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Three loop soft running, benchmark points and semi-perturbative unification

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We consider three-loop β-function corrections to the sparticle spectrum in the MSSM, with particular emphasis on Snowmass Benchmark points. The three loop running has little effect on the weakly interacting particle spectrum, but for the squark masses the effect can be comparable to, or greater than, that of two loop running. We extend the analysis to the semi-perturbative unification scenario, where the impact of the three loop corrections becomes even more dramatic.

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1. Introduction

Softly broken supersymmetry remains a well motivated and popular playground for Beyond the Standard Model practitioners. Calculations of sparticle spectra resulting from given assumptions about the underlying theory have become increasingly refined, with several public programs available that incorporate two-loop Renormalisation Group Equations (RGEs) and one-loop radiative corrections. For a recent comparison of the output of some of these programs see the paper by Allanach, Kraml and Porod (AKP) [1]. Two loop corrections are also available to, for example, the effective potential [2]. One area where there has been considerable progress in the last few years is in the calculation of the RGE β-functions. For an arbitrary supersymmetric theory the chiral supermultiplet anomalous dimension γ is known to three loops[3] and the gauge β-function(s) β_g to four loops [4]. For the MSSM, both β_{g_i} and the various γs were given through three loops in Ref. [5].

The β-functions in a general theory and the MSSM for the "standard " soft breaking terms were given to two loops in Ref. [6] and for the "non-standard" terms also to two loops in Ref. [7]. Recently, however, it has been realised[8] –[10] that in the case of the "standard" terms it is possible to express the associated β-functions exactly in terms of simple differential operators acting on β_{g_i} and γ. It is therefore a straightforward matter to derive the three-loop "standard" soft β-functions for the MSSM and variations thereof. In this paper we present three-loop running results for the MSSM with the addition of n_5 and n_{10} sets of SU_5 $5(\overline{5})$ and $10(\overline{10})$ representations respectively. A motive for grouping the additional matter in this way is that complete SU_5 representations do not (at one loop) change the prediction of $\sin^2\theta_W$ (or alternatively of $g_3^2(M_Z)$) that follows from imposing $g_{1,2,3}$ gauge unification. Also unchanged at one loop is the gauge unification scale, M_X; but at higher loops this scale increases and can approach the string scale. (For a recent account of unification at the string scale via the addition of *incomplete* SU_5 multiplets, see Ref. [11].)

At three loops each soft β-function has many terms and some of the coefficients are quite large; given this it is worthwhile checking whether perturbation theory remains good in the MSSM. It is generally believed that the perturbation series for QFT β-functions are asymptotic in nature. The exact β_g in the NSVZ scheme [12] for a pure (no matter) $N=1$ theory is clearly an exception, but in the presence of matter the perturbation series for γ (and hence also the one for β_g) is probably asymptotic (for a discussion see Ref. [13]). It is interesting that even for $n_5 = n_{10} = 0$ we find that, for the squark masses, three loop running corrections are typically larger than the two loop ones. We show explicitly how the three loop corrections affect the spectrum for some of the Snowmass benchmark points (SPS) [14].

Another motive for the calculation and in particular for extending it to $n_5, n_{10} \neq 0$ is that it enables us to explore the phenomenon of "semi-perturbative unification" as described by Kolda and March-Russell (KMR)[15]. In this scenario, the gauge couplings increase at high energies but do not quite reach a Landau pole at gauge unification (contrasting with the non-perturbative unification of Maiani et al[16], where the Landau pole occurs at M_X). It is then possible to argue that there is a regime where perturbation theory remains reliable, but the resulting physics differs markedly from that obtained in the MSSM case.[2]

Our calculations improve on those of KMR by including one loop threshold corrections and the complete three loop running corrections; we check that (for $n_5 = n_{10} = 0$) our results are consistent with those presented for the SPS points in Ref. [1] and Ref. [18]. While we provide more precise results, we support the conclusions of KMR by demonstrating that $n_5, n_{10} \neq 0$ can lead to changes which are in some cases non-negligible, but are consistent with perturbation theory (modulo issues associated with the squark masses which we will discuss later), and can be readily distinguished from the MSSM.

