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Yukawa Textures and Anomaly Mediated Supersymmetry Breaking

I. Jack and D.R.T. Jones

Dept. of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, U.K.

We present a detailed analysis of how a mixed-anomaly-free U_1 symmetry can be used to both resolve the slepton mass problem associated with Anomaly Mediated Supersymmetry Breaking and generate the fermion mass hierarchy via the Froggatt-Nielsen mechanism. Flavour changing neutral currents problems are evaded by a specific form of the Yukawa textures.

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1. Introduction

An explanation for the existence of the three generations of quarks and leptons with their widely dispersed masses remains one of the most significant problems in fundamental physics. One plausible way they may emerge from a more fundamental theory is via Yukawa textures associated with a U_1 symmetry, either global or gauged at some higher scale [1]. This idea has been much studied both with anomaly-free and anomalous U_1's. In this paper we investigate Yukawa textures in the context of a specific framework for the origin of soft supersymmetry breaking within the MSSM, known as Anomaly Mediated Supersymmetry Breaking (AMSB) [2]–[4]. Direct application of the AMSB solution to the MSSM leads, unfortunately, to negative $(\text{mass})^2$ sleptons. A number of possible solutions to this problem have been discussed; here we concentrate on proposals [5] [6] which require the existence of an additional U_1 symmetry; in the first case (Case (FI)) a "normal" U_1' (commuting with supersymmetry), and in the second case (Case ($\mathcal{R}$)) a U_1' associated with an $\mathcal{R}$-symmetry [7]. Both cases permit additional contributions to the scalar masses which preserve the exact RG invariance of the AMSB solution, providing the U_1' has no linear mixed anomalies with any gauge group of the theory. In the MSSM context this amounts to the requirement that the $U_1'(SU_3)^2$, $U_1'(SU_2)^2$ and $U_1'(U_1)^2$ anomalies cancel. The U_1' need not in fact be gauged, though of course the vanishing of these anomalies suggests that it may be. We will therefore also impose cancellation of $U_1(U_1')^2$ anomalies, so that a MSSM singlet sector would suffice to render U_1' anomaly free. It is very natural to use the same U_1' symmetry to both solve the slepton mass problem *and* generate the Yukawa textures.

With regard to Case (FI), the MSSM in fact admits two generation-independent, mixed-anomaly-free U_1 groups, the existing U_1^Y and another (which could be chosen to be U_1^{B-L} [8] or a linear combination of it and U_1^Y[5]). The existence of these two independent U_1's indeed enables us to resolve the slepton problem and predict a distinctive sparticle spectrum with characteristic mass sum rules, as described in Ref. [5]. This scenario has the advantage of incorporating natural suppression of flavour-changing effects in both hadronic and leptonic sectors. It provides no insight into the flavour problem, however, and does not accommodate neutrino masses in an elegant way.

The MSSM does not admit a generation-independent $\mathcal{R}$ symmetry (although one can be arranged using additional matter fields, see Ref. [9]), and so there is no analogous treatment to Case (FI). In Ref. [6] we argued that in conjunction with the Froggatt-Nielsen (FN) mechanism[1], use of a generation-dependent $\mathcal{R}$-symmetry could combine the desirable features of the AMSB scenario with an explanation for the flavour hierarchy. The form of Yukawa textures adopted in Ref. [6] was motivated by the limits imposed by flavour changing neutral currents (FCNC); it required, however, some fine tuning to reproduce the flavour hierarchy and the CKM matrix. In Ref. [10], motivated by the now overwhelming evidence for massive neutrinos [1], we considered a generation-dependent U_1 in the Case (FI) context, and found a different texture which both naturally reproduces the flavour hierarchy and leads to an acceptable CKM matrix. Our purpose here is to describe this scenario in more detail, explore alternatives, and extend the discussion to encompass Case ($\mathcal{R}$). We show how the existence of a mass sum involving the Higgs bosons constrains our U_1' charge assignments, and exhibit sparticle mass spectra for the various possible scenarios. In section 2 we review AMSB, in section 3 we describe briefly the MSSM generalised to incorporate massive neutrinos via the see-saw mechanism (which we term the MSSM$^\nu$); in sections 4-7 we analyse the constraints imposed by anomaly cancellation and in section 8 we pursue the experimental consequences of our preferred choice of textures.

2. Anomaly mediation

Consider a supersymmetric theory with superpotential

$$W(\phi) = \tfrac{1}{2}\mu^{ij}\phi_i\phi_j + \tfrac{1}{6}Y^{ijk}\phi_i\phi_j\phi_k, \tag{2.1}$$

and soft supersymmetry-breaking terms as follows:

$$V_{\text{soft}} = \left(\tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \tfrac{1}{2}M\lambda\lambda + \text{c.c.}\right) + (m^2)^i{}_j\phi_i\phi^j. \tag{2.2}$$

The anomaly mediation approach to the MSSM begins with the following relations:

$$M = M_0\frac{\beta_g}{g}, \tag{2.3a}$$

$$h^{ijk} = -M_0\beta_Y^{ijk}, \tag{2.3b}$$

$$(m^2)^i{}_j = \tfrac{1}{2}|M_0|^2\mu\frac{d\gamma^i{}_j}{d\mu}, \tag{2.3c}$$

[1] Direct extension of Case (FI) to include massive neutrinos is possible using Dirac mass terms, but this is very unattractive as it provides no explanation for the extreme lightness of the neutrinos. The seesaw mechanism provides just such an explanation, and most elegantly; but involves Majorana masses for right-handed neutrinos, which evidently break U_1^{B-L}. For some alternative ideas about massive neutrinos in this context see Ref. [8].

which are RG invariant to all orders of perturbation theory. (In appendix A we provide a summary of the most general set of such relations, for a theory including gauge singlets.)

Eq. (2.3c) leads to tachyonic sleptons; most studies have dealt with the so-called mAMSB, produced by replacing it (at the unification scale) with

$$m^2 = m^2_{\rm AMSB} + m_0^2, \tag{2.4}$$

that is

$$(m^2)^i{}_j = \tfrac{1}{2}|M_0|^2\mu\frac{d\gamma^i{}_j}{d\mu} + m_0^2\delta^i{}_j, \tag{2.5}$$

where m_0^2 is constant. This procedure, however, destroys the RG invariance (and hence the UV insensitivity) of the relation. Much more elegant, in our opinion, are the following two possibilities:

2.1. Case (FI): The Fayet-Iliopoulos solution

Here we replace Eq. (2.3c) with:

$$(m^2)^i{}_j = \tfrac{1}{2}|M_0|^2\mu\frac{d\gamma^i{}_j}{d\mu} + \sum_a \zeta_a(\mathcal{Y}_a)^i{}_j, \tag{2.6}$$

where ζ_a, $(\mathcal{Y}_a)^i{}_j$ are constants, satisfying the following relations:

$$(\mathcal{Y}_a)^i{}_l Y^{ljk} + (\mathcal{Y}_a)^j{}_l Y^{ilk} + (\mathcal{Y}_a)^k{}_l Y^{ijl} = 0 \tag{2.7a}$$

$$\mathrm{tr}[\mathcal{Y}_a C(R)] = 0. \tag{2.7b}$$

Here $C(R)$ is the quadratic gauge Casimir for the chiral multiplet. These constraints follow from demanding that m^2 be RG invariant, but clearly correspond to requiring that each $\mathcal{Y}$ correspond to an abelian symmetry of the superpotential (Eq. (2.7a)), such that all anomalies linear in $\mathcal{Y}$ and quadratic in *gauged* symmetries vanish (Eq. (2.7b)). It is interesting that the latter requirement derives from the X-function in the β-function for m^2; this function, whose existence was first remarked in Ref. [11], was related recently to anomalies in Ref. [12]. The ζ_a-terms in Eq. (2.6) correspond precisely to the contributions to the scalar masses from Fayet-Iliopoulos (FI) D-terms, after elimination of the auxiliary D-fields using their equations of motion. Note that the mixed anomaly cancellation requirement rules out anomaly cancellation via the Green-Schwarz mechanism [13]. The simplest realisation of the FI scenario is to have two ζ's, ζ, ζ', the first corresponding to the standard model U_1 and the second to a U_1' the possible form of which we will discuss in detail later.

2.2. Case ($\mathcal{R}$): The $\mathcal{R}$-symmetry solution

Here we have

$$(m^2)^i{}_j = \tfrac{1}{2}|M_0|^2\mu\frac{d\gamma^i{}_j}{d\mu} + m_0^2(\gamma^i{}_j + \overline{q}_i\delta^i{}_j) \tag{2.8}$$

where m_0^2 and $\overline{q}_i$ are constants, as long as a set $\overline{q}_i$ exists that satisfy the following constraints:

$$(\overline{q}_i + \overline{q}_j + \overline{q}_k)Y^{ijk} = 0 \tag{2.9a}$$

$$2\mathrm{Tr}\,[\overline{q}C(R)] + Q = 0, \tag{2.9b}$$

where Q is the one loop β_g coefficient. It is easy to show[7] that Eq. (2.9) corresponds precisely to requiring that the theory have a non-anomalous $\mathcal{R}$-symmetry (which we denote $\mathcal{R}$, to avoid confusion with our notation R for group representations), where if we set

$$\overline{q}_i = 1 - \tfrac{3}{2}r_i, \tag{2.10}$$

then Eq. (2.9a) corresponds to $(r_i + r_j + r_k)Y^{ijk} = 2Y^{ijk}$, which is the conventional $\mathcal{R}$-charge normalisation. In the rest of the paper we will work with the fermionic $\mathcal{R}$ charges, $q_i = r_i - 1$. The relation Eq. (2.8) generalises easily to the case of several $\mathcal{R}$ symmetries but the simplest possibility, which suffices to deal with the slepton mass problem, is to have one only, which we will call $U_1^{\mathcal{R}}$.

3. The superpotential and neutrino masses

The MSSM$^\nu$ is defined by the superpotential

$$\begin{aligned} W = {} & H_2QY_uu^c + H_1QY_dd^c + H_1LY_ee^c + H_2LY_\nu\nu^c \\ & + \mu H_1H_2 + \tfrac{1}{2}(\nu^c)^TM_{\nu^c}\nu^c, \end{aligned} \tag{3.1}$$

where Y_u, Y_d, Y_e are 3×3 Yukawa matrices, and Y_ν is $3\times n_\nu$, where n_ν is the number of RH neutrinos. We will neglect CP-violation and assume that all parameters in Eq. (3.1) are real. The light neutrino mass matrix m_ν is generated by the seesaw mechanism,

$$m_\nu = m_DM_{\nu^c}^{-1}(m_D)^T \tag{3.2}$$

where $m_D = v_2Y_\nu$ is the Dirac ν-mass matrix.

