LTH 468
hep-ph/9911491

Renormalisation of the Fayet-Iliopoulos D-term

arXiv:hep-ph/9911491v3 22 Dec 1999

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We consider the renormalisation of the Fayet-Iliopoulos D-term in a softly-broken Abelian supersymmetric theory. We show that there exists (at least through three loops) a renormalisation group invariant trajectory for the coefficient of the D-term, corresponding to the conformal anomaly solution for the soft masses and couplings.

December 1999

1. Introduction

In Abelian gauge theories with $N = 1$ supersymmetry there exists a possible invariant that is not allowed in the non-Abelian case: the Fayet-Iliopoulos D-term,

$$L = \xi \int V(x, \theta, \bar{\theta})\, d^4\theta = \xi D(x). \tag{1.1}$$

In this paper we discuss the renormalisation of ξ in the presence of the standard soft supersymmetry-breaking terms

$$L_{\rm SB} = (m^2)^j_i \phi^i \phi_j + \left(\tfrac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \tfrac{1}{2} b^{ij} \phi_i \phi_j + \tfrac{1}{2} M \lambda\lambda + \text{h.c.}\right) \tag{1.2}$$

Let us begin by reviewing the position when there is no supersymmetry-breaking, i.e. for $L_{\rm SB} = 0$. Many years ago, Fischler et al[1] proved an important result concerning the renormalisation of the D-term (see also Ref. [2]). Since it is a $\int d^4\theta$-type term, one may expect that the D-term will undergo renormalisation in general. Moreover, by simple power-counting it is easy to show that the said renormalisation is in general quadratically divergent. Evidently this poses a naturalness problem since (if present) it would introduce the cut-off mass scale into the scalar potential. At the one loop level it is easy to show that the simple condition $\mathrm{Tr}\mathcal{Y} = 0$ (where $\mathcal{Y}$ is the U_1 hypercharge and the trace is taken over the chiral supermultiplets) removes the divergence. Remarkably, although one may of course easily draw individual diagrams proportional (for example) to $\mathrm{Tr}\mathcal{Y}^5, \mathcal{Y}^7 \cdots$ etc., this condition suffices to all orders.

In the presence of supersymmetry breaking, however, it is clear that ξ will suffer *logarithmic* divergences. If calculations are done in the component formalism with D eliminated by means of its equation of motion, then these divergences are manifested via contributions to the β-function for m^2. It is in this manner that the results for the soft β-functions were given in, for example, Ref. [3]. Here we prefer to consider the renormalisation of ξ separately; an advantage of this is that it means that the exact results for the soft β-functions presented in Refs. [4]–[6] (see also Ref. [7]) apply without change to the Abelian case. The result for β_ξ is as follows:

$$\beta_\xi = \frac{\beta_g}{g}\xi + \hat{\beta}_\xi \tag{1.3}$$

where $\hat{\beta}_\xi$ is determined by V-tadpole (or in components D-tadpole) graphs, and is independent of ξ. In the supersymmetric case, we have $\hat{\beta}_\xi = 0$, whereupon Eq. (1.3) is

equivalent to the statement that the D-term, Eq. (1.1), is unrenormalised. In the presence of Eq. (1.2), however, $\hat{\beta}_\xi$ depends on m^2, h and M (it is easy to see that it cannot depend on b.) It is interesting that the dependence on h and M arises first at the three loop level.

Although in this paper we restrict ourselves to the Abelian case, it is evident that a D-term can occur with a direct product gauge group $(G_1 \otimes G_2 \cdots)$ if there is an Abelian factor: as is the case for the MSSM. In the MSSM context one may treat ξ as a free parameter at the weak scale[8], in which case there is no need to know $\hat{\beta}_\xi$. However, if we know ξ at gauge unification, then we need $\hat{\beta}_\xi$ to predict ξ at low energies. Now in the D-uneliminated case it is possible to express all the β-functions associated with the soft supersymmetry-breaking terms given in Eq. (1.2) in terms of the gauge β-function β_g, the chiral supermultiplet anomalous dimension γ and a certain function X which appears only in β_{m^2}; moreover in a special renormalisation scheme (the NSVZ scheme), β_g can also be expressed in terms of γ. It is clearly of interest to ask whether an analogous exact expression exists for β_ξ. Moreover, there exists an exact solution to the soft RG equations for m^2, M and h corresponding to the case when all the supersymmetry-breaking arises from the conformal anomaly[9] and it is also interesting to ask whether this solution can be extended to the non-zero ξ case.

