LTH 677

Two loop correction to the Gribov mass gap equation in Landau gauge QCD

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Abstract. We determine the two loop correction to Gribov's mass gap equation for quantum chromodynamics in the Landau gauge in the $\overline{\text{MS}}$ scheme by computing the two loop correction to the horizon condition derived from Zwanziger's local renormalizable Lagrangian which incorporates the Gribov parameter. We verify that with the explicit result, the two loop ghost propagator is enchanced in the infrared.

In 1978 Gribov demonstrated that globally fixing a gauge in a non-abelian gauge theory could not be performed in a unique way, [1]. This is due to the fact that different gauge configurations could satisfy the same gauge fixing condition. Related to this was the observation that the first zero mode of the Faddeev-Popov operator defined the edge or horizon of a region, known as the Gribov volume, inside which one had to restrict the region of integration of the path integral defining the quantum field theory, [1]. One consequence of the existence of this restricted domain was that the form of the gluon and ghost propagators in the infrared regime differed significantly from that usually used in perturbative, and therefore ultraviolet, calculations. More concretely, the ghost propagator was enhanced and the gluon propagator suppressed as the momentum decreases. Such behaviour is believed to be an element of the confinement mechanism which is nonperturbative. Another feature of Gribov's original work was the definition of the volume of the Gribov region*. This was quantified in terms of a dimensionful parameter, known as the Gribov volume or mass, which was not independent but satisfied a gap equation. Gribov computed the explicit form of the gap equation to one loop and related it to the running coupling constant, [1]. Subsequently, Zwanziger examined the Gribov path integral formulation of the problem and managed to construct a localized renormalizable Lagrangian, which included the Gribov mass explicitly, in addition to several new ghost fields over and above the usual Faddeev-Popov ghosts originating from the standard gauge fixing procedure, [3, 4, 5, 6]. When these new ghosts are eliminated by their equations of motion one recovers the non-local Lagrangian introduced by Gribov. The renormalizability properties of the Zwanziger Lagrangian have been studied in detail in [7, 8] in the Landau gauge and there is an interesting structure. First, the wave function renormalization constants of the extra Zwanziger ghosts are identical to those of the Faddeev-Popov ghosts in the Landau gauge despite being spin-1 fields with a different colour structure. Second, the renormalization of the Gribov mass is not independent. Specifically the anomalous dimension of the Gribov mass parameter is given by a combination of the β-function and gluon anomalous dimension. These have been demonstrated to all orders in perturbation theory using the algebraic renormalization technique, [7, 8]. That Gribov's original formulation of the limitations of the quantization of a non-abelian gauge theory can be recast in a localized renormalizable Lagrangian means that it ought to be possible to use the Zwanziger Lagrangian, [3, 4, 5, 6], to study problems in the presence of the Gribov mass. For example, it should be possible as an initial exercise in this direction to compute the *two* loop correction to Gribov's mass gap equation and therefore refine the relation between the Gribov mass and the coupling constant. This is the main aim of this article where we will concentrate on establishing the two loop correction to Gribov's mass gap equation in the Landau gauge in quantum chromodynamics, QCD.

First, we recall the Zwanziger Lagrangian in the conventions and notation we will use. For QCD we have in d spacetime dimensions, [3, 4, 5, 6],

$$\begin{aligned} L^Z &= L^{\mathrm{QCD}} + \bar{\phi}^{ab\,\mu} \partial^\nu \left(D_\nu \phi_\mu \right)^{ab} - \bar{\omega}^{ab\,\mu} \partial^\nu \left(D_\nu \omega_\mu \right)^{ab} \\ &\quad - g f^{abc} \partial^\nu \bar{\omega}^{ae} \left(D_\nu c \right)^b \phi^{ec\,\mu} - \frac{\gamma^2}{\sqrt{2}} \left(f^{abc} A^{a\,\mu} \phi^{bc}_\mu + f^{abc} A^{a\,\mu} \bar{\phi}^{bc}_\mu \right) - \frac{d N_A \gamma^4}{2g^2} \end{aligned} \tag{1}$$

where γ is the Gribov mass parameter and

$$L^{\mathrm{QCD}} = - \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{2\alpha} (\partial^\mu A^a_\mu)^2 - \bar{c}^a \partial^\mu D_\mu c^a + i \bar{\psi}^{iI} \not{D} \psi^{iI} \tag{2}$$

where A^a_μ is the gauge field, c^a and $\bar{c}^a$ are the Faddeev-Popov ghosts and it is understood that we will only consider the Landau gauge given by $\alpha = 0$. Further, the Zwanziger ghost fields,

