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Two loop $\overline{\text{MS}}$ renormalization of the Curci-Ferrari model

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Abstract. We renormalize the Curci-Ferrari model at two loops in the $\overline{\text{MS}}$ scheme in an arbitrary covariant gauge.

The Curci-Ferrari model is an extension of Yang-Mills theory where the gluon field, A^a_μ, is massive, [1, 2]. However, as is well known in such a massive non-abelian gauge theory, where there is no spontaneous symmetry breaking, the theory is not fully consistent. For instance, it does not obey unitarity, [2, 3, 4], though it has been shown to be multiplicatively renormalizable to all orders in perturbation theory, [1, 4, 5, 6]. Further, as the mass for the gluon is included in the Lagrangian in the simplest possible manner, there is the possibility that gauge invariance is also broken. However, as was shown in the original work of Curci and Ferrari, [1, 2], if one also includes a particular form of mass term for the ghost fields when the gauge is fixed covariantly in a non-linear way, the mass term in the Lagrangian in fact preserves BRST symmetry, though without restoring unitarity which remains broken. Despite the lack of unitarity the Curci-Ferrari model has been of interest recently for a variety of reasons. First, there have been several investigations into how a mass gap for the gluon can arise, [7, 8, 9]. For example, in [8, 9] a gluon and ghost mass term akin to that which occurs in the Curci-Ferrari model has been argued to be important in generating a mass gap in ordinary Yang-Mills theories. For instance, a non-zero vacuum expectation value for $\langle \frac{1}{2}(A^a_\mu)^2 + \alpha \bar{c}^a c^a \rangle$ emerges under certain assumptions where α is the covariant gauge parameter and c^a and $\bar{c}^a$ are the respective ghost and anti-ghost fields. Such an operator represents the simplest *local* dimension two operator which can be constructed in non-abelian gauge field theories. Also, a similar approach has been considered in [7] where the effective potential of the Landau gauge condensate is constructed. There it was shown that at two loops the non-perturbative vacuum favoured a non-zero value for the vacuum expectation value. Other studies include [10]. Second, the model itself has provided a useful laboratory for studying gauge theories with a non-linear gauge fixing, [5, 11]. Third, massive Yang-Mills models have been used to model the phenomenology of the strong interactions. For example, such a model was used in [12] to try and understand diffractive scattering. Finally, the main motivation for our interest in the Curci-Ferrari model stems from the role of the gluon mass as a *natural* infrared regulator for Feynman integrals which arise in loop calculations, [13, 6, 14].

In the renormalization of QCD at high loop orders the standard method of extracting the ultraviolet divergence structure of a Feynman integral with respect to the regularization, such as dimensional regularization, is to perform the calculation with massless propagators. However, for two and higher loop integrals one has to be careful that spurious infrared infinities are not generated. These can occur when one reduces the class of Feynman integrals to vacuum integrals by expanding in powers of the external momenta after one or more external momenta have been set to zero first. To circumvent this potential problem the technique of infrared rearrangement, [15, 16], is applied where a temporary mass is added to the appropriate propagators in an infrared divergent integral. Whilst this has been a hugely successful technique it suffers from the drawback that it is performed by hand and hence limited when considering the large number of diagrams which will arise when the loop order increases. Moreover, it was not clear until recently how one could develop the method for application in an automatic multiloop computer algebra programme. One recent approach to address this problem is that of [17, 18]. There each propagator is systematically given an infrared cutoff mass so that the resulting vacuum integrals are automatically infrared finite. Provided all the vacuum diagrams are computable at that loop order then one can automatize the process. Indeed [17, 18] provides a full two loop calculation of various anomalous dimensions. Given that the Curci-Ferrari model *naturally* incorporates a mass akin to that introduced for infrared rearrangement and *remains* renormalizable it seems appropriate to consider and develop that model as a useful and alternative tool to compute the ultraviolet structure of Yang-Mills theories. This is important since the efficient programmes such as MINCER, [19], compute the ultraviolet structure of only massless two-point functions at three loops, but for n-point functions with $n > 2$ one cannot naively nullify all bar two of the external momenta, similar to infrared rearrangement, and apply MINCER. This is because this procedure will inevitably give rise to spurious infrared infinities which, in dimensional regularization, cannot be distinguished from the desired ultraviolet divergences. Hence, it would seem more appropriate to us to apply the Curci-Ferrari model in these circumstances. However, it turns out that currently the model has only been renormalized to one loop in [4]. Moreover, an earlier calculation, [14], appears to incorrectly determine the renormalization constants which must satisfy the appropriate Slavnov-Taylor identities. Therefore, the purpose of this letter is to renormalize the Curci-Ferrari model at two loops in the $\overline{\mathrm{MS}}$ scheme using an arbitrary (non-linear) covariant gauge fixing term and dimensional regularization. This is necessary given that the renormalization constants of the fields, coupling, gauge parameter and masses are required for the subsequent renormalization of any Green's function. As a consequence of our calculations we will verify the result of [4] that five renormalization constants are necessary to render the model finite as opposed to the three suggested in [3].

