Excited B mesons from the lattice

A. M. Green,[1] J. Koponen,[1] C. Michael,[2] C. McNeile,[2] and G. Thompson[2]

(UKQCD Collaboration)

[1]*Department of Physical Sciences and Helsinki Institute of Physics, P.O. Box 64, FIN-00014 University of Helsinki, Finland*

[2]*Department of Mathematical Sciences, University of Liverpool, L69 3BX, United Kingdom*

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We determine the energies of the excited states of a heavy-light meson $Q\bar{q}$, with a static heavy quark and light quark with mass approximately that of the strange quark from both quenched lattices and with dynamical fermions. We are able to explore the energies of orbital excitations up to $L=3$, the spin-orbit splitting up to $L=2$ and the first radial excitation. These $b\bar{s}$ mesons will be very narrow if their mass is less than 5775 MeV—the BK threshold. We investigate this in detail and present evidence that the scalar meson ($L=1$) will be very narrow and that altogether 6 $b\bar{s}$ excited states will have energies close to the BK threshold and all will be relatively narrow.

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I. INTRODUCTION

The spectroscopy of excited B and D mesons is important for our understanding of QCD. Moreover, as stressed recently by Rosner [1,2], B^* states also have important applications to CP studies of neutral B mesons by the identification of their flavor (b versus $\bar{b}$) through the decay chain $B^* \to B^0 \pi^{\pm}$. Hence narrow B^* resonances will be valuable for this.

In the heavy quark limit, the $\bar{Q}q$ meson, which we refer to as a "B" meson, will be the "hydrogen atom" of QCD. Since the meson is made from non-identical quarks, charge conjugation is not a good quantum number. States can be labeled by $L_{\pm}$, where the coupling of the light quark spin to the orbital angular momentum gives $j_q = L \pm \frac{1}{2}$. In the heavy quark limit these states will be doubly degenerate since the heavy quark spin interaction can be neglected, so the P_- state will have $J^P = 0^+, 1^+$ while P_+ has $J^P = 1^+, 2^+$, etc.

This spectrum has been studied comprehensively by lattice methods in the quenched approximation [3] with a rather coarse lattice spacing. With $N_f = 2$ flavors of dynamical quark, the SESAM collaboration [4] have explored the P_- excited state. Lattice studies using nonrelativistic QCD (NRQCD) have also explored the heavy-light spectrum for b quarks, mainly using quenched lattices [5–7].

In the heavy quark effective theory, the leading order is just the static limit. The next correction will be of order $1/m_Q$ and will be relatively small for b quarks, but larger for c quarks. One way to predict the spectrum for b quarks is to interpolate between charmed states, where the experimental spectrum is known, and the static limit obtained by lattice QCD assuming a dependence as $1/m_Q$. Thus the splittings among B mesons should be approximately 0.33 of those among the corresponding D mesons.

The striking discovery that the $c\bar{s}$ states with $J^P = 0^+$ and 1^+ have very narrow widths [8] raises the question of whether the corresponding $b\bar{s}$ states will also be narrow. The main reason for the narrow width of D_s mesons is that the transition to DK is not energetically allowed (for the 2317 MeV state) or the state is close to threshold (for the 2457 MeV state). Thus the only allowed hadronic decay proceeds via isospin violation (since $m_d \neq m_u$) to $D_s \pi$ and will have a very small width. Likewise, if the equivalent $b\bar{s}$ states are close to or below the BK threshold, then they will be very narrow. One of our main tasks will be to determine the energy of these excited states as accurately as possible to check this.

As well as exploring this issue of great interest to experiment, we determine the excited state spectrum of the heavy-light system as fully as possible. This will help the construction of phenomenological models and will shed light on questions such as whether there is an inversion of the level ordering (with L_+ lighter than L_-) at larger L or for radial excitations as has been predicted [9,10]. We also compare with chiral models [11].

II. LATTICE EVALUATION

We investigate the heavy-light meson spectrum from lattice QCD using static heavy quarks. Previous lattice studies have explored [3] the full spectrum (i.e. S, P_-, P_+, D_-, D_+, F) in quenched QCD. There has also been a recent estimate of the P_- excitation energy in full QCD [4].

Here we present a range of different lattice studies: with different spatial volumes, lattice spacings and light quark masses—see Tables I and II. We follow the all-to-all methods used in the static-light lattice study of Michael and Peisa [3]. Keeping their parameters, we first use a larger spatial size of lattice to check for finite size effects—$Q1$ vs $Q3$. We are also able to correct the assignments of D_+ and D_- states in their work; see $Q1$ and $Q2$ in Table II. Our major study involves using lattice configurations [12,13] which include $N_f = 2$ flavors of sea quark, with two different lattice spacings. We use only the unitary points, namely, those with valence light quarks of the same mass as the sea quarks. The details are collected in Table I.

