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Structure functions and form factors close to the chiral limit from lattice QCD*

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Results for nucleon matrix elements (arising from moments of structure functions) and form factors from a mixture of runs using Wilson, clover and overlap fermions (both quenched and unquenched) are presented and compared in an effort to explore the size of the chiral 'regime', lattice spacing errors and quenching artefacts. While no run covers this whole range of effects the partial results indicate a picture of small lattice spacing errors, small quenching effects and only reaching the chiral regime at rather light quark masses.

1. INTRODUCTION

Chiral extrapolations of lattice data to the physical pion mass, the continuum or $a \to 0$ limit and removal of the 'quenched approximation' remain major sources of systematic uncertainty in the determination of hadron matrix elements (see eg [1]). A study of these quantities is necessary to understand how QCD binds quarks and gluons to form hadronic states and to give an explanation about how particle mass and spin arises. Due to the importance of proton/neutron scattering and DIS experiments most is known about the nucleon. In this talk we shall detail our recent progress in studying these problems from the lattice or numerical perspective in particular with regard to the chiral extrapolation. We have computed $v_n \equiv \langle x^{n-1} \rangle$, for $n = 2$, 3 and 4 related to the three lowest unpolarised moments of the F_2 nucleon structure function, which are given by

$$\langle N(\vec{p})| \left[\widehat{\mathcal{O}}^{(q)\{\mu_1 \cdots \mu_n\}} - \mathrm{Tr} \right] |N(\vec{p})\rangle^{\overline{MS}} = 2v_n^{(q)\overline{MS}} [p^{\mu_1} \cdots p^{\mu_n} - \mathrm{Tr}] \,, \quad (1)$$

where

$$\mathcal{O}^{(q)\mu_1 \cdots \mu_n} \equiv \mathrm{i}^{n-1} \overline{q} \gamma^{\mu_1} \overset{\leftrightarrow}{D}^{\mu_2} \cdots \overset{\leftrightarrow}{D}^{\mu_n} q \,, \quad (2)$$

($q = u, d$). In this report we shall restrict attention to the $n = 2$ moment only. We shall also consider here form factors arising from lepton-nucleon scattering,

$$\langle N(\vec{p}\,')|\widehat{\mathcal{V}}_\mu^{(\frac{2}{3}u - \frac{1}{3}d)}(\vec{q})|N(\vec{p})\rangle = \overline{u}_N(\vec{p}\,', \vec{s}\,') \left[\gamma_\mu F_1^p + \mathrm{i}\sigma_{\mu\nu} \frac{q^\nu}{2m_N} F_2^p \right] u_N(\vec{p}, \vec{s}) \,, \quad (3)$$

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where $\mathcal{V}_\mu^{(q)} \equiv \overline{q}\gamma_\mu q$ is the vector current and the momentum transfer $q = p' - p$.

Specifically we shall consider a variety of actions and quark mass ranges,

1. Wilson fermions at one fixed lattice spacing, $a^{-1} \sim 2.12$GeV in the quenched approximation at pseudoscalar masses down to ~ 310MeV, in order to try to match to chiral perturbation theory (χPT).

2. $O(a)$-improved Wilson fermions ('clover fermions') in the quenched approximation at three lattice spacings $a^{-1} \sim 2.12$ – 3.85GeV with pseudoscalar masses between 580MeV and 1200MeV to check finite lattice artefacts.

3. Unquenched clover fermions at pseudoscalar masses down to ~ 560MeV in order to see if there are any discernable quenching effects.

4. Overlap fermions, in the quenched approximation at one lattice size $a^{-1} \sim 2.09$GeV down to pseudoscalar masses of about 440MeV. These have a chiral symmetry even with finite lattice spacing and hence have better chiral properties than either Wilson or clover fermions.

Note that the physical pion mass is about $m_\pi \sim$ 140MeV and we use $r_0 = 0.5\text{fm} \equiv (394.6\text{MeV})^{-1}$ to set the scale.

