Hadronic decays from the lattice

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We discuss strategies to determine hadronic decay couplings from lattice studies. As an application, we explore the decay of a vector meson to two pseudoscalar mesons with $N_f = 2$ flavours of sea quark. Although we are working with quark masses that do not allow a physical decay, we show how the transition rate can be evaluated from the amplitude for $\rho \to \pi\pi$ and from the annihilation component of $\pi\pi \to \pi\pi$. We explore the decay amplitude for two different pion momenta and find consistent results. The coupling strength we find is in agreement with experiment. We also find evidence for a shift in the ρ mass caused by mixing with two pion states.

1. INTRODUCTION

The recent renaissance of experiments to study the spectroscopy of light hadrons is partly driven by the goal to understand confinement. Any analysis of experimental data requires both a calculation of decay widths as well as the masses[1]. Although computing the masses of resonances is (currently) not part of the program to compute "gold plated" observables with high precision from lattice QCD, dealing with decay widths is an inherent part of studying hadrons which are under current experimental scrutiny, such as "exotics", and gluonic components of scalar mesons. Here we compute the coupling of the ρ meson to two pions to validate our procedure.

One limitation of the lattice approach to QCD is in exploring hadronic decays because the lattice, using Euclidean time, has important contributions from low lying thresholds [2] which can obstruct the study of decay widths. The finite spatial size of the lattice implies that two-body states are actually discrete. By measuring their energy very precisely as the spatial volume is varied, it is possible [3] to extract the scattering phase shifts and hence decay properties. For on-shell transitions, it is possible to estimate hadronic transition strengths more directly and this approach has been used to explore [4] hybrid meson decay rates. Here we explore this approach further for the case of ρ meson decay to $\pi\pi$, following ref. [5].

2. TRANSITIONS

The situation we shall analyse is represented by the energy spectrum shown in fig. 1, here neglecting any interactions among the states. We evaluate correlations between lattice operators creating both a ρ meson and a $\pi\pi$ state, using a stochastic method to evaluate the quark diagrams shown in fig. 2 from 20 gauge configurations [6] (double the number used in our initial calculation) with $N_f = 2$ sea quarks of mass corresponding to about 2/3 of the strange-quark mass. These correlations, normalised by the two point functions as appropriate, are illustrated in fig. 3. Here the off-diagonal case (labelled $\rho_1 \to \pi_1\pi_0$) shows the important feature that it grows approximately linearly with increasing t. This linear growth will only occur for on-shell transitions [4] and this is essentially the case here.

To extract an estimate of the transition amplitude, we introduce a parameter $x = \langle\rho|\pi\pi\rangle$ where these states are normalised on the lattice (to unity). If higher excited states are neglected, one can make a two state model (with basis states ρ and the lightest $\pi\pi$ state) with this transition amplitude and evaluate the contributions shown in fig. 3. Unlike weak decays, there is no specific operator that causes the transition in strong decays, hence the specific time (between 0 and t) of

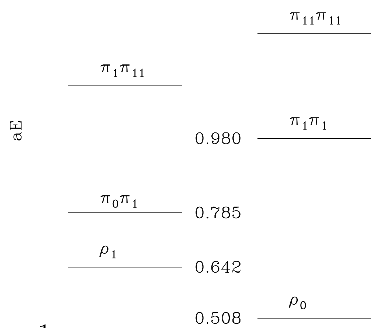

Figure 1.

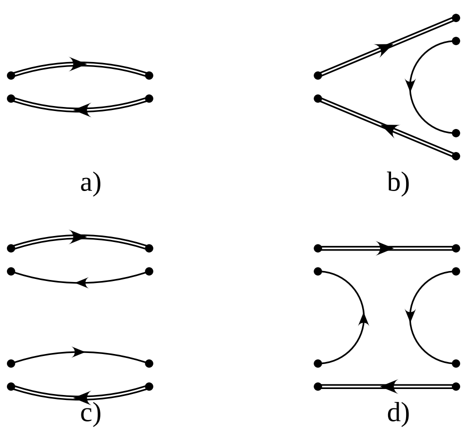

Figure 2.

any transition, such as $\rho \to \pi\pi$, is not known. In general this makes it hard to control the contributions from excited states. As emphasized previously [4], these excited state contributions can be avoided if the transition is approximately on-shell, when the linear dependence on t of the off-diagonal transition amplitude (with the slope x) is a unique signature of this on-shell transition.

Note that for the case of $\rho_0 \to \pi_1\pi_1$, the on-shell condition is much less well satisfied but the relative momentum of the pions in the centre of mass is twice as large as for $\rho_1 \to \pi_0\pi_1$ and hence x should be approximately twice as large, since for a P-wave decay there will be a momentum factor in the transition amplitude.

A further check of the extraction of x comes from the box diagram, fig. 2d, which will have a contribution behaving as $x^2t^2/2$ arising from a ρ intermediate state - see fig. 3.