To extract mass values, we use operators with the appropriate representations of the cubic group (as described by [3]) with different degrees of non-locality. In addition to op-

TABLE I. Lattice results for the energies of $Q\overline{q}$ states in units of r_0 for dynamical fermions with $N_f=2$. Here r_0 is taken to be 0.525(25) fm.

	DF1	DF2	DF3	DF4
β	5.2	5.2	5.2	5.2
C_{SW}	1.76	1.76	2.0171	2.0171
Number	20	78	20	40
Volume	$12^3\times24$	$16^3\times24$	$16^3\times32$	$16^3\times32$
κ	0.1395	0.1395	0.1350	0.1355
r_0/a	3.435	3.444	4.754	5.041
$r_0m(0^{-+})$	1.92(4)	1.94(3)	1.93(3)	1.48(3)
$1S$	3.00(5)	2.90(2)	3.68(7)	3.73(8)
$2S$	4.24(11)	4.10(5)	5.61(8)	5.60(14)
$1P_-$	4.01(6)	4.02(3)	4.71(8)	4.75(6)
$2P_-$	5.52(7)	5.57(5)	7.1(2)	7.38(9)
$1P_+$	4.18(11)	4.19(14)	5.4(3)	5.5(2)
$2P_+$	5.9(2)	5.57(5)	8.0(2)	8.35(14)
$1D_-$	5.32(12)	5.13(10)	6.6(2)	6.85(10)
$2D_-$	6.5(2)	6.35(14)	8.4(2)	8.9(2)
$1D_+$	5.73(8)	5.2(2)	7.05(14)	7.39(8)
$2D_+$	6.61(8)	6.7(3)	8.84(12)	8.99(7)
$1D_{+-}$	5.22(5)	5.17(4)	6.69(11)	7.22(6)
$2D_{+-}$	5.99(8)	6.06(10)	8.0(2)	8.47(10)
$1F_{+-}$	6.60(4)	6.25(4)	8.08(9)	7.94(12)
$2F_{+-}$	7.03(4)	6.97(3)	9.17(5)	9.53(8)

TABLE II. Lattice results for the energies of $Q\overline{q}$ states in units of r_0 in the quenched case. Here results $Q1$,$Q2$ are from Ref. [3] with their D_+ and D_- corrected.

	$Q1$	$Q2$	$Q3$
β	5.7	5.7	5.7
C_{SW}	1.57	1.57	1.57
Number	20	20	20
Volume	$12^3\times24$	$12^3\times24$	$16^3\times24$
κ	0.14077	0.13843	0.14077
r_0/a	2.94	2.94	2.94
$r_0m(0^{-+})$	1.555(6)	2.164(6)	1.555(6)
$1S$	2.57(2)	2.68(2)	2.555(12)
$2S$	3.74(3)	3.78(3)	3.70(2)
$1P_-$	3.57(13)	3.86(5)	3.62(10)
$2P_-$	5.1(2)	5.28(9)	5.0(2)
$1P_+$	3.7(2)	4.08(8)	3.82(6)
$2P_+$	5.0(2)	5.36(7)	5.0(2)
$1D_-$	4.80(10)	4.89(4)	4.67(7)
$2D_-$	5.7(2)	5.67(4)	5.60(11)
$1D_+$	4.8(2)	4.91(4)	4.98(5)
$2D_+$	5.8(3)	5.78(6)	5.69(5)
$1D_{+-}$	4.57(4)	4.64(3)	4.54(3)
$2D_{+-}$	5.37(10)	5.37(9)	5.29(6)
$1F_{+-}$	5.44(11)	5.60(7)	5.45(9)
$2F_{+-}$	6.04(13)	6.2(2)	6.04(7)

erators for S, P_-, P_+, D_-, and D_+ states that are combinations from space and spin representations appropriate for states of good total angular momentum, we also consider operators D_{+-} and F_{+-} that reflect only the $L=2$ and 3 spatial symmetry. We find, as a check, that the D_{+-} operator approximately gives the expected spin average of the D_- and D_+ levels. Therefore, we interpret the F_{+-} operator as representing the expected spin average of the two F levels. Our choice of operators enables us to determine $N\times N$ matrices of correlations for each case, where N can vary from 2 to 5. We then perform a fit to these correlations over a suitable t range with a number of states allowed. The requirement is then that the χ^2 per degree of freedom is reasonable (not much greater than 1). We always use at least 2 states so that we have a reliable estimate of the ground state mass. To extract the first excited state as well, it is preferable to use at least a 3 state fit. We check that using a subset of our largest matrix of correlators, using different t ranges, using one more or less state, etc. gives stable results.