The Lagrangian of the Curci-Ferrari model is, [1],

$$\begin{aligned} L ~=~ & -~ \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} ~-~ \frac{1}{2\alpha} (\partial^\mu A^a_\mu)^2 ~-~ \frac{m^2}{2} A^a_\mu A^{a\,\mu} ~+~ \partial_\mu \bar{c}^a \partial^\mu c^a \\ & -~ \alpha m^2 \bar{c}^a c^a ~-~ \frac{g}{2} f^{abc} A^a_\mu \, \bar{c}^b \, \overset{\leftrightarrow}{\partial^\mu} \, c^c ~+~ \frac{\alpha g^2}{8} f^{eab} f^{ecd} \bar{c}^a c^b \bar{c}^c c^d \end{aligned} \qquad (1)$$

where $1 \leq a \leq N_A$ with N_A the dimension of the adjoint representation, $D_\mu = \partial_\mu + igT^a A^a_\mu$, $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$, g is the coupling constant and m is the gluon mass. The ghost mass is related to the gluon mass by the covariant gauge fixing parameter α. The structure constants of the Lie group are f^{abc} and $\bar{c}^a \, \overset{\leftrightarrow}{\partial_\mu} \, c^b = \bar{c}^a \partial_\mu c^b - (\partial_\mu \bar{c}^a) c^b$. Following the usual procedures the gluon and ghost propagators are respectively

$$-~ \delta^{ab} \left[\frac{\eta^{\mu\nu}}{(k^2 + m^2)} ~-~ \frac{(1-\alpha) k^\mu k^\nu}{(k^2 + m^2)(k^2 + \alpha m^2)} \right] ~~~ \text{and} ~~~ \frac{\delta^{ab}}{(k^2 + \alpha m^2)} ~. \qquad (2)$$

The unphysical pole at αm^2 in the ghost propagator had led to the hope that it would counteract the same pole in the gluon propagator to establish a unitary theory. However, it has been shown, [1, 2, 3, 4], that this is not the case. To renormalize (1) the quantities of the bare Lagrangian, denoted by the subscript o, are replaced by the renormalized ones, through,

$$\begin{aligned} A^{a\,\mu}_{\mathrm{o}} ~&=~ \sqrt{Z_A} \, A^{a\,\mu} ~~,~~ c^a_{\mathrm{o}} ~=~ \sqrt{Z_c} \, c^a ~~,~~ \bar{c}^a_{\mathrm{o}} ~=~ \sqrt{Z_c} \, \bar{c}^a \\ g_{\mathrm{o}} ~&=~ Z_g \, g ~~,~~ m_{\mathrm{o}} ~=~ Z_m \, m ~~,~~ \alpha_{\mathrm{o}} ~=~ Z^{-1}_\alpha Z_A \, \alpha ~. \end{aligned} \qquad (3)$$

We have assumed initially that there are five independent renormalization constants. In ordinary Yang-Mills one has three renormalization constants since clearly there is no need for Z_m and the Slavnov-Taylor identities ensure that $Z_\alpha = 1$ in our notation to all orders to preserve gauge symmetry. Here we will allow for an independent Z_α as in [4]. To find each of the renormalization constants to two loops the gluon and ghost two-point functions are computed first which determine Z_A, Z_α, Z_c and Z_m. It is worth pointing out that since the gluon two-point function gives Z_A, Z_α and Z_m the structure of the ghost mass renormalization is determined prior to calculating it. This in fact provides a strong check on our results. In addition we have computed the ghost-gluon and triple gluon vertex renormalizations to verify that the correct scheme independent two loop β-function emerges in both the gluon and ghost sectors. The renormalization group functions are defined by, (see, for example, [20]),

