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An estimate of the chiral condensate from unquenched lattice QCD.

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Abstract

Using the parameters in the chiral Lagrangian obtained by MILC from their unquenched lattice QCD calculations with 2+1 flavours of sea quarks, I estimate the chiral condensate. I obtain the result $\langle\overline{\psi}\psi\rangle(2 \text{ GeV})/n_f$ $= -(259 \pm 27 \text{ MeV})^3$ in the $\overline{MS}$ scheme. I compare this value to other determinations.

1 Introduction

The spontaneous breaking of chiral symmetry plays an important role in the dynamics of low energy QCD. The non-zero value for the chiral condensate is caused by spontaneous chiral symmetry breaking. The chiral condensate is a basic parameter in the QCD sum rule approach to computing hadronic quantities [1, 2] so a numerical value from lattice QCD is a valuable check on that formalism.

There have been many quenched lattice QCD calculations that have reported a value for the chiral condensate [3, 4, 5, 6]. The MILC collaboration [7, 8, 9] have been performing unquenched lattice QCD calculations with the most realistic set of parameters used to date. The results from MILC's lattice calculation have been successfully compared against experiment for many quantities that are stable to strong decay [10]. MILC's lattice calculations use the improved staggered fermion action. The method of performing unquenched calculations with improved staggered quarks has potential problems with non-locality (see [11] for a review), however this problem has not shown up in the comparison of results currently computed against experiment. The unquenched calculations use 2 light quarks in the sea and one sea quark fixed at approximately the strange quark mass. The data set from MILC has two lattice spacings (0.09 fm and 0.125 fm). All the volumes were larger than 2.5 fm. The lightest pion mass used in MILC's calculation is 250 MeV.

Although the data from MILC's unquenched calculations has been used do much important phenomenology, it has not been used to estimate a physical value for the chiral condensate. In this paper, I estimate the chiral condensate from the latest published MILC data.

2 Extracting the chiral condensate using chiral perturbation theory.

In an extensive calculation [9] the MILC collaboration fitted the squared masses of pseudo-scalar mesons and pseudo-scalar decay constants to the expressions from chiral perturbation theory [12, 13, 14, 15]

$$M_{PS}^2/(m_x + m_y) = \mu(1 + ...) \tag{1}$$

where M_{PS} is the mass of the pseudo-scalar meson made of quarks x and y with masses m_x and m_y respectively. In equation 1 the dots represent higher order terms that MILC included in the fits.

Chiral perturbation theory relates the chiral condensate $\langle\overline{\psi}\psi\rangle/n_f$ to μ

$$\langle\overline{\psi}\psi\rangle/n_f = -\frac{1}{2}\mu f^2 \tag{2}$$

where f is the pion decay constant in the chiral limit. I use the normalisation of the axial current such that the physical pion decay constant is 131 MeV. When there are no correction terms in equation 1, these two equations are known as the Gell-Mann-Oakes-Renner formulae (GMOR). Strictly speaking the GMOR relation can only be used to extract the chiral condensate if there are no higher order corrections to equation 1. Chiral perturbation theory is the generalisation of GMOR to higher orders in the pion mass. The extraction of the chiral condensate from the parameters of the chiral Lagrangian obtained by fits to lattice data in the continuum large volume limits may be an empirical approach, but I believe it is valuable.

In equation 2, $\langle\overline{\psi}\psi\rangle$ is the sum of the chiral condensates for each sea quark. In the appropriate limit where the masses of all the sea quarks go to zero in the infinite volume limit, then the chiral condensate of each sea quark is the same [16, 17].

Another technique to extract the chiral condensate from lattice QCD calculations is to compute the scalar correlator directly [3, 5, 6]. The results are then extrapolated to the zero quark mass limit. In a quenched lattice QCD calculation, Bećirević and Lubicz used both the GMOR and scalar correlators to extract a consistent result for the chiral condensate [3].

The use of the parameters from the chiral Lagrangian that have been fitted from lattice data to estimate the chiral condensate may miss some interesting physics. Stern and collaborators [18, 19, 20] have proposed a scenario where the chiral condensate for three flavours is very small, so the higher order terms in equation 1 contribute the majority of the meson mass. The possibility that the two flavour chiral condensate is small has been ruled out in the theory with $n_f = 2$ by comparison of chiral perturbation theory with $\pi\pi$ scattering [21].

3 Extracting the result

MILC have recently reported a fit of the numerical data for light pseudo-scalar meson masses and decay constants to expressions derived from staggered chiral perturbation theory [9]. A simultaneous fit was done to the data at the two different lattice spacings. The fit functions for the masses and decay constants used NNLO analytic terms, as well as lattice artifact terms arising from working at fixed lattice spacing and fixed lattice volume.

