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Two-loop β-functions and their effects for the R-parity Violating MSSM

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We present the full two-loop β-functions for the MSSM including R-parity violating couplings. We analyse the effect of two-loop running on the bounds on R-parity violating couplings, on the nature of the LSP and on the stop masses.

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1. Introduction

The minimal supersymmetric standard model (MSSM) consists of a supersymmetric extension of the standard model, with the addition of a number of dimension 2 and dimension 3 supersymmetry-breaking mass and interaction terms. It is well known that the MSSM is not, in fact, the most general renormalisable field theory consistent with the requirements of gauge invariance and naturalness; the unbroken theory is augmented by a discrete symmetry (R-parity) to forbid a set of baryon-number and lepton-number violating interactions, and the supersymmetry-breaking sector omits both R-parity violating soft terms and a set of "non-standard" (NS) soft breaking terms. There is a large literature on the effect of R-parity violation; a recent analysis (with "standard" soft-breaking terms) and references appears in Ref. [1]; for earlier relevant work see in particular [2]. The need to consider NS terms in a model–independent analysis was stressed in Ref. [3]; for a discussion of the NS terms both in general and in the MSSM context see Ref. [4]–[8]; however in this paper we shall ignore the NS terms.

The unification of the three gauge couplings in the MSSM at a scale of around $M_X \sim 10^{16}$GeV provides compelling evidence both for supersymmetry and for the existence of an underlying unified theory. We shall consider the standard mSUGRA scenario where we assume just three parameters at the unification scale, namely universal scalar and gaugino masses, and a universal trilinear scalar coupling, m_0, $m_{1/2}$ and A respectively. The remaining parameters are $\tan\beta$ and $\mathrm{sgn}(\mu)$. The complete mass spectrum is determined by running the couplings and masses from M_X to M_Z (taking the quark masses and gauge couplings at M_Z as additional inputs). There is an extensive literature on this process in the R-parity conserving (RPC) case and some pioneering work in the R-parity violating (RPV) case. In particular, Ref. [1] contained a comprehensive analysis of RPV effects on various scenarios, using full one-loop β-functions for RPV parameters, and additionally including 2-loop RPC corrections for RPC parameters. Our purpose here is to make available the full 2-loop β-functions for both RPC and RPV parameters and to explore the effect of incorporating these full β-functions on a representative sample of the scenarios considered in Ref. [1]; in particular neutrino masses and the nature of the lightest supersymmetric particle (LSP). We shall not actually present the β-functions explicitly but rather refer the reader to a website [9] where they can be accessed for the most general case (including a general 3×3 matrix of Yukawa couplings).

2. The Soft β-functions

For a general $N=1$ supersymmetric gauge theory with superpotential

$$W(\phi) = \tfrac{1}{2}\mu^{ij}\phi_i\phi_j + \tfrac{1}{6}Y^{ijk}\phi_i\phi_j\phi_k, \tag{2.1}$$

the standard soft supersymmetry-breaking scalar terms are as follows

$$V_{\text{soft}} = \left(\tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\right) + (m^2)^i{}_j\phi_i\phi^j, \tag{2.2}$$

where we denote $\phi^i \equiv \phi_i^*$ etc.

The complete exact results for the soft β-functions are given by[10] –[12]:

$$\begin{aligned}
\beta_M &= 2\mathcal{O}\left[\frac{\beta_g}{g}\right], \\
\beta_h^{ijk} &= h^{l(jk}\gamma^{i)}{}_l - 2Y^{l(jk}\gamma_1{}^{i)}{}_l, \\
\beta_b^{ij} &= b^{l(i}\gamma^{j)}{}_l - 2\mu^{l(i}\gamma_1{}^{j)}{}_l, \\
(\beta_{m^2})^i{}_j &= \Delta\gamma^i{}_j,
\end{aligned} \tag{2.3}$$

where γ is the matter multiplet anomalous dimension, and

$$\mathcal{O} = Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial Y^{lmn}}, \tag{2.4a}$$

$$(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j, \tag{2.4b}$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2MM^*g^2\frac{\partial}{\partial g^2} + \left[\tilde{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}} + \text{c.c.}\right] + X\frac{\partial}{\partial g}. \tag{2.4c}$$

