LTH 590 HU-EP-03/50
Edinburgh 2003/17 DESY 03-143
LU-ITP 2003/016

Chiral Perturbation Theory and Finite Size Effects on the Nucleon Mass in unquenched QCD *

A. Ali Khan[a], T. Bakeyev[b], M. Göckeler[c,d], T.R. Hemmert[e], R. Horsley[f], A.C. Irving[g], D. Pleiter[h], P.E.L. Rakow[g], G. Schierholz[h,i], and H. Stüben[j] (QCDSF and UKQCD Collaborations)

[a]Institut für Physik, Humboldt-Universität zu Berlin, 12489 Berlin, Germany

[b]Joint Institute for Nuclear Research, 141980 Dubna, Russia

[c]Institut für Theoretische Physik, Universität Leipzig, 04109 Leipzig, Germany

[d]Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany

[e]Physik-Department, Theoretische Physik T39, TU München, 85747 Garching, Germany

[f]School of Physics, The University of Edinburgh, Edinburgh EH9 3JZ, UK

[g]Theoretical Physics Division, Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK

[h]John von Neumann-Institut für Computing NIC, 15738 Zeuthen, Germany

[i]Deutsches Elektronen-Synchrotron DESY, 22603 Hamburg, Germany

[j]Konrad-Zuse-Zentrum für Informationstechnik Berlin, 14195 Berlin, Germany

We calculate finite size effects on nucleon masses in chiral perturbation theory. We confront the theoretical predictions with $N_f = 2$ lattice results and discuss chiral extrapolation formulae.

1. INTRODUCTION

Finite size effects, in particular on dynamical configurations, can be a serious impediment to precision lattice calculations of hadron masses and matrix elements. It is of interest to find a theoretical description of finite size effects, to be able to determine the finite volume error at each lattice size and, possibly, to find an extrapolation formula to infinite volume.

On the lattice sizes used in production runs, the nucleons and pions are the relevant degrees of freedom for understanding the finite size effects on the nucleon mass. To calculate them, we use two-flavor relativistic baryon chiral perturbation theory (χPT) at $O(q^3)$ in the chiral counting (one-loop order).

*presented by A. Ali Khan

2. LATTICE PARAMETERS

We base our finite volume study on QCDSF and UKQCD configurations, using a plaquette gauge action with two flavors of non-perturbatively $O(a)$-improved Wilson fermions (a denotes the lattice spacing). The pion masses are in the interval $0.4-1$ GeV. Valence and sea quark masses are equal. Lattices volumes are $1-2.2$ fm. The scale is set with r_0, using the value at the physical point $r_0 = 0.5$ fm $\simeq 1/(395\text{MeV})$. We compare our results to JLQCD data with the same lattice actions and range of simulation parameters on varying lattice sizes [1]. In Fig. 1 we plot nucleon masses of both groups. It is found that the masses increase when the lattice size is decreased. The data from lattices of an extent ≥ 1.8 fm lie approximately on the same

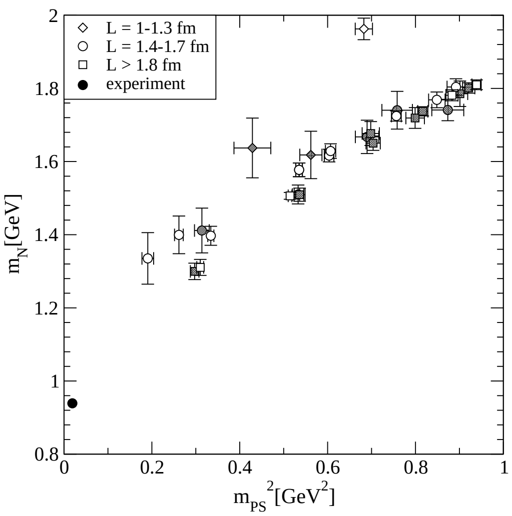

Figure 1. Nucleon masses from UKQCDSF (white symbols) and JLQCD (gray symbols).