2. The Soft Beta functions

The procedure for calculating the soft β functions from β_{g_i} and γ is described in Ref. [8]. The only subtlety relates to the X-function which arises in the soft scalar mass β-function; expressions for the leading and sub-leading contributions to this appear in Ref. [9]. Armed with these results it is straightforward to calculate the three-loop MSSM soft β-functions from the three-loop expressions for the β_{g_i}, γ given in Ref. [5]. We have generalised this whole calculation to $n_5, n_{10} \neq 0$. The resulting expressions are very unwieldy; as an example we give the one, two and three loop results for $\beta_{m^2_{Q_t}}$, in the approximation that we retain only g_3 and the top quark Yukawa coupling λ_t (in what follows we denote the third generation squarks as Q_t, t^c, b^c, the first or second generation squarks as Q_u, u^c, d^c, and we suppress $16\pi^2$ loop factors.):

[2] For other related work on unification at strong coupling see Ref. [17]

$$\beta^{(1)}_{m^2_{Q_t}} = 2\lambda_t^2(\Sigma_t + A_t^2) - 8(\tfrac{1}{60}g_1^2M_1^2 + \tfrac{3}{4}g_2^2M_2^2 + \tfrac{4}{3}g_3^2M_3^2) \tag{2.1a}$$

$$\begin{aligned}\beta^{(2)}_{m^2_{Q_t}} &= -20\lambda_t^4(\Sigma_t + 2A_t^2) + 16g_3^4M_3^2(n_5 + 3n_{10} - \tfrac{8}{3})\\ &+ \tfrac{16}{3}g_3^4(2m^2_{Q_t} + m^2_{t^c} + m^2_{b^c} + (n_{10}+2)(m^2_{u^c} + 2m^2_{Q_u}) + (n_5+2)m^2_{d^c})\end{aligned} \tag{2.1b}$$

$$\begin{aligned}\beta^{(3)}_{m^2_{Q_t}} &= [(1280k + \tfrac{20512}{9} + 16n_5^2 + (\tfrac{6224}{9} + \tfrac{320}{3}k)(n_5 + 3n_{10})\\ &+ 96n_{10}n_5 + 144n_{10}^2)M_3^2 + (\tfrac{320}{9} - \tfrac{16}{3}(n_5 + 3n_{10}))(m^2_{t^c} + m^2_{b^c} + 2m^2_{Q_t})\\ &+ (2m^2_{Q_u} + m^2_{u^c})(\tfrac{640}{9} - \tfrac{32}{3}n_5 + \tfrac{32}{9}n_{10} - \tfrac{16}{3}n_5n_{10} - 16n_{10}^2)\\ &+ m^2_{d^c}(\tfrac{640}{9} + \tfrac{224}{9}n_5 - 32n_{10} - 16n_5n_{10} - \tfrac{16}{3}n_5^2)]g_3^6\\ &- [(288 + \tfrac{544}{3}k + 48(n_5 + 3n_{10}))M_3^2 - (192 + \tfrac{1088}{9}k + 32(n_5 + 3n_{10}))A_tM_3\\ &+ (\tfrac{272}{9}k + \tfrac{176}{3} + 8(n_5 + 3n_{10}))(\Sigma_t + A_t^2)]\lambda_t^2g_3^4\\ &+ (\tfrac{160}{3} + 32k)\left[M_3^2 - 2A_tM_3 + \Sigma_t + 2A_t^2\right]\lambda_t^4g_3^2 + (6k + 90)(\Sigma_t + 3A_t^2)\lambda_t^6,\end{aligned} \tag{2.1c}$$

where $k = 6\zeta(3)$, and $\Sigma_t = m^2_{Q_t} + m_2^2 + m^2_{t^c}$. For this special case, and also with $n_5 = n_{10} = 0$, the three loop result, Eq. $(2.1c)$, was given in Ref. [19], except that in the corresponding expressions in this reference the squark masses of different generations are not clearly distinguished (as they must be since the third generation evolves differently from the other two). Complete results for the three loop β-functions including all three gauge couplings and $n_g \times n_g$ Yukawa matrices may be obtained by application to the authors.