To compare different lattice simulations, we form the dimensionless combination of r_0m, where m is a mass or energy, and where r_0/a is determined relatively accurately from the static quark potential. Our results are shown in Tables I and II and some comparisons for the P- and D-wave states are shown in Figs. 1–3 versus lattice spacing and versus quark mass.

In order to relate our lattice results to experiment we have to discuss three different extrapolations.

(i) *Finite size effects*. The lattice spatial volume should be large enough. There are several related criteria: the wave functions of the heavy light mesons should be small compared to the spatial size L_S, the exchange of the lightest particle (the pseudoscalar meson) around the periodic boundary should be small and mixing of the heavy-light mesonic states with two body states (e.g. $B\pi$ where the π has a low momentum) should be small.

We can estimate the size of the heavy-light mesons from the Bethe-Salpeter wave functions measured for ground and excited states [3] and also from the more physical charge and matter distributions evaluated for the ground state ($1S$) me-

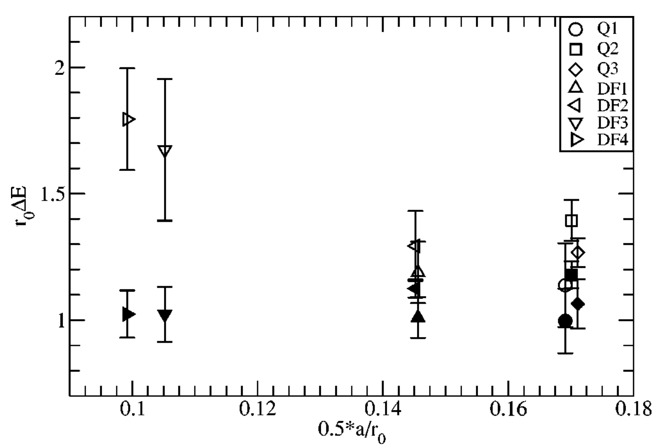

FIG. 1. The energies in units of r_0 of the P_+ (open symbols) and P_- (filled symbols) levels with respect to the $1S$ energy for different lattice spacings (approximately in femtometers with our preferred value of $r_0=0.525$ fm).

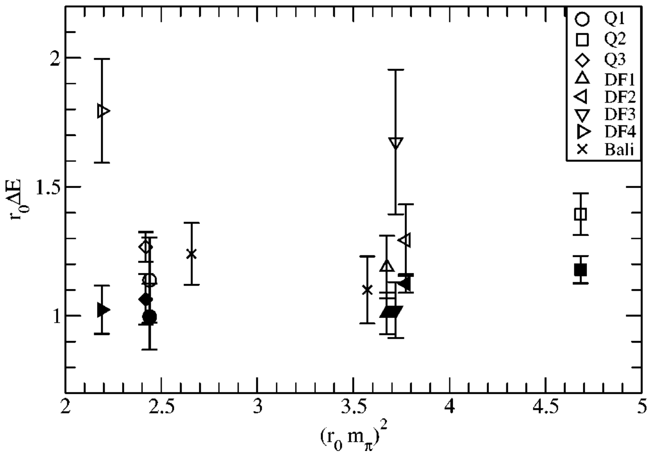

FIG. 2. The energies in units of r_0 of the P_+ (open symbols) and P_- (filled symbols) levels with respect to the $1S$ energy for different quark masses (as given by $[r_0 m(0^{-+})]^2$). Strange quarks correspond to a value of about 3.4.

son [14]. These results suggest that 2 fm is a sufficient size for quenched evaluations, which is confirmed by our results which extend the spatial volume of the previous quenched measurements but do not show any statistically significant differences.

Dynamical fermion configurations are more sensitive to finite size effects since more loop effects are included, in particular pion exchange around the boundary becomes important. The leading correction [15] for the ground state is a relative energy shift of order ce^{-mL_S}, where m is the pseudoscalar mass and c a coefficient given by the $B^*B\pi$ coupling. For the excited states, the possibility of the decay to (or mixing with) nearby two-body energy levels becomes relevant. The only excited state that couples to a low-lying two-body energy level is the P_- which has a mixing with $B\pi$ where the pion has momentum zero. Thus we expect an enhanced finite size effect may arise for P_-. We investigate this by using two spatial sizes (called DF1 and DF2, with L_S of 1.7 fm and 2.3 fm, corresponding to $m_\pi L_S = 6.7$ and 9.0 respectively). We see no evidence for a finite volume shift for P_-, although some sign of a shift for D_+ and F_{+-} which, however, is not very significant statistically.