Here M is the gaugino mass and $\tilde{Y}^{ijk} = (m^2)^i{}_lY^{jkl} + (m^2)^j{}_lY^{ikl} + (m^2)^k{}_lY^{ijl}$. Eq. (2.3) holds in a class of renormalisation schemes that includes DRED$'$[13], which we will use throughout. Finally the X function above is given (in the NSVZ scheme [14]) by

$$X^{\text{NSVZ}} = -2\frac{g^3}{16\pi^2}\frac{S}{[1-2g^2C(G)(16\pi^2)^{-1}]} \tag{2.5}$$

where

$$S = r^{-1}\text{tr}[m^2C(R)] - MM^*C(G), \tag{2.6}$$

$C(R), C(G)$ being the quadratic Casimirs for the matter and adjoint representations respectively. There is no corresponding exact form for X in the DRED$'$ scheme[13]; however we only require here the leading contribution which is the same in both schemes; the subleading DRED$'$ contribution is given in Ref. [15]. These formulae can readily be specialised to the case of the RPV MSSM and their implementation can be automated; in our case we used the FORM package. (We have also implemented this procedure up to three loops for the RPC MSSM[16], and made the results available on another website[17].)

In our analysis we also include "tadpole" contributions, corresponding to renormalisation of the Fayet-Iliopoulos (FI) D-term at one and two loops. These contributions are not expressible exactly in terms of β_{g_i}, γ; for a discussion see Ref. [18]. For universal boundary conditions, the FI term is very small at low energies if it is zero at gauge unification.

3. The R-parity Violating MSSM

The unbroken $\mathcal{N} = 1$ theory is defined by the superpotential

$$W = W_1 + W_2, \tag{3.1}$$

where

$$W_1 = Y_u Q u^c H_2 + Y_d Q d^c H_1 + Y_e L e^c H_1 + \mu H_1 H_2 \tag{3.2}$$

and

$$W_2 = \tfrac{1}{2}(\Lambda_E) e^c LL + \tfrac{1}{2}(\Lambda_U) u^c d^c d^c + (\Lambda_D) d^c LQ + \kappa_i L_i H_2. \tag{3.3}$$

In these equations, generation $(i, j \cdots)$, $SU_2(a, b \cdots)$, and $SU_3(I, J \cdots)$ indices are contracted in "natural" fashion from left to right, thus for example

$$\Lambda_D d^c LQ \equiv \epsilon_{ab} (\Lambda_D)^{ijk} (d^c)_{iI} L^a_j Q^{bI}_k. \tag{3.4}$$

For the generation indices we indicate complex conjugation by lowering the indices, thus $(Y_u)_{ij} = (Y_u^*)^{ij}$.

We now add soft-breaking terms as follows:

$$\begin{aligned} L_1 &= \sum_\phi m_\phi^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} \tfrac{1}{2} M_i \lambda_i \lambda_i + \text{h.c.} \right] \\ &\quad + \left[h_u Q u^c H_2 + h_d Q d^c H_1 + h_e L e^c H_1 + \text{h.c.} \right], \\ L_2 &= m_R^2 H_1^* L + m_K^2 L H_2 + \tfrac{1}{2} h_E e^c LL + \tfrac{1}{2} h_U u^c d^c d^c + h_D d^c LQ + \text{h.c.} \end{aligned} \tag{3.5}$$

We shall also use the notation

$$\lambda^{ijk} \equiv (\Lambda_E)^{kij}, \quad \lambda'^{ijk} \equiv (\Lambda_D)^{kij}, \quad \lambda''^{ijk} \equiv (\Lambda_U)^{ijk}, \tag{3.6}$$

with h, h' and h'' defined similarly in terms of h_E, h_D and h_U respectively. Note that

$$\lambda^{jik} = -\lambda^{ijk}, \quad \lambda''^{ikj} = -\lambda''^{ijk}, \tag{3.7}$$

with similar symmetry properties for h and h''.

