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One-loop renormalisation for the second moment of GPDs with Wilson fermions

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We calculate the non-forward quark matrix elements for operators with two covariant derivatives in one-loop lattice perturbation theory using Wilson fermions. These matrix elements are needed in the renormalisation of the second moment of generalised parton distributions measured in lattice QCD. For some commonly used representations of the hypercubic group we determine the sets of all mixing operators and find the matrices of mixing and renormalisation factors.

In recent years generalised parton distributions (GPDs) have been intensively studied both in experiments and theoretically [1] to extend our knowledge of hadron structure. However, the direct experimental access to GPDs beyond the limiting cases of distribution functions and simple form factors is limited. Therefore, it is indispensable to obtain complementary information from lattice QCD calculations.

On the lattice we can compute matrix elements of local composite operators, and moments of GPDs can be related to such matrix elements taken between states of different nucleon momenta and spins. First results for moments of GPDs have been published recently [2], see also [3], and soon results from improved calculations should become available.

In order to relate lattice measurements to continuum quantities we have to investigate the renormalisation and mixing of the operators involved. When non-forward matrix elements are studied, new features arise, which make a reconsideration of the renormalisation problem necessary. In particular, the mixing with "external ordinary derivatives", i.e. with operators of the form $\partial_\mu \partial_\nu \cdots (\bar{\psi} \cdots \psi)$, needs to be taken into account. On the lattice the mixing patterns are usually more complicated than in the continuum, because covariance under the hypercubic group $H(4)$ imposes less stringent restrictions than $O(4)$ covariance. The necessity to consider also external ordinary derivatives enlarges the set of contributing operators even further. These complications do not yet arise for operators with zero or one covariant derivative. Hence for these moments the renormalisation factors can be taken over from the forward case.

Here we investigate the renormalisation problem for operators with two covariant derivatives sandwiched between off-shell quark states at different momenta within the framework of one-loop lattice perturbation theory and report on first results [4]. We use the Wilson plaquette gauge action with Wilson fermions and perform the calculations in Feynman gauge.

Let us first discuss renormalisation and mixing in general. Denote by $\Gamma_j(p', p, \mu, g_{\rm R}, \epsilon)$ (with $j = 1, 2, \ldots, N$) the dimensionally regularised ampu-

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tated vertex functions of N mixing operators $\mathcal{O}_j$, p and p' are the quark momenta. The corresponding Born terms are denoted by $\Gamma_j^{\rm B}(p',p)$. The renormalised coupling constant $g_{\rm R}$ is related to the bare coupling g by $g_{\rm R}^2 = \mu^{-2\epsilon} g^2 \left(1 + O(g^2)\right)$, where μ is the renormalisation scale. In the $\overline{\rm MS}$ scheme the corresponding renormalised vertex functions are given by

$$\begin{aligned}\Gamma_j^R(p',p,\mu,g_{\rm R}) = \Gamma_j^{\rm B}(p',p) + g_{\rm R}^2 \Big[\sum_{k=1}^{N} (-\gamma_{jk}^V) \\ \times \ln \frac{(p'+p)^2}{4\mu^2} \Gamma_k^{\rm B}(p',p) + f_j(p',p) \Big] ,\end{aligned} \tag{1}$$

where f_j is the finite one-loop contribution. All $O(g_{\rm R}^4)$ contributions are neglected.

In the absence of mixing with lower-dimensional operators the vertex functions regularised on a lattice with lattice spacing a are

$$\begin{aligned}\Gamma_j^L(p',p,a,g_{\rm R}) = \Gamma_j^{\rm B}(p',p) + g_{\rm R}^2 \Big[\sum_{k=1}^{N} (-\gamma_{jk}^V) \\ \times \ln \left(\tfrac{1}{4} a^2 (p'+p)^2\right) \Gamma_k^{\rm B}(p',p) + f_j^L(p',p) \Big] .\end{aligned} \tag{2}$$

The relation between Γ_k^L and the $\overline{\rm MS}$ renormalised vertex functions Γ_j^R is given by

$$\Gamma_j^R = \sum_{k=1}^{N} \left(\delta_{jk} + g_{\rm R}^2 \zeta_{jk}\right) \Gamma_k^L , \tag{3}$$

where the matrix ζ is found to be

$$\zeta_{jk} = \gamma_{jk}^V \ln\left(a^2\mu^2\right) - c_{jk}^V \tag{4}$$

with the constants c_{jk}^V determined from

$$f_j^L(p',p) - f_j(p',p) = \sum_{k=1}^{N} c_{jk}^V \Gamma_k^{\rm B}(p',p) . \tag{5}$$

Mixing with lower-dimensional operators leads to the appearance of additional terms on the r.h.s. of Eq. (2), for details see [4].

