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Three loop $\overline{\rm MS}$ tensor current anomalous dimension in QCD

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Abstract. The anomalous dimensions of the general flavour non-singlet quark bilinear currents $\bar{\psi}\gamma^{[\mu_1} \ldots \gamma^{\mu_n]}\psi$ are computed at three loops in quantum chromodynamics in the minimal subtraction scheme. The dimension of the tensor current emerges for the case $n = 2$ and the anomalous dimension for the general flavour singlet current is also discussed.

In recent years the construction of perturbative results at large numbers of loops in quantum chromodynamics, (QCD), has advanced substantially. For example, the β-function and various other renormalization group functions are now known to four loops in the $\overline{\rm MS}$ scheme, [1, 2, 3]. One motivation for such calculations rests in essence with the improvement in precision of experiments relating to the strong nuclear force. Whilst such four loop results represent the current limit of our ability to calculate in perturbative QCD, various quantities remain to be determined at three loops. For instance, whilst the anomalous dimension of various quark bilinear currents such as the axial vector current are known to at least three loops, [4, 5, 6], that of the closely related flavour non-singlet tensor current $\bar{\psi}\sigma^{\mu\nu}\psi$ where $\sigma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]$ and ψ is the quark field, is only known at two loops. This anomalous dimension occurs in the matching between currents in QCD heavy quark effective theory, [7]. In [7], the two loop $\overline{\rm MS}$ result was deduced indirectly as opposed to performing an explicit two loop renormalization. In this letter we complete the gap in the literature by providing the value of the three loop $\overline{\rm MS}$ anomalous dimension of the current $\bar{\psi}\sigma^{\mu\nu}\psi$. While this may appear to represent a moderate progression in this area, we choose to determine it not by explicit calculation of the current on its own but deduce it as a corollary of the renormalization of a set of generalized currents. These are $j^{\mu_1\cdots\mu_n}_{(n)} = \bar{\psi}\Gamma^{\mu_1\cdots\mu_n}_{(n)}\psi$, where $\Gamma^{\mu_1\cdots\mu_n}_{(n)}$ is the antisymmetric product of n γ-matrices. Clearly for QCD only the first five currents are non-zero but our motivation for considering the larger set relates to other issues. First, it is clear that for $n = 2$ the result we seek for the tensor current will emerge simply. However, by calculating with the general currents the results for $n = 0$ and 1 and the naive results for $n = 3$ and 4 will provide non-trivial checks on the calculation from the point of view of, say, symmetry factors and correctly including all the three loop Feynman diagrams. These three loop results have been determined in [8, 9, 5]. We have refered to the results for $n = 3$ and 4 as naive since in these cases there is a connection with the γ^5 problem of dimensional regularization. In particular the anomalous dimensions which emerge for $n = 3$ and 4 do not correspond directly with the anomalous dimensions of the axial vector and pseudoscalar currents respectively. The correct anomalous dimensions for these currents are determined by including a finite renormalization to ensure that chiral symmetry is preserved, [5]. Such a procedure was elaborated on in detail in [5] but this issue in the context of the matrices $\Gamma_{(n)}$ needs to be addressed due to its potential application to other operators such as those which contain four quark fields and one or more γ^5-matrices. One reason for this is that the algebra of the $\Gamma_{(n)}$-matrices has been widely studied, [10, 11, 12, 13], and also provides a simpler way to programme in a symbolic manipulation language than say the split γ^5 algebra of [14]. Indeed the renormalization of $j_{(n)}$ at three loops which we carry out here is performed with the use of the MINCER package, [15], written in the language FORM, [16]. While the above motivations for this work have been in relation to QCD, we note that knowledge of the anomalous dimension of the currents $j_{(n)}$ are necessary for other problems. For instance, the relation between QCD and the non-abelian Thirring model, (NATM), in d-dimensions has been developed in [17] based on the earlier observations of [18]. To understand the connection further, one needs not only to have knowledge of the fundamental renormalization group functions such as the β-function, but also the anomalous dimensions of the operators in both theories as well. By providing these for the n-dependent operators additional information is being determined for this area. For example, in the strictly two dimensional NATM the currents with $n = 3$ and 4 would be zero. However, in the d-dimensional context of the fixed point equivalence with QCD, [17, 18], they would be evanescent and important for the connection since from the QCD point of view they do not vanish in the limit to four dimensions.

