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Searching for chiral logs in the static-light decay constant.

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ABSTRACT: Using the clover fermion action in unquenched QCD with pion masses as low as 420 MeV, we look for evidence for chiral logs in the static-light decay constant. There is some evidence for a chiral log term, if the original static theory of Eichten and Hill is used. However, the more precise data from the static action of the ALPHA collaboration do not show any evidence for non-linear dependence of the static-light decay constant on the light quark mass. We make some comments on the connection between chiral perturbation theory for decay constants of the pion and static-light meson.

KEYWORDS: Lattice QCD.

Contents

1. Introduction

The data from experiments such as BaBar and Belle is helping to measure the CKM matrix better (and hopefully see a breakdown of the standard model formalism). To extract information about the quarks, QCD must be solved for various non-perturbative matrix elements. In particular, the ratio of the decay constants of the B_s to B mesons ($\frac{f_{B_s}}{f_B}$) is a crucial QCD quantity for the unitarity checks of the CKM matrix. It will become more important once B_s mixing is directly measured at run II of the Tevatron. Ali [1] and Lubicz [2] review the dependence of the QCD matrix elements $\frac{f_{B_s}}{f_B}$ on the determination of $\mid V_{td} \mid$ and $\mid V_{ts} \mid$.

There used to a complaisant view (with perhaps a few exceptions [3, 4]) that the ratio of $\frac{f_{B_s}}{f_B}$ could easily be computed reliably from lattice QCD, because systematic errors would be reduced in ratios of decay constants.

The error on the ratio of the $\frac{f_{B_s}}{f_B}$ has recently been increased, however, because the uncertainty due to the long extrapolation in the quark mass was underestimated [5]. For example, the JLQCD [6] collaboration quote $f_{B_s}/f_{B_d} = 1.13(3)(^{+13}_{-\ 2})$, where the first error is statistical and the second error is that from the systematic uncertainties. The dominant systematic uncertainty in JLQCD's result is from the chiral extrapolation of f_B to light quark mass. There has also been work where the ratio of the B meson decay constant to the pion decay constant is used to control the log terms [9, 10].

The problem is extrapolating the value of the f_B decay constant from the masses in lattice calculations to the physical point. In particular heavy-light chiral perturbation theory predicts a log term in the light quark mass dependence of f_B. All previous lattice calculations, apart from some preliminary evidence from the HPQCD group [7, 8], have only seen linear dependence of the heavy-light decay constant on the quark mass.

There have been a number of attempts to estimate the error from extrapolating down to the light quark masses, using some physically modified form of chiral perturbation theory [6, 5, 9, 10]. These have been criticised by Sanz-Cillero et al. [11] who claim that the systematic uncertainty due to the chiral extrapolation may have been overestimated by JLQCD [6] and Kronfeld and Ryan [5]. Rather than blindly introducing the chiral log term to lattice data when those data do not show any sign of a departure from a linear behaviour, it would be better to resolve these issues by explicit looking for non-linear dependence of the static-light decay constant on the mass of the light quark and this is the goal of our unquenched lattice QCD calculations.

The UKQCD collaboration have recently finished a calculation that used sea quarks with masses around a third of the strange quark mass [12]. This is lighter than the sea quark masses used by the JLQCD calculation [6]. As part of that study UKQCD claimed to see some evidence for the chiral log term in the pion decay constant [12]. As has been noted by many authors [9, 10] the chiral log structure of f_π and f_B is rather similar at one loop. Hence, a detection of chiral logs in f_π is an indication that the parameters of the unquenched calculation are close to where chiral logs may occur in the heavy-light decay constant. The value of the lightest pion in this calculation is roughly 420 MeV [12]. The different treatments of the heavy-light chiral perturbation theory of Sanz-Cillero et al. [11] show that a deviation of linearity is expected at these pion masses.

The improved staggered formalism has produced gauge configurations with pion masses as light as 320 MeV [13]. Heavy-light staggered chiral perturbation theory can produce non-intuitive results [14]. It is valuable to perform cross checks on results from improved staggered calculations with another fermion formalism, irrespective of any theoretical concerns about the improved staggered formalism [15]. The huge computational costs of unquenched calculations with Wilson and domain wall fermions makes this a tough goal [15, 16].

