LTH 592
hep-ph/0309165

Yukawa Textures and the mu-term

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We show how with an anomaly-free U_1, simple assumptions concerning the origin of Yukawa textures and the Higgs μ-term lead to the prediction of a new physics scale of 10^8GeV and automatic conservation of baryon number.

September 2003

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1. Introduction

One of the most appealing approaches to the fermion mass hierarchy problem is provided by the Froggatt-Nielsen (FN) mechanism [1]. According to FN, the hierarchy is produced from Yukawa textures produced by higher dimension terms involving MSSM singlet "flavon" fields θ via terms such as $H_2 Q_i u_j^c (\frac{\theta}{M_\theta})^{a_{ij}}$, where M_θ represents the scale of new physics, and $a_{ij} = 0, 1, 2...$ We consider here the case when the MSSM gauge group is extended by a single U_1' group which is broken by $\langle\theta\rangle \neq 0$. An exhaustive analysis of this general approach has been performed recently [2] by Dreiner and Thormeier (DT); this paper also contains a comprehensive list of references. Our assumptions here differ from DT in two critical respects:

* We impose cancellation of all mixed U_1' anomalies [2] without invoking the Green-Schwarz mechanism [4].
* We relax the assumption that there is only a single flavon field.

It might appear that our second assumption would rob us of most if not all predictive power; we will show however, that there is a very simple naturalness criterion which results in a constrained framework resulting in definite predictions. This arises as follows. Each Yukawa matrix $Y_{u,d,e}$ gains its texture from a *particular* flavon, $\theta_{u,d,e}$ with U_1' charges $-Q_u$, $-Q_d$, and $-Q_e$, and we will choose $Q_u = 1$[3]. Our naturalness criterion is simply that this state of affairs arises by virtue of the charges of the fields, and is not imposed. Since we assume that the vevs of the various flavons are approximately the same, with

$$\langle\theta_{u,d,e}\rangle / M_\theta \approx \lambda \approx 0.22, \tag{1.1}$$

then if we want the (11) entries of Y_u and Y_d to be of order λ^8 and λ^4 respectively, we could not have $Q_d = 2Q_u$ since evidently were that so the Y_u entry could be made $O(\lambda^4)$ by using θ_d instead of θ_u. In imposing this criterion we will allow for possible flavon contributions to the Kahler potential. The kinetic term for the quark doublets Q will be, for example,

$$L = \Phi_i^* K_Q^{ij} \Phi_j$$

[2] For a recent account of how an anomaly free family-dependent U_1' might be embedded in a replicated gauge group, see Ref. [3].

[3] We might want to assume that each flavon is accompanied by an oppositely charged $\overline{\theta}$-partner; the simplest way to obtain a U_1' D-flat direction, i.e. preventing the quadratic D-terms for the U_1' from generating large masses for all the MSSM fields [5], is to assume the $\overline{\theta}$s exist and have vevs approximately equal to the corresponding θs. We will indicate when this issue affects our discussion subsequently.

in superspace, where

$$K_Q \sim \begin{pmatrix} 1 & \lambda^{k_1} & \lambda^{k_2} \\ \lambda^{k_1} & 1 & \lambda^{k_3} \\ \lambda^{k_2} & \lambda^{k_3} & 1 \end{pmatrix} \tag{1.2}$$

and $k_1 = k_2 + k_3$ (or a cyclic permutation). Then we define $\Phi' = C_Q \Phi = D_Q U_Q \Phi$, where U_Q is the unitary matrix that diagonalises K_Q, so that $U_Q K_Q U_Q^{-1} = K_{\text{diag}}$, and D_Q is the diagonal matrix whose entries are the square roots of the eigenvalues of K_Q. Evidently

$$\Phi_i^* K_Q^{ij} \Phi_j = \Phi_i'^* \Phi_i', \tag{1.3}$$

and the Yukawa matrix Y_u, for example, will be replaced by $Y_u' = (C_Q^{-1})^T Y_u C_{u^c}^{-1}$. It is important to realise that while the Yukawa terms are holomorphic, so that powers of $\theta^*_{u,d,e}$ cannot contribute to them, the Kahler terms are not. Note also, as remarked by DT, that the textures of Y_u and Y_u' may well differ, with, for example, texture zeroes being "filled in". We, however, will restrict ourselves to cases when $Y_{u,d,e}$ already have our desired texture, and this texture is preserved by the canonicalisation.

