DESY 04-169
Edinburgh 2004/16
LU-ITP 2004/027
LTH 633

Determination of Light and Strange Quark Masses from Full Lattice QCD

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We compute the light and strange quark masses $m_\ell = (m_u + m_d)/2$ and m_s, respectively, in full lattice QCD with $N_f = 2$ flavors of light dynamical quarks. The renormalization constants, which convert bare quark masses into renormalized quark masses, are computed nonperturbatively, including the effect of quark-line disconnected diagrams. We obtain $m_\ell^{\overline{MS}}(2\,\mathrm{GeV}) = 4.7(2)(3)$ MeV and $m_s^{\overline{MS}}(2\,\mathrm{GeV}) = 119(5)(8)$ MeV.

PACS numbers: 12.15.Ff,12.38.Gc,14.65.Bt

The light and strange quark masses are among the least well known parameters of the Standard Model. The reason is that quarks are confined, so that the masses must be determined indirectly through their influence on hadronic observables. This requires nonperturbative techniques. One such technique is lattice QCD.

The quark masses obtained directly in lattice calculations are bare quark masses at the cut-off scale a^{-1}, where a denotes the lattice spacing. For the lattice numbers to be useful for phenomenology, it is necessary to convert the bare quark masses to renormalized masses in some standard renormalization scheme. Because lattice perturbation theory converges badly, and the expansion coefficients are generally known to one loop order only, this ought to be done nonperturbatively. In full QCD a one-loop perturbative renormalization of the mass operator is totally inadequate even, as it does not account for the disconnected (flavor singlet) contribution shown in Fig. 1, which turns out to be comparable with the connected contribution at present lattice spacings.

In quenched QCD, in which the effect of sea quarks is neglected (and hence quark-line disconnected fermion loops are absent), several groups [1] have carried out an entirely nonperturbative calculation of the light and strange quark masses. Remarkably consistent results have been found. Previous calculations in full QCD, both with $N_f = 2$ [2] and $N_f = 3$ [3] flavors of sea quarks, employ perturbative renormalization to compute the relation between the bare and renormalized quark masses. These authors, except perhaps Eicker *et al.*, find rather small values for the strange quark mass, which lie substantially below the central value quoted by the Particle Data Group [4].

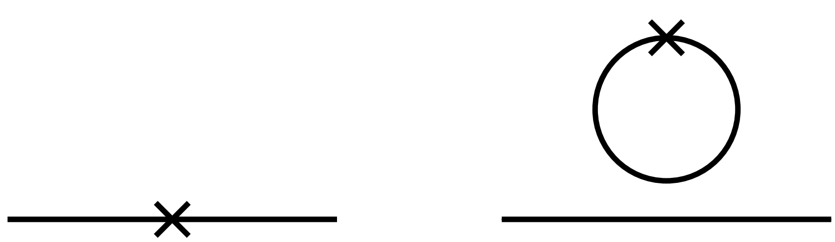

FIG. 1: Quark diagrams contributing to the renormalization of the mass operator (×). The left figure shows the connected (nonsinglet) contribution, the right figure the disconnected (singlet minus nonsinglet) contribution. Gluon lines have been omitted.

TABLE I: Hopping parameters of sea and valence quarks used in this calculation, together with their critical values.