The key to the derivation of the exact results for the soft β-functions is the spurion formalism. The obstacle to deriving an analogous result for β_ξ is the fact that individual superspace diagrams are (as already mentioned) quadratically divergent. This means that if, for example, we represent a h^{ijk} vertex in superspace by $h^{ijk}\theta^2$, then we cannot simply factor the θ^2 out, because it can be "hit" by a superspace D-derivative. The simple relationship between a graph with a h^{ijk} and the corresponding one with a supersymmetric Yukawa vertex which holds for the soft breaking β-functions is thereby lost. We are therefore unable to construct an exact formula for β_ξ; we do, however, present a solution for ξ related to the conformal anomaly solution, but which must be constructed order by order in perturbation theory.

2. The β-function for ξ

In this section we derive Eq. (1.3), and show how the contributions to $\hat{\beta}_\xi$ proportional to m^2 can be related in a simple way to β_g.

We take an Abelian $N = 1$ supersymmetric gauge theory with superpotential

$$W(\Phi) = \tfrac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \tfrac{1}{2} \mu^{ij} \Phi_i \Phi_j, \tag{2.1}$$

and at one loop we have

$$16\pi^2\beta_g^{(1)} = g^3 Q = g^3 \mathrm{Tr}\left[\mathcal{Y}^2\right], \tag{2.2a}$$

$$16\pi^2\gamma^{(1)i}{}_j = P^i{}_j = \tfrac{1}{2}Y^{ikl}Y_{jkl} - 2g^2(\mathcal{Y}^2)^i{}_j. \tag{2.2b}$$

Let us consider the D-term renormalisation. We define renormalised and bare quantities in the usual way:

$$\xi_B \int V_B\, d^4\theta = \mu^{-\frac{\epsilon}{2}} \xi Z_\xi Z_V^{\frac{1}{2}} \int V\, d^4\theta \tag{2.3}$$

where the μ factor establishes the canonical dimension for ξ in $d = 4-\epsilon$ dimensions. Then writing

$$\xi Z_\xi Z_V^{\frac{1}{2}} = \xi + \sum \frac{a_n}{\epsilon^n} \tag{2.4}$$

and using

$$\begin{aligned} \xi_B &= \mu^{-\frac{\epsilon}{2}} \xi Z_\xi \\ g_B &= \mu^{\frac{\epsilon}{2}} g Z_V^{-\frac{1}{2}} \end{aligned} \tag{2.5}$$

it is straightforward to show, using

$$\mu\frac{\partial g}{\partial \mu} = -\frac{\epsilon}{2}g + \beta_g \tag{2.6}$$

that

$$\mu\frac{\partial \xi}{\partial \mu} = \frac{\epsilon}{2}\xi + \beta_\xi \tag{2.7}$$

where

$$\beta_\xi = \frac{\beta_g}{g}\xi + \hat{\beta}_\xi, \tag{2.8}$$

and where

$$\hat{\beta}_\xi = \sum_L L a_1^L. \tag{2.9}$$

Here a_1^L is the contribution to a_1 from diagrams with L loops. On dimensional grounds,

$$\hat{\beta}_\xi = m^2 A_1(g, Y, Y^*) + hh^* A_2(g, Y, Y^*) + MM^* A_3(g, Y, Y^*) + (Mh^* + M^* h)A_4(g, Y, Y^*), \tag{2.10}$$

where we have suppressed $(i, j \cdots)$ indices for simplicity.

By considering the relationship between the original theory and the one obtained by elimination of the D-field, we can prove a remarkably simple result for A_1 above. The relevant part of the supersymmetric Lagrangian is as follows:

$$L = \tfrac{1}{2}D^2 + \xi D + gD\phi^*\mathcal{Y}\phi - \phi^* m^2 \phi + \cdots. \tag{2.11}$$

