c c_{s,t+k} + \phi s_{t+k}$$

And from $S_{i,t+k} = \left(\frac{W^*_{s,i,t}}{W_{s,t+k}}\right)^{-\epsilon_w} S_{t+k}$ we can obtain, $s_{i,t+k} = -\epsilon_w(w^*_{s,i,t} - w_{s,t+k}) + s_{t+k}$. Hence,

$$mrs_{s,i,t+k} = mrs_{s,t+k} + \phi(s_{i,t+k} - s_{t+k}) = mrs_{s,t+k} - \epsilon_w \phi(w^*_{s,i,t} - w_{s,t+k})$$

Substituting above equation into Equation 65,

$$w^*_{s,i.t} = \mu_w + (1 - \beta\theta^s_w)\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{mrs_{s,t+k} - \epsilon_w\phi(w^*_{s,i,t} - w_{s,t+k}) + p_{t+k}\Big\}$$

$$\rightarrow$$

$$(1 + \epsilon_w\phi)w^*_{s,i.t} = (1 - \beta\theta^s_w)\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{\mu_w + mrs_{s,t+k} + \epsilon_w\phi w_{s,t+k} + p_{t+k}\Big\}$$

$$\rightarrow$$

$$w^*_{s,i.t} = \frac{1 - \beta\theta^s_w}{1 + \epsilon_w\phi}\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{\mu_w + mrs_{s,t+k} + \epsilon_w\phi w_{s,t+k} + p_{t+k}\Big\}$$

Let $\hat{\mu_t}^s = \mu^s_t - \mu^w$ denotes the deviations of the economy's (log) average wage markup as $\mu^s_t = w_{s,t} - p_t - mrs_{s,t}$ from its steady state level $\mu^w$, hence above equation can be rewritten as,

$$w^*_{s,i.t} = \frac{1 - \beta\theta^s_w}{1 + \epsilon_w\phi}\sum_{k=0}^{\infty}(\beta\theta^s_w)^k E_t\Big\{(1 + \epsilon_w\phi)w_{s,t+k} - \hat{\mu_{t+k}}^s\Big\}$$

$$\rightarrow$$

$$\begin{aligned} w^*_{s,i.t} &= \frac{1 - \beta\theta^s_w}{1 + \epsilon_w\phi}\left(\Big((1 + \epsilon_w\phi)w_{s,t} - \hat{\mu_t}^s\Big) + \sum_{k=0}^{\infty}(\beta\theta^s_w)^{k+1} E_t\Big\{(1 + \epsilon_w\phi)w_{s,t+k+1} - \hat{\mu^s_{t+k+1}}\Big\}\right) \\ &= (1 - \beta\theta^s_w)w_{s,t} - \frac{1 - \beta\theta^s_w}{1 + \epsilon_w\phi}\hat{\mu_t}^s + \frac{1 - \beta\theta^s_w}{1 + \epsilon_w\phi}\sum_{k=0}^{\infty}(\beta\theta^s_w)^{k+1} E_t\Big\{(1 + \epsilon_w\phi)w_{s,t+k+1} - \hat{\mu^s_{t+k+1}}\Big\} \\ &= (1 - \beta\theta^s_w)w_{s,t} - \frac{1 - \beta\theta^s_w}{1 + \epsilon_w\phi}\hat{\mu_t}^s + \beta\theta^s_w E_t\{w^*_{s,i,t+1}\} \end{aligned}$$

Rearranging.

$$w^*_{s,i.t} = \beta\theta^s_w E_t\{w^*_{s,i,t+1}\} + (1-\beta\theta^s_w)(w_{s,t} - (1-\epsilon_w\phi)^{-1}\hat{\mu_t}^s)$$

As $1-\theta^s_w$ fraction of household chooses the same wage, $w_(s,j,t)^* = w_(s,t)^*$, and the remaining household $\theta_w$ set their wage equal to the wage observed in the previous period. thus, above equation can be rewritten as,

$$w^*_{s.t} = \beta\theta^s_w E_t\{w^*_{s,t+1}\} + (1-\beta\theta^s_w)(w_{s,t} - (1-\epsilon_w\phi)^{-1}\hat{\mu_t}^s) \tag{66}$$