It can be convenient to define $\mathcal{L}^a_{\alpha=0\ldots3} = \{H^a_1, L^a_{i=1,2,3}\}$. The couplings $\lambda_{\alpha\beta k}$, $\lambda'_{ij\alpha}$ are then defined so as to subsume Λ_E, Y_e and Λ_D, Y_d respectively; i.e $\lambda_{i0k} = Y_{eik}$, $\lambda'_{ij0} = -Y_{dij}$. $h_{\alpha\beta k}$, $h'_{ij\alpha}$ are defined similarly. In the same spirit we define $\mu_\alpha = \{\mu, \kappa_i\}$ and $b_\alpha = \{m_3^2, \left(m_K^2\right)_i\}$; and finally $m^2_{\mathcal{L}}$ incorporates m^2_L, m^2_R and $m^2_{H_1}$.

4. RGE Running and the Mass Spectrum

The $\overline{\mathrm{DR}}$ dimensionless couplings at M_Z are determined from the $\overline{\mathrm{MS}}$ gauge couplings and the physical quark masses by incorporating supersymmetric threshold corrections. The boundary conditions on the soft parameters and masses are imposed at the unification scale M_X. As mentioned earlier we adopt mSUGRA boundary conditions at M_X, so we take

$$\begin{aligned} m_{\tilde{Q}}(M_X) = m_{\tilde{u}}(M_X) =& m_{\tilde{d}}(M_X) = m_{\tilde{L}}(M_X) = m_{\tilde{e}}(M_X) = m_0 1, \\ m_{H_1} =& m_{H_2} = m_0, \end{aligned} \tag{4.1}$$

where 1 is the 3×3 unit matrix in flavour space.

$$\begin{aligned} \kappa_i(M_X) =& (m_R^2)_i(M_X) = (m_K^2)_i(M_X) = 0, \\ M_1(M_X) =& M_2(M_X) = M_3(M_X) = m_{\frac{1}{2}}. \end{aligned} \tag{4.2}$$

Finally we define

$$\begin{aligned} h_u(M_X) =& A_0 Y_u(M_X), \qquad h_d(M_X) = A_0 Y_d(M_X), \qquad h_e(M_X) = A_0 Y_e(M_X), \\ h_U(M_X) =& A_0 \Lambda_U(M_X), \qquad h_D(M_X) = A_0 \Lambda_D(M_X), \qquad h_E(M_X) = A_0 \Lambda_E(M_X). \end{aligned} \tag{4.3}$$

After running all the couplings from M_X to M_Z, the sparticle spectrum can be computed. Because of the interdependence of the boundary conditions at M_Z and M_X (the threshold corrections depend on the sparticle spectrum; the unification scale depends on the dimensionless couplings) we determine the couplings by an iterative process, reimposing the respective boundary conditions at each iteration. We define gauge unification to be the scale where α_1 and α_2 meet; we speed up the determination of this by (at each iteration) adjusting the unification scale using the solution of the one-loop β-functions for the gauge couplings from the previous value of the scale. We employ one-loop radiative corrections as detailed in Ref. [19]. A particular subtlety in the RPV case is that the RGE evolution of κ depends on μ, and that of m_K^2 on μ and $\tilde{B}$. Therefore it is not sufficient (as in the RPC case) to determine $\mu(M_Z)$ and $\tilde{B}(M_Z)$ after the iteration, from the electroweak breaking conditions; rather, μ and $\tilde{B}$ must be included in the iteration process to establish values of $\mu(M_X)$ and $\tilde{B}(M_X)$ which are compatible with the other boundary conditions. A second complication in the RPV case is the possibility of sneutrino vevs v_i, which satisfy