What must one achieve to be able to compare numerical results with QCD? One hopes that after taking the continuum limit, we can match our results to known chiral perturbation theory and then take the limit $m_{ps} \to m_\pi$. Practically we might thus expect that for a quantity Q of interest

$$Q = F_\chi^Q(r_0 m_{ps}) + O((a/r_0)^n) \,. \tag{4}$$

$F_\chi^Q(r_0 m_{ps})$ describes the (chiral) physics and the discretisation errors are $O(a^n)$ where $n = 1$ for Wilson fermions and $n = 2$ for clover and overlap fermions. The $O(a)$ errors may be split into terms that remain in the chiral limit, $O(a/\Lambda)$, and $O(am_q) \sim O(ar_0 m_{ps}^2)$. While $O(a)$ improvement will remove both these terms, near the pion mass this second set of lattice errors will be negligible anyway, so we do not have to worry about this term. Thus the variant we shall try here for clover fermions is

$$Q = F_\chi^Q(r_0 m_{ps}) + d_a^Q (a/r_0)^2 + d_m^Q a r_0 m_{ps}^2 \,, \tag{5}$$

where we shall take d_a^Q and d_m^Q to be constant.

Naively one might expect a Taylor series expansion for F_χ^Q to be sufficient, ie

$$F_\chi^Q(x) = F_\chi^Q(0) + c^Q x^2 + \ldots \,. \tag{6}$$

Over the last few years expressions for F_χ^Q have been found

$$\begin{aligned} F_\chi^{m_N}(x) &= r_0 m_N(0) + [c_{\frac{1}{2}}^{m_N} x] + c_1^{m_N} x^2 + \ldots \quad (7) \\ F_\chi^{v_n}(x) &= v_n(0)\left(1 + c_1^{v_n} x^2 \ln(x/r_0\Lambda_\chi)^2\right) + \ldots \end{aligned}$$

In general leading order χPT introduces non-analytic terms to the naive Taylor expansion of F_χ^Q, either odd powers, such as x or x^3 in $F_\chi^{m_N}$ or logarithmic as in $F_\chi^{v_n}$. Additional terms may appear when using the quenched approximation; these are shown in the above equations inside square brackets. The chiral scale, Λ_χ, is usually taken to be ~ 1GeV. Expressions are known for some of the coefficients in the above expansions (using for example the results of [2,3,4,5,6]) and rough numerical estimates give

$$\begin{aligned} c_{\frac{1}{2}}^{m_N} &= -\frac{3\pi}{2}(D-3F)^2\delta \sim -0.5 \,, \\ c_1^{m_N} &= 2(b_D - 3b_F) \sim 1.3 \,, \end{aligned} \tag{8}$$

and similarly

$$c_1^{v_n} = -\frac{3g_A^2 + 1}{(4\pi r_0 f_\pi)^2} \sim -0.66 \,. \tag{9}$$

For quenched QCD an expression for $c_1^{v_n}$ in terms of D and F can be found in [7] giving an approximate result of

$$c_1^{v_n} \sim -0.28 \,. \tag{10}$$

2. LATTICE DETAILS

2.1. Run parameters

Details of the statistics and parameter values used in the simulations for the nucleon matrix elements are given in tables 1, 2, 3 and 4, (where for

β	κ	Volume	Confs.
6.0	0.1515	$16^3 \times 32$	$O(980)$
6.0	0.1530	$16^3 \times 32$	$O(1130)$
6.0	0.1550	$16^3 \times 32$	$O(1360)$
6.0	0.1550	$24^3 \times 32$	$O(220)$
6.0	0.1558	$24^3 \times 32$	$O(220)$
6.0	0.1563	$24^3 \times 32$	$O(220)$
6.0	0.1563	$32^3 \times 48$	$O(250)$
6.0	0.1566	$32^3 \times 48$	$O(530)$

Table 1
Parameters for quenched Wilson fermions with Wilson glue. (The smallest pseudoscalar mass is $\sim$ 310MeV.)