Given the assumed Calvo wage setting rules, the evolution of the aggregate wage can be written as [50],

$$W^{1-\epsilon_w}_{s,t} = \int_0^{\theta^s_w} W^{1-\epsilon_w}_{s,t-1} di + \int_{\theta^s_w}^1 (W^*_{s,t})^{1-\epsilon_w} di$$

$$W^{1-\epsilon_w}_{s,t} = [W^{1-\epsilon_w}_{s,t-1}]_0^{\theta^s_w} + [(W^*_{s,t})^{1-\epsilon_w}]^1_{\theta^s_w}$$

$$W^{1-\epsilon_w}_{s,t} = \theta^s_w W^{1-\epsilon_w}_{s,t-1} + (1-\theta^s_w)(W^*_{s,t})^{1-\epsilon_w}$$

$$W_{s,t} = \left[\theta^s_w W^{1-\epsilon_w}_{s,t-1} + (1-\theta^s_w)(W^*_{s,t})^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$$

$$W^{1-\epsilon_w}_{s,t} = \theta^s_w W^{1-\epsilon_w}_{s,t-1} + (1-\theta^s_w)(W^*_{s,t})^{1-\epsilon_w}$$

Log-linearize the previous equation around the zero wage inflation steady state yield

$$(1-\epsilon_w)W_s^{-\epsilon_w}(W_{s,t} - W_s) = \theta^s_w(1-\epsilon_w)W_s^{-\epsilon_w}(W_{s,t-1} - W_s) + (1-\theta^s_w)(1-\epsilon_w)W_s^{-\epsilon_w}(W^*_{s,t} - W_s)$$

$$(W_{s,t} - W_s) = \theta^s_w(W_{s,t-1} - W_s) + (1-\theta^s_w)(W^*_{s,t} - W_s)$$

$$\frac{W_{s,t} - W_s}{W_s} = \frac{\theta^s_w(W_{s,t-1} - W_s)}{W_s} + \frac{(1-\theta^s_w)(W^*_{s,t} - W_s)}{W_s}$$

As $\frac{X_t - X}{X} \approx log(1 + \frac{X_t - X}{X}) = log(\frac{X_t}{X}) = log(X_t) - log(X)$, thus above equation can be rewritten as,

$$log(W_{s,t}) - log(W_s) = \theta^s_w\Big(log(W_{s,t-1}) - log(W_s)\Big) + (1-\theta^s_w)\Big(log(W^*_{s,t} - log(W_s))\Big)$$

$$log(W_{s,t}) = \theta^s_w log(W_{s,t-1}) + (1-\theta^s_w)log(W^*_{s,t})$$

$$w_{s,t} = \theta^s_w w_{s,t-1} + (1-\theta^s_w)w^*_{s,t} \tag{67}$$

From Equation 67 to obtain $w^*_{s,t} = \frac{w_{s,t} - \theta^s_w w_{s,t-1}}{1-\theta^s_w}$, and combine with Equation 66 gives,

[50]Given the fact that all households who re-setting wages will choose an idential wage $W^*_{s,t}$, it eliminated $i$ footscript in the following expression.

$$\frac{w_{s,t}-\theta_w^s w_{s,t-1}}{1-\theta_w^s} = \beta\theta_w^s E_t\Big\{\frac{w_{s,t+1}-\theta_w^s w_{s,t}}{1-\theta_w^s}\Big\} + (1-\beta\theta_w^s)(w_{s,t}-(1-\epsilon_w\phi)^{-1}\hat{\mu_t}^s)$$

$$w_{s,t}-\theta_w^s w_{s,t-1} = \beta\theta_w^s E_t\{w_{s,t+1}-\theta_w^s w_{s,t}\} + (1-\theta_w^s)(1-\beta\theta_w^s)(w_{s,t}-(1-\epsilon_w\phi)^{-1}\hat{\mu_t}^s)$$

$$w_{s,t}-\theta_w^s w_{s,t-1} = \beta\theta_s^s E_t\{w_{s,t+1}\} - \beta\theta_w^s\cdot\theta_w^s w_{s,t} + (1-\theta_w^s)(1-\beta\theta_w^s)w_{s,t} - \frac{(1-\theta_w^s)(1-\beta\theta_w^s)}{1+\epsilon_w\phi}\hat{\mu_t^s}$$

$$\frac{w_{s,t}}{\theta_w^s} - w_{s,t-1} = \beta E_t\{w_{s,t+1}\} - \beta\theta_w^s w_{s,t} + \frac{(1-\theta_w^s)(1-\beta\theta_w^s)}{\theta_w^s}w_{s,t} - \frac{(1-\theta_w^s)(1-\beta\theta_w^s)}{\theta_w^s(1+\epsilon_w\phi)}\hat{\mu_t^s}$$

$$\frac{w_{s,t}}{\theta_w^s} - w_{s,t-1} = \beta E_t\{w_{s,t+1}\} - \beta E_t\{w_{s,t}\} + \beta E_t\{w_{s,t}\} - \beta\theta_w^s w_{s,t} + \frac{(1-\theta_w^s)(1-\beta\theta_w^s)}{\theta_w^s}w_{s,t} - \frac{(1-\theta_w^s)(1-\beta\theta_w^s)}{\theta_w^s(1+\epsilon_w\phi)}\hat{\mu_t^s}$$