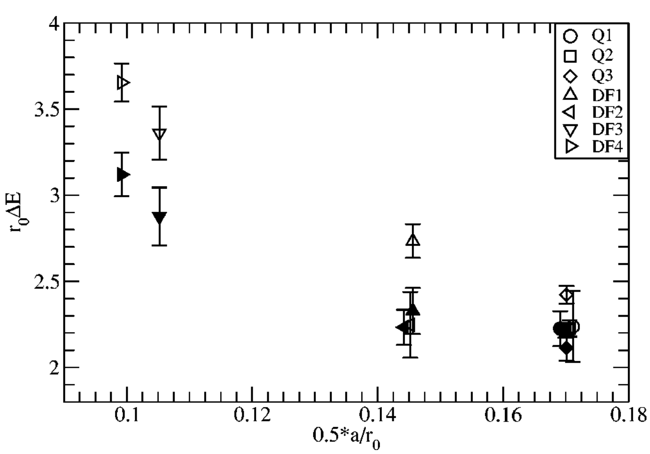

FIG. 3. The energies in units of r_0 of the D_+ (open symbols) and D_- (filled symbols) levels with respect to the $1S$ energy for different lattice spacings.

Our data set with the finest lattice spacing has a relatively small volume (1.6 fm with $m_\pi L_S = 4.5, 6.5$) and, for this situation, some evidence of finite size effects for the nucleon has been presented [16]. Some of our results from this finest lattice spacing are significantly different from the larger volume results described above. Since the order a improvement used is different (non-perturbative clover rather than tadpole-improved clover) we cannot vary the lattice spacing and volume separately while keeping the implementation the same. We would need to use a larger spatial volume with the NP clover approach to evaluate finite size effects fully.

(ii) *Quark mass dependence*. Let us first discuss the dependence on the light valence quark mass. Experimental data on the heavy light mesons with c or b quarks suggest that there is little quark mass dependence of excitation energies (i.e. energy differences from the ground state pseudoscalar meson) when going from strange quarks to lighter quarks. For instance the mass splittings $D^*(1^-)-D$ and $D_s^*(1^-)-D_s$ are 141 and 144 MeV respectively, while $D^*(2^+)-D$ and $D_s^*(2^+)-D_s$ are 593 and 604 MeV respectively [17].

We can also explore this on a lattice and quark masses (characterized by $[r_0 m(0^{-+})]^2$ where 3.4 corresponds to strange quarks as discussed below) are varied in the range of 0.6 to 1.5 times the strange quark. This is shown for the P-wave states in Fig. 2 which confirms that there is no significant slope. This means that interpolation to the strange quark mass is not delicate in any way. Extrapolation to light valence quarks is less straightforward and one issue that must be addressed is that some of the excited heavy-light mesons are unstable to strong decay. Since an excited $L_\pm$ state will have a decay to $B\pi$ with angular momentum given by $L\pm 1$, only the P_- state can decay into an S wave which then gives the lowest threshold energy on a lattice because the pion can have momentum zero. In our case, because of the discrete momentum and unphysical light-quark mass values, we do not have any open decay channels in our lattice evaluation, but they will open when extrapolating to light quark mass and to large spatial volume.

The issue of the extrapolation in the sea quark mass is difficult to resolve. We cover the range from no sea quarks (i.e. quenched) to $N_f = 2$ flavors of sea quark with mass corresponding to 0.6 times the strange quark. The evaluation with even lighter sea quarks is computationally too demanding in a Wilson-like approach.

(iii) *The continuum limit*. It is feasible to study the continuum limit in quenched studies, but for dynamical fermions we have access to only a relatively narrow range of lattice spacing (a from 0.15 fm to 0.1 fm). To make best use of this limitation, we use an order a improved clover formulation of the fermion action. The coarser lattice has a tadpole-based improvement coefficient while the finer lattice uses a non-perturbatively improved value. Because of this difference in implementation, it is not straightforward to extrapolate from these two data sets to the continuum limit. We take this into account in assigning errors.

A. Lattice spectrum

We average the values discussed above of the various excitation energies, weighting relatively more small lattice spacing, large volume and quark masses close to strange. Thus we obtain $r_0\Delta E$ of 1.07(7) for P_-; 1.33(13) for P_+. The next excited level is the $2S$ which is at 1.25(-13, $+50$); this is an average based on the larger volume studies but with the error reflecting our results at finer lattice spacing. For the D waves there is also a large spread so we quote a range: for D_- from 2.2 to 3.1, while for D_+ from 2.2 to 3.5. For the F wave, we only have an operator which excites both F_+ and F_- so that our result is for an average of these two states, with an excitation energy around 3.4 to 4.4.