Note that in our analysis we do not include "tadpole" contributions, corresponding to renormalisation of the Fayet-Iliopoulos (FI) D-term. These contributions are not expressible exactly in terms of β_{g_i}, γ; for a discussion, and three loop results for the MSSM, see Ref. [20]. For universal boundary conditions, the FI term is very small at low energies if it is zero at gauge unification; in this paper we restrict ourselves to universal boundary conditions and ignore these contributions.

3. The Running Analysis

In this section we examine the effect of the three loop corrections on the standard running analysis. We will focus on the standard treatment with universal boundary conditions at gauge unification, often termed CMSSM or MSUGRA. Thus we assume that at M_X we have universal soft scalar masses (m_0), gaugino masses ($m_{\frac{1}{2}}$) and A-parameters (A), and work in the third-generation-only Yukawa coupling approximation. This is for ease of comparison with existing results rather than because we find the scenario particularly compelling. In the MSSM the corrections to the dimensionless coupling running analysis due to two and three loop corrections are comparatively small[5]. As emphasised by KMR, however, this becomes less true for $n_5, n_{10} \neq 0$. In particular, for $n_5 + 3n_{10} = 6$, $\beta_{g_3} = 0$ at one loop, so that we need to consider at least two loop corrections, and also three loops to verify that we remain within the perturbative domain.

We will present results as a function of n_{10} (for $n_5 = 0$); the dependence of physical quantities on n_5 (for $n_{10} = 0$) is qualitatively similar though not identical. As remarked by KMR, the mass scale of these additional multiplets being unknown it makes sense to parametrise their effects by taking n_5, n_{10} to be continuous variables.

