Hadronic transitions from the lattice

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I discuss strategies to determine hadronic decay couplings from lattice studies. As a check of the methods, I explore the decay of a vector meson to two pseudoscalar mesons with $N_f = 2$ flavours of sea quark. Although we are working with quark masses that do not allow a physical decay, I show how the transition rate can be evaluated from the amplitude for $\rho \to \pi\pi$ and from the annihilation component of $\pi\pi \to \pi\pi$. I explore the decay amplitude for two different pion momenta and find consistent results. The coupling strength found is in agreement with experiment. I also find evidence for a shift in the ρ mass caused by mixing with two pion states. I also present results for the decay of a hybrid meson, for the case of heavy valence quarks.

1. INTRODUCTION

Rather few hadronic states are actually stable under strong interactions. Because of this, a full understanding of hadrons from QCD will involve developing techniques to treat unstable hadrons and to evaluate their hadronic transition strengths.

An area of hadronic physics that is of considerable interest is that of gluonic excitations: the exploration of hadrons with a non-trivial gluonic content. The most clear case is that of glueballs, however, any glueball states will mix with quark-antiquark mesons in practice. This mixing can be thought of as mediated by a hadronic transition. Moreover, the eventual hadronic decays of such mixed mesons also need to be evaluated to give a clearer picture of the relationship between the physical spectrum and the ingredients (glueballs and quark-antiquark mesons).

The other natural area to explore gluonic excitations is that of hybrid mesons. These are quark-antiquark mesons with an additional non-trivial gluonic component. The benchmark example is a spin-exotic meson (so-called because it cannot be made from combining a quark and antiquark). These spin-exotic hybrid mesons will also be unstable, and study of their decay transitions will guide the interpretation of experimental candidates.

An understanding of methods to treat purely hadronic transitions will also help in developing methods to explore weak decays to hadronic final states (e.g. $K \to \pi\pi$).

The lattice QCD approach is the only quantitative non-perturbative method but the formulation in euclidian time causes problems with the treatment of decays as I now discuss.

1.1. Euclidian time

Hadron masses are extracted from lattice QCD calculations using two point correlators, however, their behaviour as e^{-Mt} in euclidian time will not be appropriate for hadrons that can decay. The most naive modification of the lattice QCD formalism caused by the introduction of decay widths is the replacement $M \to M - i\Gamma/2$ where Γ is the decay width. This gives a behaviour as $e^{-Mt}e^{i\Gamma t/2}$ but this is not consistent: the correlation must be positive definite yet this expression oscillates. Here I discuss this apparent paradox more fully [1].

The euclidian time propagator of an unstable particle which can decay into two stable particles of mass μ can be expressed as

$$G(t) = \frac{1}{\pi} \int_{2\mu}^{\infty} dE e^{-Et} \rho(E) \tag{1}$$

where we can express the spectral density for a resonance of mass M as

$$\rho(E) = \frac{1}{2M} \frac{\Gamma(E)/2}{(M-E)^2 + (\Gamma(E)/2)^2} \tag{2}$$

As an illustration, consider taking $\Gamma(E)$ as a constant with $\Gamma \ll (M-2\mu)$ and $(M-2\mu)t \gg 1$. Then we have, in this approximation,

$$2MG(t) = e^{-Mt}\cos(\Gamma t/2) + \frac{e^{-2\mu t}\Gamma}{2\pi(M-2\mu)^2 t} \quad (3)$$

This expression, when evaluated, shows that the threshold contribution (the second term) dominates at larger t and hence the oscillating behaviour of the first term will be obliterated. This clarifies the apparent paradox mentioned above concerning the sign of $G(t)$: the contribution from the lower lying (in energy) component of the resonance is enhanced in euclidian time. Indeed this can be seen directly from eq. 1.

By contrast, the Minkowski behaviour of the two particle correlator under the same assumptions is

$$2MG_M(t) = e^{-iMt}e^{-\Gamma t/2} - \frac{ie^{-2i\mu t}\Gamma}{2\pi(M-2\mu)^2 t} \quad (4)$$

for which the second term is almost always negligible.