The quark and charged lepton matrices are diagonalised as follows:

$$Y_u^{\rm diag} = U_u^T Y_u V_u \tag{3.3}$$

(similarly for Y_d, Y_e), so that the CKM matrix is given by

$$CKM = U_u^T U_d. \tag{3.4}$$

The neutrino mixing matrix U_{MNS} relating the mass eigenstate basis to the basis in which the leptonic charged currents are flavour-diagonal, i.e.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{MNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{3.5}$$

is given by

$$U_{MNS} = U_e^T U_\nu, \tag{3.6}$$

where

$$m_\nu^{\rm diag} = U_\nu^T m_\nu U_\nu. \tag{3.7}$$

Existing oscillation data suggests non-zero neutrino masses with large mixing. Super-Kamiokande results are consistent with $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with $\delta m^2 \sim 10^{-3}{\rm eV}^2$ and maximal mixing; while the large-angle MSW solution to the solar oscillation data is consistent with $\nu_\mu \leftrightarrow \nu_e$ mixing with $\delta m^2 \sim 10^{-5}{\rm eV}^2$ and large (not quite maximal) mixing. Finally the CHOOZ reactor experiment and the Palo Verde experiment suggest that U_{e3} is small.

Thus we seek a U_{MNS} such that

$$U_{MNS} = \begin{pmatrix} c_{12}c_{31} & s_{12}c_{31} & s_{31} \\ -s_{12}c_{23} - c_{12}s_{23}s_{31} & c_{12}c_{23} - s_{12}s_{23}s_{31} & s_{23}c_{31} \\ s_{12}s_{23} - c_{12}c_{23}s_{31} & -c_{12}s_{23} - s_{12}c_{23}s_{31} & c_{23}c_{31} \end{pmatrix} \tag{3.8}$$

with $\sin^2 2\theta_{12} \sim 0.75$ and $\sin^2 2\theta_{23} \sim 1$ and $\sin^2 2\theta_{31} \sim 0$.

Examples from the literature of favoured structures are:

$$U_{MNS} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi/\sqrt{2} & \cos\phi/\sqrt{2} & -1/\sqrt{2} \\ \sin\phi/\sqrt{2} & \cos\phi/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \tag{3.9}$$

or[14]

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\phi & \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}}\sin\phi \\ -\frac{\cos\phi}{\sqrt{6}} - \frac{\sin\phi}{\sqrt{2}} & \sqrt{\frac{1}{3}} & \frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}} \\ -\frac{\cos\phi}{\sqrt{6}} + \frac{\sin\phi}{\sqrt{2}} & \sqrt{\frac{1}{3}} & -\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}} \end{pmatrix} \tag{3.10}$$

We will discuss later to what extent our framework predicts (or at least accommodates) results of this general nature.

4. The Yukawa Textures

In this section we will discuss the form of $Y_{u,d,e}$, postponing Y_ν till later. We seek to reproduce the well-known hierarchies [15]

$$m_\tau : m_\mu : m_e = m_b : m_s : m_d = 1 : \lambda^2 : \lambda^4, \quad \text{and} \quad m_t : m_c : m_u = 1 : \lambda^4 : \lambda^8 \tag{4.1}$$

(where $\lambda \approx 0.22$), an acceptable CKM matrix (without too much fine tuning), and neutrino masses and mixings consistent with current observations.

The fundamental assumption we shall make is that most of the Yukawa interactions are generated via the Froggatt-Nielsen (FN) mechanism[1]: specifically, from higher dimension terms involving MSSM$^\nu$ singlet fields $\theta_{u,d,e}$ with U_1' or $U_1^{\mathcal{R}}$ charges $-Q_u$, $-Q_d$, and $-Q_e$, via terms such as $H_2 Q_i u_j^c (\frac{\theta_u}{M_\theta})^{a_{ij}}$, where M_θ represents the scale of new physics; we will assume that $M_\theta \geq M_{\nu^c}$. We choose to normalise charges so that $Q_u = 1$. For our principal development we will assume that each Yukawa matrix $Y_{u,d,e}$ gains its texture from a *particular* θ-charge and that the vevs of the various θ-charges are approximately the same. Assigments such that this scenario is (in a sense we will define) *natural* will be discussed presently. [2] It follows at once from gauge invariance that the textures take the following form:

$$\begin{aligned} Y_u &\sim \begin{pmatrix} \lambda^{p_u} & \lambda^{x_u} & \lambda^{y_u} \\ \lambda^{a_u} & \lambda^{q_u} & \lambda^{z_u} \\ \lambda^{b_u} & \lambda^{c_u} & 1 \end{pmatrix}, \quad Y_d \sim \lambda^{\alpha_d} \begin{pmatrix} \lambda^{p_d} & \lambda^{x_d} & \lambda^{y_d} \\ \lambda^{a_d} & \lambda^{q_d} & \lambda^{z_d} \\ \lambda^{b_d} & \lambda^{c_d} & 1 \end{pmatrix}, \\ Y_e &\sim \lambda^{\alpha_e} \begin{pmatrix} \lambda^{p_e} & \lambda^{x_e} & \lambda^{y_e} \\ \lambda^{a_e} & \lambda^{q_e} & \lambda^{z_e} \\ \lambda^{b_e} & \lambda^{c_e} & 1 \end{pmatrix} \end{aligned} \tag{4.2}$$

[2] If the U_1' is in fact gauged then this implicitly assumes that there exists a D-flat direction corresponding to the M_θ scale with all the vevs of the θs involved in texture generation approximately the same.

where (in both Case (FI) and Case ($\mathcal{R}$))

$$
\begin{aligned}
a_u &= p_u + q_u - x_u \\
b_u &= p_u - y_u \\
c_u &= x_u - y_u \\
z_u &= q_u + y_u - x_u,
\end{aligned}
\tag{4.3}
$$

with similar relations for $a_{d,e}$ etc. Given that $m_\tau \sim m_b$ and $m_b/m_t \sim \lambda^3$ we might expect $\alpha_d \sim \alpha_e$ and $\tan\beta \sim \lambda^{\alpha_d - 3}$. Hence for $\tan\beta \sim 10$, for example, we would then expect $\alpha_d = 1$ or $\alpha_d = 2$. It might also be, however, that the hierarchy m_b/m_t has a different origin.

Gauge invariance of the Yukawas also provides relationships among the various charges. We will first give these relationships for the U_1' charges in Case (FI). Denoting the U_1' charges of the supermultiplets $Q_i, L_i, u_i^c, d_i^c, e_i^c, H_1, H_2$ as $q_i, L_i, u_i, d_i, e_i, h_1, h_2$, we have:

$$
\begin{aligned}
q_1 &= p_u - u_1 - h_2 \\
q_2 &= a_u - u_1 - h_2 \\
q_3 &= b_u - u_1 - h_2 \\
L_1 &= (p_e + \alpha_e)Q_e - e_1 - h_1 \\
L_2 &= (a_e + \alpha_e)Q_e - e_1 - h_1 \\
L_3 &= (b_e + \alpha_e)Q_e - e_1 - h_1,
\end{aligned}
\tag{4.4}
$$

$$
\begin{aligned}
u_2 &= u_1 + q_u - a_u \\
u_3 &= u_1 - b_u \\
d_1 &= (\alpha_d + p_d)Q_d - p_u + u_1 - h_1 + h_2 \\
d_2 &= (\alpha_d + q_d)Q_d - a_u + u_1 - h_1 + h_2 \\
d_3 &= \alpha_d Q_d - b_u + u_1 - h_1 + h_2 \\
e_2 &= (q_e - a_e)Q_e + e_1 \\
e_3 &= -Q_e b_e + e_1
\end{aligned}
\tag{4.5}
$$

and also the following relations:

$$\begin{aligned} a_d &= (a_u - p_u + Q_d p_d)/Q_d \\ b_d &= (b_u + Q_d p_d - p_u)/Q_d \\ c_d &= (b_u + Q_d q_d - a_u)/Q_d \\ x_d &= (p_u + Q_d q_d - a_u)/Q_d \\ y_d &= (p_u - b_u)/Q_d \\ z_d &= (a_u - b_u)/Q_d. \end{aligned} \tag{4.6}$$

Case ($\mathcal{R}$) differs because the superpotential has non-zero $\mathcal{R}$-charge which as usual we take to be 2. Then, in terms of the $U_1^{\mathcal{R}}$ *fermionic* charges, we have (instead of Eq. (4.4)):

$$\begin{aligned} q_1 &= p_u - u_1 - h_2 - 1 \\ q_2 &= a_u - u_1 - h_2 - 1 \\ q_3 &= b_u - u_1 - h_2 - 1 \\ L_1 &= (p_e + \alpha_e)Q_e - e_1 - h_1 - 1 \\ L_2 &= (a_e + \alpha_e)Q_e - e_1 - h_1 - 1 \\ L_3 &= (b_e + \alpha_e)Q_e - e_1 - h_1 - 1, \end{aligned} \tag{4.7}$$

while Eqs. (4.5), (4.6) are unaffected.

Cancellation of mixed anomalies for $(SU_3)^2U_1'$, $(SU_2)^2U_1'$ and $(U_1)^2U_1'$ leads to the conditions (in Case (FI))

$$A_3 = \sum_{i=1}^{3}(2q_i + u_i + d_i) = 0 \tag{4.8a}$$

$$A_2 = \Delta + \sum_{i=1}^{3}(L_i + 3q_i) = 0 \tag{4.8b}$$

$$A_1 = 3\Delta + \sum_{i=1}^{3}(3L_i + q_i + 8u_i + 2d_i + 6e_i) = 0 \tag{4.8c}$$

where we have set $h_1 = \Delta - h_2$, or in Case ($\mathcal{R}$):

$$A_3 = 6 + \sum_{i=1}^{3}(2q_i + u_i + d_i) = 0 \tag{4.9a}$$

$$A_2 = 4 + \Delta + \sum_{i=1}^{3}(L_i + 3q_i) = 0 \tag{4.9b}$$

$$A_1 = 3\Delta + \sum_{i=1}^{3}(3L_i + q_i + 8u_i + 2d_i + 6e_i) = 0, \tag{4.9c}$$

where the additional contributions in Eq. $(4.9a, b)$ are due to gauginos.