We now turn to the discussion of the computation. The procedure to renormalize the currents $j^{\mu_1\cdots\mu_n}_{(n)}$ is to insert the operator in the quark two point function, use dimensional regularization and determine the poles in ϵ, where $d = 4 - 2\epsilon$, before minimally subtracting them. In particular

the set of Feynman diagrams to three loops are generated with the QGRAF package, [19], and converted into FORM input for processing with the MINCER procedures, [15]. As this is a well documented method we merely note the major issues in relation to the operators we are interested in. First, by constructing currents with the generalized γ-matrices $\Gamma^{\mu_1 \ldots \mu_n}_{(n)}$ defined by

$$\Gamma^{\mu_1 \ldots \mu_n}_{(n)} \;=\; \gamma^{[\mu_1} \ldots \gamma^{\mu_n]} \tag{1}$$

there is no mixing under renormalization between any of the $j^{\mu_1 \ldots \mu_n}_{(n)}$. Next to apply the MINCER routines the Green's function must be converted to scalar integrals. This requires multiplying the quark two point function by $\Gamma_{(n)\,\mu_1 \ldots \mu_n}$, taking the spinor trace and dividing by the normalization $\mathrm{tr}(\Gamma^{\mu_1 \ldots \mu_n}_{(n)} \Gamma_{(n)\,\mu_1 \ldots \mu_n})$. As we are only interested in flavour non-singlet currents for the moment, the γ-matrix strings contain either two $\Gamma_{(n)}$'s or none. For those with none the trace operations is readily performed. For the former strings one has to use the properties of the $\Gamma^{\mu_1 \ldots \mu_n}_{(n)}$ which are given in [11, 12, 13]. It transpires that we only need a subset of those results. By considering the Feynman diagrams which arise at three loops it is clear that the two $\Gamma_{(n)}$'s are separated by at most six ordinary γ-matrices. Therefore, from the general expression, [12],

$$\Gamma^{\mu_1 \ldots \mu_n}_{(n)} \Gamma^{\nu_1 \ldots \nu_m}_{(m)} \Gamma_{(n)\,\mu_1 \ldots \mu_n} \;=\; f(n,m) \Gamma^{\nu_1 \ldots \nu_m}_{(m)} \tag{2}$$

it is easy to deduce

$$\Gamma^{\mu_1 \ldots \mu_n}_{(n)} \gamma^\mu \gamma^\nu \Gamma_{(n)\,\mu_1 \ldots \mu_n} \;=\; f(n,2) \gamma^\mu \gamma^\nu \;+\; (f(n,0) - f(n,2)) \eta^{\mu\nu} \tag{3}$$

and

$$\begin{aligned} \Gamma^{\mu_1 \ldots \mu_n}_{(n)} \gamma^\mu \gamma^\nu \gamma^\sigma \gamma^\rho \Gamma_{(n)\,\mu_1 \ldots \mu_n} \;=\; & f(n,4) \gamma^\mu \gamma^\nu \gamma^\sigma \gamma^\rho \\ & + (f(n,2) - f(n,4)) \left[\eta^{\mu\nu} \gamma^\sigma \gamma^\rho - \eta^{\mu\sigma} \gamma^\nu \gamma^\rho + \eta^{\mu\rho} \gamma^\nu \gamma^\sigma \right. \\ & \qquad \left. + \eta^{\sigma\rho} \gamma^\mu \gamma^\nu - \eta^{\nu\rho} \gamma^\mu \gamma^\sigma + \eta^{\nu\sigma} \gamma^\mu \gamma^\rho \right] \\ & + (f(n,4) - 2f(n,2) + f(n,0)) \left[\eta^{\mu\nu} \eta^{\sigma\rho} - \eta^{\mu\sigma} \eta^{\nu\rho} + \eta^{\mu\rho} \eta^{\nu\sigma} \right] . \end{aligned} \tag{4}$$