In principle, a lattice determination of the spectral function $\rho(E)$ from measurements of $G(t)$ using eq. 1 would be sufficient to give a full description of hadronic decays. This is not a simple task: the inverse Laplace transform need to obtain $\rho(E)$ from $G(t)$ is not numerically stable. It has to be stabilised by making model assumptions about $\rho(E)$, for example as discussed in refs. [1,2]. Another possibility is that the maximum entropy method may provide sufficient numerical stability and this has been tested in model cases [3].

The same observation that the region near to threshold dominates is at the root of the conclusions [4] of Maiani and Testa that two body states have unappetizing properties in euclidian time.

1.2. Particles in a box

In practice this issue is less relevant since the lattice evaluations are performed in a finite spatial volume with periodic boundary conditions. This finite spatial size of the lattice implies that two-body states are actually discrete. By measuring their energy very precisely as the spatial volume is varied, it is possible [5] to extract the scattering phase shifts and hence decay properties.

Consider the two body states of two pions of mass μ with momentum $\pm\mathbf{k}$ which will have total energy $E = 2(\mu^2 + \mathbf{k}^2)^{1/2}$. Then if they are non-interacting, their momentum on a spatial hyper-torus of size L will be discrete with $\mathbf{k} = 2\pi\mathbf{n}/L$ where $\mathbf{n}$ has integer components (and for future use we define N such that there are N values of $\mathbf{n}$ having a given $n = |\mathbf{n}|$, so $N = 6$ if $n = 1$). If the two pions interact with a finite range of interaction, compared to the spatial volume, this implies that the finite size effect will result in an momentum shift of order L^{-3}, provided that L is large enough to contain one pion without undue distortion. The detailed analysis of Lüscher [5] expresses the shift in this momentum value when there are interactions among the pions with phase-shift $\delta(k)$, provided E is below the inelastic threshold, as

$$d(E^2) = 4d(k^2) = \frac{-16\pi N \tan\delta(k)}{L^3 k} \quad (5)$$

to leading order in L^{-1}, and indeed a much more precise expression with sub-leading terms is available through Lüscher's work [5].

Now when there is a resonance at m_R near to a two particle state, there will be an opportunity to deduce the resonance width. Then, one can describe the phase shift $\delta(k)$ by using the expression for elastic $\pi\pi$ scattering dominated by this resonance pole:

$$\tan\delta(k) = \frac{\Gamma(k)}{2(m_R - E)} \quad (6)$$

so that determinations of $\delta(k)$ at two or more values of k will enable the resonance parameters m_R and $\Gamma(k_R)$ to be evaluated, where $m_R^2 = 4(\mu^2 + k_R^2)$. The most promising way to determine the two particle energy accurately is by using several lattice operators (including some built like two pions and some built like the resonance under study) as sources and sinks to get the best determination of the energy levels from fitting the matrix of correlators. Note that in practice it will be difficult to obtain sufficiently accurate energy determinations of the lowest two-particle state and it will be even more difficult to determine the energy level of the next heavier state with larger momenta.

1.3. A direct approach

It should be possible to extract the hadronic transition amplitude directly from the lattice (rather than via energy level shifts), and this we now discuss.

Using suitable lattice operators to create R at $t = 0$ and annihilate a two-pion state with momenta k and $-k$ at time t, the contribution to the correlator from a R state with mass m_R and a two-body state with energy $E_{\pi\pi}$ is given to leading order in x by

$$C_{R-\pi\pi}(t) = \sum_{t_1} h e^{-m_R t_1} x e^{-E_{\pi\pi}(t-t_1)} b \qquad (7)$$

where there is a summation over the intermediate t-value t_1 and where h and b are the amplitudes to make each state from the lattice operators used and x is the required transition amplitude $\langle R|\pi\pi\rangle$. Here we are assuming that the states R and $\pi\pi$ are normalised to unity. By obtaining h and b from the $R \to R$ and $\pi\pi \to \pi\pi$ correlators, one can attempt to extract x.

The complication, however, is that removal of excited state contributions is tricky, even if variational methods are used to construct improved lattice operators to create the ground states. For example, if $m_R - E_{\pi\pi} > 0$ then the transition time t_1 will be preferentially near 0 (since the heavier state then propagates less far in time) and one can complete the sum over t_1 obtaining a t-dependence of eq. 7 as $e^{-E_{\pi\pi}t}$. This same t-dependence would be obtained if the state with mass m_R were to be replaced with an excited state with an even heavier mass. Thus one cannot separate the ground state and excited state contributions even in principle. See ref. [6,7] for a fuller discussion.