$$v^2 = v_u^2 + v_d^2 + \sum_{i=1}^{3} v_i^2 = \frac{2M_W^2}{g_2^2}, \tag{4.4}$$

where $v_{d,u}$ are the $H_{1,2}$ vevs, $\tan\beta$ is defined as usual to be

$$\tan\beta = \frac{v_u}{v_d} \tag{4.5}$$

and with our conventions $v = 174\text{GeV}$. Then at each iteration, $\mu(M_Z)$ and $\tilde{B}(M_Z)$ are determined from[1]

$$\begin{aligned}
|\mu|^2 =& \frac{\left[\overline{m}^2_{H_1} + (m_R^2)_i \frac{v_i}{v_d} + \kappa_i^* \mu \frac{v_i}{v_d}\right] - \left[\overline{m}^2_{H_2} + |\kappa_i|^2 - \frac{1}{2}(g^2+g_2^2)v_i^2 - (m_K^2)_i \frac{v_i}{v_u}\right]\tan^2\beta}{\tan^2\beta - 1} \\
&- \frac{1}{2}M_Z^2, \\
\tilde{B} =& \frac{\sin 2\beta}{2}\Big\{ \left[\overline{m}^2_{H_1} + \overline{m}^2_{H_2} + 2|\mu|^2 + |\kappa_i|^2\right] \\
&+ \left[(m_R^2)_i + \kappa_i^*\mu\right]\frac{v_i}{v_d} - (m_K^2)_i \frac{v_i}{v_u}\Big\},
\end{aligned} \tag{4.6}$$

where

$$\begin{aligned}
\overline{m}^2_{H_2} =& m^2_{H_2} + \frac{1}{2v_u}\frac{\partial \Delta V}{\partial v_u}, \\
\overline{m}^2_{H_1} =& m^2_{H_1} + \frac{1}{2v_d}\frac{\partial \Delta V}{\partial v_d},
\end{aligned} \tag{4.7}$$

with ΔV being the one-loop corrections to the scalar potential (we assume the sneutrino vevs are real). Next the sneutrino vevs may be determined from

$$(M^2_{\tilde{\nu}})_{ij} v_j = -\left[(m_R^2)_i + \mu^*\kappa_i\right] v_d + (m_K^2)_i v_u - \frac{1}{2}\frac{\partial \Delta V}{\partial v_i}, \tag{4.8}$$

where

$$\begin{aligned}
(M^2_{\tilde{\nu}})_{ij} =& (m_L^2)_{ji} + \kappa_i\kappa_j^* + \tfrac{1}{2}M_Z^2 \cos 2\beta\, \delta_{ij} \\
&+ \frac{g^2+g_2^2}{2}\sin^2\beta (v^2 - v_u^2 - v_d^2)\delta_{ij}.
\end{aligned} \tag{4.9}$$

Here g is the U_1 electroweak coupling (usually written g'). The one-loop corrections to the effective potential for the RPV MSSM which appear in $\frac{\partial \Delta V}{\partial v_i}$ were obtained from Ref. [20]. We have included the squark contributions from Ref. [20], correcting an obvious typo (a missing "ln"); the next most significant corrections, from charged slepton/Higgs, given there seem clearly wrong on dimensional grounds and we have omitted them; they are much smaller in any case. If (as we do in the neutrino mass calculation) we impose electroweak symmetry breaking at the supersymmetry scale M_{SUSY} (defined here as the geometric mean of the stop masses) then the effect even of the squark contributions from Ref. [20] is negligible. For $\frac{\partial \Delta V}{\partial v_{u,d}}$ we have used the RPC corrections given in Ref. [19]. (For the calculations of selectron, stau and stop masses given later we incorporate one-loop threshold corrections and therefore the choice of EWSB scale should be less significant; and in fact we choose to evaluate the sparticle masses at their own scale.)

Our philosophy throughout is to investigate qualitative effects, particularly of using two-loop rather than one-loop β-functions. Therefore we have made various simplifications in our procedures. ADD consider three standard forms for the relation between the weak-current and quark-mass bases for the couplings, where there is either no mixing, or the mixing is all in the down-quark sector, or all in the up-quark sector. We have assumed that the Yukawa matrices are diagonal in the weak-current basis both at the GUT scale and at the weak scale. This corresponds to assuming a trivial CKM matrix, $V_{\mathrm{CKM}} = 1$ at the weak scale. We are also neglecting the generation of off-diagonal Yukawa couplings in the evolution from M_Z to M_X (an effect which we believe is negligible to the accuracy at which we are working).