$$\begin{aligned} \gamma_A(a) ~&=~ \mu \frac{\partial \ln Z_A}{\partial \mu} ~~,~~ \gamma_c(a) ~=~ \mu \frac{\partial \ln Z_c}{\partial \mu} ~~,~~ \beta(a) ~=~ \mu \frac{\partial a}{\partial \mu} \\ \gamma_m(a) ~&=~ \frac{\partial \ln m}{\partial \ln \mu} ~~,~~ \gamma_\alpha(a) ~=~ \frac{\partial \ln \alpha}{\partial \ln \mu} \end{aligned} \qquad (4)$$

where $a = g^2/(16\pi^2)$. From (3) these imply

$$\begin{aligned}
\gamma_A(a) &= \beta(a)\frac{\partial \ln Z_A}{\partial a} ~+~ \alpha\gamma_\alpha(a)\frac{\partial \ln Z_A}{\partial \alpha} \\
\gamma_\alpha(a) &= \left[\beta(a)\frac{\partial \ln Z_\alpha}{\partial a} ~-~ \gamma_A(a)\right]\left[1 ~-~ \alpha\frac{\partial \ln Z_\alpha}{\partial \alpha}\right]^{-1} \\
\gamma_m(a) &= -~\mu\frac{\partial \ln Z_m}{\partial \mu} ~.
\end{aligned} \tag{5}$$

Ordinarily in massless Yang-Mills theory one has the simple relation $\gamma_\alpha(a) = -~\gamma_A(a)$. When the condition $Z_\alpha = 1$ is not satisfied the more general relation, (5), emerges.

To compute the ultraviolet structure of the two and three point functions we have followed the strategy of [17, 18] of expanding out the Feynman integrals to the terms involving two or three external momenta respectively. Given that the Curci-Ferrari model is renormalizable, [1, 4, 5, 6], it follows that terms involving more external momenta will not contribute to the ultraviolet divergences. Hence one is left with massive vacuum two loop Feynman integrals to compute where, unlike the calculation of [17, 18], the masses in the propagators are not all the same. Such integrals have been widely studied before and we quote the result of [21], for instance, for the basic two loop topology. In general we have

$$\begin{aligned}
\int_{kl} \frac{1}{(k^2+m_1^2)(l^2+m_2^2)[(k-l)^2+m_3^2]} &= -~(m_1^2+m_2^2+m_3^2)\left(\frac{1}{2\epsilon^2} ~+~ \frac{3}{2\epsilon} ~+~ \frac{1}{\epsilon}\ln(4\pi e^{-\gamma})\right) \\
&+~ \left(m_1^2\ln m_1^2 ~+~ m_2^2\ln m_2^2 ~+~ m_3^2\ln m_3^2\right)\frac{1}{\epsilon} ~+~ O(1)
\end{aligned} \tag{6}$$

for arbitrary masses m_i where $\int_k = \int d^dk/(2\pi)^d$, $d = 4 - 2\epsilon$ with ϵ the regularizing parameter and γ is the Euler-Mascheroni constant. The integrals involving different powers of a propagator are given by differentiating with respect to the appropriate masses. It is important to realise the role of the logarithmic terms. For instance, specifying various values for m_i^2 we have

$$\begin{aligned}
\int_{kl} \frac{1}{(k^2+m^2)(l^2+m^2)[(k-l)^2+\alpha m^2]} &= -~(\alpha+2)m^2\left(\frac{1}{2\epsilon^2} ~+~ \frac{3}{2\epsilon} ~+~ \frac{1}{\epsilon}\ln(4\pi e^{-\gamma})\right) \\
&+~ \left((\alpha+2)\ln m^2 ~+~ \alpha\ln\alpha\right)\frac{m^2}{\epsilon} ~+~ O(1)
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
\int_{kl} \frac{1}{(k^2+m^2)(l^2+\alpha m^2)[(k-l)^2+\alpha m^2]} &= -~(2\alpha+1)m^2\left(\frac{1}{2\epsilon^2} ~+~ \frac{3}{2\epsilon} ~+~ \frac{1}{\epsilon}\ln(4\pi e^{-\gamma})\right) \\
&+~ \left((2\alpha+1)\ln m^2 ~+~ 2\alpha\ln\alpha\right)\frac{m^2}{\epsilon} ~+~ O(1) ~.
\end{aligned} \tag{8}$$

