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R-parity Violation and General Soft Supersymmetry Breaking

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We consider the most general class possible of soft supersymmetry breaking terms that can be added to the MSSM, with and without R-parity violation, consistent with the sole requirement that no quadratic divergences are induced. We renormalise the resulting theory through one loop and give an example of how a previously ignored term might affect the sparticle spectrum.

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The minimal supersymmetric standard model (MSSM) consists of a supersymmetric extension of the standard model, with the addition of a number of dimension 2 and dimension 3 supersymmetry-breaking mass and interaction terms. It is well known that the MSSM is not, in fact, the most general renormalisable field theory consistent with the requirements of gauge invariance and naturalness; the unbroken theory is augmented by a discrete symmetry (R-parity) to forbid a set of baryon-number and lepton-number violating interactions, and the supersymmetry-breaking sector omits both R-parity violating soft terms and a set of "non-standard" (NS) soft breaking terms. There is a large literature on the effect of R-parity violation; a recent analysis (with "standard" soft-breaking terms) and references appears in Ref. [1]; for earlier relevant work see in particular [2]. The need to consider NS terms in a model–independent analysis was stressed in Ref. [3]; for a discussion of the NS terms both in general and in the MSSM context see Ref. [4], [5], and for model-building applications see for example Ref. [6]. For application of NS R-parity violating terms to leptogenesis, see Ref. [7], and for a review of general soft breaking see Ref. [8].

In this paper we describe the renormalisation of the most general possible softly-broken version of the MSSM incorporating both RPV and NS terms. It is interesting that, as we shall see, with the generalisation to the RPV case the connection between the NS terms and cubic scalar interactions involving supersymmetric mass terms is not universal.

The unbroken $\mathcal{N}=1$ theory is defined by the superpotential

$$W = W_1 + W_2, \tag{1}$$

where

$$W_1 = Y_u Q u^c H_2 + Y_d Q d^c H_1 + Y_e L e^c H_1 \tag{2}$$

and

$$W_2 = \tfrac{1}{2}(\Lambda_E) e^c LL + \tfrac{1}{2}(\Lambda_U) u^c d^c d^c + (\Lambda_D) d^c LQ. \tag{3}$$

In these equations, generation $(i, j\cdots)$, $SU_2(a, b\cdots)$, and $SU_3(\alpha, \beta\cdots)$ indices are contracted in "natural" fashion from left to right, thus for example

$$\Lambda_D d^c LQ \equiv \epsilon_{ab}(\Lambda_D)^{ijk}(d^c)_{i\alpha} L_j^a Q_k^{b\alpha}. \tag{4}$$

For the generation indices we indicate complex conjugation by lowering the indices, thus $(Y_u)_{ij} = (Y_u^*)^{ij}$.

We omit possible mass terms H_1H_2 and LH_2 because, as we shall see, the consequent terms in the Lagrangian will be included as a special case of our general structure.

We now add soft-breaking terms as follows:

$$
\begin{aligned}
L_1 &= \sum_{\phi} m_{\phi}^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} \tfrac{1}{2} M_i \lambda_i \lambda_i + \text{h.c.} \right] \\
&\quad + \left[h_u Q u^c H_2 + h_d Q d^c H_1 + h_e L e^c H_1 + \text{h.c.} \right], \\
L_2 &= m_R^2 H_1^* L + m_K^2 L H_2 + \tfrac{1}{2} h_E e^c L L + \tfrac{1}{2} h_U u^c d^c d^c + h_D d^c L Q + \text{h.c.}, \\
L_3 &= m_4 \psi_{H_1} \psi_{H_2} + R_5 H_2^* L e^c + R_7 H_2^* Q d^c + R_9 H_1^* Q u^c + \text{h.c.}, \\
L_4 &= m_r \psi_L \psi_{H_2} + R_1 L^* Q u^c + R_2 H_1 H_2^* e^c + R_3 u^c e^c d^{c*} + \tfrac{1}{2} R_4 Q Q d^{c*} + \text{h.c.}
\end{aligned} \tag{5}
$$

Thus $L_{1\ldots 4}$ correspond to SRPC, SRPV, NSRPC and NSRPV respectively, where SRPC $\equiv$ Standard R-parity Conserving etc. All the scalar terms in Eq. (5) were first listed (as far as we are aware) in Ref. [7]. It is easy to verify that if we set $m_4 = \mu$, $m_r = \kappa$,

$$
\begin{aligned}
&(R_1)_k^{ij} = \kappa_k (Y_u)^{ij}, \quad (R_2)^i = \kappa_j (Y_e)^{ji}, \quad R_3 = R_4 = 0, \\
&(R_5)^{ij} = -\mu (Y_e)^{ij} + \kappa_k (\Lambda_E)^{jki}, \\
&(R_7)^{ij} = -\mu (Y_d)^{ij} + \kappa_k (\Lambda_D)^{jki}, \\
&(R_9)^{ij} = \mu (Y_u)^{ij},
\end{aligned} \tag{6}
$$

and

$$
\begin{aligned}
&m_1^2 = \mu^2, \quad m_2^2 = \mu^2 + \kappa^i \kappa_i, \quad (m_L^2)^i_j = \kappa^i \kappa_j, \quad (m_R^2)^i = \mu \kappa^i, \\
&m_3^2 = h_{u,d,e,U,D,E} = M_i = m_Q^2 = m_{e^c}^2 = m_{d^c}^2 = m_{u^c}^2 = m_K^2 = 0,
\end{aligned} \tag{7}
$$

then the theory becomes supersymmetric, with mass and interaction terms corresponding to the inclusion in Eqs. (2), (3) of the terms $\mu H_1 H_2$ and $\kappa L H_2$ respectively. (We have assumed for simplicity that μ is real). This limiting case provides a useful check for our results. We have separated the soft terms into Eqs. (6), (7) because those appearing in the latter do not contribute to the β-functions for those appearing in the former. Note that, as we indicated earlier, we have two interactions ($R_{3,4}$) which cannot be generated by supersymmetric mass terms. These interactions violate L, B respectively; thus we may expect their phenomenological consequences to be comparable to $h_{E,D}$ and h_U respectively. In Ref. [4] we gave the general results for the one-loop soft β-functions incorporating NS soft terms, and corresponding results for the MSSM in the RPC case. Here we generalise the latter results to include RPV, and take the opportunity to correct some errors in Ref. [4].