2. Numerical methods

The basis of our calculation is unquenched gauge configurations generated with the non-perturbatively improved clover action and the Wilson gauge action. The lattice parameters are: volume 16^3 32, $\beta = 5.2$, and the clover coefficient was the non-perturbative value of 2.0171. We only use the same sea and valence quark mass. The full details of the action and results on the hadron spectroscopy have been published [12, 17].

We use static quarks for the heavy mesons. The static formalism is the ideal framework for investigating the log form. It has fewer parameters in the effective Lagrangian (because the parameters due to $1/M_Q$ terms are obviously absent). As noted by Arndt and Lin [18], finite size effects are reduced in the static limit. Also the exploration of chiral logs in the light quark sector by CP-PACS [19] and qq+q collaboration [20], essentially used a relatively coarse lattice spacing. It is very difficult to apply these techniques for heavy quark actions because of large aM_Q errors.

UKQCD has already published [21] an extensive analysis of the spectrum of static-light mesons and a paper on the mass of the bottom quark [22]. In this paper we look for chiral logs in the heavy-light decay constant.

As the aim of this work is to look for chiral logs in the f_B decay constant that are a small effect, it is important to reduce the statistical errors. In our previous calculations we were already using all-to-all and fuzzing techniques, so we needed a new method to reduce the statistical errors. The number of available gauge configurations is fixed. The ALPHA collaboration [23] have developed a new variant of the static formalism that reduces the $1/a$ mass renormalisation that is thought to be the reason for the poor signal to noise ratio of static-light calculations.

The static action is given by

$$S_h = a^4 \sum_x \overline{\psi}_h(x) D_0 \psi_h(x) \tag{2.1}$$

$$D_0 \psi_h(x) = \frac{1}{a}[\psi_h(x) - W^\dagger(x - a\hat{t})_t] \psi_h(x - \hat{t}) \tag{2.2}$$

The original static action written down by Eichten and Hill used $W(x)_t = U(x)_t$. The version of the ALPHA [23] static action that we investigated was:

$$V(x)_t = \frac{1}{c} \sum_{j=1}^{3} [U(x)_j U(x + a\hat{j})_t U(x + a\hat{t})_j^\dagger + U(x - a\hat{j})_j^\dagger U(x - a\hat{j})_t U(x + a\hat{t} - a\hat{j})_j] \tag{2.3}$$

where c is a normalisation constant. This is their A variant of the static-light action, here labelled by suffix A or called "fuzzed static".

The f_B decay constant is defined by the matrix element below:

$$\langle 0 \mid A_\mu \mid B(p) \rangle = i p_\mu f_B \tag{2.4}$$

The f_B matrix element is extracted from the amplitudes in the two point correlator.

$$C(t) \; = \; \sum_x \langle 0 \mid A_4(x,t) \Psi_B^\dagger(x,0) \mid 0 \rangle \tag{2.5}$$

$$\rightarrow Z_L Z_{\Psi_B} \exp(-a\mathcal{E}t) \tag{2.6}$$

where Ψ_B is the interpolating operator for static-light mesons and ground state dominance is shown in equation 2.6. The Z_L amplitude is related to the f_B^{static} decay constant

$$f_B^{static} = Z_L \sqrt{\frac{2}{M_B}} Z_A^{static} \tag{2.7}$$

We discuss the renormalisation factor Z_A^{static} later. We [21] used all-to-all propagators [24] and fuzzed sources to get accurate correlators. We fit a 5 exponential model to a 5 by 5 smearing matrix. We discuss the fit strategy in more detail in section 5.

3. Improvement and perturbative matching

To extract f_B^{stat} we need the renormalisation and $O(a)$ improvement terms for the static-light axial current.

$$A_0^{stat} = \overline{\psi} \gamma_k \gamma_5 \psi_Q \tag{3.1}$$

The improved static current is:

$$(A_I^{stat})_0 = A_0^{stat} + ac_A^{stat}\overline{\psi}\gamma_k\gamma_5\frac{1}{2}(\overleftarrow{D}_k + \overleftarrow{D}_k^{\dagger})\psi_Q \tag{3.2}$$

in the ALPHA formalism [25]. $\overleftarrow{D}_k$ are covariant derivatives acting on the light quark fields (ψ). The improvement term in equation 3.2 was first introduced by Morningstar and Shigemitsu [26].