*For the reader interested in more detailed background to the Gribov problem, the lectures of [2] give detailed working of the calculations of [1]

$\{\phi^{ab}_\mu, \bar{\phi}^{ab}_\mu\}$, are complex conjugate commuting fields and $\{\omega^{ab}_\mu, \bar{\omega}^{ab}_\mu\}$ are complex conjugate anti-commuting fields. The remaining conventions are that $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\nu - g f^{abc} A^b_\mu A^c_\nu$, g is the coupling constant, T^a are the colour group generators whose structure constants are f^{abc}, ψ^{iI} is the (massless) quark field, $1 \leq a \leq N_A$, $1 \leq I \leq N_F$ and $1 \leq i \leq N_f$ with N_F and N_A the dimensions of the fundamental and adjoint representations respectively, and N_f is the number of quark flavours. The covariant derivatives are defined by

$$\begin{aligned} D_\mu c^a &= \partial_\mu c^a - g f^{abc} A^b_\mu c^c \quad , \quad D_\mu \psi^{iI} = \partial_\mu \psi^{iI} + i g T^a A^a_\mu \psi^{iI} \\ (D_\mu \phi_\nu)^{ab} &= \partial_\mu \phi^{ab}_\nu - g f^{acd} A^c_\mu \phi^{db}_\nu \, . \end{aligned} \tag{3}$$

It is worth noting the notational contrasts between this version of the Lagrangian and that of other articles. We have defined the Gribov mass parameter, γ, to be of mass dimension one. Some authors use γ^2 or γ^4, in our notation, as the equivalent Gribov parameter with respective mass dimension two and four. Second, we do not include a coupling constant with γ in the mixed mass term for the same reasons one does not include it for, say, a quark mass term. Third, the appearance of the factor $\frac{1}{\sqrt{2}}$ in the mixed quadratic term plays a key role not only in the derivation of the propagators of (1) but in verifying the correctness of the gap equation we will determine. With this version of the Gribov-Zwanziger Lagrangian, we follow [6] in noting that the Gribov horizon condition, which leads to the mass gap, is

$$f^{abc} \langle A^{a\,\mu}(x) \phi^{bc}_\mu(x) \rangle = \frac{d N_A \gamma^2}{\sqrt{2} g^2} \tag{4}$$

where all quantities are bare and (4) is derived from the equation of motion of $\bar{\phi}^{ab}_\mu$ in (1).

At one loop it is elementary to see that this condition leads to Gribov's original gap equation of [1]. For instance, from the terms quadratic in the fields of (1), the gluon and commuting ghost propagators are given by

$$\begin{aligned} \langle A^a_\mu(p) A^b_\nu(-p) \rangle &= - \frac{\delta^{ab} p^2}{[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\ \langle A^a_\mu(p) \bar{\phi}^{bc}_\nu(-p) \rangle &= - \frac{f^{abc} \gamma^2}{\sqrt{2}[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\ \langle \phi^{ab}_\mu(p) \bar{\phi}^{cd}_\nu(-p) \rangle &= - \frac{\delta^{ac}\delta^{bd}}{p^2} \eta_{\mu\nu} + \frac{f^{abe} f^{cde} \gamma^4}{p^2[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\ \langle \omega^{ab}_\mu(p) \bar{\omega}^{cd}_\nu(-p) \rangle &= - \frac{\delta^{ac}\delta^{bd}}{p^2} \eta_{\mu\nu} \end{aligned} \tag{5}$$