The various one-loop anomalous dimensions were given in, for example, Ref. [1]; we reproduce them below for convenience (we suppress a $16\pi^2$ loop factor throughout):

$$
\begin{aligned}
(\gamma_L)^i_j &= (Y_e)^{ik}(Y_e)_{jk} + (\Lambda_E)^{kim}(\Lambda_E)_{kjm} + 3(\Lambda_D)^{kim}(\Lambda_D)_{kjm} - 2C_H\delta^i_j, \\
(\gamma_{e^c})^i_j &= (Y_e)^{ki}(Y_e)_{kj} + (\Lambda_E)^{ikm}(\Lambda_E)_{jkm} - 2C_{e^c}\delta^i_j, \\
(\gamma_Q)^i_j &= (Y_d)^{im}(Y_d)_{jm} + (Y_u)^{im}(Y_u)_{jm} + (\Lambda_D)^{qmi}(\Lambda_D)_{qmj} - 2C_Q\delta^i_j, \\
(\gamma_{d^c})^i_j &= 2(Y_d)^{mi}(Y_d)_{mj} + 2(\Lambda_D)^{ikm}(\Lambda_D)_{jkm} + 2(\Lambda_U)^{kim}(\Lambda_U)_{kjm} - 2C_{d^c}\delta^i_j, \\
(\gamma_{u^c})^i_j &= 2(Y_u)^{mi}(Y_u)_{mj} + (\Lambda_U)^{ikm}(\Lambda_U)_{jkm} - 2C_{u^c}\delta^i_j, \\
\gamma_{H_1} &= 3(Y_d)^{ij}(Y_d)_{ij} + (Y_e)^{ij}(Y_e)_{ij} - 2C_H, \\
\gamma_{H_2} &= 3(Y_u)^{ij}(Y_u)_{ij} - 2C_H, \\
(\gamma_{LH_1})^i &= -3(\Lambda_D)^{kim}(Y_d)_{mk} - (\Lambda_D)^{kim}(Y_e)_{mk},
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
&C_Q = \tfrac{4}{3}g_3^2 + \tfrac{3}{4}g_2^2 + \tfrac{1}{60}g_1^2, \quad C_{u^c} = \tfrac{4}{3}g_3^2 + \tfrac{4}{15}g_1^2, \quad C_{d^c} = \tfrac{4}{3}g_3^2 + \tfrac{1}{15}g_1^2, \\
&C_{e^c} = \tfrac{3}{5}g_1^2, \quad C_H = \tfrac{3}{4}g_2^2 + \tfrac{3}{20}g_1^2.
\end{aligned}
\tag{9}
$$

The one loop results for the various R-terms follow from Eq. (2.6) of Ref. [4] and are given as follows:

$$
\begin{aligned}
(\beta_{R_1})^{ij}_k &= (\gamma_L)^m_k(R_1)^{ij}_m + (\gamma_Q)^i_m(R_1)^{mj}_k + (\gamma_{u^c})^j_m(R_1)^{im}_k + (\gamma_{LH_1})_k(R_9)^{ij} + 4C_H(R_1)^{ij}_k \\
&+ 6(R_1)^{lm}_k(Y_u)_{lm}(Y_u)^{ij} + 2(Y_e)_{km}(Y_d)^{il}(R_3)^{jm}_l - 2(R_3)^{jl}_n(\Lambda_E)_{lkm}(\Lambda_D)^{nmi} \\
&- 2(\Lambda_D)_{lkm}(\Lambda_D)^{lni}(R_1)^{mj}_n + 4(\Lambda_D)_{lkm}(\Lambda_U)^{jln}(R_4)^{im}_n - 2(\Lambda_D)_{mkl}(Y_u)^{lj}(R_7)^{im} \\
&- 8(m_r)_kC_H(Y_u)^{ij} + 2(R_9)^{lj}(\Lambda_D)_{mkl}(Y_d)^{im} \\
&- 4m_4(Y_d)^{im}(Y_u)^{lj}(\Lambda_D)_{mkl} + 4(m_r)_n(Y_u)^{lj}(\Lambda_D)^{mni}(\Lambda_D)_{mkl},
\end{aligned}
\tag{10a}
$$

$$
\begin{aligned}
(\beta_{R_2})^i &= (\gamma_{H_1} + \gamma_{H_2})(R_2)^i + (\gamma_{e^c})^i_k(R_2)^k + (\gamma_{LH_1})_k(R_5)^{ki} + 4C_H(R_2)^i \\
&+ 2(R_2)^k(Y_e)^{ji}(Y_e)_{jk} + 6(Y_e)^{ji}(\Lambda_D)_{ljk}(R_7)^{kl} + 2(R_5)^{kl}(Y_e)^{ji}(\Lambda_E)_{ljk} \\
&+ 6(Y_d)^{lk}(Y_u)_{lj}(R_3)^{ji}_k - 8(m_r)_jC_H(Y_e)^{ji},
\end{aligned}
\tag{10b}
$$