β	κ	Volume	Confs.
6.0	0.1320	$16^3 \times 32$	$O(445)$
6.0	0.1324	$16^3 \times 32$	$O(560)$
6.0	0.1333	$16^3 \times 32$	$O(560)$
6.0	0.1338	$16^3 \times 32$	$O(520)$
6.0	0.1342	$16^3 \times 32$	$O(735)$
6.2	0.1333	$24^3 \times 48$	$O(300)$
6.2	0.1339	$24^3 \times 48$	$O(300)$
6.2	0.1344	$24^3 \times 48$	$O(300)$
6.2	0.1349	$24^3 \times 48$	$O(470)$
6.4	0.1338	$32^3 \times 48$	$O(220)$
6.4	0.1342	$32^3 \times 48$	$O(120)$
6.4	0.1346	$32^3 \times 48$	$O(220)$
6.4	0.1350	$32^3 \times 48$	$O(320)$
6.4	0.1353	$32^3 \times 64$	$O(260)$

Table 2
Parameters for quenched clover fermions with Wilson glue.

Wilson or clover fermions, $am_q = (1/\kappa - 1/\kappa_c)/2$ with κ_c to be determined). Note that these refer to the number of configurations used for finding the matrix elements; for the nucleon/pion mass sometimes a higher statistic was used.

2.2. Hadron masses

To give some idea of where these parameter values lie, in Fig. 1 we plot the various nucleon masses against the square of the pion mass. Also shown for comparison is the position of a hypothetical $\overline{s}s$ pseudoscalar meson and the physical pion, as well as the chiral limit. The data lie in a narrow band tending to the physical nucleon (denoted by a star in the plot). In general there do not appear to be large discretisation effects. For the quenched data (in particular) taking the

β	κ_{sea}	Volume	Trajs.	Group
5.2	0.1342	$16^3 \times 32$	5000	QCDSF
5.2	0.1350	$16^3 \times 32$	8000	UKQCD
5.2	0.1355	$16^3 \times 32$	8000	UKQCD
5.25	0.1346	$16^3 \times 32$	2000	QCDSF
5.25	0.1352	$16^3 \times 32$	8000	UKQCD
5.25	0.13575	$24^3 \times 48$	1000	QCDSF
5.26	0.1345	$16^3 \times 32$	4000	UKQCD
5.29	0.1340	$16^3 \times 32$	4000	UKQCD
5.29	0.1350	$16^3 \times 32$	5000	QCDSF
5.29	0.1355	$24^3 \times 48$	2000	QCDSF
5.4	0.1350	$24^3 \times 48$	1000	QCDSF

Table 3
Parameters for unquenched clover fermions with Wilson glue.

β	am_q	Volume	Confs.
8.45	0.140	$16^3 \times 32$	$O(50)$
8.45	0.098	$16^3 \times 32$	$O(50)$
8.45	0.056	$16^3 \times 32$	$O(50)$
8.45	0.028	$16^3 \times 32$	$O(50)$

Table 4
Parameters for quenched overlap fermions with (tadpole improved) Lüscher-Weisz glue.

y-axis to be $\propto m_N^2$ seems phenomenologically to lead to linear lines (although possibly the lightest point in a data set might be bending slightly upwards). A pseudoscalar mass of $\sim$ 400MeV corresponds to $(r_0 m_{ps})^2 \sim 1.0$ while $m_{ps} \sim$ 600MeV gives $(r_0 m_{ps})^2 \sim 2.3$. Most of the lattice data lies above the region $m_{ps} \sim$ 600MeV, with the exception of the quenched Wilson fermion results. In what region might the leading order χPT be applicable? In Fig. 1 we also plot a representative curve (for the quenched nucleon mass). A breakdown is seen at about $(r_0 m_{ps})^2 \sim 1$ or $m_{ps} \sim$ 400MeV, as the gradient of the χPT result is too steep. Higher order χPT might improve the situation [8,9].

