Renormalization of supersymmetric gauge theory in the uneliminated component formalism

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We show that the renormalization of the $\mathcal{N} = 1$ supersymmetric gauge theory when working in the component formalism, without eliminating auxiliary fields and using a standard covariant gauge, requires a nonlinear renormalization of the auxiliary fields.

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I. INTRODUCTION

The renormalization of $\mathcal{N} = 1$ supersymmetric gauge theory is certainly well understood in the superfield formalism both in terms of formal analysis (for example Ref. [1]) and practical calculations (for example Ref. [2]. In accordance with the nonrenormalization theorem the superpotential is unrenormalized, leading to the standard expression for the Yukawa-coupling β-function in terms of the chiral superfield anomalous dimension. However, a feature of the superfield formalism which is often overlooked is the necessity for a nonlinear renormalization of the vector superfield [3].

In fact, as we shall see, the renormalization program is perhaps most straightforwardly implemented in terms of component fields and in the case where the auxiliary fields F and D are eliminated using their equations of motion. It is well documented in this case that the Lagrangian is multiplicatively renormalizable. From a practical point of view, moreover, although a softly-broken supersymmetric theory can be treated using superfields via spurion techniques, calculations in the minimal supersymmetric standard model are generally carried out using the eliminated component formalism. Now since the elimination of F and D gives rise to nonlinear terms in the supersymmetry transformations of the physical fields, one might expect that the renormalization program would be at least as simple in terms of the uneliminated formalism. Indeed, the uneliminated formalism has been employed for effective potential calculations [4] and in calculations of the β-function for soft (mass)^2 terms [5]. Our purpose here is simply to show how the uneliminated formalism requires some care in that (in a conventional covariant gauge) the theory is once again not multiplicatively renormalizable in the conventional sense; additional counterterms are required which do not correspond to terms in the original Lagrangian but which can be generated by nonlinear field renormalizations. However, these nonlinear field renormalizations appear to be distinct from those of Ref. [3], since they only appear in the presence of chiral matter whereas the latter arise even in the pure gauge case.

We also consider what happens in the light-cone gauge, which is, in a sense we shall explain, "more supersymmetric" than the conventional covariant gauge [6].

II. RENORMALIZATION

The Lagrangian is given in components by

$$S_{\rm unel} = \int d^4x\Big[-\frac{1}{4}F^{\mu\nu A}F^A_{\mu\nu} - i\bar\lambda^A\bar\sigma^\mu(D_\mu\lambda^A) + \frac{1}{2}(D^A)^2 + F^iF_i - i\bar\psi\bar\sigma^\mu D_\mu\psi - D^\mu\bar\phi D_\mu\phi + g\bar\phi R^A\phi D^A + i\sqrt{2}g(\bar\phi\lambda\psi - \bar\psi\bar\lambda\phi) + F^iW_i + F_iW^i - \frac{1}{2}W_{ij}\psi^i\psi^j - \frac{1}{2}W^{ij}\psi_i\psi_j\Big], \tag{2.1}$$

where

$$W(\phi) = \frac{1}{6}Y^{ijk}\phi_i\phi_j\phi_k \tag{2.2}$$

is the superpotential, assumed cubic in ϕ for renormalizability, $W_i \equiv \frac{\partial W}{\partial\phi^i}$, and the lowering of indices indicates complex conjugation, so that $W^i = (W_i)^*$. For simplicity we omit possible linear and quadratic terms. The chiral fields transform according to a representation R of the gauge group and we write $\lambda = \lambda^A R^A$. If we eliminate the auxiliary fields F and D using their equations of motion:

$$D^A + g\bar\phi R^A\phi = 0, \qquad F^i + W^i = 0, \tag{2.3}$$

we obtain the eliminated Lagrangian, given in components by

$$S_{\rm el} = \int d^4x\Big[-\frac{1}{4}F^{\mu\nu A}F^A_{\mu\nu} - i\bar\lambda^A\bar\sigma^\mu(D_\mu\lambda^A) - i\bar\psi\bar\sigma^\mu D_\mu\psi - D^\mu\bar\phi D_\mu\phi - \frac{1}{2}g^2(\bar\phi R^A\phi)(\bar\phi R^A\phi) + i\sqrt{2}g(\bar\phi\lambda\psi - \bar\psi\bar\lambda\phi) - W^iW_i - \frac{1}{2}W_{ij}\psi^i\psi^j - \frac{1}{2}W^{ij}\psi_i\psi_j\Big]. \tag{2.4}$$