To get from the static theory on the lattice to the static theory in the continuum a renormalisation factor is required.

$$(A_R^{stat})_0 = Z_A^{stat}(1 + b_A^{stat} am_q)(A_I^{stat})_0 \tag{3.3}$$

where m_q is the light quark mass.

The improvement coefficients b_A^{stat} and c_A^{stat} have been computed to one loop in perturbation theory [26, 25].

$$b_A^{EH;stat} = \frac{1}{2} - 0.056g^2 \tag{3.4}$$

We use the tree level value of $1/2$ for $b_A^{A;stat}$, because the one loop calculation has not yet been completed.

$$c_A^{EH;stat} = -\frac{1}{4\pi}1.0g^2 \tag{3.5}$$

$$c_A^{A;stat} = -0.1164g^2 \tag{3.6}$$

UKQCD [27] have recently written down the Z factors in the static limit. These were obtained from Kurth and Sommer's calculation [25]. This was an update on the Borrelli and Pittori [28] calculation.

$$Z_A^{stat} = 1.0 + (\frac{\log(a\mu)}{4\pi^2} - 0.137)g^2 \tag{3.7}$$

As noted by Hernandez and Hill [29], there is no effect on the value of Z_A from tadpole improvement if the standard exponential fit model is used to extract the amplitude from the correlators. In principle the improvement term could be tadpole improved.

As we are interested in the chiral logs in the leading order heavy-light chiral Lagrangian, we don't include a matching factor from the continuum static theory to QCD [29, 30]. The MILC collaboration [9] have recently discussed the appropriate scale for Z_A^{stat} using the Lepage-Mackenzie scale setting procedure [31].

The perturbative expression for Z_A for the ALPHA static action is not yet available. Experience with other "fuzzed" fermion actions suggests that the value of Z_A will be closer to one than for the static heavy quark action of Eichten and Hill [9, 32]. The "smearing" out of the gauge links can be thought of as averaging over the fields. This reduces the perturbative corrections. Alternatively, the "smearing" can be thought of as smoothing the potential in which the light quark moves, so reducing the wave-function at the origin, related to f_B.

It was found by the MILC collaboration [9, 32], for a smeared version of the clover action, that too much smearing of the fields can drastically change the decay constant. For the static action introduced by the ALPHA collaboration, only one level of smearing is used. So the heavy quark is restricted to lie within $\pm a$ of the origin. At our lattice spacing, the spatial extent of the light quark in a heavy-light meson is typically $3a$ (for example the node in the excited wave function is at that distance [24]). Thus we expect this smearing of the heavy quark to retain the same qualitative features, but it should affect the renormalisation factor for the action. This picture is consistent with the reported comparison of f_{B_s} computed in quenched QCD at $\beta = 6.2$ by Abada et al. [33], using both the Eichten and Hill static action and the static action introduced by ALPHA. However, our results using the Eichten-Hill static quark action should be less contaminated by excited states due to the use of all-to-all propagators and variational smearing techniques.

4. Chiral perturbation theory

The static limit is the ideal place to study the chiral logs predicted by heavy-light chiral perturbation theory.

$$\Phi_{f_{B_d}} \equiv f_{B_d}\sqrt{M_{B_d}} \tag{4.1}$$

The one loop correction to the static-light decay constant for 2 flavours of degenerate fermions with lightest pseudoscalar mass labelled m_π is [6, 34, 35]

$$\frac{\Phi_{f_{B_d}}}{\Phi_{f^0_{B_d}}} = 1 - \frac{3(1+3g^2)}{4}\frac{m_\pi^2}{(4\pi f)^2}\log(\frac{m_\pi^2}{\mu^2}) \tag{4.2}$$

where g is the $B^\star B\pi$ coupling. There have been a number of calculations [36, 37, 33] of the coupling g using quenched lattice QCD. We use the nominal value of $g^2 = 0.35$ in this analysis.