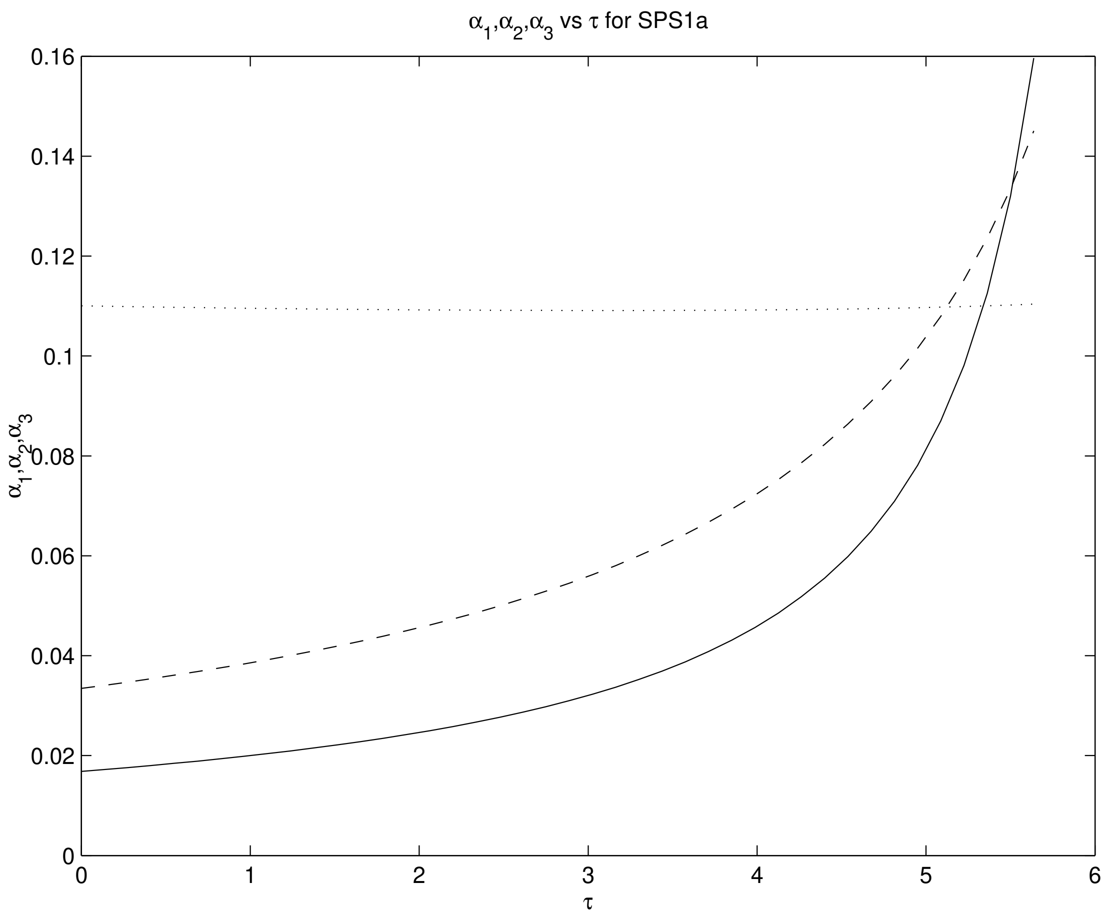

Fig.1: Gauge coupling unification for $n_{10} = 1.7$. Solid, dashed, and dotted lines correspond to $\alpha_1, \alpha_2, \alpha_3$ respectively.

In Fig. 1 we show the evolution of the gauge couplings $\alpha_i = g_i^2/(4\pi)$ for $n_{10} = 1.7$, using three loop β-functions for all couplings. The couplings are plotted against $\tau = \frac{1}{2\pi}\ln(Q/M_Z)$; evidently we are still in the perturbative regime. The input parameters at

M_Z correspond to a typical supersymmetric mass spectrum; specifically, the Benchmark point SPS1a, the details of which will be given later. We adjust input parameters according to the supersymmetric spectrum in order to account for threshold corrections in the manner of Ref. [21] [3], and run up from M_Z using the full supersymmetric β-functions; thus the input values for the gauge couplings depend on the sparticle spectrum, and are determined iteratively. We have set $n_{10} = 1.7$ because around this value the electroweak vacuum fails for the SPS1a input parameters; at this value it is interesting that (see Fig. 1) the (small) one-loop contribution to β_{α_3} is almost precisely cancelled by the two and three-loop ones. As already mentioned, if we have gauge coupling unification then this (and the scale at which it occurs) is unaffected by taking $n_5, n_{10} \neq 0$ in the one loop running approximation. This ceases to be true for two and three loop running, and we must assume the existence of quite large GUT-scale threshold corrections to ensure unification. The unification scale corresponding to Fig. 1 (defined as where α_1 and α_2 meet) is $M_U \approx 1 \times 10^{17}$GeV, significantly higher than in the MSSM.

Turning to the soft parameters, note that there are some large coefficients in the expression for $\beta^{(3)}_{m^2_{Q_t}}$ in Eq. (2.1c); the coefficient of the $M_3^2 g_3^6$ term, for example is $O(10^4)$, even in the MSSM when $n_5 = n_{10} = 0$. For weakly-interacting particles and the gluinos, the three loop effects are quite small for zero or small n_5, n_{10}; but for the squark masses the three loop β-function coefficients are (at M_Z) typically (while smaller than the corresponding 1-loop coefficients) larger than the corresponding two loop coefficients (even at $n_{5,10} = 0$) if $m_{\frac{1}{2}} > m_0$, as will be so, in fact, in the cases we shall present. One might well be tempted to interpret this as evidence for the asymptotic nature of the β-function series, as we mentioned in the Introduction. We will see the effects of this in the next section.