$$
\begin{aligned}
(\beta_{R_3})^{ij}_k &= (\gamma_{u^c})^i_l(R_3)^{lj}_k + (\gamma_{e^c})^j_l(R_3)^{il}_k + (\gamma_{d^c})^l_k(R_3)^{ij}_l + 4C_{d^c}(R_3)^{ij}_k \\
&+ 4(R_1)^{mi}_l\left[(Y_d)_{mk}(Y_e)^{lj} + (\Lambda_D)_{knm}(\Lambda_E)^{jnl}\right] + 4(R_2)^j(Y_d)_{lk}(Y_u)^{li} \\
&- 4(R_3)^{lj}_m(\Lambda_U)_{lkn}(\Lambda_U)^{imn} - 4(R_5)^{mj}(Y_u)^{li}(\Lambda_D)_{kml} \\
&+ 4(R_9)^{li}(Y_e)^{mj}(\Lambda_D)_{kml} - 8m_4(Y_u)^{li}(Y_e)^{mj}(\Lambda_D)_{kml} \\
&- 8(m_r)_n(Y_u)^{li}\left[(\Lambda_E)^{jmn}(\Lambda_D)_{kml} + (Y_d)_{lk}(Y_e)^{nj}\right],
\end{aligned}
\tag{10c}
$$

$$
\begin{aligned}
(\beta_{R_4})^{ij}_k &= (\gamma_Q)^i_l (R_4)^{lj}_k + \tfrac{1}{2}(\gamma_{d^c})^l_k (R_4)^{ij}_l + 2C_{d^c}(R_4)^{ij}_k \\
&- 2(R_1)^{il}_n (\Lambda_D)^{mnj}(\Lambda_U)_{lkm} - 2(R_4)^{il}_m \left[(Y_d)_{lk}(Y_d)^{jm} + (\Lambda_D)_{knl}(\Lambda_D)^{mnj}\right] \\
&- 2(R_7)^{il}(Y_u)^{jm}(\Lambda_U)_{mkl} + 2(R_9)^{il}(Y_d)^{jm}(\Lambda_U)_{lkm} - 4m_4 (Y_d)^{il}(Y_u)^{jm}(\Lambda_U)_{mkl} \\
&+ 4(m_r)_m (Y_u)^{jn}(\Lambda_D)^{lmi}(\Lambda_U)_{nkl} + (i \leftrightarrow j), \qquad (10d) \\
(\beta_{R_5})^{ij} &= \gamma_{H_2}(R_5)^{ij} + (\gamma_L)^i_k (R_5)^{kj} + (R_5)^{ik}(\gamma_{e^c})^j_k + 4C_H \left[(R_5)^{ij} + 2m_4 (Y_e)^{ij}\right] \\
&+ (\gamma_{LH_1})^i R_2^j + 6R_7^{kl}(Y_d)_{kl}(Y_e)^{ij} + 2R_5^{kl}(Y_e)_{kl}(Y_e)^{ij} \\
&- 6R_7^{kl}(\Lambda_D)_{lmk}(\Lambda_E)^{jim} + 2R_5^{kl}(\Lambda_E)_{lkm}(\Lambda_E)^{jim} \\
&- 2R_2^l (Y_e)_{kl}(\Lambda_E)^{jik} - 6(\Lambda_D)^{lik}(Y_u)_{km}(R_3)^{mj}_l + 8(m_r)_k C_H (\Lambda_E)^{jik}, \qquad (10e) \\
(\beta_{R_7})^{ij} &= \gamma_{H_2}(R_7)^{ij} + (\gamma_Q)^i_k (R_7)^{kj} + (R_7)^{ik}(\gamma_{d^c})^j_k + 4C_H \left[(R_7)^{ij} + 2m_4 (Y_d)^{ij}\right] \\
&+ 6(R_7)^{mn}(Y_d)_{mn}(Y_d)^{ij} + 2(R_5)^{mn}(Y_e)_{mn}(Y_d)^{ij} + 6R_7^{kl}(\Lambda_D)_{lmk}(\Lambda_D)^{jmi} \\
&- 2R_5^{lm}(\Lambda_E)_{mlk}(\Lambda_D)^{jki} - 2(R_7)^{kj}(Y_u)_{kl}(Y_u)^{il} + 2(R_9)^{ik}(Y_u)_{lk}(Y_d)^{lj} \\
&- 2(R_1)^{im}_l (Y_u)_{km}(\Lambda_D)^{jlk} + 2(R_2)^k (Y_e)_{lk}(\Lambda_D)^{jli} + 4(R_4)^{ik}_l (\Lambda_U)^{mjl}(Y_u)_{km} \\
&- 4m_4 (Y_u)^{il}(Y_u)_{kl}(Y_d)^{kj} + 4(m_r)_l (Y_u)^{im}(\Lambda_D)^{jlk}(Y_u)_{km} \\
&- 8(m_r)_k C_H (\Lambda_D)^{jki}, \qquad (10f) \\
(\beta_{R_9})^{ij} &= \gamma_{H_1}(R_9)^{ij} + (\gamma_Q)^i_k (R_9)^{kj} + (R_9)^{ik}(\gamma_{u^c})^j_k + 4C_H \left[(R_9)^{ij} - 2m_4 (Y_u)^{ij}\right] \\
&+ (\gamma_{LH_1})^k (R_1)^{ij}_k + 6(R_9)^{mn}(Y_u)_{mn}(Y_u)^{ij} - 2(R_9)^{mj}(Y_d)_{mn}(Y_d)^{in} \\
&+ 2(R_7)^{im}(Y_d)_{nm}(Y_u)^{nj} + 4(R_4)^{ip}_m (\Lambda_U)^{jmn}(Y_d)_{pn} + 2(R_3)^{jp}_m (Y_e)_{kp}\Lambda_D^{mki} \\
&+ 2(R_1)^{mj}_p (\Lambda_D)^{npi}(Y_d)_{mn} + 4m_4 (Y_d)^{ik}(Y_d)_{lk}(Y_u)^{lj} \\
&- 4(m_r)_l (\Lambda_D)^{mli}(Y_d)_{nm}(Y_u)^{nj}. \qquad (10g)
\end{aligned}
$$