After eliminating D this becomes

$$L = -\phi^* \bar{m}^2 \phi - \tfrac{1}{2} g^2 (\phi^* \mathcal{Y} \phi)^2, \tag{2.12}$$

where

$$\bar{m}^2 = m^2 + g\xi\mathcal{Y}. \tag{2.13}$$

RG invariance of this result gives (using Eq. (2.8))

$$\beta_{\bar{m}^2}(\bar{m}^2, \cdots) = \beta_{m^2}(m^2, \cdots) + 2\beta_g \xi \mathcal{Y} + g\mathcal{Y}\hat{\beta}_\xi(m^2, \cdots). \tag{2.14}$$

Now β_{m^2} is calculated in the uneliminated Lagrangian and hence does not contain the "D-tadpole" contributions. It is, in fact, given precisely by the previously derived formula:

$$(\beta_{m^2})^i{}_j(m^2, \cdots) = \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i{}_j. \tag{2.15}$$

where

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2MM^* g^2 \frac{\partial}{\partial g^2} + \tilde{Y}_{lmn}\frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}}, \tag{2.16}$$

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial Y^{lmn}}\right), \tag{2.17}$$

$$\tilde{Y}^{ijk} = (m^2)^i{}_l Y^{ljk} + (m^2)^j{}_l Y^{ilk} + (m^2)^k{}_l Y^{ijl} \tag{2.18}$$

and (in the NSVZ scheme)

$$16\pi^2 X^{\mathrm{NSVZ}} = -2g^3 \mathrm{Tr}\left[m^2 \mathcal{Y}^2\right]. \tag{2.19}$$

Now $\beta_{\bar{m}^2}$ is given by

$$\beta_{\bar{m}^2} = \beta_{m^2}(\bar{m}^2, \cdots) + g\mathcal{Y}\hat{\beta}_\xi(\bar{m}^2, \cdots). \tag{2.20}$$

In other words, if we calculate in the D-eliminated formalism, we obtain both "normal" contributions (the ones that would be single-particle irreducible in the D-uneliminated case) and the D-tadpole contributions, which appear in β_ξ in the D-uneliminated case.

The key now is the result that

$$\beta_{m^2}(\bar{m}^2, \cdots) = \beta_{m^2}(m^2, \cdots) \tag{2.21}$$

This follows simply by substituting for $\bar{m}^2$ from Eq. (2.13) and then using the facts that

$$(\mathcal{Y})^i{}_l Y^{ljk} + (\mathcal{Y})^j{}_l Y^{ilk} + (\mathcal{Y})^k{}_l Y^{ijl} = 0 \tag{2.22}$$

by gauge invariance, and

$$\mathrm{Tr}(\mathcal{Y}^3) = 0 \tag{2.23}$$

for anomaly cancellation.[1] We then find immediately that:

$$\hat{\beta}_\xi(\bar{m}^2, \cdots) = 2\frac{\beta_g}{g}\xi + \hat{\beta}_\xi(m^2, \cdots) \tag{2.24}$$

whence

$$\mathrm{Tr}(\mathcal{Y}A_1) = 2\frac{\beta_g}{g^2}. \tag{2.25}$$

So if we take the D-tadpole contributions to β_ξ, then the terms proportional to m^2 will reduce to $2\beta_g/g$ if we replace m^2 by $g\mathcal{Y}$. This result is, in fact, clear from a diagrammatic point of view, since the aforesaid replacement converts the diagrams into D self-energy graphs, and hence indeed gives rise to β_g.

3. The three loop results

Through two loops we have that

$$16\pi^2\hat{\beta}_\xi = 2g\mathrm{Tr}\left[\mathcal{Y}m^2\right] - 4g\mathrm{Tr}\left[\mathcal{Y}m^2\gamma^{(1)}\right] + \cdots \tag{3.1}$$

so we see that in fact only A_1 is non-zero through this order. Moreover, since

$$16\pi^2\beta_g = g^3\mathrm{Tr}\left[\mathcal{Y}^2\right] - 2g^3\mathrm{Tr}\left[\mathcal{Y}^2\gamma^{(1)}\right] + \cdots \tag{3.2}$$

we see that Eq. (3.1) is indeed consistent with Eq. (2.25).