Cancellation of $U_1(U_1')^2$ or $U_1(U_1^{\mathcal{R}})^2$ anomalies leads to the further condition (valid in both cases)

$$A_Q = -h_1^2 + h_2^2 + \sum_{i}^{3}(e_i^2 - L_i^2 + q_i^2 - 2u_i^2 + d_i^2). \tag{4.10}$$

MSSM singlet fields (such as ν_i^c and the θ-fields) do not contribute to Eqs. (4.8)-(4.10). They *do* contribute to U_1'-gravitational and $(U_1')^3$ anomalies, which are proportional to the following expressions (which we include for completeness but the vanishing of which we do not impose). In the FI case:

$$A_C = 2(h_1^3 + h_2^3) + \sum_{i=1}^{3}(e_i^3 + 2L_i^3 + 6q_i^3 + 3u_i^3 + 3d_i^3) + \sum s_j^3, \tag{4.11}$$

and

$$A_G = 2\Delta + \sum_{i=1}^{3}(e_i + 2L_i + 6q_i + 3u_i + 3d_i) + \sum s_j, \tag{4.12}$$

and in the $\mathcal{R}$ case:

$$A_C = 2(h_1^3 + h_2^3) + 16 + \sum_{i=1}^{3}(e_i^3 + 2L_i^3 + 6q_i^3 + 3u_i^3 + 3d_i^3) + \sum s_j^3, \tag{4.13}$$

and

$$A_G = 2\Delta - 8 + \sum_{i=1}^{3}(e_i + 2L_i + 6q_i + 3u_i + 3d_i) + \sum s_j, \tag{4.14}$$

where we have assumed a singlet sector with charges s_i which would include ν_i^c and the θ-fields. Note the gravitino contributions to $A_{C,G}$ in the $\mathcal{R}$ case[16].

5. The Wolfenstein textures

Here we explore whether we can obtain the Wolfenstein texture for the CKM matrix,

$$CKM_W \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \tag{5.1}$$

in the light of the constraints imposed in the previous section. There is a considerable literature on this subject, both with anomaly-free and anomalous U_1' symmetries; for a recent example see Ref. [17].

A matrix of the form e.g. Y_u in Eq. (4.2) has one eigenvalue $O(1)$ and two of orders $O(\lambda^{\min\{p_u,q_u,x_u,a_u\}})$, $O(\lambda^{\max\{p_u,q_u,x_u,a_u\}})$ respectively. Moreover, the Wolfenstein texture for the CKM matrix is obtained if the right-hand columns of Y_u and Y_d are both of the form $(\,\lambda^3 \quad \lambda^2 \quad 1\,)^T$, and if $x_{u,d} \geq 3$. If we require mass hierarchies of the form Eq. (4.1), then the only possible textures satisfying these conditions (together with Eqs. (4.3)) are of the form

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim \lambda^{\alpha_d} \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}. \tag{5.2}$$

All the relevant conditions from Eq. (4.6) are then satisfied by taking $Q_d = 1$.

We can then solve the linear anomaly constraints Eq. (4.8) for Δ, e_1 and Q_e, yielding (in case (FI)):

$$\Delta = \alpha_d + 6 \tag{5.3a}$$

$$Q_e = 2\alpha_d/(3\alpha_e + p_e + q_e) \tag{5.3b}$$

$$u_1 = -2\alpha_d/9 + 16/3 - 2h_2/3 - e_1/3 + Q_e(p_e + a_e + b_e + 3\alpha_e)/9 \tag{5.3c}$$

while in case $(\mathcal{R})$ we find from Eq. (4.9) that

$$\Delta = \alpha_d + 6 \tag{5.4a}$$

$$Q_e = 2(3 + \alpha_d)/(3\alpha_e + p_e + q_e) \tag{5.4b}$$

$$u_1 = 40/9 - 2\alpha_d/9 - 2h_2/3 - e_1/3 + Q_e(p_e + a_e + b_e + 3\alpha_e)/9 \tag{5.4c}$$

In both cases we necessarily have a texture-generated μ-term, related to M_θ by $\mu \sim M_\theta(\lambda)^{\alpha_d+6}$, assuming that the θ responsible for it has the same U_1' charge as $\theta_{u,d}$.

Imposing Eq. (5.3) or Eq. (5.4) renders Eq. (4.10) linear in h_2, so we solve Eq. (4.10) for h_2, obtaining in Case (FI):

$$\begin{aligned} h_2 = (&116 - 2Q_e p_e \alpha_d - 2Q_e a_e \alpha_d + 32\alpha_d + 12e_1 - 4Q_e p_e \\ &- 12Q_e\alpha_e - 4Q_e a_e - 4Q_e b_e + Q_e^2 p_e^2 + 3Q_e^2\alpha_e^2 + 2Q_e^2 p_e \alpha_e \\ &- 2Q_e p_e e_1 - 6Q_e \alpha_e e_1 + 2Q_e^2 a_e \alpha_e + 2Q_e^2 b_e \alpha_e + 2Q_e^2 q_e a_e \\ &- 2Q_e q_e e_1 - Q_e^2 q_e^2 + 4\alpha_d^2 - 6Q_e\alpha_e\alpha_d - 2Q_e b_e \alpha_d + 6e_1\alpha_d)/(4(6+\alpha_d)) \end{aligned} \tag{5.5}$$

and in Case ($\mathcal{R}$):

$$
\begin{aligned}
h_2 = (&-6Q_e a_e \alpha_d - 6Q_e b_e \alpha_d - 18 Q_e \alpha_e \alpha_d + 6Q_e^2 p_e \alpha_e \\
&+ 118\alpha_d - 6Q_e p_e \alpha_d - 6Q_e p_e e_1 - 20 Q_e a_e - 60 Q_e \alpha_e \\
&- 20 Q_e p_e - 20 Q_e b_e + 3Q_e^2 p_e^2 + 9Q_e^2 \alpha_e^2 + 60 e_1 - 18 Q_e \alpha_e e_1 \\
&+ 6Q_e^2 a_e \alpha_e + 6Q_e^2 b_e \alpha_e + 6Q_e^2 q_e a_e - 6Q_e q_e e_1 - 3Q_e^2 q_e^2 \\
&+ 12\alpha_d^2 + 18 e_1 \alpha_d + 304)/(12(4+\alpha_d))
\end{aligned}
\tag{5.6}
$$

We can thus achieve cancellation of mixed anomalies while still retaining considerable freedom in the leptonic sector. Among the possible textures for Y_e we have

$$
Y_e^I \sim \lambda^{\alpha_e} \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}, Y_e^{II} \sim \lambda^{\alpha_e} \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}, Y_e^{III} \sim \lambda^{\alpha_e} \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda \\ \lambda^4 & \lambda^2 & \lambda \\ \lambda^3 & \lambda & 1 \end{pmatrix}.
\tag{5.7}
$$

All these textures correspond to the hierarchy $m_\tau : m_\mu : m_e = 1 : \lambda^2 : \lambda^4$. From Eq. (5.3$b$) we see that for this class of textures

$$
Q_e = \frac{2\alpha_d}{3(\alpha_e + 2)} \quad \text{in the FI case} \tag{5.8}
$$

or

$$
Q_e = \frac{2(3+\alpha_d)}{3(\alpha_e + 2)} \quad \text{in the } \mathcal{R} \text{ case,} \tag{5.9}
$$

so that, for example, in the FI case with with $\alpha_d = \alpha_e = 2$ we have $Q_e = 1/3$. Notice that in the $\mathcal{R}$ case we can have $Q_e = 1$, if $\alpha_d = 3\alpha_e/2$.

6. DD Textures

In this section we consider an alternative texture solution, as described in Ref. [10] (see also Ref. [18] This takes the form:

$$
Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^4 & 1 \\ \lambda^8 & \lambda^4 & 1 \\ \lambda^8 & \lambda^4 & 1 \end{pmatrix}, \quad Y_d, Y_e \sim \lambda^{\alpha_{d,e}} \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}. \tag{6.1}
$$

We will term these textures "Doublet Democracy" because (assuming as before a specific θ-field is responsible for each texture) it corresponds to generation-independent charges for quark and lepton doublets. Unlike the Wolfenstein case, the above textures do not lead naturally to a Wolfenstein texture for CKM; in fact the entries in $U_{u,d}$, and hence CKM, are generically of $O(1)$. However, if we suppose that in fact

$$Y_u \propto \begin{pmatrix} a_u\lambda^8 & d_u\lambda^4 & 1+O(\lambda^2) \\ b_u\lambda^8 & e_u\lambda^4 & 1+O(\lambda^2) \\ c_u\lambda^8 & f_u\lambda^4 & 1+O(\lambda^2) \end{pmatrix} \quad \text{and} \quad Y_d \propto \begin{pmatrix} a_d\lambda^4 & d_d\lambda^2 & 1+O(\lambda^2) \\ b_d\lambda^4 & e_d\lambda^2 & 1+O(\lambda^2) \\ c_d\lambda^4 & f_d\lambda^2 & 1+O(\lambda^2) \end{pmatrix} \tag{6.2}$$

(in other words that the unsuppressed Yukawa couplings are approximately the same in both cases) then we obtain for the CKM matrix the texture

$$CKM_{DD} \sim \begin{pmatrix} 1 & 1 & \lambda^2 \\ 1 & 1 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix} \tag{6.3}$$

which is not of the form of the standard Wolfenstein parametrisation, Eq. (5.1). It does, however, reproduce the most significant feature, which is the smallness of the couplings to the third generation. We obtain CKM_{DD} although the entries in $U_{u,d}$ are generically still of $O(1)$, via a cancellation between U_u and U_d. This observation will be important when we come to consider neutrino masses in this scenario.

Unlike the Wolfenstein case, these textures do not dictate the value of Q_d. The anomaly constraints become (in Case (FI)):

$$A_3 = 12 + 3\alpha_d Q_d + 6Q_d - 3\Delta = 0 \tag{6.4a}$$

$$A_2 = -2\Delta - 6h_2 + 72 - 9u_1 + 12Q_e + 3Q_e\alpha_e - 3e_1 = 0 \tag{6.4b}$$

$$A_1 = -120 + 6\alpha_d Q_d + 12Q_d + 27u_1 + 18h_2 + 9Q_e\alpha_e + 9e_1 - 12\Delta = 0 \tag{6.4c}$$

$$\begin{aligned} A_Q = &-48u_1 - 144h_2 - 96Q_d - 48\alpha_d Q_d - 28Q_e^2 - 24Q_e^2\alpha_e + 12Q_e e_1 \\ &- 3Q_e^2\alpha_e^2 + 18u_1h_2 + 6Q_e\alpha_e e_1 + 6\alpha_d Q_d u_1 + 12\alpha_d Q_d h_2 + 48\Delta + 12h_2^2 \\ &+ 3\alpha_d^2 Q_d^2 + 12\alpha_d Q_d^2 + 20Q_d^2 + 12Q_d u_1 + 24Q_d h_2 + 224 + 24Q_e\Delta \\ &- 24Q_e h_2 - 6e_1\Delta + 6e_1h_2 - \Delta^2 - 4\Delta h_2 - 12Q_d\Delta - 6u_1\Delta \\ &+ 6Q_e\alpha_e\Delta - 6Q_e\alpha_e h_2 - 6\alpha_d Q_d\Delta = 0 \end{aligned} \tag{6.4d}$$

and in Case ($\mathcal{R}$):

$$A_3 = 12 + 3\alpha_d Q_d + 6Q_d - 3\Delta = 0 \tag{6.5a}$$

$$A_2 = 64 - 2\Delta - 6h_2 - 9u_1 + 12Q_e + 3Q_e\alpha_e - 3e_1 = 0 \tag{6.5b}$$

$$A_1 = -132 + 6\alpha_d Q_d + 12Q_d - 12\Delta + 18h_2 + 27u_1 + 9Q_e\alpha_e + 9e_1 = 0 \tag{6.5c}$$

$$
\begin{aligned}
A_Q = {} & 176 + 6Q_e\alpha_e e_1 + 42\Delta - 132h_2 - 42u_1 - 96Q_d - 48\alpha_d Q_d + 6Q_e\alpha_e \\
& - 28Q_e^2 - 24Q_e^2\alpha_e + 12Q_e e_1 + 24Q_e\Delta - 24Q_e h_2 - 3Q_e^2\alpha_e^2 - 6e_1\Delta \\
& + 6e_1 h_2 - \Delta^2 - 4\Delta h_2 + 24Q_e - 6e_1 + 6Q_e\alpha_e\Delta - 6Q_e\alpha_e h_2 + 12Q_d u_1 \\
& + 24Q_d h_2 - 12Q_d\Delta - 6u_1\Delta + 6\alpha_d Q_d u_1 + 12\alpha_d Q_d h_2 - 6\alpha_d Q_d\Delta \\
& + 20Q_d^2 + 12h_2^2 + 18u_1 h_2 + 3Q_d^2\alpha_d^2 + 12Q_d^2\alpha_d = 0.
\end{aligned} \tag{6.5d}
$$