The one loop results for the various $\phi\phi^*$ mass-terms follow from Eq. (2.7c) of Ref. [4]:

$$
\begin{aligned}
(\beta_{m_Q^2})^i_j &= (m_Q^2)^i_k \left[(Y_u)^{kl}(Y_u)_{jl} + (Y_d)^{kl}(Y_d)_{jl} + (\Lambda_D)^{mlk}(\Lambda_D)_{mlj}\right] \\
&+ (m_Q^2)^k_j \left[(Y_u)_{kl}(Y_u)^{il} + (Y_d)_{kl}(Y_d)^{il} + (\Lambda_D)_{mlk}(\Lambda_D)^{mli}\right] \\
&+ 2\Big[m_1^2 (Y_d)^{il}(Y_d)_{jl} + m_2^2 (Y_u)^{il}(Y_u)_{jl} + (m_{u^c}^2)^k_l (Y_u)^{il}(Y_u)_{jk} \\
&+ (m_{d^c}^2)^k_l \left[(Y_d)^{il}(Y_d)_{jk} + (\Lambda_D)^{lmi}(\Lambda_D)_{kmj}\right] + (\Lambda_D)^{mli}(\Lambda_D)_{mkj}(m_L^2)^k_l\Big] \\
&- 2(m_R^2)_k (Y_d)_{jl}(\Lambda_D)^{lki} - 2(m_R^2)^k (Y_d)^{il}(\Lambda_D)_{lkj} \\
&+ 2\left[(h_u)^{ik}(h_u)_{jk} + (h_d)^{ik}(h_d)_{jk} + (h_D)^{kli}(h_D)_{klj}\right] \\
&+ 2\left[(R_1)^{ik}_l (R_1)^l_{jk} + (R_7)^{ik}(R_7)_{jk} + (R_9)^{ik}(R_9)_{jk} + 2(R_4)^{ik}_l (R_4)^l_{jk}\right] \\
&- 4\Big[m_4^2 \left[(Y_u)^{ik}(Y_u)_{jk} + (Y_d)^{ik}(Y_d)_{jk}\right] + (m_r)^l (m_r)_l (Y_u)^{ik}(Y_u)_{jk}
\end{aligned}
$$

$$
\begin{aligned}
&+ (m_r)^m (m_r)_l (\Lambda_D)^{kli} (\Lambda_D)_{kmj}\big] + 4m_4 \left[(\Lambda_D)^{kli} (Y_d)_{jk} (m_r)_l + (\Lambda_D)_{klj} (Y_d)^{ik} (m_r)^l\right] \\
&- \left[\tfrac{32}{3} g_3^2 M_3^2 + 6 g_2^2 M_2^2 + \tfrac{2}{15} g_1^2 M_1^2 - \tfrac{1}{5} g_1^2 \mathcal{S}\right] \delta^i_j, \qquad (11a)
\end{aligned}
$$

$$
\begin{aligned}
(\beta_{m^2_{u^c}})^i_j &= (m^2_{u^c})^i_m \left[2(Y_u)^{lm} (Y_u)_{lj} + (\Lambda_U)^{mkl} (\Lambda_U)_{jkl}\right] + 4(m^2_Q)^k_l (Y_u)^{li} (Y_u)_{kj} \\
&+ (m^2_{u^c})^m_j \left[2(Y_u)_{lm} (Y_u)^{li} + (\Lambda_U)_{mkl} (\Lambda_U)^{ikl}\right] + 4(m^2_2)(Y_u)^{li} (Y_u)_{lj} \\
&+ 4(m^2_{d^c})^k_m (\Lambda_U)^{iml} (\Lambda_U)_{jkl} + 4(h_u)^{ki} (h_u)_{kj} + 2(h_U)^{ikl} (h_U)_{jkl} \\
&+ 4\left[(R_1)^{ki}_l (R_1)^l_{kj} + (R_9)^{ki} (R_9)_{kj}\right] + 2(R_3)^{ik}_l (R_3)^l_{jk} - 8m_4^2 (Y_u)^{li} (Y_u)_{lj} \\
&- 8(m_r)^k (m_r)_k (Y_u)^{li} (Y_u)_{lj} - \left[\tfrac{32}{3} g_3^2 M_3^2 + \tfrac{32}{15} g_1^2 M_1^2 + \tfrac{4}{5} g_1^2 \mathcal{S}\right] \delta^i_j, \qquad (11b)
\end{aligned}
$$