Therefore, the final renormalization constants could potentially contain $\ln\alpha$ terms. Moreover, in writing two and three point functions in the basic form (8), one uses partial fractions so that

$$\frac{1}{(k^2+m^2)(k^2+\alpha m^2)} = \frac{1}{(1-\alpha)m^2}\left[\frac{1}{(k^2+\alpha m^2)} ~-~ \frac{1}{(k^2+m^2)}\right] \tag{9}$$

which introduces, in addition, powers of $1/(1-\alpha)$ which also could appear in the renormalization constants. Given that $\ln\alpha$ and $1/(1-\alpha)^n$ for $n \geq 1$ are singular at $\alpha = 0$ and $\alpha = 1$ respectively, it might be expected that at two loops there will be a problem in the Landau and Feynman gauges respectively. However, since the original gluon propagator, (2), can be rewritten as

$$- \delta^{ab} \left[\frac{\eta^{\mu\nu}}{(k^2+m^2)} ~-~ \frac{k^\mu k^\nu}{m^2} \left(\frac{1}{(k^2+\alpha m^2)} ~-~ \frac{1}{(k^2+m^2)} \right) \right] \tag{10}$$

there ought to be no problems at $\alpha = 1$ which provides an internal consistency check on the computation. Moreover, the potential $\alpha = 0$ singularity is avoided by the fact that the basic one loop integral has the following ϵ-expansion

$$\int_k \frac{1}{(k^2+\alpha m^2)} ~=~ -~ \frac{\alpha m^2}{\epsilon} \left[1 ~+~ [1 - \gamma - \ln(\alpha m^2)]\epsilon ~+~ O(\epsilon^2) \right] ~. \tag{11}$$

Hence, the counterterms which arise from the one loop diagrams ought to cancel out the $\ln(\alpha)/\epsilon$ poles arising in integrals of the type we have discussed. Indeed in this context we avoid the usual approach of subtractions by using the method of [22]. There one computes the Green's functions in terms of the bare parameters and then rescales them at the end of the calculation in terms of the coupling constant expansion of the renormalization constants, (3), where each term of the expansion has already been determined. This strategy is appropriate given that we have carried out the calculation automatically using a symbolic manipulation programme written in FORM, [23], in order to handle the tedious amount of algebra. The Feynman diagrams for such an approach were generated using QGRAF, [24].

Consequently, we find the following $\overline{\mbox{MS}}$ renormalization constants for the Curci-Ferrari model, (1),

$$\begin{aligned}
Z_A &= 1 ~+~ C_A \left(\frac{13}{6} - \frac{\alpha}{2} \right) \frac{a}{\epsilon} \\
&\quad +~ C_A^2 \left[\left(\frac{3\alpha^2}{16} - \frac{17\alpha}{24} - \frac{13}{8} \right) \frac{1}{\epsilon^2} ~-~ \left(\frac{\alpha^2}{16} + \frac{11\alpha}{16} - \frac{59}{16} \right) \frac{1}{\epsilon} \right] a^2 ~+~ O(a^3) \\
Z_\alpha &= 1 ~-~ C_A \left(\frac{\alpha}{4} \right) \frac{a}{\epsilon} \\
&\quad +~ C_A^2 \left[\left(\frac{\alpha^2}{16} + \frac{3\alpha}{16} \right) \frac{1}{\epsilon^2} ~-~ \left(\frac{\alpha^2}{32} + \frac{5\alpha}{32} \right) \frac{1}{\epsilon} \right] a^2 ~+~ O(a^3) \\
Z_c &= 1 ~+~ C_A \left(\frac{3}{4} - \frac{\alpha}{4} \right) \frac{a}{\epsilon} \\
&\quad +~ C_A^2 \left[\left(\frac{\alpha^2}{16} - \frac{35}{32} \right) \frac{1}{\epsilon^2} ~-~ \left(\frac{\alpha^2}{32} - \frac{\alpha}{32} - \frac{95}{96} \right) \frac{1}{\epsilon} \right] a^2 ~+~ O(a^3) \\
Z_m &= 1 ~+~ C_A \left(\frac{\alpha}{8} - \frac{35}{24} \right) \frac{a}{\epsilon} \\
&\quad +~ C_A^2 \left[\left(- \frac{\alpha^2}{128} - \frac{53\alpha}{192} + \frac{1435}{384} \right) \frac{1}{\epsilon^2} ~+~ \left(\frac{\alpha^2}{64} + \frac{11\alpha}{64} - \frac{449}{192} \right) \frac{1}{\epsilon} \right] a^2 ~+~ O(a^3) \\
Z_g &= 1 ~-~ \frac{11}{6} C_A \frac{a}{\epsilon} ~+~ C_A^2 \left[\frac{121}{24} \frac{1}{\epsilon^2} ~-~ \frac{17}{6} \frac{1}{\epsilon} \right] a^2 ~+~ O(a^3)
\end{aligned} \tag{12}$$