In either case we use the standard gauge-fixing term

$$S_{\rm gf} = \frac{1}{2\alpha}\int d^4x(\partial.A)^2 \tag{2.5}$$

with its associated ghost terms. The theory in the eliminated case is rendered finite by replacing fields and couplings in Eq. (2.4) by their corresponding bare versions. We have

$$\lambda_B^A = Z_\lambda^{1/2}\lambda^A, \qquad A_{\mu B}^A = Z_A^{1/2}A_\mu^A, \qquad \phi_B = Z_\phi^{1/2}\phi,$$
$$\psi_B = Z_\psi^{1/2}\psi, \qquad g_B = Z_g g, \tag{2.6}$$
$$Y_B^{ijk} = (Z_\Phi^{-1/2})_l^i (Z_\Phi^{-1/2})_m^j (Z_\Phi^{-1/2})_n^k Y^{lmn}.$$

Here Z_Φ is the renormalization constant for the chiral superfield Φ so that the result for Y_B is the consequence of the nonrenormalization theorem. In general, however, when working in a standard covariant gauge in components, $Z_{\phi,\psi,\Phi}$ are all different; at one loop, in fact, we have

$$\begin{aligned} Z_\lambda &= 1 - 2g^2L[\alpha C(G) + T(R)], \\ Z_A &= 1 + g^2L[(3-\alpha)C(G) - 2T(R)], \\ Z_g &= 1 + g^2L[T(R) - 3C(G)], \\ Z_\phi &= 1 + L[-Y^2 + 2(1-\alpha)g^2C(R)], \\ Z_\psi &= 1 + L[-Y^2 - 2(1+\alpha)g^2C(R)], \\ Z_\Phi &= 1 + L[-Y^2 + 4g^2C(R)], \end{aligned} \tag{2.7}$$

where

$$(Y^2)^i{}_j = Y^{ikl}Y_{jkl}, \qquad C(R) = R^AR^A,$$
$$T(R)\delta^{AB} = \mathrm{Tr}[R^AR^B], \tag{2.8}$$

$C(G)$ is the adjoint Casimir and (using dimensional regularisation with $d = 4 - \epsilon$) $L = \frac{1}{16\pi^2\epsilon}$. But now what happens if we work with the uneliminated form of the action? We might expect the theory to be rendered finite by replacing fields and couplings in Eq. (2.1) by corresponding bare versions (now we also need $F_B = (Z_F)^{1/2}F$, $D_B = (Z_D)^{1/2}D$ of course). It is not difficult to see, however, that there are one-loop diagrams with 2ϕ and $2\bar\phi$ external fields for which there are no counterterm diagrams in this case (while in the eliminated case, counterterms are supplied by the W^iW_i term). We also find that the $F\phi^2$ and $D\bar\phi\phi$ terms are not rendered finite by the renormalization constants given above.

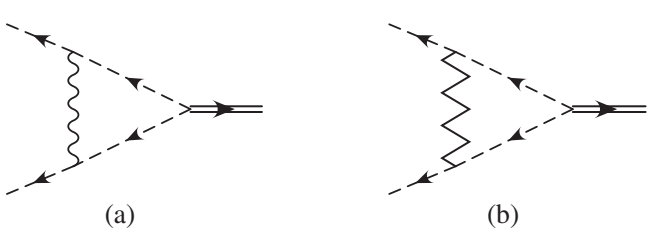

FIG. 1. Diagrams with one F, two scalar lines. Dashed, full, double full, wavy, full/wavy, zigzag lines represent ϕ, ψ, F, A, λ, D propagators, respectively.

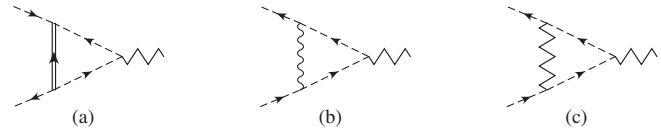

FIG. 2. Diagrams with one D, two scalar lines.