in momentum space where

$$P_{\mu\nu}(p) = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \tag{6}$$

and $f^{acd} f^{bcd} = C_A \delta^{ab}$. Concerning conventions here the appearance of C_A in the denominator factors is a direct consequence of our choice of the coefficient of the mixed quadratic term of (1). In [4] the convention was to define the mixed quadratic term to have an additional factor proportional to $\sqrt{C_A}$ whence the corresponding term of (5) would be merely γ^4. However, the key point in the derivation of (5) is the role played by the factor of $\frac{1}{\sqrt{2}}$ in the mixed mass term of (1). Ordinarily in deriving the propagators from a Lagrangian one isolates the terms quadratic in the fields in momentum space and inverts the associated operator or matrix. In the case of (1) this procedure is followed but not only is one dealing with a mixing between the gluon and commuting ghost fields but the former is real whilst the latter is complex. To deal with this consistently, one must write this matrix with respect to the basis $(\frac{1}{\sqrt{2}} A^a_\mu, \phi^{ab}_\mu)$. In the absence of the mixing term one would not need to do this as the matrix to be inverted is block diagonal. If, by contrast, the factor of $\frac{1}{\sqrt{2}}$ was omitted from (1), then the common factor in the propagators of (5) would be $[(p^2)^2 + 2C_A\gamma^4]$ as was used in [8]. Since the original Gribov article has the factor $[(p^2)^2 + C_A\gamma^4]$ we choose to include the normalization of $\frac{1}{\sqrt{2}}$ explicitly in (1) to have propagators which are consistent with those of [1]. We will comment further on the significance of this factor later, except to note here that the appearance of $\frac{1}{\sqrt{2}}$ in (5) derives from the form of the basis chosen. Finally, concerning the mixed propagator of (5), we note that this mixing disappears in either the limit as $\gamma \to 0$ or in the abelian limit which is formally defined as $f^{abc} \to 0$ implying $C_A \to 0$.

Whilst this type of mixed propagator will complicate the problem of performing loop calculations with (1), at one loop it is straightforward to use the mixed A^a_μ-ϕ^{bc}_ν propagator itself to evaluate the vacuum expectation value of (4). Thoughout our loop calculations we will use dimensional regularization in $d = 4 - 2\epsilon$ dimensions and subtract the divergences using the $\overline{\mbox{MS}}$ scheme. Using partial fractions and the standard one loop massive integral

$$\int_k \frac{1}{[k^2 + m^2]} ~=~ \frac{(m^2)^{\frac{1}{2}d-1}}{(4\pi)^{\frac{1}{2}d}} \Gamma \left(1 - \tfrac{1}{2}d\right) \tag{7}$$

where $\int_k = \int \frac{d^dk}{(2\pi)^d}$ and m is a general mass argument, we reproduce the finite Gribov mass gap equation in four dimensions of [1] as

$$1 ~=~ C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A\gamma^4}{\mu^4} \right) \right] a ~+~ O(a^2) \tag{8}$$

where $a = g^2/(16\pi^2)$ and μ is the $\overline{\mbox{MS}}$ renormalization scale introduced to retain a dimensionless coupling constant in d-dimensions and incorporates the usual numerical factor of $4\pi e^{-\gamma_{\mbox{E}}}$ with $\gamma_{\mbox{E}}$ the Euler-Mascheroni constant. In deriving this finite expression we have introduced renormalization constants for the bare objects appearing in (4). Specifically these are defined by

$$\begin{aligned} A^{a\,\mu}_{\mbox{o}} ~&=~ \sqrt{Z_A}\, A^{a\,\mu} ~~,~~ c^a_{\mbox{o}} ~=~ \sqrt{Z_c}\, c^a ~~,~~ \phi^{ab}_{\mu\,\mbox{o}} ~=~ \sqrt{Z_\phi}\, \phi^{ab}_\mu ~~,~~ \omega^{ab}_{\mu\,\mbox{o}} ~=~ \sqrt{Z_\omega}\, \omega^{ab}_\mu \\ \psi_{\mbox{o}} ~&=~ \sqrt{Z_\psi}\psi ~~,~~ g_{\mbox{o}} ~=~ Z_g\, g ~~,~~ \gamma_{\mbox{o}} ~=~ Z_\gamma\, \gamma \end{aligned} \tag{9}$$

where the subscript $_{\mbox{o}}$ denotes the bare quantity here. Since the main aim of this article is to extend (8) to the next order, we have made use of the symbolic manipulation language Form, [9], to first reproduce the original mass gap of [1] before considering the two loop calculation. In order to check that we have used a consistent set of Feynman rules in the Form programme, we have first explicitly renormalized (1) to two loops in $\overline{\mbox{MS}}$ and verified that the Slavnov-Taylor identities derived in [7, 8] are correctly reproduced. However, such a two loop renormalization can be performed with the massless version of (1). This means that the Mincer algorithm for massless 2-point functions, [10], was the tool used for this particular computation. For completeness we note that the explicit expressions for the renormalization constants for the quantities which do not ordinarily arise in the treatment of the usual QCD Lagrangian are