In order to compare with our lattice results, we use the expressions derived by Sanz-Cillero et al. [11] with a fixed cut-off Λ since this emphasises the region which is reliable in chiral perturbation theory. As well as the one loop correction from the lowest order chiral Lagrangian, there will be a term from a higher contribution to the chiral Lagrangian, namely $\alpha_1 m_\pi^2$. Thus there are three free parameters: Λ, α_1 and Φ_0. In order to establish reasonable values for these parameters, we fit this chiral expression at our two heavier quark masses, for each of the two values of $\Lambda = 0.4$ and 1.0 GeV, so determining a range of predictions for lighter quark masses.

5. Results

In table 1 we present our results for the static-light decay constant as a function of sea κ value. These values come from fitting our 5×5 matrix or correlators over the t-range 4-15 with a 5 exponential expression. This sophisticated treatment is optimal to deal with the excited state spectrum and to extract cleanly the ground state contribution needed to determine Z_L. The χ^2/d.o.f. is acceptable (indeed for $\kappa = 0.1350$, we also get an acceptable χ^2 for the naive static case for t-range 3-15, and that result is tabulated).

Our results show clearly that the fuzzed static source advocated by the ALPHA collaboration does give a much smaller statistical error.

Name	No.	$r_0 m_{PS}$	κ	Z^0_L	$r_0^{3/2} Z_L$	$\hat{Z}^0_L$	$r_0^{3/2}\hat{Z}_L$
DF3	160	1.93(3)	0.1350	0.304^{+15}_{-10}	2.87(11)	0.215^{+4}_{-4}	1.99(3)
DF4	119	1.48(3)	0.1355	0.282^{+14}_{-15}	2.89(15)	0.179^{+6}_{-6}	1.79(6)
DF6	139	1.06(3)	0.1358	0.202^{+22}_{-17}	2.24(21)	0.159^{+6}_{-5}	1.71(6)

Table 1: Decay constants (excluding any factors of Z_A) for the naive static source (Z) and the smeared source ($\hat{Z}$) for each data set. The uncorrected lattice value is Z^0_L (in lattice units), while the improved decay constant is Z_L (in units of r_0).

We compare in fig. 1 the chiral prediction discussed above with our lattice data for different quark masses, where the heaviest quark mass corresponds approximately to the strange mass. This chiral prediction is arranged to have a slope such that it goes through the values at our two larger quark masses for the fuzzed-static case. As can be seen, the curvature expected from the chiral logarithm should have set in at our lightest quark mass. The statistical errors are not sufficiently small to verify this accurately, although there is intriguing sign of a substantial curvature for the naive static case. Since this curvature is not reproduced by the relatively more accurate data from the fuzzed static case, we caution that it may be a statistical fluctuation.

If we use the chiral model as a guide to the possible extrapolation to lighter quark masses, as discussed above and shown in fig. 1, we would obtain $\frac{Z_L(B_s)}{Z_L(B)}$ from 1.31 to 1.46 as Λ is varied from 0.4 to 1.0 GeV. Thus we have a systematic error of $\pm 5\%$ arising from the chiral extrapolation. We also have a statistical error on the slope which is larger at around $\pm 35\%$. So we obtain $\frac{f_{B_s}}{f_B} = 1.38 \pm 0.13 \pm 0.08$. This is to be compared with Kronfeld's [5] best guess from JLQCD and HPQCD results of $\frac{f_{B_s}}{f_B} = 1.25 \pm 0.10$.

Our primary aim is explore the light quark mass dependence of the heavy-light decay constant, rather than to produce new values for the decay constant. As a cross-check on our work, we compute the f_{B_s} decay constant for static (EH) b quarks. The $DF3$ data set has sea quarks with masses that are close to the mass of the strange quark. As discussed in the previous section we use $Z_A = 0.68$. Using $r_0 = 0.55$ (0.05) fm, we get $f_{B_s}^{stat} = 256$ MeV with errors: 4% statistical, 14% from the scale r_0 and 10% from the uncertainties in Z_A. This value is reasonably consistent with other determinations of $f_{B_s}^{stat}$. ALPHA obtain $f_{B_s} = 225$ MeV in the continuum limit of quenched QCD using $r_0 = 0.5$ fm with a non-perturbative renormalisation scheme. Duncan et al. [30] obtain $f_{B_s}^{stat} = 304$ MeV at $a^{-1} = 1.78$ GeV in quenched QCD using the Wilson action for the light quarks.