2.3. Nucleon matrix elements

There are two distinct steps in determining nucleon matrix elements. First the bare matrix element must be determined from the ratio of three point nucleon-operator-nucleon correlation functions to two point nucleon-nucleon correlation functions. Secondly this matrix element must

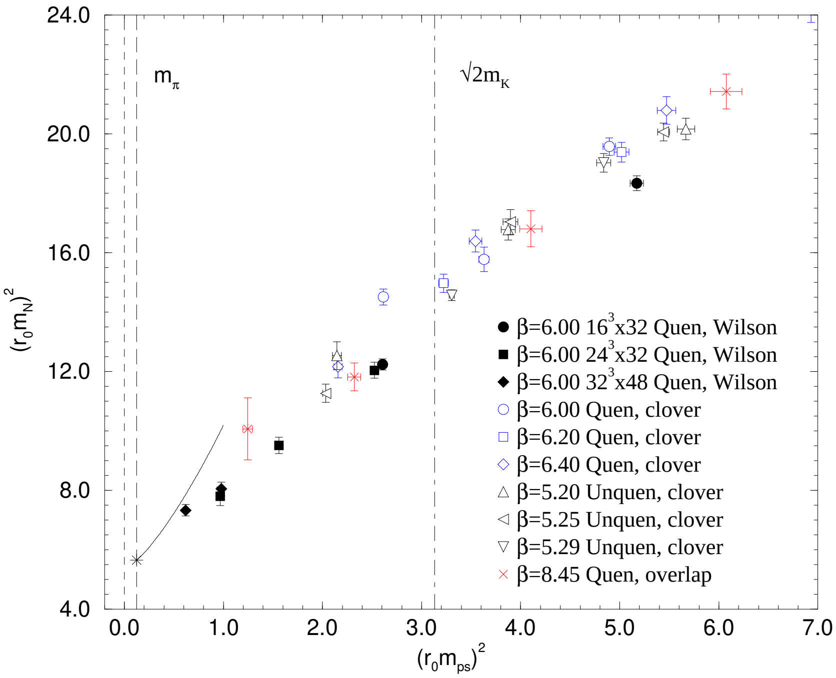

Figure 1. $(r_0m_N)^2$ versus $(r_0m_{ps})^2$ for the four sets under consideration. The experimental nucleon mass is denoted by a star. The chiral limit $m_{ps}^2 = 0$ is shown as a short dashed line, while the physical pion mass is denoted by the long dashed line. Also shown as a dot-dashed line is the mass of a hypothetical $\overline{s}s$ meson calculated as $\sim \sqrt{2}m_K$. The (quenched) χPT line uses the numerical values from eq. (8).

be renormalised. Techniques for both steps are standard, eg [10]. In general, one calculates non-singlet, NS, matrix elements because then the difficult to compute one-quark-line-disconnected part of the matrix element cancels (although for the vector current this additional piece is probably very small, [11], so we shall ignore this point here).

For all the data sets for v_2 and the vector current the appropriate renormalisation constant is known to one loop perturbation theory (for overlap fermions the results are given in [12,13]). Either one can tadpole improve, TI, the result, for example TRB-PT, [14,15] or use a non-perturbative method, NP, [16]. (For our variant of the method see [17].) For overlap fermions and some of the unquenched clover results this non-perturbative method has not yet been implemented, so we must use TI-PT. As we wish to compare all our results, for consistency we shall present them all here using TI-PT. The fact that the vector current is conserved however allows a more direct NP determination of Z_V, [11,12]. For overlap fermions in particular this appears to indicate that there are significant differences between perturbative based results and NP results; however for Wilson or clover results there is often only a couple of percent (or less) difference between TI and NP results.