The way forward is that if $m_R = E_{\pi\pi}$, the ground state contributions to eq. 7 have a t-dependence as $te^{-E_{\pi\pi}t}$ whereas any excited state contributions behave as $e^{-E_{\pi\pi}t}$ as above. So now we do have a way to isolate the required ground state contribution:

$$xt = \frac{C_{R-\pi\pi}(t)}{[C_{R-R}(t)\ C_{\pi\pi-\pi\pi}(t)]^{1/2}} + \mathcal{O}(x) + \mathcal{O}(x^3t^3) \qquad (8)$$

Note that this separation is only by a power of t which is less than the case for two-point correlations where the excited state contributions are suppressed by an exponential $e^{-(m'_R-m_R)t}$.

In practice the requirement of energy equality can be relaxed. Defining $\Delta = m_R - E_{\pi\pi}$, then the ground state contribution to the expression of eq. 8 evaluates to $2x\sinh(t\Delta/2)/(t\Delta) = x(1 + (t\Delta)^2/24 + \ldots)$. So this will be equivalent to the expression with $\Delta = 0$ provided

$$(m_R - E_{\pi\pi})t \ll 5 \qquad (9)$$

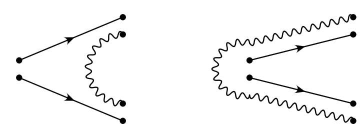

Figure 1. Quark pair production (wiggly lines) for the three point function $R \to \pi + \pi$ in euclidian time (running horizontally). The left hand diagram has the interpretation of a transition (our x) at an intermediate time while the right hand diagram can be thought of as some intrinsic mixing in the R state.

So far we have described the behaviour of the $C_{R-\pi\pi}(t)$ in the limit of small x. The first correction term from multiple transitions (to eq. 8) will be of order $x^3t^3/6$ so we need $xt \ll 1$. As well as these corrections which are of higher order in xt, one must also consider the intrinsic mixing of the initial R state with $\pi\pi$ (and vice versa). This intrinsic mixing (i.e. coming from the lattice operators used to create the states having an admixture - see fig. 1 for an example) is expected be of order $x/\mathcal{E}$ where $\mathcal{E}$ is the energy of the quark pair and so will contribute a term like $xe^{-Et}/\mathcal{E}$. This is a contribution similar to that from excited states and so will be dominated at large t by the xte^{-Et} term we are looking for. So we need both xt to be small and t to be large. This implies that x must be small for this direct approach. And indeed, it is only when x is small that the more rigorous approach of determining the energy shifts will be computationally difficult to implement.

In summary, when $t(m_R - E_{\pi\pi}) \ll 5$; $xt \ll 1$ and $(E'-E)t \gg 1$ (with $E'-E$ the energy gap to the first excited state), it is possible to estimate the hadronic transition amplitude directly, and this we will explore in detail for ρ decay.

1.4. Comparison

In order to show the relationship of this direct approach with Lüscher's approach, consider the case when a two particle state is close in energy to the resonance. It should be possible to arrange that energy levels are sufficiently close by choosing the lattice spatial length L. Then once x has been determined in this situation where the resonance does not decay significantly (because the two particle level has the same energy as the resonance), one can assume that the coupling strength determined can be used in a larger spatial volume where decay is energetically allowed.

The most direct way to evaluate resonance decay is using first order perturbation theory (Fermi's Golden Rule) which implies a transition rate

$$\Gamma = 2\pi \langle x^2 \rangle \rho(E) \tag{10}$$

where the angle brackets indicate that an average over spatial directions will be needed. For a decay from the centre of mass with relative momentum k, the density of states $\rho(E) = L^3 kE/(8\pi^2)$.

Now to compare with Lüscher's approach, we need to evaluate the energy shift of the two particle state in a finite box (which is exactly $N-$fold degenerate). This can be estimated from the mixing of the two energy levels which are close to each other, using second order perturbation theory (in x) which gives an energy shift, on the lattice, of

$$dE = -\frac{Nx^2}{m_R - E}, \tag{11}$$

Thus when the resonance lies above the two particle state, the mixing will move the resonance mass up and the two particle energy down, in the usual way as mixing causes repulsion of levels.