Thus far our analysis of the Kahler term mirrors that of DT. We differ from them in the following respect, however. We claim that quite generally the canonicalisation matrix C can always be chosen (without fine-tuning) to have the *same* texture as K. DT present an apparent counterexample, based on the matrix

$$K = \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \tag{1.4}$$

but it is easy to construct a further unitary transformation that reduces their canonicalisation matrix to our claimed form; and a unitary transformation obviously preserves the canonical kinetic form. Consider a simple $2 \otimes 2$ example,

$$K = \begin{pmatrix} 1 & \lambda \\ \lambda & 1 \end{pmatrix}. \tag{1.5}$$

This matrix is diagonalised by the transformation

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \tag{1.6}$$

which is not close to the unit matrix; however the matrix

$$C = U^{-1} D_Q U, \quad \text{where} \quad D_Q = \begin{pmatrix} \sqrt{1+\lambda} & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix} \tag{1.7}$$

is a perfectly valid canonicalisation matrix, and takes the form

$$C = \begin{pmatrix} 1+O(\lambda^2) & \frac{\lambda}{2}+O(\lambda^3) \\ \frac{\lambda}{2}+O(\lambda^3) & 1+O(\lambda^2) \end{pmatrix}, \tag{1.8}$$

in accordance with our assertion.

Let us turn now to a realistic example. Consider the "Wolfenstein" textures (see for example Ref. [6]):

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim \lambda^{\alpha_d} \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}, Y_e \sim \lambda^{\alpha_e} \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \tag{1.9}$$

The $Y_{u,d}$ textures lead to the Wolfenstein texture for the CKM matrix, and appropriate hierarchies for the quark masses. There is considerable freedom in the choice of Y_e texture; the above decision relates to the incorporation of neutrino masses, as will become clear anon. To avoid fine tuning of the leading order contributions we would expect $\alpha_d \sim \alpha_e$ and $\tan\beta \sim \lambda^{\alpha_d - 3}$; so we will restrict our attention to $3 \geq \alpha_{d,e} \geq 0$. Denoting the U_1' charges of the multiplets $Q_i, L_i, u_i^c, d_i^c, e_i^c, H_1, H_2$ as $q_i, L_i, u_i, d_i, e_i, h_1, h_2$, it is easy to show that the mixed anomalies for $(SU_3)^2 U_1'$, $(SU_2)^2 U_1'$, $(U_1)^2 U_1'$ and $(U_1')^2 U_1$ all cancel and the above textures are obtained if the following relations are satisfied:

$$Q_d = 1 \tag{1.10a}$$

$$\Delta = \alpha_d + 6 \tag{1.10b}$$

$$Q_e = 2\alpha_d/(3\alpha_e + 6) \tag{1.10c}$$

$$u_1 = -2\alpha_d/9 + 16/3 - 2h_2/3 - e_1/3 + Q_e(10 + 3\alpha_e)/9 \tag{1.10d}$$

$$\begin{aligned} e_1 = &-(116 - 12Q_e\alpha_e + 32\alpha_d - 24h_2 - 40Q_e + 24Q_e^2 + 20Q_e^2\alpha_e \\ &+ 3Q_e^2\alpha_e^2 - 6Q_e\alpha_e\alpha_d - 20Q_e\alpha_d + 4\alpha_d^2 - 4\alpha_d h_2)/(2(\alpha_d + 6)). \end{aligned} \tag{1.10e}$$

Here $\Delta = h_1 + h_2$. We have not substituted for Q_e in Eq.(1.10e) and for e_1 and Q_e in Eq.(1.10d) because the resulting expressions are unwieldy. Note that $Q_d = Q_u$ so we only need two flavons at this stage. All the remaining charges are determined in terms of h_2, α_e and α_d.