To be precise, the results for the graphs in Fig. 1 are:

$$\Gamma_a^1 = -\frac{1}{2}\alpha Lg^2(Y_{ijk}[C(R)F]^i\phi^j\phi^k - 2Y_{ijk}F^i[C(R)\phi]^j\phi^k), \tag{2.9a}$$

$$\Gamma_b^1 = -\frac{1}{2}Lg^2(Y_{ijk}[C(R)F]^i\phi^j\phi^k - 2Y_{ijk}F^i[C(R)\phi]^j\phi^k), \tag{2.9b}$$

and the results for the graphs in Fig. 2 are

$$\begin{aligned} \Gamma_a^2 &= L\bar\phi Y^2 D\phi, \\ \Gamma_b^2 &= 2\alpha Lg^2\bar\phi\left[C(R) - \frac{1}{2}C(G)\right]D\phi, \\ \Gamma_c^2 &= -2Lg^2\bar\phi\left[C(R) - \frac{1}{2}C(G)\right]D\phi, \end{aligned} \tag{2.10}$$

where $D = D^AR^A$. The results for the graphs in Fig. 3 are

$$\begin{aligned} \Gamma_a^3 &= LY_{imn}Y_{jpq}Y^{kmp}Y^{lnq}\phi^i\phi^j\phi_k\phi_l, \\ \Gamma_b^3 &= -LY_{imn}Y_{jpq}Y^{kmp}Y^{lnq}\phi^i\phi^j\phi_k\phi_l, \\ \Gamma_c^3 &= -\frac{1}{2}\alpha Lg^2Y_{ijm}Y^{kln}[C(R)]^m{}_n\phi^i\phi^j\phi_k\phi_l, \\ \Gamma_d^3 &= \frac{1}{2}Lg^2Y_{ijm}Y^{kln}[C(R)]^m{}_n\phi^i\phi^j\phi_k\phi_l, \\ \Gamma_e^3 &= -2Lg^2Y_{ijm}Y^{kln}[C(R)]^m{}_n\phi^i\phi^j\phi_k\phi_l. \end{aligned} \tag{2.11}$$

The results for the graphs in Fig. 4 are

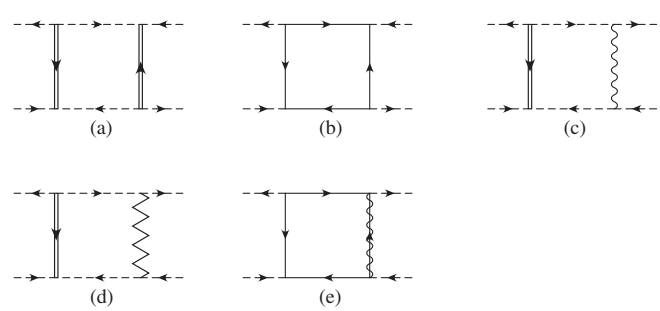

FIG. 3. Diagrams with 2 ϕ, 2 $\bar\phi$ lines and 2 or 4 Yukawa vertices.

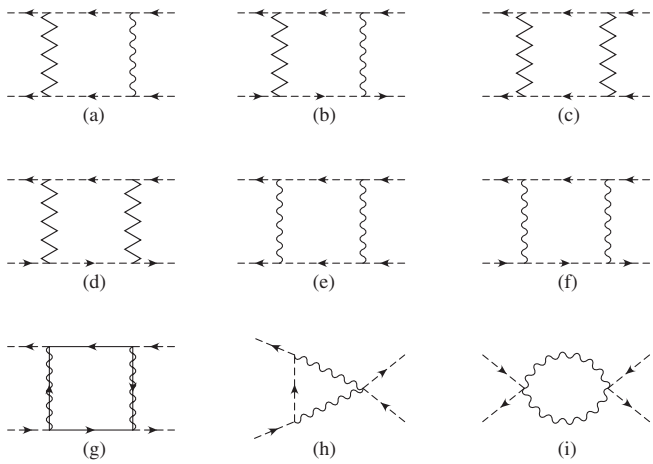

FIG. 4. Diagrams with 2 ϕ, 2 $\bar{\phi}$ lines and 4 gauge vertices.