$$
\begin{aligned}
(\beta_{m^2_{d^c}})^i_j &= 2(m^2_{d^c})^i_m \left[(Y_d)^{lm} (Y_d)_{lj} + (\Lambda_U)^{kml} (\Lambda_U)_{kjl} + (\Lambda_D)^{mkl} (\Lambda_D)_{jkl}\right] \\
&+ 2(m^2_{d^c})^m_j \left[(Y_d)_{lm} (Y_d)^{li} + (\Lambda_U)_{kml} (\Lambda_U)^{kil} + (\Lambda_D)_{mkl} (\Lambda_D)^{ikl}\right] \\
&+ 4(m^2_Q)^k_l (Y_d)^{li} (Y_d)_{kj} + 4(\Lambda_U)^{kil} \left[(m^2_{u^c})^m_k (\Lambda_U)_{mjl} + (m^2_{d^c})^m_l (\Lambda_U)_{kjm}\right] \\
&+ 4m_1^2 (Y_d)^{li} (Y_d)_{lj} + 4(\Lambda_D)^{ikl} \left[(m^2_L)^m_k (\Lambda_D)_{jml} + (m^2_Q)^m_l (\Lambda_D)_{jkm}\right] \\
&- 4(m^2_R)_k (Y_d)_{lj} (\Lambda_D)^{ikl} - 4(m^2_R)^k (Y_d)^{li} (\Lambda_D)_{jkl} \\
&+ 4\left[(h_d)^{ki} (h_d)_{kj} + (h_U)^{kil} (h_U)_{kjl} + (h_D)^{ikl} (h_D)_{jkl}\right] \\
&+ 2\left[(R_3)^i_{kl} (R_3)^{kl}_j + 2(R_4)^i_{kl} (R_4)^{kl}_j + 2(R_7)^{ki} (R_7)_{kj}\right] - 8m_4^2 (Y_d)^{ki} (Y_d)_{kj} \\
&- 8(m_r)^k (m_r)_l (\Lambda_D)^{ilm} (\Lambda_D)_{jkm} + 8(m_r)^l m_4 (Y_d)^{ki} (\Lambda_D)_{jlk} \\
&+ 8(m_r)_l m_4 (Y_d)_{kj} (\Lambda_D)^{ilk} - \left[\tfrac{32}{3} g_3^2 M_3^2 + \tfrac{8}{15} g_1^2 M_1^2 - \tfrac{2}{5} g_1^2 \mathcal{S}\right] \delta^i_j, \qquad (11c)
\end{aligned}
$$

$$
\begin{aligned}
(\beta_{m^2_L})^i_j &= (m^2_L)^i_m \left[(Y_e)^{mk} (Y_e)_{jk} + (\Lambda_E)^{kml} (\Lambda_E)_{kjl} + 3(\Lambda_D)^{kml} (\Lambda_D)_{kjl}\right] \\
&+ (m^2_L)^m_j \left[(Y_e)_{mk} (Y_e)^{ik} + (\Lambda_E)_{kml} (\Lambda_E)^{kil} + 3(\Lambda_D)_{kml} (\Lambda_D)^{kil}\right] \\
&+ 2m_1^2 (Y_e)^{ik} (Y_e)_{jk} - (m^2_R)^i (Y_e)^{lk} (\Lambda_E)_{kjl} - (m^2_R)_j (Y_e)_{lk} (\Lambda_E)^{kil} \\
&+ 2(m^2_R)^k (Y_e)^{il} (\Lambda_E)_{ljk} + 2(m^2_R)_k (Y_e)_{jl} (\Lambda_E)^{lik} - 3(m^2_R)^i (Y_d)^{kl} (\Lambda_D)_{ljk} \\
&- 3(m^2_R)_j (Y_d)_{kl} (\Lambda_D)^{lik} + 2(m^2_{e^c})^k_l \left[(Y_e)^{il} (Y_e)_{jk} + (\Lambda_E)^{lim} (\Lambda_E)_{kjm}\right] \\
&+ 2(m^2_L)^m_l (\Lambda_E)^{kil} (\Lambda_E)_{kjm} + 6(\Lambda_D)^{kim} \left[(m^2_Q)^l_m (\Lambda_D)_{kjl} + (m^2_{d^c})^l_k (\Lambda_D)_{ljm}\right] \\
&+ 2(h_e)^{ik} (h_e)_{jk} + 2(h_E)^{kil} (h_E)_{kjl} + 6(h_D)^{kil} (h_D)_{kjl} + 6(R_1)^i_{kl} (R_1)^{kl}_j \\
&+ 2(R_5)^{ik} (R_5)_{jk} - 4\big[m_4^2 (Y_e)^{ik} (Y_e)_{jk} + (m_r)^k (m_r)_l (\Lambda_E)^{mil} (\Lambda_E)_{mjk} \\
&+ (m_r)^k m_4 (Y_e)^{il} (\Lambda_E)_{ljk} + (m_r)_k m_4 (Y_e)_{jl} (\Lambda_E)^{lik}\big] \\
&- 8(m_r)^i (m_r)_j C_H - \left[6 g_2^2 M_2^2 + \tfrac{6}{5} g_1^2 M_1^2 + \tfrac{3}{5} g_1^2 \mathcal{S}\right] \delta^i_j, \qquad (11d)
\end{aligned}
$$

$$
\begin{aligned}
(\beta_{m^2_{e^c}})^i_j &= (m^2_{e^c})^i_m \left[2(Y_e)^{km} (Y_e)_{kj} + (\Lambda_E)^{mkl} (\Lambda_E)_{jkl}\right] \\
&+ (m^2_{e^c})^m_j \left[2(Y_e)_{km} (Y_e)^{ki} + (\Lambda_E)_{mkl} (\Lambda_E)^{ikl}\right] + 4m_1^2 (Y_e)^{ki} (Y_e)_{kj} \\
&+ 4(m^2_R)^k (Y_e)^{mi} (\Lambda_E)_{jmk} + 4(m^2_R)_k (Y_e)_{mj} (\Lambda_E)^{imk}
\end{aligned}
$$