In UKQCD's work on chiral logs in the pion decay constant, there was a concern about finite volume effects [12]. At $\kappa_{sea} = 0.1358$ it was argued from chiral perturbation theory that the finite volume effects were of the order of 8% in f_π. A similar order of magnitude effect was also estimated by Colangelo and Haefeli [38]. The volume of the lightest data set $DF6$ is $(1.5\text{fm})^3$. Recently Arndt and Lin [18, 39] have studied the effect of the finite volume on the ratio of heavy light decay constants and bag parameters. For a pion mass of around 400 MeV in a box of size $(1.6\text{fm})^3$, they obtain a finite volume effect in the ratio

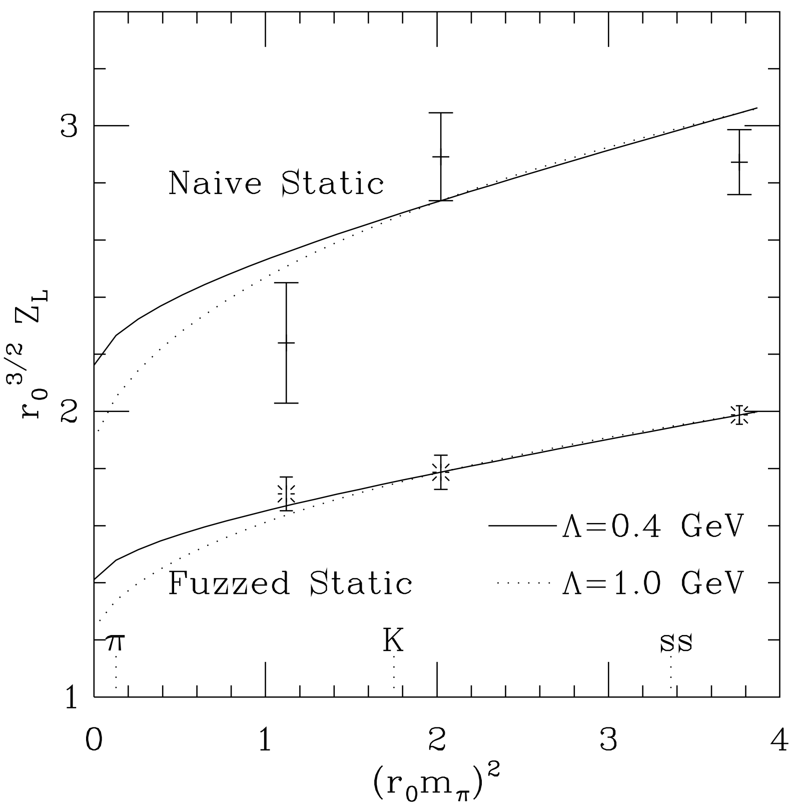

Figure 1: Static-light decay constant as a function of quark mass.

$\frac{f_{B_s}}{f_B}$ of 0.006, suggesting that the finite size effects are small. However, the next to leading order estimate of finite size effects in f_π was significant [38]. Unfortunately, Colangelo and Haefeli [38] claim that there is not enough information to make a similar estimate for f_B.

Because of computational limitations we are forced to work at a fixed lattice spacing. It is difficult to estimate the uncertainty due to not doing a continuum extrapolation. There are now variants of chiral perturbation theory that include the effects of the leading lattice spacing errors (see Bar [40] for a review). Aubin and Bernard [14] have developed a formalism for heavy-light chiral perturbation theory, at non-zero lattice spacing, with staggered fermions as the light quarks.

6. Conclusions

We have looked for the effects of the chiral log in the heavy-light decay constant using the lightest unquenched clover sea quarks used to date. Unfortunately, even with pions as light as 420 MeV, we do not see any compelling evidence for the chiral log in the static-light decay constant.

We have seen that the quark mass dependence of the static-light decay constant can be different to that of the pion decay constant. Both quantities can in principle have a different volume dependence and different higher order terms in the chiral Lagrangian. The similarity of the one loop expressions for f_π and f_B seems to be of limited value [9, 10] in guiding extrapolation.