The operators potentially contributing to the mixing have to transform identically according to a given irreducible representation of $O(4)$ or $H(4)$ and should have the same charge conjugation parity.

In terms of the quark wave function renormalisation constant Z_ψ and the matrix Z_{jk} of mixing and renormalisation coefficients, the connection between the bare lattice vertex functions and the $\overline{\rm MS}$ renormalised vertex functions can be written as

$$\Gamma_j^R = Z_\psi^{-1} \sum_{k=1}^{N} Z_{jk} \Gamma_k^L . \tag{6}$$

Using the known Z_ψ,

$$Z_\psi = 1 - \frac{g_{\rm R}^2\, C_F}{16\pi^2} \left[\ln\left(a^2\mu^2\right) + 1 + \sigma_L\right] \tag{7}$$

with $\sigma_L = 11.8524$ for Wilson fermions, we find

$$Z_{jk} = \delta_{jk} - \frac{g_{\rm R}^2\, C_F}{16\pi^2} \left[\gamma_{jk} \log(a^2\mu^2) + c_{jk}\right] \tag{8}$$

with $\gamma_{jk} = \delta_{jk} - (16\pi^2/C_F)\gamma_{jk}^V$ and $c_{jk} = \delta_{jk}(1 + \sigma_L) + (16\pi^2/C_F) c_{jk}^V$.

The one-loop computation has been performed symbolically, adopting and significantly extending a *Mathematica* program package developed originally for the case of moments of structure functions using Wilson [5], clover [6] and overlap fermions [7]. Using that approach we have full analytic control over pole cancellation. The Lorentz index structure of the matrix elements is left completely free, so that we are able to construct all representations of $H(4)$ for the second moments in the non-forward case.

We have to deal with operators of the following general types ($\overleftrightarrow{D} = \overrightarrow{D} - \overleftarrow{D}$):

$$\begin{aligned}
\mathcal{O}_{\mu\nu\omega}^{DD} &= (-1/4)\, \bar\psi \gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\omega\, \psi , \\
\mathcal{O}_{\mu\nu\omega}^{\partial D} &= (-1/4)\, \partial_\nu \left(\bar\psi \gamma_\mu \overleftrightarrow{D}_\omega\, \psi\right) , \\
\mathcal{O}_{\mu\nu\omega}^{\partial\partial} &= (-1/4)\, \partial_\nu \partial_\omega \left(\bar\psi \gamma_\mu \psi\right) , \\
\mathcal{O}_{\mu\nu\omega}^{\partial} &= (-i/2)\, \partial_\omega \left(\bar\psi [\gamma_\mu, \gamma_\nu] \psi\right)
\end{aligned} \tag{9}$$

and similarly for operators with $\gamma_\mu\gamma_5$. In lattice momentum space we realise the operators with non-zero momentum transfer q (showing as an example an operator with one covariant derivative) through

$$\begin{aligned}\Big(\bar\psi \overleftrightarrow{D}_\mu\, \psi\Big)(q) = \frac{1}{a} \sum_x {\rm e}^{iq\cdot(x + a\hat\mu/2)} \times \\ \left[\bar\psi(x) U_{x,\mu} \psi(x + a\hat\mu) - \bar\psi(x + a\hat\mu) U_{x,\mu}^\dagger \psi(x)\right] .\end{aligned} \tag{10}$$

Eq. (10) basically defines the Feynman rules in lattice perturbation theory.

Consider now the following operator:

$$\mathcal{O}_1 = \mathcal{O}_{\{114\}}^{DD} - \frac{1}{2}\left(\mathcal{O}_{\{224\}}^{DD} + \mathcal{O}_{\{334\}}^{DD}\right) \tag{11}$$

$$c_{jk} = \begin{pmatrix} -12.1274 & 1.4913 & 0.3685 & -0.4160 & 0.0156 & 0.1498 \\ 0 & 20.6178 & 0 & 0 & 0 & 0 \\ 3.3060 & -8.01456 & -14.8516 & 4.3023 & -0.9285 & 0.7380 \\ 0 & 0 & 0 & 20.6178 & 0 & 0 \\ 0 & 3.2644 & 0 & 0 & 0.3501 & 0.0149 \\ 0 & 3.2644 & 0 & 0 & 0.0050 & 0.3600 \end{pmatrix}$$

Table 1
The finite mixing part for the operators $\mathcal{O}_1, \ldots, \mathcal{O}_6$ in (11), (13).