We need a value of the scale r_0 appropriate to light quark spectroscopy, since the dynamics of the light quark is the main aspect of heavy-light mesons. Thus we do not use values of r_0 from heavy-heavy studies (which tend to give somewhat smaller values of r_0 and hence larger energy gaps in GeV) but an average of those from light-light mesons which span the range from 0.5 to 0.55 fm, namely 0.525 ± 0.025; for a discussion see Ref. [18]. This value of r_0, combined with the estimate of the mass of the pseudoscalar meson made from strange quarks [19] of 687 to 695 MeV yields $r_0 m_\pi \approx 1.84$ which sets the scale for the strange quark.

In our application to the heavy-light mesons with $N_f=2$ flavors of sea quark, we have used valence quarks identical to the sea quarks, which is the case where the theory is fully unitary. This can be interpreted in two ways—first as applying to the spectrum of excited $b\bar{n}$ states (where n is u or d) with quark masses heavier than the physical values. Indeed in the tables we give the pseudoscalar meson mass obtained by combining these light quarks. On the other hand, for our application to the $b\bar{s}$ system, we have also used valence quark masses identical to the sea-quark mass. In the real world, however, there is only one flavor of strange quark, so we can interpret our results as from one flavor of strange valence quark propagating in a sea with two flavors of light quarks whose mass happens to correspond to the valence quark mass. This is effectively treating the strange quark as partially quenched and further studies would be needed to treat fully all three flavors of light quark in the sea.

In principle one can calculate corrections to the heavy quark limit from the lattice, as discussed later. Here, however, we adopt a more modest strategy and make partial use of experimental data. Thus to interpolate to b quarks we combine our results in the static limit with experimental data [8,17] for the $c\bar{s}$ system as shown in Fig. 4. See Table III for a summary. For the P_- state the experimental excitation energy for charm quarks is 349 MeV while we obtain for static quarks 404(31) MeV. Thus the interpolation to b quarks involves only small shifts—leading to 386(31) MeV. This is close to the threshold for decay emitting a kaon (a mass gap of 404 MeV) and probably below it. So we do expect this $b\bar{s}$ scalar meson to be very narrow, as was found for the $c\bar{s}$ counterpart [8]. The associated axial meson at 434(31) MeV above the B_s will be close to the B^*K threshold (at 450 MeV) and should also be very narrow. The P_+ states lie above the BK threshold but since these states decay in a D wave, the centrifugal barrier effects will cause them to have narrow widths.

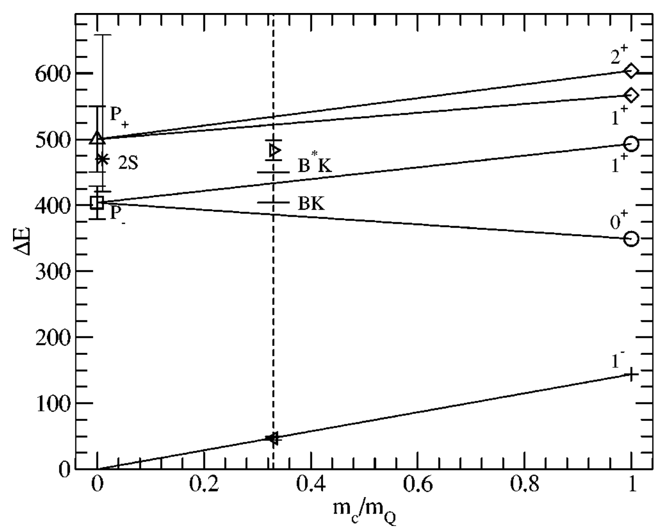

FIG. 4. The energies in MeV of P-wave excited states relative to the ground state ($J^P=0^-$) heavy-light meson with a heavy quark mass m_Q and a light quark which is strange. Data from experiment are plotted for charm and for b quarks while our lattice results are shown for static quarks. The $2S$ excitation (from our larger volume results) is also shown. The dotted vertical line gives the interpolated value appropriate for b quarks. The BK and B^*K thresholds are also shown. These are the lightest isospin-conserving decay modes allowed by strong interactions.

For the $2S$, D, F states, we do not have any $c\bar{s}$ counterpart available from experiment to allow this interpolation. Assuming, however, that the slopes versus m_c/m_Q are similar to those for the P-wave case, then the static energy values will be a good approximation to those for b quarks. Our central mass estimates (see Table III) for the $2S$ pseudoscalar (and vector) states are that they will be sufficiently light that they lie close to the B^*K (BK for vector) threshold at 450 MeV (404 MeV) and so are very narrow.