β	$\kappa_{\rm sea}$	$\kappa_{\rm val}$									$\kappa^c_{\rm val}$	$\kappa^c_{\rm sea}$
	0.13420	0.13340	0.13380	0.13420	0.13470	0.13530	0.13560	0.1360	0.13620		0.137550(49)	
5.20	0.13500	0.13320	0.13370	0.13420	0.13450	0.13500	0.13530	0.1355	0.13570		0.136889(32)	0.136008(15)
	0.13550	0.13320	0.13360	0.13400	0.13430	0.13480	0.13500	0.13530	0.13550	0.13570	0.136457(23)	
	0.13460	0.13370	0.13400	0.13460	0.13490	0.13530	0.13550	0.13590	0.13610		0.137237(19)	
5.25	0.13520	0.13370	0.13410	0.13450	0.13480	0.13520	0.13550	0.13580	0.13590		0.136883(13)	0.136250(7)
	0.13575	0.13360	0.13390	0.13430	0.13460	0.13500	0.13520	0.13550	0.13575	0.13600	0.136553(9)	
	0.13400	0.13400	0.13440	0.13490	0.13520	0.13550	0.13570	0.13590	0.13620		0.137516(33)	
5.29	0.13500	0.13400	0.13430	0.13470	0.13500	0.13550	0.13570	0.13600	0.13610		0.137045(16)	0.136410(9)
	0.13550	0.13390	0.13430	0.13460	0.13490	0.13530	0.13550	0.13580	0.13600	0.13630	0.136816(11)	
	0.13500	0.13420	0.13480	0.13500	0.13530	0.13560	0.13590	0.13610	0.13630	0.13660	0.137131(14)	
5.40	0.13560	0.13460	0.13500	0.13530	0.13560	0.13570	0.13595	0.13610	0.13650		0.136966(12)	0.136690(22)
	0.13610	0.13470	0.13500	0.13530	0.13560	0.13580	0.13595	0.13610	0.13645		0.136836(14)	

In this Letter we shall present a first fully nonperturbative calculation of the light and strange quark masses in full QCD, including the effect of flavor singlet renormalization factors. We consider nonperturbatively $O(a)$ improved Wilson fermions with $N_f = 2$ flavors of degenerate dynamical quarks and the Wilson gauge field action. The calculation is done in two steps. We simulate dynamical gauge field configurations at four different values of the coupling, β, and at three different sea quark masses each. The latter are specified by the hopping parameter $\kappa_{\rm sea}$. The actual parameters, as well as the corresponding lattice spacings and pseudoscalar mass values, are shown in Fig. 2. We use the force parameter r_0 to set the scale. On these configurations we then perform a partially quenched calculation of the pseudoscalar mass, allowing for different sea and valence quark masses, from which we derive the physical quark masses. In Table 1 we list the hopping parameters of valence ($\kappa_{\rm val}$) and sea quarks considered in this calculation.

The bare sea and valence quark masses are given by $am_{\rm sea} = 1/(2\kappa_{\rm sea}) - 1/(2\kappa^c_{\rm sea})$ and $am_{\rm val} = 1/(2\kappa_{\rm val}) - 1/(2\kappa^c_{\rm sea})$, respectively. We consider the case of degenerate valence quarks only. In Fig. 3 we show the partially quenched pseudoscalar mass $m_{PS}(\kappa_{\rm sea}, \kappa_{\rm val})$. The critical hopping parameter $\kappa^c_{\rm sea}$ is found by keeping β fixed and varying $\kappa_{\rm sea}$ until $m_{PS}(\kappa_{\rm sea}, \kappa_{\rm sea}) = 0$. Similarly, we introduce a critical hopping parameter of the valence quarks, $\kappa^c_{\rm val}$, which is found by varying $\kappa_{\rm val}$ until $m_{PS}(\kappa_{\rm sea}, \kappa_{\rm val}) = 0$, while keeping β, $\kappa_{\rm sea}$ fixed. Our calculation requires a precise determination of $\kappa^c_{\rm sea}$ and $\kappa^c_{\rm val}$. We perform a global fit of the form

$$\begin{aligned} a r_0 m_{PS}^2 = {} & u\left(\frac{1}{\kappa_{\rm sea}} - \frac{1}{\kappa^c_{\rm sea}}\right) + v\left(\frac{1}{\kappa_{\rm val}} - \frac{1}{\kappa_{\rm sea}}\right) + w\left(\frac{1}{\kappa_{\rm sea}} - \frac{1}{\kappa^c_{\rm sea}}\right)^2 \\ & + x\left(\frac{1}{\kappa_{\rm val}} - \frac{1}{\kappa_{\rm sea}}\right)^2 + y\left(\frac{1}{\kappa_{\rm sea}} - \frac{1}{\kappa^c_{\rm sea}}\right)\left(\frac{1}{\kappa_{\rm val}} - \frac{1}{\kappa_{\rm sea}}\right) \end{aligned} \tag{1}$$