3. MATRIX ELEMENT RESULTS

3.1. v_2^{NS}

We now show a selection of results. From the quenched clover fermion data we shall first check for lattice spacing effects. In Fig. 2 we show the chiral and continuum extrapolation for v_{2b} (the 'b' just denotes the lattice represention used) using the formula given in eq. (5), together with a linear function in the quark mass, eq. (6). The upper graph represents the chiral physics and the lower graph represents the discretisation er-

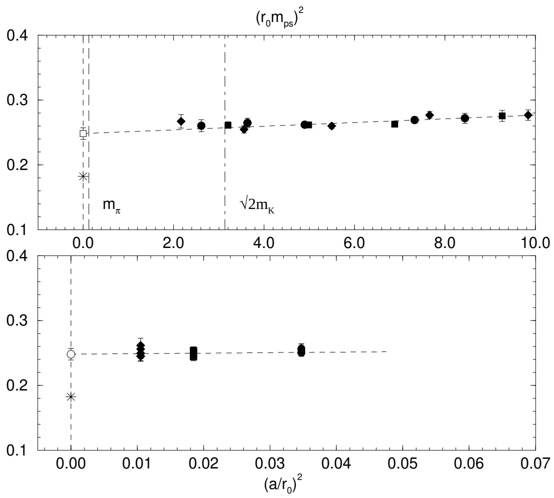

Figure 2. $v_{2b}^{NS;\overline{MS}} - d_a(a/r_0)^2 - d_m a r_0 m_{ps}^2$ at a scale of 2GeV versus $(r_0 m_{ps})^2$, upper picture, and $v_{2b}^{NS;\overline{MS}} - (F_\chi^{v_2}(0) + c(r_0 m_{ps})^2) - d_m a r_0 m_{ps}^2$ against $(a/r_0)^2$, lower picture, for quenched clover fermions. Filled circles, squares and diamonds represent $\beta = 6.0$, 6.2 and 6.4 respectively. The MRS phenomenological value of $v_2^{NS;\overline{MS}}$ is represented by a star.

rors. First we note that there seem to be very small a discretisation errors and indeed the numerical effect of additional operators needed to ensure $O(a)$ improvement seems to be negligible, [15]. Secondly a linear chiral extrapolation seems to describe the data adequately. (Again, as for the nucleon mass, we might expect χPT to be only valid for $m_{ps} \lesssim 400$MeV.) A result of $v_{2b}^{\overline{MS}}(2\text{GeV}) = 0.25(1)$ is found. This is to be compared to the phenomenological result of ~ 0.18, so we see about a 40% discrepency.

Turning now to the unquenched data, in Fig. 3 we show the results. Again similar conclusions hold as for the quenched case, although it should be noted that we are not so close to the continuum limit. We find $v_2^{\overline{MS}}(2\text{GeV}) = 0.27(2)$. So at least in this quark mass range quenching effects seem to be small.

Is the use of Wilson or clover fermions bad for the investigation of chiral properties and the chiral limit? Overlap fermions having a chiral invariance on the lattice in the massless limit, are in principle better. In Fig. 4 we show preliminary results for v_2 using quenched overlap fermions,

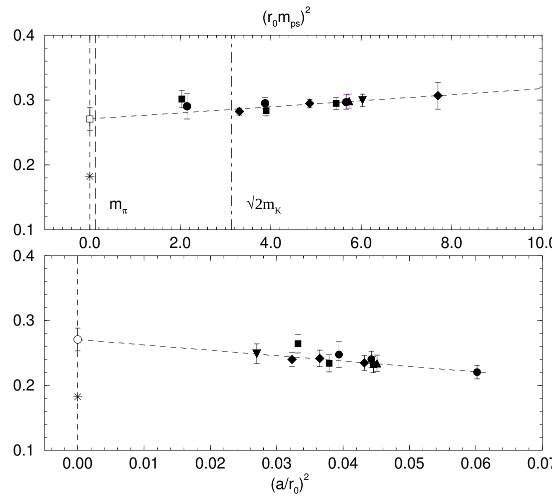

Figure 3. Same as Fig. 2 for unquenched clover fermions. Filled circles, squares, up triangle, diamonds and down triangle represent $\beta = 5.20$, 5.25, 5.26, 5.29 and 5.40 respectively.

[12]. Again we see the linear behaviour in the quark mass confirmed. (The result is in surprisingly good agreement with the phenomenological value. At present we have to use the TI perturbation result for the renormalisation constant. As noted previously when comparing a NP evaluation of Z_V with a TI perturbative evaluation the differences are greater than in the Wilson or clover case. So perhaps a NP evaluation of the renormalisation constant will give it a large value and so lead to a larger result for v_2, more in line with other values presented here.)