Although we have been unable to find unambiguous evidence of the chiral logarithm, the approach we have tried to take here is potentially a good way to study the light quark mass dependence of the heavy-light decay constant. The improvements in the numerical techniques for static-light calculations means that quite precise results are achievable. With current computer resources, the only hope of working with lighter sea masses with Wilson like quarks is to work at relatively coarser lattice spacings. Using the static limit for the heavy quarks is more controlled. The use of heavy-light perturbation theory at nonzero lattice spacing would also be required.

References

[1] A. Ali, *Ckm phenomenology and b-meson physics: Present status and current issues*, hep-ph/0312303.

[2] V. Lubicz, *The role of lattice qcd in flavor physics*, hep-lat/0410051.

[3] D. Atwood and A. Soni, *Cp asymmetry measurements in psi k0 and the ckm paradigm*, *Phys. Lett.* **B508** (2001) 17–24, [hep-ph/0103197].

[4] S. R. Sharpe and Y. Zhang, *Quenched chiral perturbation theory for heavy-light mesons*, *Phys. Rev.* **D53** (1996) 5125–5135, [hep-lat/9510037].

[5] A. S. Kronfeld and S. M. Ryan, *Remark on the theoretical uncertainty in b0 anti-b0 mixing*, *Phys. Lett.* **B543** (2002) 59–65, [hep-ph/0206058].

[6] **JLQCD** Collaboration, S. Aoki *et. al.*, *B0 anti-b0 mixing in unquenched lattice qcd*, *Phys. Rev. Lett.* **91** (2003) 212001, [hep-ph/0307039].

[7] M. Wingate, C. T. H. Davies, A. Gray, G. P. Lepage, and J. Shigemitsu, *The b_s and d_s decay constants in 3 flavor lattice qcd*, hep-ph/0311130.

[8] A. Gray *et. al.*, *B leptonic decays and b - anti-b mixing with 2+1 flavors of dynamical quarks*, hep-lat/0409040.

[9] **MILC** Collaboration, C. Bernard *et. al.*, *Lattice calculation of heavy-light decay constants with two flavors of dynamical quarks*, *Phys. Rev.* **D66** (2002) 094501, [hep-lat/0206016].

[10] D. Becirevic, S. Fajfer, S. Prelovsek, and J. Zupan, *Chiral corrections and lattice qcd results for f(b/s)/f(b/d) and delta(m(b/s))/delta(m(b/d))*, *Phys. Lett.* **B563** (2003) 150–156, [hep-ph/0211271].

[11] J. J. Sanz-Cillero, J. F. Donoghue, and A. Ross, *Long distance chiral corrections in b meson amplitudes*, *Phys. Lett.* **B579** (2004) 86–98, [hep-ph/0305181].

[12] **UKQCD** Collaboration, C. R. Allton *et. al.*, *Improved wilson qcd simulations with light quark masses*, hep-lat/0403007.

[13] C. Aubin *et. al.*, *Light hadrons with improved staggered quarks: Approaching the continuum limit*, hep-lat/0402030.

[14] C. Aubin and C. Bernard, *Staggered chiral perturbation theory with heavy-light mesons*, hep-lat/0409027.

[15] K. Jansen, *Actions for dynamical fermion simulations: Are we ready to go?*, *Nucl. Phys. Proc. Suppl.* **129** (2004) 3–16, [hep-lat/0311039].

[16] A. D. Kennedy, *Algorithms for lattice qcd with dynamical fermions*, hep-lat/0409167.

[17] **UKQCD** Collaboration, C. R. Allton *et. al.*, *Effects of non-perturbatively improved dynamical fermions in qcd at fixed lattice spacing*, *Phys. Rev.* **D65** (2002) 054502, [hep-lat/0107021].

[18] D. Arndt and C. J. D. Lin, *Heavy meson chiral perturbation theory in finite volume*, hep-lat/0403012.

[19] **CP-PACS** Collaboration, Y. Namekawa *et. al.*, *Light hadron spectroscopy in two-flavor qcd with small sea quark masses*, hep-lat/0404014.