As mentioned above, we adjust the input dimensionless parameters to accommodate threshold corrections[21]. We also incorporate the one-loop radiative corrections from this reference. For the input top mass at the weak scale we use[22]:

$$\begin{aligned} m_t(Q) = m_{t_{\rm pole}}\Big[1 - \frac{\alpha_3}{3\pi}(5-3L) - \alpha_3^2\Big(0.538 - \frac{43L}{24\pi^2} + \frac{3L^2}{8\pi^2}\Big) \\ + O(\alpha_3^3) + \text{electroweak, sparticle contributions}\Big] \end{aligned} \tag{3.1}$$

[3] In the first line of Eq. 37 of Ref. [21], the first term in the square bracket should read $-(m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})B_0(m_{\tilde{t}_2}, m_{\tilde{t}_1}, 0)$: i.e. it should have a minus sign. The corresponding exact result in Eq. D49 is correct, however.

where $L = \ln[m_t(Q)^2/Q^2]$. This formula is identical to one from the up-to-date version of Allanach et al[1].

For definiteness we analyse in detail particular SPS benchmark points[14], and verify that our results (for $n_5 = n_{10} = 0$) are in accord with those obtained with existing computational tools[1], [18].

3.1. Benchmark point SPS 1a

This point is a "typical" point in MSUGRA parameter space, with $m_0 = 100$GeV, $m_{\frac{1}{2}} = 250$GeV, $A_0 = -100$GeV, $\tan\beta = 10$ and $\mu > 0$. In Table 1 we compare our results for a selection of sparticle masses (at $n_5 = n_{10} = 0$) with the spread of results quoted in AKP (note our convention that the predominantly R-handed top squark is $\tilde{t}_2$).

$mass$	1*loop*	2*loops*	3*loops*	AKP
$\tilde{g}$	630	615	612	594 – 626
$\tilde{t}_2$	404	403	395	379 – 410
$\tilde{u}_L$	571	563	555	536 – 570
$\tilde{u}_R$	551	547	538	520 – 569
LSP	105	97	97	96.4 – 97.6

Table 1: Sparticle masses (in GeV) for the SPS1a point

We would expect our two loop results to correspond most closely to AKP and we see that they are indeed consistent. The effect of inclusion of three loop running is never greater than 2%; note, however, that the shift caused by three loop running effects is comparable for $\tilde{u}_L$ and larger for $\tilde{t}_2, \tilde{u}_R$ than that produced by two loop running effects. In Fig. 2 we plot the ratio of the light stop mass to the gluino mass as a function of n_{10} (for $n_5 = 0$), and we see that both two and three loop effects increase dramatically as n_{10} increases. (NB the increase in the squark/gluino mass ratio with $n_{5,10}$ observed by KMR applies to the squarks of the first two generations).

In Fig. 3 we plot the ratio of the LSP mass to the gluino mass as a function of n_{10} (for $n_5 = 0$). Here we see that the impact of the three loop running corrections is less marked but still appreciable at large n_{10}.

