Chiral Nonanalytic Behaviour: The Edinburgh Plot

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The Edinburgh Plot is a scale independent way of presenting lattice QCD calculations over a wide range of quark masses. In this sense it is appealing as an indicator of how the approach to physical quark masses is progressing. The difficulty remains that even the most state of the art calculations are still at quark masses that are too heavy to apply dimensionally-regulated chiral perturbation theory. We present a method allowing predictions of the behaviour of the Edinburgh plot, in both the continuum, and on the lattice.

1. INTRODUCTION

It is now well established that the one- or two-loop truncated expansion of dimensionally-regulated (dim-reg) chiral perturbation theory (χPT) is unable to reach the masses at which current dynamical lattice QCD calculations are made. In our previous works we have presented improved regulation schemes that re-sum the series in a way that suppresses higher-order terms and increases its range of applicability. These functional forms for the nucleon, Δ [1] and ρ meson [2] not only reproduce the nonanalytic behaviour of dim-reg χPT in the small quark mass region, but have the same limit exhibited by lattice simulations at larger quark masses.

2. Extrapolation Forms

Goldstone Boson loops are implicitly included in all lattice calculations. The most important sources of nonanalytic behaviour include those self-energy processes which vary most rapidly with pion mass near the chiral limit. In the specific cases of the nucleon and rho meson the self-energy diagrams that contribute the dominant nonanalytic behaviour are shown in Figs. 1 and 2.

The construction of such functional forms relies on the realisation that the source of the Goldstone boson field is not point-like, as taken in dim-reg χPT. The source is in fact a complex system of quarks and gluons, and we use an optimal regulator based on this extended object to introduce an additional scale into χPT. This optimal regulation scheme extends the applicable range of χPT into the region where the lattice is now making calculations. The approach is systematic in that it can be improved by calculating the higher-order terms in the chiral expansion. This additional regulator scale also has a simple physical basis. If the mass of the Goldstone boson is larger than the regulator scale the Compton wavelength of the pion will be less than the extended size of the source, and so the induced effects are suppressed, whilst if the pseudoscalar mass is less than that of the source regulator the rapid non-linear behaviour is evidenced.

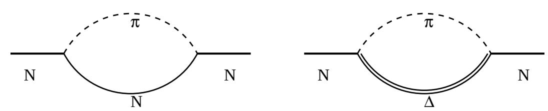

Figure 1. The one-loop pion induced self-energy of the nucleon.

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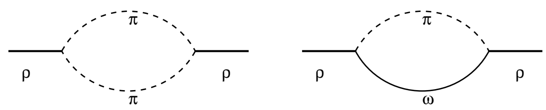

Figure 2. The one-loop pion induced self-energy of the rho meson.

The one-loop functional forms for the nucleon and ρ-meson are

$$m_N = \alpha + \beta m_\pi^2 + \sigma_N(\Lambda_N, m_\pi)\,, \quad (1)$$
$$m_\rho = A + B m_\pi^2 + \sigma_\rho(\Lambda_\rho, m_\pi)\,, \quad (2)$$

where the self-energy contributions of Figs. 1 and 2 are absorbed in the σ_N and σ_ρ, respectively, and Λ is the regulator parameter. In previous works we have shown that at light quark mass ($m_q \sim m_\pi^2$) we reproduce the exact nonanalytic behaviour of dim-reg χPT, with the correct *model independent* coefficients of the lowest nonanalytic terms[1,2]. As the mass of the quarks increases, toward those masses now being accessed on the lattice, the regulator parametrized by the additional scale Λ suppresses the Goldstone boson loops, resulting in the linear behaviour seen on the lattice.

It is important to not only attempt to respect the constraints of χPT in an extrapolation function, but to consider the differences between calculations performed on the lattice as compared to those done in the continuum. The process of putting quarks on the lattice changes the induced effects as a result of the discretisation and finite volume of space-time. The available momenta for the self-energy terms are both discretised and limited in value. Any extrapolation method ignoring this fundamental difference from the continuum is flawed. A simple replacement of the integral over loop momentum in the self-energy terms with a sum over the available momenta provides an estimate of this important insight.

Finally, the nature of the rho meson is considered. The rho is an unstable particle in nature and yet on the lattice we see no evidence for this decay. It is clear that an extrapolation form (in particular for unstable particles) must allow for decays in the right kinematic region. The preferred rho decay is p-wave to two pions, and we simply do not see this on the lattice because of the heavy quarks and large minimum non-vanishing momentum. However, in the continuum limit and at light quark masses, Eq. (2) does allow this decay.