Let us now discuss the issue of *naturalness* we described above (ignoring at first the Kahler potential). Our system will be unnatural if there are solutions for $\alpha, \beta \in \{0, 1, 2, 3 \cdots\}$ to any of the following system of equations:

$$\alpha + \beta Q_e = 8, \qquad \alpha + \beta \leq 7 \tag{1.11a}$$

$$\alpha + \beta Q_e = (4 + \alpha_d), \qquad \alpha + \beta \leq 3 + \alpha_d \tag{1.11b}$$

$$\alpha + \beta Q_e = (4 + \alpha_e)Q_e, \quad \alpha + \beta \leq 3 + \alpha_e \tag{1.11c}$$

Note that for $\alpha_d \leq 4$ all the solutions to Eq. (1.11b) are solutions to Eq. (1.11a). For any particular choice of $\alpha_{d,e}$ it is straightforward to classify the unnatural solutions for Q_e. Thus from Eq. (1.11a) we obtain unnatural Q_e values

$$8, 7, 6, 5, 4, 3, 2, \tfrac{3}{2}, \tfrac{5}{2}, \tfrac{7}{2}, \tfrac{4}{3}, \tfrac{5}{3}, \tfrac{7}{3}, \tfrac{8}{3}, \tfrac{5}{4}, \tfrac{7}{4}, \tfrac{6}{5}, \tfrac{7}{5}, \tfrac{8}{5}, \tfrac{7}{6}, \tfrac{8}{7}, \tag{1.12}$$

while with, for example, $\alpha_e = 1$, $\alpha_d \leq 4$ we also have from Eq. (1.11c) the additional unnatural values

$$\tfrac{1}{2}, \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{1}{4}, \tfrac{3}{4}, \tfrac{1}{5}, \tfrac{2}{5}, \tfrac{3}{5}, \tfrac{4}{5}. \tag{1.13}$$

If we were to assume the existence of $\overline{\theta}$ flavon partners (with similar vevs) then Eq. (1.11a), for example, would be replaced by

$$\alpha - \overline{\alpha} + (\beta - \overline{\beta})Q_e = 8, \quad \alpha + \overline{\alpha} + \beta + \overline{\beta} \leq 7. \tag{1.14}$$

In that case an additional set of Q_e values would be unnatural: Eq. (1.12) would now also include the set

$$9, 10, 11, 12, 13, 14, \tfrac{9}{2}, \tfrac{11}{2}, \tfrac{13}{2}, \tfrac{10}{3}, \tfrac{11}{3}, \tfrac{9}{4}, \tfrac{11}{4}, \tfrac{9}{5} \tag{1.15}$$

and Eq. (1.13) the set

$$\tfrac{1}{6}, \tfrac{1}{7}, \tfrac{2}{7}, \tfrac{1}{8}, \tag{1.16}$$

and the corresponding negative charge would also be unnatural in every case in Eqs. (1.12), (1.13), (1.15), (1.16).

Note that for $\alpha_d = \alpha_e = 1$ we have from Eq. (1.10c) that $Q_e = 2/9$, which value appears in none of Eqs. (1.12), (1.13), (1.15), (1.16). It is easy to establish that the possibilities $(\alpha_d, \alpha_e) = (1,0), (1,2), (2,0), (2,2), (3,1), (3,2), (3,3)$ are all unnatural, while $(\alpha_d, \alpha_e) = (1,1), (1,3), (2,1), (2,3), (3,0)$ are natural. This conclusion continues to hold if we take into account the $\overline{\theta}$-flavons. Note that $(3,0)$ gives $Q_e = 1$ so in this case we could have a single flavon; however (since $m_b > m_\tau$) this would manifestly require fine-tuning[2]. If we restrict to $\alpha_d \leq \alpha_e$, then we have three possible solutions. In what follows we will concentrate on $\alpha_d = 2$, $\alpha_e = 3$.