$$
\begin{aligned}
&+ 4(m_L^2)^k_l \left[(Y_e)^{li}(Y_e)_{kj} + (\Lambda_E)^{ilm}(\Lambda_E)_{jkm}\right] + 4(h_e)^{ki}(h_e)_{kj} \\
&+ 2(h_E)^{ikl}(h_E)_{jkl} + 4(R_2)^i(R_2)_j + 4(R_5)^{ki}(R_5)_{kj} + 6(R_3)^{ki}_l(R_3)^l_{kj} \\
&- 8\big[m_4^2(Y_e)^{ki}(Y_e)_{kj} + (m_r)^k(m_r)_l(\Lambda_E)^{iml}(\Lambda_E)_{jmk} \ + (m_r)^k m_4(Y_e)^{li}(\Lambda_E)_{jlk} \\
&+ (m_r)_k m_4(Y_e)_{lj}(\Lambda_E)^{ilk} + (Y_e)^{ki}(Y_e)_{lj}(m_r)^l(m_r)_k\big] - \left[\tfrac{24}{5}g_1^2M_1^2 - \tfrac{6}{5}g_1^2\mathcal{S}\right]\delta^i_j, \quad (11e)
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_1^2} &= 2m_1^2\left[(Y_e)^{ij}(Y_e)_{ij} + 3(Y_d)^{ij}(Y_d)_{ij}\right] + 2(m_L^2)^k_i(Y_e)^{ij}(Y_e)_{kj} + 2(m_{e^c}^2)^j_k(Y_e)^{ik}(Y_e)_{ij} \\
&+ 6(m_Q^2)^j_i(Y_d)^{ik}(Y_d)_{jk} + 6(m_{d^c}^2)^j_k(Y_d)^{ik}(Y_d)_{ij} + 2(h_e)^{ij}(h_e)_{ij} + 6(h_d)^{ij}(h_d)_{ij} \\
&+ 2(R_2)^i(R_2)_i + 6(R_9)^{ij}(R_9)_{ij} - (m_R^2)^j(\Lambda_E)_{ljk}(Y_e)^{kl} - (m_R^2)_j(\Lambda_E)^{ljk}(Y_e)_{kl} \\
&- 3(m_R^2)^j(\Lambda_D)_{ljk}(Y_d)^{kl} - 3(m_R^2)_j(\Lambda_D)^{ljk}(Y_d)_{kl} - 8m_4^2C_H \\
&- 4(m_r)^i(m_r)_j(Y_e)^{jk}(Y_e)_{ik} - \left[6g_2^2M_2^2 + \tfrac{6}{5}g_1^2M_1^2 + \tfrac{3}{5}g_1^2\mathcal{S}\right], \qquad (11f)
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_2^2} &= 6\left[m_2^2(Y_u)^{ij}(Y_u)_{ij} + (m_Q^2)^i_j(Y_u)^{jk}(Y_u)_{ik} + (m_{u^c}^2)^i_j(Y_u)^{kj}(Y_u)_{ki}\right] \\
&+ 6(h_u)^{ij}(h_u)_{ij} + 2(R_2)^i(R_2)_i + 2(R_5)^{ij}(R_5)_{ij} + 6(R_7)^{ij}(R_7)_{ij} \\
&- 8C_H\left[m_4^2 + (m_r)^i(m_r)_i\right] - \left[6g_2^2M_2^2 + \tfrac{6}{5}g_1^2M_1^2 - \tfrac{3}{5}g_1^2\mathcal{S}\right], \qquad (11g)
\end{aligned}
$$

$$
\begin{aligned}
(\beta_{m_R^2})^i &= -m_1^2(Y_e)_{jl}(\Lambda_E)^{lij} - (m_L^2)^i_m(\Lambda_E)^{lmj}(Y_e)_{jl} \\
&- 2(\Lambda_E)^{kij}\left[(Y_e)_{nk}(m_L^2)^n_j + (Y_e)_{lj}(m_{e^c}^2)^l_k\right] \\
&+ (m_R^2)^k\left[(\Lambda_E)^{mij}(\Lambda_E)_{mkj} - (Y_e)_{km}(Y_e)^{im} + 3(\Lambda_D)^{mij}(\Lambda_D)_{mkj}\right] \\
&- 3(Y_d)_{jk}\left[m_1^2(\Lambda_D)^{kij} + (m_L^2)^i_l(\Lambda_D)^{klj} + 2(m_Q^2)^j_l(\Lambda_D)^{kil} + 2(m_{d^c}^2)^k_l(\Lambda_D)^{lij}\right] \\
&+ (m_R^2)^i\left[(Y_e)^{km}(Y_e)_{km} + 3(Y_d)^{km}(Y_d)_{km}\right] - 2h_E^{kij}(h_e)_{jk} - 6h_D^{kij}(h_d)_{jk} \\
&+ 6(R_1)^i_{jk}(R_9)^{jk} + 2(R_5)^{ij}(R_2)_j - 8m_4(m_r)^iC_H \\
&+ 4m_4(m_r)^l(Y_e)^{in}(Y_e)_{ln} + 4(m_r)_j(m_r)^l(\Lambda_E)^{nij}(Y_e)_{ln}, \qquad (11h)
\end{aligned}
$$

where

$$
\mathcal{S} = m_2^2 - m_1^2 + \mathrm{Tr}\left[m_Q^2 + m_{d^c}^2 + m_{e^c}^2 - m_L^2 - 2m_{u^c}^2\right]. \qquad (12)
$$

$\mathcal{S}$ arises as a renormalisation of the Fayet-Iliopoulos D-term. It is one-loop RG invariant in the absence of NS terms, and is then small at all relevant scales if it is zero at gauge unification; however this RG invariance no longer holds in the presence of NS terms[9].