to all 100 data points, where the parameters are taken to be second order polynomials in β. The fit gave $\chi^2/\mathrm{dof} = 0.67$. The resulting values of $\kappa^c_{\rm sea}$ and $\kappa^c_{\rm val}$ are given in Table 1. The force parameter r_0/a was computed from the static potential.

Taking the derivatives of the quark propagator with respect to $am_{\rm sea}$ and $am_{\rm val}$, we obtain for the renormalized sea and valence quark masses [5]

$$m^R_{\rm sea} = Z^S_m m_{\rm sea}\,, \tag{2}$$

$$m^R_{\rm val} = Z^{NS}_m (m_{\rm val} - m_{\rm sea}) + Z^S_m m_{\rm sea}\,, \tag{3}$$

where Z^S_m and Z^{NS}_m are singlet and nonsinglet renormalization constants of the mass operator. Partially quenched chiral perturbation theory to NLO predicts [6]

$$m_{PS}^2 = [A + (B + C\ln m^R_{\rm val})\, m^R_{\rm sea}]\, m^R_{\rm val} + (D + E\ln m^R_{\rm val})\,(m^R_{\rm val})^2\,. \tag{4}$$

From (4) follows that $m^R_{\rm val}$ vanishes where the partially quenched pseudoscalar mass vanishes, which happens at the value $\kappa_{\rm val} = \kappa^c_{\rm val}$. If we insert this value into (3) we obtain the ratio

$$\frac{Z^S_m}{Z^{NS}_m} = \left.\frac{m_{\rm sea} - m_{\rm val}}{m_{\rm sea}}\right|_{\kappa_{\rm val} = \kappa^c_{\rm val}} = \left(\frac{1}{2\kappa_{\rm sea}} - \frac{1}{2\kappa^c_{\rm val}}\right)\left(\frac{1}{2\kappa_{\rm sea}} - \frac{1}{2\kappa^c_{\rm sea}}\right)^{-1}\,. \tag{5}$$

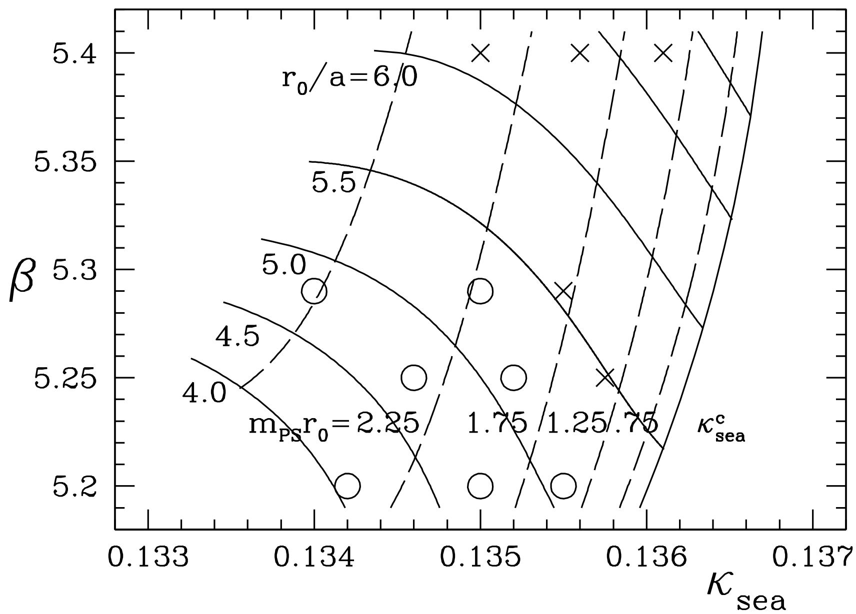

FIG. 2: Parameters of our dynamical gauge field configurations, together with lines of constant r_0/a (solid lines) and lines of constant $m_{PS}r_0$ (dashed lines). The simulations are done on $24^3\,48$ ($\times$) and $16^3\,32$ ($\bigcirc$) lattices, respectively.