We have calculated several distinct gauge invariant contributions to $\hat{\beta}_\xi^{(3)\mathrm{DRED}}$, namely the sets of terms that are $O(gY^4m^2)$, $O(gY^2h^2)$, $O(g^3Y^2m^2)$ and $O(g^3h^2)$. We find that:

$$\begin{aligned}(16\pi^2)^3\frac{\hat{\beta}_\xi^{(3)\mathrm{DRED}}}{g} &= 7(Y^2)^i{}_jY^{jkl}Y_{ikm}(m^2\mathcal{Y})^m{}_l + 4(Y^2)^i{}_jY^{jkl}Y_{imn}(m^2)^m{}_k\mathcal{Y}^n{}_l \\ &\quad - \tfrac{3}{2}\mathrm{Tr}\left[Y^2Y^2m^2\mathcal{Y}\right] - \tfrac{5}{2}Y^{ikl}Y_{imn}h_{jkl}h^{pmn}\mathcal{Y}^p{}_j - 2\mathrm{Tr}\left[Y^2h^2\mathcal{Y}\right] \\ &\quad + (10 - 24\zeta(3))g^2\mathrm{Tr}\left[Y^2m^2\mathcal{Y}^3\right] - 12g^2Y^{ikl}Y_{imn}(m^2\mathcal{Y})^m{}_k(\mathcal{Y}^2)^n{}_l \\ &\quad + (8 - 24\zeta(3))g^2\left[2Y^{ikl}Y_{imn}(m^2)^m{}_k(\mathcal{Y}^3)^n{}_l + h^{ikl}h_{jkl}(\mathcal{Y}^3)^j{}_i\right] + \cdots\end{aligned} \tag{3.3}$$

[1] A remark on scheme dependence. The result for X, Eq. (2.19), applies in the NSVZ scheme, which is one of a class of schemes (which include the standard perturbative method, DRED), related by redefinitions of g and M, the ramifications of which are described in Refs. [6]. Now X transforms non-trivially under these redefinitions, but it can be shown using Eqs. (2.22), (2.23) that X is unchanged by the replacement $m^2 \to \bar{m}^2$ in any member of this class of schemes; consequently Eq. (2.21) always applies.

where $(Y^2)^i{}_j = Y^{ikl}Y_{jkl}$, $(h^2)^i{}_j = h^{ikl}h_{jkl}$. Then replacing m^2 by $g\mathcal{Y}$, we obtain

$$g(16\pi^2)^3\mathrm{Tr}(\mathcal{Y}A_1^{(3)}) = (6X_1 + 12X_3 + 2X_4) + \cdots, \tag{3.4}$$

where

$$\begin{aligned} X_1 &= g^2 Y^{klm}P^n{}_l(\mathcal{Y}^2)^p{}_m Y_{knp}, \\ X_3 &= g^4\mathrm{Tr}\left[P\mathcal{Y}^4\right], \\ X_4 &= g^2\mathrm{Tr}\left[P^2\mathcal{Y}^2\right], \end{aligned} \tag{3.5}$$

in precise agreement with the result for $\beta_g^{(3)}$, given in [10], which for an Abelian theory is

$$(16\pi^2)^3\beta_g^{(3)\mathrm{DRED}} = g\left\{3X_1 + 6X_3 + X_4 - 6g^6 Q\mathrm{Tr}\left[\mathcal{Y}^4\right]\right\}. \tag{3.6}$$

We have not calculated the $O(g^5m^2)$ contributions to $\hat{\beta}_\xi^{(3)}$, which will produce the $O(g^7)$ terms in Eq. (3.6).

4. The conformal anomaly trajectory

The following set of equations provide an exact solution to the renormalisation group equations for M, h, b and m^2:

$$M = M_0\frac{\beta_g}{g}, \tag{4.1a}$$

$$h^{ijk} = -M_0\beta_Y^{ijk}, \tag{4.1b}$$

$$b^{ij} = -M_0\beta_\mu^{ij}, \tag{4.1c}$$

$$(m^2)^i{}_j = \tfrac{1}{2}|M_0|^2\mu\frac{d\gamma^i{}_j}{d\mu}. \tag{4.1d}$$

Moreover, these solutions indeed hold if the only source of supersymmetry breaking is the conformal anomaly, when M_0 is in fact the gravitino mass.

This set of soft breakings has caused considerable interest; but there are clear difficulties for the MSSM, since it is easy to see that sleptons are predicted to have negative $(\mathrm{mass})^2$. Most studies of this scenario have resolved this dilemma by adding a constant m_0^2, presuming another source of supersymmetry breaking. A non-zero ξ alone is not an alternative, unfortunately, as is easily seen from Eq. (2.13); the two selectrons, for example, have oppositely-signed hypercharge so one of them at least remains with negative $(\mathrm{mass})^2$.