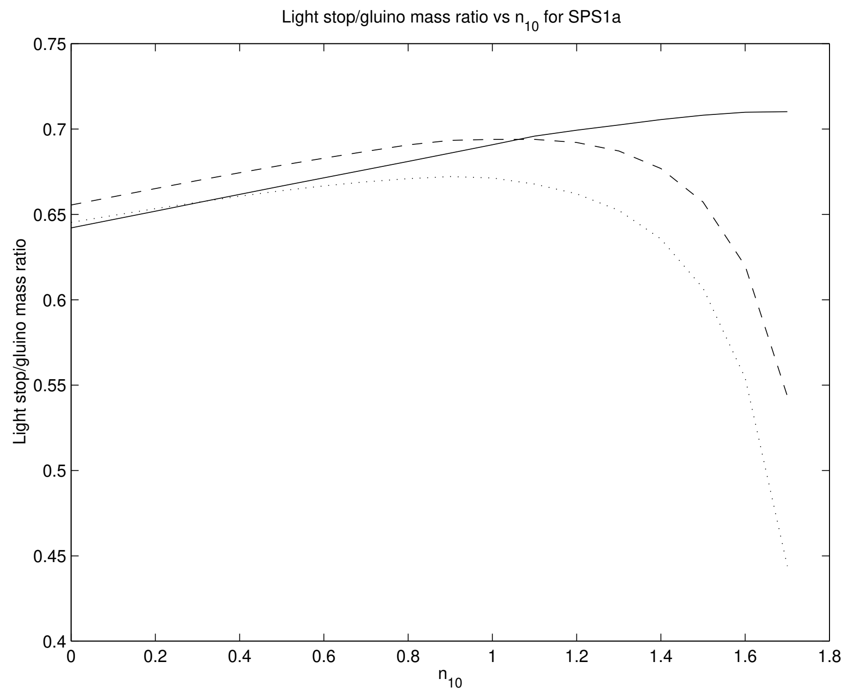

Fig.2: Plot of the light stop/gluino mass ratio against n_{10} for SPS1a. Solid, dashed and dotted lines correspond to one, two and three loop running respectively.

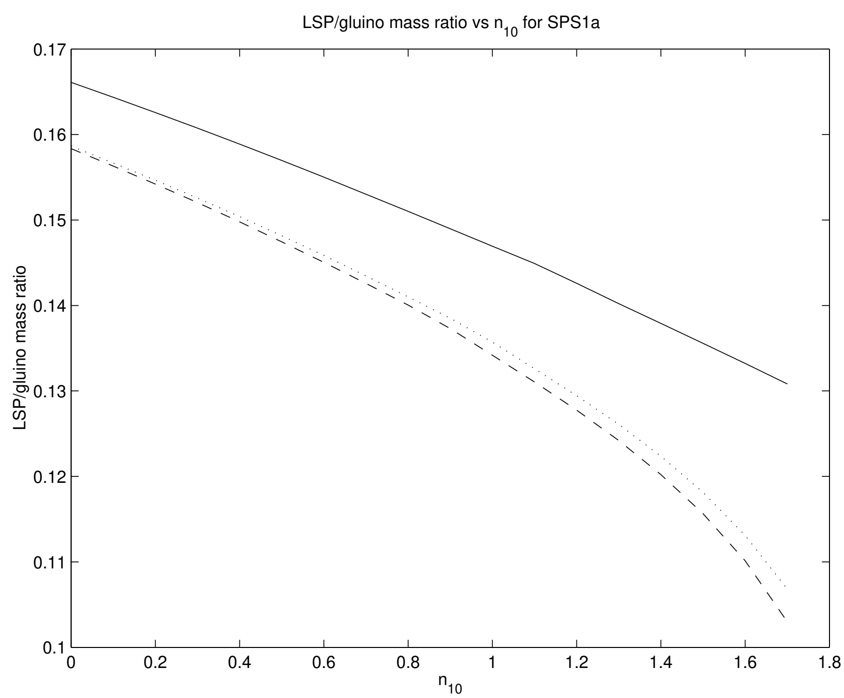

Fig.3: Plot of the LSP/gluino mass ratio against n_{10} for SPS1a. Solid, dashed and dotted lines correspond to one, two and three loop running respectively.

3.2. Benchmark point SPS 5

This point differs from the previous one in having a large value of the A-parameter: with $m_0 = 150\text{GeV}$, $m_{\frac{1}{2}} = 300\text{GeV}$, $A_0 = -1\text{TeV}$, $\tan\beta = 5$ and $\mu > 0$. The contributions of μ, A_0 to the off-diagonal term in the stop mass matrix have the same sign, and the magnitude of A_0 is large, resulting in a light stop. For this point we obtain in the MSSM (with $n_5 = n_{10} = 0$) the results shown in Table 2.