$$\begin{aligned} Z_\phi ~=~ Z_\omega ~&=~ 1 ~+~ \frac{3C_A}{4} \frac{a}{\epsilon} \\ &+~ \left[\left(\frac{1}{2} C_A T_F N_f - \frac{35}{32} C_A^2 \right) \frac{1}{\epsilon^2} ~+~ \left(\frac{95}{96} C_A^2 - \frac{5}{12} C_A T_F N_f \right) \frac{1}{\epsilon} \right] a^2 ~+~ O(a^3) \end{aligned} \tag{10}$$

and

$$
\begin{aligned}
Z_\gamma &= 1 + \left[\frac{1}{3} T_F N_f - \frac{35}{48} C_A \right] \frac{a}{\epsilon} \\
&+ \left[\left(\frac{7385}{4608} C_A^2 + \frac{5}{18} T_F^2 N_f^2 - \frac{193}{144} C_A T_F N_f \right) \frac{1}{\epsilon^2} \right. \\
&+ \left. \left(C_F T_F N_f + \frac{35}{48} C_A T_F N_f - \frac{449}{384} C_A^2 \right) \frac{1}{\epsilon} \right] a^2 + O(a^3)
\end{aligned}
\tag{11}
$$

where $T^a T^a = C_F I$ and $\mathrm{tr} \left(T^a T^b \right) = T_F \delta^{ab}$. The pole structure can be encoded in the renormalization group functions as

$$
\begin{aligned}
\gamma_\phi(a) &= \gamma_\omega(a) = - \frac{3}{4} C_A a + \left[40 C_A T_F N_f - 95 C_A^2 \right] \frac{a^2}{48} + O(a^3) \\
\gamma_\gamma(a) &= \left[16 T_F N_f - 35 C_A \right] \frac{a}{48} + \left[280 C_A T_F N_f - 449 C_A^2 + 192 C_F T_F N_f \right] \frac{a^2}{192} + O(a^3)
\end{aligned}
\tag{12}
$$

where, in the Landau gauge,

$$
\gamma_\phi(a) = \beta(a) \frac{\partial \ln Z_\phi}{\partial a} \quad , \quad \gamma_\omega(a) = \beta(a) \frac{\partial \ln Z_\omega}{\partial a} \quad , \quad \gamma_\gamma(a) = \mu \frac{\partial \ln \gamma}{\partial \mu}
\tag{13}
$$

and in (13) we have used the fact that the renormalization constants do not depend on γ. Clearly $\gamma_\phi(a)$ and $\gamma_\omega(a)$ are equivalent to $\gamma_c(a)$ in agreement with the expectation of [7, 8] and $\gamma_\gamma(a)$ satisfies the Slavnov-Taylor identity derived in [7, 8].

Concerning the explicit expression (8) we note that it agrees with that of [1] when evaluated in dimensional regularization in $\overline{\mathrm{MS}}$. In this respect it is important to note the role played by the explicit appearance of the dimension d in (8). In evaluating (4) we have not set d to be 4 on the right hand side at the outset. Whilst this may appear to be a trivial point here, since the contribution from the $O(\epsilon)$ term of $d = 4 - 2\epsilon$ is present in the constant term of (8), it will turn out that to have a *finite two* loop gap equation one must retain d as $4 - 2\epsilon$ in (4) prior to renormalization in $\overline{\mathrm{MS}}$. Having established (8) it is worth noting that for an abelian theory the equation is trivially satisfied since N_A is formally zero in the original defining horizon condition, (4).