3. The Edinburgh Plot

We present our prediction for the behaviour of the Edinburgh plot in Fig. 3. We have used data from the UKQCD [3] (open symbols) and CP-PACS [4] (filled symbols) collaborations. Two additional points are included in the figure. The physical point is indicated by the star at the experimentally measured ratios of the nucleon, rho and pion masses. The other star is the prediction from heavy quark theory, where the masses of the hadrons are proportional to the sum of their constituent quarks. The solid curve is the infinite volume, continuum prediction based on Eqs. (1) and (2) for the extrapolation of the nucleon and rho meson masses. The excellent agreement with the lattice calculations at reasonably large values of m_π/m_ρ is expected. The heavy quark mass suppresses the Goldstone effects and we reproduce the well known linear behaviour. The point of inflexion at $m_\pi/m_\rho = 1/2$ is an effect of the opening of the rho decay channel. The lowest CP-PACS point is below the opening of this channel. However, as discussed in previous work [2], the kinematics of their lattice do not allow the rho to decay. As a consequence, the self-energy contributions at this point are noticeably different between the continuum and lattice.

As we have the ability to make predictions for the extrapolated masses on both the lattice and in the continuum we consider a simple investigation of the behaviour of a lattice with 16 sites in the spatial directions with a lattice spacing of 0.13fm. We have used exactly the same fit parameters as previously, and present the results as the dashed line in Fig. 4. In addition, we overlay new data from the MILC collaboration [5] calculated on a similarly sized lattice. The agreement between our prediction and the data

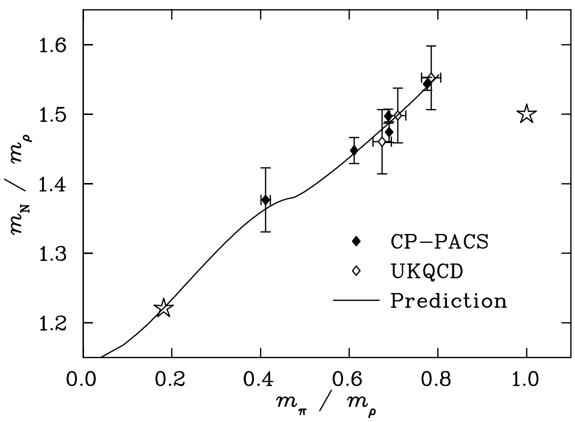

Figure 3. Edinburgh Plot for UKQCD [3] and CP-PACS [4] dynamical QCD calculations. The stars represent the known limiting cases, at the physical and heavy quark limits respectively. The solid line is the infinite volume, continuum limit behaviour predicted by our functional forms for the extrapolation of the N and ρ masses.

is encouraging as there is a suggestion of a divergence away from the continuum prediction at the lighter quark masses. We also note the significant difference between our prediction and the physical point at realistic quark masses is yet another indication that to get accurate calculations at light quark masses, large volume lattices are a necessity.

4. CONCLUSION

We have shown that a physically-motivated regulator leads to a well behaved chiral expansion that provides a method for extrapolating current lattice QCD calculations to light quark masses. The functional forms presented not only respect the constraints of traditional dim-reg χPT at light quark masses but also have the correct functional form as the quark masses tend to the region now inhabited by lattice QCD.

It has proved necessary to not only include the correct chiral behaviour, but to also allow decays where relevant and incorporate the modified physics of the lattice, in particular the discretisation of the available momenta.

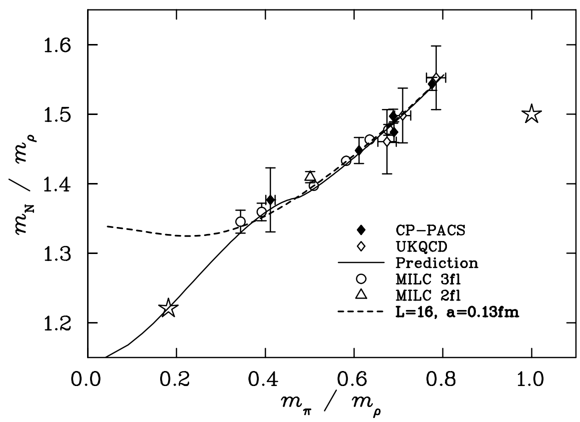

Figure 4. Edinburgh Plot as described in Fig. 3. The dashed curve is the predicted behaviour of the mass ratios on a finite lattice. The MILC data is from [5].

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REFERENCES

1. D. B. Leinweber, A. W. Thomas, K. Tsushima, S. V. Wright, Phys. Rev. D61 (2000) 074502.
2. D. B. Leinweber, A. W. Thomas, K. Tsushima, S. V. Wright, Phys. Rev. D64 (2001) 094502.
3. **UKQCD** Collaboration, C. R. Allton, et al., Phys. Rev. D60 (1999) 034507.
4. **CP-PACS** Collaboration, S. Aoki, et al., Phys. Rev. D60 (1999) 114508.
5. **MILC** Collaboration, D. Toussaint Private communications.