$mass$	$1loop$	$2loops$	$3loops$	AKP
$\tilde{g}$	745	731	729	$705 - 730$
$\tilde{t}_2$	236	250	231	$232 - 248$
$\tilde{u}_L$	682	674	666	$642 - 681$
$\tilde{u}_R$	657	655	645	$622 - 681$
LSP	128	120	120	$118.7 - 121.1$

Table 2: Sparticle masses (in GeV) for the SPS5 point

Once again the inclusion of three loop effects causes mass shifts of less than 2%, except for the light stop where there is an effect of about 8%. The light stop mass comes from a 2×2 matrix with large off-diagonal entries; the large three loop shift is caused essentially by the changes in all the entries due to the comparatively large contributions to the three loop soft β-functions, as we described earlier. The mass of the light stop is also very sensitive indeed to the input $m_t(M_Z)$, which in turn depends on the sparticle spectrum and the input top pole mass (which we have taken to be 174.3GeV). However (for a given $m_{t_{\text{pole}}}$), $m_t(M_Z)$ changes very little when we include the 3 loop corrections. Note that our two loop result is slightly above the range obtained by AKP. It is also very sensitive to n_5, n_{10}; in Figure 4 we again plot the light stop/gluino mass ratio against n_{10}: this time the electroweak vacuum fails for $n_{10} \approx 0.3$. Remarkably, the three loop and two loop corrections cancel almost exactly near $n_{10} = 0$, while at $n_{10} \sim 0.3$ the two loop corrections are rather small.

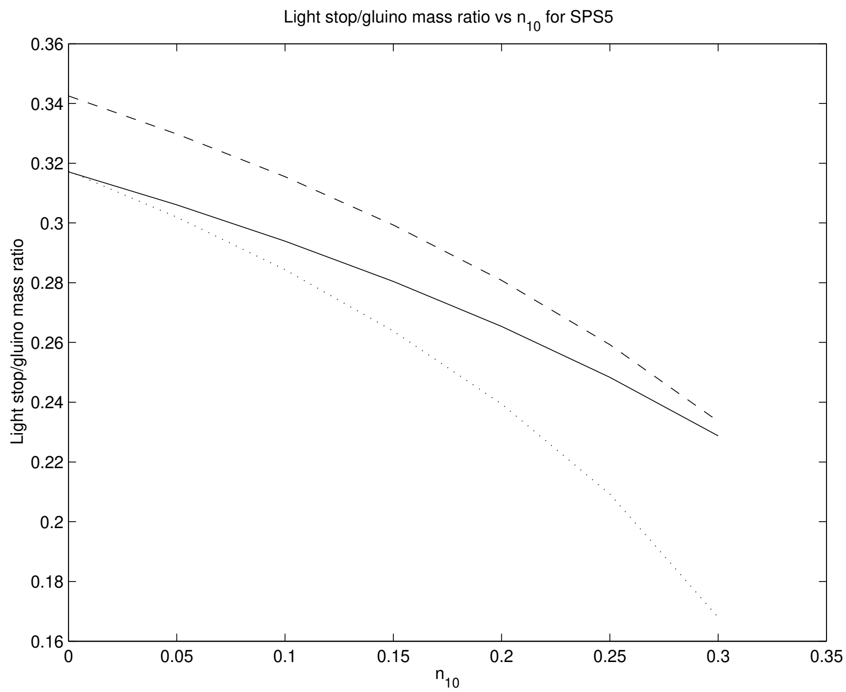

Fig.4: Plot of the light stop/gluino mass ratio against n_{10} for SPS5. Solid, dashed and dotted lines correspond to one, two and three loop running respectively.