I use equation 2 with the results from MILC's extensive fits to chiral perturbation theory. One important issue is the renormalisation of equation 2. The MILC collaboration used a conserved axial current, so no renormalisation factor is needed for the pion decay constant in the chiral limit (f). The μ term does need to be renormalised. The μ term is renormalised with Z_S that is related to the renormalisation of the mass via $Z_S = \frac{1}{Z_m}$. The Z_m renormalisation factor computed by the HPQCD, MILC, and UKQCD collaborations is reported in [22].

$$Z_m(\Lambda) = \frac{1}{u_0}\left(1 + \alpha\left(b - \frac{4}{3\pi} - \frac{2}{\pi}\ln(a\Lambda)\right)\right) \tag{3}$$

where α is the QCD coupling, $b = 0.5432$, and u_0 is the tadpole factor. I always quote numbers at the scale 2 GeV in the $\overline{MS}$ scheme.

The MILC analysis [9] is a combined fit to data at two lattice spacings. The convention for the quark mass renormalisation is to convert the quark masses to the lattice scheme on the fine lattice. Hence, the Z_S factor for the quark masses on the fine lattice must be used to convert μ into the $\overline{MS}$ scheme at a scale of 2 GeV. Using the numbers quoted by MILC [9] I get Z_m(2 GeV) = 1.195. The two loop computation of Z_m in lattice perturbation theory is underway [23]. A non-perturbative estimate of Z_m, using similar techniques to those used by JLQCD [24] to renormalise the quark mass with Kogut-Susskind fermions, would be useful. I use the estimate of 9% from MILC [9] for the error due to the truncation of the perturbative series.

One advantage of the MILC collaboration's calculation is that a consistent lattice spacing is obtained from many different quantities that are stable against strong decay [10]. The chiral condensate involves the third power of the lattice spacing, so any errors in the choice of scale are amplified. I used MILC's value [8] $r_1 = 0.317(7)$ fm to convert the μ and f parameters into physical units.

Fit	Points	Coarse	Fine	$\langle\overline{\psi}\psi\rangle(2\text{ GeV})/n_f$
A	94	$m_x + m_y < 0.40 m_s'$	$m_x + m_y < 0.54 m_s'$	$(269 \pm 17\text{ MeV})^3$
B	240	$m_x + m_y < 0.70 m_s'$	$m_x + m_y < 0.80 m_s$	$(250 \pm 21\text{ MeV})^3$
C	316	$m_x + m_y < 1.10 m_s'$	$m_x + m_y < 1.14 m_s'$	$(249 \pm 15\text{ MeV})^3$

Table 1: Results for chiral condensate in the $\overline{MS}$ scheme at a scale of 2 GeV for various fits that MILC did to their data. The coarse and fine columns correspond to the mass ranges used in the fits with the data on the coarse and fine lattice spacing. The m_s' is the mass of the strange sea quark in MILC's unquenched calculation. The points column is the number of data used in the fit.

MILC's analysis [22] of their data used a number of fits that included various subsets of their data. In table 1, I compute $\langle\overline{\psi}\psi\rangle(2\text{ GeV})/n_f$ using equation 2 and the perturbative matching factor with the values for μ and f in table IV of [9]. The results for the three main fits (called A, B and C) are in table 1. The errors for the chiral condensate are dominated by the error on the pion decay constant in the chiral limit (f).

MILC use combinations of the results from the fits A,B, and C to estimate the central values and the systematic errors. I take the average of the result for fit A and B as the central value.

$$\langle\overline{\psi}\psi\rangle(2\text{ GeV})/n_f = -0.018(5)\text{ GeV}^3 \tag{4}$$

$$= -(259 \pm 27\text{ MeV})^3 \tag{5}$$

I now discuss the important issue as to whether 2+1=3. There are two main possibilities, that the chiral perturbation theory analysis is sensitive to the three flavour chiral condensate or to the two flavour chiral condensate with one sea quark fixed at the strange quark mass. In the latter case the three flavour chiral condensate could be extracted with the help of chiral perturbation theory [25]. In the MILC calculation the mass of the strange quark is fixed. However, the data is analysed with three flavour chiral perturbation theory. The mass of the strange quark was slightly incorrect on the coarse lattice, so some small extrapolation in the data is done for the strange quark mass. It is hard to give a definitive answer to the flavour dependence of the condensate extracted in this paper without further analysis.

I could not find any recent calculations of the chiral condensate from unquenched lattice QCD calculations with two flavours of sea quarks. Early work is reviewed in [26]. I have used the chiral perturbation theory approach to extract the the chiral condensate from a recent unquenched lattice QCD calculation by the JLQCD collaboration [27]. This lattice calculation was done at a fixed lattice spacing of 0.089 fm. The lightest sea quark mass was half the strange quark mass. The axial vector current used in JLQCD's calculation needs to be renormalised. I used tadpole improved perturbation theory with a simple