Finally we investigate the chiral limit, using light Wilson quarks on large lattices. In Fig. 5 we show the results for $(r_0 m_{ps})^2$ between about 0.5 and 2.5 or $m_{ps} \sim 300 - 600$MeV. First it seems that the numerical values are about the same as for the clover quenched case, which might indicate that finite lattice effects are again small. All the results are again rather flat (and this continues to heavy quark masses [10,14]). Also shown is a linear fit from eq. (6) together with χPT results, eq. (7), starting from the MRS phenomenological value and using the quenched result in eq. (10) for chiral scale 1GeV. (Using larger values of $c_1^{v_2}$ leads to a steeper slope, while using a smaller value for Λ_χ reduces the slope.) For the present

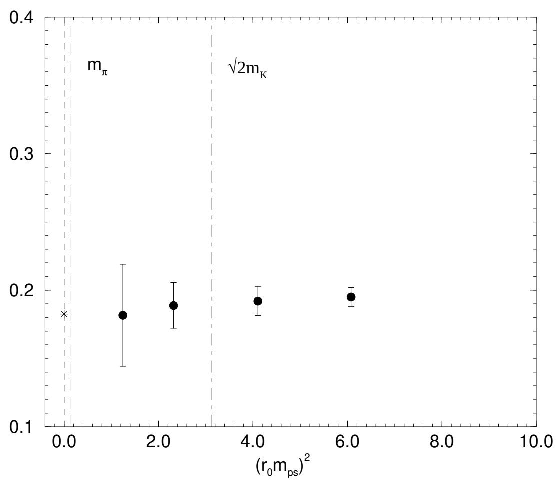

Figure 4. $v_{2b}^{NS;\overline{MS}}(2\text{GeV})$ using quenched overlap fermions at $\beta = 8.45$.

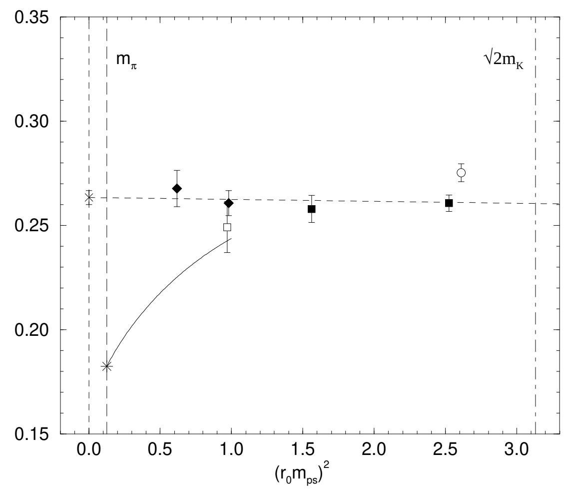

Figure 5. $v_{2b}^{NS;\overline{MS}}(2\text{GeV})$ using quenched Wilson fermions at $\beta = 6.0$. The $16^3 \times 32$ results are shown as circles, the $24^3 \times 32$ results as squares and the $32^3 \times 48$ results as diamonds. A linear fit, eq. (6), is also shown as a dashed line. The (quenched) χPT line uses the numerical values from eq. (10).

numerical results to reach the phenomenological value seems difficult, as down to ~ 300MeV they seem to be rather linear. Of course we would not necessarily expect complete agreement with the phenomenological result, however past experience with the quenched approximation does suggest that it can lead to values only a few percent away from experimental values.