Turning to the Kahler terms, one sees easily that since $q_2 = q_1 - 1, L_1 = L_2 + Q_e$ etc., we have

$$K_{Q,e^c} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, K_{u^c} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^5 \\ \lambda^3 & 1 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, \quad K_{d^c,L} \sim \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, \tag{1.17}$$

providing the Kahler textures are generated by θ_u for Q, u^c, d^c and by θ_e for L, e^c, and that in each case only one flavon can contribute. For this to be natural we must exclude solutions to (once again for $\alpha, \beta, \overline{\alpha}, \overline{\beta} \in \{0, 1, 2, 3 \cdots\}$)

$$\alpha - \overline{\alpha} + (\beta - \overline{\beta})Q_e = 5, \qquad \alpha + \overline{\alpha} + \beta + \overline{\beta} \leq 4 \tag{1.18a}$$

$$\alpha - \overline{\alpha} + (\beta - \overline{\beta})Q_e = 3Q_e, \quad \alpha + \overline{\alpha} + \beta + \overline{\beta} \leq 2 \tag{1.18b}$$

Thus for example $Q_e = -2$ is now also seen to be unnatural (this would, of course be unnatural in any case if we were assuming the existence of $\overline{\theta}$-partners). Note, however, that our anomaly cancellation conditions preclude $Q_e < 0$.

It is easy to verify that, as we asserted earlier, the canonicalisation matrices corresponding to all the K-matrices in Eq. (1.17) have precisely the same texture as the corresponding K-matrix, and that the engendered transformations preserve the form of the textures in Eq. (1.9).

We turn now to the Higgs μ-term. If we suppose that it is generated in the same way as the Yukawa textures, that is via a term of the form $M_\mu H_1 H_2 \lambda^{a_\mu}$, can we place any constraint on a_μ? Clearly we have $a_\mu = \alpha_\mu + \beta_\mu$ where

$$\Delta = \alpha_\mu Q_u + \beta_\mu Q_e. \tag{1.19}$$

Now in our example, we see from Eq. (1.10c) that to obtain $Q_e > 1$ we would need $\alpha_d > 3\alpha_e/2 + 3$, which would again be difficult to reconcile with the fact that $m_b > m_\tau$. So we may assume $Q_e < 1$, and hence manifestly the smallest attainable value of a_μ is obtained for $\beta_\mu = 0$ and is $a_\mu = \Delta = \alpha_d + 6$. So in our favoured case $(\alpha_d, \alpha_e) = (2, 3)$ we have $a_\mu = 8$ corresponding to $M_\mu \approx 10^8$GeV if we set $\mu = 500$GeV. This conclusion is not altered if we assume the existence of $\overline{\theta}$ flavon partners.

For $\alpha_{d,e} = (2, 3)$ we list the various U_1' charges in Table 1. An immediate consequence is that the dimension 3,4 R-parity violating operators of the form $LH_2, LLe^c, QLd^c, u^c d^c d^c, L^* H_1$ are all forbidden, not only in the sense that they are not U_1' invariant, but also in that they cannot be flavon generated: for example $L_1 + h_2 = 1223/300$, which manifestly can not be produced by a linear combination of $Q_u = 1$ and $Q_e = 4/15$.

Q_1	Q_2	Q_3	L_1	$L_{2,3}$	H_1
$\frac{h_2}{3} - \frac{2309}{2700}$	$\frac{h_2}{3} - \frac{5009}{2700}$	$\frac{h_2}{3} - \frac{10409}{2700}$	$\frac{1223}{300} - h_2$	$\frac{381}{100} - h_2$	$8 - h_2$

u_1	u_2	u_3	d_1	$d_{2,3}$
$\frac{23909}{2700} - \frac{4}{3}h_2$	$\frac{15809}{2700} - \frac{4}{3}h_2$	$\frac{10409}{2700} - \frac{4}{3}h_2$	$\frac{2}{3}h_2 - \frac{3091}{2700}$	$\frac{2}{3}h_2 - \frac{5791}{2700}$

e_1	e_2	e_3
$2h_2 - \frac{1021}{100}$	$2h_2 - \frac{3143}{100}$	$2h_2 - \frac{1101}{100}$

Table 1: The $U_1{}'$ hypercharges.