As $\beta E_t\{w_{s,t+1}\} - \beta E_t\{w_{s,t}\} = \beta E_t\{w_{s,t+1} - w_{s,t}\} = \beta E_t\Big\{log\Big(\frac{W_{s,t+1}}{W_{s,t}}\Big)\Big\} = \beta E_t\{log(\Pi_{t+1}^s)\} = \beta E_t\{\pi_{t+1}^s\}$, and denote $\lambda_s = \frac{(1-\theta_w^s)(1-\beta\theta_w^s)}{\theta_w^s(1+\epsilon_w\phi)}$, and above equation can be rewritten as,

$$\frac{w_{s,t}}{\theta_w^s} - w_{s,t-1} = \beta E_t\{\pi_{t+1}^s\} + \beta E_t\{w_{s,t}\} - \beta\theta_w^s w_{s,t} + \frac{(1-\theta_w^s)(1-\beta\theta_w^s)}{\theta_w^s}w_{s,t} - \lambda_s\hat{\mu_t^s}$$

$$\frac{w_{s,t} - (1-\theta_w^s)(1-\beta\theta_w^s)w_{s,t} - \beta\theta_w^s w_{s,t} + \beta(\theta_w^s)^2 w_{s,t}}{\theta_w^s} - w_{s,t-1} = \beta E_t\{\pi_{t+1}^s\} - \lambda_s\hat{\mu_t^s}$$

$$\frac{w_{s,t} - w_{s,t} + \beta\theta_w^s w_{s,t} + \theta_w^s w_{s,t} - \beta(\theta_w^s)^2 w_{s,t} - \beta\theta_w^s w_{s,t} + \beta(\theta_w^s)^2 w_{s,t}}{\theta_w^s} - w_{s,t-1} = \beta E_t\{\pi_{t+1}^s\} - \lambda_s\hat{\mu_t^s}$$

Rearrange gives,

$$w_{s,t} - w_{s,t-1} = \beta E_t\{\pi_{t+1}^s\} - \lambda_s\hat{\mu_t^s}$$

Since $w_{s,t} - w_{s,t-1} = log\Big(\frac{W_{s,t}}{W_{s,t-1}}\Big) = log(\Pi_t^s) = \pi_t^s$, thus,

$$\pi_t^s = \beta E_t\{\pi_{t+1}^s\} - \lambda_s\hat{\mu_t^s}$$

where

$$\lambda_s = \frac{(1-\theta_w^s)(1-\beta\theta_w^s)}{\theta_w^s(1+\epsilon_w\phi)}$$

and

$$\hat{\mu_t^s} = \mu_t^s - \mu_w = w_{s,t} - p_t - mrs_{s,t} - \mu_w = w_{s,t} - p_t - mrs_{s,t} - (w_s - p - mrs_s) = \hat{w_{s,t}} - \hat{p_t} - \hat{mrs}_{s,t}$$

Symmetrically, for household supplies unskilled labour service the wage inflation equation is,

$$\pi_t^u = \beta E_t\{\pi_{t+1}^u\} - \lambda_u\hat{\mu_t}^u$$

where $\lambda_u = \frac{(1-\theta_w^u)(1-\beta\theta_w^u)}{\theta_w^u(1+\epsilon_w\phi)}$

### 7.4.2 Final Goods Producer

The model for the final goods producer in this context aligns with the structure outlined in the previously discussed generalized Calvo pricing model.

### 7.4.3 Wholesalers (Intermediate Goods Producers)

The model for the final goods producer in this context aligns with the structure outlined in the previously discussed generalized Calvo pricing model.