Zwanziger's elegant reformulation of the Gribov problem and the horizon definition, (4), in his Lagrangian immediately opens up the path to computing the two loop correction to (8) which we now detail. One simply has to evaluate the two loop corrections to the vacuum expectation value $f^{abc} \langle A^{a\,\mu} \phi^{bc}_\mu \rangle$. Since this will involve two loop massive vacuum bubbles it remains to determine the set of Feynman diagrams and the procedure to evaluate them. One problem with the former is the complication of having a mixed A^a_μ-ϕ^{bc}_ν propagator. To correctly establish the Feynman graphs we used the QGRAF package, [11], using an adaptation provided by the author† of [11]. Consequently there are 17 two loop Feynman diagrams to determine. These were computed using a symbolic manipulation programme written in FORM which took the QGRAF output, substituted the Feynman rules for (1) and broke up the vacuum bubbles into a common form, which was

$$
I_2 \left(m_1^2, m_2^2, m_3^2; a, b, c \right) = \int_{kl} \frac{1}{[k^2 + m_1^2]^a [l^2 + m_2^2]^b [(k-l)^2 + m_3^2]^c}
\tag{14}
$$

†We are very much indebted to Dr P. Nogueira for his elegant solution of the mixed propagator problem in QGRAF.

in addition to the product of two one loop integrals of the form of (7). Here $\{a,b,c\}$ are strictly positive integers and the two loop vacuum bubbles have potentially three mass scales after one applies partial fractioning. In other words the set of masses $\{m_1^2, m_2^2, m_3^2\}$ can take any combination of values in the set $\{0, i\gamma^2, -i\gamma^2\}$. Since the general form of (14) has been evaluated to the finite part in d-dimensions for arbitrary m_1, m_2 and m_3, it was a straightforward exercise to make the appropriate identifications in a FORM routine using the results of [12, 13, 14]. Though it is worth noting that whilst the combinations of $\{m_1^2, m_2^2, m_3^2\}$ may appear to lead to a complex value for the integrals, in the overall sum for $f^{abc}\langle A^{a\,\mu}\phi^{bc}_\mu\rangle$ the final result remained real which was a useful check. Further checks resided in the fact that the correct divergence structure resulting from combining the basic integrals (7) and (14) in summing up all the contributions from all the Feynman diagrams was correctly cancelled by the available renormalization constants. In this respect we have followed the standard procedure of [15] for renormalization in automatic calculations. In this the two loop calculation is performed for bare parameters before their renormalized versions are introduced which automatically introduce the appropriate counterterms. Consequently, one is left with the main result of this article which is the finite two loop correction to (8),

$$\begin{aligned}
1 &= C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right] a \\
&+ \left[C_A^2 \left(\frac{2017}{768} - \frac{11097}{2048} s_2 + \frac{95}{256} \zeta(2) - \frac{65}{48} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) + \frac{35}{128} \left(\ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right)^2 \right. \right. \\
&\qquad \left. + \frac{1137}{2560} \sqrt{5} \zeta(2) - \frac{205\pi^2}{512} \right) \\
&\qquad \left. + C_A T_F N_f \left(- \frac{25}{24} - \zeta(2) + \frac{7}{12} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) - \frac{1}{8} \left(\ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right)^2 + \frac{\pi^2}{8} \right) \right] a^2 \\
&+ O(a^3)
\end{aligned} \tag{15}$$

where $s_2 = (2\sqrt{3}/9)\mathrm{Cl}_2(2\pi/3)$, $\mathrm{Cl}_2(x)$ is the Clausen function and $\zeta(n)$ is the Riemann zeta function. For the interested reader s_2 and $\sqrt{5}\zeta(2)$ arise from the finite part of integrals such as $I_2(m^2, m^2, m^2; 1, 1, 1)$ and $I_2(m^2, m^2, -m^2; 1, 1, 1)$ respectively. We also note that taking d as 4 in (4) would have resulted in an extra divergence which could not be cancelled.

Having established the two loop gap equation for the Gribov mass, we now comment on its implication for the ghost propagator. In [1] Gribov showed that the one loop condition (8) ensured that the one loop correction to the ghost propagator was enhanced in the infrared. More specifically if one writes the full ghost propagator as

$$G^{ab}_c(p^2) = \frac{\delta^{ab}}{p^2[1 + u(p^2)]} \tag{16}$$

then in the ultraviolet, the ghost propagator has the usual (perturbative) behaviour of $1/p^2$. However, in the infrared the propagator behaves as $1/(p^2)^2$ as $p^2 \to 0$, [1]. Gribov indicated that this was a property of a confining non-abelian gauge theory. Moreover, he computed the one loop correction to the ghost propagator to $O(p^2)$ and found that $1 + u(0) = 0$ provided that γ satisfied (8). This observation of $1 + u(0) = 0$, which is known as the Kugo-Ojima confinement condition, [16], should also hold at two loop in (1) if the Gribov-Zwanziger Lagrangian is a consistent formulation of the Gribov horizon condition. Therefore, we have computed the two loop correction to $u(p^2)$ by generating the 31 Feynman diagrams using QGRAF, expanding them to $O(p^2)$ and evaluating the resulting vacuum bubbles. It transpires that this $O(p^2)$