Let us now explore the economical possibility that $M_\mu \sim M_\theta$. The objection to this is the possibility of flavon-generated baryon and lepton number violation: we would prefer not to impose these symmetries. We have already seen that dangerous dimension 3,4 operators are absent; but with M_θ so low we must obviously also consider higher dimension operators such as (here we list B-violating operators only) [4]

$$\text{dimension } 5 : QQQL, QQQH_1, u^c u^c d^c e^c, QQd^{c*}$$

or

$$\text{dimension } 6 : QQQQu^c, d^c d^c d^c LL, d^c d^c d^c LH_1, u^c u^c u^c e^c e^c, u^c d^c d^c LH_2, u^c d^c d^c H_1 H_2,$$

$$QQQH_2^*, QQu^{c*}e^{c*}, Qu^{c*}d^{c*}L, Qu^{c*}d^{c*}H_1, Qu^{c*}d^{c*}H_2^*, Qd^{c*}d^{c*}H_2,$$

$$Qd^{c*}d^{c*}H_1^*, Qd^{c*}d^{c*}L^*, d^c d^c d^c e^{c*}.$$

In all cases, given Table 1, these operators are not U_1' invariant; moreover (like the R-parity violating dimension 3,4 operators) they cannot cannot be flavon generated. In fact because our scale of new physics is so (comparatively) low, B-violating operators with dimension up to at least 8 are potentially dangerous. The number of such operators is large so we do not list them. We have, however, verified that there are no operators through dimension 9 violating *either* B or L that can be generated by any combination of our two flavon charges. Remarkably enough, this conclusion was reached by simply examining all B, L violating operators with U_1 hypercharge zero, without worrying whether they are

[4] For a listing of holomorphic higher dimension operators see Ref. [7]

$SU_3 \otimes SU_2$ invariant: this set manifestly contains the genuine $SU_3 \otimes SU_2 \otimes U_1$ operators. We conclude therefore that with θ-charges $Q_{u,d,e} = 1, 1, \frac{4}{15}$ and a physics scale $M_\theta \sim 10^8$GeV, we can explain the matter mass hierarchy, the CKM matrix texture, and the magnitude of the Higgs μ-term. So B and L-violation associated with M_θ are highly suppressed; but since this includes the dimension 5 operator $\chi_{ij} = H_2 L_i H_2 L_j$ associated (generally via the see-saw mechanism) with neutrino masses, at this stage we have no explanation for the origin of the neutrino masses. However, the matrix of charges corresponding to χ_{ij} is easily constructed:

$$Q_\chi = \begin{pmatrix} \frac{1223}{150} & \frac{1183}{150} & \frac{1183}{150} \\ \frac{1183}{150} & \frac{381}{50} & \frac{381}{50} \\ \frac{1183}{150} & \frac{381}{50} & \frac{381}{50} \end{pmatrix}. \tag{1.20}$$

Then it is easy to see that if we introduce one more flavon with charge $-Q_\nu$ such that $Q_\nu = \frac{13}{150}$, we obtain a neutrino mass matrix with texture

$$M_\nu \sim \frac{v_2^2}{M_\theta} \lambda^{10} \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, \tag{1.21}$$

where the λ^{10} arises because $7 + 2.(4/15) + 13/150 = 381/50$. This texture, as shown in Ref. [8], is compatible with current knowledge of the neutrino spectrum and mixing angles, without excessive fine-tuning (for a recent review of neutrino mass patterns see Ref. [9]).

Moreover, even with the introduction of this new flavon, it remains the case that B-violation remains suppressed to at least the dimension 9 level. Because of the λ^{10} factor in Eq. (1.21), we are thus able to generate neutrino masses with the same scale, M_θ, as both the Yukawa couplings and the Higgs μ-term. It is easy to check from Eq. (1.21) that the largest neutrino mass is compatible with the "normal hierarchy" neutrino spectrum.

We turn now to an alternative texture form which we previously employed in the context of Anomaly Mediation [10][11]:

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^4 & 1 \\ \lambda^8 & \lambda^4 & 1 \\ \lambda^8 & \lambda^4 & 1 \end{pmatrix}, \quad Y_d, Y_e \sim \lambda^{\alpha_{d,e}} \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}. \tag{1.22}$$