For the $\phi\phi$-type terms:

$$
\begin{aligned}
\beta_{m_3^2} &= (\gamma_{H_1} + \gamma_{H_2})\, m_3^2 + (\gamma_{LH_1})_i(m_K^2)^i - 2(R_5)_{ij}(h_e)^{ij} \\
&- 6(R_7)_{ij}(h_d)^{ij} + 6(R_9)_{ij}(h_u)^{ij} + m_4(6g_2^2M_2 + \tfrac{6}{5}g_1^2M_1), \qquad (13a)
\end{aligned}
$$

$$
\begin{aligned}
\beta^i_{m_K^2} &= (\gamma_L)^i_j(m_K^2)^j + \gamma_{H_2}(m_K^2)^i + (\gamma_{LH_1})^i m_3^2 + 6(R_1)^i_{jk}(h_u)^{jk} \\
&+ 2(R_2)_j(h_e)^{ij} + 2(R_5)_{jk}(h_E)^{kij} + 6(R_7)_{jk}(h_D)^{kij} \\
&+ (m_r)^i\left(6g_2^2M_2 + \tfrac{6}{5}g_1^2M_1\right), \qquad (13b)
\end{aligned}
$$

and for the $\psi\psi$ terms:

$$\beta_{m_4} = (\gamma_{H_1} + \gamma_{H_2})\, m_4 + (m_r)^i (\gamma_{LH_1})_i \tag{14a}$$

$$(\beta_{m_r})^i = (m_r)^i \gamma_{H_2} + (m_r)^j (\gamma_L)^i_j + m_4 (\gamma_{LH_1})^i. \tag{14b}$$

In the supersymmetric limit described in Eqs. (6),(7), Eqs. (14) become the β-functions for the corresponding supersymmetric mass terms. As indicated earlier, this limit gives a useful check on all our results. A further check is provided by the fact that, as often discussed in the literature (for example Refs. [1],[2],[8]) in the presence of general R-parity violation the distinction between the lepton doublets L_i and the Higgs doublet H_1 is artificial. This means that by, for example, "promoting" m_L^2 to be a 4×4 matrix, we can extract from Eq. ($11d$) the results for both $\beta_{m_1^2}$ (Eq. ($11f$)) and $\beta_{m_R^2}$ (Eq. ($11h$)). In general our results reduce to and agree with those of Ref. [1] when all NS terms are removed, up to very minor typos.

In the unbroken theory, the simplified case is often considered when each dimensionless coupling matrix is assumed to have only one non-zero entry: $(Y_u)^{33} = \lambda_t, (Y_d)^{33} = \lambda_b, (Y_e)^{33} = \lambda_\tau, (\Lambda_E)^{323} = \lambda, (\Lambda_D)^{333} = \lambda^{'}, (\Lambda_U)^{323} = \lambda^{''}$ (see for example Ref. [10] for an analysis of the associated infrared fixed point structure, and Ref. [11] for a more general discussion). This is evidently phenomenologically sensible for $Y_{u,d,e}$, but is less obviously justifiable for $\Lambda_{E,D,U}$. Moreover this set of couplings is not closed under renormalisation [12]; at one loop a minimal set that *is* would also include $(Y_e)^{23}$ and $(\Lambda_D)^{323}$.

In the R-parity conserving case when $\Lambda_{E,D,U} = 0$ the single generation approximation does close under renormalisation and is naturally extended to the soft breaking case by setting $(R_9)^{33} = r_9 = m_9\lambda_t$, $(R_7)^{33} = r_7 = -m_7\lambda_b$ and $(R_5)^{33} = r_5 = -m_5\lambda_\tau$, the signs being chosen so that the supersymmetric fixed point corresponds to $m_4 = m_5 = m_7 = m_9$ (see Eq. (6)). We then obtain:

$$\beta_{m_4} = (\lambda_\tau^2 + 3\lambda_b^2 + 3\lambda_t^2 - 4C_H)m_4, \tag{15a}$$

$$\beta_{m_5} = (\lambda_\tau^2 - 3\lambda_b^2 + 3\lambda_t^2)m_5 + 6m_7\lambda_b^2 + (4m_5 - 8m_4)C_H, \tag{15b}$$

$$\begin{aligned}\beta_{m_7} = {} & (-\lambda_\tau^2 + 3\lambda_b^2 + \lambda_t^2)m_7 + 2m_5\lambda_\tau^2 + 2\lambda_t^2(2m_4 - m_9) \\ & + (4m_7 - 8m_4)C_H,\end{aligned} \tag{15c}$$

$$\beta_{m_9} = (\lambda_\tau^2 + \lambda_b^2 + 3\lambda_t^2)m_9 - 2m_7\lambda_b^2 + 4m_4\lambda_b^2 + (4m_9 - 8m_4)C_H. \tag{15d}$$

Eqs. ($15a, b$) above agree with Eq. (3.5) of Ref. [4], but Eqs. ($15c, d$) differ. The error made in Ref. [4] was neglecting possible minus signs associated with SU_2 contractions involving

ϵ^{ab} (in the RPV case one must be similarly careful with regard to the SU_3 tensor $\epsilon^{\alpha\beta\gamma}$). The analysis of the fixed point $m_4 = m_5 = m_7 = m_9$ given in section 4 of Ref. [4] is changed somewhat. The stability matrix for the evolution of $\frac{m_5}{m_4}$, $\frac{m_7}{m_4}$ and $\frac{m_9}{m_4}$ is given by:

$$S = \begin{pmatrix} 8C_H - 6\lambda_b^2 & 6\lambda_b^2 & 0 \\ 2\lambda_\tau^2 & 8C_H - 2\lambda_\tau^2 - 2\lambda_t^2 & -2\lambda_t^2 \\ 0 & -2\lambda_b^2 & 8C_H - 2\lambda_b^2 \end{pmatrix} \tag{16}$$

which has eigenvalues $8C_H, 8C_H + \Lambda_{1,2}$ where $\Lambda_{1,2}$ are the roots of the quadratic

$$\Lambda^2 + 2(\lambda_t^2 + \lambda_\tau^2 + 4\lambda_b^2)\Lambda + 4(3\lambda_b^2 + 3\lambda_t^2 + \lambda_\tau^2)\lambda_b^2 = 0. \tag{17}$$

Near the quasi-infra-red fixed point (QIRFP) for λ_t, $\lambda_t(M_Z) \approx 1.1$ (corresponding to $\tan\beta \approx 1.7$), we can neglect λ_b and λ_τ, and it is easy to see that our fixed point is stable. In the trinification region such that $\lambda_t(M_U) \approx \lambda_b(M_U) \approx \lambda_\tau(M_U) \approx 0.6$, the eigenvalues of S are all positive at unification but $8C_H + \Lambda_{1,2}$ are both negative at M_Z. Thus in this case we would expect substantial deviation from the supersymmetric limit for $m_{4,\cdots 9}$.