The only experimental observation [17] of an excited B_s state is the $B_s(5850)$ which lies 483 MeV heavier than the B_s and has a width of 47(22) MeV. This mass value is indeed in the region where we predict a rich spectrum of excited $b\bar{s}$ states.

As we see no sign of a significant light quark mass dependence in our excitation energies, we can use our results to

TABLE III. Lattice results for the energies of $b\bar{s}$ orbital ($L=1$) and radial ($2S$) excited states.

J^P	$M(B_s^*)-M(B_s)$ MeV
0^+	386 ± 31
1^+	434 ± 31
1^+	522 ± 52
2^+	534 ± 52
0^-	$470+188-52$
1^-	$470+188-52$

predict the spectrum of excited $b\bar{n}$ states albeit with a somewhat larger systematic error from the extrapolation to light quarks which we are assuming to be a constant. The only experimental observation [17] of an excited B state is the $B_J^*(5732)$ which lies 419 MeV heavier than the B and has a width of 128(18) MeV. This may indeed be composed of several states. The mass value is indeed in the region where we predict a rich spectrum of excited $b\bar{n}$ states, even though they should not be especially narrow since the $B\pi$ and $B^*\pi$ decay channels are open.

As well as predicting the spectrum, lattice methods can be used to evaluate decay amplitudes [21] and this is feasible for the heavy-light systems too [22].

III. DISCUSSION

We first discuss the issue of the theoretical relationship between the static limit and realistic heavy quarks. Then we can use this discussion to organize our comparison with other lattice determinations of the heavy-light spectrum.

A precise description of heavy-light mesons is provided by the heavy quark effective theory. The leading ($1/m_Q$) corrections [23] to the static (i.e. heavy quark) limit arise from two sources: kinetic and magnetic terms. The magnetic contribution splits each static energy level H (with total light quark angular momentum $j_q=L\pm\frac{1}{2}$, called $L_\pm$ above) into two with total angular momentum $j_1=j_q+\frac{1}{2}$ and $j_2=j_q-\frac{1}{2}$. They have masses given by

$$m_{H_1}=m_Q+\Lambda_H+\frac{\lambda_{H,K}}{2m_Q}+(2j_1+1)\frac{\lambda_{H,B}}{2m_Q}, \tag{1}$$

$$m_{H_2}=m_Q+\Lambda_H+\frac{\lambda_{H,K}}{2m_Q}-(2j_2+1)\frac{\lambda_{H,B}}{2m_Q}, \tag{2}$$

where m_Q is the mass of the heavy quark and Λ_H is the binding energy. On the lattice there is a self-energy term proportional to $1/a$, but as we only discuss mass differences, this will cancel.

Here $\lambda_{H,K}$ arises from the insertion of the heavy quark kinetic energy for state H i.e.

$$\lambda_{H,K}=\langle H|\bar{Q}D_T^2 Q|H\rangle. \tag{3}$$

As the transverse kinetic energy is expected to be positive, this implies that $\lambda_{H,K}$ should be positive also. However, it is the difference of kinetic energies between states that we need. In a simple approach with a confining potential, the excited state would have larger kinetic energy than a ground state, so the mass differences between the P- and S-wave states would increase as m_Q is decreased, but this is only a qualitative indication.

The coefficient $\lambda_{H,B}$ arises for state H from the insertion of the $\boldsymbol{\sigma}\cdot\boldsymbol{B}$ term, where $\boldsymbol{\sigma}$ is the heavy quark spin and $\boldsymbol{B}$ is the chromomagnetic field from the light quark. For the S-wave states ($B^\star$,B), the $\lambda_{S,B}$ parameter can be estimated from the experimental $B^\star$ to B mass splitting

$$\lambda_{S,B}\sim\frac{1}{4}(M_{B^\star}^2-M_B^2)=0.12\ \ \mathrm{GeV}^2. \tag{4}$$

The NRQCD lattice formalism allows these $1/m_Q$ expressions to be evaluated. The results from several recent studies [5–7] show that essentially all excitation energies increase as m_Q is decreased from the static limit. This is what would be expected from the kinetic energy correction above. A note of caution, however, is that the magnetic contribution from these studies can be compared with experimental data on the $B^\star$,B splitting and underestimates it by almost a factor of two [5,6]. This suggests that NRQCD, as currently implemented, is not reproducing the magnetic contribution accurately. Thus predictions from NRQCD of hyperfine splitting may be underestimated. In the NRQCD method one does not take a continuum limit, but the approach can be systematically improved [20] by including more terms in the effective action and by computing the coefficients of these terms (such as $\boldsymbol{\sigma}\cdot\boldsymbol{B}$) non-perturbatively, although this is yet to be carried out to a level such that systematic errors on the hyperfine splittings can be established.