It is immediately obvious that, given Eq. (4.1), there is a RG invariant solution for ξ through two loops (for $\hat{\beta}_\xi$) given by:

$$16\pi^2\xi = g|M_0|^2 \mathrm{Tr}\left[\mathcal{Y}(\gamma - \gamma^2)\right], \tag{4.2}$$

since differentiating with respect to μ and using Eq. (4.1d) leads at once to Eqs. (2.8), (3.1). Interestingly, however, this result for ξ *vanishes* at leading and next-to-leading order, since one easily demonstrates that

$$\mathrm{Tr}\left[\mathcal{Y}\gamma^{(1)}\right] = 0 \tag{4.3}$$

and

$$\mathrm{Tr}\left[\mathcal{Y}\gamma^{(2)}\right] = \mathrm{Tr}\left[\mathcal{Y}(\gamma^{(1)})^2\right], \tag{4.4}$$

using the result for $\gamma^{(2)}$, which is

$$(16\pi^2)^2\gamma^{(2)i}{}_j = \left[-Y_{jmn}Y^{mpi} - 2g^2(\mathcal{Y}^2)^p{}_j\delta^i{}_n\right]P^n{}_p + 2g^4(\mathcal{Y}^2)^i{}_jQ. \tag{4.5}$$

It is interesting to ask whether the trajectory can be extended beyond two loops, and whether it in fact continues to vanish order by order. We have shown that there is indeed a generalisation of Eq. (4.2) to at least three loops (for $\hat{\beta}_\xi$), and that at this order the result for ξ is non-zero.

Our result is as follows:

$$\begin{aligned}\frac{16\pi^2\xi^{\mathrm{DRED}}}{g|M_0|^2} &= \mathrm{Tr}\left[\mathcal{Y}(\gamma-\gamma^2)\right] - 24\zeta(3)g^2(16\pi^2)^{-3}\left[\mathrm{Tr}\left[\mathcal{Y}^3P^2\right] + (\mathcal{Y}^3)^i{}_jY^{jkl}Y_{ikm}P^m{}_l\right] \\ &\quad + 2(16\pi^2)^{-3}\left[(\mathcal{Y})^i{}_jY^{jkl}Y_{imn}P^m{}_kP^n{}_l - 2\mathrm{Tr}\left[\mathcal{Y}P^3\right] - 4g^2\mathrm{Tr}\left[\mathcal{Y}^3P^2\right]\right] + \cdots.\end{aligned} \tag{4.6}$$

Note that our partial $\hat{\beta}_\xi$ expression determines only the $O(Y^6)$ and (Y^4g^2) terms on the RHS of Eq. (4.6); it is interesting that they can be written in terms of invariants involving P. After this is done we have chosen to "promote" remaining Y^2 factors to $2P$; the difference thereby introduced depends only on terms we have not calculated. It is easy to verify that (for Y^8 and Y^6g^2 terms) the result of taking $\mu\frac{\partial}{\partial\mu}$ of this expression is identical to that obtained by substituting Eq. (4.1b,d) in Eqs. (3.1), (3.3). This is a non-trivial result in that the number of candidate terms for inclusion in Eq. (4.6) is considerably less than the number of distinct terms which arise when Eq. (3.1), (3.3) are placed on the RG trajectory. We therefore conjecture that the trajectory holds for the full $\hat{\beta}_\xi^{(3)}$ calculation, and extends to all orders.

Using the result[2] for $\gamma^{(3)}$ from Ref. [11], we find that

$$(16\pi^2)^3 \mathrm{Tr}\left[\mathcal{Y}(\gamma-\gamma^2)\right] = I_1 + 12\zeta(3) I_2 + O(Y^8, Y^6 g^2, \cdots g^8), \tag{4.7}$$

where

$$\begin{aligned} I_1 &= \mathrm{Tr}\left[\mathcal{Y} P^3\right] - \tfrac{1}{2}(\mathcal{Y})^i{}_j Y^{jkl} Y_{imn} P^m{}_k P^n{}_l + 2g^2 \mathrm{Tr}\left[\mathcal{Y}^3 P^2\right] - 2g^4 Q \mathrm{Tr}\left[\mathcal{Y}^3 P\right] \\ I_2 &= g^2 (\mathcal{Y}^3)^i{}_j Y^{jkl} Y_{ikm} P^m{}_l + g^2 \mathrm{Tr}\left[\mathcal{Y}^3 P^2\right] + 2g^4 \mathrm{Tr}\left[\mathcal{Y}^5 P\right]. \end{aligned} \tag{4.8}$$