The analogous relation involving six γ-matrices between two contracted $\Gamma_{(n)}$'s is too large to quote here though it clearly will involve the function $f(n,6)$. The values of $f(n,m)$ we require are deduced from the general formula given in [12]. We find

$$\begin{aligned} f(n,2) \;=\; & \left[d^2 - d - 4dn + 4n^2 \right] \frac{f(n,0)}{d(d-1)} \\ f(n,4) \;=\; & \left[d^4 - 8d^3 n - 6d^3 + 24d^2 n^2 + 24d^2 n + 11d^2 - 32dn^3 - 24dn^2 \right. \\ & \left. - 32dn - 6d + 16n^4 + 32n^2 \right] \frac{f(n,0)}{d(d-1)(d-2)(d-3)} \\ f(n,6) \;=\; & \left[1 \;-\; 12 \frac{(d-n)}{d} \;+\; 60 \frac{(d-n)(d-n-1)}{d(d-1)} \;-\; 160 \frac{(d-n)(d-n-1)(d-n-2)}{d(d-1)(d-2)} \right. \\ & + \; 240 \frac{(d-n)(d-n-1)(d-n-2)(d-n-3)}{d(d-1)(d-2)(d-3)} \\ & - \; 192 \frac{(d-n)(d-n-1)(d-n-2)(d-n-3)(d-n-4)}{d(d-1)(d-2)(d-3)(d-4)} \\ & \left. + \; 64 \frac{(d-n)(d-n-1)(d-n-2)(d-n-3)(d-n-4)(d-n-5)}{d(d-1)(d-2)(d-3)(d-4)(d-5)} \right] f(n,0) \; . \end{aligned} \tag{5}$$

They have been written in terms of $f(n,0)$ since this also occurs in the normalizing factor and hence will cancel in the computation. With these lemmas the $\Gamma_{(n)}$'s can be removed from all γ-strings and the spinor trace evaluated in the normal fashion which then means the integrals are in the correct format for applying the MINCER procedures.

Taking into account the three loop wave function renormalization, [20], of the quark fields present in the original operator, we find the following result for the anomalous dimension of $j_{(n)}$ for arbitrary n,

$$\begin{aligned}
\gamma_{(n)}(a) &= -(n-1)(n-3)C_F a \\
&+ \Big[4(n-15)T_F N_f + (18n^3-126n^2+163n+291)C_A \\
&\quad - 9(n-3)(5n^2-20n+1)C_F\Big]\frac{(n-1)C_F a^2}{18} \\
&+ \Big[\Big(144n^5-1584n^4+6810n^3-15846n^2+15933n+11413 \\
&\quad - 216n(n-3)(n-4)(2n^2-8n+13)\zeta(3)\Big)C_A^2 \\
&\quad - \Big(3(72n^5-792n^4+3809n^3-11279n^2+15337n+1161) \\
&\quad\quad - 432n(n-3)(n-4)(3n^2-12n+19)\zeta(3)\Big)C_A C_F \\
&\quad + \Big(8(3n^3+51n^2-226n-278) + 1728(n-3)\zeta(3)\Big)C_A T_F N_f \\
&\quad - \Big(18(n-3)(17n^4-136n^3+281n^2-36n+129) \\
&\quad\quad + 864n(n-3)(n-4)(n^2-4n+6)\zeta(3)\Big)C_F^2 \\
&\quad - \Big(12(17n^3+n^2-326n+414) + 1728(n-3)\zeta(3)\Big)C_F T_F N_f \\
&\quad + 16(13n-35)T_F^2 N_f^2\Big]\frac{(n-1)C_F a^3}{108} + O(a^4) \; . \qquad (6)
\end{aligned}$$

where $a = \alpha_s/4\pi = g^2/(16\pi^2)$ is the coupling constant, T_F, C_F and C_A are the usual colour group Casimirs and N_f is the number of quark flavours. There are various checks on this result. First, as the operator itself is gauge invariant, its anomalous dimension must be independent of the covariant gauge parameter, ξ. Therefore, in the calculation we used a gluon propagator of the form $(\eta_{\mu\nu} - \xi p_\mu p_\nu/p^2)/p^2$ where p is the momentum, and observed that ξ cancelled in the final result, (6). Second, the known three loop $\overline{\rm MS}$ results for $n = 0$, [8, 9], and 1 emerge. In the latter case the anomalous dimension vanishes as the current corresponds to the conserved electric current. Moreover, the value of (6) for $n = 3$ and 4 corresponds to the naive values obtained by Larin in [5]. Hence, we can deduce that the anomalous dimension for the tensor current $\bar{\psi}\sigma^{\mu\nu}\psi$ is