Group	n_f	$\langle\overline{\psi}\psi\rangle(2\ \text{GeV})/n_f$	$\langle\overline{\psi}\psi\rangle(2\ \text{GeV})/n_f$
This work, MILC	2+1	$-0.017(5)\text{GeV}^3$	$-(259 \pm 27\ \text{MeV})^3$
This work, JLQCD	2	$-0.009(1)\text{GeV}^3$	$-(209 \pm 8\ \text{MeV})^3$
Bećirević & Lubicz [3]	0		$-(273 \pm 19\ \text{MeV})^3$
Bećirević & Lubicz [3]	0		$-(312 \pm 24\ \text{MeV})^3$
Giusti et al. [4]	0	$-0.0147(8)(16)(12)\ \text{GeV}^3$	$-(245(4)(9)(7)\ \text{MeV})^3$
Gimenez et al. [5]	0		$-(265 \pm 5 \pm 22\ \text{MeV})^3$
Hernandez et al. [30]	0		$-(268(12)\ \text{MeV})^3$
DeGrand [6]	0		$-(282(6)\ \text{MeV})^3$
Giusti et al. [31]	0		$-(267(5)(15)\ \text{MeV})^3$
Blum et al. [32]	0		$-(256(8)\ \text{MeV})^3$

Table 2: Results for chiral condensate in the $\overline{MS}$ scheme at a scale of 2 GeV.

boosted coupling to estimate the required renormalisation (using the summary of results in the appendix of [28]). The numerical value of renormalisation factor is 0.45, so some kind of non-perturbative renormalisation is required for a definitive answer, hence the error for JLQCD estimate is unreliable. Dürr [29] has previously noted the problems with extracting the chiral condensate from unquenched calculations done by the CP-PACS and UKQCD collaborations.

4 Conclusion and comparison to other work

In table 2, I compare my analysis of the MILC and JLQCD data to a selection of recent lattice results for the chiral condensate.

Giusti et al. [4] note that their numbers are comparable to estimates of the chiral condensate from sum rules [1, 2]. The first entry in table 2 from Bećirević & Lubicz comes from a GMOR analysis and the second is from the pseudo-scalar vertex. Pennington [33] reviews various calculations of the chiral condensate from lattice, and sum rules and estimates the size of the the chiral condensate to be $\sim -(270\ \text{MeV})^3$. Jamin [34] obtained a value for the chiral condensate of $\sim -(267 \pm 16\ \text{MeV})^3$ from QCD sum rules.

From table 2, I note that the result from MILC is essentially consistent with the other results. Descotes et al. have argued that the chiral condensate with three sea quarks should be less than that from QCD with two light sea quarks [19]. Given the assumptions in this analysis it does not look as though the chiral condensate has a strong dependence on the number of quarks in the sea. The Columbia [35, 36] group claimed to see a reduction in chiral symmetry breaking from unquenched calculations with 0, 2, and 4 flavours of sea quarks, but the analysis of their data was complicated by finite size effects. From lattice QCD calculations Iwasaki et al. [37] find that the theory becomes deconfined for $n_f > 6$.

From equation 1, the higher value for the chiral condensate is correlated with smaller quark masses. This trend was noted by Gupta and Bhattacharya in a review of lattice data before 1997 [26]. Although when higher order mass corrections are included in equation 1 this is not so obvious.

From the same data set, the MILC, HPQCD and UKQCD collaborations [22] have obtained the mass of the strange quark to be $m_s^{\overline{MS}}(2 \text{ GeV}) = 76(0)(3)(7)(0)$ MeV, This value for the strange quark mass is low relative to other determinations [38, 39, 40], however, this calculation is the first large scale lattice calculation with 2+1 flavours of dynamical light quarks. The only other group to have published results for the strange quark mass from unquenched lattice QCD calculations with $2+1$ flavours of sea quarks is the JLQCD/CP-PACS collaboration. At a fixed lattice spacing, they obtain $m_s(2\text{GeV})$ between 80 and 90 MeV [41]. The JLQCD/CP-PACS collaboration are planing to compute the strange quark mass at other lattice spacings to do a continuum extrapolation.

Unfortunately, the data in table 2 are not precise enough to understand the systematics of quark mass determinations between lattice QCD calculations with 2 and 3 flavours of sea quarks. A reduction in the size of errors in the estimates of the chiral condensate from lattice calculations with a different number of sea quarks would help compare the results for quark masses from different calculations.

The main theoretical concern with unquenched calculations with improved staggered fermions is that the formalism requires taking the fourth of the determinant that controls the sea quark dynamics. There have been a number of theoretical papers on this topic [42, 43, 44, 45, 46, 47, 48] (the issues are adroitly explained by DeGrand [11]). None of the theory papers on the locality of improved staggered fermions have satisfactorily resolved the issue for QCD. One of the main tests of the fourth root trick is comparison of the lattice data with the results from chiral perturbation theory [9], hence it is important to fully understand all aspects of chiral perturbation theory applied to the MILC data. Crosschecks on the chiral perturbation theory analysis of MILC's data are also very valuable, because of the important phenomenology extracted from their work. An independent computation of the magnitude of the chiral condensate using different correlators would be a useful crosscheck [49, 50] on the estimate from the chiral perturbation theory fits. MILC do study the chiral condensate in their work on finite temperature [51].

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