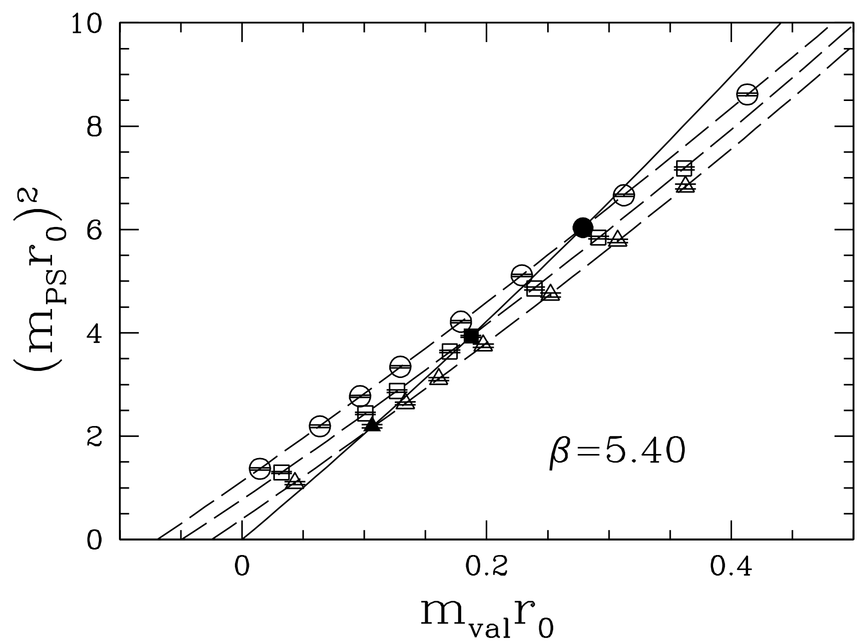

FIG. 3: The partially quenched pseudoscalar mass as a function of $m_{\rm val}$ at $\beta = 5.40$ for $\kappa_{\rm sea} = 0.1350$ ($\bigcirc$), 0.1356 ($\Box$) and 0.1361 ($\triangle$), together with the fit (1). The solid line and symbols refer to the case $\kappa_{\rm val} = \kappa_{\rm sea}$.

