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Finite Size Effects in Nucleon Masses in Dynamical QCD *

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For lattice calculations with light dynamical quarks, finite size effects have become an important aspect. We study finite size effects in nucleon masses on $N_f = 2$ dynamical lattices of $1 - 2$ fm. Predictions for the finite size effects are obtained in one-loop chiral perturbation theory.

1. INTRODUCTION

For a lattice calculation of physical quantities such as hadron masses it is desirable to have a theoretical understanding of finite size effects and to be able to extrapolate to infinite volume. On lattices used in typical quenched calculations, finite size effects of hadron masses seem to be small, but they may be considerable in the dynamical case [1]. A source of finite size effects is a virtual pion going around the boundary of the lattice before it is absorbed again. At sufficiently small pion masses and large volumes this effect can be described by chiral perturbation theory, the low-energy effective theory of nucleons and pions, in a finite box. Here, we discuss finite size effects on nucleon masses generated by the QCDSF and UKQCD collaborations.

2. THE SIMULATION

The simulations were done using the plaquette gauge action and $N_f = 2$ dynamical non-perturbatively $O(a)$-improved Wilson fermions. We consider lattices of size ~ 1 fm, ~ 1.6 fm, and ~ 2 fm. Pion masses are in the range of about $0.6 - 1$ GeV. The lattice spacing is $a \approx 0.1$ fm. We use $r_0 = 0.5$ fm to set the scale. Valence and sea quark masses are taken to be equal.

The lattice data for the nucleon mass are plotted in Fig. 1. We see significant finite size effects. We expect $O(a^2)$ effects to be small. Indeed, our

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two mass points at $r_0 m_{PS} \simeq 2$ on the 1.6 fm lattice differ by less than a percent, whereas a^2 differs by $O(20)\%$.

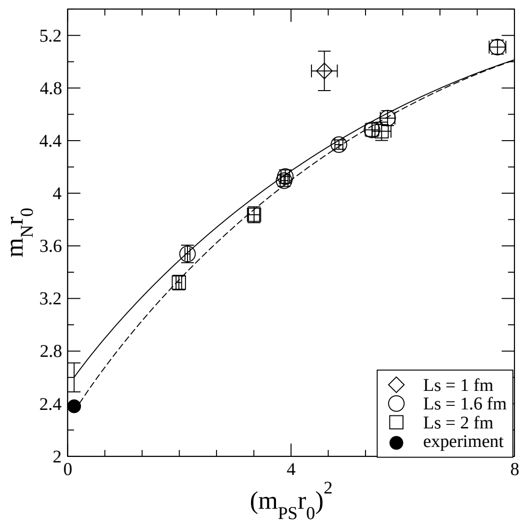

Figure 1. Nucleon mass as a function of the pion mass with extrapolation to the physical value according to Eq. (1). The solid line denotes a fit to the 1.6 fm lattices. The dashed line corresponds to the 2 fm lattices, without error bar on the corresponding chiral extrapolation, since there are only three points. The heaviest point at 1.6 fm is not included in the fit.

Chiral perturbation theory in the infinite volume predicts

$$m_N r_0 \quad = \quad a + b(m_{PS} r_0)^2 + c(m_{PS} r_0)^3, \qquad (1)$$

where the cubic term reflects the leading non-analytic (LNA) behaviour of the nucleon self-energy. The chiral extrapolation is performed separately for different lattice sizes using Eq. (1). At 1.6 fm, the chiral extrapolation gives a value that is 8% or 2σ higher than experiment, at 2 fm no discrepancy is found. The coefficient c disagrees significantly with the infinite volume LNA prediction.

3. THEORETICAL PREDICTIONS

The finite size effects are calculated from the difference of the nucleon self-energy Σ in finite and infinite spatial volume,

$$\delta m_N = \Sigma(L_s) - \Sigma(\infty). \qquad (2)$$

In the perturbative calculation, the x_4 direction is taken to be of infinite extent. We calculate the self-energy at one loop in Heavy Baryon χPT according to the lattice prescription given in Ref. [2] adapted to the case of 2-flavours. Additionally, we include Δ intermediate states. For this we discretise the $N\Delta\pi$ interaction Lagrangian given in Ref. [3]. The numerical values for f_π, $g_A = \mathcal{D}+\mathcal{F}$, and the $N\Delta\pi$ coupling c_A are also taken from this reference. The relevant Feynman diagrams are given in Fig. 2.