5. Neutrino Masses

Here we set bounds on the couplings λ, λ' from the cosmological neutrino bound. Combining the 2dFGRS data[21] with the WMAP measurement[22] one gets a bound on the neutrino mass

$$\sum_i m_{\nu_i} < 0.71\mathrm{eV}. \tag{5.1}$$

The neutrino mass is given by

$$m_\nu = \frac{\mu(M_1 g_2^2 + M_2 g^2)\sum_{i=1}^3 \Lambda_i^2}{2\left(v_u v_d(M_1 g_2^2 + M_2 g^2) - \mu M_1 M_2\right)}, \tag{5.2}$$

where

$$\Lambda_i = v_i - v_d \frac{\kappa_i}{\mu}. \tag{5.3}$$

A single non-zero RPV coupling at M_X will generate non-zero κ, m_R^2 and m_K^2 leading to a non-zero neutrino mass. We follow Ref. [1] in choosing the SPS1a mSUGRA point, which has the following parameter values at M_X:

$$m_0 = 100\text{GeV}, \quad m_{\frac{1}{2}} = 250\text{GeV}, \quad A_0 = -100\text{GeV} \quad \tan\beta = 10, \quad \text{sign}(\mu) = +. \tag{5.4}$$

Eq. (5.1) then leads to an upper bound on the given RPV coupling. We assume that only one out of the set of couplings

$$S_\lambda = \{\lambda'_{333}, \lambda'_{322}, \lambda'_{311}, \lambda_{233}, \lambda_{232}, \lambda_{131}\} \tag{5.5}$$

is non-zero at M_Z, and that only these couplings are non-zero in the running; these very nearly form a closed set in any case, since the only additional couplings which could be generated (at one loop) are λ'_{211}, λ'_{222} and λ_{121}. Looking at the form of the β-functions one can see that these couplings could not in any case be generated at a level close to their limiting values, since the coupling in S_λ responsible for generating them has a much smaller limiting value and is additionally suppressed by small (1st or 2nd generation) RPC Yukawa couplings. Moreover (if we start with just one of them non-zero) these couplings do not generate any off-diagonal contributions to $Y_{u,d,e}$ so our assumption about the form of these matrices at M_X is justified.

The bounds on these couplings are shown in Table 1.

Coupling	1 loop	2 loop RPC	2 loop
$\lambda'_{333}(M_Z)$	1.0×10^{-5}	8.7×10^{-6}	8.4×10^{-6}
$\lambda'_{322}(M_Z)$	4.0×10^{-4}	3.4×10^{-4}	3.2×10^{-4}
$\lambda'_{311}(M_Z)$	7.0×10^{-3}	5.9×10^{-3}	5.6×10^{-3}
$\lambda_{233}(M_Z)$	6.5×10^{-5}	5.3×10^{-5}	5.4×10^{-5}
$\lambda_{232}(M_Z)$	1.1×10^{-3}	1.0×10^{-3}	9.2×10^{-4}
$\lambda_{131}(M_Z)$	2.2×10^{-1}	1.9×10^{-1}	1.8×10^{-1}

Table 1: Upper bounds on $\lambda(M_Z)$, $\lambda'(M_Z)$

The "2 loop RPC" column corresponds to the procedure followed in Ref. [1], where the full one-loop β-functions were used and also the two-loop RPC corrections were included in the β-functions for the RPC couplings and masses. Our results (in the 2-loop RPC case) agree well with those of ADD, particularly for the λ' limits where we agree to better than 2%.