Using the relationship between x and Γ from eq. 10, we obtain

$$dE = -\frac{4\pi N\Gamma}{(m_R - E)kEL^3} \tag{12}$$

which is exactly the same expression as would be obtained at leading order in L^{-1} from Lüscher's formalism as presented above. This is as it should be, but does illustrate that an estimate of x directly from the lattice may provide a useful way to evaluate these decay effects.

Indeed the mass shifts are of second order in x so will be small in general, and hence difficult to determine with sufficient precision. The direct measurement of the transition giving x from the lattice is thus an attractive prospect.

2. VECTOR MESON DECAY

2.1. Transitions

For on-shell transitions, it is possible to estimate hadronic transition strengths as described above and here we explore this approach further for the case of ρ meson decay to $\pi\pi$, following ref. [8].

The situation we shall analyse is represented by the energy spectrum shown in fig. 2, here neglecting any interactions among the states. Note that no decay can actually take place with these lattice parameters. This is , however, optimum for our analysis since when the energy levels are approximately degenerate, the hadronic transition amplitude can be evaluated more effectively.

We evaluate correlations between lattice operators creating both a ρ meson and a $\pi\pi$ state, using a stochastic method with sources on time-slices [9] to evaluate the quark diagrams shown in fig. 3 from 20 gauge configurations [10] with $N_f = 2$ sea quarks of mass corresponding to about 2/3 of the strange-quark mass. These correlations, normalised by the two point functions as appropriate, are illustrated in fig. 4. Here the off-diagonal case (labelled $\rho_1 \to \pi_1\pi_0$) shows the important feature that it grows approximately linearly with increasing t. As emphasized above, this linear growth will only occur for on-shell transitions and this is essentially the case here.

To extract an estimate of the transition amplitude, we wish to evaluate $x = \langle \rho | \pi\pi \rangle$ where these states are normalised on the lattice (to unity). If higher excited states are neglected, one can make a two state model (i.e. ρ and lightest $\pi\pi$ state) with this transition amplitude and evaluate the

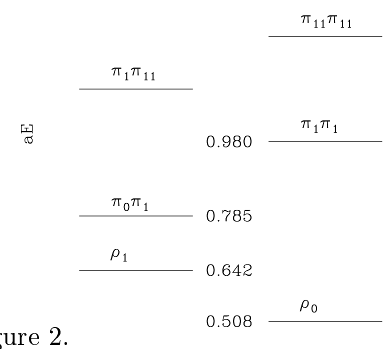

Figure 2.

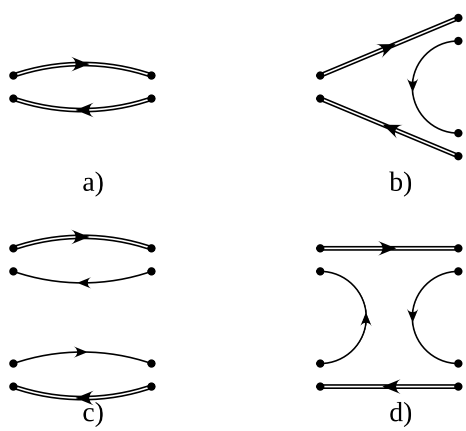

Figure 3.

contributions as shown by the curves in fig. 4. Because the location (at t_1 between 0 and t) of any transition is not known, one has potentially very serious contributions from excited states. As discussed above, these excited state contributions can be avoided if the transition is approximately on-shell, when the linear dependence on t of the off-diagonal transition (with value xt) is a unique signature of this on-shell transition that we wish to extract.

Note that for the case of $\rho_0 \to \pi_1\pi_1$, the on-shell condition is much less well satisfied but the relative momentum of the pions in the centre of mass is twice as large as for $\rho_1 \to \pi_0\pi_1$ and hence x should be approximately twice as large (as we find), since for a P-wave decay there will be a momentum factor in the transition amplitude.

A further check of the extraction of x comes from the box diagram, fig. 3d, which will have a contribution behaving as $x^2t^2/2$ arising from a ρ intermediate state - see the data labelled $\pi_1\pi_0 \to \pi_1\pi_0$ in fig. 4. Again this contribution does seem to be present at the required level.