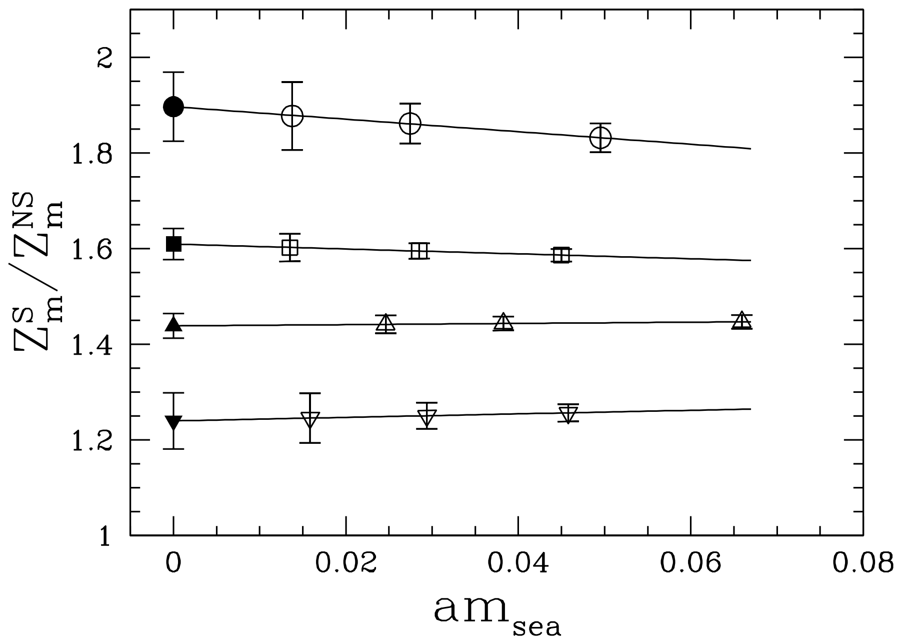

FIG. 4: The ratio Z_m^S/Z_m^{NS} at $\beta = 5.20$, 5.25, 5.29 and 5.40 (from top to bottom), together with a linear extrapolation to the chiral limit.

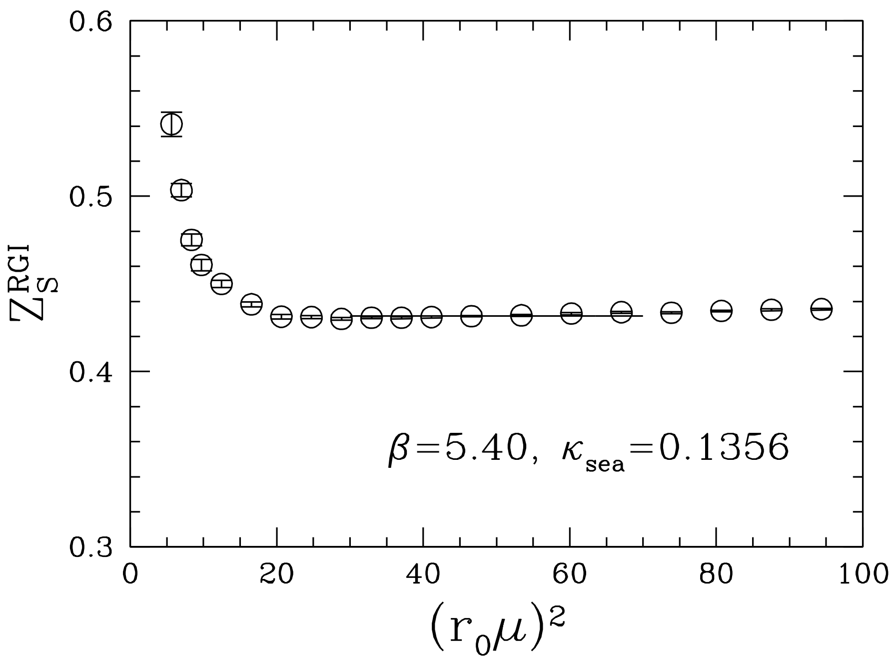

FIG. 5: The nonsinglet renormalization group invariant Z_S^{RGI} at $\beta = 5.40$, $\kappa_{\rm sea} = 0.1356$ as a function of the renormalization scale μ, together with the fit to the plateau.