[20] **qq+q** Collaboration, F. Farchioni, I. Montvay, and E. Scholz, *Quark mass dependence of pseudoscalar masses and decay constants on a lattice*, hep-lat/0403014.

[21] **UKQCD** Collaboration, A. M. Green, J. Koponen, C. McNeile, C. Michael, and G. Thompson, *Excited b mesons from the lattice*, hep-lat/0312007.

[22] **UKQCD** Collaboration, C. McNeile, C. Michael, and G. Thompson, *An unquenched lattice qcd calculation of the mass of the bottom quark*, hep-lat/0408025.

[23] **ALPHA** Collaboration, M. Della Morte *et. al.*, *Lattice hqet with exponentially improved statistical precision*, *Phys. Lett.* **B581** (2004) 93–98, [hep-lat/0307021].

[24] **UKQCD** Collaboration, C. Michael and J. Peisa, *Maximal variance reduction for stochastic propagators with applications to the static quark spectrum*, *Phys. Rev.* **D58** (1998) 034506, [hep-lat/9802015].

[25] **ALPHA** Collaboration, M. Kurth and R. Sommer, *Renormalization and o(a)-improvement of the static axial current*, *Nucl. Phys.* **B597** (2001) 488–518, [hep-lat/0007002].

[26] C. J. Morningstar and J. Shigemitsu, *One-loop matching of lattice and continuum heavy-light axial vector currents using nrqcd*, *Phys. Rev.* **D57** (1998) 6741–6751, [hep-lat/9712016].

[27] **UKQCD** Collaboration, K. C. Bowler *et. al.*, *Decay constants of b and d mesons from non-perturbatively improved lattice qcd*, *Nucl. Phys.* **B619** (2001) 507–537, [hep-lat/0007020].

[28] A. Borrelli and C. Pittori, *Improved renormalization constants for b decay and b anti-b mixing*, *Nucl. Phys.* **B385** (1992) 502–526.

[29] O. F. Hernandez and B. R. Hill, *Tadpole improved perturbation theory for heavy - light lattice operators*, *Phys. Rev.* **D50** (1994) 495–500, [hep-lat/9401035].

[30] A. Duncan *et. al.*, *Properties of b mesons in lattice qcd*, *Phys. Rev.* **D51** (1995) 5101–5129, [hep-lat/9407025].

[31] G. P. Lepage and P. B. Mackenzie, *On the viability of lattice perturbation theory*, *Phys. Rev.* **D48** (1993) 2250–2264, [hep-lat/9209022].

[32] C. W. Bernard and T. DeGrand, *Perturbation theory for fat-link fermion actions*, *Nucl. Phys. Proc. Suppl.* **83** (2000) 845–847, [hep-lat/9909083].

[33] A. Abada *et. al.*, *Lattice measurement of the couplings g-hat(infinity) and g(b* b pi)*, *JHEP* **02** (2004) 016, [hep-lat/0310050].

[34] B. Grinstein, E. Jenkins, A. V. Manohar, M. J. Savage, and M. B. Wise, *Chiral perturbation theory for f d(s) / f d and b b(s) / b b*, *Nucl. Phys.* **B380** (1992) 369–376, [hep-ph/9204207].

[35] J. L. Goity, *Chiral perturbation theory for su(3) breaking in heavy meson systems*, *Phys. Rev.* **D46** (1992) 3929–3936, [hep-ph/9206230].

[36] **UKQCD** Collaboration, G. M. de Divitiis *et. al.*, *Towards a lattice determination of the b* b pi coupling*, *JHEP* **10** (1998) 010, [hep-lat/9807032].

[37] A. Abada *et. al.*, *First lattice qcd estimate of the g(d* d pi) coupling*, *Phys. Rev.* **D66** (2002) 074504, [hep-ph/0206237].

[38] G. Colangelo and C. Haefeli, *An asymptotic formula for the pion decay constant in a large volume*, *Phys. Lett.* **B590** (2004) 258–264, [hep-lat/0403025].

[39] J. L. Goity, *Analysis of finite size effects on the decay constants of heavy mesons*, *Phys. Lett.* **B249** (1990) 495–504.

[40] O. Bar, *Chiral perturbation theory at non-zero lattice spacing*, hep-lat/0409123.