Solving Eqs. $(6.4a-c)$, we obtain

$$
Q_d = \frac{\Delta - 4}{\alpha_d + 2} \tag{6.6a}
$$

$$
Q_e = \frac{2(\Delta - 6)}{3(2 + \alpha_e)} \tag{6.6b}
$$

$$
\begin{aligned}
u_1 &= -2\Delta/9 - 2h_2/3 + 8 + 4Q_e/3 + Q_e\alpha_e/3 - e_1/3 \\
&= \frac{4\Delta - 12h_2 - 6h_2\alpha_e + 96 + 60\alpha_e - 6e_1 - 3e_1\alpha_e}{9(2 + \alpha_e)}.
\end{aligned} \tag{6.6c}
$$

It would clearly be desirable to have $Q_d = Q_e = 1$, since then we could have a single θ-charge only. However, imposing this would lead to $\alpha_d = \frac{3}{2}(\alpha_e + 2)$ which would not accord with the hierarchy of masses between leptons and quarks. If we instead make the simplifying assumption $\alpha_d = \alpha_e = 0$, we find $Q_d = -2 + \frac{1}{2}\Delta$, $Q_e = -2 + \frac{1}{3}\Delta$. We also find

$$
A_Q = \Delta\left[\tfrac{26}{9}\Delta + 4h_2 - 2e_1 - 128/3\right]. \tag{6.7}
$$

We are clearly led to $\Delta = 0$ since this gives $A_Q = 0$ and also $Q_d = Q_e$, so that θ_e may be identified with θ_d. Moreover, the fact that the θ_u and θ_d charges have *opposite* signs makes this assignment *natural*, in the sense that we do not have to forbid higher dimensional terms involving powers of $\theta_{d,e}$ from contributing to Y_u. It is this case we will concentrate on later. (After an exhaustive search we have been unable to find any other "natural" θ-charge assignments; it is worth mentioning that simply requiring $Q_d = Q_e$ and $\alpha_d = \alpha_e$ leads inevitably to the solution we have described.)

Correspondingly from Eqs. $(6.5a-c)$ we obtain instead (for Case $(\mathcal{R})$)

$$
Q_d = \frac{\Delta - 4}{\alpha_d + 2} \tag{6.8a}
$$

$$
Q_e = \frac{2(\Delta - 3)}{3(2 + \alpha_e)} \tag{6.8b}
$$

$$
\begin{aligned}
u_1 &= 64/9 - 2\Delta/9 - 2h_2/3 + 4Q_e/3 + Q_e\alpha_e/3 - e_1/3 \\
&= \frac{104 + 58\alpha_e + 4\Delta - 12h_2 - 6h_2\alpha_e - 6e_1 - 3e_1\alpha_e}{9(2 + \alpha_e)}.
\end{aligned} \tag{6.8c}
$$

Interestingly in this case we *can* achieve $Q_d = Q_e = Q_u = 1$, as follows. From Eq. (6.8a, b) we have at once that $\alpha_d = \frac{3}{2}\alpha_e$, which suggests setting $\alpha_d = \alpha_e = 0$ to avoid inverting the hierarchy between lepton and quark masses; we then have $\Delta = 6$. Now for $\alpha_d = \alpha_e = 0$ we obtain

$$A_Q = \Delta\left[\tfrac{26}{9}\Delta + 4h_2 - 2e_1 - 34\right] - 8h_2 + 4e_1 + 44/3 \tag{6.9}$$

so that for $\Delta = 6$ the constraint $A_Q = 0$ becomes $2h_2 - e_1 = 32/3$. (Once again it turns out that this is the only solution even if we don't impose $\alpha_d = \alpha_e = 0$ from the outset.) Imposing the above constraint on e_1, we then have leptonic charges $L_i, e_1, e_2, e_3 = \frac{23}{3} - h_2, 2h_2 - \frac{32}{3}, 2h_2 - \frac{38}{3}, 2h_2 - \frac{44}{3}$. Then the loop unsuppressed term from Eq. (2.8) will be positive for each of the set L_i, e_j if $m_0^2 < 0$ and $8 > h_2 > \frac{43}{6}$, which means the term will in each case help us eliminate the tachyonic slepton. We will analyse this case later; unfortunately it runs into difficulties because it leads to a comparatively light charged Higgs mass. The alternative scenario which we described as natural in the FI case was to arrange that $Q_d = Q_e < 0$. It is easy to demonstrate from Eqs. (6.8a, b) that this is incompatible with $\alpha_d \leq \alpha_e$, and moreover the alternative natural solutions $Q_d = Q_u = 1$, $Q_e < 0$, or $Q_e = Q_u = 1$, $Q_d < 0$ are also impossible; since $Q_d = 1$ gives at once $\Delta \geq 6$ and hence $Q_e \geq 0$, and $Q_e = 1$ gives $\Delta \geq 9$ and hence $Q_d \geq 0$.

An alternative which we will consider later is $\Delta = \alpha_d = \alpha_e = 0$, leading to $Q_d = -2$, $Q_e = -1$. This is clearly less satisfactory since we now must suppose that the physics responsible for the higher dimension terms does not permit θ_d to couple to the leptons.

7. Choice of Textures

In the previous two sections we have exhibited two distinct choices of Yukawa texture and showed how they both could arise from U_1' (or $U_1^{\mathcal{R}}$) charge assignments compatible with mixed-anomaly cancellation.

The Wolfenstein texture has the advantage of explaining in a completely natural way the origin of the CKM matrix; however the DD texture has an overriding advantage which is specific to our AMSB scenario. This advantage derives from the fact that for the corresponding textures $Y_{u,d,e}$ shown in Eq. (6.1), the right-handed diagonalisation matrices $V_{u,d,e}$ are close to the unit matrix. Specifically,

$$V_u \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^8 \\ \lambda^4 & 1 & \lambda^4 \\ \lambda^8 & \lambda^4 & 1 \end{pmatrix}, \quad V_d, V_e \sim \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}. \tag{7.1}$$

The significance of this becomes apparent when we consider the effect of rotating to the quark/lepton mass diagonal basis the fundamental relations Eq. (2.6) or Eq. (2.8). The AMSB contributions to the scalar masses are diagonalised to a good approximation when we transform to the fermion mass-diagonal basis, as are the contributions proportional to ${\gamma^i}_j$ in Eq. (2.8). Clearly the danger lies in the terms linear in the U_1' (or $U_1^{\mathcal{R}}$) charges. However if we choose the DD textures, then on the one hand there is no problem with the LH squarks and sleptons, because of the universal doublet U_1' charges; and on the other hand, the induced off-diagonal contributions to the RH squark and slepton mass matrices are small because of Eq. (7.1) above.

For the rest of this paper we will concentrate on the DD textures.

8. Experimental Consequences

The gaugino spectrum is to leading order independent of the mechanisms used here to resolve the slepton mass problem; and is characterised by an approximately degenerate triplet of light winos ($\tilde{W}^{\pm,0}$) The neutral wino is, in a substantial region of parameter space, the LSP; the resulting characteristic decay $\tilde{W}^{\pm} \to \tilde{W}^0\pi^0$ has been described in a number of papers.

However the LSP can also be a scalar neutrino, $\tilde{\nu}_l$, in which case the dominant decay modes of $\tilde{W}^{\pm,0}$ will be $\tilde{W}^{\pm} \to \tilde{\nu}_l l$ and $\tilde{W}^0 \to \tilde{\nu}_l \nu_l$ respectively (if the masses are ordered $\tilde{l} > \tilde{W}^{\pm,0} > \tilde{\nu}_l$) with the possibilities $\tilde{W}^{\pm} \to \tilde{l}\nu_l, \tilde{W}^0 \to \tilde{l}l$ also available if $\tilde{W}^{\pm,0} > \tilde{l} > \tilde{\nu}_l$

The fact that M_3 and M_2 have opposite signs disfavours at first sight a supersymmetric explanation of the well-known discrepancy between theory and experiment for the anomalous magnetic moment of the muon, a_μ. This is because if sign (μM_2) is chosen so as to create a positive $a_\mu^{\rm SUSY}$ then sign (μM_3) leads to constructive interference between various supersymmetric contributions to $B(b \to s\gamma)$, and consequent restrictions on the allowed parameter space[3]. It is worth noting, however, that gloomy conclusions here have generally been reached in the context of the mAMSB model, Eq. (2.5), and the issue is therefore perhaps worth revisiting, because the nature of our solution to the tachyonic slepton problem means that the squark/slepton mass difference is typically much higher in our case, so we might hope (by choosing positive μM_2, and arranging for heavy squarks) to find a negligible (even if constructive) $B(b \to s\gamma)$ contribution from squark loops.

[3] In this context *deflected* anomaly mediation[19] is worthy of consideration[20].

We must be careful, however, of the charged Higgs contribution to $B(b\to s\gamma)$, which can be quite large, is independent of squark masses, and *adds* to the SM contribution. Ignoring other supersymmetric contributions, we estimate that a limit $m_{H^\pm} > 400\mathrm{GeV}$ is required. This limit provides a useful constraint on our final choice of charge assignments, via mass sum rules, which we will discuss below.