Thus we have qualitative agreement from the three-point evaluations with two different relative momenta and from the four-point evaluation that $xa \approx 0.06$.

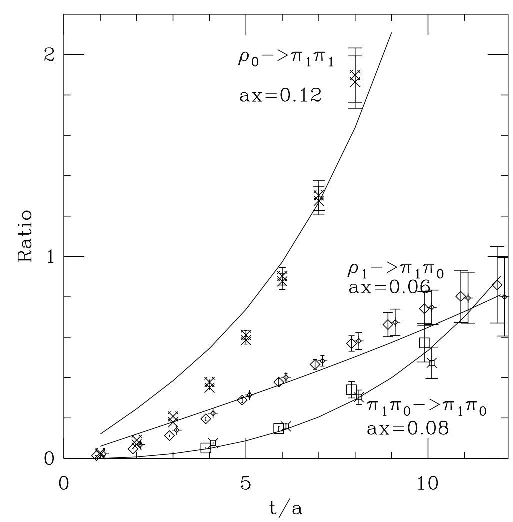

Figure 4. Ratios of 3 and 4-point correlators.

A more quantitative estimate of x can be made [8] by reducing excited state contributions as much as possible. Thus one uses fits to remove excited state contributions from the normalising two-point functions and then evaluates the slope of the ratio of eq. 8 versus t to remove the $\mathcal{O}(x)$ term also.

Note that the above methods can be used in the quenched approximation [11] to estimate hadronic transition amplitudes. They depend on assuming that xa is fairly weak, as we indeed find.

2.2. Energy shifts

A more rigorous approach is to focus on energy values. When two levels are close (our on-shell condition), then they will mix and the resultant energy shifts give relevant information [5] as discussed above. Moreover we can estimate these energy shifts from our x-value which provides more cross checks. These energy shifts can only be studied using dynamical fermions.

From a full analysis (i.e. measuring all the quark diagrams illustrated in fig. 3 and fitting the matrix of correlators) we obtain the energy shift of the $\pi_1\pi_0$ state (i.e. the un-binding energy), as needed in Lüscher's approach, as 0.02(2) (in lattice units) upward which is consistent but not sufficiently accurate to be of significant use. The energy shift of the ρ_1 state can, however, be determined in this case because of a lattice artifact. The ρ with momentum 1 (in lattice units of $2\pi/L$) can have its spin aligned parallel to the momentum axis (P) or perpendicular to it (A). Because the $\pi_0\pi_1$ state has relative momentum along a lattice axis and the transition from ρ to $\pi\pi$ has orbital angular momentum L=1 (so a distribution like $\cos\theta$ in the centre of mass), only the parallel state (P) can mix with this two pion state. This situation is not typical - it occurs because the ρ with momentum 1 only has a transition to a $\pi_0\pi_1$ state with a pion having momentum in the same direction. This mixing will not be present in the quenched approximation, so this provides a direct opportunity to see the effect of the two pion channel on the ρ in unquenched studies. We do indeed find [8] such a mass splitting between the P and A orientations of the ρ due to mixing for $N_f = 2$ but not for $N_f = 0$ as shown in fig. 5. Moreover the magnitude of this energy shift (0.026 ± 7 in lattice units) is consistent with other determinations of the transition strength.

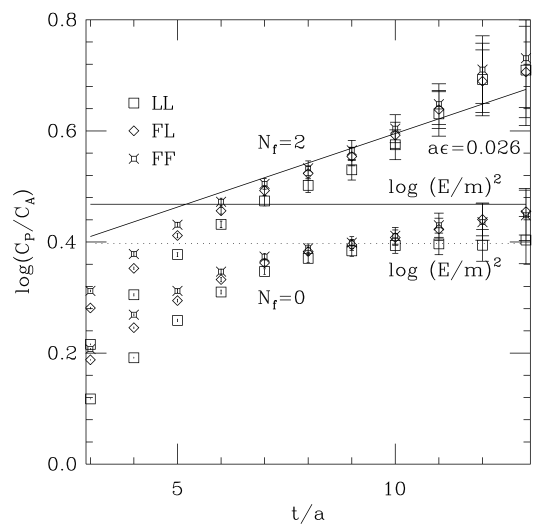

Figure 5. The ratio of Parallel to Antiparallel ρ correlators.