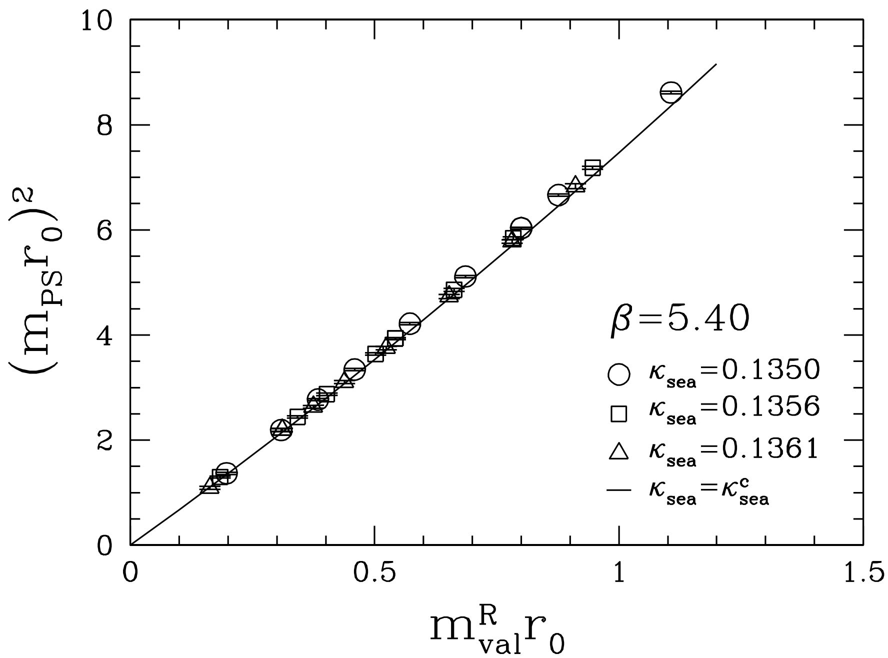

FIG. 6: The partially quenched pseudoscalar mass m_{PS} as a function of $m^R_{\rm val}$ at $\beta = 5.40$ for our three sea quark masses. The solid line shows the result of the fit for $m^R_{\rm sea} = 0$.

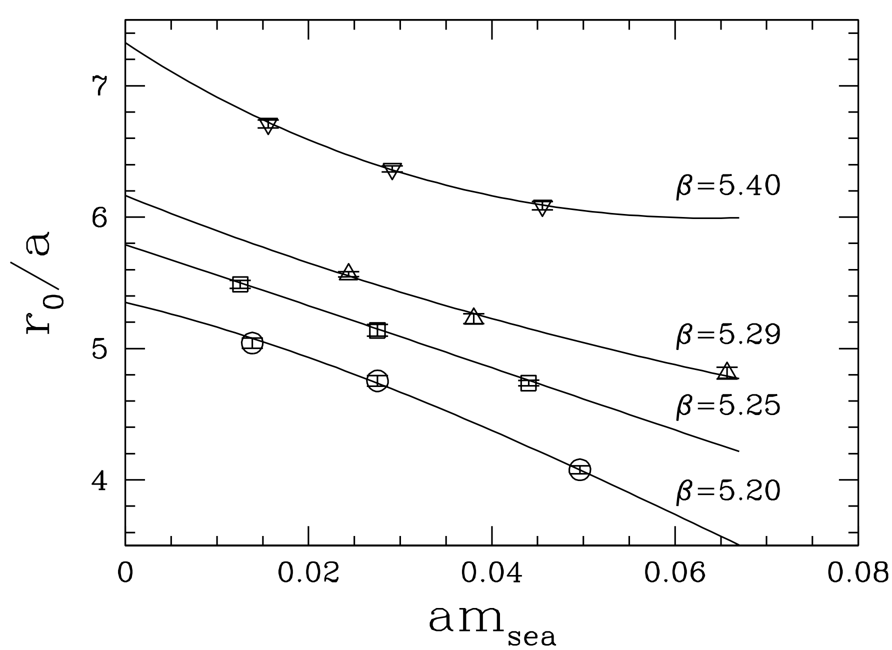

FIG. 7: The lattice spacing a extrapolated to the chiral limit.

In Fig. 4 we plot Z_m^S/Z_m^{NS} for all data sets. The effect of the quark-line disconnected diagram is found to be significant. The numbers depend only mildly on the sea quark mass.

It remains to determine Z_m^{NS}. We compute $Z_S^{NS} = (Z_m^{NS})^{-1}$ nonperturbatively [7] in the *RI-MOM* scheme. The result is converted to the more popular $\overline{MS}$ and *RGI* schemes by a three-loop perturbative calculation [8]. In Fig. 5 we show the nonsinglet Z_S^{RGI} as a function of the renormalization scale μ. We find that the nonperturbative scale dependence of Z_S^{RI-MOM} is matched by the three-loop conversion factor for $(r_0\mu)^2 \gtrsim 20$. We obtain Z_S^{RGI} from a fit to the plateau as indicated by the solid line. The result varies by a few percent only over our range of sea quark masses at any given β value. In the $\overline{MS}$ scheme at $\mu = 2\,\mathrm{GeV}$ we have [8] $Z_S^{\overline{MS}}(2\,\mathrm{GeV}) = 1.461\, Z_S^{RGI}$. At our smallest lattice spacing, $a \approx 0.07$ fm, $Z_S^{\overline{MS}} \approx 0.6$, which is certainly beyond the range of one-loop perturbation theory, tadpole-improved or not.