Returning to the NSRPV terms, as an example of their possible effect we will investigate the effect of a nonzero $(R_4)_3^{33} = r_4$ only; thus for this exercise we will assume $\Lambda_{U,D,E} = 0$ as well as all the other R-couplings, so that baryon number is violated but not lepton number. The R_4 interactions are similar to Λ_U in violating baryon number by one unit; but of course R_4 involves only sparticles and so we could expect the upper limit on any particular component to be less severe than the limit on a corresponding component of Λ_U. For interactions other than $(\Lambda_U)_{121}$ and $(\Lambda_U)_{131}$ these limits are not very strict in any case [13]; so if we assume no flavour mixing then $r_4(M_Z)$ could be as large as the susy scale. Therefore even if direct detection of the interaction is difficult it will influence the sparticle spectrum via Renormalisation Group evolution.

Since r_4 only contributes to the β-functions for $(m_Q^2)_3^3$ and $(m_{d^c}^2)_3^3$ we may expect the main effect to be on the 3rd generation squark masses. As an example of its effect, in Figure 1 we plot the light stop mass against $r_4(M_Z)$ for the SPS5 benchmark point[14]. The SPS5 point is characterised by the fact that one of the stop masses is rather light and sensitive to small changes in input parameters such as the top quark mass. We use the one-loop β function for r_4, two loop β-functions for all other couplings and masses (including r_4 only at one loop), and adjust input parameters according to the supersymmetric spectrum in order to account for threshold corrections in the manner of Ref. [15]. Our two-loop result for the light stop mass at $r_4(M_Z) = 0$ is now 257GeV, a change from that reported in Ref. [16], due to use of the exact rather than the approximate form of the stop mass matrix from Ref. [15]. We see that as $r_4(M_Z)$ approaches 0.5TeV (corresponding to a value at gauge unification $r_4 \approx 0.3$TeV) the light stop mass varies quite significantly.

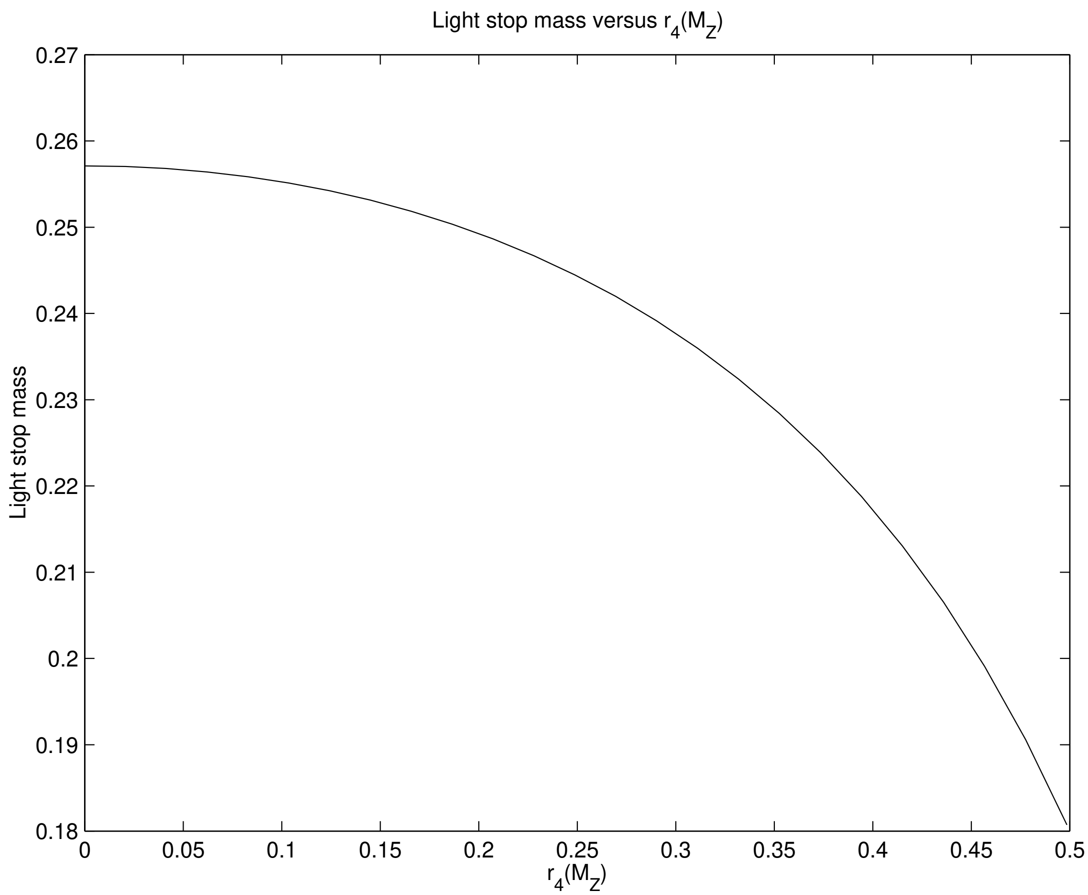

Figure 1: The light stop mass (in TeV) as a function of $r_4(M_Z)$ (also in TeV) for the SPS5 Benchmark Point

In conclusion: we have presented the one-loop renormalisation of the R-parity violating extension of the MSSM with the *most general possible* set of soft breaking terms consistent with naturalness. If the amount of flavour mixing is small then the effect of non-standard soft R-parity violating terms on the sparticle spectrum might be considerable.

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