6. The Nature of the LSP

In the RPV case the LSP is no longer stable and therefore no longer subject to cosmological constraints on stable relics. Also the LSP need not be electrically and colour neutral. Once again we follow Ref. [1] in taking for this analysis the case of "no-scale" supergravity, which corresponds to taking $A_0 = m_0 = 0$. We shall consider the variation of the nature of the LSP with λ_{231}. In this case the LSP is either a stau or a selectron. The computation of selectron masses is in general more complex than the RPC case, since the charged Higgs mix with the charged sleptons, giving mass terms of the form

$$\mathcal{L}_{\text{ch}} = -\left(h_2^- \quad \tilde{e}_{L\gamma} \quad \tilde{e}_{Rk} \right) M^2_{\text{ch}} \begin{pmatrix} h_2^+ \\ \tilde{e}^*_{L\delta} \\ \tilde{e}^*_{Rl} \end{pmatrix}, \tag{6.1}$$

where M^2_{ch} is an 8×8 matrix given by

$$M^2_{\text{ch}} = \begin{pmatrix} (m^2)_{11} + D & b^*_\delta + D_\delta & \lambda_{\beta\alpha l}\mu^* v_\beta \\ b_\gamma + D^*_\gamma & (m^2)_{\delta\gamma} + \lambda_{\alpha\gamma l}\lambda_{\beta\delta l} v_\alpha v_\beta + D_{\gamma\delta} & h_{\alpha\gamma l} v_\alpha - \lambda_{\alpha\gamma l}\mu^*_\alpha v_u \\ \lambda^*_{\beta\alpha k}\mu_\alpha v_\beta & h^*_{\alpha\delta k} v_\alpha - \lambda_{\alpha\delta k}\mu_\alpha v_u & (m^2_E)_{lk} + \lambda_{\alpha\beta k}\lambda_{\alpha\gamma l} v_\beta v_\gamma + D_{lk} \end{pmatrix}, \tag{6.2}$$

where

$$\begin{aligned}
(m^2)_{11} =& \overline{m}^2_{H_2} + |\mu_\alpha|^2, \\
D =& \tfrac{1}{4}(g_2^2 + g^2)(v_u^2 - \sum_\alpha v_\alpha^2) + \tfrac{1}{2} g_2^2 v_\alpha^2, \\
D_\delta =& \tfrac{1}{2} g_2^2 v_u v_\delta, \\
(m^2)_{\gamma\delta} =& \left(m^2_{\mathcal{L}}\right)_{\delta\gamma} + \mu_\gamma \mu^*_\delta, \\
D_{\gamma\delta} =& \tfrac{1}{4}(g_2^2 - g^2)(v_u^2 - \sum_\alpha v_\alpha^2)\delta_{\delta\gamma} + \tfrac{1}{2} g_2^2 v_\gamma v_\delta, \\
D_{lk} =& \tfrac{1}{2} g^2 (v_u^2 - \sum_\alpha v_\alpha^2)\delta_{lk}.
\end{aligned} \tag{6.3}$$

However if λ_{231} is the only RPV coupling, the matrix is still diagonal except for the standard stau mixing. We have included the one-loop corrections to the slepton masses as given in Ref. [23]. Of course these omit any corrections from RPV couplings but presumably these will be extremely small.

In Fig. 1 we show the variation of the nature of the LSP with $\tan\beta$ and $\lambda_{231}(M_X)$. Here we have used the two-loop RG evolution equations but in fact the results using one-loop evolution are almost identical. Moreover it is easy to check that (at least at one loop) if λ_{231} is the only non-zero RPV coupling at M_Z then it will remain so at all scales, so we can use a simplified set of β-functions in which we only retain λ_{231}.