Having unscrambled renormalized valence and sea quark masses, we are now able to fit our data by the partially quenched chiral formula (4) and determine the physical quark masses from it. In the process we replace all masses m by dimensionless quantities mr_0. It turns out that the data are not sensitive to logarithmic variations in the renormalized valence quark mass (parameters C and E). We therefore have chosen to fit our partially quenched pseudoscalar masses by the formula

$$(m_{PS}\, r_0)^2 = [A + B\, m^R_{\mathrm{sea}} r_0]\, m^R_{\mathrm{val}} r_0 + D\, (m^R_{\mathrm{val}} r_0)^2\,. \tag{6}$$

In Fig. 6 we plot our data for our largest β value. The slope of the data depends only rather weakly on the renormalized sea quark mass. Perhaps most of the effect is washed out by having used r_0 to set the scale. The solid curve shows

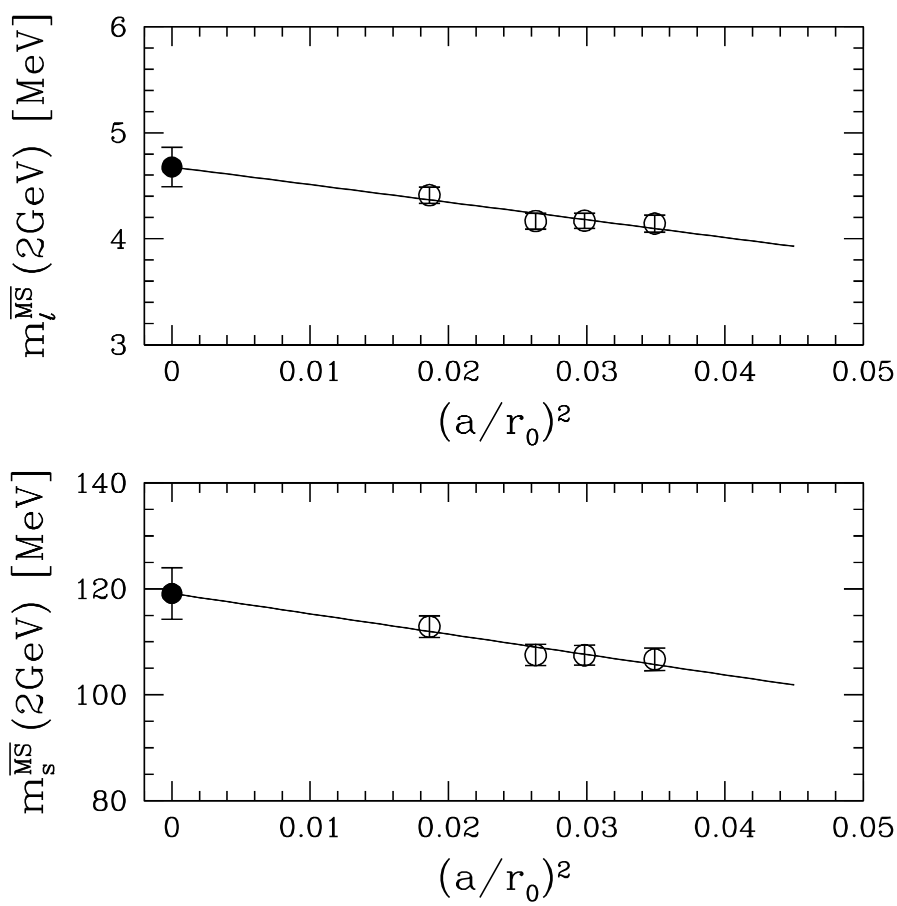

FIG. 8: The light and strange quark masses, together with the extrapolation to the continuum limit. The errors shown are statistical only.

the result of the fit in the limit of vanishing sea quark mass. We find good scaling properties. The fit parameter A varies by less than 5% over our range of β values.