Comparing Eq. (4.6) with Eq. (4.8) leads us to the conjecture that on the RG trajectory ξ^{DRED} is given at leading order by the expression

$$\frac{\xi^{\mathrm{DRED}}}{g|M_0|^2} = (16\pi^2)^{-4}\left(-3I_1 - 12\zeta(3) I_2 + \nu_1 Q g^6 \mathrm{Tr}\left[\mathcal{Y}^5\right] + \nu_2 Q g^4 \mathrm{Tr}\left[P\mathcal{Y}^3\right]\right) \tag{4.9}$$

where ν_1, ν_2 are undetermined. Note that other invariants which might in principle have appeared in Eq. (4.9) are in fact ruled out from consistency with $\hat{\beta}_\xi^{(3)\mathrm{DRED}}$; and in fact our conjecture is equivalent to the following form for $\hat{\beta}_\xi^{(3)\mathrm{DRED}}$:

$$\begin{aligned} (16\pi^2)^3 \frac{\hat{\beta}_\xi^{(3)\mathrm{DRED}}}{g} &= -6(16\pi^2)^2 \mathrm{Tr}\left[\mathcal{Y} m^2 \gamma^{(2)}\right] - 4\mathrm{Tr}\left[WP\mathcal{Y}\right] - \tfrac{5}{2}\mathrm{Tr}\left[HH^*\mathcal{Y}\right] \\ &\quad + 2\mathrm{Tr}\left[P^2 m^2 \mathcal{Y}\right] - 24 g^2 \zeta(3) \mathrm{Tr}\left[W\mathcal{Y}^3\right] \\ &\quad + \left(12\zeta(3) - \tfrac{1}{2}\nu_2\right) g^2 \mathrm{Tr}\left[M^* H \mathcal{Y}^3 + \mathrm{c.c.}\right] \\ &\quad + 6\left(-48\zeta(3) + \nu_1\right) g^4 M M^* \mathrm{Tr}\left[\mathcal{Y}^5\right] + 4\nu_2 g^2 M M^* \mathrm{Tr}\left[P\mathcal{Y}^3\right] \end{aligned} \tag{4.10}$$

where[3]

$$W^i{}_j = (\tfrac{1}{2} Y^2 m^2 + \tfrac{1}{2} m^2 Y^2 + h^2)^i{}_j + 2Y^{ipq} Y_{jpr} (m^2)^r{}_q - 8 g^2 M M^* (\mathcal{Y}^2)^i{}_j \tag{4.11}$$

and

$$H^i{}_j = h^{ikl} Y_{jkl} + 4 g^2 M (\mathcal{Y}^2)^i{}_j. \tag{4.12}$$

It is interesting to note that the particular value $\nu_1 = 48\zeta(3)$ makes $\hat{\beta}_\xi^{(3)\mathrm{DRED}}$ vanish if $P^i{}_j = 0$, $(m^2)^i{}_j = \frac{1}{3} M M^* \delta^i{}_j$, and $h^{ijk} = -M Y^{ijk}$. These relations arise (at leading order) in finite softly-broken theories (of course in the Abelian case considered here we cannot achieve a finite theory as $Q \neq 0$).

[2] Notice that $\mathrm{Tr}\left[\mathcal{Y}\gamma^{(3)\mathrm{DRED}}\right] = \mathrm{Tr}\left[\mathcal{Y}\gamma^{(3)\mathrm{NSVZ}}\right]$ in the Abelian case, by virtue of Eq. (2.23).