3.2. Form Factors

Finally we present our results for electromagnetic form factors, as given in eq. (3). We have $F_1(0) = 1$ (charge conservation leading to a NP evaluation of Z_V on the lattice) and $F_2(0) = \mu - 1$, the anomalous magnetic moment in magnetons. It is usual to give the results for the Sachs form factors

$$\begin{aligned} G_e(q^2) &= F_1(q^2) + \frac{q^2}{(2m_N)^2} F_2(q^2)\,, \\ G_m(q^2) &= F_1(q^2) + F_2(q^2)\,. \end{aligned} \qquad (11)$$

Previous experimental results give phenomenological dipole fits, for the proton, p or neutron, n,

$$\begin{aligned} G_e^p(q^2) &\sim \frac{G_m^p(q^2)}{\mu^p} \sim \frac{G_m^n(q^2)}{\mu^n} \\ &\sim \frac{1}{\left(1 + \frac{-q^2}{m_V^2}\right)^2}\,, \end{aligned} \qquad (12)$$

and

$$G_e^n(q^2) \sim 0\,, \qquad (13)$$

with $m_V \sim 0.82$GeV, $\mu^p \sim 2.79$ and $\mu^n \sim -1.91$. (Present experimental results indicate however that, [18], $G_e^p(q^2)/G_m^p(q^2)$ is decreasing with increasing $-q^2$.) In Fig. 6 we show G_e^p and G_m^p for the proton and in Fig. 7 for the neutron. Note that we have made simple linear chiral extrapolations here, see eq. (6), but at present we can do little more than this, as in particular on the large $32^3 \times 48$ lattice, we only have two quark mass values. The technique used here is the same as described in [19,20]. Previous lattice calculations described in these papers seem to show little or no lattice artifacts. Due to the larger lattice size, we are now able to go to smaller momentum transfer $-q^2_{min} \sim (2\pi/32a)^2 \sim 0.17\text{GeV}^2$ than was previously possible. We find that the lattice result tracks the experimental values reasonably well, although with rather large errors. Finally we note that as usual the electric form factor of the neutron remains difficult to measure on the lattice (see [21] for a recent attempt).

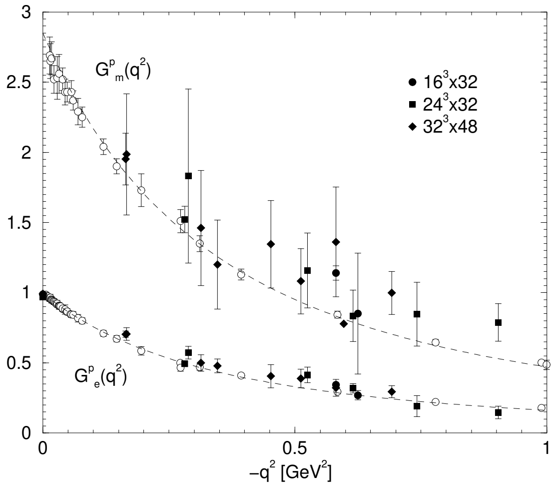

Figure 6. G_e^p and G_m^p for the proton as functions of $-q^2$ using quenched Wilson fermions at $\beta = 6.0$. The previous experimental results are given by the open circles.

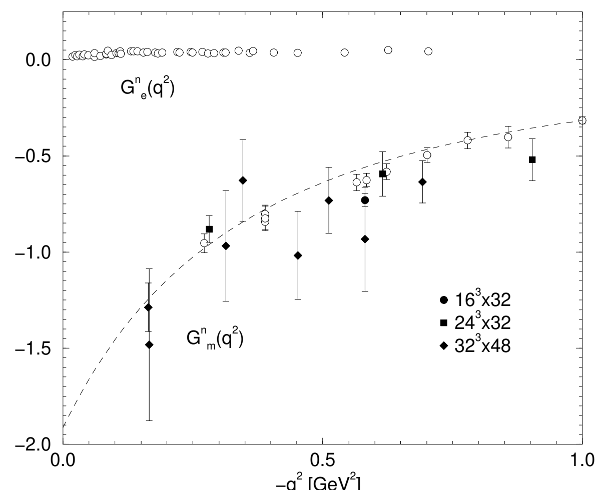

Figure 7. G_e^n and G_m^n for the neutron as functions of $-q^2$ using quenched Wilson fermions at $\beta = 6.0$.

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