To fix the scale r_0 in physical units, we extrapolate recent dimensionless nucleon masses, $m_N r_0$, found by the CP-PACS, JLQCD and QCDSF-UKQCD collaborations jointly to the physical pion mass, following [9]. This gives the value $r_0 = 0.467$ fm, which we will use here. A similar result was quoted in [10]. The average mass of the up and down quarks, $m_\ell = (m_u+m_d)/2$, is found from extrapolating m_{PS} to the physical π^0 mass, setting $m^R_{\rm val} = m^R_{\rm sea}$ in (6). We obtain $m_\ell^{\overline{MS}}(2\,{\rm GeV})\, r_0 = 0.00981(19)$, $0.00987(17)$, $0.00986(18)$ and $0.01044(19)$ at $\beta = 5.20$, 5.25, 5.29 and 5.40, respectively. Similarly, the strange quark mass, m_s, is obtained from the lattice value of $m^R_{\rm val}$ that brings m_{PS} to the physical K^0 mass, while $m^R_{\rm sea}$ is kept fixed at the corresponding physical sea quark mass m^R_ℓ. Owing to the fact that the valence quarks are degenerate, we then have [6] $m^R_s = 2m^R_{\rm val} - m^R_\ell$. This finally gives $m_s^{\overline{MS}}(2\,{\rm GeV})\, r_0 = 0.2525(50)$, $0.2544(45)$, $0.2545(47)$ and $0.2671(48)$ at $\beta = 5.20$, 5.25, 5.29 and 5.40, respectively.

To be able to extrapolate our results to the continuum limit, we need to know a/r_0 in the chiral limit. In Fig. 7 we show our data for r_0/a together with a fit (fitting function: exponential of a polynomial in β, $m_{\rm sea}$), and in Fig. 8 we show the light and strange quark masses as a function of the chirally extrapolated lattice spacing. Because our fermionic action is nonperturbatively $O(a)$ improved, we expect the error due to the finite cut-off to be at most of $O(a^2)$. A linear extrapolation in $(a/r_0)^2$ to the continuum limit is therefore appropriate. We estimate the systematic error on r_0 to be of the order of 7%. We then obtain

$$m_\ell^{\overline{MS}}(2\,{\rm GeV}) \;=\; 4.7(2)(3)\ {\rm MeV}\,, \tag{7}$$
$$m_s^{\overline{MS}}(2\,{\rm GeV}) \;=\; 119(5)(8)\ {\rm MeV}\,, \tag{8}$$

where the first error is statistical and the second systematic. In particular, $m_s/m_\ell = 26(1)$, in good agreement with leading order chiral perturbation theory.

To summarize, we have performed a lattice calculation of the light and strange quark masses in full QCD with $N_f = 2$ flavors of light dynamical quarks. Our calculation differs from previous calculations in several respects. We use nonperturbatively $O(a)$ improved Wilson fermions and perform simulations at four different couplings with $0.07 \le a \le 0.12$ fm, which allows an extrapolation to the continuum limit. Furthermore, an entirely nonperturbative scheme of mass renormalization, both for sea and valence quark masses, is devised, including the effect of quark-line disconnected contributions. The identification of renormalized sea and valence quark masses greatly facilitates the extrapolation to the chiral limit. We believe that our method will be useful in other applications of partially quenched chiral perturbation theory as well.

This work is supported by DFG under contract FOR 465 (Forschergruppe Gitter-Hadronen-Phänomenologie). The numerical calculations have been performed at LRZ (München) [9], EPCC (Edinburgh) [11], DESY (Zeuthen) and NIC (Jülich).

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