where $f^{acd} f^{bcd} ~=~ C_A \delta^{ab}$. As an additional check on our two loop results we note that the $O(1/\epsilon^2)$ pole terms of each renormalization constant agrees with the value predicted through the renormalization group from the one loop pole. Using (5), these values for the renormalization constants lead to the renormalization group functions

$$\gamma_A(a) ~=~ C_A (3\alpha - 13) \frac{a}{6} ~+~ C_A^2 \left(\alpha^2 + 11\alpha - 59 \right) \frac{a^2}{8} ~+~ O(a^3)$$

$$
\begin{aligned}
\gamma_\alpha(a) &= -\,C_A(3\alpha - 26)\frac{a}{12} \;-\; C_A^2\left(\alpha^2 + 17\alpha - 118\right)\frac{a^2}{16} \;+\; O(a^3) \\
\gamma_c(a) &= C_A(\alpha - 3)\frac{a}{4} \;+\; C_A^2\left(3\alpha^2 - 3\alpha - 95\right)\frac{a^2}{48} \;+\; O(a^3) \\
\gamma_m(a) &= C_A(3\alpha - 35)\frac{a}{24} \;+\; C_A^2\left(3\alpha^2 + 33\alpha - 449\right)\frac{a^2}{96} \;+\; O(a^3) \\
\beta(a) &= -\,\frac{11}{3}C_A a^2 \;-\; \frac{34}{3}C_A^2 a^3 \;+\; O(a^4)\;. \qquad (13)
\end{aligned}
$$

The result for the β-function agrees with the original two loop scheme independent result of [25, 26]. As the gluon mass in (1) corresponds to the coupling of a gauge variant, and therefore unphysical, operator its mass anomalous dimension is gauge dependent. Further, the result for $\gamma_m(a)$ in the Landau gauge agrees with the calculation of [7] as an additional check on our computation. It is also worth noting that the explicit sum of the gluon wave function and gauge fixing parameter anomalous dimensions gives

$$
\gamma_A(a) \;+\; \gamma_\alpha(a) \;=\; \alpha\left[C_A\frac{a}{4} \;+\; C_A^2\,(\alpha + 5)\,\frac{a^2}{16}\right] \;+\; O(a^3)\;. \qquad (14)
$$

As a final check on our results we note that in the Landau gauge $\gamma_\alpha(a) = -\,\gamma_A(a)$ as in the massless Yang-Mills theory and it is trivial to verify that the Landau gauge renormalization group functions coincide precisely with the two loop results of [25, 26, 27, 22]. In this instance the gluon propagator, (2), takes the simple transverse form

$$
-\;\frac{\delta^{ab}}{(k^2 + m^2)}\left[\eta^{\mu\nu} \;-\; \frac{k^\mu k^\nu}{k^2}\right]\;. \qquad (15)
$$

Further, if we define the ghost mass as

$$
m_c^2 \;=\; \alpha m^2 \qquad (16)
$$

then in general, using $Z_{m_c} = Z_m Z_A^{\frac{1}{2}} Z_\alpha^{-\frac{1}{2}}$,

$$
\gamma_{m_c}(a) \;=\; \gamma_m(a) \;-\; \frac{1}{2}\gamma_A(a) \;+\; \frac{1}{2}\left[\beta(a)\frac{\partial \ln Z_\alpha}{\partial a} \;+\; \alpha\gamma_\alpha(a)\frac{\partial \ln Z_\alpha}{\partial \alpha}\right] \qquad (17)
$$

giving to two loops

$$
\gamma_{m_c}(a) \;=\; -\;\frac{3}{8}C_A a \;-\; C_A^2\,(18\alpha + 95)\,\frac{a^2}{96} \;+\; O(a^3)\;. \qquad (18)
$$

To conclude we have explicitly constructed all the basic renormalization group functions for the Curci-Ferrari model, (1), at two loops in $\overline{\mathrm{MS}}$. These will be important for using this theory to renormalize Green's functions in Yang-Mills theories where one cannot readily apply current automatic multiloop programmes to determine the ultraviolet structure of n-point functions with $n > 3$ and where an infrared regularization is also necessary. Further, given that QCD is more appropriate for practical phenomenology it would be interesting to extend our computations to include massive quarks in addition to the massive gluons and ghosts. Although that theory would remain non-unitary, multiplicative renormalizability ought still to be preserved and hence the quark extended Curci-Ferrari model would provide a useful tool for renormalizing full QCD.

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