$$
\begin{aligned}
\Gamma_a^4 &= 2\alpha g^4 L(\bar{\phi}R^A R^B\phi)(\bar{\phi}R^A R^B\phi),\\
\Gamma_b^4 &= -2\alpha g^4 L(\bar{\phi}R^A R^B\phi)(\bar{\phi}R^B R^A\phi),\\
\Gamma_c^4 &= g^4 L(\bar{\phi}R^A R^B\phi)(\bar{\phi}R^A R^B\phi),\\
\Gamma_d^4 &= g^4 L(\bar{\phi}R^A R^B\phi)(\bar{\phi}R^B R^A\phi),\\
\Gamma_e^4 &= \alpha^2 g^4 L(\bar{\phi}R^A R^B\phi)(\bar{\phi}R^A R^B\phi),\\
\Gamma_f^4 &= \alpha^2 g^4 L(\bar{\phi}R^A R^B\phi)(\bar{\phi}R^B R^A\phi),\\
\Gamma_g^4 &= -8 g^4 L(\bar{\phi}R^A R^B\phi)(\bar{\phi}R^B R^A\phi),\\
\Gamma_h^4 &= -2\alpha^2 g^4 L(\bar{\phi}R^A R^B\phi)(\bar{\phi}R^A R^B\phi + \bar{\phi}R^B R^A\phi),\\
\Gamma_i^4 &= (3+\alpha^2) g^4 L(\bar{\phi}R^A R^B\phi)(\bar{\phi}R^A R^B\phi + \bar{\phi}R^B R^A\phi).
\end{aligned}
\tag{2.12}
$$

The results for the graphs contributing to the remaining interaction terms in Eq. (2.1) are the same as in the eliminated case so we shall not give detailed results. The renormalization constants $Z_{\phi,\psi,A}$ are also the same as in the eliminated case, and in addition we have

$$
Z_F = 1 - LY^2, \qquad Z_D = 1 - 2Lg^2 T(R). \tag{2.13}
$$

We find that

$$
\begin{aligned}
&\Big[\Gamma_1 + \frac{1}{2} Y_{Bijk} F_B^i \phi_B^j \phi_B^k + (\text{c.c.})] + \Gamma_2 + \Gamma_3 + \Gamma_4\\
&\quad + g_B \bar{\phi}_B D_B \phi_B\\
&= \Big\{\frac{1}{2} Y_{ijk} F^i \phi^j \phi^k - \frac{1}{2}(\alpha+3) g^2 L[Y_{ijk}[C(R)F]^i \phi^j \phi^k\\
&\quad + \frac{1}{2} Y_{ijm} Y^{kln} [C(R)]^m{}_n \phi^i \phi^j \phi_k \phi_l\Big] + (\text{c.c.})\Big\}\\
&\quad + g\bar{\phi}D\phi - (\alpha+2) C(G) g^3 L[\bar{\phi}D\phi\\
&\quad + g(\bar{\phi}R^A\phi)(\bar{\phi}R^A\phi)].
\end{aligned}
\tag{2.14}
$$

The residual divergence cancels if we substitute the equations of motion, Eq. (2.3), for D^A and F^i, as we would expect.

Alternatively, it is clear that these remaining divergences can all be canceled by making the nonlinear renormalizations

$$
\begin{aligned}
(F_B)_i &= (Z_F^{1/2} F)_i + \frac{1}{2}(\alpha+3) g^2 L[C(R)]^l{}_i Y_{ljk} \phi^j \phi^k,\\
(D_B)^A &= (Z_D^{1/2} D)^A + (\alpha+2) C(G) g^3 L \bar{\phi} R^A \phi.
\end{aligned}
\tag{2.15}
$$

A similar phenomenon was observed in a study of the renormalization of $\mathcal{N} = \frac{1}{2}$ theories, presented in Refs. [7]; though in the case without a superpotential considered there, application of the equation of motion for F is rather trivial, since the equation of motion for $\bar{F}$ gives $F = 0$. In the $\mathcal{N} = \frac{1}{2}$ case, however, a further field redefinition (of the gaugino field λ) is necessary, and this redefinition has no analogy in the $\mathcal{N} = 1$ case considered here.