To regularize the lattice integrals we vary the cut-off Λ between $\pi/a = 6$ GeV and ∞. The difference between using $\Lambda = 6$ GeV and the continuum limit is around 10% for $m_{PS} \simeq 600$ MeV. The values presented here are for $\Lambda \approx 30$ GeV, which practically amounts to infinite cut-off. Results for the smallest pion mass used in the simulation at 1.6 fm are given in Fig. 3. For $L_s \geq 1.6$ fm, the L_s dependence can be approximated by an exponential decay,

$$\delta m_N = (c_0/L_s)\exp(-c_1 L_s), \qquad (3)$$

where the fit with Eq. (3) describes the χPT result with an accuracy of several percent.

The finite size effect of the nucleon mass has been estimated previously, relating the mass shift to the πN scattering amplitude [4]. A comparison with our one-loop result is given in Fig. 3.

In Fig. 4, we show $m_N - \delta m_N$. We see that the masses on the smaller volumes are brought down towards a universal curve. In particular, 50% of the finite size corrections at $L_s = 1$ fm, and 60% at $L_s = 1.6$ fm, are accounted for by chiral perturbation theory. We consider this a remarkable result, considering that the pion mass is relatively heavy.

4. CONCLUSIONS AND DISCUSSION

We find indications for finite size effects in the nucleon mass on lattices of $1.6 - 2$ fm size. We

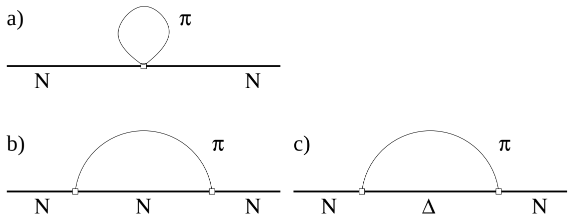

Figure 2. Self-energy diagrams relevant for the calculation at one loop.

calculated the finite volume corrections in lattice-regularised χPT at one loop. At $L_s \geq 1.6$ fm, the correction is of the order of a few percent for the pion masses considered. χPT describes about $50 - 60\%$ of the finite size effects of the data. The mass dependence is exponential rather than power-like even for intermediate lattices and pion masses. Higher order corrections to the one-loop calculation may be important at present pion masses.

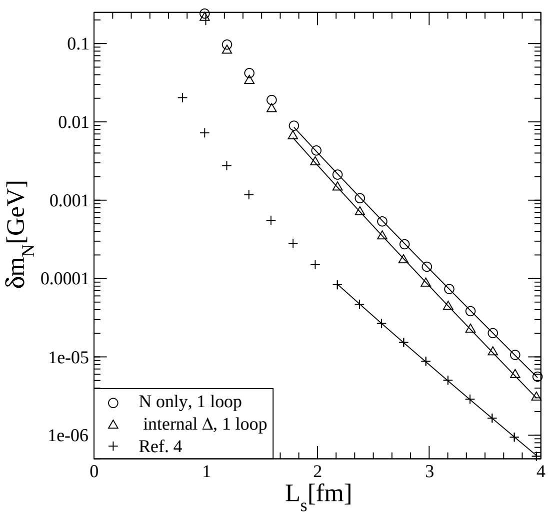

Figure 3. L_s dependence of δm_N for $m_{PS} = 578$ MeV. Circles denote the sum of diagrams a) and b) of Fig. 2, triangles the contribution of diagram c) of Fig. 2, and plusses the result of Ref. [4]. Fits to $\delta m_N = (c_0/L_s)\exp(-c_1 L_s)$ are drawn as solid lines.

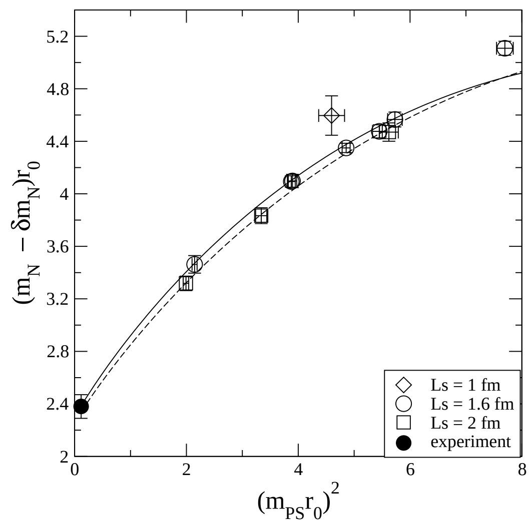

Figure 4. Nucleon masses minus the finite size effect calculated in one-loop χPT. As in Fig. 1, lines denote fits according to Eq. (1).

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