correction to $u(p^2)$ is *exactly* the same as the two loop part of (15) and therefore the Kugo-Ojima condition is satisfied at two loops *precisely* for all colour groups and N_f massless quarks. Hence, the ghost propagator is enhanced at two loops in the Gribov-Zwanziger Lagrangian and has a $1/(p^2)^2$ behaviour in the infrared which is in qualitative agreement with other approaches. For instance, ghost enhancement has also been observed in explicit (non-perturbative) studies using the Schwinger-Dyson formalism and on the lattice. More specifically in the Schwinger-Dyson computations estimates for the exponent of the power-law behaviour of the gluon and ghost propagators in the infrared have been extracted and give values which are different from those of the ultraviolet form of the propagator, [17, 18]. Also, in several lattice studies the Kugo-Ojima confinement condition itself has also been examined. See, for instance, [19, 20, 21, 22], where one recent lattice estimate for $u(0)$ is $-$ 0.83, [22].

It is worth commenting on one final aspect of the two loop calculation of (15). If one uses the propagators of (1) without the factor of $\frac{1}{\sqrt{2}}$ in (5) then not only would the ghost propagator not be enhanced but at an earlier point of the study, the ghost 2-point function would not actually be *finite* either. This further justifies the derivation of (5) from (1) and what would maybe initially appear as a peculiar convention in the definition of the mass term. More importantly, that (15) is exactly what is required for ghost enhancement provides a strong check on the result (15) as well as ensuring that one has a consistent set of Feynman rules for non-zero γ. Next, in concentrating on the consistency aspect of the ghost with respect to enhancement, the gluon propagator is also expected to have infrared behaviour different from the usual ultraviolet form, [1, 18]. Whilst we have not examined the two loop corrections to the gluon propagator as it is technically more difficult to analyse than the ghost in the $p^2 \to 0$ limit, we do not believe its behaviour in the Gribov-Zwanziger Lagrangian, (1), to be inconsistent with the expectation that the propagator vanishes, [1, 18].

In conclusion, we have provided the $O(a^2)$ correction to the Gribov mass gap equation for QCD in the Landau gauge. Reassuringly the explicit form, (15), guarantees the enhancement of the ghost propagator at two loops. Given the recent interest in both the Zwanziger approach to incorporating the Gribov problem in a localized renormalizable Lagrangian and in gauges such as linear covariant, [23], and maximal abelian gauge, [24], it would appear plausible that one could extend the one loop mass gap equations in those gauges to two loops as well. Moreover, given that [8, 25, 26] also examined the Gribov problem using the local composite operator formalism to include the dimension two composite operator $\frac{1}{2}A^{a\,\mu}A^a_\mu$ it would be interesting to extend that one loop analysis to see whether the operator condenses and lowers the vacuum energy as it does in the case when the Gribov volume is regarded as infinite. Finally, we note that we believe that this is the first non-trivial loop computation performed with Zwanziger's Lagrangian. Given that the Gribov mass can be incorporated in calculations now, it would be interesting to examine the effects the presence of γ has in phenomenological analyses. For instance, using QCD to examine deep inelastic scattering problems one will inevitably wish to extend such analyses towards the infrared. It is important that the Gribov limitations are taken into account because theoretical predictions may no longer be relevant. One case in point is the extraction of estimates for the dimension four condensate $\langle G^a_{\mu\nu} G^{a\,\mu\nu} \rangle$. Since the expansion of the gluon propagator in the presence of the Gribov mass naturally gives rise to a power correction of dimension four, it would seem important that the consequences of a non-zero γ are understood in the corresponding underlying operator product expansion.

Acknowledgement. The author thanks Prof. S. Sorella and Dr D. Dudal for discussions concerning the Gribov problem. Also the author is grateful to Dr P. Nogueira for solving the problem of mixed propagators in the QGRAF package [11] and the organisers of the ACAT05 workshop where the latter discussions took place.

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