III. THE LIGHT-CONE GAUGE

It is interesting to reconsider the above calculations in the light-cone gauge, corresponding to the $\alpha \to 0$ limit of

$$
S_{\text{gf}} = \frac{1}{2\alpha} \int d^4x (n.A)^2, \quad \text{with} \quad n^2 = 0. \tag{3.1}
$$

In the light-cone gauge one again has a choice between an eliminated and an uneliminated formalism, distinct from that associated with the auxiliary fields of supersymmetry. Choosing $n = n^-$, the light-cone gauge corresponds to $A^+ = 0$ and the field A^- is nonpropagating and can be eliminated by its equation of motion. Moreover, the condition $A^+ = 0$ is preserved by the subset of supersymmetry transformations corresponding to setting the infinitesimal spinor ϵ governing these transformations to be $\epsilon = \epsilon^+$. (This is reminiscent of $\mathcal{N} = \frac{1}{2}$ supersymmetry [8], where the action is invariant under supersymmetry transformations with respect to ϵ, but with $\bar{\epsilon} = 0$). As a consequence, one finds in the light-cone gauge that

$$
\begin{aligned}
Z_\lambda &= 1 - 2g^2 L[T(R) - 3C(G)],\\
Z_\phi &= Z_\psi = 1 + L[-Y^2 + 4g^2 C(R)],
\end{aligned}
\tag{3.2}
$$

reflecting the preservation of (half the) supersymmetry by the gauge.

Light-cone gauge QCD was discussed in Ref. [9], where it was shown that a computation of the gauge two-point function in the A^--uneliminated formalism leads to divergent structures not corresponding to terms in the Lagrangian, which however vanish if the equation of motion for A^- is applied. So this is completely analogous to the situation we found above.

Returning to the supersymmetric theory, we have recalculated Eq. (2.9) in the uneliminated light-cone gauge; Eq. (2.9b) is manifestly unchanged but $\Gamma_a^1 = -\Gamma_b^1$ so that there is no 1PI divergence, as in the superfield case. Z_ϕ

now corresponds to the supersymmetric result [as indicated above in Eq. (3.2)], but Z_F remains the same as in the covariant gauge case and so we obtain

$$\Gamma_1 + \frac{1}{2} Y_{Bijk} F_B^i \phi_B^j \phi_B^k + (\text{c.c.}) = \frac{1}{2} Y_{ijk} F^i \phi^j \phi^k - g^2 L Y_{ijk} [C(R)F]^i \phi^j \phi^k + (\text{c.c.}), \tag{3.3}$$

or more generally [instead of Eq. (2.14)]

$$\begin{aligned} &\left[\Gamma_1 + \frac{1}{2} Y_{Bijk} F_B^i \phi_B^j \phi_B^k + (\text{c.c.})\right] + \Gamma_2 + \Gamma_3 + \Gamma_4 \\ &\quad + g_B \bar{\phi}_B D_B \phi_B \\ &\quad = \left\{\frac{1}{2} Y_{ijk} F^i \phi^j \phi^k - g^2 L \left[Y_{ijk} [C(R)F]^i \phi^j \phi^k \right.\right. \\ &\qquad \left.\left. + \frac{1}{2} Y_{ijm} Y^{kln} [C(R)]^m{}_n \phi^i \phi^j \phi_k \phi_l \right] + (\text{c.c.})\right\} \\ &\qquad + g \bar{\phi} D \phi - C(G) g^3 L [\bar{\phi} D \phi + g(\bar{\phi} R^A \phi)(\bar{\phi} R^A \phi)]. \end{aligned} \tag{3.4}$$

Once again the residual divergence vanishes upon application of the equations of motion for F, D, or via a nonlinear renormalization corresponding to setting $\alpha = -1$ in Eq. (2.15).

V. CONCLUSIONS

We have seen that for $\mathcal{N} = 1$ theories the renormalization program, when carried out in the F, D uneliminated formalism, contains some subtlety in that divergent terms of a form not present in the original Lagrangian are generated. These terms can, in fact, be eliminated either by means of nonlinear field redefinitions (or renormalizations) or by imposing the equations of motion for F, D. We also recalled how an analogous phenomenon occurs in the light-cone gauge, where the role of the nonpropagating F, D fields is played by the A^- gauge field. We believe that there is some pedagogical justification for clarifying these somewhat subtle features of the uneliminated form of the familiar $\mathcal{N} = 1$ supersymmetric theory. Moreover, this renders unsurprising the nonlinear redefinition of $\bar{F}$ found necessary in the $\mathcal{N} = \frac{1}{2}$ case [7]. In particular, it is interesting that in both cases the nonlinear redefinition is gauge-parameter dependent. The phenomenon may also help to elucidate the additional redefinition of λ found to be required in the $\mathcal{N} = \frac{1}{2}$ case.

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