with charge conjugation parity $C = -1$ and corresponding to the representation $\tau_1^{(8)}$ of $H(4)$. We use the notations [8]:

$$\begin{aligned} &\mathcal{O}_{\{\nu_1\nu_2\nu_3\}} = (1/6)\,(\mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} \\ &\quad + \mathcal{O}_{\nu_2\nu_1\nu_3} + \mathcal{O}_{\nu_2\nu_3\nu_1} + \mathcal{O}_{\nu_3\nu_1\nu_2} + \mathcal{O}_{\nu_3\nu_2\nu_1})\,, \\ &\mathcal{O}_{\|\nu_1\nu_2\nu_3\|} = \mathcal{O}_{\nu_1\nu_2\nu_3} - \mathcal{O}_{\nu_1\nu_3\nu_2} + \mathcal{O}_{\nu_3\nu_1\nu_2} \\ &\quad - \mathcal{O}_{\nu_3\nu_2\nu_1} - 2\,\mathcal{O}_{\nu_2\nu_3\nu_1} + 2\,\mathcal{O}_{\nu_2\nu_1\nu_3}\,, \\ &\mathcal{O}_{\langle\langle\nu_1\nu_2\nu_3\rangle\rangle} = \mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} \\ &\quad - \mathcal{O}_{\nu_3\nu_1\nu_2} - \mathcal{O}_{\nu_3\nu_2\nu_1}\,. \end{aligned} \qquad (12)$$

The following operators (transforming identically) mix in one-loop order with $\mathcal{O}_1$:

$$\begin{aligned} &\mathcal{O}_2 = \mathcal{O}^{\partial\partial}_{\{114\}} - \left(\mathcal{O}^{\partial\partial}_{\{224\}} + \mathcal{O}^{\partial\partial}_{\{334\}}\right)/2\,, \\ &\mathcal{O}_3 = \mathcal{O}^{DD}_{\langle\langle114\rangle\rangle} - \left(\mathcal{O}^{DD}_{\langle\langle224\rangle\rangle} + \mathcal{O}^{DD}_{\langle\langle334\rangle\rangle}\right)/2\,, \\ &\mathcal{O}_4 = \mathcal{O}^{\partial\partial}_{\langle\langle114\rangle\rangle} - \left(\mathcal{O}^{\partial\partial}_{\langle\langle224\rangle\rangle} + \mathcal{O}^{\partial\partial}_{\langle\langle334\rangle\rangle}\right)/2\,, \\ &\mathcal{O}_5 = \mathcal{O}^{5,\partial D}_{||213||}\,, \quad \mathcal{O}_6 = \mathcal{O}^{5,\partial D}_{\langle\langle213\rangle\rangle} \end{aligned} \qquad (13)$$

together with the lower-dimensional operator

$$\mathcal{O}_8 = \mathcal{O}^{\partial}_{411} - \left(\mathcal{O}^{\partial}_{422} + \mathcal{O}^{\partial}_{433}\right)/2\,. \qquad (14)$$

An additional operator with the same symmetry properties does not contribute in the one-loop approximation. The anomalous dimension matrix is obtained as

$$\gamma_{jk} = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{7}{6} & -\frac{5}{6} & 1 & -\frac{3}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & \frac{2}{3} \end{pmatrix} \qquad (15)$$

and the finite mixing matrix part is given in Table 1. There is an additional mixing of $\mathcal{O}_1$ with the lower-dimensional operator $\mathcal{O}_8$ (in Born approximation):

$$\mathcal{O}_1\Big|_{\frac{1}{a}-\mathrm{part}} = \frac{g_{\mathrm{R}}^2\,C_F}{16\pi^2}(-0.5177)\,\frac{1}{a}\,\mathcal{O}_8^{\mathrm{B}}\,. \qquad (16)$$

This mixing leads to a contribution which potentially diverges like $1/a$ in the continuum limit. The operator $\mathcal{O}_8$ has to be subtracted non-perturbatively from the matrix element of the operator $\mathcal{O}_1$ which might be a difficult task in simulations.

In summary, we have presented first results for the one-loop quark matrix elements of operators needed for the second moments of GPDs. This allows us to determine the mixing matrix of Z-factors in the $\overline{\mathrm{MS}}$-scheme for operators commonly used in simulations. In the case of the representation $\tau_1^{(8)}$, a sizeable mixing with operators of the same dimension appears. Moreover, mixing with a lower-dimensional operator has to be taken into account. Additional results, in particular for *transversity* operators, will be presented in [4].

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