The Kahler textures are now given by:

$$K_{Q,L} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, K_{u^c} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^8 \\ \lambda^4 & 1 & \lambda^4 \\ \lambda^8 & \lambda^4 & 1 \end{pmatrix}, \quad K_{d^c, e^c} \sim \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \tag{1.23}$$

providing the Kahler textures are generated by θ_u for u^c, θ_d for d^c and by θ_e for e^c. Once again we have that canonicalisation does not alter the form of the textures. Cancellation of mixed anomalies leads to the following results:

$$Q_d = \frac{\Delta - 4}{\beta_d} \tag{1.24a}$$

$$Q_e = \frac{2(\Delta - 6)}{3\beta_e} \tag{1.24b}$$

$$u_1 = (60\beta_e - 6h_2\beta_e - 3e_1\beta_e + 4\Delta - 24)/(9\beta_e) \tag{1.24c}$$

$$\begin{aligned} e_1 = 2[&144(2\beta_e^2 - \beta_e^2\beta_d^2 + 2\beta_d^2) + \Delta^2(18\beta_e^2 + 8\beta_d^2 - 3\beta_e^2\beta_d^2 + 6\beta_e\beta_d^2) \\ &+ \Delta(9h_2\beta_e^2\beta_d^2 - 96\beta_d^2 - 144\beta_e^2 - 36\beta_e\beta_d^2 - 18\beta_e^2\beta_d^2)]/(9\beta_e^2\beta_d^2\Delta) \end{aligned} \tag{1.24d}$$

where $\beta_{d,e} = \alpha_{d,e} + 2$. Here we have assumed $\Delta \neq 0$; for $\Delta = 0$ we require instead of Eq. (1.24d) that

$$2(\beta_e^2 + \beta_d^2) = \beta_e^2\beta_d^2. \tag{1.25}$$

It is easy to show that (using the fact that $\alpha_{d,e} \in \{0, 1, 2, 3\cdots\}$) $\alpha_d = \alpha_e = 0$ is the only possible solution to Eq. (1.25). This case was analysed in Ref. [11] in the AMSB context. Unfortunately, however, since for $\alpha_d = \alpha_e = \Delta = 0$, we have that $Q_u = Q_d = 1, Q_e = -2$, it is easy to show that this case is clearly unnatural when we take into account the Kahler textures (or introduce $\overline{\theta}$-flavons), so from the point of view of the present paper is unsatisfactory. It is interesting, therefore, that in the AMSB context we again find ourselves driven to $\Delta \neq 0$, and hence a texture-generated μ-term.

Reverting to $\Delta \neq 0$, it is straightforward to enumerate the unnatural flavon charge assignments in the same way as we did for the Wolfenstein texture. Thus Yukawa unnaturalness will follow given a solution to any of:

$$\alpha + \beta Q_d + \gamma Q_e = 8, \qquad \alpha + \beta + \gamma \leq 7 \tag{1.26a}$$

$$\alpha + \beta Q_d + \gamma Q_e = (4 + \alpha_d)Q_d, \quad \alpha + \beta + \gamma \leq 3 + \alpha_d \tag{1.26b}$$

$$\alpha + \beta Q_d + \gamma Q_e = (4 + \alpha_e)Q_e, \quad \alpha + \beta + \gamma \leq 3 + \alpha_e \tag{1.26c}$$

(with once again an obvious generalisation if we assume there are $\overline{\theta}$ flavon partners). It is easy to show that, for example, for $\alpha_d = \alpha_e = 0$, the following values of Δ are unnatural due to Eq. (1.26$a - c$):

$$30, 27, 24, 21, 20, 18, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, -12,$$

$$\frac{9}{2}, \frac{11}{2}, \frac{13}{2}, \frac{15}{2}, \frac{17}{2}, \frac{19}{2}, \frac{21}{2}, \frac{27}{2}, \frac{33}{2}, \frac{14}{3}, \frac{16}{3}, \frac{19}{3}, \frac{20}{3}, \frac{22}{3}, \frac{26}{3}, \frac{28}{3}, \frac{32}{3}, \frac{15}{4}, \frac{27}{4}, \frac{33}{4}, \frac{39}{4}, \frac{45}{4},$$

$$\frac{12}{5}, \frac{18}{5}, \frac{21}{5}, \frac{24}{5}, \frac{32}{5}, \frac{33}{5}, \frac{34}{5}, \frac{36}{5}, \frac{38}{5}, \frac{39}{5}, \frac{42}{5}, \frac{44}{5}, \frac{48}{5}, \frac{51}{5}, \frac{54}{5}, \frac{66}{5}, \frac{72}{5},$$