Another way to estimate the $1/m_Q$ corrections from lattice studies is to compare static results, such as ours, with results from relativistic propagating quarks, where a continuum limit may be taken. Here recent results for charm quarks [18] do give a spectrum of $c\bar{s}$ mesons substantially in agreement with experiment and hence support the pattern of $1/m_Q$ corrections we show in Fig. 4. Note that Bali [4] used an estimate of $1/m_Q$ effects by taking the difference of the quenched results in the static limit (from Michael and Peisa [3]) and for charm (from Boyle [24]) and using this to correct the $N_f=2$ static result from the SESAM collaboration [4]. This different procedure explains why his result for the scalar P-wave meson is heavier than ours (by 2σ) even though he obtains a similar value for the P_- energy excitation in the static limit.

We illustrate some of the above discussion by presenting a compilation of relevant lattice results in Fig. 5. Some older lattice calculations of the mass spectrum of P-wave heavy-light mesons have been reviewed recently [4,18]. Improved lattice calculations with reduced systematic and statistical errors are required to get definitive answers [25,26].

Having discussed the heavy quark effective theory, we now discuss the implications of our results for other models of heavy-light mesons.

A traditional way to understand such spectra would be using a quark model with an underlying potential description [27]. This is not strictly justified for a light quark, but may be of qualitative use. For the experimentally observed excited D_s states, it is difficult to understand why the hyperfine splitting is sufficiently big to give a $J^P=0^+$ meson which is so light in such an approach [28]. Our results enable us to discuss the possible inversion of the level ordering (with L_+ lighter than L_-) at larger L or for radial excitations. This inversion has been predicted [9,10] from consideration of the spin-orbit force, which at larger separation would come more from the confining interaction than the short-ranged contribution from gluon exchange. We find no evidence of a sign

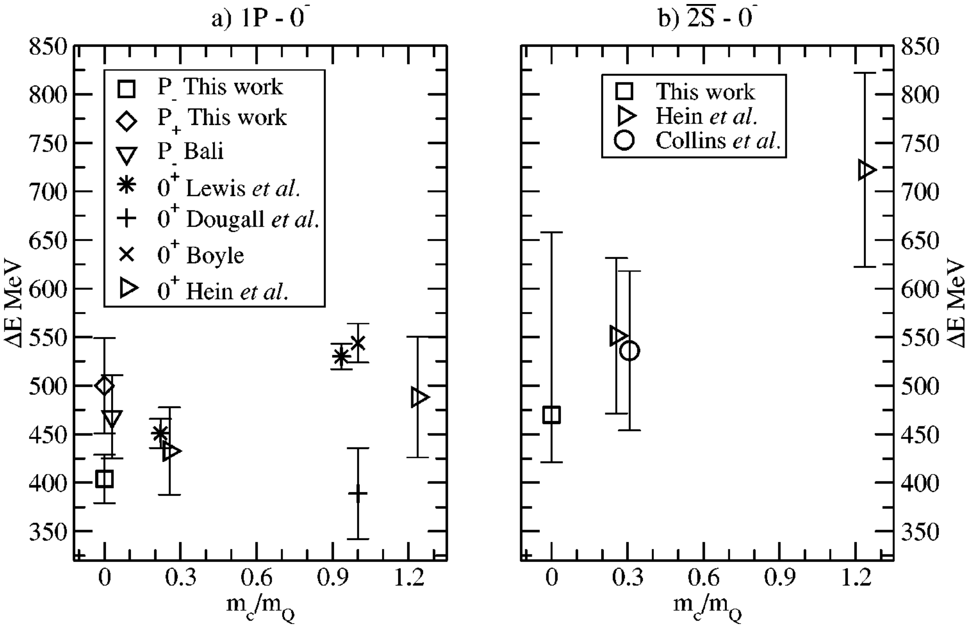

FIG. 5. The energies from lattice studies of heavy-light excited states relative to the ground state ($J^P=0^-$) heavy-light meson with a heavy quark of mass m_Q and a light quark which is strange. For clarity we have displaced some of the numbers on the x axis; the graph should be viewed as three clumps of numbers with heavy quarks at static, bottom, and charm.

change in the spin-orbit splitting of P or D waves. Thus conventional short-distance spin-orbit effects are still relevant up to radii appropriate for D-wave states.