8.1. The FI case

In section 6 we showed how with $\Delta = \alpha_d = \alpha_e = 0$ anomaly cancellation led to the economical and natural result $Q_d = Q_e = -2$ so that two θ-fields θ_u, θ_d suffice to generate the quark and lepton masses. Moreover, as mentioned earlier, there is no need to invoke any further discrete symmetry to forbid θ_d from contributing to Y_u since θ_u, θ_d have opposite charges. If one assumed that only these two fields received vevs at the M_θ-scale, and if U_1' were gauged, then there would be a D-flat direction corresponding to $\langle\theta_u\rangle = \sqrt{2}\langle\theta_d\rangle$ leading to two distinct λ-parameters, with $\lambda_u = \sqrt{2}\lambda_d$. However, we retain an agnostic attitude to the gauging of the U_1' and we choose correspondingly to stick with the basic assumption that there is a universal $\lambda \approx 0.22$ for all the Yukawa matrices.

In Ref. [10] we presented a preliminary analysis of this scenario. The main distinguishing feature of the sparticle spectrum is the large splitting among right-handed fields caused by the generation-dependent charge assignments. This is in complete contrast to the constrained MSSM (CMSSM), where, for example, $\tilde{d}_R, \tilde{s}_R$ are almost degenerate as are $\tilde{d}_L, \tilde{s}_L$. Here while $\tilde{d}_L, \tilde{s}_L$ remain degenerate, $\tilde{d}_R$ and $\tilde{s}_R$ may differ in mass by a factor of two or more.

In order to pin down an appropriate set of U_1' charge assignments (and also to explore what features of the outcome are independent of these assignments) it is useful to begin by introducing some sum rules. It is easy to show that in Case (FI) we have, after imposing the anomaly constraints Eqs. (6.6),

$$\mathrm{Tr}\left(m_L^2 + 3m_Q^2\right) = \mathrm{Tr}\left(m_L^2 + 3m_Q^2\right)|_{\mathrm{AMSB}} - \Delta\zeta', \tag{8.1a}$$

$$\mathrm{Tr}\left(m_{u^c}^2 + m_{d^c}^2 + 2m_Q^2\right) = \mathrm{Tr}\left(m_{u^c}^2 + m_{d^c}^2 + 2m_Q^2\right)|_{\mathrm{AMSB}}, \tag{8.1b}$$

$$\mathrm{Tr}\left(m_{u^c}^2 + m_{e^c}^2 - 2m_Q^2\right) = \mathrm{Tr}\left(m_{u^c}^2 + m_{e^c}^2 - 2m_Q^2\right)|_{\mathrm{AMSB}}, \tag{8.1c}$$

where the masses on the RHS correspond to pure AMSB contributions, i.e. they are calculable from Eq. (2.3*c*), which (apart from the overall scale M_0), depends only on the unbroken theory. We hence obtain sum rules for the particle masses, for example from Eq. (8.1b) we have that:

$$\sum_{\tilde{q}} m_{\tilde{q}}^2 = 2m_t^2 + j_1(\tan\beta)m_{\tilde{g}}^2 \tag{8.2}$$

where the sum includes all twelve squarks, and we have neglected quark masses apart from m_t. The function $j_1(\tan\beta)$ is a slowly varying function of $\tan\beta$, given to a good approximation by $j_1 = 9.84 - 0.013\tan\beta$ for $5 < \tan\beta < 40$. For $\tan\beta < 5$ and $\tan\beta > 40$, j increases, for example $j_1(2) = 9.91$ and $j_1(60) = 10.02$. This sum rule is very robust, being independent of Δ and any feature of the charge assignments. Note how it clearly distinguishes the FI case from the mAMSB variant defined by Eq. (2.5), which would lead to an additional term $12m_0^2$ on the RHS of Eq. (8.2), which would also hold only at high energies because of the loss of RG invariance.

Adding Eqs. (8.1a, c)we obtain

$$\sum_{\tilde{t},\tilde{c},\tilde{u}} m_{\tilde{q}}^2 + \sum_{\tilde{\tau},\tilde{\mu},\tilde{e}} m_{\tilde{l}}^2 = 2m_t^2 + j_2(\tan\beta)m_{\tilde{g}}^2 - \Delta\zeta' \tag{8.3}$$

so for the class of charge assignments such that $\Delta = 0$ (corresponding to an allowed μ-term $\mu H_1 H_2$) we have another sum rule. The function $j_2(\tan\beta) \approx 4.6$ is again insensitive to $\tan\beta$.

There is a further sum rule involving the CP odd Higgs. Using the tree minimisation conditions we obtain

$$m_A^2 = (m_2^2 - m_1^2)\sec 2\beta - M_Z^2 \tag{8.4}$$

whence

$$m_A^2 = \sec 2\beta\left(\frac{2}{3}\sum_{\tilde{\tau},\tilde{\mu},\tilde{e}} m_{\tilde{l}}^2 + j_3(\tan\beta)m_{\tilde{g}}^2 + (8 - \Delta/3)\zeta'\right) \tag{8.5}$$

where j_3 increases from $j_3(5) \approx -0.45$ to $j_3(40) \approx 0.01$.

This sum rule is particularly useful in the $B(b \to s\gamma)$ context, because m_A is linked to $m_{H^\pm}$ via the tree relation $m_{H^\pm}^2 = m_A^2 + M_W^2$, and as we described above, we want to ensure $m_{H^\pm} > 400\text{GeV}$. We must also ensure that the tachyonic mass problem is solved. For $\Delta = \alpha_d = \alpha_e = 0$ so that from Eq. (6.6c) we have $3u_1 = 16 - 2h_2 - e_1$, we have U_1' charge assignments as shown in Table 1,

Q_i	u_2	u_3	d_1	d_2	d_3
$8-u_1-h_2$	u_1-4	u_1-8	$2h_2+u_1-16$	$2h_2+u_1-12$	u_1+2h_2-8

L_i	e_1	e_2	e_3
$3u_1+3h_2-24$	$16-2h_2-3u_1$	$20-2h_2-3u_1$	$24-2h_2-3u_1$

Table 1: The U_1'-charges

where we have written the charges in terms of u_1 instead of e_1 using Eq. (6.6c). It is then easy to show that with the charge assignments shown in Table 1, there exists some range of ζ, ζ' leading to positive FI contributions for both $m_{e^c}^2$ and m_L^2 if and only if

$$3u_1 + 4h_2 < 24, \quad \text{if} \quad \zeta' < 0, \tag{8.6}$$

or

$$3u_1 + 4h_2 > 32 \quad \text{if} \quad \zeta' > 0. \tag{8.7}$$

Now in Ref. [10] we chose $h_2 = 12$ and $u_1 = -7/2$, which evidently satisfies Eq. (8.7). Throughout the corresponding allowed region in the ζ, ζ' plane, however, this gives rise to an unacceptably light $H^{\pm}$ mass from the point of view described above. The reason is easy to see from Eq. (8.5); for $\tan\beta > 1$ we have $\sec 2\beta < 0$, and so the last term in this equation reduces m_A^2 (and hence $m_{H^{\pm}}$) if $\zeta' > 0$. If, however, we choose, for example, $h_2 = 1$ and $u_1 = 1/2$ (corresponding to $e_1 = 25/2$), then we have instead $\zeta' < 0$ and duly obtain a spectrum with a significantly heavier $H^{\pm}$.

For these charge assignments, $\tan\beta = 5$, $\mu > 0$ and $M_0 = 40$TeV, we show in Figure 1 the triangular region in the $\zeta_{1,2}$ plane which corresponds to an acceptable vacuum. The LSP can be the neutral wino, or the $\tilde{\nu}_\tau$; for alternative charge assignments (such as those employed in Ref. [10]) the LSP can be a charged lepton, but we find that the constraint of a heavier $H^{\pm}$ that we favour here excludes this possibility.

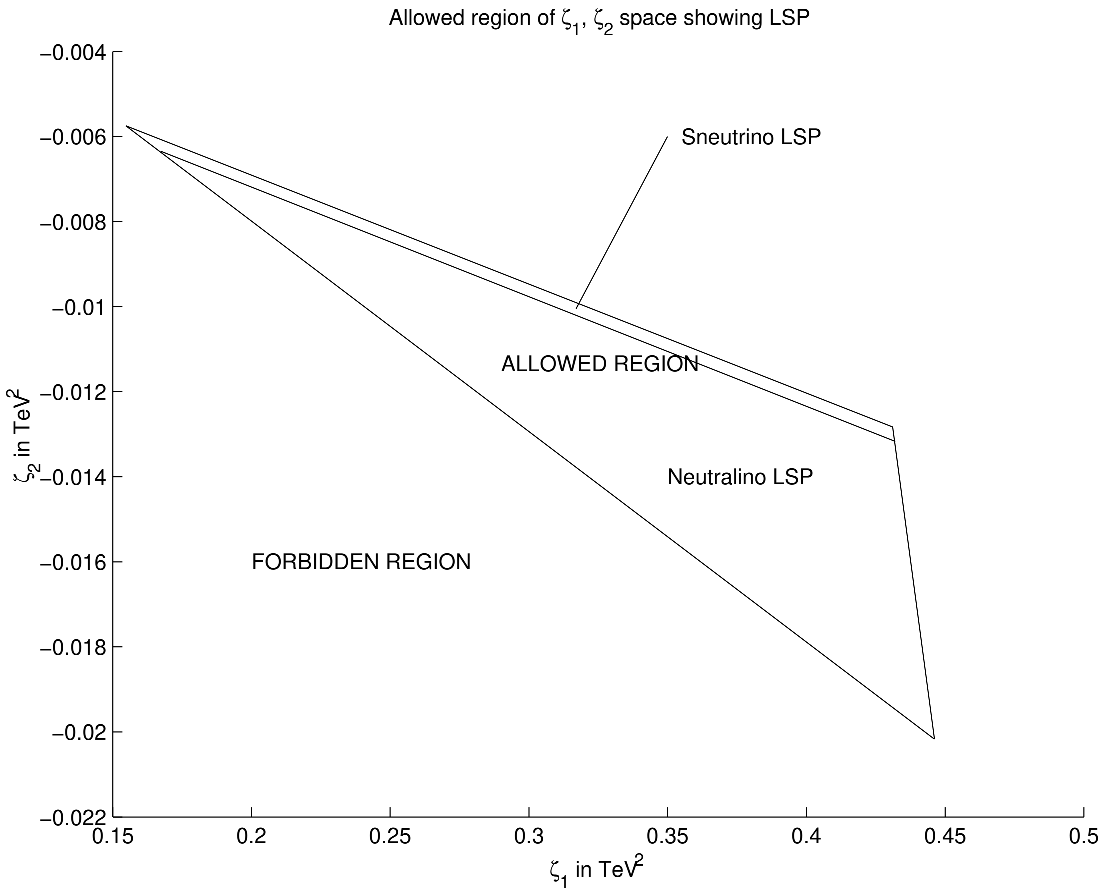

Fig.1: Allowed values of $\zeta_{1,2}$ *for* $\tan\beta = 5$, $m_0 = 40\mathrm{TeV}$ *and* $\mu > 0$.