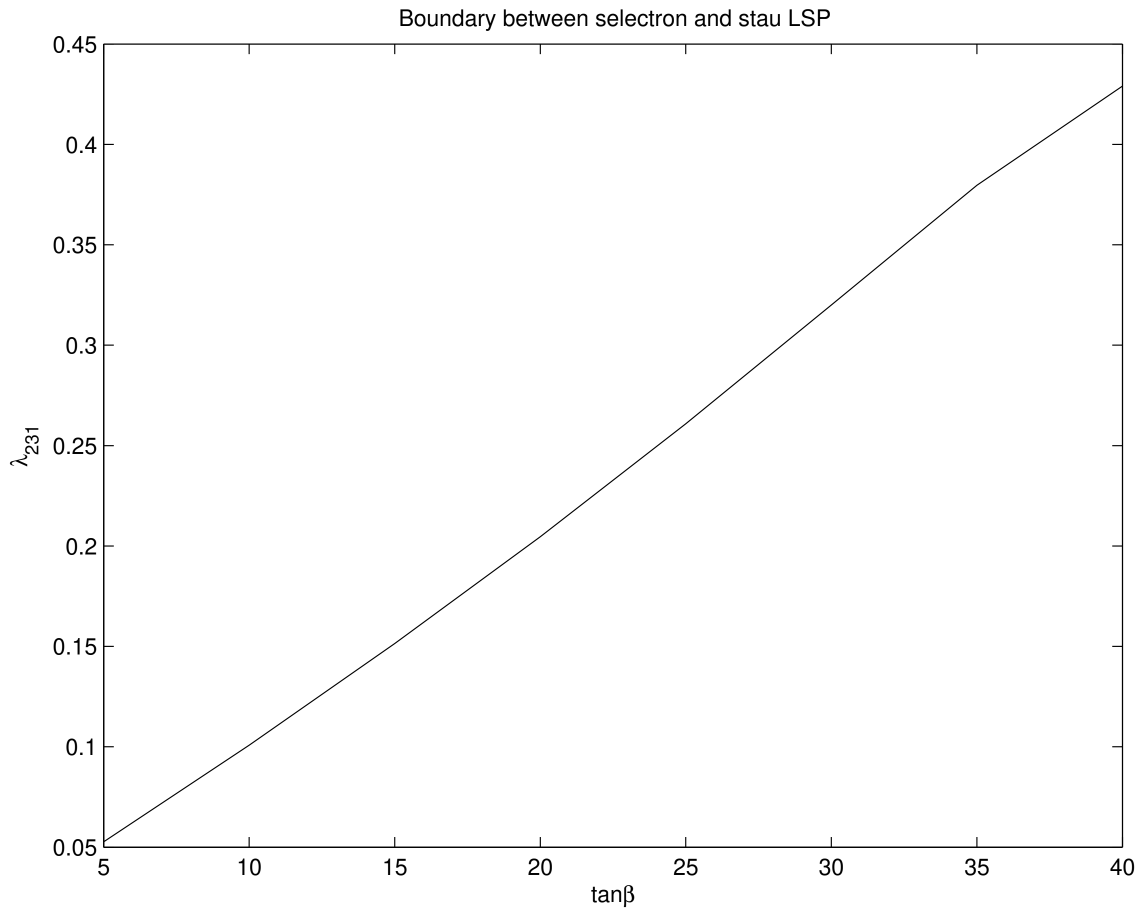

Fig. 1: The variation of the nature of the LSP (stau LSP below the line, selectron LSP above).

Once our results agree pretty with those of Ref. [1], although our demarcation line is slightly lower, particularly for larger values of $\tan\beta$.

7. The stop masses

The bounds on the λ'' couplings are much weaker than for the λ and λ' couplings and in general are only set by perturbativity of the top Yukawa coupling. The situation changes if a particular form is assumed for the quark mixing, such as mixing only in the up-quark or only in the down-quark sector. The bounds are particularly stringent in the down-quark mixing case. Although, as described earlier, we assume the no-mixing case, we expect our results to be qualitatively valid in the general case and therefore we shall display our results up to the perturbativity bound. We consider the dependence of the stop masses on λ''_{323}. (In the no-mixing case it is clearly consistent to consider a single non-zero coupling at all scales.) The mass matrix for up-type quarks has no explicit dependence on the RPV couplings and so the dependence on λ''_{323} is purely an implicit effect due to the RG evolution. The stop masses are very sensitive to the value of the top mass; here as elsewhere in the paper we take $m_{\rm top} = 174.3$GeV. We see that the variation of the stop masses, especially the light one, on λ''_{323} is considerable.