2.3. Phenomenology

The basic assumption is that the transition from ρ to $\pi\pi$ is given by an effective interaction with a finite spatial extent, this is usually summarised by an effective lagrangian where we normalise the coupling as $\bar{g}^2 = \Gamma ME/k^3$ in terms of the decay width. Then, provided the lattice spatial size is big enough that the hadrons are not distorted, our lattice situation (where no decay occurs) can be used to determine $\bar{g}$ and this can then be used to predict decay widths when the lattice volume is increased so that the minimum momentum becomes small. In our case we will also need to reduce the quark mass to allow decay since we have $m_\pi/m_\rho = 0.58$.

One complication is that the transition which is closest to being on-shell is $\rho_1 \to \pi_1\pi_0$ which is not in the centre of mass. A generalisation of Lüscher's approach has been made [12] which allows for this. The more direct method we have described here can also be extended to this case [8]. There are possible problems in this treatment of decays of moving particles. For instance, on the lattice a boost to bring the ρ to rest will not bring the two pion state to have zero net momentum also. This is because the lattice situation allows energy non-conservation which, upon boosting, will imply some momentum non-conservation.

From our lattice studies, we deduce that $ax = 0.06^{+2}_{-1}$ for $\rho_1 \to \pi_1\pi_0$. Translating [8] this lattice transition amplitude to the continuum normalisation, gives $\bar{g} = 1.40^{+27}_{-23}$. Using the observed ρ_1 energy shift gives another estimate, namely $\bar{g} = 1.56^{+21}_{-13}$. These two values agree well, which is good support for our scheme of extracting hadronic transition strengths.

Note that our lattice values would need to be extrapolated to light sea-quarks and to the continuum limit to allow all sources of systematic error to be explored in the comparison with the experimental situation. Nevertheless, these two values agree well with the values extracted from experimental data on decays of ρ, K^* and ϕ mesons, namely $\bar{g} \approx 1.5$. This further underlies the viability of our methods.

3. HYBRID MESON DECAY

Hybrid mesons are those with non-trivial gluonic excitations. The spectrum of such mesons has been determined from lattice studies, both for the case of heavy quarks and light quarks. Hybrid mesons have allowed couplings to meson meson systems. If these coupling are large, then the hybrid mesons decay widths will be big and such mesons will be difficult to identify experimentally. Here we present recent lattice results on these hadronic transitions.

3.1. Hybrid mesons with heavy quarks

One well defined way to treat heavy quarks (such as the b-quark) on a lattice is to use the leading order of the HQET which corresponds to the static approximation. Here we take the heavy quarks to be fixed at locations R apart in the z-direction.

Consider $Q\bar{Q}$ states with static quarks in which the gluonic contribution may be excited. We classify the gluonic fields according to the symmetries of the system. This discussion is very similar to the description of electron wave functions in diatomic molecules. The symmetries are (i) rotation around the separation axis z with representations labelled by J_z (ii) CP with representations labelled by $g(+)$ and $u(-)$ and (iii) C$\Re$. Here C interchanges Q and $\bar{Q}$, P is parity and $\Re$ is a rotation of 180^0 about the mid-point around the y axis. The C$\Re$ operation is only relevant to classify states with $J_z = 0$. The convention is to label states of $J_z = 0, 1, 2$ by Σ, Π, Δ respectively. The ground state (Σ_g^+) will have $J_z = 0$ and $CP = +$.

The exploration of the energy levels of other representations has a long history in lattice studies [13,14]. The first excited state is found to be the Π_u. This can be visualised as the symmetry of a string bowed out in the x direction minus the same deflection in the $-x$ direction (plus another component of the two-dimensional representation with the transverse direction x replaced by y), corresponding to flux states from a lattice operator which is the difference of U-shaped paths from quark to antiquark of the form $\sqcap - \sqcup$.

Recent lattice studies [15] have used an asymmetric space/time spacing which enables excited states to be determined comprehensively. These results confirm the finding that the Π_u excitation is the lowest lying and hence of most relevance to spectroscopy.