As $\tan\beta$ is increased the allowed region shrinks, becoming very small for $\tan\beta > 20$. In Table 2 we give representative spectra for a point from each of the two allowed regions in Fig. 1.

If we consider the muon anomalous magnetic moment for the case, for example, of the second column of Table 2, then we obtain[21]

$$a_\mu^{\mathrm{SUSY}} \approx 25 \times 10^{-10}. \tag{8.8}$$

This result (the dominant contribution to which comes from the charged wino/Higgsino diagram) is in the right region to explain the difference between the recent Brookhaven E821 result [22] and the SM prediction[4]

$$\delta a_\mu = 33.9(11.2) \times 10^{-10} \tag{8.9}$$

[4] We use the e^+e^- result from Ref. [23]; see also Ref. [24]

Thus (by choosing $\mu > 0$) we are indeed able to generate a significant positive contribution to a_μ while simultaneously suppressing the contribution to $B(b \to s\gamma)$ thanks to the large squark and Higgs masses.

$\tan\beta(\text{sign } \mu_s)$	$5(+)$	$5(+)$
$\zeta_1(\text{TeV})^2$	0.3	0.325
$\zeta_2(\text{TeV})^2$	-0.01	-0.01
$M_0\text{TeV}$	40	40
$\text{mod}(\mu)\text{TeV}$	0.677	0.667
$\tilde{t}_{1,2}$	$863, 606$	$864, 594$
$\tilde{c}_{L,R}$	$931, 852$	$933, 842$
$\tilde{u}_{L,R}$	$931, 828$	$933, 818$
$\tilde{b}_{1,2}$	$825, 1024$	$827, 1028$
$\tilde{s}_{L,R}$	$934, 1047$	$936, 1051$
$\tilde{d}_{L,R}$	$934, 1066$	$936, 1069$
$\tilde{\tau}_{1,2}$	$150, 268$	$101, 311$
$\tilde{\mu}_{L,R}$	$152, 335$	$104, 370$
$\tilde{e}_{L,R}$	$152, 390$	$104, 421$
$\tilde{\nu}_\tau$	129	64
$\tilde{\nu}_{\mu,e}$	130	66
h	117	117
H	539	514
A	538	512
$H^\pm$	544	519
$\tilde{\chi}_1^\pm$	104	104
$\tilde{\chi}_2^\pm$	681	671
$\tilde{\chi}_1$	103	103
$\tilde{\chi}_2$	367	367
$\tilde{\chi}_3$	680	670
$\tilde{\chi}_4$	689	680
$\tilde{g}$	1008	1008

Table 2: The sparticle masses (in GeV) for the FI U_1' case

8.2. The $\mathcal{R}$ case

Here also we will be guided by the Higgs mass sum rule. The sum rule analogous to Eq. (8.2) is

$$\sum_{\tilde{q}} m_{\tilde{q}}^2 = 2m_t^2 + j_1(\tan\beta)m_{\tilde{g}}^2 + k_1^{\mathcal{R}}(\tan\beta)m_0^2 \tag{8.10}$$

where $k_1^{\mathcal{R}}(5) \approx 2.7$ and is likewise slowly varying over a wide range of $\tan\beta$. Thus there is explicit dependence on the mass scale m_0; this term is small compared to the $m_{\tilde{g}}$ term, however.

Similarly we have a sum rule like Eq. (8.3):

$$\sum_{\tilde{t},\tilde{c},\tilde{u}} m_{\tilde{q}}^2 + \sum_{\tilde{\tau},\tilde{\mu},\tilde{e}} m_{\tilde{l}}^2 = 2m_t^2 + j_2(\tan\beta)m_{\tilde{g}}^2 + (\tfrac{3}{2}\Delta - k_2^{\mathcal{R}}(\tan\beta))m_0^2 \tag{8.11}$$

where $k_2^{\mathcal{R}}(5) \approx 6.2$. Finally the analogue of the Higgs mass sum rule, Eq. (8.5) is

$$m_A^2 = \sec 2\beta \left(\frac{2}{3} \sum_{\tilde{\tau},\tilde{\mu},\tilde{e}} m_{\tilde{l}}^2 + j_3(\tan\beta)m_{\tilde{g}}^2 + \mathcal{C}m_0^2 \right) \tag{8.12}$$

where

$$\mathcal{C} = \gamma_{H_2} - \gamma_{H_1} - \tfrac{2}{3}\mathrm{Tr}(\gamma_L + \gamma_{e^c}) + \mathcal{C}_q \tag{8.13}$$

and

$$\mathcal{C}_q = 2 + \sum_{i=1}^{3}(L_i + e_i) - \tfrac{3}{2}(h_2 - h_1). \tag{8.14}$$

Here there is a degree of cancellation between the first two terms on the RHS of Eq. (8.12) and hence the sign and magnitude of $\mathcal{C}_q$ becomes important. For the case $\Delta = 6$, $Q_u = Q_d = Q_e = 1$ described at the end of section 6, we find (independent of h_2) that $\mathcal{C}_q = -4$, so that since we needed $m_0^2 < 0$ to resolve the tachyonic slepton problem we can anticipate light $m_A, m_{H^\pm}$. We indeed find that for $M_0 = 40\mathrm{TeV}$, $m_{H^\pm} \approx 160 - 250\mathrm{GeV}$, and that this continues to hold even if we raise M_0 to $80\mathrm{TeV}$, giving squark masses in the region of $2\mathrm{TeV}$ and $|\mu| \approx 1.2\mathrm{TeV}$, which is at if not beyond the limit of acceptable fine tuning for the Higgs minimisation. Thus this scenario, while ideal in terms of the θ-charges, is, we believe, ruled out.

An alternative for which we will again provide detailed results is $\Delta = \alpha_d = \alpha_e = 0$, leading to $Q_d = -2$, $Q_e = -1$. Unfortunately as we already described, here we have to forbid θ_d from coupling to the leptons in order to prevent it from giving unwanted contributions to the textures for Y_e. We now find that $\mathcal{C}_q = -7$. However, this time solving $A_Q = 0$ gives $e_1 = 2h_2 - 11/3$, whereupon the leptonic charges are $L_i, e_1, e_2, e_3 = -\frac{4}{3} - h_2, 2h_2 - \frac{11}{3}, 2h_2 - \frac{5}{3}, 2h_2 + \frac{1}{3}$. Then the loop unsuppressed term from Eq. (2.8) will be positive for each of the set L_i, e_j if $m_0^2 > 0$ and $-1 < h_2 < -1/3$. So in this case we can have $m_0^2 > 0$ with $\mathcal{C}_q < 0$ and hence a *positive* $\mathcal{C}$-contribution to m_A^2 in Eq. (8.12) leading to a larger $m_{H^\pm}$. For these charge assignments, $h_2 = -2/3$ and $\mu > 0$ we show in Figure 2 the range of values of M_0 and m_0 that lead to an acceptable vacuum and sparticle spectrum, for both $\tan\beta = 5$ and $\tan\beta = 10$. In both cases the LSP is always the $\tilde{\nu}_\tau$, which is disfavoured as a dark matter candidate.

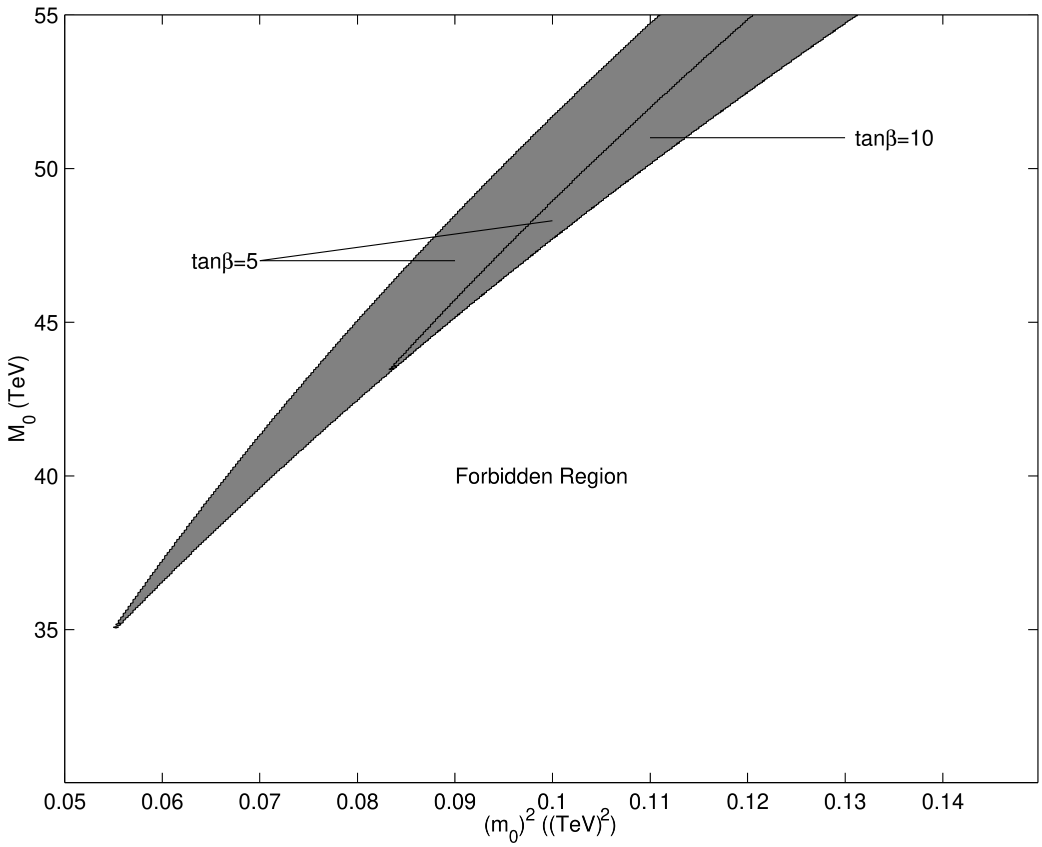

Fig.2: Range of allowed values of M_0 and m_0 for the $U_1^{\mathcal{R}}$ case

In Table 3 we give representative spectra for two points from the allowed region in Fig. 2, corresponding to $\tan\beta = 5$ and $\tan\beta = 10$ respectively.