curve. In principle there are two major sources of systematic error: finite volume and discretization effects. Among the data points, the lattice spacing varies between 0.4 and 0.6 GeV^{-1}. The $a \to 0$ limit was not performed, and we have to ascertain that there are no sizable discretization errors in the results. For example, between two data points on approximately the same volume and at the same pion mass, but with a varying by $\sim 30\%$, the nucleon mass is found to remain unchanged within the statistical errors. We also compare our large lattice results ($L \geq 1.8$ fm) with two CP-PACS data sets [2] using renormalization group improved gauge fields and tree-level tadpole-improved clover quarks at $a^{-1} \approx 1.5$ and 2 GeV respectively. Their box sizes are ≥ 2.5 fm. A compilation of the large L data is given in Fig. 2. We find a good scaling and conclude that in this data $O(a)$ uncertainties are small.

3. CHIRAL EXTRAPOLATION

The one-loop contribution is generated by the $O(q^1)$ Lagrangian $\mathcal{L}_N^{(1)}$:

$$\mathcal{L}_N^{(1)} = \bar{\Psi}\left(i\gamma_\mu D^\mu - m_0\right)\Psi + \frac{1}{2} g_A \bar{\Psi}\gamma_\mu\gamma_5 u^\mu \Psi, \quad (1)$$

with $D_\mu = \partial_\mu + \frac{1}{2}[u^\dagger, \partial_\mu u]$, $u_\mu = i u^\dagger \partial_\mu U u^\dagger$ and $u^2 = U$.

We use the infrared regularization scheme which is discussed in detail in [3]. To compute the renormalized nucleon mass m_N, we add at tree-level the $\mathcal{L}_N^{(2)}$ term $-4c_1 m_{PS}^2$ and an additional term of the form $e_1 m_{PS}^4$, which is, strictly speaking, derived from $\mathcal{L}_N^{(4)}$. The renormalization procedure is detailed in [4]. In this calculation we use $g_A = 1.2$ [5], and $F = 92.4$ MeV. We determine the nucleon mass in the chiral limit, m_0, the value of c_1 and the renormalized e_1 (e_1^r) by a fit to six lattice data points at the smallest masses, and find $m_0 = 0.85(14)$GeV, $c_1 = -0.80(18)$GeV^{-1} and $e_1^r(1\,\text{GeV}) = 2.8(1.1)$GeV^{-3}. For the definition of e_1^r see [4]. In Fig. 2, the result is compared with the lattice data on volumes > 1.8 fm. The χPT result shown here differs numerically slightly from the one quoted in [4] since they used the value $g_A = 1.267$ and a fit to a larger set of lattice points. Expanding the χPT result up to $O(m_{PS}^3)$, one obtains the non-relativistic (NR) approximation. The NR theory is valid only in the limit $m_{PS} \ll m_N$. This is reflected in the breakdown of the curves already at small pion masses, which is also shown in Fig. 2. A method to push the validity of the NR approximation to higher momentum scales within a cutoff scheme is described in [6].

4. FINITE SIZE EFFECTS

We calculate the finite size effect from the one-loop $O(q^3)$ contribution to the self energy. Putting the external nucleon line on-shell, this is given by [3]

$$\Sigma(\not{q} = m_0) = -i \frac{3 g_A^2 m_0 m_{PS}^2}{2F^2} \int_0^\infty dx$$

$$\int \frac{d^4 p}{(2\pi)^4} \left[p^2 - m_0^2 x^2 - m_{PS}^2 (1-x) + i\epsilon\right]^{-2} \quad (2)$$

in Minkowski space. We define

$$\delta = \frac{1}{m_0}\left(\Sigma(\not{q} = m_0, L) - \Sigma(\not{q} = m_0, \infty)\right) \quad (3)$$

where the temporal extent of the lattice is assumed to be infinite. Using [7], one can express δ as an integral over Bessel functions:

$$\delta = \frac{3 g_A^2 m_{PS}^2}{16\pi^2 F^2} \int_0^\infty dx \times$$

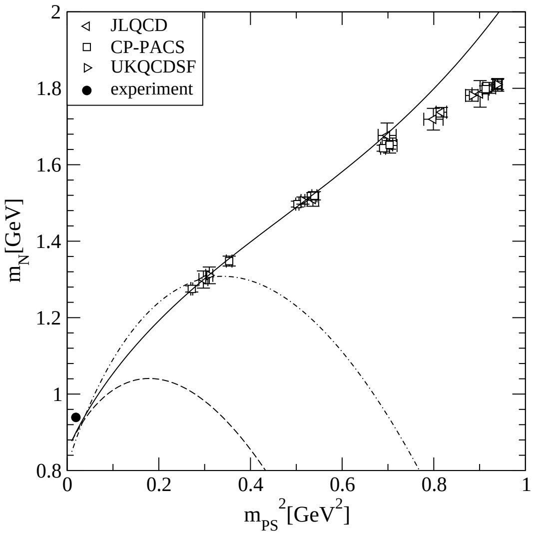

Figure 2. Nucleon mass compared to χPT. The solid curve denotes the fit with relativistic χPT, and the dashed curve the non-relativistic limit using the same values of m_0 and c_1. The dot-dashed curve shows a non-relativistic result with estimated parameters, $m_0 = 0.81$ GeV and $c_1 = -1.1$ GeV^{-1}.

$$\times \sum_{\vec{n}\neq 0} K_0\left(L|\vec{n}|\sqrt{m_0^2x^2 + m_{PS}^2(1-x)}\right). \quad (4)$$

To calculate the nucleon mass in a finite box,

$$m_N(L) = (1+\delta)m_N(\infty), \quad (5)$$

we first extrapolate to $m_N(\infty)$ by using the lattice result on the largest available lattice as input to Eq.(5). We are then able to calculate $m_N(L)$ at finite lattice extent. Lattice results for the nucleon mass at fixed values of β and κ, but different lattice sizes are compared with chiral perturbation theory in Fig. 3. It is found that the finite size effect can be well described by relativistic χPT. In contrast, in NR χPT at $O(q^3)$ we obtain only roughly $\sim 30\%$ of the finite volume effect in the lattice data [8]. At pion masses ≥ 500 MeV, large loop momenta give a substantial contribution also to the finite size effect. It is of interest to study whether this dependence is reduced in the relativistic formalism.

Acknowledgements

This work is supported in part by the Deutsche Forschungsgemeinschaft. The simulations were done on the APEmille at NIC (Zeuthen), the Hitachi SR8000 at LRZ (Munich) and the Cray T3E at EPCC (Edinburgh) and NIC (Jülich). We thank all institutions for their support.

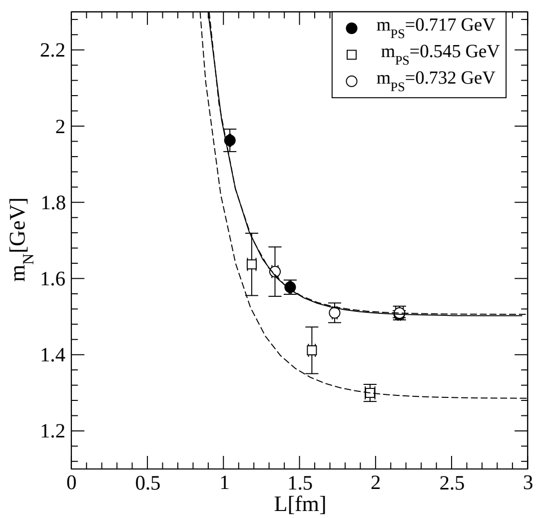

Figure 3. Comparison of the finite size dependence of lattice nucleon mass data sets at a fixed (β,κ) value and χPT. The data sets are labeled by the pion mass on the largest volume.

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