$$\frac{24}{7}, \frac{30}{7}, \frac{44}{7}, \frac{48}{7}, \frac{51}{7}, \frac{54}{7}, \frac{57}{7}, \frac{60}{7}, \frac{66}{7}, \frac{72}{7}, \frac{78}{7},$$

$$\frac{57}{8}, \frac{78}{11}, \frac{84}{11}, \frac{90}{11}, \frac{96}{11}, \frac{102}{11}, \frac{108}{11}, \frac{96}{13}, \frac{102}{13}, \frac{108}{13}, \frac{114}{13}, \frac{120}{13}, \frac{114}{17}, \frac{120}{17}, \frac{132}{17}, \frac{132}{19}.$$

It is remarkable that although (unlike in the Wolfenstein case) Δ is a free parameter, we can still limit the mass scale associated with the Higgs μ-term. This time we have $a_\mu = \alpha_\mu + \beta_\mu + \gamma_\mu$ with

$$\Delta = \alpha_\mu Q_u + \beta_\mu Q_d + \gamma_\mu Q_e. \tag{1.27}$$

Substituting for $Q_{u,d,e}$ we obtain

$$\Delta\left[3\beta_d\beta_e - 3\beta_\mu\beta_e - 2\gamma_\mu\beta_d\right] = 3\left[\beta_d\beta_e\alpha_\mu - 4\beta_e\beta_\mu - 4\gamma_\mu\beta_d\right] \tag{1.28}$$

Now manifestly if we choose β_μ, γ_μ so that

$$3\beta_d\beta_e - 3\beta_\mu\beta_e - 2\gamma_\mu\beta_d = 0 \tag{1.29}$$

then we will obtain (independent of Δ) the result

$$a_\mu = \frac{\beta_d + 4}{\beta_d}\beta_\mu + \frac{\beta_e + 4}{\beta_e}\gamma_\mu, \tag{1.30}$$

or using Eq. (1.29)

$$a_\mu = \beta_d + 4 + \frac{3\beta_e - 2\beta_d + 4}{3\beta_e}\gamma_\mu \tag{1.31}$$

whence, if $3\alpha_e \geq 2\alpha_d - 6$ (which is true given our assumption $\alpha_e \geq \alpha_d$), the dominant contribution to the μ-term is obtained by taking $\gamma_\mu = 0$. It then follows that, independent of the choice of Δ or the other unconstrained charge, the μ-term once again cannot be suppressed by a power greater than λ^{α_d+6}.

It would be logical now to reconsider the above discussion for the case when the $\overline{\theta}$-flavons are present, but we will omit this because this texture scenario has a serious problem as follows. Examining B, L violating operators, one easily finds, (for arbitrary Δ and $\alpha_{d,e}$) that there are a number of dangerous dimension 5 operators: most catastrophically $u_1^c u_3^c d_2^c e_2^c$ has U_1' charge zero and is hence suppressed only by a single power of M_θ. This happens both for $\Delta \neq 0$ and $\Delta = 0$. Although in this framework the right-handed flavour rotation is suppressed[11] it would require considerable fine-tuning to suppress it sufficiently to prevent an unacceptable proton decay rate from this operator.

In conclusion: the generalisation to several flavon fields relaxes some of the constraints on the anomaly free FN scenario, but it remains predictive if we assume a common mass scale origin for the Yukawa textures and the μ-term. The solution we have described is based on Eq. (1.9), and predicts that $M_\theta \sim 10^8\mathrm{GeV}$. Other lepton textures (for example $Y_e \sim Y_d$) are also possible; however the choice made in Eq. (1.9) enables us to also accommodate neutrino masses, albeit by means of a somewhat bizarre choice for the neutrino flavon charge. These textures are not satisfactory for the AMSB scenario described in Ref. [11] because of FCNC effects associated with the Fayet-Iliopoulos D-terms; the alternative textures which avoid this problem (Eq. (1.22)) turn out to be unsatisfactory from the point of view of naturalness that we have taken here.

Acknowledgements

DRTJ was supported by a PPARC Senior Fellowship, and was visiting the Aspen Center for Physics while part of this work was done. We thank Graham Ross and Marc Thormeier for conversations.

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