$\tan\beta(\text{sign } \mu_s)$	5(+)	10(+)
M_0(TeV)	40	50
m_0^2(TeV^2)	0.07	0.104
mod(μ)TeV	0.753	0.929
$\tilde{t}_{1,2}$	$872, 674$	$1059, 827$
$\tilde{c}_{L,R}$	$932, 667$	$1152, 836$
$\tilde{u}_{L,R}$	$932, 159$	$1152, 274$
$\tilde{b}_{1,2}$	$826, 1004$	$1011, 1228$
$\tilde{s}_{L,R}$	$935, 1198$	$1155, 1473$
$\tilde{d}_{L,R}$	$935, 1362$	$1155, 1672$
$\tilde{\tau}_{1,2}$	$108, 218$	$101, 261$
$\tilde{\mu}_{L,R}$	$114, 507$	$128, 615$
$\tilde{e}_{L,R}$	$114, 683$	$128, 831$
$\tilde{\nu}_\tau$	79	89
$\tilde{\nu}_{\mu,e}$	81	97
h	117	124
H	664	781
A	663	781
$H^\pm$	668	785
$\tilde{\chi}_1^\pm$	105	134
$\tilde{\chi}_2^\pm$	757	931
$\tilde{\chi}_1$	104	133
$\tilde{\chi}_2$	368	463
$\tilde{\chi}_3$	764	932
$\tilde{\chi}_4$	756	937
$\tilde{g}$	1008	1246

Table 3: The sparticle masses (in GeV) for the $U_1^{\mathcal{R}}$ case

Here a significant constraint on the allowed parameter space is provided by the $\tilde{u}_R$ mass, which tends to be lighter than the other squarks. Otherwise the spectrum is similar to that obtained for the $\tilde{\nu}_\tau$ region in the FI case.

9. R-parity violation

It is well-known that imposing gauge invariance alone does not forbid the addition of the following renormalisable R-parity violating terms to the superpotential of the MSSM :

$$W_R = \lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \lambda''_{ijk} u^c_i d^c_j d^c_k + m^R_i L_i H_2. \tag{9.1}$$

If all these terms are allowed then rapid proton decay results but if λ'' is forbidden then the limits on the remaining terms (which violate L but not B) are less strict. Indeed the cubic terms have been employed to provide an alternative explanation for the tachyonic slepton problem[25]. Alternatively if only the quadratic term is permitted then this results in interesting phenomenology[26]; for example the mixing between neutral gauginos and neutrinos induces neutrino masses.

When we generalise the MSSM to the MSSM$^\nu$ (Eq. (3.1)), i.e. if we consider the effective field theory at scales P such that $M_\nu < P < M_\theta$[5] then we have the further possible renormalisable R-parity violating terms

$$W^\nu_R = a_i \nu^c_i + b_i H_1 H_2 \nu^c_i + c_{ijk} \nu^c_i \nu^c_j \nu^c_k. \tag{9.2}$$

Let us now ask the following question: can our U'_1 or $U^{\mathcal{R}}_1$ be used to naturally forbid the appearance of some or all of the operators in Eqs.(9.1), (9.2) at the tree level or both at the tree level and via FN texture terms?

The answer to both these questions is in fact yes, there exist charge assignments which do precisely this. This is an attractive feature of this class of theories; the fact that a symmetry introduced to resolve the slepton mass problem can lead naturally to R-parity conservation is very economical. As an example of how this may be achieved consider the charge assignments that we studied in section 8 in the FI case (from Eq. (6.6)): $\Delta = 0, h_2 = 1, e_1 = 25/2$, with $\alpha_d = \alpha_e = 0$. This corresponds to $Q_u = 1, Q_d = Q_e = -2$ so that manifestly only operators with integer charges can be generated with the possible θ-charges. However we find that the set of R-parity violating operators of Eq. (9.1) have charges $-27/2, -35/2, -37/2, -43/2, -51/2, -59/2$. Because these are all half-integral they cannot be generated by the existing θ-charges and so R-parity conservation is exact.

[5] In fact the most natural assumption is $M_\nu = M_\theta$, in which case we would also not have to be concerned with lepton flavour violation effects generated from $Y_\nu Y_\nu^\dagger$ contributions to the running of the slepton masses between M_θ and M_ν; for a review and references see for example [27].

With regard to Eq. (9.2), the outcome depends on the ν^c U_1' charge assignments; we will reurn to this issue in the next section.

In the $\Delta = \alpha_d = \alpha_e = 0$ (so that $Q_d = -2, Q_e = -1$), $\mathcal{R}$-case analysed in Section 8.2, all the lepton number violating operators have fractional charges and are hence forbidden in the same manner, as are *tree* contributions to λ''_{ijk}. However if we allow FN couplings to the θ-fields, we can in this case generate some of these terms with powers of θ ranging from unity (for $\lambda''_{113}, \lambda''_{223}$) up to θ^7 for λ''_{312}, if we allow any of $\theta_{u,d,e}$ in each case. In fact it is λ''_{112} and λ''_{113} which are subject to the most strict experimental bounds (on double nucleon decay and $n - \overline{n}$ oscillations)[28]. These operators can be generated by θ_d^3 and θ_d (or θ_e^6 and θ_e^2) respectively, and so we need to impose that these operators cannot be produced via the $\theta_{d,e}$ spurions.

10. Neutrino Masses

There is now substantial evidence for the existence of neutrino masses, and also for a form of U_{MNS} quite different from the CKM matrix. Such a difference is not surprising in the context of the seesaw model, since although U_e is analogous to $U_{u,d}$, U_ν is quite different, involving as it does the singlet mass matrix M_{ν^c}. This issue has been discussed at length in the literature; see for example the papers of King (Ref. [29] and references therein). Specific to the the DD scenario, however, there is a reason why we might expect a distinction between CKM and U_{MNS}. The small angles of CKM are produced by cancellation between U_u and U_d in Eq. (3.4); the DD texture form produces texture suppression of the off-diagonal elements in $V_{u,d,e}$ but not in $U_{u,d,e}$. Therefore, in fact, (since U_e has a similar form to U_d), we would anticipate (even if $U_\nu \approx 1$) a non-CKM form for U_{MNS}.

It is easy to provide an explicit realisation. Suppose that m_ν is to a good approximation diagonal, so that $U_\nu \approx 1$. This is possible in the context of the explicit texture-generated construction of Ref. [10], where we had

$$m_D = \begin{pmatrix} a_\nu \lambda^n & d_\nu \lambda^m \\ b_\nu \lambda^n & e_\nu \lambda^m \\ c_\nu \lambda^n & f_\nu \lambda^m \end{pmatrix}, \tag{10.1}$$

and

$$M_{\nu^c} = \begin{pmatrix} 0 & M_1^\nu \\ M_1^\nu & 0 \end{pmatrix}. \tag{10.2}$$

If a_ν, d_ν and $b_\nu f_\nu + c_\nu e_\nu$ are small, then m_ν (which has one zero eigenvalue, because we have introduced only two right-handed neutrinos) is approximately diagonal. Now given a unitary matrix

$$U_e = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix}, \tag{10.3}$$

then trivially the matrix

$$Y_e = \begin{pmatrix} B\lambda^4 u_{11} & A\lambda^2 u_{12} & u_{13} \\ B\lambda^4 u_{21} & A\lambda^2 u_{22} & u_{23} \\ B\lambda^4 u_{31} & A\lambda^2 u_{32} & u_{33} \end{pmatrix} \tag{10.4}$$

has the appropriate texture form (see Eq. (6.1)) and is diagonalised by a left-handed transformation only:

$$U_e^T Y_e = \begin{pmatrix} B\lambda^4 & 0 & 0 \\ 0 & A\lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{10.5}$$

Thus if the neutrino mass matrix m_ν is to a good approximation diagonal and Y_e is of the form above then we obtain $U_{MNS} = U_e^T$, a natural hierarchy for the charged lepton masses, and natural suppression of leptonic FCNCs. Clearly a suitable form for U_e would be, for example (see Eq. (3.9))

$$U_e = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{10.6}$$

The coefficients u_{ij} depend on unknown physics, but this at least shows that a plausible U_{MNS} is possible within our framework. Finally we remark that if we choose (in Eq. (10.1)) $n = 2$ and $m = 1$, then with the set of FI charge assignments that we favoured in previous sections we find charges $\theta_\nu = -37/3$ and $\nu^c = \pm 37/6$ which means that all the terms in Eq. (9.2) are, like those in Eq. (9.1), forbidden at both tree and texture-generated level.

11. Conclusions

We have given a detailed analysis of the constraints that follow from imposing mixed-anomaly cancellation on the U_1' charge assignments associated with Yukawa textures (both a U_1' commuting with supersymmetry and a $U_1^{\mathcal{R}}$). The resulting texture patterns are of interest in their own right; however our specific interest is in application to the AMSB framework. Introducing the mixed-anomaly-free U_1' allows the AMSB slepton mass problem to be solved while maintaining RG invariance and UV insensitivity. In order to generate Yukawa textures the U_1' charge assignments must be generation dependent, which leads to potential FCNC problems. We have shown that a specific form for the textures solves this problem in a natural way without fine-tuning. The resulting spectrum patterns are clearly distinguished both from the CMSSM and the mAMSB by, for example, large squark and slepton mass splittings. Of the two U_1 scenarios we present, we favour the ordinary U_1' case over the $U_1^{\mathcal{R}}$ one. In the latter case although we were able to achieve the most economical θ-charge structure ($\theta_u = \theta_d = \theta_e = 1$), we find that in that case the charged Higgs mass is inevitably rather light; we present an alternative assignment ($\theta_u = 1, \theta_d = -2, \theta_e = -1$) which avoids this problem, but is unnatural in that higher dimension operators involving θ_d contributing to the lepton Yukawa matrix must be forbidden by fiat; also in this case the LSP is always a sneutrino, which is disfavoured as a dark matter candidate. Our FI case avoids all these criticisms. In addition all R-parity violating operators are naturally forbidden.

Neutrino masses and mixings consistent with current observations can be accommodated within our framework. The matrix U_e that rotates the left-handed charged leptons to the mass diagonal basis has generically large angles which can explain the difference between the CKM matrix and U_{MNS}, although specific features of preferred patterns, such as near-vanishing of the $(e3)$ element of U_{MNS}, are not predicted.

We believe that both the flavour-blind framework of Ref. [5] and the texture based frameworks of this paper are more attractive possibilities than mAMSB (and even arguably the CMSSM) and consequently worthy of attention.

Appendix A. The AMSB solution

Here we summarise the exact results for the soft supersymmetry-breaking β-functions for a general $N = 1$ supersymmetric gauge theory with a simple gauge group including the possibility of gauge singlets. We then give a set of results for the various soft parameters which form an exact RG trajectory, expressing them in terms of a single mass scale M_0 and the coupling constants of the unbroken theory. In Ref. [30], we distinguished results according to whether the auxiliary F-fields were eliminated or not; here we give only results in the F-eliminated case, which means that in the interests of notational simplicity we here represent as unbarred quantities which appeared barred ($\overline{m}^2, \overline{h} \cdots$) in Ref. [30].