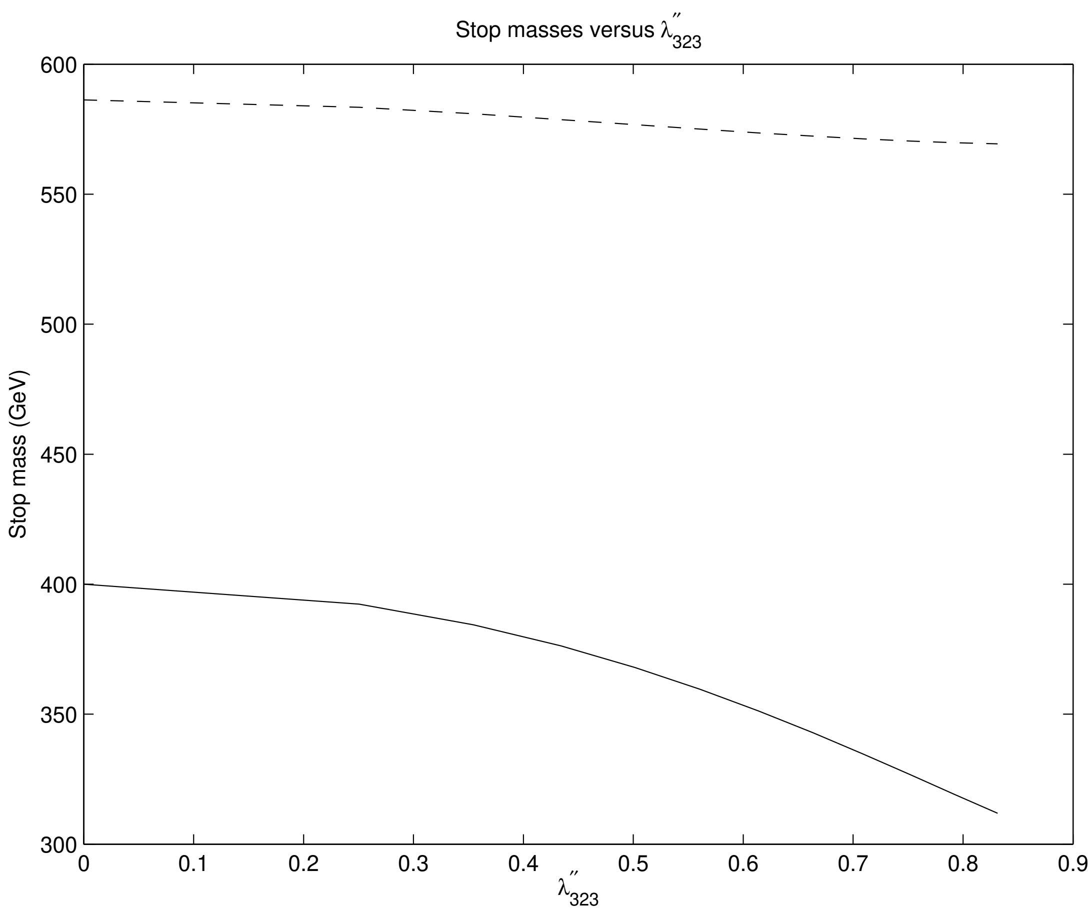

Fig. 2: Variation of the light stop mass with λ''_{323}.

8. Conclusions

We have analysed the effect of including the full set of two-loop β-functions for R-parity violating couplings in a variety of scenarios. Typically we find little difference between the effect of using the full β-functions and that of using the one-loop β-functions plus two-loop RPC corrections for RPC parameters; though as we see in Table 1 there is quite a substantial difference between the bounds on RPV couplings obtained using the full two-loop β-functions and those obtained using the full one-loop β-functions–and of course it is desirable from the point of view of consistency to use the full set of β-functions. In any event, we hope that future analysts will find the availability of the full set of β-functions for the most general R-parity violating version of the MSSM to be a useful resource[9].

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