$$\begin{aligned}
\gamma_{(2)}(a) &= C_F a + [257C_A - 171C_F - 52T_F N_f]\frac{C_F a^2}{18} \\
&+ \Big[13639C_A^2 - 4320\zeta(3)C_A^2 + 12096\zeta(3)C_A C_F \\
&\quad - 20469C_A C_F - 1728\zeta(3)C_A T_F N_f - 4016C_A T_F N_f \\
&\quad - 6912\zeta(3)C_F^2 + 6570C_F^2 + 1728\zeta(3)C_F T_F N_f \\
&\quad + 1176C_F T_F N_f - 144T_F^2 C_F^2)\Big]\frac{C_F a^3}{108} + O(a^4) \qquad (7)
\end{aligned}$$

by substituting $n = 2$ in (6). The first two terms are in agreement with [7]. When the gauge group is $SU(3)$ we have

$$\gamma_{(2)}(a) = \frac{4a}{3} - \frac{2[26N_f - 543]a^2}{27}$$

$$- \left[36N_f^2 + 1440\zeta(3)N_f + 5240N_f + 2784\zeta(3) - 52555\right] \frac{a^3}{81} + O(a^4) \qquad (8)$$

or numerically

$$\begin{aligned} \gamma_2(a) &= 1.333333a - (1.925926N_f - 40.222222)a^2 \\ &\quad - (0.444444N_f^2 + 86.061259N_f - 607.512020)a^3 + O(a^4) \; . \end{aligned} \qquad (9)$$

We note that whilst we have concentrated on the flavour non-singlet current, the expression (7) also corresponds to the anomalous dimension of the (anomaly free) flavour singlet tensor current. This follows trivially since the Feynman diagrams with a closed quark loop with one $\bar{\psi}\sigma^{\mu\nu}\psi$ operator insertion in it, has an odd number of γ-matrices and are therefore zero upon taking the spinor trace.

One question which naturally arises out of choosing to compute these anomalous dimensions with the generalized γ-matrices, $\Gamma_{(n)}$, is that of how they relate to currents which involve γ^5. As is well known its treatment in dimensional regularization is a technically difficult exercise due to the fact that it has no natural d-dimensional generalization, [14]. In [5] this issue of renormalizing the axial vector and pseudoscalar currents within the automatic multiloop computer algebra approach was addressed. To treat such currents one first of all defines γ^5 in terms of $\epsilon_{\mu\nu\sigma\rho}\Gamma_{(4)}^{\mu\nu\sigma\rho}$ where $\epsilon_{\mu\nu\sigma\rho}$ is the totally antisymmetric four dimensional pseudotensor, [5]. Then products of the ϵ-tensor are replaced by a function of the metric, $\eta_{\mu\nu}$, which does have a natural d-dimensional extension. To compensate for the lack of continuity of the γ^5 definition in d-dimensions a finite renormalization is performed in addition to the usual $\overline{\rm MS}$ subtraction of the Green's function, [5]. The condition for the finite renormalization is to impose the obvious anticommutativity of γ^5 on the finite renormalized Green's functions. We have summarized the renormalization of these quark currents of previous work, [5], to allow the interested reader to contrast their renormalization here. Clearly within the context of the complete set of currents $j_{(n)}^{\mu_1\ldots\mu_n}$ the correct anomalous dimension for the axial vector and pseudoscalar currents should somehow be present in our results. We now address this.

In choosing to work in a spacetime which is d-dimensional, with d non-integer, the problem of defining γ^5 is in some sense bypassed in that one can regard it as an object which does not exist naturally. Also the ordinary γ-matrix basis ceases to be *finite* dimensional. Indeed in this situation the $\Gamma_{(n)}$-matrices provide a more natural basis for performing d-dimensional calculations. (For a more detailed discussion on this point see, for example, [12, 13, 21].) As γ^5 is a manifestly four dimensional object one need only be concerned about incorporating its effect when projecting from the d-dimensional spacetime, with its infinite $\Gamma_{(n)}$-basis, onto the integer dimensional physical spacetime. For us this projection is not achieved by analytically continuing the product $\epsilon_{\mu\nu\sigma\rho}\epsilon^{\alpha\beta\delta\lambda}$ to d-dimensional spacetime, [5]. Instead within the context of the infinite dimensional γ-matrix basis one determines a finite renormalization constant $Z_{\rm fin}$ from a condition similar to that of [5],