We begin with a superpotential of the form:

$$W(\phi) = \tfrac{1}{2}\mu^{ij}\phi_i\phi_j + \tfrac{1}{6}Y^{ijk}\phi_i\phi_j\phi_k, \tag{A.1}$$

and soft supersymmetry-breaking scalar terms as follows:

$$V_{\text{soft}} = \left(c^i\phi_i + \tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\right) + (m^2)^i{}_j\phi_i\phi^j, \tag{A.2}$$

Note that V_{soft} contains a linear term, so we are allowing for gauge singlets in general. As remarked in Ref. [30], in the presence of soft breakings we can without loss of generality omit the linear term from the superpotential W. This statement follows (in the single field case) simply from the identity

$$V = |a + \mu\phi + y\phi^2/2|^2 = |\mu\phi + y\phi^2/2|^2 + (c\phi + b\phi^2/2 + h.c.) + a^*a, \tag{A.3}$$

where $c = a^*\mu$ and $b = a^*y$, which shows how a linear term can be “removed” from the superpotential. In the absence of explicit supersymmetry breaking this particular toy model has, of course supersymmetric ground states, corresponding to $V = 0$ (i.e. it is not of the O’Raifeartaigh [31] type).

The complete exact results for the soft β-functions are given by:

$$\begin{aligned}
\beta_M &= 2\mathcal{O}\left[\frac{\beta_g}{g}\right], \\
\beta_h^{ijk} &= h^{l(jk}\gamma^{i)}{}_l - 2Y^{l(jk}\gamma_1{}^{i)}{}_l, \\
\beta_b^{ij} &= b^{l(i}\gamma^{j)}{}_l - 2\mu^{l(i}\gamma_1{}^{j)}{}_l + Y^{ijl}\sigma_l, \\
\beta_c^i &= c^j\gamma^i{}_j + \Delta Z^i + \mu^{il}\sigma_l - (m^2)^i{}_k Z^k, \\
(\beta_{m^2})^i{}_j &= \Delta\gamma^i{}_j,
\end{aligned} \tag{A.4}$$

where

$$\mathcal{O} = Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial Y^{lmn}} - b^{lm}\frac{\partial}{\partial \mu^{lm}}, \tag{A.5a}$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2MM^*g^2\frac{\partial}{\partial g^2} + \left[\tilde{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}} + \tilde{\mu}^{lmn}\frac{\partial}{\partial \mu^{lm}} + \text{c.c.}\right] + X\frac{\partial}{\partial g}, \tag{A.5b}$$

$$(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j, \tag{A.5c}$$

$$\tilde{Y}^{ijk} = (m^2)^{(i}{}_l Y^{jk)l} \quad \text{and} \quad \tilde{\mu}^{ij} = (m^2)^{(i}{}_l \mu^{j)l}, \tag{A.6}$$

and σ, Z^i are defined as follows:

$$Z_i = Y_{imn} K^{mn}{}_{pq} \mu^{pq}, \tag{A.7a}$$

$$\sigma_i = -2\mathcal{O}\left(Z_i\right) \tag{A.7b},$$

where $Y_{imn} = (Y^{imn})^*$, with $K^{mn}{}_{pq}$ defined by the condition

$$Y_{imn} K^{mn}{}_{pq} Y^{pqj} a_j = \gamma^j{}_i a_j. \tag{A.8}$$

Finally the X function above is given (in the NSVZ scheme)

$$X^{\mathrm{NSVZ}} = -2\frac{g^3}{16\pi^2} \frac{S}{[1 - 2g^2 C(G)(16\pi^2)^{-1}]} \tag{A.9}$$

where

$$S = r^{-1}\mathrm{tr}[m^2 C(R)] - MM^* C(G), \tag{A.10}$$

For a discussion of X in the DRED$'$ scheme, see Ref. [32].

From Eq. (A.7a), (A.8) we see that Z is obtained from the subset of contributions to the anomalous dimension $\gamma^j{}_i$ where the external lines are attached to Yukawa (as opposed to gauge) couplings, by replacing the "outer" Y^{jpq} by a supersymmetric mass insertion μ^{pq}. Manifestly Z is only nonzero if there are gauge-singlet chiral superfields.

The following set of relations are RG invariant to all orders of perturbation theory:

$$M = M_0 \frac{\beta_g}{g}, \tag{A.11a}$$

$$h^{ijk} = -M_0 \beta_Y^{ijk}, \tag{A.11b}$$

$$(m^2)^i{}_j = \tfrac{1}{2}|M_0|^2 \mu \frac{d\gamma^i{}_j}{d\mu}, \tag{A.11c}$$

$$b^{ij} = -M_0 \beta_\mu^{ij} - M_0 Y^{ijk} Z_k, \tag{A.11d}$$

$$c^i = \tfrac{1}{2}|M_0|^2 \left[\mu \frac{d}{d\mu} Z^i + (\gamma Z)^i\right] - M_0 \mu^{il} Z_l. \tag{A.11d}$$

In this paper, in common with most of the AMSB literature, we focus on Eqs. (A.11$a-c$), by choosing theories such that $Z_i = 0$, and assuming an alternative source for b^{ij}, so that it can be treated as a free parameter and determined in the usual way by the minimisation conditions at the weak scale. Because the relevant β-functions do not involve b^{ij} and c^i, Eqs. (A.11$a-c$) form a RG-invariant subset.

For a U_1 theory we can also consider the possibility of a FI term, ξD. In the unbroken theory ξ is unrenormalised, as long as $\mathrm{Tr}\mathcal{Y} = 0$, where $\mathcal{Y}$ is the hypercharge matrix[33]. In the presence of soft breaking, the renormalisation of ξ has been studied up to three loops [34]–[36]; however there exists no exact relation expressing its β-function in terms of β_g and γ of the type Eq. (A.4).

For a U_1 theory, the AMSB solution for m^2 in the D-eliminated case may be generalised to either

$$(m^2)^i{}_j = \tfrac{1}{2}|M_0|^2 \mu \frac{d\gamma^i{}_j}{d\mu} + g\xi^{\mathrm{RG}}(\mathcal{Y})^i{}_j, \tag{A.12}$$

where ξ^{RG} is the RG solution for ξ, or

$$(m^2)^i{}_j = \tfrac{1}{2}|M_0|^2 \mu \frac{d\gamma^i{}_j}{d\mu} + \zeta(\mathcal{Y})^i{}_j, \tag{A.13}$$

where ζ is a constant. It is the latter possibility that forms Case (FI) of this paper.

If the theory admits a $\mathcal{R}$ symmetry then there is an alternative which is also exactly RG invariant,

$$(m^2)^i{}_j = \tfrac{1}{2}|M_0|^2 \mu \frac{d\gamma^i{}_j}{d\mu} + m_0^2(\gamma^i{}_j + \overline{q}^i \delta^i{}_j), \tag{A.14}$$

where

$$\overline{q}^i = 1 - \tfrac{3}{2} r^i, \tag{A.15}$$

and r^i are the $\mathcal{R}$-charges. This is Case $\mathcal{R}$ in this paper.

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References

[1] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. **B**147 (1979) 277 ;
M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. **B**398 (1993) 319 ;
P. Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. **B**406 (1993) 19

[2] L. Randall and R. Sundrum, Nucl. Phys. **B**557 (1999) 79

[3] G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, JHEP 9812 (1998) 27

[4] I. Jack and D.R.T. Jones, Phys. Lett. **B**465 (1999) 148

[5] I. Jack and D.R.T. Jones, Phys. Lett. **B**482 (2000) 167

[6] I. Jack and D.R.T. Jones, Phys. Lett. **B**491 (2000) 151

[7] A. Pomarol and R. Rattazzi, JHEP 9905 (1999) 013;
M.A. Luty and R. Rattazzi, JHEP 9911 (1999) 001

[8] N. Arkani-Hamed et al, JHEP 0102 (2001) 041

[9] N. Kitazawa, N. Maru and N. Okada, Phys. Rev. **D**63 (2001) 015005; Nucl. Phys. **B**586 (2000) 261; Phys. Rev. **D**62 (2000) 077701

[10] I. Jack, D.R.T. Jones and R. Wild, Phys. Lett. **B**535 (2002) 193

[11] I. Jack, D.R.T. Jones and A. Pickering, Phys. Lett. **B**426 (1998) 73

[12] E. Kraus and D. Stockinger, Phys. Rev. **D**65 (2002) 105014

[13] M. Green and J. Schwarz, Phys. Lett. **B**149 (1984) 117

[14] P.F. Harrison and W.G. Scott, Phys. Lett. **B**535 (2002) 163

[15] P. Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. **B**406 (1993) 19

[16] A.H. Chamseddine and H.K. Dreiner, Nucl. Phys. **B**458 (1996) 65;
D.J. Castano, D.Z. Freedman and C. Manuel, Nucl. Phys. **B**461 (1996) 50

[17] S. Tanaka, Phys. Lett. **B**480 (2000) 296

[18] L.L. Everett, G.L. Kane and S.F. King, JHEP 0008 (2000) 012

[19] R. Rattazzi, A. Strumia and J.D. Wells, Nucl. Phys. **B**576 (2000) 3

[20] N. Abe and M. Endo, hep-ph/0212002

[21] T. Moroi, Phys. Rev. **D**53 (1996) 6565 [Erratum-*ibid* Phys. Rev. **D**56 (1997) 4424];
S.P. Martin and J.D. Wells, Phys. Rev. **D**64 (2001) 035003

[22] G.W. Bennett *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. 89 (2002) 101804 [Erratum- *ibid.* 89 (2002) 129903]

[23] M. Davier, S. Eidelman, A. Hocker and Z. Zhang, hep-ph/0208177.

[24] S. Narison, Phys. Lett. **B**513 (2001) 53 [Erratum-*ibid.* Phys. Lett. **B**526 (2002) 414];
J.F. De Troconiz and F.J. Yndurain, Phys. Rev. **D**65 (2002) 093001;
K. Hagiwara, A.D. Martin, D. Nomura and T. Teubner, hep-ph/0209187

[25] B.C. Allanach and A. Dedes, JHEP 0006 (2000) 017

[26] F. de Campos et al, Nucl. Phys. **B**623 (2002) 47

[27] R.N. Mohapatra, hep-ph/0211252

[28] H.K. Dreiner, hep-ph/9707435;
B.C. Allanach, A. Dedes and H.K. Dreiner, Phys. Rev. **D**60 (1999) 075014

[29] S.F. King, hep-ph/0208266

[30] I. Jack, D.R.T. Jones and R. Wild, Phys. Lett. **B**509 (2001) 131

[31] L. O'Raifeartaigh, Nucl. Phys. **B**96 (1975) 331

[32] I. Jack, D.R.T. Jones and A. Pickering, Phys. Lett. **B**432 (1998) 114

[33] W. Fischler et al, Phys. Rev. Lett. 47 (1981) 757

[34] I. Jack and D.R.T. Jones, Phys. Lett. **B**473 (2000) 102

[35] I. Jack, D.R.T. Jones and S. Parsons, Phys. Rev. **D**62 (2000) 125022

[36] I. Jack and D.R.T. Jones, Phys. Rev. **D**63 (2001) 075010