### 7.4.4 Market Clearing, Stochastic Process

$$ln(A_t) = \rho_A ln(A_{t-1}) + e_t$$

$$Y_t = C_{s,t} + C_{u,t} + I_{s,t} + I_{u,t}$$

$$K_{s,t+1} = (1-\delta)K_{s,t} + I_{s,t}$$

$$K_{u,t+1} = (1-\delta)K_{u,t} + I_{u,t}$$

$$K_t = K_{s,t} + K_{u,t}$$

# 8 Appendix B

| | |
|---|---|
| 1 | wood and cork (20) |
| 2 | basic metals and fabricated metal (27t28) |
| 3 | machinery nec (29) |
| 4 | electrical and optical equipment (30t33) |
| 5 | transport equipment (34t35) |
| 6 | manufacturing nec; recycling (36t37) |
| 7 | food beverages and tobacco (15t16) |
| 8 | textiles textile leather and footwear (17t19) |
| 9 | pulp paper paper printing and publishing (21t22) |
| 10 | coke refined petroleum and nuclear fuel (23) |
| 11 | chemicals and chemical products (24) |
| 12 | rubber and plastics (25) |
| 13 | other nonmetallic mineral (26) |
| 14 | sale maintenance and repair of motor vehicles (50) |
| 15 | wholesale trade and commission trade (51) |
| 16 | retail trade except of motor vehicles (52) |
| 17 | transport and storage (60t63) |
| 18 | post and telecommunications (64) |
| 19 | real estate activities (70) |
| 20 | renting of m&eq and other business activities (71t74) |
| 21 | agriculture hunting forestry and fishing (AtB) |
| 22 | mining and quarrying (C) |
| 23 | electricity gas and water supply € |
| 24 | construction (F) |
| 25 | hotels and restaurants (H) |
| 26 | financial intermediation (J) |
| 27 | public admin and defense; compulsory social security (L) |
| 28 | education (M) |
| 29 | health and social work (N) |
| 30 | other community social and personal services (O) |

Table 46: Industry list

Notes: The code used in parentheses corresponds to the code used in the World KLEMS database.

Conclusion

This thesis encompasses a series of scholarly papers that investigate a wide range of topics pertaining to the intricate interplay among technology shocks, labour and capital utilisation, and capital-skill complementarity in the context of labour market dynamics.

Chapter 11 builds upon the research conducted by Basu (2006) and introduces a method for measuring utilisation-adjusted neutral technology shocks that remains robust when considering different skill levels. This approach estimates the utilisation series separately for high-skilled and low-skilled labour, enabling a more comprehensive examination of the cyclical variations in utilisation at both the aggregate and sector levels. Our study reveals several important findings. Firstly, at the aggregate level, our model aligns with the standard Real Business Cycle (RBC) model, yet we observe that a utilisation-controlled technology shock induces less pronounced fluctuations in output, inputs, employment, and total hours compared to Total Factor Productivity (TFP) shocks. Secondly, our analysis demonstrates that in the short run, firms tend to utilise resources of existing high-skilled workers and extend the working hours of capital associated with high-skilled workers in response to technological advancements. Simultaneously, there is a reduction in the utilisation of both labour and capital associated with low-skilled workers. Thirdly, we find that the responses of utilisation for high-skilled and low-skilled workers to technological advancements vary across different sectors. And these findings provide insights into the nature of capital-skill complementarity/substitution across different industry sectors within the U.S. economy. These insights serve as the inspiration for the subsequent exploration of this issue in Chapter 2.

Chapter 2 examines the validity of Griliches' capital-skill complementarity hypothesis across various industries in the US economy. Our analysis yields four key findings. Firstly, using Hansen's data-splitting methodology, we find evidence supporting parameter heterogeneity and non-linearity in the capital-skill complementarity/substitution relationship. Industries with higher capital-to-output ratios exhibit capital-skill substitution, while industries with lower ratios demonstrate complementarity. Secondly, we observe complementarity between skilled and unskilled labour in both industry groups, with the level of complementarity being more pronounced in the lower capital-to-output ratio group. Additionally, industries with lower capital to output ratios have a higher average education level, suggesting a stronger complementarity between skilled and unskilled labour in industries with a more highly educated workforce.

Chapter 3 builds upon the empirical findings discussed in Chapter 11. After estimating the utilisation-technology shock series while considering the heterogeneity in labour inputs for two skill levels, this chapter proceeds to examine the Impulse Response Functions (IRFs) of total hours worked for both high and low skilled workers in response to the utilisation-adjusted technology shock series. Remarkably, the results reveal distinct directional responses of hours worked for workers with different skill levels, resulting in a significant decline in the skilled-to-unskilled ratio immediately after the shock. Based on these empirical findings, the objective of this chapter is to develop a stochastic dynamic general equilibrium (DSGE) model which incorporates capital-skill complementarity in production, Calvo prices, and nominal wage rigidity to offer valuable insights into the negative dynamics observed in the skilled-to-unskilled working hours ratio and the differential responses of hours worked by skilled and unskilled workers. We found that generalized RBC and monopolist competition models fail to explain the divergent responses of hours worked for the two skill levels. In contrast, a generalized model incorporating Calvo (1983) pricing is capable of providing an explanation. Sticky prices induce firms to reduce hours worked of skilled workers in the short run in response to technological advancements. This reduction occurs due to both cost considerations and the higher efficiency of skilled workers when capital-skill complementarity is present. In addition, we also consider a model that incorporates Calvo wage and Calvo pricing settings to further enrich our analysis.

This thesis makes contributions to the field of labour economics and macroeconomics by examining the implications of technology shocks, labour and capital utilisation, and capital-skill complementarity for labour market dynamics.

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