$$G^{(2)}(p^2, n) = Z_{\rm fin} G^{(2)}(p^2, 4-n) \qquad (10)$$

where $G^{(2)}(p^2, n)$ is the finite part of the quark two point Green's function after minimal subtraction where the current $j_{(n)}^{\mu_1\ldots\mu_n}$ has been inserted. In other words it is the renormalized Green's function. Also in (10) it is understood that all the contributing diagrams have been multiplied by $\Gamma_{(n)\,\mu_1\ldots\mu_n}$ before taking the spinor trace and dividing by the appropriate normalization mentioned earlier. Choosing the argument of the right side of (10) to be $(4-n)$ ensures that for instance the finite renormalization constant which emerges for the $n = 4$ current will give the same anomalous dimension as the $n = 0$ current with no finite renormalization. Therefore from

our calculations we find that

$$\begin{aligned}
Z_{\rm fin} &= 1 + 4(n-2)C_F a \\
&- \left[(36n^2-144n+1)C_A - 18n(5n-16)C_F + 4T_F N_f\right]\frac{(n-2)C_F a^2}{9} \\
&+ \Big[216(6n^4-48n^3+134n^2-152n+39)\zeta(3)C_A^2 \\
&\quad - 3(144n^4-1152n^3+3292n^2-3952n-479)C_A^2 \\
&\quad + 216(6n^4-48n^3+134n^2-152n+39)\zeta(3)C_A^2 \\
&\quad + 6(108n^4-1080n^3+4169n^2-6314n+200)C_A C_F \\
&\quad - 432(9n^4-72n^3+200n^2-224n+57)\zeta(3)C_A C_F \\
&\quad - 8(6n^2-24n+107)C_A T_F N_f - 1728\zeta(3)C_A T_F N_f \\
&\quad + 54(17n^4-76n^3-32n^2+352n-76)C_F^2 \\
&\quad + 2592(n^4-8n^3+22n^2-24n+6)\zeta(3)C_F^2 \\
&\quad + 12(34n^2-148n+145)C_F T_F N_f \\
&\quad + 1728\zeta(3)C_F T_F N_f - 208T_F^2 N_f^2\Big]\frac{(n-2)C_F a^3}{81} + O(a^4)
\end{aligned} \tag{11}$$

and we note that it is independent of the gauge fixing parameter. Moreover, by construction it evaluates to unity for $n = 2$ as it ought. Hence, to deduce the anomalous dimension of the currents $\bar{\psi}\gamma^\mu\gamma^5\psi$ and $\bar{\psi}\gamma^5\psi$ themselves by this method, one must therefore add the piece

$$\begin{aligned}
\mu\frac{d\ln Z_{\rm fin}}{d\mu} &= -\frac{4C_F(11C_A-4T_F N_f)(n-2)a^2}{3} \\
&+ 2C_F\Big[(396n^2-1584n-601)C_A^2 - 198(5n^2-20n+8)C_F C_A \\
&\quad - 16T_F^2 N_f^2 - 16(9n^2-36n-25)C_A T_F N_f \\
&\quad + 72(5n^2-20n+11)C_F T_F N_f\Big]\frac{(n-2)a^3}{27} + O(a^4)
\end{aligned} \tag{12}$$

to $\gamma_{(3)}(a)$ and $\gamma_{(4)}(a)$ respectively for $n = 3$ and 4 where μ is the renormalization scale. This effectively projects out the true component of (6) for four dimensional spacetime and correctly reproduces the known results of [4, 5]. We note that the three loop contribution of (12) only involves the two loop term of (11). Also if one was working with reference to two dimensions the argument of the right side of (10) would instead be $(2-n)$.