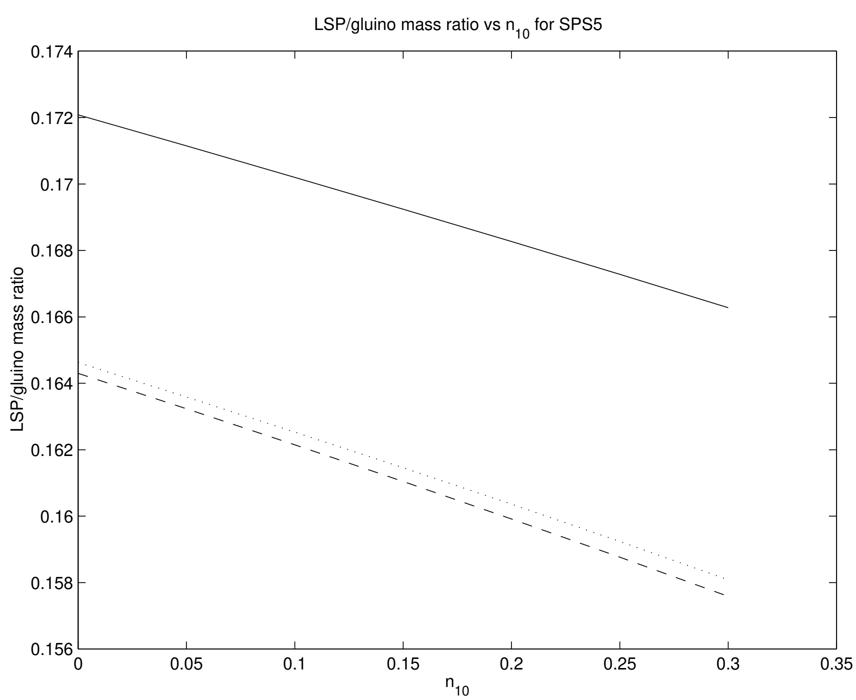

Fig.5: Plot of the LSP/gluino mass ratio against n_{10} for SPS5. Solid, dashed and dotted lines correspond to one, two and three loop running respectively.

In Fig. 5 we plot the ratio of the LSP mass to the gluino mass for SPS5 as a function of n_{10} (for $n_5 = 0$). Here we see that the impact of the three loop running corrections is less marked but still appreciable as n_{10} increases.

4. Conclusions

We have extended typical detailed running coupling analyses for a selection of MSSM SPS benchmark points to incorporate three loop β-function corrections for the running masses and couplings. Generally speaking the effect of the three loop running corrections is at most 2% and of the same size or smaller than that of the two loop corrections, except for squark masses where it can be larger; simply because the three-loop β-function coefficients are larger than the two-loop ones. For the light stop mass for the SPS5 point, we see an 8% effect; this happens because this mass results from the diagonalisation of a matrix with large off diagonal entries which all change as described above. We have also performed the same analysis for the MSSM extended to incorporate additional matter in the form of SU_5 5 and 10 representations. As the amount of such matter is increased the effect of two and three loop corrections becomes more dramatic, as the one-loop β-function for α_3 decreases.

Given some detail of the sparticle spectrum, it will be comparatively easy to distinguish, for example, the CMSSM and AMSB scenarios; however in the context of the former, disentangling the possible impact of additional matter and the effect of radiative corrections will be more difficult.

One may expect in general that two loop threshold/pole mass corrections will be competitive with the three loop running corrections that we have described, and so for accurate predictions one should include both. At this level one is also sensitive to the experimental uncertainty in m_t (for the light stop sometimes very sensitive, as we have described) and the strong coupling $\alpha_3(M_Z)$. It is feasible that by the time sparticles are discovered complete two loop threshold corrections will be available, and that both these uncertainties will also be reduced, so that significantly more accurate sparticle spectrum predictions will be possible. It appears, however, from the apparently asymptotic nature of the squark mass β-functions that squark mass predictions with an accuracy greater than around 2% will not be possible using perturbation theory.

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