It is possible to discuss chiral symmetry in the heavy quark limit. This allows relationships [11] between energy levels and also predictions for coupling strengths. A stronger assumption of the form of chiral symmetry breaking allows to obtain results away from the static limit, such as that the 1^- to 0^- splitting is the same as that from 1^+ to 0^+. Chiral symmetry in the heavy quark limit relates the S state to P_- and the D_- state to P_+, etc. This does not seem to be a very good approximation: the spectrum is closer to being dependent on L alone. Indeed our spectrum shows an approximately linear rise in excitation energy with L.

IV. CONCLUSION

We have used lattice QCD to explore the spectrum of heavy-light mesons. Our results are evaluated for static quarks but they are very relevant to b quarks (B^* states) as we have argued. We have concentrated our studies on light quarks which are of the mass of a strange quark, although we find that excitation energies are consistent with being independent of the light quark mass, and hence will apply also to light quarks which are u and d.

We have determined the spectrum up to F waves and including radial excitations; see Fig. 6. This gives a rich texture for model building of heavy-light mesons. We find no evidence of a sign change in the spin-orbit splitting of D waves. Thus conventional short-distance spin-orbit effects are still relevant up to radii appropriate for D-wave states. Rather than the pattern given by chiral symmetry (which relates S to P_- and D_- to P_+ etc. [11]) we find a spectrum which is closer to being dependent on L alone. Indeed we see an approximate linear rise in excitation energy with L, up to $L=3$, as 0.45L GeV, reminiscent of Regge or string models.

We have discussed corrections to the heavy quark limit appropriate to B^* states and have used experimental data on D^* states to establish this. Our results for the P-wave excitations confirm those obtained previously [3] that the excitation energies are relatively small. This implies that the $b\bar{s}$ P-wave states will be close to the lightest hadronic decay thresholds (namely BK and B^*K). The P_- states ($J^P=0^+,1^+$) have an S-wave decay but are light enough that there is little or no phase space for decay. The P_+ states ($J^P=1^+,2^+$) are heavier but have D-wave decays and so will also have narrow widths since centrifugal barrier effects will reduce them. We also see evidence that the $2S$ states ($J^P=0^-,1^-$) are close to the lightest thresholds and so may be narrow too. Our results for $b\bar{s}$ states are summarized in Fig. 4 and in Table III. Our central values for these energy levels imply that there will be 6 narrow excited B_s states to be found experimentally. Taking account of the error estimates, we predict at least 4 narrow excited states.

Since we see no significant dependence of the excitation

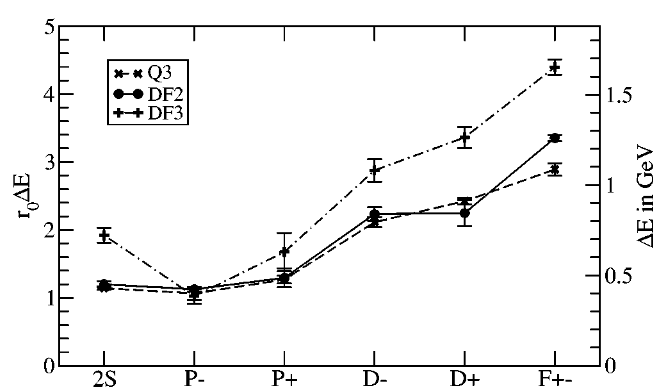

FIG. 6. The energies from some of our lattice studies with $N_f=2$ (DF) and $N_f=0$ (Q) in units of r_0 of L-wave excited states and the S-wave radial excited state relative to the ground state ($1S$) heavy-light meson with a static heavy quark and a light quark which is strange.

energies on light quark mass, our predictions from Table III can also be used as estimates for orbital and radial B^*-B excitation energies, although these mesons will not be especially narrow since the $B\pi$ and $B^*\pi$ thresholds are open.

In our lattice studies, we have pushed toward light sea quarks, toward small lattice spacing and toward large volume, but not toward all three requirements simultaneously. This leaves some room for systematic errors in our predictions that are difficult for us to quantify. These systematic errors can in principle be reduced by further studies with unquenched configurations with parameters closer to the physical ones. The prospects for the generation of significantly more physical unquenched gauge configurations with Wilson quarks [29] are not so good in the next few years, because of the high computational cost. Calculations of the D_s spectrum with improved staggered quarks, that can reach sea quark masses of a fifth of the strange quark mass, have already started [25,26]. Also, to study states near threshold it may be necessary to use a more complicated lattice QCD formalism that explicitly includes the decay products in the lattice measurement [21].

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