From the potential corresponding to these excited gluonic states, one can determine the spectrum of hybrid quarkonia using the Schrödinger equation in the Born-Oppenheimer approximation. This approximation will be good if the heavy quarks move very little in the time it takes for the potential between them to become established. More quantitatively, we require that the potential energy of gluonic excitation is much larger than the typical energy of orbital or radial excitation. This is indeed the case [13], especially for b quarks. Another nice feature of this approach is that the self energy of the static sources cancels in the energy difference between this hybrid state and the $Q\bar{Q}$ states. Thus the lattice approach gives directly the excitation energy of each gluonic excitation.

The Π_u symmetry state corresponds to excitations of the gluonic field in quarkonium called magnetic (with $L^{PC} = 1^{+-}$) and pseudo-electric (with 1^{-+}) in contrast to the usual P-wave orbital excitation which has $L^{PC} = 1^{--}$. Thus we expect different quantum number assignments from those of the gluonic ground state. Indeed combining with the heavy quark spins, we get a degenerate set of 8 states:

L^{PC}	J^{PC}	J^{PC}	J^{PC}	J^{PC}
1^{-+}	1^{--}	0^{-+}	1^{-+}	2^{-+}
1^{--}	1^{++}	0^{+-}	1^{+-}	2^{+-}

Note that of these, $J^{PC} = 1^{-+}$, 0^{+-} and 2^{+-} are spin-exotic and hence will not mix with $Q\bar{Q}$

states. They thus form a very attractive goal for experimental searches for hybrid mesons.

3.2. Hybrid decays

Within this static quark framework, one can explore the decay mechanisms. One special feature is that the symmetries of the quark and colour fields about the static quarks must be preserved exactly in decay, hence the light quark-antiquark pair produced must respect these symmetries. This has the consequence that the decay from a Π_u hybrid state to the open-b mesons ($B\bar{B}$, $B^*\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$) will be forbidden [16,7] if the light quarks in the B and B^* mesons are in an S-wave relative to the heavy quark (since the final state will have the light quarks in either a triplet with the wrong CP or a singlet with the wrong J_z). The decay to B^{**}-mesons with light quarks in a P-wave is allowed by symmetry but not energetically.

In the heavy quark limit, the only allowed decays are when the hybrid state de-excites to a non-hybrid state with the emission of a light quark-antiquark pair. Since the Π_u hybrid state has the heavy quark-antiquark in a triplet P-wave state, the resulting non-hybrid state must also be in a triplet P-wave since the heavy quarks do not change their state in the limit of very heavy quarks. Thus the decay for b quarks will be to $\chi_b + M$ where M is a light quark-antiquark meson in a flavour singlet. This proceeds by a disconnected light quark diagram and it would be expected [17] that the scalar or pseudoscalar meson channels are the most important (i.e. they have the largest relative OZI-rule violating contributions). This transition can be estimated on a lattice when the initial and final energies are similar. This is the case [7] for the Π_u de-excitation to ground state gluonic field plus f_0 meson when the interquark separation is around 0.2 fm which allows a lattice evaluation of the hadronic transition strength - see fig. 6.

Note that in fig. 6 for $R = 0.1$ fm the Π_u state is unstable to decay and, since we are using $N_f = 2$ flavours of sea-quark, this decay is enabled and we might expect to see the lower energy (i.e. $\Sigma_g^+ + f_0$) instead for this energy level. In practice, as we shall find later, the transition

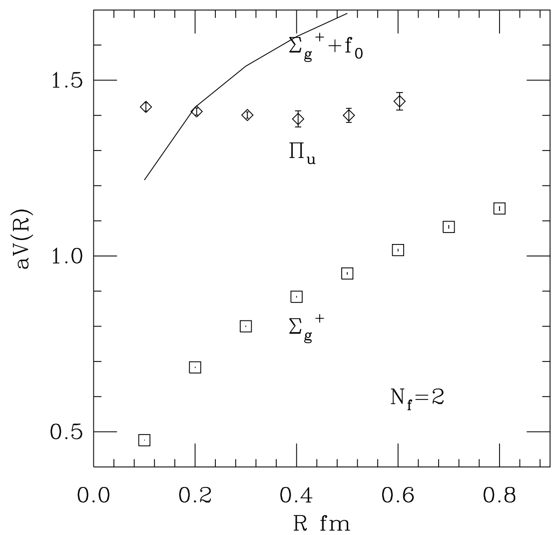

Figure 6. The potential energy $V(R)$ (in lattice units with a=0.0972 fm) versus quark separation R in fm for 2 flavours of sea quark. The energies are given [7] for the ground state and first excited gluonic state and for the two body state of ground state potential plus scalar meson (f_0) in a P-wave with the minimum non-zero momentum. The on-shell transition can be evaluated when $R \approx 0.2$ fm.

strength is quite small with $x \approx 0.01$ so the admixture of the lighter state is very small indeed (it is of order x^2) and hence it will only be significant at very large t-values, much larger than those used here.