Finally, we briefy comment on this approach to flavour singlet current anomalous dimensions which we have also analyzed. As noted earlier the flavour singlet and non-singlet currents coincide for even n. However, if one were to use the $\Gamma_{(n)}$-basis to study the anomalous dimension of singlet currents for n odd then there are two further issues to be dealt with. The first is the evaluation of γ-strings with only one $\Gamma_{(n)}$-matrix in the spinor trace. To handle these diagrams the following results are necessary. First, one decomposes the γ-string arising from the propagator and vertices into the $\Gamma_{(n)}$-basis and then uses the general property that $\mathrm{tr}(\Gamma^{\mu_1\ldots\mu_n}_{(n)}\Gamma^{\nu_1\ldots\nu_p}_{(p)})\mathrm{tr}(\Gamma_{(n)\,\mu_1\ldots\mu_n}\Gamma^{\sigma_1\ldots\sigma_q}_{(q)})$ is proportional to δ_{pq}. The tensor of proportionality involves a linear combination of products of $(p+q)/2$ η-tensors which respect the antisymmetry of the Lorentz indices of $\Gamma_{(p)}$ and $\Gamma_{(q)}$. At three loops the largest value of p or q is 5 which means that a small set of traces needs to be deduced. This is achieved by noting that, [12, 13],

$$\mathrm{tr}\left(\Gamma^{\mu_1\ldots\mu_m}_{(m)}\Gamma^{\nu_1\ldots\nu_n}_{(n)}\right) = 4(-1)^{n(n-1)/2}\delta_{mn}n!\left[\eta^{\mu_1\nu_1}\ldots\eta^{\mu_n\nu_n} + \text{antisymmetric permutations}\right]. \tag{13}$$

With these identities the graphs with a single operator insertion can be determined and the basic anomalous dimension analogous to (6) deduced as

$$\begin{aligned}\gamma^{\mathrm{singlet}}_{(n)}(a) &= \gamma_{(n)}(a) + 12\delta_{n,3}C_FT_FN_fa^2 \\ &+ \left[\delta_{n,3}C_F\left(\frac{218}{3}C_AT_FN_f + \frac{8}{3}T_F^2N_f^2 - 36C_FT_FN_f\right)\right. \\ &\left. + \delta_{n,5}(480\zeta(3)+80)\frac{d_F^{abc}d_F^{abc}}{N_{\mathrm{fund}}}\right]a^3 + O(a^4) \end{aligned} \tag{14}$$

where

$$d_F^{abc} = \mathrm{Tr}\left(T^{(a}T^bT^{c)}\right) , \tag{15}$$

T^a are the colour group generators and N_{fund} is the dimension of its fundamental representation. The δ-symbols arise from the spinor traces with one $\Gamma_{(n)}$-matrix. Whilst an additional colour group Casimir enters for the case $n = 5$, this current is evanescent with respect to four dimensions and is therefore not physically important. Clearly to relate the $n = 3$ current to the four dimensional axial vector current anomalous dimension one requires an additional finite renormalization and this arises from two places. The first occurs by imposing the condition (10) and deserves little more comment only to record that an extra term arises in (12) when $n = 3$ which is

$$2C_F\left[77C_AT_FN_f - 28T_F^2N_f^2\right]a^3 . \tag{16}$$

However, this additional finite renormalization will not ensure the correct singlet anomalous dimension emerges since the (four dimensional) chiral anomaly will not be preserved. This requires another finite renormalization and is achieved here by the method developed in [5] but with the anomaly equation treated inside a quark two point function in contrast to the gluon two point function considered in [5]. As this procedure only involves the gluonic operator inserted in a one loop diagram to the order we are interested in, we note that the additional contribution will be

$$2C_F\left[-66C_AT_FN_f + 24T_F^2N_f^2\right]a^3 \tag{17}$$

which gives the correct singlet axial vector current anomalous dimension, [4, 5].

In conclusion we have provided a new term in the series for the anomalous dimension of the tensor current in QCD in the $\overline{\mathrm{MS}}$ scheme. Also by considering the generalized γ-matrix basis, $\Gamma_{(n)}$, we have demonstrated how the anomalous dimensions of the currents which involve γ^5 emerge. Indeed we believe this is an important aspect of the calculation as it in principle provides a more systematic and alternative way of renormalizing currents or other composite operators in QCD which involve the purely four dimensional object γ^5. For example, one need only calculate Green's functions with a general insertion and then the anomalous dimensions for a variety of operators will emerge by choosing the appropriate value of the parameter of the inserted operator. It would be interesting, for instance, to develop this approach for more complicated and physically important operators such as the four quark operators which are fundamental to evaluating QCD corrections to weak processes.

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