It is natural to ask what the result for $\hat{\beta}_\xi^{(3)}$ is in the NSVZ scheme, which is obtained (at the relevant order) by the redefinitions[4]

$$\begin{aligned}(16\pi^2)^2\delta g &= -\tfrac{1}{2}g^3\mathrm{Tr}\left[P\mathcal{Y}^2\right]\\ (16\pi^2)^2\delta M &= -Mg^2\left\{\mathrm{Tr}\left[P\mathcal{Y}^2\right]-2g^2\mathrm{Tr}\left[(\mathcal{Y}^2)^2\right]\right\}+\tfrac{1}{2}g^2h^{ikl}Y_{jkl}(\mathcal{Y}^2)^j{}_i.\end{aligned} \tag{4.13}$$

It is straightforward to show that in order to obtain the results Eqs. (2.8) and (2.25) in the NSVZ scheme, we must also redefine ξ as follows:

$$(16\pi^2)^2\delta\xi = -\tfrac{1}{2}g^2\mathrm{Tr}\left[P\mathcal{Y}^2\right]\xi - g\mathrm{Tr}\left[m^2P\mathcal{Y}\right]. \tag{4.14}$$

The effect of this is to replace Eq. (4.9) by

$$\frac{\xi^{\mathrm{NSVZ}}}{g|M_0|^2} = (16\pi^2)^{-4}\left(-4I_1-12\zeta(3)I_2+\nu_1Qg^6\mathrm{Tr}\left[\mathcal{Y}^5\right]+\nu_2Qg^4\mathrm{Tr}\left[P\mathcal{Y}^3\right]\right), \tag{4.15}$$

and Eq. (4.10) by

$$\begin{aligned}(16\pi^2)^3\frac{\hat{\beta}_\xi^{(3)\mathrm{NSVZ}}}{g} &= -4(16\pi^2)^2\mathrm{Tr}\left[\mathcal{Y}m^2\gamma^{(2)}\right]-\tfrac{5}{2}\left(2\mathrm{Tr}\left[WP\mathcal{Y}\right]+\mathrm{Tr}\left[HH^*\mathcal{Y}\right]\right)\\ &\quad-24g^2\zeta(3)\mathrm{Tr}\left[W\mathcal{Y}^3\right]+\left(12\zeta(3)-\tfrac{1}{2}\nu_2\right)g^2\mathrm{Tr}\left[M^*H\mathcal{Y}^3+\mathrm{c.c.}\right]\\ &\quad+6\left(-48\zeta(3)+\nu_1\right)g^4MM^*\mathrm{Tr}\left[\mathcal{Y}^5\right]+4\nu_2g^2MM^*\mathrm{Tr}\left[P\mathcal{Y}^3\right].\end{aligned} \tag{4.16}$$

It is interesting to note that with the particular values $\nu_1 = 48\zeta(3)$, $\nu_2 = -24\zeta(3)$, Eqs. (3.1), (4.16) are consistent with the following expression

$$\begin{aligned}16\pi^2\frac{\hat{\beta}_\xi^{\mathrm{NSVZ}}}{g} &= 2\mathrm{Tr}\left[\mathcal{Y}m^2\right]-4\mathrm{Tr}\left[\mathcal{Y}m^2\gamma\right]\\ &\quad+\left(\Delta+X\frac{\partial}{\partial g}\right)\left(-\tfrac{5}{2}\mathrm{Tr}[\gamma^2\mathcal{Y}]+24\zeta(3)g^2(16\pi^2)^{-1}\mathrm{Tr}[\gamma\mathcal{Y}^3]\right),\end{aligned} \tag{4.17}$$

where Δ and X are defined in Eqs. (2.15)–(2.19).

We hope to test our conjectures Eq. (4.9) and (4.10) by completing the calculation of $\hat{\beta}_\xi^{(3)}$; we also plan to discuss scheme dependence in more detail, and extend our results to a gauge group with direct product structure and one or more abelian factors (such as the MSSM).

Note added

Since this paper was submitted we have calculated the contributions to $\hat{\beta}_\xi^{(3)\mathrm{DRED}}$ of the form $g^3\mathrm{Tr}\left[M^*H\mathcal{Y}^3+\mathrm{c.c.}\right]$ and $g^3MM^*\mathrm{Tr}\left[P\mathcal{Y}^3\right]$. These are *both* consistent with the

result $\nu_2 = 0$. Although this means that the $\zeta(3)$ terms cannot, in fact, be written in the form suggested in Eq. (4.17), it does provide strong evidence in favour of our result Eq. (4.10). We feel that our demonstration that ξ has a RG trajectory related to the conformal anomaly one is intriguing, and worthy of further investigation.

Acknowledgements

This work was supported in part by a Research Fellowship from the Leverhulme Trust. We thank John Gracey for conversations.

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