Indeed the dominant transition is found to be with M as a scalar meson and the signal for the hadronic transition strength x is shown in fig. 7. Making use of the transition strength evaluated for a range of R-values around 0.2 fm and folding in the wavefunction overlaps, we can estimate the decay width of the hybrid spin-exotic meson H (e.g. $J^{PC} = 1^{-+}$) which has Π_u excited glue. This corresponds to a width of around 100 MeV for the transition $H \to \chi_b + f_0$, whereas when M is an η or η' meson the transition strength is found to be less than a few MeV. There will be modifications to this analysis coming from cor-

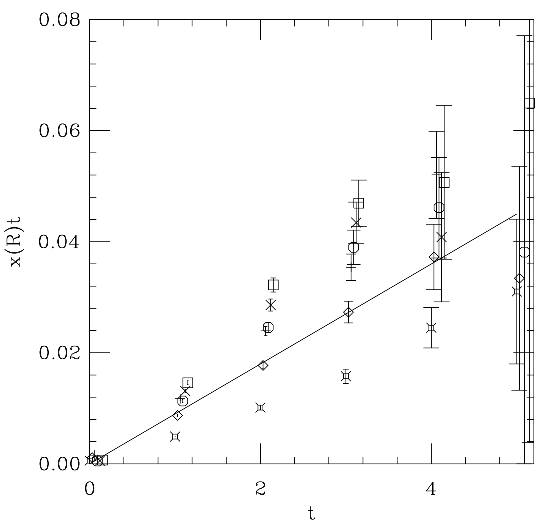

Figure 7. The transition matrix element xt for $\Pi_u \to \Sigma_g^+ f_0$ with momentum $\mathbf{n} = (1,0,0)$ versus t/a. Here $R = 0.1$ to 0.6 fm is represented by symbols: fancy square, diamond, +, octagon, ×, square. The line represents a linear fit to the $R = 0.2$ fm case.

rections to the heavy quark limit (of order $1/m_Q$ where m_Q is the heavy quark mass) which might allow hybrid meson transitions to $B\bar{B}$, etc, but these have not been evaluated yet.

In this heavy quark (or static) limit, the spin-exotic and non spin-exotic hybrid mesons are degenerate. For the latter, however, the interpretation of any observed states is less clear cut, since they could be conventional quark antiquark states. Moreover, the non spin-exotic hybrid mesons can mix directly (i.e. without emission of any meson M) with conventional quark antiquark states once one takes into account corrections (of order $1/M_Q$) to the static approximation applicable for heavy quarks with physical masses.

It is encouraging that the decay width comes out as relatively small, so that the spin-exotic hybrid states should show up experimentally as sufficiently narrow resonances to be detectable. This decay analysis does not take into account heavy quark motion or spin-flip and these effects will be significantly more important for charm quarks than for b-quarks.

4. CONCLUSIONS

It is possible to extract information about hadronic decays from euclidian lattice studies. The most rigorous method is to measure the small energy shifts in the two body states precisely as the lattice volume is varied. This then allows the phase shift to be extracted which gives information on resonance properties. This energy shift is only enabled in dynamical fermion simulations if quark-antiquark production is needed in the decay. This makes this a very difficult route to follow in practice.

We have also sketched an alternative which is less rigorous but will enable estimates to be obtained more readily. This direct method was tested in the case of $\rho \to \pi\pi$ where it gave consistent results and, moreover, results in agreement with experiment. This is a good indication that the method is reliable in practice. The method works best when the lattice transition amplitude ,x, is relatively small and it is encouraging that the method gives good indications when $ax \approx 0.06$. We then presented an application to hybrid meson decays: a case where the experimental result is not known, but where the magnitude of x is indeed small. Our result is of experimental relevance: the knowledge of the magnitude of the decay width confirms that these hybrid states are likely to be accessible to straightforward study.

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