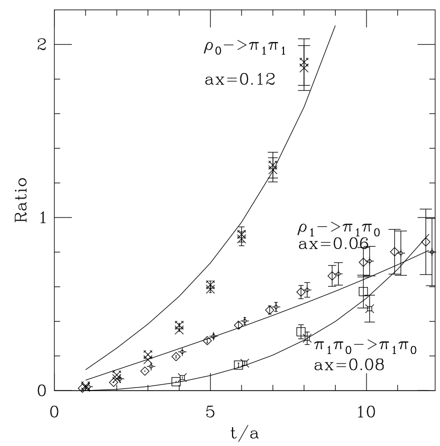

Figure 3. Ratios of 3 and 4-point correlators.

For a more quantitative estimate of x, one can suppress excited state contributions in analysing the data of fig. 3, as discussed in ref. [5].

Note that the above methods could be used, in principle, even in the quenched approximation. They depend on assuming that xa is fairly weak, as we indeed find.

3. ENERGY SHIFTS

A more rigorous approach is to focus on energy values. When two levels are close (our on-shell condition), then they will mix and the resultant energy shifts give relevant information [3]. Moreover we can estimate these energy shifts from our x-value which provides more cross checks. These shifts can only be studied using dynamical fermions.

From a full variational analysis we obtain the energy shift of the $\pi_1\pi_0$ state (i.e. the un-binding energy), as needed in Lüscher's approach, as 0.02(2) upward which is consistent but not sufficiently accurate to use. The energy shift of the ρ_1 state can however be determined because of a lattice artifact. The ρ with momentum 1 (in lattice units of $2\pi/L$) can have its spin aligned parallel to the momentum axis (P) or perpendicular to it (A). Because the $\pi_0\pi_1$ state has relative momentum along a lattice axis and the transition from ρ

to $\pi\pi$ has orbital angular momentum L=1 (so a distribution like $\cos\theta$), only the parallel state (P) can mix with this two pion state. This mixing will not be present in the quenched approximation, so this provides a direct opportunity to see the effect of the two pion channel on the ρ in unquenched studies. We do indeed find such a mass splitting between the P and A orientations of the ρ due to mixing for $N_f = 2$ but not for $N_f = 0$ as shown in fig. 4. Moreover the magnitude of this energy shift (0.026 ± 7 in lattice units) is consistent with other determinations of the transition strength.

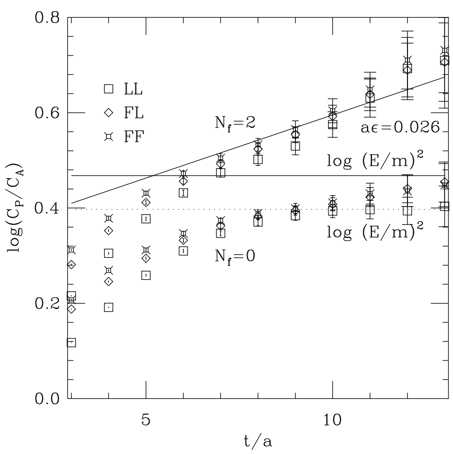

Figure 4. The ratio of Parallel to Antiparallel ρ correlators.

4. PHENOMENOLOGY

The basic assumption is that the transition from ρ to $\pi\pi$ is given by an effective interaction with a finite spatial extent, this is usually summarised by an effective lagrangian where we normalise the coupling as $\bar{g}^2 = \Gamma ME/k^3$ in terms of the decay width. Then, provided the lattice spatial size is big enough that the hadrons are not distorted, our lattice situation (where no decay occurs) can be used to determine $\bar{g}$ and this can then be used to predict decay widths when quark masses are varied, assuming that the coupling is largely independent of the quark masses.

From our lattice studies, we deduce that $ax = 0.06^{+2}_{-1}$ for $\rho_1 \to \pi_1\pi_0$. Translating [5] this lattice transition amplitude to the continuum normalisation, gives $\bar{g} = 1.40^{+27}_{-23}$. Using the observed ρ_1 energy shift gives another estimate, namely $\bar{g} = 1.56^{+21}_{-13}$. Note that our lattice values would need to be extrapolated to light sea-quarks and to the continuum limit to allow all sources of systematic error to be explored. Nevertheless, these two values agree well with the values extracted from decays of ρ, K^* and ϕ mesons, namely $\bar{g} \approx 1.5$.

Encouraged by the success of the calculation of the decay width of the ρ meson, we are starting to look at the decays of scalar mesons on the lattice. This is important for studying the glue components of singlet scalar mesons and to study $a_0(980)$ and $f_0(980)$ mesons which are believed to have important 4 quark contributions. There has been one earlier calculation of the decay width of the 0^{++} glueball from lattice QCD, but this has not been exploited by phenomenology [1].

For decays of singlet scalar mesons there are additional diagrams to those in figure 2 from disconnected graphs. The “noise” from the additional diagrams makes the extraction of a signal hard. In the real world the non-singlet $a_0(980)$ meson has the strong decays $a_0 \to \pi\eta$ and $a_0 \to K\overline{K}$. A direct comparison with experiment will be non-trivial because the $\pi\eta$ decay involves a noisy disconnected loop and the $a_0 \to K\overline{K}$ decay requires $N_f = 2 + 1$ sea quarks for a theoretically clean calculation in this channel.

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